



Error involved in isolating one floor of a building frame for design purposes
by Pete Boyaci

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University
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Abstract:

This thesis presents an investigation of the errors made in calculating the design moments of the beams in a continuous frame, when this frame is analyzed in accordance with article 702 of the American Concrete Institute (A.C.I.) specifications which is followed in the designs of multiple story building frames.

In Part I of this thesis the analysis of the frame is based on the A.C.I. specifications where every floor is treated independently, as if it were a complete structure in itself. Thus, the design moments for the beams of the frame are determined.

In Part II the frame is analyzed as a single unit and the effects of all loading combinations are included in the design moments for the beams. The comparison of the moments obtained in Part I, where each floor is treated as an isolated unit, to the moments obtained in Part II, where the frame is treated as a unit, reveals the errors made when analysis is based on the A.C.I. specifications.

The results of the two parts show discrepancies up to twenty percent in the design moments. The frame proves to be underdesigned if analyzed according to the A.C.I. code.

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OF A BUILDING FRAME
FOR DESIGN PURPOSES

by

PETE BOYAGI

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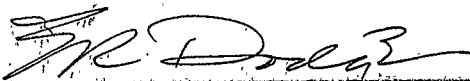
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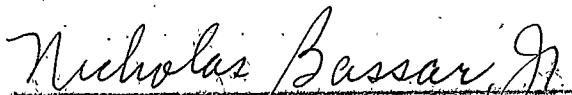
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Montana State College

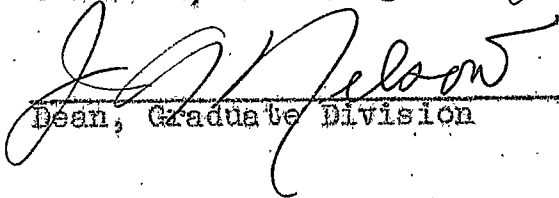
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Bozeman, Montana
June, 1952

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENT.	2
LIST OF TABLES.	5
ABSTRACT.	8
INTRODUCTION	
Object	9
History.	9
Importance	11
PROCEDURE	
Part I	
Specifications.	13
Outline of Procedure Followed	15
Design of Slabs	15
Design of Columns	18
Design of Beams	21
Stiffness Factors.	21
Distribution Factors	22
Fixed End Moments.	24
Moment Distribution.	25
Part II	
Analysis of the Frame as a Single Unit.	39
Approach.	39

TABLE OF CONTENTS

	Page
CONCLUSIONS	61
LITERATURE CITED AND CONSULTED.	64

LIST OF TABLES

TABLE NO.	DESCRIPTION	Page
I	Design of Roof Slabs S1 and S2 . . .	16
II	Design of Slabs S1 and S2 for Floors 1, 2 and 3	17
III	Design of Columns C1 through C8. . .	19
IV	Design of Columns C9 through C16 . .	20
V	Fixed End Moments for Maximum Condi- tions.	26
VI	Moment Distribution for Isolated Roof	28
VII	Moment Distribution for Isolated 3. Floor, Part a.	29
VIII	Moment Distribution for Isolated 3. Floor, Part b.	30
IX	Moment Distribution for Isolated 2. Floor, Part a.	31
X	Moment Distribution for Isolated 2. Floor, Part b.	32
XI	Moment Distribution for Isolated 1. Floor, Part a.	33
XII	Moment Distribution for Isolated 1. Floor, Part b.	34
XIII	Balanced Moments for Frame, Loaded Span AB.	40
XIV	Balanced Moments for Frame, Loaded Span BC.	41
XV	Balanced Moments for Frame, Loaded Span CD.	42

LIST OF TABLES

TABLE NO.	DESCRIPTION	Page
XVI	Balanced Moments for Frame, Loaded Span EF. * * * * *	43
XVII	Balanced Moments for Frame, Loaded Span FG. * * * * *	44
XVIII	Balanced Moments for Frame, Loaded Span GH. * * * * *	45
XIX	Balanced Moments for Frame, Loaded Span KL. * * * * *	46
XX	Balanced Moments for Frame, Loaded Span LM. * * * * *	47
XXI	Balanced Moments for Frame, Loaded Span MN. * * * * *	48
XXII	Balanced Moments for Frame, Loaded Span PR. * * * * *	49
XXIII	Balanced Moments for Frame, Loaded Span RS. * * * * *	50
XXIV	Balanced Moments for Frame, Loaded Span ST. * * * * *	51
XXV	Final Moments when Frame is Treated as a Single Unit * * * *	52
XXVI	Final Moments when Frame is Treated as a Single Unit * * * *	53
XXVII	Final Moments when Frame is Treated as a Single Unit * * * *	54
XXVIII	Final Moments when Frame is Treated as a Single Unit * * * *	55
XXIX	Final Moments when Frame is Treated as a Single Unit * * * *	56

LIST OF TABLES

TABLE NO.	DESCRIPTION	Page
XXX	End Moments for Maximum Positive Conditions * * * * *	57
XXXI	End Moments for Maximum Positive Conditions * * * * *	58
XXXII	Error in Isolating One Floor of a Frame * * * * *	59

ABSTRACT

This thesis presents an investigation of the errors made in calculating the design moments of the beams in a continuous frame, when this frame is analyzed in accordance with article 702 of the American Concrete Institute (A.C.I.) specifications which is followed in the designs of multiple story building frames.

In Part I of this thesis the analysis of the frame is based on the A.C.I. specifications where every floor is treated independently, as if it were a complete structure in itself. Thus, the design moments for the beams of the frame are determined.

In Part II the frame is analyzed as a single unit and the effects of all loading combinations are included in the design moments for the beams. The comparison of the moments obtained in Part I, where each floor is treated as an isolated unit, to the moments obtained in Part II, where the frame is treated as a unit, reveals the errors made when analysis is based on the A.C.I. specifications.

The results of the two parts show discrepancies up to twenty percent in the design moments. The frame proves to be underdesigned if analyzed according to the A.C.I. code.

INTRODUCTION

Object

The primary objective of this paper is to investigate the percentage error involved in isolating one floor of a continuous building frame and treating every floor independently. The application of the moment distribution concept provided the most direct and convenient method in determining this error.

History

The multiple story building has been utilized for many years in the building history, and was the most successful solution of the centralization problem and, at the same time, the best use of high-priced land.

History reveals that the building materials used, from the time our ancestors emerged from caves to the present, played definitely the most important role in construction. From the branches and leaves to structural steel and reinforced concrete, from art and experience to modern methods of analyzing a structure, the years that elapsed were a challenge to creative minds.

The building materials used thousands of years ago like stone, brick, wood and many others -- then in a primitive form -- are still used.

The development of the converter by Henry Bessemer

in England, and of the open-hearth furnace by William Siemens in America between 1860 and 1870, made possible the production of a material whose properties could be properly controlled and could be cast and rolled into desired shapes. The new material, known as steel, is one of the greatest achievements in building history. Steel has contributed a material that enables not only speed, strength, rigidity and lightness of erections, but also rapidity of demolition. Another material, equally important, was used by the Egyptians and Romans¹ early in the history of construction, in a massive form and in the nineteenth century reinforced with steel, was adapted as one of the major building materials. This material is known as "concrete", and with steel in the body of the concrete, known as "reinforced concrete".

As far as design is concerned, in the early years art and experience replaced computations. The Post and Lintel construction² that was started in Egypt and Persia became the most widely used method of construction. Later, the simple supported beam idea, based on the post and lintel started to grow. Multiple story buildings were

1. "Materials and Methods of Architectural Construction," by C. M. Gay and H. Parker, published by John Wiley and sons, Inc., N.Y. p. 27

2. Ibid p. 3

based on these simple supported beam principles and the statically determinate analysis.

In the year 1915, George A. Maney developed the widely known Slope Deflection Method which provided an economical design, compared to the previous ones, with the required theoretical analysis of the statically indeterminate frame. However, the tedious design, requiring the solution of numerous simultaneous equations, was too complicated compared to the analysis of determinate structures. A new technique, based on previous theories, like the Theory of Relaxation, was introduced by Hardy Cross in 1932. This new technique, known as "The Moment Distribution" was adopted by the engineers.

Then the American Concrete Institute, with numerous assumptions, made the analysis of a reinforced concrete frame comparatively shorter, and the A.I.S.C. reduced the work involved in steel design.

Importance

Article 702 of the A.C.I. specifications states "The live load may be considered to be applied only to the floor under consideration, and the far ends of the columns may be assumed as fixed."

The importance of this thesis is to investigate how much error is involved in the analysis of a frame if the

frame is designed according to this Article 702 of the specifications

PROCEDURE

Part I

A plan view of a continuous reinforced concrete frame is shown in Fig. 1. The section I-I of this frame is to be analyzed according to A.C.I. specifications.

Given loads:

Live load for Roof, 50 pounds per square foot

Live load for all other floors, 150 pounds per square foot

Specifications

The 28 day ultimate strength of concrete is 2500 pounds per square inch

Two way slabs are to be used in the analysis.

Outline of the Procedure followed

- A. The thickness of the slabs is determined
- B. Columns are designed
- C. Beams are designed.

In Fig. 2 all joints are lettered; columns and beams are numbered and they will be referred to in this manner.

Design of slabs

The design of slabs for roof and all other floors is shown in Tables I and II. The two governing factors that have to be given consideration in the design are as follows:

- a. The minimum thickness to satisfy deflection

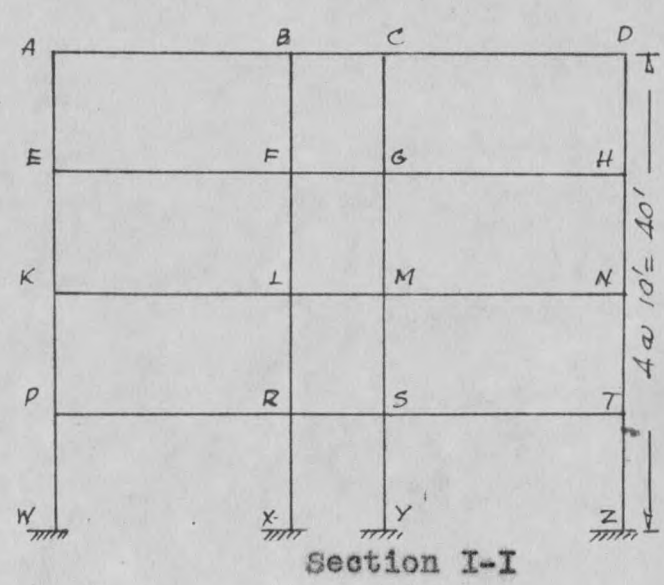
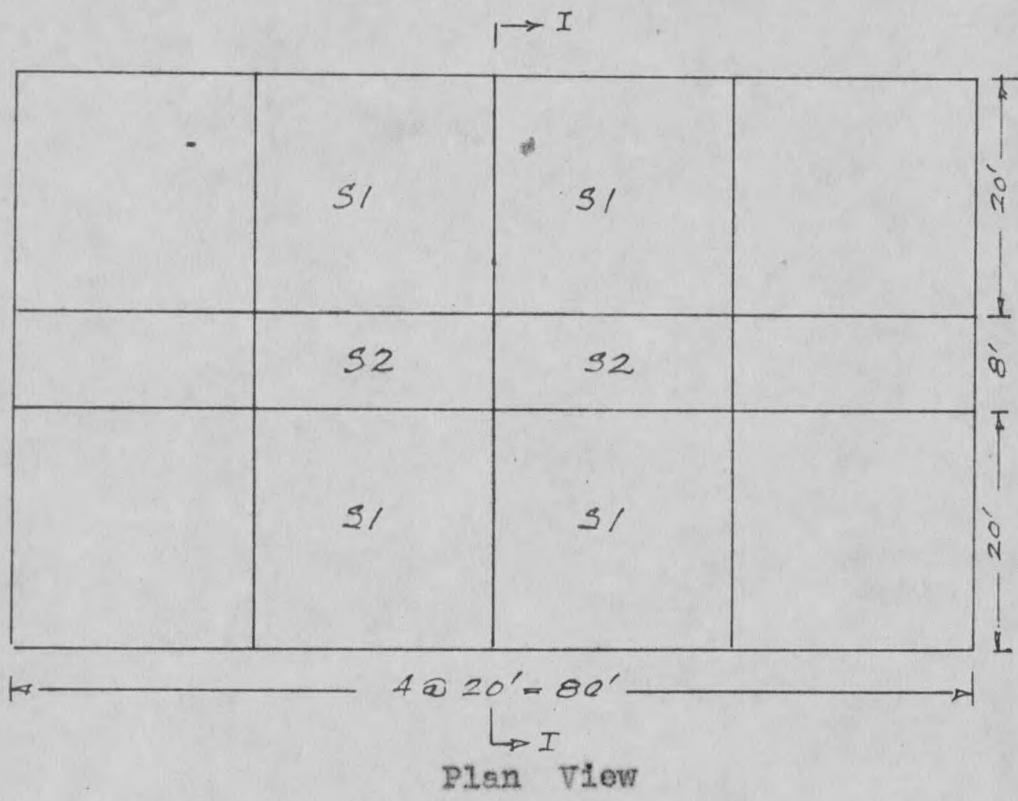


Fig. 1 Plan View and Section I-I of the Frame to be Analyzed.

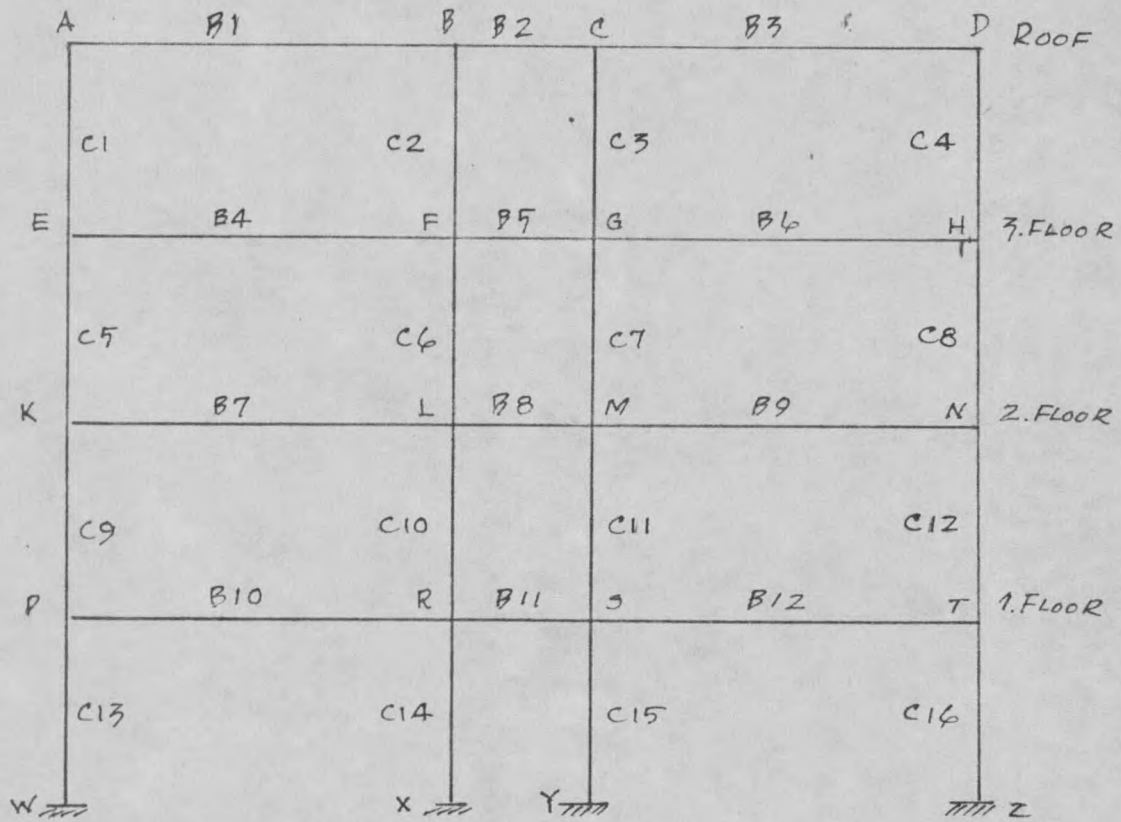


Fig. 2 Identification of the Members to be Analyzed.

TABLE I: DESIGN OF ROOF SLABS S1 & S2.

S1	S2
<p>L.L. = 50 psf D.L. = 70 psf assumed $W = 120 \quad S = 20' \quad m = 1$ $t = [20 + 20 - \frac{N}{10}] \frac{12}{72} \sqrt[3]{\frac{2500}{3000}}$ where $N = 60'$ $t = 5.33 \text{ in.}$</p>	<p>L.L. = 50 psf D.L. = 50 psf $W = 100 \quad S = 8 \quad m = .4$ $t = [8 + 20 - \frac{N}{10}] \frac{12}{72} \sqrt[3]{\frac{2700}{3000}}$ where $N = 56'$ $t = 3.54 \text{ in.}$</p>
<p>Moment coefficients C_1 coef. for Neg. Mom. at disc. Edge C_2 " " " " " Cont. " C_3 " " Pos. " " Midspan $M = CWS^2$ $M_1 = 12.10 \text{ in. kips}$ $M_2 = 23.40 \text{ " "}$ $M_3 = 17.80 \text{ " "}$ Design Mom = 16.47 in kips $d = \sqrt{\frac{M}{Kb}} = 2.42 \text{ in.}$ Use $t = 5.5 \text{ inches}$</p>	<p>Moment coefficients C_1 coef. for Neg. Mom. at Cont. Edges C_2 " " Pos " " Midspan $M = CWS^2$ $M_1 = 2.54 \text{ in. kips}$ $M_2 = 1.92 \text{ " "}$ Design Mom = 0.947 in. kips $d = \sqrt{\frac{M}{Kb}} = 1.85 \text{ in.}$ Use $t = 4 \text{ inches.}$</p>

TABLE II: DESIGN OF SLABS S1 & S2 FOR FLOORS 1, 2, & 3.

S1	S2
L.L. = 150 psf D.L. = 75 psf assumed $W = 229 \quad S = 20 \quad m = 1$ $t = 5.33''$	L.L. = 150 psf D.L. = 50 psf assumed $W = 200 \quad S = 8 \quad m = .4$ $t = 3.54 \text{ in.}$
Moment coefficients C_1 coef. for Neg. Mom. at disc. Edge C_2 " " " " " Cont. " C_3 " " Pos. " " Midspan $M = cws^2$ $M_1 = 23.20 \text{ in. kips}$ $M_2 = 44.50 \text{ " "}$ $M_3 = 34.50 \text{ " "}$ Design Mom = 51,360 in. pou. $d = \sqrt{\frac{M}{Kb}} = 3.33 \text{ in.}$ Use $t = 5.5 \text{ in.}$	Moment coefficients C_1 coef. for Neg. Mom. at cont. edge C_2 " " Pos " " Midspan $M = cws^2$ $M_1 = 5.08 \text{ in. kips}$ $M_2 = 3.84 \text{ " "}$ Design Mom = 18,220 in. pou. $d = \sqrt{\frac{M}{Kb}} = 2.52 \text{ in.}$ Use $t = 4 \text{ in.}$

requirements of the specifications

- b. The minimum thickness to satisfy the design moment requirements.

The design moments shown in the tables for the slabs are obtained in accordance with the specifications. The maximum moments for two adjacent slabs are computed based on the moment coefficients given by the A.C.I. code.

These moments are balanced assuming that the supporting beam between the two adjacent slabs takes one-third of the unbalanced moment and the remaining two-thirds balanced based on the stiffness of the slabs.

Design of Columns:

With the determined thickness of slabs a very close approximation of the loads on the columns of the frame is made. These loads and the design of the columns are shown in Tables III and IV. All columns are designed based on axial loading conditions. The dimensions of columns C1 through C4 are determined by the 120 square inches minimum area requirement of the specifications. The dimensions of columns C5 through C16 after being determined, based on the loads, are arbitrarily increased two inches in each direction to satisfy the bending moments. The design, though not a thorough one, is satisfactory for the purpose. The mere object being to determine the stiffness of the columns needed in the analysis.

TABLE III : DESIGN OF COLUMNS C1 - C8																							
<p><u>Loads on C1</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Roof slab S1</td> <td style="text-align: right;">24 K</td> </tr> <tr> <td>Assumed B1</td> <td style="text-align: right;">1.5</td> </tr> <tr> <td>Assumed girders C1</td> <td style="text-align: right;">3.0</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">1.5</td> </tr> <tr> <td></td> <td style="text-align: right;">30.0 K</td> </tr> </table> <p>to have short column $\frac{h}{t} \leq 10 \quad \frac{10 \times 12}{10} = 12 \quad t \geq 12''$ for short assumed $p = .01$ $N = A [.18f'_c + .72f_{yp}] = 700A$ Required $A = 42$ sq. in Min. Col. dimensions $10'' \times 12''$ $N_L = N (1.7 - .03 \frac{h}{t})$ $N = 32$ K short col. Required $A = 32 \div 0.70 = 43$ sq. in C1 identical to C4 Use $10'' \times 12''$ for C1 and C4</p>	Roof slab S1	24 K	Assumed B1	1.5	Assumed girders C1	3.0		1.5		30.0 K	<p><u>Loads on C5</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">1. Floor slab S1</td> <td style="text-align: right;">45 K</td> </tr> <tr> <td>Assumed B4</td> <td style="text-align: right;">2</td> </tr> <tr> <td>Assumed girders C5</td> <td style="text-align: right;">4</td> </tr> <tr> <td>Loads for C5</td> <td style="text-align: right;">30</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">2</td> </tr> <tr> <td></td> <td style="text-align: right;">87 K</td> </tr> </table> <p>Assumed $p = .01$ try 10×12 col Carrying cap. $700A = 84$ C8 identical to C5 dimensions increased 2 inches in both directions to account for bending Mom. Use $12'' \times 14''$ for C5 & C8</p>	1. Floor slab S1	45 K	Assumed B4	2	Assumed girders C5	4	Loads for C5	30		2		87 K
Roof slab S1	24 K																						
Assumed B1	1.5																						
Assumed girders C1	3.0																						
	1.5																						
	30.0 K																						
1. Floor slab S1	45 K																						
Assumed B4	2																						
Assumed girders C5	4																						
Loads for C5	30																						
	2																						
	87 K																						
<p><u>Loads on C2</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Roof slabs S1 & S2</td> <td style="text-align: right;">32 K</td> </tr> <tr> <td>Assumed girders C2</td> <td style="text-align: right;">3</td> </tr> <tr> <td>Assumed B1 & B2</td> <td style="text-align: right;">1.5</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">1.5</td> </tr> <tr> <td></td> <td style="text-align: right;">38 K</td> </tr> </table> <p>C3 identical to C2 Min Area required by Specs. 120 Use $10'' \times 12''$ for C2 & C3</p>	Roof slabs S1 & S2	32 K	Assumed girders C2	3	Assumed B1 & B2	1.5		1.5		38 K	<p><u>Loads on C6</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">1. Floor slabs S1 & S2</td> <td style="text-align: right;">61 K</td> </tr> <tr> <td>Assumed B4</td> <td style="text-align: right;">2</td> </tr> <tr> <td>Assumed girders C6</td> <td style="text-align: right;">4</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">2</td> </tr> <tr> <td>Load for C6</td> <td style="text-align: right;">38</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">107 K.</td> </tr> </table> <p>Try 10×12 Long Col. $N = \frac{107}{.94} = 114$ K Assumed $p = .025$ Required $A = 114 \div 0.94 = 121$ in² C7 identical to C6 Use $12'' \times 14''$ for C6 & C7</p>	1. Floor slabs S1 & S2	61 K	Assumed B4	2	Assumed girders C6	4		2	Load for C6	38		107 K.
Roof slabs S1 & S2	32 K																						
Assumed girders C2	3																						
Assumed B1 & B2	1.5																						
	1.5																						
	38 K																						
1. Floor slabs S1 & S2	61 K																						
Assumed B4	2																						
Assumed girders C6	4																						
	2																						
Load for C6	38																						
	107 K.																						

TABLE IV: DESIGN OF COLUMNS C9 - C16													
<p><u>Loads on C9</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Loads for C5</td> <td style="text-align: right;">83^K</td> </tr> <tr> <td>Loads from 2. Floor</td> <td style="text-align: right;">53</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">136^K.</td> </tr> </table> <p>Assume $p = .021$ Assume short Column. $N = A (540 + 336)$ Required $A = 155$ sq. in. C12 identical to C9 Use 14" x 15" for C9 and C12</p>	Loads for C5	83 ^K	Loads from 2. Floor	53		136 ^K .	<p><u>Loads on C13</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Loads for C9</td> <td style="text-align: right;">136^K</td> </tr> <tr> <td>Loads from 1. Floor</td> <td style="text-align: right;">53</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">189^K.</td> </tr> </table> <p>Assume $p = .0225$ Assume short Column $N = A (540 + 360)$ Required $A = 210$ sq. in. C16 identical to C13 Use 16" x 17" for C13 & C16</p>	Loads for C9	136 ^K	Loads from 1. Floor	53		189 ^K .
Loads for C5	83 ^K												
Loads from 2. Floor	53												
	136 ^K .												
Loads for C9	136 ^K												
Loads from 1. Floor	53												
	189 ^K .												
<p><u>Loads on C10</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Loads for C6</td> <td style="text-align: right;">69^K</td> </tr> <tr> <td>Loads from 2. Floor</td> <td style="text-align: right;">107</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">176^K</td> </tr> </table> <p>Assume $p = .025$ Assume short column $N = A (540 + 400)$ Required $A = 187$ sq. in. C11 identical to C10 Use 15" x 17" for C10 & C11</p>	Loads for C6	69 ^K	Loads from 2. Floor	107		176 ^K	<p><u>Loads on C14</u></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Loads for C10</td> <td style="text-align: right;">176^K</td> </tr> <tr> <td>Loads from 1. Floor</td> <td style="text-align: right;">69</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">245^K</td> </tr> </table> <p>Assume $p = .0253$ Assume short Column $N = A (540 + 405)$ Required $A = 255$ sq. in. C15 identical to C14 Use 17" x 19 for C14 & C15</p>	Loads for C10	176 ^K	Loads from 1. Floor	69		245 ^K
Loads for C6	69 ^K												
Loads from 2. Floor	107												
	176 ^K												
Loads for C10	176 ^K												
Loads from 1. Floor	69												
	245 ^K												

Design of Beams:

There is no direct method by which the dimensions of the beams can be determined. The best and most convenient one is to assume the dimensions and, after the analysis is complete, check the assumed dimensions. With the following tentative beam dimensions in inches

B1 and B3	10 x 18
B2	8 x 15
B4, B6, B7, B9, B10 and B12	12 x 24
B5, B8 and B11	10 x 14

The analysis is made by Moment Distribution applied to isolated floors. The information needed is as follows:

- A. Stiffness factors for all members
- B. Distribution factors for all joints
- C. Fixed end moments for all beams.

Stiffness factors:

The stiffness factor, commonly known as k-value, is the ratio of the moment of inertia to the length of the member. Thus k for member AB equals

$$k_{AB} = I_{AB}/L_{AB} = \frac{1}{12}bh^3/L = \frac{1}{12} 10 \times 18^3/20 \times 12 = 20.2 \text{ in}^5$$

The stiffness factor for all the members are given below, obtained as the stiffness factor for AB was computed in the example

<u>Member</u>	<u>k</u>
B1 and B3	20.2
B2	25.1
C1, C2, C3 and C4	12
B4, B6, B7, B9, B10 and B12	67
B5, B8, and B11	25.8
C5, C6, C7 and C8	22.8
C9 and C12	32.6
C10 and C11	51.2
C13 and C16	54.5
C14 and C15	81.0

Distribution factors:

Based on the stiffness factor k obtained in the preceding paragraph, the distribution factors for all members at all joints are determined as follows:

The distribution factors for the member B4 at the joint E is $\frac{k_{B4}}{\sum k}$ where $\sum k$ represents the sum of the k values for all members meeting at the joint, or the distribution factor B4 at joint E is $\frac{67}{12 + 22.8 + 67} = 66\%$.

In a similar way, distribution factors for all members at all joints were determined and are given below:

<u>Joint</u>	<u>Member</u>	<u>Joint</u>	<u>Member</u>	<u>D.F.</u>
A	C1	D	C4	37%
A	B1	D	B5	63%
B	B1	C	B3	56%
B	B2	C	B2	42%
B	C2	C	C3	22%
E	C1	H	C4	12%
E	C5	H	C8	22%
E	B4	H	B6	66%
F	B4	G	B6	53%
F	C2	G	C3	10%
F	C6	G	C7	18%
F	B5	G	B5	19%
K	C5	N	C8	19%
K	C9	N	C12	26%
K	B7	N	B9	55%
L	B7	M	B9	41%
L	C6	M	C7	14%
L	C10	M	C11	31%
L	B8	M	B8	14%
P	C9	T	C12	21%
P	C13	T	C16	35%
P	B10	T	B12	44%
R	B10	S	B12	30%
R	C10	S	C11	23%
R	C14	S	C15	36%
R	B11	S	B11	11%

To simplify the moment distribution process when symmetrical loading conditions exist and since the floors are "symmetrical"³ another set of distribution factors are obtained for the interior joints. The exterior joints do not differ at all.

3. "Theory of Modern Steel Structures," Vol. II, by L. E. Grinter, published by the Macmillan Co., N.Y. pp.116-117.

<u>Joint</u>	<u>Member</u>	<u>Joint</u>	<u>Member</u>	<u>D.F.</u>
B	B1	C	B5	46%
B	C2	C	C3	27%
B	B2	C	B2	12%
F	B4	G	B6	59%
F	C2	G	C3	11%
F	C6	G	C7	20%
F	B5	G	B5	10%
L	B7	M	B9	44%
L	C6	M	C7	15%
L	C10	M	C11	35%
L	B8	M	B8	8%
R	B10	S	B12	32%
R	C10	S	C11	24%
R	C14	S	C15	38%
R	B11	S	B11	6%

Fixed End Moments⁴

There are three parts combined to give the total fixed end moments

- a. Fixed End moment due to dead load of the beam
- b. Fixed End moment due to dead load of slab per A.C.I. specifications article 813.
- c. Fixed End moment due to live load on the floor per A.C.I. specifications article 813.

Fixed end moments for roof member will always include all three parts as live load cannot be omitted.

4. "Fixed End Moment Coefficients for Beams of Uniform Cross Section," by L. F. Oesterling, Civil Engineering Vol. 4, No. 12, pp. 648-649.

For all other floor members the fixed end moments will include part 'a' and 'b', but part 'c' will be included or omitted depending on the loading conditions chosen to produce desired Maximum Moments at different parts of the structure. These three parts are computed and combined to give fixed end moments for all members in Table V.

Moment Distributions

To obtain the design moments for the beams, the fixed end moments are distributed in Tables VI through XII.

For every isolated floor, shown in Fig. 3, except for the roof where live load cannot be omitted from any span, two operations of balancing moments are performed. It is important to note that from now on moments producing compression at the top and tension at the bottom of a beam will be referred to as positive moments, while moments producing tension at the top and compression at the bottom of a beam will be referred to as negative moments.

The sign convention followed in balancing moments is arbitrarily chosen. A clockwise moment is given a '+' sign, and a counterclockwise moment, a '-' sign. These signs will be of help in determining whether a moment is positive or negative, referring to the above paragraph.

In part 'a' of moment distributions, the loading conditions are so chosen to give maximum positive moments

TABLE V : FIXED END MOMENTS FOR MAXIMUM CONDITIONS

F.E.M. for B1

$$a. \frac{1}{12} w l^2 = \frac{1}{12} 188 \times 20^2 \times 12 = 75.00 \text{ inch-kips}$$

$$b. \frac{5}{96} W l^2 = \frac{5}{96} 65 \times 20 \times 20^2 \times 12 = 325.00$$

$$c. \frac{5}{96} W l^2 = \frac{5}{96} 50 \times 20 \times 20^2 \times 12 = \underline{250.00}$$

$$\text{F.E.M} = 650.00 \text{ in-kips}$$

F.E.M for B2

$$a. \frac{1}{12} w l^2 = \frac{1}{12} 125 \times 8^2 \times 12 = 8.00 \text{ in-kips}$$

$$b. \frac{5}{96} W l^2 = \frac{5}{96} 65 \times 8 \times 8^2 \times 12 = 20.80$$

$$c. \frac{5}{96} W l^2 = \frac{5}{96} 50 \times 8 \times 8^2 \times 12 = \underline{16.00}$$

$$\text{Assumed F.E.M} = 50.00 \text{ in-kips}$$

F.E.M for B4

$$a. \frac{1}{12} w l^2 = \frac{1}{12} 250 \times 20^2 \times 12 = 100.00 \text{ in-kip. } a$$

$$b. \frac{5}{96} W l^2 = \frac{5}{96} 65 \times 20 \times 20^2 \times 12 = 325.00$$

$$c. \frac{5}{96} W l^2 = \frac{5}{96} 150 \times 20 \times 20^2 \times 12 = \underline{750.00}$$

$$\begin{array}{r} a \\ + b \\ \hline 435.00 \text{ IN.K.} \end{array}$$

$$\text{F.E.M.} = 1175.00 \text{ in-kips}$$

F.E.M for B5

$$a. \frac{1}{12} w l^2 = \frac{1}{12} 120 \times 8^2 \times 12 = 7.70 \text{ in-kips } a$$

$$b. \frac{5}{96} W l^2 = \frac{5}{96} 65 \times 8 \times 8^2 \times 12 = 20.80$$

$$c. \frac{5}{96} W l^2 = \frac{5}{96} 150 \times 8 \times 8^2 \times 12 = \underline{47.00}$$

$$\begin{array}{r} a \\ + b \\ \hline 40.00 \text{ assumed} \end{array}$$

$$\text{Assumed F.E.M} = 80.00 \text{ in.kips}$$

F.E.M for B7 and B10 same as F.E.M for B4

F.E.M for B8 and B11 same as F.E.M for B5

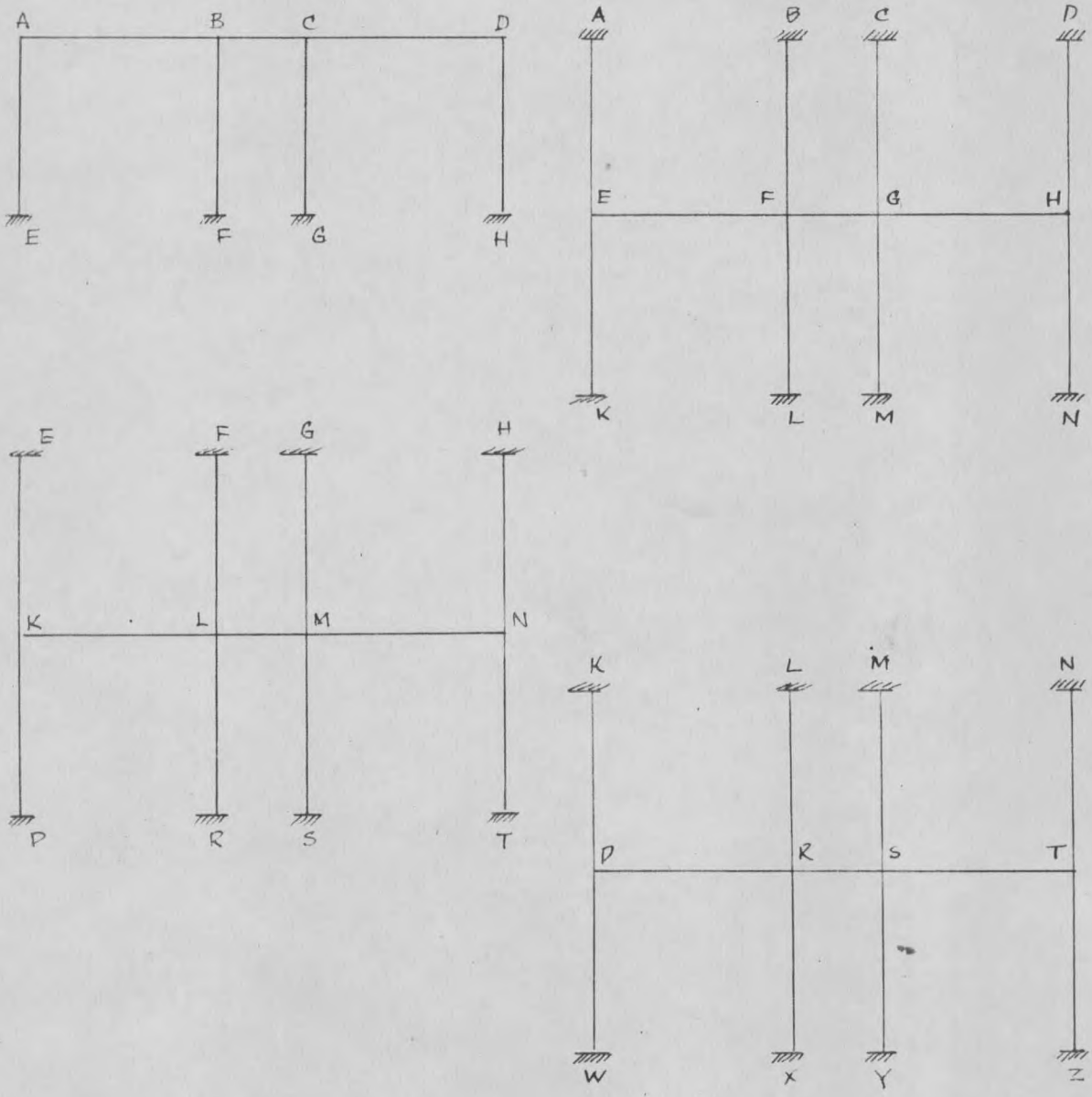


Fig. 3 Isolated Floors for Design Purposes

TABLE VI : MOMENT DISTRIBUTION FOR ISOLATED ROOF

AE	AB		BA	BF	BC
37	63		46	27	27
	A				B
	-650		+650		-50
	-138		-276	-162	-162
+292	+496		+248		
	-57		-114	-77	-77
+21	+36		+18		
	-4		-8	-5	-5
+1	+3		<u>-8</u>	<u>-5</u>	<u>-5</u>
<u>+314</u>	<u>-314</u>		+528	-244	-294
E			F		

TABLE VII: MOMENT DISTRIBUTION FOR ISOLATED 3. FLOOR.

PART α:

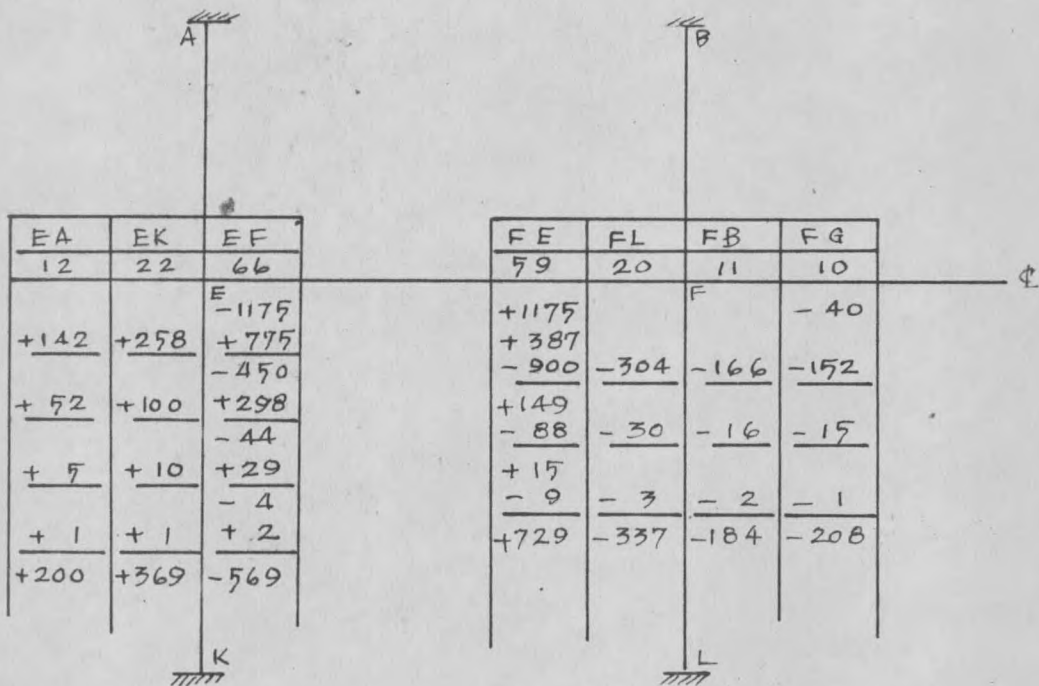
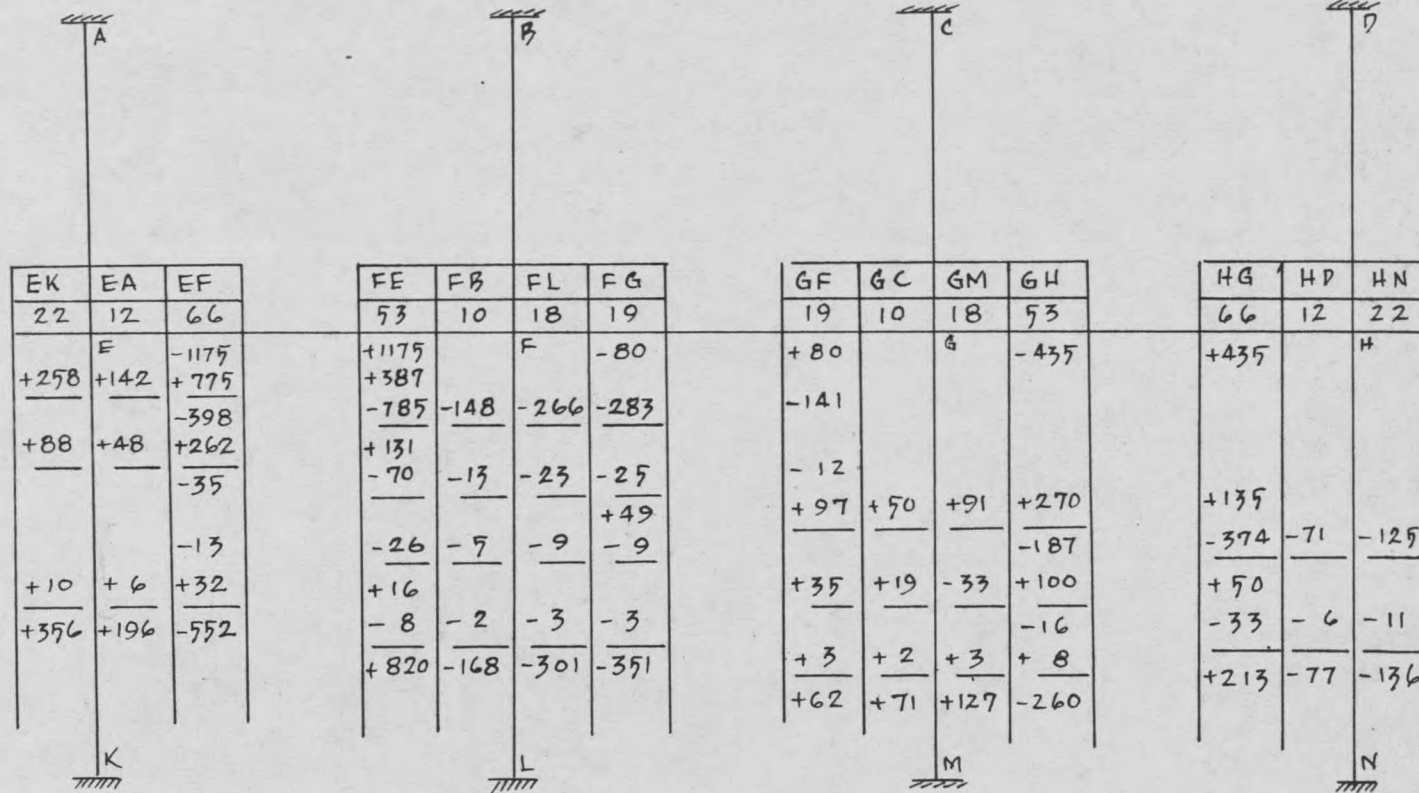


TABLE VIII: MOMENT DISTRIBUTION FOR ISOLATED 3. FLOOR.

PART B:



EK	EA	EF
22	12	66
	E	-1175
+258	+142	+775
		-398
+88	+48	+262
		-35
		-13
+10	+6	+32
+356	+196	-552

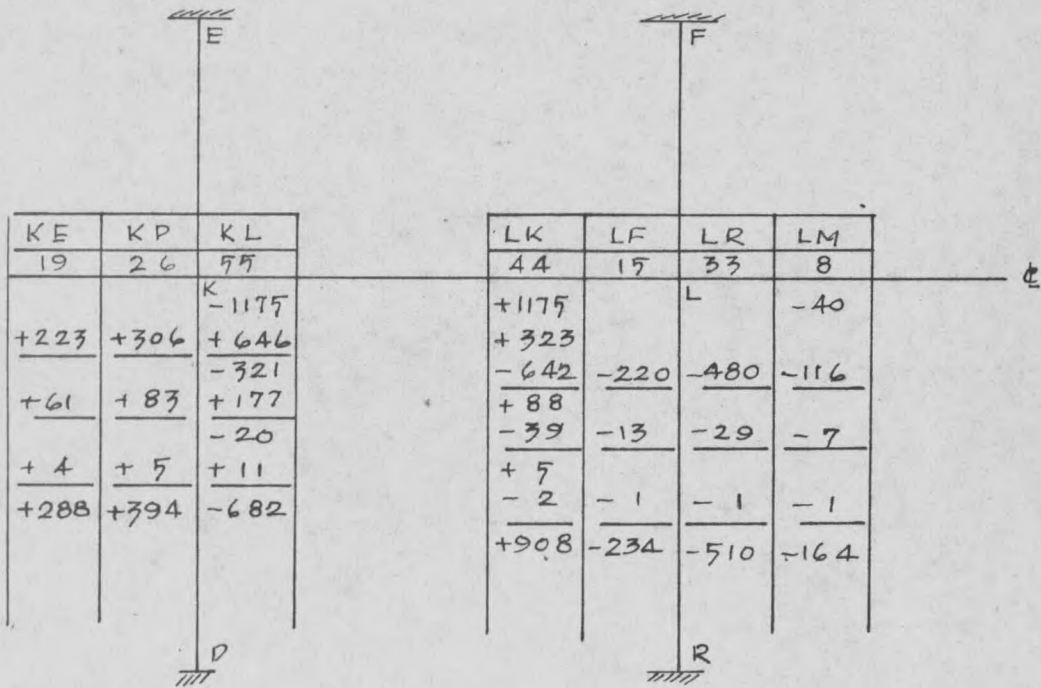
FE	FB	FL	FG
53	10	18	19
		F	-80
+1175			
+387			
-785	-148	-266	-283
+131			
-70	-13	-23	-25
			+49
-26	-7	-9	-9
+16			
-8	-2	-3	-3
+820	-168	-301	-351

GF	GC	GM	GH
19	10	18	53
		G	-435
+80			
-141			
-12			
+97	+50	+91	+270
			-187
+35	+19	-33	+100
			-16
+3	+2	+3	+8
+62	+71	+127	-260

HG	HD	HN
66	12	22
		H
+435		
+135		
-374	-71	-125
+50		
-33	-6	-11
+213	-77	-136

TABLE IX : MOMENT DISTRIBUTION FOR ISOLATED 2. FLOOR

PART α :



KE	KP	KL
19	26	55
+223	+306	K -1175
+61	+83	+646
+4	+5	-321
+288	+394	+177
		-20
		+11
		-682

LK	LF	LR	LM
44	15	33	8
+1175		L	-40
+323			
-642	-220	-480	-116
+88			
-39	-13	-29	-7
+5			
-2	-1	-1	-1
+908	-234	-510	-164

TABLE XI : MOMENT DISTRIBUTION FOR ISOLATED 1. FLOOR

PART α:

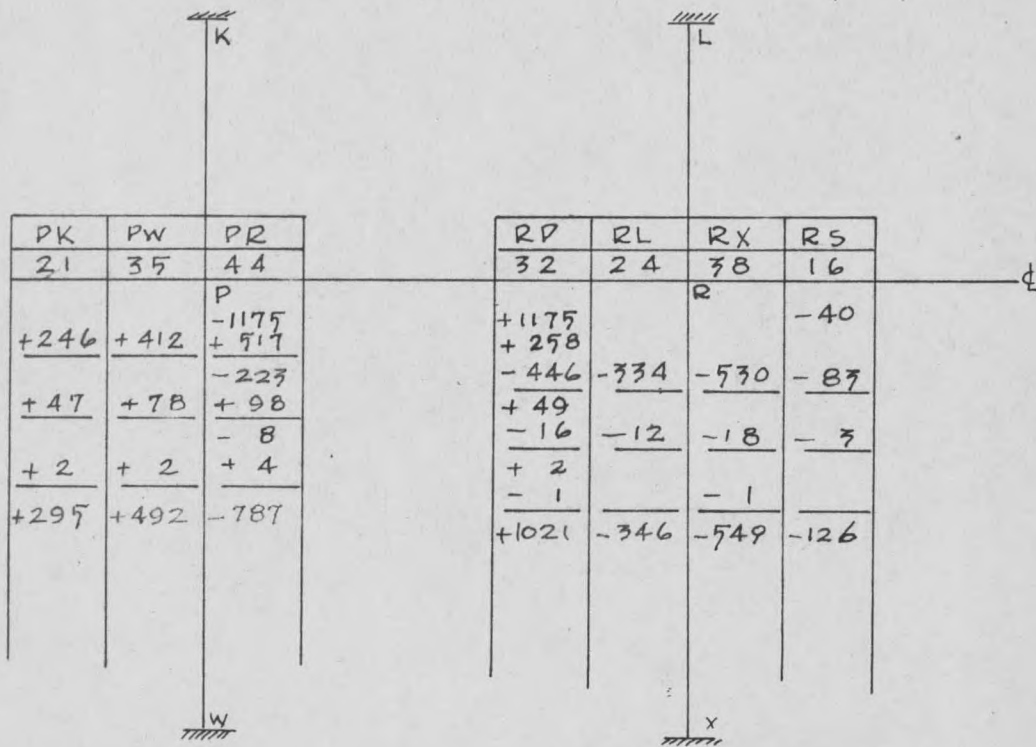
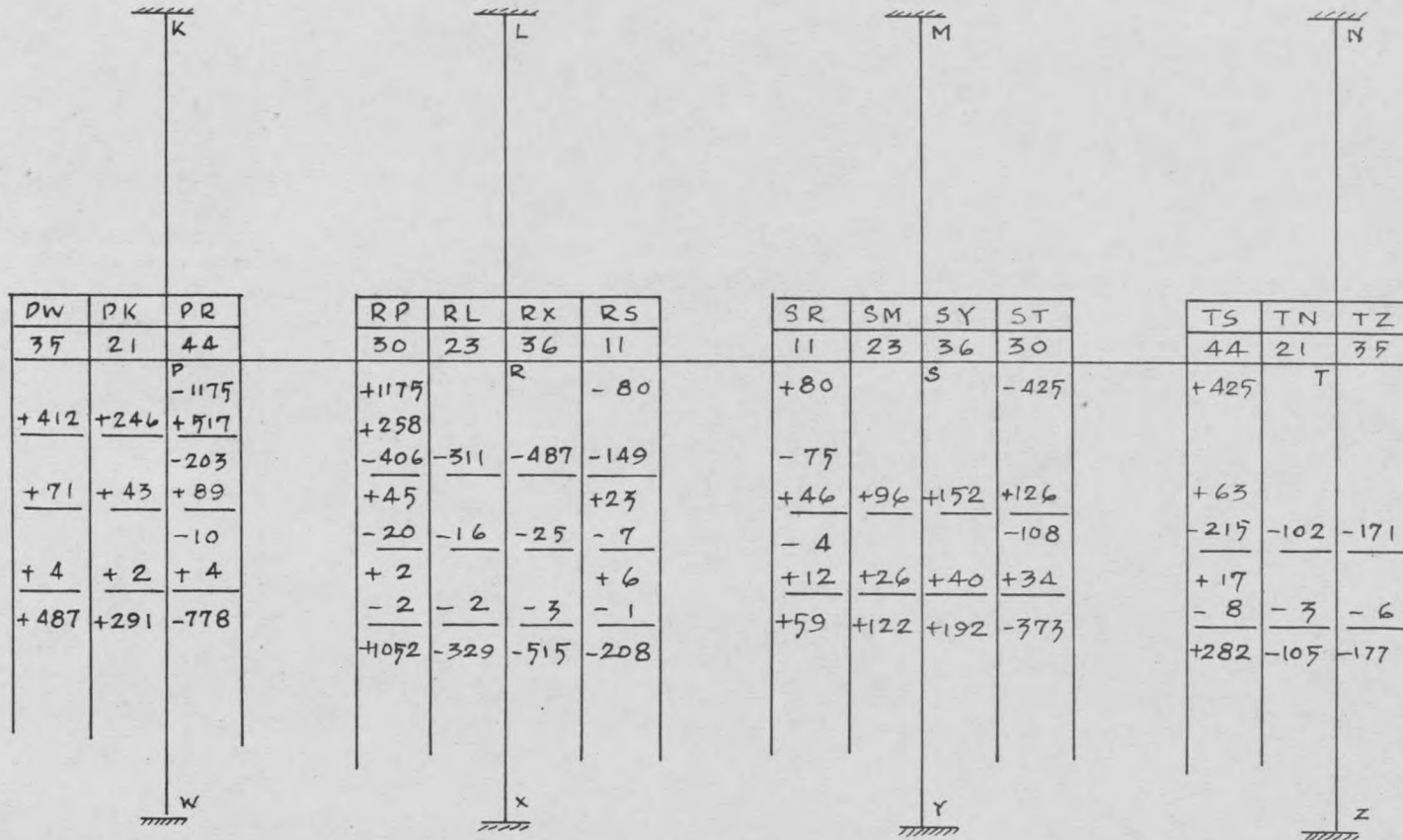


TABLE XII : MOMENT DISTRIBUTION FOR ISOLATED 1. FLOOR

PART β :



in the exterior beam and at the same time give maximum negative moments in the exterior beams, where the exterior beams meet the exterior columns.

In part 'b', the loading conditions are chosen to give maximum negative moments for both exterior and interior beams where both beams meet the interior column.

The maximum positive moments are obtained by simple statics from the balanced end moments obtained in part 'a' and the loads on the beams. For example, to obtain the maximum positive moment in B4, the end moments are -569 inch kips at E and 729 inch kips at F, both producing tension at the top. By taking moments around F the reaction at E is determined, and by taking moments at the center of the beam, the positive moment is determined. This positive moment at the center of the beam is 763 inch kips and for all practical purposes is the maximum positive moment in the beam.

The maximum positive moments for the interior beams are not considered, because the interior beams, having a short span of eight feet, will never produce positive moments to be the governing design moments for the beams.

Both maximum positive and negative moments for all beams given in inches kips are shown below and the design moments are chosen which will determine the beam dimensions.

<u>Beam</u>	<u>Max.Neg.Mom.</u>	<u>Max.Posit.Mom.</u>	<u>Design Mom.</u>
B1	528	613	613
B2	294	-	294
B3	528	613	613
B4	820	763	820
B5	351	-	351
B6	820	763	820
B7	951	638	951
B8	254	-	254
B9	951	638	951
B10	1052	533	1052
B11	208	-	208
B12	1052	533	1052

The design moments for the beams are used in the following design where the determined dimensions are compared with the assumed ones.

Design Moment $M = \frac{1}{2} f_c k j b d^2$ where $f_c = 0.45 f'_c$, $k = 0.40$, $j = 0.87$, b is the width of the beam and d is the distance to the center of reinforcing steel.

Beam B1 assumed dimensions 10 x 18
 $M = 613,000$, $b = 11$, $d^2 = \frac{613,000}{196 b}$, $d = 16.8$ in.

Beam B2 assumed dimensions 8 x 15
M = 294,000, b = 8, $d^2 = \frac{294,000}{196 b}$, d = 13.6 in.

Beam B4 assumed dimensions 12 x 24
M = 820,000, b = 12, $d^2 = \frac{820,000}{196 b}$, d = 18.7 in.

Beam B5 assumed dimensions 10 x 14
M = 351,000, b = 10, $d^2 = \frac{351,000}{196 b}$, d = 13.4 in.

Beam B7 assumed dimensions 12 x 24
M = 951,000, b = 12, $d^2 = \frac{951,000}{196 b}$, d = 20.1 in.

Beam B8 assumed dimensions 10 x 14
M = 254,000, b = 10, $d^2 = \frac{254,000}{196 b}$, d = 11.9 in.

Beam B10 assumed dimensions 12 x 24
M = 1052,000, b = 12, $d^2 = \frac{1,052,000}{196 b}$, d = 21.2 in.

Beam B11 assumed dimensions 10 x 14
M = 208,000, b = 10, $d^2 = \frac{208,000}{196 b}$, d = 10.3 in.

Beam B3 identical to B1
Beam B6 identical to B4
Beam B9 identical to B7
Beam B 12 identical to B10.

A study of the required beam dimensions shows that these dimensions are for all practical purposes very close to the assumed ones, as the 'd' dimensions are only the distance to the center of reinforcement and not the total depth of the beams. The assumed dimensions will be used in the second part of the paper where the frame is

analyzed as a single unit.

PART II

Analysis of the Frame as a Single UnitApproach

If convenient loads are assumed such that, applied to one beam at a time produce fixed end moments equal to 1000 inch pounds and if these fixed end moments are balanced separately for the entire frame, the balanced moments for fixed end moments of any magnitude can be computed at any place in the frame by proportion.

Such fixed end moments are assumed, and the final balanced moments are indicated in Tables XIII through XXIV. Based on the results, the final maximum negative moments for the beams are obtained by combining specific favorable loading conditions in Tables XXV through XXIX. Also, the end moments, which in turn will give maximum positive moments by simple statics as shown on page 35 are obtained by combining desirable loading conditions in Tables XXX and XXXI. It is obvious that for every beam three maximum moments are computed; two negative at the two ends and a positive moment very close to the center. However, again positive moments for the interior beams are omitted as in the first part of the analysis.

In Table XXXII which is a summary of the results of

TABLE XIII : BALANCED MOMENTS FOR THE FRAME, LOADED SPAN: AB

A			B				C				D																
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH															
37	63		36	22	42		42	22	36		63	37															
-1000			+1000																								
+455		-455		+865		-314		-551		-169		+70		+99		+21		-21									
E			K		K		G		G		H		H			H											
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN														
12	22	66	53	10	18	19	19	10	18	53	66	12	22														
+201		-63		-138		+41		-133		+47		+45		+12		+29		-10		-31		+2		-7		+9	
K			L		L		M		M		N		N			N			N								
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT														
19	26	55	41	14	31	14	14	14	31	41	55	19	26														
-26		+8		+18		-3		+17		-10		-4		-4		+2		+2									
P			R		R		S		S		T		T			T			T								
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ														
21	35	44	30	23	36	11	11	23	36	30	44	21	35														
W			X				Y				Z																

TABLE XIV : BALANCED MOMENTS FOR THE FRAME. LOADED SPAN: BC

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
					-1000		+1000						
-88	+88		+424	+284	-708		+708	-284	-424		-88	+88	
	E				K				G				H
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
-32	+21	+11	-76	+127	-34	-17	+17	-127	+34	+76	-11	+32	-21
	K				L				M				N
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
+7	-3	-4	+6	-13	+7			+13	-7	-6	+4	-7	+3
	P				R				S				T
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
	W				X				Y				Z

TABLE XV : BALANCED MOMENTS FOR THE FRAME. LOADED SPAN: CD

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
									-1000		+1000		
+21	-21		-99	-70	+169		+551	+314	-865		+455	-455	
	E			K				G				H	
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
+7	-5	-2	+31	-29	+10	-12	-45	+133	-47	-41	+138	-201	+63
	K			L				M				N	
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
			-2	+4	-2		+4	-17	+10	+3	-18	+26	-8
	P			R				S				T	
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
	W			X				Y				Z	

TABLE XVI : BALANCED MOMENTS FOR THE FRAME, LOADED SPAN: EF

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
<u>+59</u>	<u>-59</u>		<u>+14</u>	<u>-50</u>	<u>+36</u>		<u>+9</u>	<u>+1</u>	<u>-10</u>		<u>-2</u>	<u>+2</u>	
E			K				G				H		
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
		-1000	+1000										
<u>+156</u>	<u>+300</u>	<u>-456</u>	<u>+671</u>	<u>-140</u>	<u>-260</u>	<u>-271</u>	<u>-112</u>	<u>+13</u>	<u>+29</u>	<u>+70</u>	<u>+13</u>	<u>-4</u>	<u>-9</u>
K			L				M				N		
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
<u>+120</u>	<u>-50</u>	<u>-70</u>	<u>+23</u>	<u>-104</u>	<u>+57</u>	<u>+24</u>	<u>+8</u>	<u>+11</u>	<u>-8</u>	<u>-11</u>		<u>-2</u>	<u>+2</u>
P			R				S				T		
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
<u>-19</u>	<u>+11</u>	<u>+8</u>	<u>-5</u>	<u>+21</u>	<u>-12</u>	<u>-4</u>	<u>-1</u>	<u>-3</u>	<u>+2</u>	<u>+2</u>			
W			X				Y				Z		

TABLE XVII: BALANCED MOMENTS FOR THE FRAME LOADED SPAN; FG

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
-9	+9		-23	+42	-19		+19	-42	+23		-9	+9	
E			K				G				H		
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
						-1000	+1000						
-35	-70	+105	+558	+116	+213	-887	+887	-116	-213	-558	-105	+35	+70
K			L				M				N		
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
-26	+17	+9	-39	+90	-40	-11	+11	-90	+40	+39	-9	+26	-17
P			R				S				T		
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
+5	-3	-2	+6	-15	+8	+1	-1	+15	-8	-6	+2	-5	+3
W			X				Y				Z		

-44-

TABLE VIII : BALANCED MOMENTS FOR THE FRAME, LOADED SPAN: GH

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
-2	+2		+10	-1	-9		-36	+50	-14		+59	-59	
E			K				G				H		
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
										-1000	+1000		
-4	-9	-13	-70	-13	-29	+112	+271	+140	+260	-671	+456	-156	-300
K			L				M				N		
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
+2	-2		+11	-11	+8	-8	-24	+104	-57	-23	+70	-120	+50
P			R				S				T		
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
			-2	+3	-2	+1	+4	-21	+12	+5	-8	+19	-11
W			X				Y				Z		

TABLE XIX : BALANCED MOMENTS FOR THE FRAME, LOADED SPAN: KL

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
-6	+6		-1	+5	-4		-1		+1				
E			K			G			H				
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
-18	+90	-72	+26	+15	-70	+29	+11	-2	+2	-11	-2	+1	+1
K			L			M			N				
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
		-1000	+1000										
+235	+324	-559	+784	-189	-405	-190	-84	+15	+30	+39	+10	-4	-6
P			R			S			T				
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
+128	-72	-56	+33	-154	+93	+28	+8	+8	-9	-7	-1	-1	+2
W			X			Y			Z				

-46-

TABLE XX : BALANCED MOMENTS FOR THE FRAME, LOADED SPAN; LM

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	67		36	22	42		42	22	36		63	37	
+1	-1		+3	-4	+1		-1	+4	-3		+1	-1	
	E				K				G				H
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
+6	-11	+5	-42	-11	+63	-10	+10	+11	-63	+42	-5	-6	+11
	K				L				M				N
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
						-1000	+1000						
-42	-60	+102	+415	+157	+340	-912	+912	-157	-340	-415	-102	+42	+60
	P				R				S				T
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
-20	+22	-2	-49	+133	-72	-12	+12	-133	+72	+49	+2	+20	-22
	W				X				Y				Z

TABLE XXI : BALANCED MOMENTS FOR THE FRAME, LOADED SPAN: MN

A			B				C				D			
AE	AB		BA	BF	BC			CB	CG	CD		DC	DH	
37	63		36	22	42			42	22	36		63	37	
			-1		+1			+4	-5	+1		-6	+6	
	E				K					G				H
EA	EK	EF	FE	FB	FL	FG		GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19		19	10	18	53	66	12	22
			+11	+2	-2	-11		-29	-15	+70	-26	+72	+18	-90
	K				L					M				N
KE	KP	KL	LK	LF	LR	LM		ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14		14	14	31	41	55	19	26
											-1000	+1000		
			-39	-15	-30	+84		+190	+189	+405	-784	+559	-335	-324
	P				R					S				T
PK	PW	PR	RP	RL	RX	RS		SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11		11	23	36	30	44	21	35
			+7	-8	+9	-8		-28	+154	-93	-33	+56	-128	+72
	W				X					Y				Z

48

TABLE XXII: BALANCED MOMENTS FOR THE FRAME. LOADED SPAN: PR

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
+1	-1			-1	+1								
	E				K				G				H
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
+3	-11	+8	-4	-2	+9	-3	-1			+1			
	K				L				M				N
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
-28	+80	-52	+35	+27	-89	+27	+9	-3	+2	-8	-1	+1	
	P				R				S				T
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
		-1000	+1000										
+233	+425	-658	+878	-272	-467	-139	-62	+14	+27	+21	+6	-2	-4
	W				X				Y				Z

-10-

TABLE XXIII: BALANCED MOMENTS FOR THE FRAME, LOADED SPAN: RS

A			B				C				D		
AE	AB		BA	BF	BC		CB	CG	CD		DC	DH	
37	63		36	22	42		42	22	36		63	37	
E			K				G				H		
EA	EK	EF	FE	FB	FL	FG	GF	GC	GM	GH	HG	HD	HN
12	22	66	53	10	18	19	19	10	18	53	66	12	22
-1	+2	-1	+5	+1	-7	+1	-1	-1	+7	-5	+1	-2	+1
K			L				M				N		
KE	KP	KL	LK	LF	LR	LM	ML	MG	MS	MN	NM	NH	NT
19	26	55	41	14	31	14	14	14	31	41	55	19	26
+8	-5	-3	-49	-19	+79	-11	+11	+19	-79	+49	+3	-8	+5
P			R				S				T		
PK	PW	PR	RP	RL	RX	RS	SR	SM	SY	ST	TS	TN	TZ
21	35	44	30	23	36	11	11	23	36	30	44	21	35
W			X				Y				Z		
-31	-61	+92	+304	+231	+401	-936	+936	-231	-401	-304	-92	+31	+61

-50-

TABLE XXV : FINAL MOMENTS WHEN FRAME IS TREATED AS A SINGLE UNIT

FINAL MOMENTS FOR B1

LOAD ON	FINAL MOMENT AT A		FINAL MOMENT AT B	
B1	$\frac{47}{100} 650$	= - 292.00	$\frac{86}{100} 650$	= + 540.00
B2	$\frac{9}{100} 50$	= + 4.50	$\frac{4}{100} 50$	= + 20.80
B3	$\frac{2}{100} 650$	= - 13.00	$\frac{10}{100} 650$	= - 65.00
B4	$\frac{6}{100} 1175$	= - 70.50	$\frac{1.4}{100} 1175$	= + 16.40
B6			$\frac{1}{100} 1175$	= + 11.75
	INCH-KIPS	- 371.00	INCH-KIPS	+ 505.15

FINAL MOMENTS FOR B3 by symmetry

At D + 371.00 At C - 505.15

FINAL MOMENTS FOR B2

LOAD ON	FINAL MOMENT AT B		FINAL MOMENT AT C	
B1	$\frac{55}{100} 650$	= - 358.00	$\frac{17}{100} 650$	= + 110.00
B2	$\frac{70}{100} 50$	= - 35.00	$\frac{70}{100} 50$	= - 35.00
B3	$\frac{17}{100} 650$	= + 110.00	$\frac{55}{100} 650$	= + 358.00
B4			$\frac{1}{100} 1175$	= + 11.75
B5	$\frac{2}{100} 80$	= - 1.60	$\frac{2}{100} 80$	= + 1.60
B6	$\frac{1}{100} 1175$	= - 11.75		
	INCH-KIPS	- 296.35	INCH-KIPS	+ 296.35

TABLE XXVI : FINAL MOMENTS WHEN FRAME IS TREATED AS A SINGLE UNIT

FINAL MOMENTS FOR B4

LOAD ON	FINAL MOMENT AT E	FINAL MOMENT AT F
B1	$\frac{14}{100} 650 = - 91.00$	$\frac{4}{100} 650 = + 26.00$
B3	$\frac{0.2}{100} 650 = - 1.30$	$\frac{3}{100} 650 = + 19.50$
B4	$\frac{45.6}{100} 1175 = - 537.00$	$\frac{67.1}{100} 1175 = + 790.00$
B5		$\frac{56}{100} 80 = + 45.00$
B6	$\frac{1.3}{100} 1175 = - 35.20$	
B7	$\frac{7.2}{100} 1175 = - 84.20$	$\frac{2.6}{100} 1175 = + 30.50$
B9		$\frac{1.1}{100} 1175 = + 12.90$
	INCH-KIPS - 749.00	INCH-KIPS + 923.90

FINAL MOMENTS FOR B6 by symmetry

At H + 749.00 At G - 923.90

FINAL MOMENTS FOR B5

LOAD ON	FINAL MOMENT AT F	FINAL MOMENT AT G
B1	$\frac{45}{100} 650 = + 29.20$	$\frac{1.2}{100} 650 = + 7.80$
B3	$\frac{1.2}{100} 650 = - 7.80$	$\frac{4.5}{100} 650 = - 29.20$
B4	$\frac{27.1}{100} 1175 = - 318.00$	
B5	$\frac{88.7}{100} 80 = - 7.10$	$\frac{88.7}{100} 80 = + 7.10$
B6		$\frac{27.1}{100} 1175 = + 318.00$
B7		$\frac{1.1}{100} 1175 = + 12.90$
B9	$\frac{1.1}{100} 1175 = - 12.90$	
	INCH-KIPS - 316.60	IN-KIPS + 316.60

TABLE XXVII: FINAL MOMENTS WHEN FRAME IS TREATED AS A SINGLE UNIT

FINAL MOMENTS FOR B7

LOAD ON	FINAL MOMENT AT K		FINAL MOMENT AT L	
B1	$\frac{1.8}{100} 650$	= + 11.70	$\frac{.3}{100} 650$	= - 1.90
B3	$\frac{.2}{100} 650$	= - 1.30	$\frac{3.1}{100} 650$	= + 20.20
B4	$\frac{7}{100} 1175$	= - 82.20	$\frac{2.3}{100} 1175$	= + 27.00
B6			$\frac{1.1}{100} 1175$	= + 12.90
B7	$\frac{55.9}{100} 1175$	= - 658.00	$\frac{78.4}{100} 1175$	= + 922.00
B8	$\frac{10.2}{100} 80$	= - 8.10	$\frac{41.5}{100} 80$	= + 33.20
B9	$\frac{1}{100} 1175$	= - 11.75		
B10	$\frac{5.2}{100} 1175$	= - 61.20	$\frac{3.5}{100} 1175$	= + 41.20
B12			$\frac{.8}{100} 1175$	= 9.40
	IN-KIPS	- 810.85	IN-KIPS	+ 1063.40

FINAL MOMENTS FOR B9 by symmetry

At N + 810.85 At M - 1063.40

TABLE XVIII: FINAL MOMENTS WHEN FRAME IS TREATED AS A SINGLE UNIT

FINAL MOMENTS FOR B8

LOAD ON	FINAL MOMENT AT L		FINAL MOMENT AT M	
B4			$\frac{.8}{100} 1175$	= + 9.40
B5	$\frac{1.1}{100} 80$	= - .80	$\frac{1.1}{100} 80$	= + .80
B6	$\frac{.8}{100} 1175$	= - 9.40		
B7	$\frac{19}{100} 1175$	= - 223.00		
B8	$\frac{91.2}{100} 80$	= - 73.00	$\frac{91.2}{100} 80$	= + 73.00
B9			$\frac{19}{100} 1175$	= + 223.00
B10			$\frac{.9}{100} 1175$	= + 10.50
B11	$\frac{1.1}{100} 80$	= - .80	$\frac{1.1}{100} 80$	= .80
B12	$\frac{.9}{100} 1175$	= - 10.50		
	INCH-KIPS	- 317.50	INCH-KIPS	+ 317.50

TABLE XXIX : FINAL MOMENTS WHEN FRAME IS TREATED AS A SINGLE UNIT

FINAL MOMENTS FOR B10

LOAD ON	FINAL MOMENT FOR B10 AT P	FINAL MOMENT FOR B10 AT R
B7	$\frac{5.6}{100} 1175 = - 65.80$	$\frac{3.3}{100} 1175 = + 38.80$
B9		$\frac{.7}{100} 1175 = + 8.20$
B10	$\frac{65.8}{100} 1175 = - 773.00$	$\frac{87.8}{100} 1175 = + 1030.00$
B11		$\frac{30.4}{100} 1175 = + 24.40$
	<u>INCH-KIPS</u> <u>- 838.80</u>	<u>IN-KIPS</u> <u>+ 1101.40</u>

FINAL MOMENTS FOR B12 by symmetry

AT T + 838.80 AT S - 1101.40

FINAL MOMENTS FOR B11

LOAD ON	FINAL MOMENT FOR B11 AT R	FINAL MOMENT FOR B11 AT S
B4	$\frac{.4}{100} 1175 = - 4.70$	
B6		$\frac{.4}{100} 1175 = + 4.70$
B7		$\frac{.8}{100} 1175 = + 9.40$
B8	$\frac{1.2}{100} 80 = - 1.00$	$\frac{1.2}{100} 80 = + 1.00$
B9	$\frac{.8}{100} 1175 = - 9.40$	
B10	$\frac{13.9}{100} 1175 = - 163.00$	
B11	$\frac{93.6}{100} 80 = - 75.00$	$\frac{93.6}{100} 80 = + 75.00$
B12		$\frac{13.9}{100} 1175 = + 163.00$
	<u>INCH-KIPS</u> <u>- 253.10</u>	<u>IN-KIPS</u> <u>+ 253.10</u>

