

An economic analysis of grain cropping sequences in Montana by James Howard Nybo

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Agricultural Economics Montana State University

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Abstract:

The purpose of this study is to provide a summary and an application, of a relatively new methodology for performing an economic analysis of grain cropping sequences. The primary objective is to analyze viable grain cropping sequences in Montana using a dynamic programming framework.

A model is formulated using long-term crop experiment data and the results of a recent Cost-Returns Survey to generate an optimal cropping decision rule. This rule, which is conditional on land use the preceding year and available soil moisture at planting time, tells the individual farm decision-maker whether he should plant spring wheat, plant barley, or fallow.

Production data are from the Havre branch of the Montana Agriculture Experiment Station, from 1917 to 1947. Cost data are taken from a study by Dr. Walter G. Heid, Jr., E.R.S., Bozeman, Montana.

Some historical factors affecting grain cropping decisions are discussed, as are the mathematical applications used in the study. The primary model utilizes stochastic Dynamic Programming. Multiple regression is used to explain the grain production relationships.

Three optimal policies were developed based on different sets of assumptions. Two of these optimal policies were compared with fixed decision rules. It was demonstrated that the optimal policies would, on the average, provide higher expected returns than either a rigid wheat-fallow or continuous barley alternative policy.

A qualitative discussion of several environmental factors is included.

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AN ECONOMIC ANALYSIS OF GRAIN CROPPING SEQUENCES IN MONTANA

Ъу

JAMES HOWARD NYBO

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

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Agricultural Economics

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Head //Major Department

Chairman, Examining Committee

Graduate Dean

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ABSTRACT

The purpose of this study is to provide a summary and an application of a relatively new methodology for performing an economic analysis of grain cropping sequences. The primary objective is to analyze viable grain cropping sequences in Montana using a dynamic programming framework.

A model is formulated using long-term crop experiment data and the results of a recent Cost-Returns Survey to generate an optimal cropping decision rule. This rule, which is conditional on land use the preceding year and available soil moisture at planting time, tells the individual farm decision-maker whether he should plant spring wheat, plant barley, or fallow.

Production data are from the Havre branch of the Montana Agriculture Experiment Station, from 1917 to 1947. Cost data are taken from a study by Dr. Walter G. Heid, Jr., E.R.S., Bozeman, Montana.

Some historical factors affecting grain cropping decisions are discussed, as are the mathematical applications used in the study. The primary model utilizes stochastic Dynamic Programming. Multiple regression is used to explain the grain production relationships.

Three optimal policies were developed based on different sets of assumptions. Two of these optimal policies were compared with fixed decision rules. It was demonstrated that the optimal policies would, on the average, provide higher expected returns than either a rigid wheat-fallow or continuous barley alternative policy.

A qualitative discussion of several environmental factors is included.

CHAPTER I

INTRODUCTION

THE PROBLEM

More and more, in Montana and elsewhere, the dryland grain farmer is questioning the determinants of his cropping decisions, and considering alternatives other than the now-traditional wheat-fallow-wheat-fallow-...system. As in all forms of competitive productive activity there are many pressures exerted on the dryland grain farmer to induce him to be more efficient in the totality of his operations.

One of these forces, a problem of large proportions and farreaching implications, is that of the saline seep areas now appearing
and growing in many of Montana's grain producing regions. The magnitude and recent growth of the saline seep problem facing the dryland
grain farmer in parts of Montana and other parts of the U.S. and
Canada appears to be directly related to the way he uses soil moisture
as reflected in the cropping sequences to which he subjects his land. 1

Recent developments in technology including better tractors, bigger and more efficient implements, pesticides (to include fungicides, herbicides, and insecticides), chemical fertilizers, and new

¹c.f. Proceedings, Saline Seep-Fallow Workshop, Feb. 22-23, 1971, Great Falls, Montana, Published by Montana Cooperative Extension Service.

plant varieties all contribute to the relevance of the investigation.

With uncertainty in grain markets, in government programs, in actual production activities, and in environmental implications, it is both timely and necessary to better understand the economics of cropping decisions.

THE PURPOSE

The purpose of this study is to provide a summary and an application of a relatively new methodology for performing an economic analysis of grain cropping sequences. By including available soil moisture as a decision variable, a change in the cropping system can be suggested which will more fully utilize soil moisture and at the same time maintain or improve the economic position of the farm firm. The use of dynamic programming permits the problem to be considered over a time horizon of any length, and data needs for future analysis can be estimated.

THE OBJECTIVES

The primary objective of this study is to analyze viable grain cropping sequences in Montana using a dynamic programming framework in a manner similar to that presented by Burt and Allison

in 19632. By using the dynamic programming model to look at costs of production and production functions it is possible to generate a conditional decision rule which will tell the individual operator what cropping sequence he should follow in order to maximize the present value of his net returns for an infinite time horizon. The term "viable" restricts analysis to those alternatives which are deemed realistic by the producer for the time period and location studied. Crops other than grain could conceivably help alleviate the saline seep problem more rapidly, but would create other management problems with regard to enterprise combinations, machinery investment, and managerial knowhow. "Grain cropping sequences" refers to the cropping pattern of the individual cash-grain farmer, i.e. what crops or fallow periods (which for purposes of analysis are treated as crops) he subjects his land to over time. Data limitations have restricted the analysis to a consideration of the cropping alternatives of spring wheat, barley, and fallow. It is immediately clear that these three alternative uses of land are not all-inclusive. Certainly purely livestock-supporting alternatives such as range, pasture, or hay are possibilities, as is winter wheat. Unavailability of data and the

²Oscar R. Burt and John R. Allison, "Farm Management Decisions with Dynamic Programming", <u>Journal of Farm Economics</u>, Vol. XLV, NO. 1, February, 1963

complexities of the model have served to constrain the study to preclude these alternatives. They should be considered if and when the model is put to practical use.

SOURCES OF DATA

Production data and transition probabilities were generated from experiments done at the Havre branch of the Montana Agricultural Experiment Station, from 1917 to 1947. Cost data are taken from a study done by Heid.³

³Dr. Walter G. Heid, Jr., Farm Production Economics Division, Economic Research Service, U.S.D.A., Stationed at Montana State University, Bozeman, Montana

CHAPTER II

HISTORICAL PERSPECTIVE

HISTORICAL SUMMARY

To gain perspective on the problem, it is helpful to look at the historical determinants of cropping decisions in Montana dryland grain producing regions. Farming in Montana before the 1900's was for the most part practiced on a small scale adequate enough to provide for the needs of the placer mining communities and the fur traders. The real expansion of agriculture into Montana, however, did not take place until the early 1900's. At that time farmers came from the humid areas of the east; first in small numbers, and then after the passage of the Enlarged Homestead Act of 1909, they came in great numbers. Although the weed-reducing benefits of occasional summer fallow had been recognized by earlier farmers in some of the valleys of the region, most of the new farmers brought with them the agricultural capital and techniques of their former homes. They came to Montana prepared to farm as they had before, and for a number of years many did. The rains in the first years after the passage of the

Mary Wilma M. Hargreaves, <u>Dry Farming in the Northern Great Plains</u> (Cambridge: Harvard University Press, 1957), p. 29.

Merrill G. Burlingame, K. Ross Toole, Robert G. Dunbar, A History of Montana (New York: Lewis Historical Publishing Co., 1957)

Homestead Act were very good. Then things changed, and a five-year drought began in 1917; 1918 was drier, and 1919 even drier still.

Many were disillusioned and left; those who stayed adapted to the semi-arid nature of the region and changed their farming techniques.

The technique which was adopted came to be known as "dry farm-Burlingame characterizes dry farming as "the culture of droughting". resistant plants by means of moisture-conserving tillage practices."3 The major element in dry farming was the inclusion of the practice of summer fallow. With the establishment of the Montana Agricultural Station and many railroad field stations, new techniques were speedily developed, tested, and disseminated. 4 The first half of the twentieth century has been characterized by the use of many different tillage techniques and many varieties of equipment. Basic to virtually all of them since the early twenties has been the practice of summer fallow. Through time, agronomic research has shown that summer fallow not only conserves soil moisture, it permits soil nitrification (thus replenishing the nitrogen removed by cropping), it controls weeds, it controls certain soil-borne plant diseases, and in the case of some operations it allows the operator to take care of much more acreage than he could

^{3&}lt;sub>Tbid</sub>

⁴ Ibid

if he were to crop it every year. An increasingly important factor favoring an alternate crop-fallow sequence has been the advent of the U.S. Government wheat and barley programs, which require the setting aside of certain percentages of one's wheat or barley "allotment." By taking land out of production and subsidizing the farmer, the government can both limit the supply of grain and help to provide the farmer a better return. When land is set aside it is commonly left fallow; and cropped on alternate years.

CURRENT CROPPING SYSTEM

The choice of whether to plant wheat or barley has been and is one of economics, i.e. the most profitable alternative is generally taken. The broad acceptance of the summer fallow technique has already been mentioned. While there are available data from several long-term cropping experiments in various parts of the state, which provide meaningful information relating to production relationships, there is a great shortage of information dealing with what decisions have been made by individual farmers on a specific plot of land. These decisions are not explained or recorded in the Montana Agricultural Statistics, as they have been lost in the aggregation process. The aggregate figures of land in wheat and barley are of value to reflect the magnitude of the dryland grain industry in Montana. Table 1 shows the total harvested dryland acres of barley, spring wheat, and winter

wheat for the state of Montana in the years 1948 to 1969.5

Although the cropping decision is an economic one, it is generally not made in a purely competitive free market environment. One very significant "non-market" factor which has served to limit alternatives and to change the profitablility of alternatives has been the U.S. Government Farm Program.

TABLE 1: Harvested Acres of Dryland Barley, Winter Wheat, Spring Wheat*; Montana, 1948-1969

Year	Barley	Winter Wheat	Spring Wheat
48	750,100	1,505,000	3,146,700
49	409,800	1,382,300 .	3,699,500
50	723,600	1,111,300	3,645,600
51	357,600	1,303,400	4,379,800
5 2	367,600	1,610,700	3,959,000
53	441,800	1,487,100	4,292,800
54	1,153,500	1,641,600	2,911,500
55	1,270,800	1,989,100	2,254,900
56	965,600	1,183,100	2,496,900
57	1,626,500	1,811,200	1,748,800
58 .	1,489,600	2,310,000	1,903,200
59	1,770,600	1,702,500	2,058,300
60	1,628,200	1,958,200	1,687,600
61	1,374,000	2,022,500	1,449,300
62	1,699,000	1,653,000	1,474,300
63	1,420,500	1,854,400	1,820,800
64	1,443,000	1,797,000	1,804,900

^{*}Durum included in Spring Wheat from 1948-1954.

Table from various years of Montana Agricultural Statistics

Table 1 (continued)

Year	Barley	Winter Wheat	Spring Wheat
65	1,214,400	2,288,600	1,645,500
66	1,548,700	2,110,800	1,398,700
67	1,161,600	2, 776,700	1,641,000
68	1,062,000	2,720,000	1,395,000.
69	1,510,000	2,282,000	1,072,000

In the absence of published information as to historical time-patterns of dryland agricultural land use, Mr. Norris Hanford, an established dryland farmer of Fort Benton, Montana, was consulted regarding those practices which he has followed in the past several years. The following table shows his cropping practices for four pieces of land for the years 1955 to 1971. Although this table is not meant to be a picture of what all dryland grain farmers are doing in Montana, it does provide an indication of what is being done in the absence of broader published information.

TABLE 2: Historical Cropping Sequences - Norris Hanford Ranch, Fort Benton, Montana

Year	Field A (102.1 Acres)	Field B (97.8 Acres)	Field C (151.5 Acres)	Field D (119.5 Acres)
1971	Barley	Barley	Barley	Barley
1970	Barley	Barley	Wheat	Barley
1969	Fallow	Wheat	Fallow	Fallow
1968	Fallow	Barley	Wheat	Wheat
1967	Fallow	Barley	Wheat	Fallow
1966	Barley	Fallow	Fallow	Wheat
1965	Wheat	Wheat	Wheat '	Fallow .
1964	Fallow	Fallow	Fallow	Wheat
1963	Wheat	Barley	Wheat	Fallow
1962	Fallow	Fallow	Fallow .	Barley
1961	Wheat	Wheat	Barley	Fallow
1960	Fallow	Fallow	Fallow	Wheat
1959	Barley	Wheat	Wheat	Fallow
1958	Wheat	Fallow	Fallow	Barley
1957	Fallow	Barley	Wheat	Barley
1956	Barley	Fallow	Fallow	Wheat
1955	Wheat	Wheat	Barley	Fallow

While no attempt is made to analyze the above set of cropping decisions, Table 3 shows the frequency of choosing each of the three alternatives in the period 1955 through 1971.

TABLE 3: Array of Cropping Decisions made on Norris Hanford Ranch, Fort Benton, Montana, 1955-1971.

Cropping Decision	Number	Percentage of Total
Barley	19	28.0%
Wheat	22	32.4%
Fallow	27	39.6%
Total	68	100.0%

Plant Disease

The mathematical portion of this analysis is so structured as to rule out the consideration of any alternatives other than spring wheat, barley, and summer fallow. Although data and technical constraints were the primary determinants of such restriction of the analysis, another factor is important to winter wheat growers. In areas of high moisture a wheat fungus, known as Cephalosporium Stripe, preys on winter wheat. According to Mathre⁷, winter wheat, in a wheat-fallow-... sequence, is very susceptible to the fungus, which is formally known as Cephalosporium Gramineum. In affected areas, bot-anists are recommending that spring crops can be grown for 5 or 6 years until the fungus is killed off. This conveniently supports the exclusion of winter wheat as an alternative. Because of plant pathogenic problems associated with the continuous cropping of wheat, this analysis includes a consideration of the cropping system when wheat following wheat is not permitted.

From a personal discussion with Dr. Don Mathre, Botany-Micro-biology Department, Montana State University, Bozeman, Montana.

CHAPTER III

THE MODEL

DYNAMIC PROGRAMMING AND THIS APPLICATION

This study is concerned with the management decision facing the individual farmer who for any reason has made the decision to raise either dryland spring wheat, dryland barley, or let the land lie fallow. The analytical framework used is an adaptation of the methodology first presented by Burt and Allison. The methodology can be called "Dynamic Programming," and is consistent with the definition presented by Bellman.

In order for a problem to be validly considered in a dynamic programming framework it must meet certain requirements. One of these is that it must be a multi-stage process. In the case of the cropping decision problem, it is clearly multi-stage. The stage is the time interval into which the process is divided. At each stage a decision must be made. Since this analysis has been restricted to the three mentioned alternatives, there is only one time each year--planting time in the spring--that a decision must be made regarding what to

¹Oscar R. Burt and John R. Allison, "Farm Management Decisions with Dynamic Programming," <u>Journal of Farm Economics</u>, Vol. XLV, No. 1, February, 1963

²Richard Bellman, <u>Dynamic Programming</u>, (Princeton University Press, 1957)

plant. The interval between stages, then, is taken to be one year, beginning and ending at planting time in the spring.

If a system is to be considered using dynamic programming, then it must fit the definitional limitations of a Markov process. Fundamental to Markov processes are the concepts of the "state" of a system, and "state transition." A system is said to occupy a state when it is completely described by the values of variables that define the state. When those state variables change to values describing another state, the system makes a state transition. Changes in state variables can be taken to be a continuous process, or a discrete-time process. In this analysis, although conceptually it is realized that the physical system is continuous, it shall be treated as a discrete-time process. A process can be treated as either deterministic or stochastic. The stochastic nature of the soil moisture and crop response has necessitated that this analysis be carried out in a stochastic framework.

Evaluation of the dynamic programming problem requires a precise statement and definition of the problem including all relevant variables. It has been shown above that the cropping decision problem

Ronald A. Howard, <u>Dynamic Programming and Markov Processes</u>, (The M.I.T. Press, Cambridge, Massachusetts, 1960)

is a multi-stage process. If the Markov requirement is met, then each state is fully defined by its state variables, and is independent of any state variables at any other stage. The problem is one of solving a sequential decision process. It can be evaluated using what Howard calls value iteration.

After defining the recurrence relation and the relevant variables, the value iteration process can be better conceptualized. The decision process requires solution of the recurrence relation

$$V_{\mathbf{i}}(\mathbf{n}) = \max_{\mathbf{k}} \left[q_{\mathbf{i}}^{\mathbf{k}}(\mathbf{n}) + \beta \sum_{\mathbf{j}=\mathbf{i}}^{\mathbf{m}} P_{\mathbf{i}\mathbf{j}}^{\mathbf{k}}(\mathbf{n}) V_{\mathbf{j}}(\mathbf{n}-1) \right]$$

where

V_i(n) is the present value of the total expected net return in n stages, starting from state i, if an optimal policy is followed.

k is the decision alternative variable; in this model, k = 1, 2, 3, where k = 1 = fallow; k = 2 = plant barley; k = 3 = plant wheat.

 $q_i^k(n)$ is the term for expected immediate returns, given the ith state, the kth decision alternative, and the nth stage.

i = 1, 2,...,m is the index for the state occurring in stage n.

⁴Ibid.

- j = 1,2,...,m is the index for the states occurring in stage N-1.
- P_{ij}^k is the probability of making the transition from state i in stage N to state j in stage N-l given the k^{th} decision alternative.
- β is the discount factor; $\beta = 1/(1+r)$, where r is the relevant periodic interest rate.

The value iteration process uses a technique of iteration of the recurrence relation to generate a policy, which defines the decision to be made for a given state, at each stage for all possible combinations of stages and states. Dynamic programming yields the optimal policy for decision processes of any length. In this case, that optimal policy is defined as one which maximizes the present value of net returns over the entire planning horizon. An optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

In the value iteration process, n, the stage variable, represents the number of stages remaining in the planning horizon. If a planning horizon of 20 years is being considered, at the beginning n would assume a value of 20, and at the beginning of the 20th year would assume a value of 1. From a conceptual viewpoint this is the reverse of the normal t, t+1, t+2,..., t+n convention of treating a time variable. The value iterative process, however, begins at

the end of the planning horizon, and works back to the present. In so doing, an optimal policy is generated for all time periods up to the total number considered in the iteration.

Prior comments have emphasized the potential pathogenic problems associated with the inclusion of the continuous wheat alternative.

This problem has been considered in two frameworks: (1) A sixteenstate (i = 1,2,...,16) model where the continuous wheat alternative
is allowed, and (2) a twenty-four state (i = 1,2,...,24) model where
the alternative of planting wheat after wheat is not permitted. The
flexibility of the model is one of its pleasing qualities. As new
information becomes available, or as institutional structures such
as government programs change, the model can be adapted to handle a
wide range of changes.

The foundation of the dynamic programming analysis lies in the recurrence relation, and the economic foundation of the recurrence relation lies in the $q_i^k(n)$ term, the expected immediate returns in stage n given the i^{th} state and the k^{th} decision alternative. The model as structured for this application has either sixteen or twenty-four states. This analysis assumes that a state is completely defined by the cropping decision in the preceding stage, and the level of available moisture in the soil profile. The sixteen state model assumes that the decision in the preceding state could have been one

of two alternatives: (1) fallow, or (2) crop. Wheat or barley are treated the same in this case. In the sixteen state model this is sufficient as it describes the transition with respect to soil moisture and the previous crop. With eight levels of available soil moisture, it is possible to completely describe all possible states with sixteen states, as table 4 shows.

TABLE 4: Descriptive Matrix of States in the Sixteen State Model

•		Land Use in the Preceding Stage			
		Fallow	Crop	(wheat or	barley)
	1	1	9		
	2	2	10		
loisture	3	3	11		
1	4	4	12		
evel	5	. 5	13		
	6	6	14	•	
	7	7	15		
	8	. 8 .	16		

The twenty-four state model does not allow the wheat-wheat alternative, and requires an additional eight states beyond the 16 state model in order to do this. In the twenty-four state model there are again eight levels of available soil moisture, but the state is also described by the decision in the preceding stage, to include fallow, wheat, or barley as the decision, rather than just crop or fallow. The following matrix shows the 24 states in this model.

TABLE 5: Descriptive Matrix of States in the Twenty-four State Model.

		Land use	Land use in the preceding stage		
		Fallow	Barley	Wheat	
	1	1	. 9	17	
	2	2	10	18	
Moisture	3	3 .	11	19	
T 1	4	4	12	20	
Leve1	5	5	13	21 .	
	6	6	14	22 .	
	7	7	15	23	
	8	8	16	24	

Production Functions

Expected immediate returns are assumed to be unrelated to the stage of the process, and hence are a function of the state and the decision alternative at any stage. Expected immediate returns is the difference between gross returns and variable costs. Once a commodity price has been determined, price times yield determines gross returns The problem is to determine yield for each state. In order to determine yield in this study it was necessary to analyze production data from the Havre Branch of the Montama Agricultural Experiment Station, and generate production functions which would provide this information. The production data was analyzed in a multiple regression model, and was finally fitted into a logarithmic function where yield was estimated as a function of the logarithm of available soil moisture and the logarithm of five different precipitation variables. The state variable is soil moisture, so it was necessary to take the expectation with respect to precipitation in order to isolate the relationship between yield and soil moisture. The soil moisture variable was included as the state variable, and physically measured each year at planting/decision time. The regression values and matrix of expected immediate returns can be found in the next chapter.

Costs of Production

The second important element in the determination of the expected immediate returns is the costs of production. In the case of the production functions, it was only necessary to estimate the functions for wheat and barley. However, because the fallow alternative does carry with it real out-of-pocket costs it is necessary to estimate costs of production for all three alternatives.

The analysis is carried out using variable costs only to meet the short run assumption of economic analysis. The determination of costs of production is explicitly internal to the individual farm firm. Chapter 6 contains a qualitative discussion of some potential external costs. Chapter 4 contains a table showing the complete breakdown of costs of production.

Transition Probabilities

The recurrence relation contains the term P_{ij}^k , which is the probability of making the transition from state i to state j given decision alternative k. This term is essential to the model, as it represents our knowledge of the historical relationship between soil

moisture level and cropping decisions. It has been pointed out that the definition of a Markov process requires that a state be completely defined by its state variables. The transition probabilities provide a means to estimate those state variables given an initial state.

States i and j depend on moisture level and on cropping decision in the preceding stage, we are really concerned with the changes in moisture levels associated with states i and j. With the stated assumptions regarding homogeneity of crop water use, it is necessary to analyze the four cropping combinations to determine the probabilities of changing moisture levels. Considering time t-1 and time t, and assuming that wheat and barley have identical soil moisture consumption patterns, there are four possible crop sequences that can occur. They are crop-crop, crop-fallow, fallow-crop, and fallow-fallow.

Experimental information was available for the combinations of crop-crop, crop-fallow, and fallow-crop. Since the experiment which has provided the data for this study did not include the fallow-fallow alternative, it was assumed that the fallow-crop experiment would provide the same information as the fallow-fallow experiment. The

⁵It should be mentioned that the state variable is inches of available water at different levels in the soil broken down by feet. To a degree, then, this discrete breakdown by feet ignores the continuous distribution of the water in the soil profile.

soil moisture reading is taken at seeding time, before crop consumption of moisture. Hence, only three separate linear regressions over available soil moisture variables in different times were required. These regressions, for each experiment, gave the relationship

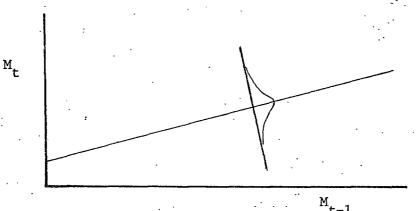
$$M_t = f(m_{t-1})$$

where

 M_t = available soil moisture in time period t . M_{t-1} = available soil moisture in the preceding time period.

In a linear regression, the function takes the form $M_t = a + b(M_{t-1}) + e$, where a is the y intercept, b is the regression coefficient, and e is an error term. Figure 1 shows a hypothetical plotting of this regression line.

FIGURE 1: Hypothetical Regression of Moisture in Time t over Moisture in Time t-1.



One of the assumptions of linear regression is that the population being estimated is normally distributed around the regression line. The regression line is, then, the locus of the means of these normal distributions. The normal distribution around one point of the hypothetical regression line is represented by the familiar bell-shaped curve in figure 1.

Once the mean and standard deviation of a normal distribution are known, the distribution can be transformed into standard form using the transformation equation

$$Z = \underbrace{M_t - \mu}_{\sigma}$$

where

 μ = the mean of the M_t observations.

Z = the standardized value for M₊.

In a standardized normal distribution the mean is equal to zero, the standard deviation is equal to one, and the area under the curve is equal to one. By standardizing the distribution of M_t 's, it is possible to measure the area under the distribution curve, and therefore approximate the probability of observing some M_t given M_t -1.

The regression parameters are tabulated in the following chapter.

CHAPTER IV

DATA PREPARATION

The preceding chapter dealt with the model without discussing the analysis of actual data. The present chapter is concerned with the steps involving data preparation and analysis which precede the running of the dynamic programming model. The theory and assumptions behind these steps have been discussed.

Production Functions

Using multiple regression, data were analyzed from four experiments extending over the 31 years from 1917 to 1947. Those experiments were continuous spring wheat, continuous barley, barley-fallow, and spring wheat-fallow. Regressions were structured so as to give

$$Y = f (ASM, P_0, P_1, P_2, P_3, P_4),$$

where

Y = Yield in bushels

ASM = Adjusted soil moisture (observed soil moisture minus four inches)

 P_0 = Precipitation in inches from seeding to emergence

 P_1 = Precipitaion in inches from emergence to tillering

 P_2 = Precipitation in inches from tillering to heading

-P, =-Precipitation in inches from heading to soft dough

 P_{L} = Precipitation in inches from soft dough to harvest

Soil moisture is read at planting time in the spring, to a depth of four feet. Because all observed values of soil moisture were greater than four inches the value used in the analysis is know as adjusted soil moisture (ASM), and is equal to actual soil moisture minus four inches.

The relationship used in estimating crop yields for the matrix of expected immediate returns was in the form

$$Y = f[ln(ASM+1), ln(P_0+1), ln(P_1+1), ln(P_2+1), ln(P_3+1), ln(P_4+1)]$$

By taking the natural logarithm of the variable plus one [ln(variable +1)], the possibility of having to take the logarithm of zero was ruled out. This was done because the logarithm of zero is not defined.

Table 6, 7, 8, and 9 are the regression values for the four experiments analyzed.

TABLE 6: Regression values for Production Functions Continuous Wheat Experiment

Variable	Mean	Standard Deviation	Correlation X VS Y	Regression Coefficient	Standard Error of Reg. Coefficient	Computed T Value	/
ln(ASM+1)	1.185	0.3023	0.4147	8.572	2.394	3.581	
ln(P ₀ +1)	0.6667	0.3502	0.3413	2.625	2.121	1.238	
ln(P ₁ +1)	0.8458	0.3804	0.7331	7.717	2.026	3.809	
1n(P ₂ +1)	1.031	0.4203	0.5363	8.145	1.737	4.689	N
ln(P ₃ +1)	0.6617	0.3361	0.2335	3.885	1.916	2.027	29
ln(P ₄ +1)	0.5370	0.2948	0.2664	0.8246	2.214	0.3725	
Dependent Yield	7.833	7.037					
Intercept		_22.02	R Squared	0.8164	Std Error - SY•X	3.371	

TABLE 7: Regression values for Production Functions
Continuous Barley Experiment

Variable	Mean	Standard Deviation	Correlation X VS Y	Regression Coefficient	Standard Error of Reg. Coefficient	Computed T Value	<i>'</i>
1n(ASM+1)	1.185	0.3023	0.2153	5.584	3.977	1.404	
ln(P ₀ +1)	0.6667	0.3502	0.2104	1.665	3.52 3	0.4727	
ln(P ₁ +1)	0.8458	0.3804	0.6848	10.79	3.366	3.207	
ln(P ₂ +1)	1.031	0.4203	0.5797	10.05	2.886	3.481	
ln(P ₃ +1)	0.6617	0.3361	0.8959E=01	0.9027	3.184	0.2835	33 60
1n(P ₄ +1)	0.5370	0.2948	0.7705E-01	-3.467	3.678	-0.9427	
Dependent Yield	8.780	8.609					
Intercept		-17.18	R Squared '	0.6615	STD Error - SY'X	5.601	<i>:</i>

TABLE 8: Regression values for Production Functions Continuous Barley-Fallow

Variable	Mean	Standard Deviation	Correlation X VS Y	Regression Coefficient	Standard Error of Reg. Coefficient	Computed T Value	/
.ln(ASM+1)	1.775	0.2088	0.3441	24.00	10.56	2.273	
1n(P ₀ +1)	0.6667	0.3502	0.1163	-0.6539	6.242	-0.1048	٠
1n(P ₁ +1)	0.8458	0.3804	0.6262	15.16	6.306	2.404	
ln(P ₂ +1)	1.031	0.4203	0.5062	15.46	5.302	2.916	(
ln(P ₃ +1)	0.6617	0.3361	-0.2565E-02	1.648	5.988	0.2752	i
ln(P ₄ +1)	0.5370	0.2948	0.7441E-01	-5.515	6.565	-0.8401	
Dependent Yield	18.49	14.28					
Intercept		-50.56	R Squared .	0.5937	STD ErrorSY'X	10.17	

TABLE 9: Regression values for Production Functions Continuous Wheat-Fallow Experiment

Variable	Mean	Standard Deviation	Correlation X VS Y	Regression Coefficient	Standard Error of Reg. Coefficient	Computed T Value	,
ln(ASM+1)	1.775	0.2088	0.3358	18.77	7.079	2.652	
ln(P ₀ +1)	0.6667	0.3502	0.2255	1.526	4.185	0.3646	
ln(P ₁ +1)	0.8458	0.3804	0.6465	10.34	4.228	2.446	
ln(P ₂ +1)	1.031	0.4203	0.4837	10.37	3.555	2.917	ն
ln(P ₃ +1)	0.6617	0.3361	0.2240	8.127	4.014	2.025	32
ln(P ₄ +1)	0.5370	0.2948	0.1396	-2.005	4.401	-0.4554	
Dependent Yield	15.79	10.25		·			
Intercept		-42.28	R Squared .	0.6456	STD ErrorSY · X	6.821	

As the individual farm operator has no control over the precipitation variables, and no prior knowledge beyond what history has given him, the precipitation variables became parameters in the yield relationships. This gives

Yield =
$$f(ASM / P_0, ..., P_4)$$

where

Y = Yield in bushels

ASM = Adjusted soil moisture

P₀,...,P₄ = expected value of the five precipitation variables = the means of the five variables for the period of the experimental data.

@~·

Thus yield becomes a function of adjusted soil moisture and the mean values of the five precipitation variables.

The next step was to make the change from a continuous production function to a discrete production schedule which would be compatible with the eight discrete moisture levels defining the states of the process. Table 10 shows this schedule for the midpoints of the eight moisture levels and for each of the four cropping possibilities.

TABLE 10: Schedule of Yields for Four Treatment Combinations and Eight Moisture Levels

Actu	al Soil Moistur	e Level		Yield (Bus	hels)		
i'	Midpoint (inches)	Range (inches)	Wheat Continuous	Fallow	Barl Continuou	•	
1	4.5	0 5.0	1.1437	0.0	4.4174	0.0	
2	5.5	5.0 - 6.0	5.5225	0.0	7.2699	0.0	
3	6.5	6.0 - 7.0	8.4067	5.9896	9.1487	5.9607	
4	7.5	7.0 - 8.0	10.5610	10.7068	10.5520	11.9923	
5	8.5	8.0 = 9.0	12.2812	14.4734	11.6726	16.8083	(
6	9.5	9.0 - 10.0	13.7131	17.6090	12.6054	20.8176	
7	10.5	10.0 - 11.0	14.9398	20.2950	13.4045	24.2521	
8	11.5	11.0 -	16.0127	22.6443	14.1034	27.2560	

The assumed midpoint of 4.5 for the 0-5.0 range and 11.5 for the open-ended range bounded below by 11.0 were chosen on the basis of an inspection of the distribution of sample observations.

Upon inspection of the schedule of yields for both barley and wheat, there is an apparent inconsistency. This can be best pictured by observing Figure 2 which is a graph of the two schedules (continuous and fallow) for wheat, and Figure 3 which is a graph of the two schedules for barley. In both cases, the lower moisture levels produce a greater crop yield for the continuous crop than for the crop preceded by fallow. Because there is no apparent answer to this inconsistency, the decision was made to use the upper envelope of the two curves as a production schedule for both crops, and to adjust the matrix of expected immediate returns by assuming different levels of nitrogen fertilizer application and the corresponding changes in costs. This will be mentioned further in the discussions of costs of production, and the matrix of expected immediate returns.

FIGURE 2: Yield Schedules for Wheat, based on Regression Analysis of Experimental Data

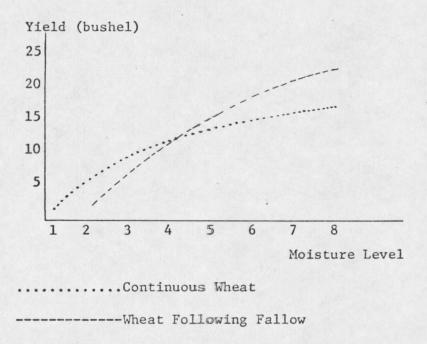


FIGURE 3: Yield Schedules for Barley, based on Regression Analysis of Experimental Data

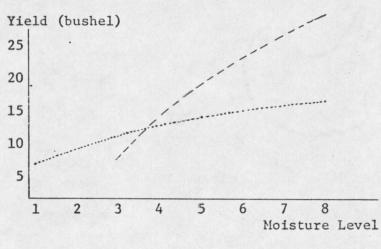


 Table 11 is a schedule of those yields and actual soil moisture levels which were used in developing the matrix of expected immediate returns.

TABLE 11: Yields and moisture levels for wheat and barley, as used in the Dynamic Programming Model.

i	Moisture Level Midpoint (Inches)	Wheat Yield (Bushels)	Barley Yield (Bu s hels)		
1	4.5	1.1437	4.4174		
2	5.5	5.5225	7.2699		
3	6.5	8,4067	9.1487		
4	7.5	10.7068	11.9923		
5	8.5	14.4734	16.8083		
6	9.5	17.6090	20.8176		
7	10.5	20.2950	24.2521		
8	11.5	22.6443	27.2560		

Costs of Production

Table 12 shows the breakdown of costs for the three decision alternatives. Although there are only three decision alternatives, there are two costs each for the alternatives of wheat and barley. As was mentioned in the previous section, this was done in order to include the differential between crop following crop and crop following fallow in the expected immediate returns. By using the same production schedules for continuous and fallow treatments, there is no allowance for the beneficial effects of fallow other than the storage of soil Thus it was necessary to adjust the costs of production to reflect this beneficial effect. The assumption was made that the effect of fallow was to increase the available nitrogen in the soil by 20 pounds per acre. The difference in costs, then, between continuous cropping and crop-fallow, is in the cost of 20 pounds of nitrogen fertilizer. It is assumed that the operator applies 20 pounds to each acre at planting time after a year of fallow, and applies 40 pounds per acre at planting time after cropping, when he has made the decision to crop.

TABLE 12: Variable Costs of Production 1

	Whe	at	Bar	ley	Fallow
·	After Fallow	After Crop	After Fallow	After Crop	
Tractor, Operating	.92	.92	.93	.93	1.45
Equipment, Operating ²	.79	. 79	.82	. 82	.13
Hired Labor	.52	.52	.51	.51	.90
Material	1.70	1.70	1.30	1.30	
Hired Labor - Hauling	.38	.38	.38	.38	
Fertilizer	1.76	3.52	1.76	3.52	
Other Operating Costs					
Pickup	.14	.14	.14	. 14	.14
Car	.13	.13	.13	.13	.13
Service Trucks	• 04	.04	.04	-04	• 04
. Shop Tools	.16	-16	.16	.16	.16
Hauling (Truck)	.46	.46	.46	.46	
Auger	.01	.01	.01	.01	
Indirect Costs ⁴	.70	.88	•66	.84	. 29
Interest on Operating Capital	.31	.39	.29	.37	.13
Total	8.02	10.04	7.59	9.61	3.37

Cost data are taken from an ERS Costs and Return Survey conducted by Dr. Walter G. Heid, Jr., Farm Production Economics Division - Economic Research Service, U.S.D.A., Stationed at Montana State University, Bozeman, Montana.

 $^{^{2}}$ Includes .03 for top-dressing of fertilizer for wheat and barley.

 $^{^3\!\!}$ Assumes 20 pounds of N following fallow, and 40 pounds of N following crop. N assumed to cost 8.8c/pound.

 $^{^4}$ Assumed to be 10% of costs (Excl. interest on operating capital).

 $^{^{5}}$ Includes charge for all operating capital, borrowed for 6 months at 8%.

Expected Immediate Returns

By multiplying price by yield for each state, and subtracting the costs of production for the activity under consideration the expected immediate returns are determined. This uses the relationship

$$q_{i}^{k} = [(Y_{i}^{k}P_{y}) - C_{k}]$$

where

 q_i^k = Expected immediate returns for the Kth alternative in the ith state.

 Y_{i}^{k} = Yield for the Kth alternative in the ith state.

 P_k = Assumed market price of K^{th} commodity.

 c_k^i = Variable production costs associated with the Kth alternative in the ith state.

i = The state index

K = The decision index; K = 1,2,3, = Fallow, Barley, Wheat.

The model was run three times. The first was the sixteen state model using price set 1. The second was the twenty four state model using price set 1. The third was the twenty four state model using price set 2. Table 13 show price sets 1 and 2.

TABLE 13: Prices Used in Analysis

Price Set	Price of Wheat	Price of Barley
1	\$1.25 per bu.	\$.90 per bu.
2	\$1.00 per bu.	\$.80 per bu.

Table 14 lists the expected immediate returns for runs 1, 2 and 3.

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TABLE 14: Expected Immediate Returns, Runs 1,2 and 3, Run Number One State Run Number Two Run Number Three Fallow Barley Wheat Fallow Barley Wheat Fallow Barley Wheat -3.37 -3.61 -6.59 -3.37 -3.61 -6.59 -3.37 -4.05 **-6**•88 -1.05 -3.37 -1.05 -1.12 -3.37 -1.12 -3.37 -1.77 -2.50 -3.37 •64 2.49 -3.37 2.49 •64 -3.37 -0.27 • 39 -3.37 3.20 5.36 2.00 -3.37 3.20 5.36 -3.37 2.69 -3.37 7.54 10.07 -3.37 7.54 10.07 -3.37 5.86 6.45 -3.37 11+15 13.99 -3.37 11.15 13.99 -3.37 9.07 9.59 -3.37 14.24 17.35 -3.37 14.24 17.35 -3.37 11.81 12.27 -3.37 16.94 20.29 -3.37 16.94 20.29 -3.37 14.22 14.62 -3.37 **-**5∙63 -8.61 -3.37 -5.63 -8.61 -3.37 -6.07 -8.90 10. -3.37 ~3.07 -3.14 -3.37 -3.07 -3.14 -3.37 -3.79 ~4.52 11. -3°37 -1.38 •47 -3.37 -1.38 -3.37 •47 -2.29 -1.63 12. -3.37 1 4 18 3 • 34 -3.37 1.18 3.34 -3.37 \ -0.02 •67 13. -3.37 5.52 8.05 -3.37 5.52 8.05 -3.37 3.84 4.43 14. -3.37 9.13 11.97 -3.37 9.13 11.97 -3.37 7.05 7.57 15. -3.37 12.22 15.33 -3.37 12.22 15.33 -3.37 9.79 10.25 16. 17. -3.37 14.92 18:27 -3.37 14.92 18.27 -3.37 12.20 12.60 -3.37 -5•63 -3.37 -6.07 18. -3.37 -3.07 -3.37 -3.79 19. -3.37 -1.38 -3.37 -2.29 20. -3.37 1.18 -3.37 -0.02 21. -3.37 5.52 -3.37 3.84 22. -3.37 9:13 -3.37 7,05 23. -3,37 12.22 -3.37 9.79 24. -3.37 14.92 -3.37 12.20

CHAPTER V

GENERATION OF AN OPTIMAL CROPPING DECISION RULE USING DYNAMIC PROGRAMMING

Up to this point the theory underlying dynamic programming has been discussed, and the assumptions underlying the analysis have been specified. This chapter presents the findings of the model itself as well as a discussion of the results.

In Tables 15,16 and 17 the letters F, B and W represent the decision which is recommended in the optimal policy, and denote the decisions to fallow, plant barley, and plant wheat. The eight moisture levels, when combined with the land use in the preceding stage, define the state variable. These moisture levels are defined in Table 10 in Chapter 4. The state variable is that variable whose value is determined at decision time, and upon which the decision is based. In the decision process, the producer knows what he did with the land in the previous stage, and he measures the moisture in the first four feet of soil at planting time. Knowing the values of these variables, the decision-maker uses the model and its optimal policy to tell him whether to fallow, plant wheat, or plant barley.

The optimal policy is a set of conditional decision rules.

The decision is based on the state of the system at the time a decision must be made. If the land is moist and fertile the decision will likely be to plant a crop. If the land is very dry and short of

Nitrogen, the decision will likely be to try to improve the conditions for the following year by allowing the land to lie fallow for a year. The optimal policy developed here is optimal in a stochastic sense. That is to say that the model is one that allows the farm decision—maker to make optimal use of his moisture on a probabilistic basis. By studying the physical cropping system it is possible to better understand the likelihood of increasing soil moisture through the process of fallow. By using this understanding in the formulation of a decision—making model, it is possible to develop a flexible cropping policy. Such a policy allows continuous cropping in moist years, but also allows for fallow where it is likely to be most advantageous. A decision based on the optimal policy could turn out wrong, but it is the decision most likely to be successful based on the available knowledge of the system.

SPECIFIC RESULTS OF THE THREE RUNS

The stage variable, n represents the number of years left in the planning horizon. In interpreting Tables 15, 16 and 17, n=1 represents that one point in time when the planning horizon of the decision-maker is one year; n=2, 2 years, and so on.

¹c.f. M.S. Stauber and Oscar R. Burt, "Crop-Fallow or Continuous Cropping: Which is More Profitable?" Big Sky Economics. Cooperative Extension Service, Montana State University, Bozeman, Montana, April 1971.

From a conceptural standpoint the results of the three runs are what would be intuitively expected with one major exception. The model, at stage n = 1, tells the decision-maker that he should choose alternatives which are expected to yield negative immediate returns. Those decisions are starred in each of the three tables of results. Stage n = 1 represents that point in the decision process when only one year remains. He is therefore, not likely to fallow, since fallow incurs a cost and is only included in his set of possible alternatives because it can better his position in future stages by increasing soil fertility and available moisture. The decisionmaker is equally unlikely to plant a crop when he can be reasonably sure of incurring a loss because of low expected yield. In the case of those decisions which are starred, his logical decision is to do nothing at all. In so doing he foregoes alternatives which are expected to have negative immediate returns, maximizing his return function by choosing zero returns. The fourth alternative, to do nothing, can easily be included in the model. This alternative should be included when this model is further developed or applied.

Run Number One. Run number one is over the sixteen state model, where the alternative of spring wheat following spring wheat is allowed.

Price set 1 is used to determine the expected immediate returns.

Table 15 summarizes the optimal policy for stages one through

five. Burt and Allison have demonstrated that dynamic programming will converge to a uniform optimal policy for an infinite planning horizon as n becomes large. While it is not obvious from Table 15, the model has converged to that policy in stage n = 3. The policy did remain constant throughout the remainder of the fifty iterations that were computed. The policy at n = 5 is that policy which the model suggests is optimal for all stages, n > 2. In state set (1-8) the policy is to fallow at the three lowest moisture levels, and plant wheat at all other levels. In states 9 - 16 the policy is to fallow at the four lowest moisture levels, and again plant wheat at the remaining higher moisture levels. Over a period of time then, one might see any combination of wheat and fallow alternatives chosen. In the optimal policy for an infinite time horizon the alternative of barley was not chosen in any state or stage. Computer print-out for $n = 1, \dots, 20$ can be found in the Appendix. Included in the print-out is the net present value of expected returns for all states and all. stages up to stage n = 20.

Run Number Two. Run number two is the twenty-four state model, where the wheat-wheat alternative is not allowed. Price set 1 is used to determine the expected immediate returns.

²Oscar R. Burt and John R. Allison, "Farm Management Decisions with Dynamic Programming", <u>Journal of Farm Economics</u>, Vol. XLV, No. 1, February, 1963.

TABLE 15: Optimal Decision Rule in Stage N = 1,...,5; Run Number One

Sta	te Descript	ors	Stage of the Decision Process					
State i	Moisture Level	Land Use in Preceding Stage	Stage N = 1	N = 2	N = 3	N = 4	N = 5	
1	1	P	F*3	F	F	F	F	
2	2	P	B*	P	F	F	F	
3	3	F	W	P	F	F	F	
4	4	F	W	F	W	W	W	
5	5	F	W	W	W	W	W	
6	6	F	W	W	W	W	W	
7	7	F	W	W	W	W	W	
8	8	F	W	W	W	W	W	
9	1	C	p*	F	F	F	F	
10	2	C	B*	F	F	P	F	
11	3	C	W	F	F	F	F	
12	4	C	W	F	F	F	F	
13	5	C	W	W	W	W	W	
14	6	C	W	W	W	W	W	
15	7	C	W	W	W	W	W	
16	8	C	W	W	W	W	W	

The eight moisture levels are defined in the preceding chapter; the moisture level is based on the number of inches of available soil moisture in the first four feet of the soil profile at the time of spring planting.

 2 In this sixteen state model, for state definition, wheat and barley in the preceding stage are treated identically and labeled crop. This assumption is lifted in the twenty-four state model.

 3 Starred decision alternatives provide negative immediate returns. In those cases the optimal policy is to do nothing at all.

Table 16 summarizes the optimal policy for stages one through six and ten. This run converged at the tenth iteration. The optimal decision rule for an infinite planning horizon is found in the far right column, n = 10. When the decision in the preceding stage was to fallow, the rule says to fallow again at the three lowest moisture levels, and plant wheat at all other levels. When the decision in the preceding stage was to fallow, the rule says to fallow again at the three lowest moisture levels, and plant wheat at all other levels. When the decision in the preceding stage was to plant wheat the model says to fallow at the five lowest moisture levels, but now says to plant barley at the three highest levels.

There are two characteristics of this decision rule which are particularly valuable to decision-makers. One is that in states following fallow the model says to plant wheat at moisture level four, while in both alternatives following a crop the decision is to fallow at the same moisture level. The other characteristic is the presence of the barley alternative in the optimal policy for states 22 through 24. Because the wheat-wheat sequence was not allowed, the model chose the next best alternative, which in this case was barley.

Run Number Three. Run number three is another twenty-four state model—where the wheat-wheat alternative is not allowed. Price set 2 is used to determine the expected immediate returns.

TABLE 16: Optimal Decision Rule in Stages N = 1,...6, and 10: Run Number Two

Sta	te Descripto	ors		Stage	of the	Decision	Process		
State i	Moisture Level	Land Use in Preceding Stage	N = 1	N = 2	N = 3	N = 4	N = 5	N = 6	N = 10
1	1	F	F*1	F	F			-	
2	2	F	B*	F	F	F	7	F	F
3	3	P	W	F	P	P	F	F	F
4	,	P	W	F	W		F	F	F
	5	F	W	W		F	W	W	W
5	2	F			W	W	W	W	W
7	0	P	W	W	W	W	W	W	W
8	,		W	W	W	W	W	W	W
0	8	F	W	W	W	W	W	W	W
9	1	В	F*	F	F	· F		-	
	2	В	B*	F	F	F	F	F	F
10	3	В	W	F	F	F	F	F	F
12	4	. В	W	F	F		F	F	F
13	5	B	W	F		F	F	F	F
14	6	В	W		W	W	W	W	. W
15	7	В		W	W	W	W	W	W
16	8	В	W .	W	W	W	W	W	W
10	0	В	W -	W	W	W	W	W	W
17	1	W	F*	F	F	F	F		
18	2	W	B*	F	F	F	F	F	F
19	3	W	R*	F	F	F	F	F	F
20	4	W	В	F	F	F	F	F	F
21	5	W	В	F	В			F	F
22	6	W	B	В	В	F	В	F	F
23	7	W		В		В	В	В	В
24	8	W	B	В	В	В	B	В	В
		"	B	В	В	В	() B	В	В

 $^{^{\}rm 1}{\rm Starred}$ decision alternatives provide negative immediate returns. In those cases the optimal policy is to do nothing at all,

Table 17 summarizes the optimal policy for the first five stages for run number three. The model converged at the fifth iteration, so the optimal decision rule for an infinite planning horizon can be found in stage n = 5. This optimal rule is slightly different than that for run number two. The difference in model structure between runs two and three was a slight change in the assumed prices received for wheat and barley. The changes had the effect of increasing the relative price of barley in run number three, and hence making barley a slightly more attractive alternative.

The optimal policy for an infinite planning horizon is: to fallow at the first three moisture levels and plant wheat at all others following fallow; to fallow at the first four moisture levels and plant wheat the remaining four following barley; and fallow at the first four moisture levels and plant barley the remaining four following wheat.

Of particular interest on run number three was the optimal policy in stage n=2. Here, in states 6, 7, 8, 15, and 16, the policy is to plant barley so that wheat (which cannot follow wheat) can be planted in stage n=1.

The Appendix contains computer output up to stage n=20 for runs 1, 2, and 3.

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TABLE 17: Optimal Decision Rule in Stages N = 1,...,5; Run Number Three

Stat	te Descripto	ors		Stag	e of the	Decision	Process		
tate i	Moisture Level	Land Use in Preceding Stage	N =	1 N = 2	N = 3	N = 4	N = 5		
1	1	F	F*	F	F	¥	F		
2	2	P	B*	F	F	F	F		
3	3	F	W	F	F	F	F		
4	4	F	W	F	W	F	W		
5	5	F	W	W	W	W	W		
6	6	F	W	В	W	W	W		
7	7	F	W	В	W	W	W		
8	8	F	W	В	W	W	W		
9	1	F	F	F	F	F	F		
10	2	F	F.	F	F	F	F		
11	3	F	F,	F	F	F	F		
12	4	F	W	F	F	F	F		
13	5	F	W	F	W	W	W	1	
14	6	F	W	W	W	W	W		
15	7	F	W	В	W	W	W		
16	8	F	W	В	W	W	W		
17	1	F	F,	F	F	F	F		
18	2	F	F'	F	F	F	F		
19	3	F	B,	F	F	F	F		
20	4	F	B	F	F	F	F		
21	5	F	B	F	В	F	В		
22	6	F	B	В	В	В	В		
23	7	F	ll B	В	В	В	В		
24	8	F	B	В	В	В	В		

 $^{^{1}\}mathrm{Starred}$ decision alternatives provide negative immediate returns. In those cases the optimal policy is to do nothing at all.

COMPARING THE OPTIMAL POLICIES WITH FIXED DECISION RULES

In order to provide a meaningful basis for comparison with the optimal policies, two models were formulated and computed using fixed decision rules. They were structured and run in a dynamic programming framework so that the same moisture level transition probabilities could be used.

The first comparison model uses the fixed decision to continuously plant barley, regardless of the soil moisture level.

The second comparison model uses the fixed decision to follow a rigid wheat, fallow, wheat, fallow,... sequence.

Table 18 shows the computer print-out for stage n=20, for runs 1 and 2 and the two comparison models.

TABLE 18: Fixed or Optimal Policies; A Comparison of Present Value of Expected Returns at stage n = 20.

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There is no evidence to suggest that the dryland grain farmer strictly adheres to such rigid decision rules as those considered in the two comparison models. They do, however, provide some very interesting results for comparison purposes. Table 19 summarizes some figures from Table 18. First, the four models are compared when each begins a twenty year period at the highest level of moisture. Then, each is compared when the period is initiated at the lowest level of moisture. In each case the returns follow this order: run number one, run number two, wheat-fallow, and continuous barley.

TABLE 19: Fixed or Optimal Policies; a comparison of present value of Expected Returns at Stage n = 20, i = 8 and i = 1.

Initial State	Run #1	Run #2	Continuous Barley	Wheat- Fallow
i = 8	\$75.71	\$70.53	\$5.17	\$64.31
i = 1	\$54.09	\$50.02	-\$17.82	\$35.38

From a comparison based on identical conditions, the optimal policies yield the farmer a higher set of expected returns than the fixed policies. It is significant to note that if the individual were to follow the policy of strictly continuous barley he would most likely go broke. It should be pointed out, however, that the results are valid only for the area from which the data were generated.

CHAPTER VI

OUR ENVIRONMENT

EXTERNALITIES

This analysis has been a study of the internal decision process of the individual producer. Assumptions have included constant commodity prices, constant technology, constant costs, and a fully internal accounting of the costs of production. This chapter is concerned with the last assumption.

The current widespread environmental concern, and the substantial environmental problems affecting agriculture in Montana and elsewhere, suggest that there may be some aspects of a purely internal (i.e. internal to the individual firm under consideration) cost accounting system which should be considered from a broader perspective. Such costs may result from the use of herbicides, pesticides, commercial fertilizers; from individual farming practices that create areawide problems such as saline seep and erosion; from other causes.

If the future should show that significant external costs accrue when any or all of the decision alternatives facing the individual producer are chosen, then the principle of social optimality dictates that they should be considered in the analysis of his decision process. A broader definition of costs (and gross returns as well) could have a significant effect on the optimal policy. This

effect would be felt in a situation where the decision-maker paid the full cost, and where costs of one activity changed at a different rate than the costs of other activities. If the costs associated with fallow should go up, ceteris paribus, we would expect him to fallow less. If the costs of producing wheat relative to barley should increase, the optimal policy would reflect the change.

This chapter discusses the potential existence of costs assotiated with (1) the fallow alternative, and (2) using chemicals in any alternative. This discussion utilizes a familiarity with the production activity, but is conceptual rather than empirical due to the lack of relevant information. Without trying to discount this discussion, it is suggested that normative conclusions not be drawn until more information is available.

FALLOW

It is generally accepted today that in certain situations allowing land to lie fallow contributes to the problem of saline seeped lands. If saline seep takes productive land out of production, or if it causes changes in the productive activity which increase operating costs, then it constitutes a cost in itself. If its appearance is totally a function of the management decisions of an individual operator, and if it appears only on his land, then it is an internal cost. That is, he necessarily bears the full cost of his

activity. If, on the other hand, one man bears the cost due to the actions of another man adjacent or even far-removed from his property, that cost is an external cost. It is generated outside his sphere of activity and control. It was apparent from the Saline Seep-Fallow Workshop that the hydrology of many of the affected areas is very complex. That means that if the saline seep difficulties are due to producers' cropping decisions, they probably overlap from one man's operation onto another's. When that hydrologic system is better understood, the economist will be in a position to estimate the true cost of a cropping decision. Even with this estimate, however, the institutional framework will likely dictate who bears that cost.

THE USE OF CHEMICALS

The multiple benefits of summer fallow practices have been mentioned earlier. Today, through the use of modern chemicals, the producer can overcome many of the problems faced by the early grainfarmer without having to resort only to summer fallow. In fact, the practice of summer fallow can be made even more efficient using

Highwood Alkali Control Association Saline Seep-Fallow Workshop, Proceedings of a Workshop Held at the Rainbow Hotel, Great Falls, Montana, February 22-23, 1971. (Bozeman, Montana: Cooperative Extension Service, 1971).

modern chemicals. According to Smika, the use of herbicides has made it possible to greatly increase fallow period water storage efficiency. The purpose of this section is to discuss the scope of pesticide use in Montana agriculture, and to suggest that there may be real costs associated with the use of chemical pesticides which are not being reflected in the market price which the farmer pays for the chemicals.

Although the use of inorganic materials for pest control has been seen for over a century, the greatest use and rate of increase have come since World War II. The following quote shows the status in 1952. "Newer and more effective pesticides continually come into use. With these new developments, acreages of farm crops and farmland treated for pests have expanded markedly. Purchases of power sprayers and power dusters in recent years have been large, more than six times the average annual purchases of the prewar period." From a U.S.D.A. study, the following figures were given as percentages of small grain crop acres treated with herbicides to control weeds, for the 48 adjacent states: 1952, 12%; 1958, 20%; 1964, 23%; 1966, 29%.

D.E. Smika, Summer Fallow for Dryland Winter Wheat in the Semiarid Great Plains, Paper No. 2413, Journal Series, Nebraska Agricultural Experiment Station

³Albert P. Brodell, and others. Extent and Cost of Spraying and Dusting on Farms - 1952. Statistical Bulletin No. 156, U.S.D.A., Agricultural Research Service.

From the same study, comes the report that in the "mountain" region of the U.S., which includes Montana, 50% of wheat crop acres were treated with herbicides in 1966.4

Inspection of data from a survey by Heid in 1969 showed that even greater increases in pesticide use have taken place in the three or four years since 1966. In data from a random sample of 31 wheat farmers in the triangle area of north-central Montana, 100% of those surveyed reported using herbicides on their wheat crops for weed control. All but one of those 31 reported using the herbicide 2,4-D. Preliminary analysis of the farmers using herbicides shows between 90 and 95% of the acreage being covered. From that same survey in the triangle area, 74% of the farmers reported that they use treated seed. The seed is treated with a chemical compound for protection against ground diseases such as smut.

Chemical pesticides are widely used in Montana. If one listens to the voices of doom, our downfall rides with the sprayplane. Much has been said about the evils of chemical pesticides in recent years.

Austin Fox, and others. Extent of Farm Pesticide Use on Crops in 1966. Agricultural Economic Report No. 147, U.S.D.A., Economic Research Service.

Data from a survey by Walter G. Heid, Jr., Farm Production Economics Division, Economic Research Service, U.S.D.A., Montana State University, Bozeman, Montana.

If even a fraction of what has been said about them is true, there are external costs associated with their use. The level at which they should be used depends on the nature of the costs and benefits associated with their use, and should be expected to vary with many other variables.

In a statement which may not be totally valid today, Rachel Carson has very eloquently discussed the economics of pesticide use as seen through her eyes.

The chemical weed killers are a bright new toy. They work in a spectacular way; they give a giddy sense of power over nature to those who wield them, and as for the long-range and less obvious effects--these are easily brushed aside as the baseless imaginings of pessimists. The "agricultural engineers" speak blithely of "chemical plowing" in a world that is urged to beat its plowshares into spray guns. The town fathers of a thousand communities lend willing ears to the chemical salesman and the eager contractors who will rid the roadsides of "brush"--for a price. It is cheaper than mowing is the cry. So, perhaps, it appears in the neat rows of figures in the official books; but were the true costs entered, the costs not only in dollars but in the many equally valid debits we shall presently consider, the wholesale broadcasting of chemicals would be seen to be more costly in dollars as well as infinitely damaging to the long-range health of the landscape and to all the varied interests that depend on it.6

^{---- 6} Rachel Carson, Silent Spring; (Boston; Houghton Mifflin, 1962)

CHAPTER VII

SUMMARY

The seed of this study was a concern over the saline seep problem. The concern grew and took shape as an economic analysis of the determinants of grain cropping decisions. The analysis included a survey of historical determinants of cropping decisions, and a quantitative analysis designed to provide a cropping policy which would be optimal for a planning horizon of any length.

This study is an effort to take a relevant problem of our time, consider it in its economic environment, and in a positive sense apply current economic technology in the form of dynamic programming to provide understanding to help alleviate that problem. Many factors in addition to saline seep add to the pertinence of the study.

While much of the study is very technical in nature, the result is a product which should be understood by most people involved in grain production. An optimal cropping decision policy has been developed which is not constant, but is conditional. A conditional policy is one which says that if you observe A, then you should do B; if you do not observe A or observe something else, then do

C. The set of optimal policies which are presented in this study tell the individual producer: "If soil moisture is a certain level

at planting time, and the field was cropped the preceding year, you should plant a crop, or you should not." The policy is a flexible one, as opposed to the traditional crop-fallow-crop-fallow... fixed decision rule. It allows the farmer to make the best use of his soil moisture, and maximize his expected returns over any length planning horizon as well.

CONCLUSIONS

It must be pointed out, however, that the comparisons were made with experimental data, and on a rigidly fixed bias. Although, there is no reason to assume that the average dryland grain farmer in Montana is likely to use such a rigid decision rule, neither is there an indication that he is, in a probabilistic sense, taking optimal advantage of the soil moisture resource in his production activities.

Knowledge of soil moisture at spring planting time is a practical decision variable. Currently there is a technique using a buried neutron source, which provides a measure of soil moisture with very little effort.

Government programs now allow greater flexibility to the individual producer in farming his non-program acres. An optimal cropping policy would be of value in such situations today, even if there were no changes in the programs. However, should there be

more restrictive program changes, an optimal policy of cropping decision-making could be used to determine the economic value of program participation. For example, lowering the maximum payment limitation could have the effect of forcing producers out of the assistance program. Table 18, in chapter 5, is a summary of expected returns following four different decision policies, two fixed and two optimal. From this comparison, it is clear that the optimal policy generated through the use of dynamic programming does in fact provide a better decision rule than the fixed policies with which it was compared.

RECOMMENDATIONS

The major difficulty encountered in formulating this analysis was balancing the trade-off between the many years of data needed to estimate the transition probabilities, and the desire to use the most current information regarding the production activity, such as current production techniques, and a broader set of alternatives.

In addition, when the model is considered for application to an individual farmer's decision-making process, how realistic are the data needs? The major problem to be found in applying this to an individual farmer's operation is going to be the generation of the transition probabilities. In future development and application of this model, an effort should be made to develop a methodology for generating transition probabilities in the absence of long-term physical

data for the specific plot under consideration.

This analysis has not fully considered the government programs. With the potential of many modifications of the government programs, such as greater payment limitations, the implications of such changes on the policies of grain producers should be considered.

To the best of the knowledge of the author, this optimal policy methodology has never been tested in the field. With the growing interest in programmed decision-making models, field experiments should be using them to determine their relevance to real-world applications.

BIBLIOGRAPHY .

- Aasheim, Torlief S. "Interrelationships of Precipitation, Soil Moisture and Spring Wheat Production in Northern Montana." Huntley Branch Station, June 1954. (Mimeographed Circular 101.)
- Army, T.J., and W.D. Hanson. "Moisture and Temperature Influences on Spring Wheat Production in the Plains Area of Montana." Production Research Report No. XXXV. Agricultural Research Service, U.S.D.A., in cooperation with M.A.E.S.
- Bauer, Armand. "For Dry Land Wheat Production Evaluation of Fallow to Increase Water Storage." North Dakota Agricultural Experiment Station, Reprint No. 666 from May June, Farm Research, Vol. XXV, pp. 6 9.
- Bellman, Richard. <u>Dynamic Programming</u>. Princeton University Press, 1957.
- Brodell, Albert P., and others. Extent and Cost of Spraying and

 Dusting on Farms 1952. Statistical Bulletin No. 156, U.S.D.A.,

 Agricultural Research Service.
- Brown, Paul L. "Water Use by Dryland Winter Wheat and Barley. Influence of Fertilization Process." Soil Fertility Forum. December 4 5, 1969, Great Falls, Montana.
- Burke, Edmund, and Rueben M. Pinckney. "The Influence of Fallow on Yield and Protein Content of Wheat." University of Montana Agriculture Experiment Station, Bulletin No. 222, July 1929, Bozeman, Montana.
- Burlingame, Merrill G., K. Ross Tool, and Robert G. Dunbar. A History of Montana. New York: Lewis Historical Publishing Co., 1957.
- Burt, Oscar R. "Operations Research Techniques in Farm Management:
 Potential Contribution." Journal of Farm Economics, Vol. XXXXVII,
 No. 5, December 1965, pp. 1418-1426.
- , and Ralph D. Johnson. "Strategies for Wheat Production in the Great Plains." Journal of Farm Economics, Vol. XXXXIX, No. 4, November 1967.

- Candler, Wildred. "Reflections on 'Dynamic Programming Models'."

 Journal of Farm Economics, Vol, XLII, No. 4, November 1960.
- Carson, Rachel. Silent Spring. Boston: Houghton Mifflin, 1962.
- Fox, Austin, and others. Extent of Farm Pesticide Use on Crops in 1966.

 Agricultural Economic Report No. 147, U.S.D.A., Economic Research Service.
- Hargreaves, Mary Wilma M. Dry Farming in the Northern Great Plains. Cambridge: Harvard University Press, 1957.
- Heid, Walter G. Jr. Fertilizer Use in Montana With Comparison 1954-1967. Montana Agricultural Experiment Station Bulletin No. 628, May 1969, Bozeman, Montana.
- Highwood Alkali Control Association Saline Seep-Fallow Workshop.

 Proceedings of a Workshop Held at the Rainbow Hotel, Great Falls,

 Montana, February 22 23, 1971. Bozeman, Montana: Cooperative
 Extension Service, 1971.
- Howard, Ronald A. <u>Dynamic Programming and Markov Processes</u>. The M.I.T. Press, Cambridge, Massachusetts, 1960.
- Hutton, Robert F. "Operations Research Techniques in Farm Management: Survey and Appraisal." <u>Journal of Farm Economics</u>, Vol. XXXXVII, No. 5, December 1965, pp. 1400 - 1414.
- Johnson, Ralph D., and Robert M. Finley. "Yield-Precipitaion Relation-ships and Some Suggested Strategies for the Transition Area of Nebraska." <u>Journal of Farm Economics</u>, Vol. XXXXV, No. 5, December 1963, pp. 1232 1237.
- Johnson, W.C. "Some Observations on the Contribution of an Inch of Seeding-Time Soil Moisture to Wheat Yield in the Great Plains." Agronomy Journal, 56: 29-35, 1964.
- Kmuch, H.S., and others. "Root Development of Winter Wheat as Influenced by Soil Moisture and Nitrogen Fertilizer." Agronomy Journal, 49: 20-25, 1957.

- Knetsch, Jack L. "Moisture Uncertainties and Fertility Response Studies." Jouranl of Farm Economics, Vol. XXXXI, February 1959, pp. 70 - 76.
- Leggett, G.E. "Relationships Between Wheat Yield, Available Moisture and Available Nitrogen in Eastern Washington Dry Land Areas."
 Bulletin 609, Washington Agricultural Experiment Stations,
 December 1959.
- Mathews and Brown. "Winter Wheat and Sorghum Production in the Southern Great Plains Under Limited Rainfall." U.S.D.A. Circular No. 477, 1938.
- Meinzer, Oscar E. (ed.). <u>Hydrology</u>. New York, Dover Publications, . 1949.
- Morris, W.H.M. "Discussion: Operations Research Techniques in Farm Management: Potential Contribution." <u>Journal of Farm Economics</u>, Vol. XXXXVII, No. 5, December 1965, pp. 1427-1432.
- Smika, D.E. "Summer Fallow for Dryland Winter Wheat in the Semiarid Great Plains." Nebraska Agricultural Experiment Station, Journal Series, Paper No. 2413.
- Stauber, M.S., and Oscar R. Burt. "Crop-Fallow or Continuous Cropping: Which is More Profitable?" <u>Big Sky Economics</u>, Cooperative Extension Service, Montana State University, Bozeman, Montana, April 1971.

APPENDIX

Transition Probabilities, I = 1,...,24