Fluid dynamic study of confluent and stratified flow in vascular replicas

by Dale Allen Valach

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Chemical Engineering
Montana State University

Abstract:
In view of the extremely limited knowledge of the magnitude of the effects of the merging of two vessels (venous flow) on pressure drops in the system, a vascular replica of such a system was created and the junction's effect on pressure drop was studied.

The main objective of the study was to determine if a finite increment of pressure drop was caused by the existence of confluent flow in the system. If it was found to exist, some mathematical model to describe the magnitude of the extra pressure drop was to be found.

Using both Newtonian (distilled water and 50 wt% glycerol-water solution) and non-Newtonian (red blood cell suspensions in plasma) fluids, differential pressures could be measured across the junction and the effects of flow rates and viscosities on the pressure drop could be determined.

Based on experimental results presented, it was concluded that there exists a small but finite additional pressure drop in the system which could only be attributed to the existence of the confluent flow. This pressure drop was found to be proportional to the empirical function which used the geometric average of the upstream Reynolds numbers.

The secondary objective of the study was to test the applicability of the Yu-Sparrow technique for predicting the pressure drop for stratified laminar flow in a system which had dimensions similar to those of blood vessels.

Based on the results of tests using a 50 wt% glycerol solution and water and tests using blood and plasma in stratified laminar flow, it was concluded that the Yu-Sparrow model will satisfactorily predict the pressure drop even in biological flow applications.

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ABSTRACT

In view of the extremely limited knowledge of the magnitude of the effects of the merging of two vessels (venous flow) on pressure drops in the system, a vascular replica of such a system was created and the junction's effect on pressure drop was studied.

The main objective of the study was to determine if a finite increment of pressure drop was caused by the existence of confluent flow in the system. If it was found to exist, some mathematical model to describe the magnitude of the extra pressure drop was to be found.

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INTRODUCTION

The desire to understand the intricacies associated with the flow of blood in biological systems has long been one of the motivating forces connected with biomedical research. The acquisition of this information is necessary in that the usual transfer of essential substances which normally occur in the various organs may be substantially altered if the flow characteristics change thus having a pronounced effect on the health of the entire organism. Therefore, a knowledge of the fluid dynamic effects on different systems would be extremely beneficial in the treatment of various maladies.

Blood and Its Characteristics

Human blood is a suspension of several formed components in a suspending fluid called plasma. The suspended particles consist of red blood cells, white blood cells and platelets. The red blood cells, also called erythrocytes, constitute the largest portion of the cellular volume occupying approximately 97% of said volume. The red blood cell (RBC) is basically a thin membrane filled with a fluid which when in its undeformed state is in the shape of a biconcave disc. This shape has the approximate dimensions of eight microns for the diameter and two microns for the thickness at the widest point. The construction of the average cell is such that it is very flexible and therefore readily deformable during flow.

The liquid portion of blood, plasma, occupies roughly 55 per cent of the total volume in a typical sample of blood from a healthy human being. The composition of plasma is primarily water, 90 per cent, with the remaining portion being carbohydrates, proteins, lipids, and inorganic materials in solution.

The final portion of the blood to be discussed is that of the cellular volume filled by the white blood cells, or leukocytes, and the platelets. The platelets are the smallest of the cells in the suspension while the white blood cells are the largest. However, since the effects of leukocytes on flow under normal physiological conditions has been shown to be negligible, this portion along with the platelets were removed from the blood samples used in this study. The removal of the platelets significantly lessens the micro emboli formation in the blood.

The hematocrit is a measure of the amount of red blood cells in a given sample of blood. More specifically, it is the volume per cent of red blood cells in the whole blood. This value is obtained by centrifuging a capillary tube filled with blood and then comparing the relative length of the column of packed red blood cells to the overall length of the sample in the tube. From this it can be seen that when different hematocrit specimens are desired, they may be obtained by combining different amounts of plasma and red cells.

Flow System Choice

In attempting to choose what system is to be used in conducting any blood flow investigation, one has to weigh the advantages of in vivo and in vitro studies. In vivo experiments have the advantages of the added certainty of knowing that the vessel geometry and other characteristics are representative of biological situations. On the other hand, a lack of ability to control given parameters and the ever present possibility of other controls such as hormonal or neurological responses introducing interference into the results represent some of the major disadvantages. In vitro studies also have their pro's and con's. The added ability to regulate specific parameters has to be weighed against the forced neglection of terms such as those introduced by the permeability and elasticity of vessel walls, pulsatile flow, and inertial effects resulting from the curvature and taper of the actual vessel.

A technique which makes use of hollow vascular replicas that are cast in a polyester resin enables a researcher to exploit the best characteristics of both the in vivo and in vitro methods of investigation (see Cokelet and Meiselman (1975)).

> The advantages of this type of flow system over the comparable in vivo preparation are its stability, transparency, the constancy of the geometry of the network of vessels under varying flow conditions, the impermeability of the walls of the flow channels, and the realistic, real-size representation of the vascular system. Not only can experimental blood flow data obtained from such a replica be compared with theoretical

models of vascular systems, but contributions to flow resistance from such in vivo factors as vessel distensibility and vessel wall permeability can be assessed by the comparison of data obtained with in vivo and replica systems.

The utilization of vascular replicas will quite probably prove to be very beneficial in obtaining information as to what parameters are important in describing the correlations between the microscopic behavior of blood flowing through isolated sections of a system and the overall macroscopic rheological properties.

Statement of Problem

The primary objective of this investigation was to test the feasibility of using the vascular technique in studying the pressure drop which is associated with the confluence of two vessels in venous flow. This was performed in the hopes of adding some quantitative evidence to either help prove or disprove the claim that the existence of a junction of two streams should produce an additional pressure drop which is not otherwise accounted for when assuming Poiseuillean flow in the vessel system.

Until this time there had been speculation about the claim both pro and con. On one side, the results of Vawter, Fung, and Zweifach indicated that although in the immediate vicinity of the junction there would be a pressure fluctuation, as the distance along the channel increased this pressure would be recovered. This meant that there was no overall effect on the pressure drop of the system due to the presence of the vessel junction (D. Vawter, Y.C. Fung, and B.W. Zweifach, 1974). In opposition to these results were the findings of H. S. Lew, who through his mathematical modeling found evidence which indicated a definite pressure drop which was attributable to the entry characteristics of confluent flow (Hyok Sang Lew, 1971).

The attempt to collect data on the magnitude of the junction effect on pressure was to help settle the controversy. In addition, if the pressure drop was found to exist, some mathematical model was to be found which could describe the observed values and predict the incremental pressure drop which would be found in other systems of a similar nature.

The secondary objective of this study was to obtain quantitative measurements of the pressure drop which existed when two different fluids were simultaneously flowing through a duct of smaller diameter in stratified laminar flow. This phenomenon had been noted in previous experiments (Cokelet and Meiselman), but little information of a quantitative nature had yet been amassed. Therefore, it was the intent of this investigation to collect data about the pressure drop and compare the results to a model for stratified laminar flow which was originally developed for ducts of much larger cross section (Yu and Sparrow (1967)). This comparison was to resolve whether or not the Yu-Sparrow model would remain valid when considering ducts with dimensions similar to those of blood vessels.

EXPERIMENTAL APPARATUS AND PROCEDURE

This experiment was conducted using a vascular replica through which several fluids were pumped in order to obtain the pressure versus flow relationships that were desired. These fluids ranged from Newtonian fluids such as water and a glycerol solution to a fluid with definite non-Newtonian characteristics, blood. The stratified flow data was collected using the same model as was used in the confluent flow study. The details of each of these segments, along with discussion about the fabrication techniques used, will be elaborated upon separately.

Replica Fabrication

The fabrication techniques used to produce the vascular model for this investigation are presented in detail by Cokelet and Meiselman (1975) in another source. Although modified slightly for this study, a summary of the procedure is as follows:

The selected tissue specimen, the forepaw of a mongrel dog, was first perfused with isotonic saline. This removed the blood from the vascular system. Next, silicone oil was injected into the vessels to provide a favorable interface for the gallium. Finally, gallium was perfused through the system and solidified in the exact geometry of the original vascular network. This last step is made possible by the fact that the melting point of gallium is 29.8°C. Therefore, it may be injected in its molten state without destroying much tissue, yet it will still solidify at room temperature.

After the gallium had solidified, the entire tissue specimen was subjected to enzymatic treatments which removed the tissue from around the metal network. The solutions to which the tissue was exposed were a pancreatin solution to dissolve the tissue, and a urea solution to act as a denaturant. In the case of the dog's paw, it took approximately five to six months to complete the process. At this point the desired sections of the metal network were removed and cast into a polyester resin (EPX-145-11 Clear Casting Resin, Delvies Plastics, Inc., with methyl ethyl ketone peroxide catalyst, Sarafan Corp.).

After allowing sufficient time for the resin to set up, holes were drilled into the casting such that the ends of the major branches of the metal network had been tapped. The gallium was flushed out of the cast using warm water first, following by dilute hydrochloric acid to remove the metal from the dead end branches. The result was a three-dimensional replica of the original vessel which was transparent and impermeable. This procedure has such a high degree of reproducibility that electron micrographs of the models have revealed the vessel wall shapes created by the actual endothelial cells.

Calibration Techniques

Due to the fact that the syringes (Hamilton Company) used in the syringe pumps (Harvard Apparatus) were different from those intended for use in the pumps, it was necessary to calibrate the volumetric flow rates for each syringe size and pump setting. This was accomplished by measuring the displacement of water during a given time interval at several pump settings.

The calibration of the pressure transducer was performed by using a Dwyer Instruments Hook Gage. This instrument had a much greater degree of precision than a conventional mercury manometer in that it could be read to approximately ±0.01 mm Hg. The hook gage reading was recorded along with the corresponding digital voltmeter reading from the transducer. The calibration results were then placed in a statistical least-squares computer program which correlated the information and determined a straight line equation for translating transducer readings to actual pressures.

Blood Preparations

The human blood samples were received in ACD anti-coagulant after being drawn by normal blood bank procedures. The specific blood type of the sample, however, was deemed unimportant and therefore no preference was given to any certain type. The whole blood was first centrifuged (using a Sorvall RC2-B automatic refrigerated centrifuge) at 3000 rpm for 15 minutes. The plasma was decanted off and saved leaving the buffy coat (platelets and leukocytes) and the packed red blood cells. The buffy coat was then poured from the top of the remaining sample and discarded. This left the RBC's and the plasma which were then mixed in the appropriate ratio to give the desired hematocrit.

Hematocrit Measurement

The hematocrits were measured using the micro-hematocrit technique. By using standard techniques for centrifuging samples in micro-hematocrit tubes (about 10,000g for ten minutes), the measuring of the volume of red blood cells per volume of cells and plasma produced the desired information.

Viscosity Measurements

The viscometric data was collected by utilizing a Wells-Brookfield Micro Viscometer which was of the cone and plate type. The calibration was made using the S-3 viscosity standard at the recorded temperature at shear rates of 75, 150, 300, 750, and 1500 inverse seconds.

Equipment and Apparatus Assembly

The data for the research was collected from the apparatus that is shown in schematic form in Figure 1. The system is comprised of syringe pumps, two pressure dampening and mixing reservoirs, a hollowed out replica of a section of actual vascular network cast in polyester resin, teflon tubing, and a pressure transducer and amplifier. Each item of equipment will be elaborated on separately below.

The syringe pumps were of the infusion/withdrawal type from Harvard Apparatus Company, Model 902. These pumps were capable of producing steady volumetric flow rates ranging from 0.00393 to 153 cc/min. when equipped with gas tight syringes between 5 and 30 ml in size inclusive.

The combination pressure dampening and mixing reservoir was incorporated into the system for two reasons. First, the syringe pumps did not produce a perfect steady flow but rather one with a slight sinusoidal fluctuation due to an aberation in the screw mechanism that advanced the syringe. Therefore, this reservoir was placed on line to assist in reducing the pressure variations to an acceptable level. A mixing device was incorporated into the design of the reservoir to prevent the settling of the red blood cells when blood was the fluid being studied. This helped to insure a constant hematocrit and thereby hold the viscosity at a steady value. The reservoirs were fabricated out of acrylic plastic and 13 gauge hypodermic needles.

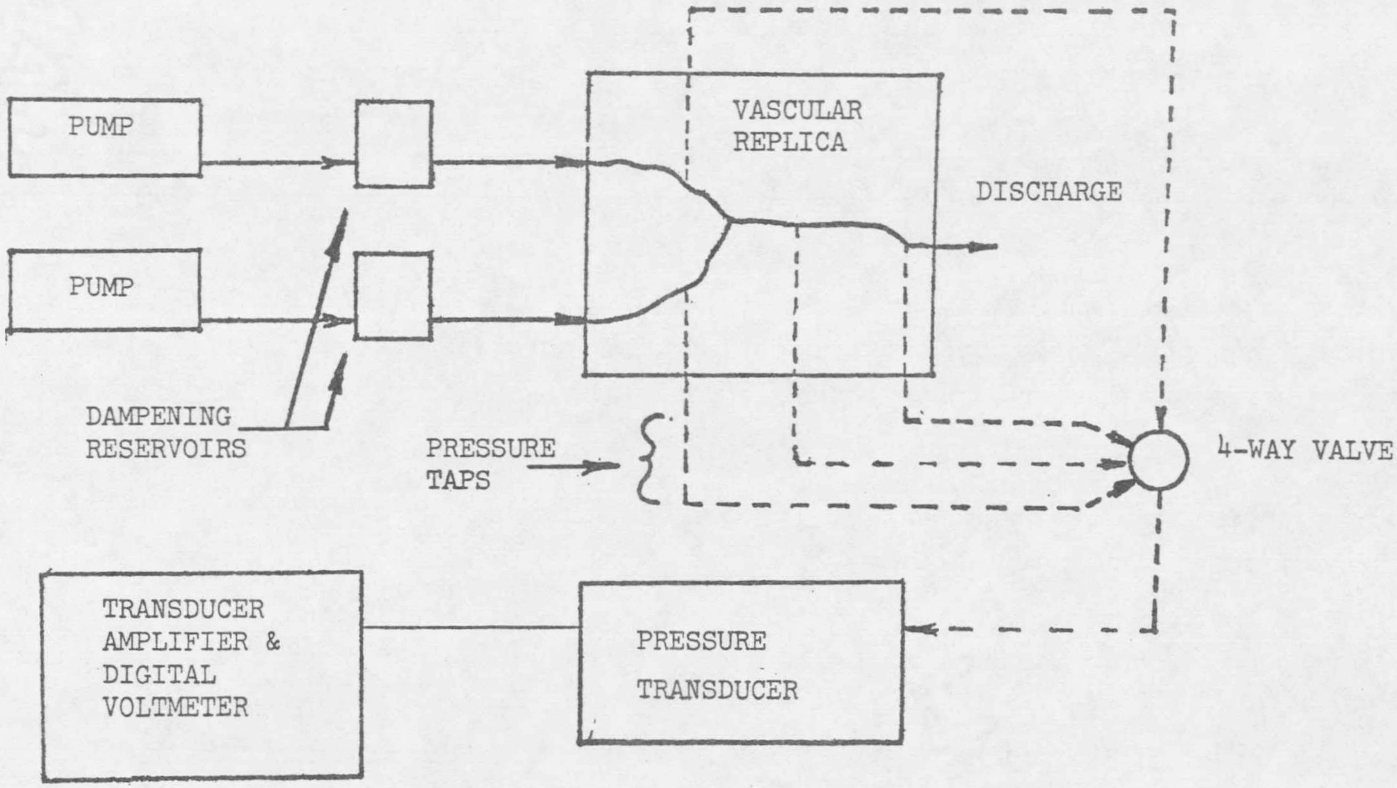


Figure 1. Schematic of research apparatus.

They were constructed such that there would be a sufficient volume of air above the fluid surface to dampen out most of the pressure variations. The device is depicted in Figure 2.

The tubing used to connect the difference pieces of equipment to the vascular replica was 1.5 mm I.D. x 3.0 mm O.D. teflon chromatography tubing (Altex Scientific Inc.). This tubing provided an essentially inert ducting for the fluids being studied and was readily adapted to luer type fittings by utilizing male and female luer adapters which were also obtained from the same company. The same type of tubing and connectors were used as the leads between the pressure taps and the pressure transducer.

Pressure readings from the system were gathered through the use of a differential pressure transducer Model MP45 (Validyne Engineering Corp.), equipped with a diaphragm designed for a range of ±50 mm Hg. The readings were facilitated by incorporating a 3-1/2 digit readout in the transducer indicator Model CD12-1003 (Validyne Engineering Corp.).

A number of two, three, and four-way valves were placed in strategic positions with the purpose of simplifying the purging the system of trapped air bubbles (Hamilton Company). Also, the four-way valve functioned as a switch which controlled which tap pressure was being displayed on the transducer indicator.

The connection of the teflon tubing to the vascular replica was made through the use of 19 and 28 gauge custom fabricated hypodermic

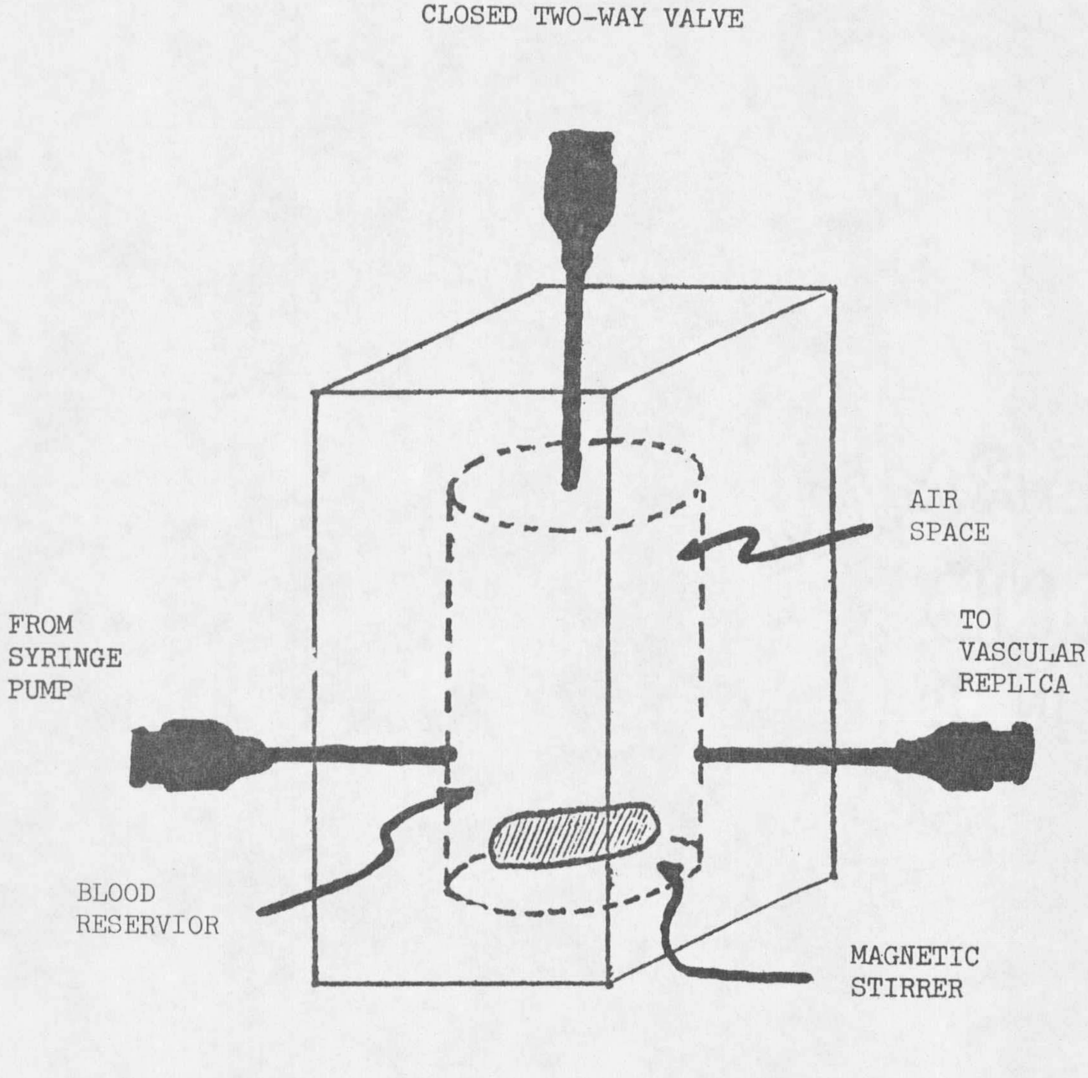


Figure 2. Pressure dampening and stirring reservoir.

needles (Popper & Sons, Inc.). These needles were mounted in holes drilled into the plastic cast and sealed into position using epoxy resin.

A bubble trap type of device was added in line between the four-way valve and the pressure transducer to allow for a flat interface between the fluid being studied and the fluid in contact with the transducer membrane. This removed the problem of corrosion which was present when water and plasma came in contact with the membrane. This also reduced greatly the need for cleaning and therefore re-calibrating of the pressure transducer. The trap was constructed from polyester resin with a chamber being hollowed out in the center and luer type fittings being mounted on the side, as seen in Figure 3. The fluid chosen for the transducer side of the trap was 5.0 cp silicone oil (Harwick) which had a specific gravity less than all of the fluids being studied.

Syringes used in the syringe pumps were 5 ml, 20 ml, and 30 ml gas tight syringes, 1000 series (Hamilton Company). These three sizes of syringes when used in the Harvard syringe pumps provided a fairly uniform distribution of volumetric flow rates.

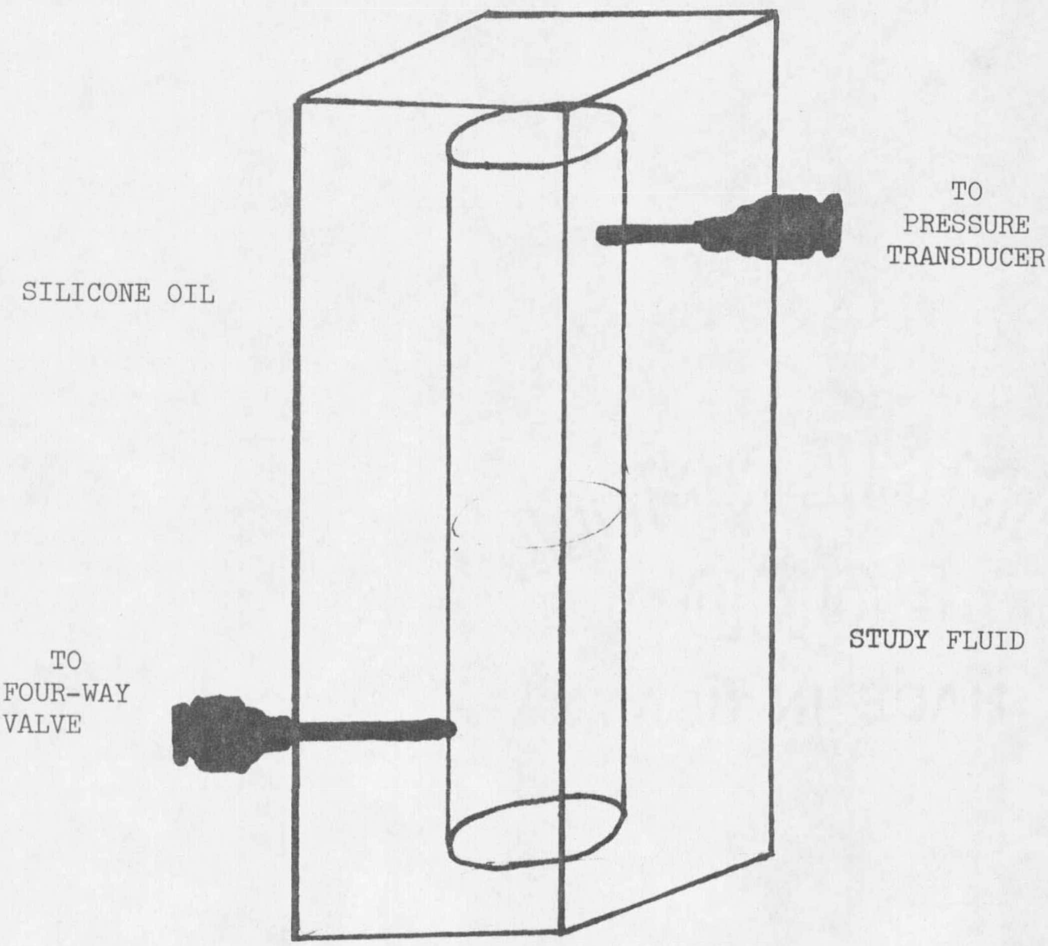


Figure 3. Bubble trap device.

EXPERIMENTAL MEASUREMENTS

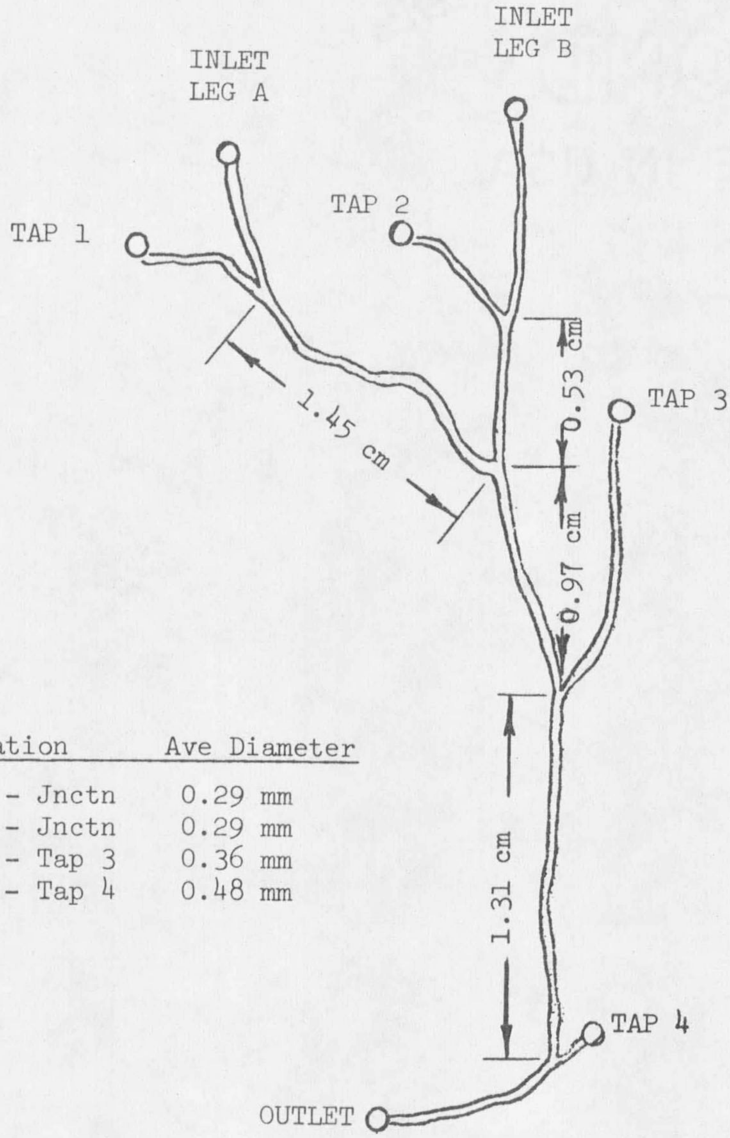
The method of data collection developed for this study was influenced largely by the manner in which the system was set up. Due to the size of the ducts and connecting tubing, it was imperative that all air bubbles were purged from the system. This was accomplished by repeated back flushing and bleeding of the incoming lines and the pressure tap lines. By adding several two-way and three-way Hamilton valves, it was possible to sectionalize the system and thereby facilitate the removal of the air pockets. Failure to remove any of the bubbles often resulted in highly erratic pressure readings.

Owing to the limitation of having only one pressure transducer, it was necessary to have a method of changing from one pressure tap line to another during a run. This was accomplished by having one side of the differential pressure transducer attached to a four-way valve which was in turn attached to each of the four pressure taps while the other side of the transducer was left open to the atmosphere. With this arrangement the plan was to measure the difference between the system pressure at each point and atmospheric pressure. The difference between the above mentioned pressure readings was then to be found yielding the pressure drop along each section of the vascular replica. From this, however, it can readily be seen that it is critical that at zero flow conditions the differential pressure readings for each tap be identical. The zeroing technique developed for this problem was also the most reliable method of determining if all of the air bubbles had been purged from the system. The procedure was to pump the study fluid through the system until a quasi steady state had been achieved. The pumping was then discontinued and the pressure in the system allowed to decay. When the pressure decline ceased, the reading of that particular tap was noted and the position of the four-way valve was changed. A moment was allowed for the reading to reach a steady value then it too was noted. When all readings were collected and compared, if they were equal it was relatively certain that the system was air bubble free and the actual data collection could begin. If the taps readings varied, the lines were flushed and the procedure repeated.

The data were collected by recording the reading on the transducer indicator at given time intervals. The length of the time interval ranged from five to fifteen seconds, depending on the rotation rate of the screw mechanism on the syringe pump. This was done because of the fact that there still existed a slight sinusoidal pressure fluctuation and this technique provided a time averaged value for each setting.

In order to obtain rough approximations of the dimentions of the model, a set of calibration operations was performed. The flow of water through one leg of the junction was set to zero while the flow rate through the other leg was set at various rates. By reading the pressures at all four pressure taps for each flow rate and repeating the procedure for the case when the other upstream leg had a zero flow rate, it was possible to determine the pressure drop through each segment of the vascular replica as a function of flow rate. Using this information along with segmental lengths obtained by manual measuring, values for the radii were calculated through the Hagen-Poiseuille equation. This information made it possible to compare future data against a crude approximation and obtain some ideas as to the reliability of the data. Figure 4 shows these approximate values for the dimensions of the vascular model.

The confluent flow data which was collected was intended to be classified into one of the following groups: 1) equal leg flow rates of Newtonian fluids, 2) unequal leg flow rates of Newtonia fluids, and 3) equal leg flow rates of non-Newtonian fluids (blood). The data for groups one and three (i.e., equal leg flow rate data) was obtained by equipping the two syringe pumps with the same sized gas-tight syringe. The pumps were then set such that the plungers for the two syringes were advanced at the same rate. The upper and lower limits for the flow rates which were studied were governed by the pressure range that could accurately be measured by the pressure transducer when equipped with the particular transducer membrane. The upper flow rate limits were 0.382 cc/min for water, 0.286 cc/min for blood, and 0.153 cc/min for glycerol while the lower flow rate limits were 0.029, 0.038 and 0.011 cc/min for water, blood and glycerol, respectively.



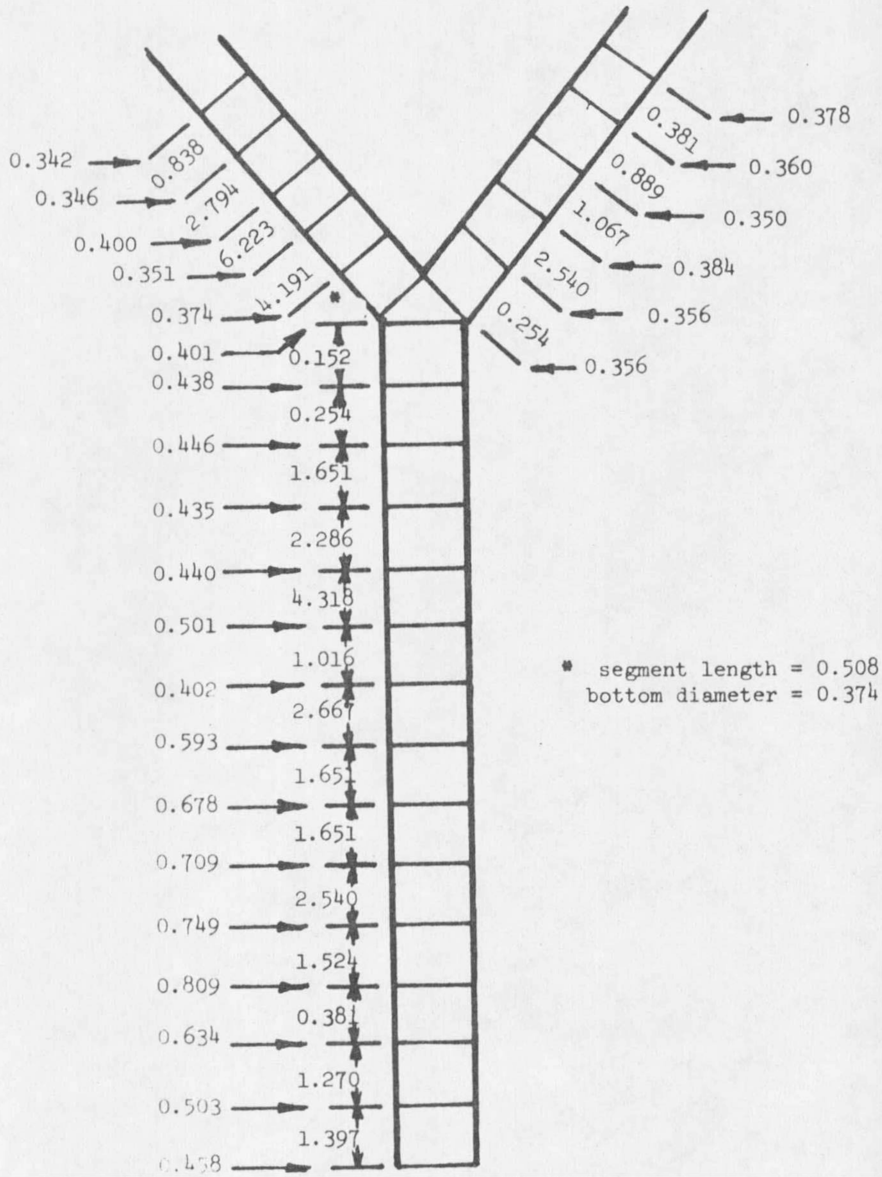
Location	Ave Diameter
Tap 1 - Jnctn	0.29 mm
Tap 2 - Jnctn	0.29 mm
Jnctn - Tap 3	0.36 mm
Tap 3 - Tap 4	0.48 mm

Figure 4. Approximate dimensions of vascular model.

The unequal flow rate information was collected by holding the volumetric flow rate through one of the upstream legs constant while varying the flow rate through the other leg. The flow rate in the varied leg ranged from approximately 0.2 to 2.5 times the volumetric flow rate in the constant leg, which was held at 0.153 cc of water/min. After data from all possible flow rates had been collected, the flow through the other leg was held constant and the complete range of flow rates were again studied. It was hoped that this set of data points would, when combined with the equal flow rate data, enable this experimenter to define the parameters which influence the pressure drop attributable to the existence of a junction of two vessels if it exists.

The stratified flow data was obtained only for cases in which equal amounts of the two fluids were being passed through the system simultaneously. The two sets of fluids which were placed in stratified laminar flow were a 50 wt% glycerol-water solution and water and blood and plasma. The first set of fluids was used primarily to check the validity of the Yu-Sparrow model for pressure drop in stratified laminar flow in channels of small cross section. The second set of fluids were included to determine if in extreme limiting cases if different hematocrit blood samples still followed the Yu-Sparrow model. As a method of calibrating the system, a third set of data points were collected to insure accuracy. In cases when water was being used as the study fluid in the confluent flow runs, the pressure drop from tap 3 to tap 4 was also recorded. This information was then to be used as a sort of hypothetical stratified flow case to determine the magnitude of any error which was brought about by the design of the system.

Upon completion of the collection of the actual pressure drop versus flow rate data, the next task was one of accurately measuring the length and radius of the various segments of the vascular replica. The most feasible technique of accomplishing this is by slicing off a known length of vessel, normal to the direction of flow, then placing the exposed end of the vessel on the platform of a metallurgical microscope and taking a photograph of the cross section. Then another length of vessel is milled off and the process is repeated. Once the photographs of the cross sections are obtained, they are graphically integrated to yield the cross-sectional area and eventually the equivalent diameter of the vascular replica at that point. Placing all these diameters in sequence along with the lengths between each measurement yields a fairly accurate picture of how the radii vary with downstream displacement. Figure 5 presents a schematic of the results of the measuring of the radii and lengths. Note that in areas where a junction of any two vessels was present, a finite length had to be machined off before any measurement could be made. In these instances it was assumed that the change in radius in that section was negligible.



* segment length = 0.508
bottom diameter = 0.374

Figure 5. Schematic of Lengths and Diameters of Various Sections of the Vascular Replica. (All Measurements Given in mm)

DATA ANALYSIS FOR CONFLUENT FLOW

The attempt to manipulate the data into a form which would be understandable proved to be as time consuming and more difficult than the actual collection of the data. The proposed method of analysis was to use the information on the geometry of the vascular replica in a mathematical model. This would then be used to calculate a predicted value for the pressure drop which should have been observed at those particular flow conditions. The predicted value was to be compared to the observed value for the pressure drop and these differences studied for possible trends due to variations in flow rate, viscosity, or some other parameter. This procedure was complicated by several factors which eventually led to a trial and error attempt to find an empirical function that would correlate the results.

The most accurate of the mathematical models which were considered for the calculating of the predicted pressure drop was the model developed by Walawender and Chen which treated blood vessels as a series of consecutive tapering tubes (Walawender and Chen, 1974). The utilization of this method had to be abandoned, however, when it was observed that the short distances between each radius measurement caused the function to diverge. The next model that was turned to was that of considering the blood vessel as being a series of cylindrical sections and assuming that the pressure drop followed that predicted by the Poiseuille equation, namely:

$$-\Delta P = \frac{128\ V\ \eta\ L}{\pi\ D^4} \qquad (1)$$

where V = bulk velocity of fluid; η = viscosity; L = tube length; and D = tube diameter. This equation utilizes the assumptions that the flow is steady and uniform and that the viscosity remains constant. One slight modification was made to the equation in that since the value of the diameter generally changes between the beginning and end of a section, the diameter was assumed to be the arithmetic average of the measured value of the diameter at each end of the segment. The incremental pressure drops from each section were then computed using the data in Figure 5. The results were then summed to yield the theoretical pressure drop between any two pressure taps.

Having computed the predicted value for the pressure drops at different flow conditions, it was a simple matter to determine the difference between the observed and predicted pressure drop. This difference was called the delta value. At this point a few trends were noticeable such as the delta values were generally 5 to 10 per cent of the observed pressure drop. However, no quantitative relationship could be found to correlate all the data. The next task therefore was to find some expression which would predict delta values similar to those observed in this study.

Several different parameters had to be considered when the choice for the functional form was being made. The varying volumetric flow rates, different viscosities, different diameters, and vessel lengths appeared to be the predominant parameters governing the magnitude of delta values. From the onset it seemed obvious that the most advantageous manner of combining these parameters would be in the form of a Reynolds number (Re). This dimensionless parameter is given as:

$$Re = \frac{D \rho V}{\eta} \tag{2}$$

where D = the diameter of the duct; V = the bulk fluid velocity; ρ = the density of the fluid; and η = the viscosity. The attractiveness of the Reynolds number was that since it essentially incorporated all the major possible flow parameters, it could possibly lead to an empirical expression which would be applicable to any confluent flow situation, provided the proper functional form was found

The development of a suitable functional form was a metamorphosis which started at the point where it was assumed that the delta values were linear functions of the upstream Reynolds number. Plots of the delta values versus the upstream Reynolds number revealed what seemed to be a relatively large amount of scatter. In attempting to eliminate some of the scatter, it was proposed that a line be placed through the points in a plot of the observed values versus the Reynolds number using a statistical least squares fit. The same was to be done to a plot of the predicted pressure drop values and the corresponding upstream Reynolds numbers. With these two lines computed, it would then be possible to compute the difference in slopes which would then be equal to the slope of a line through the points on a delta value versus Re plot. This technique gave satisfactory results when applied to the data from the equal flow rate studies, but failed to yield any worthwhile information when applied to the unequal flow rate data. The drawback of this model for the unequal flow rate study was that the delta values for one set of flow rate were plotted against two separate abscissas. In other words, the delta value from tap one to tap three was plotted against the Reynolds number in leg 'A', while the value from tap two to tap three was plotted against the Reynolds number in leg 'B'. Also, there was no dependence upon the flow conditions which existed in the opposite leg of the junction.

The next variation in the data analysis scheme was to take the arithmetic average of the two upstream Reynolds numbers. Plotting the observed pressure drops against this abscissa produced a moderate degree of success, especially when applied to the equal flow rate information. Figure 6 displays the linear correlation of the observed pressure drop values to the arithmetic Reynolds number average for equal volumetric flow information.

When the above mentioned abscissa was applied to unequal flow rate data, the results were less than satisfactory. Graphs of this group of information exhibited varying amounts of curvature as can be

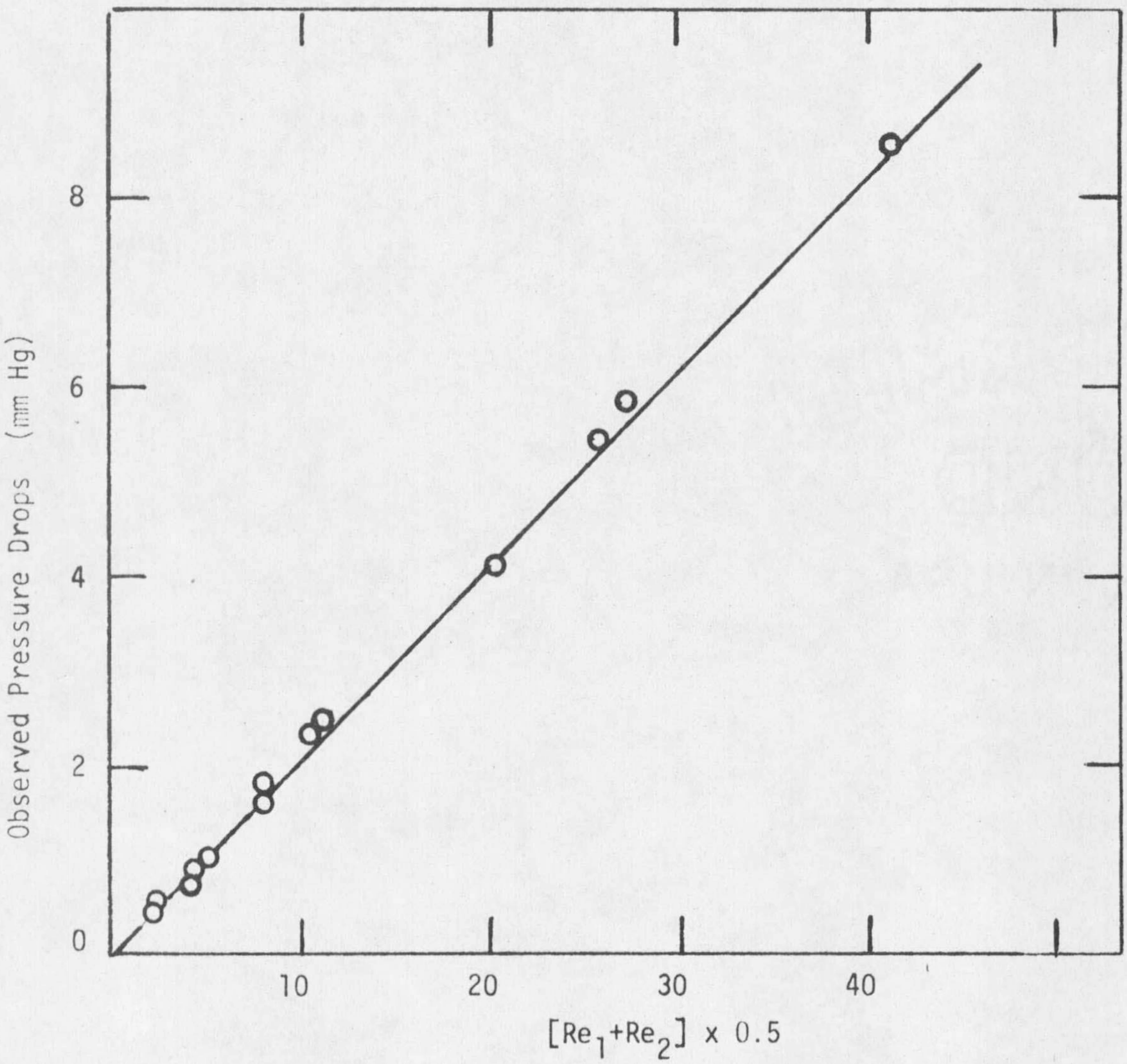


Figure 6. Plot of Observed Pressure Drop Values from Tap 1 to Tap 3 against the Arithmetic Average of the Upstream Reynolds Numbers for Water with Equal Volumetric Flow Rates in Each Upstream Leg.

seen in Figure 7, which is representative of these plots. The distinct non-linearity of these figures was partially due to the fact that the different lengths of the various legs of the vascular system were not being taken into consideration in using the Reynolds number average. Although this could be corrected by adding length terms into the proposed functional form, this step was not taken. Since the flow immediately upstream from the junction was assumed to be steady, there should be no upstream length dependency for the junction pressure drop. Therefore, instead of adding necessary terms to the empirical expression for the delta values, it was hoped that some other expression could be found to correlate the observed and predicted pressure drop values. Additional error was thought to be introduced by assuming that the pressure drop in one of the upstream legs was proportional to the sum of the two Reynolds numbers upstream from the junction. A more accurate approximation might utilize the fraction of total downstream flow which was contributed by one of the upstream legs.

The next variation in the functional form which was to express the observed pressure drop incorporated the above mentioned fraction in the form of a weighting factor. This fraction was obtained by taking the appropriate upstream volumetric flow rate and dividing it by the sum of both upstream volumetric flow rates. The weighting factors were then used in the assumption that the observed pressure drop would be correlated by the following expression:

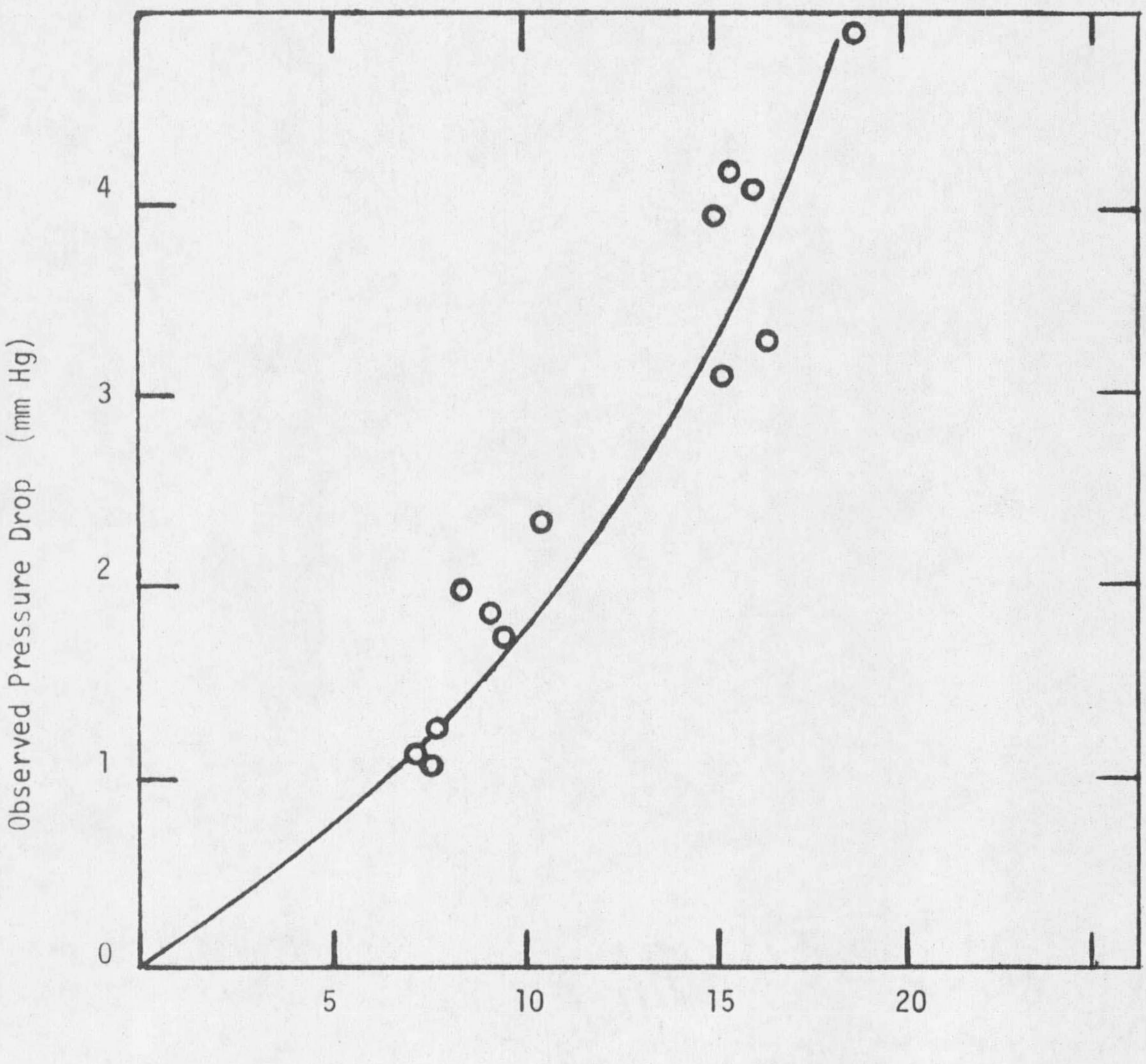


Figure 7. Plot of Observed Pressure Drop Values from Tap 1 to Tap 3 against the Arithmatic Average of the Reynolds Numbers for water when the Volumetric Flow Rate Through Leg B is Held Constant.

$$P = K \times \left[\frac{Q_1}{(Q_1+Q_2)} Re_1 + \frac{Q_2}{(Q_1+Q_2)} Re_2\right] \times 0.5 \qquad (3)$$

where K = a proportionality constant; Q_1 and Q_2 are volumetric flow rates in legs one and two, respectively; and P = the observed pressure change. The resulting plots of the observed pressure values against the weighted Reynolds number average provided some improvement in the correlation over the previous model tested. As was expected, the only change in the equal volumetric flow rate plots was a doubling of the slope values since the values along the abscissa had been reduced by a factor of one half. The degree of curvature in the unequal flow rate plots generally decreased from that noted in the previous attempt as can be observed in Figure 8. Figure 9 indicates, however, that use of the arithmetic average of the weighted Reynolds numbers in a linear function is not acceptable in all cases.

Further consideration of the use of the functional form which was discussed above revealed another major flow. Assuming that the function correlated the observed and predicted pressure drops well enough such that the difference in their slopes could be computed, a delta value function would result which would have the form of Equation (3). This function would yield a delta value that would be the same for an equal flow situation where the upstream Reynolds numbers were equal as for the situation where one leg had a zero flow rate and the

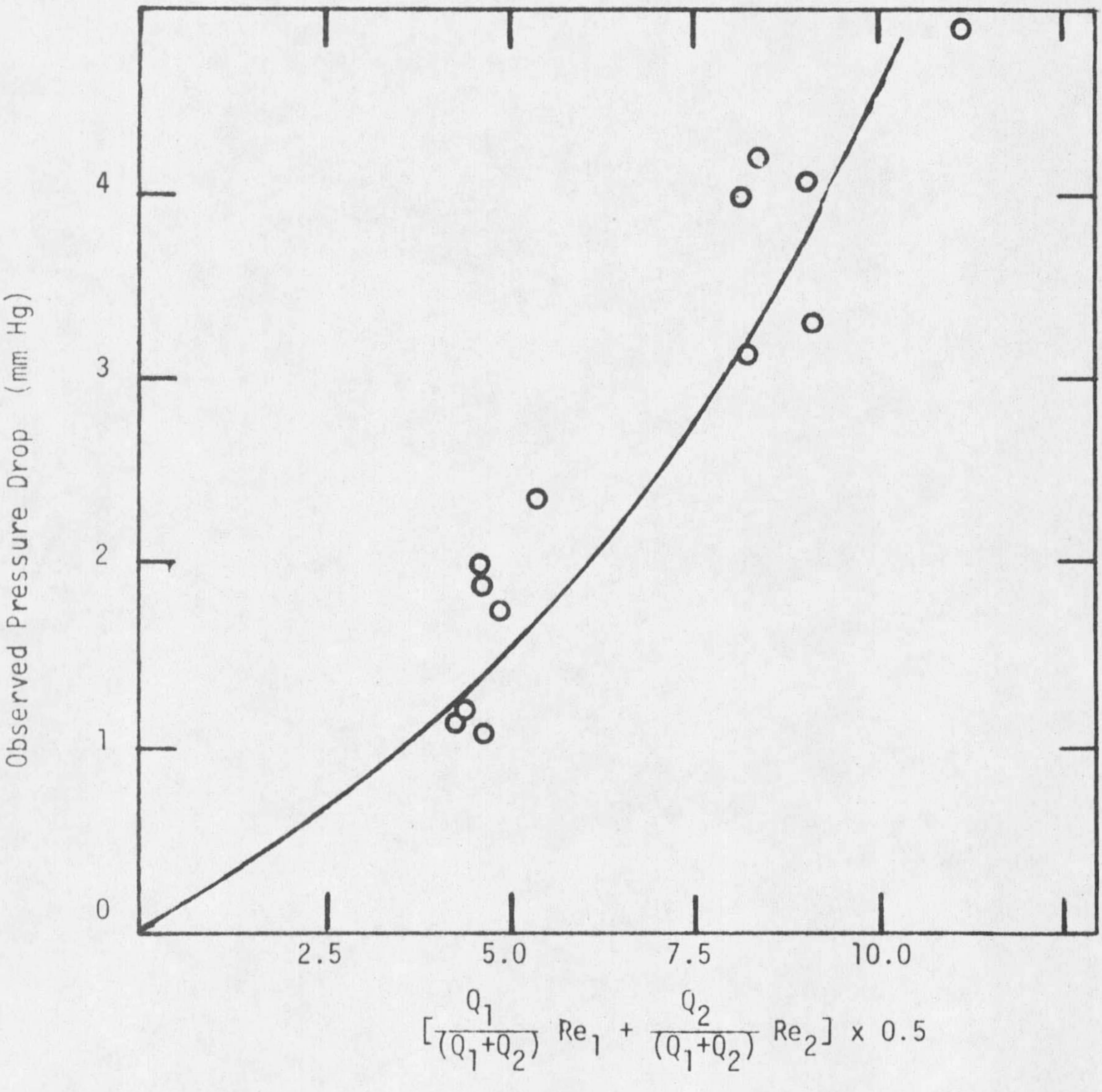


Figure 8. Plot of the Observed Pressure Drop Values from Tap 1 to Tap 3 against the Arithmetic Averate of the Weighted Reynolds Numbers for Water when the Volumetric Flow Rate Through Leg B is Held Constant.

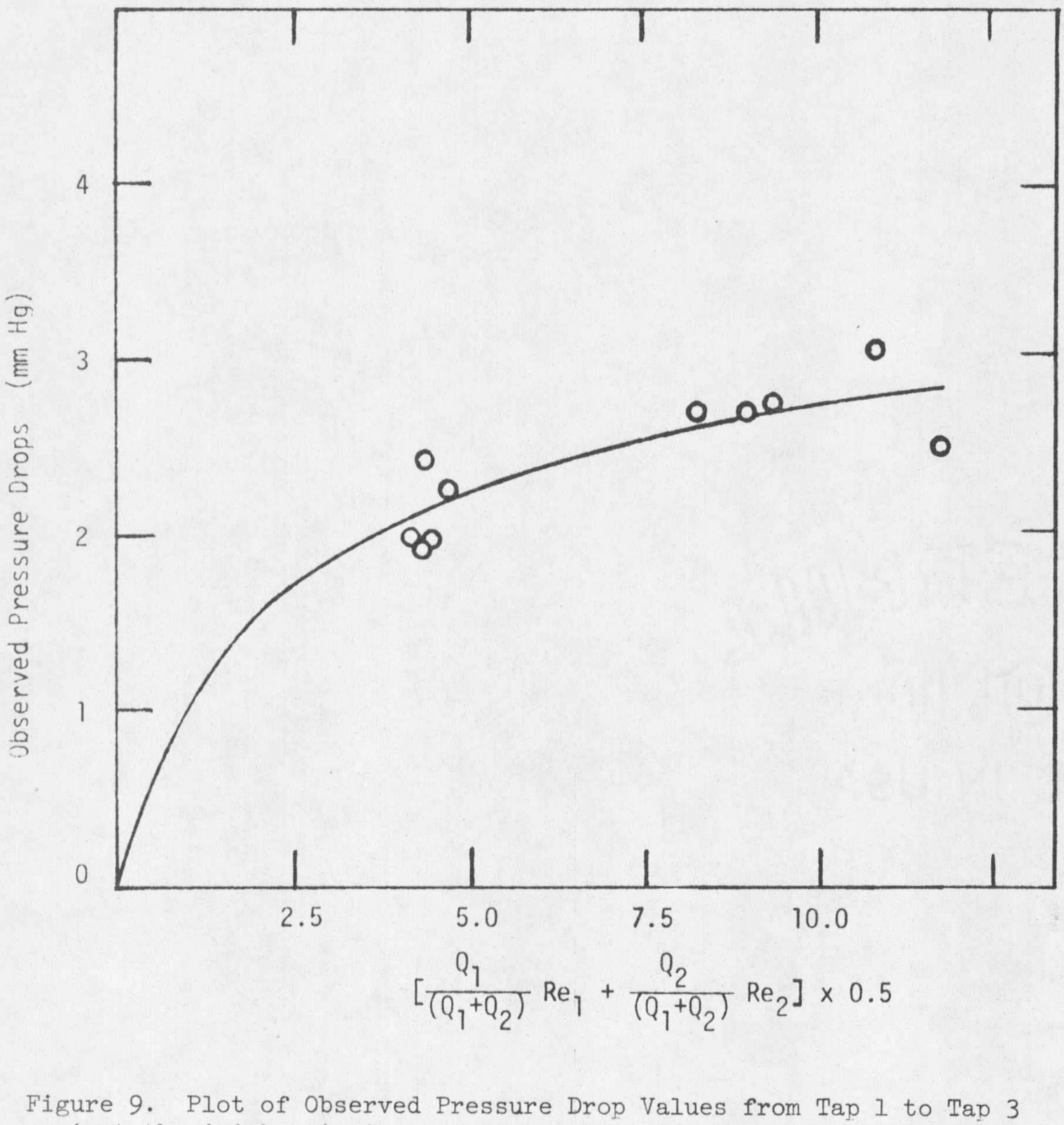


Figure 9. Plot of Observed Pressure Drop Values from Tap 1 to Tap 3 against the Arithmetic Average of the Weighted Reynolds Numbers when the Volumetric Flow Rate for Leg A is Held Constant.

other leg had the same flow rate as in the equal flow rate example alluded to above. Or to state it more simply, the function would predict a non-zero junction effect when there was no confluent flow.

To alleviate the possible false junction effect, the arithmatic average of the weighted Reynolds numbers was changed to the geometric average of the same. This then meant that as one of the flow rates went to zero, the entire function went to zero as can be determined by looking at the functional form:

$$P = K \times [\frac{Q_1}{(Q_1+Q_2)} Re_1 \times \frac{Q_2}{(Q_1+Q_2)} Re_2]^{0.5} \qquad (4)$$

where the variables are the same as in Equation (3). This attempt to correlate the observed pressure drops was partially successful in that it did prevent the prediction of a "false junction effect" without any major loss of statistical confidency, but the model failed to remove the non-linear distribution of the points on the various plots of the information. This fact is substantiated by comparing the following graphs (Figures 10 and 11) against the results of the weighted arithmetic average model discussed earlier (Figures 8 and 9).

At this point it was decided that the magnitude of the effect of the different duct lengths was too great to be correlated by the simple functions which were being tested by the experimenter. Therefore, instead of changing the functional form drastically to accommodate the length effects and in turn complicate the resulting delta

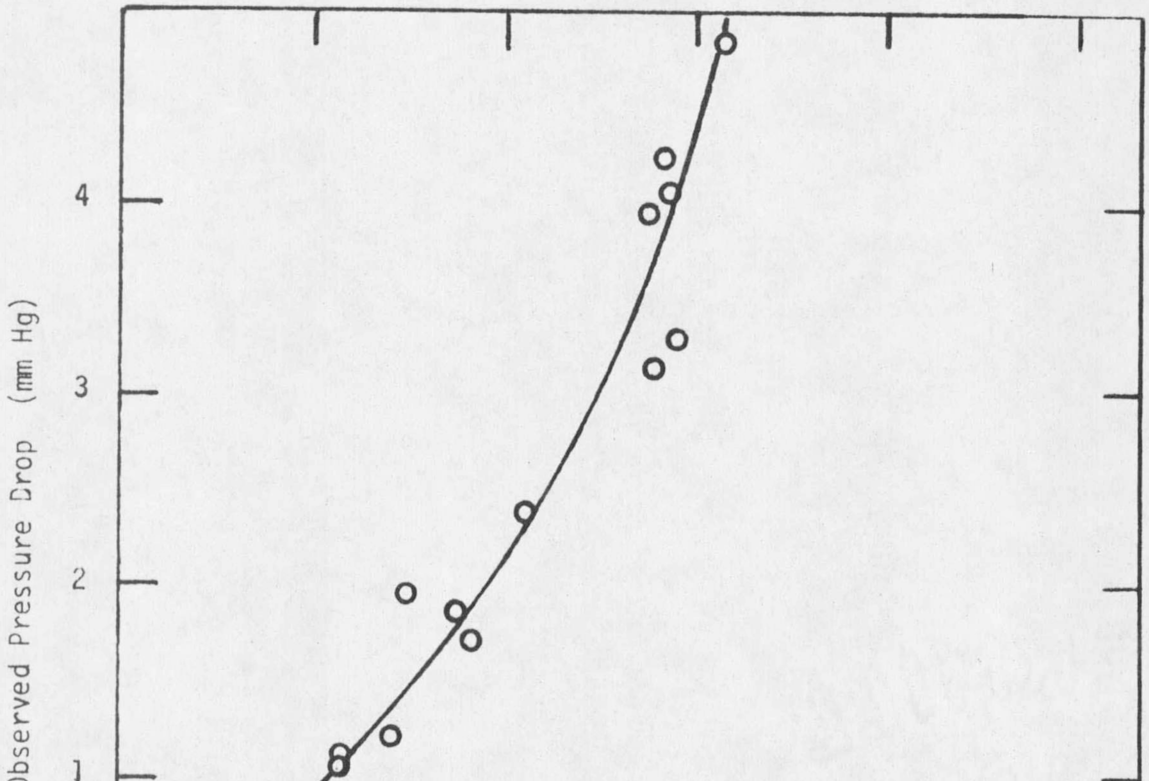


Figure 10. Plot of Observed Pressure Drop Values from Tap 1 to Tap 3 against the Geometric Average of the Weighted Reynolds Numbers for Water when the Volumetric Flow Rate through Leg B is Held Constant.

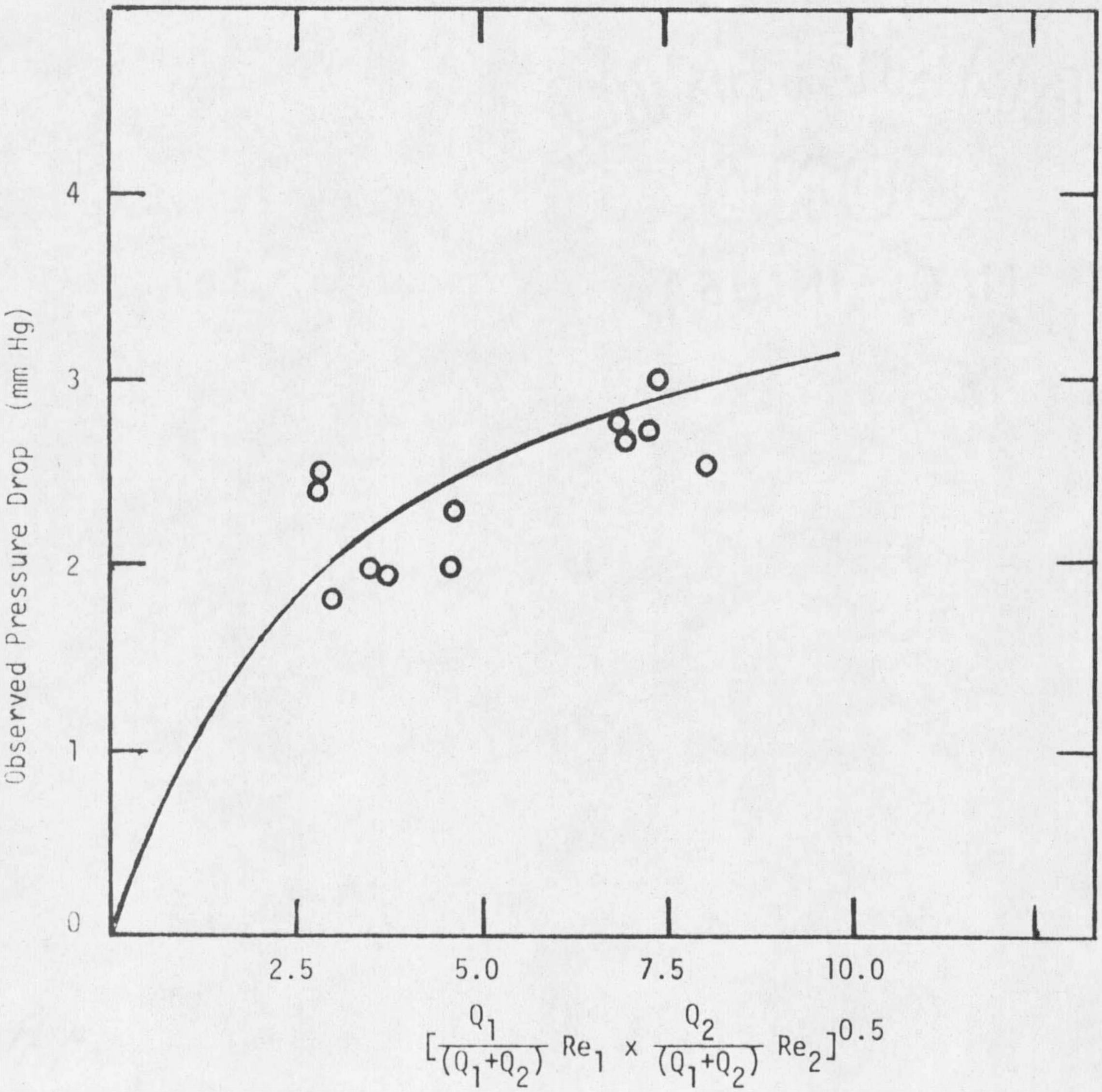


Figure 11. Plot of Observed Pressure Drop Values from Tap 1 to Tap 3 against the Geometric Average of the Weighted Reynolds Numbers for Water when the Volumetric Flow Rate through Leg A is Held Constant.

value function unnecessarily, further work along this line was ceased. The analysis method turned to was essentially the one which had been abandoned initially, that of attempting to correlate the delta values.

The first model chosen to be tested for its ability to correlate the delta values was that of the geometric average of the weighted Reynolds numbers. This model was attractive due to the fact that the possible resulting function would meet the necessary constraints, such as avoiding "false junction effects." The outcome of these efforts proved to be very encouraging. Linear regressions comparing the delta values against that abscissa exhibited high degrees of statistical confidence with the resulting lines passing through the origin in all cases. Two figures which are representative of the cases which displayed larger amounts of experimental scatter follow (Figures 12 and 13).

One flaw still existed in the design of this function if it was to be an accurate empirical model for confluent flow. The combination of the weighting factor and the Reynolds number produces a function which is dependent on the flow rate to the second power. On the other hand, if laminar flow is to be assumed to exist in this flow situation, the incremental pressure effect due to the merging of the two streams should be proportional to the volumetric flow rate to the first power. The removal of the weighting factor from the abscissa, which was no longer necessary since the observed pressure drop correlations were

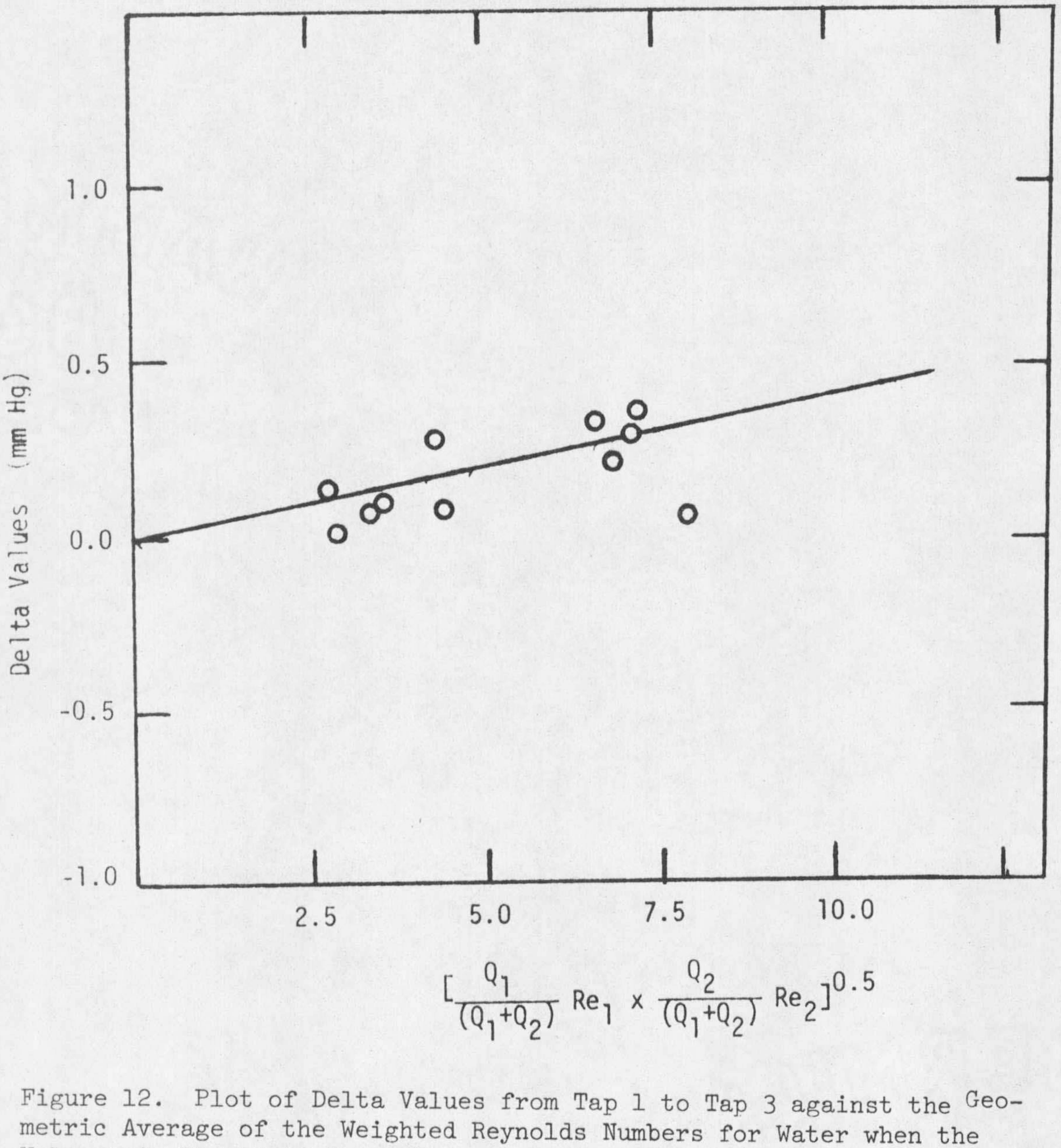


Figure 12. Plot of Delta Values from Tap 1 to Tap 3 against the Geometric Average of the Weighted Reynolds Numbers for Water when the Volumetric Flow Rate through Leg A is Held Constant.