



Optimal replacement of alfalfa stands
by Glen Irvin Goodman

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
in Applied Economics
Montana State University
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Abstract:

This study investigates the optimal stand length of irrigated alfalfa hay. A dynamic decision model is solved which determines the optimal decision (grain or continuation of an existing alfalfa stand) for 275 possible states of the system (25 price states for each of 11 land use states). The problem is solved for specified production practices and interest rates. The decision criterion is to maximize the net present value of future returns from irrigated crops in Montana river valleys. Both pure stands and companion crop stands of alfalfa are examined. Alfalfa replacement is found to take place between seven and eight years of stand life. The decision rule is relatively insensitive to interest rates and nearly identical for both the pure and companion crop stands. A pure stand of alfalfa is found to be slightly more profitable than an alfalfa stand seeded with a companion crop.

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Date May 10, 1982

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by

GLEN IRVIN GOODMAN

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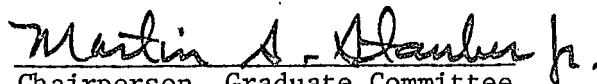
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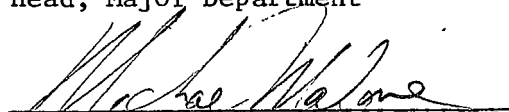
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TABLE OF CONTENTS

Chapter		Page
	VITA.	ii
	AKNOWLEDGEMENTS	iii
	TABLE OF CONTENTS	iv
	LIST OF TABLES.	vi
	LIST OF FIGURES	vii
	ABSTRACT.	viii
1	INTRODUCTION.	1
	Problem Statement	2
	Objectives.	2
2	LITERATURE REVIEW	3
	Optimum Replacement Patterns and Principles	3
	Dynamic Programming Decision Models	6
	Alfalfa Management.	8
3	THE DECISION MODEL: FORMULATION AND IMPLEMENTATION	12
	General Decision Model for Alfalfa Replacement.	12
	Application of the Model to the Determination of Optimal Age of Alfalfa Stands	14
	Crop Production Relationships	15
	Price State Variables and Transition.	24
	Crop Enterprise and Yield Data.	32
	The Empirical Model	32
	State Variables	34
	Decision Alternatives	34
	Expected Immediate Returns.	35
	Recurrence Relation	35
4	RESULTS	37
	Optimal Policies for Model I.	37
	Price State Equilibrium Probabilities and Land Use Transition Probabilities.	42
	Land Use Equilibrium Probabilities.	47

Chapter		Page
5	SUMMARY, CONCLUSIONS AND RECOMMENDATIONS.	49
	Conclusions	50
	Recommendations	50
	BIBLIOGRAPHY.	52
	APPENDICES.	55
	Appendix A: Production Cost and Yield Data	56
	Appendix B: Production Cost Data - Alfalfa	68
	Appendix C: Deterministic Alfalfa Replacement Model	71

LIST OF TABLES

Table Number		Page
1	Variable Production Costs - Irrigated Barley (Previous land use state = Alfalfa)	16
2	Variable Production Costs - Irrigated Barley (Previous land use state = Barley).	17
3	Alfalfa Yield Trial Observations.	19
4	Regression Results - Alfalfa Production Functions . .	21
5	Pure Alfalfa vs. Companion Crop Alfalfa Yields (Adjusted to irrigated farm level conditions) . . .	25
6	Parameter Estimate and Associated Statistics for Price Equations	28
7	Discrete Intervals for Alfalfa Hay and Barley Prices in 1980 Dollars per Ton	29
8	Joint Distribution of Random Price Variables.	33
9	Optimal Policies for Model I for Stages 10 and 42 at a 6 percent interest rate.	38
10	Price State Equilibrium Probabilities	42
11	Land Use Transition Probabilities	44
12	Land Use Transition Probability Matrix for Model I. .	47
13	Land Use Equilibrium Probability Vector	48

LIST OF FIGURES

Figure Number		Page
1	Optimization Over Time.	5
2	Deterministic Model for Dynamic Programming	9
3	Stochastic Model for Dynamic Programming.	10
4	Functional Forms (Models 1 and 2)	23
5	Production Function Graphs.	26
6	Illustration of Price States for Alfalfa and Barley .	30

ABSTRACT

This study investigates the optimal stand length of irrigated alfalfa hay. A dynamic decision model is solved which determines the optimal decision (grain or continuation of an existing alfalfa stand) for 275 possible states of the system (25 price states for each of 11 land use states). The problem is solved for specified production practices and interest rates. The decision criterion is to maximize the net present value of future returns from irrigated crops in Montana river valleys. Both pure stands and companion crop stands of alfalfa are examined. Alfalfa replacement is found to take place between seven and eight years of stand life. The decision rule is relatively insensitive to interest rates and nearly identical for both the pure and companion crop stands. A pure stand of alfalfa is found to be slightly more profitable than an alfalfa stand seeded with a companion crop.

Chapter 1

INTRODUCTION

Alfalfa (*Medicago sativa* L.) is the most important hay crop in Montana with approximately 1,260,000 acres presently seeded. This accounts for 53 percent of the total hay acreage in the state (10). Its primary use in state is for the feeding of livestock with the residual being transported out of the state for sale.

Alfalfa's popularity is derived from the fact that it is a perennial crop (can sustain harsh winters) with a greater yield and higher nutrient content per acre than most competing forage crops (8). Also, in cooperation with bacteria, alfalfa has the ability to utilize atmospheric nitrogen. The amount of nitrogen has been conservatively estimated at 74 to 90 pounds per acre, making it a desirable crop in rotations (8).

The 1980 Montana alfalfa crop was valued at \$173,000,000, trailing only spring and winter wheat in value (10). Because of these desirable qualities alfalfa has been designated "queen of the forage crops" by research agronomists.

Because of alfalfa's significant economic value, improvement in its management offers substantial monetary returns. One of the important questions facing alfalfa producers is when to replace an established stand of alfalfa with a new stand. The purpose of this study was to determine the optimal age to replace alfalfa stands.

Problem Statement

Since alfalfa was first introduced in Montana, farmers have used rather intuitive "rules of thumb" in deciding when to replace alfalfa stands. The aim of this research is to develop an economic decision model, which will allow farm managers to make optimal decisions regarding replacement of alfalfa stands.

The goal of replacement theory is to maximize the economic returns from an asset over time. This is accomplished by selecting a particular production period which yields the maximum net present value of future returns. The decision model developed should achieve this goal and also be general enough to allow for the production of alternative crops such as barley, if the returns from the alternative crops are superior to the returns from an alfalfa stand of optimal duration.

Objectives

The primary objective of this thesis is to develop a decision making model for Montana farmers, with the intended purpose of increasing the economic returns from alfalfa production in Montana's irrigated crop areas. An economic replacement model cast in a dynamic programming framework will be used to determine the optimal policy for replacing irrigated alfalfa stands. The dynamic replacement model will be solved for specified production practices, interest rates and product prices representing most conditions encountered on irrigated crop farms in Montana.

Chapter 2

LITERATURE REVIEW

The principles and techniques of the articles reviewed in this section are essential for the model development of this study.

The reviewed literature is divided into three categories:

1) optimum replacement patterns and principles; 2) dynamic programming decision models; and 3) alfalfa management.

Optimum Replacement Patterns and Principles

Anthony Chisholm (2), R. K. Perrin (13) and Martin Upton (17) have contributed to the understanding and implementation of asset replacement theory. (It should be noted that a German forester, Martin Faustman [1849] is credited with the first application of discounted cash flows to a replacement problem.)

Chisholm [1966] asserts the general premise of replacement as being able to select the particular production period over a pre-determined planning horizon which will maximize the net present value of future returns.

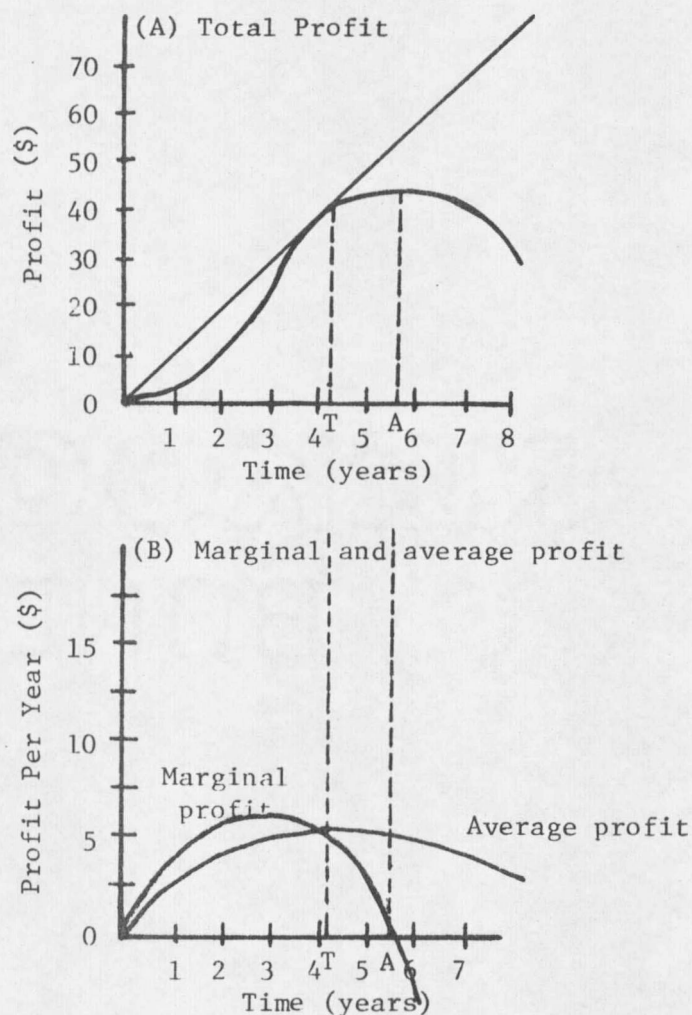
Perrin [1972] characterizes replacement as the goal of an asset manager wanting to maximize the net present value of a future stream of income associated with the asset. The actual replacement problem according to Perrin is choosing the replacement age which maximizes the net present value of cash flows attributed to an asset.

Upton [1976] illustrates replacement theory graphically. An illustration of his graphical analysis in production theory framework is depicted in Figure 1.

Chisholm [1966] addresses the opportunity cost considerations of replacement theory. He defines the relevant opportunity costs in replacement as ones which are involved in the fixed and variable costs as well as the funds tied up in the actual replacement asset under study. Chisholm concludes by stating that the relevant opportunity costs should be compounded at an appropriate rate of interest in order to compare costs and returns incurred at differing points in time.

Perrin [1972] elaborates on the choice of a discount rate. The cost of capital, return on alternative investment possibilities, or timing of personal consumption are alternative choices when deriving a discount rate according to Perrin. He further states that although neither of these are universally recognized, the choice of discount rate depends on the circumstances at hand. That is, the presence of perfect capital markets, "destitute" owners who value future earnings quite low relative to present earnings, or a fixed capital stock with no accessible external capital markets are all factors which affect the appropriate rate of discount.

Perrin also makes the distinction between continuous and discrete time. He believes that in most replacement problems, net revenues and



"Profit is represented as a function of the length of the production period in the upper graph (A). The lower graph (B) shows the average and marginal net profits per year over the production period. Maximum profit for a single non-repeated project is found where the marginal profit from prolonging it is zero (i.e. at project life of OA years). However, maximum profit per unit of time is found where marginal profit from prolonging the project is equal to the average profit per year (i.e. at a project life of OT years)."

Figure 1. Optimization Over Time

Source: Upton, Martin. Agricultural Production Economics and Resource Use. Oxford University Press [1976].

market values are observed as discrete annual levels.

Perrin also notes that evaluating the present values of the returns associated with an asset is often a better search procedure than evaluating the marginal criteria. The reason he states, is that there is a possibility of encountering a one year error when making a marginal decision.

It should be recognized that not one application of replacement theory to the optimal replacement of alfalfa stands was found in the literature.

Dynamic Programming Decision Models

Optimum replacement models can be formulated using the procedures of dynamic programming. Early applications of dynamic programming to replacement problems were made by Ronald Howard (5), with early applications to agricultural production problems by Oscar R. Burt and John R. Allison (1). Frederick S. Hillier and Gerald J. Lieberman (4) have published a contemporary textbook which contains a good discussion of dynamic programming.

Hillier and Lieberman [1980] define dynamic programming as a mathematical search procedure often useful for making a sequence of interrelated decisions. They conclude their description by characterizing dynamic programming as a systematic procedure which can determine the combination of decisions that maximizes overall efficiency.

Burt and Allison [1963] clarify the concept of dynamic programming in their work, by describing it as a multistage decision process which involves finding a sequence of decisions which maximizes an appropriately defined objective function. The stages are intervals into which the process is divided, with a decision being made at each stage. A sequence of these stages comprises the decision process. Burt and Allison go on to define a state as the condition of a process at a particular stage. This is defined by the magnitude and/or qualitative characteristics of the variables involved. Burt and Allison conclude their conceptual description of dynamic programming by placing it in a decision framework. That is, the state of a process in the following stage is controlled by the decision making at the present stage. The control can be either deterministic or stochastic.

Hillier and Lieberman [1980] describe the Markovian requirement of dynamic programming by stating that the optimal policy starting in a given state depends only on the state of the process in that stage and not on the state at preceding stages. They also comment on the recursive relationship that is present in dynamic programming models. Hillier and Lieberman note that the relationship allows the solution procedure to move backward stage by stage, finding an optimal policy at each state of the stage, until an optimal policy is found when starting at the initial stage.

Hillier and Lieberman graphically illustrate the basic structures

of deterministic and stochastic dynamic programming models. The basic structures are presented in Figures 2 and 3.

The differences between the models are clearly illustrated. That is, the state of the next stage is not solely determined by the present state and policy decision of the current stage, in the stochastic model. There is, instead, a probability distribution for determining the next state in the new stage. Hillier and Lieberman note that the probability distribution is a function of the current state and policy alternative at the current stage only.

Howard [1960] sets up an automobile replacement problem using dynamic programming. The planning horizon is designated as ten years with a replacement decision being made every three months. He describes the state of the system (i), as the age of the car in three month periods, with (i) running from 1 to 40. In order to keep the number of states finite, Howard considered a car of age 40 essentially worn out. Howard defines the decision alternatives available in each state as $S = 1$, keep the present car for another quarter, or $S > 1$, buy a car of age $S - 2$. Thus, Howard defines a replacement problem with 40 states and 41 alternatives in each state.

Alfalfa Management

Data involving alfalfa production has been drawn on from Extension research studies in Montana. Specific references are: Montana

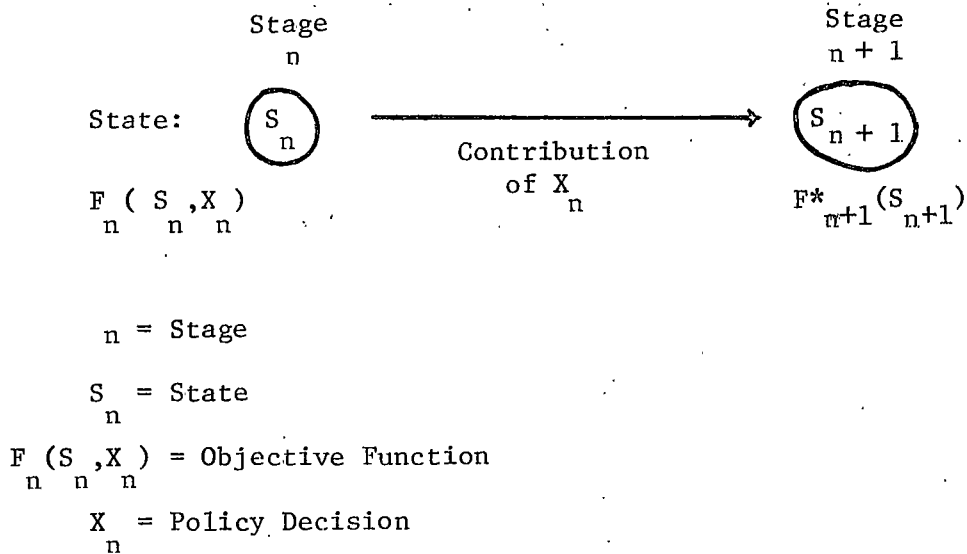
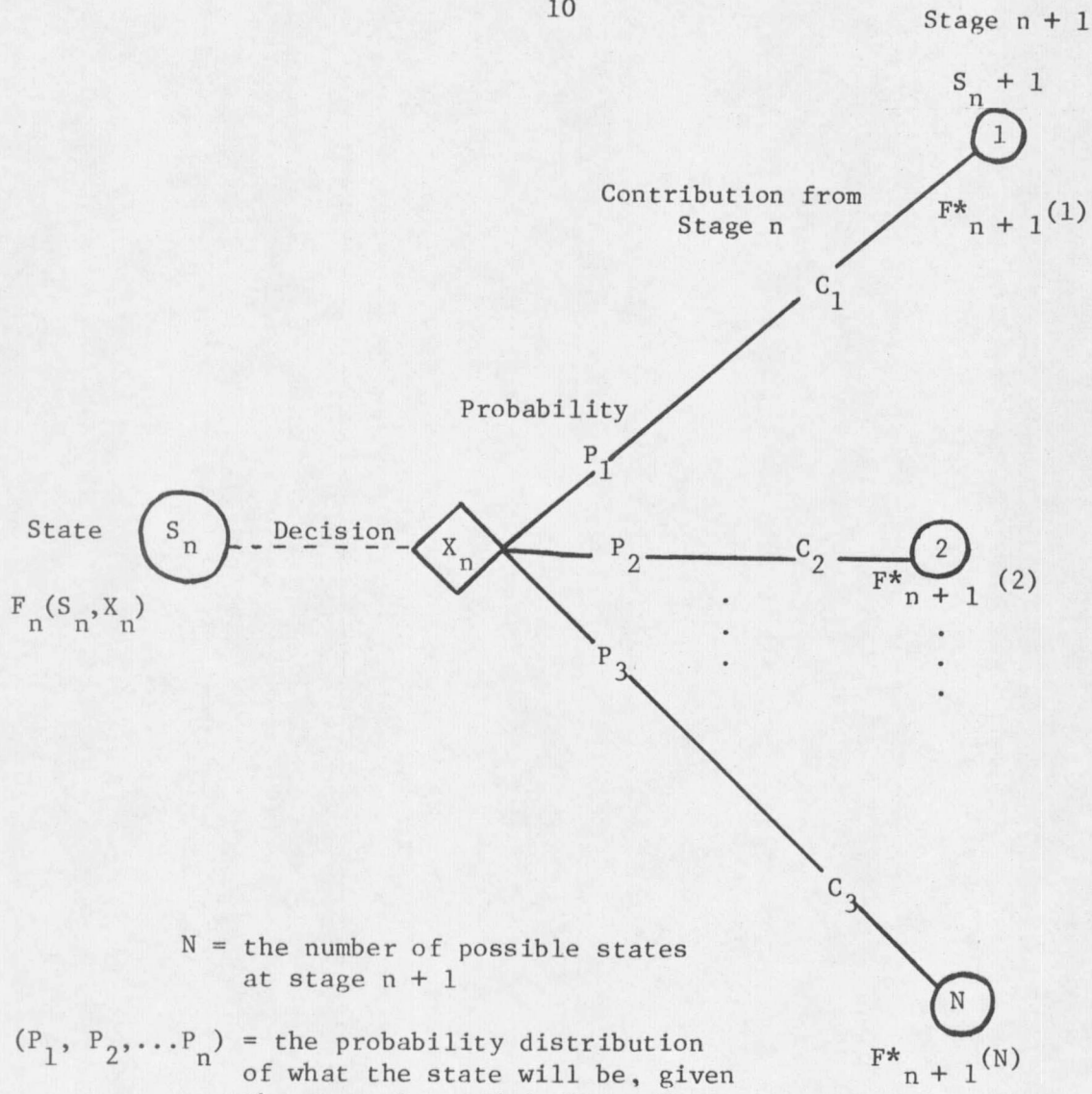


Figure 2. Deterministic Model for Dynamic Programming.

Source: Hillier, Frederick S. and Lieberman, Gerald J. Introduction to Operations Research. Holden-Day, Inc. [1980].



N = the number of possible states at stage $n + 1$

(P_1, P_2, \dots, P_n) = the probability distribution of what the state will be, given the state S_n , and decision X_n at stage n .

C_i = the resulting contribution to the objective function from stage n if the state turns out to be static i .

Figure 3. Stochastic Model for Dynamic Programming.

Source: Hillier, Frederick S. and Lieberman, Gerald J., Introduction to Operations Research. Holden-Day, Inc. [1980].

Agricultural Experiment Station Bulletin 684, April 1979, Growing Alfalfa in Montana (8); Montana Agricultural Experiment Station Bulletin 603, April 1974, The Establishment and Production of Birds-foot Trefoil-Grass Compared to Alfalfa-Grass Mixtures (7); and the Montana State University Cooperative Extension Service Fertilizer Guide (11).

Fertilization and irrigation requirements were also cited from the Cooperative Extension Service Fertilizer Guide. They were used as a reference point for alfalfa production recommendations. Individual publications selected for use from the guide were: Topics in Soil and Water Resource Management - "Irrigation - When and How Much," Management Guide Series - "Legume - Irrigated", and Fertilizer Guide - "Cereal Grain - Irrigated."

Alfalfa yield reductions due to the planting of companion crops is addressed in Tables 9 and 10 of Bulletin 603 (8). This was the only companion crop yield reduction data found for the state of Montana.

Chapter 3

THE DECISION MODEL: FORMULATION AND IMPLEMENTATION

A general formulation of an alfalfa replacement model is developed in this chapter. The general model is adapted to a specific cropping situation utilizing a discrete dynamic programming model. The data necessary to apply the model are then presented.

General Decision Model for Alfalfa Replacement

The optimal length of an alfalfa stand cannot logically be separated from the more general problem of selecting the optimal crop rotation for a given farming area. The decision model should consider both fall and spring planting seasons (decision periods), all feasible crop alternatives, the present land use state and expected future prices for the feasible crop alternatives. The principal objective of the model is to generate a decision rule which will specify the crop to plant at each decision period which will maximize the expected present value of net returns over the selected planning horizon. The decision rule will be conditional on the present land use state and expected price states of the crop alternatives.

The usual convention of dynamic programming is followed when specifying the decision model. Stages are counted from the end of the planning horizon rather than the beginning. The following notation and definitions are introduced with the stage of the process denoted

by n where $n = 0, 1, \dots, N$. Financial and physical measures are on a per acre basis.

S = the set of possible land uses or crop alternatives (decision variables) at the present stage, i.e. certain age alfalfa stand, grain crops, fallow, etc.,

u = the particular decision variable selected from the set S at a given stage, that is, the decision u must be an element of the set of possible decisions S ,

s = the state variable which designates land use at the present stage, i.e. certain age alfalfa stand, grain crops, fallow, etc.,

\vec{p} = the set of expected product prices and/or production costs for stage n . The elements of \vec{p} are state variables.

Land use transition is deterministic and does not involve the price state variables, that is

$$s(n-1) = h(u, s)$$

Transition of the price vector \vec{p} is stochastic and does not involve the decision variable, u , or the land use state variable, i.e.

$$\vec{p}(n-1) = \vec{g}(\vec{p}, \vec{v})$$

where

\vec{v} = a vector of random variables where there is an element of \vec{v} associated with each element of \vec{p} .

\vec{g} = the vector of functions associated with the elements of \vec{p} and \vec{v} .

With these definitions, the recurrence equation of dynamic programming can be written as:

$$f_n(s, \vec{p}) = \text{Max}_{u \in S} [R(u, s, \vec{p}) + \beta E f_{n-1}((h(u, s), \vec{g}(\vec{p}, \vec{v})))]$$

$$n = 0, 1, \dots, N.$$

$$\text{and } f_0(s, \vec{p}) = 0,$$

where

$f_n(s, p)$ = the expected value of discounted net returns from a n -stage process under an optimal policy when the initial state is described by s , the land use state variable, and \vec{p} , the vector of price state variables,

$R(u, s, \vec{p})$ = the expected immediate returns in stage n . The returns are a function of the crop selected, the land use state, and the vector of expected prices,

$$\beta = \text{the appropriate discount factor, i.e., } \beta = 1/(1+r)$$

where

r is the interest rate,

E = the expectation operator.

Application of the Model to the Determination of Optimal Age of

Alfalfa Stands

The dynamic programming model is used to determine the optimal age of irrigated alfalfa stands in Montana.

The principal cultivated crops in these areas include winter and spring wheat, barley, corn for silage, sugar beets, alfalfa and grass hay. This set of land use alternatives can be reduced to barley and alfalfa hay without loss of realism. Barley is selected as a proxy for the returns from any of the grain crops. Barley is selected rather than winter wheat in order to reduce the decision periods to one past year -- the spring planting period. Barley will accurately reflect the opportunity cost of establishing or continuing an alfalfa

stand. On an individual farm, a decision to plant barley can be interpreted as a decision to plant the most profitable spring grain crop.

Alfalfa hay is recognized as being superior to other hay crops in Montana under irrigated conditions, therefore, in most situations it would dominate other hay crops if they were included in the feasible set of crop alternatives.

Sugar beets and corn silage are important crops to some producers but represent only a small percentage of the irrigated cropland in Montana and are concentrated close to processing plants for beets and the cattle feeding industry for silage. Therefore, these crop alternatives are omitted. The model becomes increasingly complex with the inclusion of additional crop alternatives. In summary, it was felt that the options to grow barley and alfalfa hay sufficiently represent the irrigated cropping alternatives.

Crop Production Relationships

Before an economic analysis can proceed, the production functions for barley and alfalfa must be specified. The production relationship for barley is given a concise treatment in this study. An average annual yield, a set of fixed cultural practices and fixed input prices are developed. Two enterprise budgets for barley production under these assumptions are presented in Tables 1 and 2.

Table 1. Variable Production Costs - Irrigated Barley (Previous land use state = Alfalfa)

Item	Unit	Price	Quantity	Cost/Acre
Herbicide (2,4-DB):	Gallon	\$30.00	1/4 Gallon	\$7.50
Fertilizer (11-51-0):	Ton	\$253.00	200 lbs.	\$25.30
Fertilizer (0-0-64):	Ton	\$164.00	90 lbs.	\$7.38
Fertilizer Spread:	Acre	\$2.50	2 times	\$5.00
Seed:	100 lbs.	\$13.55	90 lbs.	\$12.20
Irrigation Water:	Acre/foot	\$5.00	1.0*	\$5.00
Irrigation Labor:	Hour	\$5.00	36 mins.**	\$3.00
Implements (VC):	Acre	\$4.72	1.0	\$4.72
Swather (VC):	Acre	\$2.35	1.0	\$2.35
Tractors (VC):	Acre	\$16.35	1.0	\$16.35
Labor:	Hour	\$5.00	1.5	\$7.50
Custom Harvest:	Acre	\$23.00	1.0	\$23.00
TOTAL VARIABLE COST				\$119.30

*2 irrigations at 6 inches per irrigation

**18 minutes labor per acre per irrigation

Table 2. Variable Production Costs - Irrigated Barley (Previous land use state = Barley)

Item	Unit	Price	Quantity	Cost/Acre
Herbicide (2,4-DB):	Gallon	\$30.00	1/4 Gallon	\$7.50
Fertilizer (11-51-0):	Ton	\$200.00	500 lbs.	\$50.00
Fertilizer (0-0-64):	Ton	\$164.00	100 lbs.	\$8.20
Fertilizer Spread:	Acre	\$2.50	2 times	\$5.00
Seed:	100 lbs.	\$13.55	90 lbs.	\$12.20
Irrigation Water:	Acre/foot	\$5.00	1.0*	\$5.00
Irrigation Labor:	Hour	\$5.00	36 mins.**	\$3.00
Implements:	Acre	\$4.72	1.0	\$4.72
Swather:	Acre	\$2.35	1.0	\$2.35
Tractors:	Acre	\$16.35	1.0	\$16.35
Labor:	Hour	\$5.00	1.5	\$7.50
Custom Harvest:	Acre	\$23.00	1.0	\$23.00
TOTAL VARIABLE COST				\$144.82

* 2 irrigations at 6 inches per irrigation

** 18 inches labor per acre per irrigation

The derivation of the yield relationship for alfalfa receives more attention because of the crucial role it plays in the economic analysis. The annual average yields of an alfalfa stand are dependent on the age of the stand. There is very little empirical data available relating annual alfalfa yields to age of stand. In addition, of the empirical data available, there is only a limited amount of information that reports alfalfa yields beyond four years of age.

Two publications summarizing alfalfa variety trials containing annual yield information in relationship to stand age were available. The data from these yield results are presented in Table 3.

The yield data consisted of 64 observations. Two multiple regression models depicting the relationship between alfalfa yields and time were formulated. Ordinary least squares was used to estimate the parameters of these models. The models specified were:

$$1) \quad Y_a = \beta_0 + \beta_1 D + \beta_2 V_1 + \beta_3 V_2 + \beta_4 V_3 + \beta_5 V_4 + \beta_6 a + \beta_7 a^2 + \beta_8 a^3 + e_a$$

$$2) \quad Y_a = \text{EXP}(\beta_0 + \beta_1 D + \beta_2 V_1 + \beta_3 V_2 + \beta_4 V_3 + \beta_5 V_4 + \beta_6 a + \beta_7 a^2 + \beta_8 a^3 + e_a)$$

where:

Y_a = annual yield of alfalfa of age a in tons per acre,

β_i = an unknown parameter that reflects the effect of the i th independent variable,

Table 3. Alfalfa Yield Trial Observations^{1/}

Variety	Years	Stand Age							
		1	2	3	4	5	6	7	8
		(tons/acre)							
Ladak	1933-39	^{2/}	8.87	7.42	8.18	7.49	8.03	8.90	
Ladak	1940-47	3.71	7.32	6.16	9.19	5.65	7.94	5.96	4.21
Ladak	1941-46	2.06*	5.38	7.39	6.51	7.97	6.28	4.83	
Ladak	1944-50	2.66*	9.32	7.84	7.05	7.14	7.94	5.64	
Ranger, Variety 1	1940-47	3.53	6.67	5.50	6.28	6.80	8.94	6.32	4.82
Ranger, Variety 1	1941-47	2.04	4.81	6.83	7.67	8.49	7.28	5.56	
Ranger, Variety 2	1941-47	1.84*	4.95	7.01	7.38	8.54	7.11	5.53	
Ranger, Variety 3	1944-50	2.24*	7.71	7.58	6.28	6.40	7.78	5.19	
Buffalo	1941-47	1.85*	5.03	6.83	6.92	9.36	6.81	5.45	

^{1/}Two cuttings per season except where noted by asterisk (*) which is only one cutting per season.

^{2/}No recorded yield observation for this season.

Source: (Report of Progress June 30, 1947, Department of Agronomy and Soils, Project: Alfalfa Varieties and Strains 262, Agronomy 73. and Report of Progress June 30, 1948, Department of Agronomy and Soils, Project: Alfalfa Varieties and Strains, M.S. 262, Agronomy 73.)

D = cutting dummy for alfalfa of age one,

V_1 = dummy for variety 1,

V_2 = dummy for variety 2,

V_3 = dummy for variety 3,

V_4 = dummy for variety 4,

a = age,

a^2 = age squared,

a^3 = age cubed,

e_a = random error term associated with alfalfa of age a .

The exponential model (equation 2) was converted to natural logarithms in order to estimate the parameters using the technique of ordinary least squares. In logarithmic form, equation 2 is expressed as:

$$\ln Y_a = \beta_0 + \beta_1 D + \beta_2 V_1 + \beta_3 V_2 + \beta_4 V_3 + \beta_5 V_4 + \beta_6 a + \beta_7 a^2 + e_a$$

The estimated parameters and associated statistics for the specified production functions are presented in Table 4.

The dummy variable for number of cuttings in year one of the yield cycle was significant at the 95 percent level for both models. The dummy variables for varieties were not significant in either model indicating no influential variety effect between the control (Ladak) and other varieties.

The exponential model was selected for the optimal replacement analysis. The significant t-values for the estimates associated with

Table 4. Regression Results - Alfalfa Production Functions.

Coefficient	Associated Variable	Polynomial			Exponential		
		β -Estimate	(SE(β))	t-Value	β -Estimate	SE(β)	t-Value
b_0	1	0.000**			1.184		
b_1	D	-1.353	.5278	-2.564*	-.7516	.1017	-7.390*
b_2	V_1	-.3815	.3646	-1.046	-.07159	.05386	-1.329
b_3	V_2	-.4281	.4852	-.8822	-.08025	.07127	-1.126
b_4	V_3	-.4438	.4852	-.9146	-.08453	.07127	-1.186
b_5	V_4	-.3109	.4852	-.6408	-.03739	.07127	-.5246
b_6	a	4.437	.3513	12.63*	.3984	.06053	6.852*
b_7	a^2	-.7415	.1306	-5.678*	-.04395	.006599	-6.660*
b_8	a^3	.03353	.0120	2.793*			
Degree of Freedom			56				56
Coefficient of Determination			.6850				.8442

* t-value significant at 95 percent level.

** intercept was forced through the origin.

variables D , a and a^2 , plus the larger R^2 were instrumental in selecting the exponential model.

It appears unlikely that the subsequent analysis will be sensitive to the specification of production function model. This can be seen by comparing the functional forms generated in the two models. They are presented in graphical form in Figure 4. It is clear that the models are quite similar with respect to estimated yields.

The exponential model was modified to derive production functions for two different methods of production:

- 1) an alfalfa stand seeded without a companion crop;
- 2) an alfalfa stand seeded with a companion crop (barley).

Yield levels differed between the production methods because of assumed differences in cultural practices and yield response.

- 1) Only one cutting of alfalfa was harvested the first year of production with a pure alfalfa stand.
- 2) The alfalfa seeded with a companion crop was not harvested until age two of the alfalfa stand.
- 3) A companion crop is postulated to decrease yields by 20 percent the second year of alfalfa stand age and 6.8 percent each year thereafter for the life of the stand (7).

Management practices and production conditions were also a factor in deriving the total product curves. The yield trials were performed under relatively ideal experimental conditions, therefore the estimated

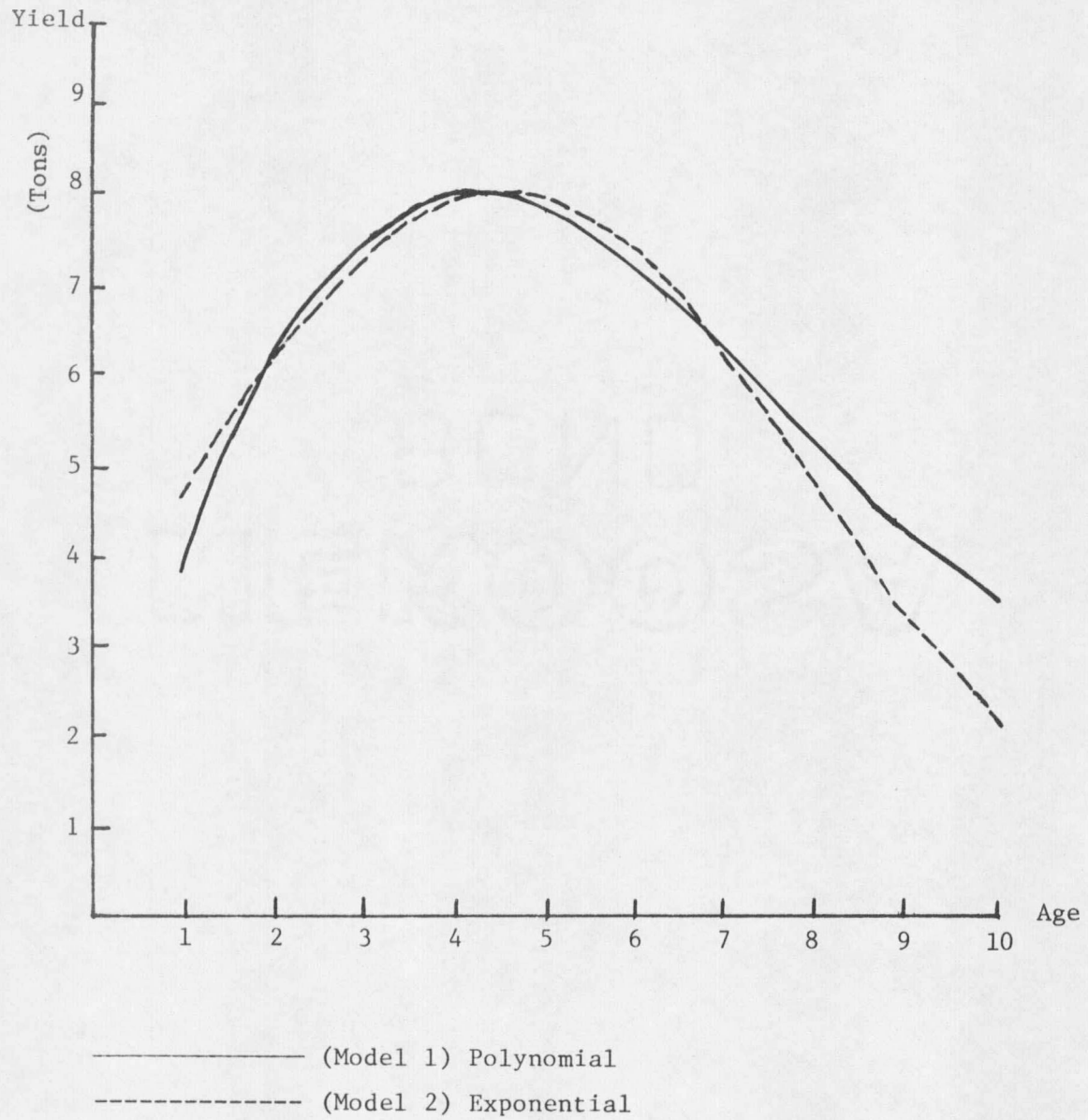


Figure 4. Functional Forms (Models 1 & 2)

yields were arbitrarily adjusted by multiplying by .67 to produce levels more likely to be attained under actual production methods.

The yields for a pure alfalfa and alfalfa companion crop stand production methods are presented in Table 5. Figure 5 presents a graphical representation of the two production functions.

Price State Variables and Transition

Two price state variables are essential to the implementation of the empirical model -- the expected price for barley and the expected price of alfalfa hay. It was postulated that a specific product price in time t was a function of the price of the relevant product prices in time $t-1$. Specifically the two price relationships were specified as:

$$(1) \quad PA_t = \alpha_0 + \alpha_1 PA_{t-1} + \alpha_2 PB_{t-1} + v_{1t},$$

$$(2) \quad PB_t = \beta_0 + \beta_1 PA_{t-1} + \beta_2 PB_{t-1} + v_{2t},$$

where:

PA_t = the natural log of the real price of alfalfa hay in time t
in dollars per ton;

PB_t = the natural log of the real price of barley in time t in
dollars per ton;

α_i = the net effect of the i th term of equation (1), $i = 0, 1, 2$;

β_i = the net effect of the i th term of equation (2), $i = 0, 1, 2$.

Table 5. Pure Alfalfa vs. Companion Crop Alfalfa Yields (Adjusted to irrigated farm level conditions)

Alfalfa Age	Pure Alfalfa	Companion Crop Alfalfa
1	1.47	--
2	4.07	3.26
3	4.87	4.54
4	5.33	4.97
5	5.35	4.99
6	4.91	4.58
7	4.13	3.85
8	3.18	2.96
9	2.25	2.10
10	1.45	1.35

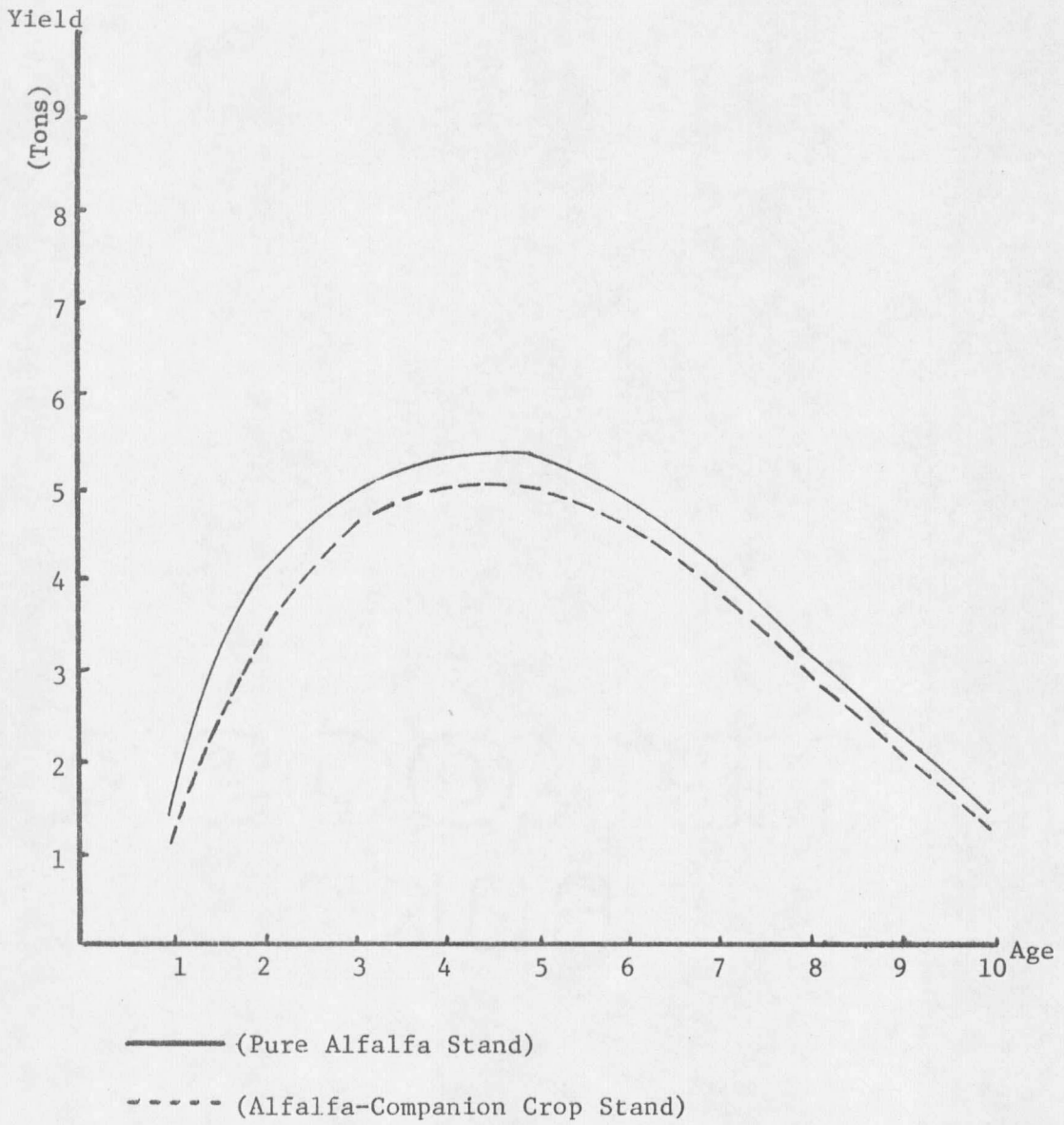


Figure 5. Production Function Graphs

v_{1t} = the random error term for equation (1);

v_{2t} = the random error term for equation (2).

The parameters of these two equations were estimated from an annual price series 32 years in length (9). Seemingly unrelated regression techniques were used to obtain the variance-covariance matrix for v_1 and v_2 , the error terms, of the two equations. The parameter estimates and associated statistics are presented in Table 6.

The price equation regression results produced no unusual results. The signs on the parameters were positive and the coefficients on the lagged dependent variable in each equation were significant at the 95 percent level. One surprising result was the small coefficient on the lagged price of barley in the alfalfa price equation. Its t-value was also insignificant at the 95 percent level. This contrasts with the larger and significant coefficient on the lagged price of alfalfa in the barley price equation.

The correlation of the error terms of the price equation was calculated to be .6107. Therefore, v_1 and v_2 are considered to be jointly dependent and this dependence must be considered in the transition of the price state variables.

In order to obtain a numerical solution to the model a discrete approximation of the stochastic nature of the process must be specified. In order to accomplish this, five different price states were defined for both price variables. The price state variables are approximated

Table 6. Parameter Estimate and Associate Statistics for Price Equations

Alfalfa Price Equation				Barley Price Equation			
Coefficient	Associated Variable	β -Estimate	t-value	Coefficient	Associated Variable	β -Estimate	t-value
b_0	1	1.5378	2.4997*	b_0	1	.52236	1.0203
b_1	PA_{t-1}	.46204	2.069*	b_1	PA_{t-1}	.42986	2.3130*
b_2	PB_{t-1}	.16044	.92057	b_2	PB_{t-1}	.49091	3.3845*

Degrees of freedom: 29

*t-values significant at 95 percent level

Sample correlation: .6107

by discrete intervals on their continuous scales of measurement. Each of these intervals is represented by its approximate midpoint value. All possible combinations of these price states result in 25 price states involving both variables jointly.

The price states were defined as indicated in Figure 6. The mean and standard deviation of alfalfa hay price were \$72.15 and \$14.37 respectively, while the mean and standard deviation for barley were \$107.42 and \$25.27, respectively. All figures are in 1980 dollars. The intervals for the two price variables are presented in Table 7.

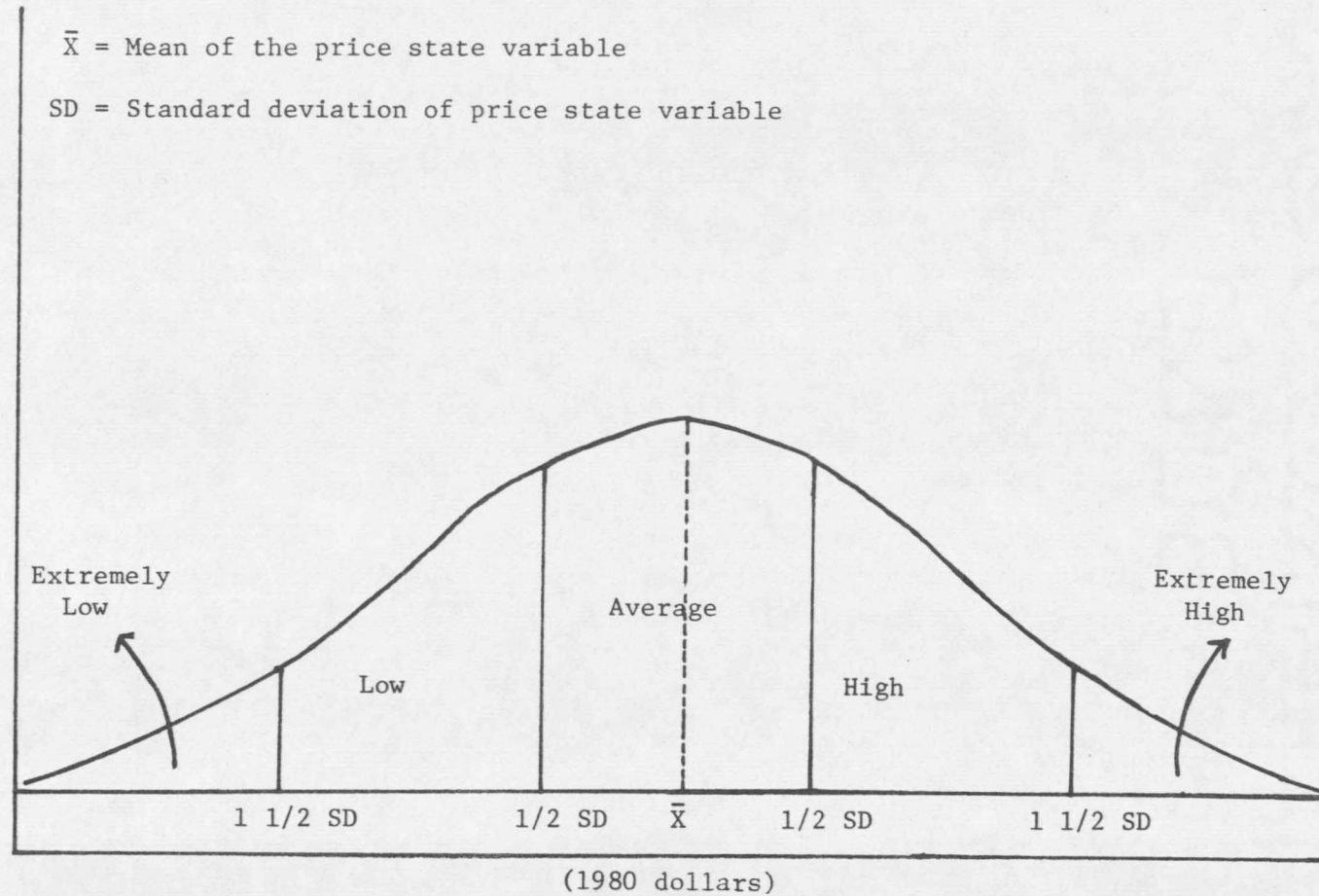
Table 7. Discrete Intervals for Alfalfa Hay and Barley Prices in 1980 Dollars per Ton.

Description of Price State	Alfalfa Hay Price			Barley Price		
	Lower Limit	Approx Midpoint	Upper Limit	Lower Limit	Approx Midpoint	Upper Limit
Extremely Low	0	43.30	50.50	0	56.60	69.25
Low	50.51	57.70	64.90	69.26	82.00	94.74
Average	64.91	72.10	79.30	94.75	107.40	120.05
High	79.31	86.50	93.69	120.06	132.80	145.54
Extremely High	93.70	100.90	∞	145.54	158.20	∞

In the discrete formulation of the model, a finite set of conditional probabilities, referred to as transition probabilities must be generated. A transition probability (p_{ij}) is the probability

\bar{X} = Mean of the price state variable

SD = Standard deviation of price state variable



30

Figure 6. Illustration of Price States for Alfalfa and Barley

that the process will occupy the j th state in stage $(n-1)$ given that the process occupied the i th state in stage n . The general transformation function analogies of price equations (1) and (2) are given by:

$$(3) PA(n-1) = g_1(PA(n), PB(n), v_1)$$

$$(4) PB(n-1) = g_2(PA(n), PB(n), v_2)$$

where time is measured by the dynamic programming stage n rather than by time t . As v_1 and v_2 are jointly dependent, this pair of transformation functions implies that a joint probability density function exists for v_1 and v_2 . We can specify the probability density function (pdf) as $d(v_1, v_2)$. The transformation functions (3) and (4) make it clear that PA and PB are parameters in the pdf. The parameters of price equations (1) and (2) and r_{v_1, v_2} are also parameters of the pdf.

For example, if i -state conditions are specified, i.e. given values for $PA(n)$ and $PB(n)$, price equations (1) and (2) can be used to derive conditional expected values (means) for $PA(n-1)$ and $PB(n-1)$ one period later. These conditional expected values, the estimates of v_1 , v_2 , and r_{v_1, v_2} can be used to evaluate the probability of any combination of $(PA(n-1), PB(n-1))$ occurring.

The pdf, $d(v_1, v_2)$ was approximated by a bivariate log normal. The transition probabilities were derived using an IMSL computer program from the Montana State University Statistical Library. Due

to the volume of the transition probabilities necessary to implement the model they are not presented.

A representation of the joint distribution of alfalfa and barley prices is presented in Table 8.

Crop Enterprise and Yield Data

An integral part of the development of the dynamic replacement model was the calculation of immediate annual returns from a specific crop. Production cost and yield data needed for these calculations are presented in detail in Appendix A.

The Empirical Model

With the necessary relations specified to generate yields, expected prices and transition probabilities, the empirical decision model can be formulated.

In the irrigated crop model, the state of the system is described by the land use in the previous year (barley or alfalfa of given age) and price expectations for alfalfa hay and barley. The land use transition is deterministic while the price state transition is stochastic and driven by the joint pdf, $d(v_1, v_2)$. The criterion is the maximization of expected present value of net returns.

The basic model formulated is for an alfalfa stand seeded without companion crop. It is also assumed that straw is a source of revenue for the barley crop. The detailed production cost data for alfalfa are

Table 8. Joint Distribution of Random Price Variables

Frequency of Pairs of Prices (Barley and Alfalfa) 1948-80 - Dollars/Ton						
Barley Price	Alfalfa Price					f_2
	50.51 Extremely Low	50.51-64.91 Low	64.91-70.30 Average	79.30-93.70 High	93.70 Extremely High	
<69.26 Extremely Low						0
69.26-94.75 Low		10	2	1		13
94.75-120.05 Average			9	1		10
120.05-145.54 High			3	2	2	7
>145.54 Extremely High				2	1	3
Frequency f_i	0	10	14	6	3	

presented in Appendix B. Each component of the decision models will be discussed and then the recurrence relation of dynamic programming will be specified.

State Variables

In the simplified model, eleven states exist which designate the land use at the decision point. The land use could be barley, alfalfa of age one, alfalfa of age two, ..., alfalfa of age ten. It was assumed that alfalfa of age ten would be plowed up and reseeded to barley the following state.

For each land use state, 25 possible combinations of price state variables exist. That is, all possible combinations of the five possible price states for each price variable are defined for each land use state. Therefore, the model consists of 275 (11 x 25) states.

Decision Alternatives

The model includes only two decision alternatives in most states. The first alternative which is possible at all states is to plant barley. The second alternative involving the establishment or continuation of an alfalfa stand is more complex. In the first land use state (barley the previous crop) an alfalfa stand can be established. In land use states two through ten the second decision alternative designates the continuation of an existing alfalfa stand; that is, alfalfa of age a is followed by alfalfa of age $a + 1$. In land use

state 11, alfalfa of age ten must be followed by barley.

These decision alternatives will be designated as A for a decision to establish or continue an alfalfa stand and B as a decision to plant barley.

Expected Immediate Returns

The expected immediate returns function is expressed as:

$$R(u,s,PA,PB).$$

This simply states that the expected annual return from a specific crop alternative in the next production period is dependent on the crop selected, the state of the process and the price expectation for the selected crop. The selection of crop u and the land use state s define the expected crop yield and production costs. The crop yields are determined from the estimates yield response function for alfalfa and from the yield assumptions made for barley. The production costs are specified in the crop enterprise budgets.

Designation of PA and PB will, of course, imply the appropriate product prices and allow the calculation of gross crop revenues. The calculation of the immediate returns was done with a computer program. Again, due to the volume the data are not presented.

Recurrence Relation

The dynamic programming recurrence relation for this application is

$$f_n(s, PA, PB) = \text{Max}_{A, B} [R(u, s, PA, PB) +$$

$$BEf_{n-1}(h(u, s), g_1(PA, PB, v_1), g_2(PA, PB, v_2))]]$$

$$n = 1, 2, \dots,$$

where $f_0(s, PA, PB) = 0$, which implies that crop price expectations and previous land use have no effect on the terminal value of the land resource at the end of the decision process.

This recurrence relation was solved for four models with slightly different production functions. The models solved were: (I) pure alfalfa stand with straw revenue from barley, (II) pure alfalfa stand with no straw revenue from barley, (III) alfalfa seeded with a companion crop with straw revenue from barley, (IV) alfalfa seeded with a companion crop with no straw revenue from barley.

A six percent interest rate was used to calculate the discount factor. In addition, model I was solved with a three percent interest rate. All models were solved for planning horizons from one to fifty years in length. Therefore, the results provide the optimal decision (A or B) for four models for all possible combinations of land use state and price expectation states for planning horizons of from one to fifty years in length. The results are summarized in the following chapter.

Chapter 4

RESULTS

The optimal policies derived by solution of the dynamic programming decision models are presented in this chapter. Implications of these policies are examined.

The solutions provide the optimal decision and net present value of return over variable costs for all combinations of stages and states. The volume of output contained in the solutions prohibits detailed presentation of the final solution.

Emphasis will be placed on the optimal policies obtained from the solutions of Model I. All models were solved for planning horizons (stages) from one to fifty years in length at a six percent interest rate.

The results from the other models closely parallel those for Model I. The similarity in the solutions will be verified later by the use of summary measures, e.g., land use equilibrium probabilities.

Optimal Policies for Model I

The optimal policy or decision with respect to crop selection for Model I is presented in Table 9. The optimal policies are presented for two planning horizon lengths -- ten years and 42 years. The selection of planning horizons was arbitrary. Ten years was selected because it had personal appeal as a reasonable organizational

Table 9. Optimal Policies for Model I for Stages 10 and 42 at a 6 percent Interest Rate^{1/}

Land Use States	ALFALFA PRICE - BARLEY PRICE STATES ²																								
	Extremely Low ³					Low					Average					High					Extremely High				
	EL	L	A	H	EH	EL	L	A	H	EH	EL	L	A	H	EH	EL	L	A	H	EH	EL	L	A	H	EH
G	A	A	(B)	B	B	A	A	A	(B)	(B)	A	A	A	(B)	B	A	A	A	A	(B)	A	A	A	A	(B)
A ₁	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
A ₂	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
A ₃	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
A ₄	A	A	A	(A)	B	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
A ₅	A	A	(A)	B	B	A	A	A	(A)	B	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
A ₆	A	(A)	B	B	B	A	A	(A)	B	B	A	A	A	(A)	B	A	A	A	A	(A)	A	A	A	A	A
A ₇	(A)	B	B	B	B	A	(A)	B	B	B	A	A	B	B	B	A	A	A	B	B	A	A	A	A	B
A ₈	B	B	B	B	B	(A)	B	B	B	B	(A)	B	B	B	B	A	(A)	B	B	B	A	(A)	B	B	B
A ₉	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B
A ₁₀	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B

¹ A equals establish or continue alfalfa stand; B equals plant barley (grain); circled symbols imply the decision changes from A to B or vice versa for stage 42.

² Symbols for barley price:
 EL equals extremely low
 L equals low
 A equals average
 H equals high
 EH equals extremely high

³ The probability of the process actually being in these states is less than 0.1 percent.

length planning period. The planning period of 42 years was selected because at this point the optimal policies had converged, i.e., the optimal decision had become a function of state alone. That is, for any planning period longer than 42 years in length, the optimal policy is identical to that for a 42 year planning horizon. The differences in the optimal policies for the two planning horizons are minor and are indicated by a circle around the decisions which change due to differences in planning horizon length.

As noted in Table 9, the probability of some of the price states actually occurring is nil. This is due to the relatively high positive correlation in the two price variables. An occurrence of extremely high barley price and extremely low alfalfa hay price is zero based on the relationships estimated from 32 years of historical data. However, all states are retained in the model and the optimal decision is valid in a conditional sense. This only emphasizes the conditional nature of the model, that is, if such a situation (state) occurs, then the given decision is the optimal course of action in order to maximize the criterion function.

Examination of the optimal policy for Model I reveals that under the costs assumptions used in this study alfalfa is a superior income producing crop to barley. This becomes evident by noting that if the land use state is barley, alfalfa is the optimal decision unless grain prices are average or above. In fact, except for the state defined

by extremely low hay price with an average barley price, barley price must be high or extremely high in order for barley to be the optimal choice. This implies that alfalfa is economically superior to continuous barley.

It is important to remember that these conclusions are drawn in the context of determining the optimal duration of an alfalfa stand. Resource constraints such as labor availability will influence the optimal mix of alfalfa and other crops in a comprehensive examination of optimal crop rotations.

Price State Equilibrium Probabilities and Land Use Transition

Probabilities

Analysis of the results with respect to the optimal time to replace an existing alfalfa stand is best done by computing land use transition probabilities. That is, the probability of transition from one land use state to another land use state.

In order to derive these probabilities, the concept of equilibrium probabilities is utilized. An equilibrium probability vector (EPV), given an optimal policy is followed, could be calculated by solving the system of linear equations given by

$$\Pi = \Pi P$$

where

Π = a 1 x 275 vector of the unknown equilibrium probabilities,

$P =$ a 275 x 275 regular stochastic matrix.

The matrix P is specified by taking the transition probabilities associated with the optimal decision at each state in the model. Once the equilibrium probabilities are known for each state the land use probabilities can be calculated by summing over all price states within a given land use state.

This approach to obtaining land use probabilities was abandoned for two reasons. First, the computational cost of obtaining a solution to the system of 275 equations with 275 unknowns was prohibitive. Second, problems with excessive rounding errors prevented an exact solution.

An alternative method was used which exploited the independence of the price state variables and land use states. Also, recall the price state transition probabilities are independent of the decision variables in the model. That is, the crop selected effects only the land use states, not the expected price states.

The first step in arriving at a land use EPV was to calculate an EPV for price states. This was accomplished by solving a set of equations of the form $\Pi = \Pi P$. In this case, Π is equal to a (1 x 25) vector of unknowns and P is a (25 x 25) regular stochastic matrix specified by the transition probabilities from the 25 price states within a given land use stage. The solution vector contains the equilibrium probabilities of the 25 price states. The solution to

the set of equations is presented in Table 10. It is arranged in a (5 x 5) square matrix for ease of interpretation.

Table 10. Price State Equilibrium Probabilities

Alfalfa Price States	Barley Price States				
	Extremely Low	Low	Average	High	Extremely High
Extremely low	.0426	.0266	.0006	.0000	.0000
Low	.0476	.2436	.0612	.0020	.0000
Average	.0034	.1452	.1957	.0329	.0013
High	.0000	.0132	.0820	.0529	.0074
Extremely high	.00000	.0003	.0090	.0206	.0119

Land use transition probabilities were derived by using the optimal decisions for a given land use state and the price state EPV. This was accomplished by summing the probabilities associated with a given decision. Recall that there are only two land use transitions possible in a given land use state. A decision to plant barley implies a transition to the barley land use state and the decision to continue the alfalfa stand implies transition from the land use state of alfalfa of age a to the land use state of alfalfa of age $a + 1$. Thus, the probability of transition from alfalfa of age a to alfalfa of age

$a + 1$ is the sum of the price state equilibrium probabilities associated with the decision to continue alfalfa in a specified land use state. In the same land use state the probability of transition from alfalfa of age a to barley is simply one minus the probability of transition from alfalfa of age a to alfalfa of age $a + 1$. Land use transition probabilities for all models are exhibited in Table 11. The information in Table 11 reveals two things more clearly than can be deduced from the optimal policies presented in Table 9. First, the four different models are essentially identical in terms of land use transition. This implies that use of companion crops versus pure seeding when establishing stands and having a market for straw from the barley produced in the cropping system have virtually no impact on the optimal time to replace an alfalfa stand. Secondly, the land use transition probabilities present a much clearer picture of the age to terminate an alfalfa stand than Table 9 can provide with respect to termination or continuation of the alfalfa stand.

The data of Table 11 indicate that more than 99 percent of the time alfalfa should be seeded following barley. This was pointed out earlier in the discussion on optimal policies. The land use transition probabilities show clearly that alfalfa of age one through age four should always be continued. Also, the probability of terminating alfalfa of age 5 is virtually zero (.0026). The probability of continuing alfalfa of age eight is zero. This implies that an alfalfa

Table 11. Land Use Transition Probabilities

Land Use State	Model I		Model II		Model III		Model IV	
	Barley	Alfalfa	Barley	Alfalfa	Barley	Alfalfa	Barley	Alfalfa
G	.0014	.9986	.0000	1.0000	.0000	1.0000	.0000	1.0000
A ₁	.0000	1.0000	.0000	1.0000	.0000	1.0000	.0000	1.0000
A ₂	.0000	1.0000	.0000	1.0000	.0000	1.0000	.0000	1.0000
A ₃	.0000	1.0000	.0000	1.0000	.0000	1.0000	.0000	1.0000
A ₄	.0000	1.0000	.0000	1.0000	.0000	1.0000	.0000	1.0000
A ₅	.0026	.9974	.0026	.9977	.0039	.9961	.0039	.9961
A ₆	.1321	.8679	.1246	.8754	.1321	.8679	.1321	.8679
A ₇	.6787	.3213	.6787	.3213	.7812	.2188	.6787	.3213
A ₈	1.0000	.0000	.9997	.0003	1.0000	.0000	.9963	.0037
A ₉	1.0000	.0000	1.0000	.0000	1.0000	.0000	1.0000	.0000

stand should usually be terminated at age six, seven or eight. The probabilities of termination are .1321, .6787 and 1.0000 respectively. These probabilities imply a probability of termination of .1321 at age six, .5904 at age seven, and .2775 at age eight. These probabilities simply mean that termination is most likely to occur at age seven while termination at age eight is twice as likely as termination at age six. The preceding discussion relates specifically to Model I, although results for the other models are similar.

It is interesting to compare the similarity between this solution and the solution to a deterministic replacement model patterned after R. K. Perrin's article. Such a comparison can be found in Appendix C.

Land Use Equilibrium Probabilities

The probability of occupying the given land use states under an optimal policy can be calculated using the land use transition probabilities. The set of equations which must be solved is

$$\Pi = \Pi P$$

where

Π = a 1 x 9 vector of the unknown equilibrium probabilities,

P = a 9 x 9 regular stochastic matrix.

The matrix P is specified by the land use transition probabilities. The land use states for alfalfa of ages nine and ten can be dropped because in Model I the probability of terminating the alfalfa stand

prior to these states is one. The actual P matrix for Model I is depicted in Table 12.

The solution vectors are presented in Table 13 for all four models. These probability values indicate the percentage of time a given tract of land will be in a given land use under an optimal policy over a long planning horizon. These probabilities provide essentially the same information as derived earlier using the land use probabilities and is presented without further elaboration.

Although alternative cultural practices had virtually no effect on the optimal policies for the four models, the net present value of returns over variable costs was higher where a market for straw was available. In addition, establishment of alfalfa stands using pure seedings produced higher returns than establishment with companion crops. This results follow directly from the assumptions specifying yield reductions over the life of the stand for companion crop seedings.

One additional situation was examined. Model I was solved using an interest rate of three percent. The optimal policy was identical for a planning horizon 42 years in length to the optimal policy obtained using a six percent interest rate.

Table 12. Land Use Transition Probability Matrix for Model I.

Land Use State (η)	Land Use State ($\eta-1$)									
	G_1	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
G_1	.0014	.9986								
A_1			1.0							
A_2				1.0						
A_3					1.0					
A_4						1.0				
A_5	.0026						.9974			
A_6	.1321							.8679		
A_7	.6787								.3213	
A_8	1.0									0

Table 13. Land Use Equilibrium Probability Vector

Land Use State	Model			
	I	II	III	IV
G	.1230	.1227	.1242	.1229
A ₁	.1228	.1227	.1242	.1229
A ₂	.1228	.1227	.1242	.1229
A ₃	.1228	.1227	.1242	.1229
A ₄	.1228	.1227	.1242	.1229
A ₅	.1228	.1227	.1242	.1229
A ₆	.1225	.1225	.1238	.1223
A ₇	.1063	.1072	.1075	.1061
A ₈	.0342	.0341	.0233	.0341
A ₉	.0000	.0000	.0000	.0001

Chapter 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study was to determine the optimal replacement pattern for a stand establishment of alfalfa hay in Montana's irrigated crop areas. The decision model was developed with the purpose of increasing the economic returns from alfalfa production in the state.

Two alfalfa production functions were estimated in order to cover alternative production procedures for Montana producers. The production functions for alfalfa yields were estimated with the use of experimental alfalfa yield trial data from the Plant and Soil Science Department at Montana State University. The relationships regressed age on alfalfa yield. A pure stand of alfalfa and an alfalfa-barley companion crop were contrasted to determine if replacement was affected by the companion crop. Also, barley was a cropping alternative to alfalfa at every decision point.

Four replacement models were developed from the two production functions estimated. The models were:

- I. Pure alfalfa stand with straw revenue from barley.
- II. Pure alfalfa stand with no straw revenue from barley.
- III. Alfalfa seeded with a companion crop with straw revenue from barley.
- IV. Alfalfa seeded with a companion crop with no straw revenue from barley.

The production relationships for barley were generated through the use of average annual yield data, a set of fixed cultural practices and fixed input prices.

Dynamic programming was used to solve the irrigated cropping models defined. The dynamic decision model considered two crop alternatives (alfalfa and barley), the present land use state and the expected joint distribution of prices for the crop alternatives. Solution of the models were accomplished by using the general recurrence relationship of dynamic programming.

Conclusions

The significant findings of this thesis were that optimal replacement would not take place before five or after eight years of an alfalfa stand. The probability of terminating alfalfa at age five is virtually zero, while replacement is most likely to occur at age seven. Also, termination at age eight is twice as likely than at age six. It was also found that a pure alfalfa stand had a higher net present value than an alfalfa-companion crop stand. These findings were based on an interest rate of six percent. Earlier work using Perrin's formulation and alternative discount rates suggested that policies were invariant to the choice of discount.

Recommendations

Research was performed with the best data available for the state

of Montana. The obvious limitations of the data indicated a definite need for increased experimentation with alfalfa yield response functions over time. Alfalfa yield trials (involving several varieties) extending in length for ten years should be tested, in order to provide information for the estimation of alfalfa yield production functions. The number and timing of cuttings per season should be varied along with the types and seeding rates of companion crops. This research data is essential to further efforts to determine optimal alfalfa stand replacement policies.

The potential of redefining the replacement model to incorporate different and additional cropping alternatives is a region where increased research could be warranted. Furthermore, adapting the model to dryland areas (given an adequate date source) would be valuable.

In closing, the findings of this study should be made available to participants in the agricultural industry across Montana. With its proper dissemination and explanation, it could become a useful decision making tool for Montana farmers. This, in turn, could encourage agriculturalists throughout the state to increase their demand for improved agricultural research.

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APPENDICES

APPENDIX A

PRODUCTION COST AND YIELD DATA

TABLE A-1. Estimated Farm Size

Total Farm Acreage	:	<u>570</u>
1.) Dryland	:	160 grain <u>160</u> fallow 320 total acres
2.) Irrigation	:	160 grain-row crop 50 alfalfa <u>40</u> pasture 250 total acres

TABLE A-2. Machinery Lists for Crop Budgets

100 h.p. two-wheel drive tractor
130 h.p. two-wheel drive tractor

Four bottom 18" spinner plow
Roller Harrow 16'
Disk-15' offset
Land Plane 12'
Triple-K Harrow 20'

Grain Drill - [Two-8' drills with grass seed boxes]
Sprayer, Tractor Mounted [300 gal. tank - 42 ft. boom]

Swather Conditioner 12'
Baler (14" x 18" bales)
Pull-type Bale Wagon [Two-wide bed, 70 bale capacity]

Ditcher
Ditch Closer

TABLE A-3. Annual Machinery Use (Hrs.)

	<u>Hrs.</u>
130 h.p. tractor	500
100 h.p. tractor	325
Baler	104
Bale Wagon	104
Plow	72
Drill	52
Roller Harrow	49
Swather	49
Ditcher	42
Land Plane	35
Ditch Closer	21
Disk	21
Triple-K Harrow	16
Sprayer	4

TABLE A-4. Annual Machinery Repairs (% of initial cost)

	<u>Percent</u>
Plow	3.0
Bale Wagon	3.0
130 h.p. tractor	2.5
100 h.p. tractor	2.0
Disk	1.5
Roller Harrow	1.5
Triple-K Harrow	1.5
Baler	1.5
Ditch Closer	1.5
Drill	1.5
Land Plane	1.0
Swather	1.0
Ditcher	1.0
Sprayer	1.0

TABLE A-5. Machinery Efficiency

$$\left[\text{Speed (M.P.H.)} \times \text{Width (Feet)} \times \frac{\text{Efficiency \%}}{825} \right]$$

	<u>Acres/hr.</u>
Sprayer	26.73
Triple-K Harrow	13.58
Ditch Closer	10.00
Roller Harrow	8.53
Disk	7.27
Swather	6.11 (Straw)
Land Plane	6.00
Drill	5.82
Ditcher	5.00
Swather	4.07 (Alfalfa)
Plow	2.91
Baler	2.77 (Alfalfa-Straw)
Bale Wagon	2.77 (Alfalfa-Straw)
Baler	2.49 (Alfalfa)
Bale Wagon	2.49 (Alfalfa)
Baler	2.16 (Straw)
Bale Wagon	2.16 (Straw)

TABLE A-6. Energy Use Performance

Fuel Consumption : Diesel \rightarrow gallons/hr. = (PTO Maximum H.P. \times .054)

Oil and Lubricants : Diesel = (15% of fuel costs)

Oil and Lubricants : Implements = (15% of repair costs/acre)

TABLE A-7. Miscellaneous Cost Information

Baler Twine	:	[400 bales/sack], \$23.00/sack
Custom Combine	:	[\$18.00-18.50/acre irrigated]
Custom Haul	:	[\$.05/bushel] on farm storage
Harvesting Costs		
Alfalfa	:	Tractors = \$8.82/acre
		Baler = \$1.04/acre
		Bale Wagon = \$1.89/acre
		Swather = \$2.78/acre
		Twine = <u>\$3.69/acre</u>
		\$18.22 acre at 2.25 tons/acre = <u>\$8.10/ton</u>
Harvesting Costs		
Straw	:	Tractors = \$10.18/acre
		Baler = \$ 1.04/acre
		Bale Wagon = \$ 1.89/acre
		Twine = <u>\$ 7.19/acre</u>
		\$20.30 acre at 2.5 tons/acre = <u>\$8.12/ton</u>
Harvesting Costs		
Alfalfa-Straw	:	Tractors = \$ 7.93/acre
		Baler = \$ 1.04/acre
		Bale Wagon = \$ 1.89/acre
		Twine = <u>\$ 5.61/acre</u>
		\$16.47 acre at 1.95 tons/acre = <u>\$8.45/ton</u>
Harvesting Costs		
Average Straw	:	$\frac{\$8.12 + \$8.45}{2} = \underline{\underline{\$8.28}} / \text{ton}$

TABLE A-8. Crop Yield Calculations

Average Alfalfa Yield: 4.5 tons/acre/year

(This was calculated by taking the mean of the pure alfalfa stand two cutting years. Stand ages two through eight.)

Average Alfalfa Yield/Cutting: 4.5 tons/year @ 2 cuttings yearly =
2.25 tons/cutting

Average Barley Yield/Acre: 91 bushel/acre

(A ratio approach was used to calculate the average barley yield.)

Ratio 1: Average Alfalfa Yield from Regression Analysis
Montana Crop and Livestock Reporting Service
Southwestern Montana Alfalfa Yields, 10. yr. average

The resulting Ratio was: $\frac{4.5 \text{ tons/year}}{2.89 \text{ tons/year}} = \underline{1.56}$

Ratio 2: X = Estimated Barley Yield (unknown)
Montana Crop and Livestock Reporting Service
Southwestern Montana Barley Yields, 10 yr. average

The resulting Ratio was: $\frac{X}{58.3 \text{ bushel/year}}$

An equality was then set up: $\frac{1.56}{1} = \frac{X}{58.3}$

X = the Estimated Barley Yield 91.3 bushel/year

Average Barley Weight: 50 lbs/bushel

(Representative of irrigated barley.)

Average Companion Crop - Barley Yield: 60 bushel/acre

(Based on the seeding rate of 60 pounds, two-thirds of normal)
Yield = $2/3(91) \approx 60$ bushel/acre

TABLE A-8 continued

Average Straw Yield: 2.5 tons/acre

(Based on the ratio rule of 1.1 pounds of straw produced for every pound of barley.)

$$4550 \text{ lbs.} \times 1.1 = 5005 \text{ lbs. straw/acre or } \approx 2.5 \text{ tons/acre}$$

Average Alfalfa Straw Yield: 1.95 tons/acre

(Based on the ratio of 1.3 pounds of alfalfa-straw produced for every pound of companion crop-barley.)

$$3000 \text{ lbs.} \times 1.3 = 3900 \text{ lbs. straw/acre or } \approx 1.95 \text{ tons/acre}$$

Table A-9. Revenue Function Definitions

BY	=	Barley Yield (bushels per acre)
BW	=	Barley Weight (pounds per bushel)
PA	=	Price of Alfalfa (dollars per Ton)
BS	=	Barley Straw (tons per acre)
SHC	=	Straw Harvesting Costs (dollars per ton)
PB	=	Price of Barley (dollars per ton)
BPCAA	=	Barley Production Costs After Alfalfa (dollars per acre)
BPCAB	=	Barley Production Costs After Barley (dollars per acre)
SR	=	Straw Revenue (dollars per acre)
CCSR	=	Companion Crop Straw Revenue (dollars per acre)
AY	=	Alfalfa Yield (tons per acre)
AIC	=	Alfalfa Irrigation Costs (dollars per acre)
AHC	=	Alfalfa Harvesting Costs (dollars per ton)
AECPS	=	Alfalfa Establishment Costs Pure Stand (dollars per acre)
CCR	=	Companion Crop Revenue (dollars per acre)
ACCEC	=	Alfalfa With Companion Crop Establishment Costs (dollars per acre)

Revenue Functions

- 1) Straw Revenue:

$$(1.1 \times BY \times BW/B) \div (2000) \times (Pa/3) - (BSTA) \times (SHCT)$$

- 2) Companion Crop Straw Revenue:

$$(1.3 \times BY \times BW/B) \div (2000 \times (Pa/3 - SHCT))$$

- 3) Net Revenue from Barley and Straw (Previous Land Use State = Alfalfa:

$$(BY \times Pb) - (BPCAA) + (SR)$$

- 4) Net Revenue from Barley and Straw (Previous Land Use State = Barley):

$$(BY \times Pb) - (BPCAB) + (SR)$$

- 5) Net Revenue from Companion Crop and Straw:

$$(BY = Pb) _ (CCSR) - (ACCEC)$$

- 6) Net Revenue from Alfalfa Establishment Year:

$$(AY \times Pa) - (AEC) - (AY \times AHCT) + (CCR)$$

- 7) Net Revenue from Alfalfa After Establishment Year:

$$(AY \times Pa) - (IC) - (AY \times AHCT)$$

APPENDIX B

PRODUCTION COST DATA-ALFALFA

Table B-1. Establishment Costs - Irrigated Alfalfa (Previous land use state = Barley)

Item	Unit	Price	Quantity	Cost/Acre
Herbicide (Eptam):	lb.	\$.70	30	\$21.00
Fertilizer (11-51-0):	Ton	\$253.00	300 lbs.	\$37.95
Fertilizer (0-0-64):	Ton	\$164.00	100 lbs.	\$8.20
Fertilizer Spread:	Acre	\$2.50	2 times	\$5.00
Seed:	lb.	\$2.32	11	\$25.52
Irrigation Water:	Acre/foot	\$5.00	1.5*	\$7.50
Irrigation Labor:	Hour	\$5.00	1.5**	\$7.50
Implements:	Acre	\$4.44	1	\$4.44
Tractors:	Acre	\$14.99	1	\$14.99
Labor:	Hour	\$5.00	.5	\$2.50
VARIABLE COST				\$134.60

*5 irrigations at 3.6 inches per irrigation

**18 minutes labor per acre per irrigation

Table B-2. Variable Production Costs - Irrigated Alfalfa - Companion Crop (Previous land use state = Barley)

Item	Unit	Price	Quantity	Cost/Acre
Herbicide (Eptam):	lb.	\$.70	15 lbs.	\$8.50
Fertilizer (11-51-0):	Ton	\$253.00	400 lbs.	\$50.60
Fertilizer (0-0-64):	Ton	\$164.00	100 lbs.	\$8.20
Fertilizer Spread:	Acre	\$2.50	2 times	\$5.00
Barley Seed:	100 lbs.	\$10.00	60 lbs.	\$6.00
Alfalfa Seed:	lb.	\$2.32	10 lbs.	\$23.20
Irrigation Water:	Acre/foot	\$5.00	1.77*	\$8.85
Irrigation Labor:	Hour	\$5.00	1.20**	\$6.00
Implements:	Acre	\$5.38	1.0	\$5.38
Swather:	Acre	\$2.35	1.0	\$2.35
Tractors:	Acre	\$23.57	1.0	\$23.57
Custom Combine:	Acre	\$21.00	1.0	\$21.00
Labor:	Hour	\$5.00	.38	\$1.90
TOTAL VARIABLE COST				\$170.55

*2 irrigations at 6 inches per irrigation plus 2 irrigations at 3.6 inches per irrigation

**18 minutes labor per acres per irrigation

APPENDIX C

DETERMINISTIC ALFALFA REPLACEMENT MODEL

Deterministic Alfalfa Replacement Model

A non-stochastic alfalfa replacement model was solved in order to compare the results with the dynamic replacement model. The solution criteria utilized in the non-stochastic model was the evaluation of the present values of net crop returns summed over time. Table C-1 displays this solution criteria.

As can be seen from the results, seven years was the optimum replacement age for an alfalfa stand. This is similar to the dynamic replacement results, but it doesn't contain the flexibility or stochastic nature of the dynamic programming methodology. Alternative solutions aren't available in the deterministic model, unlike the dynamic model.

Table C-1 Deterministic Alfalfa Replacement Model

Year	Crop	Yield	Crop Revenue	Total *Production: Costs	Net Crop Revenue	Discount** Factor 6%	Present Value NCR	Sum PV NCR	Capital Recovery Factor	CRF x SPVNCR
1	Barley Grain	91 bu.								
1	Barley Straw	2.5 tons	269.18	140.00	129.18	.9434	121.87	121.87	1.0600	129.18
2	Alfalfa ₁	1.47	73.50	146.51	-73.01	.8900	-64.98	56.89	.5454	31.03
3	Alfalfa ₂	4.07	203.50	47.97	155.53	.8396	130.58	187.47	.3741	70.13
4	Alfalfa ₃	4.87	243.50	54.69	188.81	.7921	149.56	337.03	.2886	97.27
5	Alfalfa ₄	5.33	266.50	58.17	208.33	.7473	155.69	492.72	.2374	116.97
6	Alfalfa ₅	5.35	267.50	58.34	209.16	.7050	147.46	640.18	.2034	130.21
7	Alfalfa ₆	4.91	245.50	54.77	190.73	.6651	126.85	767.03	.1791	137.38
8	Alfalfa ₇	4.13	206.50	48.45	158.05	.6274	99.16	866.19	.1610	[139.46]
9	Alfalfa ₈	3.18	159.00	40.76	118.24	.5919	69.99	936.18	.1470	137.62
10	Alfalfa ₉	2.24	112.00	33.14	78.86	.5584	44.04	980.22	.1358	133.11
11	Alfalfa ₁₀	1.45	72.50	26.75	45.75	.5268	24.10	1004.32	.1268	127.35

* Barley at \$100/ton; straw at \$16.67/ton; hay at \$50.00/ton.

** The solution was invariant for real interest rates of 0 - 10%.

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