A method for optimizing a network of pipelines for transporting woodchips
by Irving Campos Hoffman

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
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Abstract:
A method is presented in this study for optimizing an economic model of a pipeline network transporting woodchips hydraulically by determining the concentration of woodchips and pipe diameter for each line of the system which minimize the cost.

A cost function for a single pipeline is investigated by defining a response surface whose characteristics provide a method for reducing to two the number of pipe diameters which could minimize the cost. The optimum concentration and cost for each size is determined, the costs for the two are compared, and the pipe giving the lowest cost is selected.

Optimization of three- and five-line networks utilizes the cost function of single lines and, in addition, requires that the continuity of flow of the two-phase fluid be satisfied at the junctions.

The optimization technique is applied to an existing area.

Costs of pipeline transportation of woodchips from chipping areas to a processing plant are compared with costs of moving the chips by rail and truck. The comparison shows rail costs are lowest in all cases.

A METHOD FOR OPTIMIZING A NETWORK OF PIPELINES FOR
TRANSPORTING WOODCHIPS

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CHAPTER I

INTRODUCTION

Transportation of solids in pipelines is not a recent innovation.
Successful applications have been made for over one hundred years in
fields ranging from placer mining to grain handling. High costs of
labor and maintenance in other transportation systems have intensified
interest in pipelines in recent years. Presently, many successful pipe-
line installations exist, most of which occur in the mineral and mining
industry (1), (2)*. A 72-mile pipeline is transporting 800 tons per
day of gilsonite from a mine in northeastern Utah to a refinery in western
Colorado. Copper concentrate is pumped 14 miles in Chile. The mines in
South Africa have several pipelines, some up to 16 miles long, success-
fully transporting uranium-bearing gold tailings.

Since 1957, the Pulp and Paper Research Institute of Canada (3)
has been investigating the possibility of using pipelines to transport
woodchips to processing plants. In 1961, the U.S. Forest Service began
a program to examine pipelines as a means of conveying woodchips. The
Forest Service is seeking more economical methods of transporting wood
to stimulate greater utilization of woodlands in this country. Reduc-
tion in transportation costs will allow low-value wood (cull and dead
trees, slash, and residue from sawmills) now being discarded to be moved
to the processing plant. Private processors are continuously searching

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*Numbers in parentheses refer to numbered references in the Litera-
ture Cited.

for methods to lower handling and transportation costs of chips to increase production and profit. Pipelines may offer a means of reducing these costs.

The economic advantages of pipeline transportation are quite attractive:

Automation. Gas and oil pipelines have been automated for approximately twenty years. Once the fluid has been injected into the pipeline it is left unattended until the next input or discharge point. Pumping stations are controlled automatically from a central master station.

Dependability. Dependability of pipelines has been proven. The gilsonite pipeline in Utah has been operating for seven years (1). The Consolidated Coal Company operated a 108-mile pipeline in Ohio without a shutdown for three years (4).

Operating Costs. Operating costs are low; other costs are mostly fixed and remain nearly constant over the life of the installation. Other transportation systems have higher operating costs and are more easily affected by the rising costs of labor and personnel.

Maintenance Costs. Maintenance costs are low since pumping stations have few moving parts and the pipeline is buried and subject to little wear.

These advantages have been confirmed in the transport of single-phase fluids such as gas and oil. The advantages may potentially be applied to the transportation of solids by defining the hydraulics of two-phase flow.

The hydraulics of coal and gilsonite presently being transported long distances in pipelines are defined well enough to permit the design of pipeline systems. Although the mechanisms of flow for solids are not fully understood it is known that the small, uniform size of crushed coal and gilsonite produces a homogenous two-phase fluid at high velocities giving well defined friction loss relationships and allowing power requirements to be calculated and operating costs to be predicted. The hydraulic properties of woodchip mixtures are less well defined. Woodchips are relatively large and nonuniform in shape and size. The specific gravity of woodchips is lower than that of coal or gilsonite. Accurate relationships among head loss, which is shown to be the greatest economic factor, and other flow parameters, such as pipe size, velocity, and woodchip concentration, are required to predict the power requirements and, accordingly, the economics of woodchip pipeline transportation. Research is being conducted to investigate the mechanisms of motion of woodchips and to describe more accurately the head loss relationships involved.

A large research program sponsored by a group of ten interested companies was conducted in Marathon, Ontario (5). Friction loss tests for woodchip-water mixtures were conducted on 2,000 feet of 6-, 8-, and 10-inch steel pipe. Woodchip pipeline research projects at Montana State University (MSU) have been conducted to investigate the moisture absorptive properties of woodchips under pressure (6), the energy losses of woodchip-water mixtures passing through expansions and valves (7), the effect of woodchips on the performances of centrifugal pumps (8), and the economic feasibility of woodchip pipelines (9). Tests are

currently being conducted at MSU to define the energy losses due to fric-
tion by various woodchip mixtures. Queen's University in Kingston,
Ontario (10), the Pulp and Paper Research Institute of Canada (3), and
the Shell Pipeline Corporation have conducted head loss studies on trans-
porting woodchips in pipelines.

Several equations exist for head loss in two-phase flow. Durand
(11) proposed an equation giving head loss for sand and gravel. Elliott
and de Montnorency (3) modified Durand's equation to express head loss
for woodchip-water mixtures in pipelines. Faddick (12) developed an
equation for woodchips quite similar to Durand's from tests conducted
at Queen's University on 4-inch pipeline.

A mathematical model developed by Hunt (9) to investigate the
feasibility and economics of woodchip pipelining uses the head loss equa-
tion proposed by Faddick (12) for determing energy requirements, number
and size of the pumping units, and costs of the variable salaries and
wages. The analysis gives the best operating concentration and pipe
size along with costs for a given throughput and length of pipeline.
Investigation of this economic model showed that pump efficiency, fric-
tional loss coefficient, capital recovery factor, and the chip concentra-
tion are the variables having the greatest effect on pipeline economics.
The results of the model for a single pipeline with the chip source at
one end and the processing plant at the other show pipeline transporta-
tion costs to be competitive with rail and truck. The analysis was applied
to an area in Alaska which had no existing transportation facilities;

a savings of 58 percent was anticipated over road construction and haul.

The disadvantage of Hunt's model is that it cannot be applied to a network of pipelines which many of its applications will require. A network model is more complicated than a single-line model since the total costs are influenced by the operating characteristics of each line. A model analysis which gives the pipe size and concentration of woodchips for each line of a pipeline network producing the lowest cost for the system is needed.

This thesis develops a technique for determining these optimum conditions and predicting the lowest cost for pipeline networks. The method of analysis uses the response surface (13) generated by the mathematical expression developed by Hunt (9) describing the economics of the pipeline system. This expression, called the objective function, contains all the information required for a rational decision and when minimized gives the optimum operating conditions for a pipeline network.

CHAPTER II

SINGLE-LINE OPTIMIZATION

A technique of analyzing a response surface to determine the optimum

operating conditions for a single-line pipeline system is described in

this chapter. The response surface is produced from the objective func-

tion for the mathematical model developed by Hunt to describe the economics

of a woodchip pipeline. This objective function contains three decision

variables: (1) investment costs, (2) operating expenses, and (3) overhead

costs. The three decision variables, in turn, are expressed by seven

cost groups: (1) energy cost, x_1, (2) installed cost of the pipeline,

x_2, (3) installed cost of the pump stations, x_3, (4) installed cost of

injection and separation equipment, x_4, (5) cost of fixed salaries and

wages, x_5, (6) cost of variable salaries and wages, x_6, and (7) cost

of water treatment, x_7. The units for each of these groups are dollars

per ton-mile which is the total expense of moving one ton of woodchips

(oven-dry basis) one mile in a given transportation system. This unit

was selected because it provides a basis for easily comparing rates of

other transportation systems, such as rail and truck. The objective

function for the single-line system is the summation of the seven cost

groups and is expressed as

$$X_t = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

The equations developed by Hunt (9) and used in this thesis for

these seven cost groups are summarized in this chapter on page 8 and

are functions of the variables listed below:

C   = Concentration of chips in the mixture by volume

crf   = Capital recovery factor

D   = Pipe diameter, feet

e   = Combined efficiency of motor-pump drivers

f   = Friction factor for Weisbach equation

H_t = Head due to friction and difference in elevation, feet/mile

L   = Length of pipeline, miles

R_1 = Cost of electrical energy, \$/kwh

R_2 = Installed cost of pipeline, including right-of-way, \$/(in-mile)

R_3 = Cost of pump station and controls, \$/(installed horsepower)

R_4 = Cost of chip injection system \$/(ton per day of oven-dry chips)

R_5 = Cost of separation system \$/(ton per day of oven-dry chips)

R_6 = Annual cost of fixed wages, salaries, operation maintenance; exclusive of pipeline maintenance and pump station operations, \$/year

R_7 = Annual wages, salaries, etc. for pump station, \$/(pump station)

R_8 = Annual maintenance cost of pipeline, \$/mile

R_9 = Cost of water and treatment, \$/million gallons

S_m = Specific gravity of water-chip mixture

S_{odc} = Specific gravity of oven-dry chips

W   = Throughput, tons per day of oven-dry chips (TPD)

The variables S_m and H_t, which are developed in Appendix B, are functions

of the following additional variables listed in Appendix A:

$$g \quad = \quad \text{Gravitational constant, ft/sec}^2$$

$$M \quad = \quad \text{Moisture content of chips, decimal fraction of oven-dry chips}$$

$$Z_t \quad = \quad \text{Difference in elevation between the ends of pipe, feet}$$

The seven cost groups, expressed in units of dollars per ton-mile, are developed in Appendix B as functions of the above-listed variables and summarized by the following equations:

1. Energy cost,

$$x_1 \quad = \quad 0.000753 \; \left(\frac{R_1}{e \, S_{odc}}\right) \; \left(\frac{S_m H_t}{C}\right) \tag{1}$$

2. Installed cost of pipeline,

$$x_2 \quad = \quad \left(\frac{R_2 \, D}{365 \, W}\right) crf \tag{2}$$

3. Installed cost of pump stations,

$$x_3 \quad = \quad 0.000000115 \; \left(\frac{R_3}{e \, S_{odc}}\right) \; \left(\frac{S_m H_t}{C}\right) crf \tag{3}$$

4. Installed cost of injection and separation systems,

$$x_4 \quad = \quad \left(\frac{R_4 + R_5}{365 \, L}\right) crf \tag{4}$$

5. Cost of fixed salaries and wages,

$$x_5 \quad = \quad \frac{R_6}{365 \, W \, L} \tag{5}$$

6. Cost of variable salaries and wages,

$$x_6 = \frac{1}{365\ W} \left(\frac{R_7\ H_t}{H_{sa}} + R_8 \right) \tag{6}$$

7. Cost of water and water treatment,

$$x_7 = 0.00024 \left(\frac{1 - C}{C} \right) \left(\frac{R_9}{S_{odc}\ L} \right) \tag{7}$$

The analytical expressions are based on the system operating 24 hours per day for 365 days per year. The optimization technique which is presented in the following pages determines the values of C and D which give the minimum cost; all other variables must be specified. The variables R_3 and R_7 for this analysis have been modified from those used by Hunt. R_3 and R_7 were defined as functions of additional variables by Hunt; in this analysis they are assigned a constant value determined from economic data recently acquired from Continental Pipe Line Company (4).

The objective function, X_t, for the single-line system gives the total cost per ton-mile and can be expressed as a function of C and D in polynomial form by combining the seven cost groups algebraically. The polynomial expression, developed in Appendix C, is given by

$$X_t = (K_1 C^{1.84} + K_2 C^{1.84})\ D^{2.10} + (K_3 C^{-2} + K_4 C^{-3})\ D^{-5} + K_5 D + \frac{K_6}{C} + K_7 , \tag{8}$$

where the coefficients K_i, i = 1 to 7, are combinations of the variables other than C and D.

The absolute minimum cost for the single-line system is obtained by solving the simultaneous equations

$$\frac{\partial X_t}{\partial C} = 0$$

and

$$\frac{\partial X_t}{\partial D} = 0$$

Hunt investigated the solution of these equations by plotting $\frac{\partial X_t}{\partial C}$ and $\frac{\partial X_t}{\partial D}$ for different pipe sizes versus concentration as shown in Figure 1. He observed that the $\frac{\partial X_t}{\partial C}$ and $\frac{\partial X_t}{\partial D}$ curves intersected close to but never exactly on the zero ordinate and interpreted the intersection of the $\frac{\partial X_t}{\partial C}$ curves with the zero ordinate as sufficiently close to zero to describe a possible minimum condition although the equations for an absolute minimum were not satisfied. The reason these curves are not zero at the same concentration will be discussed later. Hunt was correct in selecting the points where $\frac{\partial X_t}{\partial C} = 0$ as the points of possible minimum costs. He determined which combination of concentration and diameter produced the minimum cost by using a digital computer for:

1. Solving the value of concentration at which $\frac{\partial X_t}{\partial C} = 0$ for a given pipe size

2. Computing the cost for this diameter at its optimum concentration using the seven cost groups

3. Repeating steps 1 and 2 for a given array of pipe sizes and comparing costs at the optimum conditions for each diameter.

Figure 1. PARAMETRIC CURVES OF $\dfrac{\partial X_t}{\partial D}$ and $\dfrac{\partial X_t}{\partial C}$

Analysis of the response surface (13) generated by plotting X_t, cost per ton-mile, as a function of the concentration, C, and diameter, D, in a three-coordinate system as shown in Figure 2 offers an improvement of Hunt's method of solution. All surfaces for the single-line model were found to have similar shapes which decend to lower costs with smaller diameters and higher concentrations. The shape of the surface shows no node (a point at which $\frac{\partial X_t}{\partial D}$ and $\frac{\partial X_t}{\partial C}$ are both equal to zero) in the region of physical meaning; therefore, the conditions for an absolute minimum do not occur which indicates why the intersection of the curves $\frac{\partial X_t}{\partial D}$ and $\frac{\partial X_t}{\partial C}$ plotted by Hunt and shown in Figure 1 do not occur at the zero ordinate.

The response surface must be limited to a feasible region describing physical applications with all their limitations and constraints. Such a region is necessary since the concentration of woodchips in a pipeline has a limiting maximum above which it may not be increased without compressing the chips and packing the pipe so that transport is stopped. Faddick found this limit to be 43 percent for four-inch pipe; however, Equation B-5*, which he suggested and on which the economic pipeline model is based, does not contain constraints. This physical limitation on the concentration requires that the feasible region of the response surface be bounded by the planes C = 0, D = 0, and C = C_{max} where C_{max} is the maximum allowable operating concentration. The value

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*Equation numbers which contain letters refer to equations in the Appendix corresponding to the letter.

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Figure 2. Single-Line Response Surface with
Intersecting Planes

of C_{max} must be chosen sufficiently conservative to prevent any local

concentrations occuring during operation from approaching the limiting

concentration physically possible and stopping the flow of the mixture.

The lowest point on the response surface in the feasible region

occurs at $\left.\frac{\partial X_t}{\partial D}\right|_{C\,=\,C_{max}} = 0$ as shown in Figure 2. The pipe size which

corresponds to the lowest cost is indicated as D_t in Figure 2. This

theoretical diameter will seldom occur at a commercially available size.

The minimum cost will then occur with a pipe whose diameter is either

the next commercial size larger or the next one smaller than D_t. These

particular diameters are determined by considering the slope, $\frac{\partial X_t}{\partial D}$, of

the curve formed by the intersection of the response surface and the

plane $C = C_{max}$ in the vicinity of D_t. The slope, $\left.\frac{\partial X_t}{\partial D}\right|_{C\,=\,C_{max}}$, is

negative for all diameters smaller than D_t and positive for all diameters

larger than D_t as shown in Figure 2. The diameters giving the lowest

negative and positive values of $\left.\frac{\partial X_t}{\partial D}\right|_{C\,=\,C_{max}}$ are the only two sizes

which could possibly give the lowest cost and are shown as D_2 and D_3 in

Figure 2. The planes defined by these two diameters intersect the re-

sponse surface and describe curves shown in both Figure 2 and Figure 3.

A computer program, listed in Appendix E, was developed to select the

commercially available pipe diameter with the lowest positive value and

the one with the lowest negative value of the first partial derivative

of the objective function with respect to diameter. This derivative,

developed by the author in Appendix C, is given by

Figure 3. Relationship Between Cost and
Concentration for Particular
Pipe Sizes

$$\frac{\partial X_t}{\partial D} = 2.10\,(K_1 C^{1.84} + K_2 C^{0.84})\; D^{1.10} - 5\,(K_3 C^{-2} + K_4 C^{-3})\; D^{-6} + K_5$$

The response surface analysis shortens the procedure used by Hunt by eliminating all but two possible pipe diameters.

Figures 2 and 3 show that the optimum concentration for the smaller size, D_2, will always occur at C_{max} and for the larger size, D_3, at a concentration, C, less than C_{max} and where $\left.\frac{\partial X_t}{\partial C}\right|_{D_3} = 0$. The computer program was developed to find the optimum concentration for the latter case using the Newton-Raphson method of extracting real roots, as outlined by Scarborough (14), of the first derivative of X_t with respect to concentration for the larger diameter set equal to zero;

$$\frac{\partial X_t}{\partial C} = (1.84\,K_1 C^{0.84} + 0.84\,K_2 C^{-0.16})\; D^{2.10} - (2K_3 C^{-3} + 3K_4 C^{-4})\; D^{-5} -$$

$$K_6 C^{-2} = 0 \tag{9}$$

Sample results for the optimization of a pipeline with a specified length in miles, L, throughput of woodchips in tons per day, W, and difference in elevation of the pipe ends in feet, Z_t, are shown in Table I. The other specified variables used in this sample are listed in Table VIII. Table I is divided into three sections.

The first section is a list of the variables appearing in the table and their definitions.

The second section lists in the second, third, and fourth columns the given quantities L, W, and Z_t for a designated line. The computed optimum diameter and concentration are listed in the fifth and sixth

columns. Using the optimum D and C, values are computed and listed in the remaining columns for (1) the velocity, V, given by Equations B-1 and B-2, (2) the make-up water, Q_w, using Equation B-25 with units converted to a gallons per minute basis, (3) the energy loss due to friction, H_f, from Equation B-9, and (4) the horsepower required per mile, HPPM, from Equation B-12.

The third section of the table gives a summary of the first costs for (1) pipeline installation computed by Equation B-15, (2) pumps, controls, and pump station construction given by Equation B-17, (3) the injection-separation equipment given by Equation B-21, and (4) the total for these three items. The sixth column in the third section lists the annual operating expense, ANOP, given by

$$ANOP = 365 \ (W)(L)(x_1 + x_5 + x_6 + x_7)$$

where x_1, x_5, x_6, x_7 are the operating expenses expressed by Equation 1, 5, 6, and 7. The final column presents the total costs on a ton-mile basis computed from the objective function for the optimum concentration and diameter given by Equation 8.

The logic and techniques used to investigate the single pipeline can be extended to analyze the economic feasibility of a three-line network which is considered in the next chapter.

TABLE I

SAMPLE RESULTS, SINGLE-LINE

L =LENGTH OF LINE, MILE
W =THROUGHPUT, TONS OF OVEN-DRY CHIPS PER DAY
ZT=DIFFERENCE IN ELEVATION BETWEEN INLET AND OUTLET END, FEET
D =DIAMETER OF PIPE, INCHES
C =CONCENTRATION OF WOODCHIPS IN MIXTURE BY VOLUME, PERCENT
V =AVERAGE VELOCITY OF FLOW, FEET PER SECOND
QW=VOLUMETRIC FLOW RATE OF MAKE-UP WATER, GALLONS PER MINUTE
HF=HEAD LOSS DUE TO FRICTIONAL RESISTANCE, FEET PER MILE
HPPM=INPUT POWER REQUIRED, HORSEPOWER PER MILE OF PIPELINE.

	-GIVEN QUANTITES-			-COMPUTED OPTIMUM CONDITIONS-					
LINE	L	W	ZT	D	C	V	QW	HF	HPPM
	MILE	TPD	FT	IN.	PERCENT	FPS	GPM	FT/MI	HP/MI
1	50.0	1000.	0.	10.0	.2791	6.09	1074.	273.5	106.7

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

LINE	PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER EXP $	TOTAL COSTS $/TON-MI
1	1292500.	480229.	22500.	1795229.	567374.	.05356

CHAPTER III

THREE-LINE NETWORK OPTIMIZATION

The objective function describing the transport of woodchips in a three-pipe network containing a junction with three branches is a linear combination of the objective function for each line of the system and will be optimized in this chapter by applying the method used for the analysis of a single pipe and satisfying the continuity relationships for the flow at the junction. The latter imposes an additional restriction which makes the analysis of pipeline networks more complex than for single lines.

A typical three-line network has two injection locations supplying chips to one plant as shown in Figure 4. The variables are subscripted to designate the line they describe.

The continuity conditions at the junction must be satisfied by the mass flow rates of both woodchips and the mixture of woodchips and water. The continuity relation for mass flow rate of the woodchips, developed in Appendix D, can be expressed as

$$W_3 = W_1 + W_2$$

that for the mixture as

$$Q_{m_3} = Q_{m_1} + Q_{m_2}$$

The equation of continuity for the mixture can be combined with Equation B-2 to give

$$\frac{W_3}{C_3} = \frac{W_1}{C_1} + \frac{W_2}{C_2}$$

SOURCE 2

SOURCE 1

W_2

LINE 2

W_1 LINE 1

LINE 3

W_3

$W_3 = W_1 + W_2$

PROCESSING PLANT

Figure 4. Schematic of the Three-Line Network

which is rearranged to give C_3 as a function of C_1 and C_2 :

$$C_3 = \frac{C_1 C_2 W_3}{C_1 W_2 + C_2 W_1} \qquad .$$ (9)

The objective function for the three-line network, which contains all the information required to make a rational decision about the optimum economic conditions, is now a function of the throughput and length of each line and is written as a "weighted" expression of the single-line costs to represent an average cost per ton-mile of all woodchips delivered in the system. The three-line objective function is expressed by

$$X_t = \frac{X_1 W_1 L_1 + X_2 W_2 L_2 + X_3 W_3 L_3}{W_1 L_1 + W_2 L_2 + W_3 L_3}$$ (10)

or more compactly

$$X_t = \frac{\sum_{n=1}^{3} X_n W_n L_n}{\sum_{n=1}^{3} W_n L_n}$$ (11)

where X_n is the objective function for the nth line of the system and is the same as defined for a single pipe in the previous chapter. The X_n are functions of the concentration and diameter in line n:

$$X_1 = \Phi_1(C_1, D_1)$$

$$X_2 = \Phi_2(C_2, D_2)$$

$$X_3 = \Phi_3(C_3, D_3)$$

but from Equation 9, C_3 is a function of C_1 and C_2; therefore,

$$X_3 = \Phi_3(C_1, C_2, D_3)$$

which allows Equation 10 to be expressed as

$$X_t = \Psi_1(C_1, C_2, D_1, D_2, D_3) \qquad (12)$$

The three-pipe network is optimized by determining the combination of C_1, C_2, D_1, D_2, and D_3 (C_3 is obtained from C_1 and C_2 by Equation 9) which reduces the objective function to a minimum.

Conditions for the absolute minimum cost can be found by solving the five simultaneous equations obtained by setting the first partial derivatives of the objective function with respect to the independent variables, C_1, C_2, D_1, D_2, and D_3 equal to zero:

$$\frac{\partial X_t}{\partial C_1} = \left(\frac{\partial X_1}{\partial C_1} W_1 L_1 + \frac{\partial X_3}{\partial C_3} \frac{\partial C_3}{\partial C_1} W_3 L_3 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0 \qquad (13)$$

$$\frac{\partial X_t}{\partial C_2} = \left(\frac{\partial X_2}{\partial C_2} W_2 L_2 + \frac{\partial X_3}{\partial C_3} \frac{\partial C_3}{\partial C_2} W_3 L_3 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0 \qquad (14)$$

$$\frac{\partial X_t}{\partial D_1} = \left(\frac{\partial X_1}{\partial D_1} W_1 L_1 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0 \qquad (15)$$

$$\frac{\partial X_t}{\partial D_2} = \left(\frac{\partial X_2}{\partial D_2} W_2 L_2 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0 \qquad (16)$$

$$\frac{\partial X_t}{\partial D_3} = (\ \frac{\partial X_3}{\partial D_3}\ W_2 L_2)\ \frac{1}{\sum\limits_{n\ =\ 1}^{3}\ W_n L_n} = 0\ . \qquad (17)$$

The above simultaneous equations were solved by an iteration procedure on a digital computer and found to give concentrations far beyond the maximum physically possible and diameters which did not correspond to commercially available sizes. Because the solution for the theoretical minimum occurred at nonfeasible values of C and D, the constraints $C_n \leq C_{max}$ and the limitation that diameters must be sizes commercially available were applied.

A method for reducing to two commercial sizes the number of diameters to be considered for each of the three lines is based on the following analysis. The objective function for the system, X_t in Equation 11, will be in the minimum-value region when the objective function for each of the n lines, X_n, is in its respective minimum-cost regions. As shown in Figure 2, these minimum values will occur in the vicinity of $C_n = C_{max}$. The two diameters for each line which could give the lowest cost are found by finding the commercial pipe sizes which give the lowest positive and negative values of

$$\left. \frac{\partial X_t}{\partial D_1}\right|_{C_1\ =\ C_{max}} , \qquad (18)$$

$$\left. \frac{\partial X_t}{\partial D_2}\right|_{C_2\ =\ C_{max}} , \qquad (19)$$

and

$$\left.\frac{\partial X_t}{\partial D_3}\right|_{C_3 = C_{max}}$$ (20)

as was shown for the single-line system.

All combinations formed by the two pipe sizes for each of the three lines must be examined to find the one giving the lowest cost for the network. The number of combinations to be considered is determined by the mathematics of combinations and permutations described by Riordan (15) and is expressed by the equation

$$(N_D)^{N_L} = N_C$$ (21)

where N_D is the number of diameters tried for each line, N_L is the number of lines in the network, and N_C is the number of combinations of diameters which must be examined. Eight combinations are formed for the three-line network as computed by Equation 21:

$$(2)^3 = 8$$.

The optimum concentration must be found for each line with each combination of diameters. The combination formed by the smallest size for each line will have optimum concentrations equal to C_{max}. For the other combinations, the optimum concentrations are found by solving Equation 13 and 14 simultaneously with the constraints $C_n \leq C_{max}$, giving the optimum concentrations at either $\frac{\partial X_t}{\partial C_n} = 0$ or at C_{max}. These two partial differential equations can be put in general polynomial form:

$$\frac{\partial X_t}{\partial C_i} = \left[(1.84\ K_{1,i}\ C_i^{0.84} + 0.84\ K_{2,i}\ C_i^{-0.16})\ D_i^{2.10} - (2K_{3,i}\ C_i^{-3} \right.$$

$$\left. + 3K_{4,i}\ C_i^{-4})\ D_i^{-5} - K_{6,i}\ C_i^{-2} \right] \frac{W_i L_i}{\sum_{n=1}^{3} W_n L_n} + \left\{ \left[1.84\ K_{1,3} \right. \right.$$

$$\left. \left(\frac{C_1\ C_2\ W_3}{C_1 W_2 + C_2 W_1} \right)^{0.84} + 0.84\ K_{2,3} \left(\frac{C_1\ C_2\ W_3}{C_1 W_2 + C_2 W_1} \right)^{-0.16} \right] D_3^{2.10}$$

$$- \left[2K_{3,3} \left(\frac{C_1\ C_2\ W_3}{C_1 W_2 + C_2 W_1} \right)^{-3} - 3K_{4,3} \left(\frac{C_1\ C_2\ W_3}{C_1 W_2 + C_2 W_1} \right)^{-4} \right] D_3^{-5}$$

$$\left. - K_{6,3} \left(\frac{C_1\ C_2\ W_3}{C_1 W_2 + C_2 W_1} \right)^{-2} \right\} \left[\frac{C_i^2\ W_i\ W_3}{(C_j W_i + C_i W_j)^2} \right] \frac{W_3 L_3}{\sum_{n=1}^{3} W_n L_n} = 0$$

$$(22)$$

where for the first equation the subscript i is one and j is two; for
the second equation the value of these subscripts are interchanged. The
values for the diameters are given by the combination being examined and,
therefore, are known.

These two equations can be solved simultaneously by any suitable
method which imposes the constraints $C_n \leq C_{max}$. The computer program
developed for this analysis is listed in Appendix F and uses an iterative
procedure combined with the Newton-Raphson method of extracting real
roots. A general outline fo the procedure used in the computer program
is as follows:

1. Assume starting trial values of C_1 and C_2 for the first diameter combination at values less than C_{max} to prevent the application of the concentration constraints and compute C_3 using Equation 9.

2. Solve for C_1 which satisfies $\frac{\partial X_t}{\partial C_1} = 0$ (Equation 22) by the Newton-Raphson method while holding C_2 constant.

3. Solve for C_2 which satisfies $\frac{\partial X_t}{\partial C_2} = 0$ holding C_1 constant at the value found in step 2.

4. With these "better" approximations for C_1 and C_2, begin the iteration again.

5. The constraints, $C_n \leq C_{max}$, are applied when either C_1 or C_2 becomes larger than C_{max}.

6. The process is continued until the change in the concentrations C_1 and C_2 for successive iterations are less than a predetermined value.

7. With C_1 and C_2 found, C_3 and the total costs are computed for each combination of diameters and compared to select the conditions which give the lowest cost.

Results from the optimization of a three-line network are shown in Table II which is divided into four sections. The first three sections are of the same format used in Table I and are explained in the previous chapter. The fourth section lists the total first costs and annual operating expense for the pipeline network. The final column in the fourth section gives the total cost of the system on a dollars per ton-

TABLE II

SAMPLE RESULTS, THREE-LINE NETWORK

L =LENGTH OF LINE, MILE
W =THROUGHPUT, TONS OF OVEN-DRY CHIPS PER DAY
ZT=DIFFERENCE IN ELEVATION BETWEEN INLET AND OUTLET END, FEET
D =DIAMETER OF PIPE, INCHES
C =CONCENTRATION OF WOODCHIPS IN MIXTURE BY VOLUME, PERCENT
V =AVERAGE VELOCITY OF FLOW, FEET PER SECOND
QW=VOLUMETRIC FLOW RATE OF MAKE-UP WATER, GALLONS PER MINUTE
HF=HEAD LOSS DUE TO FRICTIONAL RESISTANCE, FEET PER MILE
HPPM=INPUT POWER REQUIRED, HORSEPOWER PER MILE OF PIPELINE

	-GIVEN QUANTITES-			----COMPUTED OPTIMUM CONDITIONS----					
LINE	L	W	ZT	D	C	V	QW	HF	HPPM
	MILE	TPD	FT	IN.	PERCENT	FPS	GPM	FT/MI	HP/MI
1	100.0	500.	0.	6.0	.3000	7.87	485.	328.3	59.5
2	50.0	1000.	0.	10.0	.3000	5.66	971.	294.2	106.7
3	50.0	1500.	0.	10.0	.3000	8.50	1456.	296.8	161.6

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

LINE	PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER EXP $	TOTAL COSTS $/TON-MI
1	1551000.	536283.	7500.	2094783.	707924.	.06489
2	1292500.	480547.	7500.	1780547.	530618.	.05141
3	1292500.	727380.	15000.	2034880.	786108.	.04643

TOTAL COST FOR THE PIPELINE TRANSPORTATION SYSTEM

PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER EXP $	TOTAL COSTS $/TON-MI
4136000.	1744211.	30000.	5910211.	2024650.	.05313

mile basis, as given by the objective function expressed in Equation 10. The line numbers and layout for the sample are shown in Figure 4. Variables describing the woodchips and the various costs were taken from Table VIII. The intermediate results from the computer program for the sample described in Table II are given in Appendix E for the eight combinations resulting from using two commercial diameters for each line and for the 27 combinations resulting from taking three sizes for each line, two of which are the next sizes larger than D_t for the given line.

These techniques can be applied to higher order networks. Chapter IV outlines the optimization technique for the five-line network.

CHAPTER IV

FIVE-LINE NETWORK OPTIMIZATION

The procedure for optimizing the objective function describing the economics of transporting woodchips in a five-line network is outlined in this chapter using methods of analysis discussed in the preceding chapters for optimizing the single- and three-line objective functions.

The five-line objective function must satisfy two continuity relationships describing the mass flow rates at the two junctions which occur in the five-line network shown in Figure 5.

The continuity conditions at junction 1 are expressed by equations of the same form as given for the three-line network and are used to relate the mass flow rates of the woodchips and the woodchip-water mixture entering the junction to that leaving the junction. By changing the subscripts to conform to Figure 5, Equation 9 can be rewritten to express the concentration of woodchips in the mixture for line 4 which leaves junction 1 as:

$$C_4 = \frac{C_1 C_2 W_4}{C_1 W_2 + C_2 W_1} \qquad (23)$$

The continuity relationship at junction 2, which is fully developed in Appendix D, similarly gives the concentration of woodchips in line 5 leading from the junction as:

$$C_5 = \frac{C_3 C_4 W_5}{C_3 W_4 + C_4 W_3} \qquad (24)$$

SOURCE 2

LINE 2

W_2

SOURCE 1

LINE 1

W_1

Junction 1

LINE 4

SOURCE 3

W_4

W_3

LINE 3

$W_4 = W_1 + W_2$

$W_5 = W_3 + W_4$

Junction 2

LINE 5

W_5

PROCESSING PLANT

Figure 5. Schematic of the Five-Line Network

and can be written as a function of C_1, C_2, C_3, W_1, W_2, W_3, and W_5 by substituting

$$W_5 = W_3 + W_4$$

and the expression for C_4, given by Equation 23, into Equation 24 giving:

$$C_5 = \frac{C_1 \, C_2 \, C_3 \, W_5}{C_1 C_2 W_3 + C_2 C_3 W_1 + C_1 C_3 W_2} \qquad . \tag{25}$$

The objective function for the five-line network is a weighted expression of the single-line costs and is written in the same form as the three-line objective function given in Equation 10:

$$X_t = \frac{X_1 W_1 L_1 + X_2 W_2 L_2 + X_3 W_3 L_3 + X_4 W_4 L_4 + X_5 W_5 L_5}{W_1 L_1 + W_2 L_2 + W_3 L_3 + W_4 L_4 + W_5 L_5} \tag{26}$$

or

$$X_t = \frac{\sum\limits_{n=1}^{5} X_n W_n L_n}{\sum\limits_{n=1}^{5} W_n L_n} \tag{27}$$

where X_t is given in dollars per ton-mile and X_n is the single-line objective function. The X_n are functions of the concentration and diameters in line n:

$$X_1 = \Phi_1(C_1, D_1)$$
$$X_2 = \Phi_2(C_2, D_2)$$

$$X_3 = \Phi_3(C_3, D_3)$$
$$X_4 = \Phi_4(C_4, D_4)$$
$$X_5 = \Phi_5(C_5, D_5)$$

but from Equations 23 and 25, C_4 can be expressed as a function of C_1 and C_2 but C_5 is expressed as a function of C_1, C_2, and C_3; therefore, the objective function for the five-line network is expressed as

$$X_t = \Psi_2 (C_1, C_2, C_3, D_1, D_2, D_3, D_4, D_5) \qquad . \tag{28}$$

The five-line network is optimized by determining the combination of C_1, C_2, C_3, D_1, D_2, D_3, D_4, and D_5 which reduce the objective function to a minimum. C_4 and C_5 are found by solving the continuity relationships given in Equation 23 and 25.

The conditions for the absolute minimum were examined by the same method used for the three-line objective function discussed in the previous chapter by solving the eight simultaneous equations formed by setting the first partial derivatives of the five-line network objective function with respect to the independent variables C_1, C_2, C_3, D_1, D_2, D_3, D_4, and D_5 equal to zero. The solution gives concentrations much greater than the maximum physically possible and diameters which were not sizes commercially available. Constraints were required to limit the concentration to C_{max} and the diameters to commercial sizes.

The number of commercially available pipe sizes for each line were reduced to two by the same method used for the three-line network by finding the commercial sizes which give the lowest positive and negative values of

$$\frac{\partial X_t}{\partial D_n}\bigg| \quad C_n = C_{max} \tag{29}$$

for the five individual lines.

The number of diameter combinations formed by these two pipe sizes for each line and which must be investigated to determine the lowest costs are found from Equation 21 to be 32.

The concentration which gives the lowest cost is C_{max} for the combination formed by the smaller pipe size for each line. The optimum concentrations for the other combinations are found by solving simultaneously the following three equations obtained by taking the partial derivatives with respect to C_1, C_2 and C_3 of the five-line network objective function:

$$\frac{\partial X_t}{\partial C_i} = \frac{\partial X_i}{\partial C_i} \frac{W_i L_i}{5 \sum\limits_{n=1} W_n L_n} + \frac{\partial X_4}{\partial C_4} \frac{\partial C_4}{\partial C_i} \frac{W_4 L_4}{5 \sum\limits_{n=1} W_n L_n} + \frac{\partial X_5}{\partial C_5} \frac{\partial C_5}{\partial C_i} \frac{W_5 L_5}{5 \sum\limits_{n=1} W_n L_n} = 0 \tag{30}$$

for i = 1 to 3 and where

$$\frac{\partial C_4}{\partial C_3} = 0$$

and

$$C_i \overset{<}{=} C_{max}$$

These quations, giving the optimum concentration, can be put in polynomial form similar to Equation 22 in terms of the unknowns C_1, C_2, and C_3; the diameters are given by the combination of sizes being investigated.

These three equations are solved simultaneously in the manner out-
lined in the previous chapter by using a numerical iterative procedure
which converges to the optimum value of concentration.

Results from the optimization of a five-line network are shown in
Table III using the same format at Table II for the three-line network.
The line numbers and layout for the sample are shown in Figure 5.

The rational methods described in the last three chapters for de-
termining the optimum concentrations and diameters and predicting the
costs for pipeline networks can be extended to networks of more than
five lines. The objective functions are constructed by using the
pattern and form suggested by the similarity of the three- and five-
line objective functions shown in Equations 11 and 27. The continuity
relationships for each junction are developed to give the concentration
in the line leaving in terms of the concentrations of the lines entering
the junction in a manner similar to the method of developing the five-
line continuity relationships. The two commercial pipe sizes for each
line used in the combinations of diameters to be tried for lowest cost
are found in the same manner used in the one-, three-, and five-line
networks. The equations to be solved for optimum concentration show a
pattern of development seen in Equations 13, 14 and 30 which can be used
for networks of more than five lines.

The optimization techniques presented in the last three chapters to
determine the best operating concentrations and pipe diameters to give
the probable minimum costs for which woodchips can be transported are
applied to a specific area in the next chapter.

TABLE III

SAMPLE RESULTS, FIVE-LINE NETWORK.

L =LENGTH OF LINE, MILE
W =THROUGHPUT, TONS OF OVEN-DRY CHIPS PER DAY
ZT=DIFFERENCE IN ELEVATION BETWEEN INLET AND OUTLET END, FEET
D =DIAMETER OF PIPE, INCHES
C =CONCENTRATION OF WOODCHIPS IN MIXTURE BY VOLUME, PERCENT
V =AVERAGE VELOCITY OF FLOW, FEET PER SECOND
QW=VOLUMETRIC FLOW RATE OF MAKE-UP WATER, GALLONS PER MINUTE
HF=HEAD LOSS DUE TO FRICTIONAL RESISTANCE, FEET PER MILE
HPPM=INPUT POWER REQUIRED, HORSEPOWER PER MILE OF PIPELINE

	-GIVEN QUANTITES-			----COMPUTED OPTIMUM CONDITIONS----					
LINE	L MILE	W TPD	ZT FT	D IN.	C PERCENT	V FPS	QW GPM	HF FT/MI	HPPM HP/MI
1	50.0	800.	0.	8.0	.3000	7.08	776.	290.3	84.3
2	25.0	400.	0.	6.0	.3000	6.29	388.	292.4	42.4
3	70.0	800.	0.	8.0	.3000	7.08	776.	290.3	84.3
4	50.0	1200.	0.	10.0	.3000	6.80	1165.	285.5	124.3
5	100.0	2000.	0.	12.0	.3000	7.87	1942.	285.9	207.5

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

LINE	PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER EXP $	TOTAL COSTS $/TON-MI
1	1034000.	379355.	7500.	1420855.	450024.	.05307
2	387750.	95515.	7500.	490765.	174170.	.07741
3	1447600.	531097.	7500.	1986197.	612034.	.05216
4	1292500.	559718.	0.	1852218.	589524.	.04657
5	3102000.	1867961.	20000.	4989961.	1801991.	.04109

TOTAL COST FOR THE PIPELINE TRANSPORTATION SYSTEM

PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER EXP $	TOTAL COSTS $/TON-MI
7263850.	3433648.	42500.	10739998.	3627745.	.04599

CHAPTER V

CONCEPTUAL APPLICATION OF THE ECONOMIC MODEL

The study of the economics of pipelines as a method of transporting

woodchips in a specific location in the northern Rocky Mountain area is

presented in this chapter to demonstrate the technique developed in the

preceding chapters for establishing the optimum conditions and predicting

the cost of a pipeline system.

The location of the study area is in western Montana and northern

Idaho as shown in Figure 6.

This area is under active consideration as a possible site for a

woodpulp processing plant. The location is desirable because of the

abundant volume of wood and the access to water for processing. The

area is enhanced by the availability of rail transportation for shipment

of the finished product. The Forest Service has available for this loca-

tion data on timber volumes, timber growth, and local sawmills with their

rated production capacity (16), (17), (18), (19).

The location is divided into four areas. For all federal, state,

and private commercial forest lands, Table IV gives the timber volumes

presently available and the estimated net annual growth (i.e. the net

annual change in the volume of wood).

Existing sawmills are located throughout the study area as shown

in Figure 6. Chip production from residue created by the mills operat-

ing at their rated capacities is calculated by applying the approximate

conversion that 1,000 board feet mill production yields 1.68 tons of

oven-dry chips from the mill residue. The data on residue production

Figure 6.   Vicinity of Conceptual Application

TABLE IV

ESTIMATE OF TIMBER VOLUME AND CHIP PRODUCTION

Area	Proposed Injection Location	Volume of Timber	Estimated Net Annual Growth	Estimated Possible Harvest	Chips Available from Harvest Residue	Possible Sustained Production of Chips
		M cu ft	M cu ft	M cu ft	Ton OCD/year	Ton ODC/day
I	Thompson Falls	1,218,908	158,458	75,268	234,836	643
II	Sandpoint	1,391,354	180,876	85,916	268,058	734
III	Bonners Ferry	1,358,855	176,651	83,909	261,796	717
IV	Libby	763,409	99,243	47,140	147,077	403
	Totals	4,732,526	615,228	292,233	764,690	2,497

at the existing sawmills are tabulated in Table V.

A hypothetical chip processing mill is assumed to be located at Thompson Falls with a rated capacity of 1,250 tons of product per day. Chips must be delivered at a rate of 2,500 tons per day for the mill to operate at capacity.

The study consists of analyzing the movement of woodchips in this area to the hypothetical processing plant at Thompson Falls. To allow the application of the single-, three-, and five-line optimization techniques, three delivery schedules will be examined and are given as follows:

1. Delivery of chips from Libby to Thompson Falls

2. Delivery of chips from Libby and Sandpoint to Thompson Falls

3. Delivery of chips from Libby, Sandpoint, and Bonners Ferry to Thompson Falls.

All chips are assumed to be produced from sawmill residue for this study. Only 47.5 percent of the annual growth must be harvested from the four areas to meet the supply demand of 2,500 tons per day. The amount of chips made from the residue from this percentage of harvest are computed by using the conversion that 1,000 cubic feet of wood delivered to the sawmill yields enough residue to produce 3.12 tons of chips. Table IV gives the chip yield for each area and the combined yield of 2,497 tons of oven-dry chips per day for all the areas.

Woodchips are assumed to be available at the injection points for each area at the design throughput rate shown in Table VI. Chips in

TABLE V

PRESENT CHIP PRODUCTION CAPABILITY OF SAWMILLS IN THE APPLICATION AREA

Area	Estimated Lumber Production	Chip Production Capability Using Residue	Chip Production Capability Using Residue
	MBF/year	Tons ODC/year	Tons ODC/day
I	63,875	107,310	294
II	61,300	102,984	282
III	58,000	89,040	244
IV	16,100	27,048	74
		Total	894

TABLE VI

INJECTION POINTS AND THROUGHPUT

Injection Point	Location	Design Throughput, W
		tons/day
E	Sandpoint	734
F	Bonners Ferry	717
D	Libby	403
—	Thompson Falls	643
	Total chips	2,497

the Thompson Falls area are expected to be delivered directly from the sawmill to the processing plant. Rail, truck, and pipeline costs are not involved in the delivery of these chips.

The sawmills in the area could provide all the residue required to produce chips at the rate of 2,500 tons per day if their capacities were increased by a factor of three. This capability is shown by comparing the total estimated production available in Table IV with the present estimated mill output in Table V. A higher rate of chip supply could be obtained by increasing the percentage of harvest or by converting sawtimber into chips. Data verifying that enough wood is available for this increase in chip supply is shown in Table IV by the large volume of unharvested annual growth.

The data used in the computer program developed to optimize the various pipeline systems are given in Tables VII and VIII. Table VII gives the pipeline lengths and the difference in elevation of the ends of each line. These data were taken from Army Map Service topographic maps with a scale of 1:250,000. Table VIII gives the values of the variables used in each of the networks analyzed.

Tables IX, X, and XI give the results, in the same format used in Table II, of the analysis on each network whose locations are shown in Figure 6.

Single-Line. The single line, which begins at Libby and transports 403 tons of chips per day to Thompson Falls, has the optimum operating conditions of 30 percent concentration and a 6-inch pipe. The cost is

TABLE VII

PHYSICAL LAYOUT OF PIPELINES IN THE STUDY AREA

Line	Length	Elevation Difference, Z_t
	miles	ft
DCBA	90.6	+365
BA	40.4	+260
DCB	50.2	+105
EB	44.4	+144
CB	34.5	+200
FC	35.5	+100
DC	15.7	- 95

—43—

TABLE VIII

VALUES OF VARIABLES TO BE USED IN THE CONCEPTUAL APPLICATION

Symbol of the Variable	Value Used
C_{max}	30. percent
crf	0.20
e	65. percent
f	0.018
g	32.2 ft/sec^2
H_{sa}	1,200. ft
M	1.80
MTPU	1,500. TPD
R_1	$ 0.007/kwh
R_2	$ 2,585./in-mile
R_3	$ 90./installed HP
R_4	to be computed
R_5	to be computed
R_6	$45,000./year
R_7	$ 7,000./(pump station)
R_8	$ 100./mile-year
R_9	$ 80./million-gal
S_{odc}	0.40
U_1	$ 7,500./MTPU
U_2	$15,000./MTPU

TABLE IX

SINGLE-LINE PIPELINE SYSTEM

L =LENGTH OF LINE, MILE
W =THROUGHPUT, TONS OF OVEN-DRY CHIPS PER DAY
ZT=DIFFERENCE IN ELEVATION BETWEEN INLET AND OUTLET END, FEET
D =DIAMETER OF PIPE, INCHES
C =CONCENTRATION OF WOODCHIPS IN MIXTURE BY VOLUME, PERCENT
V =AVERAGE VELOCITY OF FLOW, FEET PER SECOND
QW=VOLUMETRIC FLOW RATE OF MAKE-UP WATER, GALLONS PER MINUTE
HF=HEAD LOSS DUE TO FRICTIONAL RESISTANCE, FEET PER MILE
HPPM=INPUT POWER REQUIRED, HORSEPOWER PER MILE OF PIPELINE

	-GIVEN QUANTITES-			-----COMPUTED OPTIMUM CONDITIONS-----					
LINE	L	W	ZT	D	C	V	QW	HF	HPPM
	MILE	TPD	FT	IN.	PERCENT	FPS	GPM	FT/MI	HP/MI
DCBA	90.6	403.	365.	6.0	.3000	6.34	391.	293.0	43.4

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

LINE	PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER. EXP $	TOTAL COSTS $/TON-MI
DCBA	1405206.	354344.	22499.	1782050.	539024.	.07004

TABLE X

THREE-LINE PIPELINE NETWORK

L =LENGTH OF LINE, MILE
W =THROUGHPUT, TONS OF OVEN-DRY CHIPS PER DAY
ZT=DIFFERENCE IN ELEVATION BETWEEN INLET AND OUTLET END, FEET
D =DIAMETER OF PIPE, INCHES
C =CONCENTRATION OF WOODCHIPS IN MIXTURE BY VOLUME, PERCENT
V =AVERAGE VELOCITY OF FLOW, FEET PER SECOND
QW=VOLUMETRIC FLOW RATE OF MAKE-UP WATER, GALLONS PER MINUTE
HF=HEAD LOSS DUE TO FRICTIONAL RESISTANCE, FEET PER MILE
HPPM=INPUT POWER REQUIRED, HORSEPOWER PER MILE OF PIPELINE

	-GIVEN QUANTITES-			COMPUTED OPTIMUM CONDITIONS					
LINE	L	W	ZT	D	C	V	QW	HF	HPPM
	MILE	TPD	FT	IN.	PERCENT	FPS	GPM	FT/MI	HP/MI
DCB	50.2	403.	105.	6.0	.3000	6.34	391.	293.0	43.1
EB	44.4	734.	144.	8.0	.3000	6.50	712.	286.1	77.1
BA	40.4	1137.	260.	10.0	.3000	6.44	1104.	286.6	120=9

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

LINE	PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER. EXP $	TOTAL COSTS $/TON-MI
DCB	778602.	195056.	7499.	981158.	307910.	.07110
EB	918192.	308104.	7499.	1233796.	381858.	.05562
BA	1044340.	439744.	14999.	1499084.	523812.	.05193

TOTAL COST FOR THE PIPELINE TRANSPORTATION SYSTEM

PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER. EXP $	TOTAL COSTS $/TON-MI
2741134.	942905.	29999.	3714039.	1213581.	.05707

TABLE XI

FIVE-LINE PIPELINE NETWORK

L =LENGTH OF LINE, MILE
W =THROUGHPUT, TONS OF OVEN-DRY CHIPS PER DAY
ZT=DIFFERENCE IN ELEVATION BETWEEN INLET AND OUTLET END, FEET
D =DIAMETER OF PIPE, INCHES
C =CONCENTRATION OF WOODCHIPS IN MIXTURE BY VOLUME, PERCENT
V =AVERAGE VELOCITY OF FLOW, FEET PER SECOND
QW=VOLUMETRIC FLOW RATE OF MAKE-UP WATER, GALLONS PER MINUTE
HF=HEAD LOSS DUE TO FRICTIONAL RESISTANCE, FEET PER MILE
HPPM=INPUT POWER REQUIRED, HORSEPOWER PER MILE OF PIPELINE

	-GIVEN QUANTITES-			----COMPUTED OPTIMUM CONDITIONS----					
LINE	L	W	ZT	D	C	V	QW	HF	HPPM
	MILE	TPD	FT	IN.	PERCENT	FPS	GPM	FT/MI	HP/MI
DC	15.7	403.	-95.	6.0	.3000	6.34	391.	293.0	41.9
FC	35.5	717.	100.	8.0	.3000	6.35	696.	285.7	75.0
EB	44.4	734.	144.	8.0	.3000	6.50	712.	286.1	77.1
CB	34.5	1120.	200.	10.0	.3000	6.34	1087.	287.1	119.0
BA	40.4	1854.	260.	12.0	.3000	7.29	1800.	285.8	196.6

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

LINE	PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER EXP $	TOTAL COSTS $/TON-MI
DC	243507.	59320.	7499.	310327.	124999.	.08375
FC	734140.	239903.	7499.	981543.	309070.	.05716
EB	918192.	308104.	7499.	1233796.	381858.	.05562
CB	891825.	369770.	0.	1261595.	409663.	.04974
BA	1253208.	715081.	18540.	1986829.	768425.	.04544

TOTAL COST FOR THE PIPELINE TRANSPORTATION SYSTEM

PIPE INSTLN $	PUMP STATIONS $	INJ-SEP SYSTEM $	TOTAL FST COST $	ANNUAL OPER EXP $	TOTAL COSTS $/TON-MI
4040872.	1692181.	41039.	5774093.	1994017.	.05128

the highest of the three networks at $0.070 per ton-mile.

Three-Line Network. The three-line network has injection units at Libby and Sandpoint and delivers 1,137 tons of chips per day to Thompson Falls. The optimum concentration is 30 percent, and the best pipe sizes are 6-, 8-, and 10-inch. The cost is the median value of $0.057 per ton-mile.

Five-Line Network. The five-line network has three injection stations: Libby, Sandpoint, and Bonners Ferry. This network delivers 1,854 tons of chips per day to Thompson Falls at a concentration of 30 percent in pipe sizes of 6-,8-, 10-, and 12-inch. The cost is the lowest of the three networks at $0.051 per ton-mile.

The lowest costs given in Tables IX, X, and XI for each of the three networks will be compared with costs for rail and truck in the next chapter.

CHAPTER VI

COMPARISON OF TRANSPORTATION COSTS

This chapter compares the costs of transporting woodchips in the single-, three-, and five-line network applications of the preceding chapter with transportation costs of rail and truck.

Freight rates for rail shipment of woodchips in western Montana can be obtained from either the railroad companies engaged in chip transport (20) or the Public Service Commission. The rates are posted in units of dollars per 200 cubic feet of chips but can be converted to dollars per ton-mile by using he conversion that 200 cubic feet of oven-dry chips weigh 2,400 pounds and dividing by the distance for the posted rates in miles. The rates for a variety of hauls ranged from $0.023 to $0.031. The latter rate will be used since it is the highest of a small sample and represents a conservative value.

Truck-haul costs are less easily determined. Because of the type of vehicle and the distances the freight is being moved, chip haulers are not required to register their rates with the Public Service Commission. Truck rates used in this comparison were taken from a 1961 Highway Research Board publication (21). These rates were adjusted to present rates, based on the rise in freight rates for the period 1961-1965 (22), (23), by increasing them 19.5 percent. With the adjustment the rate of $0.053 is used. The rate represents an operation in which the vehicle, a diesel engine tractor-trailer combination of 44,000 pounds gross weight, is loaded in one direction and returns empty.

Table XII gives the costs for each mode of transportation for each of the three networks. The line and lengths describe the routes along which each different mode of transportation must travel. The cost in dollars per ton-mile is given for each line and its throughput. The cost is converted to dollars per day for each line and then summarized for each method of transportation. The final column, listing total cost for each mode, gives a comparison of cost on a daily basis.

All modes of transportation in the study area have approximately the same route and distance to travel to deliver chips to Thompson Falls. Therefore, the mode which has the lowest cost in dollars per ton-mile will also give the lowest cost in dollars per day. In each of the three networks analyzed, rail transportation is the most economical. Although pipeline transportation costs are close to truck costs in all three networks, the pipeline shows an economic advantage only in the five-line system.

Transportation cost studies in areas where the methods of transportation do not parallel each other may produce results which show the pipeline to be most economical. Such an area could occur when the pipeline takes a route "cross country" and thereby reduces the distance chips must be moved. Pipelines with "cross country" routes may have a high rate on a dollars per ton-mile basis; but on a daily basis, reflecting the shortened route, the costs may be low and competitive with other modes of transportation. The distance the chips must be moved is an important consideration when comparing costs of several transportation systems.

TABLE XII

COMPARISON OF COSTS

Network	Mode of Transportation	Line	L Miles	W Tons/day	X $ton-mile	(L)(W)(X) $/day	Total cost $/day
Single	Rail	DCFEBA	171.0	403	0.031	2,136	2,136
	Truck	DCBA	90.6	403	0.051	1,935	1,935
	Pipeline	DCBA	90.6	403	0.070	2,556	2,556
3-line	Rail	DCFEBA	86.2	403	0.031	1,077	
		EBA	84.8	1,137	0.031	2,989	4,066
	Truck	DCBA	90.6	403	0.053	1,935	
		EBA	84.8	734	0.053	3,299	5,234
	Pipeline	DCB	50.2	403	0.071	1,436	
		EB	44.4	734	0.056	1,825	
		BA	40.4	1,137	0.052	2,389	5,650
5-line	Rail	DCF	51.2	403	0.031	640	
		FE	35.0	1,120	0.031	1,215	
		EBA	84.8	1,854	0.031	4,874	6,729
	Truck	DCBA	90.6	403	0.053	1,935	
		FCBA	110.4	717	0.053	4,195	
		EBA	84.4	734	0.053	3,283	9,413
	Pipeline	DC	15.7	403	0.084	531	
		FC	35.5	717	0.057	1,451	
		EB	44.4	734	0.056	1,825	
		CB	34.5	1,120	0.050	1,932	
		BA	40.4	1,854	0.045	3,371	9,110

Hunt showed that areas where no existing transportation facilities occured the pipeline is very competitive with other systems. When the high installation and maintenance costs are considered for other transportation systems the economic advantages of pipeline transport of woodchips are quite attractive.

CHAPTER VII

CONCLUSION AND RECOMMENDATIONS

Conclusions

The following statements can be concluded from the technique presented for optimizing the economics and operating conditions for the transportation of woodchips in pipelines:

1. The response surface generated by using the objective function offers a positive, visual analysis of the economic characteristics for the pipeline system.

2. The optimization technique presents a rational method using numerical procedures for determining the optimum pipe sizes and operating woodchip concentrations which can be extended to pipeline networks characterized by series of branching lines that do not loop back.

3. Any nonclosed-loop pipeline system can be described by properly defining the objective function for each network.

4. The seven cost groups and the objective functions are basic in describing the various costs and can be easily altered when new cost rates and head loss equations are developed.

The following conclusions are obtained from the economic analysis of the conceptual application of woodchip pipelines in northwestern Montana:

1. The proposed pipeline networks in northwestern Montana are not competitive with railroads.

2. The five-line network for transporting woodchips is competitive with trucking.

3. In general, woodchip pipelines cannot compete economically with rail when the routes are of the same distance.

4. Woodchip pipelines may be more economical than highway trucking for high throughputs and long distances.

Recommendations

The following recommendations are given for further refinement of the economic model:

1. Since the head loss equation is very significant in the development of the economic model, any improvements or developments of the head loss relationships should be incorporated in the model.

2. Tests should be conducted to define C_{max} more closely for various size pipes.

3. The variables used in the mathematical expressions for costs should constantly be updated to reflect current prices.

APPENDICES

APPENDIX A

LIST AND DEFINITION OF VARIABLES

ANOP = Annual operating expense, \$/year.

C = Concentration of chips in mixture by volume.

C_{max} = Maximum concentration of chips permissible during operation.

crf = Charge on capital investment to cover interest, depreciation, etc., capital recovery factor.

D = Pipe diameter, inches.

D_t = Theoretical pipe size at which the lowest cost occurs in the feasible region of the response surface.

d = Pipe diameter, feet.

e = Combined efficiency of motor-pump drivers.

f = Friction factor for Weisbach equation.

g = Gravitational constant, ft/sec^2.

H_e = Head due to difference in elevation, ft/mile.

H_f = Head loss due to frictional resistance, ft/mile.

H_{sa} = Total head developed per pump station, ft.

H_t = Head due to friction and difference in elevation, ft/mile.

HPPM = Power requirements, horsepower per mile of pipeline.

h_m = Frictional head loss of the mixture, ft/ft of pipe.

h_w = Frictional head loss of clear water, ft/ft of pipe.

I_{pl} = Initial investment in the pipeline, \$.

I_{tps} = Initial pump station investment, \$.

I_{is} = Initial injection-separation equipment investment, \$.

K = Constants used in the development of the objective functions.

L = Length of line, miles.

l =   Length of line, feet.

M =   Moisture content of chips, decimal fraction of oven-dry chips.

$MTPU$ =   Minimum throughput unit for injection and separation units.

Q_m =   Volumetric discharge of mixture in the pipeline, cfs.

Q_w =   Volumetric discharge of make-up water, cfs.

R_1 =   Cost of electrical energy, $/kwh.

R_2 =   Installed cost of pipeline, including right-of-way, $/(in-mile).

R_3 =   Cost of pump station and controls, $/(installed hp).

R_4 =   Cost of chip injection system, $/(ton per day oven-dry chips).

R_5 =   Cost of separation system, $/(ton per day oven-dry chips).

R_6 =   Annual cost of fixed wages, salaries, operation maintenance, exclusive of pipeline maintenance and pump station operation.

R_7 =   Annual wages, salaries, etc. for pump station, $/(pump station).

R_8 =   Annual maintenance cost of pipeline, $/mile.

R_9 =   Cost of water and treatment, $/million gallons.

S_m =   Specific gravity of water-chip mixture.

S_{odc} =   Specific gravity of oven-dry chips.

U_1 =   First cost of injection device $/unit of MTPU.

U_2 =   First cost of separation system $/unit of MTPU.

V =   Average velocity of flow, fps.

V_m =   Average velocity of the mixture, fps.

W =   Tons per day of oven-dry chips, TPD.

X =   Total cost of a pipeline, $/ton-mile.

X_t =   Objective function giving total cost of pipeline network, $/ton-mile.

x = One of seven cost groups appearing as a term in X, $/ton-mile.

Z_t = Difference in elevation between inlet and discharge ends of pipe, ft.

λ = Square root of pipe flow Froude Number.

APPENDIX B

DEVELOPMENT OF ECONOMIC COST GROUPS x_1, x_2, ... x_7

Parts of this Appendix are taken from Hunt (9).

The detailed development of each cost group (x_1, x_2, ... x_7) and
other relationships among the variables describing the hydraulics and
economics of a proposed woodchip pipeline are included in this section.

Volumetric discharge of mixture in the pipeline, Q_m, is the number
of cubic feet per second of the woodchip-water mixture flowing through
a pipeline and is given by the equation

$$Q_m = 0.00545 \, D^2 \, V_m \tag{B-1}$$

where D is the pipe diameter in inches and V_m is the average velocity of
the flowing mixture.

Throughput rate of woodchips on an oven-dry basis, W, is the number
of ton per day of oven-dry chips carried by a pipeline and is given by
the equation

$$W = \frac{62.4 \, (86,400)}{2000} \, C \, Q_m \, S_{odc} \tag{B-2}$$

where C is the concentration of wood chips by volume in the mixture and
S_{odc} is the specific gravity of the oven-dry chips.

The specific gravity of the mixture, S_m, is given by the equation

$$S_m = C \left[(M + 1) \, S_{odc} - 1 \right] + 1 \tag{B-3}$$

where M is the moisture content of the chips on an oven-dry basis.

Frictional head loss per mile of line, H_f, in feet per mile is

the energy dissipated as heat by the fluid drag on the pipe walls and
is expressed in foot-pounds per pound of fluid flowing. For water this
is given by the Weisbach equation (given in any standard hydraulics or
fluid mechanics text).

$$h_w = f \frac{1}{d} \frac{V^2}{2g} \qquad\qquad (B-4)$$

where f is the friction factor, l and d are the pipeline length and dia-
meter, respectively, in feet, and V is the average flow velocity.

The relation between the head loss, h_m, for fluid transporting
solids at a concentration C and that for water, h_w, was found by Faddick
(12) to be

$$\frac{h_m - h_w}{C\,h_w} = 18 \left(\frac{V_m^2}{gd} \right)^{-1.42} \qquad\qquad (B-5)$$

where g is the gravitational constant in feet per second squared. Solving
equation B-1 and B-2 for V_m in terms of W, C, S_{odc} and using the diameter
in inches (i.e. D = 12d), a parameter, λ, is defined as

$$\lambda = \frac{V_m^2}{gD} = \left(\frac{1}{216\,g\,D^5} \right) \left(\frac{W}{C\,S_{odc}} \right)^2 \qquad . \qquad (B-6)$$

Equation (B-5) may now be transposed and rewritten as

$$h_m = (0.547\,C\,\lambda^{-1.42} + 1)\,h_w \qquad . \qquad (B-7)$$

With the diameter in inches Equation B-4 becomes

$$h_w = 6f1 \, \frac{V_m^2}{gD} = 6f1\lambda \qquad \qquad (B-8)$$

With the length of pipeline in miles (i.e. $1 = 5280L$), the head loss per mile for the mixture, $H_f = h_m/L$ becomes

$$H_f = 31,680 \, (0.547 \, C \, \lambda^{-1.42} + 1) \, f\lambda \qquad \qquad (B-9)$$

Elevation head per mile of line, H_e, in feet per mile, is the difference in elevation between the input and output installations of the pipeline system averaged over the length of the line and is given by the equation

$$H_e = Z_t/L \qquad \qquad (B-10)$$

where Z_t is the total difference in elevation between the two ends of the pipeline. If the output end is higher than the input, this is a positive quantity; if lower, a negative quantity.

The total head per mile supplied by the pumps, H_t, in feet per mile, is the sum of the frictional head loss and the elevation head and is given by the equation

$$H_t = H_e + H_f = 31,680 \, (0.547 \, C \, \lambda^{-1.42} + 1) \, f\lambda + Z_t/L \quad . \quad (B-11)$$

The algebraic expressions for each of the seven cost groups (x_1, x_2, ... x_7) are developed in the following paragraphs using the above-listed equation and relationships.

The energy cost per ton-mile for pumping, x_1, is based on the input

horsepower to the pumping stations required for each mile of pipeline.
The latter is given by the equation

$$HP_m = \frac{62.4 \ Q_m S_m H_t}{550 \ e}$$

(B-12)

where e is the combined motor-pump efficiency. This is reduced to cost
per ton-mile by the following set of converstion factors:

$$\$/\text{ton-mi} = HP_m (\frac{hp}{mi})(0.746 \frac{kw}{hp}) \ (R_1 \frac{\$}{kwh}) \ (24 \frac{hr}{day}) \ / \ (W \frac{tons}{day})$$

(B-13)

where R_1 is the rate at which electrical energy may be obtained. Using
this conversion and Equation B-2 and B-12, the cost per ton-mile of elec-
trical energy becomes

$$x_1 = 0.000753 \ (\frac{R_1}{e \ S_{odc}}) \ \frac{S_m H_t}{C}$$

(B-14)

The cost per ton-mile for pipeline materials and installation, x_2,
is based on the initial investment in the pipeline, including appropriate
hardware. The latter is expressed by the equation

$$I_{pl} = R_2 \ D \ L$$

(B-15)

where R_2 is cost of the installation in dollars per inch of diameter for
each mile of pipeline. Representative values of R_2 were obtained from
Continental Pipe Line Company, Billings, Montana (4).

If the annual return on the investment, amortization costs, taxes
and insurance are combined together into one capital recovery factor,
crf, the cost per ton-mile for capital investment in the pipeline itself

becomes

$$x_2 = (\frac{R_2 D}{365 \ W}) (crf) \qquad . \qquad (B\text{-}16)$$

The cost per ton-mile for pump stations, x_3, includes the initial investment in pumps, controls, and station construction costs. Cost data obtained from Continental Pipe Line Company indicate this investment to be a function of the installed horsepower at each station.

The pump station investment is

$$I_{tps} = R_3 \ (\frac{62.4 \ Q_m S_m H_t}{550 \ e}) \ L \qquad (B\text{-}17)$$

where R_3 is the cost of the pump station and controls in dollars per installed horsepower. Using Equation B-2 and the previously-defined capital recovery factor, the cost per ton-mile for pump stations becomes

$$x_3 = 0.000000115 \ (\frac{R_3}{e \ S_{odc}})(\frac{S_m H_t}{C})(crf) \qquad . \qquad (B\text{-}18)$$

The cost per ton-mile for chip injection and separation systems, x_4, is based on the cost of fabricating a minimum capacity unit for each of these minimum capacity units, the cost of installation per ton per day for these is

$$R_4 = \frac{U_1}{MTPU} \qquad for \qquad \frac{W}{MTPU} > 1 \qquad (B\text{-}19a)$$

$$R_4 = \frac{U_1}{W} \qquad for \qquad \frac{W}{MTPU} < 1 \qquad (B\text{-}19b)$$

$$R_5 = \frac{U_2}{MTPU} \qquad \text{for} \qquad \frac{W}{MTPU} > 1 \qquad (B\text{-}20a)$$

$$R_5 = \frac{U_2}{W} \qquad \text{for} \qquad \frac{W}{MTPU} < 1 \qquad (B\text{-}20b)$$

where R_4 and R_5 are the costs per ton per day of capacity of the injection and separation units respectively, U_1 and U_2 are the first costs of installing injection and separation systems, respectively, of a minimum throughput capacity, MTPU.

The initial investment for the injection and separation equipment is given by

$$I_{is} = (R_4 + R_5) \, W \qquad (B\text{-}21)$$

The cost per ton-mile for this phase of the pipeline system becomes

$$x_4 = (R_4 + R_5) \frac{crf}{365 \, L} \qquad (B\text{-}22)$$

The cost per ton-mile for fixed wages, salaries, and operational expenses, x_5, includes those items of this nature which are necessary regardless of the length or size of the pipeline installed and is given by the equation

$$x_5 = R_6 / (365 \, W \, L) \qquad (B\text{-}23)$$

where R_6 is the total annual cost of these fixed charges.

The cost per ton-mile for variable wages, salaries, and operational expenses, x_6, includes those items of this nature which vary with the length of the line and number of pumping stations required. If H_{sa} is

the total head developed per pump station in feet and the annual cost of maintaining the pipeline is R_8 dollars per mile, the total cost per ton-mile of the variable salaries and wages becomes

$$x_6 = \frac{1}{365\ W} (R_7\ \frac{H_t}{H_{sa}} + R_8) \qquad \qquad (B\text{-}24)$$

The cost per ton-mile for treatment of the water, x_7, includes purchase of water, if so required, in addition to any treatment which may be necessary to the influent and effluent to the pipeline. The cost of this depends upon the make-up water required for the mixture which is determined from the equation

$$Q_w = (1 - C)\ Q_m \qquad \qquad (B\text{-}25)$$

If the cost of the water and treatment is R_9 dollars per million gallons, the cost per day is

$$C_{wt} = 24(3600)\ 7.48(10^{-6})(R_9)(1 - C)\ Q_m \qquad (B\text{-}26)$$

The cost per ton-mile using Equation B-2 then is

$$x_7 = 0.000240\ (\frac{1 - C}{C})\ \frac{R_9}{S_{odc}\ L} \qquad \qquad (B\text{-}27)$$

R_9 is taken as zero except for the lines which terminate at a processing plant and have effluent.

APPENDIX C

DEVELOPMENT OF THE OBJECTIVE FUNCTION, X_t

The objective function for the single-line, X_t, is the sum of the seven cost groups given on page 8. These cost groups can be simplified for algebraic manipulation by redefining the constant coefficients:

$$x_1 = a_1 \frac{S_m H_t}{C}$$ where $$a_1 = \frac{0.000753 \ R_1}{e \ S_{odc}}$$

$$x_2 = a_2 D$$ $$a_2 = \frac{R_2 \ crf}{365 \ W}$$

$$x_3 = a_3 \frac{S_m H_t}{C}$$ $$a_3 = \frac{0.000000115 \ R_3 crf}{e \ S_{odc}}$$

$$x_4 = a_4$$ $$a_4 = \frac{(R_4 + R_5)crf}{365 \ L}$$

$$x_5 = a_5$$ $$a_5 = \frac{R_6}{365 \ W \ L}$$

$$x_6 = a_6 H_t + a_{6A}$$ $$a_6 = \frac{R_7}{365 \ W \ H_{sa}}$$

$$a_{6A} = \frac{R_8}{365 \ W}$$

$$x_7 = a_7 \left(\frac{1}{C} - 1 \right)$$ $$a_7 = \frac{0.00024 \ R_9}{S_{odc} \ L}$$

From Appendix B, Equation B-11

$$H_t = 31,680\ f\ (0.527\ C\ \lambda^{-0.42} + \lambda) + Z_t\ /\ L$$

where

$$\lambda = \frac{W^2}{216\ g\ S_{odc}\ C^2\ D^5} \qquad\qquad ;$$

let

$$\lambda = \frac{b_1}{C^2\ D^5} \qquad\qquad \text{where} \qquad b_1 = \frac{W^2}{216\ g\ S_{odc}^2}$$

giving

$$H_t = 31,680f\ (0.527)\ \frac{C^{1.84}\ D^{2.10}}{b_1^{0.42}} + \frac{31,680fb_1}{C^2\ D^5} + \frac{Z_t}{L} \qquad ;$$

let

$$b_2 = \frac{31,680f(0.527)}{b_1^{0.42}} \qquad\qquad ;$$

$$b_3 = 31,680\ f\ b_1 \qquad\qquad ;$$

and

$$b_4 = \frac{Z_t}{L} \qquad\qquad ;$$

giving

$$H_t = b_2 C^{1.84}\ D^{2.10} + b_3\ C^{-2}\ D^{-5} + b_4 \qquad\qquad .$$

From Appendix B, Equation B-3,

$$S_m = C\ \left[(M + 1)\ S_{odc} - 1\right] + 1$$

let

$$b_5 = (M + 1) S_{odc} - 1$$

giving

$$S_m = C \, b_5 + 1$$

The objective function, X_t, can now be expressed in terms of these newly defined constants:

$$X_t = a_1 \frac{S_m H_t}{C} + a_2 D + a_3 \frac{S_m H_t}{C} + a_4 + a_5 + a_6 H_t + a_{6A} + a_7 \left(\frac{1}{C} - 1 \right)$$

rearranging and combining terms,

$$X_t = (a_1 + a_3) \frac{S_m H_t}{C} + a_2 D + a_6 H_t + \frac{a_7}{C} + a_4 + a_5 + a_{6A} - a_7$$

Substituting for H_t and S_m from Equations B-11 and B-3 gives

$$X_t = (a_1 + a_3)(b_5 + \frac{1}{C})(b_2 \, C^{1.84} \, D^{2.10} + b_3 \, C^{-2} \, D^{-5} + b_4) + a_2 D +$$

$$a_6(b_2 \, C^{1.84} \, D^{2.10} + b_3 \, C^{-2} \, D^{-5} + b_4) + \frac{a_7}{C} + a_4 + a_5 +$$

$$a_{6A} - a_7$$

Let

$$d_1 = a_1 + a_3$$

and multiply through:

$$X_t = d_1 b_5 b_2 C^{1.84} D^{2.10} + d_1 b_5 b_3 C^{-2} D^{-5} + d_1 b_5 b_4 + a_2 D +$$

$$d_1 b_2 C^{0.84} D^{2.10} + d_1 b_3 C^{-3} D^{-5} + d_1 b_4 C^{-1} + a_6 b_2 C^{1.84} D^{2.10} +$$

$$a_6 b_3 C^{-2} D^{-5} + a_6 b_4 + \frac{a_7}{C} + a_4 + a_5 + a_{6A} - a_7 \qquad .$$

These terms can be combined in like powers of D, giving:

$$X_t = \left[(d_1 b_5 b_2 + a_6 b_2) C^{1.84} + d_1 b_b C^{0.84} \right] D^{2.10} + \left[(d_1 b_5 b_3 + \right.$$

$$\left. a_6 b_3) C^{-2} + d_1 b_3 C^{-3} \right] D^{-5} + a_2 D + \frac{d_1 b_4 + a_7}{C} + d_1 b_5 b_4 +$$

$$a_6 b_4 + a_4 + a_5 + a_{6A} - a_7 \qquad .$$

The objective function can be put in the polynomial form $X_t = \Psi(K_i C^{m_i} D^{n_i})$ by defining the following constants

$$K_1 = d_1 b_5 b_2 + a_6 b_2$$

$$K_2 = d_1 b_2$$

$$K_3 = d_1 b_5 b_3 + a_6 b_3$$

$$K_4 = d_1 b_3$$

$$K_5 = a_2$$

$$K_6 = d_1 b_4 + a_7$$

$$K_7 = d_1 b_5 b_4 + a_6 b_4 + a_4 + a_5 + a_{6A} - a_7$$

The objective function may now be written

$$X_t = (K_1 \, C^{1.84} + K_2 \, C^{0.84}) \, D^{2.10} + (K_3 \, C^{-2} + K_4 \, C^{-3}) \, D^{-5} + K_5 D +$$

$$\frac{K_6}{C} + K_7 \qquad\qquad\qquad \text{(C-1)}$$

The first and second derivative of X_t with respect to C are:

$$\frac{\partial X_t}{\partial C} = (1.84 \, K_1 \, C^{0.84} + 0.84 \, K_2 \, C^{-0.16}) \, D^{2.10} - (\, 2K_3 \, C^{-3} +$$

$$3K_4 \, C^{-4}) \, D^{-5} - K_6 \, C^{-2} \qquad\qquad \text{(C-2)}$$

$$\frac{\partial^2 X_t}{\partial C^2} = (1.54 \, K_1 \, C^{-0.16} + 0.134 \, K_2 \, C^{-1.16}) \, D^{2.10} + (6K_3 \, C^{-4} +$$

$$12K_4 \, C^{-5}) \, D^{-5} + 2K_6 \, C^{-3} \qquad\qquad \text{(C-3)}$$

The first and second derivatives of X_t with respect to D are:

$$\frac{\partial X_t}{\partial D} = 2.10 \, (K_1 \, C^{1.84} + K_2 \, C^{0.84}) \, D^{1.10} - 5(K_3 \, C^{-2} + K_4 \, C^{-3}) \, D^{-6} +$$

$$K_5 \qquad\qquad\qquad \text{(C-4)}$$

$$\frac{\partial^2 X_t}{\partial D^2} = 2.31(K_1 \, C^{1.84} + K_2 \, C^{0.84}) \, D^{0.10} + 30(K_3 \, C^{-2} +$$

$$K_4 \, C^{-3}) \, D^{-7} \qquad\qquad\qquad \text{(C-5)}$$

APPENDIX D

DEVELOPMENT OF THE CONTINUITY RELATIONSHIPS

The continuity equations for the mass flow rates of the woodchips and the woodchip-water mixture must be satisfied at the junctions in pipeline networks shown in Figures 4 and 5. The continuity relationships developed to express these conditions are used to determine the concentration of woodchips in the mixture flowing out of the junction in terms of the concentrations of the lines flowing into the junction.

Three-Line Network

The volumetric flow of woodchip-water mixture entering a junction as shown in Figure 4 must equal that leaving. This statement can be expressed mathematically by

$$Q_{m_3} = Q_{m_1} + Q_{m_2} \qquad \text{(D-1)}$$

where Q_{m_i} is the volumetric flow rate in cubic feet per second for line i. Similarly, the weight rate of flow of chips entering must equal that leaving:

$$W_3 = W_1 + W_2 \qquad \text{(D-2)}$$

where W_i is the weight rate of flow of chips in tons per day. Equation B-2, giving the throughput in terms of concentration, flow rate of the mixture, and the specific weight of oven-dry chips, can be solved for Q_m and substituted in Equation D-1 giving

$$\frac{W_1}{2695.7 \, C_1 \, S_{odc}} = \frac{W_2}{2695.7 \, C_2 \, S_{odc}} + \frac{W_3}{2695.7 \, C_3 \, S_{odc}} \qquad ;$$

multiplying by $2695.7\ S_{odc}$:

$$\frac{W_3}{C_3} = \frac{W_1}{C_1} + \frac{W_2}{C_2}$$

which can be solved for C_3 giving

$$C_3 = \frac{C_1\ C_2\ W_3}{C_1 W_2 + C_2 W_1} \qquad\qquad (D-3)$$

expressing the concentration of woodchips in line 3 as a function of

the concentration in the other two lines and the throughputs.

The first-and second partial derivatives of C_3 with respect to

C_1 are found to be:

$$\frac{\partial C_3}{\partial C_1} = \frac{C_2^2\ W_1\ L_3}{(C_1 W_2 + C_2 W_1)^2} \qquad\qquad (D-4)$$

$$\frac{\partial^2 C_3}{\partial C_1^2} = \frac{-2C_2^2\ W_1 W_2 W_3}{(C_1 W_2 + C_2 W_1)^3} \qquad\qquad (D-5)$$

The first and second partial derivatives of C_3 with respect to C_2 are

similar to Equation D-4 and D-5 with the subscript 2 and 1 interchanged.

The first derivatives are used in Equation 13 and 14 for the derivatives

of the objective function for the three-line network. The second deri-

vatives are used in the Newton-Raphson method for solving $\frac{\partial X_t}{\partial C_n} = 0$.

Five-Line Network

Two junctions exist in the typical five-line network shown in Figure 5. Concentration in line 4 can be expressed as a function of C_1 and C_2; likewise, C_5 can be expressed as a function of C_3 and C_4 where C_4 is a function of C_1 and C_2. The three-line continuity relationships are used in developing the five-line expressions with the subscripts changed to conform to the notation in Figure 5 which was chosen to simplify the Fortran II programming.

The equation for C_4 takes the same form as Equation D-3:

$$C_4 = \frac{C_1 C_2 W_4}{C_1 W_2 + C_2 W_1} \tag{D-6}$$

similarly, for C_5

$$C_5 = \frac{C_3 C_4 W_5}{C_3 W_4 + C_4 W_3} \tag{D-7}$$

By substituting expressions for C_4 and W_4 and manipulating the equation, Equation D-7 can be rewritten:

$$C_5 = \frac{C_1 C_2 C_3 W_5}{C_1 C_2 W_3 + C_2 C_3 W_1 + C_1 C_3 W_2} \tag{D-8}$$

The partial derivatives $\frac{\partial C_4}{\partial C_1}$, $\frac{\partial C_4}{\partial C_2}$, $\frac{\partial^2 C_4}{\partial C_1^2}$, and $\frac{\partial^2 C_4}{\partial C_2^2}$ take the same form as Equations D-4 and D-5 with subscripts 3 and 4 interchanged.

The partial derivatives of C_5 with respect to C_3 are expressed by:

$$\frac{\partial C_5}{\partial C_3} = \frac{C_4^2 \, W_3 \, W_5}{(C_3 W_4 + C_4 W_3)^2} \tag{D-9}$$

and

$$\frac{\partial^2 C_5}{\partial C_3^2} = \frac{-2C_4^2 \, W_3 \, W_4 \, W_5}{(C_3 W_4 + C_4 W_3)^2} \tag{D-10}$$

which are of the same form as Equation D-4 and D-5 with the subscripts changed to apply to Figure 5.

The first and second derivative of C_5 with respect to C_1 are found to be

$$\frac{\partial C_5}{\partial C_1} = \frac{C_2^2 \, C_3^2 \, W_1 \, W_5}{(C_1 C_w W_3 + C_2 C_3 W_1 + C_1 C_3 W_2)^2} \tag{D-11}$$

and

$$\frac{\partial^2 C_5}{\partial C_1^2} = \frac{-2C_2^2 \, C_3^2 \, W_1 \, W_5 \, (C_2 W_3 + C_3 W_2)}{(C_1 C_2 W_3 + C_2 C_3 W_1 + C_1 C_3 W_2)^3} \tag{D-12}$$

or simply

$$\frac{\partial C_5}{\partial C_1} = \frac{\partial C_5}{\partial C_4} \frac{\partial C_4}{\partial C_1} \tag{D-13}$$

and

$$\frac{\partial^2 C_5}{\partial C_1^2} = \frac{\partial^2 C_5}{\partial C_4^2} \left(\frac{\partial C_4}{\partial C_1}\right)^2 + \frac{\partial^2 C_4}{\partial C_1^2} \frac{\partial C_5}{\partial C_4} \qquad . \qquad (D\text{-}14)$$

The partial derivative expressions on the right side of Equations D-13 and D-14 are of the forms given by Equations D-4 and D-5 with appropriate changes in the subscript notation.

The first and second partial derivatives of C_5 with respect to C_2 are of the same form as Equations D-11, D-12, D-13, and D-14 with the subscripts 1 and 2 interchanged.

APPENDIX E

INTERMEDIATE RESULTS FOR THE SAMPLE PROBLEM

The computer program listed in Appendix F produces intermediate data during the optimization procedure which gives the concentration and cost for each combination of pipe diameters being tried as given by Equation 21.

The following listing gives the intermediate data for the sample given in Table II for eight combinations given by two pipe sizes and for 27 combinations given by three pipe sizes for each line. The two diameters which give the eight combinations are the next smaller and larger pipe sizes than D_t. The three pipe sizes which give 27 combinations are the next size smaller and the next two sizes larger than D_t. The optimum conditions for each case are found to be identical and are listed in Table II.

3-LINE NETWORK, 2 PIPE SIZES TRIED FOR EACH LINE
DESCRIPTION OF THE NETWORK BEING ANALYIZED

| LINE | THROUGHPUT | PIPELINE | ELEVATION | CMAX |
| | | LENGTH | DIFFERENCE | |
	TON/DAY	MILES	FEET	PERCENT
1	500.	100.0	0.	
2	1000.	50.0	0.	
3	1500.	50.0	0.	.3000

COSTS FOR EACH COMBINATION OF DIAMETERS FOR THE NETWORK

| LINE | OPTIMUM | OPTIMUM | COST FOR | TOTAL COST |
| | DIAMETER | CONCENTRATION | EACH LINE | FOR NETWORK |
	INCH	PERCENT	$/TON-MILE	$/TON-MILE
1	8.000	.24086	.06483	
2	10.000	.27855	.05091	
3	12.000	.26474	.04827	.05375
1	8.000	.27261	.06595	
2	10.000	.30000	.05141	
3	10.000	.29027	.04733	.05382
1	8.000	.23684	.06482	
2	8.000	.30000	.05185	
3	12.000	.27551	.04820	.05399
1	8.000	.27261	.06595	
2	8.000	.30000	.05185	
3	10.000	.29027	.04733	.05395
1	6.000	.30000	.06489	
2	10.000	.26717	.05086	
3	12.000	.27728	.04821	.05373
1	6.000	.30000	.06489	
2	10.000	.30000	.05141	
3	10.000	.30000	.04643	.05313
1	6.000	.30000	.06489	
2	8.000	.30000	.05185	
3	12.000	.30000	.04855	.05416
1	6.000	.30000	.06489	
2	8.000	.30000	.05185	
3	10.000	.30000	.04643	.05325

3-LINE NETWORK, 3 PIPE SIZES TRIED FOR EACH LINE
DESCRIPTION OF THE NETWORK BEING ANALYIZED

| LINE | THROUGHPUT | PIPELINE LENGTH | ELEVATION DIFFERENCE | CMAX |
	TON/DAY	MILES	FEET	PERCENT
1	500.	100.0	0.	
2	1000.	50.0	0.	
3	1500.	50.0	0.	.3000

COSTS FOR EACH COMBINATION OF DIAMETERS FOR THE NETWORK

LINE	OPTIMUM DIAMETER INCH	OPTIMUM CONCENTRATION PERCENT	COST FOR EACH LINE $/TON-MILE	TOTAL COST FOR NETWORK $/TON-MILE
1	8.000	.24086	.06483	
2	10.000	.27855	.05091	
3	12.000	.26474	.04827	.05375
1	8.000	.27261	.06595	
2	10.000	.30000	.05141	
3	10.000	.29027	.04733	.05382
1	8.000	.22706	.06494	
2	10.000	.24575	.05132	
3	14.000	.23918	.05357	.05617
1	8.000	.23684	.06482	
2	8.000	.30000	.05185	
3	12.000	.27551	.04820	.05399
1	8.000	.27261	.06595	
2	8.000	.30000	.05185	
3	10.000	.29027	.04733	.05395
1	8.000	.21796	.06529	
2	8.000	.30000	.05185	
3	14.000	.26655	.05516	.05711
1	8.000	.25103	.06501	
2	12.000	.23156	.05667	
3	12.000	.23771	.04923	.05587
1	8.000	.27544	.06613	
2	12.000	.29002	.06208	
3	10.000	.28499	.04791	.05716

1	8.000	.23783	.06482	
2	12.000	.19708	.05532	
3	14.000	.20902	.05288	.05699
1	6.000	.30000	.06489	
2	10.000	.26717	.05086	
3	12.000	.27728	.04821	.05373
1	6.000	.30000	.06489	
2	10.000	.30000	.05141	
3	10.000	.30000	.04643	.05313
1	6.000	.30000	.06489	
2	10.000	.23490	.05192	
3	14.000	.25321	.05430	.05664
1	6.000	.30000	.06489	
2	8.000	.30000	.05185	
3	12.000	.30000	.04855	.05416
1	6.000	.30000	.06489	
2	8.000	.30000	.05185	
3	10.000	.30000	.04643	.05325
1	6.000	.30000	.06489	
2	8.000	.30000	.05185	
3	14.000	.30000	.05784	.05814
1	6.000	.30000	.06489	
2	12.000	.22402	.05621	
3	12.000	.24468	.04885	.05554
1	6.000	.30000	.06489	
2	12.000	.28458	.06148	
3	10.000	.28954	.04741	.05642
1	6.000	.30000	.06489	
2	12.000	.19028	.05535	
3	14.000	.21669	.05291	.05703
1	10.000	.17385	.07197	
2	10.000	.30000	.05141	
3	12.000	.24157	.04901	.05625
1	10.000	.21218	.07581	
2	10.000	.30000	.05141	
3	10.000	.26363	.05098	.05820

1	10.000	.15877	.07162	
2	10.000	.26458	.05087	
3	14.000	.21649	.05291	.05767
1	10.000	.17383	.07197	
2	8.000	.30000	.05185	
3	12.000	.24156	.04901	.05638
1	10.000	.21218	.07581	
2	8.000	.30000	.05185	
3	10.000	.26363	.05098	.05832
1	10.000	.15436	.07171	
2	8.000	.30000	.05185	
3	14.000	.22822	.05315	.05808
1	10.000	.18269	.07256	
2	12.000	.24911	.05801	
3	12.000	.22218	.05053	.05896
1	10.000	.21218	.07581	
2	12.000	.30000	.06321	
3	10.000	.26363	.05098	.06157
1	10.000	.16717	.07170	
2	12.000	.21125	.05561	
3	14.000	.19418	.05321	.05918

APPENDIX F

COMPUTER PROGRAM FOR OPTIMIZATION

The following is the listing of the program used to determine the optimum conditions for the hypothetical application of Chapter V. The program is divided into two basic groups: the first is the general group which reads in data, computes constants, selects the two pipe sizes for each line to be tried, computes costs, and prints the results. The second group is inserted into the center of the first group and after the two diameters for each line are selected, computes the optimum concentration for each combination of pipe sizes by using iteration and the Newton-Raphson method for extracting roots. The second group, which is inserted into the general group, can be modified to describe the system being analyzed. It is easy to make modifications to the second group to describe any system of pipelines (multiline networks, series, and networks with certain lines of given diameter). The inserts used for the analysis of the single-line, three-line network and five-line network are also listed.

```
C   PROGRAM FOR OPTIMIZING PIPELINE NETWORKS
C   FORTRAN II, IBM 1620
C   BEGIN GENERAL GROUP
      DIMENSION DC3DC(9),D2C3DC(9) ,DXIDC(10), D2XIDC(10)
     1 ,D2C5DC(10),DC4DC(10), D2C4DC(10),P(10),DP(10),
     2Y(20),DOP(20),A2(9),        X(10),R9(10),ZT(10),CTINV( 10)
     3, Q1(10),Q2(10),Q3(10),Q4(10),Q5(10),Q6(10),A6(10),A7(10)
      DIMENSION  DIAM(20),         W(20),PL(20),C(9),CI(9),
     1COP(20),        DXDC(20),D2XDC(20),DXDD(20),DIA(10,10)
     2,BET2(9),ALPHA1(9),B1(9),B2(9),B3(9),B4(9)
     3,A4(9),A5(9),BET6(10),BET7(10),D(10),XT(10),DC5DC(20)
      DIMENSION HF(10),HT(10),R4(10),R5(10),U1(9),U2(10),
     1V(10),HTOP(10),HFOP(10),HPPM(10),CPI(10),CPST(10),CISS(
     210),ANOP(10),X1(10),X2(10),X3(10),X4(10),X5(10),X6(10),
     3X1OP(10),X2OP(10),X3OP(10),X4OP(10),CII(10),XTOPT(10),
     4X5OP(10),X6OP(10),X7OP(10),QW(10),X7(10)
  401 READ 2,LINES,NDIA,CDEV,CMAX
      READ 2,(DIAM(I),I=1,NDIA)
      READ 2,CRF,E,F,HSA,CM,SODC,STPU
      READ 2,R1,R2,R3,   R6,R7,R8
      READ 2,(U1(I),I=1,LINES)
      READ 2,(U2(I),I=1,LINES)
    2 FORMAT (9F8.0)
      READ 2,(R9(I),I=1,LINES)
      READ 2,(W(I),I=1,LINES)
      READ 2,(PL(I),I=1,LINES)
      READ 2,(ZT(I),I=1,LINES)
C   COMPUT CONSTANTS FOR OBJECTIVE FUNCTION
      AK1= .7535E-03*R1/(SODC*E)
      AK2=R2   *CRF/365.
      A1  =.000753*R1/(E*SODC)
      A3  =1.15E-07*R3*CRF/(E*SODC)
      B5   =(CM+1.)*SODC-1.
      D1= A1+A3
      DO 25 I=1,LINES
      A6(I)   =R7/(365.*HSA*W(I))
      A7(I)    =.00024*R9(I)/(PL(I)*SODC)
      A2(I)   = R2*CRF/(365.*W(I))
      B1(I)    =W(I)*W(I)/(216.*32.2*SODC*SODC)
      B3(I)= 31680.*F*B1(I)
      B2(I)=31680.*.527/B1(I)**.42*F
      Q1(I)=  (D1*B5+A6(I))*B2(I)
      Q2(I)= D1*B2(I)
      Q3(I)=  (D1*B5+A6(I))*B3(I)
      Q4(I)= D1*B3(I)
      Q5(I)= A2(I)
      Q6(I)= D1*ZT(I)/PL(I)+A7(I)
C   SELECT BEST TWO SIZES FOR EACH LINE
      DO 20 J=1,NDIA
```

```
      DXDD(I)=2.1*(Q1(I)*CMAX**1.84+Q2(I)*CMAX**.84)*DIAM(J)**
     11.1-5.*(Q3(I)/CMAX**2+Q4(I)/CMAX**3)/DIAM(J)**6+Q5(I)
      IF(DXDD(I))21,24,23
   21 DIA(I,1)=DIAM(J)
      DIA(I,3)=DIAM(J)
   20 DIA(I,2)=DIAM(J)
      GO TO 25
   23 IF(J-1)22,24,22
   22 DIA(I,1)=DIAM(J)
      DIA(I,2)=DIAM(J-1)
      DIA(I,3)=DIAM(J+1)
      GO TO 25
   24 DIA(I,1)=DIAM(J)
      DIA(I,3)=DIAM(J)
      DIA(I,2)=DIAM(J)
   25 CONTINUE
      XTOP =100.
      PRINT 8
    8 FORMAT (41H1INPUT VALUES FOR THIS 5 LINE NETWORK ARE/5X,
     1    4HLINE,6X,1HW,9X,1HL,7X,4HCDEV,5X,4HCMAX)
      PUNCH 111
  111 FORMAT(12X,42HDESCRIPTION OF THE NETWORK BEING ANALYIZED/
     112X,4HLINE,2X,10HTHROUGHPUT,2X,8HPIPELINE,2X,9HELEVATION,
     24X,4HCMAX/31X,6HLENGTH,3X,10HDIFFERENCE/19X,7HTON/DAY,
     35X,5HMILES,7X,4HFEET,5X,7HPERCENT / )
      DO 4 I=1,LINES
      PUNCH 112,I,W(I),PL(I),ZT(I)
    4 PRINT 5,I,W(I),PL(I)
      PUNCH 116, CMAX
  116 FORMAT (1H*,51X,F6.4  /)
  112 FORMAT (14X,I1,F10.0,F11.1,F10.0)
    5 FORMAT (7X,I1,F10.0,F10.1)
      PRINT 6 ,CDEV,CMAX
    6 FORMAT (1H*,28X,2F9.3////)
      PRINT 7
    7 FORMAT (34H THE INVESTIGATED COMBINATIONS ARE/5X,4HLINE,
     15X,4HDIAM,7X,4HCONC,9X,4HXTOL)
      PUNCH 113
  113 FORMAT (12X,40HCOSTS FOR EACH COMBINATION OF DIAMETERS ,
     115HFOR THE NETWORK/12X,4HLINE,3X,7HOPTIMUM,5X,7HOPTIMUM,
     2 5X,8HCOST FOR,4X,10HTOTAL COST/ 18X,8HDIAMETER,2X,3HCON
     310HCENTRATION,2X,9HEACH LINE,3X,11HFOR NETWORK/20X,4HINCH
     4,7X,7HPERCENT,5X,10H$/TON-MILE,3X,10H$/TON-MILE / )


C   SINGLE -LINE  INSERT

      DO 61 J=1,2
      D(1)=DIA(1,J)
```

```
      I=1
   11 C(I)=CMAX-.05
   84 CI(I)=C(I)
      IF(C(I)-CMAX)85,32,32
   32 C(I)=CMAX
      GO TO 18
   85 M=1
      DXIDC(M)=(1.84*Q1(M)*C(M)**.84+.84*Q2(M)/C(M)**.16)*D(M)
     1**2.1-(2.*Q3(M)/C(M)**3+3.*Q4(M)/C(M)**4)/D(M)**5
     2 -Q6(M)/C(M)**2
    3 D2XIDC(M)=(1.54*Q1(M)/C(M)**.16+.134*Q2(M)/C(M)**1.16)*
     1D(M)**2.1+(6.*Q3(M)/C(M)**4+12.*Q4(M)/C(M)**5)/D(M)**5
     2 +2.*Q6(M)/C(M)**3
   33 P(I)=W(I)*PL(I)*DXIDC(I)
      DP(I)=W(I)*PL(I)*D2XIDC(I)
      DELC=-P(I)/DP(I)
      IF(C(I)+DELC)80,80,81
   80 C(I)=C(I)-C(I)/2.
      GO TO 84
   81 C(I)=C(I)+DELC
      IF(C(I)-CMAX) 37,36,36
   36 C(I)=CMAX
      GO TO 18
   37 IF(ABSF(C(I)-CI(I))-CDEV)18,18,84
   18 CONTINUE

C     END SINGLE-LINE INSERT


C     CONTINUE GENERAL GROUP,  COMPUTE COSTS
      DO 64 N=1,LINES
      CONS2=ZT(N)/PL(N)
      SM=C(N)*((CM+1.)*SODC-1.)+1.
      CONS3=W(N)*W(N)/(216.*SODC*SODC*C(N)*C(N)*D(N)**5*32.2)
      HF(N)=31680.*F*((.527*C(N)*CONS3**(-0.42))+CONS3)
      HT(N)=HF(N)+CONS2
      IF(W(N)-STPU)70,71,71
   70 R4(N)=U1(N)/W(N)
      R5(N)=U2(N)/W(N)
      GO TO 72
   71 R4(N)=U1(N)/STPU
      R5(N)=U2(N)/STPU
   72 AK4=.2739E-02*(R4(N)+R5(N))*CRF
      IF(HT(N))88,88,89
   88 HT(N)=0
   89 X1(N)=AK1*SM*HT(N)/C(N)
      X2(N)=AK2*D(N)/W(N)
      X3(N)=.1150E-06*R3*SM*CRF/(E*SODC*C(N))*HT(N)
      X4(N)=AK4/PL(N)
```

```
      X5(N)=R6/(365.*W(N)*PL(N))
      X6(N)=(R7*HT(N)/HSA+R8)/(365.*W(N))
      X7(N)= .240E-03*(1.-C(N))/C(N)*R9(N)/(SODC*PL(N))
   64 XT(N)=X1(N)+X2(N)+X3(N)+X4(N)+X5(N)+X6(N)+X7(N)
      SN=0
      SD=0
      DO 63 II=1,LINES
      SN=W(II)*PL(II)*XT(II)+SN
   63 SD=W(II)*PL(II)+SD
      XTOL=SN/SD
C  SELECT LOWEST COST
      IF(XTOL-XTOP)45,45,65
   45 XTOP=XTOL
      DO 46 N=1,LINES
      XTOPT(N)=XT(N)
      HTOP(N)=HT(N)
      HFOP(N)=HF(N)
      X1OP(N)=X1(N)
      X2OP(N)=X2(N)
      X3OP(N)=X3(N)
      X4OP(N)=X4(N)
      X5OP(N)=X5(N)
      X6OP(N)=X6(N)
      X7OP(N)=X7(N)
      DOP(N)=D(N)
   46 COP(N)=C(N)
   65 DO 66 N=1,LINES
      PUNCH 114,N,D(N),C(N),XT(N)
   66 PRINT  47,N,D(N),C(N)
  114 FORMAT (14X,I1,F10.3,F12.5,F14.5 )
      PUNCH 115 ,XTOL
  115 FORMAT (1H*, 51X,F12.5/   )
      PRINT  67,XTOL
   47 FORMAT (7X,I1,F11.3,F11.5)
   67 FORMAT (1H*,33X,F11.5///)
C  SS-3 TO LEAVE LOOP
      IF(SENSE SWITCH 3) 69,61
   61 CONTINUE
   69 SUM1=0
      SUM2=0
      SUM3=0
      SUM4=0
      SUM5=0
      DO 86 N=1,LINES
      V(N)=W(N)/(14.7*COP(N)*DOP(N)*DOP(N)*SODC)
      QW(N)=(1.-COP(N))*2.447*DOP(N)*DOP(N)*V(N)
      SM=COP(N)*((CM+1.)*SODC-1.)+1.
      HPPM(N)= 6.18E-04*DOP(N)*DOP(N)*V(N)*SM*HTOP(N)
      CPI(N) =R2*DOP(N)*PL(N)
```

```
      CPST(N) = R3*HPPM(N)*PL(N)
      CISS(N) = (R4(N)+R5(N))*W(N)
      CTINV(N)= CPI(N)+CPST(N)+CISS(N)
      ANOP(N) = 365.*(X1OP(N)+X5OP(N)+X6OP(N)+X7OP(N))*W(N)*PL
     1(N)
      SUM1=SUM1 + CPI(N)
      SUM2=SUM2 + CPST(N)
      SUM3=SUM3 + CISS(N)
      SUM4=SUM4 + CTINV(N)
   86 SUM5=SUM5 + ANOP(N)
      PRINT 101
      PUNCH 101
  101 FORMAT(12X,41HOPTIMUM CONDITIONS FOR THE SPECIFIED W,L,,
     122HZT AND OTHER CONSTANTS//12X,4HLINE,4X,1HL,5X,1HW,5X,
     22HZT,5X,1HD,6X,1HC,5X,1HV,5X,2HQW,5X,2HHF,4X,4HHPPM /18X
     3,4HMILE,3X,3HTPD,4X,2HFT,4X,3HIN.,3X,4HCONC,3X,3HFPS,3X,
     43HGPM,3X,5HFT/MI,2X,5HHP/MI / )
      DO 180 N=1,LINES
      PUNCH 105 ,N ,PL(N),W(N),ZT(N),DOP(N),COP(N),V(N),QW(N),
     1HFOP(N),HPPM(N)
  180 PRINT 105 ,N ,PL(N),W(N),ZT(N),DOP(N),COP(N),V(N),QW(N),
     1 HFOP(N),HPPM(N)
  105 FORMAT (14X,I1,F7.1,F7.0,F6.0,F6.1,F7.4,F6.2,F7.0,F7.1,
     1F7.1)
      PRINT 102
      PUNCH 102
  102 FORMAT(///12X,39HFIRST COSTS AND ANNUAL OPERATING EXPENS
     1,24HE FOR OPTIMUM CONDITIONS//12X,4HLINE,4X,4HPIPE,6X,
     24HPUMP,4X,7HINJ-SEP,4X,5HTOTAL,4X,6HANNUAL,4X,5HTOTAL )
      PRINT 103
      PUNCH 103
  103 FORMAT(19X,42HINSTLN   STATIONS   SYSTEM   FST COST   OPER
     1,12H EXP   COSTS /22X,1H$,9X,1H$,8X,1H$,8X,1H$,9X,1H$,6X
     2,8H$/TON-MI/ )
      DO 181  N=1,LINES
      PUNCH 106,N,CPI(N),CPST(N),CISS(N),CTINV(N),ANOP(N),
     1XTOPT(N)
  181 PRINT 106 ,N,CPI(N),CPST(N),CISS(N),CTINV(N),ANOP(N),
     1XTOPT(N)
  106 FORMAT (14X,I1,F11.0,F10.0,F9.0,F10.0,F10.0,F9.5)
      PUNCH 104
      PRINT 104
  104 FORMAT(///18X,39HTOTAL COST FOR THE PIPELINE TRANSPORTAT
     1,10HION SYSTEM //20X,4HPIPE,6X,4HPUMP,4X,7HINJ-SEP,4X,
     25HTOTAL,4X,6HANNUAL,4X,5HTOTAL )
      PRINT 103
      PUNCH 103
      PRINT 107,SUM1,SUM2,SUM3,SUM4,SUM5,XTOP
      PUNCH 107,SUM1,SUM2,SUM3,SUM4,SUM5,XTOP
```

```
107 FORMAT (16X,F10.0,F10.0,F9.0,F10.0,F10.0,F9.5)
    GO TO 401
400 CALL EXIT
    END




C   THIS BEGINS THE INSERT FOR  THE 3-LINE NETWORK

C   THE NEXT 3 CARDS ARE CHANGED TO TRY 3 D-S DEEP
    .   DO 61 J=1,2
        DO 61 K=1,2
        DO 61 L=1,2
        D(1)=DIA(1,J)
        D(2)=DIA(2,K)
        D(3)=DIA(3,L)
        DO 11 I=1,2
11  C(I)=CMAX-.05
91  DO 12 N=1,2
12  CII(N)=C(N)
34  DO 18 I=1,2
84  CI(I)=C(I)
        IF(C(I)-CMAX)85,32,32
32  C(I)=CMAX
        GO TO 18
85  C(3)=C(1)*C(2)*W(3)/(C(1)*W(2)+C(2)*W(1))
C   PARTIAL DERIVATIVES OF THE CONTINUITY EQUATIONS
        DC3DC(1)=C(2)**2*W(1)*W(3)/(C(1)*W(2)+C(2)*W(1))**2
        DC3DC(2)=C(1)**2*W(2)*W(3)/(C(2)*W(1)+C(1)*W(2))**2
        D2C3DC(1)= -2.*C(2)*C(2)*W(2)*W(1)*W(3)/(C(2)*W(1)+C(1)
    1*W(2))**3
        D2C3DC(2)= -2.*C(1)*C(1)*W(1)*W(2)*W(3)/(C(2)*W(1)+C(1)
    1*W(2))**3
        DO 3 M=1,3
        DXIDC(M)=(1.84*Q1(M)*C(M)**.84+.84*Q2(M)/C(M)**.16)*D(M)
    1**2.1-(2.*Q3(M)/C(M)**3+3.*Q4(M)/C(M)**4)/D(M)**5
    2 -Q6(M)/C(M)**2
 3  D2XIDC(M)=(1.54*Q1(M)/C(M)**.16+.134*Q2(M)/C(M)**1.16)*
    1D(M)**2.1+(6.*Q3(M)/C(M)**4+12.*Q4(M)/C(M)**5)/D(M)**5
    2 +2.*Q6(M)/C(M)**3
33  P(I)=W(I)*PL(I)*DXIDC(I)+W(3)*PL(3)*DXIDC(3)*DC3DC(I)
        DP(I)=W(I)*PL(I)*D2XIDC(I)+W(3)*PL(3)*(DXIDC(3)*D2C3DC
    1(I)+DC3DC(I)*DC3DC(I)*D2XIDC(3))
        DELC=-P(I)/DP(I)
        IF(C(I)+DELC)80,80,81
80  C(I)=C(I)-C(I)/2.
        GO TO 84
81  C(I)=C(I)+DELC
```

```
      IF(C(I)-CMAX) 37,36,36
   36 C(I)=CMAX
      GO TO 18
   37 IF(ABSF(C(I)-CI(I))-CDEV)18,18,84
   18 CONTINUE
      DO 19 I=1,2
      IF(ABSF(CII(I)-C(I))-CDEV) 19,19,91
   19 CONTINUE
   35 C(3)=C(1)*C(2)*W(3)/(C(1)*W(2)+C(2)*W(1))

C  END 3-LINE NETWORK INSERT



C  THE FOLLOWING IS AN INSERT FOR THE 5-LINE NETWORK

      DO 61 J=1,2
      DO 61 K=1,2
      DO 61 L=1,2
      DO 61 KK=1,2
      DO 61 LL=1,2
      D(1)=DIA(1,J)
      D(2)=DIA(2,K)
      D(3)=DIA(3,L)
      D(4)=DIA(4,KK)
      D(5)=DIA(5,LL)
      DO 11 I=1,3
   11 C(I)=CMAX-.05
   91 DO 12 N=1,3
   12 CII(N)=C(N)
   34 DO 18 I=1,3
   84 CI(I)=C(I)
      JJ=21
      IF(C(I)-CMAX)85,32,32
   32 C(I)=CMAX
      GO TO 18
   85 C(4)=C(1)*C(2)*W(4)/(C(1)*W(2)+C(2)*W(1))
      C(5)=C(1)*C(2)*C(3)*W(5)/(C(1)*C(2)*W(3)+C(2)*C(3)*W(1)
     1+C(1)*C(3)*W(2))
C  PARTIAL DERIVATIVES OF THE CONTINUITY EQUATIONS
      DC4DC(1)=C(2)**2*W(1)*W(4)/(C(1)*W(2)+C(2)*W(1))**2
      DC4DC(2)=C(1)**2*W(2)*W(4)/(C(2)*W(1)+C(1)*W(2))**2
      DC4DC(3)=0
      D2C4DC(1)= -2.*C(2)*C(2)*W(2)*W(1)*W(4)/(C(2)*W(1)+C(1)
     1*W(2))**3
      D2C4DC(2)= -2.*C(1)*C(1)*W(1)*W(2)*W(4)/(C(2)*W(1)+C(1)
     1*W(2))**3
      D2C4DC(3)=0
```

```
      DC5DC(4)= C(3)*C(3)*W(4)*W(5)/(C(3)*W(4)+C(4)*W(3))**2
      DC5DC(1)= DC5DC(4)*DC4DC(1)
      DC5DC(2)= DC5DC(4)*DC4DC(2)
      DC5DC(3)= C(4)*C(4)*W(3)*W(5)/(C(3)*W(4)+C(4)*W(3))**2
      D2C5DC(3)=-2.*C(4)*C(4)*W(3)*W(4)*W(5)/(C(3)*W(4)+C(4)
     1*W(3))**3
      D2C5DC(4)=-2.*C(3)*C(3)*W(3)*W(4)*W(5)/(C(3)*W(4)+C(4)
     1*W(3))**3
      D2C5DC(1)=D2C5DC(4)*DC4DC(1)*DC4DC(1)+D2C4DC(1)*DC5DC(4)
      D2C5DC(2)=D2C5DC(4)*DC4DC(2)*DC4DC(2)+D2C4DC(2)*DC5DC(4)
      DO 3 M=1,5
      DXIDC(M)=(1.84*Q1(M)*C(M)**.84+.84*Q2(M)/C(M)**.16)*D(M)
     1**2.1-(2.*Q3(M)/C(M)**3+3.*Q4(M)/C(M)**4)/D(M)**5
     2 -Q6(M)/C(M)**2
    3 D2XIDC(M)=(1.54*Q1(M)/C(M)**.16+.134*Q2(M)/C(M)**1.16)*
     1D(M)**2.1+(6.*Q3(M)/C(M)**4+12.*Q4(M)/C(M)**5)/D(M)**5
     2 +2.*Q6(M)/C(M)**3
C   IF YOU-VE READ THIS FAR, HAVE A BEER
   33 P(I)=W(I)*PL(I)*DXIDC(I)+W(4)*PL(4)*DXIDC(4)*DC4DC(I)
     1+W(5)*PL(5)*DXIDC(5)*DC5DC(I)
      DP(I)=W(I)*PL(I)*D2XIDC(I)+W(4)*PL(4)*(DXIDC(4)*D2C4DC
     1(I)+DC4DC(I)*DC4DC(I)*D2XIDC(4))+W(5)*PL(5)*(DXIDC(5)*
     2D2C5DC(I)+D2XIDC(5)*DC5DC(I)*DC5DC(I))
      DELC=-P(I)/DP(I)
      IF(C(I)+DELC)80,80,81
   80 C(I)=C(I)-C(I)/2.
      GO TO 84
   81 C(I)=C(I)+DELC
      IF(C(I)-CMAX) 37,36,36
   36 C(I)=CMAX
      GO TO 18
   37 IF(ABSF(C(I)-CI(I))-CDEV)18,18,84
   18 CONTINUE
      DO 19 I=1,3
      IF(ABSF(CII(I)-C(I))-CDEV)19,19,91
   19 CONTINUE
   35 C(4)=C(1)*C(2)*W(4)/(C(1)*W(2)+C(2)*W(1))
      C(5)=C(1)*C(2)*C(3)*W(5)/(C(1)*C(2)*W(3)+C(2)*C(3)*W(1)
     1+C(1)*C(3)*W(2))

C   END OF INSERT
```

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