TESTS OF DISTRIBUTIONAL ASSUMPTIONS AND THE INFORMATIONAL CONTENT OF AGRICULTURAL FUTURES OPTIONS

by

Lee Ziegler

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APPROVAL

of a thesis submitted by

Lee Ziegler

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

David Buschena ______________________ ________
(Signature) Date

Approved for the Department of Agricultural Economics and Economics

Myles Watts ______________________ ________
(Signature) Date

Approved for the College of Graduate Studies

Robert Brown ______________________ ________
(Signature) Date

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Date ________________________________

VITA

Lee Zachary Ziegler was born in Billings, Montana on February 25, 1973 and is the son of Leighton and Karen Ziegler. He grew up in Sidney, Montana, where he attended school, receiving his high school diploma in May, 1991. He graduated from Rocky Mountain College in May, 1995, where he earned degrees in mathematics and economics. He began graduate study in applied economics at Montana State University in August, 1995, and received his master of science degree in May of 1997.

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ABSTRACT

It has been proposed that agricultural futures options contain information which may be used by those involved in agriculture, such as rate setting for crop (revenue) insurance. Specifically, it is proposed that these options may be used to predict the variance and perhaps higher moments of the distribution of the respective futures prices. This thesis first tests distributional assumptions maintained by the Black-Scholes analysis. It is found that many of these assumptions, such as the commonly used lognormality, are empirically rejected. Furthermore, it is found that futures price change standard deviations and futures options implied volatilities display seasonal patterns. Second, this thesis tests whether corn, soybean, and spring wheat futures options implied volatilities obtained from the Black formula are accurate predictors of futures price variance. Empirically, these implied volatilities are found to be very poor predictors of subsequent futures price variance. Furthermore, there is no empirical support to show that the agricultural futures options market has become more efficient since it first started trading in the mid-1980's.

INTRODUCTION

Futures options are now traded on a variety of agricultural commodities. Three of the largest crops traded on the Chicago Board of Trade (CBOT) futures options market are corn, soybeans, and spring wheat. Corn and soybean futures options have actively traded since 1985, while spring wheat futures options have traded since 1987.

Gardner has proposed that agricultural futures options may contain useful information which may be utilized by those involved in agriculture. This may include not only individual producers but also those involved in agricultural public policy and farm programs. Fackler and King as well as Sherrick, Garcia, and Tirupattur have also proposed that the options market may provide useful information regarding the underlying asset price distribution. The informational content of futures options prices is currently an important area of research which should be of interest to those in agriculture.

One specific application involves rate setting for crop (revenue) insurance. When considering revenue insurance, estimating second and higher moments of the distribution of futures prices is important in determining actuarially fair premiums. It is hypothesized that volatilities implied by agricultural futures options may be used to estimate this variance.

There are two general sections included in this thesis. The first section provides a review of the literature and a layout of the empirical model pertinent to this thesis. Included is an overview of futures and options markets, a discussion of the potential informational content of futures options prices, and the empirical framework which focuses on the Black-Scholes option pricing models and futures price distributional assumptions.

It is hypothesized that agricultural futures options implied volatilities may be used to estimate the variance of futures prices. There have been a number of previous studies which have tested the potential informational content of stock option implied volatilities. Beckers, Chiras and Manaster, and Canina and Figlewski have looked at stock options. The general conclusion has been that stock option implied volatilities are not good predictors of stock price variance.

Implied volatility may be numerically calculated from the Black formula for commodity futures options, which is an extension of the original and well known Black-Scholes option pricing model for non-dividend stock. The Black-Scholes model maintains several important assumptions regarding the distribution of futures prices. First, it is assumed that the futures price follows a lognormal diffusion process with a constant implied volatility parameter. It is also assumed that the current futures price is an unbiased estimate for the mean of the distribution of futures prices at some later time. These distributional assumptions are discussed in detail in the first section. The section concludes with a thorough description of the method used to test the informational content of agricultural futures options implied volatilities as predictors of futures price variance and discusses the data used, which is CBOT futures options data for corn, soybeans, and spring wheat beginning with their respective start of trade (1985 for corn and soybeans, 1987 for spring wheat) through the end of 1996.

The second section discusses empirical results. There are two important sets of empirical results discussed in this thesis. The first set of results involve tests of distributional assumptions maintained by the Black-Scholes analysis (1973) and the Black model (1976). This includes an analysis of futures price standard deviation, agricultural futures options implied volatility, and implied standard deviation within the context of the Black-Scholes model. Also included are tests of futures price lognormality and tests of beginning period futures prices as unbiased estimates for ending period futures prices. Overall, we find that assumptions maintained by the Black formula are not empirically supported. Although the futures price unbiased estimate test is supported empirically, we find that historical futures price standard deviation estimates display seasonal patterns and that futures price lognormality within a yearly time frame is widely rejected. We also find seasonality in implied volatilities, but then subsequently find implied standard deviation to be fairly well behaved, i.e. decreasing over the course of the year as predicted by the Black-Scholes model. These empirical tests are joint tests of the Black-Scholes model and the efficiency of the futures options market. Although seasonality exists in implied volatility, the market appears to be working fairly efficiently when considering implied standard deviation.

A second set of empirical results involves tests of whether corn, soybean, and spring wheat futures options implied volatilities may be used to accurately estimate the variance of futures prices. We find implied volatilities to be poor predictors of futures price variance. This section also estimates a heteroskedastic model as a test of whether corn, soybean, and spring wheat futures options have become more efficient over time. We find no empirical support for these markets becoming more efficient..

The second section summarizes empirical results and discusses implications for future research. The empirical results found in this thesis are important when discussing future research because they provide a direction which future research should take with respect to the potential for recovering probabilistic information from agricultural futures options.

Several important points are made with respect to future research. First, this thesis provides evidence that the Black-Scholes distributional assumptions may be inaccurate and suggests the need to use more flexible distributions as an alternative to the commonly used lognormal. Second, option pricing models need to directly incorporate seasonality in futures price change standard deviation. Such models may then be compared with other, more general models which do not incorporate seasonality.

LITERATURE REVIEW AND MODEL LAYOUT

An Overview of Futures and Options Markets

A *derivative security* may be defined as a security whose value depends on one or more underlying variables. Derivative securities have become important financial instruments in recent years, and many different types of derivative securities are now actively traded on the market. Examples of derivative securities include forward contracts, futures contracts, and option contracts.[1]

A derivative security is also called a *contingent claim*, since the value or price of the security is contingent on the values of one or more underlying variables. Often these variables are prices of traded securities, such as with stock options. Derivative securities can essentially be contingent on a host of underlying variables.

The most basic type of derivative security is called a *forward contract*. A forward contract is simply an agreement between two parties to transact a specified asset at a specific time in the future at a pre-determined price. A forward contract is usually between two financial or other private institutions and not traded on an exchange.

There are two positions in a forward contract between two parties. One party assumes a *short position* and agrees to sell the asset at the specified time at the specified

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[1]For a good introduction and overview of derivative securities, see Hull, John. Options, Futures, and Other Derivative Securities.

price. The other party assumes a *long position* and agrees to buy the asset at the specified time at the specified price. The predetermined price at which the asset is transacted is called the *delivery price*, and the time of transaction is generally referred to as the *maturity*.

In general when first entering into a forward contract, the delivery price is chosen such that it has zero value to both parties at that particular time. Initially, there is no explicit cost to take a long or short position in a forward contract. The forward contract is settled at maturity and can have positive or negative realized value to either of the parties. Although the contract initially has zero value to both sides, its value is contingent upon the market price of the asset under contract and thus may change over time as the asset's market price changes.

To better understand forward contracts, it is necessary to understand the potential payoffs or losses from entering into the contract. Let the cash price (or spot price) at maturity be C_T and let K be the delivery price. The payoff to the holder of a long position in the forward contract is $C_T - K$ and the payoff to the holder of a short position is $K - C_T$. This is because the holder of a long position has an obligation to buy the underlying asset at the delivery price K and thus gains if $C_T > K$. The holder of the short position similarly gains if $K > C_T$.

A *futures* contract is a specific type of forward contract. Like a forward contract, a futures contract is an agreement between two parties to transact a specified asset at a specific time in the future for a predetermined price. Unlike forward contracts, however, futures contracts are traded on an exchange. The exchange is important because it specifies certain standardized features of the contract such as delivery time (which is usually some period of time within a particular month), the amount and quality of the asset for one contract, and the method in which the futures price is quoted. It also guarantees that the contract will be honored, since the two parties involved may not necessarily know each other. For a commodity futures contract, the exchange also specifies the product quality and place of delivery. Furthermore, the exchange assures that there is a convenient and consistent method of quoting prices, and it also assures that a particular day's trading quickly becomes public information. Two common exchanges are the Chicago Board of Trade (CBOT) and the Chicago Mercantile of Exchange (CME). Together, all of the exchanges involve a wide range of assets which underlie futures contracts. These include, among others, pork bellies, cattle, sugar, wool, lumber, copper, aluminum, corn, soybeans, and a variety of financial assets such as stock indices, currencies, Treasury bills, and various types of bonds.

As with forward contracts, there are two sides to a futures contract. One side has agreed to buy the asset at maturity and has thus taken a long position. The other side has taken a short position and has agreed to sell the underlying asset.

Volume and *open interest* are two terms commonly used to characterize the "amount" of futures (or other security) trading that has occurred on a particular day. Volume represents the total number of contracts that have been traded, while the open interest represents the number of outstanding contracts. An *open contract* is defined as a contract which has neither been offset or delivered; it is a contract that remains to be acted upon. To clarify this terminology, suppose that A sells to B.[2] A has taken a short position and B a long position.

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[2]This terminology is commonly used as a simple way of saying that A has taken a short position in a futures contract with B.

There is one open contract, and thus the open interest is one. If C sells to B, then A is still short one contract, B is now long two contracts, and C is short one contract. There are two outstanding contracts and so the open interest is two. If D then sells to A, A is even, C is still short one, B is still long two, and D is now short one. The open interest is still two. Although the open interest is two, there has been a much higher volume of trading than two. As seen in this example, it generally takes a large volume to change the open interest by a slight amount.

Options are another important type of derivative security. Options are contingent securities that give the holder the privilege of entering into a contract if desired. Options are now traded throughout the world on a wide range of assets including stocks, stock indices, foreign currencies, debt instruments, commodities, and various futures contracts. This thesis will be concerned with *futures options*, which are options on futures contracts.

There are two basic types of futures options. A *call* option gives the holder the right to purchase a futures contract by a specified date at a predetermined price. A *put* option gives the holder the right to sell a futures contract by a specified date at a predetermined price. The predetermined price at which the holder may opt to transact is called the *exercise price* or the *strike price* and is typically denoted X. The specified time by which the holder may opt to transact is called the *expiration date*, the *exercise date*, or simply *maturity*, and is denoted T. The futures price, which is the price of the asset underlying a futures contract, is denoted F. The futures price may change over the life of the contract, and so it is often useful to consider the relationship of the futures price at maturity, denoted F_T, with the futures price at some time prior to maturity, denoted F_t.

When the holder of a futures call option exercises, he or she assumes a long position in a futures contract plus a cash amount equal to the excess of the futures price (F) over the strike price (X). When the holder of a futures put option exercises, he or she assumes a short futures position and receives cash equal to the excess of the strike price (X) over the futures price (F).

Futures options are further classified based on their exercise possibilities. An option is said to be *European* if it can only be exercised at maturity. If the holder may exercise the option at any time prior to maturity then the option is said to be *American*.

It is important to understand that a futures option contract is different from a futures contract. In a sole futures contract, the two sides (long and short) have entered a binding agreement and, assuming that the contracts haven't been offset beforehand, a transaction must take place at maturity. A futures option contract, on the other hand, gives the holder the choice of whether or not to transact. Thus, an option gives the holder more flexibility than does a futures contract. As a result, entering into a futures contract costs nothing outside of transactions costs (brokers' fees, etc.), whereas an investor must pay for the "privileges" provided by an option.

There are two sides to every futures option contract, and thus a total of four possible options positions when considering puts and calls. The investor who has sold the option has taken a short position, while the investor who has purchased the option has taken a long position. The four basic option positions are thus:

1. Long call position (purchase a call option)
2. Short call position (sell a call option)
3. Long put position (purchase a put option)
4. Short put position (sell a put option)

To better understand options it is important to understand the potential payoffs associated with these four options positions.[3] First, consider a simple futures option example. Suppose an investor buys a futures call option. The price of the option is $5, the current futures price (*F*) is $25, and the option strike price (*X*) is $20. Suppose the futures price rises at maturity to $30. The investor will choose to exercise and realize a payoff of $30 - $20 - $5 = $5, assuming no transactions costs. Note that this payoff includes the initial price of the option, $5. Likewise, if the futures price at maturity is less than $20, then the holder will clearly choose not to exercise. Note that losses may potentially occur even if the option is exercised. Suppose for instance the maturity futures price is $22. If the holder did not exercise, he or she would incur a loss of $5, the initial price of the option. If the holder chose to exercise, however, then he or she would only lose $3. Thus, option exercise may be optimal in order to minimize losses.

In general, the relationship of the underlying asset price and the strike price determines the potential payoffs from exercise. The payoff from a long position in a futures call option is:

$$MAX(F_T - X, 0)$$

[3]This basic option payoff analysis is similar to options on other securities (i.e. stocks) as well.

The investor in a long call option wants to purchase the underlying asset for as little as possible and will thus exercise if $F_T > X$. If $F_T < X$, then he will not exercise. Note that the cost of the option is not included in these payoffs. Likewise, the payoff from a long position in a futures put option is given by:

$$MAX(X - F_T, 0)$$

This is because the holder of a long futures put position wants to sell the underlying asset for as much as possible, so that exercise occurs if $X > F_T$ and not if otherwise.

When considering the holders of short positions in these options, payoffs are the opposite to payoffs for long positions. Table 1 summarizes payoffs for two futures-strike price situations, one in which $F_T > X$ and another in which $F_T < X$. Again, the price of the option is not included in these payoffs.

Table 1. Payoffs from Four Basic Futures Options Positions

Position	General Payoff	Payoff 1 F_T=50, X=45	Payoff 2 F_T=35, X=45
Long Call	Max (F_T - X, 0)	5	0
Short Call	-Max (F_T - X, 0)	-5	0
Long Put	Max (X - F_T, 0)	0	10
Short Put	-Max (X - F_T, 0)	0	-10

One important way in which options are classified is based upon potential payoffs from exercise. An option is said to be *in-the-money* (ITM) if, assuming negligible transactions costs, it would generate positive returns to the holder. Using previously defined notation, a futures call option is in-the-money when $F_T > X$. For a futures put option the reverse is true; it is in-the-money if $X > F_T$. An option is said to be *out-of-the money* (OTM) if exercise would result in a loss to the holder. Thus, a futures call option is out-of-the money when $X < F_T$. Third, an option said to be *at-the-money* (ATM) if the underlying asset price is equal to the option strike price, so exercise would result in zero return to the holder; a futures option is thus at-the-money when $X = F$.

A futures option (or any option) will only be exercised if it is in-the-money. As will be discussed later, this method of option classification (ITM, ATM, OTM) will be important when proposing to use futures options data empirically.

An option may also be characterized by its *intrinsic value*, which is defined as the maximum of zero and the value it would have if exercised immediately. A futures call option's intrinsic value is thus $MAX\,(F_t - X,\ 0)$. Similarly, the intrinsic value for a futures put option is $MAX\,(X - F_t,\ 0)$. This is directly analogous to the payoffs from futures put and call options exercise discussed earlier.

Futures and options markets are valuable tools with which to hedge risk. For instance, a corn producer is concerned with harvest time prices throughout the growing year. He is faced with the probability of both increases and decreases in corn prices as the growing season progresses and thus faces risk with respect to the probability of harvest corn prices. As a way to eliminate some of this risk, the producer may take a short position in a corn futures contract to effectively "lock in" his harvest-time price. Alternatively, he may purchase a corn futures option which will provide even more flexibility, since the option is not a binding contract. Keep in mind, however, that flexibility comes at a price. Whereas it costs nothing to enter into a futures contract, the hedger must pay for the option.

Potential Informational Content of Agricultural Futures Options Markets

It has been proposed that the options market for agricultural commodities contains a wealth of potentially valuable information that could be utilized by those who work in agriculture. Gardner (1977) proposed that agricultural options may be useful for three general areas: risk management by individuals, the functioning of underlying commodity markets, and the management of public policy with respect to agricultural production.[4]

It is important to point out that the term "agricultural options" is used fairly generally here. An option may be written on an agricultural commodity itself (i.e., an option on physical corn), or it may be written on an agricultural commodity futures contract (i.e., an option on a corn futures contract). While Gardner's analysis is concerned with options on physical commodities, his reasoning may be readily extended to include futures options, which is the focus of this thesis.[5]

The first area of agricultural interest with respect to the informational content of futures options involves individual producers, such as the corn producer example briefly discussed previously. Risk reduction may be done with futures contracts, options contracts, or a combination thereof, such as with futures options. While the use of futures contracts

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[4]Gardner, Bruce. "Commodity Options for Agriculture." American Journal of Agricultural Economics. December, 1977.

[5]The reader should be aware of the fact that futures on agricultural commodities have traded for several decades, while futures commodities options are relatively new, having been traded since about 1985. Thus, agricultural futures options were not traded when Gardner published the article in 1977, and options on sole physical commodities were relatively new. Nevertheless, his analysis is equally applicable to futures options.

may be effective in hedging risk, options are a more flexible way of dealing with such risks. Gardner points out that a futures contract fixes the definite price in advance for a hedger. This is because a futures contract is a binding agreement. An option contract fixes a potential price over a range of outcomes, yet confronts the producer with prices over a different range. Futures options are thus a very flexible way for producers to deal with risk.

A second agricultural area in which futures options may be useful is public policy and farm programs. Gardner has proposed that agricultural options prices contain valuable information regarding the expected variability of the underlying commodity price. In the case of a futures option, this would mean that option prices contain information not only about expected futures price variability but also about the commodity spot price variability as well. One area of farm policy in which such information may be potentially valuable is crop insurance. For instance, when considering rate setting for revenue insurance, estimating futures price variance is important for statistically determining actuarially fair premiums. Since revenue is the quantity of a commodity times the price for which it is sold, estimating expected price changes is a critical part of revenue insurance analysis.

Heifner (1996) also proposes that commodity futures options may contain potentially useful information in evaluating futures price variability for purposes of crop insurance rate setting. The underlying argument hinges on the fact that futures contracts and futures options take current information into account to provide an "optimal" forecast of future price variability. The general notion is that these markets reflect the judgments of traders who stand to make arbitrage profits if they can forecast better.

Heifner's reasoning is thus consistent with Gardner's, and, like Gardner, Heifner hypothesizes that the options market may be used to estimate expected variability in a way analagous to the use of futures markets in estimating expected prices.

In summary, futures options may contain potentially useful information that may be of interest not only to individual producers, but also to those concerned with agricultural programs and policies such as crop insurance as well as to those wishing to acquire a better understanding of risks associated with underlying commodity markets. One of the primary purposes of this thesis is to test the hypothesized usefulness of futures options for corn, soybeans, and spring wheat in predicting futures price variability. Corn, soybeans, and spring wheat are three of the largest agricultural crops in the world. Furthermore, there is a large volume of futures options traded on these three crops. This indicates that there is a sufficiently large amount of market information and lends credibility to the hypothesis that agricultural futures options markets may potentially be used to accurately assess futures price variability. Specifically, we will look at whether implied volatilities from historical futures options prices are accurate predictors of futures price variance between early (beginning) periods and later (ending) periods within a particular production year. Such a test of futures price variance predictability is fairly general yet has important economic meaning and should thus be of interest to various areas of agriculture such as those discussed above.

Modeling Futures Price Movements

A substantial amount of work has been done to model the behavior of asset prices underlying option contracts. This analysis is specifically concerned with the behavior of futures prices, which is the underlying variable in a futures option contract. Understanding the behavior of futures prices not only helps to understand the nature of the actual futures market itself, but it provides a mechanism with which to model the futures options market and is necessary when considering option pricing theory.

Asset prices (security prices, futures prices, etc.) are often assumed to follow a continuous variable, continuous time stochastic model of the form:[6]

$$\frac{dS}{S} = \mu \; dt + \sigma \; dz \tag{1}$$

S is the asset price, μ is a growth (or drift) rate parameter, and $\sigma\, dz$ is a term which adds random "noise" or variability to the growth in S via a scaled Wiener process dz. Technically, security prices aren't continuous as such but rather observed in discrete values (such as in eights) and in discrete time intervals (when the exchange is open). In practice, such discrete differences are small, and using continuous variable processes is most useful when modeling price behavior.

The general process given by (1) may be used to model futures price movements. For a futures price, however, the growth rate parameter, μ, is zero. This should be expected, since it costs nothing to enter into a futures contract agreement. As will be discussed later, this is also consistent with the use of a "beginning" period futures price as an unbiased

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[6]This model of security price motion is known as *geometric Brownian motion*, which originated in physics to model atomic phenomena. Hull provides a good overview of the model as used in finance to model asset price movements.

estimate of an "ending" period futures price, so that $E(F_T) = F_t$, where T represents an ending period time and t represents some prior time.[7]

The Lognormal Distribution of Futures Prices

The continuous variable stochastic process used to model futures prices implies important distributional assumptions regarding changes in the futures price over finite time intervals. These distributional assumptions are critical in the Black-Scholes analysis, which will be discussed in detail later.

Black and Scholes (1973, 1976) assume that the futures price follows the general process given by equation (1). Using well known results in stochastic calculus, Black (1976) further assumes that the futures price is distributed lognormally, which means that changes in the natural logarithm of the futures price F are normally distributed.

To illustrate, suppose that the futures price today is F_t and we are interested in F_T, the futures price at some later time T. The natural logarithm of the change in F between t and T is normally distributed and is written:

$$ln(F_T) - ln(F_t) \sim N[\theta(T-t),\ \sigma^2(T-t)] \tag{2}$$

$N(n,m)$ denotes a normal distribution with mean n and variance m. $T-t$ is the time horizon, σ_I is a volatility parameter eventually known as the implied volatility, and θ is a growth (or drift) rate parameter.

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[7]T is typically used to denote the contract maturity, but for illustrative purposes may be used to represent any future time.

As discussed previously, when specifically considering futures prices, there is no expected growth in the futures price. The expected futures price at some later time is the current futures price, so that $E\,(F_T - F_t) = 0$, whence $\mu = 0$ in (1) and $\theta = 0$ in (2). Using rules of logarithms, equation (2) becomes:

$$ln(\frac{F_T}{F_t}) \sim N[0, \sigma^2(T-t)] \tag{3}$$

Equation (3) reflects the fact that there is no expected growth in the futures price. By considering properties of the normal distribution, (3) may be further rewritten:

$$ln(F_T) \sim \phi[ln\,F_t, \sigma^2(T-t)] \tag{4}$$

Both (3) and (4) are consistent with the notion that $E\,(F_T) = F_t$, and any change in the futures price over time is the result of random fluctuations, the degree of which is measure by variance (or standard deviation), which is proportional to the time interval under consideration.

It is important to point out that the lognormality of futures prices applies not only to day to day (infinitesimal) changes but also to longer periods of time. Whether we are interested in the change in the futures price in the time period of a day or in two months, the relationship concerning the change in $ln\,F_t$ still has the probability distribution as given in equation (4).

Framework for Empirical Model

There are two important parts of the empirical analysis in this thesis. First, tests of Black-Scholes option pricing assumptions are performed. This may be important for several reasons. First, the Black-Scholes pricing formulas (including the Black formula) were a major breakthrough in option pricing theory and are probably the most widely used models to analyze options prices empirically. Second, it is believed that actual option market participants actively use Black-Scholes models for potential informational content. Furthermore, tests of Black-Scholes assumptions are a tool to guide future research in option pricing theory.

The second part of the empirical analysis focuses on whether corn, soybean, and spring wheat futures options contain useful information in predicting futures price changes. Specifically, it is hypothesized that volatilities implied in futures options prices may be good predictors of changes in futures price between beginning and ending periods.

These empirical tests are of interest to those involved in agriculture for a variety of reasons. As previously discussed, one of the areas in which the informational content of futures options prices may be of potential use is price analysis for crop revenue insurance. The main problem in statistically determining actuarially fair premiums is finding estimates of the second and perhaps higher moments of the distribution of possible ending period prices during a given (beginning) period. An ending period futures price on a given day during the beginning period is considered an unbiased estimate for the mean of possible futures prices during the ending period, which is later shown to be empirically supported. It is a harder task to estimate the variance and higher moments of this distribution of possible prices. Testing the informational content of agricultural futures options implied volatility is thus of direct interest with respect to revenue insurance price analysis.

When considering the potential informational content of the futures options market, it is important to understand the basic factors affecting futures options prices. Black and Scholes (1973) point out that the higher the value of the underlying security (a futures price in this case), the greater the value of a call option for a given strike price.[8] As discussed previously, a futures call option will only be exercised if it is in-the-money, i.e. will generate positive returns to the holder. Thus, for a futures call, greater chances of option exercise are present the higher the futures price (F) above the strike price (X). In such cases, the value of the option should be approximately equal to the futures price (F) minus the price of a discount bond that matures at the same time as the option, where the bond has a face value equal to the option strike price (X). A futures call option will not be exercised if it is out-of-the money, that is if $F < X$. If the futures price (F) is less than the strike price (X) by a large enough amount, then it has a value close to 0.[9] The value of an option also depends on the time to maturity as well. Black and Scholes point out that if the maturity is of sufficiently long duration, then the price of a bond paying the strike price (face value) at maturity will be very low, so that the option value will be close to the futures price. This makes sense because if the price of a futures call option exceeded the futures price F, then an arbitrage

[8]See Myron Scholes and Fischer Black. "The Pricing of Options and Corporate Liabilities." Journal of Political Economy. 81. May/June, 1973.

[9]Black and Scholes (1973) originally presented this analysis with respect to options on non-dividend paying stock. These basic price relationships are applicable to futures options prices, which is of primary concern in this thesis.

profit could be made by taking a long position in the futures contract and selling the futures call option. If the expiration time is close, on the other hand, then the value of a futures call option will be $F - X$, if $F > X$, or 0 if $F < X$. This reflects the fact that the holder of the futures call option will exercise if the option is in-the-money ($F > X$) and not if $F < X$. It is important to keep in mind that an out-of-the money option does not necessarily have zero value, because depending on the time horizon and the amount by which the option is out-of-the money, there is always a chance that the option will become in-the-money by expiration and thus be exercised.

Discussion of these price relationships is important, because it implies upper and lower bounds which the futures option price must lie between. The upper value of a futures call option is F, since the option can never be worth more than the futures price. The minimum value of the option is 0 or $F - X$, whichever is larger. Similar bounds may be found for put prices.

Option Pricing Models

To test whether options prices may be used to predict futures price variance, option pricing models must be considered. A general form for option pricing originally proposed by Cox and Ross (1976) is discussed in Sherrick, et. al. (1996) [10]. The only assumption required by this general option pricing formula is that there are no arbitrage opportunities.

[10]Sherrick, Bruce, Philip Garcia, and Viswanath Tirupattur. "Recovering Probabilistic Information from Option Markets: Tests of Distributional Assumptions." Journal of Futures Markets. Volume 16. No. 5.

As Sherrick et. al. point out, "no arbitrage" may be defined as the condition that any two portfolios with identical distributions of future payoffs have identical current prices, so that there are no riskless arbitrage opportunities. This is a reasonable assumption, since any riskless profits resulting from market distortions should be quickly dissipated.

The general option pricing formula is:

$$V_p = b(T)\int_0^{\infty} Max(X_p - Y_T, 0)\ g(Y_T)\ dY_T$$

$$V_c = b(T)\int_0^{\infty} Max(Y_T - X_p, 0)\ g(Y_T)\ dY_T \qquad (5)$$

V_p and V_c are prices of put and call options with expiration T; X_p and X_c are the respective strike prices for puts and calls; Y_T is the unknown (random) price of the underlying asset at maturity time T; $b(T)$ is an appropriate discount factor; $g(Y_T)$ is the distribution of possible security prices, Y_T, at maturity. It is important to realize that this general option pricing form gives values for put and call options at time t, which is some time prior to expiration. This is why there is a discount factor $b(T)$, and it is also why the security price at maturity, Y_T, is considered a random variable.

When using corn, soybean, and spring wheat futures options price data empirically, a formula derived by Fischer Black (1976) may be used.[11] The Black formula gives values for European call and put commodity futures options and is a modification of the original

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[11]Black, Fischer. "The Pricing of Commodity Contracts." Journal of Financial Economics. 3 (March). 1976.

Black-Scholes pricing formulas for European put and call options on non-dividend paying stock.

Several key assumptions underlie the Black formula. As with the original Black-Scholes analysis, Black assumes that the futures price F follows the general stochastic process given in (1), so that fractional changes in the futures price over any finite interval are lognormally distributed with known, constant variance rate σ^2. Lognormality is probably the most common asset price distributional assumption used empirically because it has several desirable properties. First, lognormality is intuitively simple and mathematically easy to use. Second, the lognormal distribution is bounded below by zero, so negative prices are ruled out. Furthermore, equal percentage changes either way (for example doubling and halving) are equally likely under lognormality. This is equivalent to assuming that the futures price follows a random walk process.

Although there are several advantages to using the lognormal distribution as a model for futures (or other security) price movements over time, Campbell et. al. point out that the lognormal may not be entirely consistent with historical security price movements. They suggest that historical security price behavior often shows evidence of skewness and excess kurtosis, neither of which are accounted for in the lognormal distribution.[12]

The other Black-Scholes assumptions include that there are no transactions costs or taxes, no riskless arbitrage opportunities, that the risk free interest rate r is constant, and that

[12]The normal distribution has skewness = 0 and kurtosis = 3. Excess kurtosis is defined as the sample kurtosis minus 3.

trading is default free with perfectly divisible securities. The Black commodity futures options pricing formula is:

$$C_t = e^{-r(T-t)}[F_t N(d_1) - X N(d_2)]$$

$$P_t = e^{-r(T-t)}[X N(-d_2) - F_t N(-d_1)]$$

$$d_1 = \frac{ln(\frac{F_t}{X}) + (\frac{\sigma_I^2}{2})(T-t)}{\sigma_I\sqrt{T-t}} \qquad (6)$$

$$d_2 = d_1 - \sigma_I\sqrt{T-t)}$$

C_t and P_t are the prices of calls and puts at some time t prior to maturity T; F_t is the futures price at time t of a contract expiring at T; X is the strike price, r is the risk free interest rate, T - t is the time to maturity, σ_I is the implied volatility, and $N(\cdot)$ is the cumulative normal distribution function. Sherrick, et. al. point out that the Black formula in (6) is essentially a specific form of the general option pricing formula in (5) as proposed by Cox and Ross.[13] The security price Y_T is the random security price at maturity, which is the maturity futures price. Note that this random variable does not appear directly in the Black formula but is implicit in the distributional assumptions. The term $g(Y_T)$ represents the distribution of possible security prices at maturity. In the Black formula given by (6) this distribution is specifically assumed as lognormality. The term $e^{-r(T-t)}$ is the discount factor

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[13]The Black formula for commodity options futures is a modification of the original Black-Scholes pricing formulas for European call and put options on non-dividend paying stock. Both formulas are essentially specific forms of the general Cox-Ross equation.

denoted *b(T)* in the general formula. The Black formula is essentially an application of integral calculus to the general option pricing formula given in (5).

The Black-Scholes Parameters

To better understand the Black-Scholes analysis (1973) and the subsequent Black formula (1976), it is necessary to consider the variables which affect futures options prices, looking qualitatively at how changes in a single variable affect the option price, holding all others constant. Upper and lower bounds for futures options prices were discussed before in more general terms.

The first and probably most important variable affecting the price of a futures option is the futures price F.[14] For both European and American call options with a given strike price X, option value increases as the futures price increases. This is because the higher the futures price, *ceteris paribus*, the greater the chance that the option will be in-the-money and thus exercised; hence, the more valuable the option. For both European and American put options, the reverse is true, so that the value of the option declines as the futures price increases.

The second important variable is the option strike price X. Using similar arguments as before, the value of futures call options will decrease as the strike price increases, since the holder wants to purchase the underlying futures contract for as little as possible. For

[14]Similar arguments could be made for stock options as well. In our case we are primarily concerned with futures options.

puts, futures options value increases as the strike price increases, since the holder wants to sell for as much as possible.

Next consider the time to expiration. Here the distinction between European and American options is important. For American options, it is generally argued that option holders have more opportunities the longer the time to maturity. This is due to the fact that they have more exercise opportunities available to them, since American options may be exercised at any time prior to maturity. Thus, the longer the time to maturity the more valuable the option, *ceteris paribus*.

The holders of European options do not have the same exercise possibilities available to them as do their American counterparts. Thus, it is generally considered ambiguous as to the changes in the value of the option when the time to maturity increases.

The Black formula gives analytic formulas for the value of European futures options. In practice, however, agricultural futures options are American, so that the problem of early exercise needs to be considered. Black (1976) showed that a futures price is mathematically analogous to a security which pays a continuous dividend at a rate equal to the risk free interest rate r. Merton (1973) showed mathematically that for the case of a stock option, in which the stock pays discrete dividends, it is unlikely that early exercise will occur until possibly just prior to the final dividend payment. It follows that, for a stock which pays a continuous dividend yield, early exercise will never be optimal. Since a futures option in the Black model is analogous to a security which pays a continuous dividend yield at rate r, we conclude that early exercise of agricultural futures option is never optimal. From now on, futures option will be essentially treated as European, and the distinction between American and European options will not be important.

The next parameter to consider is the risk free interest rate r. An increase in the risk free interest rate will decrease the present value of any future cash flows. Also, the higher the interest rate the higher the opportunity cost of buying futures options. If the risk free interest rate was sufficiently high, the investor would simply invest at this rate, since he could never do better in the futures options market. In the case of a stock option, an increase in the interest rate will also have the effect of increasing the expected growth rate of the stock price. For a futures option, however, this is not the case. The expected growth rate in a futures price is zero, which is consistent with using the current futures price as an unbiased estimate of the futures price at some later time. As a result it can be argued that an increase in the interest rate will decrease the values of both put and call futures options.

The parameter of primary interest in this analysis is the implied volatility, σ_I. Under the stochastic process and distributional assumptions regarding the underlying futures price maintained by the Black formula, implied volatility is the constant parameter in (1) such that, when multiplied by the square root of the time horizon, $T- t$, gives the market's forecast of the standard deviation of the change in the natural logarithm of the futures price as seen in (3).

The implied volatility is thus an "informational" parameter because it presumably incorporates all relevant market information regarding future price variance as "embedded" in the price of the option. In accordance with this, Mayhew (1995) defines implied volatility as the market's assessment of the underlying asset's volatility (in our case the volatility of the futures price) as reflected in the option price. For both put and call futures options, the option price and implied volatility should be positively correlated, so that the higher the level of implied volatility the higher the price of the option.

As discussed earlier, agricultural futures options prices are hypothesized as having valuable information regarding expected futures price variability. When proposing to test this hypothesized informational content empirically, several important qualifications must be made concerning implied volatilities with respect to Black-Scholes assumptions.

Recall that the Black-Scholes analysis assumes that the implied volatility parameter is constant. Mayhew points out that even if the underlying asset's volatility is allowed to be stochastic, then implied volatility may be interpreted as the market's assessment of the average volatility over the remaining life of the option, and thus should still have potentially valuable information as to the variability of futures prices expected by the market.

Implied volatility should properly be considered an "index" or order statistic which gives the market's best forecast as to the predicted variance of the futures price. This interpretation of implied volatility is important because, as we will empirically show later, many of the distributional assumptions maintained by the Black formula are rejected. This may lead one to question the validity of implied volatility as an efficient assessment of future volatility. This problem may be reconciled if implied volatility is interpreted as an order statistic which is directly related to the market's assessment of future volatility. Thus, despite the fact that Black-Scholes assumptions may not be supported empirically, it is still a reasonable hypothesis that implied volatilities may be potentially useful in predicting subsequent realized futures price variance.

An important point emerges from this discussion. If the Black-Scholes world was perfect, then implied volatilities from options differing only by strike price should be constant, not only across strike prices for a particular trading day but also across the life of the option contract. Mayhew points out that often times empirically, however, implied volatilities are found to vary systematically across strike prices and across time, which Mayhew refers to as the "implied volatility smiles." As we will show later, there is a systematic seasonal pattern in corn, soybean, and spring wheat implied volatilities across months within a particular year.

Although such seasonal patterns may at first glance be interpreted as strong signs that the Black-Scholes analysis is incorrect, it is inappropriate to make this conclusion without further considering the market itself. Mayhew points out that the real problem with "volatility smiles" may be a combination of imperfections in both the market and in the Black-Scholes model. First, Mayhew points out that market imperfections may prevent prices from taking their true Black-Scholes values. Second, patterns in implied volatility across time which are inconsistent with Black-Scholes assumptions may in fact be the result of a true futures price process which differs from the assumed lognormal diffusion process. Mayhew also points out that such phenomenon may be the result of actual option market participants who strongly rely on Black-Scholes implied volatility quotes when making trading decisions. It is thus important to realize that tests involving implied volatility are tests of the market and the Black-Scholes model jointly.

Seasonality of Futures Price Volatility

It has been shown in previous literature that agricultural commodity futures price volatility, which may be generally defined as the variance of futures price changes per time or the variance per time of futures prices themselves, often displays seasonal patterns. It is typically low in the early part of the year, such as at or just prior to crop planting, rises and peaks during summer months, and eventually falls as contract maturity approaches. Kenyon, Kling, et. al. (1987) found seasonality in March corn, March soybeans, and July wheat. Anderson (1985) tested a null hypothesis of seasonality against an alternative of no seasonality and found strong evidence in support of seasonality for corn, wheat, and soybeans. Anderson concludes that seasonality is an important determinant in futures price volatility over time. Hennessy and Wahl (1996) summarize a number of theories proposed in previous literature with regard to the causes of such seasonal patterns.[15] Although hypotheses about seasonality in futures price volatility are different in many aspects, there seems to be a consensus that one of the biggest factors affecting futures price volatility is a basic pattern of information flows. In an early part of the year such as February or March, little is known about the expected crop. After the crop is planted and emerges (late April, May, or early June) information becomes increasingly available to producers as to expected crop yields at harvest. It is generally argued that such information flows lead to resolution of uncertainty, which in turn results in an increase in monthly futures price change standard

[15]Hennessy, David and Thomas Wahl. "The Effects of Futures Price Volatility." American Journal of Agricultural Economics. 78 (August 1996). 591-603.

deviation. Furthermore, there is in general a greater probability that factors occurring during summer months such as adverse weather may severely hinder growing conditions. To reduce this potential risk, producers may purchase commodity futures or futures options contracts, thereby increasing the demand for futures contracts; such information "shocks" will likely lead to an increase in the standard deviation of futures price changes in these localized "shock" times.

Hennessy and Wahl (1996) hypothesize that inflexibilities in production and demand may result in seasonal futures price volatility. In general terms, their hypothesis rests on the notion that a decision made on the supply side will make future supply responses more inelastic. Similarly, decisions made on the demand side will tend to make demand responses more inelastic.

As the growing season progresses and the actual planted crop begins to grow, production decisions become more costly, primarily because there is less flexibility in decision making due to the fact that there are fewer production options available. The supply curve thus becomes more inelastic as the season progresses. As harvest approaches, there is little if any flexibility in production. It is virtually impossible to produce more output, given the limited amount of time and the lack of feasible production choices. Supply is nearly fixed at this point.

Hennessy and Wahl point out that this "inflexibility" hypothesis is not necessarily inconsistent with other hypotheses such as the generally accepted "information flows" hypothesis. In fact, it is reasonable to expect information flows and production inflexibilities to be very closely related.

Seasonality in futures price volatility is important when considering the Black-Scholes analysis, especially when looking at implied volatilities. Since seasonality in futures price volatility is a fairly well established phenomenon, it will be interesting to test whether implied volatilities display such seasonal patterns. If the futures options market was efficient and the Black-Scholes assumptions were accurate, implied volatility should be constant over the course of a year.

A Method for Testing the Informational Content of Implied Volatility

It is hypothesized that futures options on agricultural commodities contain information that could be useful in predicting the variance of respective commodity futures prices. Although there have been a number of previous studies which have tested the potential informational content of stock options, there has been only limited effort dealing with agricultural futures options. Beckers (1981), Chiras and Manaster (1977) and Canina and Figlewski (1993) have investigated the informational content of stock option implied volatility. The general conclusion is that stock option implied volatilities have not been found to be good predictors of subsequent realized stock option price variability.

As discussed earlier, the ability of implied volatilities to forecast futures price variance is of interest to those in agriculture. For example, the problem with respect to crop insurance is estimating the variance and higher moments of the distribution of ending period futures prices during the beginning period. The price of a futures contract with ending period expiration is shown an unbiased estimate for the ending period futures price. As Heifner (1996) points out, the futures market is the best source of information available during the beginning period about expected cash (spot) prices during the ending period, since the market reflects the judgment of informed traders and arbitragers who will profit if they can forecast better. Thus, it is highly unlikely that there is a better prediction of prices than the market's.[16]

To understand why the beginning period futures price is an unbiased estimate for the future cash price, it is necessary to understand the relationship between the futures price and the cash price. Denote F_t the futures price at some time t during the growing season prior to contract maturity and $E(C_t)$ the expected maturity cash price at this prior time t. It is reasonable to expect that the futures price should, on average, be equal to the expected cash price, so $F_t = E(C_T)$. If this relationship did not hold, then arbitrage profits would be possible. For instance, if $F_t < E(C_T)$, then traders could hold long positions in futures contracts and anticipate selling at the expected cash price, thereby making positive profits. If $F_t > E(C_T)$, a trader should, over a sufficiently long period of time make positive profits by holding short positions in futures contracts. In equilibrium, we thus expect $F_t = E(C_T)$, so that on average the futures price is equal to the expected spot price at maturity.[17]

An important result of this discussion is that as the expiration of the contract draws near, the futures price should converge to the spot price. Thus $F_t \rightarrow C_T$ as $t \rightarrow T$, so that

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[16] Richard Heifner, "Price Analysis for Determining Revenue Insurance Indemnities and Premiums," Report to the Office of Risk Management, Economic Research Service, USDA.

[17]See Hull for a more thorough discussion of the relationship between the spot price and futures price of a security.

$F = C$ at T. If the futures price were above the cash price, the commodity could be purchased in the market at the cash price and sold at the futures price. If the cash price were above the futures price, similar profits would result. Arbitrage between the cash and futures market will assure that the futures price will be equal to the cash (spot) price at contract maturity.[18]

At time *T*, contract expiration, the distribution of possible cash prices is the same as the distribution of possible futures prices because of arbitrage arguments discussed above. One can thus use the futures price at some earlier time *t* as an expectation for both the futures price and the cash price at maturity *T*. At *T*, cash prices and futures prices may be used interchangeably, a result that might be of particular relevance for crop insurance rate setting considerations.

When using futures prices to forecast ending period prices, it is necessary to establish the appropriate beginning and ending periods for a given commodity, which may vary from crop to crop due to production and futures option trading considerations. For corn, soybeans, and spring wheat February is used as the beginning period. Ending periods for these crops are, respectively, December, November, and September. During a particular beginning period, futures prices for contracts expiring in these ending periods are used as unbiased estimates for the ending period futures price.

In this model, a monthly average of beginning period futures prices is used for the beginning price and is denoted *BP*. The futures price on the expiration of the contract is used

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[18]This discussion is in Hieronymus, The Economics of Futures Trading."

as an ending period price and is denoted *EP*. From these futures prices, a forecast error is calculated which is defined as the absolute value of the natural logarithm of the ratio of the two prices, denoted $|ln(ep/bp)|$. It is essentially this change which we would like to predict, since the larger the variance of the beginning period price distribution the larger this absolute price change.

The parameter of primary interest is σ_I, which is the well known implied volatility given in equation (6), the Black valuation formula for commodity futures options. Given the option price (P_C *or* P_p), the strike price (X), the risk free rate of interest (r), and the time to maturity (T-t), all of which are easily observed, the correct value of implied volatility under the assumption of lognormality is the one that equates the theoretical option price and the price observed in the market.

It is not analytically possible to solve equation (6) for the implied volatility in closed form as a function of the other parameters. Numerical techniques will be used to search for the correct value of volatility, which will be discussed in more detail later.

It is again important to distinguish between implied volatility and historical futures price volatility, both of which have potential uses for predicting future variability. Historical futures price volatility is an actual variance estimate from historical data. It is an ex-post measure only, looking back in time at the actual observable behavior of futures prices. Implied volatility, on the other hand, is a very different measure of variability. Implied volatility is the constant of proportionality such that when multiplied by the square root of the time horizon gives the market's best guess for the standard deviation of the futures price at maturity. As discussed previously, Mayhew defines implied volatility as the market's assessment of the underlying futures price volatility as reflected in the option price[19]. The higher the implied volatility, *ceteris paribus*, the higher the expected futures price variance. Implied volatility is an order statistic which gives the market's perceived level of future volatility, which is "embedded" in the price of the option. The higher the perceived level of variability, the higher the implied volatility and the higher the price of the option. This is because the higher the level of implied volatility the greater the market's assessment of future variability. As this variability assessment increases, the more appealing the options market becomes as a way to reduce future risk, and the more risk writers of options take on as a result. Prices of options thus increase.

It will be shown empirically that implied volatilities calculated from futures option premia have a seasonal pattern with peaks in mid-year as information about planted crops becomes available. Mayhew points out that even if the underlying asset's volatility is stochastic over time, implied volatility may be interpreted as the market's assessment of the average volatility over the remaining life of the option.

When proposing to use implied volatilities empirically, several issues must be addressed. On a given trading day there is a futures price F_t for a particular commodity and options with several strike prices $(X_1, X_2, ..., X_n)$ traded on the same futures contract. Furthermore, both put and call options are traded. The relationship of the futures price and strike price is important when considering option theory. As defined earlier, an option is said to be *in-the-money* if, assuming negligible transaction costs, it would generate positive

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[19]Stewart Mayhew. "Implied Volatility." Financial Analysts Journal. July-August 1995.

returns to the holder if exercised immediately. Thus, a futures call option is in-the-money if $(F - X_i) > 0$, i.e. the futures price exceeds the strike price. For a put option, the reverse is true, so that a put is in-the-money if $(X_i - F) > 0$. An option is said to be *out-of-the money* if there would be negative returns upon exercise. An option is *at-the-money* if the strike price exactly equals the futures price, so there would be zero return to the holder.

An important issue when considering implied volatility is which strike price to use. Under Black-Scholes assumptions, options on a particular futures contract differing only by strike price should have the same implied volatility and should thus be equally useful in predicting future price variability. Often times empirically, however, different implied volatilities are obtained from different strike prices on the same contract, a phenomenon Mayhew (1995) calls the "implied volatility smile." Such a phenomenon raises questions as to which strike price volatilities to estimate. Mayhew discusses various weighting schemes to use with implied volatilities obtained from different strike prices. Although several different weighting schemes have been proposed, the general consensus is that using options *near-the-money* seems to work best when testing the potential predictive power of implied volatilities. Furthermore, trading tends to be most active at options that are near-the-money. In general these options will likely have higher volumes of trade and thus contain more information than options that are further in or out of the money. In this paper, options near-the-money are used: in this paper, near-the-money options are defined as those that are at-the-money or closest in-the-money.[20]

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[20]The term *near-the-money* used in this thesis may not necessarily be defined as in previous literature. There is no standard definition for the term.

Another issue is whether to use puts or calls in calculating implied volatility. In theory, the Black formula predicts that a put option and a call option on the same contract should yield the same implied volatility. In this study, both puts and calls are used, and implied volatilities from each group are tested separately as potential predictors for futures price variance. By doing this we are essentially testing the efficiency of the put and call market in relation to one another. If puts and calls were not operating "in line" with respect to one another, then arbitrage opportunities might result. Thus, we expect that implied volatilities from puts should be highly correlated with implied volatilities from calls, and that using puts and calls should lead to similar results when considering the information contained in options prices regarding future price variability.

Fortune (1995) points out that if this were not the case, then one side (puts or calls) would be overpriced relative to the other. For instance, if implied volatilities from puts tended to be higher than those from calls, then puts might be "overpriced" relative to calls. If the reverse was true, then calls might be "overpriced" relative to puts. Fortune continues to suggest that if this was the case, then arbitragers could earn potential profits by purchasing underpriced options and selling overpriced ones. In short, arbitragers would force puts and calls back into equilibrium.[21]

When looking at whether implied volatilities are useful in predicting futures price variance, alternate beginning periods are used. These beginning periods are moved up from

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[21]Fortune's article discusses and tests the Black-Scholes model with European S&P 500 stock options. Although he doesn't discuss futures options or the Black formula, the same arbitrage arguments may be used for these types of contracts as well.

February until one month prior to the month of contract expiration. Thus, the final beginning periods for corn, soybeans, and spring wheat are, respectively, November, October and August.

There are several reasons for considering alternate beginning periods. As Heifner points out, little is known about growing conditions during early months such as February. As the season progresses, more is known about the crop. These information flows may result in different forecasts of price variance.

Furthermore, by looking at alternate beginning periods, implicit tests of the Black formula can be made. One important assumption of the Black formula[22] is that fractional changes in the futures price over any interval are lognormally distributed with constant variance rate σ^2. As seen in equation (3), the variance of $ln(F_T/F_t)$ is $\sigma^2(T-t)$, so that the standard deviation is $\sigma\sqrt{(T-t)}$. It is this volatility parameter, σ which is assumed constant over time; and σ is the parameter which is estimated as the implied volatility from equation (6). By looking at implied volatilities over successive beginning periods within a particular year, we will be able to test assumptions maintained by the Black-Scholes model and the Black formula.

The final day of the contract expiration is used as the ending period price. Futures contracts normally expire around the twentieth of the month. Options on futures expire just prior to the futures contract expiration.

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[22]See Fischer Black. "The Pricing of Commodity Contracts." Journal of Financial Economics. 3 (1976).

The previous discussion suggests looking at another measure, the *implied standard deviation*.[23] As previously discussed, the lognormal assumption of futures prices and the underlying model of futures price motion implies that $ln\ F_T$ is normally distributed with standard deviation $\sigma_I \sqrt{(T\text{-}t)}$, which is defined in this paper as the implied standard deviation. The implied standard deviation may be interpreted as the market's best estimate for the standard deviation of the distribution of possible futures prices for time T from the perspective of the current (earlier) time t.

Looking at the implied standard deviation is useful for several reasons. First, we will see if seasonality exists in this implicit measure of variability. Under the Black-Scholes assumption that the volatility parameter (σ_I) is constant, the implied standard deviation [$\sigma_I \sqrt{(T\text{-}t)}$] should be a monotonic decreasing function across a given year, since $T - t$ is a monotonic decreasing function as contract maturity approaches.

Seasonality in the implied standard deviation has important implications for the Black-Scholes analysis and market performance. First, seasonality in the implied standard deviation might contradict Black-Scholes assumptions and therefore raise questions as to the validity of the distributional and motion assumptions of the futures price. Second, seasonality in the implied standard deviation suggests potential arbitrage opportunities, which means that the market is not efficiently considering all available information. The higher the expected variance of the futures price, *ceteris paribus* the higher the price of the option. If implied standard deviation did indeed peak, then a potential profit could be made

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[23]This definition only pertains to the analysis in this paper and may not necessarily be the same definition as others have used in previous literature.

by purchasing options early in the year when expected variance is relatively low and sold later when expected variance is high. Such arbitrage possibilities should force implied volatility and implied standard deviation back into a "no arbitrage" equilibrium.

If there is indeed seasonality in implied standard deviations, then mathematically it follows that there must also be seasonality in implied volatilities. This is because the implied standard deviation is the product of the implied volatility (σ_I) and the square root of the time to maturity ($\sqrt{T\text{-}t}$), and since the horizon is a monotonic decreasing function over a year, the implied volatility must unambiguously display seasonality as well, i.e. it must rise and peak also. To be complete in the analysis it is thus important to look at implied standard deviation together with implied volatility when considering seasonality, market efficiency and arbitrage, and the usefulness of the Black formula.

Description of the Data

The data used to find implied volatilities is futures options data obtained from the Chicago Board of Trade (CBOT). It contains data for futures options on trading days from 1985, when futures options began trading, through 1996. Data used here includes the day of trade (time t), the strike price traded on that day (X), a put or call indicator, the expiration month of the contract (T), the option closing price (P_t or C_t), and the volume of trade and open interest in number of contracts.

As previously discussed, on a given day there is a futures price and options with several strike prices traded on the same futures contract. From the CBOT data, the option near-the-money is chosen, which was previously defined as the option at-the-money or closest in-the-money. For instance, if the futures price is 264.50 then the near-the-money call is that with strike price of 260, while the strike price for the near-the-money put was 270. In some cases the appropriate near-the-money contract had a closing price listed yet had zero volume, which means it was not traded on that particular day.[24] These observations were removed from the data used to numerically find implied volatility. These occurances typically happen early in the year when trading is fairly thin, such as in February or March. For corn and soybean puts and calls, such zero-volume exclusions are a limited problem, as on average there are over half of the 23 possible maximum observations. Trading picks up by March for these crops.

Spring wheat poses the biggest problem with respect to thin trading. For both spring wheat puts and calls, trading is extremely thin in February, improves slightly in March, and then picks up to normal levels in April and May.

It should be pointed out that trading generally increases over time. When the futures options market was new in 1985, it is not too surprising that trading was thin in early months. By 1996, however, there tended to be more volume traded.

Sample sizes for the monthly corn, soybean, and spring wheat futures options data used to find implied volatilities are included in the appendix.

Once the appropriate futures options data has been obtained, the Black formula may be used to find implied volatilities. The interest rate ρ on U.S. Treasury bills adjusted for

[24]These occurances were typically denoted with an 'N' in the CBOT data files. Although there were no trades on that particular day, a closing price was still listed if the option traded on a previous day.

the rate of inflation I is used as the risk free rate of interest, so that $r=\rho - I$. The gdp deflator (chain-type price index) is used to deflate F and X and to calculate inflation rates with which to adjust the nominal U.S. T-bill rate.[25] The time to maturity is based on the number of calendar days from the day of trade of the option until around the twentieth of the expiration month, which is used as the expiration of both the option and the futures contract[26]. When used in the Black formula, the time to maturity is divided by 365 and expressed as a fraction of a year.

As discussed earlier, the Black formula gives the price of a call or put option as a function of several parameters, all of which are observable except the implied volatility. It is not possible to invert the formula to find volatility as a function of the other parameters. Therefore an iterative search procedure is used. The search finds the value of implied volatility such that when plugged into the Black formula, the correct option price (the one observed in the market) is obtained.

There are different search procedures that may by used to find the implied volatility. In this study, Fortran was used to try different values of implied volatility between 0 and 1 with a grid search width of .0001. The search was done over the sample of options prices in a particular month. For each value of implied volatility in the Fortran grid search interval,

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[25]Specifically, inflation rates for each year were obtained by subtracting the ratio of the gdp deflator from the subsequent year to the gdp deflator of the current year from one. To obtain the necessary inflation adjustments, averages were used. For example, for 1987, the average of the 1987 and 1986 numbers described above was taken and used for 1987 inflation adjustment.

[26]The gdp deflator and the interest rate on six month treasury bills (used here as the risk free interest rate) are obtained from Economic Report of the President for 1996.

there is a corresponding sum of squared errors for the set of options prices over the entire month. The program is designed to pick the level of implied volatility that minimizes this sum of squared errors.[27] It should be noted that implied volatility is positive and bounded below by zero in the limit since the Black formula call and put prices are not defined for zero volatility.

There are advantages to using the "brute-force" technique in finding implied volatilities. Such a technique is simple to program and may be executed fairly quickly. Furthermore, the search procedure does not rely on curvature properties of the sum of squared errors function, which may adversely affect search algorithms based on numeric derivatives. In the case in which the sum of squared errors function has more than one critical point, this numerical technique increases the likelihood that a unique minimum will be found.

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[27]This search technique has been called the "shotgun" method or the "brute-force" method.

EMPIRICAL RESULTS

Historical Futures Price Standard Deviation

The most important assumption underlying the Black-Scholes analysis (1973) and maintained by Black (1976) is that the futures price F follows a lognormal diffusion process of the general form

$$\frac{dF}{F} = \mu \, dt + \sigma \, dz \tag{1a}$$

where μ is a constant growth rate parameter and $\sigma \, dz$ is a random noise Wiener process in which the volatility parameter σ is assumed constant. The reader should be aware that the general form (1a) is given without loss of generality, as $\mu = 0$ for a futures price. As a result of this process it is further assumed that fractional changes in F over a time interval Δt are lognormally distributed with zero mean and variance $\sigma^2 \Delta t$, or equivalently a standard deviation of $\sigma\sqrt{(\Delta t)}$, which is consistent with equation (3).

To partially test these distributional assumptions, the standard deviation of the natural logarithm of fractional price changes of the form

$$ln(\frac{F_t}{F_{t-1}})$$

is calculated for a particular month, where t represents a trading day and $t-1$ the previous trading day within that month. If markets were perfect and the Black-Scholes model was accurate, the standard deviation in these monthly "log-fractional" changes should be constant across a given year.

As previously discussed, it has been shown in previous literature (i.e. Hennessy, et.al.) that historical futures price volatility often displays seasonal patterns, typically showing peaks during summer months. It is thus of interest to see if standard deviation in "log-fractional" changes over successive months within a year display similar seasonal "peaks."

Tables 2 through 4 show standard deviations for monthly "log-fractional" changes for corn, soybeans, and spring wheat futures prices over successive months. Corn and soybean futures prices show seasonal patterns. Corn has five peaks in June, four in July, and one "late" peak each in August, September, and October. Soybeans show three peaks in June, seven in July, and one each in May and August. Overall, June and July appear to be the "peak" months for corn and soybean futures prices.

As seen in table 3, spring wheat futures prices also display seasonality, although the seasonal pattern is much less strong than for corn and soybeans. Spring wheat peaks are spread out across successive months. May, July and August have two peaks; the remaining months each have one peak.

Overall, there is strong seasonality for corn and soybeans, and peaks for these crops occur primarily in June or July. Spring wheat shows less strong evidence in seasonality, as there are peaks not only in mid-year but also in early and late months as well.

This overall pattern of seasonality in monthly "log-fractional" price changes does not support assumptions maintained by the Black formula. It is again important to realize,

Table 2. Monthly Futures Price Log-Change Standard Deviation, December CBOT Corn Futures

	February	March	April	May	June	July	August	September	October	November
1985	0.0045	0.0052	0.0040	0.0063	0.0072	0.0091	0.0084	0.0120	0.0113	0.0084
1986	0.0075	0.0092	0.0144	0.0153	0.0091	0.0142	0.0099	0.0134	0.0160	0.0089
1987	0.0102	0.0093	0.0117	0.0223	0.0228	0.0158	0.0142	0.0122	0.0133	0.0076
1988	0.0102	0.0083	0.0077	0.0166	0.0269	0.0334	0.0210	0.0117	0.0128	0.0126
1989	0.0066	0.0096	0.0130	0.0120	0.0175	0.0215	0.0140	0.0111	0.0113	0.0078
1990	0.0063	0.0074	0.0071	0.0111	0.0139	0.0126	0.0116	0.0122	0.0100	0.0128
1991	0.0061	0.0083	0.0079	0.0094	0.0150	0.0204	0.0206	0.0077	0.0079	0.0077
1992	0.0089	0.0057	0.0097	0.0134	0.0152	0.0101	0.0118	0.0100	0.0075	0.0105
1993	0.0034	0.0040	0.0094	0.0084	0.0079	0.0146	0.0094	0.0109	0.0109	0.0128
1994	0.0063	0.0074	0.0109	0.0206	0.0252	0.0137	0.0068	0.0076	0.0076	0.0057
1995	0.0044	0.0052	0.0050	0.0103	0.0160	0.0117	0.0092	0.0109	0.0082	0.0084
1996	0.0050	0.0074	0.0167	0.0201	0.0165	0.0213	0.0144	0.0121	0.0100	0.0093
Mean	0.0066	0.0073	0.0098	0.0138	0.0161	0.0165	0.0126	0.0110	0.0106	0.0094
Std. Dev.	0.0022	0.0018	0.0038	0.0052	0.0064	0.0067	0.0045	0.0018	0.0026	0.0023
Minimum	0.0034	0.0040	0.0040	0.0063	0.0072	0.0091	0.0068	0.0076	0.0075	0.0057
Maximum	0.0102	0.0096	0.0167	0.0223	0.0269	0.0334	0.0210	0.0134	0.0160	0.0128

Table 3. Monthly Futures Price Log-Change Standard Deviation, November CBOT Soybeans

	February	March	April	May	June	July	August	September	October
1985	0.0071	0.0076	0.0073	0.0085	0.0114	0.0165	0.0095	0.0106	0.0081
1986	0.0052	0.0076	0.0122	0.0103	0.0078	0.0141	0.0102	0.0072	0.0098
1987	0.0053	0.0062	0.0104	0.0249	0.0224	0.0198	0.0077	0.0075	0.0111
1988	0.0096	0.0104	0.0117	0.0184	0.0270	0.0350	0.0211	0.0147	0.0138
1989	0.0101	0.0101	0.0106	0.0120	0.0153	0.0222	0.0176	0.0098	0.0083
1990	0.0053	0.0085	0.0083	0.0128	0.0135	0.0129	0.0108	0.0096	0.0104
1991	0.0063	0.0089	0.0086	0.0079	0.0127	0.0214	0.0258	0.0104	0.0133
1992	0.0093	0.0073	0.0067	0.0124	0.0137	0.0091	0.0107	0.0085	0.0063
1993	0.0049	0.0057	0.0051	0.0059	0.0127	0.0167	0.0141	0.0146	0.0070
1994	0.0051	0.0066	0.0062	0.0226	0.0231	0.0137	0.0070	0.0079	0.0076
1995	0.0042	0.0070	0.0073	0.0137	0.0137	0.0167	0.0087	0.0141	0.0070
1996	0.0063	0.0066	0.0154	0.0101	0.0140	0.0187	0.0105	0.0128	0.0134
Mean	0.0065	0.0077	0.0091	0.0133	0.0156	0.0181	0.0128	0.0107	0.0097
Std. Dev.	0.0020	0.0015	0.0030	0.0058	0.0056	0.0065	0.0058	0.0028	0.0027
Minimum	0.0042	0.0057	0.0051	0.0059	0.0078	0.0091	0.0070	0.0072	0.0063
Maximum	0.0101	0.0104	0.0154	0.0249	0.0270	0.0350	0.0258	0.0147	0.0138

Table 4. Monthly Futures Price Log-Change Standard Deviation, September CBOT Spring Wheat

	February	March	April	May	June	July	August
1987	0.0117	0.0082	0.0127	0.0197	0.0140	0.0122	0.0097
1988	0.0077	0.0092	0.0089	0.0157	0.0361	0.0304	0.0127
1989	0.0079	0.0119	0.0077	0.0081	0.0098	0.0116	0.0063
1990	0.0044	0.0049	0.0083	0.0079	0.0068	0.0093	0.0147
1991	0.0096	0.0122	0.0085	0.0080	0.0136	0.0167	0.0220
1992	0.0185	0.0111	0.0104	0.0150	0.0113	0.0116	0.0115
1993	0.0088	0.0072	0.0074	0.0086	0.0090	0.0172	0.0117
1994	0.0084	0.0085	0.0123	0.0153	0.0139	0.0106	0.0099
1995	0.0078	0.0090	0.0111	0.0136	0.0168	0.0201	0.0191
1996	0.0097	0.0121	0.0272	0.0205	0.0192	0.0177	0.0126
Mean	0.0094	0.0094	0.0114	0.0132	0.0151	0.0157	0.0130
Std. Dev.	0.0037	0.0024	0.0059	0.0049	0.0083	0.0063	0.0046
Minimum	0.0044	0.0049	0.0074	0.0079	0.0068	0.0093	0.0063
Maximum	0.0185	0.0122	0.0272	0.0205	0.0361	0.0304	0.0220

however, that such tests are not tests solely of Black-Scholes, but rather joint tests of market efficiency and Black-Scholes together. Seasonality is contradictory not only with respect to the Black-Scholes analysis, but also because it seems plausible that the market should properly adjust to such seasonal patterns. Implications of seasonality for both the Black-Scholes model and the futures options market itself will be discussed further when considering implied volatility and implied standard deviation.

Futures Price as Unbiased Mean Estimate

As discussed earlier, futures options prices are hypothesized to contain information which may be used to predict futures price variability, and as such may be of interest to different areas of agriculture. For example, crop insurance may be interested in whether futures options contain information which may be used to predict futures price variance with respect to beginning and ending periods.

When considering the distribution of possible ending period futures prices, the futures price during a particular beginning period is used as an unbiased estimate for the ending period price. If not, then arbitrage opportunities would be possible in the cash-futures market; it is unlikely that there is a better forecast than the futures market's regarding ending period prices.

To test whether beginning period futures prices are indeed unbiased estimates for the ending period futures price, ending period futures prices are regressed on average futures prices during beginning periods ranging from February until one month prior to contract expiration. The appropriate ending period price is the final trading day futures price for the particular commodity futures contract (i.e. December for corn, November for soybeans, and September for spring wheat.)[28]

The estimated empirical form is:

$$EP = \alpha + \beta(BP)$$

BP is the average futures price during a particular beginning period, for instance the average February futures price on the December corn contract. *EP* is the futures price on the final day of the contract expiration month, i.e. December 20 for corn. This regression is run for each beginning period using futures data from 1960 through 1996. It thus includes February through November on the December corn contract, February through October on the November soybean contract, and February through August on the September spring wheat contract.

If beginning period futures prices are indeed unbiased mean estimates for ending period futures prices, then we would expect *beta* not to differ significantly from one. This is because *on average* we expect the beginning period futures price to equal the ending period price. Put differently, although it is totally reasonable to expect futures prices to differ occasionally from actual realized ending prices, there should as many above as below, so that an estimate of $\beta = 1$ should still result.

Tables 5 through 7 show "OLS expected futures price" tests for corn, soybeans, and spring wheat for successive beginning periods from 1960 through 1996. Shown is the

[28]This final day is around the 20th of the contract expiration month.

estimated intercept *alpha*, the estimated slope *beta* (with standard errors in parentheses), and a t-test of whether *beta* is significantly different from 1. The appropriate t-statistic is given by:

$$t_{\beta \neq 1} = \frac{\beta^{*} - 1}{SE}$$

β^* is the estimated slope coefficient and *SE* is the standard error. A high value of this statistic indicates that *beta* is significantly different from one.

Table 5. Futures Price Unbiased Estimate Test for Corn, OLS Test

	Alpha	Beta	T-test forBeta≠1
February	.2101	.9492	-.4190
	(.5259)	(.1213)	
March	.0077	.9992	-.0068
	(.5326)	(.1234)	
April	-.0796	1.0199	.1500
	(.5713)	(.1327)	
May	-.3861	1.0975	.7558
	(.5521)	(.1290)	
June	-.5003	1.1080	.9626
	(.4884)	(.1122)	
July	-.2211	1.0350	.5132
	(.3024)	(.0682)	
August	.2812	.9157	-2.1022
	(.1810)	(.0401)	
September	.1685	.9509	-1.3016
	(.1681)	(.0377)	
October	.2438	.9359	-2.0808
	(.1376)	(.0308)	
November	.0671	.9824	-.8904
	(.0874)	(.0198)	

Note: Standard Errors in Parentheses
Sample size includes 37 years of futures data from 1960-1996

Table 6. Futures Price Unbiased Estimate Tests for Soybeans, OLS

	Alpha	Beta	Beta≠1
February	-.0560	1.0419	.3923
	(1.038)	(.1068)	
March	-.2819	1.0629	.5803
	(1.0550)	(.1084)	
April	-.4572	1.0790	.6524
	(1.1770)	(.1211)	
May	-.8842	1.1161	1.1089
	(1.026)	(.1047)	
June	.2497	.9756	-.2802
	(.8786)	(.0870)	
July	.7119	.9109	-1.6748
	(.5530)	(.0532)	
August	1.7646	.8010	-3.7688
	(.5577)	(.0528)	
September	.7853	.9072	-2.3671
	(.4075)	(.0392)	
October	1.0427	.8898	-3.2896
	(.3457)	(.0335)	

Note: Standard errors in parentheses, sample size includes 37 years, 1960-1996

Table 7. Futures Price Unbiased Estimate Tests for Spring Wheat, OLS Test

	Alpha	Beta	Beta≠1
February	.9348	.8619	-1.1042
	(.7659)	(.1251)	
March	.6141	.9341	-.4985
	(.7905)	(.1322)	
April	-.0386	1.0595	.4250
	(.8218)	(.1400)	
May	-.6314	1.1739	1.2872
	(.7841)	(.1351)	
June	-.4009	1.1213	1.0773
	(.6645)	(.1126)	
July	-.2121	1.0832	.9265
	(.5368)	(.0898)	
August	-.0824	1.0386	1.5820
	(.1510)	(.0244)	

Note: Standard errors in parentheses, sample size includes 37 years, 1960-1996

For soybeans, we find that there are more beginning months for which *beta* appears to differ significantly differ from one. There is marginal significance for May and July, and more significance for August, September, and October. Again, this may result from *alpha* estimates that are significantly different from zero.

For spring wheat, February, May, June, and August have *betas* which are marginally different from one. The alphas for these months, however aren't significantly different from zero except for February, which is only marginally different.

Overall, the notion that futures prices early in the year are optimal forecasts for prices later in the year is fairly well supported. Although there are instances in which the estimated slope differs significantly from one, it may still be reasonable to conclude that the theory holds since it could very well be the result of *alpha* estimates which differ significantly from zero.

This "futures price unbiased estimate" hypothesis is tested another way, which is discussed in Judge, et.al: Specifically, when considering the OLS empirical form discussed earlier in this section, it is hypothesized α=0 and β=1; as discussed in Judge, et.al., our null hypothesis is that the coefficients are restricted. We thus test the null (restricted) hypothesis against the alternative that the coefficients are unrestricted. The appropriate test statistic is:[29]

$$\lambda = \frac{SSE_R - SSE_U}{J\sigma_e^2} \sim F_{(J,T-K)}$$

[29]For a description of this test see Judge, et. al. Introduction to the Theory and Practice of Econometrics, 1988.

SSE_R and SSE_U are the sums of squared errors, respectively, from the restricted and unrestricted models, T is the sample size, J is the number of constraints, K is the number of parameters in model, and σ_e^2 is an estimate for the unrestricted error variance. Specifically, an estimate for σ_e^2 used here is $SSE_U / T - K$. In our case, J and K are 2, and the sample size T is 37, which represents the number of years from 1960 through 1996 in which futures data is taken.

Given a level of significance *alpha*, the null hypothesis that the coefficients are restricted is rejected if the sample statistic $\lambda \sim F_{(J,\, T\text{-}K, \alpha)}$ is larger than the critical F-value at that particular *alpha*. If the reverse is true, so that the sample statistic λ is smaller than the critical F-value, then we do not reject the null hypothesis and thus conclude that the estimated coefficients are not significantly different from their hypothesized (restricted) values.

Table 8 summarizes results of the "restrictions" test for corn, soybeans, and spring wheat for two separate cases. The first case includes the outlier 1973, the year in which prices made a seemingly discontinuous jump as a result of the grain embargo of 1972. The second case is with 1973 removed. This is done to see if results improve significantly.

For corn, results support the hypothesis that beginning period futures prices are unbiased estimates for ending period futures prices. In the case in which 1973 is included, none of the months are rejected, although October appears to be "worse" than the other months. In the no-1973 case, October is rejected, and it appears that removing 1973 did not result in significant improvement.

Table 8: Futures Price Unbiased Estimate Test for Corn, Soybeans, and Spring Wheat: Restrictions Test

	December Corn		November Soybeans		September Spring Wheat	
	With 1973	Without 1973	With 1973	Without 1973	With 1973	Without 1973
February	.0877	.1473	.8045	.3392	.7496	2.6640
March	.0007	.2769	.7857	.3007	.5293	.7609
April	.0114	.2044	.6789	.2473	.7216	.1529
May	.2916	.1897	.9600	.5480	1.7926	.8121
June	.5286	.4300	.0409	.0822	1.4012	.3140
July	.4401	.8014	1.8546	.1559	1.3620	.5436
August	2.8672	1.2124	**7.6579**	1.2317	4.0790	2.0454
September	1.0393	2.4564	3.2445	1.2701	***********	***********
October	2.2689	**5.4783**	**5.4470**	5.2703	***********	***********
November	.4179	2.1027	*********	***********	***********	***********

Note: Boldface indicates rejection of hypothesis that beginning futures price is an unbiased estimate for ending futures price
The critical χ^2 statistic is 5.285 when 1973 is included and 5.306 when 1973 is removed as an outlier

For soybeans, August and October are rejected when 1973 is included. Results improved significantly when 1973 is removed; in the no-1973 none of the months are rejected.

For spring wheat none of the months are rejected, both when 1973 is included and when 1973 is removed as an outlier.

Overall, results based on the "restrictions" tests are extremely favorable for corn and spring wheat. Results appear overall less favorable for soybeans, although throwing out 1973 makes a significant improvement: one month is rejected without 1973 as compared to three months rejected with 1973.

Futures Price Lognormality

Probably the most important assumption maintained by the Black-Scholes analysis is that futures price changes are distributed lognormally. It is thus appropriate to test the hypothesis that futures price changes are lognormally distributed for corn, soybeans and spring wheat. Rejecting lognormality has serious implications for the Black formula.

Lognormality is tested in the following way. Using deflated futures prices, the natural logarithm of the ratio of a trading day's futures price to the previous trading day's futures price is calculated within a year. These yearly futures prices range from February through the appropriate final beginning period (November for corn, October for soybeans, and September for spring wheat). The series of price changes obtained is then regressed against a constant, 1, using a no-constant model. This technique allows us to look at the normality of residuals to determine normality of the actual price changes themselves. The appropriate test statistic is the *Jarque-Bera* statistic, which is distributed χ^2 with two degrees of freedom in this case. Lognormality is rejected if the chi-squared statistic is too large.

Tests of lognormality are critical when evaluating assumptions made by the Black Formula. One reason that lognormality is rejected often may be the result of problems with *skewness* and *kurtosis*. Sherrick, et.al. point out that the lognormal distribution is not considered to be very flexible because it does not allow for varying degrees of skewness and kurtosis. As presented in Campbell, et.al., skewness, or the normalized third moment, of a random variable ϵ with mean μ and variance[30] σ^2 is:

$$S[\epsilon] \doteq E\,[\frac{(\epsilon-\mu)^3}{\sigma^3}]$$

Skewness is a measure of the degree to which the distribution "leans" to one side or the other and may be positive or negative. The normal distribution is symmetric and thus has zero skewness.

Kurtosis is the normalized fourth moment and, using similar notation, is defined by:

$$S[\epsilon] \doteq E\,[\frac{(\epsilon-\mu)^4}{\sigma^4}]$$

Kurtosis is a measure of how "fat" the tails of the distribution are. Thus, the more probability mass ("fatter") included in the tails, the higher the kurtosis of the distribution. The kurtosis of a normal distribution is 3.

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[30]Do not confuse these generic distributional parameters with those specifically defined and used throughout this thesis.

It is suspected that non-zero skewness and *excess kurtosis*, defined as sample kurtosis minus three, may be two key problems which result in rejection of lognormality (remember that when testing for lognormality we are actually testing the normality of the natural log of the fractional price changes).

Tables 9 through 11 give estimated *Jarque-Bera* statistics, coefficients of skewness, and coefficients of excess kurtosis (with t-statistics in parentheses) for corn, soybeans, and spring wheat data from 1960 through 1996. Overall, lognormality is strongly rejected. For December corn futures prices, 21 of the possible 37 years from 1960 through 1996 are rejected. For November soybeans, 25 of the years are rejected, and for September spring wheat 18 of the years are rejected. When considering all three crops collectively over these 37 years, lognormality is rejected almost sixty percent of the time.

It is worthwhile to examine the coefficients of skewness and excess kurtosis and compare years in which lognormality is rejected with years in which lognormality is not rejected. If the distribution of futures prices was truly lognormal, then the coefficients of skewness and excess kurtosis, hereafter denoted *C(s)* and *C(k)*, should be close to zero. For years in which lognormality is not rejected, C(s) and C(k) are usually not significantly different from zero. For years in which lognormality is rejected, there are fairly high t-values for C(s) and C(k). Furthermore, by inspection of the table it appears that kurtosis is likely to be the most "significant" problem that results in rejection of lognormality.

Sherrick et. al. have also tested lognormality for soybean futures prices. By looking at monthly data for 22 different months between March 1988 and March 1991, Sherrick et. al. reject lognormality about a third of the time. Sherrick et. al point out that these results

Table 9. Lognormality Tests for December CBOT Corn Futures Prices, Jarque-Bera~$\chi^2_{(2)}$

	Jarque-Bera Statistic	Coefficient of Skewness	Coefficient of Excess Kurtosis
1960	75.9851	-.0235	2.9449
		(-.1445)	(9.0948)
1961	***1.3495***	-.1215	.3291
		(-.7459)	(1.0142)
1962	433.9476	.7290	6.8923
		(4.4533)	(21.1485)
1963	1117.0126	.2781	11.2539
		(1.7030)	(34.6061)
1964	***6.4969***	.1230	.8453
		(.7551)	(2.6049)
1965	***7.9622***	.3358	.6805
		(2.0652)	(2.1016)
1966	100.9831	-.1298	3.3889
		(-.7986)	(10.4435)
1967	26.2786	.0395	1.7456
		(.2425)	(5.3794)
1968	135.9788	1.0148	3.3745
		(6.1991)	(10.3544)
1969	372.1339	-.4990	6.4672
		(-3.0427)	(19.8016)
1970	179.4807	.9046	4.1296
		(5.5531)	(12.7260)
1971	33.5260	.0139	1.9653
		(.0855)	(6.0695)
1972	160.0029	.3994	4.1934
		(2.4518)	(12.9227)
1973	***5.1144***	-.1290	-.6901
		(-.7866)	(-2.1130)
1974	***9.1232***	-.0234	-.9895
		(-.1429)	(-3.0362)
1975	***2.9412***	.2569	-.2179
		(1.5732)	(-.6700)
1976	22.3239	.1896	1.5652
		(1.1639)	(4.8234)
1977	***4.3695***	-.1860	.6183
		(-1.1418)	(1.9054)
1978	***.8833***	-.0659	.3139
		(-.4036)	(.9653)

Note: Boldface: Lognormality not rejected, $\alpha = .005$, sample size ≈ 230 per year.
T-values for skewness/kurtosis shown in parentheses.

Table 9 (cont.)

	Jarque-Bera Statistic	Coefficient of Skewness	Coefficient of Excess Kurtosis
1979	***6.0882***	-.2338	.7079
		(-1.4317)	(2.1768)
1980	***1.5135***	.1547	.2965
		(.9473)	(.9117)
1981	***.0927***	-.0176	-.0682
		(-.1082)	(-.2106)
1982	24.5629	-.1162	1.6735
		(-.7133)	(5.1572)
1983	***3.8462***	.3230	.0055
		(1.9865)	(.0170)
1984	***10.4736***	-.1162	1.0846
		(-.7146)	(3.3496)
1985	21.6373	.5857	1.1191
		(3.4822)	(3.3416)
1986	44.2118	.5515	2.0532
		(3.2711)	(6.1162)
1987	14.7366	-.2831	1.2310
		(-1.6831)	(3.6757)
1988	***3.8642***	-.0773	.6895
		(-.4617)	(2.0613)
1989	29.4426	-.0082	1.9078
		(-.0489)	(5.7103)
1990	***8.6004***	-.3747	.7003
		(-2.2330)	(2.0961)
1991	38.1278	.1794	1.0666
		(2.1413)	(6.3938)
1992	30.4704	-.4997	1.6652
		(-2.9568)	(4.9486)
1993	58.8582	.7348	2.2334
		(4.3686)	(6.6689)
1994	138.0692	-.4469	4.0054
		(-2.6570)	(11.9600)
1995	46.4168	.0103	2.3133
		(.0632)	(7.1288)
1996	***.2226***	.0394	-.1110
		(.2386)	(-.3376)

Note: Boldface: Lognormality not rejected, $\alpha = .005$, sample size $\approx$ 230 per year.
T-values for skewness/kurtosis shown in parentheses.

Table 10. Lognormality Tests for November Soybean Futures Prices, Jarque-Bera~$\chi^2_{(2)}$

	Jarque-Bera Statistic	Coefficient of Skewness	Coefficient of Excess Kurtosis
1960	62.4081	.0251	2.8141
		(.1470)	(8.2841)
1961	411.9714	-.8835	6.9545
		(-5.1757)	(20.4725)
1962	1233.1139	.7987	12.3325
		(4.6680)	(36.2188)
1963	81.8752	-.0194	3.2276
		(-.1134)	(.3405)
1964	265.9895	.6253	5.6669
		(3.6461)	(16.5990)
1965	82.8431	-.0851	3.2335
		(-.4985)	(9.5187)
1966	42.0171	.1720	2.2940
		(1.0053)	(6.7372)
1967	294.1186	.8012	5.8508
		(4.6936)	(17.2234)
1968	69.3496	.1327	2.9765
		(.7720)	(8.6981)
1969	54.8447	.0653	2.6651
		(.3790)	(7.7700)
1970	51.7057	.5197	2.3481
		(3.0303)	(6.8779)
1971	29.7326	-.2810	1.8650
		(-1.6462)	(5.4901)
1972	18.6633	.1268	1.5308
		(.7428)	(4.5063)
1973	***3.5333***	.0292	-.6345
		(.1699)	(-1.8542)
1974	***9.7699***	.0142	-1.0793
		(.0826)	(-3.1540)
1975	***1.7938***	.0715	-.4212
		(.4169)	(-1.2337)
1976	***3.5453***	-.0580	-.6245
		(-.3390)	(-1.8341)
1977	***.3970***	-.0658	.2081
		(-.3846)	(.6112)
1978	16.9068	-.6331	.7019
		(-3.7002)	(2.0614)

Note: Boldface: Lognormality not rejected, $\alpha = .005$, sample size ≈ 200 per year.
T-values for skewness/kurtosis shown in parentheses.

Table 10 (cont.)

	Jarque-Bera Statistic	Coefficient of Skewness	Coefficient of Excess Kurtosis
1979	12.7615	-.3488	1.0701
		(-2.0434)	(3.1501)
1980	***.2854***	.0554	.1825
		(.3230)	(.5346)
1981	***4.6874***	-.0244	.7938
		(-.1426)	(2.3313)
1982	***.9223***	.0759	-.2719
		(.4436)	(-.7985)
1983	***.5294***	.1255	.0078
		(.7352)	(.0230)
1984	23.2718	-.4250	1.4889
		(-2.4956)	(4.3933)
1985	67.0444	.8081	2.5360
		(4.5707)	(7.2086)
1986	19.6940	.2105	1.5988
		(1.1906)	(4.5446)
1987	61.8960	-.1388	2.8976
		(-.7851)	(8.2365)
1988	***7.5471***	-.3738	.6843
		(-2.1202)	(1.9501)
1989	10.2196	-.2641	1.0712
		(-1.4938)	(3.0449)
1990	***1.4641***	-.0329	.4701
		(-.1861)	(1.3363)
1991	82.7556	.1024	3.3550
		(.5792)	(9.5367)
1992	13.4718	-.4397	1.0434
		(-2.4744)	(2.9508)
1993	65.3678	-.0834	2.9863
		(-.4717)	(8.4886)
1994	155.7526	-.4939	4.5006
		(-2.7872)	(12.7604)
1995	121.0458	.0494	3.9067
		(.2894)	(11.5004)
1996	***7.7789***	-.2647	.8525
		(-1.5507)	(2.5096)

Note: Boldface: Lognormality not rejected, $\alpha = .005$, sample size $\approx$ 200 per year.
T-values for skewness/kurtosis shown in parentheses.

Table 11. Lognormality Tests for September Spring Wht. Futures Prices, Jarque-Bera~$\chi^2_{(2)}$

	Jarque-Bera Statistic	Coefficient of Skewness	Coefficient of Excess Kurtosis
1960	***.5561***	.1403	.1113
		(.7357)	(.2935)
1961	94.4796	-.7750	3.5855
		(-4.0386)	(9.3984)
1962	19.1245	-.4152	1.5732
		(-2.1569)	(4.1119)
1963	***8.6900***	.1518	1.1794
		(.7886)	(3.0826)
1964	***8.8466***	-.2874	1.0719
		(1.5849)	(2.8097)
1965	***3.2785***	.1634	.6775
		(.8542)	(1.7815)
1966	30.6383	.2345	2.1905
		(1.2258)	(5.7599)
1967	17.2079	-.5895	1.1700
		(-3.0719)	(3.0852)
1968	***10.4285***	-.5766	.5848
		(-2.9860)	(1.5237)
1969	39.7601	-.6151	2.2523
		(-3.1755)	(5.8501)
1970	774.3505	.7070	11.0661
		(3.6842)	(29.0068)
1971	16.5030	.6377	.9972
		(3.3335)	(2.6221)
1972	57.0802	.9928	2.2607
		(5.1898)	(5.9445)
1973	12.1355	-.2371	-1.2714
		(-1.2317)	(-3.3231)
1974	***5.2389***	.0493	-.8765
		(.2553)	(-2.2837)
1975	***5.2442***	.4206	.3609
		(2.1849)	(.9433)
1976	***.2161***	.0602	.1766
		(.3147)	(.4644)
1977	***.1046***	.0034	.1675
		(.0177)	(.4391)
1978	***1.2179***	-.0383	.4725
		(-.1996)	(1.2385)

Note: Boldface: Lognormality not rejected, $\alpha = .005$, sample size ≈ 150 per year.
T-values for skewness/kurtosis shown in parentheses.

Table 11 (cont.)

	Jarque-Bera Statistic	Coefficient of Skewness	Coefficient of Excess Kurtosis
1979	10.8455	.2685	1.2398
		(1.3984)	(3.2405)
1980	***5.6287***	.2891	.7800
		(1.5065)	(2.0446)
1981	***1.3693***	-.0901	.4683
		(-.4695)	(1.2275)
1982	***.2652***	-.0140	-.1649
		(-.0732)	(-.4336)
1983	66.0893	.2467	3.2372
		(1.2896)	(8.5122)
1984	***3.3858***	.3470	.2320
		(1.8082)	(.6081)
1985	11.2103	-.3649	1.1550
		(-1.8956)	(3.0188)
1986	83.1107	.5685	3.5048
		(2.9532)	(9.1605)
1987	19.3812	.2665	1.8073
		(1.3279)	(4.5318)
1988	24.8158	.1346	2.0988
		(.6754)	(5.2987)
1989	21.7906	-.5961	1.5647
		(-2.9805)	(3.9364)
1990	21.0227	-.6257	1.4677
		(-3.1285)	(3.6923)
1991	85.2603	.0676	3.9149
		(.3368)	(9.8167)
1992	***5.8908***	-.0859	1.0538
		(-.4252)	(2.6253)
1993	***.2981***	.0519	.2440
		(.2595)	(.2981)
1994	***7.8097***	-.1621	1.1671
		(-.8077)	(2.9265)
1995	***2.1648***	.0411	.6205
		(.2142)	(1.6265)
1996	***1.0039***	-.0390	.4309
		(-.2032)	(1.1295)

Note: Boldface: Lognormality not rejected, $\alpha = .005$, sample size ≈ 150 per year.
T-values for skewness/kurtosis shown in parentheses.

indicate that the assumption of lognormality may be inaccurate. They further point out that it may be desirable to model futures prices with probability distributions that have higher moment flexibility than the lognormal.

Overall, these results suggest that lognormality and the underlying price motion assumptions may be inappropriate and suggest the need to research techniques of estimating price variances that do not rely on assumed distributions. Several possible candidates have been proposed in the literature. One of these is the Burr III distribution discussed in Sherrick, et. al. One of the problems with lognormality is that it is a restrictive distribution in that it does not allow for varying degrees of skewness and kurtosis. As Sherrick et.al point out, the Burr III distribution allows for a wide range of skewness and kurtosis and thus would seems a reasonable alternative to the "benchmark" lognormal distribution.

The Burr III cumulative distribution is given by:

$$F = 1 - \frac{1}{1 + (\frac{Y}{\tau})^{a}}^{\lambda}$$

Y represents the appropriate random variable, which is the futures price in this case. λ, τ, and a are parameters that must be estimated. From this a PDF for the Burr III could be calculated which could then be potentially useful in predicting future price variability.

Implied Volatility

By using the techniques outlined in detail earlier, implied volatilities were calculated from monthly options data starting with February and moved forward month by month until one month prior to expiration. Thus, for each beginning (monthly) period considered, there is one average level of implied volatility using the technique described earlier. Tables 12 through 17 show implied volatilities for corn, soybean, and spring wheat futures call and put options for these alternate beginning periods. Seasonality is evident for all three crops. Implied volatility is fairly low in the beginning of the year, rises and peaks during the growing season, then gradually falls as the season progresses. For corn and soybeans, this peak occurs mostly in June, with two years in which the peak occurred in May and two years in which the peak occurred in July. It is interesting to note that for both corn and soybeans, peak implied volatilities for a given year occur in the same beginning period month for both calls and puts. This reflects the fact that put and call markets are highly correspondent to one another. Although spring wheat shows seasonality as well, it is not as strong as for corn and soybeans. In two cases, puts and calls had peak volatilities in February, the first beginning month considered. The greatest number of peaks is split between May and July for calls and April, May, and July for puts. In general, spring wheat futures options display peak implied volatilities across a wider area of the year.

This seasonality may have important implications not only for the Black formula, but for the futures options market as well. First, seasonality in implied volatility suggests that the constant volatility parameter assumption maintained by the Black formula is inappropriate. If the market was efficient and the Black-Scholes distributional assumptions accurate, implied volatility should be constant throughout a given year.

As will be discussed in the next section, it is necessary to look at the implied standard deviation, $\sigma_I \sqrt{(T-t)}$, in addition to looking at the implied volatility. Given the

Table 12: Volatilities Implied in December CBOT Corn Call Futures Options Prices

	February	March	April	May	June	July	August	September	October	November
1985	.1017*	0.1193	0.1215	0.1266	***0.1459***	0.1343	0.1284	0.1204	0.0964	0.0724
1986	0.1493	0.1595	0.1793	***0.2242***	0.1965	0.2175	0.2085	0.1842	0.1615	0.0241
1987	0.1847	0.2019	0.2132	0.2743	***0.2878***	0.2196	0.1891	0.1863	0.1835	0.0952
1988	0.2329	0.1767	0.1776	0.2047	***0.4013***	0.4020	0.3374	0.2081	0.1485	0.1110
1989	0.2201	0.2400	***0.2554***	0.2424	0.2357	0.2339	0.1758	0.1606	0.1363	0.0832
1990	0.1814	0.1808	0.1859	0.2128	***0.2217***	0.1796	0.1579	0.1705	0.1376	0.0931
1991	0.1838	0.1947	0.2088	0.1875	0.1904	***0.2081***	0.1996	0.1574	0.1215	0.0933
1992	0.2102	0.2010	0.1958	0.2199	***0.2328***	0.1666	0.1650	0.1552	0.1350	0.1036
1993	0.1732	0.1835	0.1842	0.1803	0.1744	***0.2199***	0.1792	0.1386	0.1173	0.1149
1994	0.2229	0.1855	0.1894	0.2077	***0.2458***	0.1483	0.1252	0.1230	0.1216	0.1057
1995	0.1701	0.1845	0.2013	0.2087	***0.2206***	0.2162	0.1558	0.1521	0.1276	0.0873
1996	0.2074	0.2171	0.2714	***0.2973***	0.2829	0.2655	0.2413	0.1950	0.1427	0.1267
Mean	0.1780	0.1870	0.1987	0.2155	0.2363	0.2176	0.1886	0.1626	0.1358	0.0925
Std. Dev.	0.0613	0.0297	0.0382	0.0438	0.0663	0.0691	0.0571	0.0275	0.0224	0.0261
Minimum	0.0000	0.1193	0.1215	0.1266	0.1459	0.1343	0.1252	0.1204	0.0964	0.0241
Maximum	0.2329	0.2400	0.2714	0.2973	0.4013	0.4020	0.3374	0.2081	0.1835	0.1267

Note: Boldface italic indicates peak volatility in a given year
Asterisk (*) indicates fewer than five observations
Implied volatilities are calculated from the Black formula using a Fortran search procedure

Table 13. Volatilities Implied in December CBOT Corn Futures Put Options Prices

	February	March	April	May	June	July	August	September	October	November
1985	.0965*	0.1165	0.1260	0.1293	***0.1468***	0.1306	0.1273	0.1193	0.0964	0.0630
1986	0.1524	0.1678	0.1885	***0.2273***	0.2015	0.2222	0.2100	0.1815	0.1622	0.0680
1987	0.1920	0.2040	0.2119	0.2737	***0.2918***	0.2214	0.1980	0.1974	0.1944	0.1023
1988	0.2037	0.1841	0.1825	0.2099	***0.3922***	0.4090	0.3338	0.2138	0.1548	0.1044
1989	0.2317	0.2440	***0.2580***	0.2473	0.2402	0.2369	0.1859	0.1701	0.1436	0.0816
1990	0.1892	0.1891	0.1903	0.2212	***0.2309***	0.1839	0.1631	0.1724	0.1368	0.1116
1991	0.1915	0.1986	0.2133	0.1944	0.1921	***0.2116***	0.2054	0.1613	0.1318	0.0846
1992	0.2116	0.2095	0.2013	0.2284	***0.2422***	0.1732	0.1688	0.1560	0.1293	0.0875
1993	0.1780	0.1967	0.1963	0.1919	0.1852	***0.2319***	0.1934	0.1561	0.1256	0.1054
1994	0.1884	0.1921	0.1987	0.2196	***0.2573***	0.1597	0.1334	0.1246	0.1187	0.0725
1995	0.1783	0.1927	0.2115	0.2210	***0.2299***	0.2252	0.1659	0.1570	0.1307	0.0981
1996	0.2137	0.2256	0.2796	***0.3048***	0.2871	0.2671	0.2383	0.1994	0.1418	0.1068
Mean	0.1775	0.1934	0.2048	0.2224	0.2414	0.2227	0.1936	0.1674	0.1388	0.0905
Std. Dev.	0.0595	0.0312	0.0380	0.0433	0.0632	0.0697	0.0543	0.0284	0.0243	0.0166
Minimum	0.0000	0.1165	0.1260	0.1293	0.1468	0.1306	0.1273	0.1193	0.0964	0.0630
Maximum	0.2317	0.2440	0.2796	0.3048	0.3922	0.4090	0.3338	0.2138	0.1944	0.1116

Note: Boldface italic indicates peak volatility in a given year
Asterisk (*) indicates fewer than five observations
Implied volatilities are calculated from Black formula using a Fortran search procedure

Table 14: Volatilities Implied in November CBOT Soybean Futures Call Options Prices

	February	March	April	May	June	July	August	September	October
1985	0.1508	0.1430	0.1541	0.1564	***0.1694***	0.1488	0.1423	0.1136	0.0731
1986	0.1352	0.1384	0.1583	***0.1626***	0.1402	0.1602	0.1606	0.1072	0.0917
1987	0.1069	0.1204	0.1502	0.2268	***0.2568***	0.2193	0.1286	0.0957	0.0999
1988	0.1801	0.1769	0.1935	0.2257	0.3410	***0.3594***	0.3247	0.2142	0.1155
1989	0.2151	0.2315	0.2327	0.2109	0.2120	***0.2381***	0.1771	0.1534	0.1072
1990	0.1428	0.1537	0.1603	0.1762	***0.1883***	0.1708	0.1547	0.1270	0.0916
1991	0.1642	0.1709	0.1771	0.1661	0.1553	0.1948	***0.2106***	0.1626	0.0836
1992	0.1842	0.1923	0.1821	0.2140	***0.2329***	0.1400	0.1273	0.1232	0.1014
1993	0.0641	0.1697	0.1735	0.1740	0.1761	***0.2714***	0.1944	0.1354	0.0977
1994	0.1593	0.1589	0.1565	0.1950	***0.2510***	0.1543	0.1248	0.1182	0.0976
1995	0.1543	0.1210	0.1019	0.1021	***0.1486***	0.1260	0.0656	0.0603	0.0164
1996	0.1705	0.1663	0.2169	***0.2259***	0.2149	0.2067	0.1966	0.1559	0.1025
Mean	0.1523	0.1619	0.1714	0.1863	0.2072	0.1992	0.1673	0.1306	0.0899
Std. Dev.	0.0387	0.0309	0.0338	0.0374	0.0575	0.0665	0.0635	0.0386	0.0256
Minimum	0.0641	0.1204	0.1019	0.1021	0.1402	0.1260	0.0656	0.0603	0.0164
Maximum	0.2151	0.2315	0.2327	0.2268	0.3410	0.3594	0.3247	0.2142	0.1155

Note: Boldface italic indicates peak volatility in a given year

Implied volatilities are calculated from Black formula using a Fortran search procedure

Table 15. Volatilities Implied in November CBOT Soybean Futures Put Options Prices

	February	March	April	May	June	July	August	September	October
1985	0.1505	0.1431	0.1571	0.1614	***0.1784***	0.1605	0.1455	0.1106	0.0737
1986	0.1414	0.1484	0.1713	***0.1785***	0.1528	0.1752	0.1578	0.1161	0.0670
1987	0.1091	0.1303	0.1604	0.2426	***0.2712***	0.2345	0.1486	0.1157	0.0997
1988	0.1871	0.1845	0.2001	0.2317	0.3390	***0.3797***	0.3286	0.2133	0.1212
1989	0.2170	0.2338	0.2336	0.2126	0.2179	***0.2408***	0.1851	0.1576	0.1024
1990	0.1530	0.1670	0.1714	0.1851	***0.1973***	0.1842	0.1666	0.1366	0.0857
1991	0.1696	0.1768	0.1834	0.1774	0.1702	0.2057	***0.2334***	0.1797	0.0931
1992	0.1878	0.2042	0.1901	0.2324	***0.2507***	0.1577	0.1342	0.1257	0.0523
1993	0.1657	0.1783	0.1862	0.1920	0.1982	***0.2802***	0.2181	0.1617	0.0809
1994	0.1651	0.1698	0.1700	0.2154	***0.2682***	0.1677	0.1215	0.1164	0.0650
1995	0.1678	0.1836	0.1996	0.2096	***0.2290***	0.2270	0.1577	0.1604	0.0999
1996	0.1818	0.1820	0.2307	***0.2380***	0.2238	0.2159	0.2077	0.1648	0.1022
Mean	0.1663	0.1752	0.1878	0.2064	0.2247	0.2191	0.1837	0.1466	0.0869
Std. Dev.	0.0270	0.0276	0.0249	0.0271	0.0519	0.0629	0.0571	0.0317	0.0198
Minimum	0.1091	0.1303	0.1571	0.1614	0.1528	0.1577	0.1215	0.1106	0.0523
Maximum	0.2170	0.2338	0.2336	0.2426	0.3390	0.3797	0.3286	0.2133	0.1212

Note: Boldface italic indicates peak volatility in a given year
Implied volatilities are calculated from Black formula using a Fortran search procedure

Table 16. Volatilities Implied in September CBOT Spring Wheat Futures Call Options Prices

	February	March	April	May	June	July	August
1987	.1924*	0.1506	0.1533	***0.2428***	0.2176	0.1543	0.1165
1988	.2153*	0.2015	0.1854	0.2061	0.3794	***0.3865***	0.2118
1989	***0.2235***	0.2157	0.2126	0.2000	0.1611	0.1349	0.0894
1990	0.1379	.1399*	0.1376	***0.1578***	0.1241	0.1198	0.1309
1991	.1805*	***0.1995***	0.1913	0.1661	0.1454	0.1691	0.1404
1992	***0.2479***	0.2246	0.1963	0.1987	0.1805	0.1199	0.1016
1993	.1609*	0.1761	0.1790	0.1556	0.1398	***0.1841***	0.1143
1994	.1803*	0.1706	0.1711	0.1839	***0.1942***	0.1474	0.0963
1995	0.0259	0.1520	0.1650	0.1873	0.2188	***0.2306***	0.1296
1996	0.2127	0.2029	0.2815	***0.2930***	0.2446	0.2057	0.1169
Mean	0.0848	0.1694	0.1873	0.1991	0.2006	0.1852	0.1248
Std. Dev.	0.1077	0.0647	0.0395	0.0419	0.0739	0.0794	0.0345
Minimum	0.0000	0.0000	0.1376	0.1556	0.1241	0.1198	0.0894
Maximum	0.2479	0.2246	0.2815	0.2930	0.3794	0.3865	0.2118

Note: Boldface italic indicates peak volatility in a given year
Asterisk (*) indicates five or fewer observations
Implied volatilities are calculated from Black formula using a Fortran search procedure

Table 17. Volatilities Implied in September CBOT Spring Wheat Futures Put Options Prices

	February	March	April	May	June	July	August
1987	.0001*	0.1514	0.1557	***0.2307***	0.1972	0.1556	0.1409
1988	.2098*	.1839*	.1900*	0.2150	0.3526	***0.3677***	0.2067
1989	.2246*	0.2262	***0.2385***	0.1955	0.1645	0.1378	0.0862
1990	***.1570****	0.1411	0.1402	0.1529	0.1319	0.1258	0.1411
1991	.1787*	***0.1994***	0.1971	0.1628	0.1505	0.1668	0.1322
1992	***0.2531***	0.2203	0.1988	0.2010	0.1865	0.1270	0.1156
1993	.1714*	0.1723	***0.1829***	0.1582	0.1451	0.1809	0.1276
1994	0.1831	0.1803	0.1745	0.1879	***0.1987***	0.1570	0.1055
1995	******	0.1503	0.1698	0.1884	0.2259	***0.2346***	0.1411
1996	0.2143	0.2026	0.2784	***0.2898***	0.2504	0.2138	0.1224
Mean	0.0723	0.1644	0.1736	0.1982	0.2003	0.1867	0.1319
Std. Dev.	0.1098	0.0649	0.0731	0.0406	0.0650	0.0728	0.0316
Minimum	0.0000	0.0000	0.0000	0.1529	0.1319	0.1258	0.0862
Maximum	0.2531	0.2262	0.2784	0.2898	0.3526	0.3677	0.2067

Note: Boldface italic indicates peak in a given year
* indicates five or fewer observations, there are no observations for February 1995
Implied volatilities are calculated from Black formula using a Fortran search procedure

distributional assumptions of the Black-Scholes analysis, the implied standard deviation is the futures options market's optimal forecast of the standard deviation of the distribution of futures price over the discrete interval between t and T-t. It is thus necessary to look at implied standard deviation when considering Black-Scholes assumptions and market efficiency. The reader should note that seasonality in implied volatility is a necessary but not sufficient condition for seasonality in implied standard deviation. As a result it is important to look at both implied volatility and implied standard deviation when considering Black-Scholes assumptions.

Implied Standard Deviation

The second measure of variability that is analyzed is the implied standard deviation, which is the implied volatility times the square root of the horizon, $\sigma_I \sqrt{(T-t)}$.

As discussed in the previous section, it is important to look at implied standard deviation in conjunction with implied volatility in order to provide a complete analysis of Black-Scholes assumptions. If the Black-Scholes analysis were correct, then implied standard deviation should be a monotonic decreasing function over the time span of the contract (or at "worst" a horizontal line given the seasonality in implied volatility).

These are shown in tables 18 through 23. Although there are instances in which the implied standard deviation displays peaks in mid-year, it does not appear that implied standard deviations display an overall seasonal pattern. When looking at the mean implied standard deviation, there is a peak in June for corn puts and soybean calls. These peaks are

Table 18. Corn Call Implied Standard Deviation From CBOT December Corn Futures Options, $\sigma_I\sqrt{(T-t)}$

	February	March	April	May	June	July	August	September	October	November
1985	0.0936 *	0.1045	0.1003	0.0981	***0.1047***	0.0884	0.0757	0.0617	0.0410	0.0224
1986	0.1374	0.1397	0.1481	***0.1737***	0.1410	0.1431	0.1230	0.0945	0.0687	0.0075
1987	0.1699	0.1768	0.1761	***0.2125***	0.2066	0.1445	0.1116	0.0955	0.0780	0.0295
1988	0.2143	0.1548	0.1467	0.1586	***0.2880***	0.2645	0.1990	0.1067	0.0631	0.0344
1989	0.2025	0.2102	***0.2109***	0.1878	0.1692	0.1539	0.1037	0.0824	0.0580	0.0258
1990	***0.1669***	0.1584	0.1535	0.1648	0.1591	0.1182	0.0931	0.0874	0.0585	0.0288
1991	0.1691	0.1705	***0.1724***	0.1452	0.1367	0.1369	0.1177	0.0807	0.0517	0.0289
1992	***0.1934***	0.1761	0.1617	0.1703	0.1671	0.1096	0.0973	0.0796	0.0574	0.0321
1993	***0.1594***	0.1607	0.1521	0.1397	0.1252	0.1447	0.1057	0.0711	0.0499	0.0356
1994	***0.2051***	0.1625	0.1564	0.1609	0.1764	0.0976	0.0739	0.0631	0.0517	0.0327
1995	0.1565	0.1616	***0.1663***	0.1617	0.1583	0.1422	0.0919	0.0780	0.0543	0.0270
1996	0.1908	0.1902	0.2241	***0.2303***	0.2030	0.1747	0.1423	0.1000	0.0607	0.0392
Mean	0.1716	0.1638	0.1641	0.1670	0.1696	0.1432	0.1113	0.0834	0.0577	0.0287
Std. Dev.	0.0336	0.0260	0.0316	0.0339	0.0476	0.0454	0.0337	0.0141	0.0095	0.0081
Minimum	0.0936	0.1045	0.1003	0.0981	0.1047	0.0884	0.0739	0.0617	0.0410	0.0075
Maximum	0.2143	0.2102	0.2241	0.2303	0.2880	0.2645	0.1990	0.1067	0.0780	0.0392

Note: Boldface italic indicates peak in a given year
Asterisk (*) indicates fewer than five observations

Table 19. Corn Put Implied Standard Deviation From December CBOT Corn Futures Options, $\sigma_I\sqrt{(T\text{-}t)}$

	February	March	April	May	June	July	August	September	October	November
1985	.0888*	0.1020	0.1041	0.1002	***0.1054***	0.0859	0.0751	0.0612	0.0410	0.0195
1986	0.1402	0.1470	0.1557	***0.1761***	0.1446	0.1462	0.1239	0.0931	0.0690	0.0211
1987	0.1767	0.1787	0.1750	***0.2120***	0.2094	0.1457	0.1168	0.1012	0.0827	0.0317
1988	0.1874	0.1613	0.1507	0.1626	***0.2815***	0.2691	0.1969	0.1096	0.0658	0.0323
1989	0.2132	***0.2137***	0.2131	0.1916	0.1724	0.1559	0.1097	0.0872	0.0611	0.0253
1990	***0.1741***	0.1656	0.1572	0.1713	0.1657	0.1210	0.0962	0.0884	0.0582	0.0346
1991	***0.1762***	0.1740	***0.1762***	0.1506	0.1379	0.1392	0.1212	0.0827	0.0560	0.0262
1992	***0.1947***	0.1835	0.1663	0.1769	0.1738	0.1139	0.0996	0.0800	0.0550	0.0271
1993	0.1638	***0.1723***	0.1621	0.1486	0.1329	0.1526	0.1141	0.0800	0.0534	0.0326
1994	0.1733	0.1683	0.1641	0.1701	***0.1847***	0.1051	0.0787	0.0639	0.0505	0.0225
1995	0.1641	0.1688	***0.1747***	0.1712	0.1650	0.1482	0.0979	0.0805	0.0556	0.0304
1996	0.1966	0.1976	0.2309	***0.2361***	0.2061	0.1757	0.1406	0.1023	0.0603	0.0331
Mean	0.1708	0.1694	0.1692	0.1723	0.1733	0.1465	0.1142	0.0858	0.059	0.028
Std. Dev.	0.0318	0.0273	0.0314	0.0336	0.0454	0.0459	0.0321	0.0146	0.0103	0.0051
Minimum	0.0888	0.102	0.1041	0.1002	0.1054	0.0859	0.0751	0.0612	0.041	0.0195
Maximum	0.2132	0.2137	0.2309	0.2361	0.2815	0.2691	0.1969	0.1096	0.0827	0.0346

Note: Boldface italic indicates peak in a given year
Asterisk (*) indicates fewer than five observations

Table 20. Soybean Call Implied Standard Deviation From November Soybean Futures Options, $\sigma_I\sqrt{(T-t)}$

	February	March	April	May	June	July	August	September	October
1985	***0.1318***	0.1183	0.1194	0.1125	0.1114	0.0881	0.0734	0.0483	0.0230
1986	0.1182	0.1145	***0.1226***	0.1170	0.0922	0.0949	0.0828	0.0456	0.0288
1987	0.0935	0.0996	0.1163	0.1632	***0.1689***	0.1299	0.0663	0.0407	0.0314
1988	0.1575	0.1464	0.1499	0.1624	***0.2243***	0.2128	0.1674	0.0911	0.0363
1989	0.1881	***0.1916***	0.1802	0.1518	0.1395	0.1410	0.0913	0.0652	0.0337
1990	0.1249	***0.1272***	0.1242	0.1268	0.1239	0.1011	0.0797	0.0540	0.0288
1991	***0.1436***	0.1414	0.1372	0.1195	0.1022	0.1154	0.1086	0.0691	0.0263
1992	***0.1610***	0.1591	0.1411	0.1540	0.1532	0.0829	0.0656	0.0524	0.0318
1993	0.0560	***0.1404***	0.1344	0.1252	0.1159	0.1607	0.1002	0.0576	0.0307
1994	0.1393	0.1315	0.1212	0.1403	***0.1651***	0.0914	0.0643	0.0503	0.0307
1995	***0.1349***	0.1001	0.0789	0.0735	0.0978	0.0746	0.0338	0.0256	0.0052
1996	0.1491	0.1376	***0.1680***	0.1626	0.1414	0.1224	0.1013	0.0663	0.0322
Mean	0.1331	0.1340	0.1328	0.1341	0.1363	0.1179	0.0862	0.0555	0.0282
Std. Dev.	0.0338	0.0256	0.0262	0.0269	0.0378	0.0394	0.0328	0.0164	0.0080
Minimum	0.0560	0.0996	0.0789	0.0735	0.0922	0.0746	0.0338	0.0256	0.0052
Maximum	0.1881	0.1916	0.1802	0.1632	0.2243	0.2128	0.1674	0.0911	0.0363

Note: Boldface italic indicates peak in a given year

Table 21. Soybean Put Implied Standard Deviation From November CBOT Soybean Futures Options, $\sigma_I\sqrt{(T\text{-}t)}$

	February	March	April	May	June	July	August	September	October
1985	***0.1316***	0.1184	0.1217	0.1161	0.1174	0.0950	0.0750	0.0470	0.0231
1986	0.1236	0.1228	***0.1327***	0.1284	0.1005	0.1038	0.0813	0.0494	0.0210
1987	0.0954	0.1078	0.1242	0.1746	***0.1784***	0.1389	0.0766	0.0492	0.0313
1988	0.1636	0.1527	0.1550	0.1667	0.2230	***0.2249***	0.1694	0.0907	0.0381
1989	0.1897	***0.1935***	0.1809	0.1530	0.1434	0.1426	0.0954	0.0670	0.0322
1990	0.1338	***0.1382***	0.1328	0.1332	0.1298	0.1091	0.0859	0.0581	0.0269
1991	***0.1483***	0.1463	0.1421	0.1277	0.1120	0.1218	0.1203	0.0764	0.0292
1992	0.1642	***0.1690***	0.1473	0.1672	0.1649	0.0934	0.0692	0.0534	0.0164
1993	0.1449	0.1476	0.1442	0.1382	0.1304	***0.1659***	0.1124	0.0688	0.0254
1994	0.1443	0.1405	0.1317	0.1550	***0.1764***	0.0993	0.0626	0.0495	0.0204
1995	0.1467	0.1519	***0.1546***	0.1508	0.1507	0.1344	0.0813	0.0682	0.0314
1996	0.1589	0.1506	***0.1787***	0.1713	0.1472	0.1279	0.1071	0.0701	0.0321
Mean	0.1454	0.1450	0.1455	0.1485	0.1478	0.1297	0.0947	0.0623	0.0273
Std. Dev.	0.0236	0.0228	0.0193	0.0195	0.0341	0.0372	0.0294	0.0135	0.0062
Minimum	0.0954	0.1078	0.1217	0.1161	0.1005	0.0934	0.0626	0.0470	0.0164
Maximum	0.1897	0.1935	0.1809	0.1746	0.2230	0.2249	0.1694	0.0907	0.0381

Note: Boldface italic indicates peak in a given year

Table 22. Spring Wheat Call Implied Standard Deviation From September CBOT Spring Wheat Futures Options, $\sigma_I\sqrt{(T\text{-}t)}$

	February	March	April	May	June	July	August
1987	.1487*	***0.1084***	0.1009	0.1438	0.1122	0.0661	0.0366
1988	.1664*	0.1450	0.1220	0.1221	***0.1956***	0.1656	0.0665
1989	***0.1727***	0.1552	0.1399	0.1184	0.0830	0.0578	0.0281
1990	***0.1066***	.1007*	0.0905	0.0934	0.0640	0.0513	0.0411
1991	.1395*	***0.1436***	0.1259	0.0984	0.0750	0.0724	0.0441
1992	***0.1916***	0.1616	0.1291	0.1177	0.0930	0.0514	0.0319
1993	.1243*	***0.1267***	0.1178	0.0921	0.0721	0.0789	0.0359
1994	***.1393****	0.1228	0.1126	0.1089	0.1001	0.0631	0.0302
1995	0.0200	0.1094	0.1086	0.1109	***0.1128***	0.0988	0.0407
1996	0.1644	0.1460	***0.1852***	0.1735	0.1261	0.0881	0.0367
Mean	0.1374	0.1319	0.1232	0.1179	0.1034	0.0794	0.0392
Std. Dev.	0.0481	0.0213	0.0260	0.0248	0.0381	0.0340	0.0108
Minimum	0.0200	0.1007	0.0905	0.0921	0.0640	0.0513	0.0281
Maximum	0.1916	0.1616	0.1852	0.1735	0.1956	0.1656	0.0665

Note: Boldface italic indicates peak in a given year
Asterisk (*) indicates five or fewer observations

Table 23. Spring Wheat Put Implied Standard Deviation From September CBOT Spring Wheat Futures Options, $\sigma_I\sqrt{(T-t)}$

	February	March	April	May	June	July	August
1987	.00008*	0.1089	0.1024	***0.1366***	0.1017	0.0666	0.0443
1988	.1621*	.1323*	.1250*	0.1273	***0.1818***	0.1574	0.0649
1989	***.1736****	0.1628	0.1569	0.1158	0.0848	0.0590	0.0271
1990	***.1213****	0.1015	0.0922	0.0905	0.0680	0.0539	0.0443
1991	.1381*	***0.1435***	0.1297	0.0964	0.0776	0.0714	0.0415
1992	***0.1956***	0.1585	0.1308	0.1190	0.0961	0.0544	0.0363
1993	***.1325****	0.1240	0.1203	0.0937	0.0748	0.0775	0.0401
1994	***0.1415***	0.1297	0.1148	0.1113	0.1024	0.0672	0.0331
1995	******	0.1082	0.1117	0.1116	***0.1165***	0.1005	0.0443
1996	0.1656	0.1458	***0.1832***	0.1716	0.1291	0.0915	0.0384
Mean	0.1538	0.1315	0.1267	0.1174	0.1033	0.0799	0.0414
Std. Dev.	0.0247	0.0213	0.0265	0.0241	0.0335	0.0312	0.0099
Minimum	0.1213	0.1015	0.0922	0.0905	0.068	0.0539	0.0271
Maximum	0.1956	0.1628	0.1832	0.1716	0.1818	0.1574	0.0649

Note: Boldface italic indicates peak in a given year
Asterisk (*) indicates five or fewer observations
February 1987 is not used in summary statistic calculations
There are no observations for February 1995

not significantly different from those in earlier months when considering a standard deviation of about .03 for both crops.

Overall, it cannot be concluded that there is significant evidence for seasonality in standard deviations implied in corn, soybean, and spring wheat futures options. Although evidence strongly supports seasonality in implied volatility, it appears that the market is "correcting" such seasonality through the implied standard deviation.

Relationship Between Put and Call Implied Volatilities

It is suspected that implied volatilities from puts and calls on the same futures contract should be highly correlated. If not, then potential arbitrage opportunities exist, which would force the put and call markets back into equilibrium. Implied volatilities from calls are regressed on those from puts using OLS estimation of the following empirical form:

$$\textit{Implied Call} = \alpha + \beta(\textit{Implied Put})$$

Results are summarized in table 24 below:

Table 24: Relationship between Put and Call CBOT Futures Options Implied Volatilities

	Corn	Soybeans	Wheat
α	-.00002	.0031	-.0155
	(-.0074)	(.3735)	(-3.9744)
β	.9786*	.9007*	1.0738*
	(71.9559)	(20.2860)	(52.3805)
T-statistic for $\beta \neq 1$	-1.5735	-2.2365*	3.6000*
R-Square	.9778	.7950	.9794

Note: T-values are shown in parentheses
* denotes significance at the 5 % level for a two-tailed test
Sample sizes used are 120, 108, and 60 respectively for corn, soybeans, and wheat

While corn and spring wheat implied volatilities appear to be very highly correlated, soybean seems surprisingly low. The estimated *beta* for all three crops, however, is strongly significant. When considering a test of $\beta \neq 1$, it is interesting to note that *beta* appears to be significantly different from one for soybeans and wheat.

Levels of Implied Volatilities and Price Changes

Tables 25 through 27 show estimated call implied volatilities, put implied volatilities, and ln(ep/bp), which is used as a measure of change between the average monthly futures price and the ending price. These are the relevant data for put and call implied volatilities and commodity futures price changes between beginning and ending periods which are ultimately used to test whether futures options prices are accurate predictors of futures price changes by means of estimated implied volatilities.

Table 25. Corn Implied Volatilities and Futures Price Changes Measured by Ln (ep/bp)
February

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1017	0.0965	0.0674
1986	0.1493	0.1524	0.3309
1987	0.1847	0.1920	0.0599
1988	0.2329	0.2037	0.2089
1989	0.2201	0.2317	0.1487
1990	0.1814	0.1892	0.0994
1991	0.1838	0.1915	0.0304
1992	0.2102	0.2116	0.2518
1993	0.1732	0.1780	0.1896
1994	0.2229	0.1884	0.2022
1995	0.1701	0.1783	0.2900
1996	0.2074	0.2137	0.1351
Mean	0.1865	0.1856	0.1679
Std. Dev.	0.0365	0.0346	0.0947
Minimum	0.1017	0.0965	0.0304
Maximum	0.2329	0.2317	0.3309

Table 25 (cont.)
March

	Corn Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1193	0.1165	0.0612
1986	0.1595	0.1678	0.3127
1987	0.2019	0.2040	0.0347
1988	0.1767	0.1841	0.1998
1989	0.2400	0.2440	0.1465
1990	0.1808	0.1891	0.1289
1991	0.1947	0.1986	0.0414
1992	0.2010	0.2095	0.2407
1993	0.1835	0.1967	0.1769
1994	0.1855	0.1921	0.1823
1995	0.1845	0.1927	0.2782
1996	0.2171	0.2256	0.1558
Mean	0.1870	0.1934	0.1633
Std. Dev.	0.0297	0.0312	0.0888
Minimum	0.1193	0.1165	0.0347
Maximum	0.2400	0.2440	0.3127

April

	Corn Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1215	0.1260	0.0737
1986	0.1793	0.1885	0.2769
1987	0.2132	0.2119	0.0130
1988	0.1776	0.1825	0.1706
1989	0.2554	0.2580	0.1171
1990	0.1859	0.1903	0.1674
1991	0.2088	0.2133	0.0352
1992	0.1958	0.2013	0.1874
1993	0.1842	0.1963	0.1720
1994	0.1894	0.1987	0.1518
1995	0.2013	0.2115	0.2670
1996	0.2714	0.2796	0.2094
Mean	0.1987	0.2048	0.1535
Std. Dev.	0.0382	0.0380	0.0824
Minimum	0.1215	0.1260	0.0130
Maximum	0.2714	0.2796	0.2769

Table 25 (cont.)
May

	Corn Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1266	0.1293	0.0497
1986	0.2242	0.2273	0.2812
1987	0.2743	0.2737	0.0869
1988	0.2047	0.2099	0.1442
1989	0.2424	0.2473	0.0857
1990	0.2128	0.2212	0.1957
1991	0.1875	0.1944	0.0149
1992	0.2199	0.2284	0.2143
1993	0.1803	0.1919	0.1937
1994	0.2077	0.2196	0.1409
1995	0.2087	0.2210	0.2424
1996	0.2973	0.3048	0.2553
Mean	0.2155	0.2224	0.1587
Std. Dev.	0.0438	0.0433	0.0856
Minimum	0.1266	0.1293	0.0149
Maximum	0.2973	0.3048	0.2812

June

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1459	0.1468	0.0276
1986	0.1965	0.2015	0.2328
1987	0.2878	0.2918	0.1029
1988	0.4013	0.3922	0.1408
1989	0.2357	0.2402	0.0415
1990	0.2217	0.2309	0.2198
1991	0.1904	0.1921	0.0414
1992	0.2328	0.2422	0.2253
1993	0.1744	0.1852	0.2244
1994	0.2458	0.2573	0.1592
1995	0.2206	0.2299	0.1947
1996	0.2829	0.2871	0.2682
Mean	0.2363	0.2414	0.1566
Std. Dev.	0.0663	0.0632	0.0849
Minimum	0.1459	0.1468	0.0276
Maximum	0.4013	0.3922	0.2682

Table 25 (cont.)
July

	Corn Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1343	0.1306	0.0287
1986	0.2175	0.2222	0.1496
1987	0.2196	0.2214	0.0152
1988	0.4020	0.4090	0.1905
1989	0.2339	0.2369	0.0427
1990	0.1796	0.1839	0.1686
1991	0.2081	0.2116	0.5399
1992	0.1666	0.1732	0.1158
1993	0.2199	0.2319	0.1505
1994	0.1483	0.1597	0.0226
1995	0.2162	0.2252	0.1721
1996	0.2655	0.2671	0.2539
Mean	0.2176	0.2227	0.1542
Std. Dev.	0.0691	0.0697	0.1436
Minimum	0.1343	0.1306	0.0152
Maximum	0.4020	0.4090	0.5399

August

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1284	0.1273	0.1040
1986	0.2085	0.2100	0.1118
1987	0.1891	0.1980	0.0693
1988	0.3374	0.3338	0.1110
1989	0.1758	0.1859	0.0074
1990	0.1579	0.1631	0.0875
1991	0.1996	0.2054	0.0102
1992	0.1650	0.1688	0.0487
1993	0.1792	0.1934	0.1735
1994	0.1252	0.1334	0.0140
1995	0.1558	0.1659	0.1959
1996	0.2413	0.2383	0.2248
Mean	0.1886	0.1936	0.0965
Std. Dev.	0.0571	0.0543	0.0727
Minimum	0.1252	0.1273	0.0074
Maximum	0.3374	0.3338	0.2248

Table 25 (cont.)
September

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1204	0.1193	0.1181
1986	0.1842	0.1815	0.0963
1987	0.1863	0.1974	0.0242
1988	0.2081	0.2138	0.0935
1989	0.1606	0.1701	0.0046
1990	0.1705	0.1724	0.0241
1991	0.1574	0.1613	0.0010
1992	0.1552	0.1560	0.0370
1993	0.1386	0.1561	0.1858
1994	0.1230	0.1246	0.0060
1995	0.1521	0.1570	0.1253
1996	0.1950	0.1994	0.1717
Mean	0.1626	0.1674	0.0740
Std. Dev.	0.0275	0.0284	0.0665
Minimum	0.1204	0.1193	0.0010
Maximum	0.2081	0.2138	0.1858

October

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.0964	0.0964	0.1023
1986	0.1615	0.1622	0.1131
1987	0.1835	0.1944	0.0197
1988	0.1485	0.1548	0.0779
1989	0.1363	0.1436	0.0242
1990	0.1376	0.1368	0.0255
1991	0.1215	0.1318	0.0015
1992	0.1350	0.1293	0.0065
1993	0.1173	0.1256	0.1532
1994	0.1216	0.1187	0.0140
1995	0.1276	0.1307	0.0612
1996	0.1427	0.1418	0.0517
Mean	0.1358	0.1388	0.0542
Std. Dev.	0.0224	0.0243	0.0484
Minimum	0.0964	0.0964	0.0015
Maximum	0.1835	0.1944	0.1532

Table 25 (cont.)
November

	Corn Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.0724	0.0630	0.0384
1986	0.0241	0.0680	0.1193
1987	0.0952	0.1023	0.0213
1988	0.1110	0.1044	0.0093
1989	0.0832	0.0816	0.0189
1990	0.0931	0.1116	0.0097
1991	0.0933	0.0846	0.0315
1992	0.1036	0.0875	0.0075
1993	0.1149	0.1054	0.0565
1994	0.1057	0.0725	0.0152
1995	0.0873	0.0981	0.0454
1996	0.1267	0.1068	0.0054
Mean	0.0925	0.0905	0.0315
Std. Dev.	0.0261	0.0166	0.0321
Minimum	0.0241	0.0630	0.0054
Maximum	0.1267	0.1116	0.1193

Table 26. Soybean Implied Volatilities and Futures Price Change Measured by Ln (ep/bp)
February

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1508	0.1505	0.2106
1986	0.1352	0.1414	0.0425
1987	0.1069	0.1091	0.1858
1988	0.1801	0.1871	0.1238
1989	0.2151	0.2170	0.2301
1990	0.1428	0.1530	0.0667
1991	0.1642	0.1696	0.0842
1992	0.1842	0.1878	0.0752
1993	0.0641	0.1657	0.1545
1994	0.1593	0.1651	0.1434
1995	0.1543	0.1678	0.1541
1996	0.1705	0.1818	0.0239
Mean	0.1523	0.1663	0.1246
Std. Dev.	0.0387	0.0270	0.0665
Minimum	0.0641	0.1091	0.0239
Maximum	0.2151	0.2170	0.2301

Table 26 (cont.)
March

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1430	0.1431	0.2026
1986	0.1384	0.1484	0.0468
1987	0.1204	0.1303	0.1726
1988	0.1769	0.1845	0.1076
1989	0.2315	0.2338	0.2537
1990	0.1537	0.1670	0.1018
1991	0.1709	0.1768	0.0888
1992	0.1923	0.2042	0.0944
1993	0.1697	0.1783	0.1343
1994	0.1589	0.1698	0.1461
1995	0.1210	0.1836	0.1267
1996	0.1663	0.1820	0.0278
Mean	0.1619	0.1752	0.1253
Std. Dev.	0.0309	0.0276	0.0633
Minimum	0.1204	0.1303	0.0278
Maximum	0.2315	0.2338	0.2537

April

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1541	0.1571	0.2177
1986	0.1583	0.1713	0.0348
1987	0.1502	0.1604	0.1061
1988	0.1935	0.2001	0.0454
1989	0.2327	0.2336	0.2234
1990	0.1603	0.1714	0.1168
1991	0.1771	0.1834	0.0894
1992	0.1821	0.1901	0.0596
1993	0.1735	0.1862	0.1261
1994	0.1565	0.1700	0.0971
1995	0.1019	0.1996	0.1244
1996	0.2169	0.2307	0.0949
Mean	0.1714	0.1878	0.1113
Std. Dev.	0.0338	0.0249	0.0589
Minimum	0.1019	0.1571	0.0348
Maximum	0.2327	0.2336	0.2234

Table 26 (cont.)
May

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1564	0.1614	0.1717
1986	0.1626	0.1785	0.0529
1987	0.2268	0.2426	0.0075
1988	0.2257	0.2317	0.0381
1989	0.2109	0.2126	0.1795
1990	0.1762	0.1851	0.1450
1991	0.1661	0.1774	0.0474
1992	0.2140	0.2324	0.0947
1993	0.1740	0.1920	0.1258
1994	0.1950	0.2154	0.1271
1995	0.1021	0.2096	0.1279
1996	0.2259	0.2380	0.0961
Mean	0.1863	0.2064	0.1011
Std. Dev.	0.0374	0.0271	0.0548
Minimum	0.1021	0.1614	0.0548
Maximum	0.2268	0.2426	0.1795

June

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1694	0.1784	0.1444
1986	0.1402	0.1528	0.0178
1987	0.2568	0.2712	0.0112
1988	0.3410	0.3390	0.2439
1989	0.2120	0.2179	0.1081
1990	0.1883	0.1973	0.1077
1991	0.1553	0.1702	0.0236
1992	0.2329	0.2507	0.1074
1993	0.1761	0.1982	0.1263
1994	0.2510	0.2682	0.1527
1995	0.1486	0.2290	0.1150
1996	0.2149	0.2238	0.0503
Mean	0.2072	0.2247	0.1007
Std. Dev.	0.0575	0.0519	0.0671
Minimum	0.1402	0.1528	0.0112
Maximum	0.3410	0.3390	0.2439

Table 26 (cont.)
July

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1488	0.1605	0.1209
1986	0.1602	0.1752	0.0097
1987	0.2193	0.2345	0.0702
1988	0.3594	0.3797	0.1868
1989	0.2381	0.2408	0.0995
1990	0.1708	0.1842	0.1156
1991	0.1948	0.2057	0.0322
1992	0.1400	0.1577	0.0199
1993	0.2714	0.2802	0.0332
1994	0.1543	0.1677	0.0222
1995	0.1260	0.2270	0.0796
1996	0.2067	0.2159	0.0653
Mean	0.1992	0.2191	0.0713
Std. Dev.	0.0665	0.0629	0.0526
Minimum	0.1260	0.1577	0.0097
Maximum	0.3594	0.3797	0.1868

August

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1423	0.1455	0.0500
1986	0.1606	0.1578	0.0379
1987	0.1286	0.1486	0.1174
1988	0.3247	0.3286	0.1696
1989	0.1771	0.1851	0.0066
1990	0.1547	0.1666	0.1075
1991	0.2106	0.2334	0.0246
1992	0.1273	0.1342	0.0327
1993	0.1944	0.2181	0.0176
1994	0.1248	0.1215	0.0072
1995	0.0656	0.1577	0.1173
1996	0.1966	0.2077	0.0907
Mean	0.1673	0.1837	0.0649
Std. Dev.	0.0635	0.0571	0.0535
Minimum	0.0656	0.1215	0.0066
Maximum	0.3247	0.3286	0.1696

Table 26 (cont.)
September

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.1136	0.1106	0.0425
1986	0.1072	0.1161	0.0297
1987	0.0957	0.1157	0.0788
1988	0.2142	0.2133	0.1634
1989	0.1534	0.1576	0.0028
1990	0.1270	0.1366	0.1156
1991	0.1626	0.1797	0.0496
1992	0.1232	0.1257	0.0266
1993	0.1354	0.1617	0.0687
1994	0.1182	0.1164	0.0017
1995	0.0603	0.1604	0.0662
1996	0.1559	0.1648	0.1181
Mean	0.1306	0.1466	0.0636
Std. Dev.	0.0386	0.0317	0.0492
Minimum	0.0603	0.1106	0.0017
Maximum	0.2142	0.2133	0.1634

October

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1985	0.0731	0.0737	0.0280
1986	0.0917	0.0670	0.0253
1987	0.0999	0.0997	0.0531
1988	0.1155	0.1212	0.0853
1989	0.1072	0.1024	0.0239
1990	0.0916	0.0857	0.0937
1991	0.0836	0.0931	0.0096
1992	0.1014	0.0523	0.0442
1993	0.0977	0.0809	0.1063
1994	0.0976	0.0650	0.0373
1995	0.0164	0.0999	0.0392
1996	0.1025	0.1022	0.0021
Mean	0.0869	0.0869	0.0457
Std. Dev.	0.0198	0.0198	0.0332
Minimum	0.0523	0.0523	0.0021
Maximum	0.1212	0.1212	0.1063

Table 27. Spring Wheat Implied Volatilities and Futures Change Measured by Ln (ep/bp)

March

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1987	0.1506	0.1514	0.1009
1988	0.2015	0.1839	0.2422
1989	0.2157	0.2262	0.0921
1990	0.1399	0.1411	0.2978
1991	0.1995	0.1994	0.1078
1992	0.2246	0.2203	0.1332
1993	0.1761	0.1723	0.0302
1994	0.1706	0.1803	0.1549
1995	0.1520	0.1503	0.3281
1996	0.2029	0.2026	0.0841
Mean	0.1833	0.1828	0.1571
Std. Dev.	0.0296	0.0295	0.0989
Minimum	0.1399	0.1411	0.0302
Maximum	0.2246	0.2262	0.3281

April

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1987	0.1533	0.1557	0.0690
1988	0.1854	0.1900	0.2093
1989	0.2126	0.2385	0.0620
1990	0.1376	0.1402	0.2964
1991	0.1913	0.1971	0.1025
1992	0.1963	0.1988	0.0871
1993	0.1790	0.1829	0.0308
1994	0.1711	0.1745	0.1640
1995	0.1650	0.1698	0.2877
1996	0.2815	0.2784	0.2595
Mean	0.1873	0.1926	0.1568
Std. Dev.	0.0395	0.0403	0.1002
Minimum	0.1376	0.1402	0.0308
Maximum	0.2815	0.2784	0.2964

Table 27 (cont.)
May

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1987	0.2428	0.2307	0.0127
1988	0.2061	0.2150	0.1928
1989	0.2000	0.1955	0.0816
1990	0.1578	0.1529	0.2974
1991	0.1661	0.1628	0.1089
1992	0.1987	0.2010	0.0895
1993	0.1556	0.1582	0.0216
1994	0.1839	0.1879	0.1486
1995	0.1873	0.1884	0.2460
1996	0.2930	0.2898	0.2934
Mean	0.1991	0.1982	0.1493
Std. Dev.	0.0419	0.0406	0.1047
Minimum	0.1556	0.1529	0.0127
Maximum	0.2930	0.2898	0.2974

June

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1987	0.2176	0.1972	0.0648
1988	0.3794	0.3526	0.0488
1989	0.1611	0.1645	0.0544
1990	0.1241	0.1319	0.2699
1991	0.1454	0.1505	0.1312
1992	0.1805	0.1865	0.0900
1993	0.1398	0.1451	0.0513
1994	0.1942	0.1987	0.1364
1995	0.2188	0.2259	0.1590
1996	0.2446	0.2504	0.1723
Mean	0.2006	0.2003	0.1178
Std. Dev.	0.0739	0.0650	0.0708
Minimum	0.1241	0.1319	0.0488
Maximum	0.3794	0.3526	0.2699

Table 27 (cont.)
July

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1987	0.1543	0.1556	0.0935
1988	0.3865	0.3677	0.0527
1989	0.1349	0.1378	0.0387
1990	0.1198	0.1258	0.1826
1991	0.1691	0.1668	0.1852
1992	0.1199	0.1270	0.0217
1993	0.1841	0.1809	0.0063
1994	0.1474	0.1570	0.1699
1995	0.2306	0.2346	0.0570
1996	0.2057	0.2138	0.1069
Mean	0.1852	0.1867	0.0915
Std. Dev.	0.0794	0.0728	0.0676
Minimum	0.1198	0.1258	0.0063
Maximum	0.3865	0.3677	0.1852

August

	Call Implied Volatility	Put Implied Volatility	Ln(ep/bp)
1987	0.1165	0.1409	0.0732
1988	0.2118	0.2067	0.0481
1989	0.0894	0.0862	0.0305
1990	0.1309	0.1411	0.0849
1991	0.1404	0.1322	0.1154
1992	0.1016	0.1156	0.0582
1993	0.1143	0.1276	0.0185
1994	0.0963	0.1055	0.1075
1995	0.1296	0.1411	0.0823
1996	0.1169	0.1224	0.0745
Mean	0.1248	0.1319	0.0693
Std. Dev.	0.0345	0.0316	0.0311
Minimum	0.0894	0.0862	0.0185
Maximum	0.2118	0.2067	0.1154

OLS Estimation

Despite the fact that many of the assumptions maintained by the Black-Scholes analysis are not empirically supported, it is still hypothesized that implied volatilities may nonetheless be useful predictors of futures price variability. As discussed earlier, Mayhew points out that implied volatility is still the market's assessment of volatility as reflected in the price of the option. Mayhew also points out that even if the underlying asset's volatility is considered stochastic, implied volatility would be interpreted as the market's assessment of the average volatility over the life of the option.

In a sense such a hypothesis is awkward. On one hand we are rejecting many of the assumptions maintained by the Black formula, primarily those involving the stochastic behavior of futures prices. On the other hand, we still hypothesize that implied volatilities may contain useful information in predicting futures price variance, even though their behavior across time (in our case a growing year) does not conform to Black Scholes predictions.

To test this potential predictive power, the absolute log-ratio estimate for futures price variance, $|ln(ep/bp)|$, is regressed on implied volatility. In practice, different estimates of futures price variance may be used in such a regression. For instance, it may also be feasible to use the absolute difference in the average beginning period futures price and the beginning period price. In practice, these alternate measures of futures price change should be highly correlated. This analysis will consider only $|ln\ (ep/bp)|$ as a measure of beginning-ending futures price change.

The average futures price during a particular beginning month is used as the beginning period price and thus as an unbiased estimate of the ending futures price. The futures price on the final day of the contract expiration is used as the ending price. The beginning periods under consideration are February through November for December corn, February through October on November soybeans, and February through August on September spring wheat.

The beginning period months of primary interest with respect to crop insurance are February and March, since that is the sign up period for revenue insurance. It is still worthwhile to test whether implied volatilities from successive beginning periods display better predictive power. The fact that implied volatilities display seasonal patterns would indicate that implied volatilities could potentially become more useful in predicting futures price variability as the growing season progresses.

The estimated equation has the empirical form:

$$ln(\frac{eP}{bP}) = \alpha + \beta \sigma_I$$

Ln(ep/bp) is the absolute value of the natural logarithm of the ending-beginning futures price ratio (absolute logratio), and σ_I is the corresponding beginning period implied volatility.

If the hypothesis that implied volatilities are good predictors of futures price variance (as estimated by the measure of price change *ln ep/pp*) is true, then we would expect alpha to be zero, since intuitively zero implied volatility would mean that the market is perceiving no future changes in price.[31] We would expect *beta* to be positive and significantly different from zero. It is not possible to predict what the magnitude of *beta* should be, since there is no reason to believe that futures price changes and implied volatility should be correlated one-for-one.

It should be pointed out that we regress the absolute log-ratio on the implied volatility and not the implied standard deviation. This is because in the regression, a constant beginning period is used, and within a beginning period T-t is considered a constant. It is therefore sufficient to use only implied volatility when estimating the regression.

Tables 28 through 33 show regression results for absolute log-ratio on implied volatility from put and call futures options for corn, soybeans, and spring wheat. Shown is the estimated intercept α, the estimated slope coefficient β, the R-Square statistic, and the log of the likelihood function (LLF). T-statistics for the estimated coefficients are shown in parentheses.

For corn, implied volatilities appear to be poor predictors of futures price variability. For both puts and calls, May and July are significant. Both have a *beta* estimate which is positive and significant as expected, and *alpha* for both is not significantly different from zero. In addition, October is fairly significant for corn calls.

With respect to crop insurance, we are primarily interested in February and March, since that is around the sign up time for revenue insurance. For corn calls, *beta* has the

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[31]It is important to remember that the Black formula is not defined for an implied volatility value of zero. Such an argument is thus an intuitive one and not one implied directly by the formula.

Table 28. OLS Results for Ln(ep/bp) on Corn Call Implied Volatility

	Alpha	Beta	R-Square	LLF
February	.3022	-.6449	.0324	11.1502
	(1.3133)	(-.5489)		
March	.1750	-.0631	.0004	12.5529
	(.9782)	(-.0667)		
April	.1296	.1201	.0031	13.4707
	(.9425)	(.1765)		
May	.0042	.7170	.1347	13.8583
	(.0333)	(1.2476)		
June	.1180	.1631	.0162	13.1853
	(1.1992)	(.4058)		
July	-.0161	.5965*	.2726	15.8807
	(-.2297)	(1.9361)		
August	.0357	.3228	.0644	15.3604
	(.4673)	(.8300)		
September	.0474	.1637	.0046	16.0536
	(.3777)	(.2148)		
October	-.4231	.1117	.0383	20.0657
	(-.6311)	(1.2115)		
November	.1222	-.9798	.6368	30.8409
	(5.4554)	(-4.1872)		

Note: * indicates significance at the 5% level for a one-tailed test on estimated beta
Sample includes 12 observations (1985-1996) for each month analyzed

Table 29. OLS Results for Ln(ep/bp) on Corn Put Implied Volatility

	Alpha	Beta	R-Square	LLF
February	.5125	-1.7323	.1544	11.8916
	(1.9472)	(-1.2822)		
March	.1283	.1807	.0040	12.5744
	(.7290)	(.2009)		
April	.0965	.2781	.0165	13.5516
	(.6825)	(.4092)		
May	-.0198	.8028**	.1651	14.0726
	(-.1533)	(1.4057)		
June	.1007	.2313	.0297	13.2680
	(.9673)	(.5530)		
July	-.0200	.6002*	.2815	15.9548
	(-.2837)	(1.9796)		
August	.0326	.3300	.0609	15.3380
	(.3971)	(.8055)		
September	.0309	.2570	.0120	16.0986
	(.2474)	(.3490)		
October	.0974	-.3109	.0244	19.9796
	(1.1131)	(-.5002)		
November	.1068	-.8320	.1853	25.9940
	(2.1075)	(-1.5081)		

Note: * denotes significance at the 5% level for a one-tailed test on estimated beta
** denotes significance at the 10% level for a one-tailed test on estimated beta
Sample includes 12 observations (1985-1996) for each month analyzed

Table 30. OLS Results for Ln(hp/pp) on Soybean Call Implied Volatility

	Alpha	Beta	R-Square	LLF
February	.1309	-.0414	.0006	16.0175
	(1.5364)	(-.0762)		
March	.0401	.5258	.0661	17.0284
	(.3901)	(.8415)		
April	.0767	.2018	.0134	17.5605
	(.8031)	(.3687)		
May	.1903	-.4787	.1069	19.0126
	(2.2928)	(-1.0939)		
June	-.0300	.6308*	.2926	17.9935
	(-.4511)	(2.0335)		
July	-.0117	.4164*	.2765	20.7682
	(-.2629)	(1.9549)		
August	.0271	.2261	.0720	19.0731
	(.5930)	(.8811)		
September	-.0095	.5605**	.1940	20.9419
	(-.1939)	(1.5513)		
October	.0257	.2226	.0294	24.5350
	(.6817)	(.5504)		

Note: * indicates significance at the 5% level for a one-tailed test
** indicates significance at the 10% level for a one-tailed test
Sample includes 12 observations (1985-1996) for each month analyzed

Table 31. OLS Results for Ln(Hp/pp) on Soybean Put Implied Volatility

	Alpha	Beta	R-Square	LLF
February	.1135	.0664	.0007	16.0184
	(.8664)	(.0853)		
March	.0548	.4027	.0307	16.8053
	(.4329)	(.5633)		
April	.0552	.2988	.0159	17.5758
	(.3926)	(.4024)		
May	.2565	-.7528	.1380	19.2257
	(2.0736)	(-1.2656)		
June	-.0661	.7422*	.3294	18.3143
	(-.8573)	(2.2162)		
July	-.0364	.4912***	.3442	21.3573
	(-1.3018)	(2.2910)		
August	-.0099	.4071**	.1886	19.8787
	(-.1934)	(1.5247)		
September	-.0700	.9120***	.3456	22.1917
	(-1.1785)	(2.2984)		
October	.0337	.1380	.0068	24.3966
	(.7155)	(.2609)		

Note: * indicates significance at the 5% level for a one-tailed test, sample size 12 per month
** indicates significance at the 10% level for a one-tailed test
*** indicates significance at the 2.5% level for a one-tailed test

Table 32. OLS Results for Ln(hp/pp) on Spring Wheat Call Implied Volatility

	Alpha	Beta	R-Square	LLF
March	.4469	-1.5804	.2234	10.7340
	(2.3131)	(-1.5167)		
April	.1516	.0277	.0001	9.3438
	(.8855)	(.0309)		
May	.0552	.4722	.0358	9.0818
	(.3135)	(.5446)		
June	.1810	-.3150	.1080	13.3879
	(2.6618)	(-.9844)		
July	.1314	-.2158	.0643	13.6170
	(2.2577)	(-.7413)		
August	.0689	.0032	.0000	21.0469
	(1.6764)	(.0100)		

Note: None of the months was significant for at least a 10% one-tailed significance level
Sample size includes 10 years (1987-1996) for each month

Table 33. OLS Results for Ln(hp/pp) on Spring Wheat Put Implied Volatility

	Alpha	Beta	R-Square	LLF
March	.2642	-1.2832	.0397	3.1268
	(.6399)	(-.5747)		
April	.3000	-1.5074	.0988	3.3057
	(.9494)	(-.9363)		
May	.2515	-1.2974	.0779	3.4085
	(.7889)	(-.8217)		
June	-.0771	.3874	.0310	5.9471
	(-.4807)	(.5063)		
July	-.0270	.2531	.0253	8.0266
	(-.2443)	(.4561)		
August	-.0112	.2942	.0159	12.4715
	(-.1011)	(.3592)		

Note: None of the months was significant for at least a 10% one-tailed significance level
Sample size includes 10 years (1987-1996) for each month

wrong sign (is negative) but is insignificant. For puts, *beta* for February is negative but positive for March. Neither are significant.

Results are similar for soybeans. For soybeans calls, June and July have positive and significant *beta* estimates and *alpha* estimates which don't significantly differ from zero. For soybean puts June, July, August, and September are significant. It is interesting to note that for both soybean puts and calls, May has a *beta* estimate which is negative and significant and an *alpha* which appears to be significantly different from zero. Again, February and March are not significant with respect to crop insurance rate setting purposes. For soybean calls, February has a negative *beta* which is insignificant and an *alpha* significantly different from zero. Otherwise, *beta* estimates are positive but insignificant.

Results for spring wheat appear to be the poorest of the three crops tested. For spring wheat calls, March has a negative, insignificant *beta* and an *alpha* which is significantly different from zero. For puts, none of the beginning period months have significant *beta* estimates. The "best" month, April, has the wrong sign.

Overall it can be concluded that volatilities implied in futures options premia are very poor predictors of futures price variability. Despite the fact that there are some months in which *beta* estimates are significant, there is no evidence to support that implied volatilities have useful information in predicting futures price variance. It is interesting to note, however, that the most significant beginning period seems to be around June or July (at least for corn and soybeans), which is around the peak time for implied volatility. Again, this may be attributed to the fact that early in the year such as in February, little is known about the crop. Hennessy and Wahl point out that as the season progresses, information about the crop becomes increasingly available. It seems plausible that better forecasts of futures price variance should be made during these "peak" months, which is empirically the case for corn and soybeans.

Heteroskedasticity

The options market is relatively new. Options on futures have actively traded for only a little more than a decade. For corn and soybeans, we have Chicago Board of Trade (CBOT) data beginning in 1985, thus giving us only 12 years of data. For spring wheat, we have data beginning in 1987 which is 10 years of data.

One interesting question that arises is whether the option market has improved over time. In other words, do option prices today better convey information than those from, say, 1986 when the market was young?

To test whether the options market has become more efficient, a heteroskedastic model is estimated. Heteroskedasticity occurs when the error variance is non-constant. When considering whether the options market has become more efficient over time, testing for heteroskedasticity is important since error variance should decrease if the markets are truly becoming more efficient over time.

A multiplicative heteroskedastic model is used. This model has the empirical form:

$$var = e^{(Z_t\, \alpha)}$$

Such a model specifies the variance *var* as a function of exogenous parameters Z_t and a vector of unknown parameters α. In this model, Z represents a "learning" index for market age, ranging from Z=1 in 1985 to Z=12 in 1996. Such a variable seems appropriate if we are testing whether the options market is improving over time. As such, this variable is defined as *time* in the data set.

The heteroskedastic model thus estimates an equation of the form

$$var = \alpha_o + \alpha_1 (Time)$$

where h is the variance, *time* is the exogenous variable Z which is hypothesized to affect the variance, and α_i are the parameters estimated in the variance equation.[32]

The parameter of interest is α_1. If α_1 is negative, then the error variance is decreasing over time, which means the market is becoming "better." If α_1 is positive then it is unlikely that the market is improving. If α_1 is zero then there is no effect on the market's "efficiency" over time.

A way to test for heteroskedasticity (denoted hereafter as *HET*) is to compute the log of the likelihood functions obtained from the OLS and HET models. If the two statistics do not differ significantly, then OLS cannot be rejected. If there is significant difference, then heteroskedasticity is more likely.

In this empirical framework, OLS is considered a restricted model, since it essentially assumes or "restricts" the error variance to be constant. HET is considered the unrestricted model, since error variance is allowed to change.

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[32]The heteroskedastic model is estimated using Shazam, and for models with exogenous heteroskedasticity Shazam estimates an intercept term in the variance equation.

The appropriate test to consider is discussed in Judge, et al.. The test statistic of interest is given by:

$$LR = 2[LLF(HET) - LLF(OLS)] \sim \chi^2(2)$$

LLF(HET) is the log likelihood function of the HET (unrestricted) model, while *LLF(OLS)* is from the OLS (restricted) model[33]. It can be shown that this statistic is distributed as a χ^2 with degrees of freedom k equal to the number of restrictions. In this case there is one restriction; thus there is only one degree of freedom. OLS is rejected if the test statistic *LR* is too large, that is, if it exceeds the appropriate chi-squared statistic.

Results for heteroskedasticity are summarized in tables 34 through 36. Each table summarizes results for the HET model from both puts and calls. Shown is the LR for calls, the LR for puts, and the estimated coefficient on *time*. T-statistics are shown in parentheses. If there is indeed heteroskedasticity in which the error variance is decreasing over time as a result of "learning" in the market, then it is expected that coefficient estimates on time would be negative and significantly different from zero.

For corn, OLS is rejected in August for both calls and puts based on the LR test at a significance level of alpha=.025. When considering the coefficients on *time*, however, *beta* is positive and significant, which is not the sign expected if error variance is decreasing over time.

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[33]It is actually the absolute value of the LR statistic which is of interest. If LLF(OLS) were greater than LLF(HET) then it would be necessary to multiply by -2 instead of 2.

Table 34. Heteroskedasticity for Corn, Ln(ep/bp) on Implied Volatility

	LR (Calls)	Time	LR (Puts)	Time
February	.8650	-.1349	********	********
		(-1.0000)	********	********
March	1.7078	-.1779	1.4828	-.1618
		(-1.5040)		(-1.3680)
April	1.0376	-.1426	1.1104	-.1453
		(-4.2426)		(-1.2290)
May	.8330	-.1532	.9498	-.1582
		(-1.2960)		(-1.3370)
June	.3750	-.1107	.3952	-.1173
		(-.9364)		(-.9918)
July	.4480	.0962	.5584	.1048
		(.8137)		(.8859)
August	5.9680*	.3380	5.9034*	.3393
		(2.8580)		(2.8690)
September	2.2156	.2122	2.3564	.2458
		(1.794)		(2.0780)
October	.0034	.0087	.0255	.0243
		(.0740)		(.2056)
November	1.1602	.2558	1.4308	-.1622
		(2.1630)		(-1.3720)

Note: Asymptotic t-values shown in parentheses
LR statistic is distributed $\chi^2_{(1)}$
* denotes OLS rejected based on LR test at alpha=.025 significance
Sample size includes 12 years of data for each month

Table 35. Heteroskedasticity Results for Soybeans, Ln(hp/pp) on Implied Volatility

	LR (Calls)	Time	LR (Puts)	Time
February	.2998	-.0653	.2232	-.0527
		(-.5523)		(-.4453)
March	.2278	-.0524	.3722	-.0698
		(-.4434)		(-.5900)
April	9.6886*	-.5968	7.5276**	-.4062
		(-5.046)		(-3.4350)
May	7.5438**	-.5174	5.5172***	-.4573
		(-4.3750)		(-3.8670)
June	1.1148	-.1142	2.8772****	-.1859
		(-.9652)		(-1.5720)
July	.7052	-.1348	2.2252	-.2320
		(-1.1390)		(-1.9620)
August	.1142	.0557	.0428	.0313
		(.4706)		(.2650)
September	.0678	.0399	.1562	-.0677
		(.3372)		(-.5725)
October	1.4426	.2343	2.7126****	.4377
		(1.9810)		(3.7010)

Note: Asymptotic t-values shown in parentheses
LR statistic is distributed $\chi^2_{(1)}$
Sample size includes 12 years of data for each month
* denotes OLS rejected based on LR test at alpha=.005 significance
** denotes OLS rejected based on LR test at alpha=.010 significance
*** denotes OLS rejected based on LR test at alpha =.025 significance
**** denotes OLS rejected based on LR test at alpha=.100 significance

Table 36. Heteroskedasticity Results for Spring Wheat, Ln (hp/pp) on Implied Volatility

	LR (Calls)	Time	LR (Puts)	Time
March	.0440	-.0383	.0963	-.0833
		(-.2458)		(-.5353)
April	.1688	.0846	2.1651	-.6702
		(.5434)		(-4.3040)
May	1.6152	-.2786	.0009	.0124
		(-1.7890)		(.0794)
June	6.8158*	-1.0575	4.2288**	.0751
		(-6.7920)		(.4826)
July	.0716	.0739	.1109	.0605
		(.4746)		(.3887)
August	.0440	.0534	.5052	.1248
		(.3430)		(.8018)

Note: Asymptotic t-values shown in parantheses
Sample size includes 12 years of data for each month
LR statistic is distributed $\chi^2_{(1)}$
* denotes OLS rejected based on LR test at alpha=.010 significance
** denotes OLS rejected based on LR test at alpha=.050 significance

For soybeans, April is rejected for calls based on the LR test at alpha=.005. Furthermore, the estimated coefficient on *time* is negative and significant. For April puts and May calls OLS is rejected at alpha=.01, for May puts at alpha=.025, and for June and October puts at alpha=.100. For October puts, however, the coefficient on *time* has the wrong (positive) sign.

For spring wheat, June calls are rejected based on the LR test at alpha=.010. Furthermore, the coefficient on *time* is negative and significant. June puts are rejected at alpha=.050, but *time* has the wrong sign.

Overall, the hypothesis that the futures options market is becoming better over time is not strongly supported. There are a few instances in which heteroskedasticity looks probable, such as in August for corn, April and May for soybeans, and June for spring wheat. When further considering the estimated coefficients on *time*, however, the likelihood of heteroskedasticity is narrowed down to April and May for soybeans and June for spring wheat, since it is expected that the coefficient on *time* would be negative if options markets were indeed improving over time.

Furthermore, with respect to crop insurance, there seems to be no improvement in February or March for the futures options market. Since February and March is around the sign up period, testing for heteroskedasticity seems most relevant in these months.

Overall, there is not much validity in the "learning" story for corn, soybean, and spring wheat futures options.

CONCLUSIONS

Summary of Results

This thesis tests important Black-Scholes distributional assumptions and also the informational content of corn, soybean, and spring wheat futures options prices as potential predictors of futures price changes. Data used included corn, soybean, and spring wheat futures price data from 1960 through 1996, corn and soybean futures options data from 1985 through 1996, and spring wheat futures options data from 1987 through 1996

There are several findings regarding assumptions maintained by the original Black-Scholes option pricing formulas (1973) and the subsequent extension to futures options prices by Black (1976). Black-Scholes assumes that futures prices are lognormally distributed with a constant variance rate parameter (implied volatility), and that current futures prices are unbiased estimates for later futures prices.

First, it is found that standard deviations of the natural logarithm of daily price changes calculated within months display seasonality across months. These historical standard deviations are low early in the year, rise and peak during mid-year (summer) months, and fall as the appropriate contract maturity month (December, November, and September for corn, soybeans, and spring wheat) approaches. This is consistent with futures price seasonality findings already in the literature. This seasonality contradicts Black-

Scholes constant volatility assumptions. One point that is stressed throughout this thesis is that tests of Black-Scholes assumptions are in actuality simultaneous tests of Black-Scholes model and the efficiency of the futures option market.

A second conclusion is that average futures prices for successive beginning periods starting with February are indeed unbiased estimates for ending period futures prices.

A third important finding which primarily concerns the Black-Scholes analysis and the Black formula is that the assumption of lognormality is widely rejected for corn, soybeans, and spring wheat futures prices. Lognormality is the most important distributional assumption maintained by the Black formula; rejection of lognormality is evidence which casts doubt on the validity of Black-Scholes distributional assumptions. It is also found that excess kurtosis is the main reason why lognormality is often rejected.

A fourth finding is that implied volatilities calculated from the Black formula display seasonal patterns. This seasonality is further evidence that Black-Scholes assumptions are inaccurate, since Black-Scholes assumes that the implied volatility parameter is constant.

Although there is seasonality in implied volatility, we find that implied standard deviation is fairly well behaved, i.e. decreasing over the course of a year. Although there are instances in which implied standard deviation peaks in mid-year, there is no significant evidence to support the notion that there is seasonality in these measures also. Overall, the futures options market appears to be "correcting" the seasonality in implied volatility.

A second set of results concerns the potential usefulness of agricultural futures options in predicting the variance in futures prices. Despite the fact that most Black-Scholes assumptions are statistically rejected, it is still hypothesized that implied volatilities may be useful in predicting the variability of futures prices. Mayhew points out that implied volatility may still be considered an optimal assessment of futures variability, and even if futures price volatility is known to be stochastic implied volatility may be interpreted as the market's average assessment of variability over the remaining life of the option.

First, an OLS regression of the absolute value of the natural logarithm of the ratio of beginning period to ending period futures prices on implied volatility was estimated. Overall, these implied volatilities are poor predictors of subsequent realized price changes for individual successive beginning periods starting with February and moved forward until one month prior to contract maturity.

As a possible explanation for the poor results of the "informational content" tests for corn, soybean, and spring wheat puts and calls, a heteroskedastic model is estimated, where error variance is hypothesized to be a function of market age (time). We are interested in whether the agricultural futures options market has improved over time, as these futures options have only traded for slightly more than a decade. Overall, heteroskedasticity is not significant for any of these three crops. There is no evidence to suggest that there has been "learning" or improved efficiency in these markets over time.

Implications

The empirical results in this thesis suggest the direction for future research, in order to evaluate the use of futures options prices to recover ex-ante probabilistic information about second and higher moments of futures price distributions. As discussed, applications of the general Cox and Ross (1976) option pricing formula (equation 5) may be used, and various distributions have been proposed as alternatives to the commonly used lognormal. Many of these alternate distributions have the advantage of providing more flexible parameterization than the lognormal. The lognormal is inflexible in that it does not allow for varying degrees of skewness and kurtosis, which may be better handled by other, more flexible distributions.

Sherrick, Irwin and Foster (1996) discuss the use of the BURR-XII distribution, which they point out has become a popular distribution for modeling prices and which has been utilized extensively by the insurance industry. Empirically, Sherrick, et. al. find that the BURR-XII generally performs better than the lognormal. Fackler and King also discuss the use of the BURR-XII (parametric) distribution as well as non-parametric methods.

Sherrick, Garcia, and Tirupattur (1996) propose the use of the BURR-III distribution, another "flexible" distributional candidate. They find that in depicting ex-ante price variability, the BURR-III substantially outperforms the lognormal.

Myers and Hanson (1993) suggest the use of a generalized autoregressive conditional heteroskedastic (GARCH) process. They recognize that the lognormal may be an inappropriate representation of futures price changes and also that volatility may be stochastic, thereby suggesting the use of the GARCH model. Myers and Hanson also point out that the GARCH model does a good job of modeling excess kurtosis and time-varying volatility, which are shown in this thesis to be two possible problems with the use of the Black-Scholes model and the assumption of lognormality. When testing a GARCH specification for CBOT soybean futures prices, Myers and Hanson find that the model estimates actual market prices significantly better than the Black formula, which further supports the need for more flexible distributional models.

A key point in this discussion concerns seasonality. Overall, it is found that historical futures price standard deviation and implied volatility display strong, systematic seasonal patterns. Neither the GARCH models nor the BURR distributions incorporate variables which account for such systematic seasonality; thus, seasonality presents serious challenges to the proposed models.

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APPENDIX

Table 37: Sample Size for Corn Call Futures Options Implied Volatilities

	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov
1985	2	18	18	21	18	20	22	19	22	6
1986	18	20	15	19	18	21	20	21	23	5
1987	16	22	21	20	22	22	20	21	22	15
1988	19	21	20	21	22	20	23	21	21	14
1989	17	22	19	21	21	20	23	20	22	13
1990	19	22	20	22	21	21	23	19	23	16
1991	17	20	20	21	18	22	22	20	23	11
1992	18	22	17	19	22	22	21	21	22	15
1993	14	20	20	19	21	21	22	21	21	15
1994	18	21	19	21	22	20	23	21	21	14
1995	19	23	19	22	22	20	23	20	22	13
1996	20	21	21	22	20	22	22	20	23	11

Table 38: Sample Size for Corn Put Futures Options Implied Volatilities

	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov
1985	2	14	19	21	20	22	22	20	23	6
1986	19	20	22	21	21	22	21	21	23	5
1987	18	17	20	19	19	22	21	21	22	15
1988	12	12	18	19	19	20	23	20	21	14
1989	14	19	19	22	21	20	23	20	22	13
1990	19	20	20	19	20	21	23	19	23	16
1991	10	19	19	22	10	22	22	20	23	11
1992	10	17	17	19	19	22	21	21	22	15
1993	8	13	21	20	22	21	22	21	22	15
1994	15	20	19	21	22	20	23	21	21	14
1995	19	22	19	22	22	20	23	20	20	13
1996	19	21	20	21	20	22	22	20	23	11

Table 39: Sample Size for Soybean Call Futures Options Implied Volatilities

	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct
1985	14	18	19	21	20	22	22	20	9
1986	15	19	21	21	18	20	18	19	8
1987	16	20	21	20	22	22	21	21	17
1988	19	23	20	21	22	20	23	21	15
1989	17	17	20	21	21	19	22	20	15
1990	19	22	20	22	21	21	23	19	15
1991	14	18	14	17	14	21	22	20	13
1992	16	20	17	20	22	22	21	21	17
1993	18	18	21	20	22	21	22	21	15
1994	18	23	18	21	22	20	23	21	15
1995	19	23	19	22	22	20	23	20	15
1996	20	21	21	22	20	22	22	20	14

Table 40: Sample Size for Soybean Put Futures Options Implied Volatilities

	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct
1985	13	19	13	22	19	22	22	20	9
1986	10	16	16	19	18	20	21	21	13
1987	17	15	18	18	20	22	21	21	17
1988	9	13	14	18	22	20	23	19	15
1989	9	16	18	19	22	20	23	20	15
1990	15	18	17	22	21	21	23	19	15
1991	6	8	14	22	20	21	22	20	14
1992	12	14	14	17	19	22	21	21	17
1993	15	13	12	12	21	19	22	21	16
1994	12	15	16	19	20	20	23	21	15
1995	14	12	15	22	18	16	23	20	15
1996	17	17	19	22	19	21	22	19	14

Table 41: Sample Size for Spr. Wheat Call Futures Options Implied Volatilities

	Feb	Mar	Apr	May	Jun	Jul	Aug
1987	3	13	15	19	16	11	13
1988	5	6	11	19	22	20	15
1989	10	18	17	22	19	20	13
1990	7	5	8	14	17	17	18
1991	3	11	19	18	18	20	17
1992	12	17	16	15	22	22	9
1993	4	13	19	20	21	21	15
1994	2	17	17	21	21	20	15
1995	9	18	19	22	22	20	14
1996	11	18	21	22	19	21	17

Table 42: Sample Size for Spr. Wheat Put Futures Options for Implied Volatilities

	Feb	Mar	Apr	May	Jun	Jul	Aug
1987	2	12	11	12	18	22	13
1988	2	3	3	16	19	17	15
1989	4	11	7	14	17	19	14
1990	3	7	9	17	20	20	12
1991	2	6	11	18	20	22	15
1992	11	12	13	16	21	22	15
1993	5	20	19	20	22	21	14
1994	13	8	15	18	17	19	14
1995	0	14	18	16	13	20	14
1996	8	8	15	16	19	22	17