An empirical comparison of alternative functional forms of systems of consumer demand equations
by Cathy Anne Roheim

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Applied Economics
Montana State University

Abstract:
This study considers five functional forms of consumer demand models applied to the same set of data to estimate systems of demand equations: the Rotterdam model, the Indirect Addilog model, the Linear Expenditure System, the Almost Ideal Demand System, and the double logarithmic model. These are compared to each other and the "best" one is determined on the basis of its adherence to existing theoretical restrictions on its respective parameters and on its ability to statistically summarize observed consumer behavior accurately.

Theoretical restrictions assumed to hold include weak separability of the utility function and perfect aggregation of the utility function over individuals. Homogeneity of degree zero in prices and income was imposed on all estimated demand equations, while Slutsky symmetry was tested for explicitly in each model. The Rotterdam model, Indirect AddiIog model, and the Almost Ideal Demand System accepted the symmetry conditions, whereas the remaining models did not.

To statistically compare the performance of the models, the information inaccuracy measure was used in which the actual budget shares of each commodity are compared to the budget shares predicted by the model. The functional form corresponding to the information inaccuracy closest to zero is considered best. In this study, the Rotterdam model placed first with the Almost Ideal Demand System and the Indirect Addilog in second and third places, respectively.

Data used to estimate the model are annual observations on U.S. real personal income, real prices and per capita consumption of fed and non-fed beef, pork, and chicken from 1962 through 1980. The deflator used to adjust for inflation over the observation period was the Implicit Price Deflator for Personal Consumption Expenditures.

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Cathy Anne Roheim

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ABSTRACT

This study considers five functional forms of consumer demand models applied to the same set of data to estimate systems of demand equations: the Rotterdam model, the Indirect Addilog model, the Linear Expenditure System, the Almost Ideal Demand System, and the double logarithmic model. These are compared to each other and the "best" one is determined on the basis of its adherence to existing theoretical restrictions on its respective parameters and on its ability to statistically summarize observed consumer behavior accurately.

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INTRODUCTION

An understanding of theoretical implications of consumer demand models is as important to the accurate estimation of demand equations as the correct use of econometric techniques. Not only is obtaining unbiased and efficient estimators of the parameters of concern, but so is determining whether the model in question obeys properties set forth by basic utility theory. An empirical demand equation with an excellent statistical fit is of little use to economists if it has no theoretical basis and cannot be used for instructional or predictive purposes. This study considers five functional forms applied to the same set of data to estimate systems of consumer demand equations: the Rotterdam model (Theil 1978), the Indirect Addilog model (Houthakker 1960), the Linear Expenditure System (Stone 1954), the Almost Ideal Demand System (Deaton and Muellbauer 1980), and the double logarithmic model. These will be compared to each other and the "best" one will be determined on the basis of their adherence to existing theoretical restrictions on their respective parameters and on their ability to statistically summarize observed consumer behavior accurately.

Data used to estimate the models are annual observations on U.S. real personal income, real prices, and per capita consumption of fed and non-fed beef, pork, and chicken from 1962 through 1980. The deflator used to adjust for inflation over the observation period was the Implicit Price Deflator for Personal Consumption Expenditures.[1]

Other studies have also attempted to determine the best functional form specification for demand equations. Parks (1969) applied the information inaccuracy measure, outlined in Chapter VII, to compare the Rotterdam model, the Indirect Addilog model, and the

[1] U.S. Department of Agriculture (see Appendix).

Linear Expenditure System. Data used were a time-series of prices, quantities, and total consumption in Sweden for the period 1861-1955. Eight sectors of the economy, including agriculture, manufacturing, transportation and communications, commerce and insurance, domestic services, housing services, public services, and imported goods were covered. Findings were that the Rotterdam model had the smallest information inaccuracy for the entire sample period, making it the "best" model. However, Parks mentions that an important difference among the models which may be reflected in the results is that the Indirect Addilog model and Linear Expenditure System both assume additivity and thus allow less flexibility in the adjustment of the coefficients on the price terms than does the Rotterdam model, which is constrained only by homogeneity and symmetry, properties discussed in Chapter I.

Goldman (1971) did a similar comparison with the same models using British and Dutch data from 1922-1939 and 1949-1963 for foods, "vice" (including tobacco and beverages), durables, and a remainder. His conclusions were that the Indirect Addilog had the worst fit of all the models, and that the Rotterdam performed relatively better than the other two.

Using Dutch data for food, beverages and tobacco, durables, and a remainder from 1922-1963 and British data on the same groups of commodities from 1900-1938, Theil (1975a) determined that the best demand model was the Rotterdam. Based on the average information inaccuracy for all commodity groups taken jointly, the Indirect Addilog was second and the Linear Expenditure System third.

Deaton and Muellbauer (1980) have concluded that the Almost Ideal Demand System is comparable to the Rotterdam and translog models, but has advantages over both. Bewley (1982), interested to see if this claim was in fact true, compared the Rotterdam model, the Almost Ideal Demand System, the Indirect Addilog model, and a "naive" model. He used an Australian time-series from 1960-1975 of per capita expenditure and prices for food, alcohol and tobacco, clothing, rent, durables, transportation and communication, and other expenditures. The information inaccuracy difference between the Almost Ideal Demand System and the Rotterdam model is small. Supporting earlier claims, both models performed better than the Indirect Addilog which did, however, do better than the naive model.

Yoshihara (1969) theoretically and empirically compared the Rotterdam, Indirect Addilog, double logarithmic, and Linear Expenditure System models and found that the Linear Expenditure System explained the pattern of Japanese demand best. He determined that both the Rotterdam and double logarithmic models were unsatisfactory from a theoretical viewpoint. Both models violated his empirical conditions that the income elasticities should not all be equal to one. Yoshihara believes that when a consumer is presented with rising income and constant prices, his expenditure pattern is more likely to change than stay constant. This constraint, however, implies that the utility function must be non-homogeneous which is a property not allowed in this study; homogeneity of degree zero in prices and income of the demand equations was imposed on all estimations.

The Indirect Addilog and Linear Expenditure System models satisfied the theoretical properties, and so were fitted to Japanese data on per capita consumption for 1902-1960. Based on sum of squares of residuals, the conclusion was that the Linear Expenditure System provided a better fit.

In this paper, Chapter I will outline the theoretical implications and restrictions implied by utility maximizing behavior. The next five chapters will each be devoted to an individual functional form specification, including explanations of estimation procedures and results. Chapter VII, the final chapter, provides conclusions about the five demand models based on the preceding results.

CHAPTER I

DEMAND FUNCTIONS AND RESTRICTIONS

To obtain a system of demand equations by conventional microeconomic theory, we must assume the existence of an underlying utility function,

$$U = f(q_1, q_2, \ldots, q_n) \tag{1.1}$$

which the representative consumer wishes to maximize subject to a budget constraint. The utility function is assumed to be strictly increasing, quasi-concave, and twice differentiable.

Ordinary demand equations can be obtained from the direct utility function, or from the associated indirect utility function. To arrive at the demand equations via the direct utility function, one must explicitly solve a constrained optimization problem. A Lagrangean function is formed to maximize utility subject to the budget constraint

$$L = f(q_1, q_2, \ldots, q_n) + \lambda(M - \sum_{i=1}^{n} p_i q_i) \tag{1.2}$$

where λ is the Lagrangean multiplier, M is nominal income, and p_i equals the nominal price of the ith good. First order conditions are found by differentiating L with respect to q_i and λ, resulting in

$$\partial L/\partial q_i = U_i - \lambda p_i \qquad \text{for } i = 1, 2, \ldots, n \tag{1.3}$$

$$\partial L/\partial \lambda = M - \sum_{i=1}^{n} p_i q_i \tag{1.4}$$

where $U_i = \partial U/\partial q_i$. Notice that the right hand side of (1.4) is equal to the budget constraint in implicit form. Setting (1.3) and (1.4) equal to zero, we obtain the (n+1) first order conditions

$$U_i = \lambda p_i \qquad \text{for all } i = 1, 2, \ldots, n \tag{1.5}$$

$$\sum_{i=1}^{n} p_i q_i = M \tag{1.6}$$

Solving for λ in (1.5) and equating the resulting equations establishes that

$$U_1/p_1 = U_2/p_2 = \ldots = U_n/p_n \tag{1.7}$$

which expresses mathematically the economic principle that the marginal utility per dollar of all goods $i=1, \ldots, n$ is equal when utility is maximized, assuming second order conditions are met. This can be shown graphically in a two good case, depicted in Figure 1.

At points q_1^* and q_2^*, we have obtained the highest attainable level of utility which is the highest indifference curve given our budget constraint.

Demand functions for the n commodities are found by substituting U_i/p_i into the differential of L with respect to λ and solving the resulting equation for q_i^*. The q_i^* functions form what is known as a system of "ordinary" demand equations, as opposed to "compensated" demands, denoted q_i^c, where expenditures are minimized subject to a fixed level of utility.

To insure that the first order conditions yield a global maximum, second order conditions must be checked. Assuming the first order conditions are satisfied, it is a sufficient condition for a global maximum if the utility function, U(q), exhibits strict quasi-concavity in all of the positive, or first quadrant of n dimensional space.

Given that utility is a function of quantities consumed, and that the function is strictly quasi-concave, monotonically increasing, and differentiable, a specification for demand functions to be empirically estimated can be derived from the first order conditions of utility maximization as shown above. In some cases, this may be more easily said than done because these conditions frequently cannot be solved explicitly for the demand functions, and when they are, the resulting equation may be difficult or impossible to estimate.

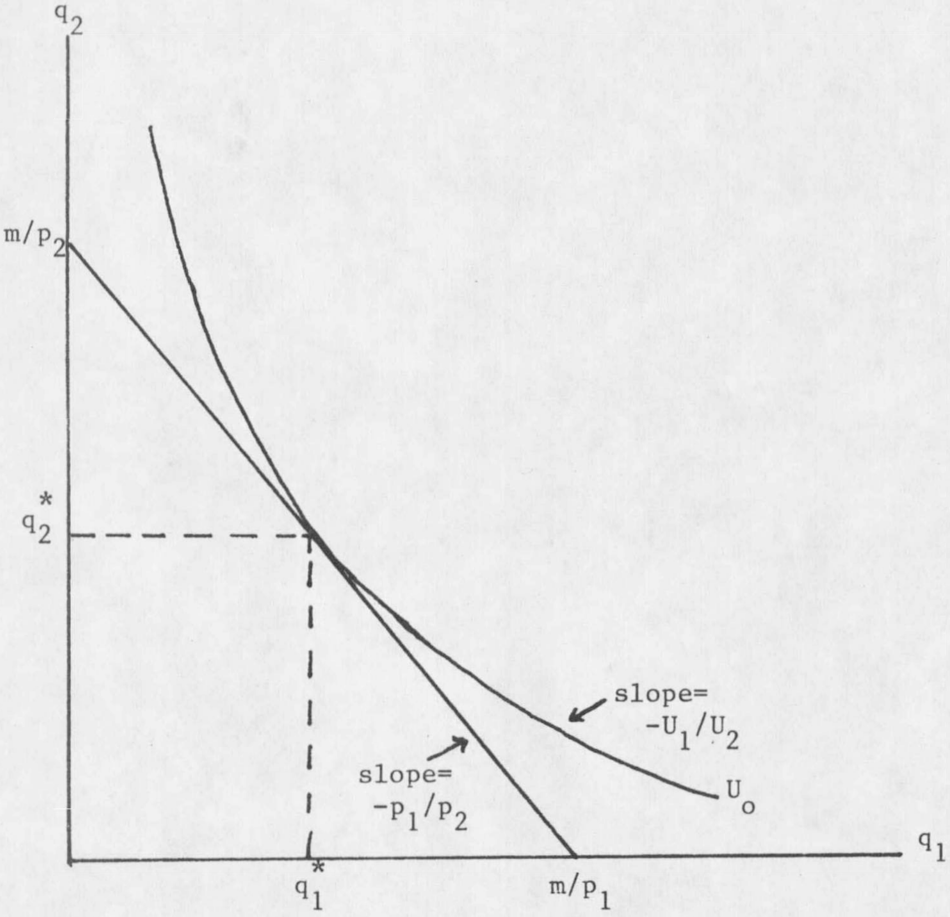

Figure 1. Two good case of utility maximization.

Duality theory provides a convenient way of obtaining a functional specification for demand functions, yet avoiding the problem of explicitly solving an optimization problem. The dual approach makes it possible to go from preferences to behavior and back again in two steps. Therefore, construction of preference consistent demand functions becomes straightforward.

Duality is based on an indirect utility function

$$\tilde{U} = g(p_1, p_2, \ldots, p_n, M) \qquad (1.8)$$

defined to be the maximum utility obtainable at the given prices and income level. The indirect function is obtained by substituting the ordinary demand equation for each good into the direct utility function; the resulting indirect utility is thus a function of prices and income rather than a function of quantities of goods consumed, $q_1, \ldots, q_n$. A system of ordinary demand equations is obtained from the indirect utility function by applying Roy's Identity,

$$q_i^* = -\partial\tilde{U}/\partial p_i \Big/ \partial\tilde{U}/\partial M \qquad (1.9)$$

Another approach to applying duality theory is to substitute U into compensated demand functions obtaining ordinary demand functions; the compensated demand functions are obtained by applying Shepard's lemma to an indirect expenditure function. However, utility, a non-estimatable variable, is an argument in the resulting demand equations, thus explaining why ordinary rather than compensated demands are estimated in empirical studies.

As a practical matter, the dual approach is sometimes easier to apply, likely to be more flexible to measure, and able to more conveniently analyze problems than the primal approach. Whichever approach is chosen to arrive at the system of demand equations, these equations must satisfy three restrictions that do not depend on functional form or the approach to obtaining specifications of a system of demand equations. Adding-up is one of these restrictions where

$$\sum_{i=1}^{n} p_i q_i = M \qquad (1.10)$$

or total expenditures sum to total income. However, this restriction is more relevant to complete systems of equations where estimates of the demand for all commodities is desired, rather than estimation of demand for a subsystem such as the one in this study.

The other two restrictions are: (1) each equation must be homogeneous of degree zero in prices and income, and (2) the demand system must satisfy what are known as Slutsky conditions, which are mathematical restrictions on derivatives of the demand equations.

Homogeneity[2] is based on the assumption that individual consumers make their decision based on relative prices, not absolute prices. The notion of money illusion is dismissed.

The Slutsky equation is written

$$\partial q_i^*/\partial p_j = \partial q_i^c/\partial p_j - q_j^*(\partial q_i^*/\partial M) \qquad i, j = 1, \ldots, n \qquad (1.11)$$

where q_i^* and q_i^c denote the ordinary and compensated demand functions, respectively. This equation shows a separation of the reaction of a consumer to a price change of good j on the quantity demanded of good i into two parts, the income effect, $q_j^*(\partial q_i^*/\partial M)$, and the substitution effect, $\partial q_i^c/\partial p_j$. The income effect takes into account the change in quantity demanded due to a change in real income of the consumer which is due to the nominal

---

[2] A function is defined to be homogeneous of degree k if

$$f(tq_1, tq_2, \ldots, tq_n) = t^k f(q_1, \ldots, q_n)$$

where k is a constant and t is any positive number. Therefore, if a function is to be homogeneous of degree zero, then

$$f(tq_1, \ldots, tq_n) = t^0 f(q_1, \ldots, q_n) = f(q_1, \ldots, q_n)$$

In other words, this means that if all prices and income are multiplied by a positive constant t, the quantity demanded of the goods i = 1, . . ., n must remain unchanged. This restriction is easily imposed by employing the necessary and sufficient condition that all prices and income are in ratio form, i.e., divide each by a price deflator.

price change. The substitution effect incorporates the change in quantity demanded of a good because of a change in its relative price.

The conditions which are derived from the Slutsky equation and which all demand functions must satisfy are

$$\partial q_i^*/\partial p_j + q_j^*(\partial q_i^*/\partial M) = \partial q_j^*/\partial p_i + q_i^*(\partial q_j^*/\partial M) \quad (1.12)$$

the cross-substitution effect k_{ij} exactly equals k_{ji}.

$$\sum_{i=1}^{n} p_i(\partial q_i/\partial p_j) + q_j = 0 \quad (1.13)$$

or, the sum of all price and income elasticities exactly equals zero. And

$$\partial q_i^c/\partial p_i < 0 \quad (1.14)$$

and own-substitution effect k_{ii} is negative.

Along with the general restrictions on demand functions discussed above, some particular properties of the utility function are required. These include separability and aggregation. A weakly separable utility function is defined as a function in which the marginal rate of substitution between two variables belonging to the same group be independent of the value of any other group (Leontief 1947). It can be written as

$$U(q) = \phi[U_1(q_1), \ldots, U_s(q_s)] \quad (1.15)$$

where $(U_1, \ldots, U_s)$ is a function of s variables and, for each s, $U_s(q_s)$ is a function of subvector q_s alone (Goldman and Uzawa 1964).

To be able to partition the consumption set into subsets which would include commodities that are closer substitutes or compliments to each other than to goods in other subsets is a desirable property, particularly in cases such as this study. Gorman (1959) proves that if specific utility functions are to be homogeneous of degree one in prices and income, we should never group luxuries, near-luxuries, and necessities together. This justifies looking at the portion of income that individuals will allocate to individual goods, such as meats, a subset of a larger group, food, which are closer substitutes than, for example, beef and furniture. For a more in depth discussion of separability definitions, see Goldman and Uzawa.

Phlips (1974) and Pollack (1971) feel that additivity is defensible only if the arguments of the utility function are taken to be broad aggregates of goods, such as food and clothing, rather than individual commodities. Additivity extends the definition of separability to items from a pair of different groups; that is, the marginal rate of substitution between dress slacks and steak should be independent of the consumption of tents.

We would also like our utility function to be applicable to both the individual and the aggregate. A representative consumer exists if the market behavior of an aggregate of different consumers is the same as the market behavior of a number of identical hypothetical consumers, each with the same level of income (Muellbauer 1976). This presents the aggregation problem. Does the representative consumer exist, and if so, does he reflect the behavior of the average consumer?

Muellbauer (1975) establishes necessary and sufficient conditions for the aggregation relations to be consistent in functional form with the individual relations. The most general condition is called "generalized linearity" (GL), where the relative marginal value shares are independent of income or utility, i.e.,

$$\partial/\partial y(\partial w_i/\partial y \big/ \partial w_j/\partial y) = 0 \qquad \text{for all } i, j \qquad (1.16)$$

More restrictively, aggregate market demand equations are consistent with individual demand equations corresponding to some level of income, which does not vary as relative prices vary if and only if "price independent generalized linearity" (PIGL) holds. This is defined by budget shares of the form

$$w_i(M, p) = \log M \, A_i(p) + B_i(p) \qquad (1.17)$$

or

$$w_i(M, p) = M\, A_i(p) + B_i(p) \qquad (1.18)$$

Therefore, according to Muellbauer, if there is no change in the distribution of relative money incomes and price independent generalized linearity holds, then market equations correspond to the individual equations at the mean income.

Polemarchakis (1983) showed that under the assumption that agents are endowed with a fixed share of the aggregate endowment vector, homotheticity of preferences is a necessary and sufficient condition for aggregation. A function is homothetic if it can be written

$$F[f(q_1, \ldots, q_n)] \qquad (1.19)$$

where $f(q_1, \ldots, q_n)$ is homogeneous of any degree and $F' \neq 0$.

As an example, consider any point on an isoquant ($L°$, $K°$) as in Figure 2 where L is labor and K is capital. Suppose each output is doubled. If the production function is homogeneous of any degree, the slope of the isoquant, $-f_L/f_K$ will be the same at ($2L°$, $2K°$) as at ($L°$, $K°$) (Silberberg 1978).

At present, the aggregation problem remains largely unsolved since rather restrictive assumptions are required to show that all individual conditions can be carried over to the aggregate level. The attitude of most applied econometricians is to ignore this problem (Phlips 1974). Aggregation error is probably of little importance as Houthakker and Taylor state in their book on dynamic demand analysis (1970, p. 200), "... we simply state as our opinion that of all the errors likely to be made in demand analysis, the aggregation error is the least troublesome."

That the utility function satisfies weak separability and aggregates perfectly over consumers will be assumed for the remainder of this study. Whether or not the demand models used in this study meet the Slutsky conditions and the adding-up condition will be discussed in the following chapters.

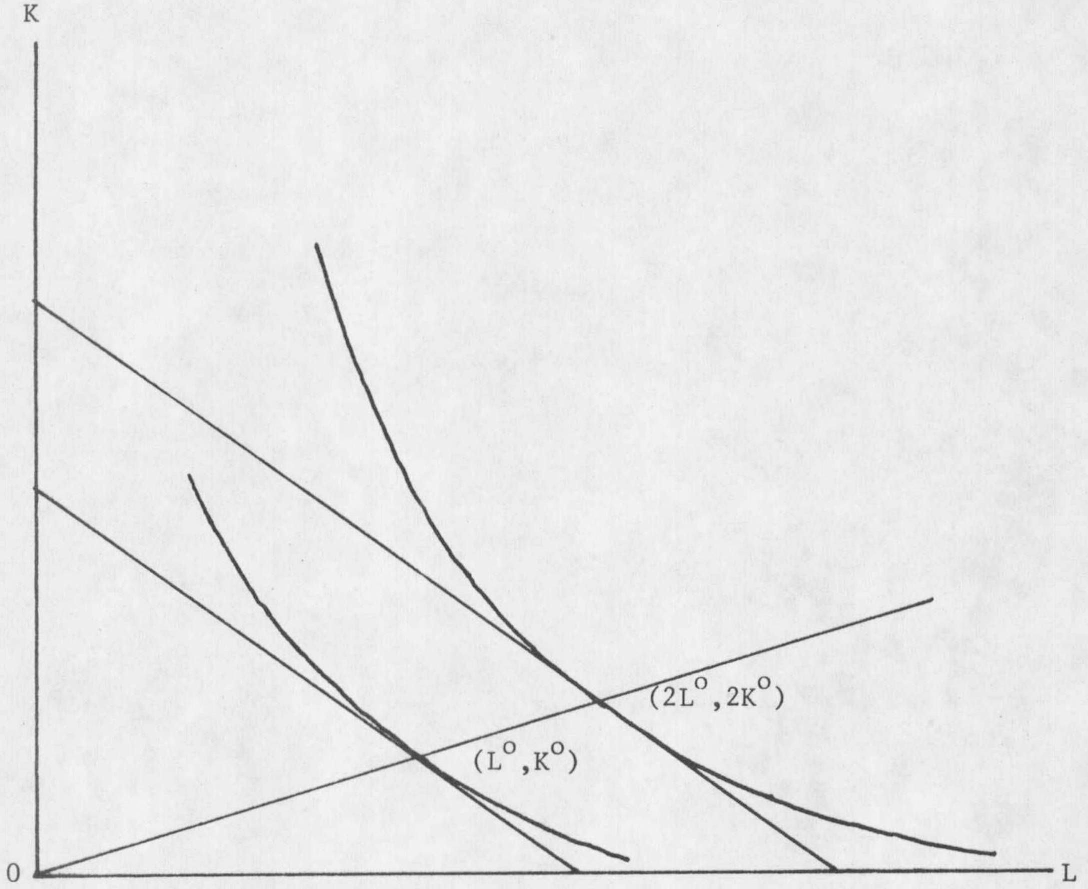

Figure 2. Example of homotheticity.

CHAPTER II

THE ROTTERDAM MODEL

While the Rotterdam model, developed by Theil and Barten, was not derived from either a direct or an indirect utility function, it satisfies the restrictions of classical demand theory. It can be shown that the model implies an underlying utility function, a member of the Bergson family, which is

$$U = \sum_{i=1}^{n} \beta_i \log q_i \qquad \beta_i > 0, \sum_{i=1}^{n} \beta_i = -1 \tag{2.1}$$

a special case of the Stone-Geary utility function $U = \sum_{i=1}^{n} \beta_i \log (q_i - \gamma_i)$. Properties of the Rotterdam model include directly additive utility, constant marginal budget shares, and constant income flexibility. Because of these properties, the parameters $\gamma_1, \ldots, \gamma_n$ are all equal to zero and for this reason, the Stone-Geary utility function specializes to $\sum_{i=1}^{n} \beta_i \log q_i$ where income flexibility equals -1.

Because of the restrictive nature of the Bergson utility family, many economists question the acceptability of the Rotterdam model. Yoshihara (1969) rejects it since it does not allow the income elasticity to vary. Nevertheless, it remains a useful tool for testing the general restrictions as a particular strength of this model is that it is possible to apply constraints to the parameters explicitly within the model, as will be done in this study with the Slutsky conditions.

The form of the Rotterdam model used was the absolute price version as applied to goods within a group (Theil 1978). The N goods a consumer purchases are divided into G groups of goods and, assuming the consumer's preferences are additively separable, the utility function can be written

$$U(q_1, \ldots, q_n) = U_1(\quad) + \ldots + U_G(\quad) \tag{2.2}$$

When we wish to consider one group, meats in this case, we define

$$\overline{W}_{gt} = \sum_{i=1}^{N_g} \overline{w}_{it} \tag{2.3}$$

and

$$M_g = \sum_{i=1}^{N_g} \mu_i \tag{2.4}$$

where $\overline{w}_{it} = 1/2(w_{i,t-1} + w_{it})$, and is the mean budget share, $\overline{W}_{gt}$ is the budget share of the group, μ_i is the marginal share of each good within the group, $\partial(p_i q_i)/\partial M$, and M_g is the group's marginal share. The latter refers to the additional amount allocated to the goods of the meat group if income increases by one dollar.

Variables in the Rotterdam model, both independent and dependent, are in log-change form, and the equation is written

$$\frac{\overline{w}_{it}}{\overline{W}_{gt}} Dq_{it} = \frac{\mu_i}{M_g} DQ_{gt} + \sum_{i=1}^{3} \Pi_{ij}(Dp_{jt} - Dp_{4t}) + e_{it} \tag{2.5}$$

where $Dq_{it} = \log q_{it} - \log q_{i,t-1} = \log \frac{q_{it}}{q_{i,t-1}}$

and $DQ_{gt} = \sum_{i=1}^{N_g} \frac{\overline{w}_{it}}{\overline{W}_{gt}} Dq_{it}$, or the log-change in the quantity of the gth group is a weighted mean of the log-changes of each good within the group. The "compensated Slutsky coefficient," Π_{ij}, measures the effect of a change in the jth price on the demand for the ith good when $DQ_{gt} = 0$, and μ_i/M_g is the conditional marginal share of the ith good within its group.

Note that only three of the four equations are to be estimated to remove the possibility of a singular covariance matrix. For the simple case, where the errors have zero means and only contemporaneous correlations, the covariance matrix must be singular. This follows from the adding-up criterion, and is accommodated by arbitrarily deleting one equation from the system. Resulting maximum likelihood estimates are invariant to the equation deleted (Barten 1969).

The following specifications are applied to (2.5) (Theil 1975):

1. For each observation in period t, DQ_t and $Dp_{it}, \ldots, Dp_{nt}$ are fixed values taken by exogenous variables and free of observational errors.

2. For each t, the error vector e_t has zero mean, $E(e_{it}) = 0$ for each pair (i, t), and a contemporaneous covariance matrix $[\sigma_{ij}]$ which is a symmetric positive semi-definite n×n matrix with rank n–1 where $\sum_{i=1}^{n} \sigma_{ij} = 0$ for all i. It is assumed that each e_t is mutually independent on each of the other observations in the time span involved and $\sum_{i=1}^{n} e_{it} = 0$ for each t.

3. Marginal shares, μ_i, sum to one.

4. The n×n matrix of Slutsky coefficients, $[\Pi_{ij}]$, is symmetric, negative semi-definite with rank n–1 and $\sum_{i=1}^{n} \Pi_{ij} = 0$ for all i.

These specifications are sufficient for identification of the model, however, the absolute price version of the model has the burden of a large number of unknown parameters, unless the number of demand equations in the system is very small.

The Rotterdam model for the system of consumer demand equations was estimated with Zellner's Seemingly Unrelated Regressions technique (Kmenta 1971). The reason for using this technique is an additional piece of information not taken into account by using ordinary least squares can be used to arrive at unbiased and more efficient estimators of the parameters. This information is the disturbance in the regression equation under consideration could be correlated with the disturbance in some other regression equation.

In matrix notation the system of equations can be written as

$$
\begin{aligned}
Y_1 &= \beta_1 X_1 + e_1 \\
Y_2 &= \beta_2 X_2 + e_2 \\
&\cdots \\
&\cdots \\
&\cdots \\
Y_m &= \beta_m X_m + e_m \qquad m = 1, 2, \ldots, M
\end{aligned}
\tag{2.6}
$$

where Y_m is a T×1 vector of sample values for the dependent variable, X_m is a T×K matrix of the sample values of the explanatory variables, β_m is a K×1 vector of the regression coefficients, and e_m is a T×1 vector of the samples values of the disturbances. It is assumed that e_m is normally distributed with mean zero and the variance-covariance matrix $E(e_m e'_m) = \sigma_{mm} I_T$ where I_T is an identity matrix of order T×T.

Suppose the regression disturbances in different equations may be mutually correlated. We have $E(e_m e'_p) = \sigma_{mp} I_T$ (m, p = 1, . . ., M). Here σ_{mp} is the covariance of the disturbances of the mth and pth equations, assumed to be constant over all observations, and is the only link between the two equations.

To estimate the system when the variance-covariance matrix is unknown, we need to find consistent estimators of the variances and covariances of the regression disturbances. Using residuals obtained from ordinary least squares, e_{mt}, is one way to do this. The result is

$$
\hat{\Omega} = \begin{bmatrix}
s_{11} I_T & s_{12} I_T & \cdots & s_{1m} I_T \\
s_{21} I_T & s_{22} I_T & \cdots & s_{2m} I_T \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
s_{m1} I_T & s_{m2} I_T & \cdots & s_{mm} I_T
\end{bmatrix}
\tag{2.7}
$$

where $s_{mp} = 1/(T-K_m) \sum_{t=1}^{T} e_{mt} e_{pt}$, $K_m \geqslant K_p$ and m, p = 1, . . ., M

Since s_{mm} is an unbiased consistent estimator of σ_{mm}, it can be shown that s_{mp}(m,p) is also a consistent estimator of σ_{mp}. The resulting estimator of β is

$$\hat{\beta} = (X' \hat{\Omega}^{-1} X)^{-1} (X' \hat{\Omega}^{-1} Y) \tag{2.8}$$

with an asymptotic variance-covariance of $\hat{\beta} = (X' \Omega^{-1} X)^{-1}$. This estimator is called a two-stage Aitken estimator after going through the first two procedures. The two-stage Aitken estimator of β is asymptotically equivalent to Aitken's generalized least squares estimator of β and is asymptotically efficient and its asymptotic distribution is normal.

Table 1 shows the resulting parameter estimates for the Rotterdam model. Standard errors are shown in parentheses.[3] Parameters for the chicken equation (i = 4) in both the constrained and unconstrained cases were estimated from the other three by using the identities

$$\sum_{i=1}^{n} \mu_i = 1$$
$$\sum_{j=1}^{n} \Pi_{ij} = 0 \tag{2.9}$$

which should hold given that symmetry and adding-up are accepted. The test for symmetry is an F test with 3 and 39 degrees of freedom. The reported statistic is 1.621, and at a 95% confidence level the test value is approximately 2.84, at a 99% confidence level it is approximately 4.31; therefore, symmetry is accepted and use of identities (2.9) to obtain equation four is justified.

A problem that can present itself in any regression using time-series is serial correlation. When observations are made over time, the effect of the disturbances occurring in one

[3] The statistical package SHAZAM, developed by Dr. Kenneth White of Rice University, was used to obtain the parameter estimates.

Table 1. Parameter Estimates of the Absolute Price Version of the Rotterdam Model for Goods Within a Group.

		μ_i/Mg	Π_{i1}	Π_{i2}	Π_{i3}	Π_{i4}
Constrained						
Fed beef	(i=1)	.69671	−.25631	.13247	.13631	−.01247
		(.11038)	(.087011)	(.067919)	(.031753)	
Non-fed beef	(i=2)	.0015835	.13247	−.19250	.039561	.02047
		(.090137)	(.067919)	(.056826)	(.022658)	
Pork	(i=3)	.26758	.13631	.039561	−.20207	.02620
		(.041457)	(.031753)	(.022658)	(.018653)	
Chicken	(i=4)	.03413	−.01247	.02047	.02620	−.03420
Unconstrained						
Fed beef	(i=1)	.68833	−.24690	.13152	.14136	−.01053
		(.11962)	(.097876)	(.069848)	(.062057)	
Non-fed beef	(i=2)	−.0061339	.14150	−.19593	.052453	.02091
		(.097938)	(.080132)	(.057186)	(.050807)	
Pork	(i=3)	.29681	.11593	.043504	−.21145	.01764
		(.0361135)	(.029566)	(.021099)	(.018746)	
Chicken	(i=4)	.02099	−.01053	.02091	.01764	−.02802

period may carry over into another period. The shorter the interval between periods, the greater the likelihood of encountering autoregressive disturbances.

The coefficient of correlation, ρ, measures the degree of the relationship between two random variables, in this case e_t and e_{t-1}, and its values range from –1 to +1. The coefficient whose value is close to –1 or +1 indicates a high degree of negative or positive relationship between variables. A value close to zero indicates a low degree of relationship. Table 2 displays the first-order approximations to the coefficient of correlation, $\hat{\rho}$, along with the adjusted R^2 values for each equation. The pork equation has the highest value, with the Durbin-Watson statistic associated with it revealing positive autocorrelation. A probable explanation for this may be that the model uses first-differencing of the variables to remove the effect of autoregressiveness. This method assumes that $\rho = 1$, at the other extreme from using no correction method which assumes that $\rho = 0$. Neither assumption may be appropriate. Explicit correction using first-order approximations for ρ would appear to be more econometrically correct.

Table 2. Coefficient of Correlation Approximations and the Adjusted Multiple Correlation Coefficient for the Equations of the Rotterdam Model.

	$\hat{\rho}$	$\bar{R}^2$
Fed-beef (i=1)	.1613	.6216
Non-fed beef (i=2)	.1385	.3726
Pork (i=3)	.3894	.9075

CHAPTER III

THE INDIRECT ADDILOG MODEL

Specification of the demand equations of the Indirect Addilog model utilizes the dual approach. Roy's Identity, explained in Chapter I, is applied to the indirect addilog utility function proposed by Houthakker (1960)

$$U(M, p) = -\sum_{i=1}^{n} A_i (p_i/M)^{\alpha_i} \tag{3.1}$$

This utility form displays constant elasticity, additivity, and homogeneity of degree zero in prices and income.

Resulting demand equations are of the following form:

$$q_i = \frac{A_i \alpha_i (p_i/M)^{\alpha_i - 1}}{\sum_{k=1}^{n} A_k \alpha_k (p_k/M)^{\alpha_k}} \tag{3.2}$$

Positive quantities require that $A_i \alpha_i > 0$ for i=1, 2, . . ., n. To transform (3.2) into budget share form, both sides are multiplied by p_i/M and we obtain

$$w_i = \frac{A_i \alpha_i (p_i/M)^{\alpha_i}}{\sum_{k=1}^{n} A_k \alpha_k (p_k/M)^{\alpha_k}} \tag{3.3}$$

which, when summed over i equals 1. In this form estimation would be non-linear and most likely serially correlated. Thus, transformations are desirable. Initially employing an approach used by Theil (1975a), w_{it} is divided by w_{nt} and logarithms are taken yielding

$$\log \frac{w_{it}}{w_{nt}} = \log \frac{A_i \alpha_i}{A_n \alpha_n} + \alpha_i \log \frac{p_{it}}{M_t} - \alpha_n \log \frac{p_{nt}}{M_t} + \eta_{it} - \eta_{nt} \tag{3.4}$$

However, in this form the residuals show strong positive autocorrelation. Assuming $\rho = 1$, we first-difference to remove this effect and obtain

$$Dw_{it} - Dw_{5t} = \alpha_i(Dp_{it} - Dm_t) - \alpha_5 (Dp_{5t} - Dm_t) + e_{it} - e_{5t} \tag{3.5}$$

where $e_{it} = \eta_{it} - \eta_{i,t-1}$ for $i = 1, \ldots, 4$.

Departing from Theil's method, since the prices $p_{1t}, \ldots, p_{4t}$ and income M_t are deflated by p_{5t}, the implicit price deflator, rather than the price of a specific commodity, $\alpha_5 Dm_t$ will be used in place of $\alpha_5 (Dp_{5t} - Dm_t)$. Notice the A's have disappeared, however, it is the α's, which are used in the determination of the Slutsky coefficients, that are of most concern.

Assume that the vector $[e_{1t}, \ldots; e_{nt}]$ has zero mean and zero lagged covariances and a constant contemporaneous matrix $[\sigma_{ij}]$. Then the disturbance combination $e_{it} - e_{5t}$ for $i = 1, 2, 3, 4$ in (3.5) will also have zero mean and zero lagged covariance, while it contemporaneous covariances are of the form

$$\text{cov}(e_{it} - e_{5t}, e_{jt} - e_{5t}) = \sigma_{ij} - \sigma_{i5} - \sigma_{5j} + \sigma_{55} \tag{3.6}$$

which is independent of t.

Unlike the Rotterdam model, the Slutsky coefficients are not explicitly estimated within the model. By differentiating (3.2) with respect to prices, the result is

$$\begin{aligned} \frac{\partial q_i}{\partial p_j} &= -\alpha_i \frac{q_i^2}{M} + (\alpha_i - 1) \frac{q_i}{p_i} \quad \text{if } i = j \\ &= -\alpha_j \frac{q_i q_j}{M} \quad \text{if } i \neq j \end{aligned} \tag{3.7}$$

The Slutsky equation, $\partial q_i^c/\partial p_j + q_j^* (\partial q_i^*/\partial M)$, when multiplied on both sides by $(p_i p_j/M)$ yields

$$\Pi_{ij} = w_i^2(1 - 2\alpha_i + \sum_{k=1}^{n} w_k \alpha_k) + w_i(\alpha_i - 1) \quad \text{if } i = j$$

$$= w_i w_j(1 - \alpha_i - \alpha_j + \sum_{k=1}^{n} w_k \alpha_k) \quad \text{if } i \neq j \tag{3.8}$$

These Π_{ij} are not constant as they depend on the w's.

That the matrix $[\Pi_{ij}]$ has to be negative semi-definite with rank n–1 is equivalent to the condition that $\alpha_i < 1$ for i = 1, . . ., n. Negative α's are not excluded, although α's equal to zero are. Notice also that if the ith and jth goods are significant luxuries, then α_i and α_j will both be small and their Slutsky coefficient will be positive. Thus, the Indirect Addilog model implies that luxuries tend to be substitutes.

Estimation of the four equations as an unconstrained system yields four significantly different estimates of α_5. To constrain the system to obtain only one estimate of α_5, which is reasonable since it appears in each equation, generalized least squares is applied to

$$\begin{bmatrix} Dw_{1t} - Dw_{5t} \\ Dw_{2t} - Dw_{5t} \\ Dw_{3t} - Dw_{5t} \\ Dw_{4t} - Dw_{5t} \end{bmatrix} = Z_t \vec{\alpha} + \begin{bmatrix} e_{1t} - e_{5t} \\ e_{2t} - e_{5t} \\ e_{3t} - e_{5t} \\ e_{4t} - e_{5t} \end{bmatrix} \tag{3.9}$$

where

$$Z_t = \begin{bmatrix} Dp_{1t} - Dm_t & 0 & 0 & 0 & Dm_t \\ 0 & Dp_{2t} - Dm_t & 0 & 0 & Dm_t \\ 0 & 0 & Dp_{3t} - Dm_t & 0 & Dm_t \\ 0 & 0 & 0 & Dp_{4t} - Dm_t & Dm_t \end{bmatrix}$$

and

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix}$$

That α_5 is identical over all four equations was tested using an F-test. The test statistic is 1.406 with 3 and 64 degrees of freedom. The null hypothesis was accepted at both 95% and 99% levels of confidence. The restriction that α_5 is the same over all equations is accepted, therefore, we may conclude that Slutsky symmetry also holds.

The parameter $\hat{\alpha}$ is defined as

$$(\sum_{t=1}^{T} Z_t' V^{-1} Z_t)^{-1} \sum_{t=1}^{T} Z_t' V^{-1} \begin{bmatrix} Dw_{1t} - Dw_{5t} \\ Dw_{2t} - Dw_{5t} \\ Dw_{3t} - Dw_{5t} \\ Dw_{4t} - Dw_{5t} \end{bmatrix} \qquad (3.10)$$

where V is the 4×4 disturbance covariance matrix whose (i, j) element is defined in (3.8).

Using Zellner's Seemingly Unrelated Regression technique, ordinary least squares is applied to the system of equations, resulting in a generalized least squares estimator of $\vec{\alpha}$. These estimates are presented in Table 3. Table 4 shows the Slutsky coefficients, as defined by (3.7). Notice the cross-price effects are all positive and the own-price coefficients are negative, giving us substitutable commodities and negatively sloped demand curves.

From Table 5, as with the Rotterdam model, the $\bar{R}^2$ are quite low, especially for non-fed beef. This is not surprising though, since these are reported $\bar{R}^2$ of equations where first differencing of the variables was employed. The adjusted R^2 for levels, not first differences, can be expected to be a good deal higher. The corresponding $\hat{\rho}$ are relatively low, indicating that first-differencing may have removed most of the serial correlation.

Table 3. Parameter Estimates of the Indirect Addilog Model.

	α_1	α_2	α_3	α_4	α_5
LS i = 1	.33910 (.19316)				−.12818 (.62926)
LS i = 2		.27249 (.20276)			1.8936 (.92108)
LS i = 3			.22905 (.073216)		1.0450 (.35533)
LS i = 4				.75785 (.061481)	.81274 (.22742)
Constrained	.53665 (.15917)	.15594 (.17907)	.19859 (.06651)	.75672 (.060089)	.81978 (.20314)

Table 4. Computed Slutsky Coefficients of the Indirect Addilog Model.

	Π_{i1}	Π_{i2}	Π_{i3}	Π_{i4}
Π_{1j}	−.01024			
Π_{2j}	.00005	−.00595		
Π_{3j}	.00010	.00007	−.01255	
Π_{4j}	−.00003	.000004	.00001	−.00124

Table 5. Coefficient of Correlation Approximations and the Adjusted Multiple Correlation Coefficient for the Equations of the Indirect Addilog Model.

	$\bar{R}^2$	$\hat{\rho}$
Fed beef (i=1)	.2179	.0311
Non-fed beef (i=2)	.0375	.4084
Pork (i=3)	.1529	−.0900
Chicken (i=4)	.8625	−.1567

CHAPTER IV

THE LINEAR EXPENDITURE SYSTEM

From the Stone-Geary utility function we obtain the Linear Expenditure System (LES). The underlying utility function is

$$U(q_1, \ldots, q_n) = \sum_{i=1}^{n} \mu_i \log(q_i - \gamma_i) \tag{4.1}$$

where μ_i and γ_i are the parameters to be estimated, and the q_i's are assumed to be positive and greater than γ_i.

Marginal utility of the ith commodity is

$$\partial u/\partial q_i = \frac{\mu_i}{q_i - \gamma_i} > 0 \tag{4.2}$$

for i = 1, . . ., n. The condition that all marginal utilities be positive implies that $\mu_i > 0$ for all i. This particular utility function presents some serious restrictions, and it is not at all clear that these will be valid. For example, (4.2) shows that the marginal utility of every good depends only on the quantity of this good, not on that of any other good.

The Linear Expenditure System is defined as

$$p_i q_i = p_i \gamma_i + \mu_i(M - \sum_{k=1}^{n} p_k \gamma_k) + \eta_i \tag{4.3}$$

for i = 1, . . ., n which describes expenditure on the ith commodity, $p_i q_i$, as a linear function of income and n prices. A serious restriction that may occur is the independence assumption where cross-price elasticities are zero (Varian 1978). However, this occurs only if the expansion path is homothetic out of the origin, or, equivalently, $\partial q_i/\partial p_j = -\mu_i \gamma_j/p_i = 0$ only if $\gamma_j = 0$.

At the beginning of the period concerned, the consumer buys γ_i units of the ith good, a minimum or subsistence purchase. For all n goods, the total amount expended will be $\sum_{k=1}^{n} p_k \cdot \gamma_k$. Having made these initial purchases, the consumer concludes that he has the amount

$$M - \sum_{k=1}^{n} p_k \gamma_k > 0 \tag{4.4}$$

available to spend an additional amount of these goods. This leftover amount of purchasing power is referred to as supernumerary income and is allocated to the n commodities with fixed proportions $\mu_1, \ldots, \mu_n$.

Estimation of the demand system using the LES model was approached in two ways: first, by explicitly correcting for serial correlation using a first-order approximation for the correlation coefficient, $\hat{\rho}$, and second, by introducing a trend variable. In the first approach, ordinary least squares was run on a linear approximation

$$p_{it}\, q_{it} = \alpha_{ii}\, p_{it} + \beta_{io}\, M_t + \sum_{j=1}^{n} \beta_{ij}\, p_{jt} + e_t \tag{4.5}$$

of all four equations to obtain $\hat{\rho}$. Resulting estimates for the parameters were used as starting values in the non-linear estimation where both independent and dependent variables were corrected for serial correlation, i.e., $q^*_{it} = q_{it} - \hat{\rho}\, q_{i,t-1}$.

Introduction of a trend variable to account for any serial correlation in the variables produces

$$p_{it}\, q_{it} = \delta_i\, p_{it} + \lambda_i(tp_{it}) + \mu_i M_t - \sum_{j=1}^{n} \mu_i \delta_j\, p_{jt} - \sum_{j=1}^{n} \mu_i \lambda_j(tp_{jt}) + u_i \tag{4.6}$$

where $\gamma_{it} = \delta_i + \lambda_{it}$ for $i = 1, \ldots, n$ and t = trend. A similar estimation procedure was used where ordinary least squares was performed on a linear approximation of (4.6) and, using the resulting estimated parameter starting values, non-linear regression yielded the final parameter estimates, presented in Table 6 along with the estimates of the first approach.

To test the acceptance of Slutsky symmetry, a likelihood ratio test was conducted on the restricted and unrestricted equations for both the trend and first-order rho corrections for serial correlation. In the trend version, the likelihood ratio was 97.24, asymptotically distributed χ^2 with 18 degrees of freedom. Slutsky symmetry is rejected since the test statistic $\chi^2_{18,.05}$ is 28.9. Similar results hold for the first-order rho correction. The likelihood ratio is 77.81 asymptotically distributed χ^2 with 24 degrees of freedom, and the test statistic $\chi^2_{24,.05}$ is 23.33.

Table 6. Parameter Estimates for the Linear Expenditure System.

Correction Using $\hat{\rho}$	μ_i	γ_i
i = 1	.0089497	43.690
	(.0093794)	(32.194)
i = 2	.0056044	6.5428
	(.0060944)	(33.552)
i = 3	-.0013447	73.382
	(.0059929)	(26.565)
i = 4	.0032883	12.860
	(.00029411)	(2.3428)

Trend Introduced	μ_i	δ_i	λ_i	γ_i
i = 1	.015812	17.860	.25068	20.24
	(.0059434)	(17.060)	(.35790)	
i = 2	.0078548	-.72187	-.26300	-3.22
	(.0015128)	(7.1124)	(.22224)	
i = 3	.015590	18.798	-1.5182	4.38
	(.0013170)	(4.9595)	(.099378)	
1 = 4	.0019943	19.511	.43609	23.65
	(.00038041)	(2.0161)	(.10205)	23.65

Notice that in the estimation using $\hat{\rho}$ to correct for serial correlation, we obtain a negative μ_3. According to our theory this is not allowable, however, notice that μ_3 is not significantly different from zero since its standard error is relatively large. The remaining parameter estimates are in accordance with the parameter restrictions.

Where the trend was introduced, the marginal budget shares, μ_i, are all between zero and one, but both δ_2 and λ_2 are negative implying γ_2 (= $\delta_2 + \lambda_2 t$) is also negative since $t > 0$. It is possible to get negative γ's if the expenditure path is homothetic out of anywhere other than the origin. In Equation 4.3, λ_3 is negative, although γ_3 is not, if the trend is taken to be an average of 9.5.

Pollack and Wales (1969) note that the use of a trend variable is not very satisfactory because it gives so little insight into the structure of the economic system. This may not be true since trend variables proxy (1) information, in particular, increasing knowledge of the sources and potentially damaging effects of cholesterol, (2) the changing age structure of the population, and (3) technical changes as in such sectors as meat processing. They also feel a trend variable implied a taste change would continue unabated (the necessary quantities would continue to increase) even if prices and income remain constant over a long period of time. Only if you believe the same market structure remains indefinitely into the future will this be true.

CHAPTER V

THE ALMOST IDEAL DEMAND SYSTEM

A model for a system of demand equations which has been developed recently is the Almost Ideal Demand System (AIDS). Deaton and Muellbauer build upon a model proposed by Working (1943) and Leser (1963) where the ith budget share was postulated to be related to the logarithm of income

$$w_i = \alpha_i + \beta_i \log M \qquad (5.1)$$

This model was extended to include prices of other goods under study, forming

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \log p_j + \beta_i \log(M/P) \qquad (5.2)$$

where α_i is the average budget share when all log prices and real expenditure equal one, γ_{ij} is the change in the ith budget share with respect to a percentage change in the jth price with real expenditure held constant, and β_i is the change in the ith budget share with respect to the percentage change in real income with prices held constant.

Rather than using real prices and real income, real prices and nominal income were used where nominal income was converted to real using P, a price index defined as

$$\log P = \alpha_0 + \sum_{k=1}^{n} \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j \qquad (5.3)$$

This is clearly non-linear in parameters. To obtain a linear approximation we use

$$\log P^* = \sum_k w_k \log p_k \qquad (5.4)$$

where $P \approx \xi P^*$, and we re-define (5.2) as

$$w_i = \alpha_i^* + \sum_j \gamma_{ij} \log p_j + \beta_i \log(M/P^*) \qquad (5.5)$$

with $\alpha_i^* = \alpha_i - \beta_i \log \xi$.

AIDS was derived by use of duality concepts from an indirect expenditure function

$$\log c(U, p) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj}^* \log p_k \log p_j + U\beta_0 \prod_k p_k^{\beta_k} \qquad (5.6)$$

where c denotes the indirect expenditure function, U represents an unobservable utility level, β_0 is a non-estimatable expenditure parameter, p_k's are prices and α_k, γ_{kj}^*, and β_k are the parameters to be estimated. This particular expenditure function was chosen because it is flexible, permits exact aggregation over consumers, and will result in a system of demand equations with desirable properties.

Using Shepard's lemma, $\partial c(U, p)/\partial p_i = q_i^c(U, p) = q_i^c$, we obtain compensated demand equations which are multiplied by $p_i/c(U, p)$ to obtain the equation in budget share form. The result is

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i U\beta_0 \prod_k p_k^{\beta_k} \qquad (5.7)$$

where $\gamma_{ij} = 1/2(\gamma_{ij}^* + \gamma_{ji}^*)$. Solving for U where the function $M = c(U, p)$ is in equilibrium, and substituting that into (5.7) the result is (5.2).

Advantages of this particular system include:

1. it gives an arbitrary first order approximation to any demand system
2. it exactly satisfies axioms of choice
3. it aggregates perfectly over consumers
4. its functional form is consistent with previous household budget data
5. it is simple to estimate in its linear approximate form, and
6. it can be used to test for symmetry and homogeneity.

In addition, AIDS is indirectly non-additive allowing the consumption of one good to affect the marginal utility of another, a characteristic the LES cannot claim. Therefore, AIDS does not impose substitution limitations as the LES does.

A limitation of the linear approximation version of the Almost Ideal Demand System is its loss of flexibility as compared to the non-linear specification. However, estimation of the model in full generality is difficult to accomplish.

The adding-up restriction where the budget share sum to one is met if $\sum_i \alpha_i = 1$, $\sum_j \gamma_{ij} = 0$, and $\sum_i \beta_i = 0$. By imposing $\sum_j \gamma_{ij} = 0$ the homogeneity condition is met, and imposing $\gamma_{ij} = \gamma_{ji}$ for all i,j implies symmetry.

The estimation procedure used was again Zellner's Seemingly Unrelated Regressions using a system of four equations. Again, since this is a subsystem of commodities, singularity of the covariance matrix is disregarded.

Also estimated as an AIDS model was a form of AIDS resembling the Rotterdam model. By using

$$\Delta w_i = \beta_i \Delta \log \left(\frac{M}{P}\right) + \sum_j \gamma_{ij} \Delta \log p_j \qquad (5.8)$$

and replacing $\Delta \log P$ by $\sum_k w_k \Delta \log p_k$ the specification for the right-hand side is identical to the Rotterdam model. The left hand side is, however, not the same.

Explicitly testing both specifications of the system of demand equations for acceptance of the null hypothesis that Slutsky symmetry conditions hold was again accomplished through use of an F-test. The test statistic for the linear approximation version was 1.830 with 6 and 48 degrees of freedom. The null hypothesis is accepted at both 5% and 1% significance levels. For the Rotterdam version, the test statistic is 2.004 with 6 and 52 degrees of freedom. Acceptance of the null hypothesis occurs at the 5% significance level, but not at the 1% level.

Tables 7 and 8 display the parameter estimates for (5.5) and (5.8), respectively. First order approximations for the coefficients of correlation, $\hat{\rho}$, and the adjusted R^2 values are found in Table 9.

Table 7. Parameter Estimates for the Almost Ideal Demand System as in Equation (5.5).

	α_0	β	γ_{1i}	γ_{2i}	γ_{3i}	γ_{4i}
Constrained						
Fed beef (i=1)	−.015559	−.0032497	.006012	.0095858	.0011287	−.0012249
	(.018408)	(.0013013)	(.010803)	(.0056595)	(.0027544)	(.00066026)
Non-fed beef (i=2)	−.012128	−.0010898	.0095858	−.0067797	.0018339	.0012529
	(.009544)	(.00064934)	(.0056595)	(.0034351)	(.0015537)	(.00036285)
Pork (i=3)	.061834	−.0053765	.0011287	.0018339	.0010761	.00036116
	(.0053917)	(.0003618)	(.0027544)	(.0015537)	(.0010586)	(.00025956)
Chicken (i=4)	−.0028189	−.00038323	−.0012249	.0012529	.00036116	.0025502
	(.0016301)	(.00011231)	(.00066026)	(.00036285)	(.00025956)	(.0002509)
Unconstrained						
Fed beef (i=1)	.04034	−.0064954	.0086299	.010572	.0061661	−.014502
	(.034242)	(.0023931	(.011461)	(.0064937)	(.0051646)	(.0074222)
Non-fed beef (i=2)	−.024682	−.00010653	.0076520	−.0065731	.00063993	:0047271
	(.018528)	(.0012949)	(.0062017)	(:0035937)	(.0027945)	(.0040161)
Pork (i=3)	.044906	−.004535	.0012362	.0015233	−.00086351	.0041881
	(.01069)	(.00074711)	(.0035782)	(.0020273)	(.0016124)	(.0023172)
Chicken (i=4)	−.0075865)	−.0010117	.0010936	−.00016284	.0035592	−.0075865
	(.0025842)	(.000865)	(.00049009)	(.00038978)	(.00056016)	(.0025842)

Table 8. Parameter Estimates for the Almost Ideal Demand System as in Equation (5.8).

	β	γ_{1i}	γ_{2i}	γ_{3i}	γ_{4i}
Constrained					
Fed beef (i=1)	-.0033026	.0034085	.0059692	.00039789	-.00088652
	(.0041041)	(.0087494)	(.0048195)	(.0020685)	(.00071720)
Non-fed beef (i=2)	-.0012463	.0059692	-.00285	.0020099	.00073008
	(.00256)	(.0048195)	(.0031374)	(.0012468)	(.00042404)
Pork (i=3)	-.0056518	.00039789	.0020099	.0031914	.0007206
	(.0015242)	(.0020685)	(.0012468)	(.00092167)	(.00030584)
Chicken (i=4)	-.00020037	-.00088652	.00073008	.0007206	.0032105
	(.00037708)	(.0007172)	(.00042404)	(.00030584)	(.00033768)
Unconstrained					
Fed beef (i=1)	-.0055294	.0048804	.0094147	.0021727	-.0067471
	(.0042686)	(.0089028)	(.0051467)	(.0041227)	(.0048091)
Non-fed beef (i=2)	-.0013525	.0028621	-.0014315	.0021083	-.0011788
	(.0026759)	(.005810)	(.0032264)	(.0025845)	(.0030147)
Pork (i=3)	-.0064925	.00021298	.0045199	.0026075	-.00079188
	(.0015941)	(.0033247)	(.0019220)	(.0015396)	(.0017959)
Chicken (i=4)	-.00028739	-.0005833	.00091055	.00049202	.0033567
	(.00038130)	(.00079526)	(.00045974)	(.00036827)	(.00042988)

Table 9. Coefficients of Correlation Approximations and Adjusted Multiple Correlation Coefficients for the Equations of the Almost Ideal Demand System with Equation (5.5) in (a) and (5.8) in (b).

	Constrained		Unconstrained	
	$\bar{R}^2$	$\hat{\rho}$	$\bar{R}^2$	$\hat{\rho}$
(a)				
Fed beef (i=1)	.3497	.6126	.4625	.5513
Non-fed beef (i=2)	.2438	.3182	.2798	.1894
Pork (i=3)	.9305	.3036	.9405	.0661
Chicken (i=4)	.8896	−.0288	.9096	.0519
(b)				
Fed beef (i=1)	.2667	.0559	.3658	.0842
Non-fed beef (i=2)	−.2429	.0683	−.1479	.2724
Pork (i=3)	.3286	−.0636	.4681	.1214
Chicken (i=4)	.8812	−.0952	.8927	−.1757

CHAPTER VI

THE DOUBLE LOGARITHMIC MODEL

Demand models may be derived either by the selection of a utility function, or by the arbitrary specifications of a system of equations which may be modified according to utility theory, as in the case of the Rotterdam model. The problem with the first approach is that there may be a difficulty in selecting the "right" functional form. The problem with the latter, however, is the difficulty in choosing which variables to parameterize, and this decision may lead to unexpected consequences (Deaton 1974). It has also been argued that a set of estimated equations not derived from utility theory cannot represent an underlying rational utility function unless the parameters of the equations are constrained to satisfy an appropriate set of conditions guaranteeing symmetry of the Slutsky terms and negative semi-definiteness of the Slutsky matrix (LaFrance 1983).

The double logarithmic functional form for consumer demand equations was not developed through the use of utility theory, but rather by an arbitrary specification with the result of ease in the estimation of the parameters. Its functional form is

$$\log q_i = \alpha_i + \gamma_i \log \frac{M}{p_n} + \sum_k \beta_{ik} \log \frac{p_k}{p_n} + u_i \, . \tag{6.1}$$

where γ_i is the income elasticity and β_{ik}'s are the price elasticities.

To constrain this functional form to comply with Slutsky symmetry and a negative semi-definite Slutsky matrix, one must impose

$$\beta_{ij} \frac{q_i}{p_j} + \gamma_i \frac{q_i q_j}{M} = \beta_{ji} \frac{q_j}{p_i} + \gamma_j \frac{q_i q_j}{M} \tag{6.2}$$

for i, j = 1, . . ., n−1. It can be shown that in this system $\gamma_i = \gamma_j$ for all i and j (LaFrance 1983). Given this,

$$\beta_{ij}\, p_i\, q_i = \beta_{ji}\, p_j\, q_j \tag{6.3}$$

is a sufficient condition for Slutsky symmetry.

Other characteristics of this model include

$$\begin{aligned} \beta_{ij} &= 1 + \beta_{jj} \\ \beta_{ji} &= 1 + \beta_{ii} \\ \beta_{ik} &= \beta_{jk} \qquad \text{for } k \neq i,j \end{aligned} \tag{6.4}$$

From these it follows that

$$\alpha_i\,(1 + \beta_{jj}) = \alpha_j\,(1 + \beta_{ii}) \tag{6.5}$$

$$\text{sign}\,(1 + \beta_{ii}) = \text{sign}\,(1 + \beta_{jj}) \qquad i \neq j \text{ such that } \beta_{ij} \neq 0.$$

The own Slutsky terms $(\beta_{ii}/p_i + \gamma q_i/m)q_i$ will be negative if and only if $(\beta_{ii}/p_i + \gamma q_i/m) < 0$, requiring that $\beta_{ii} < 0$ and/or $\gamma < 0$.

Homogeneity is satisfied because of the division of income and prices by p_n, the deflator. However, the adding-up condition cannot be accommodated within the double logarithmic specification because it implies that either all the γ's must equal unity, or at least one of them must be larger than unity, neither of which appear plausible (Deaton and Muellbauer 1980). This need not present any difficulties in this study since the estimation procedure involves a subsystem of four commodities rather than the entire budget outlay.

Estimation of this model involves an interative procedure beginning with the use of Zellner's technique on

$$\ln q_1 = \alpha_1 + \beta_{11}\ln(\frac{p_1}{p_5}) + (1+\beta_{22})\ln(\frac{p_2}{p_5}) + (1+\beta_{33})\ln(\frac{p_3}{p_5}) + (1+\beta_{44})\ln(\frac{p_4}{p_5}) + \gamma\ln(\frac{m}{p_5}) + e_1$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \qquad (6.6)$$

$$\ln q_4 = \alpha_4 + (1+\beta_{11})\ln(\frac{p_1}{p_5}) + (1+\beta_{22})\ln(\frac{p_2}{p_5}) + (1+\beta_{33})\ln(\frac{p_3}{p_5}) + \beta_{44}\ln(\frac{p_4}{p_5}) + \gamma\ln(\frac{m}{p_5}) + e_4$$

This yields initial estimates of the parameters which can be used as starting values for Zellner's non-linear estimation of

$$\ln q_1 = \alpha + (e^{\beta_1}-1)\ln(\frac{p_1}{p_5}) + e^{\beta_2}\ln(\frac{p_2}{p_5}) + e^{\beta_3}\ln(\frac{p_3}{p_5}) + e^{\beta_4}\ln(\frac{p_4}{p_5}) + \gamma\ln(\frac{m}{p_5}) + u_1$$

$$\ln q_2 = \alpha + \ln(e^{\beta_2}/e^{\beta_1}) + e^{\beta_1}\ln\frac{p_1}{p_5} + (e^{\beta_2}-1)\ln\frac{p_2}{p_5} + e^{\beta_3}\ln\frac{p_3}{p_5} + e^{\beta_4}\ln\frac{p_4}{p_5} + \gamma\ln\frac{m}{p_5} + u_2$$

$$(6.7)$$

$$\ln q_3 = \alpha + \ln(e^{\beta_3}/e^{\beta_1}) + e^{\beta_1}\ln\frac{p_1}{p_5} + e^{\beta_2}\ln\frac{p_2}{p_5} + (e^{\beta_3}-1)\ln\frac{p_3}{p_5} + e^{\beta_4}\ln\frac{p_4}{p_5} + \gamma\ln\frac{m}{p_5} + u_3$$

$$\ln q_4 = \alpha + \ln(e^{\beta_4}/e^{\beta_1}) + e^{\beta_1}\ln\frac{p_1}{p_5} + e^{\beta_2}\ln\frac{p_2}{p_5} + e^{\beta_3}\ln\frac{p_3}{p_5} + (e^{\beta_4}-1)\ln\frac{p_4}{p_5} + \gamma\ln\frac{m}{p_5} + u_4$$

where $\beta_1 = \ln(1 + \beta_{11})$ to insure that the estimates of the own-price elasticities satisifed $0 \geqslant \beta_i \geqslant -1$.

A likelihood ratio was calculated between the restricted equations (6.7) and the unrestricted (6.1) to determine a test for Slutsky symmetry. For 28 degrees of freedom, the ratio determined was 114.46, asymptotically distributed χ^2. The test statistic was 41.3 at a 5% significance level, thereby rejecting the symmetry conditions.

Final estimates are shown in Table 10. The own-price elasticities are all negative, as expected, and by addition of 1 to all own-price elasticities shows that cross-price elasticities are positive. The elasticity estimates are significant since the t-ratio values of the β_{ii}'s range from -12.235 to -27.043. The common income elasticity is positive and significant.

Table 10. Parameter Estimates of the Double Logarithmic Model.

Equation		β_{i1}	β_{i2}	β_{i3}	β_{i4}	γ	α
Fed beef	(i = 1)	–.7141					
Non-fed beef	(i = 2)		–.9063			.52812 (.18368)	–.83618 (2.4576)
Pork	(i = 3)			–.7891			
Chicken	(i = 4)				–.9332		

CHAPTER VII

COMPARISONS AND CONCLUSIONS

From a purely subjective standpoint, there is no clear cut choice as to which of the five models of systems of consumer demand equations performed "best" in adhering to the theoretical restrictions outlined in Chapter I, and in their ability to statistically summarize consumer behavior accurately. A quantitative comparison of the models is preferred. An example of such a comparison involves using $\overline{R}^2$ to contrast the fit of the equations. First, there are different dependent variables in each model. Second, application of the coefficient of multiple correlation is not unique when it is used for models consisting of several equations (Theil 1975a). For example, assume two models exist, each having the same dependent variable and each consisting of four equations. In model A there are two equations which have larger adjusted R^2 values, but model B has larger values for the remaining two equations. It is not clear on this basis alone which system to choose.

Theil's information inaccuracy measure is computed to quantitatively compare the five models presented in this study. Recall that demand theory is basically an allocation theory, concerned with the proportions of total expenditure on n commodities and the determination of the proportions by the level of total expenditure and the n prices. Since these prices and total expenditure levels are considered exogenous, all models can be used to generate predictions of budget shares. The shares are positive and sum to unity, thus they may be interpreted as probabilities. Considering the models' predictions as prior probabilities and then evaluating the expected gain in information that is obtained from the true shares, information inaccuracy is measured by

$$I_T = \sum_{i=1}^{N} w_{it} \ln \frac{w_{it}}{\hat{w}_{it}} \qquad (7.1)$$

The predicted shares for period t is $\hat{w}_{it}$, and the actual share for the ith good in the t^{th} period is w_{it}. An excellent method of measuring the relative performance of the models can be obtained by examining the average information inaccuracy over the sample period. The model associated with the information inaccuracy nearest zero is considered best.

Information inaccuracy has several advantages over multiple correlations computed for each demand equation separately. First, it refers to all n commodities and therefore all n demand equations simultaneously thus avoiding the possibility of conflicting verdicts for different equations. Secondly, it recognizes explicitly that demand theory is concerned with allocation of total expenditure in terms of budget shares for each commodity, a feature multiple correlations disregard. Third, information inaccuracy can be compared for each observation separately, whereas multiple correlation coefficients typically refer to the sample of all observations.

Table 11 summarizes the statistics for each model. According to the information inaccuracies, the absolute price of the Rotterdam model for goods within a group is superior to the others.[4] This is not surprising, however, given that the estimated Rotterdam model is less restrictive of its parameters than the other models. As seen in Chapter II, the underlying utility function of the Rotterdam model is a special case of the utility function from

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[4] Predicted budget shares for the Rotterdam model were found using

$$\hat{w}_{it} = w_{it} - \hat{e}_{it} \qquad (7.a)$$

where $\hat{e}_{it}$ is the estimated residual in

$$\frac{\bar{w}_{it}}{\bar{W}_{gt}} Dq_{it} = \frac{\mu_i}{M_g} DQ_t + \sum_{i=1}^{3} \Pi_{ij} (Dp_{it} - Dp_{4t}) + e_{it} \qquad (7.b)$$

(Goldman 1971). If one is willing in this case to assume that $.5\,(w_{it} + w_{i,t-1})$ is constant, then (7.a) is in fact true.

which the Linear Expenditure System is derived. Therefore, the Rotterdam model is also a special case of the Linear Expenditure System. In the former, each of the estimated demand equations within the system yield price coefficients for all four commodities and allows these estimates to vary from equation to equation. The LES, however, estimates only the own-price coefficients in each individual equation, which is more restrictive. In view of this, if the estimation of the Rotterdam model had been restricted as much as the Linear Expenditure System, perhaps the Almost Ideal Demand System would be the "best" model since it performed better than the rest of the model specifications in terms of information inaccuracy measures.

Table 11. Information Inaccuracies of the Seven Demand Models.

Model	I_T	I_T (goods within group)
Rotterdam	--	.00150
Indirect Addilog	.00009	.00258
LES w/$\hat{\rho}$ correction	.00026	--
LES w/ trend correction	.00024	--
AIDS (5.5)	.00010	.00189
AIDS (5.8)	.00011	.00181
Double logarithmic	.00034	--

All specifications, excluding the Rotterdam model, dealt with five categories of goods: fed beef, non-fed beef, pork, chicken, and all other goods. Within that category, the Indirect Addilog placed first, although the Almost Ideal Demand System placed a close second and third.[5] For comparison to the Rotterdam model, budget shares for each type

[5] To obtain the predictors $\hat{w}_{it}$ of the Rotterdam, leaving the α's in (7.c) are replaced by their constrained estimates.

$$Dw_{it} - Dp_{5t} = \alpha_i (Dp_{it} - Dm_t) - \alpha_5 \, Dm_t - u_{it} \qquad i = 1, 2, 3, 4 \qquad (7.c)$$

we thus get an estimator

$$\hat{y}_{it} = \ln \left(\frac{\hat{w}_{it}/\hat{w}_{5t}}{w_{i,t-1}/w_{5,t-1}} \right) \qquad i = 1, 2, 3, 4 \qquad (7.d)$$

of meat in the three model specifications were converted to budget shares within the meat group. Average information inaccuracy was then re-calculated for the Indirect Addilog model and the Almost Ideal Demand System. Each exhibited a larger value than did the Rotterdam, leaving the Rotterdam the best model of the five functional forms. Note, however, that for goods within a group, both of the Almost Ideal Demand System specifications outperformed the Indirect Addilog.

A common feature of the five functional forms outlined in this study is the small number of parameters estimated in each, although AIDS tends to have more than the others give that its cross-price coefficients are not predetermined. If it is the true functional form being estimated, then the resulting demand equations are much more accurate when there are fewer parameters involved and degrees of freedom are of less concern. However, this strength of the model may also imply a weakness; restrictiveness of preferences. Consideration of this along with the results of the study implies that further work on specification of demand models is needed.

What about the Slutsky symmetry constraints imposed on the models? The four main models which ranked highest in terms of information inaccuracies, the Rotterdam model, the Indirect Addilog model, and both forms of the Almost Ideal Demand System, accepted the symmetry constraints. Now we not only have a test of the models' statistical fit via the information inaccuracy measure, but also a test of the models' adherence to theoretical restrictions that must hold to have a viable system of demand equations. The results of the

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taking $w_{i,t-1}$ as given. The final prediction formula becomes

$$w_{it} = \frac{(w_{i,t-1}/w_{5,t-1}) \times e^{\hat{y}_{it}}}{1 + \sum_{k=1}^{4} (w_{k,t-1}/w_{5,t-1}) \times e^{\hat{y}_{kt}}} \qquad i = 1, 2, 3, 4 \tag{7.e}$$

Obtaining $\hat{w}_{it}$ for the remaining models was a matter of algebraic manipulation of both sides of the equality sign to convert the dependent variable to budget shares, the $\hat{e}_{it}$ becomes the difference between $\hat{w}_{it}$ and w_{it}.

two tests are entirely consistent with one another since the models which performed best statistically also accepted the theoretical constraints imposed.

In comparing these results with those of other studies, economists such as Barten (1969), Byron (1970), and Baldwin (1983) rejected the validity of the symmetry restrictions in view of the empirical studies conducted by each. Deaton (1972) does not and suggests that the results of Barten and Byron reflect the use of inappropriate estimation procedures. Baldwin points to other possible explanations including

1. whether or not the symmetry restrictions were valid may be dependent on whether or not homogeneity was satisfied. In those studies where homogeneity conditions have been tested separately, the outcome was rejection of these conditions and

2. the specifications of demand equations and the treatment of durable goods may have influenced the conclusions. It takes time to adjust to price changes of durable goods and a dynamic treatment of these goods would fare better than a static treatment.

A possible reason that may be the cause of the acceptance of symmetry in this study versus the rejection in other studies is the difference between the commodities studied. Substitution and complimentarity are much more likely to be testable within a system of equations that represent demand for goods within a particular budget outlay, i.e., food. When testing the null hypothesis that the Slutsky conditions hold, we are tacitly performing two hypothesis tests simultaneously: (1) the functional form and (2) aggregation over commodities. Regarding the first, this study considered five functional forms all of which explicitly tested the Slutsky conditions. Three of those accepted symmetry; the three whose functional forms are considered best from the information inaccuracies. Since Slutsky symmetry is accepted at the disaggregated level, perhaps aggregation over commodities to such as "food" and "clothing" in the other studies presents the problem. Rather restrictive assumptions are required at an aggregate level to obtain symmetry across groups, including separability, homotheticity of the individual aggregator functions, and additivity of the utility functions across the aggregated groups (Blackorby et al. 1978). Therefore, symmetry of substitution between aggregated commodities does not seem to be as easy to test as between disaggregated commodities, and may be less valid.

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APPENDIX

Table 12. Data Used in Estimations.

Year	Per Capita Consumption of Fed Beef (lbs)	Non-fed Beef (lbs)	Pork (lbs)	Chicken (lbs)	Real Price of Fed Beef (¢/lb)[1]	Non-fed Beef (¢/lb)[2]	Pork (¢/lb)	Real Price of Chicken (¢/lb)[3]	Nominal Income(¢)	Real Income(¢)	Deflator (1980=100)[4]
1962	52.52	36.35	75.0	29.8	198.78	121.05	141.85	98.54	207300.0	504380.0	41.1
1963	57.60	36.83	76.3	30.8	188.25	112.90	134.05	97.84	214400.0	514149.0	41.7
1964	61.66	38.28	76.3	31.2	180.42	110.07	130.66	91.04	229600.0	541509.0	42.4
1965	60.98	38.50	67.2	33.3	185.85	113.87	151.28	91.88	244800.0	567981.0	43.1
1966	65.81	38.32	65.7	35.6	190.52	119.26	165.69	93.91	261300.0	589842.0	44.3
1967	69.63	36.84	72.0	36.4	186.34	115.09	146.70	85.24	275700.0	607269.0	45.4
1968	73.08	36.65	73.5	36.7	187.92	116.69	141.53	86.44	295600.0	626271.0	47.2
1969	77.02	33.80	71.4	38.4	181.74	126.77	149.29	88.03	315200.0	639351.0	49.3
1970	81.53	32.02	72.6	40.4	197.09	125.64	150.00	80.81	339000.0	656977.0	51.6
1971	80.44	32.22	78.7	40.3	200.56	125.08	129.50	77.92	362000.0	671614.0	53.9
1972	86.03	29.45	70.9	41.8	212.72	133.21	148.21	76.52	386000.0	691756.0	55.8
1973	80.32	28.52	63.4	40.4	240.85	158.19	185.08	103.05	431500.0	731356.0	59.0
1974	73.49	42.24	68.5	40.7	225.08	143.85	165.85	87.69	466700.0	718000.0	65.0
1975	62.37	56.43	55.4	40.1	221.46	121.77	192.66	91.99	507500.0	726037.0	69.9
1976	72.86	54.74	58.7	42.7	201.63	113.27	182.31	83.27	547700.0	745170.0	73.5
1977	76.74	47.23	60.5	44.1	190.99	104.14	116.39	79.67	595400.0	766281.0	77.7
1978	80.65	37.26	60.3	46.7	218.63	133.32	172.60	79.93	657100.0	789784.0	83.2
1979	74.25	31.22	68.8	50.6	249.50	170.53	158.88	74.64	729300.0	804079.0	90.7
1980	70.40	32.98	73.4	50.0	237.60	156.17	139.50	71.90	800200.0	800200.0	100.0

Source: U.S. Department of Agriculture.

[1] Average choice grade prices.

[2] Average ground beef prices.

[3] Average broiler prices.

[4] Implicit Price Deflator for Personal Consumption Expenditures. Source: *Economic Report of the President*.

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An empirical comparison of alternative functional forms of systems of consumer demand equations

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