Quantum fluctuations and thermodynamic processes in the presence of closed timelike curves
by Tsunefumi Tanaka

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics
Montana State University

Abstract:
A closed timelike curve (CTC) is a closed loop in spacetime whose tangent vector is everywhere timelike. A spacetime which contains CTC's will allow time travel. One of these spacetimes is Grant space. It can be constructed from Minkowski space by imposing periodic boundary conditions in spatial directions and making the boundaries move toward each other. If Hawking's chronology protection conjecture is correct, there must be a physical mechanism preventing the formation of CTC's. Currently the most promising candidate for the chronology protection mechanism is the back reaction of the metric to quantum vacuum fluctuations. In this thesis the quantum fluctuations for a massive scalar field, a self-interacting field, and for a field at nonzero temperature are calculated in Grant space. The stress-energy tensor is found to remain finite everywhere in Grant space for the massive scalar field with sufficiently large field mass. Otherwise it diverges on chronology horizons like the stress-energy tensor for a massless scalar field.

If CTC's exist they will have profound effects on physical processes. Causality can be protected even in the presence of CTC's if the self-consistency condition is imposed on all processes. Simple classical thermodynamic processes of a box filled with ideal gas in the presence of CTC's are studied. If a system of boxes is closed, its state does not change as it travels through a region of spacetime with CTC's. But if the system is open, the final state will depend on the interaction with the environment. The second law of thermodynamics is shown to hold for both closed and open systems. A similar problem is investigated at a statistical level for a gas consisting of multiple selves of a single particle in a spacetime with CTC's.

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APPROVAL

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Tsunefumi Tanaka

This thesis has been read by each member of the thesis committee, and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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I wish to express my gratitude and appreciation to my grandparents who had set the goal of my life for which I will be always striving.

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Throughout our calculations natural units in which $c = G = \hbar = 1$ are used and the metric signature is $+2$.

ABSTRACT

A closed timelike curve (CTC) is a closed loop in spacetime whose tangent vector is everywhere timelike. A spacetime which contains CTC's will allow time travel. One of these spacetimes is Grant space. It can be constructed from Minkowski space by imposing periodic boundary conditions in spatial directions and making the boundaries move toward each other. If Hawking's chronology protection conjecture is correct, there must be a physical mechanism preventing the formation of CTC's. Currently the most promising candidate for the chronology protection mechanism is the back reaction of the metric to quantum vacuum fluctuations. In this thesis the quantum fluctuations for a massive scalar field, a self-interacting field, and for a field at nonzero temperature are calculated in Grant space. The stress-energy tensor is found to remain finite everywhere in Grant space for the massive scalar field with sufficiently large field mass. Otherwise it diverges on chronology horizons like the stress-energy tensor for a massless scalar field.

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CHAPTER 1

Introduction

In recent years the physics of time travel has been hotly debated. The study of time travel falls into two categories: the (im)possibility argument on time travel and the exploration of physical effects due to time travel if it is possible. The first part of this thesis deals with a physical process, the growth of vacuum fluctuations of quantized fields, which might be able to prevent time travel. The quantized fields are analyzed in a particular model spacetime, called Grant space. It will be shown that the vacuum fluctuations do not always diverge. In the second half simple thermodynamic processes and statistical mechanics of particles in a spacetime allowing time travel are discussed.

Closed Timelike Curves

The concept of time travel in general suggests "going back in time." However, this statement is too ambiguous. A spacetime in which time travel is allowed is one with closed timelike curves. A closed timelike curve (CTC) is defined as *a world line which is a closed loop whose tangent vector is everywhere timelike.* According to a clock carried by an observer on a CTC, time always moves forward. But since his world line is closed, he comes back to the same point in spacetime. To a second observer who is not on a CTC, the first observer appears to be traveling from the future to the past. On a CTC the choice of an event divides other events on the curve into future events and past events only locally. If the observer follows a CTC in the future direction based on his proper time starting from an event X, he will eventually reaches the same event again. This implies that events to the future of X can influence the outcome of an observation at X.

Chronology Horizons

A region of spacetime without any CTC's is called a chronal region; a region with CTC's is called a nonchronal region. At the boundary between chronal and nonchronal regions there exists a chronology horizon. The nonchronal region is bounded to the past by a future chronology horizon and to the future by the past chronology horizon (see Fig. 1). The future chronology horizon is a special type of future Cauchy horizon. It is generated by null geodesics that have no past end points but can leave the horizon when followed into the future [1]. A past chronology horizon is generated by null geodesics that have no future endpoints but can leave the horizon when followed into the past. These null geodesics, called generators, appear to originate from a smoothly closed null geodesic, called the fountain. There must be something deflecting null geodesics around the fountain in order for the generators

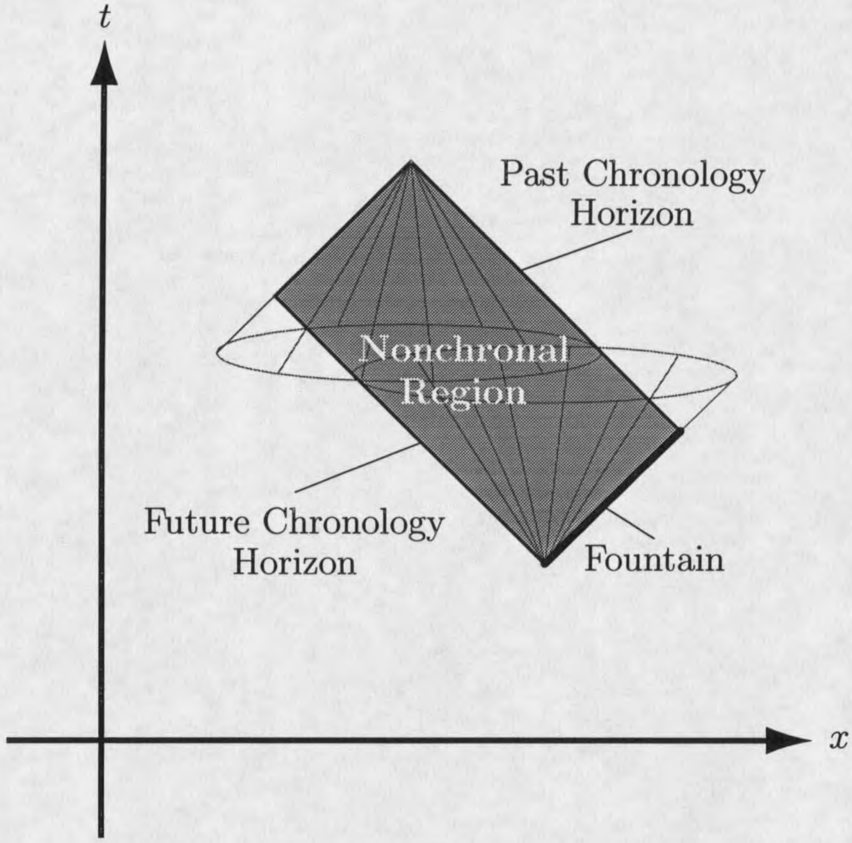


Figure 1: A spacetime with a compact nonchronal region.

to emerge from the fountain [1]. The total energy density of all matter fields around the fountain need to be negative so that a bundle of null geodesics spreads out as it travels along the fountain.

Spacetimes with CTC's

Closed timelike curves appear in some solutions of Einstein field equation, such as Van Stockum space and the Gödel universe, and also in spacetimes with nontrivial topology, for example, Gott space, Misner space, Grant space, and wormhole spacetimes. In the cases of Van Stockum space and the Gödel universe, light cones are tilted in the spatial direction due to the gravitational field. In other cases the spacetime manifold, or at least a part of it, becomes periodic in the time direction. General relativity does not impose any restrictions on the topology of spacetime. Therefore, the topology is a mathematical choice rather than a physical requirement. Misner space, Grant space, and wormhole spacetimes have a non-Hausdorff topology. A brief description of each of these spacetimes follows.

Van Stockum Space

In 1937 Van Stockum discovered a solution to Einstein field equations consisting of an infinitely long cylinder made of rigidly and rapidly rotating dust [2, 3]. The dust particles are held in position by gravitational attractions between them and the centrifugal force due to rotation. Near the surface of the cylinder inertial frames are dragged by rotation so strongly that light cones tilt over in the circumferential direction (See Fig. 2). Frame dragging tilts the light cone so strongly that a velocity vector of a timelike worldline inside the light cone can have a negative time component as seen by an observer far away from the cylinder. A particle following this trajectory can travel backward to an arbitrary point in the past by circling around the cylinder a sufficient number of times. By moving away from the cylinder the particle can start moving forward in time again and reach a point where it started originally. In Van Stockum space CTC's pass through every point in the spacetime, even through the

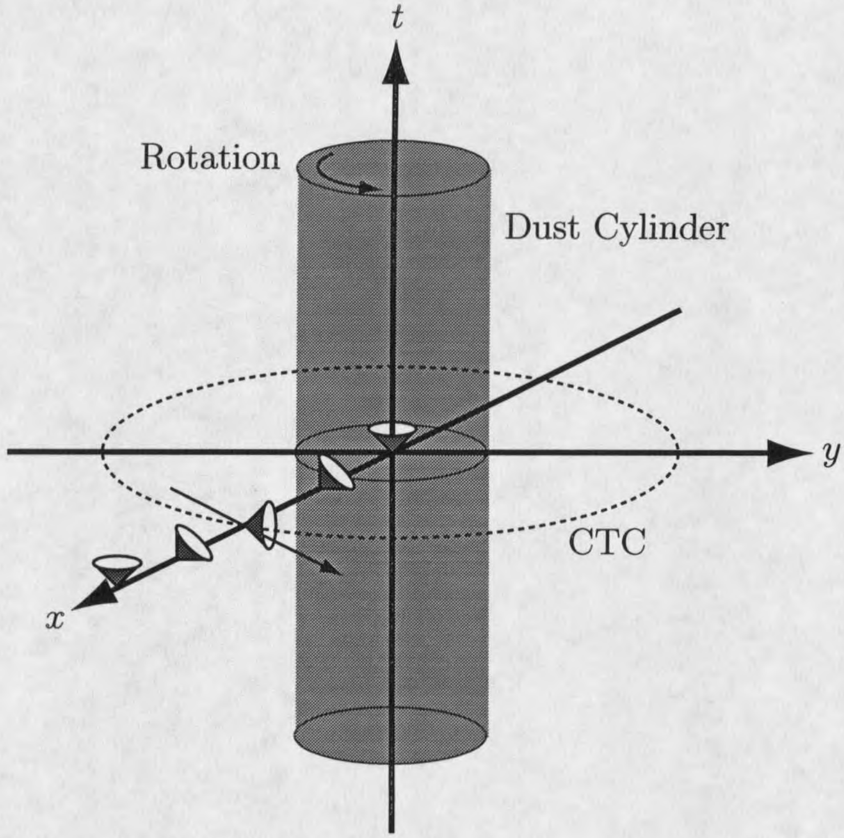


Figure 2: Light cones are tilted in the spatial direction near the surface of the cylinder in Van Stockum space.

center of the cylinder where the light cone is not tilted.

The Gödel Universe

Another solution of Einstein field equation with CTC's is the Gödel universe [4]. It is a stationary, homogeneous cosmological model with nonzero cosmological constant. The universe is filled with rotating, homogeneously distributed dust. The spacetime is rotationally symmetric about any points. Like Van Stockum space CTC's are formed by the tilting of light cones due to inertial frame dragging. On any rotational symmetry axis the light cone is not tilted; it is in the $\frac{\partial}{\partial t}$ direction. As the radial distance from the axis increases, the light cone starts to tilt in the $\frac{\partial}{\partial \phi}$ direction. For radial distances greater than a particular value, $\frac{\partial}{\partial \phi}$ becomes a timelike vector, and a circle of a constant r becomes a closed timelike geodesic. Because the spacetime is homogeneous and stationary, all points in the spacetime are equivalent and CTC's pass through every point [4].

Gott Space

In Gott space two infinitely long, parallel cosmic strings move past each other at high speed without intersecting [5]. Spacetime is flat except on the cosmic string where a conical singularity exists. A circle around the string has a circumference less than $2\pi r$. Gott space can be constructed by cutting out two wedges of a deficit angle $8\pi\mu$ from Minkowski space, where μ is the mass per unit length of the cosmic strings in Planck units, then identifying two edges of each wedge. The apexes of these wedges moves on parallel lines in opposite directions at a high speed. In the center of momentum frame of the strings a point on one side of the wedge and its identified point on the other side of the wedge do not have the same time coordinate due to the motion of the string. Therefore, a path entering the wedge from the leading side in the future exits from the trailing side in the past. By using two cosmic strings a closed timelike path can be formed (Fig. 3).

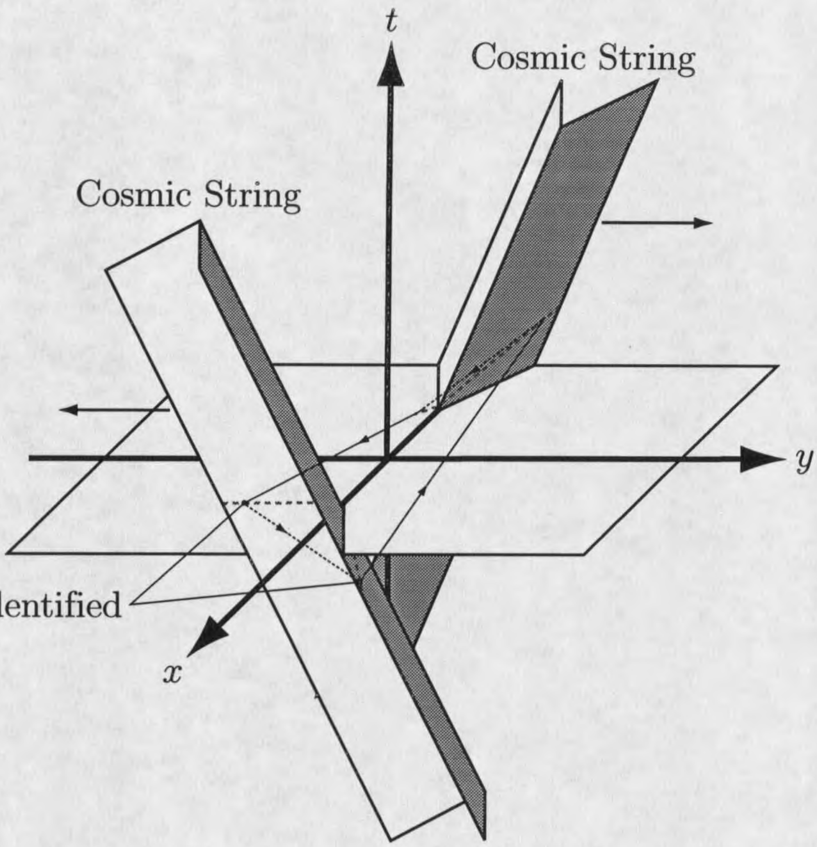


Figure 3: Closed timelike curves are formed around two cosmic strings in Gott space.

Misner Space

Misner space can be constructed from Minkowski space by imposing periodic boundary conditions in a spatial direction [4, 6]. A time shift is then introduced between the proper times of the boundary walls by moving them toward each other at a constant speed [1]. As the walls get closer the time shift becomes equal to the spatial separation between the wall. First a closed null geodesic (fountain) then CTC's are formed as shown in Fig. 4.

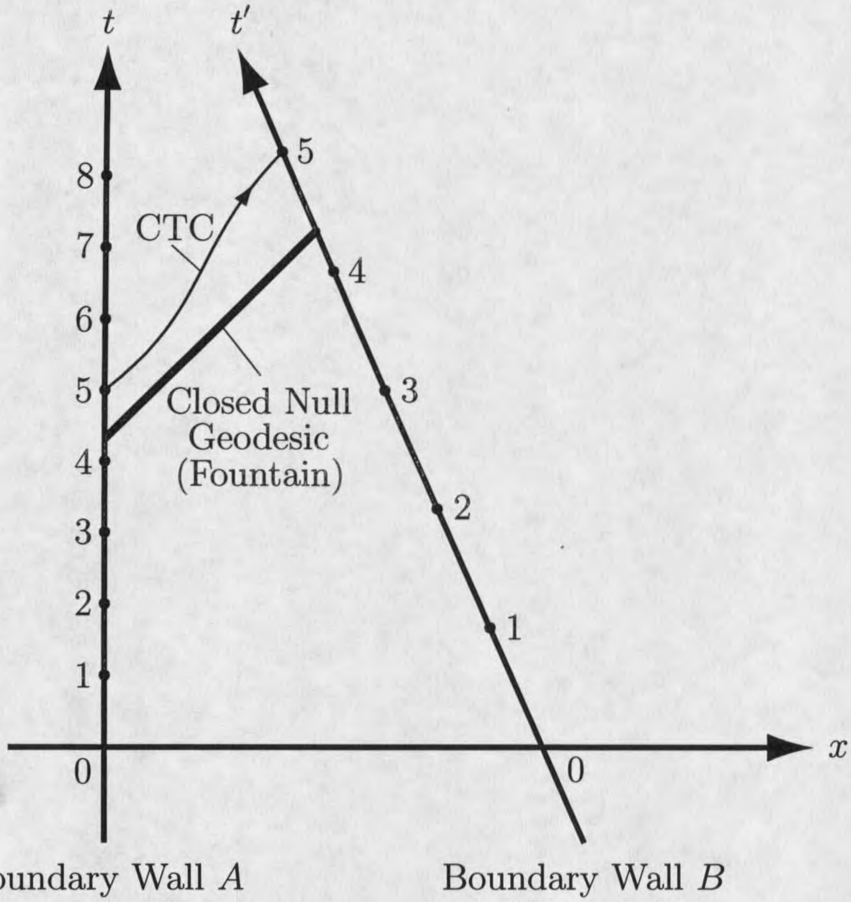


Figure 4: As the periodic boundary walls move toward each other, first a closed null geodesic then CTC's are formed in Misner space.

Wormhole Spacetime

A wormhole is a tunnel connecting two distant parts of a spacetime (Fig. 5). The length of the tunnel, or "throat," could be less than the external distance between the entrances, or "mouths." A simple wormhole spacetime could be constructed from Minkowski space by removing two spheres and identifying their surfaces. The wormhole throat length is zero in this spacetime. With advanced technology a macroscopic wormhole might be constructed by enlarging a loop of quantum gravitational space-

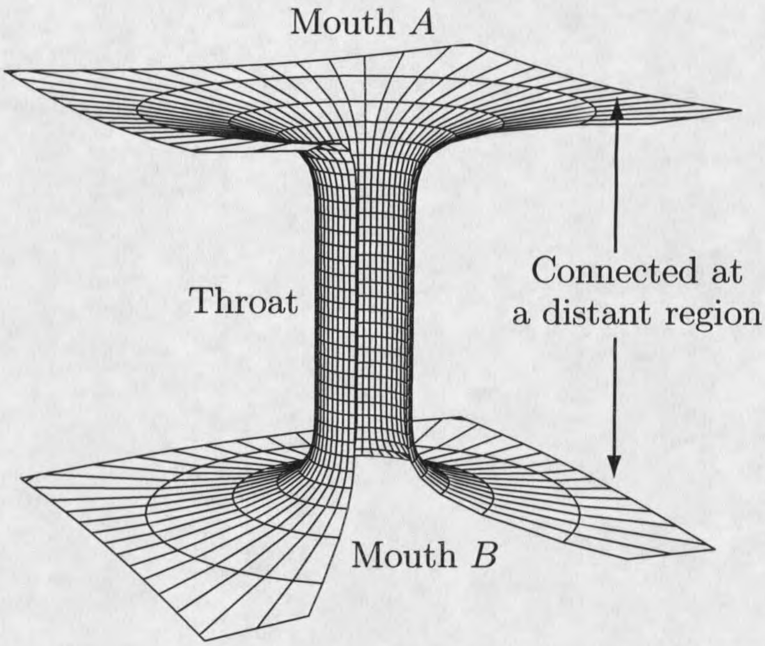


Figure 5: An embedding diagram of a wormhole connecting two distant parts of the spacetime.

time foam at the Planck scale. Wormhole spacetime can be thought as Misner space with curved boundary walls.

Moving one of the mouths relative to the other introduces a dilation of proper time on the moving mouth as seen by an observer who is stationary with respect to the second mouth. This situation is similar to the usual twin paradox but with two wormhole mouths instead of two brothers. However, there is no such time dilation between the observers moving with the mouths as seen through the wormhole throat. If the shift between the proper times of the mouths becomes greater than the spatial distance between them, then a CTC is formed (See Fig. 6).

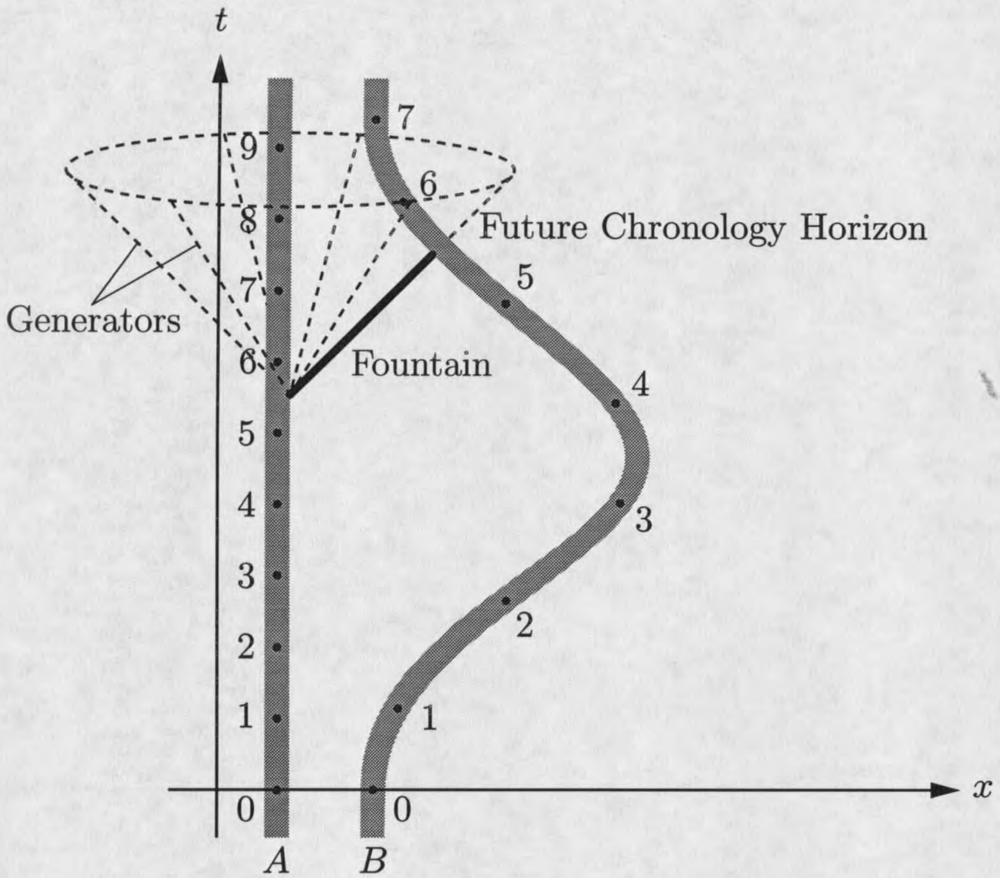


Figure 6: Construction of a wormhole time machine.

Chronology Protection

Why do most scientists feel time travel is unphysical? The main problem with CTC's is that causality might be violated. A time traveler makes a change in the past history and this change propagates in the future direction and eventually alters the present where the traveler originated. In order to avoid causality violation and its consequences in physics, Hawking has proposed the chronology protection conjecture: *The laws of physics prevent CTC's from appearing* [7]. If the formation of CTC's is forbidden, then it is expected to be by a certain physical mechanism that works in all spacetimes which might admit CTC's. There are several candidates for the chronology protection mechanism, but none of them to date have been shown to be universally effective. The following describes the possible mechanisms.

Astronomical Observations

Van Stockum space requires an infinitely long, rotating cylinder of dust, but such an object does not exist in our universe and cannot be constructed even with highly advanced technology. In the case of Gödel universe, a nonzero cosmological constant is required, but its existence has not been confirmed. Also, the observed universe is not rotating fast enough (if rotating at all) to cause significant frame dragging. Similarly, Gott space is not very realistic either. Even if cosmic strings exist, it is very difficult to have two parallel strings attain the necessary speed to form CTC's. It is possible that the strings can achieve high velocity during their collapse. But in that case their energies in the center of momentum frame will be so great that the collapsing loops will produce black holes [1]. It seems that all known solutions of Einstein field equations that contain CTC's are physically implausible according to the current observation of the universe. However, the Einstein field equations do not impose any restrictions on the global topology of spacetime. If the spacetime is multiply connected, CTC's can appear even if the spacetime is flat.

Classical Instability of Fields

Future chronology horizons are Cauchy horizons and hence classically unstable [8]. A wave approaching a chronology horizon is infinitely blue shifted. For example, a particle traveling between the periodic boundary walls in Misner space is Lorentz boosted every time it goes through the wall. The number of traverses between the walls before the particle reaches the horizon is infinite in a finite amount of time. Thus, its energy becomes infinite. The diverging energy of the particle (or of a field) acts back on the metric through the Einstein field equations and alters the spacetime geometry before CTC's could appear. However, this classical instability does not work in wormhole space. The curved walls of the wormhole mouths defocuses a bundle of rays effectively canceling an increase in energy by the blue shift (See Fig. 7) [1].

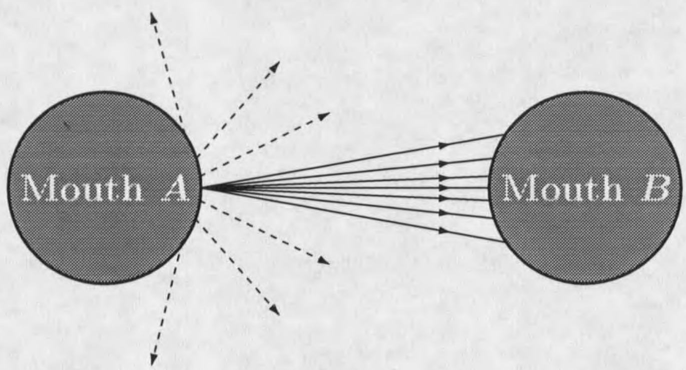


Figure 7: A bundle of rays is defocused by the wormhole mouths as it goes through the throat

Weak Energy Condition

Closed timelike curves and their construction aside, the wormhole space is not entirely free of problems. In order to keep the wormhole throat open "exotic" matter is required [9]. The most unusual property of the exotic matter is that it has a negative energy density, violating the weak energy condition which states that *the energy density cannot be negative.* However, negative energy densities have been indirectly observed in the laboratory in the form of the Casimir effect [10, 11]. Nontrivial topology of the field configuration can lower the vacuum energy density below the Minkowski value causing two flat neutral conducting plates to attract each other in vacuum.

Vacuum Fluctuations

Currently the most promising candidate for the chronology protection mechanism is the back reaction on the metric due to diverging quantum fluctuations on the chronology horizon. Vacuum fluctuations of any quantum field pile on top of each other in the vicinity of the chronology horizon. It has been shown that the vacuum expectation value of the stress-energy tensor for a massless scalar field diverges in Misner space [12], wormhole space [13], and Gott space [14]. However, this divergence is so slow that the perturbation in the metric becomes significant (i. e., the order of 1) only at a distance of the Planck scale from the chronology horizon [1]. It is difficult to conclude that this metric perturbation will definitely change the spacetime geometry.

Also at such a small length scale, the effects of quantum gravity become important, but no viable theory of quantum gravity exists today.

In Chapter 2 the calculation of the stress energy for realistic quantized scalar fields, other than free massless scalar field, in Grant space will be described. If the stress energy for these fields diverges faster than that for the massless field, then the metric perturbation on the chronology horizon is expected to become greater than the order of 1. The back reaction to the metric will definitely change the spacetime geometry via Einstein field equations before CTC's are formed. This will make the vacuum fluctuation a more credible candidate for the chronology protection mechanism. On the other hand, if the stress-energy tensor for more realistic fields diverges more slowly than that for the massless field, then the metric perturbation is going to be less than the order of 1. If so, it will cast doubt on back reaction to vacuum fluctuations as the universal chronology protection mechanism.

It will be shown whether the vacuum energy density diverges on the chronology horizon depends on the field mass. Also, it is found that the vacuum energy density in the self-interacting $\lambda\phi^4$ theory grows without bound as fast as the free field. The self-interaction has a very important role in the evolution of states in the nonchronal region, but its effect on the divergence of vacuum fluctuations is minimal. In addition, thermal effects on the stress energy of a quantized field in Grant space are explored. The thermal contribution to the energy density is found to oscillate rapidly about zero with a growing amplitude near the horizon. However the rate of growth as the horizon is approached is not as fast as the rate of divergence for the vacuum contribution to the total stress energy. Therefore, the thermal contribution will not be able to cancel the vacuum contribution, and the total stress energy still diverges on the chronology horizon.

If the back reaction on the metric due to the diverging quantum fluctuations cannot prevent the appearance of CTC's and if no other chronology protection mechanism is found, it opens a door for the study of physics in the presence of CTC's. In Chapter 3 a review of general physics, both classical and quantum mechanical, in the presence of CTC's is presented. Time travel does not violate causality if an additional condition is imposed on all classical physical processes ensuring no change to the past history. For quantum mechanics in a nonchronal region, two different generalizations (the path integral method and density matrix representation) have been proposed. Both of them reduce to ordinary quantum mechanics in the absence of CTC's.

As an example of application of the self-consistency principle to classical physics, a simple thought experiment with a box filled with an ideal gas is described in Chapter 4. The box goes back in time and interacts with its younger self. They undergo several simple thermodynamic processes. If the system is isolated, the box is always in equilibrium with its older self and there will be no change in its thermal state as it traverses the nonchronal region. On the other hand, the final state of the box cannot be determined by the initial conditions alone if the system is open. It depends on how much work is done on the environment inside the nonchronal region. For open systems, the third law of thermodynamics is violated, but the second law holds for both closed and open systems.

In Chapter 5 the statistical mechanics of particles in a nonchronal region is discussed. If a particle enters a nonchronal region, an indefinite number of its selves could appear due to time travel. The ensemble of systems consisting of these multiple selves is described by a grand canonical ensemble. The self-consistency condition is imposed on the part of the system going back in time. The density operator of the particle as it leaves the nonchronal region is sought. It is very similar to a system in contact with a heat reservoir.

CHAPTER 2

Scalar Fields in Grant Space

Although the global topology of spacetime cannot be fixed by the equations of general relativity, it has measurable local effects in quantum field theory even in a flat spacetime. When the spacetime manifold does not have a simple topology, more specifically, when a spacetime is multiply connected, only those modes of a field that satisfy boundary conditions determined by the topology are relevant in the calculation of physical quantities such as the vacuum expectation value of the stress-energy tensor. For example, in a cylindrical two-dimensional spacetime $R^1(\text{time}) \times S^1(\text{space})$, the only allowed momentum is an integer multiple of $\frac{2\pi\hbar}{a}$ where a is the circumference in the closed spatial direction. In contrast, Minkowski space is simply connected and is infinite in all four dimensions. Thus, the momentum is allowed to have any value. This restriction in the allowed modes results in a shift in the vacuum stress-energy from the Minkowski value which is identically zero. DeWitt, Hart, and Isham [15] thoroughly studied the effects of multiple connectedness of the spacetime manifold (called Möbiosity), twisting of the field, and orientability of a manifold on $\langle 0 | T_{\mu\nu} | 0 \rangle$ for a massless scalar field in various topological spaces. Their work was extended for a massive scalar field with arbitrary curvature coupling by the author and Hiscock [16].

If the spacetime is multiply connected in the time direction, CTC's will be formed. Many types of spacetimes with CTC's can be constructed by simply cutting and pasting Minkowski space. Hiscock and Konkowski [12] have shown that the shift in the vacuum energy density diverges on the chronology horizon in one of these spacetimes, Misner space. The diverging quantum fluctuations will act back on the metric through the Einstein field equations and change the spacetime geometry before CTC's could actually be formed. Their discovery prompted others to investigate the behavior of quantum fluctuations in other types of spacetimes with CTC's and to determine whether gravitational back reaction to the vacuum fluctuations could be the chronology protection mechanism. The vacuum stress energy of a massless scalar field has been shown to diverge on the chronology horizon in Gott space [14], wormhole spacetime [13], and Roman space [17]. However, in some Roman type spacetimes, where there are more than one wormhole, the divergence of the stress energy due to a pile up of quantum fluctuations can be canceled by defocusing effect by the wormhole mouths. Furthermore, the metric perturbation due to the diverging stress-energy tensor for a massless scalar field is only of the order of 1 on the chronology horizon. Then it is hard to conclude that the back reaction stops the formation of CTC's. One of the objectives of this thesis is to find out whether the stress energy of more realistic fields diverge differently on the chronology horizon of Grant space than a massless scalar field. Boulware [14] has shown that the vacuum stress energy of a massive scalar field is finite on the chronology horizon in Gott space. Since Grant space is holonomic to Gott space and contains Misner space as a special limit, it is expected that the stress energy will remain finite in Grant space. Behavior of a massive scalar field, a self-interacting ($\lambda\phi^4$) field, and nonzero temperature effects in Grant space will be examined in the following sections.

Grant Space

Grant space is interesting because it is flat yet contains CTC's. Also, it is closely related to Gott space. Grant space can be considered as a generalization of Misner space with an additional periodic boundary condition in a spatial direction. The original Misner space was developed to illustrate topological pathologies associated with Taub-NUT- (Newman-Unti-Tamburino-) type spacetimes [4, 6]. Misner space is simply the flat Kasner universe with an altered topology. Its metric in Misner coordinates (y^0, y^1, y^2, y^3) is

$$ds^2 = -(dy^0)^2 + (y^0)^2(dy^1)^2 + (dy^2)^2 + (dy^3)^2. \tag{1}$$

That Misner space is flat can be easily seen; the above metric becomes identical to the Minkowski metric via the coordinate transformation

$$x^0 = y^0 \cosh y^1, \quad x^1 = y^0 \sinh y^1, \quad x^2 = y^2, \quad x^3 = y^3. \tag{2}$$

Grant space is constructed by making topological identifications of the y^1 and y^2

directions in the flat Kasner universe:

$$(y^0, y^1, y^2, y^3) \leftrightarrow (y^0, y^1 + na, y^2 - nb, y^3). \tag{3}$$

Misner space is the special case $b = 0$. In Cartesian coordinates the above identification is equivalent to

$$\begin{aligned}(x^0, x^1, x^2, x^3) \quad \leftrightarrow \quad & (x^0 \cosh(na) + x^1 \sinh(na), \\ & x^0 \sinh(na) + x^1 \cosh(na), x^2 - nb, x^3). \end{aligned} \tag{4}$$

It can be shown that Grant space is actually a portion of (holonomic to) Gott space, which describes of two infinitely long, straight parallel cosmic strings passing by each other [1, 18]. The periodicities a and b in Grant space are related respectively to the relative speed and distance between the two cosmic strings in Gott space. As b approaches zero (the Misner space limit) the impact parameter of the two strings also approaches zero.

Grant space can be considered as a portion of toroidal spacetime ($R^2 \times T^2$) with the periodic boundaries in the x^1 direction moving toward each other at constant velocity. A spacetime diagram of the maximally extended Grant space is shown in Fig. 8. Radial straight lines represent $y^1 = na$ surfaces. Hyperbolas are constant y^0 surfaces. A set of identified points is located on a hyperbolic surface. Points A and B are identified with each other. As a particle crosses the radial boundary, $y^1 = na$,

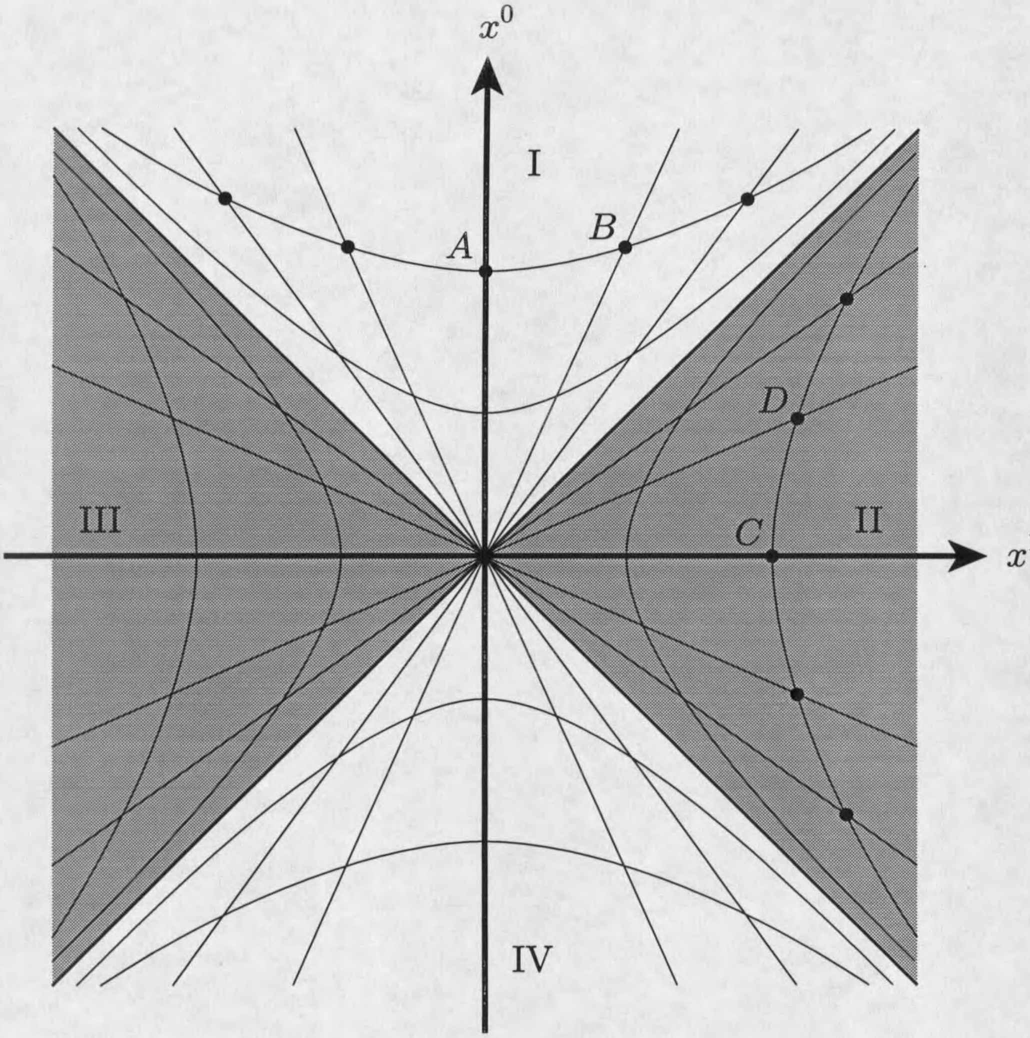


Figure 8: Spacetime diagram of the maximally extended Grant space.

it is Lorentz boosted in a new inertial frame moving at a speed $v = \tanh a$ in the x^1 direction with respect to the original frame and is translated by $-b$ in the x^2 direction. What is extraordinary about Grant space is that it contains nonchronal regions (II and III). In those regions the roles of y^0 and y^1 are switched. The radial boundaries are now spacelike and the spacetime becomes periodic in the time (y^1) direction. Two identified points C and D can be connected by a timelike curve. This topological identification creates CTC's in those regions. The boundaries ($x^0 = \pm x^1$ or equivalently $y^0 = 0$) separating chronal regions (I and IV) and nonchronal regions (II and III) are chronology horizons, which are a kind of Cauchy horizons. The origin ($x^0 = 0, x^1 = 1$) is a quasiregular singularity and is removed from the manifold. The chronological structure of Grant space is discussed in Ref. [1]. In the next section the calculation of the renormalized vacuum stress-energy tensor $\langle \bar{0} \,| T_{\mu\nu} |\, \bar{0} \rangle_{\text{ren}}$ in Grant space will be described.

Calculation of $\langle 0 \,| T_{\mu\nu} |\, 0 \rangle$ in Grant Space

The calculation of the vacuum expectation value of the stress-energy tensor in Grant space is greatly simplified by the fact that all curvature components vanish in a flat spacetime. However, the topology of the spacetime manifold makes the calculation complicated since it allows two points on the manifold to be connected by multiple geodesics. The global covering space of Grant space is Minkowski space, but Grant space does not share the same global timelike Killing vector field (i.e., $\frac{\partial}{\partial x^0}$) with Minkowski space. Actually Grant space does not have any global timelike Killing vector field, but its vacuum state will be assumed to be identical to the Minkowski vacuum. This assumption is defended later by an argument based on a particle detector carried by a geodesic observer. In order to take the topological boundary conditions of Grant space into account, point-splitting regularization (or the "method of images") is used. In this method the topological structure of spacetime is represented by the geodesic distances between image charges and in the number of geodesics connecting the points. Once the geodesic distance for Grant space is found the calculation of $\langle 0 |T_{\mu\nu}| 0\rangle$ reduces to simple differentiation of the Hadamard elementary function and taking the coincidence limit.

Stress-Energy Tensor for a Free Scalar Field

The stress-energy tensor $T_{\mu\nu}$ is formally defined as the variation of the action with respect to the metric. In a flat four-dimensional spacetime the stress-energy tensor for a general free scalar field is given by

$$\begin{aligned} T_{\mu\nu} &= (1-2\xi)\nabla_\mu\phi\nabla_\nu\phi + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi - 2\xi\phi\nabla_\mu\nabla_\nu\phi \\ &\quad + 2\xi g_{\mu\nu}\phi\Box\phi - \frac{1}{2}M^2 g_{\mu\nu}\phi^2. \end{aligned} \tag{5}$$

Note that $T_{\mu\nu}$ depends upon the value of the curvature coupling ξ, even when the curvature vanishes. For conformal coupling $\xi = \frac{1}{6}$; for minimal coupling $\xi = 0$. The value of ξ will be kept arbitrary to make the results as general as possible. The scalar field ϕ satisfies the Klein-Gordon equation $(\Box_x - M^2)\phi(x) = 0$.

Since every term in $T_{\mu\nu}$ is quadratic in the field variable $\phi(x)$, the point x could be split into x and $\tilde{x}$, and the coincidence limit as $\tilde{x} \to x$ could be taken,

$$\begin{aligned} T_{\mu\nu} &= \frac{1}{2}\lim_{\tilde{x}\to x}\Big[(1-2\xi)\,\nabla_\mu\widetilde{\nabla}_\nu + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu}\nabla_\alpha\widetilde{\nabla}^\alpha - 2\xi\nabla_\mu\nabla_\nu \\ &\quad + 2\xi g_{\mu\nu}\nabla_\alpha\nabla^\alpha - \frac{1}{2}M^2 g_{\mu\nu}\Big]\left\{\phi(x), \phi(\tilde{x})\right\}, \end{aligned} \tag{6}$$

where $\{A, B\}$ is the anticommutator of A and B. Covariant derivatives ∇_μ and $\widetilde{\nabla}_\nu$ are to be applied with respect to x and $\tilde{x}$. $T_{\mu\nu}$ is also symmetrized over $\phi(x)$ and $\phi(\tilde{x})$. Before taking the vacuum expectation value of $T_{\mu\nu}$ the vacuum state of the spacetimes needs to be defined. This is nontrivial because Grant space lacks a global timelike Killing vector field, which is required to define positive frequency.

Vacuum State of Grant Space

In Minkowski space there exists a global timelike Killing vector field (i.e. $\frac{\partial}{\partial x^0}$) which may be used to define positive frequency modes,

$$u_{\mathbf{k}} = \frac{e^{-ik_\alpha x^\alpha}}{[2\omega(2\pi)^3]^{\frac{1}{2}}}. \tag{7}$$

The usual Minkowski vacuum state $|0_M\rangle$ is then defined as that state which is annihilated by the operator $a_{\mathbf{k}}$ for all spatial momenta $\mathbf{k}$:

$$a_{\mathbf{k}}|0_M\rangle = 0. \tag{8}$$

The boundary conditions on the Klein-Gordon equation have the effect of restricting the set of modes to a discrete subset of the full Minkowski spectrum of Eq. (7).

Grant space is obtained by making topological identifications on Minkowski space as described in the previous section. However, no global timelike Killing vector exits in Grant space. Within each interval between the periodic boundaries (i.e., one period) $\frac{\partial}{\partial x^0}$ is a locally timelike Killing vector field, but it is impossible to define a global timelike Killing vector field by patching these $\frac{\partial}{\partial x^0}$'s together. Without a global timelike Killing vector field the vacuum state of the spacetime cannot be formally defined. However, the Minkowski vacuum state could be considered as a valid vacuum state of the Grant space. Each interval in Grant space is identical to a portion of Minkowski space, so a geodesic observer in the interval will not detect any particles if Grant space is in the Minkowski vacuum state. Since the only difference between one interval to its neighbors is a constant relative velocity in the x^1 direction and a translation in the x^2 direction, geodesic observers in the neighboring intervals will not find any particles in the same vacuum state. The state in which no geodesic observer detects any particles is a good candidate for the vacuum state. Moreover, it is possible to express positive frequency modes within each interval of Grant space in the Misner coordinates as superpositions of the Minkowski positive frequency modes of Eq. (7) [19].

Then the vacuum state of Grant space, $|\bar{0}\rangle$, can be defined as the state constructed from the discrete set $\{a_{\mathbf{k}_n}\}$ of modes which is annihilated by the Minkowski annihilation operator, restricted to the momenta permitted by the Grant space topology:

$$a_{\mathbf{k}_n}|\bar{0}\rangle = 0. \tag{9}$$

$\mathbf{k}_n$ is the set of momenta, labeled by the discrete index n, which are allowed by the boundary conditions of the topological identification. The renormalized expectation value of $T_{\mu\nu}$ will be calculated in the Grant space vacuum state $|\bar{0}\rangle$. Since each section of Grant space between the periodic boundaries is identical to a portion of Minkowski space, the renormalized vacuum expectation value of the stress-energy tensor is then given by

$$\langle\bar{0}|T_{\mu\nu}|\bar{0}\rangle_{\rm ren} = \langle\bar{0}|T_{\mu\nu}|\bar{0}\rangle - \langle 0_M|T_{\mu\nu}|0_M\rangle\,. \tag{10}$$

The vacuum expectation value of the stress-energy tensor can be expressed in terms of the Hadamard elementary function,

$$\begin{aligned}\langle 0\,|T_{\mu\nu}|\,0\rangle &= \frac{1}{2}\lim_{\tilde{x}\to x}\left[(1-2\xi)\,\nabla_\mu\widetilde{\nabla}_\nu + \left(2\xi-\frac{1}{2}\right)g_{\mu\nu}\nabla_\alpha\widetilde{\nabla}^\alpha - 2\xi\nabla_\mu\nabla_\nu\right.\\ &\quad\left. + 2\xi g_{\mu\nu}\nabla_\alpha\nabla^\alpha - \frac{1}{2}M^2 g_{\mu\nu}\right]G^{(1)}(x,\tilde{x}),\end{aligned} \tag{11}$$

where the Hadamard elementary function $G^{(1)}(x,\tilde{x})$ is defined as

$$G^{(1)}(x,\tilde{x}) = \langle 0\,|\{\phi(x),\phi(\tilde{x})\}|\,0\rangle \tag{12}$$

and satisfies the Klein-Gordon equation $(\Box_x - M^2)\,G^{(1)}(x,\tilde{x}) = 0$. Here $|0\rangle$ stands for the vacuum state of any spacetime. In Minkowski space $G^{(1)}(x,\tilde{x})$ for a massive scalar field is a function of the half squared geodesic distance $\sigma = \frac{1}{2}g_{\alpha\beta}(x^\alpha - \tilde{x}^\alpha)(x^\beta - \tilde{x}^\beta)$ between two points x and $\tilde{x}$ and has the form

$$\begin{aligned}G^{(1)}(x,\tilde{x}) &= \frac{M}{2\pi^2\sqrt{2\sigma}}\Theta(2\sigma)K_1(M\sqrt{2\sigma})\\ &\quad + \frac{M}{4\pi\sqrt{-2\sigma}}\Theta(-2\sigma)I_1(M\sqrt{-2\sigma}),\end{aligned} \tag{13}$$

where Θ is a step function and I_1 and K_1 are modified Bessel functions of the first and second kinds, respectively [20, 21].

Method of Images

Next, the Hadamard function for Grant space needs to be defined. It turns out that the Hadamard function has the same functional form as the Minkowski counterpart but different arguments (i.e., σ) reflecting the periodic boundary condition. Because the spacetime is multiply connected, there can be more than one geodesic connecting the two points x and $\tilde{x}$. For example, suppose the spacetime $R^3 \times T^1$ is closed in the x^1 direction. Two points in the spacetime, x and $\tilde{x}$, could be connected with a direct path, or another path can start from x and circle around in the x^1 direction once, twice, or an arbitrary number of times before arriving at $\tilde{x}$. Since the path circling around n times cannot be deformed continuously into the one which circles around $n'(n' \neq n)$ times, all inequivalent paths must be taken into account. Equivalently this situation could be treated as an electrostatic potential problem and use the method of images. The "image charges" of the point $\tilde{x}$ are located at $\tilde{x} \pm a, \tilde{x} \pm 2a, \cdots, \tilde{x} \pm na$, where a is the period (or circumference) in the closed spatial direction. All these image charges are connected to the point x by geodesics whose half squared distances σ_n are given by

$$\sigma_n = \frac{1}{2} g_{\alpha\beta}(x^\alpha - \tilde{x}_n^\alpha)(x^\beta - \tilde{x}_n^\beta), \tag{14}$$

where $\tilde{x}_n$ is the position of the nth image charge. In the case of Grant space, the image charges lie on a hyperbolic surface (See Fig. 8), and the half squared distance in Cartesian coordinates is equal to

$$\begin{aligned}\sigma_n &= \frac{1}{2}\Big\{-\Big[x^0 - \tilde{x}^0\cosh(na) - \tilde{x}^1\sinh(na)\Big]^2 \\ &\quad + \Big[x^1 - \tilde{x}^0\sinh(na) - \tilde{x}^1\cosh(na)\Big]^2 \\ &\quad + \Big(x^2 - \tilde{x}^2 + nb\Big)^2 + \Big(x^3 - \tilde{x}^3\Big)^2\Big\}. \end{aligned} \tag{15}$$

The contribution from each image charge is summed over to construct the Hadamard function $\bar{G}^{(1)}$ for Grant space. The term $n = 0$ corresponds to the case with no boundary, and so it is identical to the Minkowski Hadamard function. This term will give an infinity associated with the unrenormalized stress energy of Minkowski space and will be subtracted in Eq. (10). By excluding the $n = 0$ from the summation the renormalized Hadamard function $\bar{G}^{(1)}_{\rm ren}$ is obtained. Then using Eq. (11) it gives the renormalized stress-energy tensor $\langle\bar{0}\,|T_{\mu\nu}|\,\bar{0}\rangle_{\rm ren}$:

$$\bar{G}^{(1)}_{\rm ren}(x,\tilde{x}) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \bar{G}^{(1)}(\sigma_n), \tag{16}$$

$$\begin{aligned}\langle\bar{0}\,|T_{\mu\nu}|\,\bar{0}\rangle_{\rm ren} &= \frac{1}{2}\lim_{\tilde{x}\to x}\Big[(1-2\xi)\,\nabla_\mu\widetilde{\nabla}_\nu + \Big(2\xi - \frac{1}{2}\Big)\,g_{\mu\nu}\nabla_\alpha\widetilde{\nabla}^\alpha - 2\xi\nabla_\mu\nabla_\nu \\ &\quad + 2\xi g_{\mu\nu}\nabla_\alpha\nabla^\alpha - \frac{1}{2}M^2 g_{\mu\nu}\Big]\,\bar{G}^{(1)}_{\rm ren}(x,\tilde{x}). \end{aligned} \tag{17}$$

The calculation of $\langle \bar{0} | T_{\mu\nu} | \bar{0} \rangle_{\rm ren}$ has thus been reduced to (1) writing an appropriate σ_n for Grant space, (2) applying the derivative operator in Eq. (17), and (3) taking the coincidence limit as $\tilde{x} \to x$.

Massive Scalar Field

First the vacuum stress energy for a massive scalar field with arbitrary curvature coupling will be calculated. Only the first half of the Hadamard function (Eq. 13) is used because the behavior of $\langle \bar{0} | T_{\mu\nu} | \bar{0} \rangle_{\rm ren}$ in the chronal regions (I and IV) determines whether the gravitational back reaction will prevent the formation of CTC's. The results of the calculation are most simply expressed in the Misner coordinates (y^0, y^1, y^2, y^3):

$$
\begin{aligned}
\left\langle \bar{0} \left| T^0_0 \right| \bar{0} \right\rangle_{\rm ren} &= \frac{M^4}{2\pi^2} \sum_{n=1}^{\infty} \left[1 + 4\xi \sinh^2\left(\frac{na}{2}\right) \right] \frac{K_2(z_n)}{z_n^2}, \\
\left\langle \bar{0} \left| T^1_1 \right| \bar{0} \right\rangle_{\rm ren} &= \frac{M^4}{2\pi^2} \sum_{n=1}^{\infty} \left[1 + 4\xi \sinh^2\left(\frac{na}{2}\right) \right] \\
&\quad \times \left[\frac{K_2(z_n)}{z_n^2} - 4M^2 (y^0)^2 \sinh^2\left(\frac{na}{2}\right) \frac{K_3(z_n)}{z_n^3} \right], \\
\left\langle \bar{0} \left| T^2_2 \right| \bar{0} \right\rangle_{\rm ren} &= \frac{M^4}{2\pi^2} \sum_{n=1}^{\infty} \left\{ \left[1 + 2(4\xi - 1) \sinh^2\left(\frac{na}{2}\right) \right] \frac{K_2(z_n)}{z_n^2} \right. \\
&\quad \left. + M^2 \left[4(1-4\xi)(y^0)^2 \sinh^4\left(\frac{na}{2}\right) - n^2 b^2 \right] \frac{K_3(z_n)}{z_n^3} \right\}, \\
\left\langle \bar{0} \left| T^3_3 \right| \bar{0} \right\rangle_{\rm ren} &= \langle \bar{0} | T_{22} | \bar{0} \rangle + \frac{M^6 b^2}{2\pi^2} \sum_{n=1}^{\infty} n^2 \frac{K_3(z_n)}{z_n^3},
\end{aligned}
\tag{18}
$$

where $z_n = M\left[4(y^0)^2\sinh^2\left(\frac{na}{2}\right)+n^2b^2\right]^{\frac{1}{2}}$. The trace is equal to

$$\begin{aligned}\left\langle\bar{0}\left|T^{\mu}_{\mu}\right|\bar{0}\right\rangle_{\text{ren}} &= \frac{M^4}{2\pi^2}\sum_{n=1}^{\infty}\left\{4\left[1+(6\xi-1)\sinh^2\left(\frac{na}{2}\right)\right]\frac{K_2(z_n)}{z_n^2}\right.\\&\left.+\left[-z_n^2+8(1-6\xi)M^2(y^0)^2\sinh^4\left(\frac{na}{2}\right)\right]\frac{K_3(z_n)}{z_n^3}\right\}. \qquad (19)\end{aligned}$$

On the chronology horizon ($y^0=0$), the components of the stress-energy tensor are

$$\begin{aligned}\left\langle\bar{0}\left|T^0_0\right|\bar{0}\right\rangle_{\text{ren}} &= \frac{M^4}{2\pi^2}\sum_{n=1}^{\infty}\left[1+4\xi\sinh^2\left(\frac{na}{2}\right)\right]\frac{K_2(Mnb)}{(Mnb)^2},\\ \left\langle\bar{0}\left|T^1_1\right|\bar{0}\right\rangle_{\text{ren}} &= \left\langle\bar{0}\left|T^0_0\right|\bar{0}\right\rangle_{\text{ren}},\\ \left\langle\bar{0}\left|T^2_2\right|\bar{0}\right\rangle_{\text{ren}} &= \frac{M^4}{2\pi^2}\sum_{n=1}^{\infty}\left\{\left[1+2(4\xi-1)\sinh^2\left(\frac{na}{2}\right)\right]\frac{K_2(Mnb)}{(Mnb)^2}-\frac{K_3(Mnb)}{Mnb}\right\},\\ \left\langle\bar{0}\left|T^3_3\right|\bar{0}\right\rangle_{\text{ren}} &= \frac{M^4}{2\pi^2}\sum_{n=1}^{\infty}\left[1+2(4\xi-1)\sinh^2\left(\frac{na}{2}\right)\right]\frac{K_2(Mnb)}{(Mnb)^2}. \qquad (20)\end{aligned}$$

Figure 9 shows how the energy density $\rho = -\left\langle\bar{0}\left|T^0_0\right|\bar{0}\right\rangle_{\text{ren}}$ depends on the field mass M for the case of a conformally coupled field ($\xi=\frac{1}{6}$). The periodicities a and b are both set to 1. The contribution to ρ from the nth image charge for $n \gg 1$ is proportional to $\exp[n(a-Mb)]$. The factor of e^{na} comes from the Doppler shift as the particle is boosted in the y^1 direction n times. In Misner space this factor causes $\left\langle\bar{0}\left|T_{\mu\nu}\right|\bar{0}\right\rangle_{\text{ren}}$ to diverge on the chronology horizon, and it might prevent the formation of CTC's. However, in Grant space the exponentially decaying factor e^{-nMb}, which comes from the nonvanishing geodesic distances between image charges in the y^2 direction, b, determines the divergence of $\left\langle\bar{0}\left|T_{\mu\nu}\right|\bar{0}\right\rangle_{\text{ren}}$. For $M > \frac{a}{b}$, ρ remains finite on the chronology horizon; for $M < \frac{a}{b}$ (the shaded region in Fig. 9), ρ diverges on

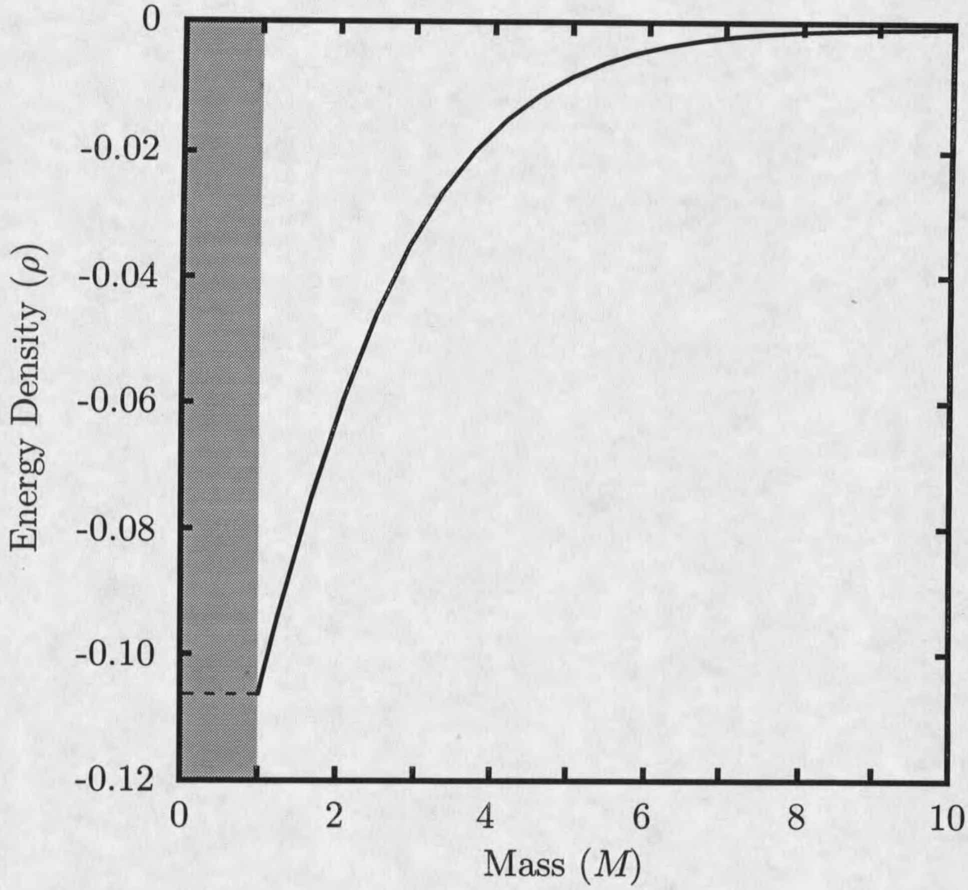


Figure 9: The energy density of a massive conformal scalar field on the chronology horizon in Grant space.

the chronology horizon. At the critical value $M = \frac{a}{b}$, the limiting value of ρ is equal to -0.106 for $a = b = 1$. This is in agreement with Boulware's similar calculation for a conformally coupled massive scalar field in Gott space [14]. It was expected since Grant space is holonomic to Gott space. This result may have significant consequences for chronology protection. It suggests that the metric back reaction from the stress energy of a massive quantized field will likely not be large enough to significantly alter the geometry and prevent the formation of CTC's. If quantized matter fields are to provide the chronology protection mechanism, the above result would indicate that only massless fields may be capable of providing a sufficiently strong back reaction to prevent the formation of CTC's. Outside the domain of quantum gravity, this would place a heavy responsibility on the electromagnetic field (and conceivably neutrino fields, should any be massless) as the sole protector of chronology.

Self-Interacting Scalar Field ($\lambda\phi^4$)

Next $\langle \bar{0} | T_{\mu\nu} | \bar{0} \rangle_{\text{ren}}$ for a self-interacting field is calculated. The self-interacting field is very important in the study of CTC's because all real fields in nature are interacting; also because the evolution of the states through a nonchronal region fails to be unitary for the self-interacting field but not for a free field [22, 23, 24, 25]. Moreover, a massless self-interacting field could gain an effective mass due to the topology of spacetime. Then the vacuum fluctuations of this field might remain finite on the chronology horizon. The self-interaction term will be treated as a perturbation (i. e., the coupling constant $\lambda \ll 1$) in the following calculation of the vacuum stress energy.

In the first order perturbation theory $\langle \bar{0} | T_{\mu\nu} | \bar{0} \rangle_{\text{ren}}$ is given by

$$\langle \bar{0} | T_{\mu\nu} | \bar{0} \rangle_{\text{ren}} = \left\langle \bar{0} \left| T_{\mu\nu}^{\text{free}} \right| \bar{0} \right\rangle_{\text{ren}} + \left\langle \bar{0} \left| T_{\mu\nu}^{\text{self-int}} \right| \bar{0} \right\rangle, \tag{21}$$

where $\left\langle\bar{0}\left|T_{\mu\nu}^{\text{free}}\right|\bar{0}\right\rangle_{\text{ren}}$ is the free particle stress-energy tensor found in the previous section [26]. It is shown by Ford [27] that the self-interaction term is equal to

$$\left\langle\bar{0}\left|T_{\mu\nu}^{\text{self-int}}\right|\bar{0}\right\rangle = -\frac{\lambda}{4!}g_{\mu\nu}\left\langle\bar{0}\left|\phi^4\right|\bar{0}\right\rangle = -\frac{\lambda}{8}g_{\mu\nu}\left\langle\bar{0}\left|\phi^2\right|\bar{0}\right\rangle^2. \tag{22}$$

The self-interacting quantum field ϕ can be represented as

$$\phi = \phi_0 + \hat{\phi}, \tag{23}$$

where ϕ_0 is the c-number background field $\langle\phi\rangle$ and $\hat{\phi}$ is a quantum fluctuation with vanishing expectation value [28]. Then $\hat{\phi}$ satisfies the Klein-Gordon equation for a free field in the one-loop approximation with variable mass $\mu^2 = M^2 + \frac{\lambda}{2}\phi_0^2$ [29].

Unlike the free field theory in a flat spacetime, the self-interaction requires the renormalization of the physical parameters:

$$\begin{aligned} M^2 &= M_{\text{ren}}^2 + \delta M^2, \\ \xi &= \xi_{\text{ren}} + \delta\xi, \\ \lambda &= \lambda_{\text{ren}} + \delta\lambda. \end{aligned} \tag{24}$$

δM^2, $\delta\xi$, and $\delta\lambda$ are counterterms of order $\hbar$ which must be fixed by renormalization. They serve to cancel the divergence in $\left\langle\bar{0}\left|\hat{\phi}^2\right|\bar{0}\right\rangle$ [28]. In addition, the wavefunction must be renormalized. In the one-loop approximation (i. e., up to the order of $\hbar$) the mass counterterm δM^2 is first order in the renormalized coupling constant λ_{ren} and is second order in the renormalized mass itself,

$$\delta M^2 = -\frac{M_{\rm ren}^2 \lambda_{\rm ren}}{16\pi^2}. \tag{25}$$

Therefore, for a massless field it is zero. The mass counterterm might be zero even at higher loop order for a massless field. Both $\delta\lambda$ and the wavefunction renormalization are the second order in the coupling constant $\lambda_{\rm ren}$ and thus can be ignored [30, 31],

$$\begin{aligned} \delta\lambda &= \frac{3\lambda_{\rm ren}^2}{16\pi^2}, \\ \hat{\phi} &= \left(1 + \frac{\lambda_{\rm ren}^2}{3072\pi^4}\right)^{\frac{1}{2}} \hat{\phi}_{\rm ren}. \end{aligned} \tag{26}$$

Furthermore, the coupling constant counterterm $\delta\xi$ vanishes for the conformal coupling $\xi_{\rm ren} = \frac{1}{6}$,

$$\delta\xi = \frac{\lambda_{\rm ren}\left(\frac{1}{6} - \xi_{\rm ren}\right)}{16\pi^2}. \tag{27}$$

Therefore, the following calculation of the stress-energy tensor will be limited only to the conformal coupling case. There is still a nonzero contribution to the stress energy from the two-loop vacuum graph (◯◯) which is still first order in $\lambda_{\rm ren}$ [32]. This is the only contribution from the two-loop effect [33]. The stress energy for the background field ϕ_0 is zero for the vacuum state so that the only contribution to the vacuum energy density comes from the quantum fluctuation $\hat{\phi}$. Since $\hat{\phi}$ satisfies the Klein-Gordon equation, the same Hadamard function can be used for $\hat{\phi}$ as for the free field (Eq. 13). In the massless limit the Hadamard function takes the form

$$G^{(1)}(x,\tilde{x}) = \left\langle \bar{0} \left| \left\{ \hat{\phi}(x), \hat{\phi}(\tilde{x}) \right\} \right| \bar{0} \right\rangle = \frac{1}{4\pi\sigma(x,\tilde{x})}, \tag{28}$$

where $\sigma(x,\tilde{x})$ is the half squared geodesic distance; it is $\sigma(x,\tilde{x})$ which contains the information about the global topology of the spacetime. Then, the renormalized contribution to the vacuum stress-energy tensor due to the self-interaction is given by

$$\left\langle \bar{0} \left| T_{\mu\nu}^{\text{self-int}} \right| \bar{0} \right\rangle = -\frac{\lambda_{\text{ren}}}{32} \lim_{\tilde{x}\to x} \left\{ \sum_n \left[G^{(1)}(x,\tilde{x}_n) \right]^2 - \left[G^{(1)}(x,\tilde{x}) \right]^2 \right\}, \tag{29}$$

where the sum is over all image charges located at $\tilde{x}_n$ and the second term inside the limit corresponds to the Minkowski vacuum term. There will be a shift in the energy density of the vacuum state of the order λ_{ren} due to the fact that the first order vacuum graph (◯◯) is nonzero when the spacetime is not globally Minkowskian [34].

On the chronology horizon of Grant space, all nonzero components of the free vacuum stress-energy tensor diverge due to the blue shift,

$$\begin{aligned}
\left\langle \bar{0} \left| T_{00}^{\text{free}} \right| \bar{0} \right\rangle &= -\frac{\pi^2}{90b^2} - \frac{1}{3\pi^2 b^4} \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{na}{2}\right)}{n^4}, \\
\left\langle \bar{0} \left| T_{11}^{\text{free}} \right| \bar{0} \right\rangle &= 0, \\
\left\langle \bar{0} \left| T_{22}^{\text{free}} \right| \bar{0} \right\rangle &= -\frac{\pi^2}{30b^2} - \frac{1}{3\pi^2 b^4} \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{na}{2}\right)}{n^4}, \\
\left\langle \bar{0} \left| T_{33}^{\text{free}} \right| \bar{0} \right\rangle &= \frac{\pi^2}{90b^2} - \frac{2}{3\pi^2 b^4} \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{na}{2}\right)}{n^4}.
\end{aligned} \tag{30}$$

But the self-interaction components remain finite in Grant space,

$$\left\langle \bar{0} \left| T^{\text{self-int}}_{\mu\nu} \right| \bar{0} \right\rangle = -\frac{\lambda_{\text{ren}}}{5760 b^4} g_{\mu\nu}, \tag{31}$$

where $g_{\mu\nu}$ is the Minkowski metric in the Misner coordinates. Therefore, the self-interaction in Grant space is incapable of keeping the stress energy finite on the chronology horizon. Also, it does not make the divergence any stronger. Therefore, the metric perturbation on the chronology horizon is still about 1.

In the Misner space limit ($b = 0$), the free field term diverges like $(y^0)^{-4}$ near the chronology horizon,

$$\begin{aligned} \left\langle \bar{0} \left| T^{\text{free}}_{00} \right| \bar{0} \right\rangle &= -\frac{1}{16\pi^2 (y^0)^4} \left(D + \frac{2}{3} E \right), \\ \left\langle \bar{0} \left| T^{\text{free}}_{11} \right| \bar{0} \right\rangle &= 3(y^0)^2 \left\langle \bar{0} \left| T^{\text{free}}_{00} \right| \bar{0} \right\rangle, \\ \left\langle \bar{0} \left| T^{\text{free}}_{22} \right| \bar{0} \right\rangle &= \left\langle \bar{0} \left| T^{\text{free}}_{33} \right| \bar{0} \right\rangle = -\left\langle \bar{0} \left| T^{\text{free}}_{00} \right| \bar{0} \right\rangle, \end{aligned} \tag{32}$$

where D and E are positive finite numbers given by

$$\begin{aligned} D &= \sum_{n=1}^{\infty} \frac{1}{\sinh^4 \left(\frac{na}{2} \right)}, \\ E &= \sum_{n=1}^{\infty} \frac{1}{\sinh^2 \left(\frac{na}{2} \right)}. \end{aligned} \tag{33}$$

However, the self-interaction term diverges at the same rate as the free field term,

$$\left\langle \bar{0} \left| T^{\text{self-int}}_{\mu\nu} \right| \bar{0} \right\rangle = -\frac{\lambda_{\text{ren}} D}{1024\pi^4 (y^0)^4} g_{\mu\nu}. \tag{34}$$

The self-interaction term cannot completely cancel the free field by choosing a particular value for λ_{ren}.

Nonzero temperature

The systems examined so far are all at zero temperature. The Hadamard function used in the calculation of the vacuum stress energy is basically an expectation value of ϕ^2 for a pure state $|0\rangle$. However, if the system is at a nonzero temperature, the expectation value should be given by an ensemble average over the expectation values of all pure states [20]. The thermal Hadamard function $G^{(1)}_{\beta}$ for a nonzero temperature system can be defined by replacing the vacuum expectation value in the definition of the zero-temperature Hadamard function $G^{(1)}$ by the ensemble average $\langle\ \rangle_{\beta}$. It can be shown that the thermal Hadamard function can be written as an infinite imaginary time image charge sum of the corresponding zero-temperature Hadamard function [20],

$$G^{(1)}_{\beta}(t, \mathbf{x}; \tilde{t}, \tilde{\mathbf{x}}) = \sum_{k=-\infty}^{\infty} G^{(1)}(t + ik\beta, \mathbf{x}; \tilde{t}, \tilde{\mathbf{x}}), \tag{35}$$

where $\beta = \frac{1}{k_B T}$. The term corresponding to $k = 0$ is the zero temperature Hadamard function defined in the previous section.

There seems to be a very important potential connection between CTC's and thermal physics. First, in a nonchronal region the spacetime becomes periodic in the real time direction, and all image charges separated in the real time direction must be summed over to find the Hadamard function. For the thermal Hadamard function, the image charges are separated in the imaginary time direction. Secondly, the number of particles in the nonchronal region is indefinite and it cannot be determined by the initial conditions on the future chronology horizon. A particle flying through the nonchronal region may go back in time an indefinite number of times before it leaves the region. Hartle shows that in order to find the net effect of the path through the nonchronal region, all paths with different numbers of time traverses must be summed over [24]. This is very similar to an equilibrium system which is described by a grand canonical ensemble of states. A relationship between quantum field theory in the nonchronal region and the nonzero temperature theory is explored by Hawking [35]. He suggests that a system inside a nonchronal region is similar to a system in contact with a heat reservoir with an imaginary temperature. In Chapter 5 simple classical thermodynamic processes in a nonchronal region are examined. In Chapter 6 a relationship between CTC's and a heat reservoir is studied in detail from quantum statistical point of view. But first, the thermal stress-energy tensor for a scalar field in Grant space is calculated.

The calculation of $\langle \bar{0} | T_{\mu\nu} | \bar{0} \rangle_\beta$ is straightforward, if not simple, due to imaginary separations. The half squared geodesic distance between a point and its images becomes complex and is equal to

$$\sigma_n = \frac{1}{2}\Big\{-\Big[x^0 + ik\beta - \tilde{x}^0\cosh(na) - \tilde{x}^1\sinh(na)\Big]^2 + \Big[x^1 - \tilde{x}^0\sinh(na) - \tilde{x}^1\cosh(na)\Big]^2 + \Big(x^2 - \tilde{x}^2 + nb\Big)^2 + \Big(x^3 - \tilde{x}^3\Big)^2\Big\}. \tag{36}$$

The renormalized Hadamard function can be found by summing over the massive Hadamard functions Eq. (13) with above argument and then subtracting the zero temperature Minkowski term corresponding to $n = k = 0$. By applying the differential operator of Eq. (17) to the renormalized Hadamard function, the thermal energy density for the conformal coupling ($\xi = \frac{1}{6}$) on the line $y^1 = 0$ in Misner space is found to be

$$\begin{aligned}\langle\bar{0}\,|T_{00}|\,\bar{0}\rangle_\beta \doteq{}& \Big\langle\bar{0}\Big|T_{00}^{\text{free}}\Big|\bar{0}\Big\rangle + \frac{2}{3\pi^2}\sum_{n,k=1}^{\infty}\Big\{(k\beta)^8\,[4 + 5\cosh(na)]\\ &+ 8(k\beta)^6\sinh^2\left(\frac{na}{2}\right)\Big[17 + 7\cosh(na) - 12\cosh^2(na)\Big](y^0)^2\\ &+ 32(k\beta)^4\sinh^4\left(\frac{na}{2}\right)\Big[9 + 4\cosh(na)\\ &- 2\cosh^2(na) - 2\cosh^3(na)\Big](y^0)^4\\ &+ 256(k\beta)^2\sinh^8\left(\frac{na}{2}\right)[7 + 6\cosh(na)]\,(y^0)^6\\ &- 256\sinh^8\left(\frac{na}{2}\right)[2 + \cosh(na)]\,(y^0)^8\Big\}\\ &\times\Big\{(k\beta)^4 + 16(k\beta)^2\sinh^4\left(\frac{na}{2}\right)\cosh(na)(y^0)^2\\ &+ 16\sinh^4\left(\frac{na}{2}\right)(y^0)^4\Big\}^{-3},\end{aligned} \tag{37}$$

where $\left\langle \bar{0} \left| T_{00}^{\text{free}} \right| \bar{0} \right\rangle$ is the zero temperature contribution given by Eq. 32 and $\beta = \frac{1}{k_{\text{B}}T}$.

Fig. 10 is a plot of the energy density due to the nonzero temperature terms vs the Misner coordinate y^0 along a constant $y^1 = 0$ line (or equivalently the Cartesian coordinate x^0) for $a = b = 1$ at $T = 0.001$ in Planck units. The energy density is

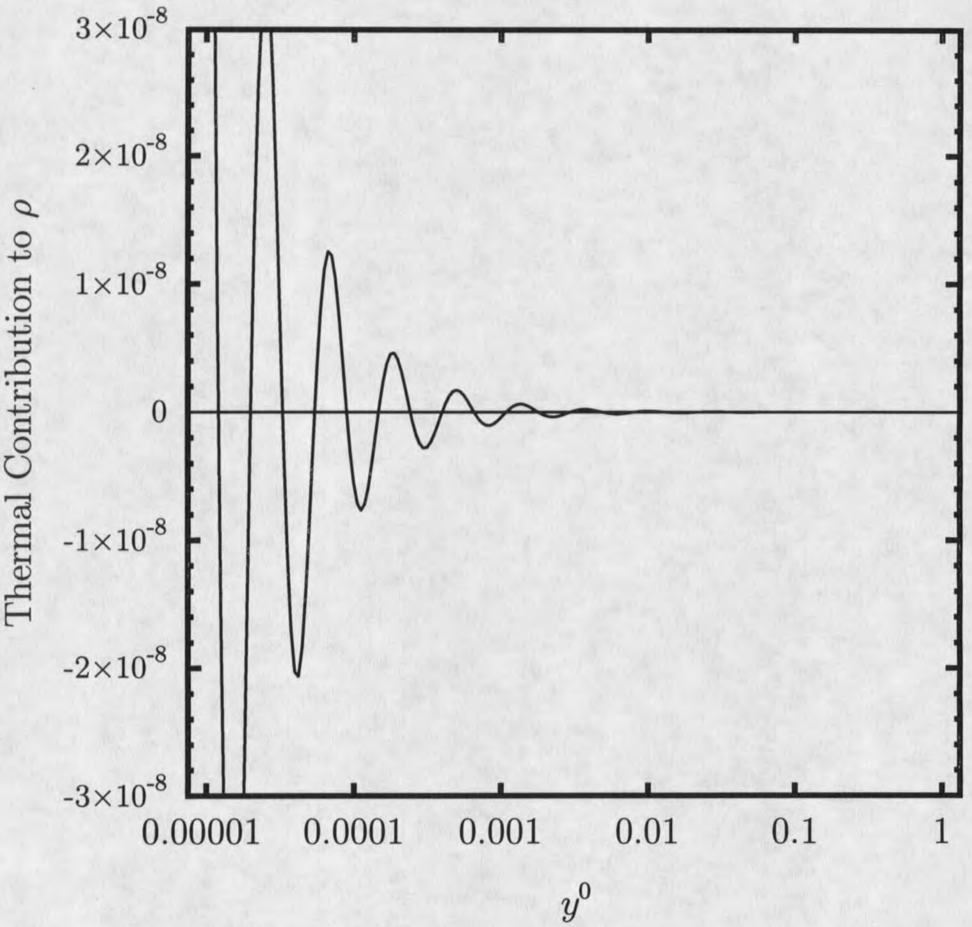


Figure 10: The thermal contribution to the energy density of a massless conformal scalar field at temperature $T = 0.001$ near the chronology horizon in Misner space.

oscillatory, and its amplitude increases in the positive direction in the vicinity of the chronology horizon located at $y^0 = 0$. However, this divergence in the energy density from the nonzero temperature term is not fast enough to cancel the zero temperature term which grows out of bound in the negative direction. Near the chronology horizon the lowest term for the thermal contribution is zeroth order in y^0. It still diverges on the chronology horizon because of the summation over all image charges, each of which carries a Doppler shift factor of e^{na}. On the other hand, the zero temperature term diverges like $(y^0)^{-4}$ in addition to the Doppler shift factor. Therefore, the total energy density still diverges in the negative direction on the chronology horizon in Misner space at the same rate as the zero temperature contribution. This seems true even in Grant space according to my numerical calculations. This means that fluctuations in a quantum scalar field at nonzero temperature neither strengthens or weakens the metric perturbation on the chronology horizon.

The calculations of $\langle\bar{0}\,|T_{\mu\nu}|\,\bar{0}\rangle_{\rm ren}$ for different types of scalar fields in Grant space shown in this chapter indicate that quantum fluctuations remain finite on the chronology horizon only if the field is massive. Although some might argue that the massive scalar field is still not very physically "realistic," it is important to notice that the quantum fluctuations do not diverge for all types of fields. That means that quantum fluctuations is not the universal chronology protection mechanism which everyone is looking for.

Even when the vacuum energy density diverges near the chronology horizon, it does so at such a slow rate that quantum gravitational effects at Planck scale must be taken into account before the metric back reaction becomes significant [1, 12].

Quantum fluctuations alone cannot create the metric perturbation larger than the order of 1, and it remains inclusive that the metric perturbation of this size can stop the formation of CTC's by changing the spacetime geometry. A viable theory of quantum gravity is definitely required to prove or disprove Hawking's chronology protection conjecture.

CHAPTER 3

Physics in the Presence of CTC's

The lack of a valid proof for the chronology protection conjecture allows us explore physics in the presence of CTC's. In this chapter general approaches to physics in the presence of CTC's by others will be reviewed. A typical objection to time travel is based on the violation of causality. A person or an object travels back in time and interacts with others thus altering the course of history. What will happen to the time traveler if he accidentally kills his parent before he was born? It has been an extremely popular paradox in science fiction movies and TV shows, and generally their solutions are logically inconsistent. Physically speaking the solution to the time travel paradox requires a better understanding of the causal structure of spacetime than what is known today. The past history can be assumed to be either fixed, even if the time travel is allowed, or unfixed and the history bifurcates every time a change is made. If the past is fixed, there must be some kind of physical principle which prevents the alteration of the history. The principle, called the self-consistency principle, states that *the only solutions to the laws of physics that can occur locally in the real universe are those which are globally self-consistent.* In other words, solutions for the local equations of motion must be consistent with the global history of spacetime. The time traveler who is determined to murder his parent will definitely fail for some reason, for example he forgets a gun in the future, because the murder never took place in the past. According to the self-consistency principle, the past history cannot be altered regardless of how hard the time traveler tries to change.

Another solution to the time travel paradox is to assume that the past is not fixed. Any change made in the past history by the time traveler will change the future history. There is a natural interpretation of this view in terms of the many-world interpretation of quantum mechanics [36, 37]. I will not follow this path. For the rest of this treatise the self-consistency principle is imposed on all physical processes.

Classical Scattering in Wormhole Spacetime

As usual the very first physical problem to be examined is a collision of classical particles (i. e., billiard balls) in a traversable wormhole spacetime [38]. A time shift between the wormhole mouths could be introduced by moving mouth B away from mouth A at a high speed then bringing it back (see Fig. 6). After the two mouths are brought together, they are stationary with respect to each other, but there exists a fixed time shift τ between them. A billiard ball entering the mouth B at $t = 0$ comes out from the mouth A at $t = -\tau$. The scattering of billiard balls becomes very complicated because of the multiple connectedness of the spacetime and because of the curved surfaces of the wormhole mouths.

In this problem a billiard ball is set in motion far away from the wormhole with some initial velocity in a general direction of the wormhole. Near the wormhole the ball is hit by its older self which appears from the mouth A and its course is changed toward the mouth B. The ball comes out of the other mouth and collides with its younger self, then it leaves to infinity. The whole process takes place inside a nonchronal region. It is possible that the ball goes through the wormhole multiple times before it collides with its younger self and that it undergoes multiple collisions. In their paper (Ref. [38]) Echeverria, Klinkhammer and Thorne ask whether Cauchy problem is well-posed in this problem or not. They define the multiplicity of an initial trajectory for the ball as the number of self-consistent solutions of the ball's equations of motion given that initial trajectory. If there is only one solution for each initial trajectory, then the multiplicity is one and the Cauchy problem is well-posed. On the other hand, an incoming ball might be scattered by the older self in such a way that younger self's new trajectory does not lead to the same collision (e. g., the older self misses the younger self) after going through the wormhole. The past history is changed in such a collision and the solution for the equations of motion is not self-consistent. In this case the multiplicity becomes zero, and the Cauchy problem is ill-posed. Echeverria, Klinkhammer and Thorne call this kind of trajectory "dangerous" [38]. However, they failed to find any "dangerous" trajectories. What they found is that for a wide class of initial trajectories the multiplicity is actually infinite. There are far too many self-consistent solutions for a given initial trajectory. Others (Ref. [39, 40]) have tried more sophisticated versions of the same problem, for example, by making the collision inelastic and by replacing the billiard ball with a bomb. They did not find any trajectory with zero multiplicity but always found multiple self-consistent trajectories. Classical physics is thus underdetermined in the presence of CTC's because additional data (called supplementary data) about what happens in the nonchronal region may be required to specify a unique solution of the equations of motion [36]. Echeverria, Klinkhammer, and Thorne expect the problem to become well posed if it is treated quantum mechanically by summing over all self-consistent trajectories [38]. Then a unique probability distribution for the outcomes of all measurements should be obtained.

Quantum Mechanics in a Nonchronal Region

Two widely different approaches to the generalization of quantum mechanics in a spacetime with CTC's have been introduced. Each of them has an unavoidable feature which does not exist in ordinary quantum mechanics. The first approach, by Hartle, is based on the path integral of all histories through a nonchronal region [24]. The other approach, by Deutsch, uses density matrices instead of state vectors [36], and it has greatly influenced my work reported in this thesis, especially thermodynamics processes and statistical mechanics in the presence of CTC's in Chapters 4 and 5. In the path integral formulation unitarity is lost; in the density matrix formulation coherence is lost. In ordinary quantum mechanics in a spacetime without CTC's, neither coherence nor unitarity is lost. However, the two approaches are not equivalent in the presence of CTC's. The path integral method predicts that an experiment conducted before the appearance of CTC's is affected by the CTC's due to nonunitary evolutions. In contrast, the density matrix approach is causal, but it allows a pure state to evolve into a mixed state by a traverse through a nonchronal region.

Path Integral Formulation

The quantum state of the matter field is defined on a spacelike hypersurface σ, and this state is evolved to a future spacelike hypersurface σ' by Hamiltonian evolution or the Schrödinger equation (Fig. 11). However, if CTC's exist, a spacetime cannot be

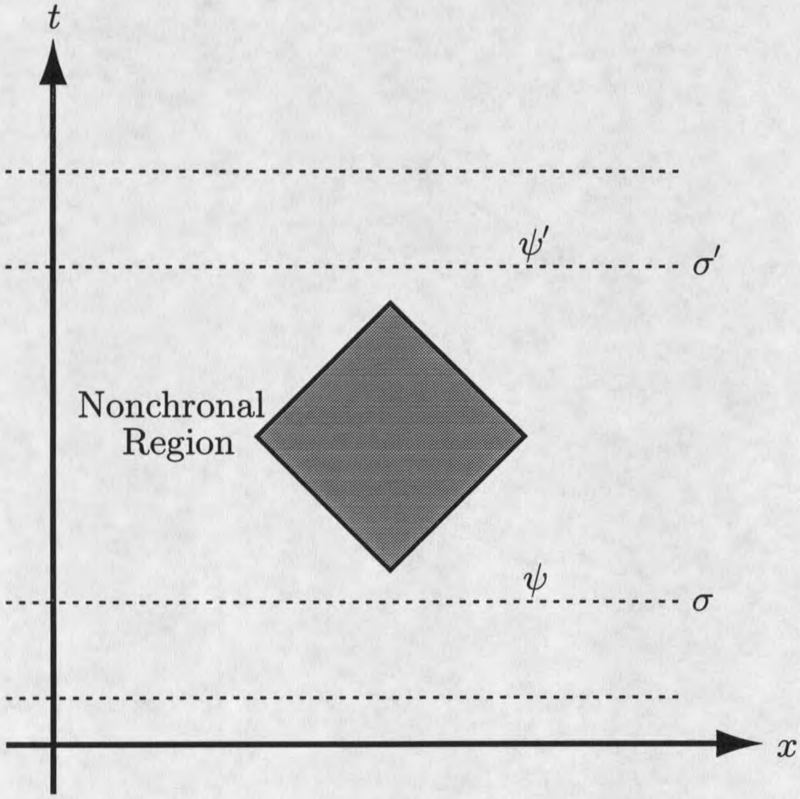


Figure 11: Foliation of a spacetime by spacelike hypersurfaces.

foliated by a family of spacelike hypersurfaces; in other words, there is no unique time ordering of those hypersurfaces. It is still possible to formulate quantum mechanics even without a notion of state vectors or a foliation of the spacetime by spacelike hypersurfaces. Hartle has shown that the Feynman path integral offers a generalization of quantum mechanics for a spacetime containing CTC's [24]. In the path integral approach to ordinary quantum mechanics the scattering matrix is constructed by summing over all paths containing an initial state $|\psi(\sigma)\rangle$ to a final state $|\psi'(\sigma')\rangle$,

$$\langle\psi'(\sigma')|\psi(\sigma)\rangle = \int_{[\psi,\psi']} \delta\psi e^{iS[\psi]}, \tag{38}$$

where $S[\psi]$ is the action. The sum is over four-dimensional field configurations between σ and σ'. The evolution of states is unitary, meaning that the states remain normalized as they evolve in time. This is true even for systems with nonlinear equations of motion.

However, in a nonchronal region nonlinearity leads to nonunitarity. It is known that the evolution of states is unitary for free fields but not, in general, for interacting fields [14, 22, 41]. Suppose an operator X evolves the state on σ to that on σ',

$$|\psi(\sigma')\rangle = X\,|\psi(\sigma)\rangle\,. \tag{39}$$

We assume that the initial state $|\psi(\sigma)\rangle$ is normalized. Then the new state $|\psi(\sigma')\rangle$, in general, will not be normalized,

$$\langle\psi(\sigma')|\,X^{\dagger}X\,|\psi(\sigma')\rangle \neq 1, \tag{40}$$

unless the opertor X is unitary; i. e., $X^\dagger X = 1$. Probability is not conserved through nonunitary evolution.

In order to recover the probability interpretation for the left hand side of Eq. (40), the sum of the probabilities for all possible outcomes for each individual initial condition must be renormalized to one. Let P_α be a projection operator corresponding to some observable α. Then the probability of finding the state in α on the hypersurface σ is given by

$$\mathcal{P}_\alpha(\sigma) = \frac{\langle\psi(\sigma)|\, P_\alpha \,|\psi(\sigma)\rangle}{\langle\psi(\sigma)|\psi(\sigma)\rangle}. \tag{41}$$

and on σ' by

$$\mathcal{P}_\alpha(\sigma') = \frac{\langle\psi(\sigma)|\, X^\dagger P_\alpha X \,|\psi(\sigma)\rangle}{\langle\psi(\sigma)|\, X^\dagger X \,|\psi(\sigma)\rangle}. \tag{42}$$

In this way we can determine the probability for any particular outcome relative to any particular initial condition [42]. However, this renormalization depends on the initial state. The above rule is not covariant with respect to the choice of spacelike surfaces unless X is a unitary operator. The probabilities given by Eq. (41) and Eq. (42) are not necessarily equal to each other. The normalized probability for a particular outcome $|\psi'(\sigma')\rangle$ given by Eq. (42) depends on a particular initial state $|\psi(\sigma)\rangle$ and how it evolves under X. Therefore, the theory becomes nonlinear in a general state vector and the superposition principle is lost.

To maintain a consistent probability interpretation, Hartle claims that the path integral must include all paths extending from a chronal region in the past of a nonchronal region to a future chronal region beyond the nonchronal region even when an observation is carried out in the past chronal region [42]. As a result, generalized quantum mechanics by the path integral through nonunitary evolution becomes acausal because information about the future is required to calculate the probabilities in the present [24].

Density Matrix Formulation

Deutsch's work on CTC's is based on the concept of quantum computational network which is regarded as a representation of a physical process [36]. An "input" to the network consists of a particle entering a nonchronal region from the past side of the future chronology horizon. This particle and its older selves which traveled back in time undergo some kind of interaction in a small region called a "gate." After the interaction, one of the selves leaves the nonchronal region as an "output," and the rest follow CTC's and go back in time. A negative time delay in the network plays the role of a time machine and its existence makes no fundamental difference to the behavior of the quantum computational network. In order to simplify this physical process, only the internal degrees of freedom of the particles are treated quantum mechanically, and each particle traveling in the network is replaced by sufficiently many "carriers" which have only 2-state internal degrees of freedom, or bits. When this process is reduced to bits and pieces it looks like Fig. 12. The input represented by m bits and the time traversing (TT) part represented by n bits enter the interaction gate where the carriers exchange their bits. The TT carriers are on CTC's. After

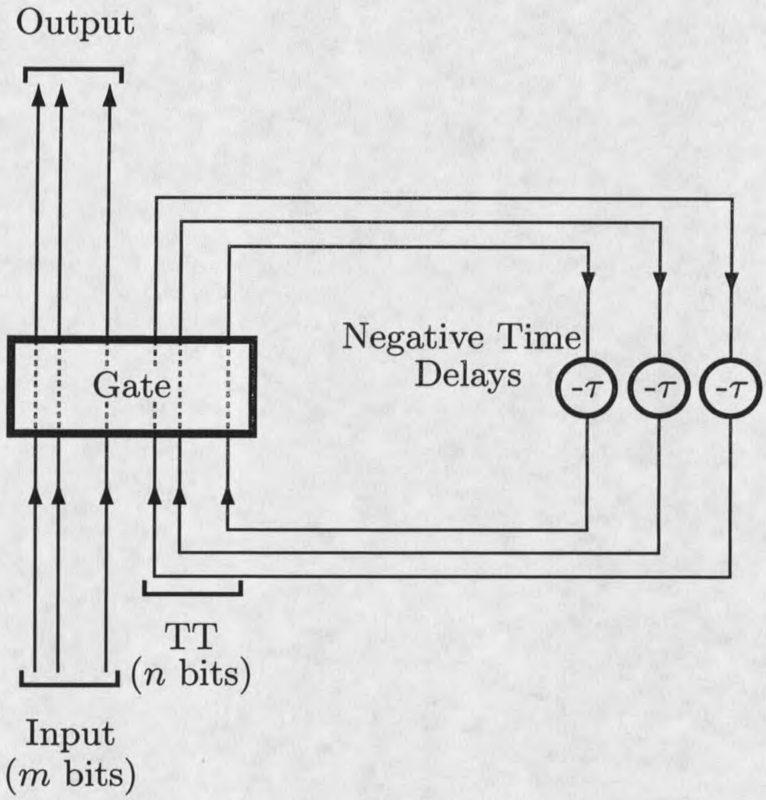


Figure 12: Quantum computational network with negative time delays.

the interaction m bits leave the nonchronal region forming the output while n bits go through the negative time delays and go back in time. It is possible to reduce all classical time travel "paradoxes" into equivalent quantum computational networks. All the special properties of the network with negative time delays are consequences of the self-consistency conditions on the TT part [36].

Suppose the input is described by a density operator $\rho_{\text{IN}} = \Sigma_k w_k^{(\text{IN})} \left|\alpha_k\right\rangle \left\langle\alpha_k\right|$ where w_k^{IN} is the statistical weight for the input systems in the state $\left|\alpha_k\right\rangle$ in an ensemble. Similarly the time traversing part is given by $\rho_{\text{TT}} \Sigma_k w_k^{(\text{OUT})} \left|\beta_k\right\rangle \left\langle\beta_k\right|$. If ρ_{TT} is not correlated to ρ_{IN}, then the density operator $\rho(t=0)$ of the system (the input plus the TT part) as it enters the gate is given by

$$\rho(t=0) = \rho_{\mathrm{IN}} \otimes \rho_{\mathrm{TT}}. \tag{43}$$

Inside the gate the system goes through a unitary evolution represented by U. The evolution operator U depends on the type of interactions between the particles. However, what happens inside the nonchronal region is not completely determined by the initial conditions on the IN part. There is the TT part on a closed loop inside the nonchronal region and it is never observed in the past chronal region. In order to predict a unique evolution of the system by the time evolution operator U, "supplementary data" on the TT part has to be specified. As the system exits the gate its density operator has evolved into

$$\rho(t=\tau) = U\left(\rho_{\mathrm{IN}} \otimes \rho_{\mathrm{TT}}\right) U^{\dagger}. \tag{44}$$

The density operator ρ_{OUT} of the output can be found by taking a partial trace of $\rho(t=\tau)$ over the TT part,

$$\rho_{\mathrm{OUT}} = \mathrm{Tr}_{\mathrm{TT}}\left[\rho(t=\tau)\right]. \tag{45}$$

Similarly the density operator of the time traversing part before it enters the negative time delay is given by tracing over the OUT part,

$$\rho_{\mathrm{TT}} = \mathrm{Tr}_{\mathrm{OUT}}\left[\rho(t=\tau)\right]. \tag{46}$$

The negative time delay does not affect the state of the TT part, so its density operator after it comes out of the gate must be identical to the one that enters the gate according to the self-consistency principle. From Eqs. (44) and (45),

$$\rho_{\mathrm{OUT}} = \mathrm{Tr}_{\mathrm{TT}}\left[U\left(\rho_{\mathrm{IN}} \otimes \rho_{\mathrm{TT}}\right) U^{\dagger}\right]. \tag{47}$$

Notice that the self-consistency condition is imposed on the density operator, not on the state of each sample in the ensemble. Deutsch shows that the physical questions raised by the classical time travel paradoxes simply disappear when they are treated quantum mechanically as described above [36]. Also, the self-consistency condition does not allow us to dump entropy, defined as

$$S = -\mathrm{Tr}\left[\rho \log \rho\right], \tag{48}$$

into the nonchronal region, thus violating the second law of thermodynamics, except inside the nonchronal region where its meaning is unclear. However, it is possible that an input in a pure state could evolve into an output in a mixed state even if the combined OUT plus TT states can be fixed, and therefore coherence will be lost. In order to prevent new information from entering the system, unknown before it entered the nonchronal region but gained from the supplementary data inside, Deutsch assumes the maximum entropy rule: *The supplementary data is in the state of greatest entropy compatible with the initial data.* Then the system inside the nonchronal region will always be in the maximally mixed states. If the system is isolated from its environment, then it will eventually evolve into an equilibrium state, which is the state with maximum entropy.

CHAPTER 4

Classical Thermodynamic Processes in a Nonchronal Region

Works by Hawking, Hartle, and Deutsch [24, 35, 36] strongly suggests that there exists a strong relation between thermodynamic processes and CTC's. Hawking points out that a part of the quantum state circles around on CTC's and never leaves the nonchronal region causing a loss of coherence [35]. When an external field interacts with a heat reservoir, it loses quantum coherence since the reservoir itself is in a mixed quantum state. The only difference is that the system with CTC's has an imaginary temperature. Thus, it seems worthwhile to study thermodynamic processes inside the nonchronal region. A system traversing through the nonchronal region is expected be in a state of equilibrium. This is because in order to fix the system's evolution from the past chronal region through the nonchronal region to the future chronal region, a supplementary condition on the time traversing part of the system is required. According to Deutsch's maximum entropy hypothesis the time traversing (TT) part must be in the state of maximum entropy. Then the TT part must be in some state of thermodynamic equilibrium.

In this chapter, I will allow a box filled with an ideal gas to undergo various thermodynamic processes with its older self in a nonchronal region until they reach equilibrium. The self-consistency condition is imposed on the box which goes back in time. My calculation shows that the box's entropy never decreases as it traverses the nonchronal region.

Apparatus

Determining trajectories through a nonchronal region is nontrivial because the number of time traverses cannot be fixed by the initial conditions on the future chronology horizon. There could be an indefinite number of identical systems on CTC's in the nonchronal region due to multiple time traverses. There are two distinct trajectories even for one time traverse as shown in Figs. 13 and 14. For trajectory

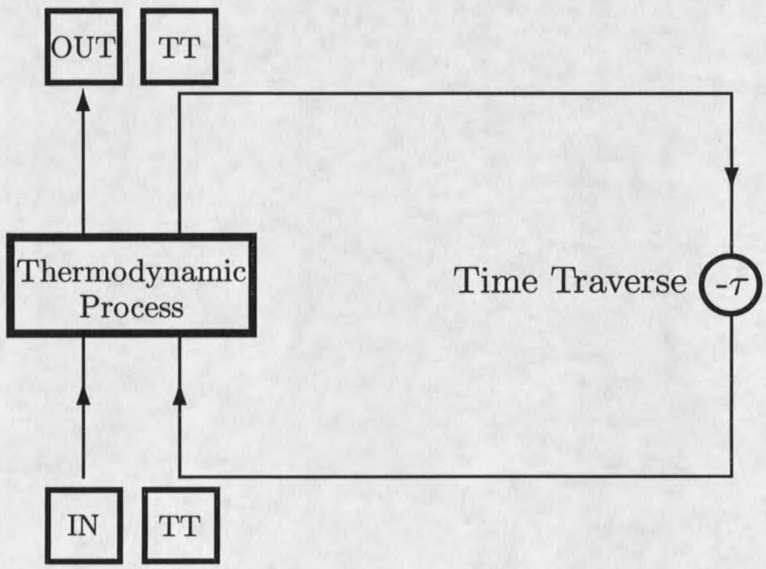
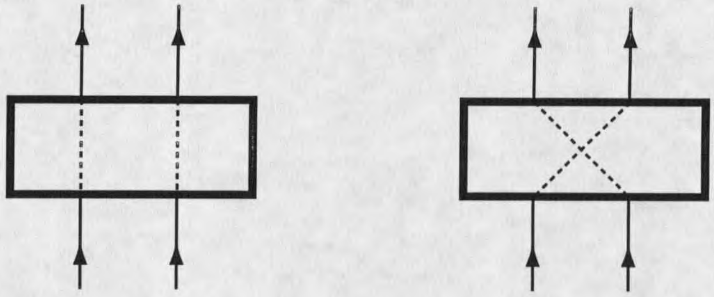


Figure 13: General trajectories of the box with one time traverse.



Trajectory 1 Trajectory 2

Figure 14: Possible trajectories of two boxes in the interaction region.

1, an IN box entering the nonchronal region encounters an identical but distinct time traversing (TT) box on a closed trajectory. In this case the IN box leaves the nonchronal region as an OUT box after the thermodynamic process without going back in time. On the other hand, for trajectory 2 the IN box after the process goes back in time and becomes the TT box and then leaves the nonchronal region as the OUT box. The two interacting boxes are the same box at different proper times: the younger self and the older self. For two time traverses, there are six distinct trajectories (See Figs. 15 and 16). The number of possible trajectories increases drastically as the number of time traverses increases. For a classical box it is $(n+1)!$ where n is the number of time traverses.

In quantum mechanics this permutation of trajectories in the interaction region is equivalent to the symmetrization (or antisymmetrization) of the state vector for a system consisting of indistinguishable boxes. To find distribution of the states of the OUT box, all allowed trajectories for all numbers of time traverses (from zero to infinity) must be summed over. In classical mechanics the boxes are distinguishable and their trajectories could be controlled by assuming an active observer who travels

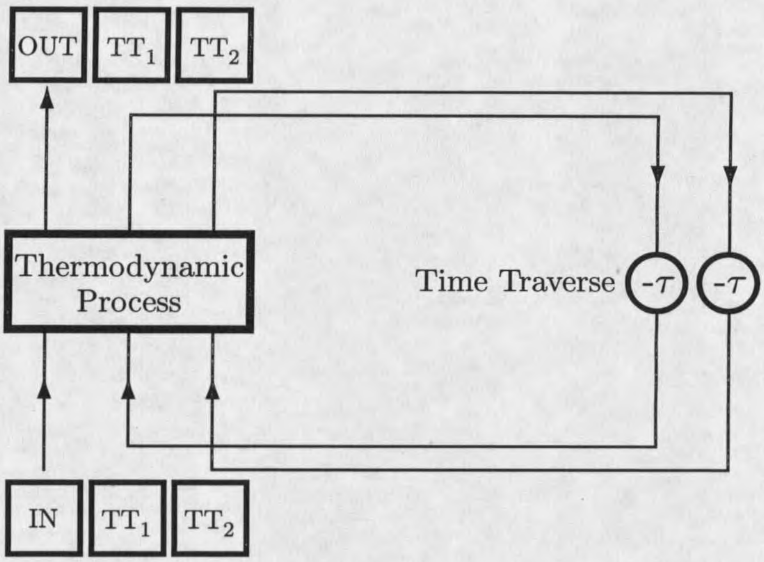


Figure 15: General trajectories of the box with two time traverses.

with the box through its entire traverse inside the nonchronal region. In this chapter only a classical box on trajectory 2 in Fig. 14 will be examined. (Thermodynamic processes for the trajectory 1 become trivial after the self-consistency condition is imposed.) According to the observer traveling with the box, the entire experiment looks as follows:

Step 1

A thermally insulated IN box filled with a monatomic ideal gas enters the nonchronal region from the past chronal region. The box does not interact with outside. In a sense, it is inert. Its thermal state is described by temperature ($T_{\rm IN}$) and volume ($V_{\rm IN}$).

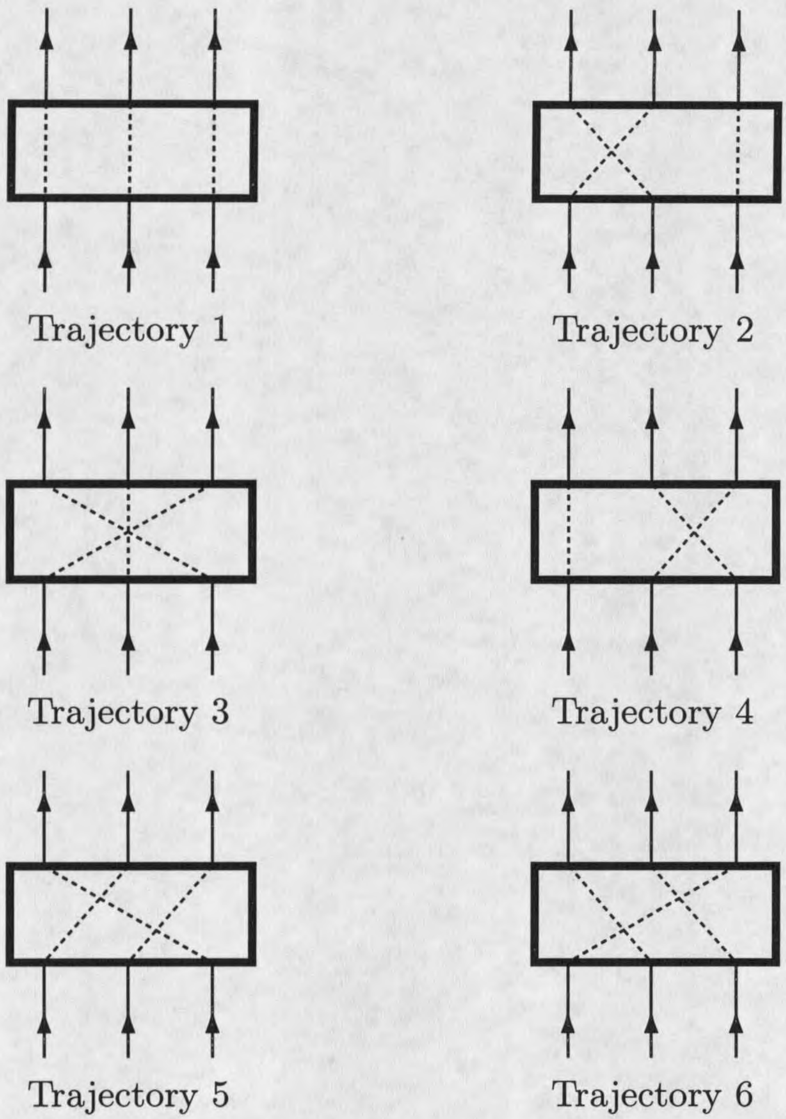


Figure 16: Possible trajectories of three boxes in the interaction region.

Step 2

The younger self (IN box) comes to a thermal contact with its older self (TT box). Depending on the process the boxes are allowed to exchange heat, do work on each other, or mix gases by removing a wall between them with another with an appropriate property. If the system (IN box plus TT box) is closed, it is insulated from the environment; if it is open, it can interact with the environment. (From here on the system refers to a collection of an object and all its older selves inside the nonchronal region, whether it is a classical box or a quantum particle.)

Step 3

After the process of Step 2 is over, the wall between becomes a fixed insulator and the boxes get separated. Again, they become inert. The younger box (according to the observer following the box) travels back in time in a state defined by $T_{\rm TT}$ and $V_{\rm TT}$ while the older box exits the chronal region to the future chronal region as an OUT box in a state $(T_{\rm OUT}, V_{\rm OUT})$.

Self-consistency condition

In order to avoid any causality violation, the self-consistency condition must be imposed on the time traversing part of the system. For this problem the condition is implemented as

$$\begin{pmatrix} \text{Thermodynamic state} \\ \text{of the younger box} \\ \text{after the interaction} \end{pmatrix} = \begin{pmatrix} \text{Thermodynamic state} \\ \text{of the older box} \\ \text{before the interaction} \end{pmatrix}. \tag{49}$$

The objective of this thought experiment is to find the thermal state of the OUT box for a given state of the IN box while satisfying the first law of thermodynamics, the ideal gas law, and above self-consistency condition. The processes which the boxes go through are not unusual: heat transfer, isothermal expansion, isobaric expansion, and mixing of the gases. These processes and their outcomes are well understood. Therefore, if the state of the OUT box is found be constrained by an unfamiliar rule, then it must be due to the self-consistency condition. It turns out that the self-consistency condition does not impose any extra rules, other than those by the first law and the ideal gas law, for closed systems. However, that is not true for open systems.

Closed Systems

In a closed system, the younger box and the older box interact only with each other. They are thermally insulated from the environment and their outside walls do not move. Only the wall between the boxes is allowed to transfer heat, to slide back and forth like a piston inside the boxes, or to be removed allowing the gases to mix.

Heat Transfer

During this process a wall between the younger and older boxes is fixed in place but allows heat conduction (see Fig. 17). If there is any temperature difference between

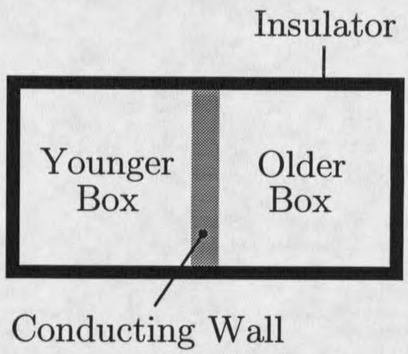


Figure 17: A wall between the boxes are allowed to conduct heat.

the boxes, there will be heat flow from the box with higher temperature to the one with lower temperature. The boundary wall remains conductive until the two boxes are at the same temperature. After separation the boxes are thermally shielded separately. The younger box goes back in time to meet the IN box, and the older box exits the nonchronal region as the OUT box.

Because the volume of each box is fixed, the change in the internal energy of the younger box is equal to heat transferred from the older box.

$$\Delta U_{\mathrm{y}} = \Delta Q_{\mathrm{y}}. \tag{50}$$

For a monatomic ideal gas, the internal energy U is equal to $\frac{3}{2}N_{\mathrm{y}}T_{\mathrm{y}}$ where N_{y} and T_{y}

are the number of particles and temperature of the younger box, respectively. Then,

$$\Delta Q_{\mathrm{y}} = \frac{3}{2} N_{\mathrm{y}} k_{\mathrm{B}} \Delta T_{\mathrm{y}}. \tag{51}$$

Similarly for the older box

$$\Delta Q_{\mathrm{o}} = \frac{3}{2} N_{\mathrm{o}} k_{\mathrm{B}} \Delta T_{\mathrm{o}}. \tag{52}$$

Since the system is isolated, the heat gained by the younger box ΔQ_{y} must be equal to the change in heat for the older box ΔQ_{o},

$$\begin{aligned} 0 &= \Delta Q_{\mathrm{y}} + \Delta Q_{\mathrm{o}} \\ &= \frac{3}{2} \left[N_{\mathrm{y}} k_{\mathrm{B}} \Delta T_{\mathrm{y}} + N_{\mathrm{o}} k_{\mathrm{B}} \Delta T_{\mathrm{o}} \right] \\ &= \frac{3}{2} k_{\mathrm{B}} \left[N_{\mathrm{y}} \left(T_{\mathrm{y}_f} - T_{\mathrm{y}_i} \right) + N_{\mathrm{o}} \left(T_{\mathrm{o}_f} - T_{\mathrm{o}_i} \right) \right]. \end{aligned} \tag{53}$$

The temperatures of the boxes in equilibrium are equal,

$$T_{\mathrm{y}_f} = T_{\mathrm{o}_f}. \tag{54}$$

From the self-consistency condition,

$$\begin{aligned} N_{\mathrm{y}} &= N_{\mathrm{o}}, \\ T_{\mathrm{y}_f} &= T_{\mathrm{o}_i}. \end{aligned} \tag{55}$$

Therefore,

$$T_{o_f} = T_{y_i}. \tag{56}$$

The temperature T_{OUT} of the OUT box and the temperature T_{IN} of the IN box are the same. The temperature of the box remains constant during its entire journey through the nonchronal region,

$$T = \text{constant}. \tag{57}$$

Since there is no work done on the boxes, a change in entropy for each box is given by

$$\begin{aligned} dS &= \frac{dQ}{T} = \frac{dU}{T} = \frac{3}{2} N k_{\text{B}} \frac{dT}{T}, \\ \Delta S &= \frac{3}{2} N k_{\text{B}} \log \left[\frac{T_{\text{f}}}{T_{\text{i}}} \right]. \end{aligned} \tag{58}$$

The final and initial temperatures of each box are the same, so there is no change in entropy for each box. Hence, the self-consistency condition requires that there will be no change in entropy between the IN box and the OUT box,

$$\Delta S = 0. \tag{59}$$

This means that the box's state does not change at all and that it is always in thermal equilibrium with its older self.

Adiabatic Expansion

After the IN box and the TT box are brought together the wall between them is allowed to slide back and forth while still keeping the two gases separate and insulated (Fig. 18). The outer walls remain insulating and immobile, so the total volume of

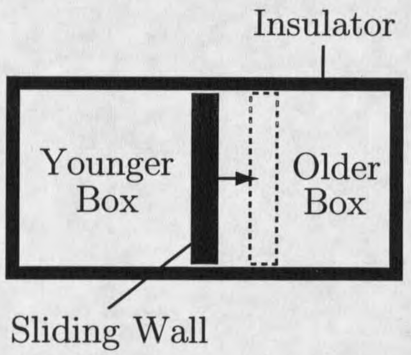


Figure 18: One of the box expands adiabatically while the other contracts adiabatically.

the system stays constant. If one box has a higher pressure than the other the wall moves toward the gas with lower pressure until the pressures become equal. If the wall move very quickly, then the gases will be in nonequilibrium states during the process.

When equilibrium is reached the boxes have the same temperature $T_{\mathrm{y}_f} = T_{\mathrm{o}_f}$ and

$$\begin{aligned}
\frac{3}{2}N_{\mathrm{y}}k_{\mathrm{B}}\left(T_{\mathrm{y}_f} - T_{\mathrm{y}_i}\right) &= \Delta Q_{\mathrm{y}} - \Delta W_{\mathrm{y}}, \\
\frac{3}{2}N_{\mathrm{o}}k_{\mathrm{B}}\left(T_{\mathrm{o}_f} - T_{\mathrm{o}_i}\right) &= \Delta Q_{\mathrm{o}} - \Delta W_{\mathrm{o}}. \qquad (60)
\end{aligned}$$

The sum of heat transfers between the boxes is equal to to zero, $\Delta Q_{\mathrm{y}} + \Delta Q_{\mathrm{o}} = 0$. Also, the works done on each other add up to zero, $\Delta W_{\mathrm{y}} + \Delta W_{\mathrm{o}} = 0$. In addition, the self-consistency condition requires that

$$N_{\mathrm{y}} = N_{\mathrm{o}},$$
$$T_{\mathrm{y}_f} = T_{\mathrm{o}_i},$$
$$V_{\mathrm{y}} = V_{\mathrm{o}}. \tag{61}$$

Then, the temperature of the box must be always equal to the initial temperature T_{IN}.

In the final equilibrium state the boxes have the same pressure so the wall between them would not move. From the ideal gas law, the IN box and OUT box must have the same volume, $V_{\mathrm{y}_f} = V_{\mathrm{o}_f}$ because their T, P, and N are the same. Using the self-consistency condition Eq. (61) and a fact that the total volume of the system does not change, it can be shown that the volumes of the boxes are all identical before and after the process. Therefore, the boxes are always in identical thermal states

$$T = \text{constant},$$
$$P = \text{constant}, \tag{62}$$

and the wall between the gases would not move. Moreover, there is no change in entropy since there is no change in states,

$$\Delta S = 0. \tag{63}$$

Mixing of the Gases

In this case the wall between the boxes is removed, and it is reinserted but not necessarily at the original location after the gases reach equilibrium,(Fig. 19). If there is a difference between the densities of the gases, they will quickly become equal in this process due to diffusion of the particles. Since each individual particle making up the gas is not tracked in this problem, it is possible for a particular particle to not time travel at all or to be on a CTC forever. Only the box's trajectory is exactly known.

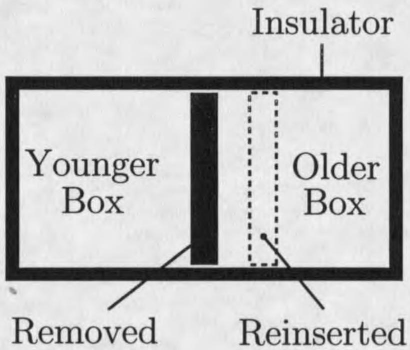


Figure 19: The wall between the wall is removed allowing gases to mix, then it is reinserted.

In mixing the gases inside a closed system, the total internal energy is conserved.

$$\frac{3}{2}N_{\mathrm{y}_i}k_{\mathrm{B}}T_{\mathrm{y}_i} + \frac{3}{2}N_{\mathrm{o}_i}k_{\mathrm{B}}T_{\mathrm{o}_i} = \frac{3}{2}N_{\mathrm{y}_f}k_{\mathrm{B}}T_{\mathrm{y}_f} + \frac{3}{2}N_{\mathrm{o}_f}k_{\mathrm{B}}T_{\mathrm{o}_f}. \tag{64}$$

Also, the total number of particles is conserved,

$$N_{y_i} + N_{o_i} = N_{y_f} + N_{o_f}. \tag{65}$$

When mixing is complete the gas has a uniform temperature, so $T_{y_f} = T_{o_f}$. The self-consistency condition requires only that $N_{y_f} = N_{o_i}$. Solving these equations gives $T_{y_i} = T_{o_f}$, so

$$T = \text{constant}. \tag{66}$$

Furthermore, it can be shown, from the self-consistency condition and from the fact that neither the total volume nor the total number of particles changes during the process, that $V_{\text{IN}} = V_{\text{OUT}}$ and $N_{\text{IN}} = N_{\text{OUT}}$. Furthermore, the pressure and density of the gas does not change at all. Since there isn't any difference in the density or pressure, the removal of the wall does not change the state of the gas. However, the volume and therefore the number of particles of the older box at the beginning of mixing remain undetermined. The time traversing box can have any volume, but the wall must be reinserted at a such position that the OUT box has the same volume as the IN box in order to satisfy the self-consistency condition.

Open Systems

An open system is allowed to interact with the environment. The only kind of interaction allowed in this study is the pressure-volume (P-V) type work. The outer wall of the older box is allowed to expand or contract. This restriction is imposed in order to keep the problem simple. It is possible to consider other kinds of interactions such as heat transfer between the system and the environment.

Isothermal Expansion

The wall between the boxes is turned into a conductor, allowing heat transfer, and the older gas expands at a constant temperature (see Fig. 20). In other words, the younger box is used as a heat reservoir from which energy is withdrawn by heat transfer, but it is not an effective one because its size is comparable to the older box. The total energy of the younger box noticeably changes as it loses energy to the older

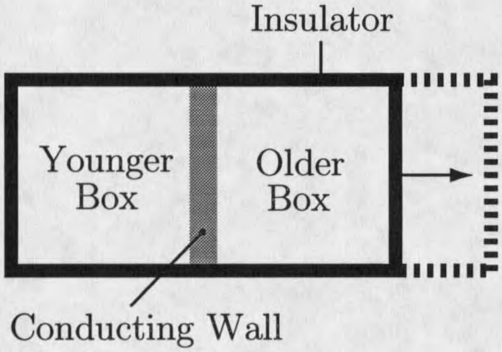


Figure 20: Heat from the younger box is used to expand the gas inside the older box at a constant temperature.

box. Also, the rate of the work done by the older box must be controlled by the observer so that the box remains at a constant temperature all the time.

For an isothermal process there is no change in the internal energy because the temperature of the ideal gas remains constant. Then from the first law of thermodynamics

$$\Delta Q_{\rm o} = W_{\rm o}. \tag{67}$$

The heat gained by the older box is equal to the heat lost by the younger box, $\Delta Q_{\rm o} = -\Delta Q_{\rm y}$. Then,

$$\frac{3}{2} N_{\rm y} k_{\rm B} \left(T_{{\rm y}_f} - T_{{\rm y}_i} \right) = N_{\rm o} k_{\rm B} T_{{\rm o}_i} \log \left[\frac{V_{{\rm o}_f}}{V_{{\rm o}_i}} \right]. \tag{68}$$

There is no mixing of the gas, so the number of particles in the box does not change.

$$N_{\rm y} = N_{\rm o}. \tag{69}$$

The younger box is used only as a reservoir and its volume does not change,

$$V_{{\rm y}_i} = V_{{\rm y}_f}. \tag{70}$$

By the self-consistency condition,

$$\begin{aligned} T_{{\rm y}_f} &= T_{{\rm o}_i}, \\ V_{{\rm y}_f} &= V_{{\rm o}_i}. \end{aligned} \tag{71}$$

Combining Eqs. (68, 69, 70, 71), and the isothermal condition $T_{o_i} = T_{o_f}$ gives

$$T_{\rm OUT} = \frac{T_{\rm IN}}{1 + \frac{2}{3}\log\left[\frac{V_{\rm OUT}}{V_{\rm IN}}\right]}. \tag{72}$$

The temperature of the OUT box ($T_{\rm OUT}$) can be less or greater than the temperature of the IN box ($T_{\rm IN}$) depending on how much work is done by the environment. Because the older box undergoes an isothermal process, its internal energy does not change. All work done by the older box on the environment comes from the younger box in the form of heat.

$$\Delta Q_{\rm y} = \frac{3}{2}N_{\rm y}k_{\rm B}\left(T_{\rm OUT} - T_{\rm IN}\right) = -N_{\rm y}k_{\rm B}T_{\rm OUT}\log\left[\frac{V_{\rm OUT}}{V_{\rm IN}}\right]. \tag{73}$$

The maximum heat that can be extracted from is $\frac{3}{2}Nk_{\rm B}T_{\rm OUT}$ when $T_{\rm IN} = 0$. Therefore, $V_{\rm OUT}$ cannot be less than $V_{\rm IN}e^{-\frac{3}{2}}$.

The younger box's volume does not change, so the change in its entropy is equal to

$$\Delta S_{\rm y} = \frac{3}{2}N_{\rm y}\log\left[\frac{T_{{\rm y}_f}}{T_{{\rm y}_i}}\right]. \tag{74}$$

On the other hand, the older box's temperature does not change, and its entropy change is given by

$$\Delta S_{\rm o} = N_{\rm y}\log\left[\frac{V_{{\rm o}_f}}{V_{{\rm o}_i}}\right]. \tag{75}$$

The change in entropy as the box traverses through the nonchronal region is the sum of ΔS_{y} and ΔS_{o}.

$$\Delta S = N\left\{\log\left[\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}}\right] - \frac{3}{2}\log\left[\frac{2}{3}\log\left[1+\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}}\right]\right]\right\}. \tag{76}$$

Entropy never decreases during this process. The change in entropy, ΔS, is zero only when there is no work done (Fig. 21). It is interesting that as $\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}}$ increases T_{OUT}

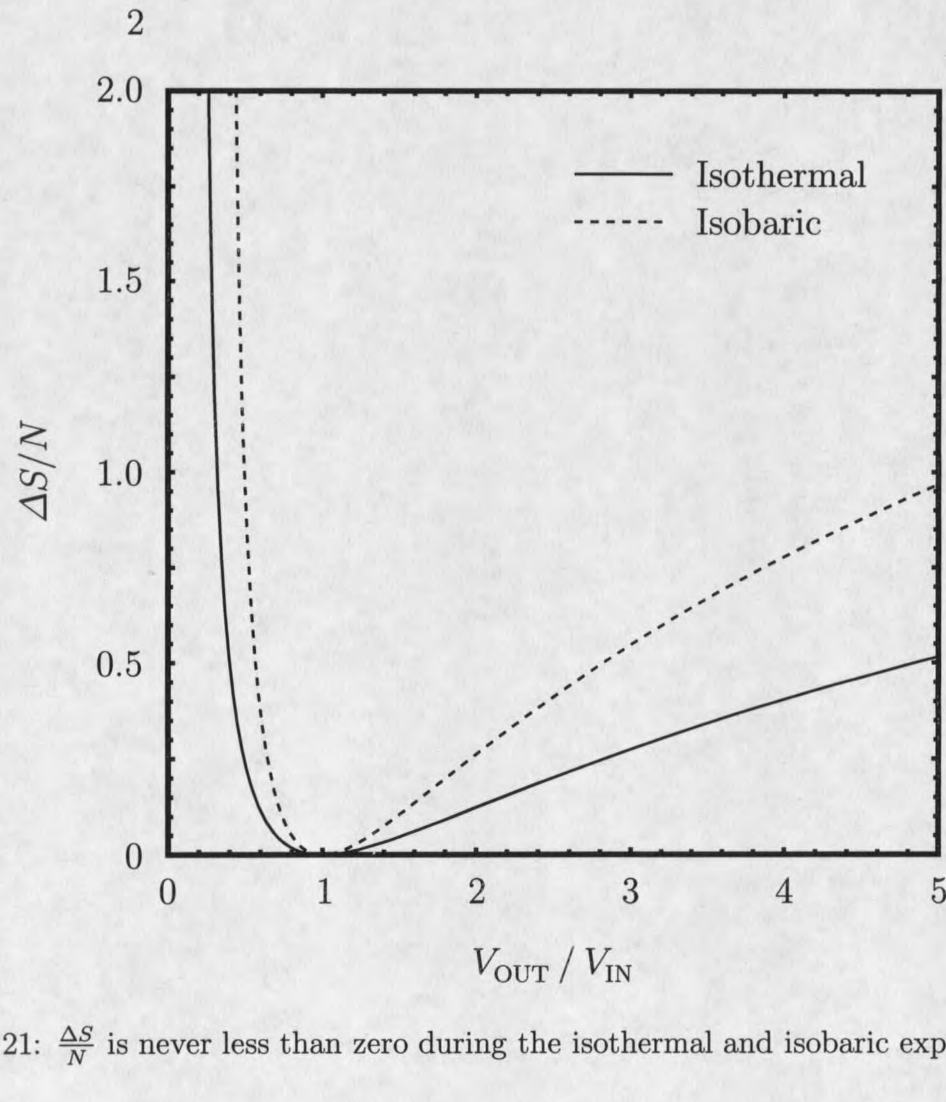


Figure 21: $\frac{\Delta S}{N}$ is never less than zero during the isothermal and isobaric expansions.

decreases toward zero, but the entropy keeps increasing. In fact, ΔS becomes infinite at $T_{\mathrm{OUT}} = 0$. This is because the volume of the OUT box becomes infinitely large at that point.

Isobaric Expansion

This process is identical to the isothermal expansion except the pressure, instead of the temperature, of the older box is kept constant by the observer (Fig. 22). The

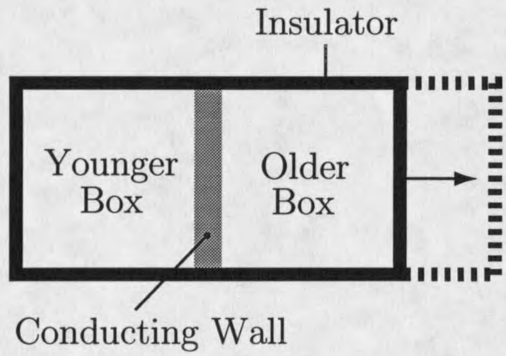


Figure 22: Isobaric expansion. A heat from the younger box is used to expand the gas inside the older box at a constant pressure.

younger box's volume is fixed, so by the first law of thermodynamics,

$$\frac{3}{2}N_{\mathrm{y}}k_{\mathrm{B}}\left(T_{\mathrm{y}_f} - T_{\mathrm{y}_i}\right) = \Delta Q_{\mathrm{y}}. \tag{77}$$

The older box expands at a constant pressure and performs work on the environment.

$$\frac{3}{2}N_{\mathrm{o}}k_{\mathrm{B}}\left(T_{\mathrm{o}_f} - T_{\mathrm{o}_i}\right) = \Delta Q_{\mathrm{o}} - P_{\mathrm{o}}\left(V_{\mathrm{o}_f} - V_{\mathrm{o}_i}\right) \tag{78}$$

The heat into the younger box is equal to the heat from the older box, $\Delta Q_{\mathrm{y}} = -\Delta Q_{\mathrm{o}}$, because the older box exchanges energy with the environment only through the P-V type work. By adding Eqs. (77) and (78),

$$\begin{aligned}&\frac{3}{2}N_{\mathrm{y}}k_{\mathrm{B}}\left(T_{\mathrm{y}_f}-T_{\mathrm{y}_i}\right)+\frac{3}{2}N_{\mathrm{o}}k_{\mathrm{B}}\left(T_{\mathrm{o}_f}-T_{\mathrm{o}_i}\right)\\&=-P_{\mathrm{o}}\left(V_{\mathrm{o}_f}-V_{\mathrm{o}_i}\right).\end{aligned} \tag{79}$$

With the self-consistency condition,

$$\begin{aligned}T_{\mathrm{y}_f} &= T_{\mathrm{o}_i},\\ V_{\mathrm{y}_f} &= V_{\mathrm{o}_i},\\ N_{\mathrm{y}_f} &= N_{\mathrm{o}_i}=N,\end{aligned} \tag{80}$$

and the constant volume for the younger box ($V_{\mathrm{y}_i} = V_{\mathrm{y}_f}$), Eq. (79) can be solved for $T_{\mathrm{OUT}} = T_{\mathrm{o}_f}$ in terms of the IN box's temperature $T_{\mathrm{IN}} = T_{\mathrm{y}_i}$ and $\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} = \frac{V_{\mathrm{o}_f}}{V_{\mathrm{y}_i}}$.

$$T_{\mathrm{OUT}} = \frac{3T_{\mathrm{IN}}}{5-2\frac{V_{\mathrm{IN}}}{V_{\mathrm{OUT}}}}. \tag{81}$$

T_{OUT} depends on how much work is done on the system by the environment, but for $V_{\mathrm{OUT}} \leq \frac{2}{5}V_{\mathrm{IN}}$, the temperature of the OUT box becomes negative. That means the work done on the environment exceeds the initial total energy of the system.

The entropy of the younger box is equal to

$$\Delta S_{\mathrm{y}} = \frac{3}{2} N k_{\mathrm{B}} \log \left[\frac{T_{\mathrm{y}_f}}{T_{\mathrm{o}_i}} \right] = \frac{3}{2} N k_{\mathrm{B}} \log \left[\frac{P_{\mathrm{y}_f} V_{\mathrm{y}_f}}{P_{\mathrm{o}_i} V_{\mathrm{o}_i}} \right]. \tag{82}$$

The older box's pressure does not change, so $P_{\mathrm{o}_i} = P_{\mathrm{o}_f}$; and the younger box's volume is fixed, so $V_{\mathrm{y}_i} = V_{\mathrm{y}_f}$. Then,

$$\Delta S_{\mathrm{y}} = \frac{3}{2} N k_{\mathrm{B}} \left\{ \log \left[\frac{T_{\mathrm{OUT}}}{T_{\mathrm{IN}}} \right] + \left[\frac{V_{\mathrm{IN}}}{V_{\mathrm{OUT}}} \right] \right\}. \tag{83}$$

For the older box,

$$\Delta S_{\mathrm{o}} = N k_{\mathrm{B}} \left\{ \log \left[\frac{3}{2} \frac{T_{\mathrm{o}_f}}{T_{\mathrm{o}_i}} \right] + \left[\frac{V_{\mathrm{o}_f}}{V_{\mathrm{o}_i}} \right] \right\} = N k_{\mathrm{B}} \left\{ \log \left[\frac{3}{2} \frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} \right] + \left[\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} \right] \right\}. \tag{84}$$

The change in the entropy of the box as it exits the nonchronal region is given by

$$\Delta S = N k_{\mathrm{B}} \left\{ \frac{3}{2} \log \left[\frac{3}{5 - 2\frac{V_{\mathrm{IN}}}{V_{\mathrm{OUT}}}} \right] + \log \left[\frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}} \right] \right\}. \tag{85}$$

The behavior of $\frac{\Delta S}{N}$ is similar to that of the isothermal process except it is always greater (see Fig. 21). The entropy always increases except when $V_{\mathrm{OUT}} = V_{\mathrm{IN}}$, then $\Delta S = 0$ because the state of the box does not change during the process.

In these simple classical thermodynamic processes inside a nonchronal region, I have shown that the second law of thermodynamics always holds if the self-consistency condition is imposed. The most obvious situation is the one in which nothing happens thermally; i. e., the state of the box never changes, so it is always in thermal equilibrium with its older self. This is always the case for the closed system. For open systems the OUT box can be in a different state than the IN box depending on how much work is done on the environment.

The analysis of classical thermodynamic processes inside the nonchronal region is far from complete. First, only a few types of processes have been examined. It might be possible to come up with a process such that entropy of the OUT box is less than that of the IN box as suggested by Deutsch (Ref. [36]). If this is true then the nonchronal region could be used as a place to dispose some of the entropy of the universe. Secondly, the environment is not monitored in this analysis. For open systems, there is an interaction between the system and the environment. In order to check the second law of thermodynamics for the universe, the entropy of the environment needs to be monitored. Also, the self-consistency condition must be imposed on the part of the environment that goes back in time. Finally, these processes should be examined at a quantum statistical level, replacing the macroscopic states of the gas with the density operator.

CHAPTER 5

Statistical Mechanics in a Nonchronal Region

This research is still in progress and only a general approach to the study of the statistical mechanics of particles in a nonchronal region is presented. First, consider the following model spacetime with CTC's. Start with a two-dimensional Minkowski spacetime and cut two spacelike slices of a finite length separated by τ in the time direction (see Fig. 23). The past side of the slice A and the future side of the slice B are identified. The future side of the slice A smoothly connects to the past side of the slice B. This part of the spacetime manifold is analogous to the wormhole throat. The spacetime is flat except at the endpoints of the slices where there are excess angles of 2π [41]. The future chronology horizons are formed by null geodesics originating from the end points of the slice A in the future direction. The generators of the past chronology horizon start from the end points of the slice B and extend in the past direction.

Formation of a Multiple-Self Gas

Suppose $n^{(\mathrm{IN})}$ identical particles cross the future chronology horizon from the past chronal region and enter the nonchronal region. They comprise the IN(put) part of