



Pressure drop for flow of air and grain mixtures in circular pipes of two diameters
by John J Cassidy

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering at Montana State College
Montana State University
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Abstract:

The results of experimental measurements made on the pressure' drop incurred during the flow of mixtures of air and various weight rates of wheat flow in circular pipes of two diameters are given in this paper in both graphical and tabular form.

The pressure drop due to the air alone and wheat alone are separated and the calculated values found for the pressure drop due to the solid particles alone is tabulated in graphical form for each of the experimental 'measurements.

An equation which seems to account for the pressure drop due to the solid particles alone is derived by dimensional analysis. Friction factors for use with this equation are calculated from the original measurements and are given graphically for use in the derived equation.

The results of this research are compared with that of Mr. J. W. Crane who conducted a similar project. The work of other researchers along the lines of this project has been reviewed and discussed.

The physical properties of the wheat used in this research are listed in an appendix.

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Approved:



Head, Major Department



Chairman, Examining Committee



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ABSTRACT

The results of experimental measurements made on the pressure drop incurred during the flow of mixtures of air and various weight rates of wheat flow in circular pipes of two diameters are given in this paper in both graphical and tabular form.

The pressure drop due to the air alone and wheat alone are separated and the calculated values found for the pressure drop due to the solid particles alone is tabulated in graphical form for each of the experimental measurements.

An equation which seems to account for the pressure drop due to the solid particles alone is derived by dimensional analysis. Friction factors for use with this equation are calculated from the original measurements and are given graphically for use in the derived equation.

The results of this research are compared with that of Mr. J. W. Crane who conducted a similar project. The work of other researchers along the lines of this project has been reviewed and discussed.

The physical properties of the wheat used in this research are listed in an appendix.

INTRODUCTION

PURPOSE

Theory concerning the pressure losses incurred in the pneumatic conveying of grain have been practically non-existent. Pneumatic grain conveying installations have been designed by empirical formulas or by rules of thumb.

The purpose of this thesis was to study the pressure drop encountered in the flow of air and wheat mixtures and to develop a theory which would account for these pressure losses. The theory as developed was substantiated over the range of experimental results.

It was planned that the initial construction of this equipment would provide the beginning of a continuing graduate research program at Montana State College in the flow of air and solid mixtures.

IMPORTANCE

The use of pneumatic methods in conveying grain and other solids has increased rapidly during the past several years.¹ The types of materials being handled by pneumatic processes has also increased widely. These materials vary widely in particle size, particle shape, and density. This increased use of pneumatic conveying will require increased research activity concerning the theory involved in the process, if pneumatic conveying systems are to be designed with the confidence which a firm knowledge of well founded theory can give to a design.

1. Rhodes, T. W., "Air As A Conveying Medium For Handling Materials." Industrial and Engineering Chemistry, (October 1953), Page 119-120.

Present empirical methods of design require the knowledge of the characteristics of some previously constructed system. For someone designing a system which will convey a material which has not been handled pneumatically prior to that time it would be difficult, if not impossible, for him to design a system which would operate as designed. If, however, a theory could be established which would explain the characteristics of pneumatic conveying through the use of the variables concerned, then the designer would need only to determine the nature of the variables concerned in his particular problem and apply this theory. This would undoubtedly save much money spent on the design of systems which may prove to be uneconomical after being put into operation.

PREVIOUS WORK

The vast majority of the work which has been done in investigating the pressure drop due to the pneumatic conveying of solids has been of an empirical nature. Many of the investigators have had a particular situation in mind during their investigations and have come up with results which for that particular situation have been sufficient for them to perform the required design.

The most comprehensive work of this nature was that of Segler.² Segler performed laboratory work over a period of several years during which he collected data on such grains as oats, wheat, barley, and flax.

2. Segler, Dr.-Ing. G. 1951. "Pneumatic Grain Conveying." National Institute of Agricultural Engineering. Braunschweig, Germany.

He measured pressure drops in straight pipes, around bends, and during periods of particle acceleration after the grain had been introduced into the air stream.

Segler presented the following empirical formula.

$$h_s = \lambda L V^2$$

In this equation, h_s is the pressure drop due to the simultaneous flow of air and grain, (L) is the pipe length, (V) is the air velocity, and (λ) is a constant depending upon the type of grain, the pipe diameter, and the weight rate of flow of the grain. He has listed values of (λ) for many different grains, pipe sizes, and air velocities. No theory has been presented in support of this equation.

Segler also listed values of equivalent pipe lengths which can be used to predict pressure drops around bends and pressure drops during acceleration of particles after they have been introduced into the air stream. He suggested that the pressure drop be calculated for these equivalent pipe lengths by the above equation.

Segler also presented a rather thorough discussion of the operating characteristics of various types of feeders.

All of Segler's work has been empirical in nature, and the values he has suggested for (λ) are the results of many tests on many pipe diameters using various grains, varying air velocities, and varying weight rates of flow of grain.

Vogt and White³ have done considerable laboratory work on the pressure loss due to the conveyance of solids in an air stream.

They have suggested the following equation for the determination of this head loss.

$$(\beta - 1) = a \left(\frac{D}{D_p} \right)^2 \left(\frac{\rho_f R}{\rho_s N_R} \right)^k$$

where β is the ratio of the head loss due to the flow of air and solids to the head loss due to the flow of air alone, and "a" and "k" are functions of the dimensionless group.

$$\sqrt{(\rho_s - \rho_f) g \rho_f D_p^3 / 3\mu}$$

Values for (a) and (k) have been listed graphically in their paper for both vertical and horizontal flow. (Dp) is the effective diameter of the particles and has been defined as the diameter of a sphere of equal surface area, (ρ_s) is the mass density of the solid material, (ρ_f) is the mass density of the conveying fluid, (D) is the diameter of the pipe, (g) is the acceleration due to gravity, (R) is the ratio of the weight rate of solid flow to the weight rate of fluid flow, and (μ) is the viscosity of the conveying fluid. The determination of (Dp) has presented considerable difficulty, since for irregular particles it is extremely difficult to measure surface area. The work of Vogt and White has been used quite extensively in the Chemical Engineering Industry.

3. Vogt, E. G., and White, R. R. "Friction in the Flow of Suspensions" Industrial and Engineering Chemistry. Vol. 40, Page 1731-1738, 1948.

Pinkus⁴ was the first to attempt to develop a theory concerning the pressure drop due to the flow of air and solid mixtures. In his work he attempted to use the Fanning Equation⁵ to account for the pressure drop. He studied the pressure drop due to the flow of air and sand mixtures in a 2.00 inch steel pipe. His observations proved that the Fanning Equation was not valid within the range of his experiment. From his observations, Pinkus concluded that the pressure drop due only to the solids phase of the flow was linearly proportional to the velocity of the solid particles. He also concluded that the ratio of the velocity of the solids to the velocity of the conveying fluid was a constant. However, he was unable to measure the velocity of the solid particles and thus was unable to prove this last conclusion. Pinkus' research was done using a single horizontal pipe of 2 inch diameter. He used an auger type feeder to feed sand into the air stream. Considerable trouble was experienced with this type feeder. The feeder was subject to plugging and binding while carrying the solid material.

Pinkus graphed the head loss due to the solid particles alone against air velocity, obtaining a linear relationship. He was able to solve for the pressure drop due to the solid particles alone by subtracting the pressure drop due to the air alone. In doing this, he assumed that the pressure drop due to the fluid phase was unaffected by the addition of the solids and thus was a function of the rate of fluid flow alone.

-
4. Pinkus, O. "Pressure Drops in Pneumatic Conveyance of Solids." *Journal of Applied Mechanics*. Vol. 19, Page 525-531, Dec. 1952.
 5. Fanning, J. T. "A Treatise on Hydraulic and Water Supply Engineering." 4th Ed. D. Van Nostrand, New York. 1884.

In his observations, Pinkus found that the pressure drop due to the acceleration of the solid particles after their introduction to the air stream occurred for only a distance of approximately 8 inches downstream in the conveying pipe.

The next attempt to derive a theory to explain pressure drop was done in 1956 by Crane⁶. Crane patterned his theory after that of Pinkus. However, in place of the Fanning Equation, Crane used the Darcy Weisbach equation to account for the pressure drop due to the grain colliding with the pipe wall and with other grain particles. He also included a term to account for the pressure drop due to the increase in elevation of the solid particles in an inclined or vertical pipe. The equations which Crane developed are as follows.

An equation for pressure drop:

$$\Delta H = \frac{f_s V_s L G_s}{2 D^3 g \gamma_{H_2O}} + \frac{G_s L \sin \theta}{V_s \gamma_{H_2O}} + \frac{f_A L_D V_A^2 \gamma_A}{2 D g \gamma_{H_2O}} \quad (1)$$

An equation for velocity of the solid particles:

$$V_s = \frac{\frac{V_A^2 \gamma_A C_{AP}}{2 W_P} - g \left(\frac{\gamma_P - \gamma_A}{\gamma_P} \right) \sin \theta}{\frac{\gamma_A C_{AP} V_A}{2 W_P} + \sqrt{\frac{\gamma_A C_{AP} g \sin \theta}{2 W_P} + \frac{V_A^2 f_s \gamma_A C_{AP}}{4 D D W_P} - g \left(\frac{\gamma_P - \gamma_A}{\gamma_P} \right) \frac{f_s \sin \theta}{2 D}}$$

And an equation for the friction factor (f_s):

$$f_s = \frac{D [\gamma_A C_{AP} (V_A - V_s)^2 - 2g V_P \gamma_P \sin \theta]}{V_P \gamma_P V_s^2}$$

6. Crane, J. W. "Predicting Pressure Drop in the Pneumatic Conveying of Grains." A Thesis. Michigan State University, 1956.

Where (Δh) is the pressure drop in feet of water, (V_s) is the velocity of the solid particles, (L) is the length of the pipe, (G_s) is the weight rate of flow of solids per square foot of the cross-sectional area of the conveying pipe, (D) is the diameter of the conveying pipe, (γ_{H_2O}) is the specific weight of water, (θ) is the angle of inclination of the conveying pipe, (f_A) is the friction factor in the Darcy Weisbach equation, (γ_A) is the specific weight of the conveying fluid, (V_A) is the velocity of the conveying fluid, (γ_p) is the absolute specific weight of the particles, (W_p) is the weight per particle of the grain, (C) is the drag coefficient for a sphere with a diameter equal to the smallest diameter of a wheat particle, (A_p) is the smallest projected area of a wheat particle, and (V_p) is the absolute volume of a wheat particle.

Crane measured the pressure drop due to the conveying of soft white spring wheat in a 3.58 inch diameter aluminum pipe at various weight rates of solid flow and at various pipe inclinations. His results seemed to substantiate his theory, except at air velocities over 100 fps.

In the derivation of these equations, Crane assumed that the drag coefficients for wheat particles in an air stream were the same as those for spheres. Crane computed Reynolds Numbers for the wheat particles by using the smallest of the minor diameters of the wheat particle for "D" in the equation, $N_R = VD\rho/\mu$. He then selected

drag coefficients for spheres using these Reynolds Numbers. This appears approximately correct if all wheat particles were oriented with their longitudinal axes parallel to the flow. However, this assumption seems difficult to justify since in the flow of the air and solid particles, the particles are continually colliding with one another and with the pipe wall.

In his thesis, Crane has given values of his coefficient (f_s) for various air velocities. These velocities are in a range of Reynolds Numbers from 80,000 to 170,000. The coefficients are for one type of light spring wheat only.

Crane used a revolving bucket type feeder with considerable success. His solid particles were introduced into the air stream through fixed plate orifices. He made attempts to measure the velocity of the solid particles by photographic means and by attempts to measure the density of dispersed particles within the pipe. However, he was unsuccessful in both attempts. In the photography attempt, he found that in the high speed photographs which he took of the flow, there were so many particles in the picture that it was impossible to trace a single one and thus compute its velocity.

His attempt to measure the density of the dispersed solids was made by installing two automatic gates which closed instantaneously and simultaneously, thus isolating a 5 foot length of pipe. He then removed this section and weighed the grain content. His results were extremely erratic. Crane assumed that his gates were not closing simultaneously;

however, the visual observations made of the solids flow, done in this thesis, show that for air velocities in the range of those reported by Crane, the particles are not dispersed uniformly within the pipe.

Hariu and Molstad⁷ measured the pressure drop due to various chemical powders in vertical conveying. However, they unknowingly included pressure losses due to the acceleration of the conveyed particles as they left a vertical bend in their apparatus. Their measurements were made in 1/4 inch and 1/2 inch diameter glass tubes which were 12 inches long.

Belden and Kassel⁸ made experimental measurements which correlated well with those of Hariu and Molstad for particles up to 0.10 inches in diameter conveyed in lines of large diameters. Here again their approach to the problem was empirical.

Many other experimenters have attacked the problem of explaining pressure losses due to the flow of air and solid mixtures. Practically all of these experimenters have attacked the problem in an empirical manner. The work of these experimenters is much too voluminous to be described here; however, a rather thorough bibliography has been included at the end of this thesis.

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7. Hariu, O. H., and Molstad, M. C., "Pressure Drop in Vertical Tubes in Transport of Solids by Gases." Industrial and Engineering Chemistry Vol. 41, Page 1148-1160, 1949.
 8. Belden, D. H., and Kassel, L. S., "Pressure Drops Encountered in Carrying Particles of Large Diameters in Vertical Transfer Lines." Industrial and Chemical Engineering. June 1949.

To date, no theory has been evolved which will explain all of the factors involved in pneumatic conveying.

Rouse⁹ has alluded to this in his treatment of the problems involved in transportation of discrete matter. The variables involved in sediment transportation are basically the same as those in the problem of pneumatic conveying. He says that generally speaking the problem is a function of the following type.

$$f(L, V, \rho, \mu, d, \sigma_d, \Delta\gamma, C) = 0$$

Where (L) is a linear term to define the boundary scale, (V) is the fluid velocity, (ρ) is the mass density of the fluid, (μ) is the viscosity of the fluid, (d) is a term to account for particle size, (σ_d) is the standard deviation about the particle size, ($\Delta\gamma$) is the effective specific weight of the discrete matter, and (C) is the concentration of the discrete matter in the fluid.

Rouse writes: "With its eight variables, such a function is obviously too complicated to study as a whole, even though it represents about as simple an aspect of such motion as can be found. The solution is seldom a complete one, however, and deviations are to be expected as the range of the investigation is exceeded. It is for this reason that such occurrences as sediment transportation along the bed of a channel have been among the last to be brought out of the realm of pure empiricism."

9. Rouse, Hunter. "Advanced Mechanics of Fluids." 1959. John Wiley and Sons, New York.

DESIGN AND CONSTRUCTION OF EXPERIMENTAL APPARATUS

COMPONENTS OF THE GENERAL SYSTEM

Almost any type of solid material which can be conveyed pneumatically could have been used in the experimental research reported here. Wheat was chosen primarily because many farming concerns have been using pneumatic conveyors to handle wheat in this western area for some time. It was also desirable to use wheat because of its relative cleanliness during conveying. The experimental equipment was set up in the Civil Engineering Laboratory in Ryon Laboratory at Montana State College. Laboratory classes and other projects were being conducted in the laboratory simultaneously with this research which made a material that was relatively clean desirable.

The equipment with which the experimental research was conducted was built during the summer of 1959. The basic system consisted of the following components:

- 1.) A conical shaped hopper which discharged the grain into the rotary feeder.
- 2.) A rotary-air-lock type feeder which received wheat from the hopper mounted above and discharged it into the conveying pipe.
- 3.) Two complete sets of test pipes, one with an inside diameter of 2.06 inches, and the other with an inside diameter of 1.63 inches.

- 4.) A 3450 rpm, electrically driven, impeller type blower, rated at 240 cfm of air, with discharge pressure of 27.7 inches of water.

The general location, arrangement, and overall dimensions of the experimental system are shown in Fig. 1 on Page 20.

Because the length of time required to make the necessary measurements during an experimental run was quite long, it was decided to construct the system so that the wheat would circulate continuously, thereby making the manual refilling of the hopper unnecessary. The system was therefore designed so that the wheat was blown through the test pipe and then back into the hopper for recirculation.

THE FEEDER

A rotary type feeder was used in the research reported here. This type was selected primarily because of its successful use by Crane¹⁰.

The characteristics of this type of feeder have been described in detail by Segler¹¹. The principle of the feeder is basically that of a revolving door. It allows the solids to enter the feeder from a low pressure area above the feeder and then discharges them into the high pressure area of the conveying pipe, theoretically with minor losses of air.

10. Crane, J. C. Op. Cit. P. 45.

11. Segler, G. Op. Cit. P. 73.

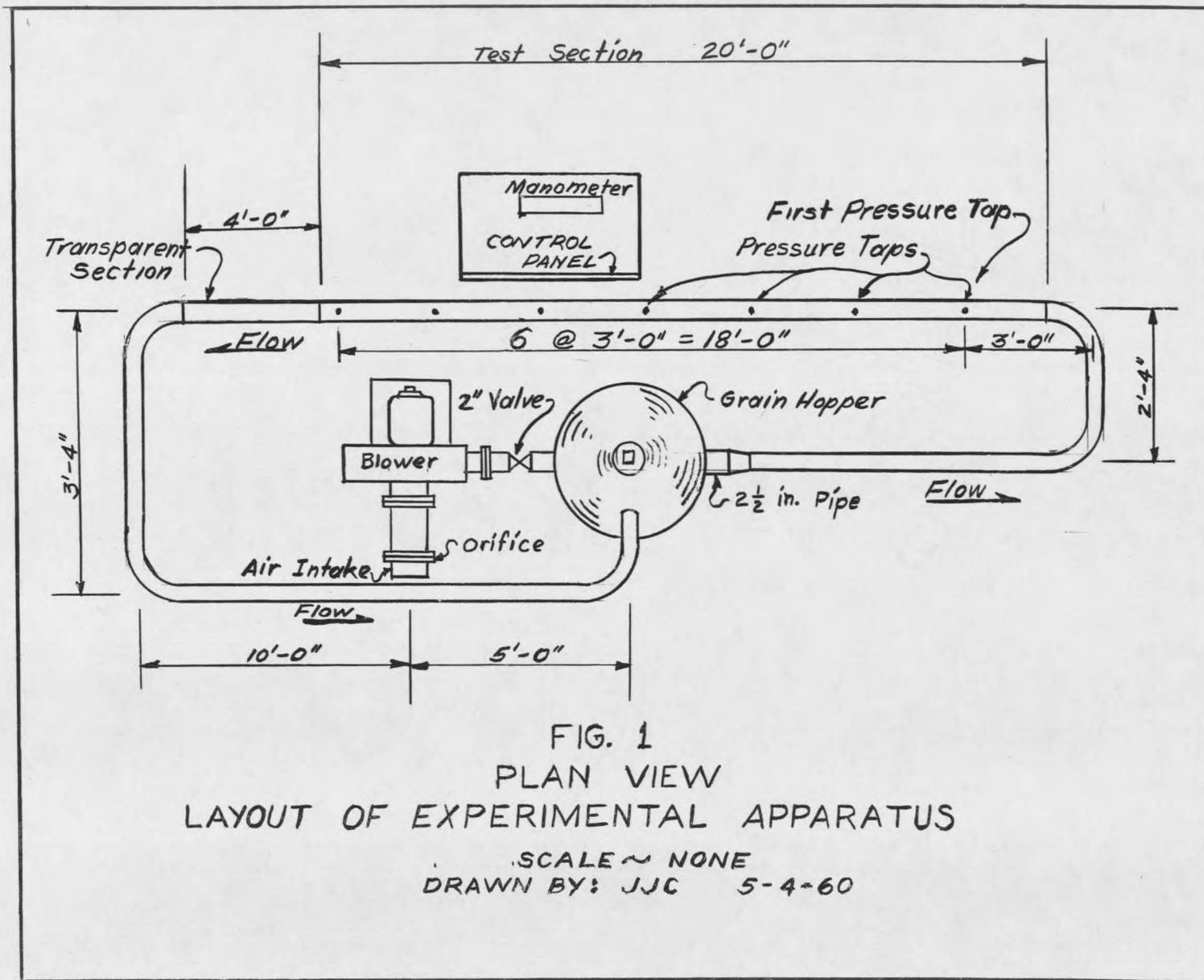


FIG. 1
 PLAN VIEW
 LAYOUT OF EXPERIMENTAL APPARATUS

SCALE ~ NONE
 DRAWN BY: JJC 5-4-60

A drawing of the general design of the feeder is shown in Fig. 2, Page 22. The assembled feeder is also shown in Fig. 3. The body of the feeder was made from a 12-inch length of 10-inch diameter standard steel pipe. The remainder of the feeder parts are described in the drawing in Fig. 2. It was decided that the opening in the top of the feeder through which the hopper discharged the grain into the feeder should be made larger than the bottom of the hopper. Any air which leaked through the feeder would thereby pass around the hopper and would not affect the weight rate of discharge of the grain from the hopper as it would if this leakage of air escaped upward through the hole in the hopper through which the grain discharged.

The feeder was propelled by a 1/2-horsepower, 110 volt electric motor. The drive connection was made by using type A V-belts. An 18-inch diameter V-belt pulley was attached to the feeder shaft. The motor drove the feeder through a reducing idler shaft. The total reduction in speed between the motor and the feeder was 19.3 to 1, with the motor turning at 1750 rpm, while the feeder turned at 91 rpm.

During operation it was found that the feeder leaked excessively and since the quantity of air flowing was measured ahead of the feeder, this was a serious problem. With this leakage, the air velocity in the pipe could not be obtained with any accuracy. The feeder was rebuilt several times, but the problem of leakage was not overcome. Finally it was decided that the amount of leakage through the feeder would have to be measured.

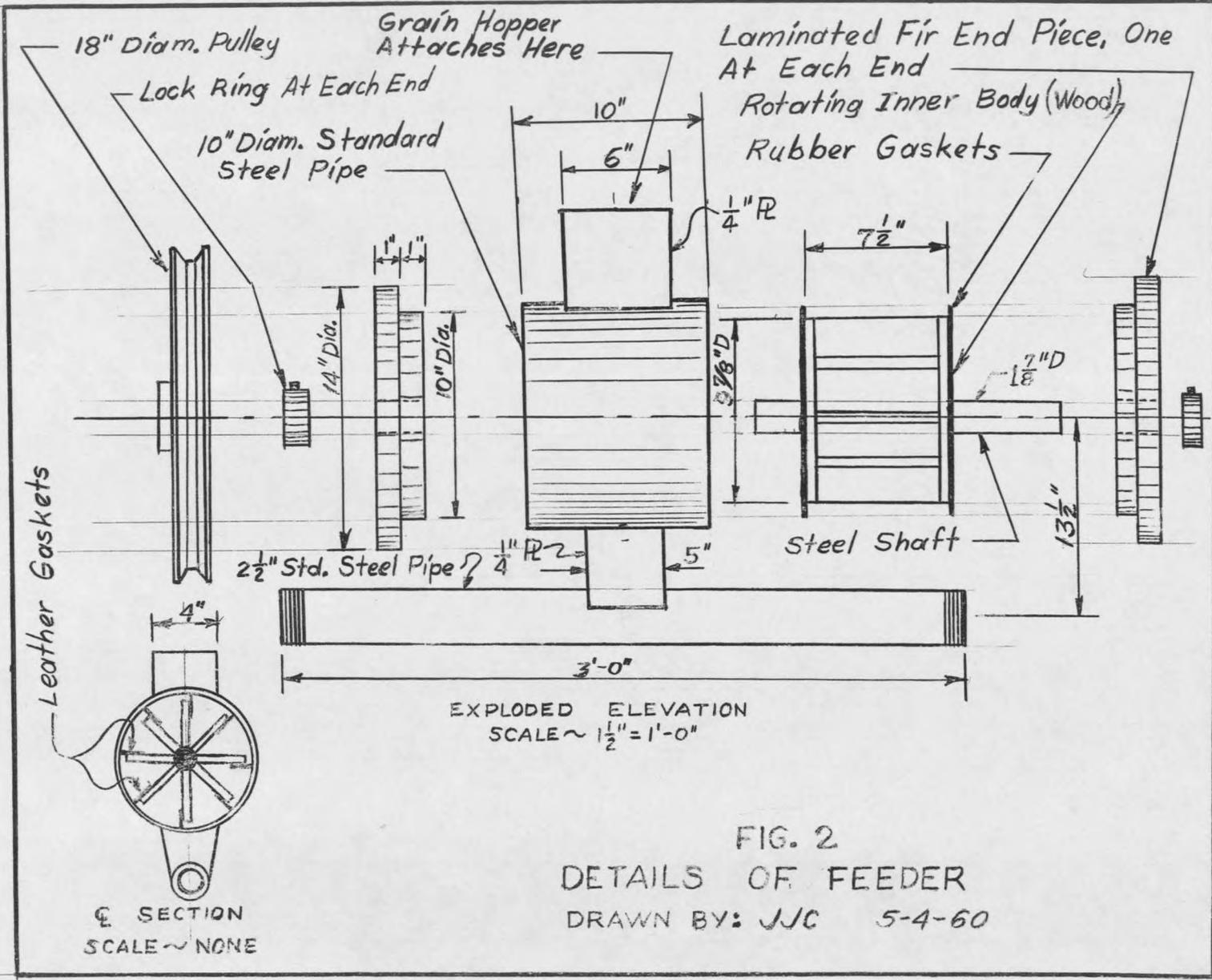


FIG. 2
 DETAILS OF FEEDER
 DRAWN BY: JJC 5-4-60

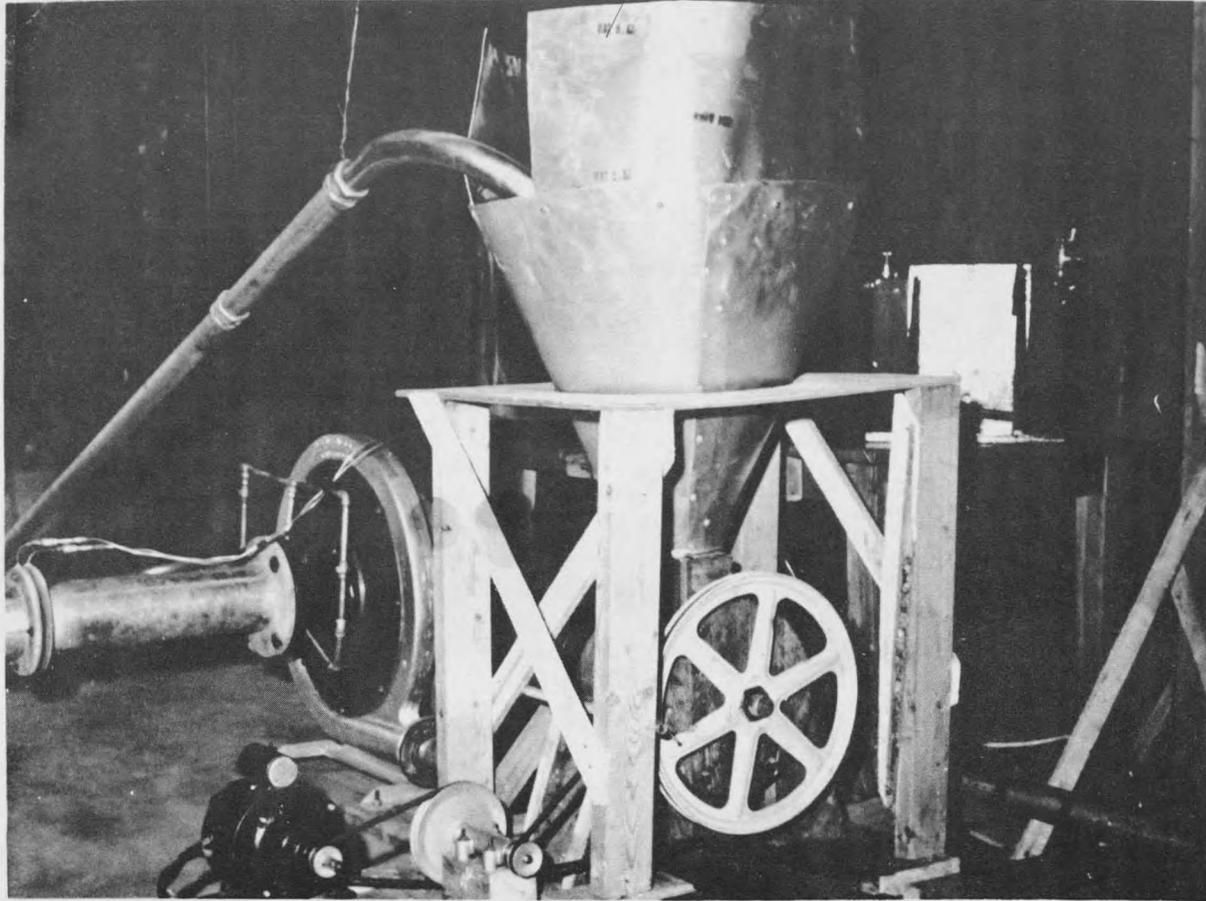


Fig. 3. Grain Hopper in Place and the Drive System for the Feeder.

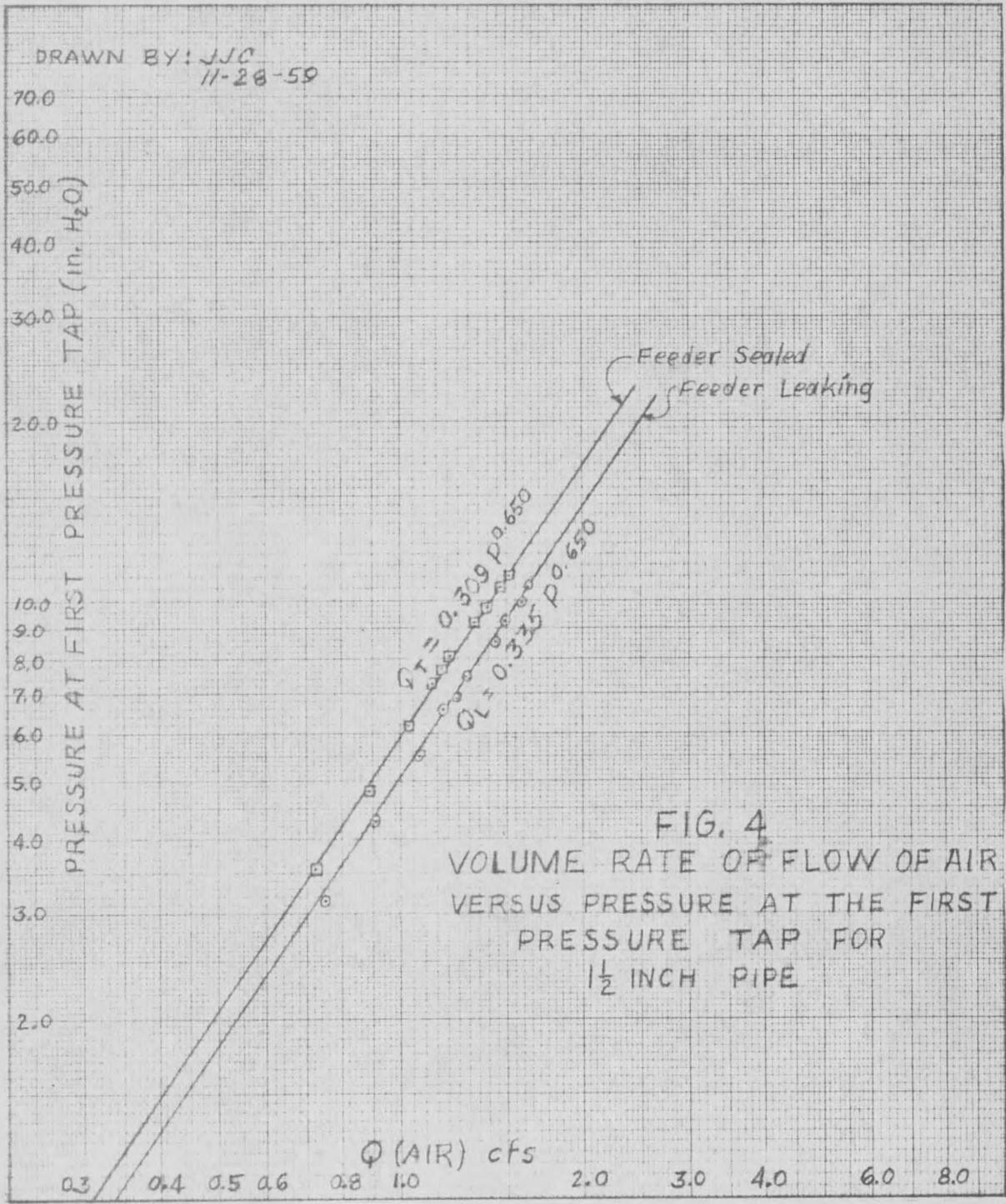
The feeder was then run under two conditions. First the top of the feeder was sealed so that no leaking could take place. Under this condition, the intake quantity of air to the blower was measured, and the pressure at the first pressure tap on the test pipe was measured. The location of this pressure tap is shown in Fig. 1. The seal on the feeder was then removed, and again the intake of air to the blower and the pressure at the first downstream pressure tap was recorded. This process was continued over the full range of possible air velocities. The intake of air with the feeder not leaking, and the intake of air with the feeder leaking, were then plotted on logarithmic paper against the pressure at the first downstream pressure tap. These graphs are shown in Fig. 4 and Fig. 5. From these graphs, equations were derived to predict the leakage of air through the feeder in terms of the pressure at the first pressure tap. These equations were derived as follows¹².

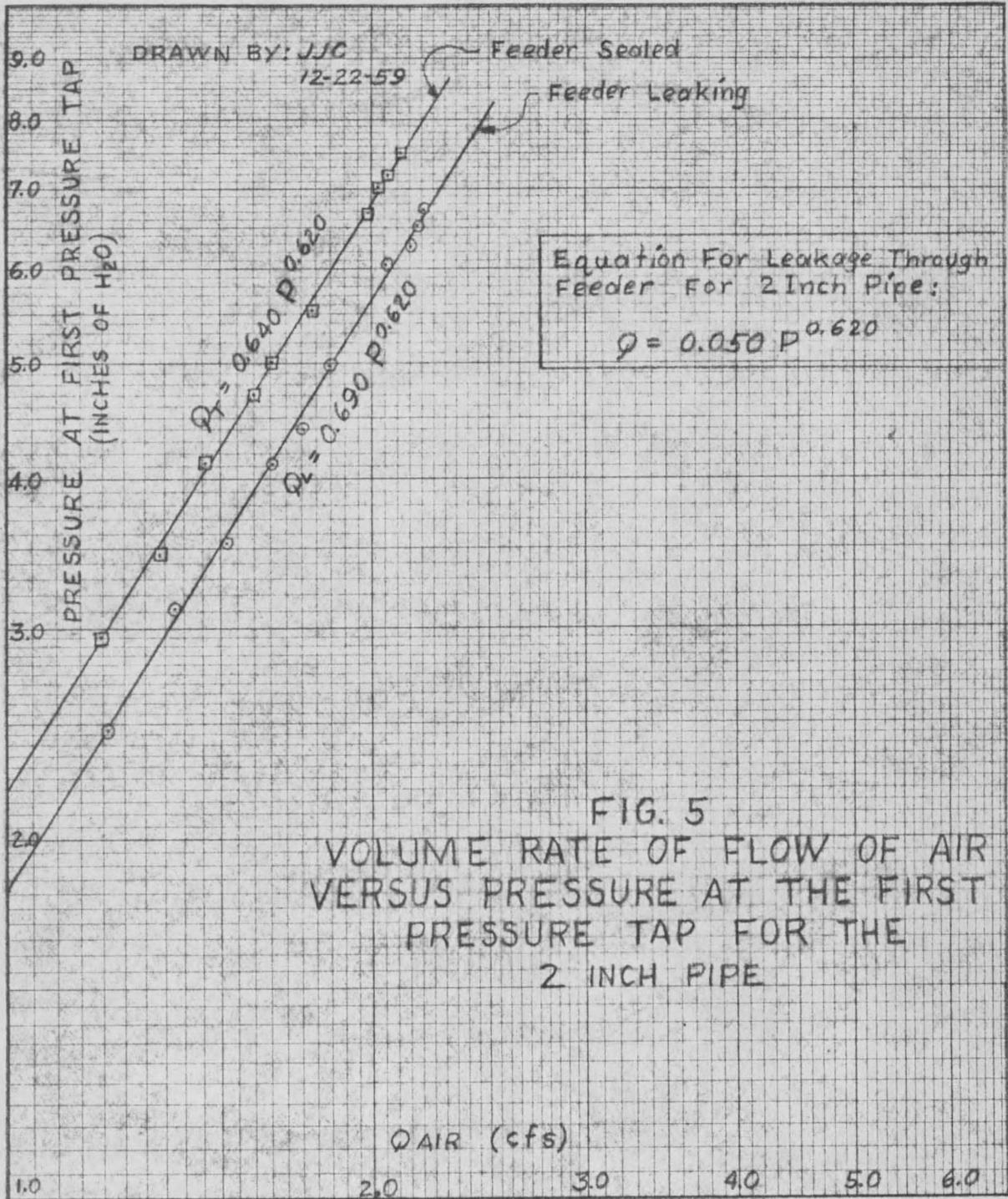
The actual slope of the two curves shown in Fig. 4 is 0.650 as measured on the graph.

The value of Q (feeder not leaking) when the pressure at the first pressure tap is equal to one inch of water, is 0.309 cfs. The similar value for Q (feeder leaking) is 0.335 cfs.

Let the pressure at tap #1 in inches of $H_2O = P$, the air intake in

12. Sokolnikoff, J. S., and E. S. "Higher Mathematics for Engineers and Physicists." McGraw-Hill. 1941.





cfs with the feeder not leaking = Q_T , and the air intake in cfs with the feeder leaking = Q_L .

The logarithmic equations for these two curves are

$$\log Q_T = 0.650 \log P + 0.309$$

$$\log Q_L = 0.650 \log P + 0.335$$

Changing these equations to rectangular form, we have

$$Q_T = 0.309 P^{0.650} \quad (2)$$

$$Q_L = 0.335 P^{0.650} \quad (3)$$

If the value of (P) is the same in both equations, then the difference in Q's for this case will be the amount of leakage through the feeder in cfs.

$$Q \text{ leakage} = Q_L - Q_T = (0.335 - 0.309) P^{0.650}$$

$$Q \text{ leakage (cfs)} = 0.026 P^{0.650} \quad (4)$$

A similar equation was derived for the 2.06 inch pipe. This equation is shown in Fig. 5. The pressure at tap #1 was measured in all experimental runs, and the volume rate of flow of air in the test pipe was computed by correcting the air intake to the blower, using these equations.

If the flow of the air leaking through the feeder had been of a turbulent nature, the exponent of P should have been approximately 0.5. If the flow had been laminar in nature, the exponent of P should have been 1.00. This feeder had clearances between the revolving bucket wheel and the feeder body which were quite small. From this standpoint, it appeared that the exponent of 0.650 for P was reasonable.

PIPE TEST SECTIONS

Standard thin-wall electrical conduit was used for both sizes of conveying pipe used in this project. Several different types of pipe were considered, including aluminum tubing and standard steel pipe. Thin-wall electrical conduit was chosen because of economy and ease of handling. The pipe came in 10-foot sections. These sections were joined, using standard water-tight "EMT" couplings. Particular attention was used in selecting the couplings which joined the two lengths of pipe which formed the test section, to insure that a smooth joint was formed.

Bends in the pipe were accomplished using commercially available 90 degree bends. These bends had a radius of curvature of 10 inches, and were of constant section throughout.

The conveying tube was joined to the 2-1/2 inch standard steel pipe which was welded to the bottom of the feeder by using a standard "EMT" water-tight connector and bell reducer couplings. The maximum operating pressure inside the pipe was approximately 20 inches of water.

The entire piping system functioned satisfactorily, and was absolutely air tight over the range of pressures to which it was exposed.

The test section of the pipe was made up of two 10-foot sections of the conduit. The location of this test section is shown in Fig. 1. Special care was exercised to make sure that the test section was perfectly horizontal and in good alignment. At the end of the test section,

a 4-foot long section of transparent acrylic tubing was installed in order that the flow could be observed during the experimentation. The inside diameter of this tubing matched the inside diameter of the pipe being tested, so that no change in the flow pattern occurred as the grain passed from the test section into the transparent section. This observation section proved to be quite valuable during the actual experimentation.

Two sizes of test pipe were used in the project. These sizes were 1-1/2 inch and 2-inch "EMT" conduit which had measured inside diameters of 1.63 inches and 2.06 inches, respectively.

Portions of the conveying pipe are visible in Figs. 6 and 7. Pressure taps were installed at intervals along the test section in order that differential pressures along the test section could be measured. These pressure taps are described on Page 49 .

THE BLOWER

To provide the necessary air supply for the experimentation, an electrically driven blower was installed in the system. The blower was a centrifugal type driven by a direct coupled two-horsepower, 220 volt, A.C. motor. The motor ran at 3450 rpm. The blower is shown in the photograph on Page 35.

This blower was loaned to the Civil Engineering Department by the Chemical Engineering Department. No specific data was available on the characteristics of this blower; however, its maximum delivery while being

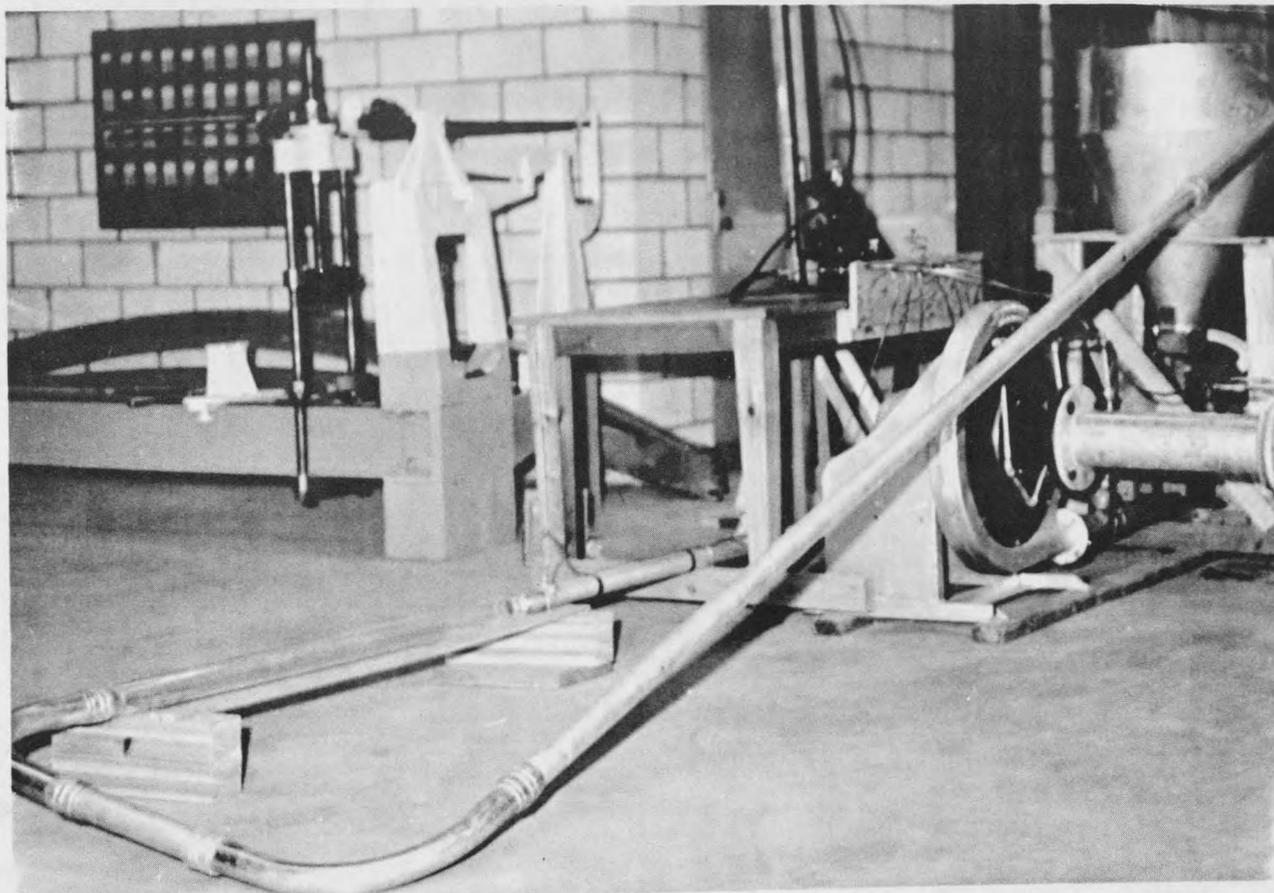


Fig. 6. Downstream end of the test pipe showing the transparent section of acrylic tubing used for observing the flow.

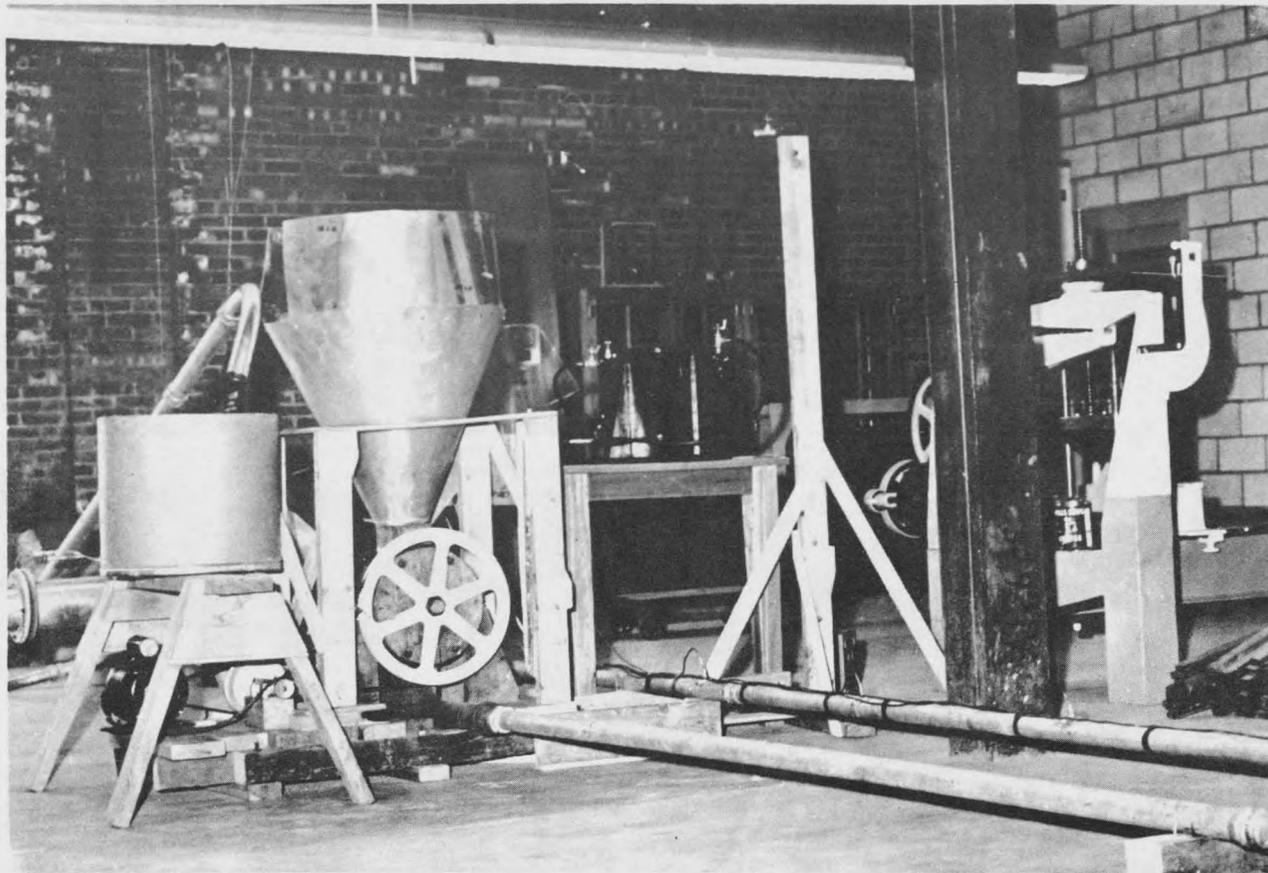


Fig. 7. Upstream end of the test section of the conveying pipe. The drum was set in position as it was used to calibrate the weight rate of flow of grain through the hopper.

used was 110 cfm at a pressure of 20 inches of water. It was learned early in the experiment that the blower did not have adequate capacity to provide the desired air velocity range; however, a blower with the desired characteristics was not available for use.

In order to measure the air intake to the system, a circular sharp edged orifice was installed on the inlet side of the blower. Considerable amount of throttling was noticed when a 2.0 inch orifice was installed at a distance of 8.0 inches from the eye of the blower impeller. It was decided that this throttling was due to the short distance between the orifice and the impeller. In a distance this short, the jet of air would not have time to expand and fill the eye of the impeller. An extension to the blower inlet was built, and the orifice reinstalled at a distance of 24 inches from the eye of the impeller. No throttling effect was observed due to this installation.

The outlet end of the blower was equipped with a 2-1/2 inch diameter threaded flange. A 2-inch brass body gate valve was installed in this outlet line between the blower and feeder in order that the rate of flow of air could be controlled.

RATE OF FLOW OF AIR, CONTROL AND MEASUREMENT

ORIFICE

A sharp-edged circular orifice was installed at the inlet to the blower in order that the rate of flow of air could be measured. This installation has been mentioned briefly in the description of the blower installation on Page 29 of this paper. The orifice installation can be seen in Fig. 8.

A 1.65 inch diameter orifice plate was installed in the 6.0 inch sheet metal pipe on the inlet side of the blower. This orifice plate was 0.10 inches thick and was sharp-edged. A section of the 6.0 inch diameter sheet metal pipe 1.0 feet long was installed in front of the orifice to eliminate any possible re-entrant effects¹³.

The orifice was installed between two 3/4 inch thick plywood flanges. A section of 1/4 inch O.D. copper tubing was installed in each flange as shown in Fig. 9. These tubes were used as flange taps in order to measure the pressure differential across the orifice plate. The tubing was soldered to the sheet metal pipe in order to provide a tight connection. The ends of the tubing led to the central control panel manifold shown in Fig. 10. A 60 inch U-tube manometer filled with oil of specific gravity 0.824 was used to measure the pressure differential across the orifice plate.

13. King, Wisler, and Woodburn. "Hydraulics." 1948, John Wiley and Sons, New York. P. 145.

THEORY OF THE ORIFICE EQUATION

An equation was developed to determine the rate of flow of air into the blower. This equation depends upon the pressure differential across the orifice plate. The orifice theory is as follows¹⁴.

In Fig. 9 on Page 36, Point 1 is taken as some point far enough up stream from the orifice plate that the stream lines of the flow are parallel. Point 2 is a point in the vena contracta downstream from the orifice plate.

Writing the continuity equation between Points 1 and 2, we have:

$$G = \gamma_1 A_1 V_1 = \gamma_2 A_2 V_2 \quad (5)$$

Assuming that $\gamma_1 = \gamma_2$, we can write the following equation for the velocity of the air at Point 1.

$$V_1 = V_2 \frac{A_2}{A_1} \quad (6)$$

Writing the Bernoulli equation¹⁵ between 1 and 2, and neglecting head loss, we have:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2$$

Assuming Points 1 and 2 are at the same elevation, and substituting the value of V_1 from (6) above, the following is obtained:

14. Ibid. P. 123.

15. Ibid. P. 94.

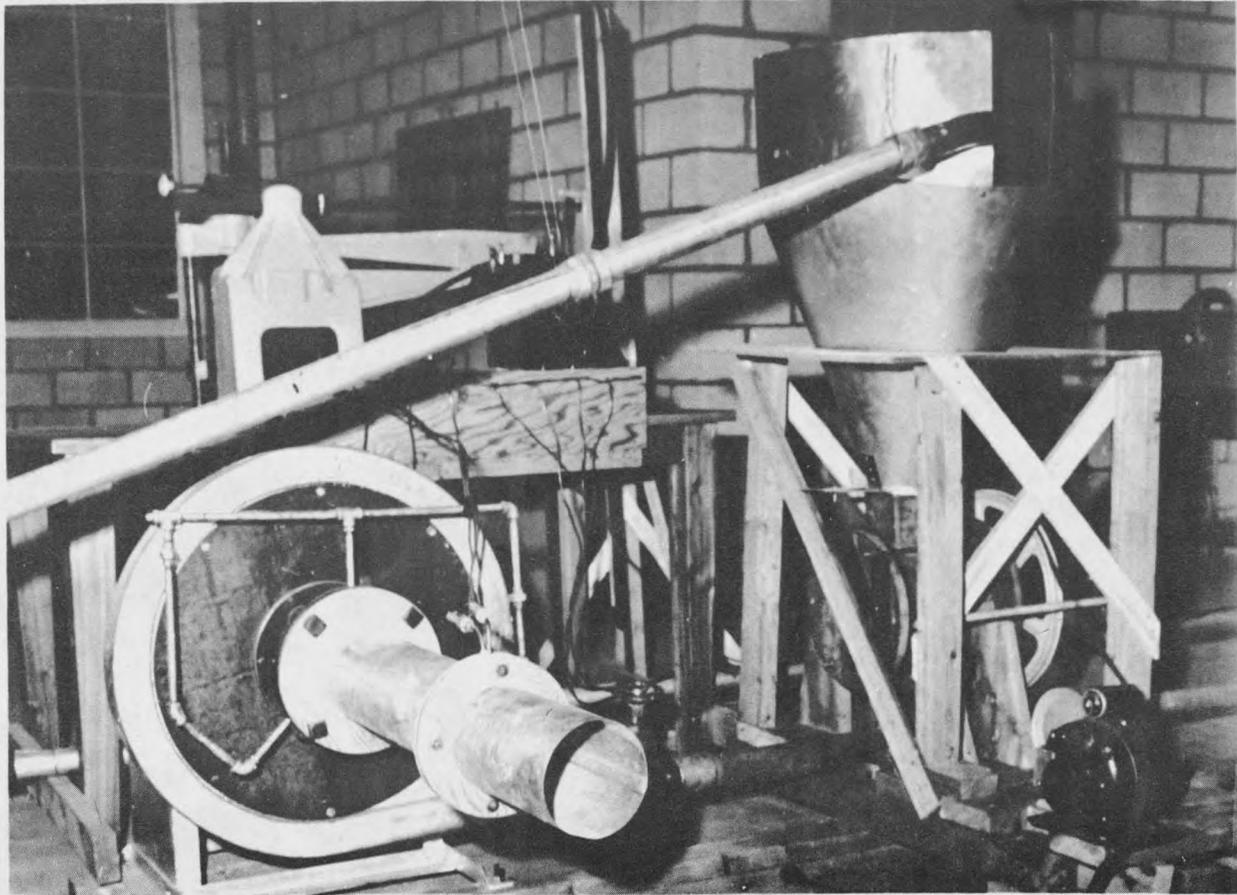


Fig. 8. Intake to the blower showing the wooden flanges used for mounting the 1.65 inch orifice.

Note:
All Flanges Bolted
With $\frac{1}{4}$ " x 2" Bolts

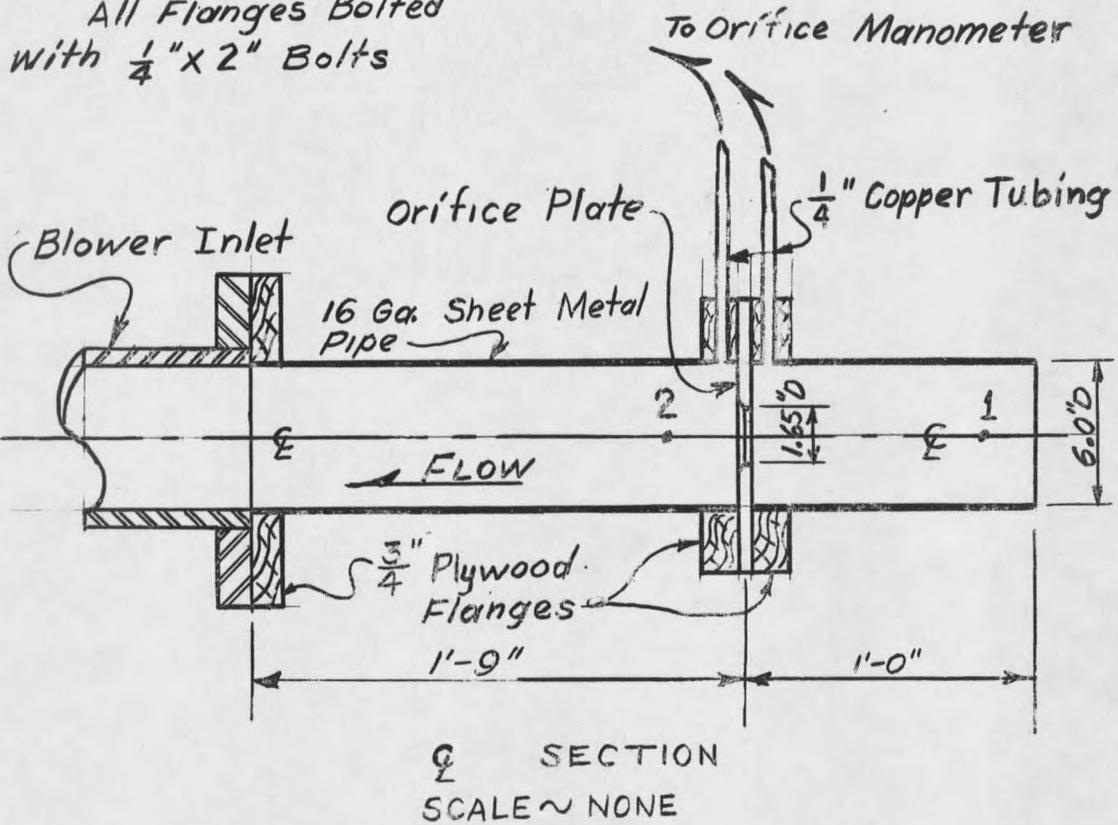


FIG. 9
DETAIL OF
ORIFICE INSTALLATION

DRAWN BY: J.C. 5-4-60

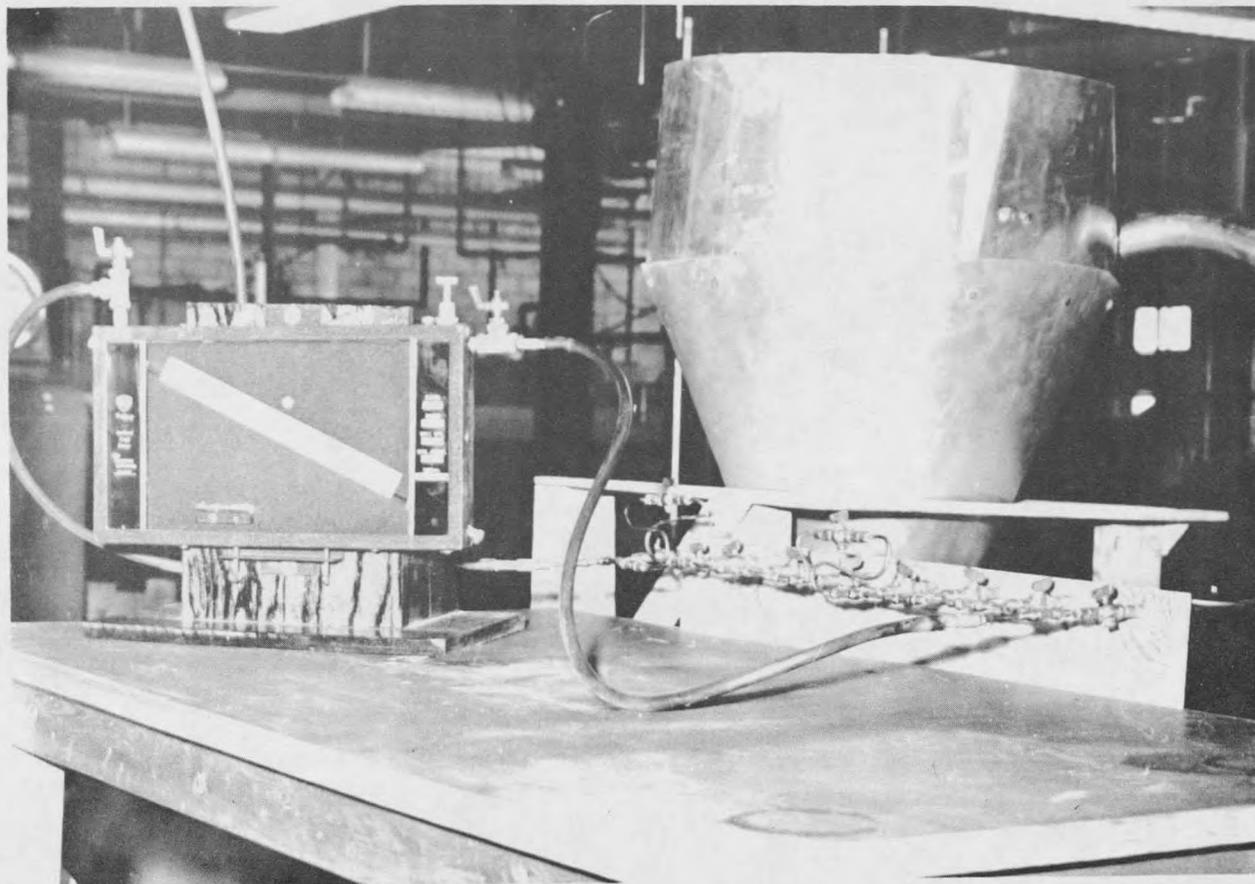


Fig. 10. At Left: The manometer used in measuring pressure differential along the test section.

At Right: The control manifold connecting to the pressure taps.

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = \frac{V_2^2}{2g} \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right]$$

which can be rewritten as

$$V_2 = \sqrt{\frac{2g \left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma} \right)}{1 - \left(\frac{A_2}{A_1} \right)^2}} \quad (7)$$

This is the equation for the theoretical velocity in the vena contracta. However, due to energy losses, V_2 (actual) will be less than V_2 (theoretical). To get V_2 (actual), we must multiply the above equation by a coefficient (C_v) called the velocity coefficient. Letting $\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma} \right) = H$, Equation 6 is rewritten to form

$$V_2 = C_v \sqrt{\frac{2gH}{1 - \left(\frac{A_2}{A_1} \right)^2}} \quad (8)$$

In order to determine Q , the volume rate of flow, we must multiply Equation (8) by the cross-sectional area of the vena contracta, since V_2 is now the actual velocity in the vena contracta. The cross-sectional area of the vena contracta will be less than that of the orifice. The area of the vena contracta is defined as $(C_c A_o)$ where A_o is the area of the orifice. Equation (1) now becomes:

$$Q = C_c C_v A_o \sqrt{\frac{2gH}{1 - \left(\frac{C_c A_o}{A_1} \right)^2}}$$

where A_o is the cross-sectional area of the orifice opening. This can be rewritten as

$$Q = \frac{C_c C_v A_o}{\sqrt{1 - C_c^2 \left(\frac{A_o}{A_1} \right)^2}} \sqrt{2gH} \quad (9)$$

Letting the quantity $\frac{C_c C_v}{\sqrt{1 - C_c^2 (A_0/A)^2}} = C$, the discharge coefficient,

we have

$$Q = C A_0 \sqrt{2gH} \quad (10)$$

where the units of H are foot pounds per pound of air, or simply feet of air. Since in this research, " H " was measured in inches of water, it was desirable to modify Equation (10) in order that Q could be computed in cfs while working with " H " in inches of water. In order to accomplish this, we write the following equation.

$$H \text{ (inches of water)} = \frac{H \text{ (feet of air)} \times 12 \times \gamma_a}{62.4} \quad (11)$$

where 62.4 is the specific weight of water. But γ_a , the specific weight of air, varies with pressure and temperature according to the gas law¹⁶.

$$\gamma_a = \frac{P}{RT} \quad (12)$$

In Equation (12), " P " the pressure is in pounds per square foot absolute, T is the temperature in degrees Fahrenheit absolute, and R (the gas constant) = 53.3 for air.

Solving Equation (11) for H (feet of air) and substituting for γ_a according to Equation (12), we have:

$$H \text{ (feet of air)} = 276.8 \frac{TH}{P} \text{ (inches of water)} \quad (13)$$

Substituting this into Equation (10), we obtain the following.

16. Vennard, J. K. "Elementary Fluid Mechanics." 1958. John Wiley and Sons, New York, P. 6.

equation for the volume rate of flow of air in terms of pressure and temperature.

$$Q = CA_0 \sqrt{2g(276.8) \frac{TH}{P}} \quad (14)$$

A thermometer and a barometer were stationed near the inlet to the blower so the air temperature and pressure could be recorded during the experiment. The barometer recorded in inches of mercury. Re-writing Equation (14) so that P' can be inserted in the equation in inches of mercury, we have

$$Q = CA_0 \sqrt{\frac{29(12)(276.8)}{62.4(13.6)} \frac{TH}{P'}}$$

where 62.4 = specific weight of water, and 13.6 = specific gravity of mercury.

Letting g equal 32.2 fps^2 , this equation reduces to

$$Q = 15.85 CA_0 \sqrt{\frac{TH}{P'}} \quad (15)$$

The cross-sectional area of the orifice used was 0.0148 square feet.

The coefficient of discharge is not only dependent upon the geometric properties of the orifice, but experimenters have shown that it is also dependent on Reynolds Number.¹⁷ Reynolds Number for the orifice was computed from the equation

$$N_R = \frac{VD}{\nu} = \frac{QD}{\left(\frac{\pi D^2}{4}\right)\nu} = \frac{4Q}{\pi D\nu}$$

where Q is the volume rate of flow, D is the diameter of the orifice, and

17. *ibid.* P. 302.

ν is the kinematic viscosity of the fluid. Reynolds numbers for the orifice ranged from 30,000 to 70,000. For this range of Reynolds numbers, with the orifice installed as shown with flange taps, Buckingham¹⁸ suggests a discharge coefficient "C" from 0.6063 to 0.6022. It was decided to use an average value of 0.6042 in Equation (15). Substituting for "C" and for the cross-sectional area in Equation (15), we obtain the following equation.

$$Q = 0.1415 \sqrt{\frac{TH}{P'}} \quad (16)$$

In Equation (16), Q is in cfs of air, T is the air temperature in degrees Fahrenheit absolute, and P' is the air pressure in inches of mercury. If the effects of compressibility are considered negligible, then Equation (16) can be used to compute the volume rate of flow through the intake orifice.

EFFECTS OF COMPRESSIBILITY

Throughout the preceding derivation, the flow of air was considered to be applicable to the same laws which apply to a non-compressible fluid. Since air is compressible, the error due to this assumption was analyzed.

It is convenient to consider the flow process through the orifice as isentropic. In actuality there is some energy loss through the orifice, and due to this loss, there will be some heat exchange. How-

18. Buckingham, E. "History of Orifice Meters and the Calibration, Construction, and Operation of Orifices for Metering." American Society of Mechanical Engineers. New York. 1935. P. 123.

ever, assuming that the fluid is ideal, will ease the work in the following development, and will not give a serious error in calculation, since the flow process through the orifice occurs rapidly and there is little time for heat exchange.

In Fig. 9, let Point 1 be a point in the orifice intake pipe far enough upstream from the orifice that the stream lines of the flow are parallel. Let Point 2 be a point in the vena contracta.

Writing the continuity equation between 1 and 2, we have

$$G = A_1 V_1 \gamma_1 = A_2 V_2 \gamma_2$$

The general form of the Bernoulli equation can be written between points 1 and 2 as ¹⁹

$$\int_2^1 \frac{dp}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{V_2^2}{2g} + Z_2 + \int_1^2 \frac{\tau dL}{\gamma R} \quad (17)$$

In the equations used here, G is the weight rate of flow, A_1 is the cross-sectional area of the pipe, A_0 is the cross-sectional area of the orifice, γ is the specific weight of the fluid, k is the adiabatic coefficient for the fluid, V is the average velocity, P is the pressure in the flow, τ is the unit shearing stress, L is the length of pipe, and R is the hydraulic radius of the pipe.

Since it has been assumed that the fluid is ideal, there will be no friction loss between 1 and 2. Therefore the last term is dropped from

19. Vennard, J. K., Op. Cit. P. 71.

the Bernoulli equation above, since this term accounts for energy loss due to friction.

Remembering that Points 1 and 2 are at equal elevations, Equation (17) is rewritten as

$$\int_2^1 \frac{dP}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad (18)$$

For an isentropic process $\frac{P}{\gamma^K} = \text{constant} = C$. Solving this for γ , the following is obtained.

$$\gamma = \left(\frac{P}{C}\right)^{\frac{1}{K}}$$

Substituting this for γ in Equation (18), integrating and rearranging terms, the following equation can be written.

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{K}{K-1} \left(\frac{P_1}{\gamma_1}\right) \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right] \quad (19)$$

When the continuity equation is solved for V_1 and this value substituted into Equation (19), the following expression is obtained.

$$\frac{V_2^2}{2g} \left[1 - \left(\frac{A_2}{A_1}\right)^2 \left(\frac{\gamma_2}{\gamma_1}\right)^2\right] = \frac{K}{K-1} \left(\frac{P_1}{\gamma_1}\right) \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right] \quad (20)$$

But $\gamma_2^K = \frac{P_2}{C}$ and $\gamma_1^K = \frac{P_1}{C}$

Substituting the above values for γ_2 and γ_1 and solving Equation (20) for V_2 , Equation (21) is obtained.

$$V_2 = \frac{\sqrt{\frac{2gK}{K-1} \left(\frac{P_1}{\gamma_1}\right) \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2 \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}}} \quad (21)$$

Substituting this value of V_2 into the equation ($G = A_2 V_2 \delta_2$), the following is obtained.

$$G = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2 \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}}} \sqrt{\frac{2gK}{K-1} \left(\frac{P_1 \delta_2^2}{\delta_1}\right) \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]} \quad (22)$$

Letting $\frac{\delta_2}{\delta_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{K}}$, Equation (22) can be rewritten as

$$G = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2 \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}}} \sqrt{\frac{2gK}{K-1} (P_1 \delta_1) \left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{K}} - \left(\frac{P_2}{P_1}\right)^{\frac{K+1}{K}}\right]} \quad (23)$$

This equation gives the theoretical value of G . As was stated earlier in this section, the fluid dealt with is not ideal, and some friction loss does exist across the orifice. In order to account for this friction loss, we must multiply Equation (23) by a velocity coefficient C_v to reduce the theoretical jet velocity to the actual jet velocity. Point 2 has been taken as a point in the vena contracta which has a cross-sectional area which is less than that of the orifice. In order to correct Equation (23) to allow for the decrease in area of the jet at the vena contracta, we must substitute $C_c A_0$ for A_2 in Equation (23), where A_0 is the area of the orifice, and C_c is the contraction coefficient. The area of the vena contracta then is defined as $C_c A_0$. This is identical to theory used in incompressible flow. For compressible flow, however, the value of C_c does not depend on the ratio A_2/A_1 alone, but depends also upon the ratio P_2/P_1 . C_c increases as the ratio P_2/P_1 decreases. Multiplying Equation (23) by $C_v C_c$ and substituting A_0 for A_2 , we have

$$G = \frac{C_c C_v A_o}{\sqrt{1 - \left(\frac{A_o}{A_1}\right)^2 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}} \sqrt{\frac{2gk}{k-1} \left(\frac{P_1 \gamma_1^2}{\gamma_1}\right) \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} \quad (24)$$

This equation is much too complex for convenient use in metering. It has been expressed in a much simpler form by Daugherty and Ingersoll²⁰ as

$$G = C Y A_o \sqrt{2g \gamma_1 \left(\frac{P_1 - P_2}{1 - (D_o/D_1)^4}\right)} \quad (25)$$

where $(C = C_v C_c)$ and has the same value as the discharge coefficient (C) for an orifice with incompressible fluid flowing. The factor "Y" is an expansion factor which accounts for the effects of compressibility. If Equation (25) is equated to Equation 24 and this equality is solved for "Y", the following result is found

$$Y = \sqrt{\frac{\left[\left(\frac{k}{k-1}\right)\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]\right]}{\left[1 - \left(\frac{P_2}{P_1}\right)\right]}} \sqrt{\frac{\left[1 - \left(\frac{D_o}{D_1}\right)^4\right]}{\left[1 - \left(\frac{D_o}{D_1}\right)^4\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}}\right]}} \quad (26)$$

For a nozzle or a venturi tube where $C_c = 1$, it can be seen that Y can be computed from the above equation; however, where the coefficient C_c is changing with the ratio P_2/P_1 , then experimental methods must be used to find the values of "Y". Buckingham²¹ suggests the following empirical equation for "Y".

$$Y = 1 - \left[0.41 + 0.35\left(\frac{D_o}{D_1}\right)^4\right] \left[\frac{P_1 - P_2}{k P_1}\right] \quad (27)$$

20. Daugherty, R.L., and Ingersoll, A.C. "Fluid Mechanics With Engineering Applications." 1954. McGraw-Hill. New York. P. 150.

21. Buckingham, E. Op. Cit. P. 28.

where D_o is the diameter of the orifice.

Buckingham states that this equation is valid for flange taps or vena contracta taps.

Equations (25) and (27) can now be used to compute the rate of flow of air through the orifice. This result can then be compared with the rate of flow of air computed by Equation (16) which neglects the effects of compressibility. To make this comparison, the maximum rate of flow was chosen, since the effects of compressibility should be a maximum at this rate. Therefore this comparison will indicate the maximum error involved in computing the rate of flow of air while neglecting the effects of compressibility. In this installation, the orifice diameter was 1.65 inches, and the pipe diameter was 6.0 inches.

For the maximum rate of flow, the following values were recorded.

Temperature = 75° F (The temperature is assumed to be the same at Points 1 and 2).

P_1 = 25.62 inches of mercury.

$P_1 - P_2$ = 10.00 inches of water.

For air the adiabatic coefficient $K = 1.40$, and the gas constant $R = 53.3$.

From the above figures, the following computations can be made.

$$A_o = (1.65)^2 \frac{\pi}{4} (144) = 0.0148 \text{ ft}^2$$

$$\frac{D_o}{D_1} = \frac{1.65}{6.00} = 0.275$$

$$\frac{P_1 - P_2}{P_2} = \frac{10}{25.62(13.6)} = 0.0287$$

$$\gamma_1 = \frac{P_1}{RT_1} = \frac{(25.62)(13.6)(62.4)}{(12)(53.3)(535)} = 0.0635 \text{ lbs. per ft}^3$$

$$Y = 1 - [0.41 + 0.35(0.275)^4] \left[\frac{10}{1.4(25.62)(13.6)} \right] = 0.99155$$

These results can now be substituted into Equation (25) to calculate the rate of flow of air. The discharge coefficient used will be taken as the same as that ~~was~~ used in the development of Equation (16) on Page 41.

$$G = 0.6042(0.99155)(0.0148) \sqrt{64.4(.0635) \left[\frac{10(62.4)}{12(1 - (0.274)^4)} \right]}$$

$$= 0.00887 \sqrt{214} = 0.130 \text{ lbs. per minute}$$

When the above values are substituted into Equation (16), the volume rate of flow is obtained for the case when compressibility effects are neglected.

From Equation (16)

$$Q = 0.1415 \sqrt{\frac{535(10)}{25.62}} = 2.05 \text{ cfs}$$

If this value of Q is multiplied by γ_1 , a weight rate of flow is found, and this value can be compared to that obtained by including effects of compressibility.

$$G = 2.05 (.0635) = 0.1305 \text{ lbs. per min.}$$

The error in G incurred by neglecting the effects of compressibility can then be expressed as

$$\text{Error} = \left[\frac{0.1305 - 0.1300}{0.1300} \right] \times 100 = 0.385 \%$$

In the 2.06 inch pipe, this is equivalent to an error in air velocity of 0.34 fps. For the 1.63 inch pipe, the error in velocity would be 0.55 fps.

Since the error in the rate of flow of air due to neglecting compressibility effects was small, Equation (16) was used to compute the volume rate of air flow throughout this project.

SYSTEM FOR MEASURING PRESSURE DROP ALONG THE TEST SECTION

PRESSURE TAPS AND CONTROL PANEL

Two complete sets of test pipes were used. The dimensions of these pipes are given on Page 29.

Pressure taps were installed in the test pipe at 3-foot intervals along the pipe. Seven taps in all were installed in the test section. The first tap was located 2 feet from the end of the 90° bend in the conveying pipe which marked the beginning of the test section. A detail photograph of one of these pressure taps can be seen in Fig. 11.

The pressure tap was made by first drilling a 1/64 inch diameter hole through the pipe wall. Special care was taken to insure that these holes were normal to the pipe wall. After the hole was drilled, a 3/16 inch brass flare type fitting was welded to the pipe over the drilled hole. Special care was used in removing the burr around the drilled hole on the inside of the pipe so that none of the velocity head would be converted to negative pressure head as the air flow passed over the burr. To accomplish this, a long dowel 1-1/2 inches in diameter was wrapped with fine emery cloth for a distance of about one foot from one end. The dowel was then inserted into the test pipe, and the burr at each pressure tap was ground smooth. After one operation with the dowel and emery cloth, the taps were redrilled, using the same size drill bit. Following this, the taps were ground smooth ~~once more~~ using the dowel and emery cloth. Next 3/16 inch O.D. copper tubing was installed from the

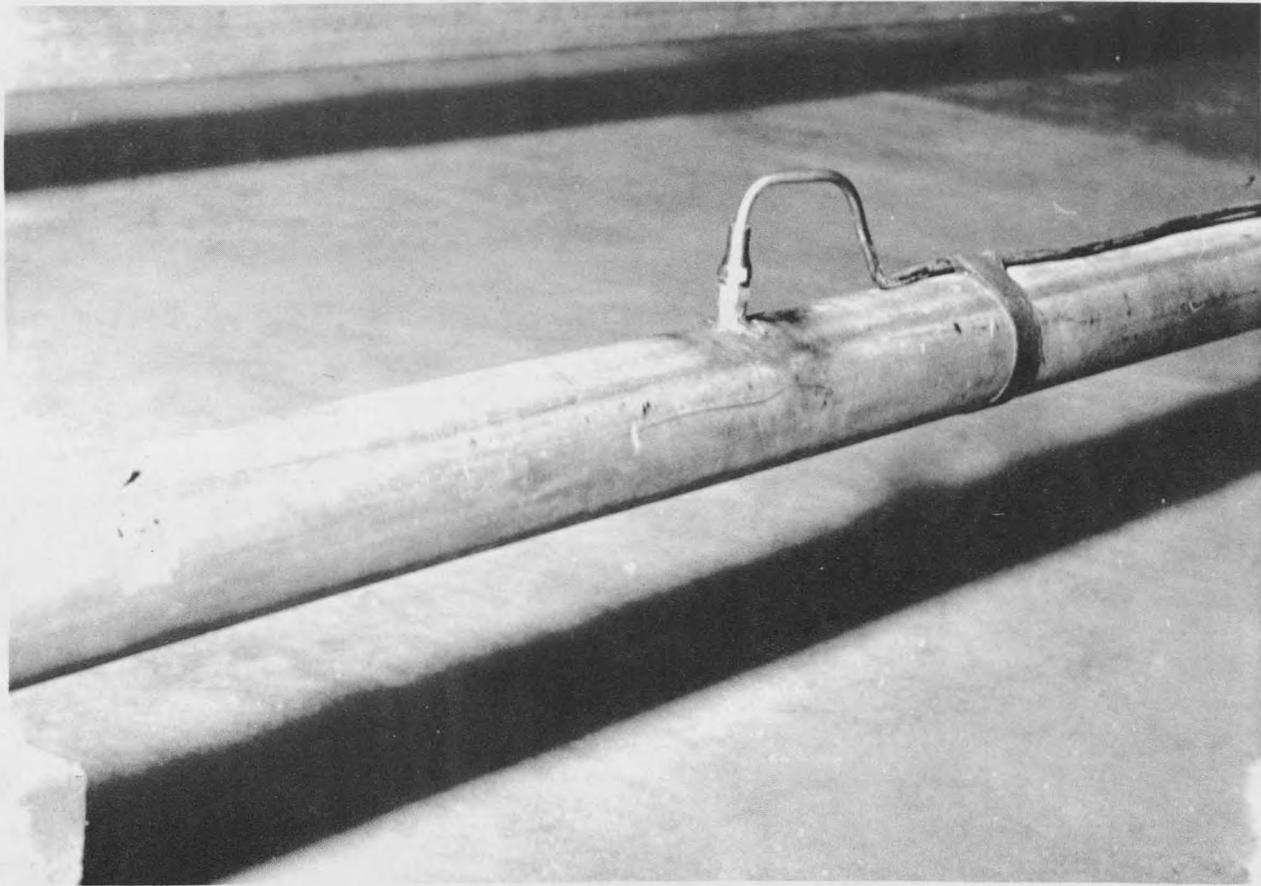


Fig. II. Pressure tap on the test pipe.

pressure tap to the control panel.

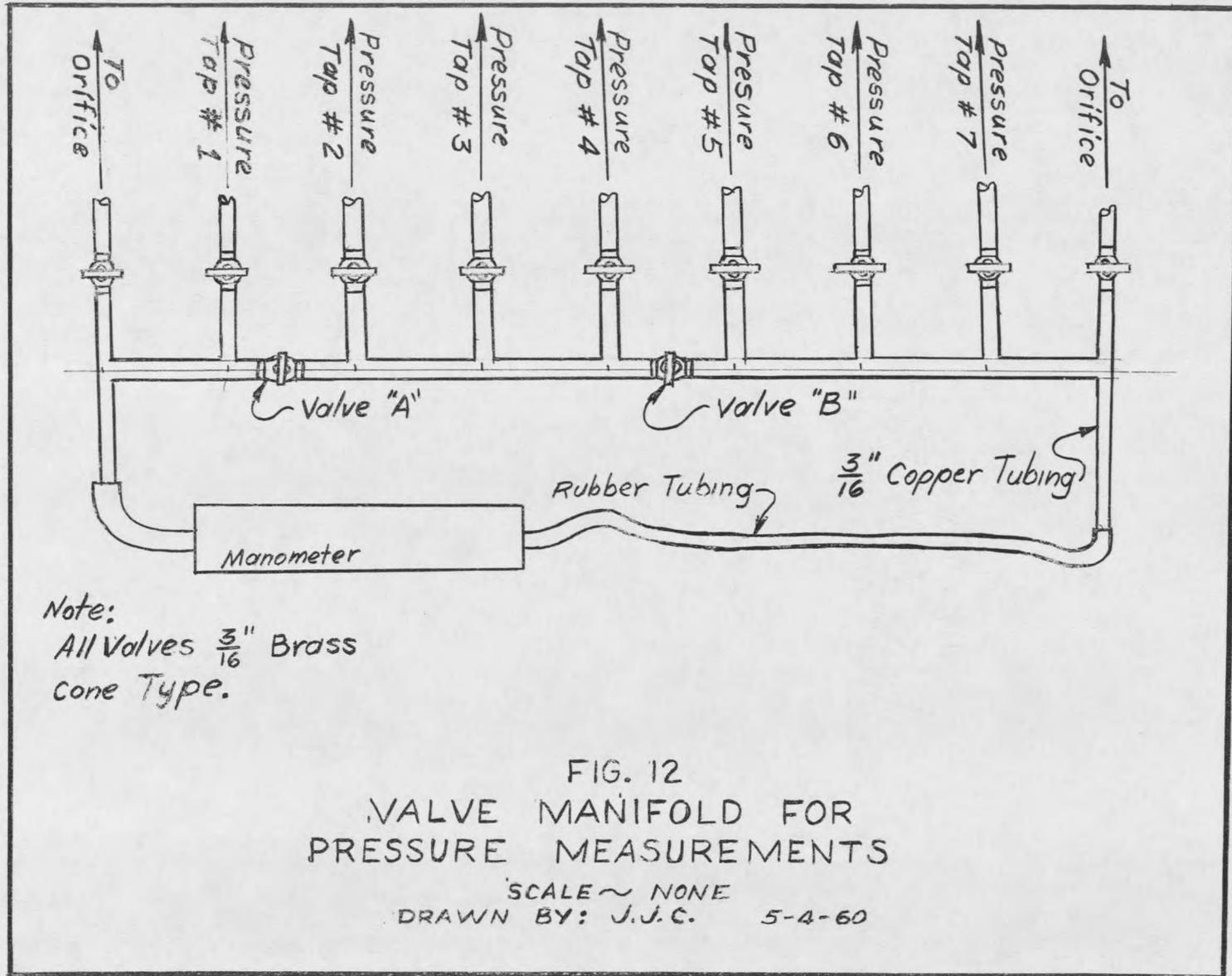
Only one pressure tap was installed at each 3-foot interval. Care was taken to see that each tap was installed exactly in the top center of the pipe. Other investigators²² have used multiple taps at each section with these taps spaced at equal intervals around the pipe diameter. However, the taps used here seemed to function quite well, and very little fluctuation was noticed on the manometer when pressure readings were being taken. The differential pressures between the taps were satisfactorily uniform when steady state conditions of flow existed.

CONTROL PANEL AND MANOMETER

A control panel was set up so that the differential pressure between pressure taps could be measured without disconnecting the manometer from each pair of consecutive taps. This control panel with the manometer attached is visible in Fig. 10. The layout of the piping for the control panel is shown in the drawing on Page 52.

An inclined tube type manometer was used to measure the differential pressures. The manometer used a red oil with a specific gravity of 0.824. The manometer scale read directly in inches of water, and was graduated to read a maximum of 4 inches of water by graduations of 0.02 inches. However, it was found that manometer deflections over 3.0 inches of water were too unstable to afford accurate readings. At the high weight rates of grain flow more than 3.0 inches of deflection was required

22. Crane, J. W. Op. Cit. P. 28.



to read the differential pressure between tap #1 and tap #7. To remedy this, the two valves marked A and B in the drawing on Page 52 were installed so that the differential pressure between tap #4 and tap #5, #6, and #7 could be measured after the differential pressures between tap #1 and tap #2, #3, and #4 had been measured. After this correction, the manometer was capable of measuring any required differential pressure with negligible fluctuation.

The two flange type pressure taps on either side of the intake orifice, previously described, were also connected to the control panel manifold. A vertical U-tube manometer filled with red oil of specific gravity 0.824 was used to measure the differential pressure across the orifice plate. Fluctuation in this manometer during measurement was negligible, and it was possible to estimate the manometer reading to the nearest 0.02 inches of oil.

CONTROL OF RATE OF FLOW OF GRAIN

The experimental pressure drops were measured at varying air velocities with a constant weight rate of grain flow.

To meter the weight rate of flow of grain, it was decided to use a variable area orifice. This orifice was constructed in the form of a sliding aluminum gate in the bottom of the grain hopper. A rectangular sharp-edged hole 1.0 inches wide and 3.0 inches long was machined in the center of this gate. This gate is shown in Fig. 13. As the gate was pushed further into the slot under the hopper, the hole opened longer, allowing more wheat to flow through the orifice.

A system of holes drilled through the plate was devised so that the gate could be locked in 6 different positions. The hopper was then calibrated in a static position off the experimental equipment. The weight rate of flow of grain through each preset orifice opening was calibrated by taking the average of 20 times rates of flow for each position. The results of this calibration are shown in Fig. 14. As can be seen, the weight rate of flow when plotted against the measured gate opening produces a straight line. The equation of this line determined by the methods used on Page 27 is

$$G_s = 14.37 X^{1.95} \quad (28)$$

where G_s is the weight rate of flow of grain in pounds per minute and X is the gate opening in inches. This equation is valid only for a gate opening 1.0 inches wide, for wheat with the properties given in the appendix, and for a "head" of at least 6 inches of wheat above the orifice.

After the hopper was placed back in the experimental system, it was calibrated again with the system running. A difference of approximately 0.11 pounds per minute greater flow per orifice opening was observed in this test. By varying the volume rate of flow of air, it was found that the air leaking from the feeder did not affect the weight rate of flow of grain from the hopper. The increase in weight rate of flow was attributed to the vibration of the equipment while running. It was noted during the calibration that when the grain was kept at a depth greater than 6 inches

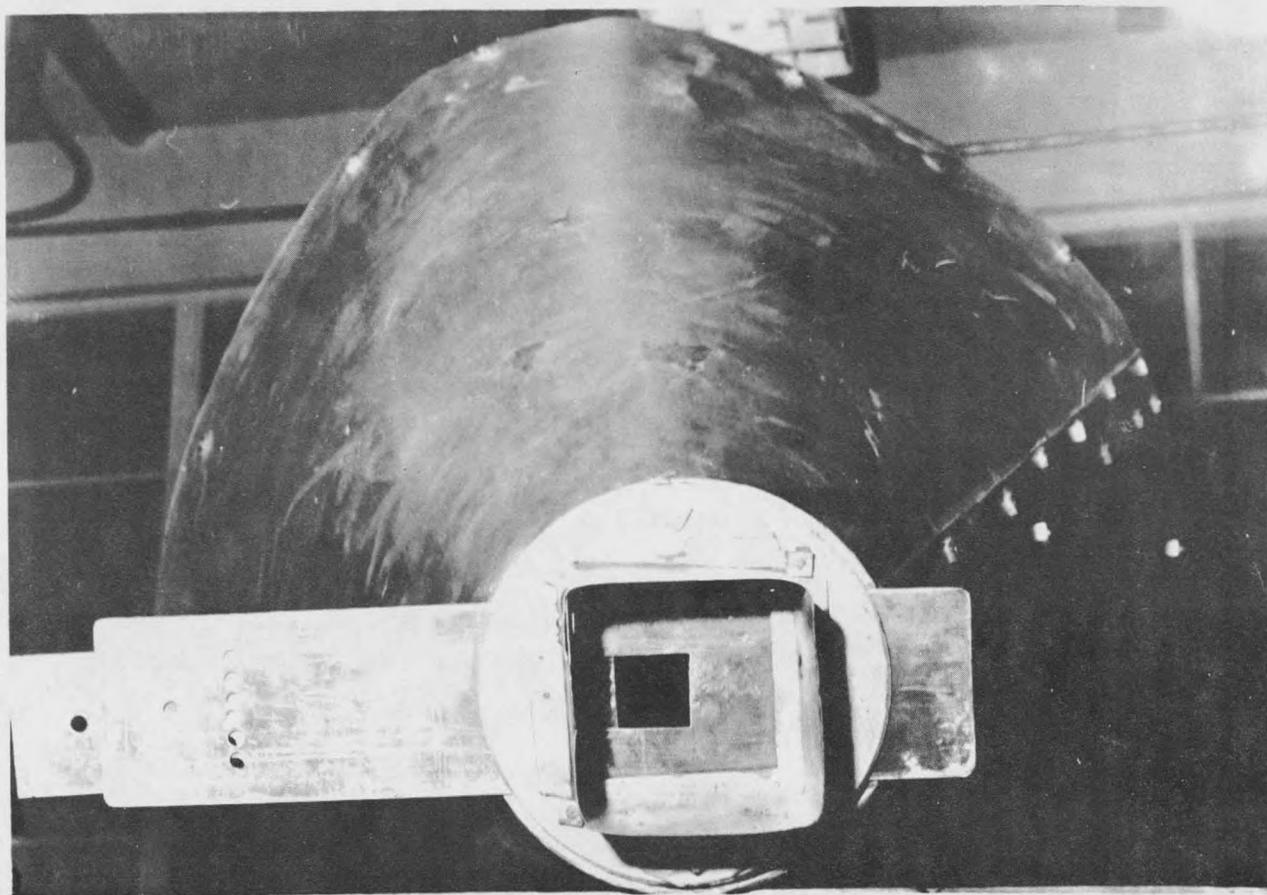
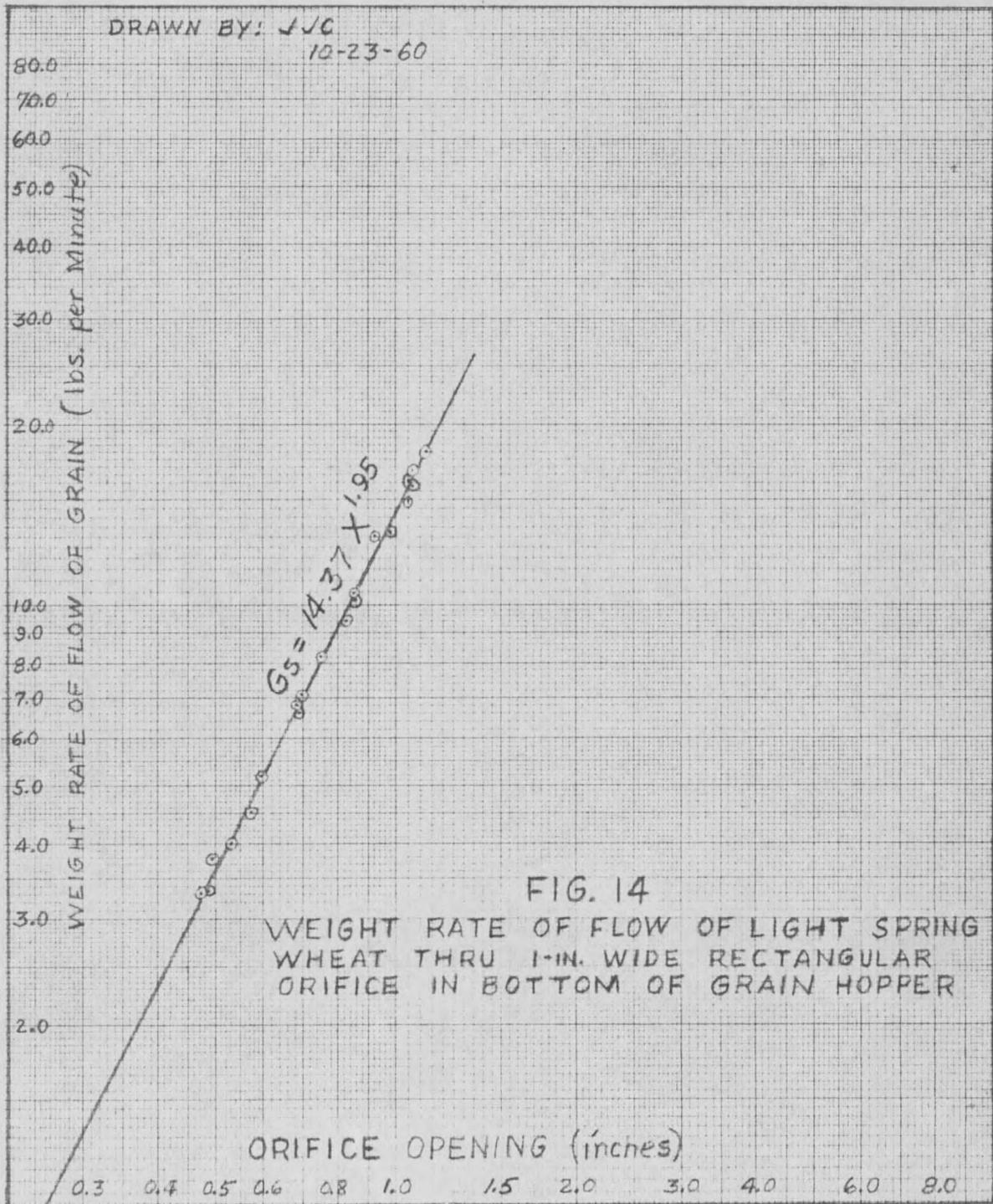


Fig. 13. Bottom view of variable orifice in bottom of the grain hopper.



above the orifice plate, the weight rate of flow of grain through the orifice was independent of the depth of grain above the orifice opening.

The grain hopper itself was conical in shape and was built from 0.125 inch thick galvanized sheet steel. The 11 inch high metal rim visible in Fig. 3 was added after experimentation began to prevent the escape of wheat grains from the hopper.

The hopper and orifice functioned well during the experimentation. Periodic checks were run on the hopper calibration to make sure that the weight rate of flow of grain had not deviated from the calibrated rate. No deviations greater than 0.10 pounds per minute were noted.

In the 1-1/2 inch test pipe, 5 different weight rates of grain flow were used, and during the test of the 2 inch pipe, 6 different rates were used. The weight rates of grain flow that were used were 3.36 pounds per minute, 6.67 pounds per minute, 10.35 pounds per minute, 15.98 pounds per minute, and 17.10 pounds per minute. The first five rates were those used while testing the 1-1/2 inch pipe, while all six were used in testing the 2 inch pipe.

EXPERIMENTAL PROCEDURE

THE GRAIN USED

Four hundred pounds of light spring wheat were obtained for use in the experimental work in this project. The wheat was obtained from a local grain elevator. All four hundred pounds were taken at one time from the same bin. The wheat was cleaned in a commercial type seed wheat fan mill at the grain elevator. To remove the foreign matter, the wheat was cleaned again, this time using a small kicker-type cleaner, which is normally used to determine the percent "dockage" in wheat. This operation cleaned the wheat satisfactorily.

The specific gravity of the wheat was determined by measuring the volume of a known weight of the wheat. The following procedure was used.

A small beaker was first filled with wheat and weighed. Then the number of particles in the beaker were counted. After counting, the wheat particles were poured into a graduate containing a known volume of water. The contents of the graduate were stirred vigorously while the wheat particles were being introduced, in order to dislodge the air bubbles which seemed to cling to the minute fibers around the wheat particle. The increase in volume due to the addition of the wheat particles was noted. From the above information, it was possible to compute the weight per particle, the volume per particle, and the absolute specific weight of the particles in pounds per cubic foot. These values for the wheat are given in the appendix. The average value obtained for

the absolute specific gravity of the wheat was compared with an average figure from the Grain Laboratory at Montana State College. The value obtained in this paper was 87.8 pounds per cubic foot. The range of values for this type of wheat grown in the Gallatin Valley as given by the Grain Laboratory were from 82.5 to 88.0 pounds per cubic foot. 87.8 pounds per cubic foot seemed reasonable in this comparison.

No data was available for comparison of the weight per particle and the volume per particle.

Ten particles were measured, using a micrometer. These particles were measured on their major axis and on both minor axes. A shape factor of C/\sqrt{ab} , as suggested by Corey²³, was determined for each particle. In this expression, "C" is the length of the major axis, and "a" and "b" are the lengths of the minor axes. The average value of this shape factor was 0.600. The deviation of the individual particles from 0.600 was quite small, even though some of the particles varied considerably in size.

Special care was used in examining the wheat during the actual test runs. When the wheat began to show signs of damage, it was removed and an unused charge of wheat was placed in the hopper. Approximately forty pounds of wheat were used in each charge. The average time of use for each charge of wheat, before noticeable damage to the particles began

23. Corey, A. T. "Drag Coefficients of Non-uniform Particles." A Thesis, Colorado State University, 1954. P. 20.

to occur, was approximately ten hours. This would have represented an average of 142 complete circulations through the system, assuming the average weight rate of flow was the average of the 6 rates used.

DESCRIPTION OF PROCEDURE USED

The step by step procedure followed during each experimental run was the same for each pipe size. For the 1.63 inch pipe, only five weight rates of flow of wheat were used, while in the 2.06 inch pipe 6 weight rates of flow of wheat were used. These rates of flow have been listed on Page 57.

The exact procedure followed during the runs was as follows.

- 1.) The blower was turned on, and the 2-inch air control valve was opened completely.
- 2.) The feeder was set into operation.
- 3.) The variable orifice in the bottom of the grain hopper was opened to the desired weight rate of flow and locked into position.
- 4.) The reading of the vertical U-tube manometer connected across the intake orifice was recorded.
- 5.) The air temperature and the barometric pressure were recorded.
- 6.) The pressure inside the test pipe at the first pressure tap was recorded.
- 7.) The differential pressures between pressure taps along the test section were recorded.

8.) The vertical U-tube was again connected to the intake orifice and the manometer reading across the orifice was checked. If this reading and that taken in step (4) differed, the run was repeated. If the reading checked that in step (4), step (9) was initiated.

9.) The opening of the two-inch air control valve was reduced until a reduction in the intake orifice manometer was noted.

Following step (9), the procedure from step (4) through step (9) was repeated throughout the full range of flow. The lower limit of flow occurred when the conveying pipe began to block. Before blocking occurred, the flow became irregular, and the sensitive manometer used to measure differential pressures along the test pipe began to fluctuate. A noticeable fluctuation of the manometer used to measure differential pressure across the intake orifice was also evident when air velocities approached the velocity at which blocking occurred.

After the completion of the experimental run for each solid rate of flow, the leakage from the feeder was checked as described on Page 22.

The procedure described above was repeated for each weight rate of solid flow for both pipe sizes. Approximately two and one-half hours of continuous testing were necessary to complete the entire run for each weight rate of solid flow.

INITIAL RUNNING

Before the experimental data could be recorded, it was found that the

system had to be operated for approximately six hours before the pressure drop along the test pipe began to be a constant between the pressure taps. This change in pressure drop was due to the constant polishing of the pipe interior by the wheat particles. No data was recorded for the pressure drop due to air alone before the pipe became polished; however, it can be reasonably assumed that the pipe was transformed from one which behaved as a rough pipe to a pipe which was hydraulically smooth within the range of Reynold's numbers reported in this research.

EXPERIMENTAL RESULTS

COLLECTION OF DATA

Pressure drop along the test section was measured throughout the full velocity range of the experimental system. This range was limited by two separate conditions. The maximum air velocity attainable was limited by the capacity of the blower. The maximum air velocity attained was 84 fps in the 1.63 inch pipe, and 80 fps in the 2.06 inch pipe. Both values were attained with a weight rate of flow of wheat of 3.36 pounds per minute. The lower limiting velocity of the experimental range was governed by the blocking tendencies of the test pipe. An experimental run was always begun using the maximum air velocity attainable. The flow of air was then throttled down in small increments until it reached the point at which the system began to block. After some experience, it was found that blocking was impending when the sensitive manometer used to measure pressure differential between the pressure taps began to fluctuate. Once this condition was reached, it was found that even a very small decrease in air flow would cause blocking. The rate at which the blocking became complete was dependent upon how far below the blocking limit the rate of air flow was cut. When blocking was taking place, the system behaved in the following manner.

First the distribution of the solids in the flow ceased to be uniform. Large slugs of grain could be seen passing the transparent section. As the extent of the blocking continued, the grain began to flow in surges. Flow

of grain would stop temporarily, causing a buildup of pressure within the test pipe. When the pressure became large enough, a surge of grain was blown violently through the system, resulting in a decrease in pressure. This process repeated itself until complete blockage occurred. To unplug the test pipe, the grain hopper orifice was closed. The cycle of blocking just described would then repeat itself in reverse process until all the grain in the system had been discharged.

The maximum grain capacity of the apparatus was 16.05 pounds per minute with an air velocity of 58.0 fps in the 1.63 inch pipe, and 18.03 pounds per minute with an air velocity of 59.9 fps in the 2.06 inch pipe.

During each experimental run, the pressure drop was measured between the following pressure taps: 1 and 2, 1 and 3, 1 and 4, 4 and 5, 4 and 6, and 4 and 7. The increment of pipe length between successive pressure taps was 3.00 feet.

Experimental data was read as accurately as possible with the equipment used. In some measurements taken during the runs, where the 15.98 and 17.10 pounds per minute solids rates were used, the manometer was fluctuating as much as 0.04 inches of water. The solid flow during these measurements was not uniform, and the air velocity was approaching the minimum necessary to sustain flow. These measurements were erratic, as can be seen in Fig. 16.

ACCURACY OF DATA RECORDED

The accuracy of the data taken during these measurements was in-

fluenced by several factors. All but one of these factors influenced the accuracy of the computation of the velocity of air in the test pipe. The one factor which was not involved in the calculation of the air velocity was the measurement of differential pressures along the test section. This measurement was dependent upon two things, the characteristics of the pressure taps, and the accuracy of the manometer. The manometer readings were estimated accurately to the closest 0.01 inch of water. The characteristics of the pressure taps were unknown; however, the differential pressures between them and not the pressures at the taps were of prime importance, so special care was taken to make the pressure taps as nearly uniform in construction as possible, so that differential pressures would be measured accurately.

An error in air velocity occurred, due to the measurement of the rate of flow of air, at the intake orifice. The accuracy of the measurement here was directly dependent upon the accuracy of the manometer reading. The manometer used here was read to the nearest 0.02 inches of oil of specific gravity 0.824. In the orifice equation (16), which was used to compute the volume rate of flow of air, this amounts to an accuracy of plus or minus 0.005 cfs at a manometer reading of ten inches of water, which was approximately the maximum reading used.

The maximum error in the volume rate of flow of air due to neglecting the effects of compressibility was computed as 0.0079 cfs on Page 47.

The method used in determining the leakage through the feeder was

also a potential source of error. The only possible estimate of the accuracy of this method is that which was inherent in the manometer readings. The manometer was read to the nearest 0.05 inches of oil of specific gravity 0.824. Using the maximum pressure measurement, which was 15 inches of water, this accuracy of measurement would have given an estimated error, calculated by Equation (3), of 0.002 cfs.

Assuming that all of these errors occur in the same direction gives an estimate of the maximum error incurred in computing the rate of flow of air. This maximum estimated error is the sum of those listed above.

$$Q \text{ (maximum error)} = 0.008 + 0.005 + .002 = 0.015 \text{ cfs}$$

This error in Q was equivalent to an estimated maximum error in air velocity in the test section of 1.04 fps in the 1.63 inch pipe, and 0.64 fps in the 2.06 inch pipe.

The influence of compressibility on the flow has already been determined on Page 47. Although this effect was negligible, the velocity of the flow in the test section was computed from the weight rate of flow of air using the continuity equation. The pressure in the test section was measured during every run, and from this value, the specific weight of the air in the test section was computed.

CALCULATION OF AIR VELOCITY

In calculating the velocity of the air in the test section, the presence of the wheat was neglected. This was an approximation. However, its effect was negligible. In order to calculate the exact air velocity,

it would have been necessary to know the velocity of the solid particles. Since it was not feasible to measure the velocity of the solid particles, it was decided to neglect their effect.

The effect of this assumption can be seen in the following sample computations. The rate of air flow in a 2.06 inch diameter pipe has been determined to be 1.95 cfs. This corresponds to an air velocity in the test pipe of 84.0 fps.

Assume that the velocity of the solid particles is 40 fps. This assumption seems reasonable after observing the flow of the solid particles in this experiment. Also assume that the weight rate of flow of solids was 17.10 pounds per minute, or 0.285 pounds per second. Assume also that the cross sectional area of a particle is 0.000255 square feet, and the weight of a particle is 7.30×10^{-5} pounds. These were properties of the average wheat particle used.

The number of particles flowing per second would be

$$\frac{0.285}{7.30 \times 10^{-5}} = 3900 \text{ particles per second.}$$

The number of particles per foot of pipe would be

$$\frac{3900 \text{ particles per second}}{40 \text{ feet per second}} = 97.5 \text{ particles/ft.}$$

The volume of the particles in one foot of pipe can be found by multiplying the number of particles in one foot of pipe by the volume of one particle, which can be assumed as 8.79×10^{-7} cubic feet.

Volume of particles per foot of pipe = $97.5 (8.79 \times 10^{-7}) = 8.57 \times 10^{-5}$ cubic feet.

The volume of air in one foot of pipe is the volume of the pipe minus the volume of the particles.

$$\begin{aligned} \text{Volume of air per foot of pipe} &= \frac{2.06^2}{12} \frac{\pi}{4} - .000857 \\ &= .0231453 - .0000857 \\ &= .0230596 \text{ cubic feet} \end{aligned}$$

The average air velocity is the velocity of this volume of air necessary to produce the given volume rate of flow of air. The net cross-sectional area of the pipe available for the flow of air is the volume of air per cubic foot of pipe divided by the length of pipe which we have considered to be one foot.

The average air velocity can be computed by dividing the volume rate of flow of air by the available cross-sectional area.

$$\text{Average Air Velocity} = \frac{1.95}{.0230596} = 84.70 \text{ fps}$$

The cross-sectional area of the pipe available for air flow, if the wheat volume is neglected, is 0.0231453 square feet. If the air velocity is computed from this cross-sectional area using the same volume rate of air flow as before, we obtain

$$\text{Average air velocity} = \frac{1.95}{.0231453} = 84.20 \text{ fps}$$

The error due to this approximation can be expressed percentage-wise as

$$\frac{84.70 - 84.20}{84.70} \times 100 = 0.590\%$$

This error is only slightly more than the potential error in the calculation of the volume rate of flow of air. The potential error due to instrumental accuracy was calculated previously on Page 65.

Since the error due to the above assumption was so small, average air velocities were calculated neglecting the grain volume throughout this paper.

ACCELERATION LOSSES

The two experimental measurements which were of prime importance were the air velocity in the pipe and the pressure drop per unit length of pipe.

In analyzing the results, it was found that a loss due to acceleration of the air and the solid particles was occurring after the flow had negotiated the 90 degree bend which entered the test section. It was observed from the experimental measurements that this effect died out somewhere between the second and third pressure taps. This was a distance of approximately eight feet from the end of the bend at the beginning of the test section. No attempt was made to measure these losses.

To prevent the inclusion of these acceleration losses in the data reported, particular care was given to analyzing the measured pressure drops between pressure taps. The pressure drops between taps became constant downstream from the #3 pressure tap. In obtaining the pressure drop per unit length of pipe, only these steady state pressure drops were averaged into the final figures. The average value for the pressure drop which was used in this analysis was obtained by dividing the average of the

pressure drops between pressure taps by the length of the pipe between the taps.

Thus the results of this research pertain only to steady state flow of air and wheat mixtures. The problems of accelerated flow of air and solid mixtures is another complex problem with which this paper is not concerned.

GENERAL THEORY

MEASUREMENT OF VELOCITY OF SOLID PARTICLES

The theory of Crane²⁴ has been mentioned previously in this paper. When this project was first begun, it was the intention of the writer to work along the same lines as Crane in developing a theory to cover the head loss.

In this light, considerable thought and planning was done toward the problem of measuring solid velocities in the flow. In order to develop any further theory using Crane's original ideas, it would have been almost a necessity to measure the velocities of the solid particles, since the drag force on the wheat particle is related to the relative velocity of the air and the grain or $(V_a - V_s)$, where the subscript, a, indicates air, and the subscript, s, indicates solids.

The possibility of coating some of the wheat particles with a radioactive material was considered. If this had been done, it would have been theoretically possible to install the necessary Geiger tubes and other electronic equipment to have timed the particle in its flight between two points. However, upon further consideration, it was decided that the conveying pipe would undoubtedly become contaminated from the radioactive material which rubbed off the wheat particles. This would have made the timing of any particular particle impossible.

Some consideration was given to the growing of radioactive wheat

24. Crane, J. W. Op. Cit. P. 40.

particles. However, it seemed that this process would only lengthen the time required for the system to become contaminated.

The possibility of photographing the particles in motion with a high speed motion picture camera was also considered. However, such equipment was found to be very expensive and was not available locally.

THEORY DEVELOPED

After the experimental measurements had been made, graphs of the pressure drop per foot of pipe were plotted against the computed air velocity. The graphs for both pipe sizes are shown in Figs. 15 and 16. These appeared to be a family of curves which closely paralleled the curve for the pressure drop due to air alone. The equation of the curve for the pressure drop due to the air alone is the familiar Darcy Weisbach equation²⁵.

$$h_{L_A} = f \frac{L}{D} \frac{V^2}{2g} \quad (29)$$

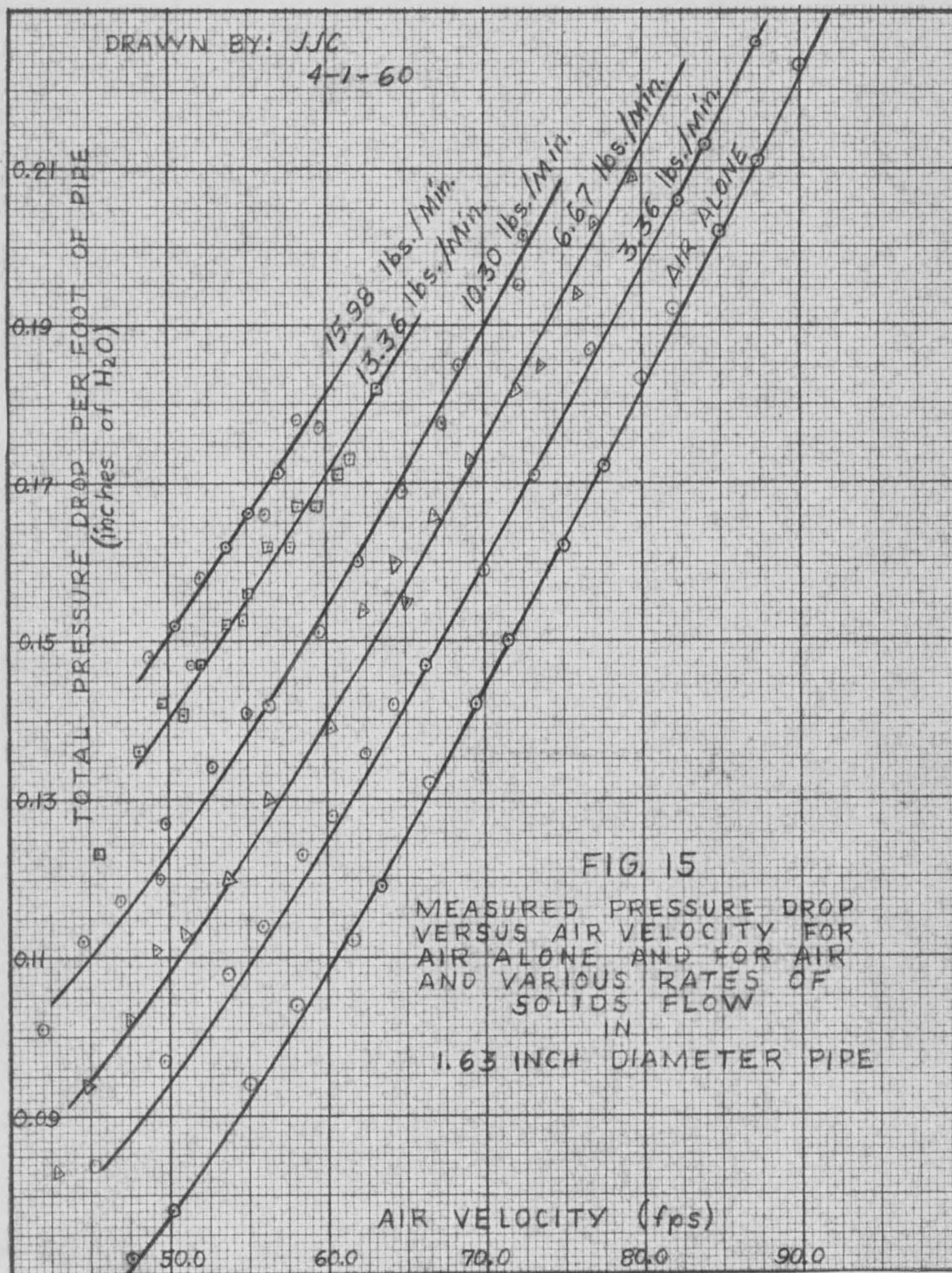
From a study of Fig. 15 and Fig. 16, it was decided that the family of curves could be represented by an equation,

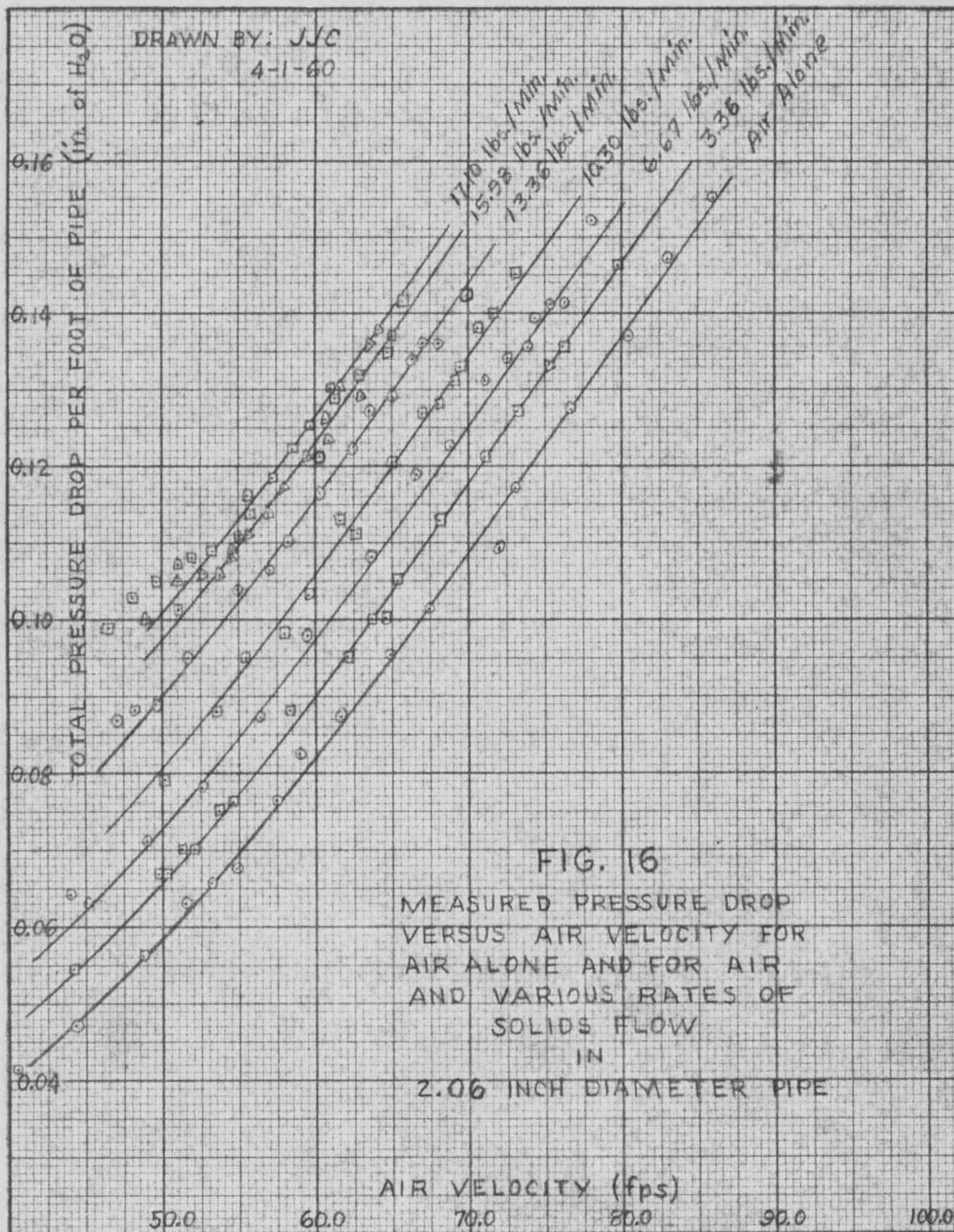
$$h_L = K f \frac{L}{D} \frac{V^2}{2g} \quad (30)$$

where K is a proportionality factor which is dependent upon the weight rate of flow of solids.

However, it appeared equally possible that the family of curves in Fig. 15 and Fig. 16 could be represented by an equation of the form,

25. Vennard, J. K. Op. Cit. P. 192.





$$h_L = f_f \frac{L}{D} \frac{V^2}{2g} + \phi(V, \rho_f, \rho_s, \mu, D, G_s, S) \quad (31)$$

In Equation (31), the first term on the right was assumed to account for the pressure drop due to the air alone, and the second term was assumed to account for the pressure drop due to the solid material alone. In order to account for the pressure drop due to the solid materials, the second term on the right of Equation (31) must be a function of at least those variables shown which are: V = velocity of the conveying fluid; ρ_f = mass density of the conveying fluid; ρ_s = mass density of the solids; μ = viscosity of the fluid; D = diameter of the conveying pipe; G_s = weight rate of flow of the solid material; and S = a factor to account for the shape and size of the solid particles.

If Equation (31) proved valid, then the following conclusion could be made.

The pressure drop due to the flow of air and solid mixtures can be divided into the following components.

- 1.) The pressure drop due to the air alone.
- 2.) The pressure drop due to the interaction of the air and the solid particles, the interaction of the solid particles, and their action upon the pipe walls.

Number 2 above is a definition of the pressure drop due to the solid material.

In regard to the conclusion above, Equation (31) implied that the

pressure drop^{due} to the air alone was unaffected by the addition of the solid material.

In order to test the validity of Equation (31), it was necessary to determine the value of the pressure drop due to the solids alone. It was assumed that this could be accomplished by subtracting values of pressure drop due to air alone from measured values of pressure drop due to air and solids. The magnitude of the pressure drop due to air alone was computed from Equation (29). Here it was assumed that the friction factor " f " was unaffected by the addition of the solids to the flow.

DIMENSIONAL ANALYSIS

With the above theory in mind, it was decided to perform a dimensional analysis of the variables involved in this particular research. It has already been pointed out by Rouse, as described on Page 17, that all of the possible variables involved in transportation of discrete matter are too numerous to afford examination.

The variables which are concerned in this research are the following.

- 1.) ρ_f -- Density of the fluid in slugs per cubic foot.
- 2.) ρ_s -- Density of the solids in slugs per cubic foot.
- 3.) μ -- Viscosity of the fluid in pound seconds per square foot.
- 4.) D -- Diameter of the pipe in feet.
- 5.) V_f -- Velocity of the fluid in fps.
- 6.) R -- Ratio of the weight rate of flow of solid material to the weight rate of flow of conveying fluid.

7.) h_{L_s} -- Pressure drop due to solids in pounds per square foot per foot of pipe.

The ratio " R " above was included after several attempts had already been made at dimensional analysis involving other variables to account for the amount of solid material which was present in the flow.

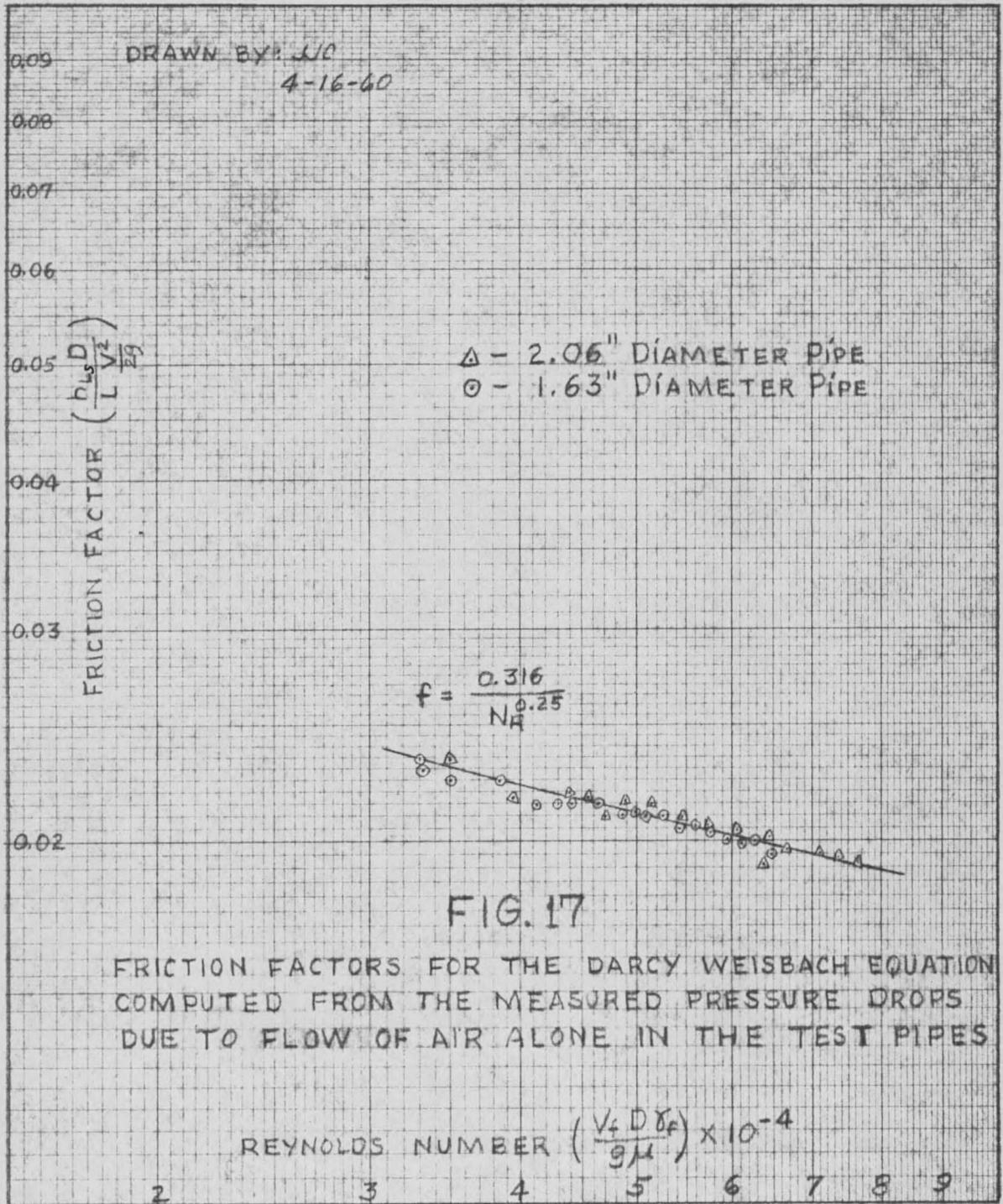
Friction factors for the Darcy Weisbach equation were computed for the pressure drop due to air alone in both pipes. These friction factors have been plotted against Reynolds number in the graph in Fig. 17. From examination of the graph, it was found that the friction factors satisfied Blasius' equation²⁶ for a hydraulically smooth pipe. Blasius' equation is written in the following form.

$$f_f = \frac{0.316}{N_R^{0.25}} \quad (32)$$

The line shown in Fig. 17 is the curve represented by Equation (32). This result meant that the pressure drop due to the air alone conveyed in the pipe was independent of the pipe roughness. Therefore it was assumed that since pipe roughness had not affected the pressure drop due to air alone, it would not affect the pressure drop due to the solid particles in the airstream, and accordingly no parameter for pipe roughness was included in the dimensional analysis.

No variable was put in this analysis to account for shape of the particles. The same wheat was used throughout the experimentation so there

26. Vennard, J. K. Op. Cit. P. 194.



was no variation in material tested. Undoubtedly the shape and size of the solid particles plays some rôle in the relations which account for the pressure drop due to pneumatic conveying of solids; however, this factor was not investigated in this paper.

The above variables were combined to form the following function.

$$h_{L_s} = \phi(P_f, P_s, R', \mu, D, V_f) \quad (33)$$

The variable, h_L , was moved to the opposite side of the equation in the following manner.

$$f(P_f, P_s, R', \mu, D, V_f, h_{L_s}) = \frac{\phi(P_f, P_s, R', \mu, D, V_f)}{h_L} = 1$$

The above equation was written in the following form

$$f(P_f^a, P_s^b, R'^c, \mu^d, D^e, V_f^f, h_{L_s}^g) = 1 \quad (34)$$

which rendered both sides of the equation dimensionless.

In the dimensional analysis, the foot-pound-second system was used. Under this system, the units of Mass (M), Length (L), and Time (T) are as follows:

M = slugs

L = feet

T = seconds

Using the above units, the variables were expressed in dimensional equalities as follows:

$$P_s = P_f = \frac{M}{L^3}$$

R' = Dimensionless variable

$$\mu = \frac{M}{LT}$$

$$D = L$$

$$V_f = L/T$$

$$h_{L^3} = \frac{M}{T^2 L^2}$$

Substituting the variables as expressed above into Equation (34) resulted in the following expression.

$$f \left[\left(\frac{M}{L^3} \right)^a \left(\frac{M}{L^3} \right)^b (R')^c \left(\frac{M}{LT} \right)^d (L)^e \left(\frac{L}{T} \right)^f \left(\frac{M}{T^2 L^2} \right)^g \right] = 1 \quad (35)$$

Since both sides of Equation (35) are dimensionless, the sum of the exponents of like quantities must be equal to zero. Equating the sum of the exponents of the mass (M), Length (L), and Time (T) to zero gave the following three equations.

$$\begin{aligned} a + b + d + g &= 0 \\ -3a - 3b - d + e + f - 2g &= 0 \\ -a - f - 2g &= 0 \end{aligned} \quad (36)$$

The above three equations together are a system of three equations in six unknowns. The first of the three equations can be solved for "a" to give

$$a = -b - d - g \quad (37)$$

The third equation can be solved for f to give the following:

$$f = -d - 2g \quad (38)$$

The second of the three equations can be solved for "e" and the values of "d" and "f" in Equations (37) and (38) substituted into this expression to obtain the following equation.

$$e = -d + g \quad (39)$$

The values of "a", "f", and "e" in Equations (37), (38), and (39) can then be substituted into Equation (34). This eliminates three of the six unknown exponents and gives the following.

$$f \left[\left(\frac{\rho_s}{\rho_f} \right)^{-b-d-g} (\rho_s)^b (R)^c (\mu)^d (D)^{-d+g} (V_f)^{-d-2g} (h_L)^g \right] = 1 \quad (40)$$

Collecting quantities with like exponents gives rise to the following equation.

$$f \left[\left(\frac{\rho_s}{\rho_f} \right)^b (R)^c \left(\frac{\mu}{\rho_f D V_f} \right)^d \left(\frac{D h_L}{\rho_f V_f^2} \right)^g \right] = 1 \quad (41)$$

This equation may be rewritten to give the following equation where "K" is a proportionality constant.

$$K \left(\frac{\rho_s}{\rho_f} \right)^b (R)^c \left(\frac{\mu}{\rho_f D V_f} \right)^d \left(\frac{D h_L}{\rho_f V_f^2} \right)^g = 1 \quad (42)$$

Equation (42) can then be solved for "h_L" to give the following.

$$h_L = K_1 \left(\frac{\rho_s}{\rho_f} \right)^{\frac{g}{b}} (R)^{\frac{g}{c}} \left(\frac{\rho_f D V_f}{\mu} \right)^{\frac{g}{d}} \left(\frac{\rho_f V_f^2}{D} \right) \quad (43)$$

Letting $\frac{g}{b} = x$, $\frac{g}{c} = Y$, $\frac{g}{d} = z$, $\frac{\rho_f}{\mu} = \rho_f$, and $R = G_s/G_f$ produces the following equation.

$$h_{L_s} = K_2 \left(\frac{P_s}{P_f} \right)^X \left(\frac{G_s}{G_f} \right)^Y \left(\frac{P_f D V_f}{\mu} \right)^Z \left(\frac{\gamma_f V_f^2}{D \cdot 2g} \right) \quad (44)$$

Since in the beginning of the analysis, pressure drop was expressed as pounds per square foot per foot of pipe, Equation (44) may be multiplied by L to give total pressure drop in pounds per square foot.

$$h_{L_s} = K_2 \left(\frac{P_s}{P_f} \right)^X \left(\frac{G_s}{G_f} \right)^Y \left(\frac{P_f D V_f}{\mu} \right)^Z \left(\frac{\gamma_f L V_f^2}{D \cdot 2g} \right) \quad (45)$$

The quantity $\frac{P_f D V_f}{\mu}$ is recognized as Reynolds number. Using that identity, Equation (45) may be rewritten to give the following.

$$h_{L_s} = K_2 \left(\frac{P_s}{P_f} \right)^X \left(\frac{G_s}{G_f} \right)^Y (N_R)^Z \left(\frac{L}{D} \frac{V_f^2}{2g} \right) \gamma_f \quad (46)$$

Equation (37) divided by γ_{H_2O} gives the pressure drop in feet of water.

$$h_{L_s} = K_2 \left(\frac{P_s}{P_f} \right)^X \left(\frac{G_s}{G_f} \right)^Y \left(\frac{\gamma_f}{\gamma_{H_2O}} \right) (N_R)^Z \left(\frac{L}{D} \frac{V_f^2}{2g} \right) \quad (47)$$

In order to evaluate the exponent "y", a graph of the pressure drop due to air plus solids versus the weight rate of flow of solids was drawn for each pipe size. These graphs are shown in Fig. 18 and Fig. 19. A study of these graphs shows that pressure drop is linearly proportional to the weight rate of flow of solids.

From this evidence, it can be assumed that the exponent "y" in Equation (47) must be equal to 1. Using this assumption, and letting $G_f = (\gamma_f V A_D)$, Equation (47) can be rewritten as

$$h_{Ls} = K_2 \left(\frac{\gamma_f}{\gamma_{H_2O}} \right) \left(\frac{\rho_s}{\rho_f} \right)^x (NR)^z \left(\frac{G_s L V_f^2}{D \gamma_f V_f A_D 2g} \right) \quad (48)$$

Cancelling out the "V" in the denominator, substituting $\frac{\pi D^2}{4}$ for the

area of the pipe, and combining constants, this equation is again re-written as

$$h_{Ls} = \frac{K_3}{\gamma_{H_2O}} \left(\frac{\rho_s}{\rho_f} \right)^x (NR)^z \frac{L V_f G_s}{D^3 g} \quad (49)$$

It was then assumed that Equation (49) was the second term on the right in Equation (31). Substituting Equation (49) for this term in Equation (31) produces the following equation.

$$h_{Ls} = f_f \frac{L}{D} \frac{V^2}{2g} + \frac{K_3}{\gamma_{H_2O}} \left(\frac{\rho_s}{\rho_f} \right)^x (NR)^z \frac{L V_f G_s}{D^3 g} \quad (50)$$

Equation (49) can be written as

$$h_{Ls} = f_s \frac{L V_f G_s}{\gamma_{H_2O} D^3 g} \quad (51)$$

where

$$f_s = K_3 \left(\frac{\rho_s}{\rho_f} \right)^x (NR)^z \quad (52)$$

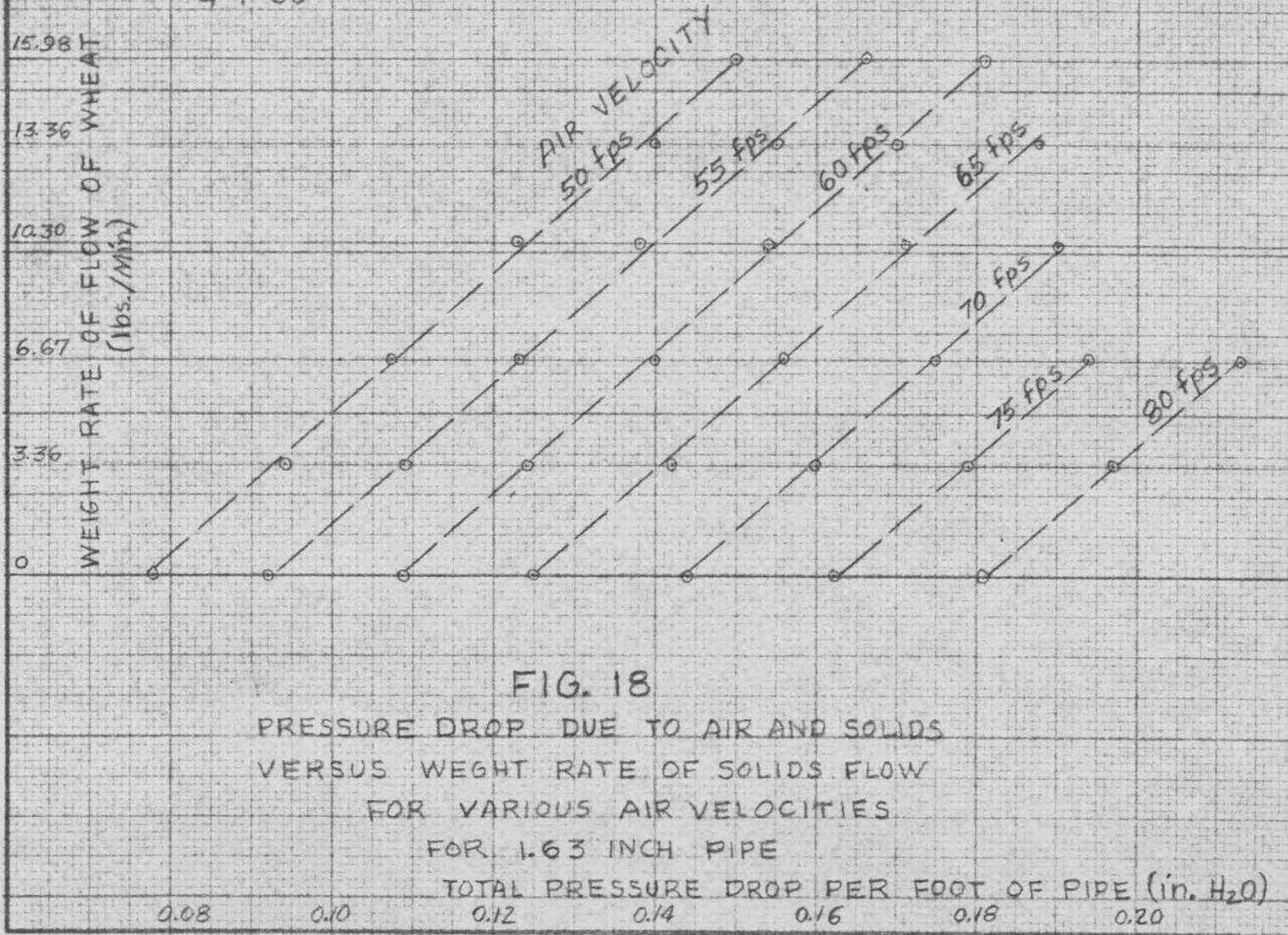
and

$K_3 = \text{constant}$

$\gamma_s = \text{specific weight of the solid particles}$

$\gamma_f = \text{specific weight of the conveying fluid}$

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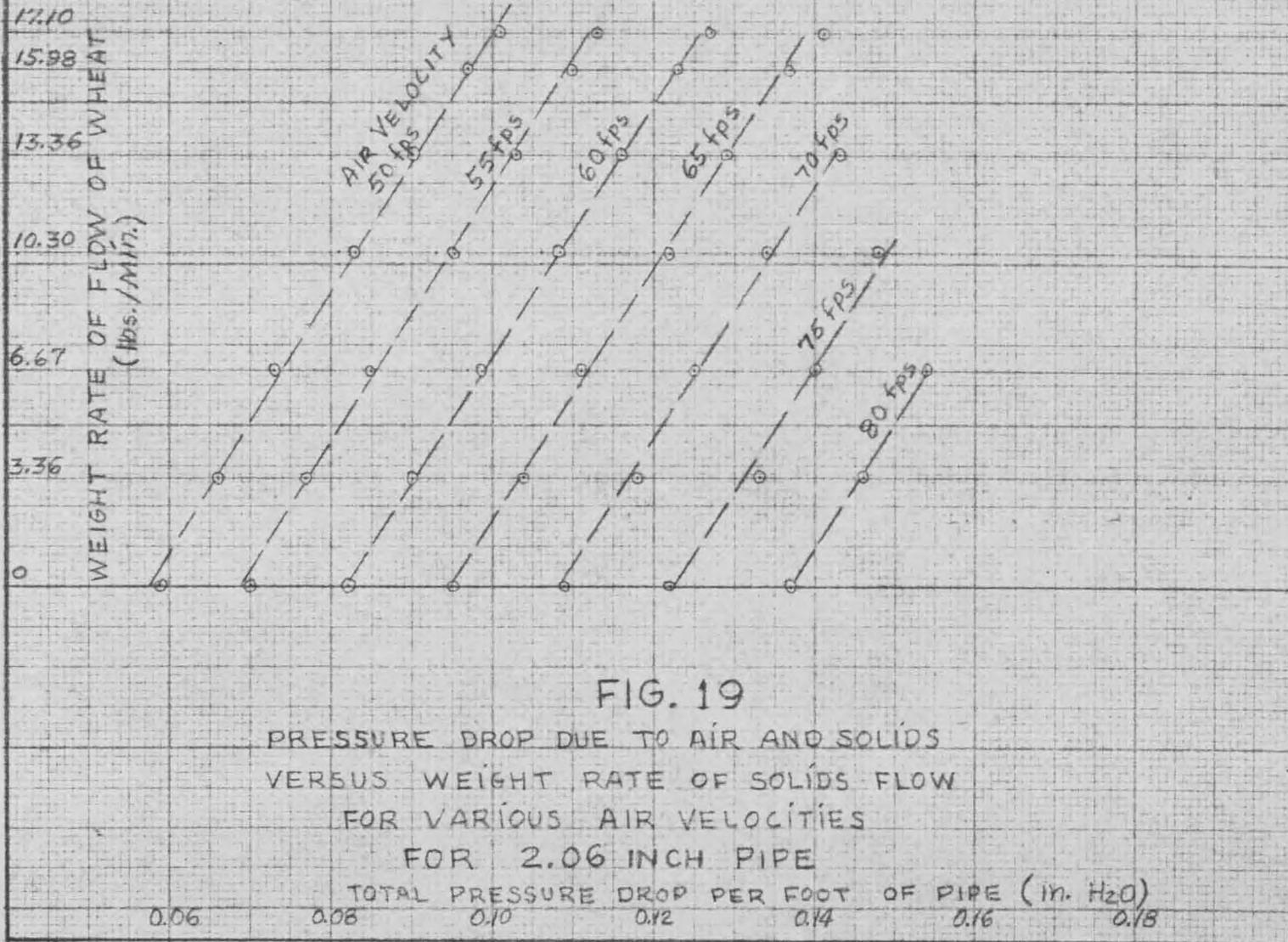


FIG. 19

PRESSURE DROP DUE TO AIR AND SOLIDS
VERSUS WEIGHT RATE OF SOLIDS FLOW
FOR VARIOUS AIR VELOCITIES
FOR 2.06 INCH PIPE

TOTAL PRESSURE DROP PER FOOT OF PIPE (in. H₂O)

0.06 0.08 0.10 0.12 0.14 0.16 0.18

Nr = Reynolds number.

The factor "fs" can be called a friction factor for the solid material. Examining Equation (51), it is evident that if the equation were valid, then the pressure drop due to the solids alone is a linear function of the velocity of the conveying fluid. It was expected then that a graph of the pressure drop due to the solids alone versus the average velocity of the conveying fluid would produce a family of straight lines.

Equation (50) can be rewritten using Equation (51) to produce

$$h_L = f_f \frac{L}{D} \frac{V^2}{2g} \frac{\gamma_F}{\gamma_{H_2O}} + f_s \frac{LV_F G_s}{D^3 g \gamma_{H_2O}} \quad (53)$$

where h_L is the pressure drop in feet of water.

This equation is similar to that of Crane given on Page 13 of this paper. For a horizontal pipe, Crane's equation for pressure drop, which is Equation (1), reduces to

$$h_L = f_f \frac{L}{D} \frac{V^2}{2g} \frac{\gamma_F}{\gamma_{H_2O}} + \frac{f'_s V_s L G_s}{2 D^3 g \gamma_{H_2O}} \quad (1)$$

where the middle term became zero, since for a horizontal pipe $\sin \theta = 0$.

In his equation, Crane used the solids velocity in the second term of his equation, while in Equation (44), the velocity of the conveying fluid is used. Crane's equation also has a factor of 2 in the denominator. This would indicate that the friction factors (f_s) in Equation (44) should be twice those for Equation (1) for the same flow conditions, provided both equations are valid.

Equation (52) shows the variables upon which the friction factor for the solids varies. A plot of the friction factor against the Reynolds number should show a definite relationship, if Equation (52) is valid.

With the above equations developed, experimental results were used to determine their validity.

ANALYSIS OF THE RESULTS

After the experimental results had been obtained, the graphs shown in Fig. 15 and Fig. 16 were drawn. Lines of best fit were constructed through the experimental points plotted for the various flows.

The curves appeared to be approximately parallel, and upon analysis as described on Page 82, were found to be spaced at intervals which were linearly proportional to the weight rates of solid flow.

In order to determine the pressure drop due to the solids alone, the ordinate of the curve for the pressure drop due to the air alone was subtracted from the ordinate of the curve for the pressure drop due to the air and wheat combined.

The curves for the pressure drop due to the grain alone are shown in Fig. 21 and Fig. 22.

A friction factor "fs" for the equation

$$h_{L_s} = f_s \frac{L V_f G_s}{D^3 g \gamma_{H_2O}} \quad (51)$$

was determined for each experimental point. An attempt was made to plot these coefficients against Reynolds number. However, it was found that

this plot formed a separate graph for each pipe diameter. This result seemed to prove that the equation

$$f_s = K_3 \left(\frac{G_s}{D_f} \right)^x (N_R)^z \quad (52)$$

was not valid, at least within the range of Reynolds numbers encountered in this research.

An attempt was then made to plot the friction factors for the solids against the air velocities in the conveying pipe. This attempt produced the graph shown in Fig. 20. This graph showed that the friction factors calculated for the two pipe diameters plotted as one line. There was considerable spread among the plotted points; however, it was apparent from this graph that within the range of these results, the friction factor "fs" was not dependent upon Reynolds number, but was dependent upon air velocity regardless of pipe diameter.

The friction factors were calculated so that using G_s in pounds per second, and the remainder of the terms in foot-pound-second units, h_L in Equation (42) is calculated in feet of water per foot of pipe.

The graph of the friction factor "fs" versus velocity shows a definite trend. The friction factor increases with decreasing velocities.

The spread of the experimental points in Fig. 20 is large, with the greatest spread occurring at the lower velocities. As has been previously stated, the flow in the test section ceased to be uniform at these lower velocities, and as a result, the measured pressure drop was subject to

considerable fluctuation.

The scatter of these points could be decreased by using a longer test section with greater spacing between pressure taps.

With the graph for "fs", it was possible to obtain predicted values of the pressure drop due to the solid particles for any size pipe. Values for the pressure drop due to the solid particles were computed for each pipe diameter and then plotted on Fig. 21 and Fig. 22.

This plot shows that the pressure drop as calculated using Equation (51) and values of "fs" from Fig. 20, agrees quite well with the measured pressure drops at all the solid flow rates. The poorest agreement occurs in the 2.06 inch pipe at the 17.10 lb./minute solid flow rate. This lack of agreement can be explained by the occurrence of non-uniform flow and "slugging" conditions at this solid flow rate.

The curve for "fs" and Equation (51) were then used to compare the results of Crane²⁷ with those of this research. The result of this comparison is shown in Fig. 23. The results do not agree. However, Crane's results were obtained with smaller, less dense wheat particles, a higher atmospheric pressure, and flow conditions at Reynolds numbers beyond the range of those reported here. If Equation (52) is valid at all for "fs", then the difference in the variables included in Equation (43) between Crane's results and those reported here could account for the lack of agreement shown in Fig. 23.

27. Crane, J. W. Op. Cit. P. 100.

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0.004

0.003

0.002

0.001

0.000

FRICION FACTOR ($f_s = \frac{h_s D^3 g}{L V G_s}$)

□ - 2.06 INCH DIAMETER PIPE
○ - 1.63 INCH DIAMETER PIPE

40.0

50.0

60.0

70.0

80.0

90.0

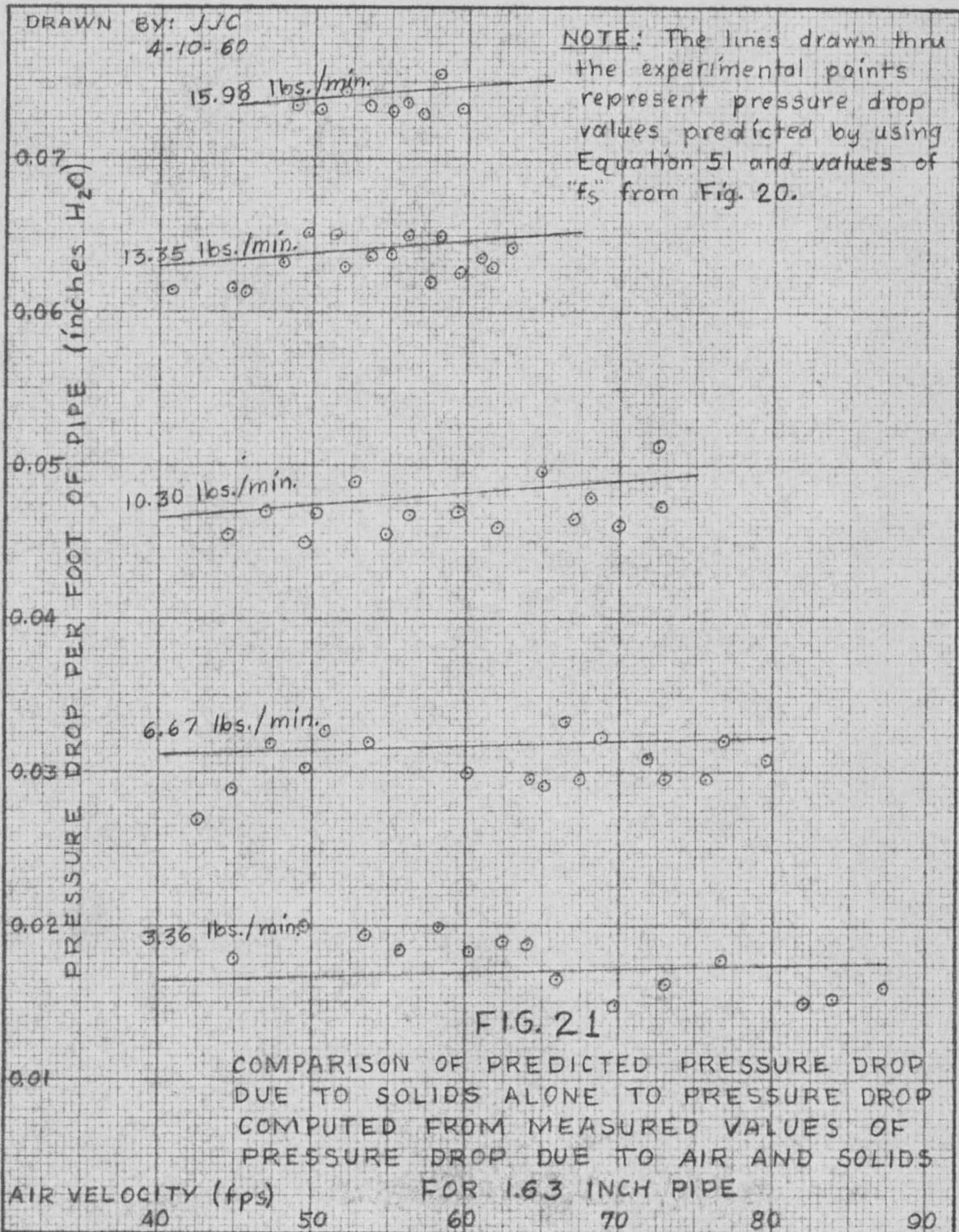
100.0

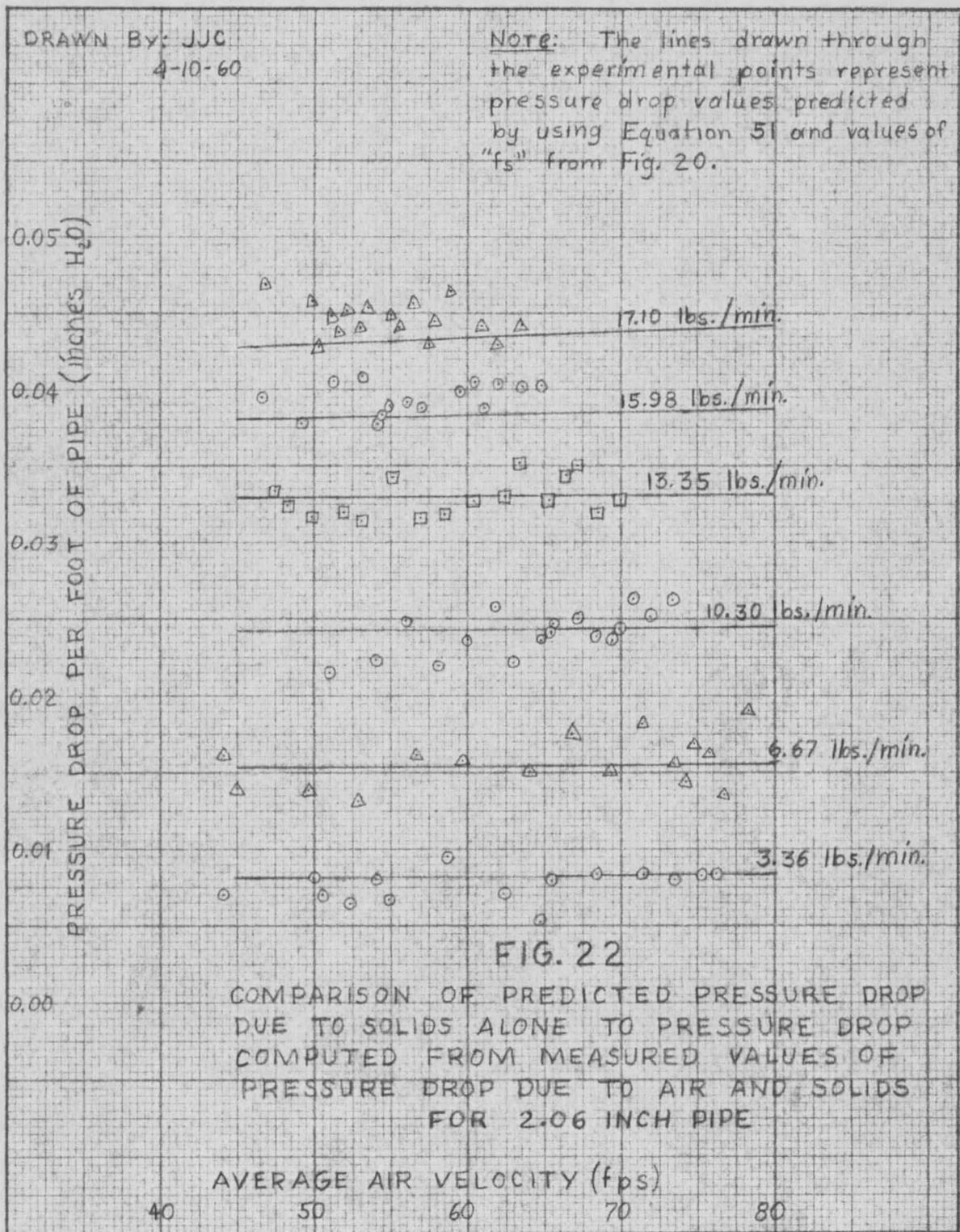
AIR VELOCITY (fps)

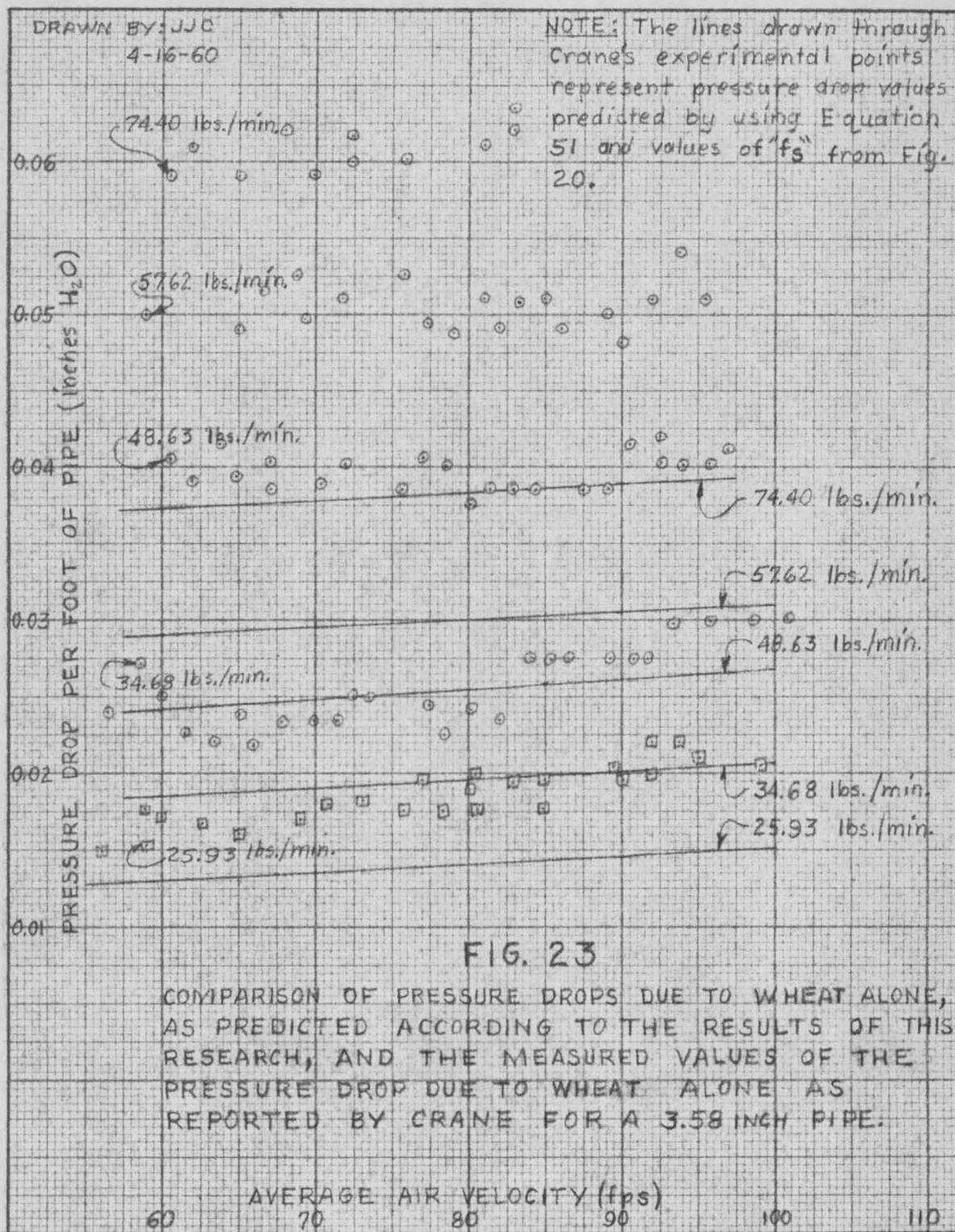
FIG. 20

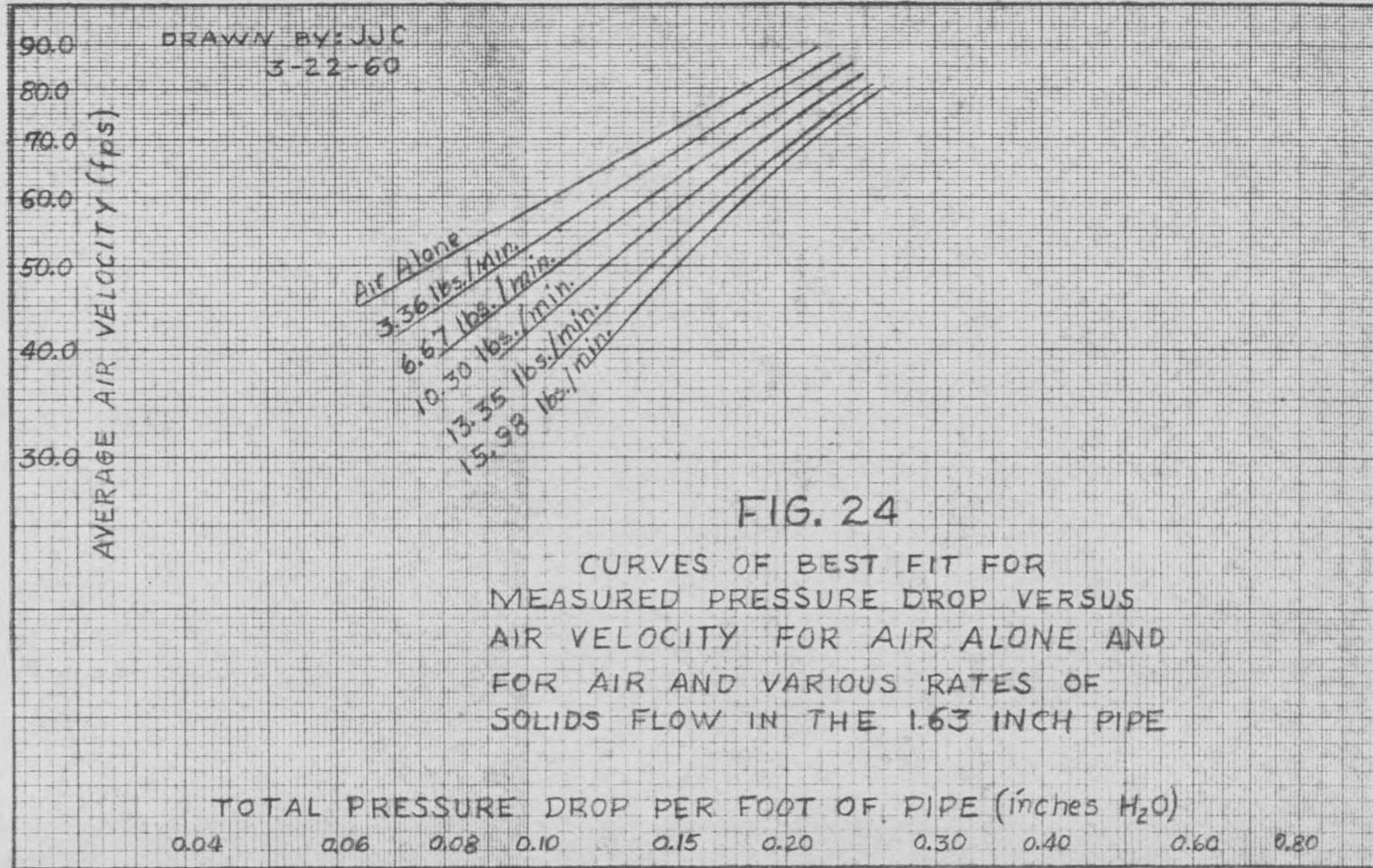
FRICION FACTORS COMPUTED FROM PRESSURE
DROP DUE TO SOLIDS ALONE VERSUS AIR
VELOCITY

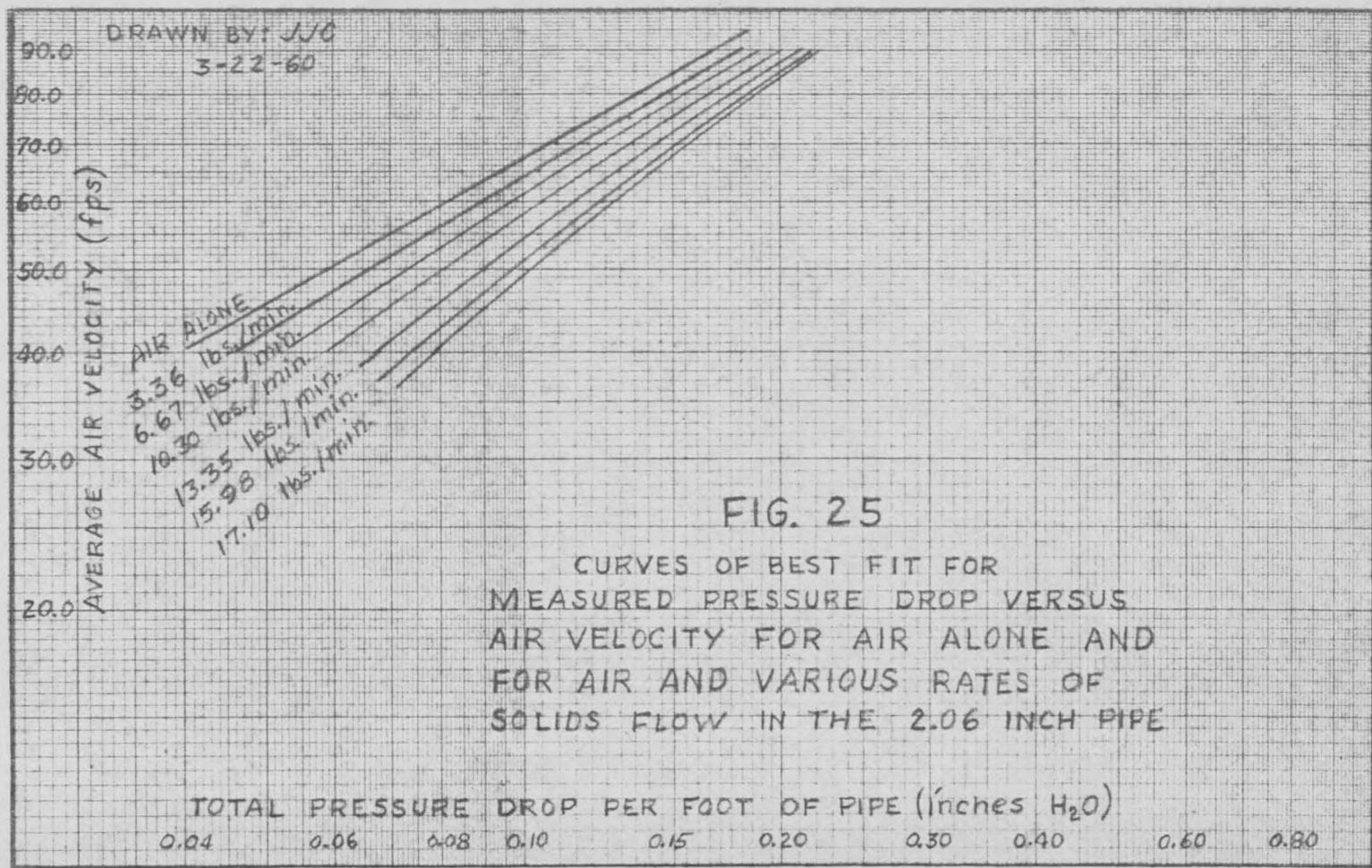
The pressure drops due to the air and solids together were also plotted against air velocity on logarithmic graphs for both pipe diameters. These graphs are shown in Fig. 24 and Fig. 25. These graphs present an interesting picture. As can be seen on the graph, the curves are approximately straight lines converging in the direction of increasing velocities. Since it seems unlikely that the pressure drop due to air plus solids will ever become less than the pressure drop for air alone, the curves in Figs. 24 and 25 would logically be asymptotic to the pressure drop curve for air alone.











CONCLUSIONS

The following conclusions can be reached as a result of this research.

1.) Within the limits of air velocities and Reynolds numbers prevailing in this research the pressure drop due to wheat particles (with the properties of those used here) being conveyed by an air stream in a hydraulically smooth pipe can be computed by the equation,

$$h_{L_s} = f_s \frac{L V_f G_s}{D^3 g \gamma_{H_2O}} \quad (51)$$

where h_L is the pressure drop in feet of water per foot of pipe.

2.) The pressure drop due to the wheat particles alone is almost independent of the air velocity. This conclusion is supported by the graphs of pressure drop due to solids in Fig. 21 and Fig. 22, which show the pressure drop curves to be almost horizontal.

3.) The friction factor " f_s " in the above equation is not expressed by the equation,

$$f_s = K_3 \left(\frac{\gamma_s}{\gamma_f} \right)^x (N_R)^z \quad (52)$$

within the range of this research since it has been shown that " f_s " is dependent not on Reynold's number, but on the air velocity. However, sufficient data was not available to determine if " f_s " is dependent on

$\frac{\gamma_s}{\gamma_f}$. Comparison of the data obtained here with that obtained by

J. W. Crane indicates that " f_s " is dependent on some variable other than

the air velocity.

4.) The friction factor " f_s " in Equation (5.1) increased with decreasing velocities. This fact can be explained in the following manner. As the velocity of the conveying fluid becomes smaller, the individual particles experience less lift. With this decrease in lift, the particles concentrate at the invert of the conveying pipe, and the spacing of the particles in the flow becomes smaller. Increasing amounts of energy are then dissipated in overcoming the friction of the particles against the pipe wall, and increasing amounts of energy are dissipated due to the increased frequency of particle contact.

5.) The quantities involved in determining the pressure drop due to the wheat particles alone are quite small, and in order to obtain better values for " f_s ", improved techniques and equipment are needed. A longer test section with greater distances between pressure taps would tend to smooth out the scatter of the experimental points in the curve for " f_s " versus air velocity in Fig. 20. Higher air velocities would also give more regular results, since the combined flow of the air and solids is more uniform at higher velocities.

SUGGESTIONS FOR ADDITIONAL RESEARCH

The research and results shown here have indicated that further research in the topic of pressure drop in the flow of air and solid mixtures is needed. The following topics were among those in which interest was aroused during the process of this project.

- 1.) A study of the flow of air and solid mixtures should be carried out over a much wider range of velocities and Reynolds numbers than those which were possible in this project.
- 2.) Study of the flow of air and solid mixtures using several types of solids involving variation in particle size, shape and density is needed to determine the effect of the physical characteristic of the solid materials upon the pressure drop.
- 3.) Study of the flow of air and solid mixtures in several diameters of pipes other than those used here should be made to determine better the relationship of pipe size to the friction loss.
- 4.) The study of the above items should be carried out, using more than one conveying fluid. This would indicate the relationship of the fluid properties in the equation for the pressure drop due to the solid phase of the conveyed material alone.

APPENDIX

SYMBOLS AND SUBSCRIPTS USED

The following symbols have been used throughout this paper.

<u>Symbol</u>	<u>Meaning</u>
h_L	Pressure drop
L	Length of conveying pipe.
V	Velocity.
γ	Specific Weight
D	Diameter of the conveying pipe.
ρ	Mass Density
g	Acceleration due to gravity.
D_p	Effective diameter of a particle.
β	Ratio of the pressure drop due to fluid plus solids, to the pressure drop due to air alone.
R'	Ratio of the weight rate of flow of solids to the weight rate of flow of fluid.
R	Gas constant.
N_r	Reynolds Number.
μ	Viscosity of fluid.
f	Friction factor.
G	Weight rate of flow.
θ	Angle of inclination of the conveying duct.
A	Cross-sectional area.
W_p	Weight per particle.

<u>Symbol</u>	<u>Meaning</u>
C	Discharge coefficient.
P	Pressure.
Q	Volume rate of flow.
Z	Elevation.
H	Differential pressure across the orifice plate.
C _c	Contraction coefficient.
C _v	Velocity coefficient.
T	Temperature.
X	Opening of variable orifice in grain hopper.
K	Proportionality constant.
k	Adiabatic coefficient.
M	Mass

The following subscripts were used in connection with the symbols above.

s	Solids.
α	Air.
D	Conveying duct.
f	Fluid.
P	Particle .

ABBREVIATIONS USED

fps	Feet per second.
cfm	Cubic feet per minute.
cfs	Cubic feet per second.

SUMMARY DATA SHEET

PHYSICAL PROPERTIES OF THE WHEAT USED

Specific gravity of the wheat	1.375
Specific weight of the wheat particles	87.8 lbs/ft ³
Weight per particle of the wheat	75.3 × 10 ⁻⁶ lbs.
Average volume per particle	8.79 × 10 ⁻⁷ ft ³
Average major axis length (c)	0.231 inches
Average minor axis length (a)	0.099 inches
Average minor axis length (b)	0.118 inches
Average shape factor	0.600
Bulk weight per bushel of wheat	56.1 pounds
Type of wheat	Light spring wheat

SUMMARY DATA SHEET

Pipe Diameter 1.63 inches

Temperature 75° F.

Barometric Pressure 25.62 inches of mercury.

Average Specific Weight of air in test section 0.0650#/ft³

Run No.	Ave. Air Velocity ft/sec.	Ave. Pressure Drop Inches H ₂ O/Ft.	Run No.	Ave. Air Velocity ft/sec.	Ave. Pressure Drop Inches H ₂ O/Ft.
"6"	Wheat =	10.30#/min.	"6"	Wheat =	6.67#/min.
1	72.2	0.195	1	79.5	0.209
2	68.3	0.185	2	75.8	0.194
3	64.9	0.169	3	73.5	0.185
4	62.2	0.160	4	72.0	0.182
5	59.6	0.151	5	69.1	0.173
6	56.4	0.142	6	66.8	0.166
7	54.8	0.141	7	64.3	0.160
8	52.7	0.134	8	60.2	0.139
9	49.8	0.127	9	56.3	0.130
10	47.0	0.117	10	53.7	0.120
11	44.6	0.112	11	51.0	0.113
12	41.8	0.101	12	47.5	0.102
13	49.5	0.120	13	44.8	0.094
14	67.3	0.178	14	42.9	0.083
15	72.5	0.201	15	62.3	0.154
16			16	49.2	0.111
17			17	77.0	0.203
18			18	65.0	0.155
19					

SUMMARY DATA SHEET

Pipe Diameter 1.63 inches

Temperature 77° F

Barometric Pressure 25.68 inches of mercury

Average specific weight of air in the test section .0653 #/ft³

Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.	Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.
"G"	Wheat =	3.36#/Min.		Air Alone	
1	87.2	0.226	1	45.0	0.066
2	84.0	0.213	2	47.5	0.072
3	82.2	0.206	3	50.0	0.078
4	76.7	0.187	4	55.0	0.094
5	73.2	0.171	5	58.0	0.104
6	70.0	0.159	6	61.5	0.112
7	66.2	0.147	7	63.3	0.119
8	64.2	0.142	8	66.6	0.132
9	62.3	0.136	9	69.5	0.142
10	60.2	0.128	10	71.3	0.150
11	58.3	0.123	11	75.0	0.162
12	55.8	0.114	12	77.5	0.172
13	53.7	0.108	13	80.0	0.183
14	49.7	0.097	14	82.0	0.192
15	45.2	0.084	15	85.0	0.202
16			16	87.3	0.211
17			17	90.0	0.223
18			18	92.3	0.230
19			19		

SUMMARY DATA SHEET

Pipe Diameter 1.63 inches

Temperature 75° F

Barometric Pressure 25.62 inches of mercury

Average specific weight of air in test section 0.0652 #/ft³

Run	Ave. Air Velocity	Ave. Pressure Drop	Run	Ave. Air Velocity	Ave. Pressure Drop
No.	Ft./Sec.	Inches H ₂ O/Ft.	No.	Ft./Sec.	Inches H ₂ O/Ft.
"G"	Wheat =	15.98 #/Min.	"G"	Wheat =	13.35#/Min.
1	58.2	0.178	1	63.2	0.182
2	57.0	0.171	2	61.5	0.173
3	55.2	0.166	3	59.4	0.167
4	56.2	0.166	4	60.5	0.171
5	53.7	0.162	5	58.2	0.167
6	52.0	0.158	6	56.3	0.162
7	50.4	0.152	7	53.7	0.152
8	48.8	0.148	8	55.0	0.156
9	59.7	0.177	9	52.0	0.147
10			10	51.4	0.147
11			11	49.7	0.142
12			12	48.2	0.136
13			13	45.5	0.123
14			14	50.9	0.141
15			15	54.8	0.153
16			16	57.7	0.162
17			17		
18			18		
19			19		

SUMMARY DATA SHEET

Pipe Diameter 2.06 inches

Temperature 73° F

Barometric Pressure 25.60 inches mercury

Average specific weight of air in test section .0653 #/ft³

Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.	Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.
"G"	Wheat =	13.35 #/Min.	"G"	Wheat =	10.30 #/Min
1	70.0	0.142	1	73.3	0.145
2	68.3	0.136	2	71.8	0.140
3	66.5	0.134	3	70.8	0.138
4	67.2	0.136	4	69.8	0.133
5	65.4	0.129	5	69.4	0.131
6	62.5	0.122	6	68.3	0.128
7	63.6	0.127	7	67.3	0.127
8	60.3	0.116	8	65.3	0.121
9	58.5	0.110	9	65.2	0.120
10	57.1	0.106	10	63.0	0.111
11	55.1	0.104	11	61.8	0.113
12	52.8	0.095	12	59.8	0.103
13	51.9	0.095	13	58.2	0.098
14	49.8	0.089	14	55.7	0.095
15	47.3	0.087	15	53.8	0.088
16	48.3	0.088	16	50.2	0.079
17			17		
18			18		
19			19		

SUMMARY DATA SHEET

Pipe Diameter 2.06 inches

Temperature 78° F

Barometric Pressure 25.70 inches of mercury

Average specific weight of air in test section 0.0651 #/ft³

Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.	Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.
"G"	Wheat =	6.67 #/Min.	"G"	Wheat =	3.36 #/Min.
1	78.3	0.152	1	80.0	0.146
2	76.7	0.141	2	76.0	0.135
3	75.7	0.141	3	75.5	0.133
4	74.7	0.139	4	73.5	0.127
5	74.2	0.135	5	71.3	0.121
6	73.3	0.134	6	68.4	0.113
7	71.3	0.131	7	65.5	0.105
8	69.2	0.122	8	62.3	0.095
9	66.8	0.119	9	58.5	0.088
10	64.0	0.108	10	53.8	0.075
11	59.8	0.098	11	50.3	0.067
12	56.7	0.087	12	44.2	0.054
13	52.9	0.078	13	50.0	0.067
14	49.3	0.071	14	52.3	0.070
15	44.2	0.064	15	54.8	0.076
16	45.1	0.063	16	64.8	0.100
17			17		
18			18		

SUMMARY DATA SHEET

Pipe Diameter 2.06 inches

Temperature 75° F

Barometric Pressure - 25.62 inches of mercury

Specific weight of air in test section 0.0652 #/ft³

Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.	Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.
"G"	Wheat =	17.10 #/Min.	"G"	Wheat =	15.98 #/Min.
1	46.8	0.102	1	64.3	0.138
2	64.9	0.135	2	63.2	0.129
3	61.5	0.130	3	61.0	0.123
4	57.5	0.118	4	59.7	0.121
5	65.1	0.137	5	57.0	0.114
6	63.0	0.127	6	56.2	0.111
7	61.3	0.128	7	54.8	0.109
8	60.3	0.121	8	52.9	0.106
9	57.9	0.117	9	51.2	0.107
10	56.3	0.110	10	49.1	0.100
11	53.5	0.109	11	46.8	0.099
12	51.0	0.101	12	61.8	0.130
13	50.0	0.105	13	60.6	0.126
14	52.0	0.108	14	55.3	0.111
15	55.5	0.116	15	51.2	0.105
16	58.5	0.122	16	53.9	0.110
17	63.7	0.136	17		

SUMMARY DATA SHEET

Pipe Diameter 2.06 inches

Temperature 80° F

Barometric pressure 25.59 inches mercury

Average specific weight of air in test section 0.0648 #/ft³

Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.	Run No.	Ave. Air Velocity Ft./Sec.	Ave. Pressure Drop Inches H ₂ O/Ft.
Air Alone					
1	40.0	0.041	1		
2	44.5	0.047	2		
3	48.3	0.056	3		
4	51.5	0.063	4		
5	53.3	0.065	5		
6	55.0	0.071	6		
7	57.5	0.076	7		
8	59.0	0.082	8		
9	61.7	0.087	9		
10	64.8	0.095	10		
11	68.5	0.105	11		
12	72.1	0.109	12		
13	73.0	0.117	13		
14	76.7	0.127	14		
15	80.3	0.138	15		
16	83.2	0.147	16		
17	86.0	0.155	17		
18			18		
19			19		

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Cassidy, J. J.

Pressure drop for flow of air
and grain mixtures in circular
pipes

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