Dynamic response model of the human leg

by Douglas Wayne Johnson

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
A three-dimensional finite element dynamic response model of the lower human leg was constructed for the purpose of simulating the vibrational response of a leg subject to harmonic excitation. The model consists of the major structural components of the leg (tibia, fibula, flesh, and interosseous ligament), and includes viscoelastic and support condition parameters. Several methods were used to reduce the system array size and speed up computation time. The results are presented in the form of eight test response runs, each of which was done altering material property parameters. Comparisons between experimental and analytical data indicated that flesh damping, end condition stiffness, and interosseous ligament stiffness all played a major role in determining leg vibrational response.

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Date 10/19/78

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Mechanical Engineering

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October, 1978

ACKNOWLEDGEMENT

The author wishes to thank Dr. Dennis Blackketter for his continuing support throughout the year. The author would also like to thank Dr. Edward Garner for his wisdom and knowledge on the subject of bone shaking.

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ABSTRACT

A three-dimensional finite element dynamic response model of the lower human leg was constructed for the purpose of simulating the vibrational response of a leg subject to harmonic excitation. The model consists of the major structural components of the leg (tibia, fibula, flesh, and interosseous ligament), and includes viscoelastic and support condition parameters. Several methods were used to reduce the system array size and speed up computation time. The results are presented in the form of eight test response runs, each of which was done altering material property parameters. Comparisons between experimental and analytical data indicated that flesh damping, end condition stiffness, and interosseous ligament stiffness all played a major role in determining leg vibrational response.

CHAPTER I

REVIEW

In recent years methods involving 'in vivo' materials testing of long bones have gained increased interest. One particular technique involves the development of response plots of frequency vs. amplitude for a bone as it is excited in the range of its first bending resonance frequency by low amplitude sinusoidal input. The shape of these plots are a function of the stiffness, geometry, and damping in the system. 'In vivo' tissue mechanical properties can be derived by accurately modeling the system and adjusting parameters until the response predicted by the model matches experimental data. Several models have been proposed for the purpose of predicting 'in-vivo' long bone response due to harmonic excitation.

Jurist [1] modeled the human ulna with three relatively simple models and compared predicted results with experimental results of previous tests. The ulna was modeled as an elastic homogeneous cylindrical beam with different end conditions comprising the three models. Jurist assumed that his response plots were measuring the resonance caused by the first bending mode of vibration, and correlated bone stiffness, geometry, and mineral content with respect to this first natural frequency.

Doherty [2] indicates that rigid body motion might be the dominant mode in the first resonance peak observed by Jurist, and concludes that these tests might be more a test of bone mass and end condition stiffness than of bone stiffness. Jurist [3], however, using a fourth rigid body model demonstrated a low correlation between rigid body motion and experimental results.

Orne [4] added viscoelastic properties to Jurist's pinned end beam model and attempted to match his analytical results with experimental impedance test data. Because of abnormally high bone damping necessary to match results, Orne concludes that considerable damping is produced by the surrounding flesh, and that further models should include flesh. In a later model, Orne [5] added a lumped mass flesh system to his model and was able to more accurately mathc experimental results.

Garner and Blackketter [6] constructed a finite element model of the entire forearm including both the ulna and radius along with surrounding flesh. Matching theoretical and experimental response plots, Garner was able to obtain both flesh and bone viscoelastic material properties. However, since only one arm was modeled and tested, no statistical verification of his results could be established.

Although a number of models have been proposed, the validity of 'in vivo' dynamic response measurements as a means of predicting biological tissue properties has not been established.

It is the intent of this research to construct a finite element dynamic response model of the human lower leg for the purpose of material property evaluation. This model will incorporate techniques utilized by Garner, with various modifications. The model will include viscoelastic bone and flesh properties as well as variable end conditions to allow rigid body motion. The proposed model will also be capable of quickly matching geometric properties with those of a given x-ray in an attempt to handle large population samples.

The following sections describe the modeling techniques in more detail as well as the effects of different viscoelastic parameters on vibrational response. Finally, general comments are made about modeling and experimental techniques involving long bone vibrational testing, with recommendations for both.

CHAPTER II

THE MODEL

The human leg is a complicated biological system consisting of two bones, tibia and fibula, connected to various types of soft tissue. For this reason, the finite element method was chosen as best suited to accurately model the lower leg. Also, because of the high degree of damping apparent from experimental response plots, the model includes viscoelastic properties.

The model can be visualized as a series of ten leg segments obtained by slicing the lower leg at eleven equally spaced cross sections from the knee to the ankle. Each segment contains two beam elements, representing the tibia and fibula, and fifteen hexahedron flesh elements surrounding the bones. Figure 1 is a typical computer generated cross section showing the bone and flesh elements as well as the flesh tie in points. These flesh tie in points are assumed to be rigidly attached to the bone.

The resulting finite element matrix equation in the Laplace domain is

$$[(s^2 \underset{\sim}{M} + \underset{\sim}{K}_b(s) + \underset{\sim}{K}_f(s)] \underline{U}(s) = \underline{F}(s) \quad (1)$$

where $\underset{\sim}{M}$, $\underset{\sim}{K}_b(s)$, and $\underset{\sim}{K}_f(s)$ are the combined bone and flesh mass array, bone stiffness array, and flesh combined stiffness array respectively. The Laplace variable is s while $\underline{U}(s)$ and $\underline{F}(s)$ are displacement and force vectors. Note that due to the viscoelastic behavior of the material,

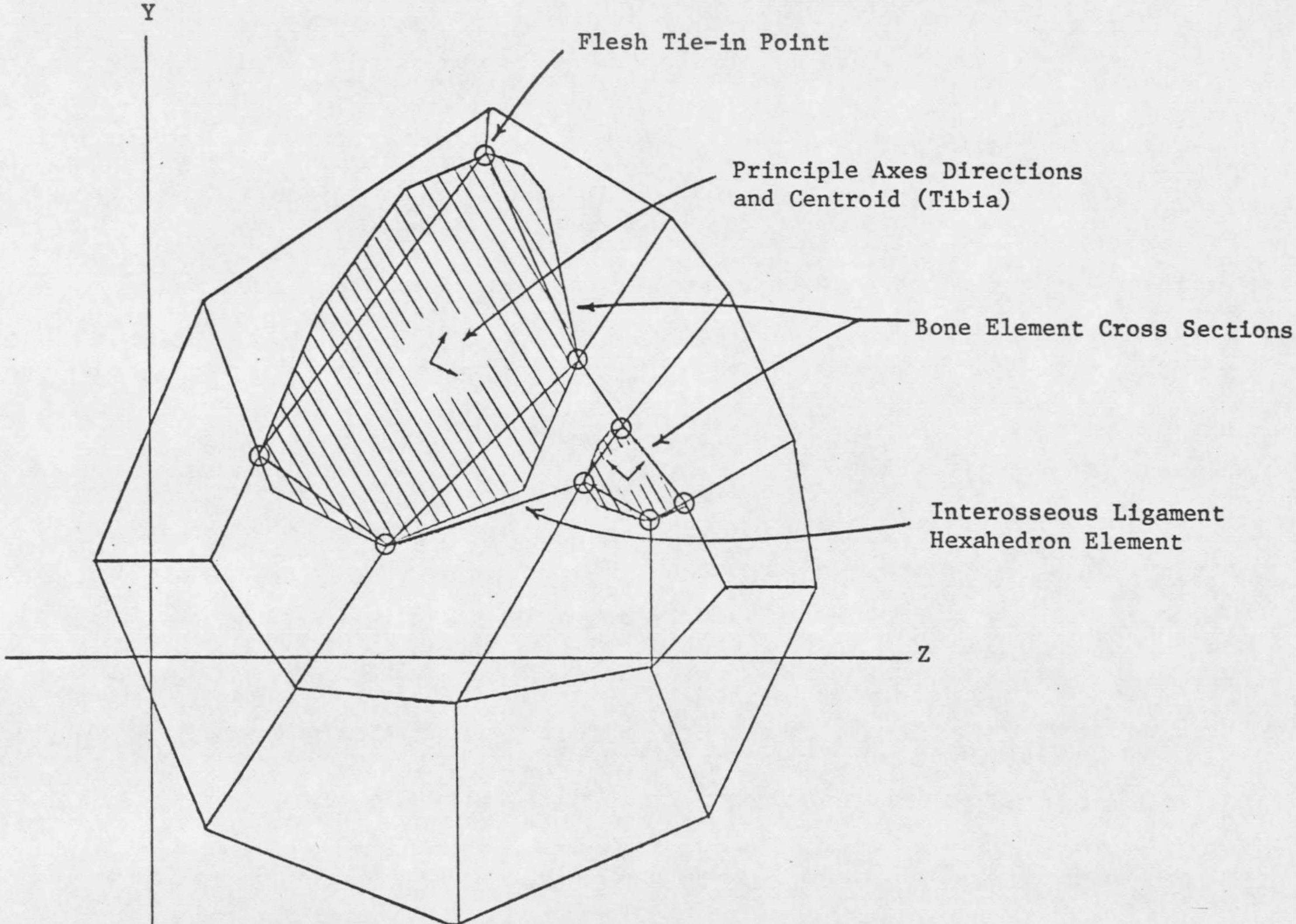


FIGURE 1.--Typical Leg Cross Section

the stiffness arrays become functions of the Laplace variable. For steady state harmonic motion, Equation 1 can be solved in the time domain by using the relationship

$$s = j\omega$$

where j is the complex indicator and ω is frequency. The result is a set of complex simultaneous equations of the form

$$\underset{\sim}{D}(j\omega)\ \underline{U}(j\omega) = \underline{F}(j\omega) \tag{2}$$

where $\underset{\sim}{D}(j\omega) = (-\omega^2 \underset{\sim}{M} + \underset{\sim}{K}_b(j\omega) + \underset{\sim}{K}_f(j\omega)]$

which can be solved for displacement amplitudes by using any complex simultaneous equation routine available on a computer.

Viscoelastic Model

The viscoelastic constituative laws for linear three dimensional viscoelastic materials in the Laplace domain may be divided into deviatoric and hydrostatic components of the form

$$S^d_{ij} = sY^d(s)\ E^d_{ij}$$

$$S^h_{kk} = sY^h(s)\ E^h_{ij}$$

where S and E denote stress and strain respectively. Subscripts indicate component directions with kk indicating summation of the 11, 22, and 33 directions. Y(s) is the relaxation modulus corresponding to a particular reological model, and superscripts denote hydrostatic (h) or deviatoric (d) components. These equations can be combined to form the total stress equation

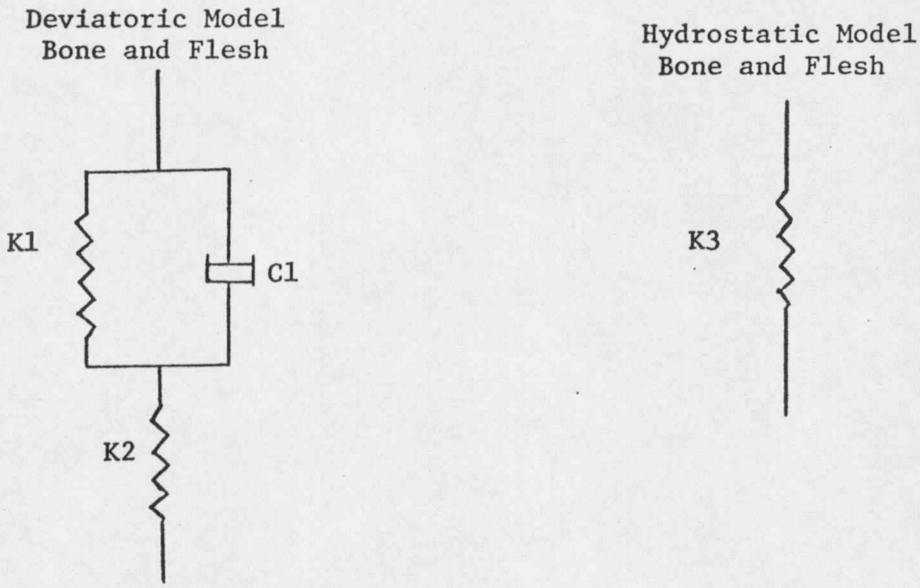
$$\sigma_{ij} = S^{d}_{ij} + \frac{1}{3}\delta_{ij}\ S^{h}_{ij}$$

where σ is the total stress and δ is the Kronecker delta.

For this study both the flesh and bone viscoelastic properties are modeled similarly. Figure 2 shows the three parameter deviatoric and simple elastic hydrostatic reological models used for either flesh or bone. The models are of the same form and differ only in parameter selection. Particular parameter selection will be discussed later in the results section. Using the correspondence principle, the finite element equations may now be developed in the Laplace domain similar to the methods used for the elastic case.

Bone Elements

Both the tibia and fibula are modeled using a series of ten beam elements, each of which is defined by eight displacement coordinates. Figure 3 shows a typical bone element along with its local, global, and displacement coordinate systems. Órne [7] determined that torsional effects as well as shear deformations have little effect on ulna response over the range of its first bending resonance. Assuming the same



$$Y^d(s) = \frac{(B1 - B2)}{(B3 + s)} + \frac{B2}{s}$$

$$Y^h(s) = \frac{B4}{s}$$

where $B1 = K2$

$B2 = K1\ K2/(K1 + K2)$

$B3 = (K1 + K2)/C1$

$B4 = K3$

B1, B2, B3, and B4 are viscoelastic parameters for either flesh or bone, and are given values in later sections.

FIGURE 2.--Rheological Model