THE USE OF COMPUTER ALGEBRA SYSTEMS IN A PROCEDURAL ALGEBRA COURSE TO FACILITATE A FRAMEWORK FOR PROCEDURAL UNDERSTANDING

by

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GLOSSARY

CAS/Framework Curriculum: Name for the redesigned curriculum that incorporates the use of CAS to implement the Framework for Procedural Understanding. This curriculum is the basis for the instructional treatment in this study.

Computer Algebra System (CAS): Computer algebra systems (CAS) are computer software or hand-held calculators that can perform the majority of the graphical, numerical, and symbolic procedures that most students have spent their mathematical careers learning (Heid, 1997).

Conceptual knowledge of procedures: One possesses conceptual knowledge of a procedure if able to address and communicate the Framework objectives effectively. See Procedural understanding defined below.

Developmental mathematics course: A course that is "beyond high school level but … is designed to fill the gaps between high school preparation and college expectations" Boylan, Bonham, & White, 1999, p. 88). By this description, Pasard and Lewis (2003) would classify developmental courses as remedial. The author consistently uses the term remedial for all classes below college level. See also Remedial mathematics course, defined below.

Discourse: Term to "denote any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system" (Sfard, 2001, p. 28).

The Framework: An abbreviation for the Framework for Procedural Understanding developed by Hasenbank (2006).

Framework-based: Indicating alignment with the objectives of the Framework.

Procedural skill: The ability to perform computations and execute algebraic procedures and algorithms.

Procedural understanding: "Refers to the degree to which one understands a mathematical procedure. It is operationally defined to mean the degree to which one can address the eight objectives of the Framework for Procedural Understanding (in the context of a specific procedure)" (Hasenbank, 2006, p. xi). This procedural understanding is the conceptual platform for the procedural work in the algebra course in this study.

Remedial mathematics course: A course in mathematics for "college-level students lacking those skills necessary to perform college-level work at the level required by the institution" (Pasard & Lewis, 2003, p. iii). With respect to this definition, courses defined as developmental in much of the literature would be classified remedial.

Treatment Instructor: Refers to the instructor who taught the course using the CAS/Framework curriculum which was the basis of the instructional treatment. Important to note is that this instructor taught three sections of introductory algebra, only one of which received the treatment.

ABSTRACT

This dissertation study evaluated the implementation and effectiveness of an introductory algebra curriculum designed around a Framework for Procedural Understanding. A Computer Algebra System (CAS) was used as a tool to focus lessons on the Framework and help students gain a deeper, well-connected understanding of algebraic procedures. This research was conducted in response to the prevalence of remedial mathematics and addresses the need for students in remedial mathematics to have a successful learning experience.

The curriculum was implemented in the Spring 2007 semester at a western land-grant university. In this quasi-experimental study, one section of introductory algebra was taught using the CAS/Framework curriculum. This treatment section was determined based on a pretest used to judge equivalency of groups. Data sources included procedural understanding assessments with follow-up student interviews, procedural skill exams, classroom observations, and a debriefing interview with the treatment instructor. Qualitative analysis of student and instructor interview transcripts was done to supplement independent observation reports to evaluate the implementation of the curriculum. Analyses of covariance and independent samples *t*-tests were used to compare treatment and control groups based on the quantitative measures.

The treatment instructor and students were able to integrate CAS technology into the classroom without difficulty. The instructor implemented the curriculum with fidelity but the discourse in the classroom did not reach the desired level. No significant difference was found between treatment and control students on the skills-based final exam, indicating that the introduction of CAS did not diminish procedural skill levels. No difference in procedural understanding based on the Framework was observed.

The data indicated that the students viewed mathematics as learning how to do procedures. This philosophy of mathematics and the limited classroom discourse impeded progress towards learning other aspects of the Framework. Recommendations include changing classroom norms to foster more discussion and placing more emphasis on Framework-based understanding in the assessment structure and course grade.

CHAPTER 1

STATEMENT OF PROBLEM

Introduction

Remedial mathematics courses are taught in postsecondary institutions across the country and have large enrollments. In a report from the National Center for Educational Statistics (NCES), it was determined that 71% of all postsecondary institutions offered remedial mathematics courses in the year 2000. While almost all public 2-year institutions (97%) offered remedial courses in 2000, 78% of public 4-year institutions offered those courses as well. Nearly a quarter (22%) of the freshmen beginning at institutions offering remedial courses in fall 2000 were enrolled in remedial mathematics (Pasard & Lewis, 2003). The term remedial is used in this study to be consistent with published research and statistics and no pejorative meaning is intended.

Students who successfully complete a remedial course in mathematics tend to be more successful in subsequent college-level courses than students who place into these courses but do not enroll in them (Johnson, 1996; Boylan, Bonham, & Rodriguez, 2000). Unfortunately, remedial mathematics has historically had the lowest completion rate among remedial subjects with about one-third of the students failing or withdrawing (Boylan et al., 2000).

Remedial courses are designed for students who lack the necessary preparation to operate at the college level and typically include high school topics such as pre-algebra and algebra. Students must repeat these algebra courses because they did not learn algebra at a level required for postsecondary mathematics courses, such as precalculus or statistics. To have a successful academic career, these students need an opportunity to learn this material at a deeper level.

To promote deeper conceptual learning among remedial students, Hasenbank (2006) developed and implemented a Framework for Procedural Understanding. The Framework was embedded in a college algebra curriculum to create a learning environment that fostered development of deeper mathematical knowledge. In the study conducted here, the implementation issues and effectiveness of using computer algebra systems (CAS) to embed the Framework in an introductory algebra curriculum was studied.

Background

The Framework for Procedural Understanding

The curriculum underlying the instructional treatment in this study was designed, in part, to incorporate Hasenbank's Framework for Procedural Understanding. The Framework is based upon guidelines that were proposed in a book published by National Council of Teachers of Mathematics (NCTM). The teaching and learning of mathematics should strive for the following goals:

1. The student understands the overall goal of the algebraic process and knows how to predict or estimate the outcome.
2. The student understands how to carry out an algebraic process and knows alternative methods and *representations* of the process.
3. The student understands and can *communicate* to others why the process is effective and leads to valid results.

4. The student understands how to evaluate the results of an algebraic process by invoking *connections* with a context or with other mathematics the student knows.
5. The student understands and uses mathematical *reasoning* to assess the relative efficiency and accuracy of an algebraic process compared with alternative methods that might have been used.
6. The student understands why an algebraic process empowers her or him as a mathematical *problem solver* (NCTM, 2001, p. 31, emphasis in original).

Hasenbank re-expressed these guidelines as eight student-centered questions for application in the classroom setting:

1. (a) What is the goal of the procedure, and (b) what sort of answer should I expect?
2. (a) How do I execute the procedure, and (b) what are some other procedures I could use instead?
3. Why is the procedure effective and valid?
4. What connections or contextual features could I use to verify my answer?
5. When is this the "best" procedure to use?
6. What can I use this procedure to do? (Hasenbank, 2006, p. 7-8)

The focus of many remedial algebra courses and students is on 2a of the Framework, "How do I execute the procedure?" It is possible that students do not retain mathematics knowledge because this knowledge is shallow, based on memorization of rote procedures and without "deep" understanding. Thus, while procedures may be mastered for the next test, the underlying concepts are weakly understood and quickly forgotten (Hasenbank, 2006). CAS was used to execute algebraic procedures, thereby allowing classroom discourse to center on the other aspects of the Framework.

Computer Algebra Systems in Undergraduate Mathematics

Computer algebra systems (CAS) are powerful software tools found on computers and calculators that are capable of symbolic manipulation of variables, function graphing, and working with numeric tables and geometric figures. The key characteristic differentiating CAS from graphing calculators is the ability to perform symbolic operations. Some claim that "the support of CAS can facilitate the implementation of a curriculum that places less emphasis on manipulation skills and more emphasis on conceptual understanding and symbol sense" (Thomas et al., 2004, p. 157). Such claims are currently being investigated by educational research.

Available evidence supports the above claim. In particular, two studies conducted at the undergraduate level are important to consider. Heid (1988) designed a calculus course that utilized CAS for the first 12 weeks of the semester leaving the last 3 weeks for development of manipulation skills. The results indicated an increased conceptual understanding without a decrease in procedural skills. O'Callaghan (1998) used the Computer-intensive Algebra curriculum in a college algebra setting and found that students gained a better understanding of the concept of a function and improved problem solving skills compared to the students in the traditional courses. In a more extensive review of research findings, evidence emerges that using CAS in mathematics instruction does not lessen the by-hand manipulation skills of students but does lead to deeper conceptual understanding (Heid, Blume, Hollebrands, & Piez, 2002).

Statement of the Problem

Remedial mathematics courses are prevalent throughout the postsecondary education system in the United States. Students in these courses did not learn the material at a deep enough level in high school or did not have a meaningful opportunity to do so. An unsuccessful attempt to complete remedial mathematics coursework has immediate and substantial consequences. An estimated 87% of all postsecondary institutions place some restrictions on the regular academic courses that students can take while they are enrolled in remedial courses (Pasard & Lewis, 2003). Johnson (1996) noted that poor performance in exit-level remedial mathematics increases the risk of failure and attrition as these students progress into college level mathematics. This is confirmed by Smith, O'Hear, Baden, Hayden, & Gorham (1996), whose observations showed that success in remedial mathematics is related to retention. They also observed that attendance and engagement were related to success in remedial mathematics.

There are many roles that CAS can assume in the classroom and benefits that come from these uses. One such role is that of a compensation tool for mathematically weak students. Kutzler (2003) and Yerushalmy (2006) advocate this use of CAS as a way to equalize the playing field and empower students. Other benefits of CAS are the reduction of mathematics anxiety and increased interest in mathematics courses (O'Callaghan, 1994). However, CAS has mostly been used in calculus courses and has been slow to find its way into other university courses (Stewart, Thomas, & Hannah, 2005).

"Teachers have the pedagogical duty to use all available resources to facilitate the learning process of their students" (Kutzler, 2003, p. 57). CAS is a tool that might address some of the needs of remedial students, specifically by empowering and engaging students and enabling deeper discussion and learning. However, this tool has not been widely used or studied in remedial courses.

Purpose of the Study

This study has three purposes, all related to implementation of an introductory algebra curriculum that incorporates the use of CAS and the Framework. The first purpose of this study is to carefully document a design and implementation process to adapt a traditional, procedure-oriented introductory algebra curriculum to better meet the needs of remedial students. The two-fold process embeds the Framework for Procedural Understanding into the curricular goals, assessment, and instruction of the remedial course and integrates CAS at these levels to facilitate the embedding of the Framework.

A second purpose of this study is to then evaluate the impact of implementing this redesigned curriculum that incorporates the Framework and CAS and an instructor's ability to do so, especially at the remedial level. The demands placed on teachers and students are explored to inform future redesign and implementation of this type of curriculum. Issues of particular interest are preparation time for the teacher, the introduction of CAS to the classroom, and meeting the constraints of the department.

The third purpose of this study is to determine the effectiveness of the instructional treatment that is a result of the redesigned curriculum and that integrates CAS into every lesson. Specifically, the researcher determined the extent to which the redesigned instructional activities lead to a deeper procedural understanding, as measured by Framework-based assessments. Also, it will be determined if students in the treatment group can perform as well on the skills-based departmental exams as the students using the traditional curriculum – which addresses the effect of CAS use on procedural skill development.

Research Questions

The following research questions refer to an instructional treatment comprised of a redesigned, remedial curriculum. This curriculum uses CAS to facilitate implementation of the Framework for Procedural Understanding.

1. What issues arise from the design and implementation of a Framework-oriented, CAS-facilitated curriculum in a remedial algebra class?
2. Do students learning through the CAS/Framework curriculum have the same level of Framework-based procedural understanding as students in the traditional courses?
3. Do students learning through the CAS/Framework curriculum have the same level of procedural skill as students in the traditional courses?

Significance of the Study

The potential of CAS in introductory mathematics has not been realized, as it is often unclear how to insert this technology into the curriculum due to its capabilities and sometimes unwieldy user interface (Nunes-Harwitt, 2005). In addition to integrating CAS into the curriculum, the curriculum designed for the study follows Hasenbank's (2006) recommendation for future research to embed the Framework into the curriculum instead of using it in a supplementary fashion. The design process was guided by the "Backward Design" method (Wiggins and McTighe, 2005) and can be applied to an existing, skills-based curriculum such as Bittinger's 10^{th} edition of Introductory Algebra (2006). Thus, a template for integrating CAS and the Framework into existing remedial courses is available for use by teachers and curriculum developers in other remedial programs.

Further research with the Framework for Procedural Understanding contributes to the body of evidence of its benefits, especially to remedial students. These benefits include learning with more understanding and breaking the "cycle of learning and forgetting" common to these students (Hasenbank, 2006). The fragmented algebra experience of students in these remedial courses calls for the use of new and innovative strategies, such as using CAS to quickly refresh their memory and increase their fluency in the language of algebra.

In a recent article, the University of Chicago School Mathematics Project (UCSMP) director, Zalman Usiskin, committed UCSMP to include CAS in the next edition of its widely used secondary curriculum. Usiskin noted that Educational Testing Service (ETS) is also developing a curriculum that heavily utilizes CAS and that CAS was a main topic at a regional NCTM conference in Chicago in September of 2006 (Usiskin, 2006-07). This study will be informative to the efforts to integrate CAS into algebra curricula.

Usiskin also establishes that CAS is "at least as valuable to the poorest algebra student as it is to the best" (Usiskin, 2006-07, p. 6). This study acknowledges that belief and is an answer to the call by Yerushalmy (2006) for research that integrates CAS in the work of beginning algebra students, especially those who need remediation. It focuses on remedial students, an area which has not been widely studied with respect to CAS. Also, the focus of the revised curriculum shifts emphasis from mastery of symbolic manipulation to the deepening of procedural understanding. The concentration of the course on the Framework for Procedural Understanding is in an effort to deepen the students' understanding and increase their chances for success and retention.

CHAPTER 2

REVIEW OF THE LITERATURE

Introduction

The review of literature begins with a characterization of remedial education in the United States, with the discussion focusing on issues pertaining to this research. This is followed by a summary of research findings on the uses, benefits, and obstacles of using CAS. Next, the Framework for Procedural Understanding is examined and aligned with theory of CAS use. Research on mathematical discourse and communication is presented to connect learning theory to the acquisition of Framework understanding. The conclusion of the chapter discusses research design and methodology used in CAS research.

Remedial Mathematics

Remedial mathematics courses are prevalent in postsecondary education with 71% of all institutions offering some form of remedial mathematics and 22% of all freshmen enrolling in such courses. Further illustrating the place of remedial education are data indicating that from 1995 to 2000 more institutions were imposing mandatory enrollment policies for students who needed remediation in mathematics. The most common type of credit given for remedial mathematics courses was institutional credit that counts for purposes of financial aid, housing, and full-time status but does not contribute to degree completion. Another trend identified from the 1995 - 2000 data is that the average time spent in remedial courses has increased (Pasard & Lewis, 2003).

It is important to understand the pressures felt by students in remedial mathematics to appreciate and study issues involving research in this area. Not only do these aspects of remedial education present a needed research agenda, but these trends, as well as student characteristics, are important to consider in research design. Factors that are particularly pertinent to this study are attendance, student motivation, mathematics anxiety, and interest. Pass/fail and withdrawal rates are also important.

Attendance

One critical issue affecting success in remedial mathematics is attendance. Waycaster (2001) conducted an observational study of remedial mathematics courses in five colleges that are part of the Virginia Community College System. Courses had a maximum enrollment of 20-25 students based on an assumption that there would be a few students who would never attend class. Observations showed that 56% to 81% of students were usually in attendance; however this number tapered off as the semester progressed, which is characteristic for remedial courses.

Another observational study at a four-year institution presented data on five sections of remedial mathematics. Over half of the students attended regularly, but a large minority attended less frequently with over 10% only showing up to take tests (Smith et al., 1996). Statistically, a significant (.05 level) relationship was found between attendance and grade, with 96% of passing students described as regularly attending.

Observers also noted that passing grades seemed to be connected to the level of engagement with the instructor and other students in the class.

Thomas and Higbee (2000) studied correlation between involvement variables and achievement variables in remedial algebra. Of the seven defined involvement variables, only number of absences was consistently and significantly related (negatively) to achievement variables. Other involvement variables included time spent in tutorial sessions, time working with others and alone outside of class, number of computer tutorials completed, and watching mathematics videos.

In summary, success in remedial mathematics is linked to attendance. Intervention research must account for the low attendance rate commonly found in remedial courses and engage students when they are in attendance.

Student Motivation, Anxiety, and Interest

"Math anxiety is a commonly known and documented negative influence, but disinterest can be equally detrimental" (Thomas & Higbee, 2000, p. 230). In a study of low-achieving, remedial mathematics students, Miller (2000) found that these students could be categorized into at least one of three groups: mathematics anxiety, responsibility overload with work and/or family, and lack of perception of the relevance of mathematics. Movement beyond these barriers occurred when students were successful early in their mathematics classes and found mathematics useful or interesting. A key finding of Miller's study is that understanding mathematics leads to increased motivation to learn.

Other factors can contribute to a student's motivation to be successful in remedial mathematics. When given an opportunity to learn mathematics other than as a passive observer of a lecture, students' attitudes and responsibility for learning improve (Vejdani-Jahromi, Hardin, & Bassler, 1994). However, institutional factors such as no degree credit for remedial courses and pressure to finish quickly can frustrate students, lowering their motivation to learn and to remain enrolled in college (Hagedorn, Siadat, Fogel, Nora, & Pascarella, 1999).

Withdrawal and Pass Rates

Research suggests that successful completion of remedial coursework leads to success in subsequent courses (Smith et al., 1996; Johnson, 1996; Boylan et al., 2000). However, data on pass rates paint a dismal picture; the majority of students in remedial courses never reach successful completion (Stage & Kloosterman, 1995). These remedial courses become gate keepers for students desiring particular careers, especially when one considers results, such as those reported by Smith and colleagues (1996), that among students who did not return in the spring semester of the study, 82% had failed their mathematics class.

Waycaster (2001) found that the passing percentages of the classes observed in the Virginia community college system ranged from 29% to 64%. Ten of the fifteen courses reached about 50% passing rates, as compared to the 61-70% completion rate that Roueche and Roueche (1993) established as “adequate” for remedial courses.

Related to withdrawal rates is the phenomenon of “stopping-out” that is common to mathematics courses. Stopping-out is the practice of allowing several semesters to pass before attempting the next course in the sequence. As the number of semesters between courses increases, the number of withdrawals dramatically increases (Johnson, 1996). This underscores the need for the students to learn mathematics at a deeper level, so that they can retain that understanding for future success.

Summary of Remedial Mathematics Characteristics

The research about remedial mathematics reveals the tendencies of students to have poor attendance and lack of motivation. Low pass rates place remedial mathematics classes in the role of gatekeeper, an obstacle to successful completion of a postsecondary degree. Efforts to increase attendance, engagement, and motivation are important to consider in research involving the improvement of remedial mathematics.

Computer Algebra Systems

This section of Chapter 2 focuses on the use of computer algebra systems (CAS) to teach and learn mathematics. First, the theoretical background of CAS in mathematics is established through an overview of the common ways that students use CAS and theories of CAS applied to curriculum. Then, findings from research on the benefits and obstacles of using CAS are discussed.

Roles of CAS

Kutzler (2003) presents four ways in which CAS can be used as a pedagogical tool: trivialization, experimentation, visualization, and concentration. Kutzler’s ideas overlap and coincide with other researchers' theories and each is discussed to provide a comprehensive overview of the roles CAS can fill when used by students.

Trivialization. CAS empowers students with the capability to handle more complex and realistic problems by manipulating complicated symbolic expressions and producing graphs of functions that would be difficult to do by hand. Although Kutzler deems trivialization as least important, Heid (2003) recognizes that using CAS in this way allows students to direct their attention to applications and an increased level of abstraction, while still using concrete examples. Students performing manipulations and graphing complicated functions may get lost in the details of by-hand work, resulting in the goal of the problem being obscured (Heid, Blume, Flanagan, Iseri, & Kerr, 1998). Thus, use of CAS to facilitate this work is justified.

Trivialization is an important way that CAS empowers less successful students. Yerushalmy (2006) observed that lower achieving students actually demonstrated more mature problem-solving skills and were able to do so by using technology to handle tedious computations, test conjectures, and correct errors (See the experimentation and concentration roles explained below.) When the need to be symbolically capable is removed, students who were weak in symbol-manipulation skills outperformed others on formulating and interpreting situations.

Experimentation. Using CAS, students can perform procedures and generate examples rapidly, in succession, and without fear of losing accuracy or becoming fatigued by repetitive paper-and-pencil work (Kutzler, 2003; Heid, 2003). This type of activity allows students to search for symbolic patterns and generalize their results (Heid, 2003; Heid & Edwards, 2001). Students can confidently and reasonably develop conjectures using reliable results and receive immediate feedback to confirm or refute their thinking, allowing them to develop a better understanding of the generality of algebraic procedures (Thomas et al., 2004). In addition to being immediate and accurate, feedback from the CAS is nonjudgmental. Thus, students need not be afraid of trial and error (Heid & Edwards, 2001; Heid, 1997). The powerful ability of CAS to handle manipulation of multivariate expressions allows students to generalize algebraic processes and uncover formulas and rules (Heid & Edwards, 2001).

Visualization. Visualization is defined as the "illustration of an object, fact, or process with results that are graphic, numeric, or algebraic" (Kutzler, 2003, p.62). Visualization is primarily used to help students use multiple representations, typically algebraic and graphical. However, research has shown that students will not always take full advantage or link representations unless specifically instructed (Thomas et al., 2004; Heid, Blume, Hollebrands, & Piez, 2002). Students have the capability to check their work with a second representation but do not always use that option (Heid et al., 2002). It has been observed, though, that students will often try a different representation when encountering obstacles in problem solving and use different representations to inform their reasoning and conjectures (Heid et al., 2002; Heid et al., 1998).

Concentration. One of the central problems in mathematics education is that "students must learn a *new* skill while practicing an *old* skill" (Kutzler, 2003, p. 65, emphasis in original). The learning process can be impeded when a student must interrupt the process to do calculations and possibly make errors in those calculations. With the support of CAS, a student can focus on the higher-level skill and concept to be learned (Kutzler, 2003). This role can also be seen in the problem solving process. By assigning the tasks of simplifying, solving, and evaluating to the CAS, students have more time to make the executive decisions of how to model the problem and craft solution strategies (Heid, 2003; Thomas et al., 2004). Students also have more time to reflect on the interpretation and validation of the solution (Thomas et al., 2004).

In this section, the major roles of CAS found in a review of the literature have been organized using Kutzler's (2003) categories of benefit as a basis. Incorporating the ideas and contributions of other researchers, the roles are renamed as: 1) trivialization, 2) experimentation and generalization, 3) visualization of multiple representations, and 4) concentration. Common to all of these roles is a shift of time and focus spent on by-hand manipulations from the student to the CAS. The implication this has for mathematics curriculum is discussed in the next section.

Mathematics Curriculum and CAS

An algebra curriculum in which CAS is available to the students allows more time for identifying structures, patterns, and the linking of representations at the expense of spending time memorizing routines (Thomas et al., 2004). Such a curriculum can also result in new ways to understand traditional procedures and more opportunities to develop connections among mathematical ideas (Heid & Edwards, 2001). These uses of

CAS in the curriculum and evidence from current research lead to the conclusion that the use of CAS can lead to deeper conceptual understandings (Heid et al, 2002).

Heid (2003, 1997) discusses two ways that CAS can function when integrated into a curriculum: as an amplifier or a reorganizer. As an amplifier, CAS helps students achieve objectives more quickly and they can work with a greater range of examples. In this way the curriculum is extended but the original goals and sequence of the curriculum are left untouched. When CAS is used to reorganize the curriculum, the sequence and fundamental nature of the curriculum and content is changed. Heid cites Kilpatrick and Davis (1993) to portray the central role of technology when used as a reorganizer:

> The computer is not merely an amplifier of general curriculum issues specific to the mathematics curriculum. … It changes certain fundamental questions one needs to consider in any attempt to revise the mathematics curriculum by making the subject matter itself more problematic. What is mathematics? What knowledge of mathematics does tomorrow's society demand? What mathematics should this pupil learn so as to be a wise and human citizen of that society? (p. 204)

Usiskin (1998) notes that as technology changes some algorithms become more important, some less important, and others retain the same level of importance. Since algebra courses typically emphasize learning and executing algorithmic procedures, Usiskin's principles about teaching algorithms are particularly relevant in this discussion. Three types of processes are involved in algorithms: those done mentally, those written on paper, and those done with the aid of computer technology. This characterization by Usiskin is at the root of much debate among CAS researchers. The question of the role of by-hand and mental work in the learning process remains unanswered (Thomas et al., 2004) but a common assumption among mathematics teachers is that mastery of procedures must precede learning concepts (Heid, 2003). Educators also fear that the use of CAS will lead to decreased by-hand and mental skill and reliance on technology (Heid et al, 2002; Thomas et al., 2004; Heid, 1997). Evidence from research is presented in the next section of this chapter to justify the use of CAS to form a deep conceptual understanding that later supports by-hand work.

Although it is possible to use CAS to develop conceptual understanding and allow students to still remain at par with procedural skills, Usiskin (1998) believes that some paper-and-pencil algorithms will be deleted from the curriculum and that those that have been removed already, such as Cramer’s rule and hand calculation of square roots, have not handicapped students. As CAS is used to reorganize the curriculum, what algorithms will be removed? To answer this question, Usiskin’s reasons for algorithms are helpful.

Algorithms are powerful, reliable, accurate, and fast, the same adjectives that might be used to describe CAS. Thus, teaching algorithms so that students can compute quickly and accurately may be reevaluated in the presence of CAS. Usiskin’s other reasons for algorithms include: they furnish a written record, establish a mental image, are instructive, and can be the object of study. It is to accomplish this last reason, algorithms as an object of study, that CAS and the Framework can be combined to accomplish.

Waits and Demana (2000) agree with Usiskin’s (1998) assessment that as technology becomes more pervasive, some mathematics will become less important, some more important, and some new mathematics will become possible. They further emphasize that the current curriculum contains techniques and methods that are taught because they were the only ones possible in the past. The days of being limited to contrived problems for students to solve are fading. Dugdale et al. (1995) envision new curricular opportunities that depart from skills practice and allow more active roles for students in applying algebraic ideas and planning, strategizing, and reasoning with and about mathematics.

Benefits of CAS in Research Findings

The research base supporting CAS in mathematics teaching and learning continues to grow. Heid (1984, 1988) performed the groundbreaking study that introduced CAS into the classroom setting as an integral part of the learning process. For the first twelve weeks of a calculus class, the computer performed all of the algorithms for the students. This use of CAS decreased the time normally spent on computational skills, provided data for students to conjecture and reason upon, provided a tool that could confirm or disprove their conjectures, and gave flexibility to explore problem situations. The last three weeks of the course were devoted to mastering the procedures by hand.

The results of the research validate the theories presented earlier in the chapter and, in fact, they form the basis for many of them. When scores on the skills-based final examination were analyzed, the students in the experimental course performed as well as those who spent the whole semester practicing their skills. Heid et al. (2002) reviewed 8 studies conducted subsequently and all but one of them confirmed this result that the proper use of CAS did not decrease students' procedural skill.

Heid found that students felt CAS helped in their conceptual understanding by lifting the manipulation burden to allow them to focus on the concepts of a problem. Also, students indicated that the CAS gave them confidence in the results they could base their reasoning on and allowed them to focus on the problem solving process. Using interviews and conceptual test questions, Heid also found that the CAS students had a better conceptual understanding in most cases. Additional studies support that the use of CAS develops a deeper conceptual understanding in calculus (Judson, 1990; Palmiter, 1991), secondary algebra (Boers-van Oosterum, 1990; Matras, 1988; Sheets, 1993), and college algebra (Campbell, 1994; O'Callaghan, 1998). The algebra studies are of particular relevance in this study.

The secondary algebra studies listed above provide examples of using CAS as a curriculum reorganizer and placing concepts and applications at the center of algebra (Heid, 2003). Matras (1988) found that students using Computer Intensive Algebra (CIA) curriculum did better on two problem solving tasks than those students using the traditional curriculum. Boers-van Oosterum (1990) also studied the effect of CIA, but focused on conceptual knowledge of variables. Using interviews and posttests, students using the CIA curriculum had a better understanding of variables as shown in their ability to use variables to model problem situations and working with graphs, tables, and equations of functions.

O'Callaghan (1994, 1998) studied students who were taught college algebra using the CIA curriculum with an emphasis on the concept of function. In this comparative study, the CIA students had a better knowledge of modeling, interpreting, and translating functions. This is another evidence of the major claim of CAS research, that using CAS to study concepts does seem to deepen understanding. There were no significant differences between the scores on the departmental final exam of the control and experimental group after adjusting for mathematics ACT scores. As already reported, withdrawal and failure rates are a concern in lower-level mathematics courses. O'Callaghan's treatment group had a higher success rate and lower withdrawal rate than the control classes using the traditional curriculum and lecture format. In addition, students in the computer group had an improved attitude, rating the class more interesting and having less anxiety. However, in O'Callaghan's study, the researcher taught the treatment section for which students had volunteered to register. This may introduce an effect due to the instructor or self-selection.

The benefits of using CAS in class, especially in O'Callaghan's study, directly tie to the needs of remedial students discussed earlier in this chapter. CAS in remedial mathematics has the potential to make mathematics more meaningful and interesting, perhaps countering attendance and anxiety issues. The ability of CAS to trivialize the manipulations can give students a sense of success which has been found to increase motivation. Most importantly, a concepts-based curriculum has been shown to increase conceptual understanding without a detrimental impact on procedural skills.

Obstacles for Students

One concern when introducing students to CAS, especially in remedial mathematics courses, is that it will be too difficult to use (Nunes-Harwitt, 2005). Drijvers (2000) refers to this inflexibility of CAS as "idiosyncrasy." With a specific syntax and possibly unfamiliar outputs, CAS may not seem like a natural tool. The benefits of CAS can be undermined when technical difficulties replace the difficulties that students normally face when doing the work with pencils and paper (Lagrange, 1999).

Detractors of CAS argue that since students are not focusing on by-hand manipulation they do not benefit from insights they would obtain through manipulation of symbols and notation (Heid, 1997). Because of the above obstacle of syntax, Dick (1992) counters with the assertion, "to realize the savings in time and to harness the power of computation that a symbolic calculator can provide, students need to pay more, not less attention to understanding the meaning of the symbols and notation they use" (p. 2). In Yerushalmy's (2006) study, the low achievers struggled with using the technology because it required symbolic fluency. While care must be taken to help students overcome the idiosyncrasy of CAS, the experience can provide an opportunity to focus on notation, symbolism, and representations.

The black box nature of CAS can also be an obstacle to using it for learning. The CAS does not show its methods for executing commands and thus no insight is gained (Drijvers, 2000). For instance, students cannot investigate some rational expressions because CAS will automatically rewrite and simplify those (Bosse & Nandakumar, 2004). Using other commands that will factor or expand parts of the expression can breakdown this process, but Drijvers (2000) has observed that students are not able to decide when and how CAS can be helpful in this manner. Thus, activities need to be carefully designed to allow students to investigate and gain insight into procedures that may be obscured by CAS. For example, when solving equations, instead of using a solve command, CAS can be used in a richer interaction that shows more steps and requires more student participation (Nunes-Harwitt, 2005). Tonisson (1999) has done work offering examples of how CAS can perform step-by-step procedures in this manner.

The CAS can do most of the operations and manipulations that students have seen in their mathematical studies. There is a danger that students may develop a false sense of security and not be fully aware of the limitations of CAS and calculator answers (Heid, 1997; Drijvers, 2000). The CAS may not do what the students expect it to do or the answer may not look as "simple" as the students expected (Drijvers, 2000). In these cases students need to know how to take extra steps to modify the input or use different commands to use the CAS effectively. Students need to reflect and interpret the output carefully to decide how the results are useful or how they can be connected to their expectations. Heid et al. (1998) observed that students would often abandon strategies due to misinterpreted or uninterpreted calculator results.

CAS can also give results in exact or approximate form, identifying another literacy that students need when working with CAS. Students need to be able to classify answers and know when and how to change these settings (Drijvers, 2000). The power of CAS to generalize in multivariate situations also brings up the issues of recognizing the difference between variable and parameter (Drijvers, 2000). Understanding that variables allow for generalizing arithmetic procedures is critical (Dugdale et al., 1995) and CAS can allow for this to happen. The obstacles described above require attention when implementing a CAS curriculum but also present opportunities to strengthen literacies and concepts.

Teachers' Needs in a CAS Environment

The introduction of CAS into a classroom will require change on the part of the teacher and introduce new needs for teacher preparation. However, not much is known about issues that arise when teachers implement CAS in classrooms (Thomas et al., 2004). Introducing CAS will challenge a teacher's knowledge of mathematics, knowledge of technology, understanding of learning, and the nature of mathematical knowing. Use of CAS can even threaten a teacher's perceived command of subject knowledge (Lumb, Monaghan, & Mulligan, 2000).

Lumb et al. (2000) discuss some issues that teachers grapple with when using CAS: time, materials, and pressure. Teachers must spend extra time to become competent with the technology as well as to plan lessons and worksheets. Materials are needed for teachers that "fit" the mathematics they were taught and are familiar with. However a re-evaluation of the mathematics teachers know must also take place. There is considerable pressure for teaching to support formally assessed work and teachers are constrained by curriculum decisions not of their own making. Stacey, Kendal, and Pierce (2002) comment that, "Until the external curriculum environment changes, teachers and students will live in an ambiguous situation about by-hand skills" (p. 124). This pressure is evident in the research climate, discussed previously, as studies are typically designed to test for undiminished computational skill paralleled with increased conceptual understanding.

The Framework for Procedural Understanding and CAS

The Framework for Procedural Understanding (Hasenbank, 2006) was designed to guide instruction so that students can develop a deep and well-connected knowledge of mathematical procedures. This is of particular importance for remedial students as this type of understanding can help recall and promote future learning. In this section, the relationship of CAS in implementing the framework is detailed. First, the interplay of procedural and conceptual knowledge is discussed as it pertains to CAS theory and the Framework. Following this discussion, the feasibility of applying CAS to implement the Framework and realizing the goals of the Framework is discussed along with an illustration of how CAS can facilitate in asking the Framework-type questions. Lastly, assessment issues with respect to the Framework are discussed.

Procedural and Conceptual Knowledge

The importance of and relationship between procedural and conceptual knowledge acquisition is an historical debate (Hiebert & Carpenter, 1992). One perspective found in the research indicates that procedural automation can free up cognitive resources for further development of conceptual knowledge (Wu, 1999). CAS can provide procedural automation (Kutzler, 2003) and research has produced evidence that CAS curricula can focus, first, on concepts and address procedural skill afterwards without hurting the development of these skills (see p. 20).

However, the idea that procedural and conceptual knowledge develop iteratively is supported by research reported in Hasenbank's (2006) review of literature. CAS can also be adapted in this environment. Kadijevich (2002) explains that to promote conceptual and procedural links CAS can "(a) facilitate the procedural work when a concept is being verified; and (b) require the user to think conceptually before a procedure is used" (p. 72). Heid (1994) reorganized a semester calculus course into 12 weeks of conceptual work and 3 weeks of procedural work. A smaller scale of this model can be applied to a curriculum unit or even an individual lesson to address concepts first but develop conceptual and procedural knowledge iteratively.

More recently, Baroody, Feil, and Johnson (2007) claimed that procedural and conceptual knowledge cannot be separated and that linking the two types of knowledge, "can make learning facts and procedures easier, provide computational shortcuts, ensure fewer errors, and reduce forgetting" (p. 127). The Framework provides practical guidelines to linking procedural and conceptual knowledge. Star (2007), in a response to Baroody et al., argues that procedural knowledge can deepen by making connections to conceptual knowledge. However, he goes on to argue, procedural knowledge can be deepened and flexibility with procedures developed without necessarily making connections to conceptual knowledge. The main agreement in this research commentary is that procedural knowledge is an important part of mathematics learning. This is particularly important in light of current remedial algebra curricula that focus heavily on procedures. The Framework can serve to both deepen students' procedural knowledge and develop conceptual connections to procedures.

The idea of repetitive practice versus varied practice put forth by Brownell (1956) is informative when considering the interplay between conceptual understanding and procedural skills. Repetitive practice is helpful in developing efficiency and competence with procedural skill but Brownell cautions that this is the last step in learning and should not be introduced too soon. Varied practice allows students to try different strategies, think about explaining and justifying the steps taken, and what the best procedure is. This type of practice and the “How” and “Why” type of questions Brownell encourages are in harmony with the Framework.

Alignment of the Framework and CAS

“Technology makes the study of algorithms more important” (Heid, 1997, p. 14), but this study of algorithms moves from skilled by-hand manipulation to a deeper analysis of patterns and relationships. Usiskin (1998) proposed that algorithms can be instructive and also notes this shift to algorithms as the objects of study. Heid and Edwards (2001) suggest that CAS can be used to provide new ways to understand traditional procedures. The Framework can guide this study of procedures and the remedial algebra curriculum, with its focus on learning procedures, seems an ideal place for such an application.

Before illustrating examples of CAS facilitating use of the Framework, it is important to establish alignment of the classroom environment that Hasenbank (2006) suggests for development of deep mathematical knowledge with the principles guiding the use of technology. Hasenbank, citing Carpenter and Lehrer (1999), recommends that students have opportunities to reflect, extend and apply knowledge, articulate their knowledge, and make mathematical knowledge their own. Similar to these guidelines, Heid (1997) gives four principles to guide decisions concerning the use of technology:

technology can make education more student-centered, give the student the experience of being a mathematician, promote reflection, and shift more responsibility for learning to the student. Thus, a Framework-oriented classroom and the use of CAS are compatible.

Lastly, Hasenbank (2006) found that teachers felt time constraints prohibited further development of Framework understanding. CAS has the ability to save time through the outsourcing of symbol manipulation and computation. In this way, the CAS seems an appropriate tool to aid in the implementation of the Framework.

CAS Facilitation of Framework Questions

In this section, CAS will be discussed with respect to each objective of the Framework as represented by Hasenbank's student-centered questions.

Framework Objective 1. What is the goal of the procedure, and what sort of answer should I expect? Using CAS in the role of experimentation and generalization by producing examples can help students recognize patterns in the outcomes of procedures. They can then conjecture and learn to estimate the expected outcome and understand what the goal of the procedure is. This type of mental algebra and estimation is thought by some to be more important than by-hand algebra (Thomas et al., 2004).

Framework Objective 2. How do I execute the procedure, and what are some other procedures I could use instead? The answer to the first question is obviously postponed and the execution of the procedure is done by CAS to allow discussion of the other framework questions. This allows a conceptual basis to be formed on which to build by-hand skills later. Using CAS to do procedures step-by-step can illustrate different procedures to use and this knowledge is critical for Brownell's (1956) varied practice. This question can also be answered in the context of other ways to symbolize the procedure, different syntax to use, or different representations that lead to the same result. CAS forces students to think of symbolism and syntax and offers quick access to various representations.

Framework Objective 3. Why is the procedure effective and valid? This question is another that Brownell (1956) would deem critical to ask. CAS can be used to perform and verify steps on general, multivariate expressions. It is critical that step-by-step activities be designed for CAS so that the black box does not obscure the procedure. Presenting misconceptions or nonexamples can lay the foundation for class discussion. Also, in this Framework objective, the student should learn how to reconstruct the procedure and be able explain the procedure to others.

Framework Objective 4. What connections or contextual features could I use to verify my answer? Access to multiple representations, particularly tables and graphs, can be used to check answers and teach the idea of equivalence. For example, graphical visualizations help students connect solutions of equations with intersection points or intercepts.

Framework Objective 5. When is this the "best" procedure to use? This objective does not seem to fit well with CAS as students can perform the operations very quickly using the technology. However, even when using CAS, a problem may have alternative approaches whose characteristics make one approach better than another in the situation at hand. Furthermore, a literacy that students should acquire is being able to use mental cues and determine when one might use a calculator, pencil and paper, or mental estimation. Usiskin (1998) warns that algorithms can be overapplied in all three settings. For example, a calculator or pencil and paper may not be appropriate when the procedure can be performed mentally. Mental mathematics can also be dangerous because no record of the input is available and so error is harder to determine and find. Students must learn to evaluate a problem to decide the best way to proceed.

Framework Objective 6. What can I use this procedure to do? This is perhaps one of the strongest applications of CAS as it allows students to concentrate on what procedures can be used for without knowing how they even work. A curriculum unit can begin with application problems as CAS trivializes the work needed to solve equations or evaluate functions. Students can spend time modeling situations and recognize that algebra can be used to generalize arithmetic procedures, allowing them to deal with entire classes of problems. With CAS students can also address a wider range of applications due to the ease of manipulation of complicated expressions.

Assessment of Framework Understanding

Assessing understanding is difficult and the temptation to be satisfied when a student can repeat the words of a generalization verbatim must be overcome (Brownell, 1956). Wiggins and McTighe (2001) suggest that, “to judge understanding of how someone makes sense of something, we typically need the person to explain it to us” (p. 80). Brownell concurs and suggests that students’ oral reports can be a fruitful way to get authentic data. Probing student responses in the face of error is another valuable tool. Interviews are the ideal choice but journal response was the tool used by Hasenbank (2006) given constraints on a feasible, valid research design.

O'Callaghan (1998) assessed students' understanding of the concept of function according to a four-component framework of competencies: modeling, interpreting, translating, and reifying. In constructing a test to measure student understanding of functions, O'Callaghan designed questions that were intended to probe one aspect of knowledge of functions. Analogous problems were used in an interview process that allowed O'Callaghan to investigate the methods and reasoning that students used to solve the problems. This method of assessing understanding according to a framework, coupled with the principles in the above paragraph, served as a basis for assessment of understanding according to the Framework for Procedural Understanding.

Summary of CAS and the Framework in Remedial Mathematics

Remedial education at universities and colleges is beneficial for students when they are successful but pass rates are low. Characteristics particularly relevant to this study are tendencies of remedial students to have low attendance, motivation, and interest. Results indicating increased interest and motivation suggest using CAS might be particularly useful in remedial mathematics. CAS can empower remedial students to feel successful early in the course and to take a more active role in learning. The Framework shows promise in helping remedial students learn mathematics at a deeper level and retain that knowledge for success in future mathematics courses.

Research builds a strong case that CAS can be used to promote deeper conceptual understanding, which makes it a natural choice as a tool in implementing the Framework. CAS alleviates time constraints so the Framework-based discussion can take place. The roles CAS plays enable students to investigate Framework questions without mastery of procedural skills and establish mathematical discourse in the classroom.

Discourse in Mathematics Classrooms

Discourse and communication have become central to mathematics education. The NCTM (1989, 1991, 2000) has repeatedly emphasized practices to establish mathematical discourse communities and outlined the roles of students and teachers in these communities. Research studies have been conducted to further define mathematical discourse, aid teachers in practices to establish mathematical discourse in the classroom, and develop theories of knowledge based on communication (Sfard, 2001; Goos, 2004; Hufferd-Ackles, Fuson, & Sherin, 2004; Ben-Yehuda, Lavy, Linchevski, Sfard, 2006).

Students' Roles in Discourse

Discourse, most broadly defined, is any specific act of communication, including verbal and nonverbal, synchronic or asynchronous, and with others or with oneself (Ben-Yehuda et al., 2006; Sfard, 2001). Students take part in discourse by making and exploring conjectures, questioning other students and their teacher, and validating knowledge using mathematical evidence (NCTM, 1991). Sfard (2000) studied the aspects of different types of discourse by examining letters that students had written to a friend to convince them of a mathematical claim. Goos (2004) found that students believed that explaining their thinking was a means to strengthen and evaluate their understanding. These explanations from students came as responses to questions like “How could we verify this?” and “What was the reason for completing this procedure?”

The Framework questions play a prominent role in organizing mathematical discourse as evidenced above. Questioning is a key aspect of creating a community of discourse and the focus of questioning should shift from finding answers to uncovering mathematical thinking behind answers (Hufferd-Ackles et al., 2004). The first step is to open the classroom up for students’ ideas. A framework for helping teachers in developing a community of discourse is discussed next.

Teachers’ Roles in Discourse

The NCTM (1991) identifies roles that teachers must take on in a community of discourse:

- Posing questions and tasks that elicit, engage, and challenge each other’s thinking;
- listening carefully to students’ ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students’ ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students’ participation in discussions and deciding when and how to encourage each student to participate (p. 35)

Silver & Smith (1996) identify two challenges that teachers face in establishing a mathematical discourse community in their classroom. First, a teacher must build an environment of trust in which students feel comfortable proposing their ideas, question others' ideas, and sharing alternative interpretations. Second, a teacher must center the discussion on mathematical ideas, posing questions and tasks that uncover students' thinking and challenge them.

Hufferd-Ackles (2004) developed a framework to guide teachers in implementing the discourse practices recommended by the NCTM into their classroom communities. Four components were identified in the study. These ideas are consistent with prior research, although the components were previously examined separately. The four components are described over a progression of 3 levels during which the students act in a more central role and the focus shifts from answers to mathematical thinking:

A. Questioning: Shift from teacher as questioner to students and teacher as questioners.
B. Explaining mathematical thinking: Students increasingly explain and articulate their math ideas.
C. Source of mathematical ideas: Shift from teacher as the source of all math ideas to students' ideas also influencing direction of lesson.
D. Responsibility for learning: Students increasingly take responsibility for learning and evaluation of others and self. Math sense becomes the criterion for evaluation (p. 88).

These components of a learning community parallel the norms of a Framework-oriented classroom and principles of technology use established in a previous section aligning the Framework and CAS use (see p. 28).

Communicational Approach to Cognition

The guiding principles of mathematical discourse communities seem to fit well with the type of classroom that is possible using CAS to facilitate the Framework. It is helpful to consider Sfard's (2001) theory of cognition as a basis for implementing the Framework. "The basic tenet of the communicational approach to the study of human cognition is that *thinking may be conceptualized as a case of communication*, that is, as one's communication with oneself" (p. 26, emphasis in original). Sfard further defines learning as being a participant in certain activities rather than acquiring generalized, context-independent knowledge.

In this way, learning can be redefined as "an initiation to mathematical discourse". Sfard identifies support from different perspectives of the sociocultural framework. For example, learning is described as "an improved participation in an interactive system" (Greeno, 1997) and "a reorganization of an activity" (Cobb, 1998). Sfard contrasts this participation approach to cognition with the metaphor of learning as the acquisition of knowledge. The emphasis on the term acquisition underscores the individual nature that may take place by passive reception or active construction, but results in a personalized knowledge of concepts and procedures.

A significant difference between these two types of learning is the "breach-in-understanding" that motivates learning. Cognitive conflict assumes that the learner gains new knowledge by adjusting his understanding to external facts and ideas. This could happen without the involvement of other people. Discursive conflict stresses the need of communication in motivating our cognitive actions, specifically the need to be consistent with other people in discourse. Sfard allows for the acquisition model of cognition and communication approach to cognition to be complementary, but the theory of learning through communication and discourse seems relevant to the validity of implementing a curriculum embedded with the Framework and CAS.

In summary, the Framework pedagogy of frequently asking questions seems to fit with the above learning theory based on discourse. Learning defined as communication and participation forms a theoretical foundation for curriculum design intending to embed the Framework.

Research Methods

Several dissertation studies have previously been conducted to examine the effectiveness of curricula that utilized CAS. Commonalities in the research designs, data collection methods and instruments are informative for the current and future CAS studies. Most notably, CAS researchers are seeking to establish gains in students' conceptual understanding without a loss of proficiency in procedural skills.

A review of past studies shows that researchers typically use separate measures of procedural and conceptual knowledge (Heid, 1984; Campbell, 1994; Palmiter, 1991; O'Callaghan, 1994). Often the researcher constructs an instrument to measure the specific type of conceptual understanding of interest in the study. For example, O'Callaghan (1994, 1998) designed an exam to measure conceptual knowledge of functions and Boers-van Oosterum (1990) developed a tool to measure understanding of variable. Interviews were also used to probe conceptual understanding and to allow students to explain their reasoning (O'Callaghan, 1994; Heid, 1984; Boers-van Oosterum, 1990; Sheets, 1993). For procedural skill, a common, skill-based final exam often served as a comparison for experimental and control groups (Heid, 1984; O'Callaghan, 1994, Judson, 1990).

As is common in educational research, a quasi-experimental design was used by researchers when a comparison was needed (Campbell, 1994; O'Callaghan, 1994; Matras, 1988; Heid, 1984). Pretest and posttests were used to show growth in students' understanding and establish initial group differences (Campbell, 1994; O'Callaghan, 1994). ANCOVA was used with pretest scores and math ability measures such as ACT/SAT scores or readiness exams as the covariate (Judson, 1990; Matras, 1998; O'Callaghan, 1994).

CHAPTER 3

METHODOLOGY

Introduction

This chapter begins with a restatement of the research questions as a backdrop for a general description of the research methodology and design. Complete description of the curriculum design process and characteristics of the course are followed by observations from a pilot of the instructional treatment. Then the specific design procedures of the study are explained, including instruments, sources of data collection, and data analysis methods that will be used to answer the research questions and judge the fidelity of implementation.

Research Questions

The following research questions refer to an instructional treatment comprised of a redesigned curriculum that uses CAS to facilitate implementation of the Framework for Procedural Understanding.

1. What issues arise from the design and implementation of a Framework-oriented, CAS-facilitated curriculum in a remedial algebra class?
2. Do students learning through the CAS/Framework curriculum have the same level of Framework-based procedural understanding as students in the traditional courses?

3. Do students learning through the CAS/Framework curriculum have the same level of procedural skill as students in the traditional courses?

Overview of Methodology

The researcher used both quantitative and qualitative methods to address the research questions. A semester-long quasi-experimental design used intact classes of introductory algebra students. One instructor was recruited to implement the experimental curriculum in one of his sections. The other sections of the course served as a control group for the planned comparisons. Data collected from assessments given in these courses was analyzed to answer questions two and three.

The qualitative component involved interviews of the instructor of the treatment group and a sample of students in the treatment group. The instructor and a sample of students were interviewed to answer the first research question. Interviews of students were used to validate the instrument used for question two. Observational data was collected to judge the faithfulness with which the instructor implemented the CAS/Framework curriculum and characterize the implementation of the curriculum to contribute to the answer of the first research question.

The Curriculum Design Process

Before detailing the design of the proposed research, a thorough understanding of the experimental curriculum is needed. The process of developing the new curriculum follows the principles of Backward Design popularized by Wiggins and McTighe (2005).

This involves three sequential areas of consideration: desired outcomes, acceptable evidence of these outcomes, and instructional activities for students that promote understanding of the goals and prepares students for assessment of their understanding.

The design of this curriculum took place over several months of the summer of 2006. A comparison of the ordering of the sections in the traditional and CAS/Framework classes can be found in Appendix A. An example of a lesson using CAS and the Framework can be found in Appendix C.

Identifying Desired Understandings

The first stage of developing the curriculum was to identify the essential understandings involved with the mathematics content of the course. The textbook (Bittinger, 2006) was obtained and, section by section, each chapter in the syllabus was carefully considered. The researcher and his advisor listed all of the main mathematical ideas, concepts, and procedures related to each section topic. This included material from the text and from teaching experience. An associated list of notation, language, and definitions that students would need to know was also compiled. In anticipation of the use of CAS, this list also included syntax and symbolism related to CAS input and output. Lastly, common misconceptions were identified that students would need to be aware of and surmount. Exams from the fall courses of the treatment instructor were collected and analyzed to make this list of misconceptions as comprehensive and applicable as possible.

The Framework questions were then considered for the main procedures in each section. The use of CAS as a tool to answer these questions was incorporated where applicable. For example, to verify that two expressions are equivalent, tables can be used to evaluate the expressions for a list of values. Alternatively, to understand what to expect when solving a quadratic equation, graphs of parabolas and their intercepts can be examined in conjunction with using the solve command to look for patterns in the solution sets to quadratic equations.

After the above procedure was applied to each section, the chapter goals and understandings were considered as a whole and essential questions for the chapter were developed. This allowed for a focused view of the most important understandings to address in the curriculum. Consequently, the ordering of the sections in some chapters was rearranged to fit the Heid (1984, 1988) model of applications and concepts first.

Determining Acceptable Evidence

The next stage of the curriculum development process was to determine the evidence students would provide to indicate that they achieved the desired understanding. Two types of knowledge were considered: procedural skill and Framework-based procedural understanding. Acceptable evidence of the first type of knowledge consisted of some of the more challenging problems representative of those that students would be required to solve on departmental examinations.

Communication of ideas is an important part of the Framework so it was incorporated in the evidence of Framework-based procedural understanding. Conceptual prompts were designed that could be asked in a journaling activity, quiz, or interview question. Some of these prompts were based on specific, relevant Framework questions such as, "Describe the procedures that could be used for solving systems and when it might be best to use each of them." Other prompts were designed to confront common misconceptions.

Planning Learning Experiences and Instruction

The final stage of the curriculum development phase was to develop lesson plans for use by the treatment instructor. The lesson plan was prefaced by information from the first two stages of the design process. Specifically, the desired outcomes were listed, followed by the applicable Framework questions to emphasize. Examples of the types of problems and questions students would be expected to answer were listed next in two groups as described above, problems that students would need to do to demonstrate procedural skill and questions indicating their conceptual understanding in accordance with the Framework. Lastly, a clear description of the role CAS would play in the lesson was included. This introductory information conveys to the instructor the critical aspects of the first phases of the curriculum design.

The roles of CAS discussed in Chapter 2 were implemented in the instructional experiences to lay foundations for Framework discussions. Namely CAS was used to trivialize, experiment and generalize, visualize, and focus concentration on new procedures. A few of these uses of CAS can be observed in the lesson plan found in Appendix C as examples of how it can be used in instructional planning.

The order of sections within a chapter of the text was often changed to follow Heid's (1984, 1988) model of focusing on applications and concepts before procedural work. For instance, applications of quadratic equations were discussed before methods of solving quadratic equations and factoring were learned. CAS trivialized the solving procedure so that students could concentrate on modeling application problems and answering the Framework question, “What can I use this procedure (solving quadratic equations) to do?”

CAS also allows students to construct different representations of expressions or equations. Equivalence of expressions is a central theme of the curriculum and tables help establish this notion of equivalency and are used throughout the curriculum. Graphs are used to visualize systems of equations and parabolas to answer the Framework questions of what sort of answer to expect and what the procedure can be used to do.

CAS can also be used to find patterns and generalize the outcomes of procedures. Its capability to work with multivariable expressions can be used to operate on and simplify general expressions such as $a(b + c)$. Lastly, CAS can empower students by offering them a way to do the majority of the exercises in their textbooks. An example of a complete lesson plan can be found in Appendix C.

Learning Theory and Revision of CAS/Framework Curriculum

One assumption critical to curriculum analysis and design is how learning occurs and is facilitated (Posner, 1995). One of the results of the pilot study, presented later in this chapter, was an indication of increased discourse behavior. This observation led to identifying Sfard’s (2001) communicational theory of learning as a foundation for the CAS/Framework curriculum.

The curriculum design of the piloted lessons incorporated the idea of questioning by emphasizing the Framework but these question/response activities were made more overt as a response to the pilot results and to ground the curriculum in Sfard’s (2001) learning theory. With the four-component guide to discourse practices of Hufferd-Ackles and her colleagues (2004) in mind, the role of Framework questions was retooled and activities were redesigned to promote the following behaviors:

1. Questioning: The instructor will use Framework questions and specifically chosen examples to organize classroom discussion. Students can use CAS to answer these questions and resolve issues of their understanding by asking questions of their own.
2. Explaining mathematical thinking: As the instructor questions the students and discussion ensues, the Framework questions “Why is the procedure valid?” and “How can I verify my answer?” will elicit explanation from the students. The instructor will prompt students to explain their reasoning.
3. Sources of mathematical ideas: The CAS can be used to experiment, conjecture, and generalize. The instructor will guide students in activities to use their ideas as a basis for the lesson using Framework questions such as “What sort of answer do I expect?” or “What alternatives procedures could I use?”
4. Responsibility for learning: The expectation of instructor will be that students will actively help each other in understanding ideas and correcting misconceptions. Activities will be chosen to create these situations by introducing misconceptions.

Importance of the Redesigned Curriculum

This redesigned curriculum is distinguished from the CAS curriculum of past studies like Heid (1984) and O'Callaghan (1994) by its basis in an existing, traditional skills-based curriculum and text in its 10^{th} edition. If implementation proves to be successful and offers evidence of improved understanding, then this procedure can be applied to other curricula of this type and allow CAS and the Framework to be integrated into existing programs.

Population and Sample

Student Population

The population for this study is comprised of the students enrolled in introductory algebra at the College of Technology in Bozeman (COT), an extension of Montana State University – Great Falls that is located on the Montana State University – Bozeman (MSUB) campus. The introductory algebra course is considered remedial in nature as it is a required prerequisite of college algebra and other college-level courses.

Instructor Selection

There are currently four full-time mathematics instructors at the COT. One of these instructors was recruited for the proposed study because of his familiarity with the Voyage 200 and his willingness to participate. This instructor, hereafter referred to as the treatment instructor, was also involved with the pilot project, taking into account Hasenbank's (2006) suggestion that the instructor be familiar teaching with the Framework before implementation. During the spring 2007 semester, the treatment instructor taught three of the eight sections of introductory algebra. The treatment section was purposively chosen from these three sections by the use of a pretest described below (see p. 58).

Instructor Qualifications. The treatment instructor holds a Master of Science degree in mathematics with a focus on mathematics education. This degree program included mathematics coursework that utilized CAS technology. At the time of implementation, the treatment instructor had held his position at COT for nearly two years. The treatment instructor had taught the introductory algebra course during three previous semesters and was familiar with the traditional curriculum and text. The treatment instructor's capability to teach the CAS/Framework curriculum is strengthened further by his ability to elicit student responses and incorporate them in each lesson, as evidenced by pilot study observations.

Student Sample

The students who received the experimental treatment were those enrolled in one section of the treatment instructor. The other sections, two taught by the treatment instructor and five taught by the other COT instructors, will be used for comparisons. Thus intact groups determined by registration procedures and teaching assignments were used. A pretest, described on page 57, will be given on the second day of class to determine the initial level of skill and knowledge of the different sections. The treatment group will then be chosen (see p. 65 for selection methods) to ensure that the students are not significantly different in their initial mathematics skills.

Students in the treatment section were notified by the instructor about the use of CAS calculators in the course and be given a written explanation on the third day of class. The CAS/Framework curriculum does not differ from the traditional courses until the Chapter 2 of the text, approximately 2 weeks after the beginning of the semester. Students can and often do change schedules in the first weeks of class so those students that do not wish to be involved in a class with CAS are free to change sections.

Course Description

Introductory algebra is a 4 credit lecture-based course offered by the COT as a lecture-based alternative to the self-paced program for introductory algebra offered at MSUB. Between the fall of 2001 and the summer of 2005, 3130 students enrolled in introductory algebra at MSUB. Of these students, 42.1% passed during this time period, some after multiple attempts. This is consistent with pass rates found in other research studies but below the “acceptable” pass rates of 61-70%. However, a significant amount of students withdrew from the course (20.6%) or received an incomplete (36.3%) on their first attempt so it cannot be assumed that there is a 57.9% failure rate. The COT is becoming a more popular alternative for students to complete introductory algebra and is the setting for the implementation of the Framework/CAS curriculum.

In the spring 2007 semester, the COT offered 8 sections with a maximum enrollment of 30 students. Each section met for 50 minutes Monday through Thursday. All sections used the 10th edition of the Bittinger (2006) text that was the basis for the revised CAS/Framework curriculum. The Bittinger text does not present the idea of a function until the last section of the last chapter which is not on the COT syllabus.

Graphical connections are mentioned but in a supplementary manner. The focus is predominately on manipulating algebraic expressions, particularly solving equations, factoring, and simplifying. Standard word problems are presented at the end of each chapter and chapters are organized around specific types of algebraic expressions such as polynomial, rational, and radical expressions (See Appendix A).

In the spring 2006 semester, the average attendance rates for each the instructor's introductory algebra classes was close to 75%. This does not include the few withdrawals of students in each section. The majority of the individual attendance rates ranged from 55% to 86 %. This describes trends consistent with research results presented in Chapter 2 (p. 11). The COT faculty implemented an attendance policy in its classes described by the following excerpt from the instructor's syllabus.

> ATTENDANCE IS REQUIRED AT ALL CLASS MEETINGS. Although attendance is required, there will be instances when students are not able to make it to class. Each student is allowed a maximum number of personal days in COT courses before your grade is effected.
>
> <u>Personal Days</u>
> Students are granted a maximum number of 4 personal days which is equivalent to missing one week of the course.
>
> <u>Excused Absences</u>
> Instructors may grant students excused absences for extraordinary reasons. Documentation maybe required. Students seeking excused absences, after using allotted personal days, for university-sponsored events require documentation at least one week in advance.
>
> <u>Unexcused Absences</u>
> Absences in excess of the maximum personal days allowed for the semester result in the loss of points for the course. Unexcused absences equivalent to one week of the course result in an 8% loss of the course grade for the course.

Examples of Attendance Deductions:
This course meets 4 days/week: Grade drops 2% per unexcused absence.
For example for a total of 9 absences—4 personal days, 5 unexcused absences would drop a grade 10%.

Pilot Study

In the fall semester of 2006, five lessons of the redesigned curriculum were piloted in an introductory algebra course of the treatment instructor. The purpose of this pilot was twofold: to inform the researcher of issues involving the introduction of CAS into a classroom setting and to familiarize the instructor with teaching using CAS and the Framework. The researcher chose five lessons from three chapters and observed as the instructor presented each lesson. A classroom set of Texas Instruments Voyage 200 calculators with an overhead projector display was provided during these pilot lessons.

A summary of observations and feedback from students and the instructor is offered here but a full report can be found in Appendix D. The most common question from the students was about the availability of CAS on the quizzes and exams. A general consensus was that they would rather focus class time on learning to do it by hand as this would be required on the exams. Hasenbank (2006) noted that students seemed to pay most attention when the step-by-step procedures were being explained as they viewed this to be the most important goal of the course. Prior to full scale implementation of the new curriculum, students were presented with the Framework-oriented goals of the course. Another modification was to allow CAS use on some quizzes and a portion of each unit exam. This was done to motivate students to learn how to use the calculator and alleviate their concerns.

Despite the desire to have the calculators on the exam, about two-thirds of the students thought the calculator was helpful, especially to check their work quickly and accurately and to visualize concepts. Most students did not indicate difficulty learning how to use the calculator. A few who did have trouble admitted the need to pay attention and ask questions. The complaint that there were too many buttons and functions can be alleviated with a poster or emulator. A poster was used during implementation for the study since an emulator for the Voyage 200 is not available.

Before each pilot lesson, the instructor was given a copy of the lesson from the experimental curriculum (See Appendix C for an example). The researcher discussed any questions the instructor had and walked through the lessons step by step. He expressed that he was able to teach the relevant material for the exams and felt that the added CAS component could have provided enrichment for the students. He particularly felt that, "Students are actively engaged (CAS work) for more of the class time than in my typical non-CAS lessons" (Appendix D). This alludes to the researcher's observation that these lessons seemed to promote discourse through the opportunity that CAS and the Framework provided.

One particular discussion in the second pilot lesson underscores the value of discourse and how the revised curriculum has the potential to enhance this discourse. Students were asked to think about the expression $\frac{m^2+9}{m+3}$ and write down what they expected from the simplification process. Then using the CAS students entered the expression to check their answer. This expression is already simplified as there are no factors common to the numerator and denominator so the CAS just returns the same expression. Students then discussed the answers they had expected and why they expected them. In explaining the reasoning they had used, the instructor and other students were able to correct mistakes and address common misconceptions regarding simplifying rational expressions.

In the third pilot lesson, the instructor had finished demonstrating a problem using by-hand manipulations on the board. A student had checked the answer using CAS and noticed a difference in the placement of the negative sign. This lead to a discussion of placement of the negative sign in a rational expression, not a trivial concept for this level of student. These behaviors and others observed in the pilot are consistent with the components of discourse that Hufferd-Ackles and colleagues (2004) expressed. Specifically, students explained their thinking, questioned the instructor and each other, and took more responsibility for their learning. The instructor noted, “Students seem more willing to participate in discussions, maybe because the lesson plans explicitly call for discussion throughout the lesson” (Appendix D).

The pilot lessons also informed the researcher of the teaching style and habits of the instructor chosen to teach the CAS curriculum. Consistent with promoting discourse, the instructor challenged his students by asking for explanations of their ideas. The Framework questions that are integrated into the lessons seemed to fit with his style of teaching. Fidelity of implementation is a critical issue and observations from the pilot indicate that the instructor is learning how to use the Framework and incorporate that specific style of questioning and discussion into his lectures.

Design of the Study

The following section describes the treatment to be given to one section of introductory algebra and the experience of the remaining sections that comprise the control group. Then the instruments used in the study and the methods of data collection and analysis for each research question are discussed.

The Instructional Treatment

One section of introductory algebra taught by the treatment instructor received instruction based on the revised CAS/Framework curriculum. These students were notified on the third day of class that their section would be taught using calculators. The review chapter and Chapter 1 of the text were covered in the same manner as the other sections. During the second week of class, a set of Texas Instruments Voyage 200 calculators with CAS capabilities were brought to class by the researcher and checked out to the students. For the next 11 weeks, the instructor taught according to the revised CAS/Framework curriculum designed by the researcher. The researcher had phone conversations or in person discussions with the instructor prior to each of the lessons. This was to clarify any questions or concerns that the instructor had about the lesson plan. The calculators were returned on the day of the last unit exam. Two significant differences set the treatment class apart from the other sections of introductory algebra, calculator activities coupled with Framework discussion prompts and modified and additional assessments.

Calculator Activities and Discussion. Each of the daily lessons in the CAS/Framework curriculum included calculator work. Often students would work several examples at the beginning of class and then report to the instructor. Other times the instructor would work on the calculator with the students while the screen was displayed by an overhead projector. Sometimes the instructor would display screens that had been preloaded in preparation for the day's lesson. The reason for these calculator activities was to provide material for discussion of the Framework questions. These activities and discussions replaced time that would have been used by the instructor to work examples and time he would have given students to practice in their seats. For an example of these types of activities see Appendix C.

Additional and Modified Assessments. Assessment of desired understandings is an integral part of a curriculum. For this reason, one quiz was given during each two-chapter unit of the curriculum to assess students' Framework-based understanding of procedures. These quizzes gave emphasis to the central role of the Framework in the curriculum. A full description is given in the next section of this chapter.

The students receiving the instructional treatment were allowed to use their Voyage 200 CAS on the Framework quizzes and portions of the unit exams. This was done in response to the pilot study results as noted above to avoid the complaint of students not being allowed to use the technology on tests. It also served to align instructional activities more closely with the course assessments.

The Control Group

The eight other sections of introductory algebra were used as a control group for comparisons in this study. These sections did not receive specialized instruction involving the Framework or CAS. Sections and chapters were covered in order (See Appendix A) with the possible exception of Chapter 3, Graphs of Linear Equations. In some sections, this chapter was covered after Chapter 6 but before Chapter 7 on systems. The students in the control group were exposed to each procedure through lecture and examples, followed by time for students to work at their seats. Homework was assigned to both the treatment and control groups to provide them with repetitive practice.

Data Sources

The primary independent variable of this study is use of the CAS/Framework curriculum. There are two main dependent variables. The first is Framework-based procedural understanding, the conceptual platform for the procedural work in Bittinger. Procedural skill was considered as a key dependent variable. Data collection for these variables is discussed below. In addition, interview and observational data used to answer research question one and judge fidelity will be described.

Procedural Understanding Exam. The researcher constructed an exam consisting of eleven hypothetical, scenario-based prompts to assess students' level of Framework-based procedural understanding. The course curriculum consists mainly of two groups of procedures, simplifying algebraic expressions and solving algebraic equations. These procedures are performed on single-variable polynomial/quadratic, rational, and radical expressions. Linear equations in two variables are discussed in terms of representing solutions in graphical form and solving systems of two equations. The test was designed to ask questions from each of the two main procedure types within each of the four topic areas listed above (see Figure 1). It was also a designed to get at least one question from each of the Framework objectives.

Construction of the exam questions was informed by the results of the Framework quizzes given to the treatment class during the semester. These quizzes were used as a pilot of this type of question and as a data source for reliability and validity. The exam's validity was also evaluated by an independent reviewer. Question 9 was changed to reflect that work rate problems were a standard procedure in the curriculum.

Figure 1 – Content of the Procedural Understanding Exam

<table>
<tr><th></th><th>Lines and Systems (Ch. 2, 3 & 7)</th><th>Polynomials and Quadratics (Ch. 4 & 5)</th><th>Rational Expressions and Equations (Ch. 6)</th><th>Radical expressions and Equations (Ch. 8)</th></tr>
<tr><td>Simplifying</td><td rowspan="2">1b, 2a, 2b</td><td>3, 5, 6</td><td>2a,3, 5</td><td>6</td></tr>
<tr><td>Solving</td><td></td><td>1b</td><td>3, 4</td></tr>
</table>

The exam was administered in one 50-minute class period during the last week of classes. A copy of the test as given to the students and the task-specific rubric can be found in Appendices H and I, respectively. Prior to exam day, the researcher assigned a random, five-digit identification number to each student in all sections. Exams were given to the instructors the day before administration but after teaching for that day was over. The researcher collected each set of exams after each class was finished and photocopied them while covering identifying information with the assigned identification code. This prepared them for later analysis.

Two of the eighteen students in the treatment section who took the final exam were not in attendance for the procedural understanding. One student was absent every day during the last week of class. The second student was only in attendance the last day of class and was unable to make up the exam. These two students were habitually absent and were the only students who had missed over 20 class periods. All other students in the treatment section had 11 or less absences.

Procedural Understanding Quizzes. Four quizzes were developed by the researcher to measure the CAS/Framework students' Framework-based procedural understanding. One quiz was given during each two-chapter unit of the curriculum (see Appendix B) and corresponded to the procedure for solving a specific type of equation found in those units, specifically linear equations in one variable, linear equations in two variables, quadratic equations, and systems of equations. Other skills in the curriculum such as simplify, factor, add, or multiply were viewed as subordinate skills to the task of solving. The solve procedure is also central to completing application problems and for these reasons was chosen to be the procedure of focus for these quizzes.

The design of the quiz questions followed the method used by O'Callaghan explained in Chapter 2 (p. 32). Each question probed one or more aspects of the Framework with respect to the "solve" procedures of algebraic equations.

Misconceptions provided centerpieces for classroom discourse and similarly provided tasks to assess understanding both on the quizzes and in interviews. See Appendices E and F for an example of a Framework quiz and grading rubric, respectively.

The students were given the last 30-35 minutes of one class period to complete each quiz. All but one of the quizzes was based on the first chapter of each two-chapter unit and were given as the class transitioned from one chapter to the next. Take-home assignments similar to the quizzes were given for the chapters not quizzed on. These were used for review by the instructor but not graded.

Follow-up Quiz Interviews. Interviews of three students were conducted after the second, third, and fourth procedural understanding quizzes. These interviews were based upon their written responses to their quizzes and an unmarked copy of their quiz was presented at the time of the interview. Students were asked to clarify or expand certain responses to explore their understanding outside of a limited-time, written exam. Additionally, students were asked Framework questions that are better suited to an interview format (see Appendix G). These interviews were also used to establish the validity of the quiz questions and the reliability of the student answers. These quiz questions were considered a pilot of the type of questions used on the procedural understanding exam. The researcher employed this design to establish that the questions were valid in the sense that students understood the wording and what the question was asking. Another consideration was to be sure the written response reliably represented the students' understanding.

Interviews were attempted after the first quiz, but no students agreed to be interviewed. This may have been because of the invitation strategy, which was to randomly select students and give them notes during the quiz explaining and asking them to participate in the interview. The day before the second quiz, the researcher's advisor and the dean from COT visited the first few minutes of class to express appreciation and emphasize the importance of the research project. The next day the researcher handed out notes to randomly selected students prior to the quiz with a verbal explanation to the class and added incentive of lunch. The researcher then waited outside of the classroom to intercept the students as they left to schedule a time. A similar process was done for quizzes three and four but the incentive of food was not used since none of the previous students took advantage of it.

The students were chosen to receive invitations for the interview based on random selection. The researcher randomly chose 6 students who had not been interviewed previously and scheduled three out of six depending on attendance on the quiz day and willingness to participate. Interviews were done within 36 hours of the quiz and before graded quizzes were handed back to the students. Students were given a sample answer sheet for each quiz along with their graded quiz so they could become more familiar with the Framework ideas.

Procedural Skill Pretest. A pretest in procedural skill was given to all students enrolled in COT introductory algebra courses. This was the first semester that COT faculty instituted a pretest but this practice will be continued by their department in all mathematics classes as part of their evaluation program. During the week prior to the beginning of the spring 2007 semester, a pretest was developed by two faculty members from the COT and the researcher. During the meeting all other COT faculty members were consulted about the use of calculators of any kind on the pretest and final. All agreed that they would be willing to forbid the use of calculators provided that the required arithmetic was manageable so that student work time would be spent largely on algebraic skills and concepts.

Each faculty member brought a copy of the final exam that they had given in the previous semester to establish a pool of questions for the pretest. Each instructor highlighted certain problems on their exams that they suggested be placed on the pretest. The first time this was done, problems were chosen that were interesting, challenging, and multi-step. Upon reflection of the purpose of the pretest, it was agreed that problems should be chosen that were acceptable for a final exam but simple enough that the pretest would provide a baseline for comparison (more variation in scores instead of all zeroes).

A pretest of 20 questions was constructed by choosing 18 previous final exam questions. The researcher, who had reviewed the curriculum extensively to incorporate the Framework and CAS, and the experienced instructors, felt that the questions represented the range of topics taught in the course. Questions were also chosen that could be asked in a multiple choice format. One question was rewritten to facilitate multiple choice answers. A question dealing with simplifying an expression with exponents was spilt into two questions so that each question could focus on just two rules of exponents. There was also one question that was not pulled from either existing final exam. The test authors felt that a graphing question needed to be asked and agreed that a multiple choice graphing question would be found in the test-generating software provided with the textbook.

Once the list of questions was chosen and agreed upon, the instructors merged the test files and set problems to the multiple choice format. Some questions had been modified from their original form in the test bank software so alternative choices were written to transform it to multiple-choice format. The test was then reviewed by the instructors and researcher. All Mathematics 101 instructors were asked to give this on the second day of class, unannounced, and without calculators. This was done to get as many students attending as possible since the first day of class was a Thursday with the class not meeting again until Monday.

Procedural Skill Posttest. The common-hour final exam was written by COT faculty from the Bozeman and Great Falls campuses. It was 40 questions in a multiple-choice format. At the request of the researcher, it contained as a subset a group of questions from the pretest. The faculty selected 13 questions for the final that were also asked on the pretest. This allowed for comparisons to be made between classes from the beginning to the end of semester. All introductory algebra students took the exam at the same time in the same room during finals week and were not allowed calculators. Both the pretest and posttest were multiple choice and graded electronically.

Instructor Debrief Interview. The treatment instructor was interviewed by the researcher about time constraints, using CAS in the classroom, and his perceived impact of using the CAS/Framework curriculum (see Appendix K). The interview was conducted after the last day of class but before the final exam was given and the procedural understanding tests were evaluated. The interview was transcribed for later analysis.

Student Interviews. Students were interviewed about their ability to use CAS, how CAS impacted their learning in the course, and suggestions for improving the course for future students. The nine students that were interviewed following the Framework quizzes were approached to be interviewed about CAS in the classroom. This was done because they were familiar with the process and with the interviewer. Also, an incentive of a $25 gift certificate was being offered and it was felt it should be offered to the previously interviewed students first. Eight interviews were conducted. It was originally planned that ten would be conducted but the class size decreased from 29 to 18 students so eight was considered a sufficient sample size. It should be noted that although the selection for these interviews was purposive, the students were originally chosen through a random process for the follow-up quiz interviews.

The interviews were conducted between April 11 and April 18, about 3 - 4 weeks before the end of the semester. The researcher was assisted by another graduate student who conducted and transcribed four of the eight interviews. The researcher was present for all eight interviews, conducting and transcribing four of them. The interview questions are listed in Appendix L. Each interview lasted from 10 – 20 minutes.

Researcher Observations and Journal. The researcher attended class every day except for test review days, test days, and a few other days on which there were scheduling conflicts. The researcher made notes about the implementation of the lesson, student engagement, and other significant events. These journal entries will be used to describe the students and classroom as well as corroborate and offer insight into the results of this study.

Fidelity Observations. Two graduate students assisted in the fidelity of implementation portion of this study. Each of them was provided with a copy of the Framework objectives and student-centered questions with supporting literature. They also were supplied with research articles on the use of CAS in the classroom and sample lesson plans from the CAS/Framework curriculum that served as the treatment in this study.   After reviewing these materials, the researcher met with these colleagues and a preliminary observation tool was designed. Five class periods were observed, two for development and training and three for data collection purposes. The instructor had advance notification of each of these observations.

The first observation was made to familiarize the independent observers with the class and pilot the preliminary observation tool. Between the first and second observations, the observers met and formalized the observation tool (See Appendix M). The main goal of these observations was to determine if the instructor was faithfully implementing the curriculum. This required judging if the appropriate Framework questions were emphasized in the lessons, whether the CAS activities were completed and used as intended, and characterizing the effect the Framework and CAS had on classroom discourse.

The second observation was videotaped so that the observers could use it for training purposes. The students and instructor were given advance notice and a copy of the video was supplied to the instructor. Between the second and third observations, the observers met and reviewed the use of the formal observation tool. The video tape of the second observation was viewed to allow for the observers to come to agreement on how the tool should be used.

Three more observations were done to collect data concerning the fidelity of implementation of the CAS/Framework curriculum. These observations were done during the eighth, ninth, and eleventh weeks of class (See Appendix B). The observers completed their observation evaluations and then met to discuss the results. Each observer submitted a report to the researcher expressing their impressions and conclusions by answering the six fidelity questions listed in the observation tool (Appendix M). The analysis of this data is discussed later in this chapter and the results are presented in Chapter 4. Data from these observations was used in conjunction with student and teacher interviews and other researcher observations to fully characterize the impact of the CAS/Framework curriculum in response to the first research question.

Data Analysis

In the following section, the methods of data analysis that were used to choose the treatment section and judge fidelity of implementation are detailed. Then the analysis procedures used to seek answers to the research questions are described. This includes the methods used to establish the validity and reliability of the procedural skill and procedural understanding instruments. In addition, the methods for analyzing the interview data are described.

Analysis of Pretest Scores

Because intact groups were used for this study, selection of the treatment section was done after testing the equivalency of the different sections based on pretest scores. A Pearson's chi-square test was used to compare the distribution of pretest scores from each of the treatment instructor's sections, 1, 3 and 6, with the pooled distribution of pretest scores from the other 7 sections in the following way.

The overall range of scores on the pretest was from a low of zero to a high of thirteen. This range was broken into four intervals 0 – 3, 4 – 6, 7 – 9, and 10 – 13. The distribution of scores for section 1, a possible treatment section, was represented in a frequency table and compared to a similar table representing the distribution of scores for the other seven sections. The Pearson Chi-square analysis tested the hypothesis that these two distributions were different. The process was repeated for section 3 and 6. Tables and results of this analysis can be found in Chapter 4 (p. 71).

Fidelity of Implementation

Three class periods were observed by two independent observers and the researcher. A qualitative summary was made by each observer and the researcher by answering the following questions from the observation tool (Appendix M).

1. Were the Framework objectives identified in the desired understandings of the lesson appropriately emphasized?
2. To what extent were the Framework questions used for discussion?
3. What was the level of student participation in the Framework discussion?

4. Were the CAS activities used as intended?
5. Were the CAS activities used to promote classroom discussion? Answer the intended Framework questions they were designed for?
6. Characterize the student and teacher behavior/interaction during the CAS activities.

For each observation, the answers that each observer provided for the six questions above were extracted verbatim and compared question by question and a conclusion was made for each question. Then the conclusion statements for question 1 were compared across the three observations to arrive at a final determination of fidelity. This process was repeated for the remaining questions and a six-point overall fidelity evaluation was achieved. This fidelity report can be seen in Chapter 4 (p. 73).

<u>Effect of Treatment on Procedural Understanding</u>

The procedural understanding exam was used to assess the degree of students' procedural understanding as it pertains to the Framework (See Appendix H). Prior to the first scoring, the researcher developed a task-specific rubric for each exam question. The tests were put in ascending order by their five-digit identification codes in effect shuffling the treatment section along with the other sections. The researcher then scored the exams one question at a time. No marks were made on the exams, but scores were kept on a separate spreadsheet listing identification codes in ascending order with a column for each question. After each question, the researcher reflected upon the responses and adjusted the rubric for more specificity and clarity. For major changes the researcher adjusted the scores.

After a week and a half, the researcher again graded the exams question by question. This was done to establish stability and consistency. The researcher also trained an independent reviewer to establish inter-rater reliability. A random sample of 35 exams (about 25%) was selected. After this was done the researcher purposively chose 6 exams to get a range of scores for questions with lower intra-rater scores and a reasonable range of scores for the remaining questions. The independent reviewer scored the 6 training exams and then consulted with the researcher to confirm that the rubric was being interpreted properly. A few clarifications were made and then the independent reviewer scored the sample of 35 exams.

Three measures were used to assess stability and inter-rater reliability for each question on the exam. First, the percentage agreement on a point-by-point basis was determined. Second, Pearson's *r* was computed to measure the association between reviewers' scores. That is, the degree to which high (low) scores of one reviewer correlated to high (low) scores of the other reviewer. Third, Cohen's *kappa* was found to measure the agreement rate of the two reviewers on each question, taking into account the expected rate of agreement due to chance. Pearson's *r* was also computed to measure the association between the average question score given to each student by the researcher and reviewer. The results of these analyses can be found in Chapter 4 (p. 83).

For each student, the average question score was computed as measure of their Framework-based procedural understanding. An independent samples *t*-test was done to test for a difference in average question score between the treatment group and the control groups. An analysis of covariance was also done using pretest scores as a covariate. In addition, the researcher also conducted independent samples *t*-test to test for differences between the treatment group's average score on each question versus the control group's average score for each question.

Effect of Treatment on Procedural Skill

The final exam scores were compared using an analysis of covariance with the pretest scores as the covariate and treatment condition as the fixed factor. However, since groups were intact prior to treatment assignment, a gain score approach was also done as recommended by Gliner and Morgan (2000).

By design, there is a subset of 13 questions that appear on both the pretest and final exam. A gain score analysis was done using an independent samples *t*-test to compare the mean change in correct answers on this subset of questions from the pretest to the final exam. This allowed the growth of procedural skill to be compared irrespective of the differences in level of skill present prior to the study.

Student Issues

The eight interviews conducted near the end of the course on the use of CAS were transcribed and two types of analyses were conducted. First, the researcher identified where each question was addressed in the interview transcript. The eight responses to question 1 were read and summarized, and then the same procedure was carried out for the other 11 questions (See Appendix L). This allowed the researcher to tally responses to yes/no type questions and also get a sense of emerging themes and a list of preliminary codes to use.

Second, the researcher excerpted the substantial snippets of text, usually a phrase, sentence, or sometimes two sentences. Then, using the broad themes from the first reading as a guide, related snippets of text were coded and grouped. The researcher read the transcripts again, assigning most relevant snippets a code and organizing the codes into the themes presented with frequencies in Chapter 4.

Instructor Perceptions

The instructor's interview was transcribed and coded similarly to the student interviews as explained above. Summary data is presented according to the emergent themes to indicate important considerations for implementing and teaching this curriculum. This presentation was reviewed by the instructor for accuracy in representing his ideas.

CHAPTER 4

RESULTS

Introduction

This chapter contains the results of the data analysis conducted for this study. The first section examines the characteristics of the treatment instructor's three sections including the results used to select the treatment section. Next, the qualitative results concerning the fidelity of implementation are presented to establish the degree to which the CAS/Framework curriculum was adhered to in the treatment section. Then there are two sections devoted to the comparison of the treatment section to the other sections regarding their level of procedural understanding and procedural skill. Lastly, the perceptions of the students and instructor are presented based on analysis of interview transcripts.

Characteristics of Treatment Instructor's Sections

Pretest and Section Selection

The treatment section for this study was chosen based on analysis of pretest results. This was done to ensure that the treatment group was equivalent to other sections according to previous procedural skill. The methodology for this analysis was described in Chapter 3 (p. 65). Table 1 shows the breakdown of scores into the specified intervals for section 3. A similar table was done for the other two sections taught by the treatment instructor, sections 1 and 6.

Table 1 – Distribution of Pretest Scores
(Section 3 vs. Other Sections)

Pretest Score Interval	Section 3	Other Sections
	n = 27	*n* = 139
0-3	2 7.4%	9 6.5%
4-6	6 22.2%	49 35.3%
7-9	16 59.3%	62 44.6%
10-13	3 11.1%	19 13.7%

A Pearson Chi-square test was done to test the null hypothesis that the distribution of pretest scores of section 3 was the same as the distribution of scores for the other sections combined. This was also done for sections 1 and 6. There was no statistical significance (see Table 2) for any of the Pearson Chi-square statistics. Section 3 was chosen as the treatment section based on its higher significance value.

Table 2 – Comparison of Sections Based on Pretest

	Section		
	1	3	6
Pearson Chi-square	2.634	2.33	2.534
Significance	0.452	0.596	0.469

Attendance

Attendance was kept each day of class due to the attendance policy that COT put in place this semester (See p. 49). A one-way analysis of variance (ANOVA) was conducted using section number as the independent variable and students' number of absences as the dependent variable. The means, standard deviations, and class size are presented in Table 3. The ANOVA ($F(2, 50) = .978$, $p = .383$) showed no significant difference in mean absence rates among the treatment instructor's sections.

Table 3 – Average Absences

Section	*M*	*SD*	*N*
1	4.21	2.119	14
3*	6.78	7.000	18
6	5.33	4.820	21

* Denotes treatment section

The students included in this sample are those that completed the course by taking the final exam. Since the sample sizes were different, a test for equality of error variances was done which was not significant at the .05 level.

Drop Rates

The drop rates for the treatment instructor's sections are displayed in Table 4. The treatment section had a higher drop rate but not alarmingly so. The instructor and researcher were able to become familiar with the circumstances of some students and also, as calculators were retrieved, had the opportunity to follow-up with some students as they dropped the course. None of these decisions seemed to be related to the treatment condition. For example, one student mentioned in an interview that she was taking 20 credits during the semester and mathematics had the lowest priority. She did not complete the course after her grades slipped beyond recovery. Another student dropped after she became ill for several weeks and felt she was too far behind. A third student dropped because he had been dating a girl in the class and the relationship had ended. The difference in drop rates was not deemed to be significant as it is consistent with the typical one-third drop rate (see p. 1).

Table 4 – Drop Rates for Treatment Instructor's Sections

	Section		
	1	3*	6
Started Course	18	29	28
Took Final Exam	14	18	20
Drop Rate	22%	38%	29%

* Denotes treatment section

Fidelity of Implementation

In this section the faithfulness with which the treatment instructor implemented the CAS/Framework curriculum will be discussed. This will be done by presenting the qualitative observation results to answer six fidelity of implementation questions. Three of these questions deal with the way the Framework was used in class and the remaining three dealt with the way CAS was used in class. The schedule for the observations is found in Appendix B. The observational data that was used to arrive at the conclusions presented in these sections can be found in Appendix N.

Faithfulness to the Framework

In this section, data will be presented to answer the first three questions on the observation tool found in Appendix M regarding implementation of the Framework portion of the curriculum. For each question, the conclusion made based on the three observer's comments will be presented for each observation and the fidelity implications will be discussed.

Fidelity Question 1. Were the Framework objectives identified in the desired understandings of the lesson appropriately emphasized? The conclusions made for each of the three fidelity observations for question 1 are presented in Table 5.

Table 5 – Conclusion Statements for Fidelity Question 1

Observation	Were the Framework objectives identified in the desired understandings of the lesson appropriately emphasized?
1	Time issues did not allow for the Framework objectives to be emphasized in as much depth as they should have been. Of what was completed, the emphasis was appropriate.
2	The Framework objectives were emphasized as appropriate.
3	The Framework objectives were appropriately emphasized, though one aspect may have been indirectly addressed.

Based on the observations, the instructor was able to appropriately emphasize the Framework objectives as required in the lesson plans. The first observation was not ideal but, as in most classrooms, the instructor needed to adapt to the needs of the students. That particular lesson was the day that the previous unit exam was passed back. Time was spent addressing questions which limited the material the instructor was able to teach. The researcher's journal for the day after observation 1 reads, "He gave a good review and tied FW [Framework] concepts together as he caught up on the graphing activity. [The instructor] used the overhead display to demonstrate the graph and had the equations already entered to save time." The instructor was aware of the need to cover all of the material to prepare the students for the quizzes. Although the lesson was not completed, the quality of the lesson plan that was completed during the first observation was acceptable for implementation purposes.

Thus, in response to the fidelity question 1, the instructor emphasized the Framework questions as appropriate and intended in the lesson plans.

Fidelity Question 2. To what extent were the Framework questions used for discussion? This question is important because the intent of the lesson was not only to focus on the Framework question but also to promote learning through classroom discourse. The conclusions made for each of the three fidelity observations for question 2 are presented in Table 6.

Table 6 – Conclusion Statements for Fidelity Question 2

Observation	To what extent were the Framework questions used for discussion?
1	The instructor was only able to get limited responses from the students with no resulting discussion. This was not because of lack of effort or attempts to probe. As a result the Framework objectives were, in large part, lectured on.
2	The instructor made good use of questions throughout the lesson. Theses questions were direct Framework questions or questions central to Framework objectives. However, some Framework questions were not asked directly, but in a rephrased form.
3	The Framework questions were used to try and promote discussion but when this failed the Framework questions were used to organize the instructor’s presentation.

The conclusions made from the observations begin to describe the atmosphere of the class. The instructor made repeated efforts to begin discussion with the students.

Most times these efforts were unsuccessful but the instructor still based the lesson presentation on Framework objectives, sometimes answering his own questions. One important point made in observation 2 was that the instructor often rephrases the Framework question. This is acceptable because the questions asked on the quizzes and exam were not the Framework questions verbatim, but questions designed to focus on Framework objectives.

In answer to the second fidelity question, the instructor faithfully endeavored to build classroom discussion but student response was limited. This is described in greater detail by the observations made for the next fidelity question.

Fidelity Question 3. What was the level of student participation in the Framework discussion? The observations made for this question shed more light on the students' reception of the Framework portion of the treatment. The conclusions made for each of the three fidelity observations for question 3 are presented in Table 7.

Table 7 – Conclusion Statements for Fidelity Question 3

Observation	What was the level of student participation in the Framework discussion?
1	The instructor mainly had to call on students to answer questions. Often it took several attempts before an answer was given.
2	Students were able to answer questions directed to them and this provided some classroom discussion to complement the instructor's lecture.
3	The instructor had to direct questions to students who often answered "I don't know" on impulse. The instructor tried to probe and rephrase to get an answer from students.

The pattern of classroom interaction is clear from the above conclusions. The instructor must direct questions to specific students. In most cases this is a drawn out process of probing and rephrasing the question for a given student or asking several students until getting an acceptable response. It seemed that students did not even think before answering "I don't know." Several instances are documented in the researcher's journal that describes the instructor's frustration with students not paying attention or taking class seriously. This resulted twice in the instructor sternly lecturing the class and leaving the room for a few minutes.

Conclusions about this apathy on the part of the students are presented in the next chapter. It is clear that the answer to fidelity question 3 is that students rarely participate in classroom discussion and answer questions involuntarily, however the instructor faithfully made the efforts to begin discussion.

Faithfulness to the Intended Use of CAS

In this section, data will be presented to answer the last three questions on the observation tool found in Appendix M regarding implementation of the CAS portion of the curriculum. For each question, the conclusion made based on the three observer's comments will be presented for each observation and the fidelity implications will be discussed.

Fidelity Question 4. Were the CAS activities used as intended? The conclusions made for each of the three fidelity observations for question 4 are presented in Table 8. The first observation was of a lesson that was not completed so the observers answered the question on the quality of the portion of the lesson that was completed. As discussed above, the lesson was completed the next day. The unanimous finding was that CAS activities were completed as intended to trivialize, experiment, and visualize.

Table 8 – Conclusion Statements for Fidelity Question 4

Observation	Were the CAS activities used as intended?
1	CAS was used as intended for the portion of the lesson that was finished.
2	CAS was used as intended in the lesson plan.
3	The CAS was used as intended by the lesson plan.

Fidelity Question 5. Were the CAS activities used to promote classroom discussion? Answer the intended Framework questions they were designed for? The conclusions made for each of the three fidelity observations for question 5 are presented in Table 9.

Table 9 – Conclusion Statements for Fidelity Question 5

Observation	Were the CAS activities used to promote classroom discussion? Answer the intended Framework questions they were designed for?
1	The teacher used the calculator activities to provide the material needed for a Framework discussion and then posed the questions in the lesson plan. However, classroom discussion was not achieved as noted previously.
2	The CAS activities were used to promote classroom discussion and discuss the ideas central to Framework understanding.
3	The CAS was used to introduce and provide a basis for Framework question discussion.

The difficulty of sustaining classroom discussion has been already been noted. Without taking into consideration whether discussion was achieved, the calculators were observed to be effective tools for providing discussion material and setting the stage for focus on Framework objectives. In relation to fidelity question 5, CAS was used as intended to lay the foundations for classroom discussion.

Fidelity Question 6. Characterize the student and teacher behavior/interaction during the CAS activities. The conclusions made for each of the three fidelity observations for question 6 are presented in Table 10. The observations support the conclusion that the students were able and willing to complete the CAS activities. This kept the students engaged at their seats and on task with the material, fulfilling one of the goals for CAS use.

Table 10 – Conclusion Statements for Fidelity Question 6

Observation	Characterize the student and teacher behavior/interaction during the CAS activities.
1	Most of the students participated in the activities and seemed to have no troubles doing so. The CAS activities were student-centered.
2	The calculator work was student-centered and it appeared that students were able to perform the calculations.
3	Most students are engaged during calculator activities. The students do computations and answer the teacher's questions. The teacher preloads graphs and tables to show the students.

Summary of Fidelity Findings

The fidelity of implementation observations were done to judge how faithfully two central aspects of the curriculum, the Framework and CAS, were implemented by the treatment instructor. For the instructor's part, he faithfully emphasized the Framework objectives and made appropriate efforts to build classroom discussion. The student's, however, took on a passive role except in calculator activities and the instructor struggled to elicit verbal responses. Thus, the conclusion is that the instructor faithfully presented the treatment, or enacted the curriculum, but the experienced curriculum was not as envisioned.

In regards to CAS, the instructor used the activities as intended, the students were able to successfully complete them, and they were effective in providing material and background for discussion. However, although students were actively engaged by the CAS activities, this did not lead to active participation in class discussion.

Effect of Treatment on Procedural Understanding

Validity and Reliability Issues

The procedural understanding quizzes given during the semester to the treatment class and the procedural understanding exam given to all students at the end of the semester were based on the Framework. Content validity was established by an independent reviewer, Hasenbank, and his validation remarks for the procedural understanding exam can be found in Appendix J.

The follow-up quiz interviews were designed and conducted to answer the following two questions:

1. Are the questions valid in the sense that students understood the wording and what is being asked?
2. Are the students' answers reliable in the sense that their written response reflected their understanding?

Continued analyses of transcripts as interviews were conducted over the course of the semester were used to strengthen subsequent quizzes and design the procedural understanding exam. These analyses also provided data to answer the above questions affirmatively. The researcher reviewed interview transcripts to evaluate why answers were left blank or appeared to be off task. For questions to be valid, it was determined that they needed to be grounded in specific examples. Instead of a verbal description or name of a procedure, examples of the procedure were needed to be sure students understood the question properly to be able to attempt an answer. Two students that were interviewed left answers blank. This was not due to the question but because the students had been absent and did not know the material required.

A specific change in wording was made to assess understanding with respect to Framework objective 6. Instead of asking for examples of "real-world applications" or "connections to other types of mathematics", the question was modified to ask for "word problems" and "homework problems" that the procedure would empower students to do.

The researcher reviewed the interview transcripts to determine if the interview probes helped students to give a more complete answer than what they had written down in the timed-test situation. The researcher found that despite given time to reflect on the answer and having different probes asked, generally students did not display more understanding than in their written responses to most questions. As the researcher was developing these types of questions, some questions were not written properly so clarification probes allowed students to give better answers. In general, students persisted in misconceptions, rephrased their answer that had been written, or restated a comment made earlier in the interview by the researcher.

In addition to the work done to develop a valid and reliable exam, intra-reliability and inter-reliability analyses of exam scoring were done as described in Chapter 3 (p. 65). The outcomes of these comparisons are noted in Table 11. The agreement and correlation of the researcher's scores over time was acceptable for all questions. However, *kappa* for the comparison of the researcher and independent reviewer on question 6 was low. This is based on benchmarks set by Landis and Koch (1977) of: < 0.0 Poor; .00-.20 Slight; .21-.40 Fair; .41-.60 Moderate; .61-.80 Substantial; .81-1.00 Almost Perfect.

After further analysis, it was determined that there was a discrepancy in the rubric. Question 6 was scored based on how many items student identified from a list. The researcher included an item while scoring that was not on the formal rubric. This explains the reason that *r* for question 6 is acceptable because the reviewer was consistently scoring students at one level less.

Questions 5 and 11 were not at ideal strength levels (Moderate) but the statistical analyses conducted to compare the treatment group to the other sections were done using the average question score of each student. The *r* value for intra-reliability was .943 and *r* was .932 for the correlation of average scores between the reviewer and researcher. These values were strong and indicated that the comparison could be made with confidence.

Table 11 – Reliability of Procedural Understanding Exam

Exam Question	Researcher: First vs. Second Grading			Researcher vs. Independent Reviewer		
	% Agree	*r*	*kappa*	% Agree	*r*	*kappa*
1	0.78	0.896	0.674	0.75	0.862	0.627
2	0.82	0.928	0.739	0.83	0.882	0.754
3	0.84	0.940	0.773	0.80	0.920	0.722
4	0.81	0.882	0.673	0.86	0.901	0.726
5	0.69	0.807	0.567	0.60	0.790	0.434
6	0.79	0.852	0.691	0.46	0.704	0.276
7	0.87	0.883	0.783	0.83	0.849	0.716
8	0.87	0.937	0.808	0.86	0.889	0.778
9	0.95	0.924	0.847	0.97	0.935	0.921
10	0.86	0.912	0.795	0.77	0.893	0.675
11	0.88	0.930	0.832	0.66	0.680	0.521

Comparison on Procedural Understanding

An independent samples *t*-test was done to compare the average question score of treatment students to control students. The test did not reveal a significant difference ($t_{133} = -.162$, $p = .871$). Table 12 displays the mean question score for students overall and for each question. Independent samples *t*-tests were used to determine that none of these differences were significant except for question 11 ($t_{133} = -2.202$, $p = .029$). This difference was in favor of the treatment group.

An analysis of covariance (ANCOVA) was conducted to control for the possible effect of procedural skill by using the pretest as a covariate. First, it was determined that there was no interaction between pretest scores and average question scores ($F(1, 127) = .899$, $p = .345$). A subsequent ANCOVA was conducted without the interaction term and did not reveal a significant difference ($F(1, 128) = .018$, $p = .893$) in average questions scores after controlling for pretest scores. The main effect of pretest scores was also not significant ($F(1, 128) = 2.900$, $p = .091$).

Table 12 – Question Score Means for Procedural Understanding Exam

Question	Treatment Mean *n* = 16	Control Mean *n* = 119
1	0.56	0.98
2	0.69	0.97
3	1.50	1.44
4	0.75	0.61
5	1.31	1.42
6	1.25	1.58
7	1.50	1.39
8	1.13	1.25
9	0.44	0.28
10	1.44	0.98
11	1.75	1.21
Overall	1.12	1.10

Based on the above analyses, it was determined that the treatment had appeared to have no effect on the procedural understanding of the students as measured by the procedural understanding exam.

Effect of Treatment on Procedural Skill

Validity and Reliability Issues

The final exam was used as the primary measure for procedural skill. It is common to use final examinations in this manner as noted in the review of other CAS studies methodology (See Chapter 2, p.37). This test was written by COT faculty to contribute to the determination of students' final grades and was procedure oriented. It was therefore assumed to be valid and reliable. The pretest was also used for a measure of procedural skill as a covariate in statistical analyses and to choose the treatment section. The construction and format of the pretest mirrored the final exam, it too was assumed to be valid and reliable.

Comparison on Procedural Skill

An analysis of covariance (ANCOVA) was conducted to test for differences in performance on the skills-based final exam. The ANCOVA model did not show a significant interaction term between treatment status and pretest scores ($F(1,133) = .024$, $p = .876$). An ANCOVA without this interaction term was done and revealed no significant difference ($F(1, 134) = 2.636$, $p = .107$) in mean final exam scores between the treatment students ($M = 21.67$, $SD = 5.401$, $N = 18$) and the control students($M = 23.87$, $SD = 6.250$, $N = 119$). The main effect for pretest was significant ($F(1, 134) = 20.609$, $p < .001$).

The final exam contained 40 questions and so the above means are interpreted as students answering, on average, 54 – 60 % of the questions correctly which is not considered passing. Although the difference between the treatment and control groups was not significant, typical students from both groups performed below the passing level. Standard deviations and individual student data indicate that the majority of students struggled on the final exam. The implications of this result are discussed in the conclusions of the next chapter.

A gain score analysis was also done since the groups were intact prior to the selection of the treatment section. The gain score was computed as the increase in number of questions correct on the subset of questions that was common to both the pretest and final exam. An independent samples *t*-test showed that there was no significant difference between the mean gain scores of treatment students and control students ($t_{135} = -.077$, $p = .938$). On average, students were able to answer 3 more questions correctly on this subset of questions (See Table 13).

To summarize the above findings, there is no evidence to suggest that the treatment students performed at a different procedural skill level than the control students.

Table 13 – Average Gain Scores

	M	*SD*	*N*
Treatment	3.00	2.849	18
Control	2.95	2.537	119

Student Perceptions and Issues

In this section, the results of interviews conducted with a sample of students will be presented. These student interviews mainly focused on the use of CAS in the classroom and discussion will be focused on common themes found through analysis of the interview transcripts. A list of the questions asked each student can be found in Appendix L.

Calculator Background

Students were asked about the use of calculators in their previous mathematics classes. Six of the eight students who were interviewed replied that they had used graphing calculators. During analysis of responses to this question the codes in Table 14 emerged.

It was evident that although students had used calculators in other courses they were not utilized by the instructor as an integral part of the curriculum. The calculator was not required by any of the teachers and was more of a convenience for the students. The uses made were occasional graphing, but more commonly to do and check problems on homework and quizzes.

Table 14 – Emergent Codes for Calculator Background

Codes and Sub codes	*n*	Codes and Sub codes	*n*
Not central to curriculum	8	Uses	10
		Check work	(2)
		Graphing	(4)
		Homework/Tests	(4)

Perception of the Voyage 200

Students were asked several questions about their thoughts about the calculator. The analysis of these questions led to the emergence of the codes and sub codes found in Table 15.

Table 15 – Emergent Codes for Student Perception of the Calculator

Codes and Sub codes	*n*	Codes and Sub codes	*n*
<u>Increased Capabilities</u>	17	<u>Benefits for learning</u>	14
Physical Differences	(3)	Visualize	(4)
Able to do more	(6)	Alternative method	(3)
Solve	(6)	Experiment	(6)
Graph	(2)	Cover more	(1)
<u>Advantages</u>	17	<u>Disadvantages</u>	13
Trivialize	(7)	No insight	(4)
Speed	(6)	Over reliance	(9)
Reassurance	(4)		

When asked about the differences between this calculator and others, most students remarked that it was more powerful and could do more. A few mentioned physical differences such as the keyboard and size. The students who had used graphing calculators before identified the solve command as a key difference. “We can type in an equation and it will solve it for us.” This comment typifies what students recognized about the calculator’s capabilities. However, this also led to one of the disadvantages that were mentioned.

One student commented, “It just kind of gave me the answer,” and, “It didn’t really go step-by-step.” This is an example of students’ tendency to focus on mathematics as learning how to do the procedures to get the answer and not on other aspects of procedures such as those of the Framework. This corresponds with caution many expressed such as, “I could see how you could become dependent on it.” It is important to make the distinction that only one student admitted to relying on it more and the rest of the comments that were coded “Over reliance” were warnings.

Students appreciated the advantages of having the calculators, especially to check their work noting, “You always have the reassurance that it’s there.” Most students responded to the question about the helpfulness of the calculator by indicating it made things a lot faster and was able to do problems for them.

In a slightly different category, codes emerged that illustrated how the students felt the calculator benefited their learning. One student remarked that, “You can do more in the lesson.” Several students also noted that they liked knowing how to do it by hand and using the calculator. The codes with the largest frequencies were “Visualize” and “Experiment.” The researcher used these terms from the literature because they fit the uses of CAS that were designed as part of the lesson plan. Each will be discussed in turn.

“It was a good visual to show you how problems work,” was one students comment. Another noted, “It helps with that so I can graph or put it in the table and see how it all works out.” This use of CAS was built into many of the activities and appealed to some of the students’ self-defined learning style.

One student remarked that CAS helps in “knowing if there’s, like, two answers that you’re supposed to find.” This illustrates using CAS to experiment, another role that was part of the CAS/Framework curriculum. This moves beyond the focus on learning how to do the procedure to the Framework objective of knowing what sort of answer to expect. However, of the six responses coded as “Experiment,” two described using the CAS to do the problems so that the by-hand manipulation process could be deduced.

Learning to Use the Voyage 200

One of the important aspects of implementing a curriculum of this nature is the possible difficulty of integrating the technology into the classroom. Students were asked in several different ways during the interview about difficulties in learning how to use the calculator and about having the calculators as part of each lesson. The emergent codes from analysis of the transcripts are presented in Table 16.

Table 16 – Emergent Codes for Learning to Use the Calculator

Codes and Sub codes	*n*	Codes and Sub codes	*n*
It was easy to learn	22	Difficulties	12
It was easy	(10)	Finding things	(4)
Familiar setup	(5)	Syntax	(6)
No difficulties	(4)		
Attention and attendance	(3)	Good intro/explanation	8

The above results confirm the informal observations made by the researcher and fidelity observers that indicated the students had little difficulty using the calculators. Every student gave a response that was coded as “It was easy to learn.” In a lot of the responses the phrase “pretty easy” was used. Some students referred to the similarities with other calculators they had used in the past as many of them had used Texas Instruments graphing calculators.

The students were probed as to why it was easy for them to use the calculator. As indicated in Table 16, this was due to the good introduction that the instructor gave the first day the calculators were brought in and continued explanation as the CAS was used in different ways throughout the semester. This is supported by the several remarks made about attending and paying attention. As one student explained it, “As long as you’re there everyday.” Another student said that it was easy “because I paid attention.”

Despite the overall ease with which students seemed to learn how to use the calculator there were a few difficulties. As anticipated, there were some difficulties with syntax issues. These typically involved parentheses errors or forgetting a parameter. Another group of remarks indicated that it could be a struggle “just finding out where to go to get the things.” There were a few other isolated difficulties observed by only one student each. The first of these was switching modes. Another student mentioned that he “didn’t know you could use insert” because his calculator had been set in the overwrite mode for most of the semester. The conclusion was made that students did not have difficulty learning how to use the calculator other than isolated difficulties.

Perceptions of CAS Use in the Classroom

The students were asked about their feelings regarding the calculator use in class. These codes that emerged for this theme are listed in Table 17, reflecting opinions of 75% of the interviewed students.

Table 17 – Emergent Codes for Perceptions of Calculator Use in Class

Codes and Sub codes	*n*
Used too much	6
Rather learn by hand	7

The general consensus of the students that were interviewed can be summed up by one student’s remark, “Personally I think he is using it too much. I’d rather learn how to do everything…him showing us on the board instead of him showing us how to do everything on the calculator." Another student expressed the opinion, "Sometimes I felt like we used them a lot, like, I would kind of rather do more problems on my own to try and learn." These remarks and the codes in Table 17 underscore the students' philosophy of mathematics as executing procedures and finding the right answer. Although this is one aspect of the Framework, the students did not show evidence of recognizing that the calculator work was not intended to teach them how to find the answer but to investigate other aspects of the Framework. In their opinion, the purpose of the lessons was to teach them how to do the problems by hand and they viewed calculator work as obscuring that purpose rather than opening up discussion avenues for other Framework topics.

Students' Suggestions and Opinions

The students were asked a few general questions about their suggestions and opinions to give them a chance to offer comments on anything not already specifically asked. Two codes emerged from these responses that are listed in Table 18.

The "More emphasis on by-hand" code is distinct from the "Rather learn by hand" code of the previous section. In the latter case, students are speaking of the experiences they had in this classroom and their preferences, whereas in this coding the students are making suggestions and comments on what should be done in the future. For example, one student commented, "When you are first learning something it would be better to work it out by hand." Another student, whose opinion was that the calculator was overused, suggested that the class have more of a balance between by-hand work and calculator work. It is interesting that one student took responsibility for not practicing by-hand execution of the procedures, suggesting that students need to take the time to become proficient in by-hand skills.

Table 18 – Emergent Codes for Overall Opinions

Codes and Sub codes	*n*	Codes and Sub codes	*n*
More emphasis on by-hand	7	Testing issues	8
		Motivation to not over use	2
		Should be allowed	6

As in the pilot study, several students indicated that if calculators are used in class to learn the material then they should also be allowed on assessments. One student remarked, "What's the point of having them in the first place if we are not gonna use them for something we need to be graded on?" The calculators were allowed on the Framework quizzes and a portion of each unit exam, but this did not seem to be enough partly because the students did not see the value of the Framework understanding questions. They wanted to be able to use the calculator on procedural skill questions.

Summary of Interview Analysis

Students mainly viewed the calculator as a tool to perform calculations for them and expressed concern that this could lead to over reliance on its capabilities. Many remarks and suggestions indicated that students viewed the focus of their class as learning how to do procedures and that they felt the calculator work detracted from that focus. Also noteworthy from the above results is that the only difficulties were minor, isolated, and easily resolved.

Instructor Perceptions and Issues

The results of the instructor end-of-course interview are presented in this section. The interview transcript can be found in Appendix K. Themes that emerged from the analysis of this transcript include benefits of having CAS in the classroom, issues that arise due to having CAS in the classroom, and then issues regarding implementation of the CAS/Framework curriculum. Each is discussed in turn.

Benefits of CAS

The key benefit of having the calculators in class was that it kept the students engaged in the lesson. On four different occasions during the interview the instructor said something similar to, "It's a really good way to get students actively involved in the lesson." The instructor also described an incident that happened after the calculators had been turned in. The instructor had been reviewing for the final and the class was unresponsive.

> I took about a 5 minute break halfway through the review due to obvious lack of attention and focus on the part of the class. It wasn't isolated cases; it seemed to be most of the people there. And I was thinking the calculators weren't there, they had been turned in. There was no physical involvement in what was going on. I wondered if that had part of…something to do with the completely dead reaction I was getting from most students.

The instructor felt that the calculator activities were beneficial not only because they kept the students involved, but also gave them visual output that could be studied and discussed. He felt that using the CAS has "a lot of potential to kind of reverse the way the material is introduced… and bring the concepts …in early in the chapter or unit and then work on the procedures that support those concepts later."

<u>CAS in the Classroom</u>

The instructor felt that the calculators were well received and mentioned several times that there did not seem to be any difficulties on the part of the student. "There were difficulties. I'd say they were fairly isolated and I could pretty much tell when people were having problems." The instructor cited the same difficulties that the students did, mainly parentheses and forgetting syntax or where to go to find certain commands or functions.

The instructor mentioned a few issues that he dealt with. One was determining the best method of presentation. "It took me a few tries to figure out what the optimal configuration was." The class switched rooms on February 20, about 3 weeks after the calculators were brought to class. The reason and details are described best by the instructor:

> We started out in one classroom in Johnstone and it's not much of a classroom. It's kind of a bowling alley arrangement where I was at one end and the students were at the other in the pin pit. Of course they all want to sit towards the back. And the real problem there though was, aside from the fact that the classroom's not much good, the board – there was very narrow space at the front of the classroom so we just had a limited amount of board space. To show the calculator projection on the screen, which you pulled down across the board, destroyed most of the board space. You couldn't use it. It was a real problem. Of course you know, we addressed that after a few weeks and moved to a couple of different classrooms.

The new classroom arrangement was such that the class met in one room on Tuesdays and Thursdays and a different room on Mondays and Wednesdays. The instructor had past experience with this classroom arrangement, and it was not an issue for him or the students.

Other issues the instructor dealt with as a result of the treatment curriculum did not specifically deal with CAS and is discussed later in this section. The instructor felt that his familiarity and training with the CAS was adequate and commented that he was constantly learning new things. The CAS did not create any obstacles in the classroom.

Implementing the CAS/Framework Curriculum

The instructor described in great detail the impact that implementation held for the students and himself. In this section the perceptions of the instructor regarding drawbacks of the curriculum, differences between treatment and control groups, issues for teachers, and suggestions for improvement are presented.

Drawbacks of the Curriculum. The instructor struggled with the time constraints of a pre-scripted lesson. He explains, “I didn’t feel that I had a whole lot of freedom to dwell on points that students appeared to be having trouble with or that I wasn’t sure they were getting.” He made similar comments twice during the interview, specifically mentioning that he did not have adequate time to answer questions or discuss homework. But there was an advantage to the structure of the lesson plans as he expressed, “Maybe it kept me on track more than slowing me down.”

This curriculum focused on the Framework which puts the by-hand execution of procedures in a subordinate role since execution is only one aspect of the Framework and CAS was utilized to handle some of the manipulations. This resulted in the treatment section missing out on the "advantage of practice or demonstration on the board that the other two sections had." This reflects the need the instructor felt to prepare the students for the skills-based final exam.

Several times the instructor mentioned the challenge it was to get students engaged in discussion. This is consistent with the fidelity observations.

<u>Differences between Treatment and Control Groups</u>. The instructor struggled to get responses from the students. The instructor noted that it might have seemed more difficult than in his other sections because he "was trying to get so much more discussion out of that class." As a result he was asking more questions that were more thoughtful and probing than in his other sections.

Other differences that were discussed in the debrief interview were that of absence and drop rates. The instructor felt that both were a little higher in the treatment class. The instructor speculated that it was "just the randomness of sections" because the attendance rate of the treatment section was comparable to other semesters before the attendance policy (see p. 49) was implemented. The other two sections had better attendance rates compared to the last three semesters.

Lastly, the instructor reported that another difference between the sections was that "there was probably less engagement, less attention, less focus on what was being discussed in my other two sections than there was in the calculator section when the calculators were being used." The instructor had a strong opinion that CAS is a good tool to keep students engaged.

Implications for Teachers. An important consideration for implementing a curriculum like this is the demand placed on a teacher's time. The instructor spent an extra one to two hours of preparation time per lesson beyond the time spent preparing for the other two sections. This time was spent familiarizing himself with the flow of the lesson, understanding the calculator operations, making notes on the lesson plan of cues for himself, and meeting with the researcher for pre-class and post-class discussions. But the instructor pointed out that, "If I were doing all my sections with CAS, I wouldn't be spending the time in preparation for those two other sections like I had to this semester."

He also thought, "You've got something that can be handed over to somebody and used for their complete plan for teaching introductory algebra." The instructor used material from the lesson plans in his other two sections because he liked the examples and the flow of the lesson. The instructor noted that as he reflected on reviewing for the final exam with the treatment section, he felt there was very little material that he had covered in the two other sections that he had not covered in the treatment section. "The order was a little different but we seemed to be on track. We covered all of the material."

Suggestions for Improvement. The instructor shared several suggestions to improve the curriculum and its delivery. The first was a request for a computer emulator that could be displayed through the video projection system. The instructor suggested that the ability of CAS to keep students' attention should be capitalized on by using it at the beginning of class to grab their attention and at the end of class to "bring people back." The middle portion of the class period could be less structured and not so dependent on the calculator.

The third recommendation that the instructor made was to modify the lesson plans to make the student and teacher roles more visually accessible from the written plan. The layout could be changed to emphasis what the students should be doing, what the teacher should put on the board, and what calculator work should be done by the teacher and what should be done by the student.

Summary of Instructor Debrief

The calculators did not pose a problem for the students or for the flow of the lesson, but were an important tool to keep the students actively engaged and keep their attention. The instructor spent more time preparing for the lessons but felt that he was able to cover the material though at the expense of time for students to ask homework questions.

Summary of Important Results

Listed below is a summary of the significant research findings from this chapter that will be used in Chapter 5 to discuss the research questions, conclusions, and implications for future practice and research.

1. It was determined that the instructor faithfully implemented the curriculum although there was difficulty establishing classroom discourse.

2. There was no significant difference in question score means detected between treatment and control students on the Procedural Understanding exam.
3. There was no significant difference detected between the treatment and control groups in procedural skill as measured by the final exam.
4. There were no difficulties expressed by the instructor or students in learning how to use the calculators or in the teaching and learning process.
5. The treatment instructor was able to implement the curriculum in line with the department syllabus and schedule.

CHAPTER 5

CONCLUSIONS

Introduction

Chapter 5 contains the conclusions to the research questions based on the results of the data analysis found in the previous chapter. After the discussion of the conclusions to the research questions, the limitations and delimitations of the study will be presented. This will be followed by recommendations for future research and concluding remarks.

Research Question Conclusions and Implications

Research Question 1 – Curriculum Implementation and Technology Issues

To answer the first research question, a characterization and description of the curriculum implementation is given from several sources. First, the observations designed to judge fidelity, along with additional researcher observations, will be used to characterize implementation of this type of curriculum in a remedial class. Second, an explanation of the instructor's perception of the impact of the CAS/Framework on student learning and the classroom environment is discussed. Then, issues that the instructor and students encountered while using CAS and the Framework are summarized.

Observations of a Remedial Classroom. The fidelity of implementation observation data were important to establish that the CAS/Framework curriculum was faithfully implemented but also to help characterize the implementation of such a curriculum at the remedial level. The finding of the observers was that the instructor faithfully enacted the CAS/Framework curriculum. However, the curriculum was not experienced fully as intended in its design. The key component of discourse was not achieved at the level intended by the curriculum.

The observers noted that the questions were difficult for the remedial students to answer. Also, many times the instructor began class with a review of previous material or tests because students had misconceptions about a procedure. The poor performance on the final exam by both the treatment and control students (p. 85) is additional evidence that the typical student struggled to demonstrate adequate knowledge about the execution of procedures. This suggests that the Framework may be difficult at the developmental level because the students needed more varied practice (Brownell, 1956) with a procedure before they could answer Framework-type questions.

The researcher noticed that students were more inclined to answer questions during lessons that were focused more on by-hand manipulation. The observers also noted that the students rarely volunteered answers. Students at the remedial level are not confident speaking up in class. The researcher observed students answering questions quietly to themselves, but when called upon, giving no response or being very cautious. The instructor commented, in an after-class debriefing session recorded in the researcher's journal, that "there seems to be an unspoken understanding among the classmates that no one should volunteer an answer." This characteristic of remedial students influenced the experienced curriculum.

The above observation also points to another characteristic common to remedial students: the lack of motivation. Often the researcher and observers noted that the calculator was the only thing that some students had on their desks. In agreement with the instructor, the observers felt that the CAS was an effective engagement tool; however, it appeared as if students would have done little else without the calculator. Student's questions focused on the easiest way to get the right answer. One observer pointed out that students were only concerned with the correct answer to test questions were and not learning how to fix their mistakes. Remedial students may not be inclined to discuss mathematical ideas beyond by-hand manipulation because they are so focused on doing well on tests that require them to attain a high level of procedural skill.

In summary, the curriculum was enacted as intended but not experienced with the level of discourse intended by curriculum design. This may be due to several characteristics of remedial students and mathematics. Specifically, these characteristics are the lack of varied practice, of confidence in sharing thoughts, and of motivation to learn.

<u>Instructor Perceptions of the CAS/Framework Curriculum in the Classroom</u>. The instructor felt that the CAS/Framework offered several positive benefits for student learning. Foremost among these was the active, physical engagement that calculators provided for students. This was observed by both the researcher and instructor. The instructor further explained that he noticed the absence of this engagement in the non-treatment sections:

> Having the calculator there in the classroom and students working on it on a fairly regular basis was a way to keep them physically involved in the instruction and learning and I didn't have that in the other classrooms…there was probably less engagement, less attention, less focus on what was being discussed in my other two sections than there was in the calculator section when the calculators were being used.

The instructor also noticed a difference after the calculators were retrieved from the treatment section (see p. 94).

The calculators were successful in providing engagement for students at their seats but discussion based on the results from the calculators and Framework questions was difficult to establish. One possible reason for this is the lack of rapport between instructor and the students because of the nature of the curriculum.

The instructor noted that he asked more questions and different types of questions "because there was so much more emphasis on probing and discussion and understanding." This may explain some of the difference the instructor observed in student responses to questions between the treatment and non-treatment sections. The instructor noted that some of the difficulty "may have been I was trying to get so much more discussion out of that class than my other two sections." Teaching according to the Framework and using CAS differs from the classroom norms these students are accustomed to.

The emphasis on this type of understanding was not reflected in the exam structure. The students took four Framework-based quizzes which were not a significant portion of their grades. Students realized that their grades were dependent on the skills-based unit and final exams and towards the close of the semester responded to the CAS activities with the question, "How do we do these by hand?" Rapport could have been damaged, making discussion difficult, if the students felt the instructor was not preparing them to be successful on the exams. In contrast, the instructor's control sections were more comfortable with answering questions, possibly because they were more in line with the expected norm of focusing on how to do a procedure.

The instructor felt that another benefit from the Framework and CAS was that the CAS gave visual output that, coupled with the Framework prompts, provided good grounds for discussion and focus on concepts first. Because of the difficulty experienced in establishing classroom discourse, however, the instructor could not qualify how these activities supported learning concepts.

Curriculum Issues for the Instructor. The CAS/Framework curriculum was provided to the instructor from an outside curriculum designer. Although the instructor felt that it was a comprehensive curriculum and beneficial for the students, there were some issues that should be noted. First, the instructor spent an additional one to two hours per lesson in preparation and meetings with the researcher. However, introductory algebra was the only course instructor taught this semester. In effect, preparing for the treatment section was similar to having to prepare to teach a different course. Had all the sections the instructor taught used the CAS curriculum, the instructor claimed the preparation time would not have been an issue.

The other central issue was the limited time that the instructor had "to dwell on points that students appeared to be having trouble with or that I wasn't sure they were getting." The instructor also expressed concern that treatment students did not receive the same time focusing on how to do procedures as control students did through demonstration on the board by the instructor or guided practice. This was intentional as by-hand execution of procedures is just one of the facets of the Framework; however, there was pressure to prepare students for the final exam to meet the department requirements and syllabus. Research question three was included in the study to address this concern (see p. 110). It is important to note here that the instructor felt he was able to keep on schedule and cover the material that was necessary.

Technology Issues for the Instructor and Students. The interview data from both the instructor and the students indicate that there were no difficulties integrating CAS technology in the classroom. There were some isolated difficulties but the instructor was able to resolve them without detracting from the lesson. The difficulties were of the type that is expected when becoming familiar with new technology, such as forgetting syntax or location of buttons. The introduction and explanation of the calculators was sufficient at the beginning of the semester and continued practice helped increase the ability of students to work on the calculators.

Summary of Conclusions and Implications. Based on observations and the responses from the instructor and students used to answer the first research question, several conclusions were made. CAS provided opportunities to keep students actively engaged and provide material for discussion. CAS was not difficult for students to learn or for the instructor to use as a teaching tool during the lesson plan.

The instructor had difficulty creating and sustaining classroom discussion. The more frequent and probing questions were different from the norm that students experienced in the past, possibly distancing them from the instructor. Daily use of the calculators to do symbol manipulation also violated the usual classroom norms. Students were further distanced from the instructor and the value of calculator activities was diminished because students could not use CAS on the exams that were a significant portion of their grade.

Implications for future use include cognizance of time constraints both in and out of the classroom. Extra preparation time may be required to fully plan for the use of CAS in the classroom. Also, prioritization of activities in the lesson plan is important so that teachers can make use of formative assessment and spend appropriate class time responding to students' needs. More overt methods of promoting discussion should also be employed such as small group discussion and student journaling. The exam structure should place more emphasis on the Framework to increase student's perception of its value.

Research Question 2 – Framework-based Procedural Understanding

The CAS/Framework curriculum was designed to expand learning how to execute procedures to include other aspects of the Framework for Procedural Understanding (see p. 2). The procedural understanding exam was designed to evaluate students' depth of Framework-based procedural understanding. The exam was given to all sections at the end of the semester but no statistically significant difference in average question scores was detected between the treatment group and control group. There are two possible explanations for this result that are supported by data. These are the lack of discourse to support the emphasis of the Framework objectives and the students' perception of mathematics as learning how to do procedures.

The CAS/Framework curriculum was grounded in Sfard's (2001) communicational theory of learning. As noted in the summary of results to the first research question, it was a challenge for the instructor to engage students in a discussion of the Framework ideas. The instructor did emphasize the Framework ideas in class, as the lesson plans specified (see Fidelity of Implementation, p. 73), but the discourse that was sparked did not achieve the level intended in the curriculum design.

The students' philosophy of learning mathematics is centered on learning the rules and steps to complete procedures. This is reinforced by the exams that they take in order to pass the class. Assessments are often skills-based, judging how many correct answers students can compute by hand in a short time period. The instructor acknowledged this when he expressed concern that the control sections were at an advantage because they had more demonstrations on the board of how to execute procedures and more guided practice during class. Thus, student attention seems to focus on procedural execution in order to achieve the desired grade on the exams.

The following is an excerpt from a follow-up quiz interview. The Framework objective being discussed is knowledge of alternative procedures and when a procedure is best to use.

> Student: For me, if I find something, I don't really, like, pay attention to other ideas because I'll get confused.
>
> Researcher: If you find something that works…

> Student: Because, math is not one of my strong points, obviously, and so once I find something that works for me I like to stay with it.

This student has learned to filter out ideas that distract from a method that can be remembered and can be used to get reliable answers. When asked about the reasoning behind the choice for a particular procedure, another student responded, “That’s just what you do. I don’t know. I’ve always been taught that.” In this case the student retained the steps of a procedure but not the reasoning behind them. Again, this may be attributable to students’ experience of skills-based assessments.

“Some of the quizzes were kind of really weird,” one student said referring to the Framework-based quizzes. “I just feel like using the calculator doesn’t help at all, I don’t know, because they were kind of factual questions usually.” Students are accustomed to tests that require execution of procedures. One student viewed learning mathematics as learning to do what the calculator can do by commenting, “It seems smarter to show how to do it in your head and on paper for like on a test and for future reference when you will not be able to use a calculator.” This importance of learning how to perform procedures by hand is pervasive throughout the student population.

A major concern that students had was over reliance on the calculator. A majority felt it was used too much and wanted explanation “by hand how to do it completely” and to be taught “how to do the actual work on paper. One student prioritized learning, “When you are first learning something it would be better to work it out on paper.” Students viewed the calculator more as a tool that could do problems quickly and easily than as a tool to help experiment and visualize.

Given this philosophy of mathematics as set of procedures to be learned, the conclusion can be made that the students did not see the Framework questions as an important part of the lesson. Thus, there was no impetus for them to join in the discussion. Even though the intended level of discussion was not reached, the instructor still presented and focused the lesson on the Framework objectives so some difference in knowledge of the Framework should be expected. The lack of increased Framework-based knowledge may be attributable to the difficulty of the Framework for novice students who lack the varied practice of procedures to be able to answer the Framework questions. This is evidenced by the results of the final exam on which a majority of students, both treatment and control, were unable to demonstrate adequate knowledge of the execution of procedures. If students are unable to grasp the execution aspect of the Framework, it may preclude their ability to address other aspects of the Framework.

Research Question 3 – Comparable Procedural Skill

The procedural skill level of students in the treatment section was of great importance to the researcher. Opponents of using CAS in the classroom, particularly in remedial algebra, would argue that students would not attain an acceptable level of skill in by-hand manipulation of algebraic expressions and equations. Research studies involving CAS often include a question similar to the third research question of this study to determine if there is a difference in procedural skill between students who have been taught with CAS and those who have not (see p. 20). In this study, the final exam was used as the instrument to measure procedural skill. This was done not only because the exam was skills-based but also because it is the common department assessment worth 25% of the students' grade.

The gain score analysis showed that students in both the control and treatment groups were able to correctly answer an additional three questions on the subset of questions common to the pretest and final. A comparison of group means on the final exam showed that, although the treatment students scored slightly lower on the final exam, this difference was not statistically significant. Therefore, the answer to research question three was concluded to be that there was no significance difference in students' procedural skill as measured by the introductory algebra final exam.

It is important to interpret this finding in the context of the CAS/Framework curriculum. The use of CAS and Framework questions took significant time away from the instructor and students that would have been spent developing by-hand procedural skills. The instructor believed his other two sections had an "advantage of practice or demonstration on the board." And students noticed this difference as well, remarking in the interview that the instructor "teaches them [the calculators] more than he teaches us how to do the actual work by hand" and "I would kind of rather do more problems on my own to try and learn." Despite the delegation of procedural execution to the CAS and significantly less time spent developing procedural skill, there was not a significant difference in treatment students' final exam scores when compared to the control students' scores.

The implication of this conclusion is that CAS can be used in this level of mathematics without detrimental effects to procedural skill. Secondary algebra curricula are being developed that utilize CAS. Curriculum can also be developed or adapted for use in remedial mathematics.

Limitations and Delimitations

The limitations of this study are:

1. The population for this study was the students who chose to enroll for lecture-based introductory algebra through COT in the spring 2007.
2. The design was quasi-experimental using intact groups of students. The treatment section was chosen from three sections taught by a specific instructor. A pretest of procedural skill was used to choose the treatment group to ensure the equivalence of the treatment and control group.
3. The treatment instructor also taught some of the control sections. The instructor may have consciously or subconsciously used material from the CAS/Framework curriculum in the control sections.
4. The pilot implementation was done by the treatment instructor but was limited in material. Although the instructor was familiar with the algebra content, during full implementation it was the first time he taught the entire course using CAS and the Framework approach.
5. The treatment and control sections were not isolated groups. Interaction may have occurred between students in the treatment and control groups.

The delimitations of this study are:

1. The treatment instructor was chosen with prior CAS experience, having more experience than other instructors that may implement a curriculum of this nature.
2. The students were informed at the beginning of the semester that the course would be taught differently as part of a research study. They were given the opportunity to change sections if they did not want to be part of the study.
3. The researcher attended class most days and was joined by independent observers throughout the semester.

Research Implications

The results of this study support previous findings in CAS research. As in the studies conducted by Heid (1984) and O'Callaghan (1998), the integration of CAS in the curriculum did not have a negative effect on students' procedural skill. However, unlike these CAS studies no increase in conceptual understanding was observed. In this study, this would have been noted as an increase of procedural understanding based on the Framework. Although Hasenbank (2006) implemented a treatment that was successful in improving Framework-based understanding in college algebra, this study does not support that finding for introductory algebra students.

Characteristics of remedial students in this study are consistent with those found in the literature (see p. 12-14). Specifically, students were not motivated, interested, or confident. The pass rates were poor with about 1/3 of the students not finishing the semester. Controls were put in place to encourage attendance and efforts were made to motivate students through the use of CAS and applications. However, the affect literature regarding remedial students is important to consider for future research. The experiential level of curriculum decision making resides with the students as they decide what their curriculum will be based on their interest and willingness (Klein, 1991). Thus, recommendations first focus on the student, followed by an outline of areas important in the professional development of teachers.

Future Research and Curriculum Development

Recommendations for future research and curriculum development are made based on several issues arising from discussion of the results. There are merits to teaching with CAS and a focus on the Framework. Future research should focus on evaluation of curriculum design efforts that are informed by the results of this study.

The CAS/Framework curriculum was designed to promote classroom discourse through carefully chosen examples and calculator activities with accompanying discussion prompts. However, this was not enough to establish and sustain the intended level of classroom discourse. Further curriculum development needs to focus on incorporating methods that more overtly nurture classroom discourse such as small group activities or student journaling. Research is needed on developing classroom norms and behaviors for remedial students that will increase their motivation and willingness to participate in discussion. Many sociological factors are not controllable by researchers, but more research on how to address these factors is important.

Another improvement suggested for the curriculum is to more heavily emphasize the Framework objectives on assessments. The students' philosophy of mathematics suggested by the results of this study prioritizes learning the by-hand execution of procedures over other ideas and types of understanding. Students did not value Framework-based knowledge because it was not a significant portion of their grade. Wilson (1993) supports this claim with research indicating that what gets graded determines what it means to know and do mathematics in the classroom. Thus balancing skill and understanding assessments may help promote classroom discourse if students are aware that demonstrating Framework-based understanding is significant in determining their final grade. The instruments in this study provide one way to assess Framework-based procedural knowledge. Other research is needed to develop and evaluate this type of assessment.

Professional Development

The CAS/Framework curriculum requires training in several areas: CAS, discourse, assessment, and the Framework. In this study, the treatment instructor was chosen because of prior experience with CAS and only needed occasional support in calculator operations. In implementing any CAS curriculum, the teachers will need opportunities to learn how to use CAS and effectively teach with CAS.

Professional development also needs to focus on classroom discourse and assessment. The level of desired discourse was not reached in this implementation of the curriculum. The instructor used the discussion prompts to begin whole class discussion as the lesson plans indicated. As different methods, such as small group activities, are implemented to increase classroom discourse, teachers will need to be knowledgeable of how to conduct and effectively use these methods. Also important will be the ability to develop Framework-based assessments suitable for formative use in the classroom and for grading purposes.

Also important in successfully implementing a curriculum of this nature is content knowledge of the appropriate procedures according to the framework. The treatment instructor had a strong mathematical background but on occasion was required to think about and discuss aspects of the Framework that were not clear. It is important that teachers understand procedures fully from the perspective of the Framework and also how to teach to the Framework. This is addressed with learning methods of discourse as described above.

Concluding Comments

CAS is not new to undergraduate mathematics with studies in calculus dating back to the early 1980's (Heid, 1984). Use and studies of CAS continued to be completed in calculus and also in college algebra and secondary algebra. With this research base and the greater availability of CAS, nationwide secondary curricula developers are now including CAS (Usiskin, 2007). The results of this study are informative to curriculum developers seeking to integrate technology into remedial, undergraduate mathematics and offer insight into how the Framework might be integrated into a skills-based remedial curriculum. Although no significant differences were found in Framework-based procedural understanding, evidence suggests that it is a promising approach to assist students in deepening their understanding of mathematical procedures (Hasenbank, 2006).

Of particular benefit to the students in this study was their ability to use CAS to perform procedures. This experience provided them with an additional method to perform procedures and access to multiple representations which are important parts of the Framework. Students also used CAS in other classes and will be prepared to use graphing calculators in higher level math courses.

The Framework was not readily learned in the context of remedial mathematics students. One explanation was the lack of discourse, possibly due to remedial students' philosophy of learning mathematics as doing procedures correctly by hand. This philosophy of math impedes progress in developing Framework-based understanding beyond execution of procedures. Important in successfully promoting discourse is changing classroom norms of students in a passive role, reluctant to share, and focus on repetitive examples and practice of procedures. It seems promising that appropriately emphasizing the Framework in the exam and grade structure may help change this philosophy and, in conjunction with more overt methods of promoting discourse, the Framework objectives may be learned at this level. Further research is needed to investigate this conjecture.

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APPENDICES

APPENDIX A

CURRICULUM FLOW AND COMPARISON

Traditional Course	CAS/FW Course
Prealgebra Review	Same coverage
Introduction and Prestest	as Traditional Course
R.1, R.2	
R.3, R.4	
R.5, R.6	
Intro to Real Numbers and Algebraic Expressions	Same coverage as Traditional Course
1.1, 1.2	
1.3, 1.4, 1.5	
1.6, 1.7	
1.8	
Solving Equations and Inequalities (1 variable)	
2.1, 2.2, 2.3	CAS Introduction
2.4, 2.5	2.1, 2.2, 2.3, 2.4
2.6	2.5, 2.6
2.7, 2.8	2.7, 2.8
Review	Review
Test 1 (Chapters 1 & 2)	Test 1
Graphs of Linear Equations	
3.1	3.1, 3.2
3.2	3.3, 3.5
3.3	3.6
3.4	3.4
3.5, 3.6	3.7
3.7	
Polynomials: Operations	
4.1	4.1, 4.2
4.2	4.3, 4.4
4.3, 4.4	4.5, 4.6
4.5, 4.6	4.7
4.7, 4.8	4.8
Review	Review
Test 2 (Chapters 3 & 4)	Test 2

Polynomials: Factoring	
5.1, 5.2	5.8 (Applications)
5.3, 5.4, 5.5	5.7 (Solving Quadratic Equations)
5.6	5.1, 5.5
5.7	5.2, 5.3
5.8	5.4, 5.6
Rational Expressions and Equations	
6.1, 6.2	6.7 (Applications)
6.3, 6.4, 6.5	6.1, 6.2
6.6	6.3, 6.4, 6.5
6.7	6.6
6.8	6.8
Review	Review
Test 3 (Chapter 5 & 6)	Test 3
Systems of Equations	
7.1	7.4 (Applications)
7.2, 7.3	7.1
7.4	7.2, 7.3
7.5	7.5
Radical Expressions and Equations	
8.1, 8.2	8.6 (Applications)
8.3	8.1, 8.2
8.4	8.3, 8.4
8.5	8.5
8.6	
Review	Review
Test 4 (Chapters 7 & 8)	Test 4
Quadratic Equations	
Sections of Chapter 9	Same Coverage
Covered at discretion of Instructor	

APPENDIX B

TREATMENT SECTION SCHEDULE

AND RESEARCH EVENTS

Spring 2007 Schedule

MON	TUE	WED	THUR
			1/18 Intro and Review
1/22 ***Pretest***	1/23 Review	1/24 Ch. 1	1/25 Ch. 1
1/29 Ch. 1	1/30 Ch. 1	1/31 Ch. 2 ***CAS Checkout/ Intro***	2/1 Ch. 2 *2.1-2.4*
2/5 Ch. 2 *2.5-2.6*	2/6 Ch. 2 *2.7-2.8*	2/7 Review for Test 1	2/8 Test 1 Ch. 1 & 2
2/12 Ch. 3 ***FW Quiz #1 (Ch. 2)***	2/13 Ch. 3 *3.1-3.2*	2/14 Ch. 3 *3.3, 3.5*	2/15 Ch. 3 *3.6*
2/19 NO CLASSES President's Day	2/20 Ch. 3 *3.4*	2/21 Ch. 3 *3.7*	2/22 Ch. 3 ***FW Quiz #2 (Ch. 3)***
2/26 Ch. 4 ***Fidelity #1*** *4.1*	2/27 Ch. 4 *4.2-4.3*	2/28 Ch. 4 *4.4*	3/1 Ch. 4 *4.6-4.6*
3/5 Ch. 4 *4.7-4.8*	3/6 Review for Test 2	3/7 Test 2 (Ch. 3 & 4)	3/8 Ch. 5 ***Fidelity #2*** *5.8* ***Video***
3/12 NO CLASSES - Spring Break!	3/13	3/14 **Observation Tool Completion and Training**	3/15
3/19 Ch. 5 ***Fidelity #3*** *5.7*	3/20 Ch. 5 *5.1-5.2*	3/21 Ch. 5 *5.2,5.3,5.6*	3/22 Ch. 5 ***FW Quiz #3 (Ch. 5)***
3/26 Ch. 6 *6.7*	3/27 Ch. 6 ***Fidelity #4*** *6.1-6.2*	3/28 Ch. 6 *6.3-6.5*	3/29 Ch. 6 *6.6*
4/2 Ch. 6 *6.8*	4/3 Review for Test 3	4/4 Test 3 (Ch. 5 & 6)	4/5 Ch. 7 *7.4*
4/9 Ch. 7 *7.1*	4/10 Ch. 7 ***Fidelity #5*** *7.2-7.3*	4/11 Ch. 7 ***Start Interviews*** *7.5*	4/12 Ch. 7 ***FW Quiz #4 (Ch. 7)***

4/16 Ch. 8 *8.6*	4/17 Ch. 8 *8.1-8.2*	4/18 Ch. 8 ***Finish Interviews*** *8.3-8.4*	4/19 Ch. 8 *8.5*
4/23 Ch. 8	4/24 Review - Test 4	4/25 Test 4 (Ch. 7 & 8)	4/26 Ch. 9
4/30 Ch. 9	5/1 Ch. 9 ***FW Understanding Exam***	5/2 Review for Final	5/3 Review for Final
5/7	5/8 ***Instructor Debrief*** FINALS WEEK	5/9 ***Final Exam*** 10-11:50	5/10

APPENDIX C

SAMPLE LESSON PLAN

Chapter 5 Lesson 2 of 5

Section(s): 5.7 Solving Quadratic Equations by Factoring

Desired Understandings

- Be able to identify solutions to factored quadratic equations using the Zero Product Principle (ZPP)
  - Must have 0 on one side of the equation – may require manipulation
  - Be able to give factorization if solutions to the equation are given
- Know how to identify the solutions of a quadratic equation by graphing
  - Understand why you will always have two factors, if factorable, but could have only 1 solution
  - Be aware that a quadratic equation may not have a solution and why (parabola w/o x-intercepts, polynomial not factorable over the reals)

Framework Questions to Emphasize:

What sort of answer should I expect from the procedure “Solve Quadratic Equation”?

How do I execute the procedure “Solve Quadratic by Factoring”?

What can I use the procedure “Solve Quadratic Equation” to do?

Why is solving by factoring an effective procedure to solve quadratic equations? When is it useful/applicable?

How can I verify solutions to quadratic equations?

Assessment

Communicate: (Framework ideas, misconceptions) **See also Quiz C**

1. Why is it important that one side of the equation is zero?
2. Why doesn't a constant factored out in front contribute a solution?
3. Given the solutions to a quadratic equation, $ax^2 + bx + c = 0$, what else can you tell me about the function and its graph? *Can determine factors, intercepts*
4. What sort of answer do you expect in solving a quadratic equation? Explain.

Calculate/Compute:

1. $y(y+5) = 0$
2. $3(4x+9)(5x-4) = 0$
3. $9x(3x-2)(2x-1) = 0$
4. $y(y+8) = 16(y-1)$
5. $(t-3)^2 = 36$
6. $(t-5)^2 = 3(5-t)$
7. $1 + x^2 = 2x$
8. $2y^2 + 12y = -10$

Problems #1-3 are expected at the end of this lesson. #4-8 will be good examples of equations the students are expected to solve after the chapter is finished.

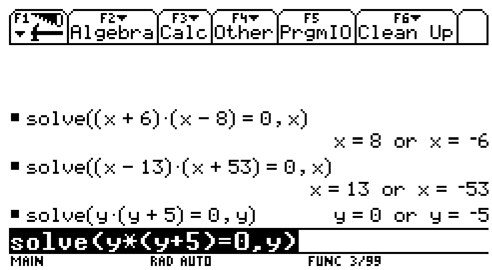
Role of CAS: In this lesson, the CAS will solve factored and unfactored quadratic equations, in addition to graphing the corresponding functions. This will allow for a discussion of the underlying concepts of solving a quadratic equation, such as the relationship between factors and solutions, the relationship between zeroes and intercepts, and what to expect in the process. Students are empowered by being able to solve any quadratic equation and know exactly what needs to be done to do so without CAS – learn how to factor.

Begin Lesson

> **#1**
> Use CAS to introduce the ZPP and discuss why factoring is effective and valid to use for solving.

Have students use the solve command to solve the following problems from 5.7:

$$(x+6)(x-8)=0$$
$$(x-13)(x+53)=0$$
$$y(y+5)=0$$



Discuss their ideas of what the "black-box" procedure could be.

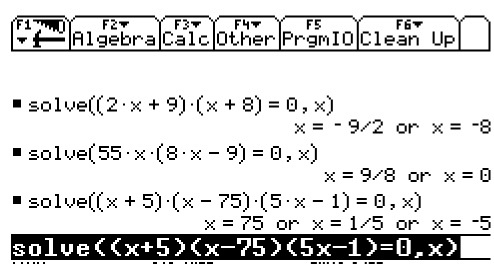
These first few examples should reveal the idea behind the Zero Product Principle.

Formally introduce the Zero Product Principle to discuss the Framework question: **Why is the procedure effective and valid?**

> **#2**
> Use CAS to look at some different situations that can occur and address common misconceptions.

Now have them solve the following problems:

$$(2x+9)(x+8)=0$$
$$55x(8x-9)=0$$
$$(x+5)(x-75)(5x-1)=0$$



Discuss the subtle differences and common misconceptions:

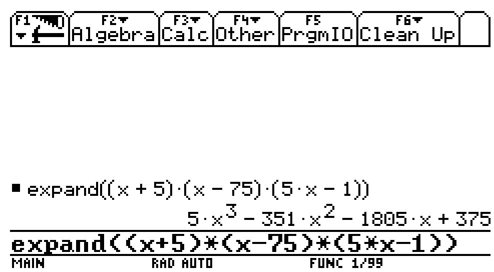
Why did we get fractions for answers to these equations? *Coefficients on the x terms in the binomial factors.*

How can we solve these mentally? *Take the opposite of the constant term divided by the coefficient of the variable term.*

Why isn't 55 an answer to the second equation? *Doesn't give a zero in the product. Monomial coefficients can't give us zeroes, only substitutions for the variable.*

Why are there three answers to the third example? *Three factors.*

Use the expand command:



What is the exponent on the polynomial? *Three, called a cubic.*

We have been working with quadratics. If we multiply out the other examples above (using FOIL or distribution) we would get polynomials with powers of 2.

How many solutions do we expect from polynomials? *The same as the degree*

(Note: we will confront them with repeated factors next to further develop their understanding.)

#3

Use CAS to look at what happens to give one or no solutions by looking at factors and graphs.

These next examples aren't factored but as I write them, guess how many solutions you expect:

$$x^2 + 7x + 6 = 0$$

$$x^2 - 3x = 0$$ *Two, because of the exponent.*

$$0 = 25 + x^2 + 10x$$

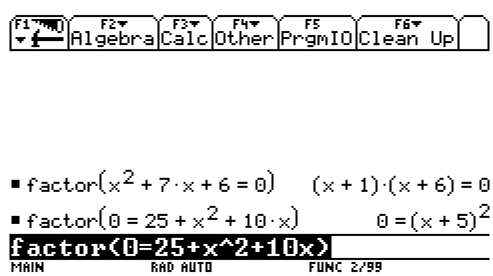
Have the students use the solve command and as they get the answers, **have them write down what they think the factors should be**. They will only get one answer for the last example, providing the following discussion opportunity.

Consider: $x^2 + 7x + 6 = 0$

What did the solve command give you? Solve command gave x = -1 or x = -6

Will the polynomial factor? What did you guess?

Use CAS factor command:



Did you get it right?

Now think of: $0 = 25 + x^2 + 10x$

Solve command gave x = -5. Why only one answer?

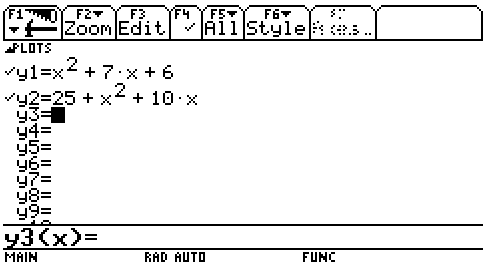
Use the factor command (see above screen)

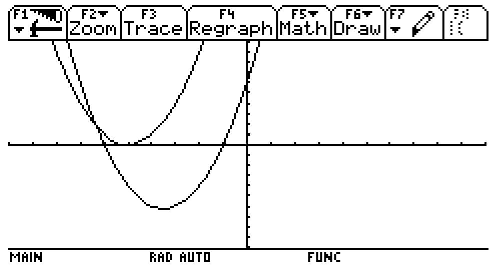
Still get the expected 2 factors because $(x+5)^2 = (x+5)(x+5)$, but they are the same so we only got one solution (zero times zero is zero just the same).

#4

Use CAS to look at relationship between solutions and intercepts and the different number of solutions to expect 0, 1, or 2.

Let's graph the above examples:





(In zoom standard <F2>:6)

Find the points where $x = -1$ and -6 on the first graph and $x = -5$ on second graph.

Notice these points are the intercepts. The picture of the second graph also shows why it only has one solution – it just touches the graph and doesn't cross twice.

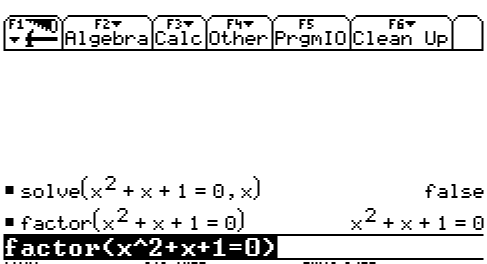
Optional

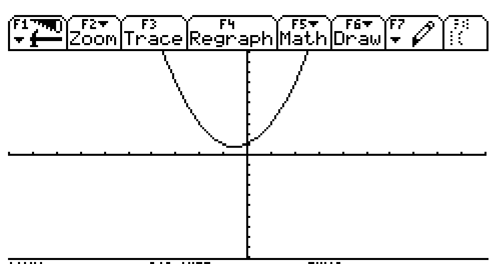
Not a focus of the text or class to discuss no solution. They will not have these problems in HW or test.

Point out that if a graph was completely over or under the axis it would have no solutions.

What would this mean for factoring?

Check with the example $x^2 + x + 1 = 0$. Calc won't solve (returns false) or factor and the graph illustrates why – there are no intercepts.





#5

Concluding framework discussion about the results we've seen and to reinforce FW objectives.

Framework discussion:

What sort of answer should I expect when solving a quadratic equation? *0,1, or 2.*

In our case 1 or 2 but always has 2 factors even if they are the same.

How do I solve a quadratic equation? *Get zero one on side and then:*

Factor and use ZPP, or Graph and find intercepts.

Why is it important to have zero on one side? Why does this make the process work so well? *We only know what one of the factors has to be if they multiply to 0. If they multiplied to 12 then we don't know what the factors should be: 2 and 6 or 4 and 3, etc.*

How can I verify my answer? *Plug in and compute to verify the equation is true, graph and look at the intercepts, write the factors based on your answers and multiply (reverse factor).*

What can I use solving quadratic equations to do?

If not already known, I can find its factorization and the intercepts of its graph.

I can solve any equation that is a product on one side and zero on the other.

Solve certain applications like we did last time: areas, consecutive integers, Pythagorean formula triangle problems

IF TIME

Work through an application problem, reinforce zero needed on one side and recall an example from the last lesson.

The length of a rectangle is 2 ft more than the width. The area of the rectangle is 48 ft^2. Find the length and width.

Set up the above problem. This problem requires multiplying the factors and then subtracting the 48. This reinforces that you must have zero on one side and that it may take some work to get there. Then use the CAS to factor (not solve). The students can then read off the answers from the factored form.

Do other examples from 5.8 if time allows.

APPENDIX D

PILOT OBSERVATION NOTES AND FEEDBACK

Pilot #1 Observation Notes – October 19th

This was the first day that the students worked with the Voyage 200 and class went smoothly. The instructor spent 20 minutes familiarizing the students with the calculator. Some students seemed bored; others clearly felt comfortable with the calculator and worked ahead or explored it on their own. The instructor specifically asked students to stay on task and ask if there were questions. There were a few conversations throughout the period but most students were attentive and a few asked questions when difficulties came up. These difficulties were mainly with command syntax: parentheses and comma. Some students forgot to put the solve command before their equation. An example in which the calculator did not recognize an implied multiplication also caused a bit of trouble.

The first question from the students was if the calculators would be allowed for the exam. Then students wondered what the point was if they were not able to. One student remarked during next class period that she did not know how to do the homework because they did it all on the calculators. The rest of the class did not seem to agree. At one point a student commented that they had already figured out how to use the calculator to do factoring. My overall impression is that the students in general had no trouble familiarizing themselves with the calculator but wondered why they were using them. Their view of the class is to learn how to do the procedures by hand, but my view is to also give more framework-based understanding.

The instructor felt that he had taught them what they needed to know for later assessments and also remarked that the calculator may have enriched the learning experience. One suggestion was to incorporate more discussion among the calculator work to help keep the students' attention. As the instructor and students become more familiar with the framework-oriented lessons this should begin to happen.

I was somewhat surprised with how well things went today. There were 24 students in attendance. The instructor covered the introductory activities in the suggested 20 minutes and followed the script closely. He wrote the problems on the board so that the students without there book could follow along. I felt that a few comments could have been used more effectively but I have had the benefit of working with the CAS and reflecting on the framework a lot longer than the instructor. He improved a few details on a few of the examples and has offered valuable feedback to improve the next version of the curriculum. He was a little rushed at the end but covered the necessary concepts.

Pilot #2 Observation Notes – October 24th

This pilot activity was an enlightening one. My initial reaction was that the instructor had not followed the plan I had drawn up and that we had discussed. Upon further reflection I realized that he had addressed the ideas we had discussed just not necessarily in the same order or time in the lesson. For example, a critical idea was using tables to discuss the equivalence of a rational expression, its simplified form, and excluded values. He did address this just not with the example I had intended and at the point in the lesson I had indicated. One reason for this was that the flow of the lesson, by the instructor's own admission, was disrupted by student responses. Time was spent trying to understand what the student was saying and how it applied. Also, the instructor did some hand work at the board that should have been done on the calculator but I had not discussed it with him. The result was too little time to cover the lesson. Important to note is that the difficulties in the discussion were not tech related.

The instructor still felt that the calculators provided enrichment, particularly with using the table function. Also, a good discussion resulted from students entering a rational expression that they thought would simplify but the calculator returned the same expression. This uncovered some common mistakes and challenged the students' thinking. One student who is more advanced than most of the class stole the CAS's thunder a few times but the discussion points were still there. I was disappointed that the CAS was not used to generalize the procedures in the lesson as I had intended.

Lessons learned: Pay close attention to the length and timing of the lessons. Be clear with the instructor about the intended application of CAS and the framework questions. Learn from the adaptations the instructor makes to the lesson plan – he has good instinct and insight about his class. The example he chose went much better than I think mine would have.

The last five minutes of class were used to give the students a short survey and receive feed back from the whole class. All of the students gave them to me with their calculator as they left the room. They were asked the following three questions. A summary of the responses is given.

1. Was using the calculator helpful in understanding the concepts of the lesson? Why or why not?

- 12/19 answered YES with the following comments:
  - Helpful to check answers
  - Could see the answer right away
  - Gave the right answers
  - Calculator did all the work but did help me understand it
  - Made the steps easier
  - Faster than checking it by hand
  - Helped me to visualize
  - Did not learn to do it by hand
  - More helpful if available on test
  - Need another day without calculator since can't use it on test

- 6/19 answered NO with the following comments:
  - Was fun and would be handy on test but didn't clarify lesson
  - I already know how to do this
  - No need for a calculator that can do everything for you
  - I still don't get it
  - Helpful but I still don't understand how to do it without the calculator
  - It did too much for me and made it difficult to see the concepts

- 1/19 answered I am not sure

2. What difficulties did you have in using the calculator and how might those be addressed?

- 14/19 answered that they had no difficulties, one comment was that it would have been great to learn more of what could be done

- 5/19 expressed some confusion:
  Too many buttons and functions (needed poster to point them out)
  Just not paying attention
  Confusing, need to ask for help

3. Describe any other feelings or thoughts you have about using the calculator in class.

- 5/19 Should be able to use them on the tests

- 3/19 I wish I had one

- 2/19 It is less boring

- 2/19 Provided a fun and helpful learning environment

- Some of the things we learned were elementary and not worth the class time

- Great tool to use

- An okay break from the ordinary

- Not sure of the purpose, would rather spend more time learning the section and going over homework

Pilot #3 Observation Notes – October 31

This third pilot activity was the best of the three. Students were able to use the calculators without any trouble. The instructor also indicated that he felt he had left the calculator alone after midway through the lesson, but he was in accordance with the plans I had given him. The lesson could have incorporated more calculator work but some students checked the answers to the by-hand examples on their own. In one instance this led to a question and discussion about where the negative of a rational expression can be put. I noticed towards the end of the lesson a student pulled out a TI-84. This could indicate that she was not comfortable or aware of the Voyage 200's capabilities. The lesson went smoothly and as scripted but a few framework questions were omitted. The last one was because of time constraints but one in the middle of the lesson is skipped. Framework discussion is as important as CAS so we need to get in a framework mode of thinking and asking.

I was able to have a discussion with two students after class. They both expressed that they did not understand why the class was using the calculators. They felt that it had weakened their ability to do them by hand and that the class time would be better spent doing more problems. A few other students gave me verbal feedback as well to confirm these feelings. One felt it was a waste of time at this level. The written feedback from the previous pilot activity represents the perspectives of the whole class and not all students are of this same opinion. Many of them felt that it should be available on the tests if they were going to use them during class. It is important to point out that these students appeared to be able to use the technology as indicated by correct solutions given in discussion.

I conveyed these ideas to the instructor and he indicated that he thought they wanted to learn the procedures in class because they did not want to do homework and felt that often they did not do homework. I think it will be important to introduce the framework to the students so that they do not narrow their focus on execution of procedures.

Written responses to reflection questions given to the instructor after the third pilot

Have you noticed anything different in …

Class discussion: *Students seem more willing to participate in discussions, maybe because the lesson plans explicitly call for discussion throughout the lesson*.

Student questions or comments: *Some are questioning the motivation for using CAS in class when it can't be used on assessments such as quizzes and tests*.

Student behavior: *Students are actively engaged (CAS work) for more of the class time than in my typical non-CAS lessons*.

Student understanding on quizzes, homework, or tests: *No differences noted yet (limited observation - one quiz)*

Coverage of the ideas and concepts in each lesson: *Lessons are well-planned and there are good connections / transitions so that the lesson flows nicely and logically.*

Student interaction and engagement: *Typical for this section – little interaction between students. (My other three sections are quite interactive with each other.) However, I think more students are engaged when using CAS than would otherwise be*.

Pilot #4 Observation Notes – November 9

There were only 10 students present at the time the class began. This concerned me because I had observed one student walk into the room, comment that it was a calculator day, and then left. She waited a few minutes then came back to get her test. She did not stay for the lesson. I wondered if students were not coming because they preferred not to use the calculators. The instructor has informed the students in advance that they will be using the calculators for each pilot lesson but it did not make a difference for the first three. Other speculations are that it was the day before a three day weekend and that it was the second day after they had taken a test. When asked, the instructor commented that the student mentioned above does not have a good attendance record.

I observed the instructor demonstrate a problem using CAS in this lesson but then explain how the procedure works before they use the CAS to investigate a group of problems. This undermines the use of CAS and we discussed how he might allow them the opportunity to conjecture and discover on their own using the CAS and trying to answer the Framework questions. He agreed to make this effort.

One concern I have is that some students that know the answers to critical questions undermine the CAS explorations and Framework discussions. I observed this today and in previous pilot lessons. Today the instructor put a system on the board that was not ideal for substitution (it was rigged to work easily with elimination). A student was able to spot this and commented on what to do. This seemed to deflate the importance of the next examples to be worked and the coming discussion questions.

Examples and the order of questions are critical to creating discussions. Asking students to estimate the output of the calculator needs to be emphasized as well.

I observed good examples of discourse behavior. Two students gave different answers to a question and the first student was asked to explain. An error was discovered this way. Numerous times the instructor asked students about what to do and required them to explain why. For example, “Which equation would you substitute in for to find the value of the other variable?” and “Why?” Not only is this discourse behavior but use of the Framework questions.

The instructor did well limiting the time students had to complete the CAS problems. He integrated Framework questions into the discussions. He again noted that he felt the calculators were a good diversion to offer some variety for the students. I feel that this curriculum can incorporate the idea of discourse.

Pilot #5 Observation Notes – November 21

Attendance was 13 students out of 25, however this was the day before the Thanksgiving break. The lesson started with the CAS solving two different equations, one with radicals and the same equation that has been squared to remove radicals. The instructor still feels the need to do some examples by hand to justify the calculator steps. He focused on why squaring removes the radical instead of quickly introducing the conflict that drives the whole lesson. Specifically, the instructor went through the solution process for a quadratic equation instead of having the calculator solve it quickly. This is a good review for the students but not the intent of the CAS lesson. There seems to be a lingering habit of demonstrating procedures that needs to be replaced with classroom discourse. This is a fundamental change that instructor and students are being asked to make and the instructor has been effective in doing so over the course of these pilots.

The instructor is making a habit of requiring students to explain their thinking, often to correct a mistake or misconception. The students struggled with multiplying binomials, puzzling me and the instructor. A few times he questioned the student and represented the square as the product of binomials so that the student would recognize their error. A great discussion took place when the instructor asked how to distinguish the two graphs. Students realized that one of them was a line and were able to draw on previous knowledge of what the graph should be like. They struggled to connect the solutions of the equation to the intersection of the graphs of each side.

The instructor and I noticed that the students did not seem motivated. They struggled with the idea of squaring a binomial and did not seem to engage in discourse as much as previously.

APPENDIX E

FRAMEWORK QUIZ: SOLVING QUADRATIC EQUATIONS

Quiz C Name: ________________________

This quiz is worth 12 points. You have 30 minutes to complete the quiz.

You have solved many quadratic equations similar to the ones that Fred is working on for homework. The first 7 of the 15 problems on his homework assignment shown below:

Homework 5.6
Solve the following quadratic equations:

1. $2y(y+5)=0$
2. $3x^2+8x=9+2x$
3. $(x-3)(x+2)=6$
4. $36x^2-25=0$
5. $x^2+10x=-25$
6. $5x^2=6x$
7. $x^2-2x-8=0$

•

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Answer the following questions that Fred has about solving quadratic equations. You may refer to any of the above equations in your explanations.

1. "On #12, I couldn't find a solution but then on problem #13, I got exactly two solutions! I thought to solve an equation I am supposed to find the one number that works." Tell Fred about the types of solutions you expect to get when solving a quadratic equation. Explain your reasoning.

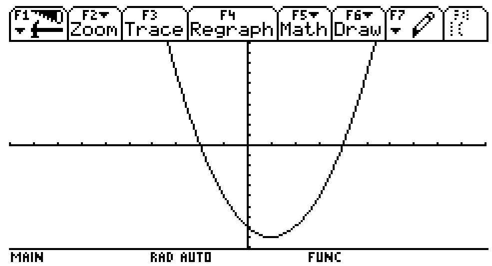
2. Fred is solving a quadratic equation **similar** to #2 above. Explain to Fred in words the steps should he take if he wanted to solve using factoring.

3. Fred is now faced with equation #3 above. He is excited that he doesn't have to do the factoring and quickly answers that $x = 3$ and $x = -2$ are solutions. Help him understand his mistake by explaining to him why factoring is an effective way to solve and the conditions necessary for the factoring method to be valid.

4. "If I make a mistake like I did on the last problem, what can I do to check my solution so I know if I got it right or wrong?" Explain to Fred at least three different ways to verify solutions to quadratic equations.

5. Suppose Fred showed you the graph of the left-hand side of an equation like #1, #4, or #7.

For example, the window to the right is the graph given when the left-hand side of equation #7 is entered in the y = editor.



"If all I have is a graph that looks like this, what can I deduce about the equation that goes along with it?" Explain to Fred the kind of information the graph tells him about the equation.

6. Share with Fred an example of what knowledge of quadratic equations and factoring can be used to do in the following areas:

(a) Real-world applications?

(b) Solving other types of equations?

APPENDIX F

FRAMEWORK QUIZ: SOLVING QUADRATIC EQUATIONS

SCORING RUBRIC

Quiz C – Sample Answers and Rubric

You have solved many quadratic equations similar to the ones that Fred is working on for homework. The first 7 of the 15 problems on his homework assignment shown below:

Homework 5.6
Solve the following quadratic equations:

1. $2y(y+5)=0$
2. $3x^2+8x=9+2x$
3. $(x-3)(x+2)=6$
4. $36x^2-25=0$
5. $x^2+10x=-25$
6. $5x^2=6x$
7. $x^2-2x-8=0$

•
•

Answer the following questions that Fred has about solving quadratic equations. You may refer to any of the above equations in your explanations.

1. "On #12, I couldn't find a solution but then on problem #13, I got exactly two solutions! I thought to solve an equation I am supposed to find the one number that works." Tell Fred about the types of solutions you expect to get when solving a quadratic equation. Explain your reasoning.

Finding solutions to any equation means finding all values for the variable that make the equation a true statement. For quadratic equations, I usually expect two solutions. This is related to the exponent of two on the variable. But there are cases though that there might only be one solution or no solution. Let me explain why.
Two solutions: This happens when we are solving by factoring and get two different factors that yield two solutions. If we visualize this on a graph, we see the curve cross the x-axis in two places, each corresponding to a solution.
One solution: This happens when the quadratic polynomial factors but the factors are the same (perfect square trinomial). The graph would show the curve touching the x-axis at one point.
No solution: This happens when I can't factor the polynomial and the curve is entirely above or below the x-axis.

Level 3 – High degree of understanding: Identifies and explains all three cases.
Level 2 – Moderate degree of understanding: Identifies all three but not fully explained OR Misses one case but explains other two.
Level 1 – Low degree of understanding: Just list some or all of the three cases with little or no explanation.
Level 0 – No Understanding Demonstrated: No answer, off topic, no merit.

2. Fred is solving a quadratic equation **similar** to #2 above. Explain to Fred in words the steps should he take if he wanted to solve using factoring.

The most important thing to do is get zero on one side of the equation and the polynomial in standard form on the other side: $ax^2 + bx + c = 0$. *This might require that you multiply, add/subtract terms on both sides, and combine like terms. After getting the equation in standard from, use your favorite technique to factor or follow the general strategy based on the number and type of terms. Don't forget to factor out the GCF first, if there is one. After factoring, set each factor equal to zero and solve. It is important to check your answer as well.*

Level 3: Complete explanation in general terms.
Level 2: Correctly explains how to solve this problem.
Level 1: Partial solves this equation by factoring, but does not solve.
Level 0: No answer, off topic, no merit.

3. Fred is now faced with equation #3 above. He is excited that he doesn't have to do the factoring and quickly answers that $x = 3$ and $x = -2$ are solutions. Help him understand his mistake by explaining to him why factoring is an effective way to solve and the conditions necessary for the factoring method to be valid.

Using factoring to solve works because of the Zero Product Principle (ZPP). If two factors multiply to zero, then one of them must be equal to zero. This is why this method is only effective if one side of the equation is zero and the other side of the equation is factored. If the product is not equal to zero, then it is not possible to narrow down what each factor should equal. In #3, factors could equal 1, 2, 3, or 6.
For this problem you need to multiply out the left-hand side then subtract the 6 from both sides and factor again. I know it seems like a lot of work and going backwards, but setting each factor equal to zero is only valid if the other side of the equation is zero.

Level 3: Gives correct way to solve and explains why it is necessary to have zero on one side and what the ZPP is.
Level 2: Tells him what he did wrong and how to fix OR explains why it is necessary to have zero on one side and what the ZPP is.
Level 1: Just solves the equation for him or simply states the mistake.
Level 0: No answer, off topic, no merit.

4. “If I make a mistake like I did on the last problem, what can I do to check my solution so I know if I got it right or wrong?” Explain to Fred at least three different ways to verify solutions to quadratic equations.

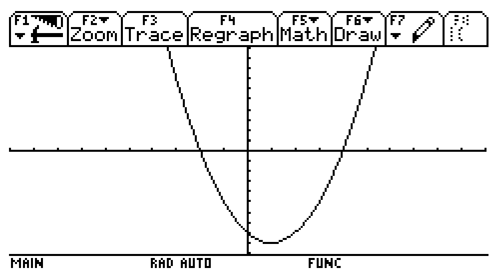
First of all, the solution(s) should make the equation true. Always plug in the value of the variable to see if the equation is a true statement. Other things that you can do are:

Graph and identify the x-intercepts. They should match your solutions.
Use the CAS solve command, or graph/table functions.

L(x): x number of ways to check identified and explained.

5. Suppose Fred showed you the graph of the left-hand side of an equation like #1, #4, or #7.

For example, the window to the right is the graph given when the left-hand side of equation #7 is entered in the *y* = editor.



“If all I have is a graph that looks like this, what can I deduce about the equation that goes along with it?” Explain to Fred the kind of information the graph tells him about the equation.

The graph actually tells me the solutions! The x-intercepts are the solutions to the equations. This also means that I can tell how many solutions to expect 0, 1, or 2. If the intercepts are easily identifiable, I can write the equation in factored form and then expand to actually come up with the equation itself.

<u>Level 3</u>: Identifies that the intercepts are the solutions and lead to the factors that can be multiplied to get the quadratic. States that the graph indicates how many solutions to expect.
<u>Level 2</u>: Identifies that the intercepts are the solutions.
<u>Level 1</u>: The student gives correct information about the graph such as intercepts but makes no connection to the original equation.
<u>Level 0</u>: No answer, off topic, no merit.

6. Share with Fred an example of what knowledge of quadratic equations and factoring can be used to do in the following areas:

(a) Real-world applications?

I can do problems that require me to find two unknown, but related quantities that are multiplied together. For example, areas require two dimensions like length and width or base and height. This gives a quadratic equation that I can solve. Other applications include products of consecutive integers and right triangle problems using Pythagorean's theorem $a^2 + b^2 = c^2$*. Motion problems are also something I can do with quadratic equations like finding the height of a rocket or other falling object.*

(b) Solving other types of equations?

The ZPP can be used to solve higher degree polynomials if I have the factorization. Working with the graphical representations of quadratics has also taught me that if I get zero on one side then I can look for solutions of any equation as x-intercepts of its corresponding graph and quickly determine the number of solutions there will be.

<u>Level 3</u>: 2 good examples, one for each part, with explanation.
<u>Level 2</u>: 1 good example with explanation, missing other example or no explanation.
<u>Level 1</u>: 2 examples without explanation.
<u>Level 0</u>: No answer, off topic, no merit.

APPENDIX G

QUIZ FOLLOW-UP INTERVIEW PROMPTS

Quiz C - Interview Prompts

Can they answer this question as I would want them to with further probes? Does their written answer represent their understanding?

Was the question clear in wording and expectations?

You have solved many quadratic equations similar to the ones that Fred is working on for homework. The first 7 of the 15 problems on his homework assignment shown below:

Homework 5.6
Solve the following quadratic equations:

1. $2y(y+5)=0$
2. $3x^2+8x=9+2x$
3. $(x-3)(x+2)=6$
4. $36x^2-25=0$
5. $x^2+10x=-25$
6. $5x^2=6x$
7. $x^2-2x-8=0$

•

•

Answer the following questions that Fred has about solving quadratic equations. You may refer to any of the above equations in your explanations.

1. "On #12, I couldn't find a solution but then on problem #13, I got exactly two solutions! I thought to solve an equation I am supposed to find the one number that works." Tell Fred about the types of solutions you expect to get when solving a quadratic equation. Explain your reasoning.

Can they answer this question as I would want them to with further probes? Does their written answer represent their understanding?

What does it mean to be a solution of any equation?

How many values could you plug in for these types of equations listed?

Explain why you listed these cases.

How do these cases relate to factoring? The graph?

Was the question clear in wording and expectations?

2. Fred is solving a quadratic equation **similar** to #2 above. Explain to Fred in words the steps should he take if he wanted to solve using factoring.

Express these steps more generally to explain the procedure for any quadratic equation.

What does this step involve? Why is this step important?

Why is it necessary to have zero on one side?

3. Fred is now faced with equation #3 above. He is excited that he doesn't have to do the factoring and quickly answers that $x = 3$ and $x = -2$ are solutions. Help him understand his mistake by explaining to him why factoring is an effective way to solve and the conditions necessary for the factoring method to be valid.

Why is it necessary to have zero on one side?

How does this make the solving process easier?

How can I make the problem fit the form I need for ZPP?

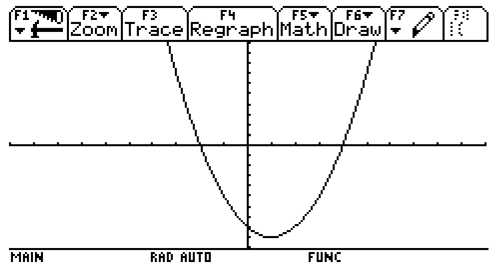
4. "If I make a mistake like I did on the last problem, what can I do to check my solution so I know if I got it right or wrong?" Explain to Fred at least three different ways to verify solutions to quadratic equations.

How would you use (this method) to check?

Any other ways to check that you can think of?

5. Suppose Fred showed you the graph of the left-hand side of an equation like #1, #4, or #7.

For example, the window to the right is the graph given when the left-hand side of equation #7 is entered in the *y* = editor.



"If all I have is a graph that looks like this, what can I deduce about the equation that goes along with it?" Explain to Fred the kind of information the graph tells him about the equation.

What does the graph tell me about solutions to the equation?

What does the graph tell me about the number of solutions to the equation?

What does the graph tell me about the factor to the equation?

6. Share with Fred an example of what knowledge of quadratic equations and factoring can be used to do in the following areas:

(a) Real-world applications?

What type of word problems have you been able to do that involve quadratic equations?

(b) Solving other types of equations?

Can graphs or factoring be used to solve other types of equations?

APPENDIX H

PROCEDURAL UNDERSTANDING EXAM

Concepts Review Test   Name ________________________

Section # __________

Read each question carefully and completely. In most questions, you may not be required to perform a procedure, but explain something about the procedure. The questions are framed in a hypothetical setting to help you relax and more easily share your mathematical ideas on the questions posed.

You and a few friends (Marcie, Dustin, and Fred) are studying for the final exam at a table in the SUB. Answer these questions that your friends bring up during the study session.

1. Fred begins the study session by making the comment, "I was looking back at my notes about how to find the graph when given an equation of a line. I had written down that it is easy to graph lines by finding the intercepts. I can't quite remember how to do that." Explain how to graph a linear equation in 2 variables (such as $3x-2y=6$) using the intercepts.

2. Marcie says, "What if one of the intercepts is a fraction like in $7x-2y=8$? It is hard for me to plot fractions accurately so Fred's method of using intercepts to graph wouldn't work well for me. Is there another way I can do it?" Explain an alternative method that Marcie could use to graph this equation of a line.

3. Dustin remarks, "Hey, aren't the systems of equations we learned about just two of these linear equations taken together? If I draw the graphs of the two linear equations on the same graph, shouldn't the graphs tell me something about the solutions to the system?" Explain to Dustin the different outcomes to expect when solving a system of two linear equations and how the outcomes relate to the graphs of the lines of the system.

4. As the group moves on to discuss the topic of exponents, Fred moans, "There are so many rules to remember. On the last test I forgot the quotient rule and said that $\frac{x^8}{x^2} = x^4$ since 8 ÷ 2 is 4. I know that instead of dividing the exponents, I should have subtracted them to get $\frac{x^8}{x^2} = x^{8-2} = x^6$. Maybe if I can understand why the quotient rule works, I'll be able to remember it next time." Explain why the quotient rule is valid mathematically.

5. Dustin wants to talk about factoring. “The first step in factoring a polynomial is to check for a greatest common factor. After that there are several methods I could use like looking for a difference of squares or perfect square trinomial, using FOIL in reverse, or factoring by grouping. How can I figure out the best way to go?” Discuss a strategy for deciding when to use one of the methods Dustin mentions to factor a given polynomial. You may refer to the examples below. Also, feel free to recommend other methods that come to mind.

a. $12x^2 + 3x - 8x - 2$

b. $4x^2 - 25$

c. $x^2 - 5x + 6$

d. $16x^3 + 32x^2 - 9x - 18$

e. $x^2 - 18x + 81$

f. $x^2 - 10x + 16$

6. Another student walks by your group’s table and overhears your conversation about factoring polynomials. He stops and makes the comment, “I didn’t like learning how to factor. What is it useful for anyway?” Explain to this passerby how factoring has been useful in doing algebra problems in your class. Give three examples of problems that you have learned about in this class that have required factoring as one of the steps.

7. Two of your friends disagree about how to reduce the rational expression $\frac{x^2 - 25}{x^2 - 10x + 25}$. Fred just cancelled the x^2 terms and the 25's and wrote $\frac{1}{10x}$. Marcie thinks Fred's answer is incorrect but is having trouble explaining it to Fred. Describe the general steps for simplifying a rational expression and why it works that way.

8. Dustin recalls that sometimes you get answers to rational equations that really are not solutions to the equations and have to be excluded. "In an equation like $\frac{2}{x-3} - \frac{3}{x+3} = \frac{12}{x^2 - 9}$, how would I be able to check if my answer was one that has to be thrown out?" Explain to Dustin how to determine if an answer to a rational equation is really a solution to the equation.

9. “We usually have word problems to solve at the end each chapter,” Marcie notes. “In this chapter about rational equations, we had to solve work problems. For example, Adam finishes a job in 4 hours that Ben can finish in 3 hours. How long would it take them together? Sometimes the answers to work problems are messy fractions like $2\frac{15}{16}$ minutes or $5\frac{27}{35}$ hours. How can I estimate a reasonable answer for these types of problems?”

Explain to Marcie how to estimate a reasonable answer to work problems. Explain why your estimation process works.

10. “We learned about radicals pretty recently so I am still trying to think of times when knowing how to simplify a radical was necessary in homework problems.” Share with Marcie two or three types of problems that you did that required you to simplify radicals.

11. Fred is working on solving a radical equation and says, "I squared both sides like we learned and got $x = 1$ and $x = 5$ as the answers to the equation. The back of the book only gives $x = 5$ as an answer. Here is the work I did on the problem. What did I do wrong?"

$$\sqrt{2x-1} = x - 2$$
$$\left(\sqrt{2x-1}\right)^2 = (x-2)^2$$
$$2x - 1 = x^2 - 4x + 4$$
$$0 = x^2 - 6x + 5$$
$$0 = (x-1)(x-5)$$
$$x = 1 \text{ and } x = 5$$

Show Fred how to check his answers. Discuss why it is necessary to check your answers after squaring both sides of an equation that contains radicals.

APPENDIX I

PROCEDURAL UNDERSTANDING EXAM SCORING RUBRIC

Coding Levels

Level 3: High Degree of Understanding

Level 2: Moderate Degree of Understanding

Level 1: Low Degree of Understanding

Level 0: No Understanding Demonstrated

Coding Guidelines

- These questions were designed to assess the depth of knowledge of Framework-based ideas.
- Coding decisions should not be influenced by poor wording or incorrect grammar/language.
- Some portions of the answer may be incorrect. Unless specifically noted in the rubric, scores should not be reduced for just that reason.

Notes about the exam

The exam was given during one 50-minute class period.

1. Fred begins the study session by making the comment, "I was looking back at my notes about how to find the graph when given an equation of a line. I had written down that it is easy to graph lines by finding the intercepts. I can't quite remember how to do that." Explain how to graph a linear equation in 2 variables (such as $3x - 2y = 6$) using the intercepts.

<u>Sample Answer</u>

I would plug in zero for y and solve the equation for x to find the coordinates of the x-intercept. Plot this point on the x-axis. Then I would plug in zero for x to find the coordinates of the y-intercept. Plot this point on the y-axis and connect these two points to get the graph of the line. I can also use a third point to check by plugging a number in for x and solving for y to get another ordered pair that should be on the same line.

<u>Note</u>: Correctly graphing the equation DOES NOT represent complete understanding.

<u>Level 3</u>: Complete explanation of finding the intercepts (Set $x = 0$ and solve for y, then set $y = 0$ and solve for x). Explains that intercepts give you two ordered pairs that you can plot and connect to graph the line.

<u>Level 2</u>: Complete, general description of finding intercepts but missing/misconceptions in explanation of how to get the graph. OR Good description of finding specific intercepts and the correct graph displayed.

<u>Level 1</u>: Explains how to find these specific intercepts (or just gives correct intercepts) but no graphical connection OR Explains how to graph using the slope-intercept form of the equation, specifically identifying the *y*-intercept and it's role as the starting place.

<u>Level 0</u>: Off task, missing response, answers wrong question, discusses another method such as must be in $y = mx + b$ form (no mention of how to use the *y*-intercept and slope) or talks about plotting points with no specific mention of zero.

2. Marcie says, “What if one of the intercepts is a fraction like in $7x - 2y = 8$? It is hard for me to plot fractions accurately so Fred’s method of using intercepts to graph wouldn’t work well for me. Is there another way I can do it?” Explain an alternative method that Marcie could use to graph this equation of a line.

Sample Answer

I can find the y-intercept by plugging in zero for x. That gives me one point to plot. I can plug in other numbers like 2 or 4 for x and not get fractions for the y-coordinate. This gives me enough points to draw and check the line.

I can solve the equation for y which gives me the slope intercept form y = mx + b. I plot the y-intercept (0,b) and then use the slope as rise over run to count up if positive or down if negative and then right. This gives me a second point so I can graph the line.

Note: If the student references question 1 then you can refer to that answer as well to judge the level of understanding for question 2.

Note: Do not penalize for arithmetic errors.

Level 3: Fully explains a valid method – finding and plotting integer points or using the slope-intercept form are the common two.

Level 2: Valid method but leaves out some details, such as “use the slope” w/o explaining how to use rise over run by counting OR “use rise over run” without explaining to start at the intercept and counting, OR build a table without discussing how to pick good points, etc.

Level 1: States valid method w/o explanation. (Use slope-intercept, $y = mx + b$, rise over run, chart)

Level 0: Off task, missing response, answers wrong question, or same method (use intercepts). Decimal point, assumes that the fractional intercept is rise over run.

3. Dustin remarks, “Hey, aren’t the systems of equations we learned about just two of these linear equations taken together? If I draw the graphs of the two linear equations on the same graph, shouldn’t the graphs tell me something about the solutions to the system?” Explain to Dustin the different outcomes to expect when solving a system of two linear equations and how the outcomes relate to the graphs of the lines of the system.

Sample Answer

If I graph two lines there are three things that can happen. The lines could intersect at one point, they could be parallel, or they may end up on top of each other. The solution to the system is where the two lines intersect. In these three cases there would be one solution point (x,y) if they intersect, no solution if they are parallel, or infinite solutions (each point on the line) if the equations lead to the same line.

Note: Sketches of the graphical situations are equivalent to the verbal description.

Level 3: Completely describes the three cases in terms of the graphs AND the number of solutions. (Perpendicular acceptable for intersection)

Level 2: Able to describe all three cases graphically OR by number of solutions OR complete descriptions (graph and solutions) for two of the three cases.

Level 1: Only talks about a few of the cases without complete descriptions OR treats it as a “parallel, perpendicular, neither” problem.

Level 0: Off task, missing response, answers wrong question, or gives information about a single line.

4. As the group moves on to discuss the topic of exponents, Fred moans, "There are so many rules to remember. On the last test I forgot the quotient rule and said that $\frac{x^8}{x^2} = x^4$ since 8 ÷ 2 is 4. I know that instead of dividing the exponents, I should have subtracted them to get $\frac{x^8}{x^2} = x^{8-2} = x^6$. Maybe if I can understand why the quotient rule works, I'll be able to remember it next time." Explain why the quotient rule is valid mathematically.

<u>Sample Answer</u>

If I write in expanded notation $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$ *I can match up factors on top and bottom* $\frac{x}{x} \cdot \frac{x}{x} \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$ *using the rule of multiplication* $\frac{a \cdot c}{b \cdot d} = \frac{a}{b} \cdot \frac{c}{d}$. *Then I can use the fact that* $\frac{x}{x} = 1$ to "cancel" the *x*'s.

<u>Level 3</u>: Shows or talks about cancelling 2 *x*'s top and bottom AND talks about dividing common factors/multiplying by 1 as the reason this works.

<u>Level 2</u>: Shows or clearly describes the cancellation but offers no reason as to why that is okay to do. (The how but not the why.)

<u>Level 1</u>: Quotes product rule and the inverse relationship of division.

<u>Level 0</u>: Off task, missing response, answers wrong question, restates quotient rule or quotes other rules, uses example to show that rule doesn't work (answering different FW question). Gives a memory device.

5. Dustin wants to talk about factoring. "The first step in factoring a polynomial is to check for a greatest common factor. After that there are several methods I could use like looking for a difference of squares or perfect square trinomial, using FOIL in reverse, or factoring by grouping. How can I figure out the best way to go?" Discuss a strategy for deciding when to use one of the methods Dustin mentions to factor a given polynomial. You may refer to the examples below. Also, feel free to recommend other methods that come to mind.

a. $12x^2 + 3x - 8x - 2$
Grouping or combine/reverse FOIL

b. $4x^2 - 25$
Difference of squares

c. $x^2 - 5x + 6$
Reverse FOIL

d. $16x^3 + 32x^2 - 9x - 18$
Grouping

e. $x^2 - 18x + 81$
Perfect square or reverse FOIL

f. $x^2 - 10x + 16$
Reverse FOIL

<u>Sample Answer</u>

After seeing if a polynomial has a GCF to factor out, the next thing I do is count the number of terms. If there are two terms, the only thing I can do is determine if it is a difference of squares and use $a^2 - b^2 = (a+b)(a-b)$. *If it is a trinomial (three terms) then I can use reverse FOIL unless I happen to notice that the first and last terms are perfect squares, then I can use a shortcut. If the polynomial has four terms, factoring by grouping is the way to go.*

<u>Note</u>: Being able to do the factorizations correctly IS NOT by itself evidence that students have a clear idea of when to use appropriate methods, nor do students need to do the factoring if they correctly identify the appropriate method.

<u>Note</u>: Perfect square trinomials can be done by reverse FOIL with efficiency so recalling the formula is not a necessary part of the strategy.

<u>Level 3</u>: Describes a general strategy applicable to any polynomial. Bases this on the number of terms.

<u>Level 2</u>: Lists the appropriate method for each of the examples but no insight into why they made those choices OR Describes a general strategy that is incomplete or has minor errors.

<u>Level 1</u>: Sticks to one method OR Gives a few correct examples OR Attempts to list methods for each polynomial but has a few errors.

<u>Level 0</u>: Off task, missing response, answers wrong question, or just gives the correct factorizations.

6. Another student walks by your group's table and overhears your conversation about factoring polynomials. He stops and makes the comment, "I didn't like learning how to factor. What is it useful for anyway?" Explain to this passerby how factoring has been useful in doing algebra problems in your class. Give three examples of problems that you have learned about in this class that have required factoring as one of the steps.

<u>Sample Factoring Uses</u>

- Solve equations (If specific, can be counted for more than one example)
  - Quadratics
  - Polynomial
  - Radical
  - Rational
  - Set equal to zero
- LCM, LCD, GCF
- Add, subtract, multiply, divide, reduce rational expressions
- Check domain of rational expression/equation
- Add, subtract, multiply, divide simplify radical expressions
- Combine like terms
- Find intercepts on graph
- Word Problems (MUST be specific)
  - Pythagorean formula/Triangles
  - Areas
- Solving formulas for specified variables
- Divide polynomials (but not add, subtract, or multiply)

<u>Note</u>: The examples don't necessarily have to illustrate the factoring (i.e. if they give a rational expression to simplify, the student does not need to simplify, nor does the problem actually have to have anything that can be simplified).

<u>Note</u>: The student must be specific about word/story problems.

<u>Note</u>: Credit should only be given for graphing if they explain that factoring gives intercepts.

<u>Level 3</u>: 3 <u>distinct</u> examples of problems in words or symbols

<u>Level 2</u>: 2 <u>distinct</u> examples of problems in words or symbols

<u>Level 1</u>: 1 example of a problem in words or symbols, include here the case of three examples of the same type of problem OR explains the factoring procedure.

<u>Level 0</u>: Off task, missing response, answers wrong question.

7. Two of your friends disagree about how to reduce the rational expression $\frac{x^2 - 25}{x^2 - 10x + 25}$. Fred just cancelled the x^2 terms and the 25's and wrote $\frac{1}{10x}$. Marcie thinks Fred's answer is incorrect but is having trouble explaining it to Fred. Describe the general steps for simplifying a rational expression and why it works that way.

Sample Answer

The first step is to factor as much as possible in the numerator and denominator. That is because if we can find common factors to both the top and bottom then they will divide to 1 and "cancel out" after using the associative property. We must remember, however, that values making the denominator zero cannot be plugged into either expression.

Level 3: Explanation of "factor and cancel common factors". Explain that this is the same as multiplying by one (something over itself)

Level 2: Explains the steps (factor completely and cancel common factors) without reasoning as to why that can be done.

Level 1: Just shows execution OR explains with errors (such as cancel like terms after factoring)

Level 0: Off task, missing response, answers wrong question, or "must have multiplication not add or subtract" but doesn't mention factoring.

8. Dustin recalls that sometimes you get answers to rational equations that really are not solutions to the equations and have to be excluded. “In an equation like $\frac{2}{x-3} - \frac{3}{x+3} = \frac{12}{x^2-9}$, how would I be able to check if my answer was one that has to be thrown out?” Explain to Dustin how to determine if an answer to a rational equation is really a solution to the equation.

<u>Sample Answer</u>

It is easy to tell which numbers would have to be excluded. If I factor the denominators first, then I can determine the numbers that would make it equal to zero causing the fraction to be undefined. These are the bad numbers.

When I am finished solving, I can plug my answer in to check. If it makes the denominator zero, then it is not a valid solution. Otherwise I can proceed and see if both sides are equal thereby checking for any other errors I made during the process.

<u>Level 3</u>: Indicates that the denominator can’t be zero AND explains a process (plug in or factor at beginning) to determine if denominator is zero.

<u>Level 2</u>: Just states that you can’t have zero in denominator but no explanation as to how check (as question asks) OR States the proper restrictions without reasoning.

<u>Level 1</u>: Plug answer back in to check but doesn’t mention zero in denominator, OR mentions zero but no connection to denominator

<u>Level 0</u>: Off task, missing response, answers wrong question , or mistakes the problem for verifying a subtraction of rational expressions.

9. “We usually have word problems to solve at the end each chapter,” Marcie notes. “In this chapter about rational equations, we had to solve work problems. For example, Adam finishes a job in 4 hours that Ben can finish in 3 hours. How long would it take them together? Sometimes the answers to work problems are messy fractions like $2\frac{15}{16}$ minutes or $5\frac{27}{35}$ hours. How can I estimate a reasonable answer for these types of problems?”

Explain to Marcie how to estimate a reasonable answer to work problems. Explain why your estimation process works.

Sample Answer

If Ben can finish in 3 hours, then with help the job would definitely be done in less than 3 hours. If there were two Bens the job would be done in 1.5 hours. If there were two Adams the job would be done in 2 hours. A good answer would be between 1.5 and 2 hours.

Level 3: Predicts a number between half of the two times (1.5 and 2)

Level 2: Predicts a number less than the shorter time (less than 3)

Level 1: Averages the two times or predicts between the two times (3.5)

Level 0: Off task, missing response, answers wrong question, or suggests rounding the sample answers or computing decimal.

10. “We learned about radicals pretty recently so I am still trying to think of times when knowing how to simplify a radical was necessary in homework problems.” Share with Marcie two or three types of problems that you did that required you to simplify radicals.

Sample Radical Problems

Simplify,
Multiply
Divide, rationalize denominator
Quadratic formula, equations
Pythagorean formula, right triangles, find missing length
Add radicals
Subtract radicals

Note: Squaring both sides of an equation is a different procedure than simplifying a radical and should not be counted as such.

Note: The examples DO NOT have to be engineered to illustrate the use of simplifying (i.e. $\sqrt{3} \cdot \sqrt{2}$ illustrates a multiplication of radicals. Usually this requires simplification even though this specific problem would not.)

Level 3: 3 distinct examples (in words or symbolically)

Level 2: 2 distinct examples

Level 1: 1 example, three examples of the same kind, explanation of simplifying

Level 0: Off task, missing response, answers wrong question, misconception of definition of radical.

11. Fred is working on solving a radical equation and says, "I squared both sides like we learned and got $x = 1$ and $x = 5$ as the answers to the equation. The back of the book only gives $x = 5$ as an answer. Here is the work I did on the problem. What did I do wrong?"

$$\sqrt{2x-1} = x - 2$$
$$\left(\sqrt{2x-1}\right)^2 = (x-2)^2$$
$$2x - 1 = x^2 - 4x + 4$$
$$0 = x^2 - 6x + 5$$
$$0 = (x-1)(x-5)$$
$$x = 1 \text{ and } x = 5$$

Show Fred how to check his answers. Discuss why it is necessary to check your answers after squaring both sides of an equation that contains radicals.

Sample Answer

You need to substitute your answer(s) back in for the variable and make sure the equation is true. There are a few things to watch out for: a negative under the radical or a radical equal to a negative number. Remember, the radical sign for us means the principle or positive square root. When you square both sides, "extra" answers might be introduced. You can see this if you graph each side before and after squaring.

Level 3: Explains how to check (substitution) AND why some answers don't work: (principle square root, negative radicand, equation changes after squaring) OR graphical explanation (lines become parabolas, etc.)

Level 2: Explains how but only states that you might get extra answers without saying why OR shows plugging in and coming to the point where $1 \neq -1$

Level 1: Plug in/substitute and check (shows or just states)

Level 0: Off task, missing response, answers wrong question, or plug in to a step after squaring or check work at some point after squaring.

APPENDIX J

PROCEDURAL UNDERSTANDING EXAM

VALIDATION

Review Test

Name ________________________

Section # __________

Read the question carefully and completely. In most questions, you may not be required to perform a procedure, but explain something about the procedure. The questions are framed in a hypothetical setting to help you relax and more easily share your mathematical ideas on the questions posed.

You and a few friends (Marcie, Dustin, and Fred) are studying for the final exam at a table in the SUB. Answer these questions that your friends bring up during the study session.

1. Fred begins the study session by making the comment, "I was looking back at my notes about how to find the graph when given an equation of a line. I had written down that it is easy to graph lines by finding the intercepts. I can't quite remember how to do that." Explain how to graph a linear equation in 2 variables (such as $3x - 2y = 6$) using the intercepts.

This seems to be Item 2a – the student can perform the procedure and explain it to others. I don't see an explicit request for an explanation for why it works (i.e. by substituting (0,y) and (x,0) and plotting these points.) Treatment students might know to write something about that from their prior work, but control students might be less inclined to do so. I could see students just saying x= 6/3 = 2 is the x-intercept without explaining why. If your intention is to address Item 3, you might consider rephrasing it to make it more explicit.

2. Marcie says, "What if one of the intercepts is a fraction like in $7x - 2y = 8$? It is hard for me to plot fractions accurately so Fred's method of using intercepts wouldn't work well for me. Is there another way I can do it?" Explain an alternative method that Marcie could use to graph this equation of a line.

This is a nice context for listing alternative methods: Item 2b.

3. Dustin remarks, "Hey, aren't the systems of equations we learned about just two of these linear equations taken together? If I draw the graphs of the two linear equations on the same graph, shouldn't the graphs tell me something about the solutions to the system?" Explain to Dustin the different outcomes to expect when solving a system of two linear equations and how the outcomes relate to the graphs of the lines of the system.

Aligned with Item 1b: predicting the answer and knowing what sort of answer to expect. Connecting it to the graphs also hits on Item 2b insofar as it refers to alternative representations for the procedure.

4. As the group moves on to discuss the topic of exponents, Fred moans, "There are so many rules to remember. On the last test I forgot the quotient rule and said that $\frac{x^8}{x^2} = x^4$ since 8 ÷ 2 is 4. I know that instead of dividing the exponents, I should have subtracted them to get $\frac{x^8}{x^2} = x^{8-2} = x^6$. Maybe if I can understand why the quotient rule works, I'll be able to remember it next time." Explain why the quotient rule is valid mathematically.

Aligned with Item 3 – why does it work, why is it valid?

5. Dustin wants to talk about factoring. "The first step in factoring a polynomial is to check for a greatest common factor. After that there are several methods I could use like looking for a difference of squares or perfect square trinomial, using FOIL in reverse, or factoring by grouping. How can I figure out the best way to go?" Discuss a strategy for deciding when to use one of the methods Dustin mentions to factor a given polynomial. You may refer to the examples below. Also, feel free to recommend other methods that come to mind.

a. $12x^2 + 3x - 8x - 2$

b. $4x^2 - 25$

c. $x^2 - 5x + 6$

d. $16x^3 + 32x^2 - 9x - 18$

e. $x^2 - 18x + 81$

f. $x^2 - 10x + 16$

Aligned with Item 5 – how do I decide which is the best method to use?

6. Another student walks by your group's table and overhears your conversation about factoring polynomials. He stops and makes the comment, "I didn't like learning how to factor. What is it useful for anyway?" Explain to this passerby how factoring has been useful in doing algebra problems in your class. Give three examples of problems that you have learned about in this class that have required factoring as one of the steps.

Aligned with Item 6 – what can I use this procedure to do?

7. Two of your friends disagree about how to reduce a rational expression like $\frac{x^2 - 25}{x^2 - 10x + 25}$. Fred just cancelled the x^2 terms and the 25's and wrote $\frac{1}{10x}$. Marcie thinks Fred's answer is incorrect but is having trouble explaining it to Fred. Describe the general steps for simplifying a rational expression and why it works that way.

Nice problem! Aligned with Item 2a (what are the steps) and Item 3 (explaining why it works).

8. Dustin recalls that sometimes you get answers to rational equations that really are not solutions to the equations and have to be excluded. "In an equation like $\frac{2}{x-3} - \frac{3}{x+3} = \frac{12}{x^2 - 9}$, how would I be able to check if my answer was one that has to be thrown out?" Explain to Dustin how to determine if an answer to a rational equation is really a solution to the equation.

Aligned with Item 4: How can I verify my answer?

9. "We usually have word problems to learn about in each chapter," Marcie notes. "In this chapter about rational equations, we talked about work problems. If I write the wrong equation, I can correctly solve and check my answer but still be incorrect. It would be great if I could estimate beforehand what a reasonable answer might be. Can you help me with this next problem?"

Help Marcie understand how to estimate a reasonable answer for the following problem without solving it:

Adam finishes a job in 4 hours that Ben can finish in 3 hours. How long would it take them together?

At a glance, this question seems to be aligned with Item 1b (predicting or estimating the outcome of a procedure), except here I think estimation is being applied to a problem, not to a procedure. In this case, I can estimate how long it will take the two of them to finish the job (between 1.5 and 2 hours) without having a procedure in mind. Therefore, while this question is asking students to estimating an answer, I'm not sure I would associate it with procedural understanding (that is, understanding of any given procedure). I'm not sure I have a good suggestion for a workaround at the moment, but I'd be happy to talk about it with you if you like.

10. "We learned about radicals pretty recently so I am still trying to think of times when knowing how to simplify a radical was necessary in homework problems." Share with Marcie two or three types of problems that you did that required you to simplify radicals.

Aligned with Item 6 – What kinds of problems can I solve with this method?

11. Fred is working on solving a radical equation and says, "I squared both sides like we learned and got $x = 1$ and $x = 5$ as the answers to the equation. The back of the book only gives $x = 5$ as an answer. Here is the work I did on the problem. What did I do wrong?"

$$\sqrt{2x-1} = x-2$$
$$\left(\sqrt{2x-1}\right)^2 = (x-2)^2$$
$$2x-1 = x^2-4x+4$$
$$0 = x^2-6x+5$$
$$0 = (x-1)(x-5)$$
$$x = 1 \text{ and } x = 5$$

Show Fred how to check his answers. Discuss why it is necessary to check your answers after squaring both sides of an equation that contains radicals.

This question hits two objectives. First, it refers to knowing how to check the answer (presumably, by substitution, or possibly by graphing given your technology emphasis). That's Item 4 – Verifying the answer.

The question of why it is necessary to check could be answered at a number of levels. "Because the answers might not both work" is a fairly superficial reason, whereas the principle that squaring both sides is not a one-to-one operation also explains why new solutions can be introduced in the process. Nonetheless, the question seems to be partially aligned with Item 1b (what sort of answer should I expect) and also with Item 3 (why is it effective & valid) in that it involves the conceptual underpinnings.

This is a rich question. It will be interesting to see how students from both the treatment & control groups respond.

<u>Overall:</u>

It looks as though you have selected a nice range of Framework-oriented procedural understanding. I did not identify any problems with Item 1a ("What is the goal of the procedure"), but there were several addressing Item 1b ("What sort of answer should I expect"). Your question 9 made me think about what Item 1b means: have a look at my comments and feel free to contact me if you want to discuss it. Overall, I agree this test does a nice job of addressing Framework-oriented procedural understanding.

APPENDIX K

INSTRUCTOR DEBRIEF INTERVIEW

11:15 am, Tuesday, May 08, 2007

Researcher: How did your students respond when you introduced computer algebra systems into your lessons? The first day you brought the calculators in?

Instructor: I think they responded fairly well, of course, we'd had the trial with that last semester and it didn't seem to be any major problems that I recall. And it went fairly well. One thing I noticed was that it's really a way to get students actively involved in the lesson. Hands-on involved, let's say.

Researcher: Did you observe any difficulties as the students were learning how to use the calculators?

Instructor: There were difficulties. I'd say they were fairly isolated and I could pretty much tell when people were having problems. I'd see some gathering of conversations in the back of the room, usually in the back of the room. I could find out what the trouble was or go and observe and give them some help. So it seemed to work pretty well. And that went throughout the semester, too. Later on in the semester, either they had forgotten some procedure or something of that nature and we would have trouble. But it wasn't a big issue. I'd say that the students did much better with the calculator than I would have done and have done in recent years.

Researcher: You don't feel like those isolated difficulties hampered the lesson or instruction?

Instructor: No.

Researcher: What difficulties, if any, did you encounter while teaching with the CAS?

Instructor: It was a bit awkward for me at times. It kind of interrupted the flow of what I felt I needed to do and wanted to do, for me to do work on the calculator. It worked better if I could leave it to the students to do the calculator work and then talk to them about the results they were getting. I think that's what I tended to gravitate towards, towards the later part of the semester. I would have them do some calculator work, poll them for what they saw – the results from the calculator or the results that they got, and work the discussion from that. Other times it did help when I had the calculator preloaded with equations that I wanted to use, for example, save a little time. Or sometimes I would put problems on the board for students to work with the calculator and then during the time they were doing that I could do my own work.

Researcher: What about the presentation for the class – the overhead projection, you mentioned having them preloaded – did that work well? Do you have recommendations for how that could be improved upon?

Instructor: Yeah, we started out in one classroom in Johnstone and it's not much of a classroom. It's kind of a bowling alley arrangement where I was at one end and the students were at the other in the pin pit. Of course they all want to sit towards the back. And the real problem there though was, aside from the fact that the classroom's not much good, the board – there was very narrow space at the front of the classroom so we just had a limited amount of board space. To show the calculator projection on the screen, which you pulled down across the board, destroyed most of the board space. You couldn't use it. It was a real problem. Of course you know, we addressed that after a few weeks and moved to a couple of different classrooms. One of those, I would say, would be ideal and that was the one that had the electronic podium with internet access so we could get to my website and use that for various things. Also, it would have been nice if there had been a Voyage 200 simulator that we could have used online to do this stuff.

Researcher: I know for other calculators they have that. The 84 – you have a simulator. The Voyage 200 that we use doesn't but I think the new computer algebra system, the Nspire, will have one. We're hoping to be able to improve upon presentation.

Instructor: Yeah, it seems like it would an important instructional tool.

Researcher: Would you favor that then if it could be projected on a computer screen?

Instructor: Absolutely, yeah.

Researcher: Hopefully we'll even get smart board capability where the picture's up there and all you have to do is push the buttons. Let's not get too starry-eyed.

Instructor: The other thing I noticed was with using the projector, the overhead projector, to project the Voyage 200, it took me a few tries to figure out what the optimal configuration was in a particular classroom. In Wilson, for example, first I was projecting on the front wall, not using the screen because there again the screen interfered with most of the black board. And then I tried the side wall which seemed to work a lot better. And in the other room, with the electronic podium, again because of the screen coming own right over the board, what I gravitated to was projecting on the side wall there. Not ideal because the lighting is not the best for projecting on those walls, too much ambient light in the room, but it worked.

Researcher: Did you have any technical difficulties or syntax difficulties for you and the students that you can think of?

Instructor: There were parentheses problems from time to time. And that was one thing that during one of the last times we used the calculators, it was a parentheses problem. And, you know, I would have those myself from time to time. I think it went pretty well.

Researcher: Do you feel that you were familiar with the computer algebra system? Could you have maybe received more support in learning that or training? Any thoughts along those lines?

Instructor: I think it was adequate. I don't feel like I'm a pro with it now, but it was adequate. I was constantly learning new things.

Researcher: Me, too. Maurice still brings up things – you never showed me that. I have no idea what you're talking about.

Instructor: But I don't think it really got in the way of my instruction.

Researcher: So, in your opinion then, how did CAS, having CAS in the class, affect your instruction and flow of the class. You taught three sections of it this semester. How does the flow of this class compare to your other sections? (Pause) Do you feel like it slowed you down? Do you feel like you were on par?

Instructor: Maybe it kept me on track more than slowing me down. I know we had trouble getting through the lesson plans, especially earlier in the semester. It seemed like we were always coming up a bit short – one or two topics short – and it seemed to improve over the semester. I think for two reasons, me adhering more to the plan and trying to stick with it and being aware of the time issue and also between us we cut some of the material out from time to time to address the time issue. As far as the flow went with that class compared to the other two, I guess my style is to react from responses I get from students and so in the other two classes it flowed more freely I would say. Responding to a student's response and then based on that or some problems I was seeing, move in a particular direction or maybe spend more time on that topic. Because of the time constraints and wanting to keep with the lesson plans I didn't feel that I had a whole lot of freedom to dwell on points that students appeared to be having trouble with or that I wasn't sure they were getting.

Researcher: Ok.

Instructor: Maybe it was more structured and that may be good.

Researcher: That's exactly the type of information I'm looking for…

Instructor: We talked about that in our debriefings after some of the lessons. I was concerned because the question came up and we went down a different road and eventually got back to some of the other things that didn't get covered. In the final analysis, a number of times we decided we got the essentials of what needed to be covered. It may not have been in the order it was in the lesson plan, but we got to the material. So I think it worked out satisfactorily.

Researcher: Some of these questions you may have already answered in some of your other comments. I am going to go through the list anyways. Maybe we'll get a few different viewpoints. How would you differentiate the lessons in your calculator section from the lessons in your other sections? You already talked about that a little bit in some respects but are there other things that you can think of that were different between the two classes?

Instructor: Yeah, there were a lot of probing questions in the calculator section. That was directly from the lesson plan. It was a challenge for the students and a challenge for me to get them engaged in discussion. I think that that flowed over into the other two sections because I am guessing that my style has changed a bit this semester from previous semesters. It's lecture-based but there's a lot more probing going on, there was a lot more probing going on, this semester and sticking with the question and working and really getting students engaged in thinking about the material and thinking about concepts. That's one difference.

Researcher: Any others?

Instructor: Yeah, one and it kind of ties into some other stuff we've already discussed. Having the calculator there in the classroom and students working on it on a fairly regular basis was a way to keep them physically involved in the instruction and learning and I didn't have that in the other classrooms. And I'm thinking that when students in our section were working with the calculators they were at least physically engaged if not completely mentally engaged. And when I compare that and contrast it to the other two sections, I didn't have that way to get them physically involved, at least not with the calculator. And I'm thinking that there was probably less engagement, less attention, less focus on what was being discussed in my other two sections than there was in the calculator section when the calculators were being used. I think that was maybe, I think we had discussed my review section, reviewing for the final exam, where I took about a 5 minute break halfway through the review due to obvious lack of attention and focus on the part of the class. It wasn't isolated cases, it seemed to be most of the people there. And I was thinking the calculators weren't there, they had been turned in. There was no physical involvement in what was going on. I wondered if that had part of…something to do with the completely dead reaction I was getting from most students. Just trying to keep their minds in the classroom seemed to be an impossible task. Of course it was the last week of school and that may have had something to do with it.

Researcher: Yeah, and spring time and what not. So one of your comments brought to mind another question I have. Your style seems to be asking questions and responding to students' remarks. Were there differences in the students responding to questions and their ability to answer those questions between the two different groups?

Instructor: It seemed like it was more of a challenge to get participation from the calculator group than it was the other two. The whole complexion of the class seemed different to me.

Researcher: Do you have any thoughts on why?

Instructor: Yeah, I thought it about it kind of throughout the semester and didn't know what to attribute it to. I've seen that before, when I've had three or four sections I've been teaching, one of them will stand out for some reason. One that made a very big impression was a group I had in Whitewater. They were very interactive and very social. Just amazingly social compared to my other two sections. And I am not sure why. I think just maybe luck of the draw, just random happenstance. I thought because of this relationship I seemed to have with the students in that class that they would perform better in the class. And, wouldn't you know it, it turned out that they were the lowest performing of my three sections.

Researcher: Interesting.

Instructor: The other two were dead in comparison. They were zombies. Very tough to get discussion going. I had to pry anything out of any of the students and in comparison this one was so social and communicative and seemed to be having fun. Maybe they were having too much fun.

Researcher: It will be interesting to see how these three shake out on the final tomorrow.

Instructor: I am very interested to see that.

Researcher: Once I have time to get to that stack of posttests, I'll have that gone through by June 6 when you're back in town. Throughout the semester though you did give section tests or unit tests. Did you observe any difference in test performance?

Instructor: I thought I did and I went back and reviewed it all last night getting my thoughts together for talking to you today. And I am not sure what I was thinking earlier on, I guess I had a preconceived notion that maybe this calculator section would do better. The results on the first, I think the first 3 tests, it looked like they were coming out inferior. And it was a concern but when I went back and looked at it there wasn't that much difference in how the 3 sections did. And on the last test, I think, the calculator section was the top-scoring section. So, I was concerned about the differences I was seeing but whether there is any significance to those differences or not, I would be surprised if there is.

Researcher: And we'll run statistical tests on the final exam and see what happens.

Instructor: One thing I did notice, Jon, I don't know if you have questions on this or not, the attendance situation was, it seemed to be different. It seemed to be poorer in the calculator section than the other two sections. And I am not sure what to attribute it to. And maybe it just the randomness of sections, this is one that just wasn't in to being in class. Of course, we have a policy that we were hoping would encourage them to be there. And when I compare the attendance this semester compared to the last 3 semesters it was best this semester overall for my sections.

Researcher: So the policy did seem to work?

Instructor: It seemed to. For some reason they were there more of the time.

Researcher: So would you say this calculator section may have been a little worse but it is probably comparable to other semester without the policy?

Instructor: Yes. And I have that data. I'll send you over what I've put together. You might be interested to see it. I had a policy last semester. It was more of a reward policy. If students had fewer than 4 absences, 4 absences or less, then they could waive, they had the right to waive the final exam. This semester it was more of a penalty plan and this is common to all the sections in math 101 and our math 065 course too. Same policy for everyone. The policy that is written in the syllabus was beyond 4 absences they would be deducted 2 points from their final grade for each additional absence. So it was punitive as opposed to rewarding like last semester. I don't have much of a sample base but my sections did better under the punitive plan. I just read some research that said reward was a better approach and you get better results.

Researcher: And you never know if you continued with that for several semesters you might notice a different pattern. That brings to mind the thought of, there's attendance and there's also withdrawals. And I know in this class, we lost, we'll see how many show up for the final, but I think now there's 18 from 29, I think. So, is that different than your other sections?

Instructor: It was a little bit higher. One other section had a comparable number at the beginning of the semester. The other section had fewer students. I think the other two sections, one of them is going to finish with 14, I believe, and the other with 19. That'll be in the data I send over to you. But it did seem like absenteeism was a little higher in this section and the dropout rate was higher too.

Researcher: Back to having the calculator in class. Do you feel like those calculator activities helped introduce some of the Framework ideas and questions? I know it was written that way in the lesson plan but did you feel like it did or did not help in that?

Instructor: I think it did. Just from the fact that they were physically involved. I think that was a help. They were with me because they had to be doing something with their hands, staying with the plan. The second part of that would be the visual output that they were able to look at and think about. So they had done something physically with the calculator and then they had a visual output that they could look at, study and talk about. Those two aspects were helpful in introducing concepts.

Researcher: There's this idea of concepts, we based those on framework, the Framework ideas, and there's also the idea of being able to do it by hand. Of course, on the exams, they are not allowed calculators. Do you feel like in your instruction that you were able to balance calculator and by-hand work or that is was heavier to one of those?

Instructor: I felt at times that that section didn't have the advantage of practice or demonstration on the board that the other two sections had, now whether they as a result they are less proficient at learning the procedures…

Researcher: That's why we're going to look at the final exams to try and answer that question. Do you think on the opposite side of that, about learning concepts, do you have any thoughts on how that might have been different with this class than the others?

Instructor: Well, with good students, I think it would be helpful for students to use the calculator to learn concepts, to have the calculator to support learning concepts. With remedial students, I'm not sure. I don't know. And I kind of make the comment based on my review for the final exam and some of the blank looks and absence of response to some of my questions which were basic.

Researcher: In all sections?

Instructor: Especially with the calculator section. But that's a good point. It may not have been that much different in the other two sections. But pretty fundamental principles it was like I was speaking another language. The idea of solutions, it's all about solutions to equations. The number of solutions you would expect to quadratic equations. Some pretty disappointing responses on those pretty basic questions. I mean there were, in fairness to the class, there were students in there who knew what was going on but didn't want to volunteer, right. And if I happened on one of those I would get more reasonable responses.

Researcher: And I wonder, too, if there's some way to correlate that with if they missed that day in class, especially with the attendance issues in the calculator section.

Instructor: Another possibility is the wide range of interest in the subject. And it could very well be the ones who have interest in it, for whatever reason, understand this, are thinking about the concepts and trying to figure out why this works the way it does. As opposed to the ones who are there somehow to get through it all. Their interest is elsewhere and maybe their abilities are elsewhere too.

Researcher: Again, feel free to speak your mind. Were you comfortable with the lesson plans?

Instructor: Yeah, I thought that they offered a lot to me. I mentioned before, I think there was probably a fair amount of spillover in the way I handled my other two sections because of the work that had gone into the lesson plans that I was getting for the calculator section. Lots of good examples, the flow of the lesson, to help me with the other section. So I like that a lot. What got in the way at times was the time spent with the specifics of the lesson and then not having time to kind of do stuff on the side and spend additional time in some areas.

Researcher: Such as following up on student remarks?

Instructor: Yeah, right. Giving them time to work on their own and get together in groups.

Researcher: Did the lesson plans address the understandings that you wanted for your assessments and the department's assessments?

Instructor: Yeah, I would say they did. And I think the test of that is when I was reviewing with the students for the 4 chapter tests, one of the things I looked at was what have I covered in the other two sections that I really haven't spent time on in this one? And there wasn't a lot of that that I saw. There was occasionally something.

Researcher: Good. My next question, with those new lesson plans compared to other intro algebra classes you've taught, this semester you had a couple of other sections and previous years, how would you compare your preparation time teaching this CAS section compared to a normal class, or your normal class?

Instructor: Well, having the lesson plan done for me was a big help. So that shortened preparation. On the other hand, incorporating CAS into the lesson plan required me to spend time going through the lesson with CAS and working with it and getting comfortable with it. So what I wound up doing was spending I estimate between 1 and 2 hours of additional time for the CAS class over what I had done for the other class. There was a trade-off there. What I was doing for the class with CAS was helping me with the other sections too. But typically what I did was go through planning for the other sections and then spend an additional two hours with the CAS lesson. So it was time-intensive from that standpoint. Now if I were doing all my sections with CAS, I wouldn't be spending the time in preparation for those other two sections like I had to this semester. In summary, the lesson plans were done for me so that was a big time saver. I still needed to prepare for an hour or two to get comfortable with it. But if that were the only instructional mode that I was using for introductory algebra it would have been fine. It would have been great. The way it worked out, it was a big time consumer.

Researcher: One thing it tried to recognize was that you were working with somebody else's lesson plan. In thinking of future projects and work with this, do you think it might be different with a lesson plan that you've written and designed compared to a lesson plan that was handed to you as far as being able to use it in a classroom?

Instructor: I don't know. I think there's a lot of value in what you've done in preparing those lesson plans. I think you've got something that can be handed over to somebody and used for their complete plan for teaching introductory algebra. All of that work has been done. It's just a question of well how does that, what's been done, fit with the style or desires of someone to teach it. As long as they're flexible on how to teach the class and don't mind adapting to somebody else's technique and instructional methods I think you've got something great there.

Researcher: So what are your thoughts on receiving something like that and adapting it to your own style or recommendations that we can give to other teachers that might try to use this in adapting it to their classroom?

Instructor: I think they would – How would they adapt it? – I think they would pick and choose which of the particular calculator pieces they wanted to use but I would want to encourage them to get too far off track from what's already been prepared there.

Researcher: What particular issues were there for you in that extra time, that hour or two with that lesson plan, to feel comfortable walking into the classroom with it?

Instructor: Ask me again please.

Researcher: What issues were there for you, or considerations, to walk in the classroom and feel comfortable using it?

Instructor: Ok. Kind of from experience working with the plans from last semester, I needed to really understand the flow. I needed to understand the calculator operations involved so I could do it myself. If it was stuff I could do in preparation for the class, that is, get the calculator loaded and primed then I wanted to do that. I needed notes on the lesson plan to key me that this is what the students are doing, this is what I've done ahead of time, this is what I want to put on the board, for example, get these examples on the board so students can take them off the board. Those kind of cues for myself. The lesson plans got really marked up with my notes and my outline.

Researcher: So do you think that's something that would be helpful for me to incorporate from the designer's position or do you think there is benefit to you doing that on your own?

Instructor: I think it may have helped if that were in the plans. And you did do that, you modified them as we moved through and made some changes to help me understand the plan and look at it and be able to tell what's going on here or there. But I'm thinking, for example, the stuff that I boarded, the examples I boarded, if they were somehow set aside very clearly it would have been more paper but it would have been clearer to me, I got to get this stuff on the board and then I move on to this stuff here. The students are working on that on the board, and then I do that work in the calculator, and we discuss this, and then I project and so on… And not necessarily with a lot of words but visually if you could get it in there to the lesson plans so it would be obvious when somebody looks at it, ah, there's board work, here's some more board work, here's some more board work the instructor has to do.

Researcher: And clearly define the instructor and student activities or responsibilities. That's helpful to know.

Instructor: And the discussion items so they really jump out visually. Now you bolded and you had FW and tried to indicate it. If it could be bolder for me to see it when I'm in front of the class.

Researcher: That's good to know. That's exactly the feedback I was hoping to get from you. Please freely add anything that comes to mind. Do you feel there was sufficient time to cover the material in class while you were trying to emphasize this framework understanding, these framework-type questions?

Instructor: It seemed like we were pushed for time. I always went up right up to the end of class and it didn't seem like I allowed time for questions and discussion, very little review of homework questions. It seemed to be all get through the lesson plan. And in some of the other classes, especially earlier in the semester, there would be 15 or 20 minutes, even a half an hour sometimes of discussion of previous homework, which I thought was a learning opportunity. Some of the students thought, hey, they didn't do their homework, we shouldn't be wasting time in class because they didn't do their homework. I got those comments from people from time to time. It was a tradeoff there too, to find the right balance. But it seemed like my nature was in the calculator class to not even ask if they had questions and I think it was probably because I knew I was going to have trouble getting through the lesson plan anyway. The same thing at the end of the class, not a whole lot of time to wrap up, to summarize, to see if there were any questions, so it was pressed for time.

Researcher: Do you feel that you were able to keep pace with the schedule as far as what was on the department syllabus?

Instructor: Yes, we did. The order was a little bit different but we seemed to be on track. We covered all of the material.

Researcher: What advice would you give to other teachers who might try to use this or be asked to teach this way?

Instructor: (Pause) A hush came over the crowd.

Researcher: If you have any thoughts about that.

Instructor: I would encourage them to try it. It's, I think, a great set of lesson plans and there's a lot of work that's gone in to those and it's work that if you're not using those plans you've got to come up with your own. So it could be a huge time saver for a teacher. And I look at it from that standpoint. It's got great structure to it, it's got excellent examples, very well thought out, and the flow of it is great.

Researcher: What would you do differently if you were to teach this curriculum again next semester?

Instructor: I'm thinking now, based on what I've said so far this morning and thoughts I've been having recently, maybe set it up so that the calculator is used early in the class and then maybe later in the class but not so consistently throughout.

Researcher: So in one class period?

Instructor: Yes, in one class period, right. Because it's great I think to keep their attention. They're working with it, they're involved. As you get further into the class period, attention wanes. It has a tendency to do that, natural tendency. I'm thinking the calculator could be a great tool to bring people back. Now how that affects the lesson plans, because they seem to use the calculator throughout or maybe use it at the beginning and then not in the later part, so maybe if it was at the beginning and then later in the class with some room in between that wasn't quite so structured with the calculator. That might be an interesting change.

Researcher: Anything else you might do differently?

Instructor: If we were talking about a 2-hour class instead of a 1-hour, we have both here, for a 2-hour that could be very effective. Intersperse the calculator work throughout, two or three times, throughout the two hours because it gets pretty tough towards the end of a two-hour class.

Researcher: What suggestions do you have for improvement of this teaching method? We've kind of already talked about that. Any others?

Instructor: The simulator would be great, the computer simulator, so we could use that electronic classroom to project. The calculators seemed to work well for most of the students. There were a few students who you could kind of count on not to have their calculators there, but for the most part I think they were well accepted. And when there was an opportunity they jumped on it. There were exceptions, people would just do it the old-fashioned way. I thought they were very well received. I had some concerns about that, what are they going to think about using calculators and not being able to use them on the test? It didn't seem to be an issue.

Researcher: You've read through and prepared and looked at some data for this discussion we're having today. What other observations or thoughts do you have that we haven't necessarily talked about yet? What else do you have to add?

Instructor: Just curiosity about maybe how the calculators would have worked in my other two sections. They were in one section and maybe I focused on the differences of that section compared to the other two. Had they been used in all three would I still have seen differences between the sections? Probably.

Researcher: Yeah, and ideally that's how we would like to run a study. But for a dissertation and a grad student there's not the capability to do that. So kind of some summary questions. What is your view of the impact of using CAS and organizing discussion around the Framework questions on student learning? The impact of CAS and Framework on student learning?

Instructor: I think it's got a lot of potential to kind of reverse the way material is introduced, like you did with your lesson plans, and bring the concepts and, for example, solutions to equations in early in the chapter or unit and then work on the procedures that support those concepts later. I think, for a lot of that, it would be pretty tough to do without the computer algebra system. I think that's got a lot of potential.

Researcher: What's your view of the impact of CAS and Framework discussion on the general classroom environment?

Instructor: Well, I've already talked about this, but it keeps the students physically involved, and somewhat involved in the class, which I think is very important. So from that standpoint it has something to offer. Trying to do the same discussions with some of them would have been impossible without the computer algebra system. For example, we had, with quadratic equations, solving them on the first day of class which you couldn't have done in the other sections. So it gave us a vehicle where we could do that and kind of get away from all of the procedures that might be involved in solving the questions and just get at quadratics equations and what kind of solutions we would expect from it and then think about how the calculator is coming up with these results. I think that was powerful.

Researcher: We as researchers thought this could be done. From the teacher's position, a practitioner, do you feel that this was a feasible curriculum? That as we develop it and make improvements that it is feasible? It can be done in an actual classroom setting?

Instructor: Yes, definitely.

Researcher: As we iron out the kinks and get it well-developed, we hoped we'd be able to pass it on to other teachers. Any other kinks that stand out that need to be ironed out that make it…stumbling blocks, obstacles.

Instructor: No, I've pretty well discussed what I thought might be some areas for improvement in the lesson plans. It was more the layout and the visual aspects, allowing the instructor to quickly differentiate what the student is doing, what's going to go up on the board, what calculator work is expected from the instructor, what calculator work are the students doing on their own, those kinds of things.

Researcher: At times it seems…trying to get students to answer questions is like pulling teeth, maybe, was that isolated to this class that I observed? Do you find that common to most sections?

Instructor: It's probably more common than I realized and maybe it was accentuated with the calculator class because there was so much emphasis on probing and discussion and understanding.

Researcher: Do you feel like you asked different types of questions in the calculator class than in the other classes?

Instructor: I think I did and I asked more of them too because of the lesson plan. I think that flowed over into the other two sections, but in the other sections I typically didn't have my calculator lesson plan with me. If I used examples from that lesson plan, and I did that on occasion in the other class, then I would put them into a separate lesson plan. So, yeah, it seemed like it was pulling teeth but part of that may have been I was trying to get so much more discussion out of that class than my other two sections.

Researcher: Rather than something like "What's the next step?" or "What do I get here?" type of questions.

Instructor: Yes.

Researcher: What ideas do you have for improving that? What might get better response in the future?

Instructor: Well, when I walked out and gave the students a 5 minute break during the review for the final exam, I walked back into the class and a couple had left like I had offered and suggested they should do – if they weren't prepared to think about math at that particular time then just leave, go back to it when they could concentrate on it. The ones that were left, the first question I asked they were tripping over each other to volunteer answers. Nobody had to be called on, I mean, answers were coming from all sections of the room.

Researcher: Do you typically have to call on other students in you other classes as well?

Instructor: I do and I think part of it is the pattern that I set up. I want to make sure everybody is involved and I think that they probably have conformed to my pattern which is I'll ask a question, give a moment for people to reflect, and then call on somebody who may be looking at me, giving me some eye contact, maybe look at them working on their paper with something or other. That's kind of random and sometimes calculated. My style has become to lecture with lots of question and then call specifically on students after I've asked the question.

Researcher: Do you feel that, I don't know what other teaching experience you've had, have you had the opportunity to teach other classes that wouldn't be considered remedial?

Instructor: I've taught college algebra here at MSU and I've taught liberal arts math a couple of times at MSU and math for elementary teachers.

Researcher: Do you find differences, still focusing on students being willing to discuss and answer questions, do you find differences among those classes at different levels?

Instructor: Yeah, my recollection is that when teaching math 130, math for elementary teachers, the students were very participative. A whole different atmosphere.

Researcher: And college algebra?

Instructor: College algebra – they were not. And I wasn't, that was 3 or 4 years ago, and I wasn't doing so much probing then. It was a big class and I left it more to the students to volunteer answers. I asked questions but looked to volunteers. So I think my style has changed fairly significantly over the past semester or two maybe.

Researcher: And I am wondering if there is any tie to the level of student.

Instructor: I suspect there is.

Researcher: I think part of it is them being motivated and engaged. Any other issues you haven't already addressed about bringing calculators into the classroom? What issues that brings for the teacher and the students?

Instructor: No, I think I've pretty well discussed my main concerns, things I liked about it, things I thought were good it brought to the classroom.

Researcher: Anything that you thought it took away from the classroom, deficiencies or detractors?

Instructor: Not that hasn’t already been discussed, no.

Researcher: All right. Just wanted to make sure I get a complete understanding of your perspective. Anything else that you have in mind that you haven’t shared yet?

Instructor: No.

Researcher: Ok.

APPENDIX L

STUDENT INTERVIEW PROTOCOL

Introduction to the Interview

The time is __________ on __________________. This is student number ___ for the CAS interview.

In the next fifteen to twenty minutes, I am going to ask you some questions about the Voyage 200 calculator you were issued for your math class this semester. This is not intended to be a test of any kind, but to help me understand your opinions about using the calculator in this class. Nothing you say here will be revealed to your instructor in a way that you can be identified, and will have no affect on your grade. I will audiotape this interview so that I can listen without having to take notes on everything. Please speak clearly and verbalize any thoughts you have.

I have a calculator here to use for demonstration and/or reference as we talk about how it was used. Please answer the specific questions as completely as possible. At the end of the interview you will be given the opportunity to share any thoughts or opinions that you feel were not addressed by any of the questions. With your assistance, we are working to find ways to enhance students' learning experiences.

Do you have any questions before we begin?

CAS Interview Protocol

Background and prior experience

1. What other types of calculators have you used for previous math classes?

    Were they required, optional, or did you choose to get one on your own?

    How were the calculators used by the teacher in class?
    How did you use the calculator in class and out of class?

2. How is the Voyage 200 that you are using this semester different from previous calculators you have used?

    Are there things that you did with this calculator that you weren't able to (or never tried) with other calculators?

Personal issues

3. What was your experience in learning how to use the Voyage 200?

    Why was it hard/easy for you to learn how to use the calculator?
    Describe, if any, the difficulties you had learning how to use the calculators.
    What could be done in a future section of this class to best introduce the calculators?

4. How did you use the calculator during class?

5. Did you use the calculator outside of class? If so, how?

    Did the calculator influence you homework habits? In what way?

6. What did you find helpful or difficult about having the calculator to use?

    How did it influence your ability to do problems?
    How did it relate to your understanding of concepts behind the problems?

<u>Classroom issues</u>

7. How did the calculator impact the lessons?

Were the lessons enhanced by including calculator work? How?
Was the presentation of the material affected by including calculator work?
Did the organization of the material make sense?
Within lessons? Within chapters?
Was the pace of the material manageable?
Do you feel you could have learned the material to the same extent without the calculators?

8. Did incorporating the calculators in the classroom change your learning experience? How?

What do you think about the ways the instructor used the calculator?
How was it used to help you learn?

9. How did you interact with the teacher and other students during calculator activities?

How did you get help with the calculator when (if) you needed it?

<u>Opinions and attitudes</u>

10. What are your opinions about the use of these calculators in the classroom?

11. What do you feel we should know from the students' perspective about using calculators in class?

12. Do you have any thoughts to add?

APPENDIX M

FIDELITY OF IMPLEMENTATION OBSERVATION TOOL

<u>Fidelity of Implementation – Overall Impressions and Conclusions</u>

The goal of the classroom observations is to answer the following questions:

<u>Framework</u>:

Were the FW objectives identified in the **desired understandings** of the lesson appropriately emphasized?

To what extent were the FW questions used for discussion?

What was the level of student participation in the FW discussion?

- Did students participate voluntarily or were they called on to answer?

<u>CAS</u>:

Were the CAS activities used as intended?

- Were they used as intended to trivialize, experiment, and visualize?
- Were they used as specified to demonstrate and/or to involve students?

Were the CAS activities used to promote classroom discussion? Answer the intended FW questions they were designed for?

Characterize the student and teacher behavior/interaction during the CAS activities.

How many students were in attendance?

How much time was spent on business and review?

<u>CAS Activity</u>: Check off the activity on the lesson plan as it is done and use the following to organize the observations made about the use of the calculators to aid in later determining if CAS was used as intended.

Describe activity:

CAS use was:

Teacher-oriented OR **Student-oriented**

#of students engaged: ________

CAS was used for:

Experimentation **Trivialization** **Visualization**

Role in subsequent discussion (Non-FW questions, other significant events):

Procedure ____________________________________

Framework Objective	**Related Question**	**Teacher-Centered** Lecture, teacher answers own questions	**Both Std and Tchr** Students briefly answer direct questions	**Student-Centered** Students discuss questions asked
1a. The **overall goal** of the procedure.	"What are we trying to accomplish?"			
1b. **Predicting & Estimating**.	"What sort of answer should we expect?"			
2a. **Performing** the procedure.	"How do we carry out this procedure? What are the steps?"			
2b. **Alternate Methods/ Representations**.	"How else could we have done this?"			
3. Why the procedure is **Effective** & **Valid**.	"Why does this work? Why is it valid?"			
4. **Evaluate Results** by using context, other procedures, etc.	"How can we verify the answer? Does it make sense?"			
5. Assess relative **Efficiency & Accuracy**.	"What is the most efficient method to use?"			
6. **Empowerment** as problem solver.	"What types of problems can we solve with this?"			

APPENDIX N

FIDELITY OF IMPLMENTATION

DATA AND ANALYSIS

<u>Analysis of Observation 1</u>

<u>Framework</u>:

Were the FW objectives identified in the desired understandings of the lesson appropriately emphasized?

Observer 1: The instructor *covered most of the Framework material*. The instructor covered what answers to expect and how to execute the procedure but the overall discussion was lacking. The first *15 minutes of class was spent on handing back the exam and answering questions related to it…* to the extent that one of the most important part[s] of the lecture was not covered at all.

Observer 2: *The framework questions stipulated in the lesson plan were used to a limited exten[t]*. Not everything that was supposed to be covered according to the lesson plan for the day was covered. I think this is because *a lot of time was used for the review*.

Researcher: Of the three most important Framework objectives, the teacher appropriately emphasized two of them, the expected answer and effectiveness/validity of the procedure, while not placing enough emphasis on how to verify the outcome of the procedure. Again, I believe this would have been emphasized to a greater extent and depth *if the lesson had been completed.*

Conclusion: <u>Time issues did not allow for the Framework objectives to be emphasized in as much depth as they should have been. Of what was completed, the emphasis was appropriate.</u> * Covered in the next lesson.

To what extent were the FW questions used for discussion?

O1: I feel that the Framework questions should have been emphasized more. Part of this is no fault of the instructor, but rather the apathy of the students that make it hard to get any discussion going. *Responses from the students were limited*. *He ended up lecturing* more and having less student participation

O2: Their use [the Framework questions] was not to stimulate discussions but mainly for the *question and answer type of session*. Without getting any answers *he probes* further asking students to look the similarities.

R: The instructor *made the effort to ask students questions but they were unresponsive*. Few questions were direct Framework questions but most the questions asked were leading to the Framework objectives. *An even amount of Framework ideas were given by students answering questions as were given by the instructor's lecturing*.

Conclusion: The instructor was only able to get limited responses from the students with no resulting discussion. This was not because of lack of effort or attempts to probe. As a result the Framework objectives were, in large part, lectured on.

What was the level of student participation in the FW discussion?

O1: Most students were *participating in the activities and answered questions when called upon*, but no discussion was generated, nor did the students seem interested in the material, they were not trying to recognize patterns, offer conjectures or do anything that would stimulate discussion. The instructor *had to call on specific students* to get answers at all.

O2: The students were not prepared to give answers and the instructor *had to call upon them* and sometimes rephrasing his questions to prompt them to respond.

R: Most questions had to be *directed to a specific student* and asked to several different students before an appropriate or helpful response was given.

Conclusion: The instructor mainly had to call on students to answer questions. Often it took several attempts before an answer was given.

CAS:

Were the CAS activities used as intended?

O1: The *CAS was mostly used as intended*. The CAS was thus used for *experimentation* and *visualization*. However, the graphical and visual aspect of CAS was not utilized due to time constraints.

O2: CAS calculators were mainly used to *trivialize* and to some extent to *experiment* by students.

R: Of the portion of the lesson that was completed, *CAS was used appropriately and as intended*. The activities done were all for *experimentation* purposes and were student-oriented.

Conclusion: CAS was used as intended for the portion of the lesson that was finished.

Were the CAS activities used to promote classroom discussion? Answer the intended FW questions they were designed for?

O1: In the three activities, the *CAS offered the answers for the students so that they could give explanations and draw conclusions* about the procedure for solving factored quadratic equations.

O2: It should be noted that the CAS activities did not promote discussions as intended but rather the *students used the calculators to answer the instructor' questions*.

R: *The script following each of the CAS activities was followed fairly closely*. The instructor *used each activity to talk about the ideas they were designed to uncover*. The level of success of these discussions was noted above.

Conclusion: The teacher used the calculator activities to provide the material needed for a Framework discussion and then posed the questions in the lesson plan. However, classroom discussion was not achieved as noted previously.

Characterize the student and teacher behavior/interaction during the CAS activities.

O1: *Most of the students participated* in the activities and the CAS use was clearly *student-centered*.

O2: It seems that *most of the students were confident with the use of the calculators* as they could easily punch in all the problems without the help of the instructor. The *instructor did not need to demonstrate anything* on the calculator though he had it set up in case he need to use it. Sadly I noted 3 students without their calculators, and I just wondered whether they forgot to bring their calculators or they don't know how to use them.

R: *Most of the students* (all but 3 or 4) had their calculators out and seemed to be doing the calculations. *Some students discussed with their neighbors* the answers, the syntax, or other calculator issues but the details and duration of these discussions is not known.

Conclusion: Most of the students participated in the activities and seemed to have no troubles doing so. The CAS activities were student-centered.

Overview notes: Next class period after a test, time was spent at the beginning to go over the test. This limited the material that the teacher was able to address. The researcher had the benefit of observing the next class where the teacher finished the required activities.

<u>Analysis of Observation 2</u>

<u>Framework</u>:

Were the FW objectives identified in the desired understandings of the lesson appropriately emphasized?

Observer 1: Overall, the desired understandings were *treated appropriately.*

Observer 2: I think they were *appropriately emphasized to a large extent*, though some times it was not very clear. For example, this desired understanding, "determine excluded values – state the domain that the expressions are equivalent to the original" was not very clear to me. A table of values was used to capture this but then I seem to have missed how one clearly states the domain that the expressions are equivalent to the original from that table.

Researcher: *Good emphasis was put on the FW questions that were the intended focus* of the lesson. (See individual report for details.)

Conclusion: <u>The Framework objectives were emphasized as appropriate.</u>

To what extent were the FW questions used for discussion?

O1: The instructor *used the FW questions throughout the lecture* to try to engage the students in the solutions to the problems. So, the FW questions were used as intended.

O2: They were *used for discussion to a very reasonable extent* though sometimes the questions are not emphasized. I also felt that, as the instructor *rephrases the FW questions* the original meaning or emphasis is somehow lost or missed.

R: The teacher asked many questions in this lesson, FW and non-FW. *Much of the class discussion was brought about by the instructor asking specific FW questions or asking questions to get at FW ideas*. I would like to see more FW questions asked in the *original wording* to draw students' attention to the FW objectives.

Conclusion: <u>The instructor made good use of questions throughout the lesson. Theses questions were direct Framework questions or questions central to Framework objectives. However, some Framework questions were not asked directly, but in a rephrased form.</u>

What was the level of student participation in the FW discussion?

O1: Students were called on to answer and only ask questions sporadically. However, the instructor was *able to generate some discussion with the students*, and in one instance one student corrected the mistake of another. So, *predominantly students only speak when called on*, but some discussion was generated.

O2: Though the *student participation was not voluntary*, most of the students called upon had something to say and they gave correct answers most of the time, which is quite commendable.

R: The students *were able to answer the questions reasonably well*. The majority of class discussion was because students were answering the instructor's questions. Mostly he called on them but *a few times students offered their own ideas*. There were a few times when the instructor was in more of a lecture mode.

Conclusion: <u>Students were able to answer questions directed to them and this provided some classroom discussion to complement the instructor's lecture.</u>

<u>CAS</u>:

Were the CAS activities used as intended?

O1: Yes. The CAS was used mostly for *experimentation and trivialization*. Students were asked to perform an operation on the CAS and then to explain why they got that particular answer. In one instance a table was used to visualize how two equivalent expression are actually equivalent. *The CAS use was student-oriented and used as intended.*

O2: I think that *CAS activities were used as intended*. In activities such as 1, 7 and 8, CAS was used to *trivialize,* since it simplified the expressions for students. The use of CAS to *experiment* is evident in activity 2. Activity 6 was meant to help students to visualize the equivalence of the two expressions from just the table of values. It is also important that it is the students who did almost all of the calculator activities on their own and as proof that they did it well they gave correct answers.

R: *The CAS was used as intended*. The use of CAS became optional towards the end as the instructor gave them the choice to work through problems by hand. Many chose to do them by hand and then check using the calculator. I feel this was an appropriate shift because the instructor observed that students could do the manual calculations.

Conclusion: <u>CAS was used as intended in the lesson plan.</u>

Were the CAS activities used to promote classroom discussion? Answer the intended FW questions they were designed for?

O1: The students used the CAS to get an answer. They then reported their answer and were *asked to give reasons for why the CAS did what it did*. In this sense, the CAS was used to promote classroom discussion, and to answer FW questions. There was also a nice balance between using the CAS for certain examples and trying to have the students perform the operation by hand in other examples.

O2: Yes, *CAS activities were used to promote classroom activities*; almost all of the instructor's *questions were based on CAS activities*. Most of the instructor's questions ranged from asking for simple answers from the calculator to explanations how the calculator did it. As for answering the intended FW questions they were designed for, this was somehow not so clear because as the instructor *rephrased the FW questions*, the original emphasis of the FW questions is somehow lost.

R: *The examples and CAS use did provide opportunities for the teacher and students to discuss the ideas of the lesson*.

Conclusion: <u>The CAS activities were used to promote classroom discussion and discuss the ideas central to Framework understanding.</u>

Characterize the student and teacher behavior/interaction during the CAS activities.

O1: *For the most part the students did the CAS activities* and then the instructor explained/discussed the answers. A couple of times the students actually engaged in a discussion with the instructor in order to explain the answers. It was interesting to note that during one of the activities where the students could choose between paper-and-pencil and the CAS, most, if not all, the students chose to do the problem by hand. A couple of students then used the CAS to verify their calculations.

O2: During the CAS activities, the teacher's role was to assign problems and then waits on the students to give him answers. Only on the last activity when they wanted to check whether they had simplified correctly that the instructor had to show students something on his calculator. He had this entered already. Apart from this, he reminded students about important things to note or do such as putting parentheses and checking the history window on their calculators. It is commendable that the *students did all the calculator manipulations unaided.*

R: The *students did a lot of the work at the beginning* and gave the instructor the answers. From this questions were asked and the discussion was carried out. Towards the end of the class period, students were still doing the work, some by hand and others with the CAS. The teacher demonstrated one preloaded table (as planned previously) to save time and complete the activity as part of the lesson.

Conclusion: The calculator work was student-centered and it appeared that students were able to perform the calculations.

<u>Analysis of Observation 3</u>

<u>Framework</u>:

Were the FW objectives identified in the desired understandings of the lesson appropriately emphasized?

Observer 1: In my opinion, *these framework questions were appropriately emphasized*. The instructor emphasized that the system will give you two line that can intersect in one, no, or infinitely many points and used graphs to illustrate this. He used tables to check solutions, and he also went back to the equations to check that the answers are solutions. He didn't actually solve a system by graphing, so I am not sure that the students realize that you can use the graphs to solve the system, but the instructor emphasized the graphical aspect enough that they should be able to make the connection.

Observer 2: *I think they were appropriately emphasized since class activities seem to have covered all of them*. However, I feel the way the graph was used seemed to be more of checking or validating the solution they had got earlier on their own. The other time they had to graph a system on their graph paper the solutions could not be discussed as the students seemed to have quit to which the instructor responded by dismissing them since it was almost time up.

Researcher: *I feel that the instructor emphasized the appropriate framework objectives applicable to this lesson*. (See report for full details.)

Conclusion: <u>The Framework objectives were appropriately emphasized, though one aspect may have been indirectly addressed.</u>

To what extent were the FW questions used for discussion?

O1: For the most part, the *instructor was the driving force* behind the discussion. *The instructor used the framework question[s],* but he was unable to generate discussion.

O2: Somehow I feel the FW question, "What sort of answer do you expect?" was well manipulated for discussion and it is one question that was asked directly. *As for the other questions, though not directly asked, activities pointed to these questions*. However, I feel the expected emphasis was not there sometimes.

R: The FW question, "What sort of answer we should expect?" was a great discussion organizer for the lesson. The instructor tried to utilize this but as is typical, the students were not very responsive. There were several, "I don't know," answers. *The instructor did his best to draw students in with FW-type questions but often used the FW questions to organize his presentation to them.*

Conclusion: The Framework questions were used to try and promote discussion but when this failed the Framework questions were used to organize the instructor's presentation.

What was the level of student participation in the FW discussion?

O1: He used the framework questions to promote discussion, but the *students did not really participate unless asked direct questions*. In addition, some students automatically answer "*I don't know*" on direct questions.

O2: Though the students *did not participate voluntarily*, when called upon to give answers they *were able to give good answers most of the time*. There were times when a student called upon could not answer, the instructor would not let him/her get away with it, instead he keeps probing till the student figures out the answer.

R: As has been observed previously, the instructor *has to direct questions* to students. In some cases, they just say "*I don't know*" so that he will move on to another student. But a lot of times he can *re-question* them in different ways to get the material out of them. I think they know more than they let on but are afraid to share and be wrong. They are very passive.

Conclusion: The instructor had direct questions to students who often answered "I don't know" on impulse. The instructor tried to probe and rephrase to get an answer from students.

CAS:

Were the CAS activities used as intended?

O1: Out of the 4 activities outlined in the plan, 3 were completed in the class. The instructor started the last activity, but cut it short since none of the students were interested. The first activity was student-oriented. The students entered three systems of equations on the calculator, and let the calculator give the solutions. The answers were then discussed by the instructor. The *CAS was used for trivialization* in that it provided the answers and for experimentation in that the answers were discussed. In the second exercise, the CAS use was more teacher-oriented, the instructor used the table function on the CAS to show that only one point was the solution to a particular system, and he also used the CAS for graphing the two equations. The main use for the *CAS was for visualization*. For the third activity, the instructor mostly used the board to draw graphs and explain the possible solution sets to the systems.

O2: *I think the CAS activities were used as intended*. Initial CAS activities involving solving systems of equation were intended to *trivialize* the calculation process and using the table of values and the graph to validate the answer were ways of *visualizing*.

R: The instructor *had the students use the calculators at the appropriate times*. He also used the calculator to demonstrate for *visualization* purposes. This has been the best way to do so as time is a precious commodity.

Conclusion: <u>The CAS was used as intended by the lesson plan.</u>

Were the CAS activities used to promote classroom discussion? Answer the intended FW questions they were designed for?

O1: Yes, the instructor *used the CAS as a means to discuss the answers*. He asked all of the FW questions emphasized in the lesson plan, and he *used the CAS as a basis for these questions*. So, the CAS was used in the way intended. However, it seems that the instructor will often go back to the board to illustrate a point, when he could use the CAS.

O2: Yes, *they were used to promote classroom discussion* because the teacher would always ask for answers from the calculators to start with and many more questions would follow after. The *CAS activities also tried to answer all the FW questions* though sometimes the activities do not emphasize the FW questions they are designed to answer.

R: The calculator was *effective in introducing the FW questions and providing discussion topics*. In one instance the instructor might have phrased a FW question differently and before showing the associated graph but elsewhere the instructor was able to ask questions directly from the lesson plan after doing the associated activities. A good example is the way the teacher used the calculator's return of "false" to talk about parallel lines. He also used the first activity of students doing three systems to ask about the expected outcome and type of answer.

Conclusion: <u>The CAS was used to introduce and provide a basis for Framework question discussion.</u>

Characterize the student and teacher behavior/interaction during the CAS activities.

O1: *Most of the students participate* in the CAS activities, but some don't participate at all. Students mostly use the calculator for visualization and experimentation, since they use it to solve equations and they are asked to reflect on the answers they get. The instructor mostly use[s] it for visualization of graphs and checking answers using tables.

O2: There was a time when the teacher gave students systems of equations to solve and then *wait upon them to give him the answers*. He also advised them to use the format they used on Thursday and even illustrates this on the board. For the visualization process the teacher does not let the students do it but he projects what he has already in the calculator onto the wall and discuss it with the students after explaining why and how he entered the second equation. But I just wonder why the teacher does not enter anything into his calculator during the class.

R: When the students were required to work on the CAS the teacher put the information on the board and gave them time to compute. He then asked them questions (to specified people) about the results. For visualization, the teacher had the functions preloaded. The *students seemed to be mostly engaged* with only a few missing their calculators.

Conclusion: <u>Most students are engaged during calculator activities. The students do computations and answer to the teacher. The teacher preloads graphs and tables to show the students.</u>

<u>**Fidelity Questions Across Observations**</u>

Were the FW objectives identified in the desired understandings of the lesson appropriately emphasized?

<u>First observation</u>: Time issues did not allow for the Framework objectives to be emphasized in as much depth as they should have been. Of what was completed, the emphasis was appropriate. * Covered in the next lesson.

<u>Second observation</u>: The Framework objectives were emphasized as appropriate.

<u>Third observation</u>: The Framework objectives were appropriately emphasized, though one aspect may have been indirectly addressed.

To what extent were the FW questions used for discussion?

<u>First observation</u>: The instructor was only able to get limited responses from the students with no resulting discussion. This was not because of lack of effort or attempts to probe. As a result the Framework objectives were, in large part, lectured on.

<u>Second observation</u>: The instructor made good use of questions throughout the lesson. Theses questions were direct Framework questions or questions central to Framework objectives. However, some Framework questions were not asked directly, but in a rephrased form.

<u>Third observation</u>: The Framework questions were used to try and promote discussion but when this failed the Framework questions were used to organize the instructor's presentation.

What was the level of student participation in the FW discussion?

<u>First observation</u>: The instructor mainly had to call on students to answer questions. Often it took several attempts before an answer was given.

<u>Second observation</u>: Students were able to answer questions directed to them and this provided some classroom discussion to complement the instructor's lecture.

<u>Third observation</u>: The instructor had direct questions to students who often answered "I don't know" on impulse. The instructor tried to probe and rephrase to get an answer from students.

Were the CAS activities used as intended?

First observation: CAS was used as intended for the portion of the lesson that was finished.

Second observation: CAS was used as intended in the lesson plan.

Third observation: The CAS was used as intended by the lesson plan.

Were the CAS activities used to promote classroom discussion? Answer the intended FW questions they were designed for?

First observation: The teacher used the calculator activities to provide the material needed for a Framework discussion and then posed the questions in the lesson plan. However, classroom discussion was not achieved as noted previously.

Second observation: The CAS activities were used to promote classroom discussion and discuss the ideas central to Framework understanding.

Third observation: The CAS was used to introduce and provide a basis for Framework question discussion.

Characterize the student and teacher behavior/interaction during the CAS activities.

First observation: Most of the students participated in the activities and seemed to have no troubles doing so. The CAS activities were student-centered.

Second observation: The calculator work was student-centered and it appeared that students were able to perform the calculations.

Third observation: Most students are engaged during calculator activities. The students do computations and answer the teacher’s qustions. The teacher preloads graphs and tables to show the students.