Forced vibration of unevenly distributed masses
by Richard Randolph Frank

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
Forced vibration has been used extensively in the transportation or separation of various sized particles. The forcing frequency is usually created with a rotating or reciprocating unbalance that impresses a periodic, harmonic motion. This motion is directed to impart both a horizontal and a vertical movement to the particle. This thesis shows the development of the equations, with their boundry conditions, that describe, the motion of unattached particles on a vibrating platform. Results from these theoretical equations were tabulated in table form and compared with the test results.

A high powered stroboscope was used with a photoelectric pickoff and a flash delay to photograph various stages in the cycle of rock particles on a vibrating platform. The position of the eccentric weights was used as a marker to determine the point in the vibrating platform's cycle that the photographs were taken. Various exciting frequencies and exciting forces were photographed for comparison with the theoretical results.

The photographs from the experimental tests are included in this thesis along with an explanation of their position in the cycle. A comparison with the theoretical results given in the tables showed that a very accurate and consistent relationship between thg two results existed. From these results, it has become evident that further analysis could be carried out using the stroboscope and possibly slow motion picture or multiple flash photography along with the single flash photography.

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Abstract

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CHAPTER 1

INTRODUCTION

Vibration could be described as the motion of a mass under a constant change of displacement, velocity, and acceleration. If this motion is repeated over a definite interval of time it is called periodic motion. The simplest periodic motion is harmonic motion, which can be represented by a sine or cosine function.

If a vibrating system is set into motion by an external exciting force, it is said to be a forced vibration. Possibly the most common external exciting force for a vibrating system would be a reciprocating or rotating unbalance. This type of exciting force would impress a harmonic motion with a periodic frequency that would dominate the system. The forces on this system are related by Newton's second law of motion; the sum of the external forces must equal the inertia force. This is indicated by the following equation of motion for a forced vibrating system:

$$m\ddot{x} + c\dot{x} + kx = f\ (t)$$

where f (t) is the exciting force and m, c, and k are the mass, damping coefficient, and spring constant of the system, respectively. The displacement of the system is given by x which is assumed positive in the upward direction. The first derivitive of x with respect to time is the velocity, denoted by $\dot{x}$, and the second derivitive is the acceleration, denoted by $\ddot{x}$. The exciting force for a rotating or reciprocating unbalance, moving at a constant angular velocity, ω, may be represented by the following:

$$f\ (t) = F \sin \omega t = me\omega^2 \sin \omega t$$

where m is the mass of the unbalanced weights, e is the eccentricity, and t represents time.

Once a forced vibrating system reaches the steady-state oscillation a particular solution to its equation of motion may be expressed as

$$x = X \sin (\omega t - \phi)$$

where X is the amplitude of vibration and ϕ is the angle at which the displacement lags the exciting force.

Figure 1 shows the vector relationship of the forces on the system as represented by the equation of motion and its solution. From the geometry of this diagram, it may be seen that the amplitude of vibration can be represented as

$$X = \frac{F}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

Further analysis shows that this may be rewritten as

$$X = \frac{\frac{m}{M} e \left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2 \zeta \frac{\omega}{\omega_n}\right]^2}}$$

Where ω_n is the natural frequency of the system and ζ is the damping factor. A plot of this equation with a very small damping factor is shown in Figure 2. When the frequency of the exciting force matches the natural frequency of the system, extremely large amplitudes result. Further study of this graph shows that as the forcing frequency approaches 4 or 5 times the natural frequency, the amplitude of the system approaches

$$X = \frac{me}{M}$$

where M is the total mass of the vibrating system.

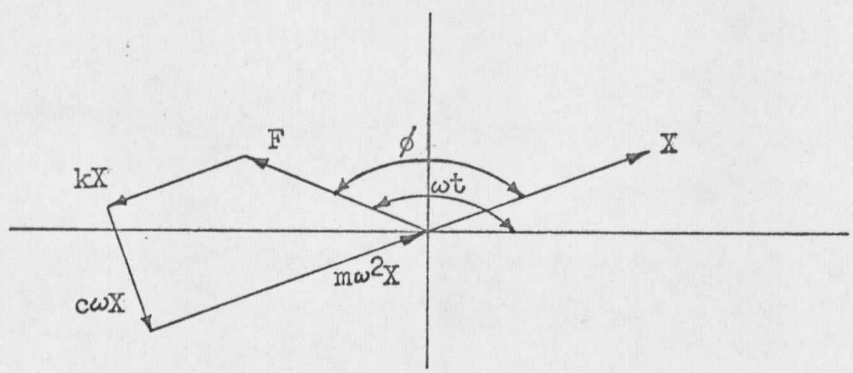

Figure 1. Vector diagram of the forces on a forced vibration system.

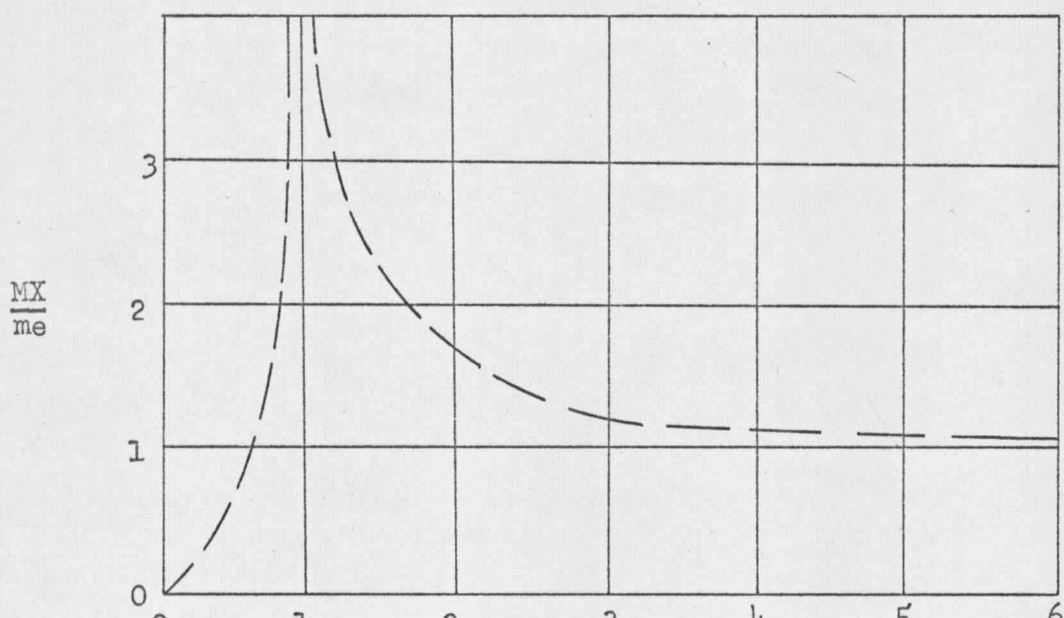

Figure 2. Plot of $\frac{MX}{me}$ vs. $\frac{\omega}{\omega_n}$ for a very small damping factor in a forced vibration.

Figure 3 shows the relationship between the frequency ratio, ω/ω_n, and the phase angle, ϕ, for a very small damping factor. When the forcing frequency approaches 4 or 5 times the natural frequency, the phase angle approaches 180 degrees. When this happens in a forced vibration, the displacement will be about 180 degrees out of phase with the exciting force.

Since the amplitude at any time, t, is represented by x, the acceleration at any time, t, is represented by

$$\ddot{x} = -\omega^2 X \sin(\omega t - \phi)$$

A graphical representation of the displacement and acceleration is shown in Figure 4. The acceleration is 180 degrees out of phase with the displacement. The equation indicates that the maximum acceleration of the vibrating system will occur when sin $(\omega t - \phi)$ is equal to unity. This will take place each time that the maximum displacement of the vibrating unit occurs. The maximum acceleration may then be expressed as

$$\ddot{x}_{max} = \omega^2 X$$

One important use of forced vibration is the separation or transportation of large quantities of various materials. Various sized particles may be separated according to size by using a vibrating screen with openings that are large enough for the smaller particles to pass through, but too small for the larger particles. Little or no research has been done on the analysis of the motion of the particles in this type of system. The particles will act as ballistic projectiles through part of their cycle and will follow the sinusoidal motion of the screening platform through the remaining portion of their cycle. The two portions of the

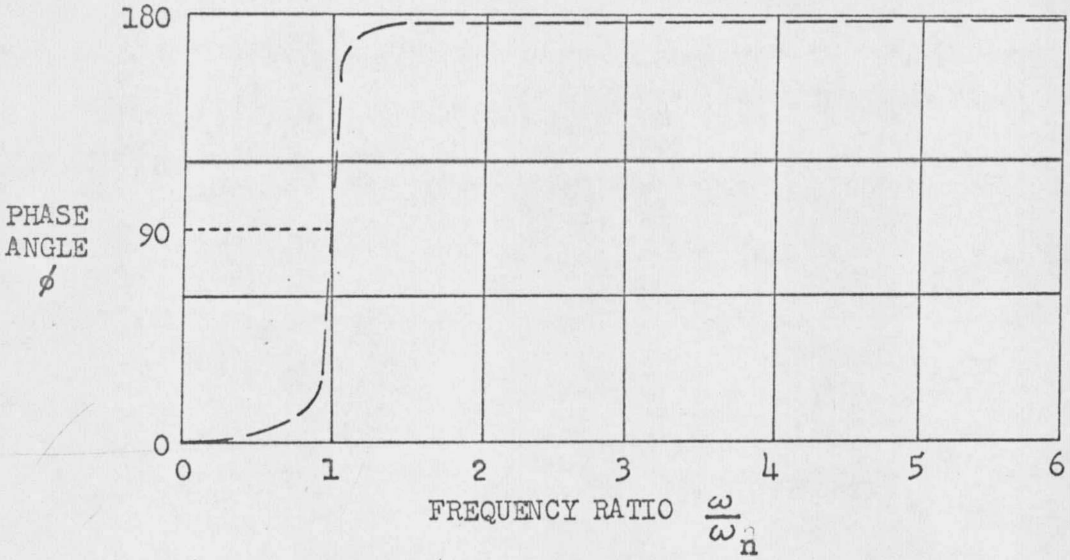

Figure 3. Plot of ϕ vs. $\frac{\omega}{\omega_n}$ for a very small damping factor in a forced vibration.

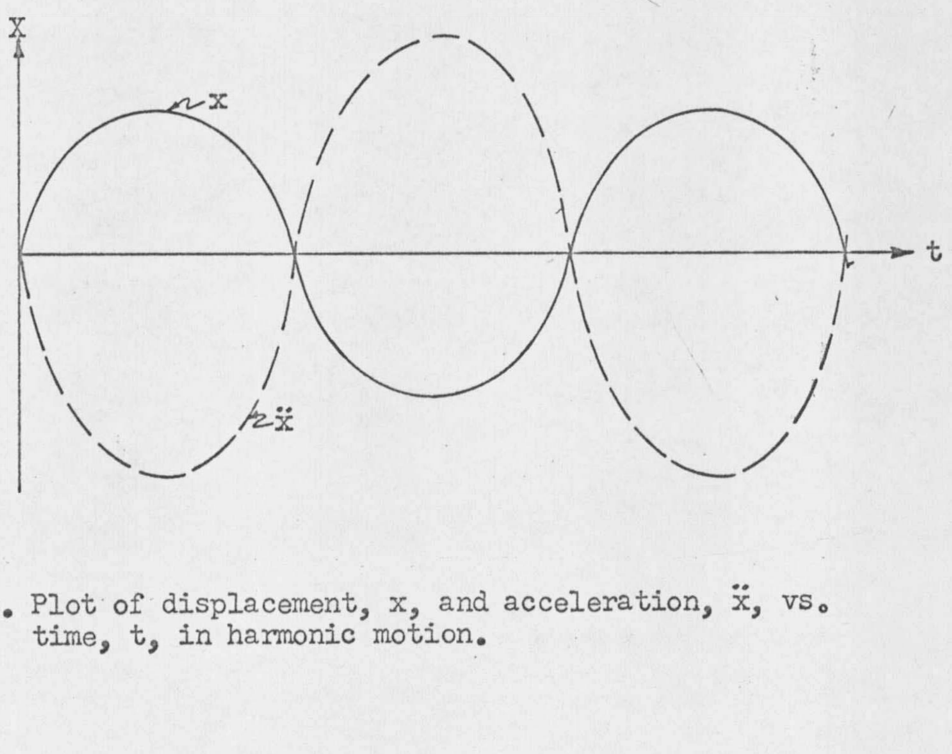

Figure 4. Plot of displacement, x, and acceleration, $\ddot{x}$, vs. time, t, in harmonic motion.

cycle must be analyzed separately and then combined to get a more accurate picture of the actual motion.

The notation used in this thesis is defined at its first appearance and is consistently used throughout the text.

CHAPTER 2

THEORETICAL DEVELOPMENT

As a preliminary analysis, consider a single particle of mass resting unattached to a platform which is set into a harmonic oscillation by an external exciting force. The motion of the particle resting on the platform would depend upon $\ddot{x}_{max}$ of the platform. If the maximum acceleration of the vibrating platform were less than or equal to the acceleration of gravity, g, the motion of the particle would be the same as the motion of the platform. As was mentioned previously, the maximum acceleration of the platform would occur at its extreme displacement positions. At the uppermost position of the platform the acceleration will be a maximum in the downward direction. If this is exactly equal to the acceleration of gravity, then the force on the particle due to the platform would be zero at that instant.

As $\ddot{x}_{max}$ is increased beyond that of g, the particle will leave the platform through part of its cycle and ride it through the remaining portion of the cycle. Figure 5 shows the relationship between the path that a particle of mass would take and that of the platform after the platform has passed the equilibrium position in its upward swing. For the conditions given in Figure 5, the particle of mass will act as a projectile for part of its cycle and will follow the motion of the screen during the remaining portion of the cycle.

If the frequency of the exciting force was such that the acceleration of the platform was exactly equal to that of gravity at time, t_1, and again at t_2, the particle would act as a projectile during this interval of time. This is shown in Figure 6. The particle will still be a projectile during the time interval between t_2 and t_3, which is the time