Run-time optimization of a radiation driven crown fire model
by Patrick Timothy Call

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering
Montana State University

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MONTANA STATE UNIVERSITY
Bozeman, Montana

April 1997

APPROVAL

of a thesis submitted by

Patrick Timothy Call

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Date APRIL 14, 1997

ACKNOWLEDGMENT

The author wishes to express his appreciation to the USDA Forest Service, Intermountain Research Station for research joint venture agreement INT-95082-RJVA and to the Canadian Forest Service for its support under Green Plan research contribution "A physical model for crown fire behavior."

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NOMENCLATURE

α	thermal diffusivity	m^2/s
b	width of flame base in a line fire	m
B	Stephan-Boltzman constant	$5.67x10^{-8}$ $W/m^2/K^4$
β	fraction of volume filled with solids in fuel layer	
Bi	Biot number of fuel particle	
C	reaction rate coefficient, opacity parameter	
C_p	specific heat	kJ/kg/K
δ	layer depth	m
d	diameter	m
D	depth of burning zone	m
Fr	Froude number	
g	acceleration of gravity	m/s^2
γ	tilt angle	
h	convective heat transfer coefficient	$W/m^2/K$
H	height of flame	m
H_a	heat-release rate of fuel per unit mass of air	W/kg
H_c	heat of combustion for the fuel	KJ/kg
I	line intensity of fire	W/m
k	thermal conductivity	W/m/K

NOMENCLATURE - Continued

l', m', n'	direction cosines	
L	flame length	m
Nu	Nusselt number	
P_f	power of the fire	W/m^2 ($Btu/ft^2/s$)
P_w	power of the wind	W/m^2 ($Btu/ft^2/s$)
ρ	mass density	kg/m^3
ρ_o	oven dry density of fuel particles	kg/m^3
ρ_{wt}	weight density	N/m^3
q_1, q_2, q_i	three regimes of heat absorption in fuel bed	W/m^3
$\dot{Q}'$	heat transfer rate per unit length	W/m
Q_d^*	dimensionless buoyancy flux	
$\dot{Q}_o$	heat release rate of the fire	W
$\dot{Q}_{rad}'''$	net rate of absorption of energy in fuel bed	W/m^3
R	rate of spread for flame geometry	m/s
RR	reaction rate	s^{-1}
R_{mix}	air / flame fluid mixing rate	s^{-1}
s	distance in direction of radiation ray	m
S	speed of ignition isotherm	m/s

NOMENCLATURE - Continued

S_L	burning velocity	m/s
t_r	thermal relaxation time	s
T	temperature of particle in fuel bed	K
T_a	ambient air temperature	K
T_b	temperature of boiling point of water	K
T_B	temperature of burning zone	K
T_f	final temperature	K
T_i	ignition temperature	K
τ_{bum}	time to burn a particle	s
τ_{fl}	fluid transit time	s
τ_{mix}	characteristic mixing time	s
τ_r	characteristic heating time	s
u	horizontal component of velocity	m/s
$\bar{U}$	mean free stream wind velocity	m/s
U_c	wind velocity at crown top	m/s
V	total velocity	m/s
w	vertical component of velocity	m/s
w_c	mass of fuel consumed per unit area	kg/m^2
w_o	dry fuel loading of a layer	kg/m^2

ABSTRACT

A numerical model of the behavior of wildland forest fires is studied. The model predicts spread rate, flame geometry, and intensity of a crown fire using the radiation transport equation. This thesis presents dimensional analyses applying the physics of fire to justify the model assumptions, and introduces a closure scheme that expedites the model convergence. The closure scheme minimizes the number of outer iterations the model performs by efficiently converging to a consistent solution. It was discovered that the number of outer iterations can be minimized through the use of a linear approximation scheme. The selected closure scheme has performed well in the limited number of test cases for which data are available.

CHAPTER 1

INTRODUCTION

For many decades, research has been conducted in an attempt to better understand the phenomena of fire. As one of mankind's first tools, fire is an invaluable source of energy and a tremendous means of destruction. Each year, fire claims on average 10-20 lives per million people population (Cox 1995). This, along with the staggering impact fire directly and indirectly has on the economy, is a motivation for scientists and engineers to try to better understand physical principles governing fire.

The fires responsible for loss of life and destruction of property can be classified into two major categories: structural and wildland. Structural fires occur in buildings or other man-made structures, and are generally confined by artificial boundaries. Wildland fires burn in grass, brush, timber or other forms of vegetation, and are not necessarily confined by artificial boundaries. With the aid of wind and topography, free-burning wildland fires can create a tremendous impact (although not always bad) on the vegetation and wildlife in the area.

Wildfires can be categorized into three major groups: ground fires, surface fires, and crown fires. Ground fires burn the organic layers found below the surface and generally creep at a very slow pace (Albini, 1984). Surface fires burn the vegetation on or near the surface and have the potential to move at very high speeds. Surface fires

consume fuels that include litter, grasses, and shrubs, along with the larger dead fall and live juvenile trees. A crown fire burns the foliage and branches in the tops of trees. Crown fires perpetuate their motion by moving from treetop to treetop, although there usually is a surface fire occurring simultaneously with the crown fire.

The most destructive of these three types of wildland fires is the crown fire. Crown fires account for less than 10 percent of the total wildland fires, but burn 90 percent of the total acreage (Albini and Stocks, 1986). All crown fires start out as surface fires with all the burning taking place below the crown. If conditions such as ample wind and dense surface vegetation are met, a surface fire can climb into the crown and continue its motion in the crown (Van Wagner, 1977).

Fire management personnel are interested in controlling and even harnessing wildland fires, thus continuing efforts are in progress to create models that can predict the speed, intensity, and direction of motion of wildland fires. Several models exist today that attempt to solve the problem of modeling a wildland fire. These models are primarily empirical in nature. They consider wind, terrain, moisture content of the fuel, and type of fuel as parameters. These models use controlled experimental fires in which all the fuel characteristics are determined prior to ignition, and the speed, shape, and size are recorded after the burn. With enough experimental data, a 'black box' model can be created that takes inputs and then estimates a solution using predicted fire behavior. Empirical models are approximate and do not attempt to explain the fluid flow processes and heat transfer mechanisms involved in a wildland fire. Understanding these processes may ultimately lead to broadly applicable models.

The phenomenology of free burning wildland fires is described in Albini (1984; 1992). Free-burning fires spread by igniting new fuel at the periphery. Vegetation fuels vary in size, moisture content, and compactness, so that the finest fuels are the agents of spread. They are most readily ignited and burn out most rapidly. Only fuels up to approximately a few millimeters in diameter participate in the spread process. Other components may interfere with the heat transfer process and add to heat required for ignition, but contribute little to the heat release.

The ignition of the fuel is controlled by piloted ignition of volatile pyrolyzate, the combustible gases released by thermally decomposing vegetable matter. Gases burn upon mingling with oxygen, releasing heat energy in the form of radiation and hot gaseous products of combustion. Radiation from the sooty flames of natural fires is almost entirely due to small (approximately less than 10^{-5} meters in diameter) solid particles in the flame that are heated by contact with the hot products of gaseous combustion and by glowing combustion of the particles.

Small vegetation fuel particles (leaves, small twigs, bark flakes, etc.) once ignited burn on the order of 10 seconds before they are reduced to mineral ash. The heat released in this process causes the mixture of gases (pyrolyzate, air, and gaseous combustion products) to rise buoyantly, creating the visible structure we identify as the flame. This structure, reaching from the surface to above the top of the highest fuel layer involved in the fire, radiates vigorously. In a crown fire, where flames may extend tens of meters above the tops of trees, exposed skin can be promptly blistered at a distance of 100 meters.

If the fire advances at a steady rate, then during the combustion of ignited fine fuels, enough heat must be transferred to the same mass of unignited fuel particles ahead of the fire front to raise their temperatures to the piloted ignition temperature, about 600K (Stockstad, 1975). In typical conifer tree crowns, the dry mass of live foliage per unit volume is on the order of 200 gm/m^3. These particles are roughly cylindrical, with length over diameter ratios very much larger than unity. It will be shown below that they can be modeled as thermally thin, or isothermal. If the foliage is thermally thin and contains water mass approximately equal to its dry mass, then the energy required simply to boil away the water is on the order of 0.5 MJ/m^3. So, if the fire front is advancing at, for example, 0.3 m/s, this is equivalent to a heat transfer rate of 150 kW/m^2. This clearly rules out thermal conduction; if the conductivity of air is assumed to be 0.05 W/mK, the thermal gradient would have to be 3×10^6 K/m. Since the maximum temperature achievable by the combustion of natural conditions is about 1500 K, the temperature difference between fuel particles at the boiling temperature of water and the hypothetical burning zone at 1500 K would have to be maintained over a fraction of a millimeter.

The required heat transfer could presumably be accomplished by convection, if the hot flame fluid passes through unignited vegetation fuel. In this work we shall be concerned with modeling the steady advance of long line fire under the influence of an aiding wind. But it will be shown below that there exists a broad regime in which the buoyant flow of the flame fluid obstructs the path of the aiding wind, allowing the free flame above the tops of the trees to stand erect, creating a zone of stagnant air immediately down stream of the burning zone. This is shown in Figure 1.

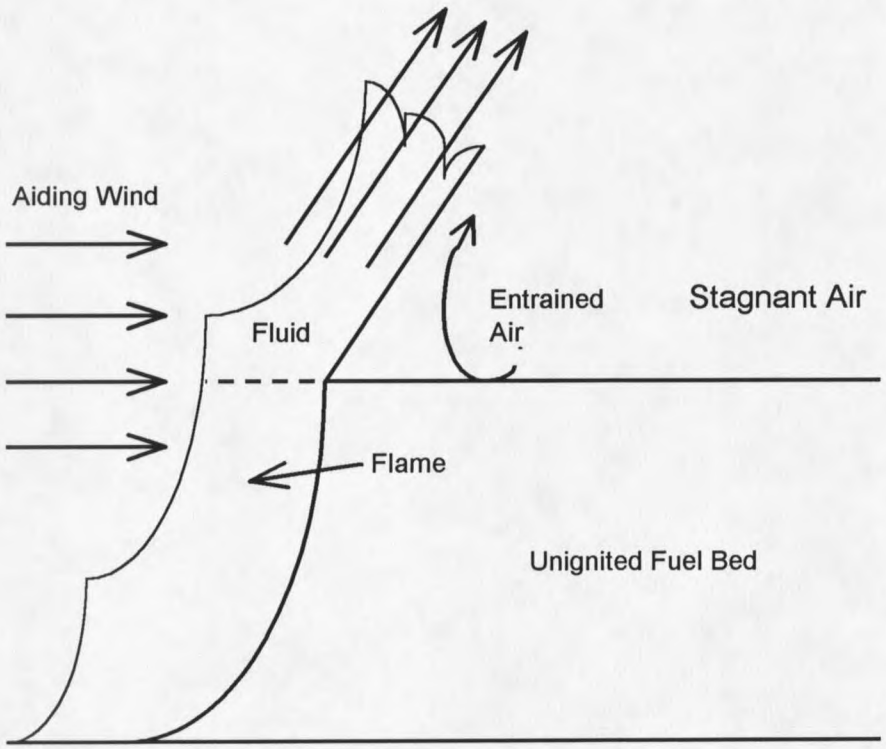

Figure 1. Air flow diagram of a free-burning fire.

In this instance, the condition being modeled, there can be no convective heat transfer to the unignited fuel because the flame fluid does not come into contact with the unignited fuel. Thus the heat transfer mechanism that causes the fire to spread is radiation, and the rate of heat transfer determines the rate of fire spread.

The fuel bed ahead of the fire can be viewed as three separate layers: surface layer, stem space layer, and crown layer (Albini, 1984). The surface layer consists of all the fuel on or near the ground and also a small portion of the duff just underneath the surface. The amount of duff contributing to the flame structure of the crown fire is only the depth the burn will reach in the initial passing of the flaming front. The remaining duff beneath this is not considered. The next layer, the stem space layer, is the area between the surface

fuels and the crown foliage. This layer is mostly comprised of boles. The top layer, the crown layer, is comprised of the small diameter fuel components along with the larger branches and boles. Each of these layers contributes to the heat energy given off in the burning zone which will determine the flame structure and hence the rate of spread of the flaming front.

A simplified fuel bed is created in order to identify the rate at which radiant heat is causing the temperature to rise in the fuel bed. There are several components used to describe the fuel. These consist of layer depth, layer opacity parameter, loading, oven dry fuel density, mineral ash fraction, moisture content, heat of combustion, and surface area to volume ratio of the fuel particles. The contributions of each fuel component to these parameters are computed according to a weighted average scheme. This scheme preserves most of the physical features of the fuel bed while combining their contributions into those of a single size class that is completely consumed by the passing fire front. This eliminates the need to classify each particle of fuel and its contribution to the flame front. The radiation driven crown fire model described by this paper uses the representative value of each of the above components as detailed in Albini (1996) and Call and Albini (in press).

Several mathematical idealizations of the situation modeled are summarized below to give an understanding of the assumptions needed to simplify the calculations. 1. The fire is modeled as a two-dimensional line fire that is aided by wind blowing in the direction of the motion of the fire. 2. The burning zone bounding surface is called the 'ignition interface', and is an isotherm at the ignition temperature, T_i. 3. The free flame is a flat

diffuse radiator. 4. The fuel bed layers are replaced by hypothetical equivalent layers made up of a single size class totally consumed by the passing of the fire. 5. The fire is characterized by spread rate and intensity, where intensity is measured as the power per unit length of the fire edge. The units of intensity in the metric system are in kW/m. 6. The incident wind field is characterized by the crown top velocity, U_c, and the log speed profile (Baughman and Albini, 1980). 7. A simplified flame structure model uses intensity, wind field and stand height to predict the flame height and tilt angle (Albini, 1981).

The two dimensional geometry of the crown fire model is illustrated in Figure 2. The spread is modeled by attaching the coordinate system to the interface, and advancing the fuel from right to left into the flame at speed S.

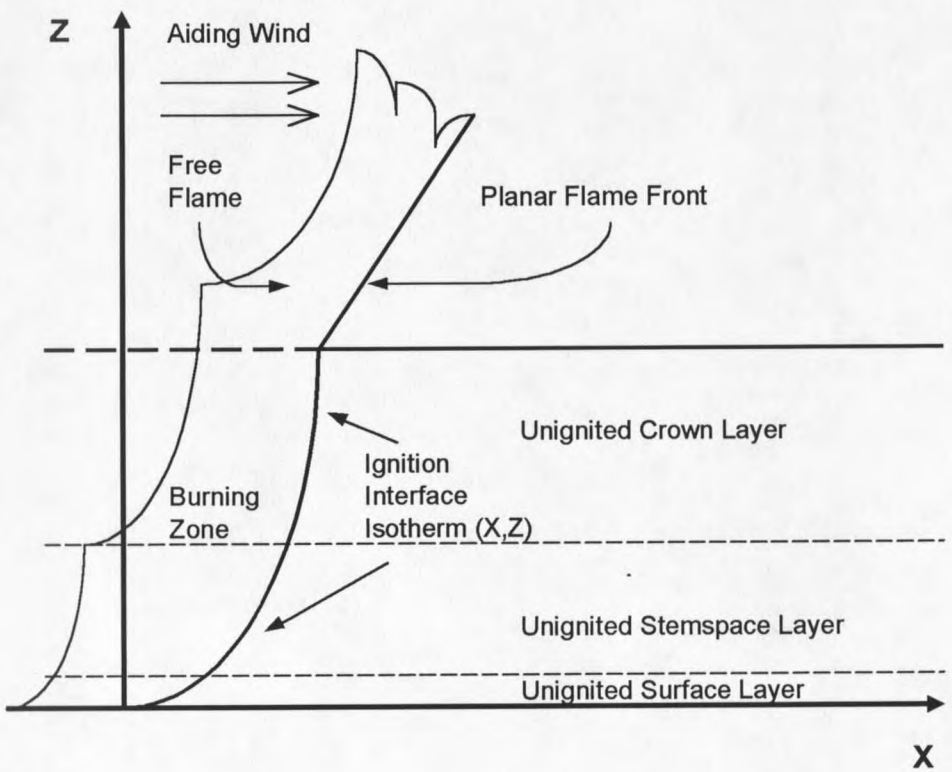

Figure 2. Geometry of flame/fuel interface.

The specific objectives of this thesis are twofold. First, to develop the physics and mathematics needed in the modeling of a radiation driven crown fire. To understand the physical mechanisms discussed briefly in this introduction, sufficient background in the temporal and spacial scales are given to support the model assumptions through the use of an order of magnitude analysis. This analysis leads into the specific development of the model by applying the governing equations of radiation heat transfer.

The model exists as FORTRAN code with four levels of iterations that are extremely process intensive. After describing the model structure, the second objective is to develop an iteration scheme that will allow the expedient convergence of the model code. This task can be accomplished either by increasing the efficiency of the iterations consuming the most time, or by minimizing the number of times the model performs these iterations. The focus of this study is to minimize the number of outer iterations the model performs which greatly decreases the time needed for convergence.

CHAPTER 2

BACKGROUND

Time Scales

Four different types of time comparisons are presented to explain the relative importance of the processes involved with a crown fire. The first time comparison is a ratio of the characteristic heating time to the thermal relaxation time which, if small enough, proves that the modeled fuel particles can be treated as thermally thin. The second comparison is the ratio of the time it takes for the fluid to transit from the base of the flame to the tip of the flame to the time it takes for the air to mix with the flame. This helps explain why the flame structure remains relatively stable as opposed to sporadic flickers of flame. The next comparison considers the time it takes to completely burn a fuel particle divided by the time for the fluid to transit the flame. This also explains how the flame maintains a stable structure. The last comparison is the ratio of the chemical reaction time to the time it takes fluid to pass through the flame from the base to the tip, also known as the Damkohler number. A very small Damkohler number will prove that the chemical reaction process need not be modeled. These four comparisons are individually analyzed below.

Characteristic heating time / thermal relaxation time

The heating characteristic time can be derived from the equation for particle heating due to convection. This equation is

$$\dot{Q}' = h\pi d(T_f - T) \tag{1}$$

where $\dot{Q}'$ is the heat transfer rate per unit length into the cylindrical fuel particle, h is the convective heat transfer coefficient, d is the diameter of the fuel particle, T is the temperature of the particle, and T_f is the final temperature. This relation can be equated to an expression for the heat storage rate in a uniform-temperature particle derived from the first law of thermodynamics. This expression is:

$$\dot{Q}' = \frac{\pi d^2}{4}\rho C_p \frac{dT}{dt} \tag{2}$$

where C_p is the specific heat of the particle and ρ is the density of the particle. When these two expressions are equated, the following differential equation is obtained:

$$\frac{dT}{dt} = \frac{4h}{\rho C_p d}(T_f - T) \tag{3}$$

The solution of this equation results in a expression that gives the exponential temperature history of a particle. This is given as:

$$\frac{T - T(0)}{T_f - T(0)} = 1 - e^{-\frac{4ht}{\rho C_p d}} \tag{4}$$

From this, the characteristic heating time is defined as:

$$\tau_h = \frac{\rho C_p d}{4h} \tag{5}$$

In this instance, the characteristic heating time is a measure of the time it takes to bring a particle of fuel to a temperature where combustion can occur.

The thermal relaxation time is the square of the characteristic dimension over the thermal diffusivity (6). This is the time a particle of fuel needs to distribute heat throughout its interior. As can be seen, this parameter is strongly dependent on the diameter.

$$t_r = \left(\frac{d}{2}\right)^2 \cdot \frac{1}{\alpha} \tag{6}$$

The relative size of these two characteristic times is very important to the assumption of thermally thin particles. It can be shown that the ratio of heating time to thermal relaxation time is the Biot number:

$$Bi = \frac{t_r}{\tau_h} \tag{7}$$

where t_r is the thermal relaxation time, and τ_h is the characteristic heating time. To show these two times are related by the Biot number, the following development is given to reduce equation (7) to the familiar expression for Biot number (9):

$$Bi = \frac{\left(\frac{d}{2}\right)^2 \cdot \frac{1}{\alpha}}{\frac{\rho \cdot C_p \cdot d}{4 \cdot h}} \tag{8}$$

$$Bi = \frac{h \cdot d}{\rho \cdot C_p \cdot \alpha} = \frac{h \cdot d}{k} \tag{9}$$

The Biot number is also equal to the Nusselt number multiplied by the ratio of the thermal conductivity of air to that of the particle (10).

$$Bi = Nu \cdot \frac{k_{particle}}{k_{air}} \tag{10}$$

The thermal conductivity values are estimated at approximately 0.045 W/mK for air, and approximately 0.1 for wood. These values can only be estimated at 600 degrees Kelvin due to the lack of experimental data for conductivity of fuel material. The Nusselt number is taken to be consistent with natural convection, 0.35, because of the lack of air movement just ahead of the fire (proof of this is given in Chapter 3). Using this relation it can be seen that the Biot number is about 0.7, implying the particles can be treated as thermally thin, or isothermal.

Time for fluid flow / time for fluid to mix

The nomenclature needed to develop this section and the following sections of the time comparisons is given in Figure 3. The figure expands upon Figure 2 by identifying the burning zone and the longitudinal depth of the burning zone, D (lower case d is always used for diameter), and the height of the fuel layer, δ. The flame height, H, is shown along with the flame length, L. These two distances are related by the tilt angle, γ. The wind is assumed to be horizontal with average speed $\bar{U}$ over the flame height.

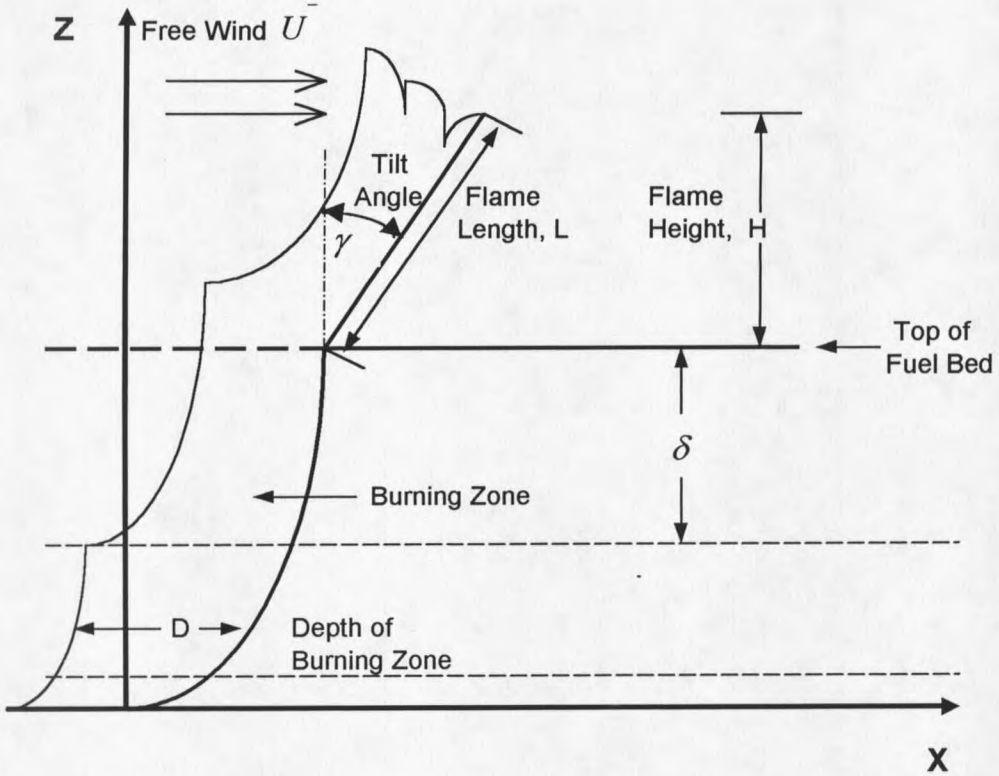

Figure 3. Geometry needed to develop the time comparisons.

The characteristic time for a fluid particle to pass from the bottom of the flame through to the tip of the flame is explained in great detail in the flame structure paper by Albini (1981). Two methods can be used to find this characteristic time. The first is to divide the height of the flame, H, by the component of the total velocity moving in the upward direction, w (11). The second is to divide the flame length, L, by the total velocity of the fluid (12). Either of these two methods could be used, but for this example the first will be shown.

Method 1: $$\tau_{fl} = \frac{H}{w} \tag{11}$$

Method 2: $$\tau_{fl} = \frac{L}{V} = \frac{L}{\sqrt{u^2 + w^2}} \tag{12}$$

The tilt angle of the flame is needed to find the component of the velocity in the upward direction. Assuming that the average flame fluid streamlines are very nearly straight (Albini, 1981), the horizontal velocity, u, divided by the vertical velocity, w, is equal to the tangent of the tilt angle γ (13).

$$\tan\gamma = \frac{u}{w} = const. \tag{13}$$

The expression for $\tan\gamma$ given in Albini (1981) is:

$$\tan^2\gamma = \frac{3\bar{U}^2}{2gH} \tag{14}$$

where g is the acceleration of gravity and H is the flame height. Using these two expressions, the average vertical velocity can be solved in terms of the flame height (15).

$$w = \sqrt{\frac{2gH}{3}} \tag{15}$$

Next, the characteristic time can be found by taking the height divided by the vertical velocity. This is shown in equation (16) after substituting w into equation (11).

$$\tau_{fl} = \sqrt{\frac{3H}{2g}} \tag{16}$$

The characteristic mixing time, τ_{mix}, can be calculated by taking the reciprocal of the rate of mixing of the air with the gaseous products given off from the heating of the fuel, or pyrolyzate. The rate of mixing, R_{mix}, is approximated as (Spalding, 1976):

$$R_{mix} = C \cdot \left| \frac{du}{dz} + \frac{dw}{dx} \right| \approx C \cdot \frac{2 \cdot w}{D} \tag{17}$$

where C is a dimensionless coefficient and D is the longitudinal depth of burn of the flame front. Spalding approximated the C coefficient as 0.35. For the purpose of this calculation, the velocity gradient in the z direction can be assumed to be very small when compared to the velocity gradient in the x direction. This is because the fluid particle is rapidly accelerated in the vertical direction in a small x distance versus a small acceleration in the horizontal direction in a larger distance in z. The gradient in the x direction is further simplified as two times the vertical velocity divided by the depth of burn (Albini, 1981). The characteristic mixing time then reduces to an expression in terms of the depth of burn and the vertical velocity (18).

$$\tau_{mix} = \frac{D}{0.7w} \tag{18}$$

Once the expressions for τ_{mix} (18) and τ_{fl} (16) are found, a ratio of the two can be taken to determine if one is significantly larger than the other. Dividing the characteristic mixing time by the time for fluid flow to pass, the following expression is obtained.

$$\frac{\tau_{mix}}{\tau_{fl}} = \left(\frac{D}{H}\right) / 0.7 \tag{19}$$

This expression is meaningful because it shows qualitatively that the mixing time is the same order of magnitude as the time it takes for a particle of fluid to traverse the

flame. This is true when the depth of burn, D, is of the same order of magnitude as the flame height, H, which is the case in the crown fires this model is intended to simulate. This conclusion is significant in that it helps give meaning to the shape of the flame in a crown fire. If the mixing rate were much higher than the time it takes for the fluid flow, the flame structure would simply flicker instead of giving a consistent shape.

Time to burn particle / transit time for fluid

The time a particle of fuel needs to completely pyrolyze can be analyzed using two different methods. One method is to use a series of simplifications in an order of magnitude analysis to show that the time it takes to burn the particle is large when compared to the mixing time. The other method comes from a publication by Albini and Reinhardt (1995) where the time can be calculated from experimental equations.

The simplest way to perform an order of magnitude analysis on the burning time of a particle is to take the mixing time from equation (18) and then substitute the spread rate multiplied by the characteristic time to burn a particle. The characteristic time to burn a particle is conceptually defined as the longitudinal depth of burn divided by the speed of the fire, S. After a few steps of algebra, the following expression with the burning time divided by the mixing time is obtained:

$$\frac{\tau_{burn}}{\tau_{mix}} = \frac{0.7H}{S\tau_{fl}} \qquad (20)$$

However, the ratio that is being analyzed is the burning time divided by the fluid transit time, not the burning time divided by the mixing time. Since the fluid transit time was

previously proven to be of the same order of magnitude as the mixing time, this ratio can be used to compare the time to burn the particle and the transit time for the fluid.

To determine the order of magnitude of this ratio, sample numbers can be substituted from observation of actual crown fires (Stocks, 1987). For example, if a flame height, H, of 7 meters, a spread rate, R, of 0.2 meters per second, and a τ_{fl} of 1 second is substituted into equation (20), a ratio of 25 is obtained. These numbers demonstrate that the time it takes for a fuel particle to completely burn is much greater than the time needed for the fluid to mix.

The second method for calculating the characteristic time to burn a fuel particle can be taken from Albini and Reinhardt (1995). An empirical equation is given in this publication that was developed from laboratory data. The equation is:

$$\tau_{burn} = \left(\frac{928-673}{T_F - T_C}\right)\cdot\left(\frac{\rho_o}{446}\right)\cdot\left(\frac{d}{h_{eff}}\right)\cdot(a'+Mb') \qquad (21)$$

where T_F is the fire environment temperature, T_C is the burning particle's surface temperature, ρ_o is the oven dry density of the particle, d is the diameter of the particle, h_{eff} is the effective heat transfer coefficient, and a' and b' are empirical constants. Plugging in numbers that are used for this model, τ_{burn} is approximately 10 seconds. The ratio then becomes approximately 10, which is consistent with the above findings.

The fact that the ratio τ_{burn} / τ_{fl} is of the order of 10 implies that the time to burn the particle is much larger than the transit time for the fluid. This gives a stable flame structure for the fire allowing the flame to be treated as stationary.

Time for the chemical reaction to complete / transit time for fluid

To understand the time needed for the gas phase chemical reaction to complete, the concept of flame speed is needed. The flame speed, or burning velocity, is defined as the velocity at which unburned premixed fuel and oxidizer gases travel through a stationary combustion wave in a direction perpendicular to the wave surface (Glassman, 1977). The flame is referred to as a wave in this definition due to the wave like motion of the flame as it propagates through a medium. To determine the flame speed, several approaches using different theories have been developed. Fortunately, they all lead to the same conclusion. For the purpose of this argument, the simplest derivation will be given.

The analysis first was done by Mallard and Le Chatelier (Glassman, 1977) who stated that the controlling mechanism of the flame propagation was the heat traveling back through the layers of gas. The analysis was done by dividing the flaming zone of the combustion into two regions: the region of heat transfer and the region of burning. The interface between these two regions is the point at which ignition occurs. The Mallard and Le Chatelier model assumed that the temperature increase from the ignition temperature to the final burning temperature was linear. This greatly simplified the analysis of determining the burning velocity. The result was the following expression:

$$S_L = \left(\frac{k}{\rho c_p} \cdot \frac{T_f - T_i}{T_i - T_o} \cdot \frac{d\varepsilon}{dt} \right)^{\frac{1}{2}} \tag{22}$$

where S_L is the burning velocity, k is the thermal conductivity, ρ is the density of the gas, c_p is the specific heat, and $\frac{d\varepsilon}{dt}$ is the time rate of change of the combustion product mole