Constitutive laws of composite materials via multi-axial testing and numerical optimization
by Dongfang Huang

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering
Montana State University

Abstract:
The use of composite materials in structural components has increased dramatically in recent years. As applications become more demanding, the need for reliable prediction of their mechanical properties is increasing as well.

Four different loading paths were applied with multi-axial testing, and the corresponding FEM modeling was created. After carrying out optimization, the optimum engineering constants were obtained. This set of optimum properties is very close to those obtained with standard unidirectional testing. Fewer tests are required, less time is taken, and more precise properties are gained with this procedure.

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APPROVAL

Of a thesis submitted by

Dongfang Huang

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

The use of composite materials in structural components has increased dramatically in recent years. As applications become more demanding, the need for reliable prediction of their mechanical properties is increasing as well.

Four different loading paths were applied with multi-axial testing, and the corresponding FEM modeling was created. After carrying out optimization, the optimum engineering constants were obtained. This set of optimum properties is very close to those obtained with standard unidirectional testing. Fewer tests are required, less time is taken, and more precise properties are gained with this procedure.

CHAPTER 1

INTRODUCTION

The use of composite materials in structural components has increased dramatically in recent years as their cost of production continues to decline and advances in composite design methodology become increasingly widespread. As applications become more demanding, the need for reliable prediction of their mechanical properties and behavior is becoming ever more important [1].

As a result of the increased demands on the structural applications of composites, efforts have been made to investigate the mechanical properties in composite materials at structural levels [1]. However, familiarity with orthotropic behaviors is needed to characterize the material behavior of composites. These behaviors require detailed and well organized test methods to determine the associated laminate properties.

The Montana State University (MSU) composite team built an In Plane Loader (or IPL). The IPL is designed and built for gaining a better understanding of material properties, especially composite material properties. The IPL is a highly computerized automatic machine which can collect the reaction forces and displacements at specific points when applying a variety of pre-selected loading paths. Moreover, the IPL built by MSU allows the low cost, simple, and easy experiments to be used for acquiring composite properties. The IPL in MSU composite lab is illustrated in Figure 1.01.

On the other hand, there are some numerical approaches for predicting composite properties as well. Finite element method (FEM) modeling is an appropriate way to gain the knowledge of material behavior by means of simulation. ANSYS, the powerful computer program, was selected to accomplish FEM modeling. [3] Besides the FEM model, two numerical techniques, bounding techniques and optimization, were also used to determine the composite properties.

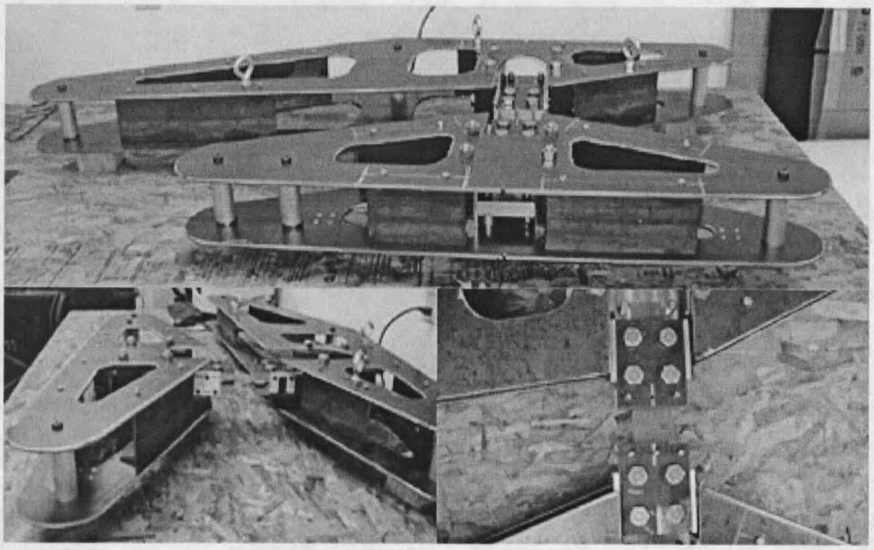

Figure 1.01 The In-Plane Loader machine

Although many numerical approaches have been developed to predict composite properties, these may not be accurate and must be verified with experimental results. Namely, the FEM modeling results have to be compared with the experimental data obtained by the IPL machine. If both data match, the FEM modeling results can be considered accurate. If not, an optimization procedure can be carried out in order to find values that optimize the difference between FEM modeling and experimental data.

This methodology may be a more effective approach than the traditional ones used to estimate a composite's properties. The current research provides an investigation into the efficiency, cost, and precision of this methodology to determine composite properties.

Motivation

Conventional approaches for determining the constitutive law of composites require conducting multiple experiments. Significant time is often spent collecting and analyzing the experimental data in order to estimate the material constitutive properties. However, convention characterization procedure is time-consuming, tedious, labor-intensive, inaccurate, and often prohibitive to complete characterization.

A more practical approach, which is more efficient and accurate, is strongly desired. This more desirable approach would involve limited experiments and would yield a more precise constitutive law by employing multiple loadings on a simple sample combined with numerical techniques, such as finite element analysis, optimization, and bounding techniques.

The constitutive behavior of a composite laminate in structural components is complicated. The lamina properties can be estimated by micromechanical analysis procedures, and lamina properties can be measured by physical means in a macromechanical analysis of the structure. In this work, micromechanical analysis deals with the individual constituent materials mechanical response to determine the equivalent combined continuum mechanical response. Real design power is realized when the micromechanical estimate of the properties of a lamina agree with the measured properties. However, it should be noted that a micromechanical analysis has inherent limitations. The micromechanical theories must be validated by careful experimental work [2].

Because this discrepancy exists between the micro-structure of the composites in practical analysis and the assumed perfect micro-structure of composites in theoretical analysis, it is inevitable that the engineering constants of composites estimated by micromechanical analysis will not agree with the experimental data of the physical tests. The purpose of this study is to find more accurate engineering constants which agree well with the experimental data. Several numerical analysis techniques are employed to explore whether or not the approach is feasible.

Objective and Approach

Our goal of this research is to find an appropriate methodology to determine the composite laminate properties. Note that this research is restricted to linear elastic behavior.

Micromechanical analysis and macromechanical analysis are used in numerical modeling and experimental approaches respectively. Furthermore, the mechanics of materials approach and elasticity approach are employed to estimate the engineering constants of a composite, to examine the feasibility of the technique, and determine the upper and lower bounds of each engineering constant.

Coupons with a notch were selected to determine the composite constitutive law by means of the IPL tests and mathematical modeling.

As mentioned above, the MSU composite team has built the IPL in the materials lab. The IPL machine is used to conduct the tests and to obtain the reaction forces and displacements of a specific point in each test specimen.

FEM modeling is used to simulate the experiments on the IPL and to gain the corresponding reaction forces and displacements at a specific point. ANSYS was the program used for the FEM modeling.

The IPL is a testing machine capable of providing any combination of in-plane loads on a sample. Four loading paths were designed and applied to the coupons in this research. These were tensile (open/close), tensile + shear (sliding), rotation (moment), and tensile + shear + rotation. Upon applying the loading sequences to the coupons, data of reaction forces and displacements were acquired. Meanwhile, the FEM model was used to calculate the corresponding reaction forces and displacements as well. The optimization procedure was needed if there was a discrepancy between the results of FEM modeling and experiments.

The reaction forces and displacements depend on the composite properties, geometry and the specific loadings. The following function depicts the relationship.

$$F_n^r, \delta_n^r = F_n^r, \delta_n^r (E, G, \upsilon, g, f, \delta) \qquad (1.01)$$

where E, G and υ are the elastic moduli of fiber reinforced composite, g stands for the specific geometry of the sample, f and δ are respectively the applied forces and applied displacements, and F_n^r and δ_n^r are the reaction forces and displacements at the specific nodes respectively.

The optimization is accomplished by minimizing the discrepancy between the FEM modeling results and the experimental results.

$$\left| \left(F_n^r, \delta_n^r\right)_{Computer} - \left(F_n^r, \delta_n^r\right)_{Experiment} \right| = e \qquad (1.02)$$

where $\left(F_n^r, \delta_n^r\right)_{Computer}$ is the data of reaction forces and displacement obtained by computer modeling, $\left(F_n^r, \delta_n^r\right)_{Experiment}$ is the reaction forces and displacement obtained by experiments, and e is the error to be minimized.

The objective of this research is to determine the constitutive law (or engineering constants) of composites by means of several numerical analysis techniques and experimental data by IPL. The operation procedure is illustrated in Figure 1.02.

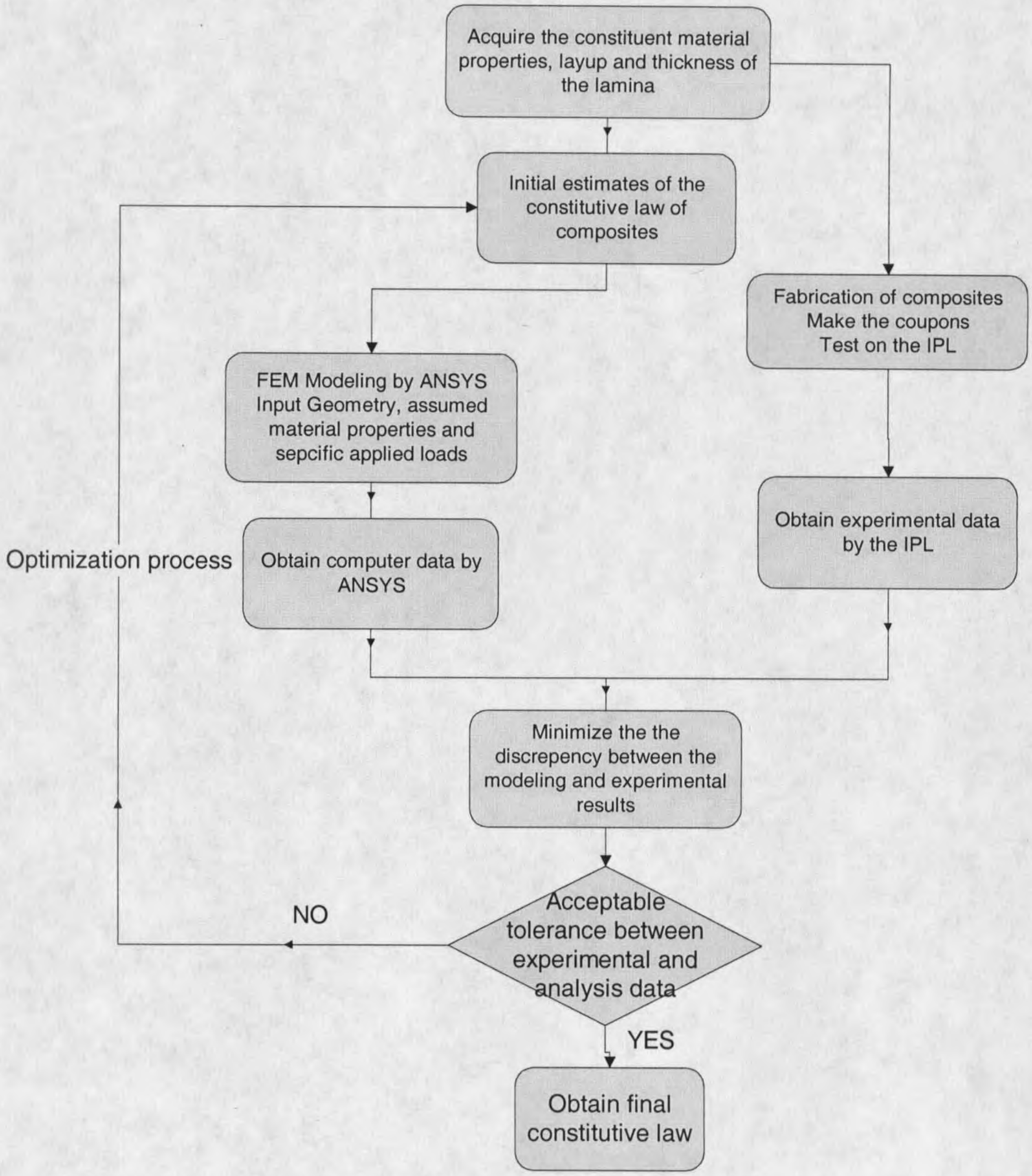

Chart 1.02 Flow chart of determining the composite constitutive properties

Organization of Thesis

In Chapter 2, theories of elasticity and mechanics of composite materials are introduced briefly as these theories are used to determine the initial constitutive law of composites with the aid of the numerical analysis techniques. In chapter 3, the fabrication of composite samples and experimental procedures are described. In Chapter 4, the details of numerical approaches of composite engineering constants are demonstrated, and finite element analysis modeling, optimization and bounding techniques are described. In chapter 5, experimental results are presented, numerical results and experimental data are compared. In chapter 6, the conclusions from this study are presented as well as a list of items that should be included in future studies.

CHAPTER 2

BACKGROUND

This research involves elasticity theory and focuses upon composite materials. The theory of elasticity and mechanics of composite materials are introduced briefly in this chapter. A key concept is that the micromechanical estimation of the engineering constants of a composite must agree with the experimental data, thus leading to an approach of determining more accurate engineering constants by means of a combined empirical and numerical analysis technique. The In-Plane Loader machine, which is designed to facilitate the testing of mechanical behaviors of composites is described in this chapter as well.

Introduction to Linear Elastic Solids

Concepts and Definitions

The current research is limited to linear elastic behavior. Generalized anisotropic Hooke's constitutive law states [3]

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2.01}$$

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} \tag{2.02}$$

where, C_{ijkl} is the elastic stiffness tensor,

S_{ijkl} is the elastic compliance tensor,

σ_{ij} is the second order stress tensor,

ε_{kl} is the second order strain tensor.