The response of an airplane to a dynamic load
by Rodney Lee Gilge

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University

Abstract:
The response of an airplane to a blast load is studied. The effects of rigid body translation and rigid body rotation are taken into account. Linear and non-linear solutions are compared. The non-linearities result from use of a non-linear stress-strain relation and from geometry changes due to large deflection.

It is concluded that the response of an airplane to a dynamic load is definitely influenced by the effects of rigid body translation and rotation. For large loads the non-linear solution predicts a larger wing deflection and a longer period of wing oscillation than does the linear solution.

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ABSTRACT

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CHAPTER I

INTRODUCTION

Design loads for accelerated airplane structures are frequently based on the assumption that the wing is perfectly rigid. This may lead to failure due to dynamic overstress. For example, a gust load may produce wing bending moments at the fuselage that are 15-20% greater than those calculated on the assumption of a rigid wing. Dynamic loads cause translation and rotation of the airplane as a whole and also cause vibrations of the structure. Dynamic overstress is produced by the additional inertia forces associated with the structure vibrations.

The load distribution on the wing is also affected by the wing deformation and vibration. Determining the load distribution on the basis of a rigid wing may lead to results that are too much in error to be useful. There may also be serious loss of aileron, elevator, and rudder control effectiveness due to deformation of the structure. In this paper only the airplane response to a dynamic load will be considered. The effect of this response on the wing load distribution and control effectiveness will not be analyzed.

The response of an airplane to dynamic loads has been frequently studied. In some studies (11)[1] the airplane was con-

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[1] Numbers in parenthesis refer to literature consulted.

sidered to be perfectly rigid. In other studies (7,10) the wings were considered to be elastic but the effects of rigid body translation and rotation of the airplane were neglected. Bisplinghoff (4) and Houbolt (5) included the effects of rigid body translation in their studies. However, they considered only dynamic loads that were uniform across the airplane so that no rigid body rotation occurred.

Most of the studies that have been made do not include the effects of both rigid body rotation and translation. The possibility of large non-linear wing deflections is considered in only very few studies.

The purpose of this paper is to study the response of an airplane to a dynamic load. The effects of rigid body rotation and translation and wing vibrations will be considered.

A combination of the modified Galerkin method and Hamming's modified predictor-corrector method will be used to solve the governing partial differential equations.

CHAPTER II

FORMULATION OF THE PROBLEM

SYSTEM DESCRIPTION

The system selected for study is shown in figure 1. It is intended to be a simple representation of an airplane. It consists of a free-free beam with a lumped mass at the center. The lumped mass simulates the fuselage of the airplane and the left and right portions of the beam represent the wings. The dynamic load acts across the entire wing cross-section, but only the rectangular portion of the wing is assumed to carry any load. The material properties are assumed to be homogeneous throughout each wing. This system is identical to one used by Bisplinghoff (10) in his study of aeroelasticity.

Three coordinant systems are used to describe the motion of the system. The fixed coordinants X-Y with unit vectors $\bar{i}$ and $\bar{j}$ are used to describe the rigid body translation and rotation of the system. The moving coordinants x_1-y_1 and x_2-y_2 with unit vectors $\bar{i}_1$, $\bar{j}_1$ and $\bar{i}_2$, $\bar{j}_2$, respectively are used to measure the deflection of the wings relative to the fuselage.

EQUATIONS OF RIGID BODY MOTION

The response of the system can be specified by four equations of motion. The equations describe the rigid body translation, the

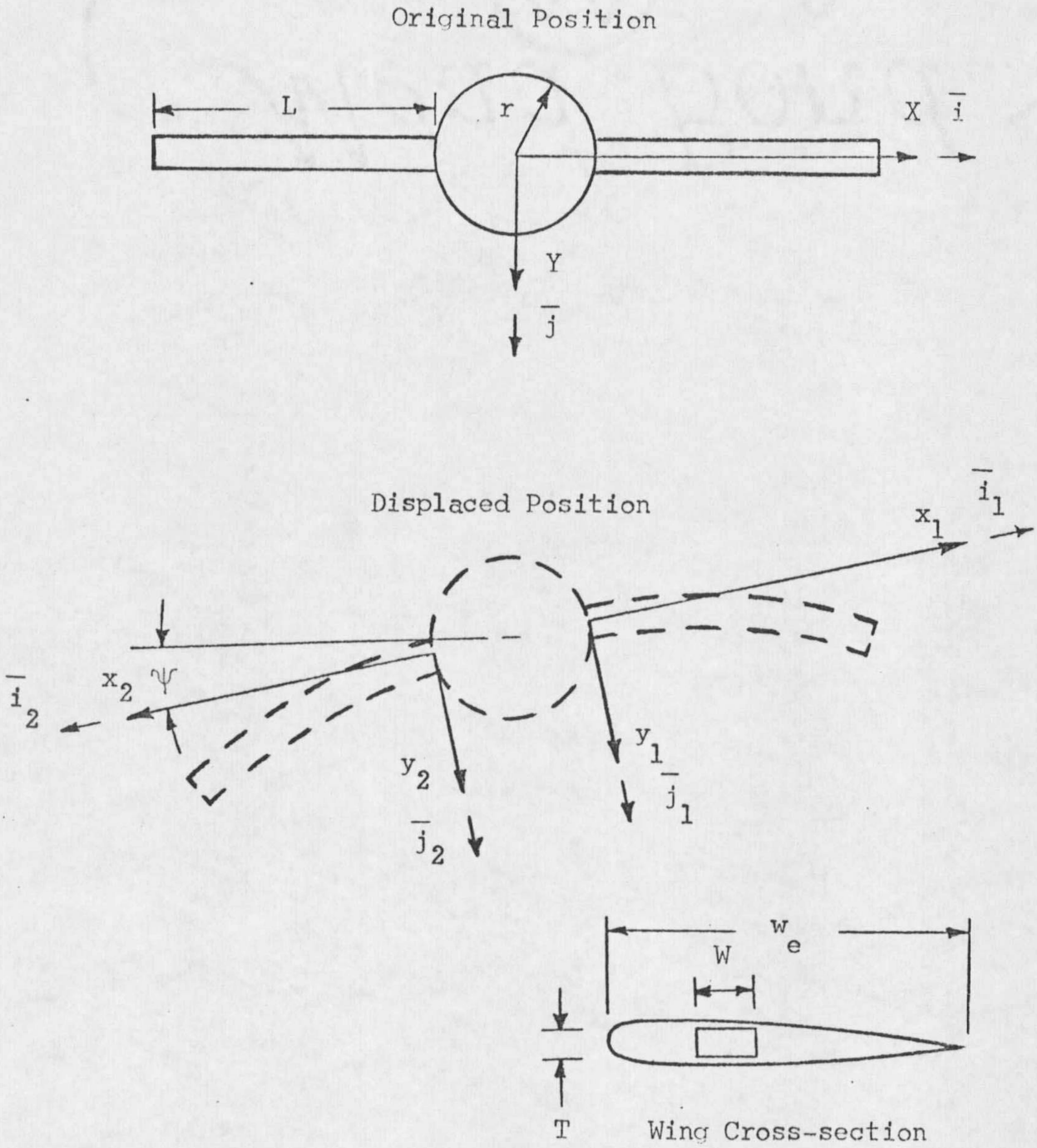

Figure 1. The Airplane Model

rigid body rotation, the relative displacement of the left wing, and the relative displacement of the right wing. The two equations of rigid body motion can be determined by considering the forces acting on the fuselage.

The forces acting on the fuselage are shown in figure 2. It is assumed that the weight of the airplane and the lift on the wings cancel each other. Therefore, these terms are not included in the derivation of the governing equations. Another simplifying approximation is that no forcing function acts on the fuselage.

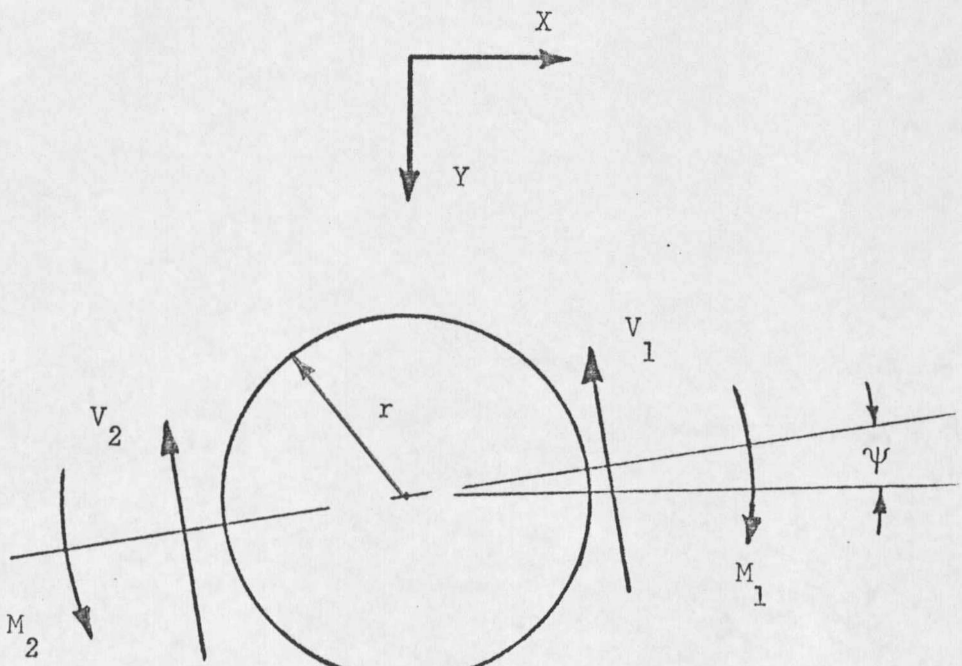

Figure 2. Forces on the Fuselage

The equation for rigid body rotation can be obtained by summing moments about the center of the fuselage. This yields:

$$\ddot{\psi} = \left[r(V_1 - V_2) + M_2 - M_1 \right] / I_b \tag{1}$$

where ψ is the rigid body rotation, r is the radius of the fuselage, V_1 and V_2 are shears, M_1 and M_2 are internal moments, and I_b is the mass moment of inertia of the fuselage. A dot , $\dot{}$, above a quantity indicates differentiation with respect to time and a prime , $'$, indicates differentiation with respect to position.

By summing forces in the $\bar{j}$ direction the equation for rigid body translation can be shown to be:

$$\ddot{Y} = -(V_1 + V_2) \cos \psi / M_b \tag{2}$$

where M_b is the mass of the fuselage.

LINEAR EQUATION OF WING MOTION

The fuselage of the airplane can both displace and rotate. Since the wings are rigidly attached to the fuselage, any motion of the fuselage will result in a corresponding motion of the wings. In addition the wings can also move relative to the fuselage.

If it is assumed that cross-sectional planes remain plane and if shear deformation and rotary inertia of bending are neglected, then the linear equations for the relative motion of the wings can be written as: (see appendix B)

$$- E\ I\ y_1'''' + w_1(x_1,t) = u\left[\ddot{Y}\cos\psi - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] \quad (3)$$

$$- E\ I\ y_2'''' + w_2(x_2,t) = u\left[\ddot{Y}\cos\psi + r\ddot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2\right] \quad (4)$$

where Y is the rigid body translation, ψ is the rigid body rotation, x_1 and x_2 are positions along the wings, y_1 and y_2 are the relative displacements of the wings, w_1 and w_2 are the forcing functions, and u is the mass per unit length of wing.

NON-LINEAR EQUATIONS OF WING MOTION

The non-linear wing equations include the effects of geometry changes due to large deflections and also account for the possibility of strains that are in the non-linear portion of the stress-strain curve.

Consider an element of the right wing as shown in figure 3. Again assume plane sections remain plane and neglect shear deformation and rotary inertia of bending.

The equations of motion of the wings with respect to the fuselage can be shown to be: (see appendix C)

$$M_1'\phi_1' \sin(2\phi_1) - M_1'' \cos^2(\phi_1) + u\ T^2\phi_1'\ddot{\psi}\sin(2\phi_1)/\ 12 + w_1(x_1,t) = u\left[\ddot{Y}\cos(\psi) - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] \quad (5)$$

$$M_2'\phi_2' \sin(2\phi_2) - M_2'' \cos^2(\phi_2) - u\,T^2\phi_2'\ddot{\psi} \sin(2\phi_2) +$$

$$w_2(x_2,t) = u\left[\ddot{Y}\cos(\psi) + r\ddot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2\right] \tag{6}$$

where T is the wing thickness, and ϕ is the rotation of the wing in the moving coordinant system.

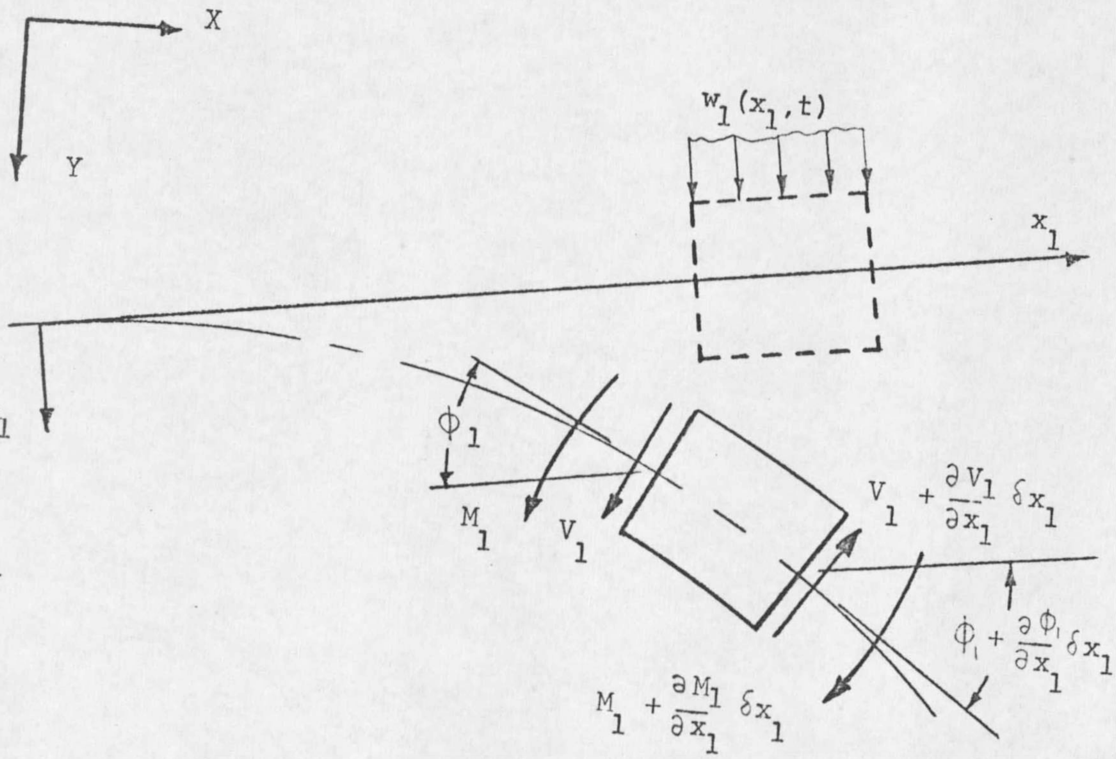

Figure 3. Beam Element

The inverse tangent function can be used to represent the true stress-strain relation of a material. The stress can be expressed as:

$$\sigma = \tan(a\varepsilon) / b \tag{7}$$

where σ is the stress, ε is the strain, and a and b are curve-fitting constants.

Assuming that the neutral plane of a wing always passes through the centroid of the area and recalling the assumption that plane sections remain plane, the strain can be written as:

$$\varepsilon = z\,\phi' \tag{8}$$

where z is the distance from the neutral axis to the fiber being strained.

The internal moment can be expressed in terms of the stress, and the bending angle, ϕ, can be expressed in terms of y'. Therefore, by proper substitutions the equations of motion of the wing elements can be written as:

$$w_1(x_1,t) - F_1\left[y_1(x_1,t)\right] - u\left[\ddot{Y}\cos(\Psi) - r\ddot{\Psi} - x_1\ddot{\Psi} + \ddot{y}_1 - y_1\dot{\Psi}^2\right] = 0 \tag{9}$$

$$w_2(x_2,t) - F_2\left[y_2(x_2,t)\right] - u\left[\ddot{Y}\cos(\Psi) + r\ddot{\Psi} + x_2\ddot{\Psi} + \ddot{y}_2 - y_2\dot{\Psi}^2\right] = 0 \tag{10}$$

where F_1 and F_2 are non-linear differential operators as defined in appendix C.

INITIAL AND BOUNDARY CONDITIONS

The motion of the system is described by four differential equations. Two of the equations are partial differential equations that are fourth order with respect to position and second order with respect to time. The remaining two equations are ordinary differential equations that are second order with respect to time. Therefore, it will be necessary to have a total of 16 initial and boundary conditions. Four initial conditions are necessary to specify the rigid body motion. The relative motion of the wings can be specified by four "natural boundary conditions", four "forced boundary conditions", and four initial conditions. The 16 initial and boundary conditions are:

Initial Conditions

Rigid Body Translation
$$\begin{cases} Y(x,0) = 0 \\ Y(x,0) = 0 \end{cases}$$

Rigid Body Rotation
$$\begin{cases} \psi(x,0) = 0 \\ \dot{\psi}(x,0) = 0 \end{cases}$$

Initial Relative Wing Displacement
$$\begin{cases} y_1(x_1,0) = 0 \\ y_2(x_2,0) = 0 \end{cases}$$

Initial Relative Wing Velocity

$$\begin{cases} \dot{y}_1(x_1,t) = 0 \\ \dot{y}_2(x_2,t) = 0 \end{cases}$$

Boundary Conditions

Relative Wing Displacement at the Fuselage

$$\begin{cases} y_1(0,t) = 0 \\ y_2(0,t) = 0 \end{cases}$$

Relative Wing Slope at the Fuselage

$$\begin{cases} y_1'(0,t) = 0 \\ y_2'(0,t) = 0 \end{cases}$$

Moment at Tip of Wing

$$\begin{cases} M\left[y_1(L,t)\right] = 0 \\ M\left[y_2(L,t)\right] = 0 \end{cases}$$

Shear at Tip of Wing

$$\begin{cases} V\left[y_1(L,t)\right] = 0 \\ V\left[y_2(L,t)\right] = 0 \end{cases}$$

DESCRIPTION OF THE FORCING FUNCTION

The forcing function is an approximation of a bomb blast. The force on the wings will be independent of position if the source of the blast is a large distance from the airplane and the wings of the airplane are parallel to the blast front.

Under these conditions a blast load can be approximated very closely by:

$$w(t) = w_e P_o e^{-\beta t} \tag{11}$$

where P_o is the maximum pressure, w_e is the effective wing width, t is time, and β is a blast decay constant.

If the wings of the airplane are not parallel to the blast front then the load on the airplane will be a function of both position and time. The loading for such a condition is represented by figure 4.

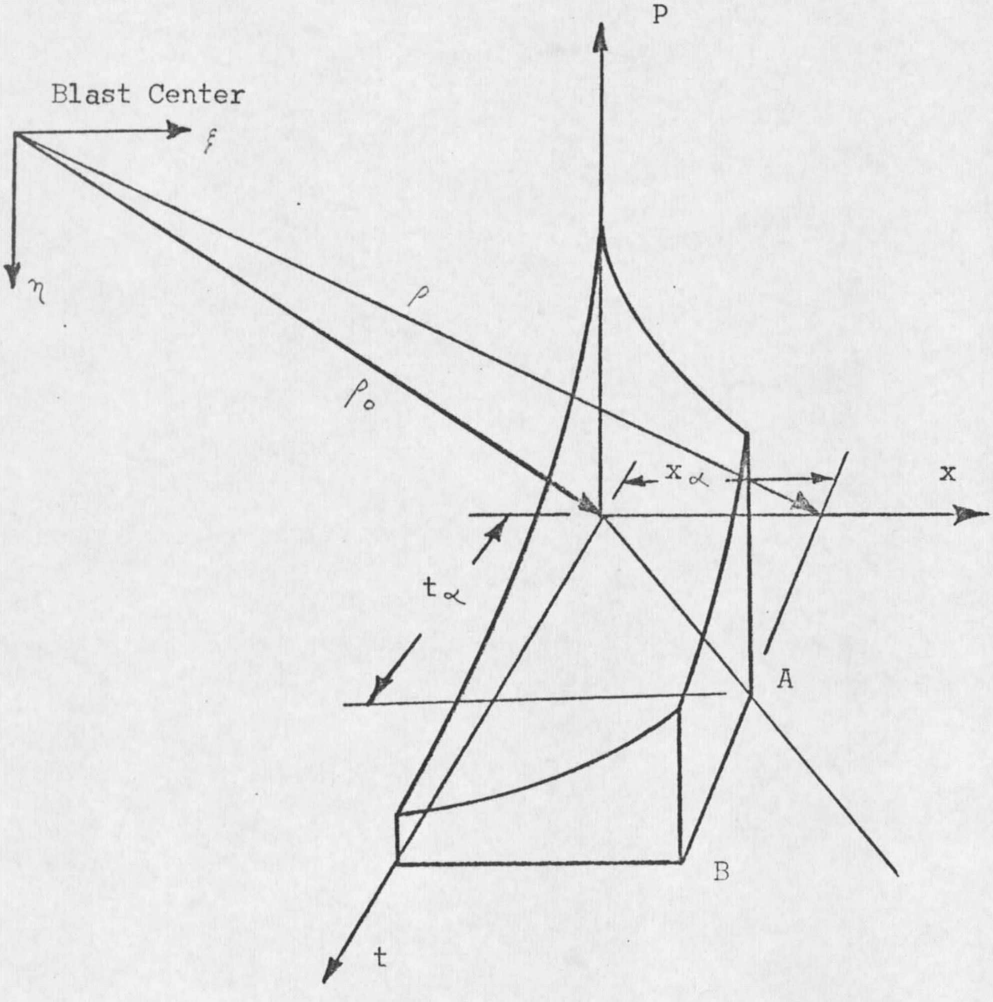

Figure 4. Blast Pressure Decay

The blast pressure decays as the square of the distance from the blast center (12). The pressure at some position A can be expressed as:

$$P_A = P_o \frac{\rho_o^2}{\rho^2} \qquad (12)$$

where P_o is the maximum pressure, ρ_o is the distance from the blast center to the end of the left wing, ρ is the distance from the blast center to the position A, and P_A is the pressure at A.

The blast pressure also decays with time. At some position B the pressure is:

$$P_B = P_A \, e^{-\beta(t - t_\alpha)} \qquad (13)$$

$$P_B = P_o \frac{\rho_o^2}{\rho^2} \, e^{-\beta(t - t_\alpha)} \qquad (14)$$

where β is the blast decay constant and t_α is the time required for the blast front to reach position A.

Referring to figure 5, t_α can be expressed as:

$$t_{\alpha_{1,2}} = \frac{x_\alpha \sin \gamma}{c}$$

where γ is the angle between the blast front and the wing, c is the speed of the blast front, and x_α is the distance from the end of the left wing to the point under consideration on the airplane. Subscript 1 refers to the right wing and subscript 2 refers to the left wing.

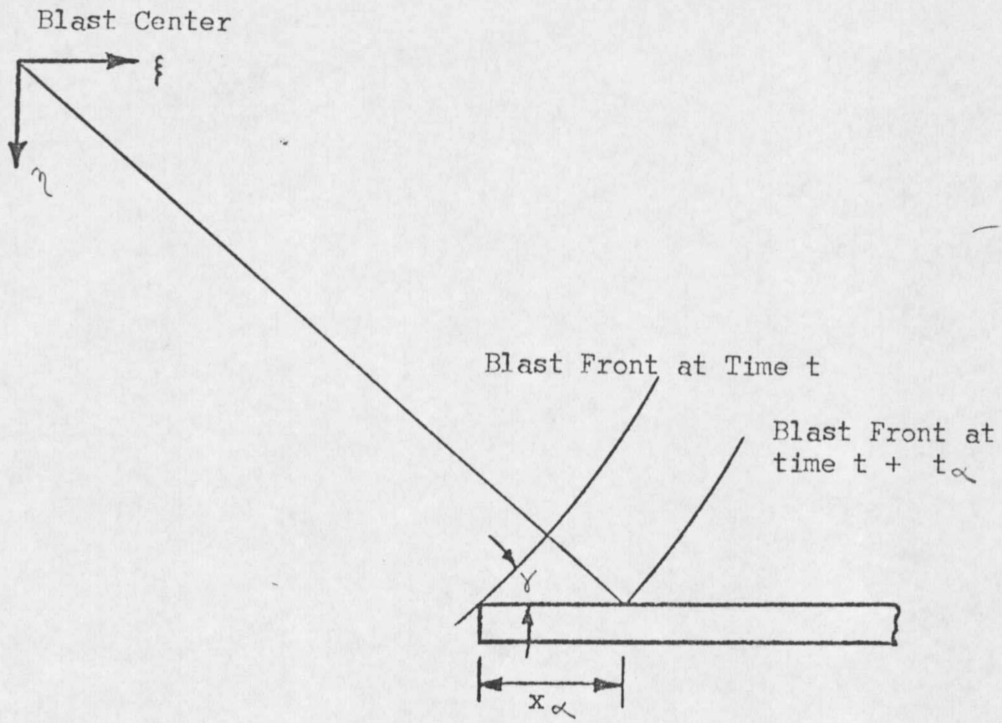

Figure 5. Arrival Time of Blast

The expression for t_α is valid only when the radius of the blast front, ρ_o, is large. By making the proper substitutions the expressions for $t_{\alpha_{1,2}}$ can be shown to be:

$$t_{\alpha_2} = \frac{(L-x_2)\cos\left[\sin^{-1}(s/\rho_o)\right]}{c} \tag{16}$$

$$t_{\alpha_1} = \frac{(L+2r+x_1)\cos\left[\sin^{-1}(s/\rho_o)\right]}{c} \tag{17}$$

where s is the vertical distance from the blast center to the fuselage.

CHAPTER III

METHOD OF SOLUTION

MODIFIED GALERKIN METHOD

One method of solution of the equations developed is the modified Galerkin method. The modified Galerkin method is a version of the method of weighted residuals as described by Finlayson and Scriven (9). To solve a set of equations that are functions of both time and position the first step in this method is to assume a complete solution to the set of equations. The complete solution will consist of individual solutions for each of the dependent variables. The individual solutions will be of the form:

$$y^*(x,t) = \sum_{i=1}^{n} q_i(t) \; \Theta_i(x) \tag{18}$$

The $\Theta(x)$ terms must satisfy the "forced boundary conditions" but do not have to satisfy the "natural boundary conditions".

The next step is to substitute the assumed solutions into the differential equations to determine the equation residuals, R(x,t). Since the assumed solutions were not required to satisfy the "natural boundary conditions" there will be boundary errors or residuals. Finally the weighted integral of the residual plus the weighted boundary errors are set equal to zero for each of the equations.

$$\int_{\text{domain}} R(x,t) \; \Theta_j(x) \; dx + E_v \, \Theta_j(x) + E_m \, \Theta_j'(x) = 0 \tag{19}$$

E_v is the boundary shear error and E_m is the boundary moment error. Equation 19 will yield a set of simultaneous differential equations in time for each of the terms of the complete assumed solution. These equations can then be solved to determine the $q_i(t)$ terms. Multiplying the $q_i(t)$ terms times the $\Theta_i(x)$ terms and summing them for each of the dependent variables will yield the complete solution for the set of equations.

APPLICATION OF THE MODIFIED GALERKIN METHOD

The four equations that describe the motion of the system can be reduced to three equations by substituting the expression for Y into the equations for the relative motion of the wings. The resulting equations are:

$$w_1(x_1,t) - N_1\left[y_1(x_1,t)\right] - u\left[-(V_1+V_2)\cos^2\psi/M_b - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] = 0 \quad (20)$$

$$w_2(x_2,t) - N_2\left[y_2(x_2,t)\right] - u\left[-(V_1+V_2)\cos^2\psi/M_b + r\ddot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2\right] = 0 \quad (21)$$

$$\ddot{\psi} = r\left[(V_1 - V_2) + M_2 - M_1\right] / I_b \quad (22)$$

where N_1 and N_2 are differential operators.

By applying the modified Galerkin method the equations for the relative motion of the wings can be reduced to equations in time alone. The solution of the equations can be greatly simplified by restricting the assumed solutions for the modified Galerkin method to one term. Under this restriction the system reduces to three coupled equations in q_1, q_2, and ψ. They are:

$$\ddot{q}_1 = C_1\dot{\psi}^2 q_1 + C_2\ddot{\psi} + C_3 \cos^2\psi + C_4 \qquad (23)$$

$$\ddot{q}_2 = D_1\dot{\psi}^2 q_2 + D_2\ddot{\psi} + D_3 \cos^2\psi + D_4$$

$$\ddot{\psi} = Q\left[q_1, q_2\right]$$

where C_1, C_2, C_3, C_4, D_1, D_2, D_3, and D_4 are constants and Q is a differential operator on q_1 and q_2. The three coupled equations can now be solved numerically using the Hamming's modified predictor-corrector method.

The deflection shape chosen for the wings was the deflection shape produced by a uniform static load on a cantilever beam. The deflection shape was normalized to 1 at the end of the wings.

The physical dimensions of the system were selected to approximate a small airplane. The wings are 250 inches long, 10 inches wide, 4.6 inches thick and are made of 6061 T6 aluminum. The wing width and thickness are small so that the wing stiffness more closely approximates that of a real wing. The blast load is assumed to act on an effective wing width of 100 inches. The mass of the body is 20.7 lb. sec./in.

The stress-strain diagram for the wing material is shown in figure 6 along with the function used to approximate it.

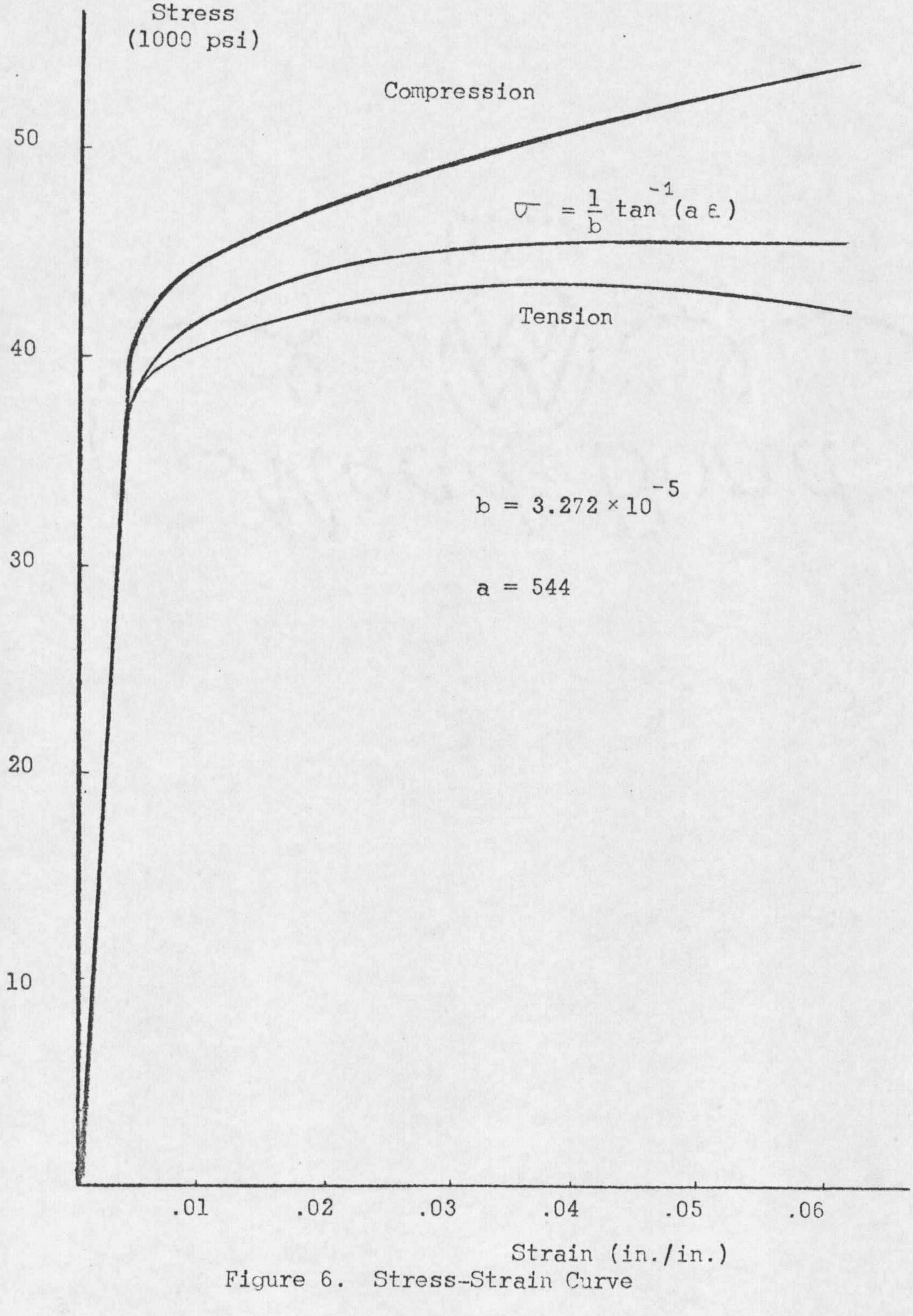

Figure 6. Stress-Strain Curve

CHAPTER IV

RESULTS AND CONCLUSIONS

UNIFORM LOAD

If the blast load is uniform across the airplane the deflection of the wings will be symmetrical and there will be no rigid-body rotation. Thus, under the restriction of a uniform load the three governing equations can be reduced to 1 equation.

For the purpose of discussion a blast load will be considered "small" if it produces a stress which is in the linear portion of the stress-strain diagram. Further it will be referred to as uniform if it is a function of time only and non-uniform if it is a function of position and time. The linear and non-linear solutions give nearly identical results for "small" loads. Figure 7 shows typical displacement, velocity, and acceleration diagrams of the wing tip for "small" loads.

Figure 7 shows that the wing response for both the linear and non-linear case is sinusoidal and has a period of approximately .85 seconds. The period for an equivalent cantilever beam vibrating in its fundamental mode is .43 seconds. It is reasonable to expect that the wing would have a longer period because it is not truly fixed at the fuselage. That is, it is allowed to translate.

When the loads become large the linear and non-linear solutions no longer give the same results. The response calculated

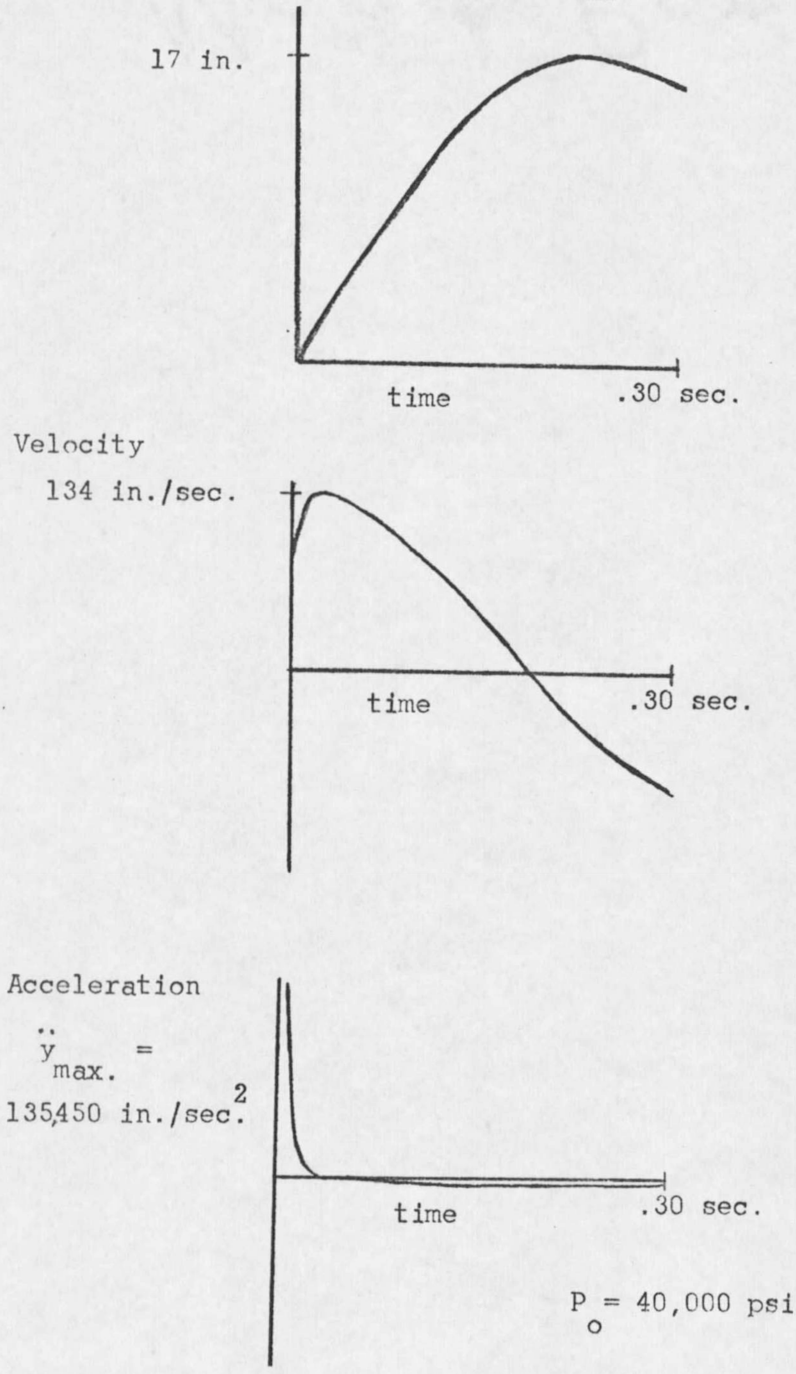

Figure 7. Typical Linear and Non-Linear Wing Tip Response for a Small Uniform Load

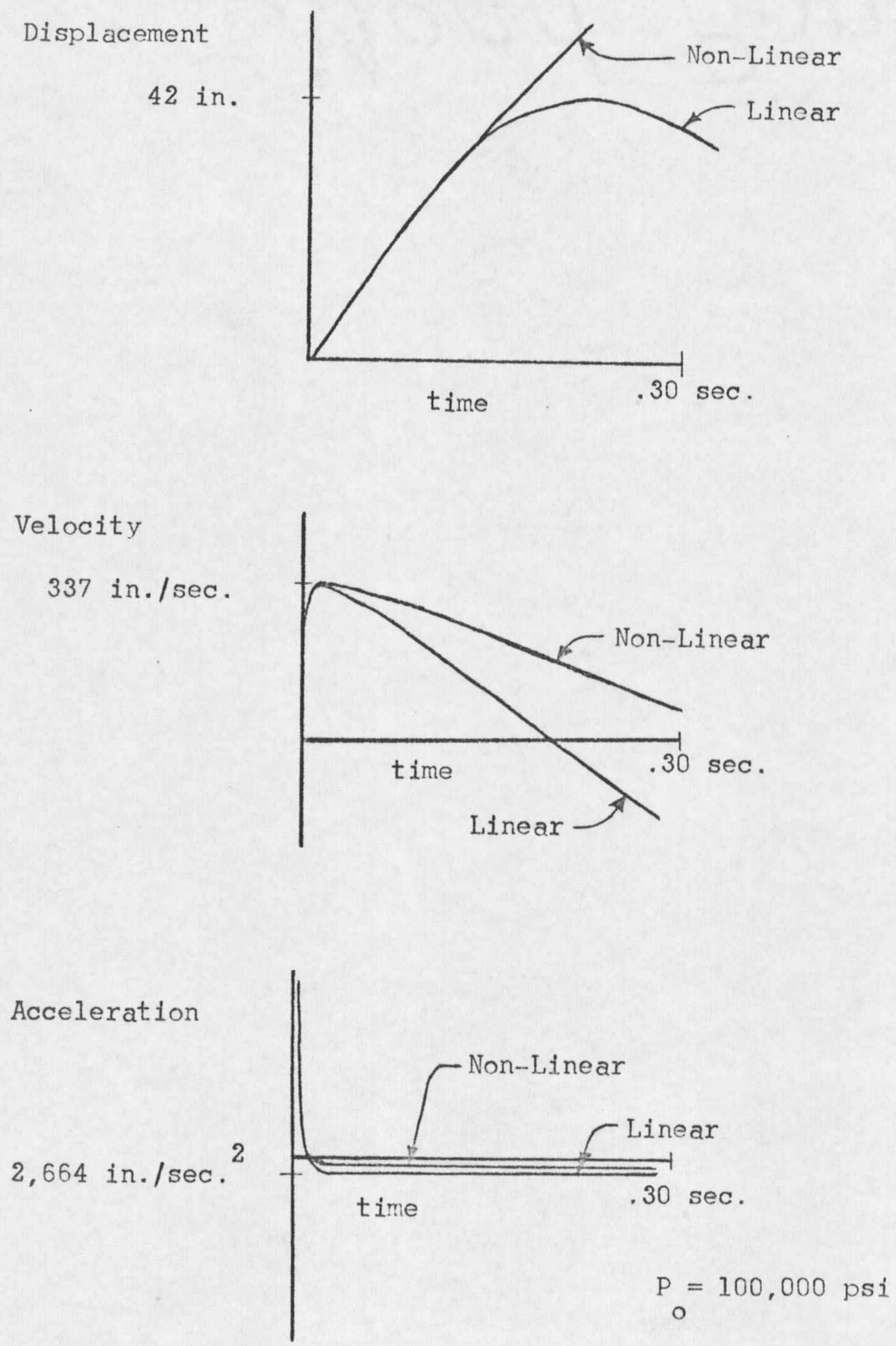

Figure 8. Comparison of Linear and Non-Linear Solutions for a Large Uniform Load

using the non-linear equation is much greater than the response indicated by the linear equation. The main reason for the difference is that the non-linear solution takes into account the flattening off of the stress-strain curve for large strain and the linear solution does not. Figure 8 compares linear and non-linear solution for large uniform loads.

Another difference is that for large loads the non-linear solution predicts a longer period than does the linear solution. This indicates that the non-linear wing stiffness decreases for large strain. The period of oscillation increases due to the reduced wing stiffness. The reduced stiffness of the non-linear wing is also due to the flattening off of the stress-strain curve.

NON-UNIFORM LOAD

The response of the airplane to a non-uniform blast load is much more complicated than its response to a uniform blast load. A non-uniform blast load causes non-symmetric wing response as well as rigid body rotation.

For small non-uniform loads the linear and non-linear solutions again, as in the case of uniform loads, are nearly identical. They, however, are not simply sinusoidal as indicated by figure 9.

There is a "flutter" super-imposed on the wing displace-

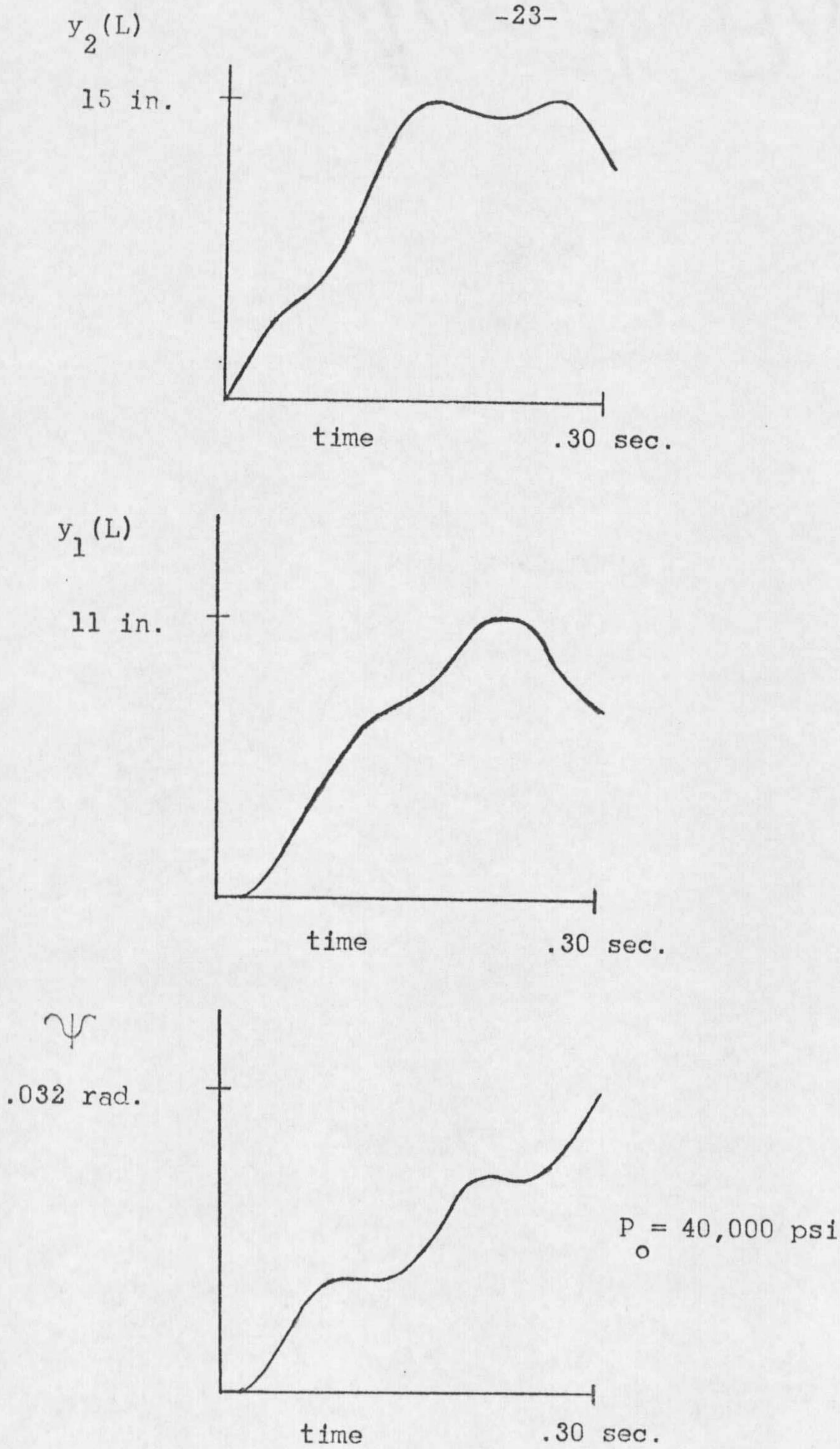

Figure 9. Typical Linear and Non-Linear System Displacements for a Small Non-Uniform Load

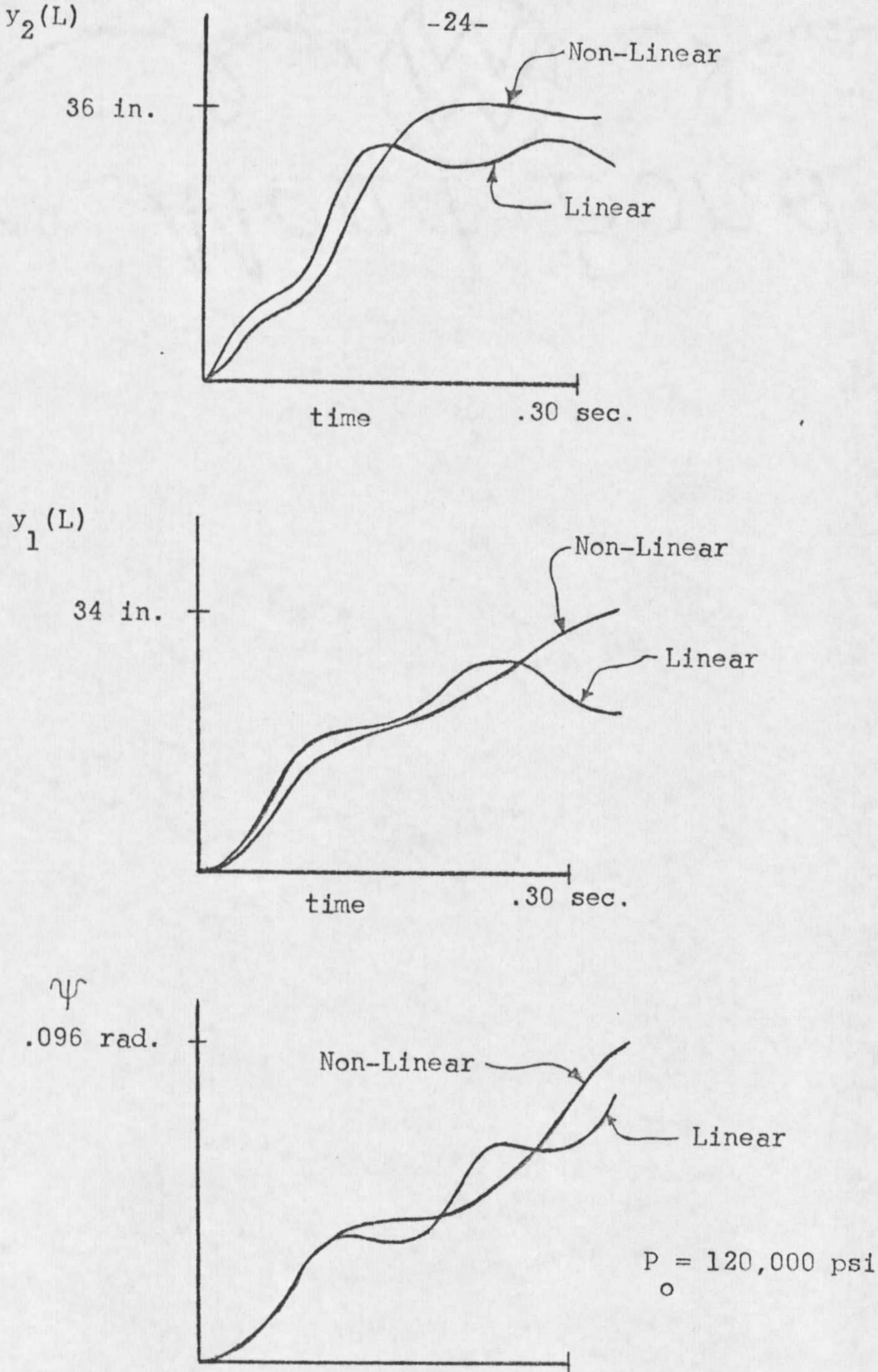

Figure 10. Comparison of Linear and Non-Linear Solutions for Large Non-Uniform Loads

ment. The flutter is apparently a result of the rigid body rotation effects.

Large non-uniform loads have the same effect as did the large uniform loads. That is, for large non-uniform loads the non-linear solution indicates a larger response and a longer period of oscillation than does the linear solution. This can be seen by looking at figure 10.

The inclusion of the effects of rigid body rotation and translation and large deflection do affect the system response.

The effect of including rigid body translation is to increase the period of wing vibration and decrease the wing displacement. Wing "flutter" is caused by the effects of rigid body rotation. The non-linear or large deflection solution indicates a greater system response and an increased period for large loads.

FURTHER STUDIES

The response of an airplane to a dynamic load was determined in this study. A logical next step would be to determine the effects of the response on the control effectiveness of the airplane. It would also be of interest to investigate the effects of damping on the system response.

The stress-strain relation used in the non-linear section is valid only for loading of the system. If the strain is large the system will not unload along the same path as the loading occurred. The cyclic response of the system for large loads could be studied if the stress-strain relation were properly defined to account for unloading.

The analysis presented in this paper could be applied to a wide variety of airplanes. It would not be valid, however, if there were a large variation in wing stiffness. Also a large lumped mass, such as an engine, on the wings would render the analysis invalid.

APPENDIX

APPENDIX A

ACCELERATION EXPRESSIONS

Consider the acceleration of some point a on the left wing. The position vector to point a is:

$$\bar{r}_a = Y\,\bar{j} - r\cos\psi\,\bar{i} + r\sin\psi\,\bar{j} + x_2\,\bar{i}_2 + y_2\,\bar{j}_2 \tag{1}$$

The acceleration of point a can be determined by differentiating the position vector twice. Doing this yields the following acceleration expression:

$$\begin{aligned}\ddot{\bar{r}}_a = {} & (\ddot{Y}+r\ddot{\psi}\cos\psi - r\dot{\psi}^2\sin\psi)\,\bar{j} + (r\ddot{\psi}\sin\psi + r\dot{\psi}^2\cos\psi)\,\bar{i} + \\ & (\ddot{x}_2 - x_2\dot{\psi}^2 - 2\dot{y}_2\dot{\psi} - y_2\ddot{\psi})\,\bar{i}_2 + (2\dot{x}_2\dot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2)\,\bar{j}_2\end{aligned} \tag{2}$$

The above expression is the total acceleration of point a. In deriving the equations of motion only the acceleration in the $\bar{j}_2$ direction is considered. The acceleration in the $\bar{j}_2$ direction can be shown to be:

$$\ddot{\bar{r}}_{a_{j_2}} = \ddot{Y}\cos\psi + r\ddot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2 \tag{3}$$

In the same manner the acceleration of some point b on the right wing can be shown to be:

$$\ddot{\bar{r}}_{b_{j_1}} = \ddot{Y}\cos\psi - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2$$

APPENDIX B

LINEAR EQUATIONS OF WING MOTION

From elementary beam theory the equation of motion of a beam can be shown to be:

$$- E\ I\ y'''' + w(x,t) = u\ a$$

where E is the modulas of elasticity of the beam material, I is the cross-sectional moment of inertia, u is the mass per unit length, and a is the acceleration. The acceleration expressions for the left and right wing are determined in appendix A. By making the proper substitutions for the acceleration terms the equations of motion for the left and right wings become:

Right Wing

$$- E\ I\ y_1'''' + w_1(x_1,t) = u\left[\ddot{Y}\cos\psi - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] \qquad (1)$$

Left Wing

$$- E\ I\ y_2'''' + w_2(x_2,t) = u\left[\ddot{Y}\cos\psi + r\ddot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2\right] \qquad (2)$$

APPENDIX C

NON-LINEAR EQUATIONS OF WING MOTION

Consider the forces on an element of the right wing as shown in figure 3. By summing the moments about the right face of the element the shear expression can be shown to be:

$$V = M' \cos \phi_1 + u\, T^2 \ddot{\psi} \cos \phi_1 \tag{1}$$

where V is the shear and M is the moment. Differentiating the shear expression yields:

$$V' = M'' \cos \phi_1 - M' \phi_1' \sin \phi_1 - u\, T^2 \ddot{\psi}\, \phi_1' \sin \phi_1 / 12 \tag{2}$$

Next sum the forces in the $\bar{j}_2$ direction. This yields:

$$V \phi_1' \sin \phi_1 - V' \cos \phi_1 + w_1(x_1, t) = u\, a \tag{3}$$

where a is the acceleration. Now substitute the expressions for V, V', and a into the above equation. This gives the equation of motion of the right wing in the form:

$$\begin{aligned} M' \phi_1' \sin(2\phi_1) + u\, T^2 \sin(2\phi_1)\, \phi_1'/12 - M'' \cos^2(\phi_1) + w_1(x_1, t) = \\ u\left[\ddot{Y} \cos(\psi) - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] \end{aligned} \tag{4}$$

Define a non-linear differential operator F_1.

$$F_1\left[y_1(x_1, t)\right] = -M' \phi_1' \sin(2\phi_1) - \phi_1' u\, T^2 \sin(2\phi_1)/12 + M'' \cos^2(\phi_1) \tag{5}$$

Then the equation of motion of the right wing can be written as:

$$w_1(x_1,t) - F_1\left[y_1(x_1,t)\right] - u\left[\ddot{Y}\cos\psi - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] = 0 \qquad (6)$$

In the same manner the equation of motion of the left wing can be shown to be:

$$w_2(x_2,t) - F_2\left[y_2(x_2,t)\right] - u\left[\ddot{Y}\cos\psi + r\ddot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2\right] = 0 \qquad (7)$$

where F_2 is the same as F_1 except it is written in terms of x_2 instead of x_1.

Now once the moment, bending angle, strain, and stress are defined the equation of motion will be completely specified. The bending angle can be expressed as:

$$\phi = \tan^{-1}\frac{\partial y}{\partial x} \qquad (8)$$

The strain can be defined as:

$$\varepsilon = z\,\frac{\partial \phi}{\partial x} \qquad (9)$$

where z is the distance from the neutral axis to the fiber being strained.

The stress can be written in terms of the strain in the form:

$$\sigma = \frac{1}{b}\tan^{-1}(a\,\varepsilon) \qquad (10)$$

Finally the moment can be written in terms of the stress.

$$M = W \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{1}{b} \tan^{-1}(a \varepsilon) \; z \; dz$$

W is the width of the beam.

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