



An investigation of the adequacy of recontoured spoil sampling regulations using geostatistics  
by Charles Kinsey Hardy

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Land Rehabilitation

Montana State University

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**Abstract:**

Surface coal mines in the western United States are required to sample regraded spoils to determine if spoil handling procedures have been effective, to screen the material for adverse physicochemical properties, and to determine how adequately premining overburden information can predict the nature of this spoil material. At the Absaloka Mine in southeast Montana the surface 2.44 meters of regraded spoils were sampled in two depth increments, 0-1.22 meters and 1.22-2.44 meters, and at a sampling interval of approximately 69 meters as required by the Montana Department of State Lands. Geostatistical procedures were used to more objectively characterize the spoil material. The purpose of this study is to show how geostatistical techniques can be used to develop a more efficient approach to sampling regraded spoil.

The objectives of this study were to determine if significant differences in physicochemical properties exist between the two spoil sampling zones, to quantify the spatial aspects of spoil physicochemical properties using the semi-variogram, and demonstrate how geostatistical techniques can be used to develop sampling strategies for characterization of the regraded spoil.

Semi-variograms were computed from 240 samples for pH, EC (mmhos/cm), saturation percentage, SAR, ESP, and percent clay and sand. The semi-variograms had large nugget variances and ranges of influence that varied between 244 meters and 366 meters. Semi-variograms were fitted with spherical models and validated using jackknifing techniques. Block kriging was used to map the spoil properties and delineate areas of spoil that are potentially phytotoxic.

To characterize the spoils a two phase sampling strategy is proposed. Spoils are first sampled at a fixed sample spacing based on semi-variogram properties and kriging techniques. At a square grid sample spacing of 140 meters, kriging estimates could be made anywhere within the sample region for all the spoil properties investigated. Second phase sampling is implemented if problem areas of spoil are found during first phase sampling. To better characterize problem areas of spoil, the sample intensity is increased over the 140 meter sample intensity set during phase one sampling. Second phase sampling is based on the average estimation variance associated with a particular sample spacing. Curves were developed to help guide this additional sampling.

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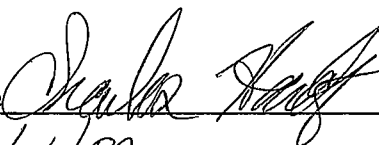
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**ABSTRACT**

Surface coal mines in the western United States are required to sample regraded spoils to determine if spoil handling procedures have been effective, to screen the material for adverse physicochemical properties, and to determine how adequately premining overburden information can predict the nature of this spoil material. At the Absaloka Mine in southeast Montana the surface 2.44 meters of regraded spoils were sampled in two depth increments, 0-1.22 meters and 1.22-2.44 meters, and at a sampling interval of approximately 69 meters as required by the Montana Department of State Lands. Geostatistical procedures were used to more objectively characterize the spoil material. The purpose of this study is to show how geostatistical techniques can be used to develop a more efficient approach to sampling regraded spoil.

The objectives of this study were to determine if significant differences in physicochemical properties exist between the two spoil sampling zones, to quantify the spatial aspects of spoil physicochemical properties using the semi-variogram, and demonstrate how geostatistical techniques can be used to develop sampling strategies for characterization of the regraded spoil.

Semi-variograms were computed from 240 samples for pH, EC (mmhos/cm), saturation percentage, SAR, ESP, and percent clay and sand. The semi-variograms had large nugget variances and ranges of influence that varied between 244 meters and 366 meters. Semi-variograms were fitted with spherical models and validated using jackknifing techniques. Block kriging was used to map the spoil properties and delineate areas of spoil that are potentially phytotoxic.

To characterize the spoils a two phase sampling strategy is proposed. Spoils are first sampled at a fixed sample spacing based on semi-variogram properties and kriging techniques. At a square grid sample spacing of 140 meters, kriging estimates could be made anywhere within the sample region for all the spoil properties investigated. Second phase sampling is implemented if problem areas of spoil are found during first phase sampling. To better characterize problem areas of spoil, the sample intensity is increased over the 140 meter sample intensity set during phase one sampling. Second phase sampling is based on the average estimation variance associated with a particular sample spacing. Curves were developed to help guide this additional sampling.

## INTRODUCTION

Sampling and analysis of regraded spoils is a regulatory requirement for surface coal mines in the western United States. As part of the premine baseline data collection process overburden is drilled and sampled for hazardous materials. One consideration of the sampling and testing program is to screen the regraded material for deleterious chemical and physical properties. This is done to determine if spoil handling procedures have been effective. Sampling regraded spoils serves to determine how adequately this overburden information can predict the nature of the regraded spoils. Regraded spoils information will also be reviewed during the bond release process.

Data for this study come from Westmoreland Resources' Absaloka Mine in southeast Montana. These data were taken from an existing data base kept on file by the Montana Department of State Lands (DSL), Reclamation Division. Of the coal mines in Montana the Absaloka Mine has the most intensively sampled regraded spoil. The mine has been operating since 1974, therefore sufficient data exist to make a geostatistical analysis.

Appropriate sampling intensity and parameters to be analyzed are determined on a site-specific basis according to mine permitting agreement. Due to this flexibility, sampling and testing requirements can vary significantly between mines. The guidelines for sampling regraded spoils require samples to be taken prior to topsoiling and

revegetation activities (Montana Department of State Lands, 1983). The spoil sampling should be conducted to a depth of 2.44 meters (8 feet) with samples taken from the surface 0-1.22 meters (0-4 feet) of spoil and the subsurface 1.22-2.44 meters (4-8 feet) of spoil. Spoils should also be sampled on approximately 91 meter (300 foot) centers. Parameters to be analyzed include: saturation percentage, pH, conductivity, SAR and/or ESP, particle size distribution and bulk density.

Geostatistics is a relatively new statistical technique developed mainly by Matheron (1963) and Krige (1966) for the estimation of ore reserves in mining. There are fundamental differences between geostatistics and classical statistics according to Matheron; classical methods are unable to adequately treat the spacial aspect of data, and neighboring samples may not be independent of each other and, as a result, samples taken close together tend to be more similar than those that are far apart. By taking into account spacial dependence in the data, geostatistics and particularly kriging procedures, will yield unbiased estimates of spoil properties and minimize the variance associated with the estimate. In the sense that it minimizes the variance associated with an estimate, kriging is an optimum means of interpolation.

Mining companies have been collecting regraded spoil data for years with little attempt to analyze these data. For many mines sufficient data have been collected to make a geostatistical analysis feasible. With this "new" analytical tool a more critical assessment of a regraded spoil sampling program can be made. It is the intent of

this study to use a geostatistical approach to assess the adequacy of regraded spoil sample spacing requirements. \*

The objectives of this study were too:

- 1) Determine whether significant differences exist between the two spoil sampling zones. Spoils are sampled from 0-1.22 meters (0-4 feet) and from 1.22-2.44 meters (4-8 feet) below the surface.
- 2) Determine if spacial dependencies in regraded spoil physical and chemical properties exist at the Absaloka Mine.
- 3) Demonstrate that geostatistical techniques can be used to improve sampling strategies for characterization of the regraded spoil.

### THEORY

Geostatistics is based on the theory of regionalized variables. A regionalized variable is a variable distributed in space and is in part dependent on the spacial position of the variable. A regionalized variable possesses two fundamental characteristics: (i) a local, random erratic component similar to that of a random variable; (ii) a general structural aspect which can be described by a samples spacial relationship to neighboring samples. Unlike most classical statistics, the assumption of independence is not made. A comprehensive review of the practical application of geostatistics to the problems of ore reserve estimation is given by Journel and Huijbregts (1978).

The semi-variogram is the basic tool of geostatistics. All other geostatistical techniques are contingent on the semi-variogram (i.e. kriging, determination of estimation variance). Semi-variance can be defined by the following equation:

$$\gamma(h) = \frac{1}{2} E[ \{ Z(x+h) - Z(x) \}^2 ] \quad [1]$$

$Z(x)$  and  $Z(x+h)$  are two numerical values separated by a vector  $h$ . The vector  $h$  has a distance and direction component and semi-variance is calculated for as many different distances as possible. The semi-variogram is a plot of semi-variance on the vertical axis and distance between sample pairs, or lag on the horizontal axis (Figure 1).



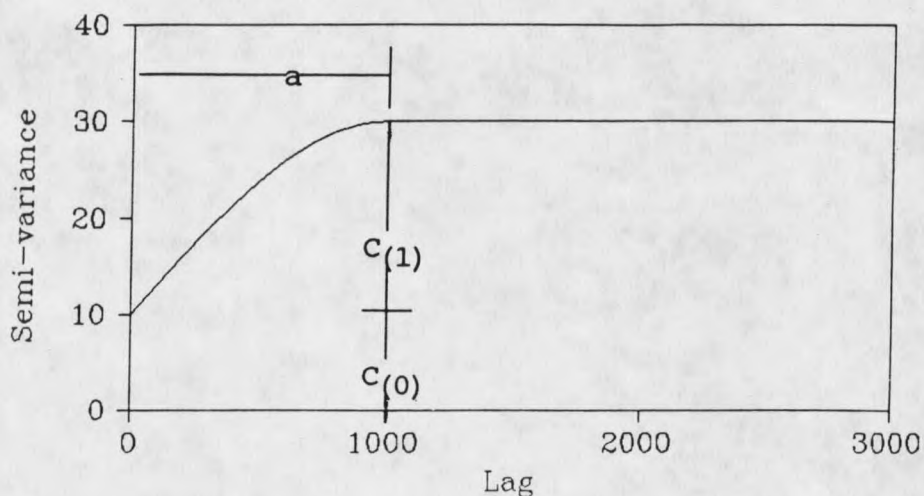


Figure 1. Idealized spherical semi-variogram model with nugget variance,  $C_{(0)}=10$ ; sill,  $C_{(0)}+C_{(1)}=30$ ; and range,  $a=1000$ .

Figure 1 shows an example of a semi-variogram model. Models are fitted to experimental semi-variograms to describe the spatial character of a regionalized variable. Common models used are spherical, exponential and linear. The model shown is a spherical model with a range of influence ( $a$ ) of 1000, a sill value ( $C_{(0)}+C_{(1)}$ ) of 30 and a nugget value ( $C_{(0)}$ ) of 10. The spherical model is described by the equations:

$$\begin{cases} \tau(h) = C_{(0)} + C_{(1)} \{1.5(h/a) - 0.5(h/a)^3\} & \text{for } 0 < h \leq a \\ \tau(h) = C_{(0)} + C_{(1)} & \text{for } h > a. \end{cases} \quad [2]$$

In these equations "h" is the separating distance between points, or lag. The nugget value represents a random component to the semi-variance. The nugget value  $C_{(0)}$  can't be explained by a samples spatial location and is due to errors in measurement and micro-

variabilities of the parameter of interest. The range of influence (a) represents the distance within which samples are spatially correlated. Beyond this range samples are independent of each other. The value at which  $\tau(h)$  levels out is called the sill. It consists of the nugget variance,  $C(0)$ , plus a component,  $C(1)$ , which represents that portion of the semi-variance which is due to spacial dependence in the data.

Kriging uses the principle of weighted local averaging which can give estimates of spoil properties at unknown locations and is therefore a type of interpolation. Kriging provides the best linear unbiased estimate of the unknown characteristic with an associated estimation variance. Kriging estimates were made using a data set of spoil parameters and the semi-variogram model describing the spacial variability in the studied zone. An excellent review of kriging, that used soil properties is given by Burgess and Webster (1980 a,b,c) and McBratney and Webster (1983).

The kriging system predicts values at unknown sites by appropriately weighing adjacent known values through the use of the semi-variogram. Each estimated value,  $Z^*(x_0)$ , at some unobserved location,  $x_0$ , is determined from a linear combination of the known values  $Z(x_i)$ ,  $i=1,2,3,\dots, n$ . Thus,

$$Z^*(x_0) = \sum_{i=1}^n \Gamma_i Z(x_i) \quad [3]$$

where  $\Gamma_i$  are the weights applied to each sample in the kriging neighborhood. The  $\Gamma_i$  associated with each  $Z(x_i)$  are chosen with the constraint that

$$\sum_{i=1}^n \Gamma_i = 1 \quad [4]$$

which insures unbiasedness (Tabor et al. 1984).

Minimizing the error variance associated with each estimate involves solving a linear system of equations. For punctual kriging this involves finding the partial derivatives with respect to each  $\Gamma_i$  and introduces a Lagrange parameter  $\mu$ . The weights,  $\Gamma_i$ ,  $i=1,2,3,\dots,n$ , and the Lagrangian multiplier,  $\mu$ , were obtained by solving the linear system

$$[A] \begin{bmatrix} \Gamma \\ \mu \end{bmatrix} = [b]. \quad [5]$$

The  $n+1$  by  $n+1$  matrix A contains the covariances between all  $n$  points within the estimation neighborhood. The  $n+1$  vector b contains the average covariances between the observed points in the estimation neighborhood and the point to be estimated. The solution vector  $\begin{bmatrix} \Gamma \\ \mu \end{bmatrix}$  contains the weights and Lagrangian multiplier. Covariances are determined from the semi-variogram model. The weights are then applied to equation [3] to obtain the punctually kriged value at each location (Trangmar 1986).

The minimum estimation variance,  $\sigma_E^2$  for each point was obtained by solving

$$\sigma_E^2 = [b]^T \begin{bmatrix} \Gamma \\ \mu \end{bmatrix}. \quad [6]$$

The weights used in the kriging system take account of the known spatial dependencies expressed in the semi-variogram and the geometric relationships among the observed points. In general, near points carry more weight than distant points, points that occur in clusters carry less weight than lone points, and points lying between the point to be

interpolated and more distant points screen the distant points so that the latter have less weight than they would otherwise (Burgess and Webster 1980a).

The mathematical models fitted to the semi-variograms are used in subsequent applications, e.g. kriging. To date, there is no foolproof, purely objective method for fitting models to sample semi-variograms. A method which is commonly used to cross-validate semi-variogram models is jackknifing or leave-one-out validation (Morkoc et al. 1987). When jackknifing, a data point is eliminated and a local estimate of the eliminated point is made, using kriging procedures, from remaining data. This process is carried out for all samples in the area of interest. The statistics of the errors between the measured value,  $Z(x)$  and the estimated value,  $Z^*(x)$  are analyzed to see if the model is acceptable.

For a model to be considered valid a number of requirements are necessary. Since kriging is an exact interpolator, an estimated value should be equal to the measured value, i.e.,  $[Z^*(x)=Z(x)]$ . The observed distribution of errors,  $[Z^*(x)-Z(x)]$  should have a mean equal to zero and the variance of actual errors should equal the kriging variance (Journel and Huijbregts 1978). The variance of actual errors, and the kriging variance have the same mathematical expectation ( $E\{[Z(x)-Z^*(x)]^2\}$ ). Also, 95% of the observed errors should fall within  $\pm 2\sigma_E$  of the mean.

As an example the histogram of errors for the spoil parameter clay percentage is shown in Figure 2. For this particular semi-variogram model clay percentage values from both spoil sampling zones are used

and 456 samples are used in the validation process. The model used is spherical with a nugget variance of 22, a sill of 45, and a range of influence of 366 meters (1200 feet). The errors are approximately normally distributed with a mean equal to  $-0.0092\%$  clay, a variance of errors equal to 37.49 and a kriging variance equal to 37.44. Also, 94.52% of these data fall within two standard deviations of the mean. This model is unbiased since the mean of the errors is essentially zero when compared to a mean percent clay of 26.8. The standard normal confidence interval ( $\pm 2\sigma_E$ ) correctly estimates the 95% confidence interval and the kriging variance is approximately equal to the error variance.

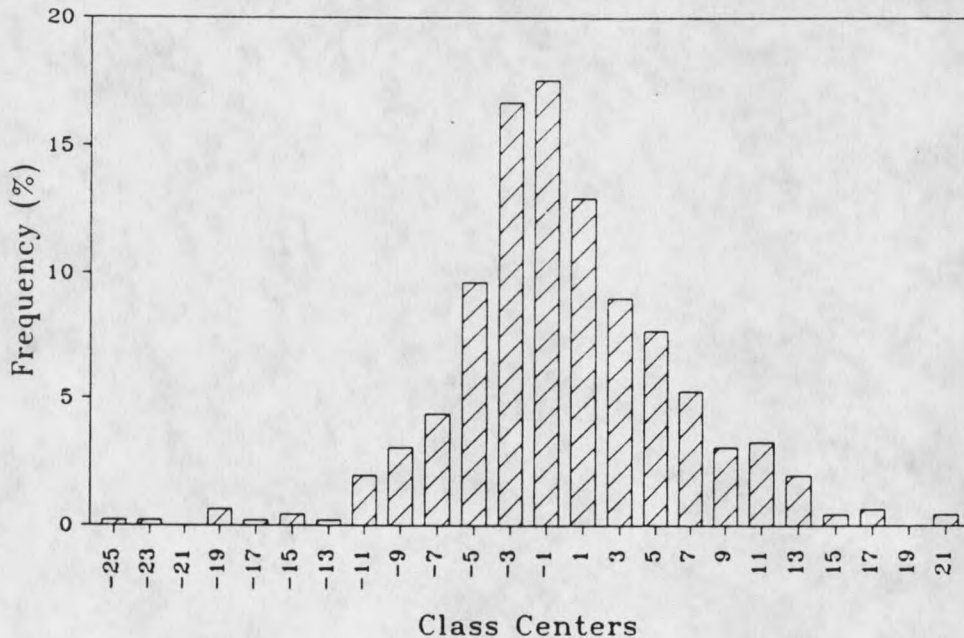


Figure 2. Histogram of errors from jackknifing for the spoil parameter percent clay.

## LITERATURE REVIEW

Geostatistics or the Theory of Regionalized Variables was first applied to problems in mining and geology. The technique is firmly established in the mining industry for the estimation of ore reserves. However, geostatistical techniques can be used whenever a sample value is expected to be affected by its position and its relation to its neighbors. Recently, geostatistics has been applied to spatially related data by plant, soil, and environmental scientists.

An early study examining the rates of spacial variability of soil properties using geostatistical methods was performed by Campbell (1978). Spacial variation of sand and pH measurements were analyzed using the semi-variogram on contiguous delineations of two soil series in Kansas. The soil property pH had random variation within both areas implying no spacial correlation between the data. In the soil series derived from glacial till, sand showed a range of influence of at least 40 m. The soil series derived from fine textured sediments produced a range of influence equal to 30 m. It was concluded that this information can be used to select optimum spacing of samples when designing soil sampling plans. \*

### Anisotropies

Anisotropies are variations in spatially related properties with direction. Anisotropies are analyzed by computing semi-variograms for different directions. The properties of the directional semi-

variograms are usually accounted for when modeling the spacial variation of the property of interest. Including anisotropies in the model will generally improve the quality of the estimates made when kriging.

Burgess and Webster (1980a) found anisotropic variation in soil stone content at the Welsh Plant Breeding Station. An anisotropic linear semi-variogram model was used. Mode of deposition of the soil was shown to have an influence on the spacial structure of the semi-variograms. The soil was fluviially deposited and a general trend in stone content was present over the study area. Semi-variograms along the strike were more similar and had a lower slope than semi-variograms in the direction of the dip. Klusman (1985) found anisotropic variation in soil molybdenum and also used a linear anisotropic semi-variogram model.

Tabor et al. (1984) compared isotropic and anisotropic semi-variogram models of nitrate in irrigated cotton petioles. Petiole sampling and analysis is useful in monitoring the nitrogen status of cotton fields. The models were compared using the jackknifing technique. The anisotropic model produced a lower error variance indicating a better fit than the isotropic model. The anisotropic model portrayed the strong influence cultural practices such as direction of rows and irrigation has on the variability of petiole nitrate.

Wollum and Cassel (1984) used semi-variograms to analyze the spacial distribution of Rhizobium japonicum in two cultivated North Carolina soils. The variance structure was found to be directional.

It was different in the direction "parallel to the row" compared to the direction "perpendicular to the row". Possible contributors to the changes in rhizobia populations observed were root densities, inherent soil properties, seasonality, and management practices.

Quantitative analysis of anisotropic spacial dependence can aid interpretation of soil genesis. Trangmar et al. (1986a) used anisotropic spacial dependencies of particle size fractions, pH, and 25% HCL-extractable P to analyze differences in the main soil-forming factors in Sitiung, West Sumatra. Directions of maximum variation coincided with the main axis of volcanic tuff fallout, deposition of alluvium and the general sequence of soil weathering in the region.

#### Kriging Procedures

Kriging procedures are used to make estimates of spatially related properties at unrecorded places with a known and minimum variance. Kriging is generally used to create maps of regionalized variables, also variances associated with the kriged estimates can be mapped. A variety of kriging techniques have been used in the environmental sciences including: ordinary kriging (punctual or block), universal kriging and co-kriging.

Burgess and Webster (1980a) used ordinary punctual kriging to map sodium content, stoniness, and cover-loam thickness in Central Wales and Norfolk. As the name implies punctual kriging is used to make point estimates of soil properties. The maps produced by punctual kriging showed intricate isarithms and substantial short range variation, caused by semi-variogram models with large nugget variances.



Another paper by Burgess and Webster (1980b) used ordinary block kriging, instead of punctual kriging, to map the above soil properties. Block kriging produces average estimates of soil properties over areas rather than point estimates. Estimation variances for block kriging are much smaller and the maps produced are considerably smoother than the punctually kriged maps, showing a more distinct and purposeful regional pattern. The large nugget variances for these soil properties make the largest contribution to the estimation variances. Maps produced by punctual kriging are erratic and have marked discontinuities near the data points. The authors concluded that block kriging is likely to prove more appropriate than punctual kriging for estimating values of soil properties. This is because land managers will usually be interested in average values over areas rather than point values.

Another kriging technique demonstrated by Webster and Burgess (1980c) is universal kriging. Universal kriging is a form of interpolation that takes account of local trends, or drift in the data, which are identified by structural analysis. Ordinary kriging operates under the assumption that drift is not present in the data. Universal kriging was applied to measurements of electrical resistivity made in the soil at 1 m intervals at Bekesbourne, Kent, United Kingdom. Structural analysis showed the data could be represented as a series of linear drifts over distances of 4 m to 8 m. Semi-variograms of residuals from drift showed negligible nugget variance, and were used to kriging missing values at the site.

It was concluded that the method is not universally applicable to soil survey. This is because large nugget variances are usually encountered in the spatial analysis of soil properties. Large nugget variances imply there is substantial "noise" or short range variation in the data which effectively prevents any distinction between constant or changing drift. Large nugget variances originate in part because measurements are made on small widely separated volumes of soil. Universal kriging is likely to be applicable only where measurements are made on contiguous volumes of soil or after substantial mixing.

As part of a study performed by Yost et al. (1982) estimates of soil P sorption were compared using ordinary and universal kriging. Universal kriging, either by polynomial trend removal or by local polynomial trend removal during estimation, was not beneficial in spite of widely varying P sorption and a significant polynomial trend in the data. Results suggest ordinary kriging is useful in summarizing and interpreting soil analysis and that ordinary kriging seems quite robust to certain degrees of nonstationarity.

McBratney and Webster (1983) applied co-kriging to soil textural properties on the Woburn Experimental Farm in England. Co-kriging extends regionalized variable theory from that of a single soil property to situations where there are two or more spatially interdependent ones. Sometimes, one variable is costly to estimate or has not been sampled sufficiently to provide estimates of acceptable accuracy. The precision of the estimate may be improved by considering the spacial relations between this variable and other better sampled variables. Co-kriging can be used to interpolate values of a poorly

sampled variable from one or more correlated properties that have been more intensively sampled. McBratney and Webster found a strong co-regionalization with common anisotropy between topsoil silt, subsoil silt and subsoil sand. This allowed topsoil silt to be estimated and mapped by co-kriging more precisely than kriging from data on topsoil silt alone.

Trangmar et al. (1986b) used co-kriging to interpolate values of topsoil 0.5 M  $\text{NaHCO}_3$ -extractable P at 234 locations in Indonesia, by exploiting its spacial covariance with a more densely sampled property, 25% HCl-extractable P. Sampling was performed in a nongeometric pattern across the 106,650 ha. region and sampling densities for  $\text{NaHCO}_3$ -P varied across the study area. The map of co-kriged values for  $\text{NaHCO}_3$ -P showed more detail than that achieved by kriging from  $\text{NaHCO}_3$ -P samples alone. Co-kriging reduced estimation variances, relative to kriging, by up to 40% in areas where sampling density of  $\text{NaHCO}_3$ -P was lowest. Co-kriging variances exceeded those of kriging by up to 10% in areas where sampling density of  $\text{NaHCO}_3$ -P was high. These results show that the benefits of co-kriging interpolation are not necessarily obtained from non-geometrically sampled variables.

#### Sampling Strategies

Geostatistics is particularly well suited to the formulation of optimum sampling strategies. Burgess, Webster, and McBratney (1981) demonstrated how geostatistics can be used to design optimum sampling schemes for soil survey. To design an optimum sampling strategy prior knowledge of the semi-variogram for the property of interest is

necessary. Then from the semi-variogram estimation variances can be found for any combination of block size and sampling density by the methods of kriging. Alternatively for a given block size the sampling density needed to achieve a predetermined precision can be estimated.

The authors also discussed optimum configurations for sample locations. An equilateral triangular scheme for sample locations is best where variation is isotropic, although a square grid pattern is nearly as efficient and more realistically implemented. Where the semi-variograms exhibit geometric anisotropic variation optimal sampling is achieved by choosing a rectangular grid. The sides of the grid are in the same proportion to one another as the slopes of the semi-variograms in the directions of maximum and minimum variation.

McBratney and Webster (1983) discussed sampling procedures for determining regional estimates of soil properties using geostatistical techniques. Also, they compared the efficiency of the geostatistical methods to classical statistical approaches. Examples show that in all instances the sampling effort determined using a geostatistical approach is less than would have been judged necessary using the classical approach. Three-and-a-half-fold to nine-fold gains in efficiency were realized over that estimated by classical theory for simple random sampling.

Lang et al. (Lang et al. 1986. The use of geostatistics to evaluate sample adequacy on regraded mine spoil. Unpublished paper. 26pp.) used geostatistics to evaluate sample adequacy on regraded mine spoil at Western Energy Company's Rosebud Mine in southeastern Montana. The purpose of the study was to use a geostatistical approach called

error bound to evaluate the utility and adequacy of regraded spoil sampling requirements. A 95% confidence interval on the variance of the estimation error (error bound) as approximated from semi-variograms, was used to assess the reliability of sampling. Error bound defines the error associated with the global estimate for an area of interest and is expressed as a percentage of that estimate. It places a confidence interval on the mean estimate of a spoil property. Sample adequacy as it pertains to this study is defined as the amount of error considered acceptable when estimating the composition of the regraded spoil. An error bound of  $\pm 20\%$  was determined to be adequate for characterization of the spoil material, and this error bound could be achieved by sampling on approximately 305 meter (1000 foot) centers.

## MATERIALS AND METHODS

### Site Description

The Absaloka Mine is an area strip mine located about 26 miles east of Hardin, Montana (Figure 3). Coal is removed from three coal seams, the Rosebud-McKay coal seams separated by a thin parting and the Robinson seam. Geographically the mine is located in the northern portion of the Powder River Basin. Stratigraphically the area is underlain by the Tongue River Member of the Fort Union Formation. The overburden is typically free of physical and chemical problems, although areas of regraded spoil with high SAR, ESP and percent clay can be found. In general, the overburden has a slightly alkaline pH and is low in salinity. The interburden also has an alkaline pH and is low in salinity, however, the interburden is generally sodic. Textures of the overburden and interburden are usually clay loam to silty clay loam, but range from clay to loamy sand (Montana Department of State Lands and the Office of Surface Mining, 1984).

### Data Set

The data base used in this study came from records kept on file by the Montana Department of State Lands (DSL). Samples were taken by the mining company from December of 1979 through July of 1986. A total of 240 sample sites were used in the analysis. A map of the sample locations is shown in Figure 4. Spoil sampling was conducted to a

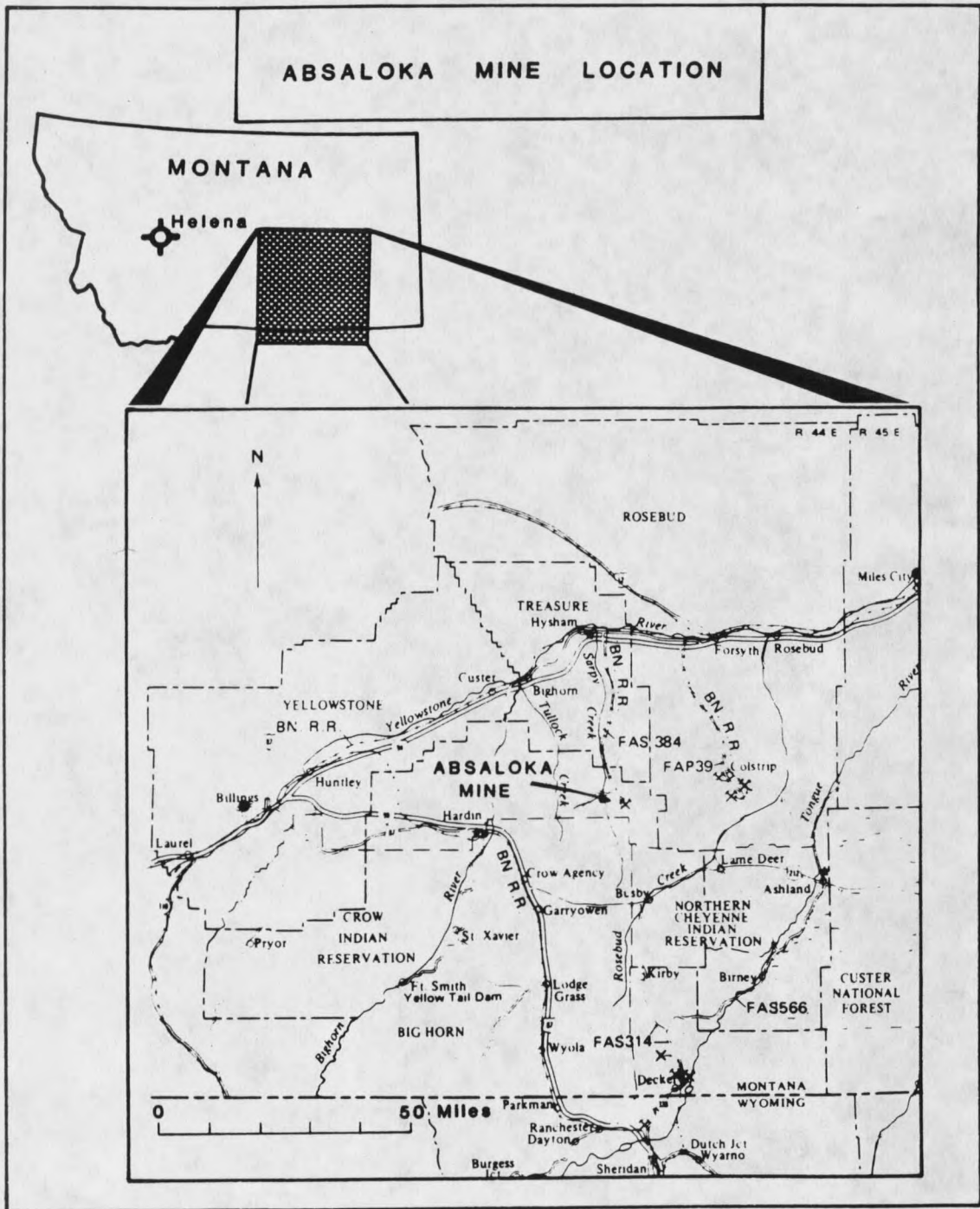


Figure 3. Site location of the Absaloka Mine (Montana Department of State Lands and the Office of Surface Mining, 1984).

depth of 2.44 m with samples taken from the surface 0-1.22 m of spoil and the subsurface 1.22-2.44 m of spoil. Samples were collected using a giddings coring device.

To perform a geostatistical analysis the spacial location of each sample site must be known. A computer digitizer was used to generate cartesian coordinates for each sample site from maps of sample locations prepared by the mining company. Samples are located according to the Montana coordinate system.

#### Laboratory Analysis

Laboratory analysis was performed according to procedures stated in the Soil and Overburden Guidelines prepared by the DSL (1983). Samples were air dried at  $\leq 35^{\circ}\text{C}$  and then ground until they were able to pass a 10 mesh (2 mm sieve). U.S.D.A. Handbook 60 (Richards 1954) was used as a reference for analysis of the following parameters: saturation percentage, pH, calcium, magnesium, sodium, and SAR (sodium absorption ratio). Conductivity, pH, and SAR were determined in the saturation extract. Exchangeable sodium percentage (ESP) and conductivity (mmhos/cm at  $25^{\circ}\text{C}$ ) were determined using U.S.D.A. Handbook 525 (Sandoval and Power 1978). Particle size analysis was performed by the hydrometer method (Black 1965).

Criteria for unsuitability of regraded spoils have been outlined by the DSL (Montana Department of State Lands, 1983). The purpose of these criteria is to set limits on acceptable spoil physical and chemical properties. Within these limits reclamation plant species



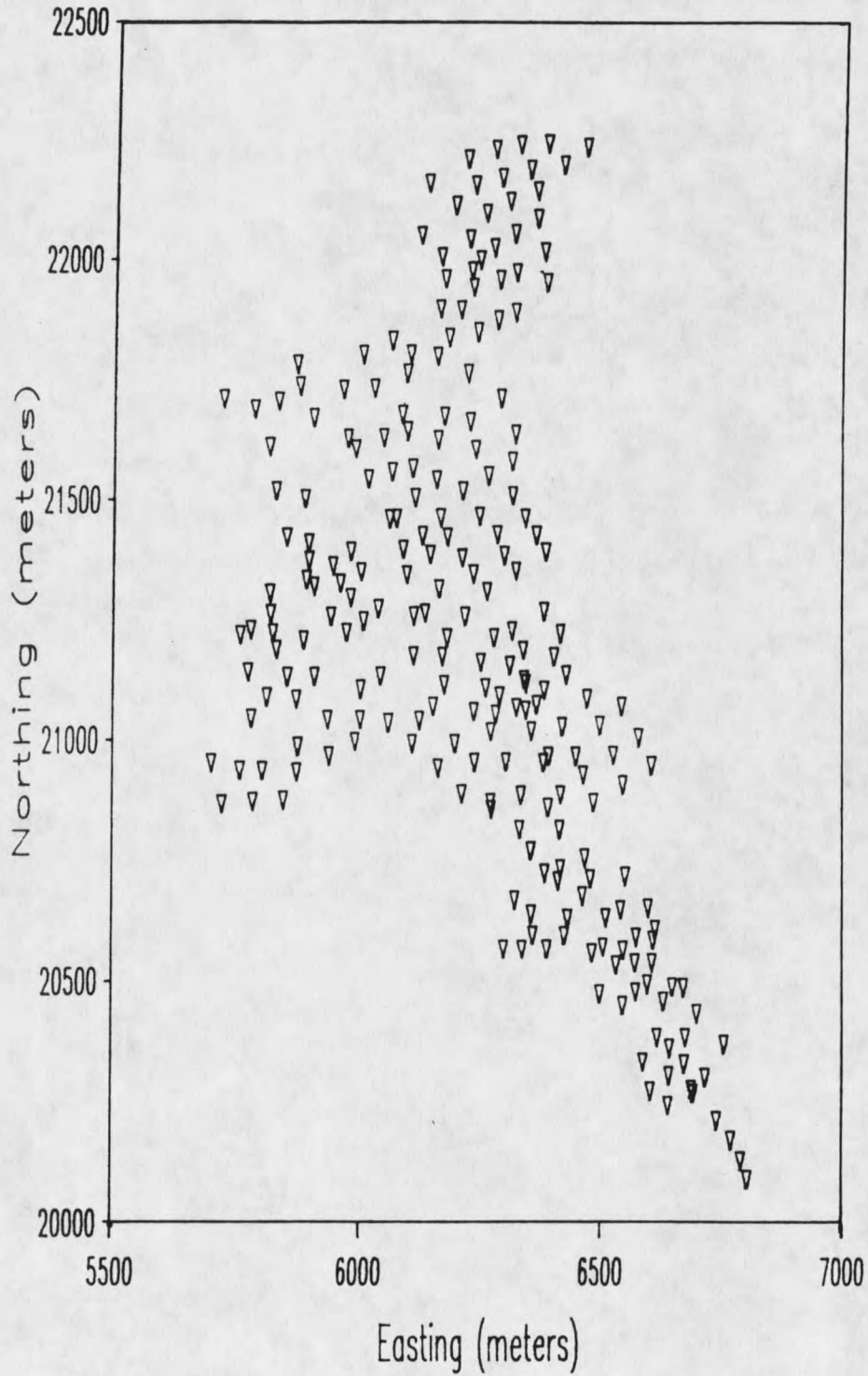


Figure 4. Location of regraded spoil sample sites at the Absaloka Mine.

will be able to function properly. Table 1 shows the present unsuitability requirements for regraded spoil.

Inconsistencies were found in these data which could bias the results. For particle size analysis two different laboratory techniques were employed. A two hour hydrometer method was used on 74 of the 240 sample sites instead of the usual eight hour hydrometer method. The two hour hydrometer method was used on samples analyzed before September 1980. A higher estimate for the clay fraction of the soil is produced using a two hour settling time. Also, lab analyses were performed by two different companies: Northern Testing Laboratories Inc., and Energy Laboratories Inc.

#### Statistical Analysis

Summary statistics were computed for each spoil property and for each sampling zone. The summary statistics computed include mean, standard deviation, skewness, kurtosis, maximum and minimum values, and coefficient of variation. To quantify differences between the two sampling zones histograms, linear regression analysis and a paired t-test analysis were used. Histograms along with the summary statistics were used to compare central tendency and data dispersion between the two zones. The linear regression analysis was used to compare the similarity between the two zones by using the sample value from the surface zone to predict the sample value of the subsurface zone. The paired difference test was used to determine if the surface zone has significantly higher or lower values than the subsurface zone for

each parameter. Summary statistics, linear regression and paired t-test were performed using MSUSTAT (Lund 1987).

Table 1. Unsuitability criteria for regraded spoil.

Parameter	Suspect Level	Comments
pH	< 5.5 > 8.5	U.S.D.A. Handbook 60, Method 21a, p. 102.
Conductivity (mmhos/cm) (EC)	> 4.0-8.0	The actual maximum acceptable salt level will depend on the plant species proposed in the revegetation plan and the potential for upward salt movement. U.S.D.A. Handbook 525, Method 1, pp.22-24.
Saturation percentage	> 90% < 25%	U.S.D.A. Handbook 60, Methods 2 and 3a, pp. 84 and 88, and Method 27a, p. 107.
Sodium absorption ratio (SAR)	> 20	If the clay content is < 35% and the saturation percentage is < 90% a SAR of < 20 is acceptable. U.S.D.A. Handbook 60, p. 26.
Exchangeable sodium percentage (ESP)	> 15 > 18	% clay<35 and % clay>20 % clay<20 Analysis of ESP not required as of June 1987. U.S.D.A. Handbook 525, No. 6A, p. 9.
Textural class		c, sc, sic, cl (> 35% c), sicl, (> 35% c, < 15% s), sil (< 15% s), si, ls, s. These textural classes are considered unsuitable.

Geostatistical analysis was performed using software developed by H. P. Knudsen (1987) and methods described by Journel and Huijbregts (1978). Semi-variograms for each spoil property were computed to determine the extent of spatially dependent variance at the mine site. Also, semi-variograms (equation [1]) were computed in four different directions (N-S, NE-SW, E-W, SE-NW), each with a direction tolerance of  $\pm 22.5^\circ$ , to examine for anisotropies. Spherical models were used to describe spacial variability (equation [2]).

Anisotropies, if present, were accounted for using a geometric model. Anisotropy ellipses were constructed showing how the ranges of influence change with direction. This model assumes that the directional graphs for the ranges of influence are elliptical and that the anisotropy can be reduced to isotropy by a linear transformation of the coordinate system. An element of subjectivity is involved when deciding whether or not to account for geometric anisotropies. Three general criteria were used; (i) the sill and nugget variances were similar between the directional semi-variograms, (ii) the directional graphs produce a distinguishable ellipse, (iii) and when jackknifing, inclusion of the anisotropy reduced the estimation variance.

To assure that the unbiased condition was met and the estimation errors conform to the 95% gaussian confidence interval a jackknifing method of semi-variogram validation was used. During jackknifing procedures point estimates of spoil parameters were made using ordinary kriging. Anisotropies, if present, were modeled during jackknifing. The validated semi-variogram models from jackknifing were used during kriging procedures.

Ordinary block kriging was used to estimate average values of 30.48 meter by 30.48 meter blocks by averaging 25 kriged points over each area. The kriged points were estimated using a minimum of 3 and a maximum of 20 closest measured points. In the theory section it was stated that the interpolated value of a property  $Z$  at any location,  $x_0$ , is a weighted average of the observed values in the kriging neighborhood, thus

$$Z^*(x_0) = \Gamma_1 \cdot Z(x_1) + \Gamma_2 \cdot Z(x_2) + \dots + \Gamma_n \cdot Z(x_n). \quad [7]$$

The weights  $\Gamma_i$ ,  $i=1,2,\dots,n$ , and a Lagrange parameter  $\mu$  for punctual kriging are obtained by solving  $[\Gamma_\mu] = A^{-1}[b]$ , where  $A$  is a matrix of semi-variances between data points and  $b$  is a matrix containing the semi-variances between the data points and the point to be estimated. In block kriging, instead of just a point  $x_0$ , we consider a region  $V$  of area  $H_V$  with its center at  $x_0$ . In this study the region  $V$  has an area ( $H_V$ ) of 30.48 meters by 30.48 meters (100 feet by 100 feet). When block kriging the weights and Lagrange parameter are found by solving  $[\Gamma_\mu] = A^{-1}[s]$ , where  $[s]$  is a matrix of the average semi-variances between the data points and all points in the region  $V$ . The estimation variance for the area  $H_V$  becomes

$$\sigma_H^2 = [s]^T [\Gamma_\mu]^{-1} \tau(v,v) \quad [8]$$

where  $\tau(v,v)$  is the average semi-variance between points within the block  $V$ , the within block variance of classical statistics (Burgess and Webster, 1980b).

From equation [8] it can be seen that as the within block variance  $\{\tau(v,v)\}$  increases the estimation variance will decrease. As block size increases,  $\tau(v,v)$  increases, so the larger the block size the

smaller the estimation variance. When calculating the estimation variance for punctual kriging the within block variance term is not present. Therefore estimates made by punctual kriging have higher estimation variances than estimates made from block kriging.

Block kriging has advantages over punctual kriging for mapping purposes. Maps drawn from point estimates are the most accurate isarithmic maps that can be made using a set of point data, but, discontinuities can seriously obscure longer range trends. Maps of block kriged values smooth the discontinuities, showing more clearly trends in the data, and providing a more meaningful and informative method of looking at data (Burgess and Webster 1980b).

Choice of an appropriate block size is important when making estimates of spoil properties. The block estimate simulates an average value over a volume, so areas within the block will have values that are higher or lower than the estimated average. Blocks with values close to the suspect limit will contain areas with values that exceed the suspect limit. A small block will give a more exact and realistic estimate than a large block and is more desirable for mapping problem areas. A block size of 30.48 meters (100 feet) per side should provide a desirable combination of exactness and smoothing of discontinuities in the data. This block size also represents a manageable unit of area. If a portion of the regraded spoils are determined to be unsuitable then removal of the affected area may be necessary.

From the kriged estimates of spoil parameters it can be determined if block estimates of regraded spoil have average values that exceed, or potentially exceed suspect levels set by the DSL. Using the

variance associated with each block estimate confidence limits can be placed about each estimate. Using a one tailed t-test the 95% upper confidence limit for each estimate can be determined by,  $Z^*(x) + 1.645 \cdot \sigma_H$ , where  $Z^*(x)$  is the kriged block estimate,  $\sigma_H$  is the standard deviation associated with each kriged estimate, and 1.645 is the one tailed critical t value for  $\alpha=0.050$  and an infinite sample size. Block estimates or 95% upper confidence limits that exceed the suspect level for any spoil parameter should be considered suspect. For computation of the kriged estimates the validated variogram model using both zones of recontoured spoil was utilized.

Sample spacing adequacy and sample size determination were evaluated using a variety of techniques. A two phase approach to sampling spoils is proposed. The purpose of first phase sampling is to characterize the spoil material using kriging procedures and a sample spacing based on semi-variogram properties. Second phase sampling should be implemented if problem areas of spoil are found. The purpose of second phase sampling is to increase sampling intensity to better characterize problem areas. Second phase sampling is based on the average estimation variance associated with a specified sample spacing. Estimation variances were determined using kriging procedures and curves were developed to define appropriate additional sampling.

## RESULTS AND DISCUSSION

### Comparing Differences Between Sampling Zones

Summary statistics, histograms, paired t-test, and a linear regression analysis were used to compare differences between the two sampling zones. Table 2 shows the summary statistics for the surface and subsurface zones of recontoured spoil. The mean, standard deviation (STD. DEV.), skewness, kurtosis, minimum (min.), maximum (max.), and coefficient of variation were computed. It is apparent that the statistics are similar between the two sampling zones. This indicates that data dispersion and central tendency are comparable between the two zones.

The summary statistics also indicate that there are values of the spoil parameters that exceed suspect levels. This occurs for the parameters; percent clay, EC, ESP, and SAR. Percent clay has a maximum value of 49.1% in the surface zone and 44.0% in the subsurface zone, these values exceed the suspect limit of 35%. Exchangeable sodium potential has maximum values of 27.6 in the surface zone and 22.6 in the subsurface zone, these values exceed the maximum suspect limit of 18. Sodium absorption ratio has maximum values of 31.9 and 27.5 in the surface and subsurface zones, respectively, and these values exceed the suspect limit of 20. Conductivity has maximum values of 6.6 and 6.9 mmhos/cm in the surface and subsurface zones respectively, these values exceed the minimum suspect limit of 4.0 mmhos/cm.



Histograms were generated for each spoil parameter and for each sampling zone to more visually represent these data. Figures 5 and 6 show histograms for percent clay. Additional histograms of spoil physicochemical properties are shown in Appendix A. Histograms also indicate that central tendency and data dispersion are analogous between the two zones.

Table 2. Summary statistics.

Surface Zone of Recontoured Spoil							
Number of samples = 240							
VARIABLE	MEAN	STD. DEV.	SKEWNESS	KURTOSIS	MIN.	MAX.	CV(%)
pH	7.31	0.3186	0.2514	3.272	6.40	8.30	4.36
EC (mmhos/cm)	3.77	1.246	-0.2695	2.758	0.42	6.60	33.09
SAT. %	43.75	7.659	0.8458	6.033	26.30	84.60	17.51
SAR	7.63	5.016	1.668	7.971	0.38	31.90	65.71
ESP	6.78	4.679	1.078	4.644	0.08	27.60	69.00
%SAND	38.50	12.06	0.6769	3.194	16.40	77.00	31.32
%CLAY	26.80	8.351	-0.2103	2.594	6.00	49.10	31.16
Subsurface Zone of Recontoured Spoil							
Number of samples = 216							
VARIABLE	MEAN	STD. DEV.	SKEWNESS	KURTOSIS	MIN.	MAX.	CV(%)
pH	7.35	0.3287	0.2719	3.440	6.30	8.50	4.47
EC (mmhos/cm)	3.72	1.257	-0.2119	2.803	0.75	6.90	33.83
SAT. %	44.02	7.125	0.6107	5.921	26.80	80.90	16.19
SAR	7.85	4.885	1.100	4.822	0.39	27.50	62.20
ESP	7.13	4.603	0.6942	2.976	0.25	22.60	64.53
%SAND	37.98	12.46	0.7415	3.235	16.00	75.00	32.81
%CLAY	26.75	7.945	-0.3252	2.644	5.00	44.00	29.70

A paired difference test was used to determine if one sampling zone had significantly higher or lower values than the other at the 95% confidence level. The values at each sample site were paired and the

subsurface zone was compared to the surface zone. Table 3 shows the results from the paired t-test. Exchangeable sodium percentage was the only parameter showing a significant difference according to this test. In the subsurface zone ESP had significantly higher values than the surface zone. The difference in means (surface zone - subsurface zone) is -0.4469, the 95% upper and lower confidence limits were both less than zero and the P-value is less than 0.05. Even though this difference was detected, ESP analysis in regraded spoil is no longer required by the DSL as of June 1987 (Montana Department of State Lands 1987).

Table 3. Paired t-test results.

	Difference in Means	SE for Difference	95% Lower Limit	95% Upper Limit	T (d.f.=215)	P-Value
Number of pairs = 216						
pH	-0.0194	0.0124	-0.0438	0.0050	-1.570	0.1163
EC	0.0292	0.0462	-0.0619	0.1205	0.632	0.5272
(mmhos/cm)						
SAT. %	-0.4037	0.3643	-1.122	0.3144	-1.108	0.2678
SAR	-0.2571	0.1725	-0.5971	0.0829	-1.490	0.1361
ESP	-0.4469	0.1976	-0.8364	-0.0575	-2.262	0.0237
%SAND	0.7435	0.5230	-0.2874	1.774	1.422	0.1551
%CLAY	-0.3949	0.3144	-1.015	0.2248	-1.256	0.2091

The linear regression analysis was used to see how values from the surface zone could predict values from the subsurface zone of regraded spoil. If a strong relationship exists between the two zones the results should produce a slope and a coefficient of determination ( $r^2$ ) of near one, and an intercept of zero. The results from this analysis are shown in Table 4. The coefficients of determination ranged from

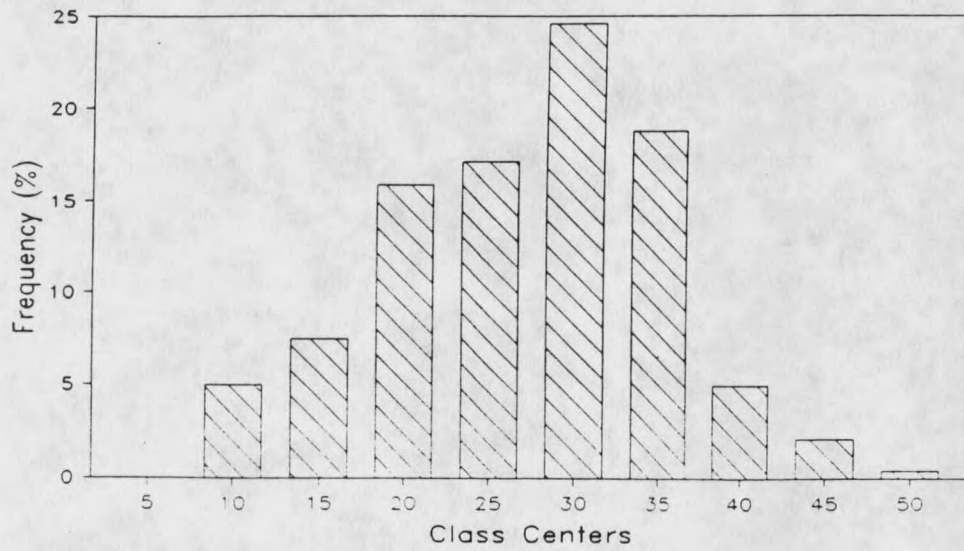


Figure 5. Histogram for percent clay using data from the surface zone.

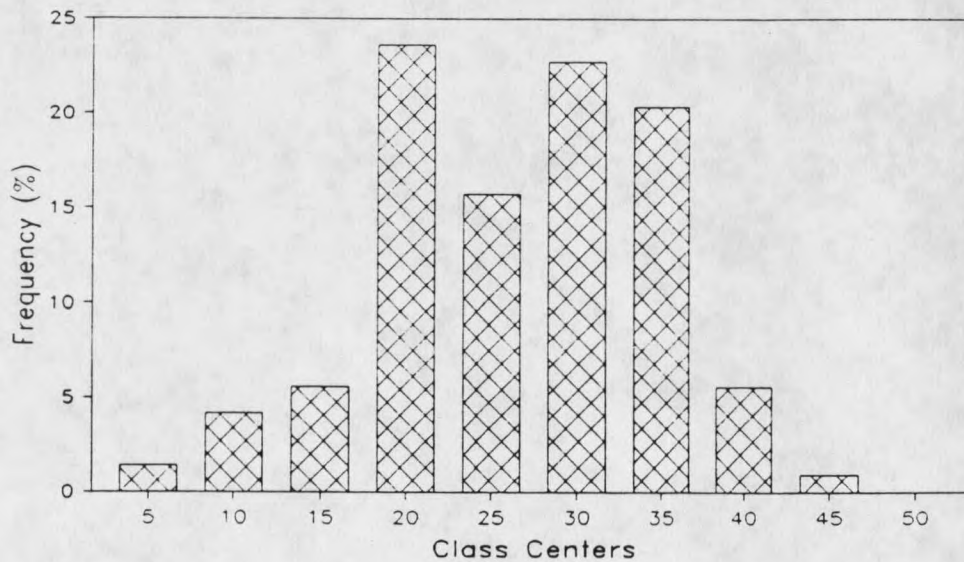


Figure 6. Histogram for percent clay using data from the subsurface zone.

0.56 for saturation percentage to 0.76 for SAR and indicate that there is considerable variation between the two sampling zones. Even though this variation exists the P-values from regression are all 0.0000, indicating that a significant relationship exists between the two sampling zones.

Table 4. Regression results, using the surface zone to predict values for the subsurface zone.

Number of samples = 216							
Parameter	pH	EC (mmhos/cm)	SAT. %	SAR	ESP	%SAND	%CLAY
SLOPE	0.8607	0.8444	0.6805	0.8291	0.7769	0.8173	0.7903
INTERCEPT	1.040	0.5537	14.34	1.556	1.938	6.331	5.923
r <sup>2</sup>	0.7123	0.7321	0.5585	0.7631	0.6561	0.6519	0.7119
P-VALUE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

#### Semi-variograms and Anisotropies

Semi-variograms for the spoil parameters were computed for eight directions 90° (N-S), 67.5°, 45°, 22.5°, 0° (E-W), -22.5°, -45°, and -67.5°. Since these data were irregularly spaced, search windows of 22.5° were used to compute each directional semi-variogram.

Directional semi-variograms computed using data from both sampling zones are shown in Figures 7 and 8 for the parameter percent clay. Appendix B contains directional semi-variograms for the remainder of the spoil properties. Only points with greater than 30 sample pairs are shown. The first graph shows the 90°, 67.5°, 45°, and 22.5° directional semi-variograms (Figure 7). The second graph shows the 0°, -22.5°, -45°, and -67.5° directional semi-variograms (Figure 8).

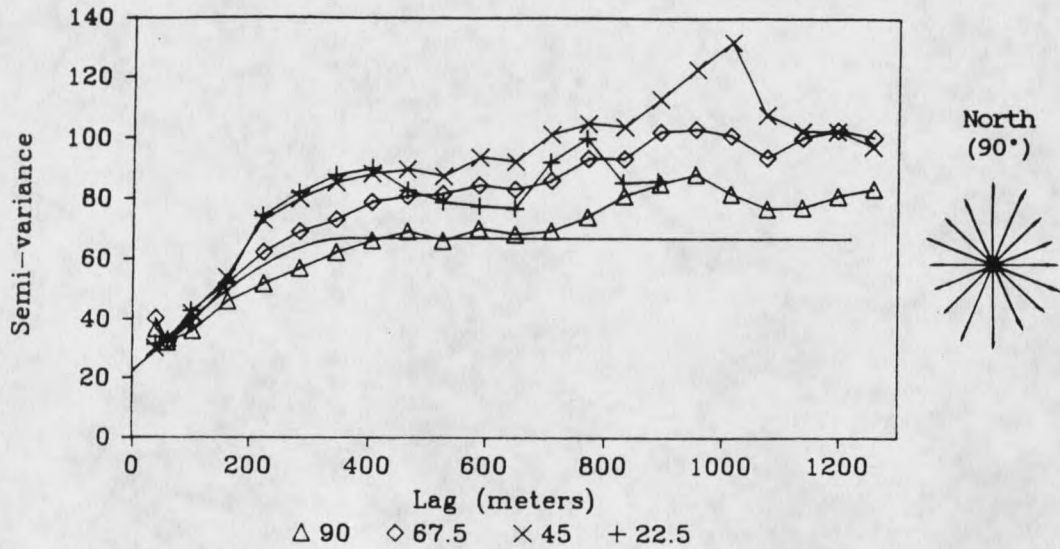


Figure 7. Directional semi-variograms for percent clay, viewing the 90°, 67.5°, 45°, and 22.5° angles. The anisotropy ellipse has a rotation of 79° and a anisotropy ratio of 1.35.

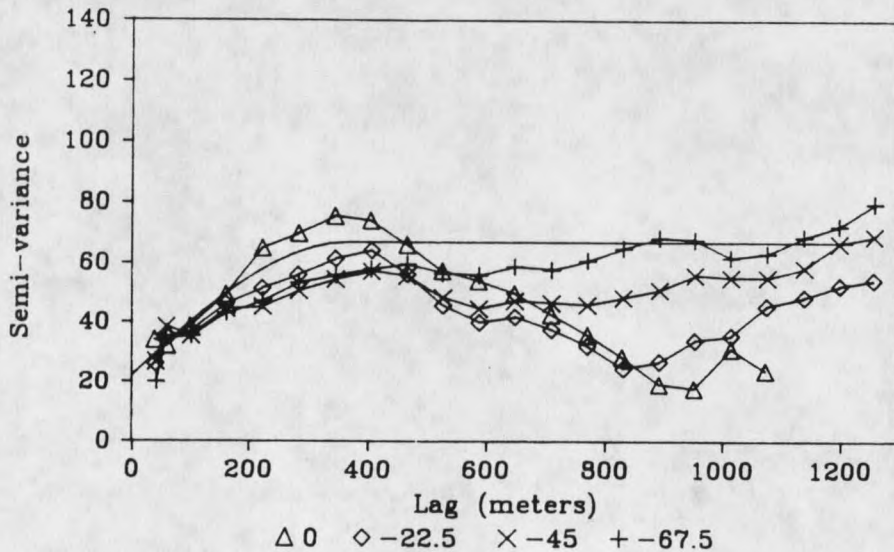


Figure 8. Directional semi-variogram for percent clay, viewing the 0°, -22.5°, -45°, and -67.5° angles.

Anisotropy ellipses are shown with the directional semi-variograms if geometric anisotropies were used when computing the spatial model. The solid line within each graph represents the validated semi-variogram model from jackknifing.

Similarities between the semi-variograms can be distinguished. In general, nugget variances tend to be high indicating substantial short range variation in these data. Nugget variances for the spoil parameters SAR and ESP are exceptionally high. This is due to erratic directional semi-variograms with angles of  $22.5^\circ$ ,  $0^\circ$ , and  $-22.5^\circ$ . Considering the validated models for both zones of spoil, the ranges of influence are similar between the spoil properties. They range from 245 meters (800 feet) for saturation percentage and ESP, 335 meters (1100 feet) for SAR and 366 meters (1200 feet) for the remainder of the parameters.

#### Validation

The validation process helps determine if the semi-variogram model used during kriging procedures is appropriate. Semi-variogram models were validated for the surface zone, the subsurface zone, and both zones of recontoured spoil. Spherical models were used in all cases. Validation results are shown in Table 5. "Variance" in Table 5 refers to the sample variance for the sampling zone(s) and should be approximately equal to the sill value. "Range" refers to the range of influence, in meters, of the semi-variogram model. "Rotation" refers to the angle of rotation of the anisotropy ellipse. The angle is

measured from the east-west ( $0^{\circ}$ - $180^{\circ}$ ) axis to the major axes of the anisotropy ellipse. Positive angles are turned in a counter clockwise

Table 5. Validated semi-variogram models.

Surface Zone of Recontoured Spoil							
Parameter	pH	EC (mmhos/cm)	SAT.%	ESP	SAR	%CLAY	%SAND
Variance	0.102	1.55	58.7	21.9	25.2	69.7	145
Sill	0.078	1.3	55	21	24	67	161
Nugget	0.048	0.6	24	14	13	22	54
C(1)	0.030	0.7	31	7	11	45	104
Range	427	366	320	305	335	366	396
Rotation	$-67.5^{\circ}$		$-79^{\circ}$			$90^{\circ}$	$90^{\circ}$
Ratio	1.58		2.2			1.3	1.7
Maximum Radius	366	366	396	305	305	366	366
Subsurface Zone of Recontoured Spoil							
Parameter	pH	EC (mmhos/cm)	SAT.%	ESP	SAR	%CLAY	%SAND
Variance	0.108	1.58	50.8	21.2	23.9	63.1	155
Sill	0.086	1.5	56	21	20	63	163
Nugget	0.034	0.6	33	11	13.5	25	80
C(1)	0.058	0.9	23	10	6.5	38	83
Range	305	488	305	244	396	396	396
Rotation	$-45^{\circ}$						$67^{\circ}$
Ratio	1.25						1.5
Maximum Radius	274	335	305	245	305	366	366
Both Zones of Recontoured Spoil							
Parameter	pH	EC (mmhos/cm)	SAT.%	ESP	SAR	%CLAY	%SAND
Variance	0.105	1.56	54.8	21.5	24.5	66.5	150
Sill	0.084	1.30	51	21	20	67	158
Nugget	0.043	0.6	23	12	13.5	22	63
C(1)	0.041	0.7	28	9	6.5	45	95
Range	366	366	245	245	335	366	366
Rotation			$-79^{\circ}$			$79^{\circ}$	$-68^{\circ}$
Ratio			2.4			1.35	1.9
Maximum Radius	274	366	305	245	305	366	366

direction. "Ratio" refers to the ratio of the major axis of the two dimensional anisotropy ellipse to the minor axis. "Maximum radius" refers to the maximum search radius, in meters, between the sample to be kriged and the points used in the kriging calculations.

In order for a model to be considered valid, certain statistical criteria must be met. The mean of the kriging errors should be approximately zero, the variance of errors should be equal to the average kriging variance, the errors should be normally distributed, and 95% of the errors should fall within two standard deviations of the mean. Summary statistics of kriging errors from validation are shown in Table 6. From Table 6 it can be seen that these criteria have been met. Skewness and kurtosis values describe the distribution of the kriging errors. A histogram of the kriging errors for percent clay using the model for both sampling zones is shown in Figure 9. Histograms for the remainder of the spoil properties are shown in appendix C. It can be seen that these data are normally distributed.

#### Comparison of the Sampling Zones Using Spacial Properties

Similarities between the two sampling zones can be compared by analyzing the spacial properties of each zone. If the two zones are spatially similar we would expect the semi-variogram parameters for each zone to be similar. Table 5 can be used to compare the semi-variogram models between the two zones. Sill values between the two zones are similar. However, differences occur between the nugget and  $C(1)$  values. Percent sand shows the most dramatic difference between nugget and  $C(1)$  values. For the surface zone the nugget variance is 54



Table 6. Summary statistics of kriging errors from semi-variogram validation.

Surface Zone of Recontoured Spoil							
Number of Samples = 240							
Parameter	pH	EC (mmhos/cm)	SAT. %	ESP	SAR	%CLAY	%SAND
Mean	0.0024	0.0052	0.057	0.075	0.034	-0.0042	0.0015
Skewness	-0.483	0.2153	-1.03	-1.28	-1.53	0.0105	-0.631
Kurtosis	4.19	3.61	7.94	7.29	9.63	4.28	5.20
Variance	0.0611	0.8421	39.83	17.59	17.67	37.08	94.27
Average Kriging							
Variance	0.0611	0.8425	39.79	17.56	17.45	37.04	94.28
Percent Errors							
Within $\pm 2\sigma_E$	95.42	95.00	96.67	96.25	95.83	94.58	94.17
Subsurface Zone of Recontoured Spoil							
Number of Samples = 216							
Parameter	pH	EC (mmhos/cm)	SAT. %	ESP	SAR	%CLAY	%SAND
Mean	0.0014	-0.0041	-0.011	0.025	-0.0076	-0.037	-0.0094
Skewness	-0.301	0.254	-1.24	-0.641	-1.03	-0.036	-0.421
Kurtosis	3.29	4.09	9.20	3.30	5.31	4.84	4.52
Variance	0.0555	0.837	43.98	16.30	16.25	36.80	113.63
Average Kriging							
Variance	0.0557	0.839	43.79	16.19	16.33	36.78	113.35
Percent Errors							
Within $\pm 2\sigma_E$	94.91	95.37	95.83	94.44	93.98	93.52	93.52
Both Zones of Recontoured Spoil							
Number of Samples = 456							
Parameter	pH	EC (mmhos/cm)	SAT. %	ESP	SAR	CLAY%	SAND%
Mean	-0.0008	-0.007	0.033	0.041	0.011	-0.009	0.0326
Skewness	-0.420	0.234	-1.08	-0.988	-1.33	0.016	-0.497
Kurtosis	3.75	3.83	8.67	5.61	7.79	4.58	4.91
Variance	0.0582	0.841	41.88	16.91	16.85	37.49	102.81
Average Kriging							
Variance	0.0582	0.845	41.95	16.87	16.65	37.44	102.81
Percent Errors							
Within $\pm 2\sigma_E$	95.18	95.18	95.83	94.96	94.52	94.52	94.30

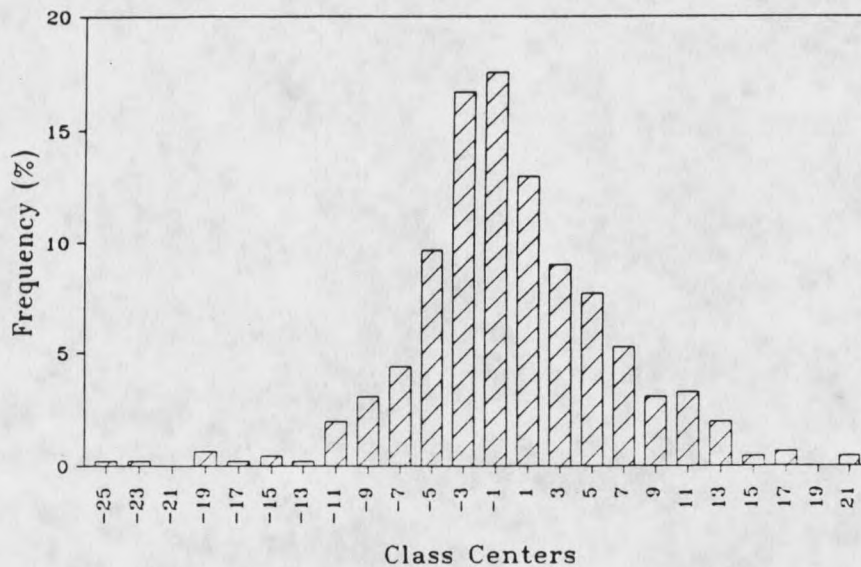


Figure 9. Histogram of kriging errors for percent clay using data from both zones of spoil.

and the  $C_{(1)}$  value is 104, compared to a nugget variance of 80 and a  $C_{(1)}$  value of 83 for the subsurface zone. Notable differences in nugget and  $C_{(1)}$  values can also be seen for the parameters ESP, saturation percentage, and pH. Ranges of influence can also be compared using Table 5. Notable differences in ranges of influence can be seen for the parameters pH and EC. The surface zone for pH has a range of influence of 427 meters (1400 feet) compared to 305 meters (1000 feet) for the subsurface zone. For EC the surface zone has a range of influence of 366 feet (1200 feet) and the subsurface zone has a range of influence of 488 meters (1600 feet). Anisotropy factors should also be considered when comparing the two zones. Anisotropy factors are similar for the parameters pH and percent sand, but, differences exist for saturation percentage and percent clay. Saturation percentage and percent clay have anisotropies modeled for

the surface zone but not the subsurface zone. This difference is particularly important for saturation percentage due to the high anisotropy ratio for the surface zone.

The variance of kriging errors can also be used to compare differences between the sampling zones. Table 6 shows the variance of kriging errors for the sampling zones. The variance of kriging errors is the variance of errors between the true sample values and values estimated by kriging. This variance measures the predictive power of the semi-variogram model. If the error variances are similar between the two zones it can be assumed that the semi-variogram models have parallel predictive strength. Since there are differences between some of the semi-variogram models, comparing the error variances is a more useful way to assess spatial correspondence between the zones. The parameter sand percentage shows the greatest difference in error variances. The surface zone has an error variance of 94.27 and the subsurface zone has an error variance of 113.63. This difference in error variances is primarily due to the large differences in nugget variances between the two zones. The error variances between sampling zones for the other spoil properties are quite similar.

An objective of this study was to statistically compare the two sampling zones to determine if sampling both zones is necessary. Comparison of the two sampling zones using classical statistical tests demonstrated that significant relationships exist. Summary statistics and histograms showed that data dispersion and central tendency were comparable between the two zones. The paired t-test indicated that, except for ESP, the surface zone does not have notably higher or lower

values than the subsurface zone. Linear regression analysis indicated that the surface zone can predict values for the subsurface zone, although this association is not strong. Similarities between the two zones are also evident from the comparison of spacial properties. This comparison is somewhat subjective, but semi-variogram properties and error variances were similar between the two zones, except for the parameter percent clay. Another important factor which should be considered is the large nugget variances shown by the semi-variogram models. The large nugget variances indicate that variability at close sampling intervals is an inherent property of the spoil.

Sampling of the two zones is conducted to assure that 2.44 meters of spoil material suitable to plant growth lies at the surface of the regraded spoil. From the linear regression analysis the surface zone can predict values for the subsurface zone, but this relationship is not strong. So values for the subsurface zone can not be predicted with a high degree of precision from values measured in the surface zone. Variation at close sample spacings is also evident from the large nugget variances found in the semi-variogram models. If sampling was conducted with the goal of being able to determine whether the top 2.44 meters of spoil is suitable for plant growth, then it would be necessary to sample both zones.

#### Block Kriging

Due to spatial similarities between the two sampling zones block kriging calculations were performed using the validated semi-variogram models for both sampling zones. Kriged estimates were made using 30.48

meter by 30.48 meter (100 foot by 100 foot) blocks for both zones of spoil. Three dimensional surface plots were made from the kriged estimates for the surface zone of spoil. Figure 10 shows the surface plot for percent clay. Appendix D shows surface plots for the remainder of the spoil physicochemical parameters using kriged values from the surface zone of spoil.

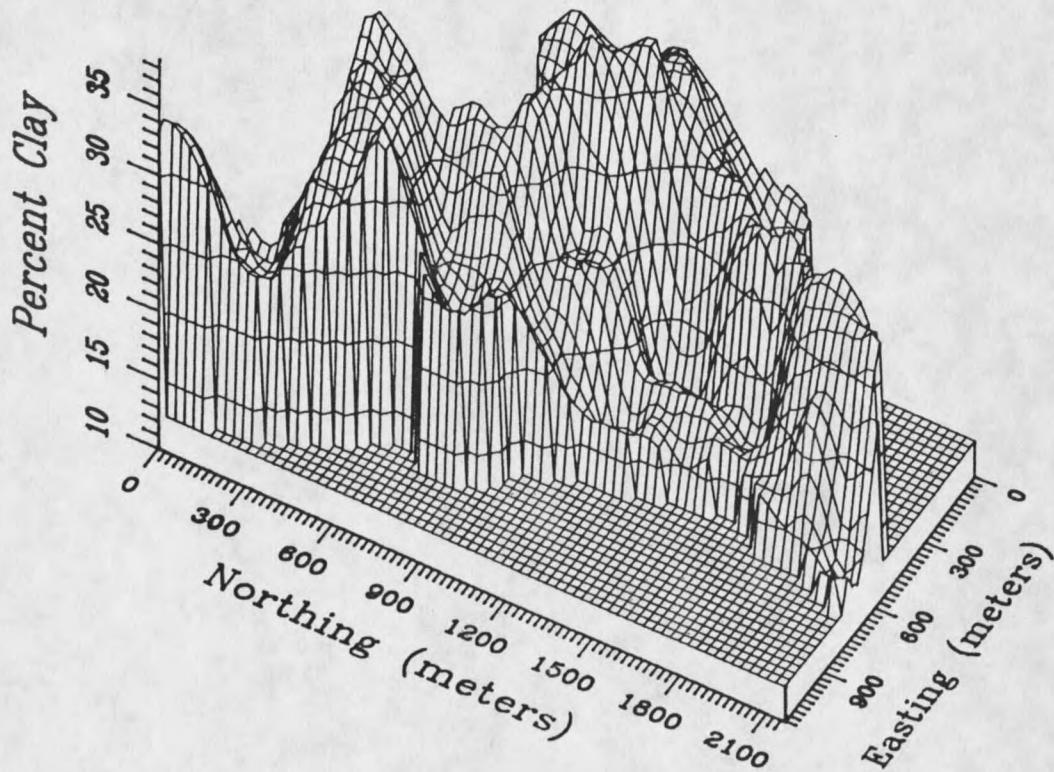


Figure 10. Surface plot for the spoil parameter percent clay. The plot is rotated  $30^\circ$  and the angle of tilt is  $50^\circ$ . Contour intervals are spaced every 5 data units.

The parameters ESP and SAR are lognormally distributed, so it should be questioned whether these data need to be transformed before the geostatistical analysis. Construction of the experimental semi-

variograms and the estimation procedures produced by geostatistics does not depend on what distribution the samples follow, so ordinary kriging can be applied to lognormally distributed data (Clark, 1979). The geostatistical technique lognormal kriging is available to deal with lognormally distributed data, but was not applied in this study. Experience has shown that, if these data are clearly lognormally distributed, lognormal kriging is a better estimator than ordinary kriging. Lognormal kriging is "better" because a lesser experimental estimation variance is produced.

One objective of kriging is to map values of spoil properties to determine if areas of spoil exceed suspect limits set by the DSL. From the surface plots and Table 7 it can be seen that the parameters EC and percent clay have values which exceed suspect levels. It should be noted that the range in kriged values is less than the range of values in the original data. From the original data the parameters SAR and ESP had values which exceeded suspect levels. From Table 7 maximum kriged values for SAR and ESP does not exceed suspect levels. This reduction in the range of kriged values as compared to the original sample values is due to the smoothing effect of kriging.

Table 7. Maximum and minimum block kriged values. Data taken from block kriged estimates made on both zones of spoil.

Parameter	pH	EC (mmhos/cm)	SAT. %	SAR	ESP	%CLAY	%SAND
Maximum	7.926	5.853	63.11	15.45	14.12	38.59	59.51
Minimum	6.685	1.309	32.96	1.719	1.764	12.59	23.64

The smoothing effect of block kriging removes short range variations in these data so regional patterns become clear. The large nugget variances in the semi-variograms indicate that considerable short range variation exists. From the surface plots it can be seen that block kriging has removed these short range data inconsistencies and longer range trends become apparent.

### Sampling Strategy

Geostatistical techniques can improve the efficiency of a regraded spoil sampling program. Methodology which makes use of semi-variogram properties, kriging procedures and estimation variance can be used to streamline spoil sampling procedures.

The semi-variogram is a model that represents the spatial variability of a spoil property. Within the range of influence, spoil properties are spatially dependent. If the spoils are sampled at a spacing less than the range of influence, kriging techniques can be used to map the spoil properties. In order for the spoils to be considered characterized, three important criteria should be considered: (i) sample spacing should allow mapping of spoil properties, (ii) sampling should be conducted so that we are assured suspect levels are not exceeded, (iii) if areas of spoil are found that exceed suspect criteria, the reliability associated with the estimate should be known. These criteria can be met using geostatistical techniques.

A two phase sampling strategy could be employed to characterize the spoil. Assuming that semi-variogram models are known from prior

sampling. The first phase would be to sample the spoil at a fixed sample spacing based on semi-variogram models and kriging techniques. Second phase sampling would be employed if problem areas of spoil were found. The purpose of the second phase sampling is to increase the sampling intensity to better characterize problem areas. Second phase sampling is based on the estimation variance associated with a particular sample spacing.

First phase sampling uses a fixed sample spacing based on semi-variogram properties and kriging techniques to characterize the spoil. Characterization of the spoils is achieved when estimates of spoil properties can be made across the entire sampling area using kriging techniques. Kriging is most effective when samples are spaced on intervals less than the range of influence.

Estimates can be made using kriging at sampling intervals greater than the range of influence, but it may not be possible to make estimates everywhere within the sampling area. To make kriged estimates at sample intervals greater than the range of influence, semi-variance must be known from prior sampling. Sampling at intervals less than the range of influence is desirable to quantify changes in semi-variance which may occur in future sampling areas. Spatially correlated estimates of semi-variance can only be made at sample intervals less than the range of influence.

Since the semi-variograms for the spoil properties are known, a first phase sample size determination can be made. The ranges of influence varied from 244 meters (800 feet) to 366 meters (1200 feet) for the semi-variogram models using both zones of spoil (Table 5). To



effectively map all the spoil parameters, samples must be taken on at least 244 meter centers.

For logistical reasons sampling on less than 244 meter centers is necessary to make kriged estimates everywhere within the sampling region. Kriging is a local estimation technique. To make kriged estimates, samples within a fixed radius of the block to be estimated are used when kriging. The fixed radius is usually equal to the range of influence. If there are not enough samples within this fixed radius then kriging estimates can not be made. For the semi-variogram models used in this study a minimum of three samples within this fixed radius are necessary before a kriged estimate can be made. If the sample spacing is set on 244 meter centers then it will not be possible to make kriging estimates everywhere within the sampling region. When making kriged estimates at the edge of the sampling region there will not be enough neighboring points to make estimates. For this logistical reason a more intense sample spacing is needed. Samples placed on 140 meter (450 foot) centers will allow blocks to be estimated anywhere within the sampling region and mapping of spoil properties by the methods of kriging.

This sample spacing determination was arrived at by trying to estimate a kriged value at the corner of a square matrix of points. The parameter saturation percentage was used because its narrow range of influence and large anisotropy ratio (Table 5) make estimation of corner values difficult. A single estimate was made for a 30.48 meter by 30.48 meter block centered on the corner of the sample point matrix. Corner values were estimated at a variety of square grid sample

spacings differing by intervals of 5 meters. At a spacing of 140 meters it was possible to estimate a corner value, but at a spacing of 145 meters it was not possible to estimate a value. Hence a value of 140 meters was chosen as the optimum spacing necessary for characterization of the spoils.

This exercise was carried out for the remaining six spoil properties. Results are shown in Table 8 for pH, EC, saturation percent, SAR, ESP and percent sand and clay. The spacing values represent the spacing above which corner values can not be estimated using the validated semi-variogram model for both zones of spoil. It should be noted that the 140 meter spacing is the smallest spacing at which corner estimates for all the parameters can be made. Parameters other than saturation percentage can be mapped at spacings greater than the 140 meter minimum. Anisotropy ratios and search radiuses are included in Table 8 to show how these parameters affect spacing. Parameters with isotropic semi-variogram models (pH, EC, SAR and ESP) have spacings equal to the maximum search radius. Parameters with anisotropic semi-variogram models have spacings substantially less than the maximum search radius.

Table 8. Corner value estimation results.

Parameter	pH	EC (mmhos/cm)	SAT. %	SAR	ESP	%Clay	%Sand
Spacing (meters)	270	365	140	305	245	270	185
Anisotropy Ratio			2.4			1.35	1.9
Search Radius (meters)	274	366	305	305	245	366	366

The purpose of phase two sampling is to better characterize potentially suspect areas of spoil. Since a variance is associated with each kriged estimate, confidence limits can be placed on each estimate. Areas with kriged values less than a suspect level of interest, but having confidence limits that exceed a suspect level, should be considered potentially hazardous. In this case additional sampling may reduce the kriging variance so the confidence limit placed upon the kriged value falls below the suspect level. To determine how much additional sampling is necessary curves can be developed using the estimation variance associated with different sample spacings.

Using geostatistics the reliability resulting from additional sampling can be determined before the sampling is done. This is because the equation for estimation variance (equation [8]) takes into account only the spacial locations of the sample values and not the values themselves. To determine the estimation variance for a block three factors are required; a semi-variogram model, the configuration of the sample values, and the block size. To determine estimation variances matrixes of sample locations were generated at a variety of square grid spacings. To be consistent a constant region size and block size were used when generating variances. A block size of 30.48 meters (100 feet) per side and a region of 1219 meters (4000 feet) per side were used. The region size is similar to that of the mining area. Sample spacings on 7.6, 15, 31, 61, 122, 152, 244, and 305 meter centers were used when determining variances. For each of these spacings and for each of the spoil parameters, estimation variances were determined by kriging procedures. The validated semi-variogram

model for the combined sampling zones was used. Figure 11 shows the orientation of the sample matrix within the 1219 meter by 1219 meter region for percent clay. A sampling interval of 61 meters (200 feet) is used. Figure 12 is a surface plot of estimation variances interpolated from this sample array. It can be seen that the variance remains constant across the central portion of the region and increases towards the margins, reaching a maximum in the corners of the region. To get an estimate of the variance associated with a particular sample spacing an average of all the estimation variances within the region was used. Figure 13 shows how estimation variance changes with sample spacing for percent clay. Appendix E contains similar curves for the remainder of the spoil parameters. Within the range of sample spacings used, estimation variance and sample spacing are linearly related. Burgess, Webster and McBratney (1981) calculated estimation variance as a function of sampling interval in a similar manner for soil thickness.

Plots of estimation variance as a function of sampling interval can aid in determining appropriate sampling strategy. Optimum sampling strategy corresponds to the sampling interval for which the average estimation variance equals the maximum permissible for characterization of spoil. Using this type of approach the amount of error associated with a given sampling effort can be used to determine sample spacing adequacy.

Confidence limits can be used to assess the reliability of an estimate and can be used to guide further sampling to achieve a specified reliability. Sample spacing adequacy can be defined as that intensity which will assure us that the 95% upper confidence limit for

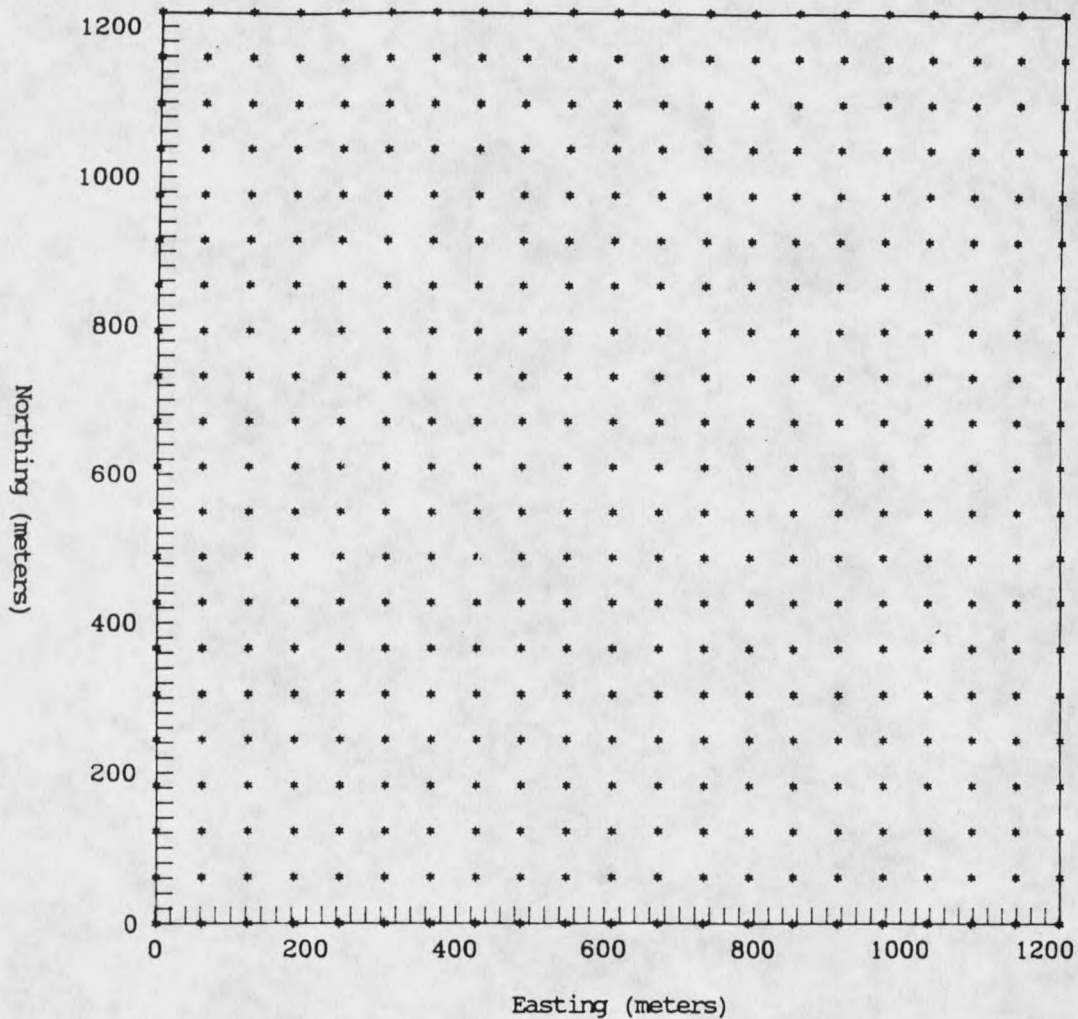


Figure 11. Sample orientation within the estimation region for percent clay. Spacing locations are shown by asterisks, 61 meter (200 foot) centers are shown.

a spoil property will not exceed the suspect limit. This determination of sample spacing adequacy applies to block estimates that have values below the suspect level, but have upper confidence limits that exceed the suspect level. Using the known suspect level for a spoil parameter, and the estimation variance associated with each sample spacing, a block estimate can be determined which has an upper

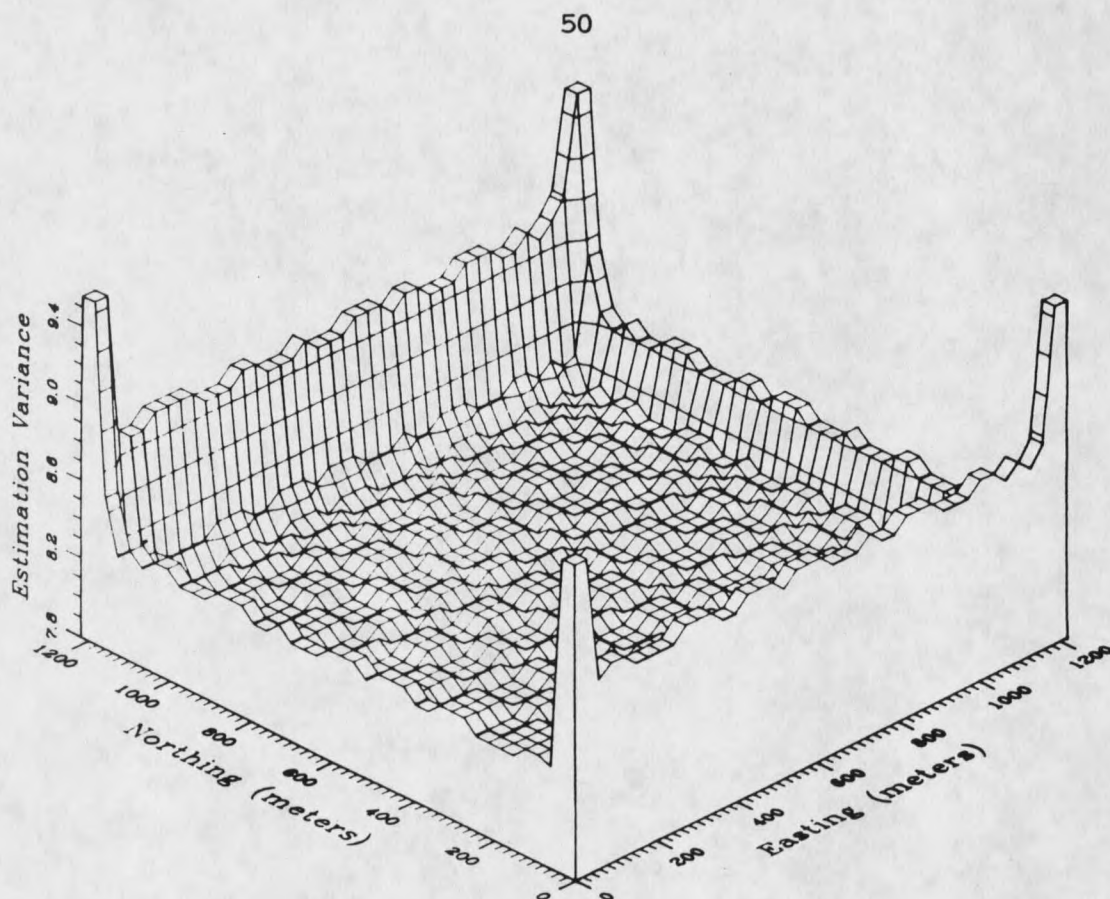


Figure 12. Surface plot of estimation variances estimated across a 1219 meter by 1219 meter region using a sample spacing of 61 meters for the parameter clay content.

confidence limit that equals the suspect level for a given sample spacing. Figure 14 shows how confidence limits determined from the block estimation variances can be used to assess sample spacing adequacy for percent clay. Given a sampling interval of 150 meters (492 feet), it can be seen from Figure 14 that block values greater than or equal to 27.7 percent clay will have upper confidence limits greater than the suspect level of 35 percent clay. Alternatively, using this type of analysis, additional sampling requirements can be determined to bring the upper confidence limit below the suspect level,

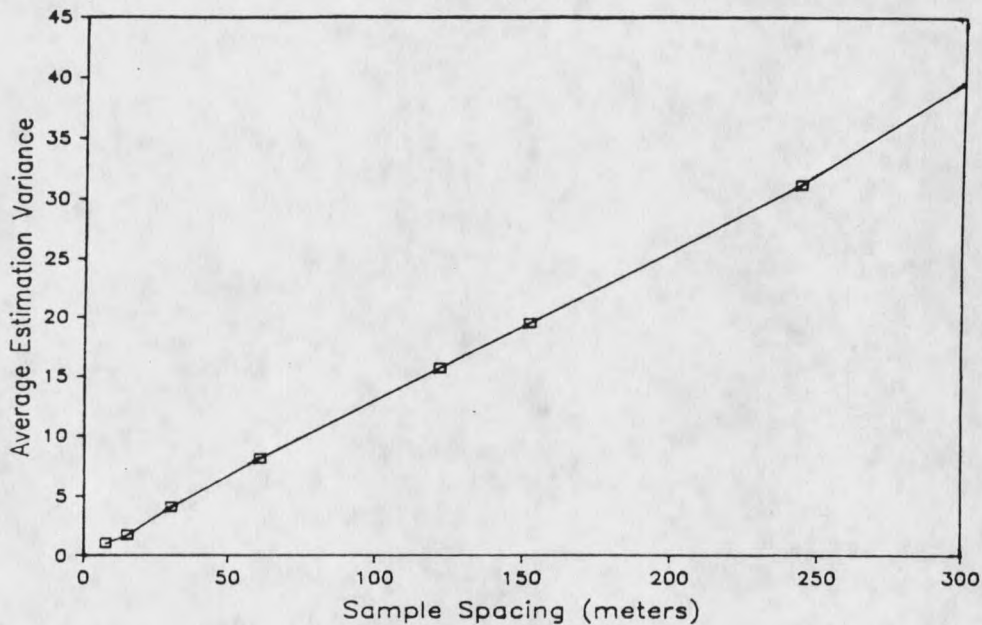


Figure 13. Curve showing the relationship between average estimation variance and sample spacing for percent clay.

if values are actually below the suspect level. For example, if an area had been sampled at a grid spacing of 150 meters and a block value was determined to be 29.8%, meaning a value with an upper confidence limit above 35, the area would have to be sampled at a grid spacing of approximately 75 meters to assure the confidence limit falls below 35%. This assumes that additional sampling will not increase the value of the block estimate.

Complications could arise using this additional sampling approach. If additional sampling has the effect of increasing block values then the sampling goal of bringing the upper confidence limit below the suspect level may not be achieved. Expanding on the above hypothetical example, the area sampled at 150 meters was resampled at 75 meters in an attempt to improve the precision of the kriging estimate. Using

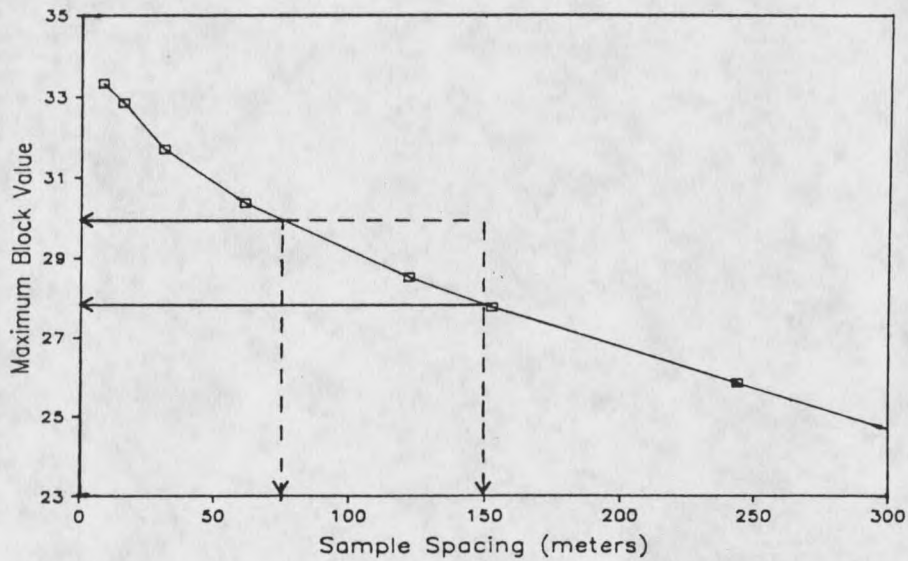


Figure 14. Maximum block value curve for percent clay. The maximum block value for a sample spacing of 150 meters is 27.7%. Dotted lines show how additional sampling can be determined.

these new samples, block values were again estimated and were found to exceed the maximum block value of 29.8% for a sample spacing of 75 meters (Figure 14). If this scenario occurs more sampling would be necessary unless the upper confidence limits for the new estimates do not exceed 35%. In a similar manner additional sampling may lower the estimated block values. In this case the sampling goal should be achieved.

The area contained in regions of the study site where average estimation variance and maximum block value curves can be applied are summarized by parameter in Table 9. Only the parameters EC and percent clay have values allowing the application of these curves. Values for area are in hectares and for EC a suspect limit of 4 mmhos/cm is assumed. Figure 15 shows areas where average estimation variance



curves for percent clay can be applied. Figure 16 is a surface plot of kriged estimates with values that potentially exceed the suspect limit of 35 for percent clay. The maximum block value curve for percent clay could be applied to these areas. Figure 16 shows one-tailed upper confidence limits for regions with kriged estimates below the suspect limit of 35 percent clay, but having upper confidence limits greater than 35 percent clay. The 95% one-tailed upper confidence limit was calculated using the following equation,  $UCL = KG + 1.645 \cdot \sigma_H$ , where UCL is the upper confidence limit, KG is the kriged grade, 1.645 is the one tailed t-value for  $\alpha=0.050$  and an infinite sample size, and  $\sigma_H$  is the standard deviation associated with a block estimate.

Table 9. Area (hectares) at the study site meeting phase II sampling criteria.

Parameter	pH	EC (mmhos/cm)	SAT. %	SAR	ESP	%CLAY	%SAND
Average Estimation Variance	0.00	41.7	0.00	0.00	0.00	7.2	0.00
Maximum Block Value	0.00	30.2	0.00	0.00	0.00	30.4	0.00

To better understand the concept of phase II sampling criteria Figures 15 and 16 should be compared to the surface plot for percent clay (Figure 10). The regions displayed in Figure 15 correspond with the peaks having values greater than 35 in Figure 10. Figure 16 shows areas which have the potential to exceed suspect criteria. These areas surround the peaks shown in Figure 15. Peaks with values close to 35 percent in Figure 10 are also present in Figure 16. An example of this

can be seen with the peak adjacent to the z-axis in Figure 10. Note that Figures 10, 15 and 16 have the same tilt and rotation.

To determine the maximum allowable block value for a given sample spacing the following equation was used,

$$\text{Maximum Block Value} = SL - (1.645 \cdot \sigma_H) \quad [9]$$

where SL is the suspect limit, 1.645 is the one-tailed t-value for  $\alpha=0.050$  and an infinite sample size, and  $\sigma_H$  is the standard deviation associated with a block estimate. Maximum block values were determined for each of the sample spacings used to calculate estimation variance curves. Additional maximum block value curves are shown in Appendix F.

A variety of assumptions have been made which should be considered when implementing this sampling strategy. The average estimation variance is assumed to be an accurate predictor of the variance associated with a particular sample spacing. From Figure 12 it is evident that estimation variance will change depending on where the estimate is made within the sampling region. Maximum block value versus sample spacing and average estimation variance versus sample spacing curves are based on average variance values. Variance at the edge of the sampling region will be higher for a given sample spacing than is indicated by the average estimation variance versus sample spacing curve. This implies that the edge of the sampling region will have to be sampled more intensively to achieve the variance value indicated by the average estimation variance curve. In a similar fashion underestimates of sample spacing requirements will be made at the edge of the sampling region for maximum block value curves.

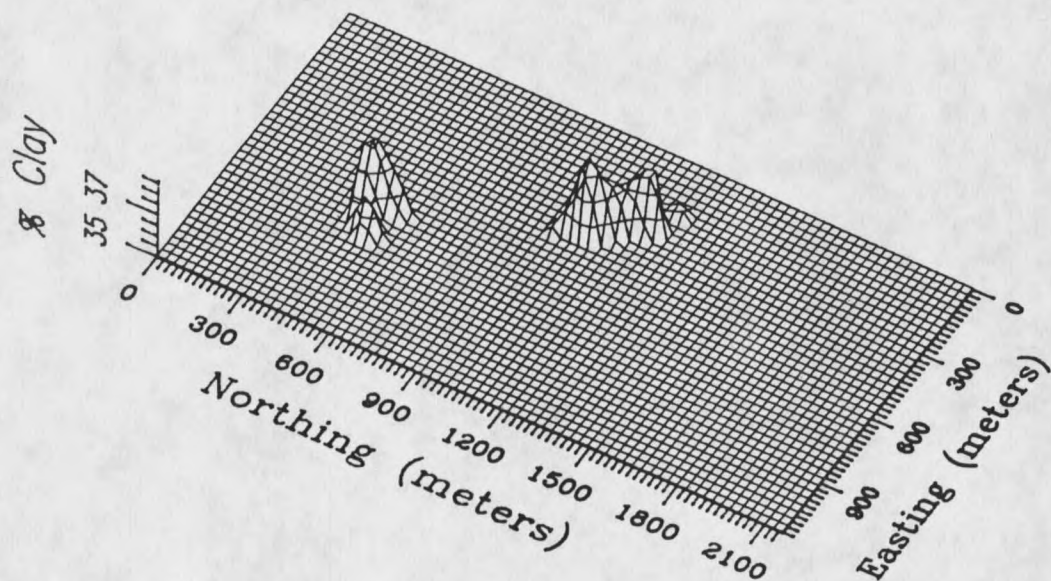


Figure 15. Surface plot of areas where average estimation variance curves can be applied for percent clay. The plot is rotated  $30^\circ$ , the angle of tilt is  $50^\circ$  and contour intervals are spaced every 1.0 data units.

The configuration of the sample points will also affect the utility of the curves. The average estimation variance computations assume a square grid sample spacing. More efficient sampling schemes, such as an equilateral triangular configuration, will lower the variance estimates.

If a geostatistical approach to sampling spoils is undertaken a number of important considerations should be made. Semi-variogram models need periodic verification as new areas are mined and additional spoil samples are taken. Additional samples are likely to change the semi-variogram models. Kriging procedures and estimation variance are dependent on the semi-variogram model of interest. A change in the

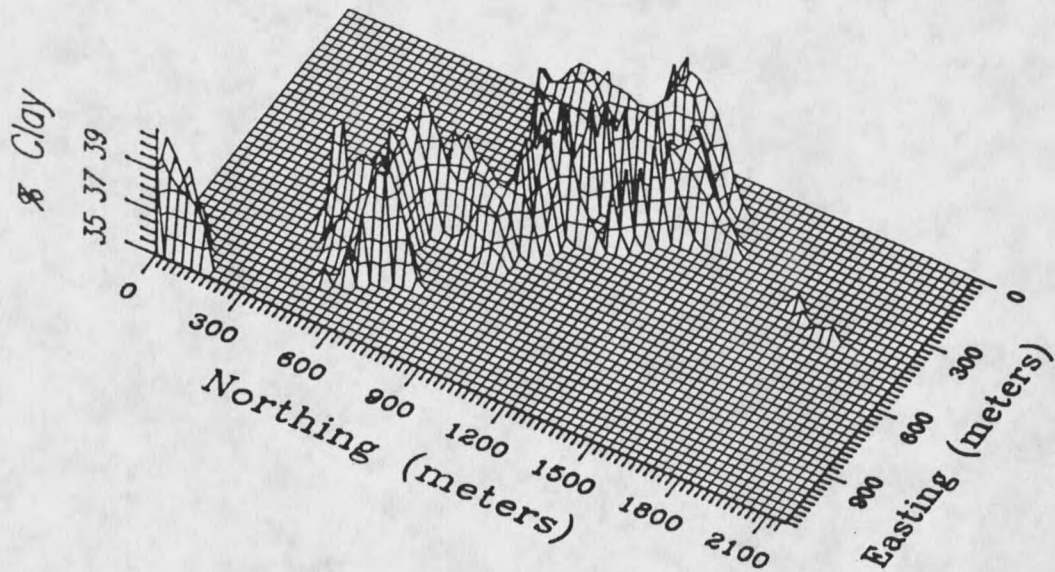


Figure 16. Surface plot of areas where maximum block value curves are applicable for percent clay. The plot is rotated  $30^\circ$ , the angle of tilt is  $50^\circ$  and contour intervals are spaced every 1.0 data units.

semi-variogram model would require recalculation of average estimation variance and maximum block value curves. These methods will require the continued use of geostatistics and a significant investment in computer time and technical skill. Also, this type of analysis is site specific, semi-variogram models will vary between mining areas, implying different sampling strategies at each mining site. As a final consideration accurate semi-variogram models require a large sample size. These methods can not be implemented until an adequate sample population has been collected.

A variety of sampling strategies have been demonstrated. Using semi-variogram properties and kriging procedures it was shown that a

sampling interval of 140 meters could be used to characterize the spoil material. Characterization of the spoil implies that maps of spoil properties using kriging techniques can be made. At this mine site there are areas of spoil with levels of physicochemical properties which exceed or potentially exceed suspect criteria. In this instance it is desirable to better characterize these areas by increasing sampling intensity. To assist in deciding what additional sampling is necessary estimation variance and maximum block value curves were developed. Maximum block value curves are used to determine additional sampling when a block value is less than the suspect level, but the confidence limit for the block exceeds the suspect level. Estimation variance curves can be useful when determining the level of reliability needed to characterize areas that exceed suspect levels. Using these techniques a more objective regraded spoil sampling program can be implemented.

### SUMMARY AND CONCLUSIONS

A geostatistical analysis using regraded spoil sampling data from the Absaloka Mine in southwest Montana was initiated to assess the adequacy of regraded spoil sampling requirements set by the Montana Department of State Lands (DSL). The objectives of this study were to determine if significant differences in physicochemical properties exist between the two spoil sampling zones, to quantify the spatial aspects of spoil physicochemical properties using the semi-variogram, and demonstrate how geostatistical techniques can be used to develop sampling strategies for characterization of the regraded spoil. The company has been sampling this spoil material for years and adequate information is available to perform a geostatistical analysis.

Classical and spatial statistics were used to determine if the two spoil sampling zones were sufficiently similar to make sampling of the surface zone adequate for characterization of the spoil. The DSL requires sampling of these two zones because 2.44 meters (8 feet) of suitable plant growth material is necessary for reclamation success. Linear regression analysis comparing the two zones showed that the surface zone could predict values for the subsurface zone, but correlations were not strong. Large nugget variances shown by the semi-variogram models indicated that variability at close sample spacings is an inherent property of the spoil. Since the surface zone can't predict values for the subsurface zone with a high degree of

precision at this site then sampling both zones is necessary for characterization of the surface 2.44 meters (8 feet) of spoil.

Semi-variogram properties and kriging procedures were used to designate an appropriate sample spacing for characterization of the spoil material. Ranges of influence for the spoil properties varied from 244 meters (800 feet) to 366 meters (1200 feet), implying that samples taken on less than 244 meter centers will be spatially dependent for all the spoil parameters. Since areas of spoil exceed suspect levels set by the DSL, characterization of the spoil should imply being able to map the spoil material with reliability using kriging techniques. A sampling interval of 140 meters (450 feet) will permit mapping of all the spoil properties using kriging procedures. Block kriging was used to make estimates for 30.48 meter by 30.48 meter (100 foot by 100 foot) blocks and produce maps of the spoil properties.

To better characterize areas of spoil that exceed suspect levels or have 95% upper confidence limits that exceed suspect levels, techniques were described to help guide additional sampling. These techniques rely on a known semi-variogram model and the average estimation variance associated with a particular sample spacing. To guide additional sampling for areas of spoil with kriging values which exceed suspect levels, curves were developed which show how average estimation variance changes with sample spacing. In this case adequate sampling intensity will correspond with the maximum allowable estimation variance deemed necessary for characterization purposes. The strategy is optimal in the sense that the sampling effort is the least possible to achieve the precision desired.

Situations may arise where areas of spoil have 95% confidence limits which exceed suspect levels, but have kriging values below the suspect criteria. For this special case curves were developed to guide additional sampling. The purpose of the additional sampling is to decrease the estimation variance so we are 95% confident that the confidence limit does not exceed the suspect level.

Since semi-variogram properties are likely to be unique between mining sites this type of analysis will be site specific. Also, as a mining area expands and more samples are taken semi-variogram properties may change. This implies that semi-variogram models will have to be reevaluated periodically. The use of these techniques will require the continued use of geostatistics, the analysis is not a one time estimate. The use of geostatistics is becoming common place in the mining industry for ore reserve estimation, so many companies have the personnel and computer facilities to perform this type of analysis. Presently, the Absaloka Mine samples the spoil on a sampling interval of about 69 meters (225 feet). If the spoil could be sampled on 140 meter (450 foot) centers substantial cost savings could be realized. Since areas with spoil properties that exceed suspect limits are localized and there is usually only one parameter that exceeds suspect criteria, additional sampling need only be employed occasionally, analyzing only a few parameters.



**LITERATURE CITED**

## LITERATURE CITED

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**APPENDICES**

APPENDIX A

HISTOGRAMS

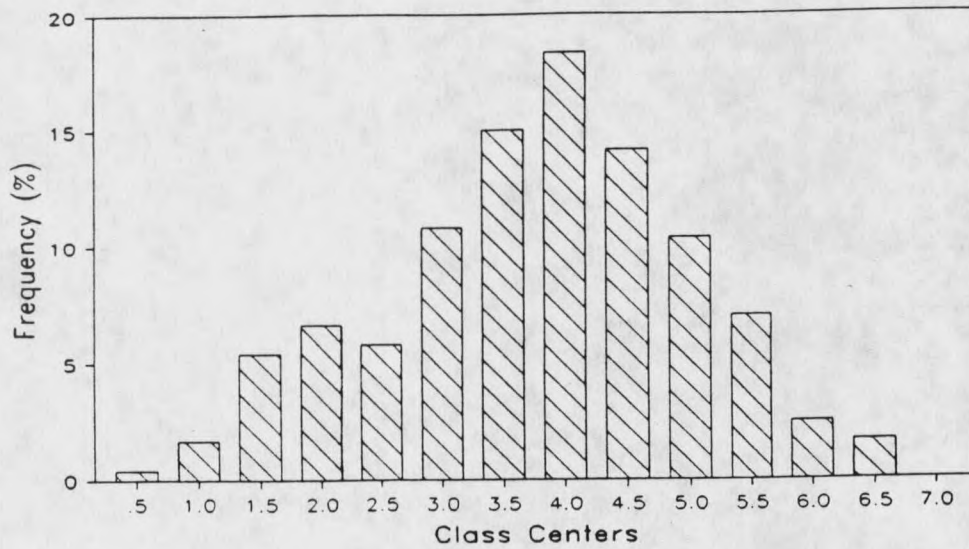


Figure 17. Histogram for EC using data from the surface zone.

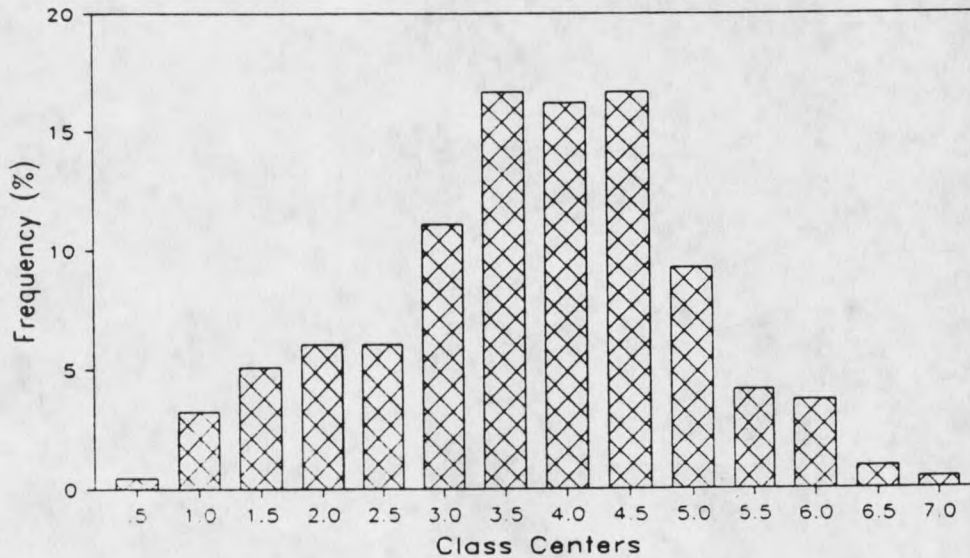


Figure 18. Histogram for EC using data from the subsurface zone.

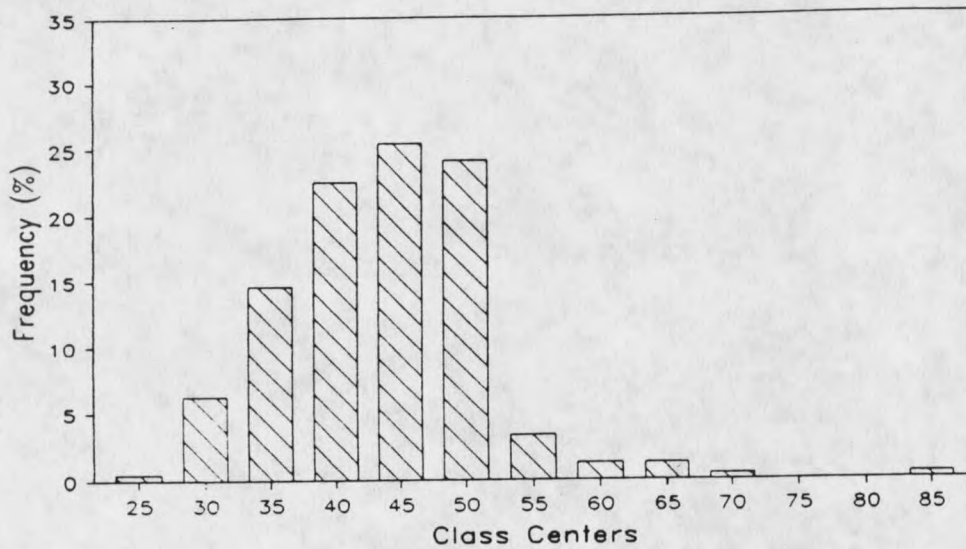


Figure 19. Histogram for saturation percentage using data from the surface zone.

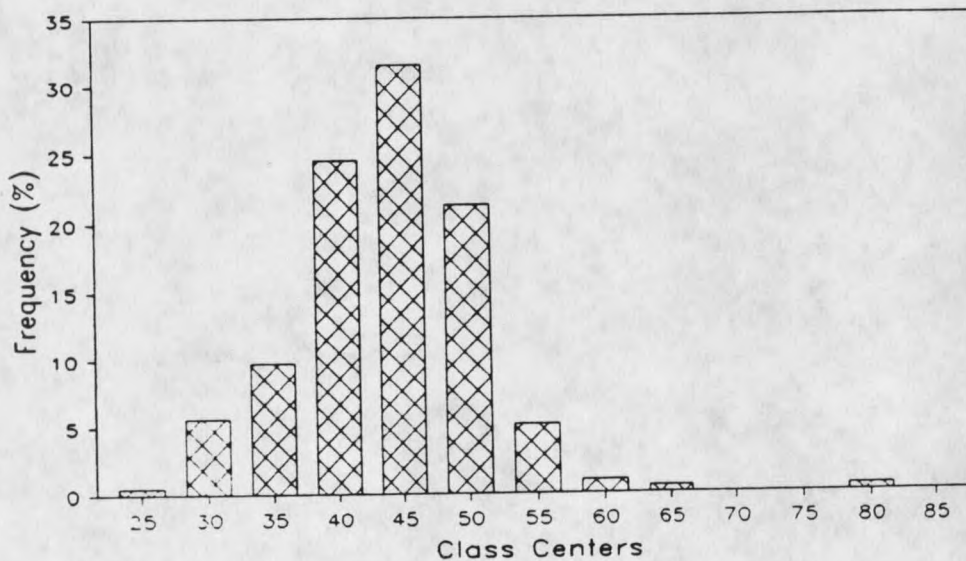


Figure 20. Histogram for saturation percentage using data from the subsurface zone.

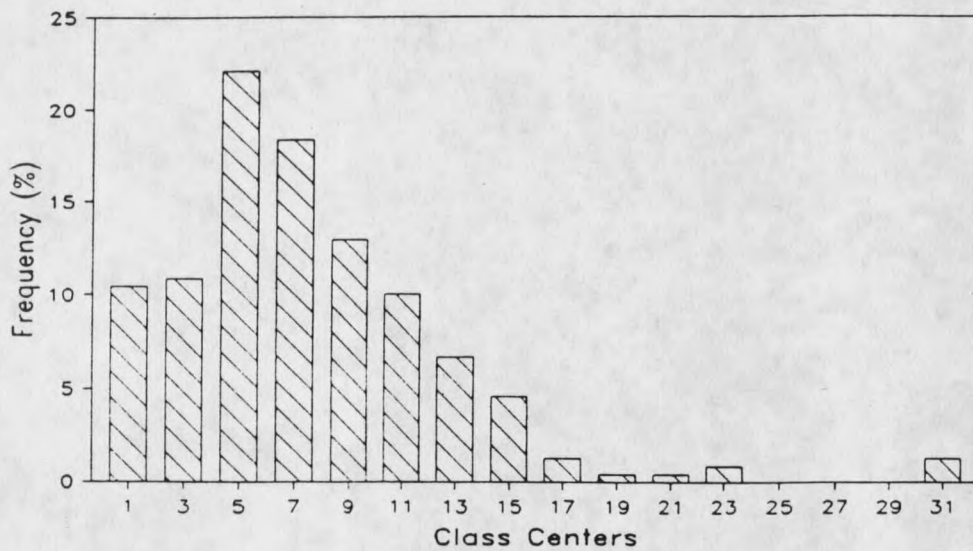


Figure 21. Histogram for SAR using data from the surface zone.

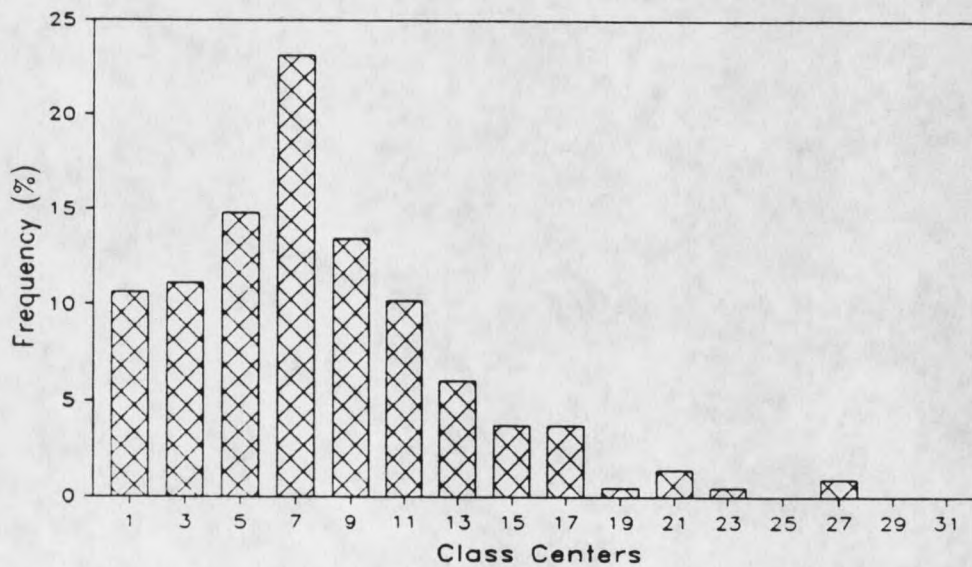


Figure 22. Histogram for SAR using data from the subsurface zone.



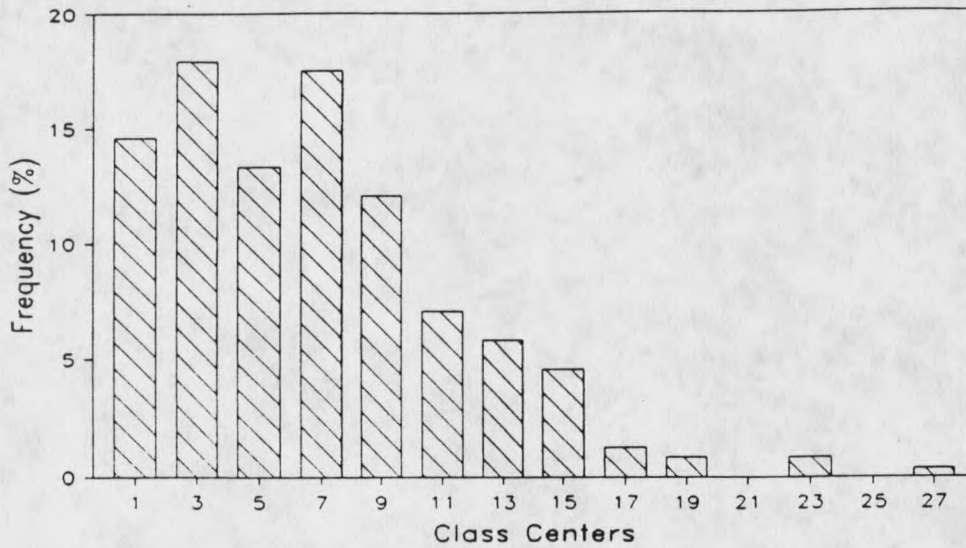


Figure 23. Histogram for ESP using data from the surface zone.

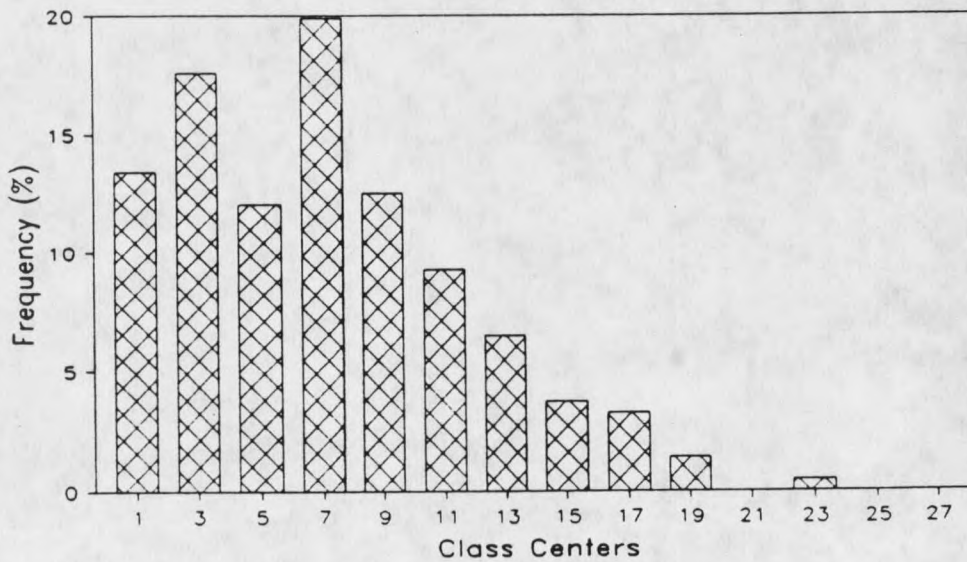


Figure 24. Histogram for ESP using data from the subsurface zone.

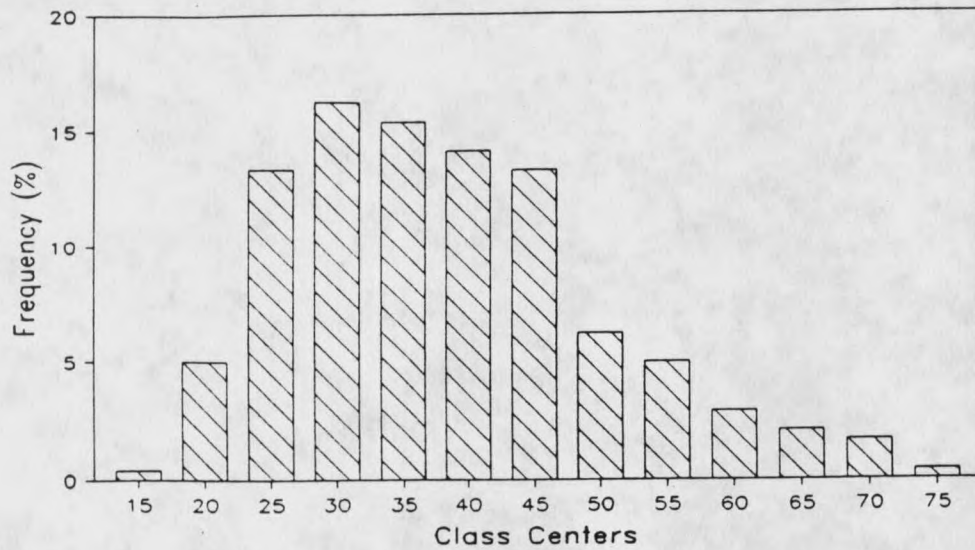


Figure 25. Histogram for percent sand using data from the surface zone.

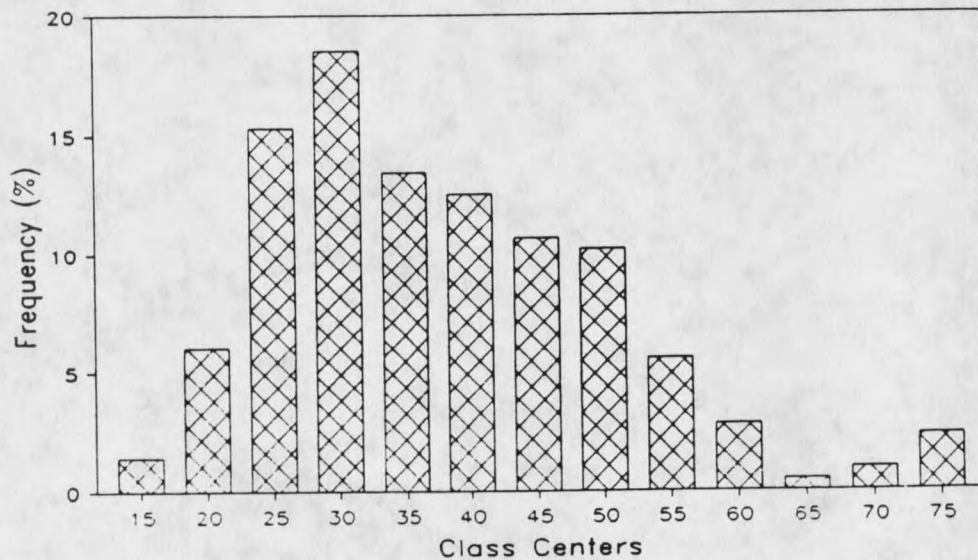


Figure 26. Histogram for percent sand using data from the subsurface zone.

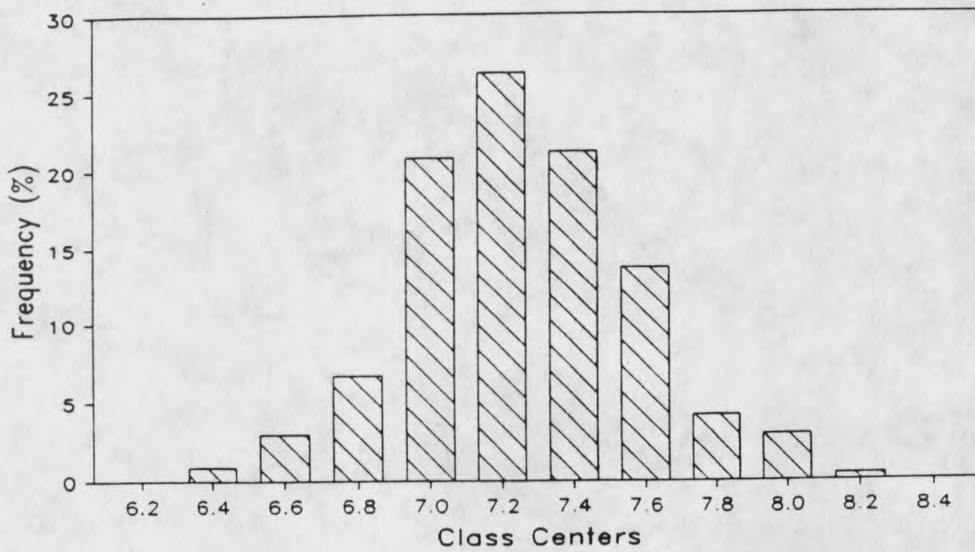


Figure 27. Histogram for pH using data from the surface zone.

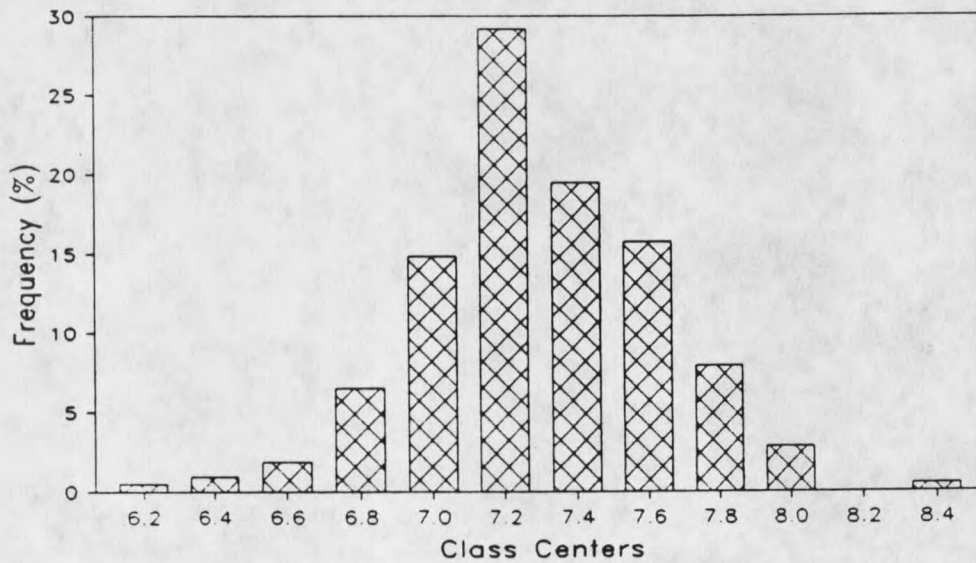


Figure 28. Histogram for pH using data from the subsurface zone.

APPENDIX B  
DIRECTIONAL SEMI-VARIOGRAMS



























































