



Generalized DeWitt-Schwinger point-splitting expansions for the charged scalar field in curved space  
by Rhett Byron Herman

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in  
Physics

Montana State University

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Abstract:

The theory of DeWitt-Schwinger point-splitting is developed for a massive, charged quantized scalar field coupled to an electromagnetic field possessing  $U(1)$  symmetry in a general, four-dimensional spacetime. The infinite regularization counterterms required to renormalize the vacuum polarization ( $\langle \phi^2 \rangle$ ), the current ( $j_\mu$ ) associated with the scalar field, and the stress-energy tensor ( $T_{\mu\nu}$ ) associated with the scalar field are presented. The DeWitt-Schwinger point-splitting expansions in this thesis are carried out until they reach terms of order  $m^{-2}$  for  $\langle T_{\mu\nu} \rangle$ , where  $m$  is the mass of the quantized field. Presented here for the first time is an analytic approximation for  $\langle T_{\mu\nu} \rangle$  for a massive, quantized charged scalar field in a general spacetime with a general electromagnetic field possessing  $U(1)$  symmetry.

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SPACE**

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**MONTANA STATE UNIVERSITY**

Bozeman, Montana

August, 1996

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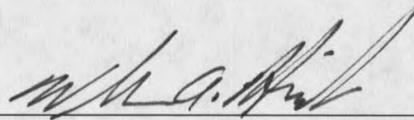
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August 1, 1996

Date



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## ACKNOWLEDGMENTS

I wish to express my gratitude and appreciation to my advisor, William A. Hiscock, for his guidance and support throughout the course of my years at Montana State University. His guidance and feedback were invaluable during my years of research and development in physics.

I would like to acknowledge the other students in the Relativity Group at Montana State University without whose continued help, feedback, and support I could not have finished this work. Their input and criticisms were greatly appreciated and, in no small way, directly responsible for my continued career in physics. I would like to acknowledge the contributions of Professor Gregory Francis, who taught me how to teach others about what I was learning.

I am grateful for the support that I received during my year as a NASA Research Fellow and during my two years as a DOE/EPSCoR Graduate Trainee. The work in this thesis was supported in part by the U. S. Department of Energy under Cooperative Agreement No. DE-FC02-91ER75681, Amendment No. 002.

Finally, I would like to thank my wife, Kirsten Herman, for her continued support throughout the course of my research. Her patience and guidance provided the strength that I, at times, could not muster. I would also like to acknowledge my children, Jonathan and Aaron Herman, without whose humor, questions, and patience graduate school would not have been such a fun experience.

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## CONVENTIONS

The sign conventions used in this thesis are those of Misner, Thorne, and Wheeler [1]. Natural units, where  $G = c = \hbar = 1$  are used throughout except where explicitly indicated. Derivatives are indicated as follows:

$$\frac{\partial}{\partial x^\mu} \text{ or } \partial_\mu \text{ or } ,\mu \quad \text{partial derivative}$$

$$\nabla_\mu \text{ or } ;\mu \quad \text{covariant derivative}$$

$$\nabla_\mu - ieA_\mu \text{ or } |_\mu \quad \text{gauge covariant derivative.}$$

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The theory of DeWitt-Schwinger point-splitting is developed for a massive, charged quantized scalar field coupled to an electromagnetic field possessing  $U(1)$  symmetry in a general, four-dimensional spacetime. The infinite regularization counterterms required to renormalize the vacuum polarization  $\langle\phi^2\rangle$ , the current  $\langle j^\mu\rangle$  associated with the scalar field, and the stress-energy tensor  $\langle T^{\mu\nu}\rangle$  associated with the scalar field are presented. The DeWitt-Schwinger point-splitting expansions in this thesis are carried out until they reach terms of order  $m^{-2}$  for  $\langle T^{\mu\nu}\rangle$ , where  $m$  is the mass of the quantized field. Presented here for the first time is an analytic approximation for  $\langle T^{\mu\nu}\rangle$  for a massive, quantized charged scalar field in a general spacetime with a general electromagnetic field possessing  $U(1)$  symmetry.

## CHAPTER 1

## Introduction

The study of quantum fields in curved space follows the typical path in physics of moving from models that are initially simple and easier to understand to models that are more complicated but more physically realistic. Successive models build upon the knowledge gained in the study of previous models. The goal of this thesis is to build the theoretical foundation for the study of quantized charged scalar fields in a general four-dimensional spacetime which has an arbitrary electromagnetic field of  $U(1)$  symmetry. The DeWitt-Schwinger point-splitting procedure [2] is developed for the case of the charged scalar field coupled to the electromagnetic field of the curved spacetime. The first set of the two major results derived here are the regularization counterterms for the vacuum expectation values (VEVs) for the vacuum polarization  $\langle\phi^2\rangle$ , the current  $\langle j^\mu\rangle$ , and the stress-energy tensor  $\langle T^{\mu\nu}\rangle$  associated with the charged scalar field.

The DeWitt-Schwinger point-splitting procedure is known to be capable of yielding analytic expressions that are approximations to both  $\langle\phi^2\rangle$  and  $\langle T^{\mu\nu}\rangle$ . The DeWitt-Schwinger point-splitting expansions in this thesis are carried out until they reach terms of order  $m^{-2}$  for  $\langle T^{\mu\nu}\rangle$ , where  $m$  is the mass of the quantized field. Presented here for the first time is an analytic approximation for  $\langle T^{\mu\nu}\rangle$  for a massive, quantized charged scalar field in a general spacetime with a general electromagnetic field possessing  $U(1)$  symmetry. After subtracting the infinities of the stress-energy tensor, which are contained in the terms in the expansion proportional to non-negative powers of  $m$ , the remaining expression, proportional to  $m^{-2}$ , serves as the “DeWitt-Schwinger

approximation" to the actual renormalized value for  $\langle T^{\mu\nu} \rangle$ . This expression, along with an analytic expression for  $\langle \phi^2 \rangle$  of order  $m^{-4}$ , constitute the second of the two major results derived here.

The theory of DeWitt-Schwinger geodesic point-splitting began with the classic work of Schwinger [3]. Schwinger calculated the Green function  $G(x, x')$  associated with a fermion current produced by and external electromagnetic field,

$$\langle j^\mu(x) \rangle = \lim_{x' \rightarrow x} ie \operatorname{tr}[\gamma^\mu G(x, x')] \quad (1)$$

where  $e$  is the charge of the fermion field and the  $\gamma^\mu$  are the Dirac matrices. He calculated the Green function by introducing a fictitious, non-quantum-mechanical Hilbert space in which calculations were performed [4]. This Hilbert space was constructed in  $4 + 1$ -dimensions, with the 4 familiar spacetime dimensions being supplemented by a fifth dimension identified as the proper time parameter  $s$  in this fictitious space. Working within this fictitious Hilbert space, Schwinger was able to isolate the divergences that appeared in the quantum field integrals involved with the fermion field current, and use those divergences to renormalize the charge of the fermion current and the strength of the external field.

While Schwinger's original work was performed in flat spacetime, DeWitt recognized that Schwinger's method could provide a way to isolate the divergences that appeared in quantum field theory calculations in curved spacetime. Consider the semiclassical Einstein-Maxwell field equations,

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle, \quad (2)$$

and

$$F^{\mu\nu}{}_{;\nu} = 4\pi \langle j^\mu \rangle. \quad (3)$$

Eqs.(2) and (3) treat the gravitational and electromagnetic fields classically, while treating the sources for these fields quantum mechanically. It is well known that,

when the transition is made from classical to quantized fields, infinities appear in the expectation values on the right hand sides of Eqs.(2)–(3) in both flat [5, 6, 7] and curved [8] spacetimes. These infinities can not be physical in the context of curved spacetime physics. For example, in non-gravitational physics, any infinities that appear in energy density calculations are considered to be “zero-point energies” and are summarily discarded. This rescaling of the zero point of energy in flat space does not change the physics and is allowed. However, in gravitational physics, all energy is a source of the gravitational field and thus a source of curvature. Zero-point energy, infinite or finite, may not be naïvely thrown away since doing so throws away a source of curvature and thus change the physics. Yet there must be a way to correctly remove any unphysical infinities from the theory.

The geodesic point-splitting regularization scheme developed by DeWitt [2, 4] is a fully covariant method whereby these unphysical infinities may be isolated, a process known as *regularization*. Then, in the process known as *renormalization*, the infinities are discarded by subtracting them from the unrenormalized field equations, leaving behind finite quantities that represent the physical universe. All of the quantities to be renormalized are VEVs constructed from products of the quantized field  $\phi(x)$  and its derivatives. Products such as  $\langle \phi(x)\phi(x') \rangle$  have their two constituent quantum fields evaluated at two spacetime points,  $x$  and the nearby  $x'$ . This product is finite so long as the two points are separated in spacetime. The infinities will be shown to arise when these two spacetime points are brought together.

DeWitt based his scheme on the earlier proper-time method method used by Schwinger to calculate the Feynman Green function associated with a quantized fermion field. While there are other regularization methods, point-splitting has proven to be the most robust and trustworthy method of the lot [8]. This is because the point-splitting procedure is well-developed for a general spacetime of arbitrary curvature.

Although algebraically quite complicated, point-splitting works in every case, fully isolating all infinite quantities.

Other regularization methods do exist [9]. Pauli-Villars regularization [10] requires the introduction of fictitious fields whose own divergences are chosen to exactly cancel the divergences of the physical field. The number of these fields introduced is chosen according to the number of divergences the physical fields possess, and these fields are allowed to either commute or anti-commute depending on whether they are required to add to or subtract from the divergences in the stress-energy tensor of the physical field.

Dimensional regularization involves the continuation of quantum field calculations to non-integral spacetime dimensions. Physical parameters requiring renormalization are shown to have bare values proportional to  $(n-4)^{-1}$ , where  $n$  is the dimension of the spacetime. This fact requires the introduction into the original physical Lagrangian terms proportional to bare coefficient of adiabatic order 4 that will serve to absorb the infinities. However, the renormalized quantities that result must have restrictions on their size in order to be consistent with observations [11, 12].

Adiabatic regularization has been used extensively in calculations of conformally flat spacetimes [13, 14]. In this method, the subtraction of infinite quantities is based on the adiabatic expansion of the modes of the quantized field. However, the subtractions necessary to renormalize the physical parameters are often too difficult to evaluate. This means that, while the infinities are isolated, they can not serve to renormalize the VEVs of interest such as  $\langle T^{\mu\nu} \rangle$ .

The technique of  $\zeta$ -function regularization allows the effective Lagrangian to be written as a derivative of a  $\zeta$  function *resembling* Riemann's  $\zeta$  function on the curved space [15]. This formal technique for regularizing the effective action uses a generalized  $\zeta$  function,  $\zeta(\nu)$ , whose argument  $\nu$  must be analytically continued from regions

where  $\zeta(\nu)$  converges into regions where  $\zeta(\nu)$  does not converge. This is necessary since  $\nu = 0$ , a value for which the generalized  $\zeta$  function does not converge, is the value of interest in quantum field calculations.

Point-splitting was chosen for the purposes of this thesis for two main reasons. First, the major advantage that point-splitting possesses over these and any other known regularization schemes is that it is the most efficient method to use when computing actual values for the quantized fields instead of working with purely formal manipulations of the effective action of the theory. Second, by using point-splitting for the present calculations of a complex scalar field in curved space, a connection may be made with the previous work of Christensen in using the point-splitting procedure for regularization of the real scalar field in curved space [31, 30]. The results of this thesis will explicitly show that, if the charge of the complex scalar field is allowed to vanish, then the regularization counterterms presented here reduce exactly to those of Christensen.

After the point-splitting calculations have been performed and the infinities of the field equations are isolated then, in principle, they may be subtracted from the unrenormalized equations, leaving a finite remainder which contains the relevant physical information. In practice, the renormalized quantities are often too algebraically complicated to evaluate analytically in the case of general spacetimes. In the case of conformally invariant fields in conformally invariant spacetimes, the renormalized quantities been evaluated analytically [16, 17, 13, 18]. These calculations have been performed by first renormalizing the Hadamard elementary function,  $G^{(1)}(x, x')$ , from which the quantities  $\langle \phi^2 \rangle$  and  $\langle T_{\mu\nu} \rangle$  are constructed. The VEV  $\langle \phi^2 \rangle$  is directly proportional to  $G^{(1)}(x, x')$  and is renormalized when  $G^{(1)}(x, x')$  is renormalized. The stress-energy tensor  $\langle T_{\mu\nu} \rangle$  is constructed from  $G^{(1)}(x, x')$  and its covariant derivatives. The presence of these derivatives requires a slight modification of the renormalization

of  $G^{(1)}(x, x')$ , yet these modifications in essence amount to carrying out certain power series expansions involving the two spacetime points  $x$  and the nearby  $x'$  to a higher order than when renormalizing  $G^{(1)}(x, x')$ .

Point-splitting renormalization calculations have been performed for specific spacetimes for both  $\langle\phi^2\rangle$  and  $\langle T_{\mu\nu}\rangle$  for the real scalar field. Candelas [19] computed a renormalized value for  $\langle\phi^2\rangle$ , or  $\langle\phi^2\rangle_{ren}$ , for the massless scalar field in Boulware, Hartle-Hawking, and Unruh vacuum states in the region exterior to the horizon of a Schwarzschild black hole. Some of the components of  $\langle T_{\mu\nu}\rangle_{ren}$  were also renormalized. Frolov [20, 21] generalized the work of Candelas such that  $\langle\phi^2\rangle_{ren}$  for the massless field could be calculated on the event horizon of a Reissner-Nordström black hole and near the pole of the event horizon of a charged Kerr black hole. Candelas and Howard [22] calculated  $\langle\phi^2\rangle_{ren}$  for the Hartle-Hawking vacuum in the Schwarzschild spacetime in the region exterior to the horizon. Candelas and Jensen [23] analytically continued the calculation of  $\langle\phi^2\rangle_{ren}$  across the event horizon of a Schwarzschild black hole, giving an expression which is valid for a range Schwarzschild radial coordinate  $r$ . They numerically evaluated  $\langle\phi^2\rangle_{ren}$  for the range of  $r$  given by  $0.5M < r < 2M$ .

Following the work of the investigators mentioned above, Anderson [24] has described a method whereby  $\langle\phi^2\rangle_{ren}$  may be numerically computed for free scalar fields in a general static spherically symmetric spacetime. This method assumes the spacetime is asymptotically flat for the purpose of defining initial conditions for the modes sums to be computed for the fields. The fields may be massive or massless, at zero or non-zero temperature, and the spacetime may have arbitrary curvature coupling  $\xi$ . This scheme is fully renormalized, and the computations may be carried to arbitrary numerical precision.

Renormalization calculations for  $\langle T_{\mu\nu}\rangle$  in specific spacetimes have also been performed. These calculations include the work by Howard and Candelas [25] wherein

$\langle T_{\mu}^{\nu} \rangle_{ren}$  was calculated for a massless, conformally invariant scalar field in a Hartle-Hawking vacuum state in a Schwarzschild spacetime. Their work allowed numerical computations of  $\langle T_{\mu}^{\nu} \rangle_{ren}$  in the region exterior to the event horizon and described methods for increasing the efficiency of the computations. Frolov and Zel'nikov [20, 26] calculated an approximate expression for  $\langle T_{\mu}^{\nu} \rangle_{ren}$  for massive scalar, spinor, and vector fields in Ricci-flat spacetimes. This was done by calculating an approximation for the effective action using the generalization of the DeWitt-Schwinger technique developed by Barvinsky and Vilkovisky [27]. Brown, Ottewill, and Page derived an analytic approximation for  $\langle T^{\mu\nu} \rangle$  in conformal spacetimes using the one-loop effective action for massless, conformally invariant scalar, spinor, and vector fields on static Einstein spaces [28].

Building on the studies of  $\langle T_{\mu\nu} \rangle$  above, Anderson, Hiscock, and Samuel [29] have recently described a method whereby  $\langle T_{\mu\nu} \rangle_{ren}$  may be calculated to arbitrary numerical precision for a general static spherically symmetric spacetime. The scalar field can be massless or massive, in a zero temperature or non-zero temperature state, and the coupling  $\xi$  to the scalar curvature can be arbitrary. While computationally quite intensive, this method is very powerful since it provides a way of using a fully renormalized stress-energy tensor in quantum field theory calculations in these static spherically symmetric spacetimes.

These renormalization calculations rely on the regularization counterterms previously derived by Christensen [30, 31]. They use these counterterms to subtract infinities from the unrenormalized expressions for  $\langle \phi^2 \rangle$  and  $\langle T_{\mu\nu} \rangle$ , leaving behind finite remainders which correctly describe the physics. These subtractions have been performed for the specific spacetimes mentioned above and not for completely general spacetimes. Also, all of these studies have been performed using uncharged fields. The first of the two major results of this thesis are the point-splitting regularization

counterterms that will be required for the study of quantized charged scalar fields interacting with the electromagnetic fields that may be present in curved spacetimes.

The quantities  $\langle\phi^2\rangle$ ,  $\langle j^\mu\rangle$ , and  $\langle T_{\mu\nu}\rangle$  are constructed from the Hadamard elementary function,  $G^{(1)}(x, x')$ , and its derivatives. The new feature present in this thesis is that these derivatives are now *gauge covariant* derivatives. As explained further at the end of this chapter, the coupling of the charged scalar field to the electromagnetic field of the curved spacetime will introduce new physics into the structure of  $G^{(1)}$  and its derivatives. This new physics will not only modify the results of Christensen for a real scalar field, but will also be responsible for the generation of the current,  $\langle j^\mu\rangle$ , associated with this charged scalar field.

The Hadamard elementary function is constructed from  $\langle\phi(x)\phi(x')\rangle$  and is finite so long as the points  $x$  and  $x'$  in the argument of  $G^{(1)}(x, x')$  are not coincident. The point-splitting scheme is actually an asymptotic expansion of the biscalar  $G^{(1)}(x, x')$  in powers of the mass  $m$  of the quantized field, an expansion known as a “DeWitt-Schwinger (DS) expansion.” The expansion of  $G^{(1)}(x, x')$  goes as,

$$G^{(1)}(x, x') = A_{+2}(x, x')m^2 + A_0(x, x')m^0 + A_{-2}(x, x')m^{-2} + \dots, \quad (4)$$

where the  $A_n$  are coefficients constructed from curvature and electromagnetic tensors, and  $m$  is the mass of the quantized field. Eq.(4) has been shown to contain infinities that arise when the points  $x$  and  $x'$  are brought together [30, 31, 8, 4]. These infinities have been shown to be contained in the first two terms of the right hand side of Eq.(4), or, in the terms proportional to nonnegative powers of  $m$ . These infinities may then be, in principle, subtracted from the unrenormalized expressions involving the charged quantized field, with the finite result containing real physical information. Unfortunately, these subtractions are difficult at best to evaluate analytically, as mentioned previously.

Fortunately, point-splitting provides a way out of this dilemma of difficult sub-

tractions, at least in the case of massive quantized fields. The magnitudes of the  $A_n$  depend directly on the strength of the curvature and electromagnetic fields of the spacetime since they are constructed from curvature and electromagnetic invariants. As the inverse power of  $m$  becomes large, the  $A_n$  contain higher and higher derivatives of both the spacetime metric and electromagnetic field functions. Yet, since Eq.(4) is an asymptotic series for  $G^{(1)}(x, x')$ , for some large value of  $n$ , the magnitude of the  $A_n$  becomes large when compared with the magnitude of the  $m^{-n}$ . Beyond this value for  $n$ , the asymptotic expansion breaks down, and the series representation of  $G^{(1)}(x, x')$  must be abandoned. Subtracting the infinite terms from the unrenormalized expression for  $G^{(1)}(x, x')$  leaves a series finite in length containing finite terms. The first few, or even the first one, of these terms may be kept as a “DeWitt-Schwinger approximation” for the actual value of  $G^{(1)}(x, x')$ . The accuracy of this approximation will depend on how large the mass  $m$  of the field is chosen when compared to the magnitude of the spacetime curvature invariants. Hereafter, the phrases “DS expansion” or “DS approximation” will refer to the large mass expansion of any quantity.

The point-splitting procedure is capable of yielding a DS approximation for both the vacuum polarization  $\langle\phi^2\rangle$  (which is directly proportional to  $G^{(1)}$ ) and the vacuum expectation value of the stress energy tensor  $\langle T_{\mu\nu}\rangle$  (which is assembled from  $G^{(1)}$  and its derivatives). Unfortunately, one major limitation of the point-splitting procedure is exposed whenever real particle production occurs. As will be shown in greater detail later, point-splitting is incapable of yielding either the imaginary part of the expansion of  $G^{(1)}$  [2], or odd powers of  $m$  in Eq.(4). These two limitations mean that point-splitting is incapable of yielding a DS approximation for the current of the quantized charged scalar field  $\langle j^\mu\rangle$ .

This thesis calculates the regularization counterterms for the three VEVs  $\langle\phi^2\rangle$ ,  $\langle j^\mu\rangle$ , and  $\langle T_{\mu\nu}\rangle$ . All of these will be derived for the case of a complex scalar field

interacting with a classical background electromagnetic in a spacetime of arbitrary curvature. A scalar field is chosen in the present work in order to make a connection with the regularization work previously done by Christensen [30, 31] and for simplicity. The added complication of spin need not be considered since essentially no new physics is expected to arise when spin effects are considered.

Chapter 2 contains a discussion of the degree of divergence in QED will be presented. The relationship between the momentum space representations of VEVs and the geodesic separation of spacetime points  $x$  and  $x'$  will be discussed. This discussion will show  $G^{(1)}(x, x')$  (and equivalently  $\langle\phi^2\rangle$ ) to have an expected quadratic divergence in the spacetime separation distance  $|x - x'|$  as the two points  $x$  and  $x'$  are brought together. The quantities  $\langle j^\mu\rangle$  and  $\langle T_{\mu\nu}\rangle$  will be shown to potentially contain cubic and quartic divergences, respectively, in the separation distance  $|x - x'|$ .

Chapter 3 begins with a discussion of the local structure of spacetime upon which point-splitting regularization depends crucially. A Hamilton-Jacobi analysis of geodesic motion in the presence of gravitational and electromagnetic fields will lead to the biscalar of the geodesic interval,  $\sigma(x, x')$ . The biscalar  $\sigma(x, x')$  will be shown to be equal to one-half the square of the geodesic distance between the points  $x$  and  $x'$ , and will be shown to carry information about the structure of spacetime along the geodesic between the two points. The Van Vleck-Morette determinant, symbolized  $D$ , arises in a discussion of caustic surfaces in curved spacetimes, and will be shown to place constraints on how the points  $x$  and  $x'$  are separated. Differential equations that define  $\sigma(x, x')$  and  $D$  for the point-splitting procedure will be derived. The bivector of parallel transport,  $g^\mu{}_{\nu'}$  will be shown as the means for conveying information from the nearby point  $x'$  back to the stationary point  $x$ . A differential equation involving  $g^\mu{}_{\nu'}$  that is required for the point-splitting procedure will be derived.

Chapter 4 presents a derivation of the geodesic point-splitting procedure using the quantities derived in Chapter 3. This chapter will start with Schwinger's original proper time method of calculating the Feynman Green function, and end with the presentation of the DS expansion of  $G^{(1)}$ . This will illustrate the need for  $G^{(1)}$  and its derivatives to be rewritten in terms of local geometric quantities. The point-splitting "recursion relations" derived by Christensen [30] are a set of differential equations that are used to construct  $G^{(1)}$  and its derivatives. In this chapter, a derivation of the recursion relations of the familiar form presented by Christensen [30] will follow. The crucial new feature in this thesis is the presence of the electromagnetic gauge field. The original recursion relations will be cast into the gauge-invariant form necessary for calculating effects due to *both* the gauge and gravitational fields. The gravitational field emerged in Christensen's work while solving the differential equations, or recursion relations, through the commutator of the covariant derivative

$$[\nabla_\alpha, \nabla_\beta]V^\mu = R^\mu{}_{\nu\alpha\beta}V^\nu, \quad (5)$$

where  $\nabla_\alpha$  is the covariant derivative,  $V^\mu$  is a purely geometric vector, and  $R^\mu{}_{\nu\alpha\beta}$  is the Riemann tensor. In this thesis, the addition of the electromagnetic field requires the use of the gauge covariant derivative whose commutator is given by

$$[\nabla_\alpha - ieA_\alpha, \nabla_\beta - ieA_\beta]W = ieF_{\beta\alpha}W, \quad (6)$$

where  $W$  is a scalar that explicitly depends on the gauge field,  $A_\alpha$  is the vector potential for the gauge field, and  $F_{\beta\alpha}$  is the Maxwell tensor. The effects of this gauge field in quantum field theory calculations in curved spacetime is the heart of the present work.

Chapter 5 is devoted to solving the recursion relations in order to construct the expansion of Eq.(4). The recursion relations can not be solved analytically and so

they must be iteratively solved order by order. This iterative process is detailed and a listing of the geometric quantities that form the  $A_n$  of Eq.(4) will be given.

In Chapter 6, the DS point-splitting expansions for  $G^{(1)}$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  will be given, with the divergent and finite parts of each clearly identified. These point-splitting expansions will be asymptotic series as in Eq.(4). The expansions for all three VEVs will explicitly show that the divergent parts are proportional to non-negative powers of  $m$ . The expansion for  $G^{(1)}$  will include terms of order  $m^{-2}$  and  $m^{-4}$ , and these terms may be used as a DS approximation to the true value for  $G^{(1)}$  as discussed above. This chapter will discuss why the coefficients  $B_n$  in the expansion for  $\langle j^\mu \rangle$ ,

$$\langle j^\mu \rangle = B_{+2}(x, x')m^2 + B_0(x, x')m^0 + B_{-2}(x, x')m^{-2} + \dots, \quad (7)$$

vanish for all negative  $n$ . The DS approximations for  $\langle \phi^2 \rangle$  and  $\langle T_{\mu\nu} \rangle$  will be presented in their most general form as combinations of the electromagnetic and curvature tensors. Previous workers derived expressions for  $\langle \phi^2 \rangle$  and  $\langle T_{\mu\nu} \rangle$  for uncharged fields that contained only geometric terms. Here, the contributions of the electromagnetic field to these expressions will be shown.

Chapter 7 will discuss the possible future use of this work in studying the evolution of charged black hole interiors. The regularization counterterms presented here for  $\langle j^\mu \rangle$  and  $\langle T_{\mu\nu} \rangle$  are a first step towards studies of both gravitational and electromagnetic backreaction effects. Also, the use of the general DeWitt-Schwinger approximation presented here for  $\langle T_{\mu\nu} \rangle$  will be discussed.

The appendices following the Bibliography contain expressions which are too large to list in the body of this thesis. The equations in these appendices are quite long. So long, in fact, that there was no practical way to apply every simplification rule in order to write them in their most compact form. Rewriting or cancelling a few terms will make no difference in the physics contained in the longer expressions. These

appendices are intended as an archive for the expressions derived in the DeWitt-Schwinger point-splitting procedure. Appendices A–E contain the coincidence limits of derivatives of the biscalars  $\sigma(x, x')$ ,  $\Delta^{1/2}(x, x')$ ,  $a_0(x, x')$ ,  $a_1(x, x')$ , and  $a_2(x, x')$ . Appendix F contains the purely geometric regularization counterterms first derived by Christensen [30] that contribute to  $\langle T_{\mu\nu} \rangle_{finite}$ . In the interest of space, Chapter 6 contains only those terms in the DS expansion of  $\langle T_{\mu\nu} \rangle$  which depend on the electromagnetic field. Appendix G contains the purely geometric terms of order  $m^{-2}$  term of the DS approximation of  $\langle T_{\mu\nu} \rangle$ .

## CHAPTER 2

## Degree of Divergence in Scalar QED

As mentioned in Chapter 1, the biscalar  $\langle \phi(x)\phi(x') \rangle$  has its two constituent quantum fields evaluated at two spacetime points,  $x$  and the nearby  $x'$ . This is finite so long as the two points are separated in spacetime, but diverges as the two points are brought together. The so-called *degree of divergence* indicates whether the quantities  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  diverge logarithmically, linearly, etc., when the points are brought together. Quantities that have a logarithmic, linear, etc., degree of divergence contain terms proportional to  $\ln(|x-x'|)$ ,  $|x-x'|^{-1}$ , etc. The DeWitt-Schwinger point-splitting procedure isolates the infinities which appear in  $\langle \phi(x)\phi(x') \rangle$  as  $x' \rightarrow x$  and in the VEVs of  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$ . This chapter discusses the origin and structure of these infinities using the familiar tool of the Fourier expansion of quantum field operators.

Relativistic quantum field theory calculations of VEVs of operators may be performed by Fourier transforming to wavenumber space. It is known that infinities arise as the integration limits over the wavenumbers involved are extended to infinity. As an example of divergences appearing due to divergent integration over wavenumbers, consider the Wightman function,

$$G^+(x, x') \equiv \langle 0 | \underline{\phi}(x) \underline{\phi}(x') | 0 \rangle, \quad (8)$$

in flat space. The Fourier transform of the operator  $\underline{\phi}(x)$  is given by,

$$\underline{\phi}(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \left[ \underline{a}(k) e^{-ik_\alpha x^\alpha} + \underline{b}^\dagger(k) e^{ik_\alpha x^\alpha} \right], \quad (9)$$

where  $k^\alpha$  is the wavenumber of the basis functions  $e^{\pm ik_\alpha x^\alpha}$ ,  $\omega_{\vec{k}} \equiv k_0 = \sqrt{\vec{k} \cdot \vec{k} + m^2}$ , and the limits of integration extend from zero to infinity in each wavenumber integral. The action of the particle annihilation and creation operators on the vacuum state  $|0\rangle$  is given by

$$\underline{a}(\vec{k})|0\rangle = 0 \quad , \quad \underline{a}^\dagger(\vec{k})|0\rangle = |1(\vec{k})\rangle_{part}, \quad (10)$$

where  $|1(\vec{k})\rangle_{part}$  is a state with one particle of wavenumber  $\vec{k}$ . The action of the antiparticle annihilation and creation operators on the vacuum state  $|0\rangle$  is given by

$$\underline{b}(\vec{k})|0\rangle = 0 \quad , \quad \underline{b}^\dagger(\vec{k})|0\rangle = |1(\vec{k})\rangle_{anti}. \quad (11)$$

where  $|1(\vec{k})\rangle_{anti}$  is a state with one antiparticle of wavenumber  $\vec{k}$ . Substituting Eq.(9) into Eq.(8), and using Eqs.(10)–(11) along with the commutation relations,

$$[\underline{a}(\vec{k}), \underline{a}^\dagger(\vec{k}')] = [\underline{b}(\vec{k}), \underline{b}^\dagger(\vec{k}')] = \delta^3(\vec{k} - \vec{k}'), \quad (12)$$

gives

$$\begin{aligned} G^+(x, x') &= \int \frac{d^3k \, d^3k'}{(2\pi)^3 \sqrt{2\omega_{\vec{k}} 2\omega_{\vec{k}'}}} e^{-ik_\alpha x^\alpha + ik'_\alpha x'^\alpha} \langle 0 | \underline{a}(\vec{k}) \underline{a}^\dagger(\vec{k}') | 0 \rangle \\ &= \int \frac{d^3k \, d^3k'}{(2\pi)^3 \sqrt{2\omega_{\vec{k}} 2\omega_{\vec{k}'}}} e^{-ik_\alpha x^\alpha + ik'_\alpha x'^\alpha} [\langle 0 | \underline{a}^\dagger(\vec{k}') \underline{a}(\vec{k}) | 0 \rangle + \langle 0 | \delta^3(\vec{k} - \vec{k}') | 0 \rangle] \\ &= \int \frac{d^3k \, d^3k'}{(2\pi)^3 \sqrt{2\omega_{\vec{k}} 2\omega_{\vec{k}'}}} e^{-ik_\alpha x^\alpha + ik'_\alpha x'^\alpha} \delta^3(\vec{k} - \vec{k}') \langle 0 | 0 \rangle \\ &= \int \frac{d^3k}{(2\pi)^3 2\omega_{\vec{k}}} e^{-ik_\alpha (x^\alpha - x'^\alpha)}. \end{aligned} \quad (13)$$

The last line is finite so long as the points  $x$  and  $x'$  are not coincident. As the points  $x$  and  $x'$  are brought together,  $G^+(x, x')$  diverges quadratically since  $G^+(x, x') \sim \int k \, dk \sim k^2$  as  $k \rightarrow \infty$ . This is where a direct correlation exists between point-splitting and wavenumber space integrations. States with high wavenumbers, or high energy values, have short wavelengths. These short wavelength states are probing the spacetime at ever-decreasing length scales. As the wavenumbers diverge, the

spacetime is probed to ever-decreasing length scales, and the vacuum self-energy diverges. Instead of investigating the behavior VEVs using divergent wavenumber integrations, point-splitting studies the behavior of VEVs explicitly in terms of small length scales.

It is straightforward to show that one and two derivatives of  $G^+(x, x')$  give the cubic and quartic divergences

$$\begin{aligned}\frac{\partial}{\partial x} G^+(x, x') &\sim \int k^2 dk \sim k^3, \quad k \rightarrow \infty, \\ \frac{\partial^2}{\partial x^2} G^+(x, x') &\sim \int k^3 dk \sim k^4, \quad k \rightarrow \infty, \end{aligned} \quad (14)$$

where spacetime indices have been suppressed for simplicity. It should be noted that  $G^+(x, x')$  is similar to the Hadamard elementary function  $G^{(1)}(x, x')$  in that they are both constructed from the VEV of products of the fields  $\phi(x)$  and  $\phi(x')$ .

While the above discussion of the divergences of the Wightman function is in flat space, it gives an intuitive picture of the physical origin of the divergences in quantum field theory. With  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  being constructed from  $G^{(1)}(x, x')$  and its derivatives, it is possible to consider an analogy between the divergences of  $G^+(x, x')$  and its derivatives and  $G^{(1)}(x, x')$  and its derivatives. For example, the stress-energy tensor  $\langle T_{\mu\nu} \rangle$  will be constructed from up to two derivatives of  $G^{(1)}(x, x')$ . With the result of Eq.(14), the stress-energy tensor would potentially contain up to quartic divergences. This will be shown to be true. The current  $\langle j^\mu \rangle$  will be constructed from up to one derivatives of  $G^{(1)}(x, x')$  and would potentially contain up to cubic divergences. However,  $\langle j^\mu \rangle$  will be shown to actually contain only a linear divergence.

While the above discussion provides a general framework within which to view the origin of the divergences in scalar QED, there exists a method for determining the specific degree of divergence for  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  individually. To calculate these degrees of divergence, it is necessary to consider the action functional for a complex

scalar field coupled to the electromagnetic field in an arbitrary curved background [1];

$$S[\phi, A_\mu, g_{\mu\nu}] = -\frac{1}{2} \int \mathcal{L} dV \quad (15)$$

$$= -\frac{1}{2} \int \mathcal{L} (-g)^{1/2} d^4x \quad (16)$$

$$= -\frac{1}{2} \int (-g)^{1/2} \left[ (D_\mu \phi)(D^\mu \phi)^* + (m^2 + \xi R)\phi\phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] d^4x, \quad (17)$$

where  $\phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$  is the complex scalar field,  $g$  is the determinant of the metric  $g_{\mu\nu}$ ,  $D_\mu \equiv (\nabla_\mu - ieA_\mu)$  is the gauge covariant derivative,  $A^\mu$  is the classical electromagnetic vector potential,  $e$  is the coupling between the complex scalar and the electromagnetic fields,  $m$  is the mass of the complex scalar field,  $\xi$  is the scalar curvature coupling, and  $R$  is the scalar curvature. The Lagrangian density  $\mathcal{L}$  may be rewritten in terms of interaction Lagrangians

$$\begin{aligned} \mathcal{L} &= \left[ (D_\mu \phi)(D^\mu \phi)^* + (m^2 + \xi R)\phi\phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] \\ &= \left[ (\nabla_\mu - ieA_\mu)\phi(\nabla^\mu + ieA^\mu)\phi^* + (m^2 + \xi R)\phi\phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] \\ &= \frac{1}{2} \{ \partial_\mu \phi, \partial^\mu \phi^* \} + \frac{1}{2} (m^2 + \xi R) \{ \phi, \phi^* \} + \\ &\quad \frac{ie}{2} [ \{ D^\mu \phi, \phi^* \} - \{ D^\mu \phi, \phi^* \}^* ] A_\mu - \frac{e^2}{2} A_\mu A^\mu \{ \phi, \phi^* \} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &= \frac{1}{2} \{ \partial_\mu \phi, \partial^\mu \phi^* \} + \frac{1}{2} (m^2 + \xi R) \{ \phi, \phi^* \} + j^\mu A_\mu - \frac{e^2}{2} A_\mu A^\mu \{ \phi, \phi^* \} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (18) \end{aligned}$$

where  $j^\mu \equiv \frac{ie}{2} [ \{ D^\mu \phi, \phi^* \} - \{ D^\mu \phi, \phi^* \}^* ]$  is the scalar field current, and the anti-commutators have arisen by symmetrizing with respect to the fields. Eq.(18) may be rewritten in terms of three interaction Lagrangian densities and the classical electromagnetic Lagrangian density;

$$\mathcal{L} \equiv \mathcal{L}_{I,1} + \mathcal{L}_{I,2} + \mathcal{L}_{I,3} + \mathcal{L}_{EM}, \quad (19)$$

where

$$\mathcal{L}_{I,1} \equiv \frac{1}{2} \{ \partial_\mu \phi, \partial^\mu \phi^* \} + \frac{1}{2} (m^2 + \xi R) \{ \phi, \phi^* \} \quad (20)$$

is the interaction Lagrangian density for the scalar field in curved space studied by Christensen [30],

$$\mathcal{L}_{I,2} \equiv j^\mu A_\mu = \frac{ie}{2} [\{D^\mu \phi, \phi^*\} - \{D^\mu \phi, \phi^*\}^*] A_\mu \quad (21)$$

is the interaction Lagrangian density for the scalar field current  $j^\mu$  and the classical background electromagnetic field  $A_\mu$ ,

$$\mathcal{L}_{I,3} \equiv -\frac{e^2}{2} A_\mu A^\mu \{\phi, \phi^*\} \quad (22)$$

is the interaction Lagrangian density for the scalar field and classical background electromagnetic field, and

$$\mathcal{L}_{EM} \equiv -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (23)$$

is the Lagrangian density for the classical background electromagnetic field. Note that Eq.(21) is an interaction first order in the coupling constant  $e$ , while Eq.(22) is second order in  $e$ .

The three VEVs  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  arise from functional variation of Eq.(17) for the total action with respect to the the fields  $\phi^*$ ,  $A^\mu$ , and  $g_{\mu\nu}$ , respectively. The degree of divergence of each VEV may be predicted from analysis of the Feynman diagrams for the interaction Lagrangian density pertaining to each. For example, the total interaction Lagrangian density for the scalar field and the electromagnetic field is

$$\mathcal{L}_{\phi, A^\mu} \equiv \mathcal{L}_{I,2} + \mathcal{L}_{I,3}. \quad (24)$$

The first term gives rise to the three-point Feynman graph of Figure 1, while the second term is the four-point graph.

In the 3-point diagram, there are two external scalar (boson) lines, one external photon line, and one vertex. For scalar QED, the degree of divergence  $D$  in four dimensions is given by [5]

$$D = 4 - P_e - Q_e, \quad (25)$$

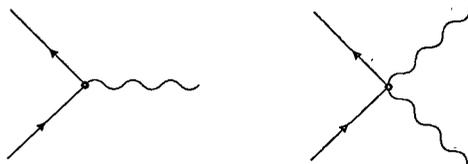


Figure 1: Feynman diagrams for scalar field-photon interaction

where  $P_e$  and  $Q_e$  are the number of external scalar and photon lines, respectively, in the Feynman diagrams for the interaction. The values  $D = 0, 1, 2, \dots$  imply logarithmic, linear, quadratic, etc., divergences, while  $D < 0$  implies the interaction is not divergent. DeWitt has pointed out for the generalized Yang-Mills field in the presence of the gravitational field that the simplest possible diagram will be the most divergent [2]. With the total interaction given by Eq.(24), the three-point graph will be the most divergent. Thus, the expected degree of divergence is  $D = 1$ , a linear divergence. The direct correspondence between the degree of divergence in the wavenumber Fourier transforms of the operators  $\phi(x)$  and the dependence upon the splitting of points in spacetime indicates that, when all calculations of the VEV of the current  $\langle j^\mu \rangle$  are complete, the infinities should be proportional to  $|x - x'|^{-1}$ .

The vacuum polarization  $\langle \phi^2 \rangle$  for a real scalar field in curved space has well-known quadratic and logarithmic divergences as the split points  $x$  and  $x'$  are brought together [30]. The degree of this divergence should not change in the case of a complex scalar field since the complex field is constructed from the complex sum of two real scalar fields. Thus,  $\langle \phi\phi^* \rangle \equiv \langle \phi^2 \rangle$  should also have a quadratic divergence. The interaction Lagrangian density of Eq.(20) gives rise to a two-point graph with a degree of divergence  $D = 2$ . This gives an expected quadratic divergence in full agreement with previous point-splitting results. With no gauge field contribution to this interaction, all of the divergences should be exactly the same as when no gauge field is present in the spacetime.

The VEV of the stress-energy tensor in curved space has well-known quartic, quadratic, linear, and logarithmic divergences in the case of a real scalar field with no gauge field present [30]. According to DeWitt [2], the addition of the gauge field should contribute additional divergences within the existing logarithmic ones. These may be considered as arising from the Lagrangian density of Eq.(22) which has the logarithmic degree of divergence  $D = 0$ .

Point-splitting will be shown capable of isolating all of the divergences of  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  predicted above.

## CHAPTER 3

## The Local Structure of Spacetime

The DeWitt-Schwinger geodesic point-splitting procedure depends crucially upon geometric quantities describing the local structure of spacetime. At the heart of point-splitting lies the idea of N-tensors [32, 33, 2, 31]. An N-tensor is simply a tensor that may be evaluated at more than one point in spacetime. For example,  $\langle \phi^2 \rangle \equiv \langle \phi(x)\phi(x') \rangle$  is a biscalar constructed from the product of two scalars, with each individual scalar being evaluated at its own spacetime point. Another example would be the bivector  $\langle \phi(x)\phi(x') \rangle^{;\mu'} \equiv \langle \phi(x)\phi^{;\mu'}(x') \rangle$ , where the derivative with respect to  $x'$  does not affect the scalar  $\phi(x)$ , a scalar which is associated with the nearby point  $x$ . (Note how the prime on the derivative index indicates the spacetime point on which the derivative acts.) With derivatives being taken with respect to a particular spacetime point, all derivatives with respect to primed indices commute with all derivatives with respect to unprimed indices, and vice versa. In the present work, we will only be concerned with bitensors. More general quantities such as  $\langle \phi(x)\phi(x')\phi(x'') \rangle$  will not arise.

A concept of importance associated with N-tensors is that of tensor weight [1]. Consider, for example, the familiar transformation involving volume elements in flat space:

$$dx^0 dx^1 dx^2 dx^3 \equiv d^4x = \left| \frac{\partial x}{\partial x'} \right| d^4x' \quad (26)$$

$$= (-g)^{1/2} d^4x', \quad (27)$$

where  $|\partial x / \partial x'|$  is the Jacobian of the transformation from the coordinates  $\{x^{\alpha'}\}$  to the coordinates  $\{x^\alpha\}$ . The number of factors of  $|\partial x / \partial x'|$  determines the weight of the

density. Here,  $d^4x'$  with a tensor density of weight +1 transforms like an ordinary tensor such that  $(-g)^{1/2}d^4x'$  is an invariant volume element. Another example of tensor weight involves the basis kets  $|x\rangle$  such as those of Eqs.(10)–(11). These transform as densities of weight  $\frac{1}{2}$  under coordinate transformations in flat space [4]:

$$|x'\rangle = (-g)^{1/4}|x\rangle. \quad (28)$$

The most important biscalar for the point-splitting procedure is  $\sigma(x, x')$ , the biscalar of the geodesic interval [2], which will be defined as one-half the square of the geodesic distance between the points  $x$  and  $x'$ . The biscalar  $\sigma(x, x')$  contains information about the local structure of spacetime and arises from a Hamilton-Jacobi analysis of motion along geodesics.

The Lagrangian describing the motion of a charged particle in combined external gravitational and electromagnetic fields is given by [34],

$$L = \frac{1}{2}g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} + eA_\mu\frac{dx^\mu}{d\lambda}, \quad (29)$$

where  $A^\mu$  is the electromagnetic gauge field and  $e$  is the charge of the particle. The invariant interval is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -d\tau^2. \quad (30)$$

Following Carter [34], we are free to choose the normalization constant, or mass  $m$  of the particle moving along the geodesic, such that

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = -\left(\frac{d\tau}{d\lambda}\right)^2 = -m^2 = \left(\frac{ds}{d\lambda}\right)^2 = \text{constant}. \quad (31)$$

Given this normalization condition, the action for the motion of the particle becomes, for the parameter  $\lambda$  along any curve  $x^\mu(\lambda)$ ,

$$S = \int_{x'(\lambda')}^{x(\lambda)} L d\lambda \quad (32)$$

$$= -\frac{1}{2}m^2 \int_{x'(\lambda')}^{x(\lambda)} d\lambda + e \int_{x'(\lambda')}^{x(\lambda)} A_\mu \frac{dx^\mu}{d\lambda} d\lambda \quad (33)$$

$$= \frac{1}{2} \left( \frac{ds}{d\lambda} \right)^2 (\lambda - \lambda') + e \int_{x'(\lambda')}^{x(\lambda)} A_\mu \frac{dx^\mu}{d\lambda} d\lambda \quad (34)$$

$$\equiv \frac{\sigma(x, x')}{(\lambda - \lambda')} + e \int_{x'}^x A_\mu dx^\mu. \quad (35)$$

The transition from Eq.(34) to Eq.(35) was made by requiring

$$\sigma(x, x') \equiv \frac{1}{2} \left( \frac{ds}{d\lambda} \right)^2 (\lambda - \lambda')^2 \quad (36)$$

showing  $\sigma(x, x')$  must be defined as one-half of the square of the geodesic distance between  $x$  and  $x'$  in the limit that  $(\lambda - \lambda')$  is an infinitesimal quantity.

It is not generally possible to write the right hand side of Eq.(36) in terms of metric functions and their derivatives. Doing so would require directly evaluating

$$ds = \int_{x'(\lambda')}^{x(\lambda)} \left( \frac{ds}{d\lambda} \right) d\lambda = \int_{\lambda'}^{\lambda} \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} d\lambda = \text{constant}(\lambda - \lambda') \quad (37)$$

and expressing the results in terms of metric functions and their derivatives. Then, one would have to solve the geodesic equations in order to also express  $(\lambda - \lambda')$  in terms of metric functions and their derivatives. Point-splitting will instead rely upon a differential equation for  $\sigma(x, x')$  that may be solved iteratively. This differential equation may be obtained by moving from the Lagrangian of Eq.(29) to a Hamiltonian study of geodesic motion. In the Hamiltonian formalism, the conjugate momenta and Hamiltonian are given by,

$$p_\mu = \frac{\partial L}{\partial \left( \frac{dx^\mu}{d\lambda} \right)} = g_{\mu\nu} \frac{dx^\nu}{d\lambda} + eA_\mu \equiv \pi_\mu + eA_\mu, \quad (38)$$

and,

$$H = p_\mu \frac{dx^\mu}{d\lambda} - L = \frac{1}{2} g_{\mu\nu} (p^\mu - eA^\mu)(p^\nu - eA^\nu). \quad (39)$$

The Hamilton-Jacobi equation is given by [35, 34],

$$\frac{\partial S}{\partial \lambda} = -H(x^\alpha, \frac{\partial S}{\partial x^\alpha}) = \frac{1}{2} g^{\mu\nu} \left( \frac{\partial S}{\partial x^\mu} - eA_\mu \right) \left( \frac{\partial S}{\partial x^\nu} - eA_\nu \right). \quad (40)$$

Using

$$\frac{\partial S}{\partial \lambda} = -\frac{\sigma(x, x')}{(\lambda - \lambda')^2}, \quad (41)$$

and

$$\frac{\partial S}{\partial x^\alpha} = -\frac{\sigma_{,\alpha}(x, x')}{(\lambda - \lambda')} + eA_\alpha, \quad (42)$$

the Hamilton-Jacobi equation becomes

$$\begin{aligned} \frac{\sigma(x, x')}{(\lambda - \lambda')^2} &= \frac{1}{2} \left[ \frac{\sigma_{,\mu}(x, x')}{(\lambda - \lambda')} + eA_\mu - eA_\mu \right] \left[ \frac{\sigma^{,\mu}(x, x')}{(\lambda - \lambda')} + eA^\mu - eA^\mu \right] \\ &= \frac{1}{2} \frac{\sigma_{,\mu}(x, x') \sigma^{,\mu}(x, x')}{(\lambda - \lambda') (\lambda - \lambda')}, \end{aligned} \quad (43)$$

where  $+eA^\mu - eA^\mu$  has been included to emphasize the explicit cancellation of the gauge field terms. Multiplying both sides by  $(\lambda - \lambda')^2$ , and defining  $\sigma \equiv \sigma(x, x')$  gives,

$$\sigma = \frac{1}{2} \sigma^{,\mu} \sigma_{,\mu} = \frac{1}{2} \sigma^{i\mu} \sigma_{;i\mu} \equiv \frac{1}{2} \sigma^\mu \sigma_\mu, \quad (44)$$

where  $\sigma^\mu \equiv \sigma^{i\mu}$ . This differential equation will be used extensively in the point-splitting procedure.

The result of Eq.(44) is very important. The insensitivity of the differential equation (44) to the presence of the gauge field demonstrates that the biscalar  $\sigma(x, x')$  is a purely geometric quantity. The first covariant derivative of  $\sigma$  is a simple partial derivative, requiring no gauge,  $A^\mu$ , or Christoffel,  $\Gamma^\rho_{\mu\nu}$ , connections. Second and higher covariant derivatives will require Christoffel connections but no gauge connections. Like all other purely geometric quantities such as curvature tensors, etc., the only connections required for the covariant derivatives of  $\sigma(x, x')$  are the Christoffel connections. Point-splitting regularization in the presence of a gauge field requires careful identification of those quantities whose covariant derivatives will require not only the Christoffel connections but also the gauge connection  $A^\mu$ .

The components on the right hand side of Eq.(44) may be given a physical meaning. Consider the geodesic connecting the points  $x$  and  $x'$  of Figure 2.

The vector  $\sigma^\mu$  is a tangent to the curve at  $x$ , being the derivative of a scalar with

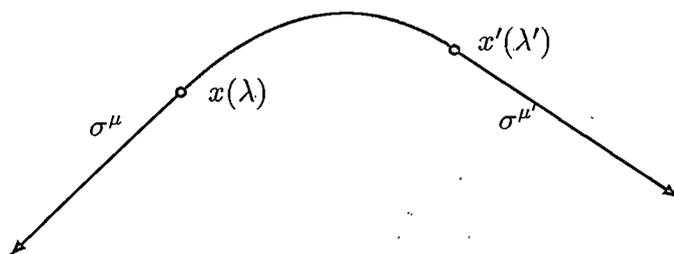


Figure 2: Geodesic connecting points  $x$  and  $x'$  with tangent vector at each endpoint respect to  $x^\mu$ . It has length equal to the arc length of the geodesic between  $x$  and  $x'$ , and points in the direction  $x' \rightarrow x$ . An equation similar to Eq.(44) may be derived for  $\sigma^{\mu'}$ , the vector tangent at the other end of the geodesic and pointing in the direction  $x \rightarrow x'$ :

$$\sigma(x, x') = \frac{1}{2} \sigma^{\mu'} \sigma_{\mu'} = \sigma(x', x). \quad (45)$$

$\sigma^{\mu'}$  is a vector tangent to the curve at  $x'$ , has length equal to the arc length of the geodesic between  $x$  and  $x'$ , and points in the direction  $x \rightarrow x'$ .

The biscalar of the geodesic interval,  $\sigma(x, x')$ , describes the spacetime geometry along a geodesic between the separated points  $x$  and  $x'$ . Point-splitting requires these points to be separated by a *unique* geodesic so that  $\sigma(x, x')$  is a single-valued function of the points  $x$  and  $x'$ . In a general Riemannian manifold, the geodesics emanating from one spacetime point begin to overlap and cross one another at some distance from that point [2]. Consider the points  $x$ ,  $x'$ , and  $x''$  and the two geodesics of Figure 3.

While there are an infinite number of geodesics emanating from the stationary point  $x$ , this diagram is only concerned with the two shown. These two are shown because the point  $x''$  is the nearest point where the geodesics emanating from  $x$  start to cross themselves. This diagram shows the points  $x$  and  $x''$  are not sufficiently

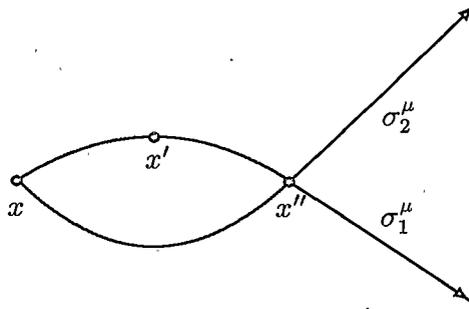


Figure 3: Two of the infinite number of geodesics emanating from  $x$  that cross at  $x''$  close in this spacetime for  $\sigma(x, x'')$  to be a single-valued function. The point  $x'$  is closer to  $x$ , close enough such that no other geodesic coming from  $x$  connects to  $x'$ , and thus  $\sigma(x, x')$  is a single-valued function as shown in Figure 3. The surface at which geodesics start to overlap is known as a *caustic surface*. It should be noted here that there are no caustic surfaces in flat spacetime. (An example of caustic surfaces occurring in two-dimensional space is the two-sphere. The infinite number of geodesics (great circles) emanating from the “north” pole cross themselves at the “south” pole of the two-sphere.) If the points  $x$  and  $x'$  are “sufficiently close,” caustic surfaces may be avoided. A way of quantifying the criterion of “sufficiently close” is provided by calculating the Van Vleck-Morette determinant [36, 37].

The Van Vleck-Morette determinant arises when considering how variation of the tangent vectors at  $x'$  relates to the single-valuedness of  $\sigma(x, x'')$ . A geodesic may be characterized either by its two endpoints  $\{x^\mu, x^{\nu'}\}$ , or by one of its endpoints along with the tangent vector at that endpoint,  $\{x^{\mu'}, \sigma_{\nu'}\}$ . This last characterization may not seem adequate to specify the geodesic at first glance, but it actually does. Consider the geodesic of Figure 2. There are an infinite number of geodesics emanating from the point  $x'$ . However, only that geodesic which has  $\sigma^{\mu'}(x, x')$  as its tangent vector is the one that connects the point  $x'$  to the point  $x$ . Any other of the infinite number of tangent vectors at  $x'$  would specify a geodesic connecting  $x'$  to a point

other than  $x$ .

The transformation from characterizing a geodesic by  $\{x^\mu, x^{\nu'}\}$  to  $\{x^\mu, \sigma_{\nu'}\}$  is given by the Jacobian,

$$\frac{\partial(\sigma_{\nu'}, x^{\rho'})}{\partial(x_\mu, x^{\tau'})} = \left| \begin{array}{cc} \frac{\partial\sigma_{\nu'}}{\partial x_\mu} & \frac{\partial\sigma_{\nu'}}{\partial x^{\tau'}} \\ \frac{\partial x^{\rho'}}{\partial x_\mu} & \frac{\partial x^{\rho'}}{\partial x^{\tau'}} \end{array} \right| = \frac{\partial(\sigma_{\nu'})}{\partial(x_\mu)} = \det(\sigma_{\mu\nu'}), \quad (46)$$

where the fact that partials with respect to different coordinates commute has been used. Variation of the tangent vector  $\sigma_{\nu'}$  produces a corresponding change in the other endpoint at  $x$ ;

$$\delta\sigma_{\nu'} = \delta(\sigma_{\nu'}) = \delta \left( \frac{\partial\sigma}{\partial x^{\nu'}} \right) = \frac{\partial}{\partial x^\mu} \left( \frac{\partial\sigma}{\partial x^{\nu'}} \right) \delta x^\mu = \sigma_{,\mu\nu'} \delta x^\mu. \quad (47)$$

Inverting this gives the variation of the endpoint at  $x$  in terms of the variation of the tangent vector  $\delta\sigma_{\nu'}$ ,

$$\delta x^\mu = -(-\sigma_{\mu\nu'})^{-1} \delta\sigma_{\nu'} \equiv -(D^{-1\mu\nu'}) \delta\sigma_{\nu'}, \quad (48)$$

where  $D_{\mu\nu'} \equiv -\sigma_{\mu\nu'}$  for a given  $x$  and  $x'$ , and  $(D^{-1\mu\nu'})$  is the inverse of  $(-\sigma_{\mu\nu'})$ . Eq.(48) means that, when  $(D^{-1\mu\nu'})$  is zero for a given  $x$  and  $x'$ , variations of the tangent vector  $\sigma_{\nu'}$  do not produce a corresponding change in the original endpoint of the geodesic located at  $x$ . Figure 3 illustrates an example wherein  $(D^{-1\mu\nu'})$  is zero. Variation of the tangent vector from the vector  $\sigma_1^\mu$  to the vector  $\sigma_2^\mu$  at point  $x''$  changes from the upper geodesic between  $x$  and  $x''$  to the lower geodesic between these points. Yet both of these geodesics still have  $x$  as their other endpoint. Thus  $\sigma(x, x'')$  is not single-valued,  $x''$  is not "sufficiently close" to  $x$ , and the point  $x''$  is not appropriate to choose for point-splitting.

With more than one geodesic emanating from  $x$  crossing at the point  $x''$ , a caustic surface has been found at  $x''$ . This occurs if  $(D^{\mu\nu'})^{-1} = 0$ , or, equivalently,  $\det(D^{-1\mu\nu'}) \equiv D^{-1} = 0$ . If  $D^{-1} = 0$ , then  $\det(D_{\mu\nu'})$  blows up. The Van Vleck-Morette determinant  $D$  is defined by

$$D(x, x') \equiv \det(D_{\mu\nu'}). \quad (49)$$

Evidently, when the Van Vleck-Morette determinant diverges, a caustic surface has been found, and the points of chosen for geodesic point-splitting are not "sufficiently close."

An important identity involving the Van Vleck-Morette determinant may be derived from Eq.(44). Taking one covariant derivative with respect to the stationary point  $x^\alpha$  gives

$$\sigma_{;\alpha} = \sigma_{;\mu\alpha}\sigma^{i\mu} = \sigma_{;\alpha\mu}\sigma^{i\mu}. \quad (50)$$

Taking one covariant derivative with respect to the nearby point  $x^{\beta'}$  gives

$$\sigma_{;\alpha\beta'} = \sigma_{;\alpha\mu\beta'}\sigma^{i\mu} + \sigma_{;\alpha\mu}\sigma^{i\mu}{}_{;\beta'} = \sigma_{;\alpha\beta'\mu}\sigma^{i\mu} + \sigma_{;\alpha}{}^\mu\sigma_{;\mu\beta'} \quad (51)$$

where the fact that covariant derivatives with respect to different spacetime points commute has been used. This may be rewritten as

$$D_{\alpha\beta'} = D_{\alpha\beta';\mu}\sigma^{i\mu} + D_{\mu\beta'}\sigma_{;\alpha}{}^\mu. \quad (52)$$

Multiplying by  $(D^{\alpha\beta'})^{-1}$ , and using  $\text{tr} \ln(A) = \ln \det(A)$  for any matrix  $A$ , gives

$$(D^{\alpha\beta'})^{-1}D_{\alpha\beta'} = (D^{\alpha\beta'})^{-1}D_{\alpha\beta';\mu}\sigma^{i\mu} + (D^{\alpha\beta'})^{-1}D_{\mu\beta'}\sigma_{;\alpha}{}^\mu \quad (53)$$

$$4 = [\text{tr} \ln(D_{\alpha\beta'})]_{;\mu}\sigma^{i\mu} + \delta^\alpha{}_\mu\sigma_{;\alpha}{}^\mu \quad (54)$$

$$4 = [\ln \det(D_{\alpha\beta'})]_{;\mu}\sigma^{i\mu} + \sigma_{;\mu}{}^\mu \quad (55)$$

$$4 = D^{-1}D_{;\mu}\sigma^{i\mu} + D^{-1}D\sigma_{;\mu}{}^\mu \quad (56)$$

$$4 = D^{-1}(D\sigma^{i\mu})_{;\mu}. \quad (57)$$

This has the form of a divergence equation for the vector  $\sigma^{i\mu}$ . Eq.(57) may be rewritten in a more convenient form. First, note the action of the operator,  $\sigma^\mu \frac{\partial}{\partial x^\mu}$  on the scalar  $f$ :

$$\sigma^\mu f_{,\mu} = (\lambda - \lambda') \frac{dx^\mu}{d\lambda} \frac{\partial f}{\partial x^\mu} = (\lambda - \lambda') \frac{df}{d\lambda}. \quad (58)$$

This operator gives the change of a function along the geodesic parameterized by  $\lambda$ , or, in essence, the change in the function as it is parallel transported along that geodesic. Also,

$$\sigma^{\mu'} f_{,\mu'} = (\lambda' - \lambda) \frac{dx^{\mu'}}{d\lambda'} \frac{\partial f}{\partial x^{\mu'}} = (\lambda' - \lambda) \frac{df}{d\lambda} \quad (59)$$

gives the change of the  $f$  when it is parallel transported in the opposite direction along the geodesic. Defining,

$$\Delta(x, x') \equiv g^{-1/2}(x) D(x, x') g^{-1/2}(x'), \quad (60)$$

and using Eq.(58), Eq.(57) may now be rewritten in terms of the divergence of the vector  $\sigma^\mu$ :

$$\sigma^\mu{}_{;\mu} = 4 - \Delta^{-1} \sigma^{;\mu} \Delta_{;\mu} \quad (61)$$

$$= 4 - \Delta^{-1} (\lambda - \lambda') \frac{d\Delta}{d\lambda} \quad (62)$$

$$= 4 - \frac{d\Delta (\lambda - \lambda')}{\Delta d\lambda} \quad (63)$$

$$= 4 - \frac{d(\ln \Delta)}{d(\ln \lambda)}, \quad (64)$$

where the transition to the last line was made by setting the parameter  $\lambda' = 0$ . In flat spacetime, there are no caustic surfaces, the Van Vleck-Morette determinant is zero, and this divergence is 4. If  $D$ , and thus  $\Delta$ , blows up, then there are an infinite number of geodesics emanating from the point  $x$  crossing at  $x'$ , and there is a caustic surface at  $x'$ .

Finally, there is the matter how to interpret N-tensors having weight at a spacetime point that is not the stationary point  $x$ . For example, consider the vector  $A^{\nu'}$  which is evaluated at the nearby point  $x'$ . A way is needed for the information contained in this vector to be conveyed to the stationary point  $x$ . This is a concept familiar in field theory in that Green functions accomplish such transport of information. Consider

the flat space Klein-Gordon equation

$$(\partial_\alpha \partial^\alpha - m^2)\phi(x) = j(x), \quad (65)$$

where  $\partial_\alpha \partial^\alpha$  is the flat space D'Alembertian and  $j(x)$  is a source for the field  $\phi(x)$ . Solutions to Eq.(65) will be generated by [41]

$$\phi(x) = \phi^{(0)}(x) + \int G(x, x') j(x') d^4 x', \quad (66)$$

where  $\phi^{(0)}$  is a solution to the homogeneous form of Eq.(65). The Green function  $G(x, x')$  takes information at  $x'$  and conveys it to the point  $x$ . In a general spacetime, the entity analogous to  $G(x, x')$  is  $g^\mu{}_{\nu'}$ , the *bivector of parallel transport*, defined by

$$A^\mu = g^\mu{}_{\nu'} A^{\nu'}, \quad (67)$$

where  $g^\mu{}_{\nu'}$  takes the information at the nearby point  $x'$  and conveys it to the stationary point  $x$ .

An important property of  $g^\mu{}_{\nu'}$  may be demonstrated by using the operator of Eq.(58) to parallel transport  $A^\mu$  along a geodesic parameterized by  $\lambda$ . Parallel transport of  $A^\mu$  along the geodesic connecting the points  $x$  and  $x'$  requires

$$\sigma^\rho A^\mu{}_{;\rho} = 0, \quad (68)$$

and thus

$$\sigma^\rho A^\mu{}_{;\rho} = g^\mu{}_{\nu'}{}_{;\rho} \sigma^{i\rho} A^{\nu'} = 0. \quad (69)$$

For this to be true for arbitrary  $\sigma^{i\rho}$  and  $A^{\nu'}$  requires

$$\sigma^{i\rho} g^\mu{}_{\nu'}{}_{;\rho} = 0. \quad (70)$$

This equation will be needed for the point-splitting procedure.

## CHAPTER 4

## DeWitt-Schwinger Geodesic Point-Splitting

The geodesic point-splitting scheme of DeWitt [2] was developed from Schwinger's [3] original method of calculating the Feynman Green function  $G_F(x, x')$  associated with a quantum field. At the heart of Schwinger's method lies the idea that the Feynman Green function is determined by the differential operator of the gauge invariant wave equation. The wave equation for the charged complex scalar field is obtained by varying the action in Eq.(17) with respect to  $\phi^*$  and is given by

$$g^{1/2}[D_\mu D^\mu - (m^2 + \xi R)]\phi(x) \equiv \phi_{|\mu}{}^\mu - (m^2 + \xi R)\phi = 0. \quad (71)$$

It will be shown that, once the Feynman Green function is determined, the Hadamard function  $G^{(1)}(x, x')$  will be determined. Then the DS point-splitting expansions for  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  may be constructed from  $G^{(1)}$  and its derivatives. This chapter will detail the steps for obtaining a DeWitt-Schwinger expansion for  $G^{(1)}(x, x')$  from Schwinger's original proper time method.

The classical current for a charged scalar field is obtained by varying the action in Eq.(17) with respect to  $A^\mu$  and is given by

$$j^\mu = \frac{1}{4\pi} F^{\mu\nu}{}_{;\nu} = \frac{1}{2}ie [\{D^\mu \phi, \phi^*\} - \{D^\mu \phi, \phi^*\}^*], \quad (72)$$

where the braces  $\{\}$  denote the anticommutator. The components of the classical stress-energy tensor are given by,

$$\begin{aligned} T^{\mu\nu} = & \frac{1}{2} \left[ \frac{1}{2} (1 - 2\xi) \{ \phi^{|\mu}, \phi^{*|\nu} \} + \frac{1}{2} (2\xi - \frac{1}{2}) g^{\mu\nu} \{ \phi_{|\sigma}, \phi^{*|\sigma} \} \right. \\ & - \xi \{ \phi^{|\mu\nu}, \phi^* \} + \xi g^{\mu\nu} \{ \phi_{|\sigma}{}^\sigma, \phi^* \} \\ & \left. + \frac{1}{2} \xi (R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R) \{ \phi, \phi^* \} - \frac{1}{4} m^2 g^{\mu\nu} \{ \phi, \phi^* \} + c.c. \right], \quad (73) \end{aligned}$$

where *c.c.* denotes the complex conjugate of all of the previous terms. The anti-commutators above arise from symmetrizing with respect to the fields  $\phi$  and  $\phi^*$ .

Consider now the transition from the classical form of  $\{\phi(x), \phi(x)\}$  to its quantized form in the point-splitting procedure. The functions  $\phi(x)$  become the field operators  $\underline{\phi}(x)$ . The first field operator is moved to the nearby point  $x'$  and evaluated between the vacuum states  $\langle 0|$  and  $|0\rangle$ . A similar expression is then obtained by taking the second field operator to the point  $x'$ . The two results are then averaged, yielding the biscalar Hadamard function  $G^{(1)}(x, x')$ , given by

$$\{\phi(x), \phi^*(x)\} \rightarrow G^{(1)}(x, x') \equiv \langle 0 | \{ \underline{\phi}(x), \underline{\phi}(x') \} | 0 \rangle. \quad (74)$$

With this definition, the vacuum polarization is given by

$$\langle \phi^2 \rangle = \lim_{x' \rightarrow x} \frac{1}{2} G^{(1)}(x, x'). \quad (75)$$

Constructing the VEV of the current and stress-energy tensor will require not only  $G^{(1)}(x, x')$ , but also gauge covariant derivatives of  $G^{(1)}(x, x')$ . Remembering that derivatives with respect to a spacetime point only act on functions of that point, it is straightforward to show the first few gauge covariant derivatives of  $G^{(1)}(x, x')$  are

$$G^{(1)|\mu} \equiv \langle 0 | \{ \underline{\phi}^{|\mu}(x), \underline{\phi}^*(x') \} | 0 \rangle; \quad (76)$$

$$G^{(1)|\mu'} \equiv \langle 0 | \{ \underline{\phi}(x), \underline{\phi}^{*|\mu'}(x') \} | 0 \rangle, \quad (77)$$

$$G^{(1)|\mu\nu} \equiv \langle 0 | \{ \underline{\phi}^{|\mu\nu}(x), \underline{\phi}^*(x') \} | 0 \rangle, \quad (78)$$

$$G^{(1)|\mu'\nu'} \equiv \langle 0 | \{ \underline{\phi}^{|\mu}(x), \underline{\phi}^{*|\nu'}(x') \} | 0 \rangle, \quad (79)$$

and

$$G^{(1)|\mu'\nu'} \equiv \langle 0 | \{ \underline{\phi}(x), \underline{\phi}^{*|\mu'\nu'}(x') \} | 0 \rangle. \quad (80)$$

In all of these, the unprimed and primed indices refer to differentiation with respect to the spacetime points  $x$  and  $x'$ , respectively. Since  $G^{(1)}(x, x')$  is understood to be a

biscalar, the  $(x, x')$  argument of  $G^{(1)}(x, x')$  will hereafter often be dropped for brevity in the long expressions to follow.

The expectation value of the current associated with the charged scalar field may be written using Eq.(72) and the definitions of Eqs.(76-(80) and is given by

$$\langle \underline{j}^\mu(x) \rangle = \lim_{x' \rightarrow x} \frac{ie}{4} \left[ \left( G^{(1)|\mu} + g^\mu{}_{\tau'} G^{(1)|\tau'} \right) - \left( G^{(1)|\mu} + g^\mu{}_{\tau'} G^{(1)|\tau'} \right)^* \right]. \quad (81)$$

The bivector of parallel transport  $g^\mu{}_{\tau'}$  has been used to transport the information from the spacetime point  $x'$  back to the stationary point  $x$ . Note that the form of Eq.(81) guarantees that the current will be real. The expectation value of the stress-energy tensor may similarly be written in terms of  $G^{(1)}$  and its derivatives as

$$\begin{aligned} \langle \underline{T}^{\mu\nu}(x) \rangle = \lim_{x' \rightarrow x} \text{Re} \left[ \frac{1}{2} \left( \frac{1}{2} - \xi \right) (g^\mu{}_{\tau'} G^{(1)|\tau'\nu} + g^\nu{}_{\rho'} G^{(1)|\mu\rho'}) + \left( \xi - \frac{1}{4} \right) g^{\mu\nu} g^{\alpha\rho'} G^{(1)}|_{\alpha\rho'} \right. \\ \left. - \frac{1}{2} \xi (G^{(1)|\mu\nu} + g^\mu{}_{\tau'} g^\nu{}_{\rho'} G^{(1)|\tau'\rho'}) + \frac{1}{4} \xi g^{\mu\nu} (G^{(1)}|_{\alpha}{}^\alpha + g^\alpha{}_{\tau'} g_{\alpha\rho'} G^{(1)|\tau'\rho'}) \right. \\ \left. + \frac{1}{2} \xi (R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R) G^{(1)} - \frac{1}{4} m^2 g^{\mu\nu} G^{(1)} \right]. \quad (82) \end{aligned}$$

Eqs.(75)-(82) are divergent for the same reasons that the Wightman function  $G^+(x, x')$  of Eq.(13) is divergent. The Hadamard elementary function,  $G^{(1)}$ , is constructed from the VEV of the product of two fields in a manner similar to  $G^+(x, x')$  as shown by Eqs.(74) and (13). Just as  $G^+(x, x')$  is quadratically divergent so, too, is  $G^{(1)}(x, x')$  quadratically divergent. By direct analogy with Eq.(14), one derivative of  $G^{(1)}$  will potentially have a cubic divergence, while two derivatives will potentially have a quartic divergence. Point-splitting will isolate these divergences in preparation for renormalizing Eqs.(75), (81), and (82).

Evaluating  $G^{(1)}$  and its derivatives requires relating  $G^{(1)}$  to the Feynman Green function  $G_F(x, x') \equiv G_F$ . The Feynman Green function for the complex scalar field is given by the VEV of the time-ordered product of the field and its conjugate,

$$iG_F(x, x') = \langle 0 | T[\phi(x)\phi^*(x')] | 0 \rangle \quad (83)$$

$$= \langle 0 | \Theta(t - t') \underline{\phi}(x) \underline{\phi}^*(x') + \Theta(t' - t) \underline{\phi}^*(x') \underline{\phi}(x) | 0 \rangle, \quad (84)$$

where

$$\Theta(t - t') = \begin{cases} 1, & t > t' \\ 0, & t' > t. \end{cases} \quad (85)$$

The Pauli-Jordan, or Schwinger function,  $G(x, x')$ , is given by the VEV of the commutator of the field operators  $\underline{\phi}(x)$ ,

$$iG(x, x') \equiv \langle 0 | [\underline{\phi}(x), \underline{\phi}^*(x')] | 0 \rangle. \quad (86)$$

The retarded and advanced Green functions are defined by

$$G_R(x, x') \equiv -\Theta(t - t')G(x, x'), \quad (87)$$

and

$$G_A(x, x') \equiv \Theta(t' - t)G(x, x'). \quad (88)$$

With these definitions, it is straightforward to derive the relationship

$$G_F(x, x') = \overline{G}(x, x') - \frac{1}{2}iG^{(1)}(x, x'), \quad (89)$$

where  $\overline{G}(x, x')$  is one-half the sum of the advanced and retarded Green functions. This relation shows that, were the Feynman Green function known, its imaginary part would be the Hadamard function from which the DS point-splitting expansions for  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T^{\mu\nu} \rangle$  may be constructed. Determining the Feynman Green function requires determining the action of the differential operator of Eq.(71) on the Feynman Green function. To do this, we now make the transition from coordinate space to Schwinger's proper time space.

The transition from classical to quantum fields is made by imposing the equal time commutation relations for the quantum operators  $\underline{\phi}(x)$  and  $\underline{\pi}(x)$  [49],

$$[\underline{\phi}(t, \vec{x}), \underline{\phi}(t, \vec{x}')] = 0 \quad (90)$$

$$[\underline{\pi}(t, \vec{x}), \underline{\pi}(t, \vec{x}')] = 0 \quad (91)$$

$$[\underline{\phi}(t, \vec{x}), \underline{\pi}(t, \vec{x}')] = i\delta^3(\vec{x} - \vec{x}'), \quad (92)$$

where  $\pi(t, \vec{x})$  is the canonically conjugate variable to  $\phi(t, \vec{x})$  defined by

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \partial_0 \phi, \quad (93)$$

where  $x^0$  is a time (temporal) coordinate. Using Eqs.(90)–(92), along with Eq.(71) and  $\partial_t \Theta(t - t') = \delta(t - t')$ , the action of the wave differential operator on  $G_F(x, x')$  may be shown to be

$$g^{1/2}[D_\mu D^\mu - (m^2 + \xi R)]G_F(x, x') = -\delta^4(x - x'), \quad (94)$$

or

$$F(x)G_F(x, x') = -\delta^4(x - x'), \quad (95)$$

where  $F(x) \equiv g^{1/2}[D_\mu D^\mu - (m^2 + \xi R)]$  operates with respect to the spacetime point  $x$ . Rewriting Eq.(95) as a matrix equation gives

$$\int \langle x | \underline{F} | x'' \rangle \langle x'' | \underline{G}_F | x' \rangle d^4 x'' = -\langle x | \underline{1} | x \rangle, \quad (96)$$

where  $\underline{F}$  and  $\underline{G}_F$  are now matrix operators. Inserting two factors of  $\underline{1} = \underline{g}^{-1/4} \underline{g}^{1/4}$  in Eq.(96) maintains the transformation properties of the bases  $|x\rangle$  as in Eq.(28). Removing the projection operator,  $\int |x''\rangle \langle x''| d^4 x''$ , along with the matrix elements  $\langle x | (\dots) | x' \rangle$  allows the matrix equation (96) to be written as,

$$\underline{g}^{1/4} \underline{G}_F \underline{g}^{1/4} = \frac{\underline{1}}{-(\underline{g}^{-1/4} \underline{F} \underline{g}^{-1/4} + i\epsilon \underline{1})} = i \int_0^\infty e^{-i\underline{H}s} ds = i \int_0^\infty e^{-i\underline{H}(s-0)} ds. \quad (97)$$

In Eq.(97), the factor  $e^{-i\underline{H}0} = 1$  has been explicitly added for clarity below, and the matrix operator  $\underline{H}$  is defined by

$$\underline{H} \equiv -(\underline{g}^{-1/4} \underline{F} \underline{g}^{-1/4} + i\epsilon \underline{1}) \quad (98)$$

The infinitesimal  $+i\epsilon \underline{1}$  has been added to make this integral analytic in the upper half of the complex plane [2, 49]. Taking matrix elements of Eq.(97) and rearranging, yields,

$$G_F(x, x') = i \int_0^\infty g^{-1/4}(x) \langle x | e^{-i\underline{H}(s-0)} | x' \rangle g^{-1/4}(x') ds. \quad (99)$$

The operator  $e^{-i\mathcal{H}(s-0)}$  was interpreted by Schwinger to act in a fictitious 4 + 1-dimensional Hilbert space where  $s$  is a proper time parameter. With the spacetime points  $x$  and  $x'$  understood to be associated with points  $s$  and  $s'$ , respectively, in this Hilbert space, the operator  $e^{i\mathcal{H}s}$  is interpreted to act on the kets  $|x, s\rangle$  as,

$$e^{i\mathcal{H}s}|x_0, s = 0\rangle \equiv |x, s\rangle. \quad (100)$$

The ket  $|x_0, s = 0\rangle$ , which is associated with the spacetime point  $x_0$  and the proper time parameter  $s = 0$ , has been taken to the ket  $|x, s\rangle$ , which is associated with the spacetime point  $x$  and the proper time parameter  $s$ . Thus,

$$G_F(x, x') \equiv i \int_0^\infty g^{-1/4}(x) \langle x, s|x', 0\rangle g^{-1/4}(x') ds. \quad (101)$$

The matrix element  $\langle x, s|x', 0\rangle$  obeys a Schrödinger-like equation in both Hilbert space form,

$$i \frac{\partial}{\partial s} \langle x, s|x', 0\rangle = \langle x, s|\mathcal{H}|x', 0\rangle, \quad (102)$$

and coordinate space form,

$$i \frac{\partial}{\partial s} \langle x, s|x', 0\rangle = g^{-1/4}(x) g^{-1/4}(x') [(m^2 + \xi R) - D_\mu D^\mu] \langle x, s|x', 0\rangle, \quad (103)$$

where the infinitesimal factor  $+i\epsilon$  has been dropped for brevity. Also, the reader should take care not to confuse the factor  $g^{-1/4}(x)g^{-1/4}(x')$  in this equation with the two factors  $g^{-1/4}(x)$  and  $g^{-1/4}(x')$  in Eq.(101). Keeping track of these factors is very important in this derivation! In Eq.(103), the operator  $g^{-1/4}$  to the right of the derivatives has been allowed to operate on the ket  $|x', 0\rangle$ . Then, since  $[(m^2 + \xi R) - D_\mu D^\mu]$  is chosen to operate with respect to the point  $x$ , the factor of  $g^{-1/4}(x')$  may be moved to the left of the derivative operator. The boundary condition,

$$\langle x, 0|x', 0\rangle = \langle x|x'\rangle = \delta(x - x'), \quad (104)$$

is imposed in accordance with Eq.(95).

At this point, solving either of the the Schrödinger-like equations exactly would completely determine  $G_F$ . Unfortunately, there is no known exact solution for  $G_F$  in a general spacetime, so an approximate solution is needed. This approximate solution is found by determining the action of the operator  $\underline{H}$  in the proper time space of  $s$ .

To determine the action of the operator  $\underline{H}$  in the 4 + 1-dimension proper time space used by Schwinger [3], the operator  $\underline{H}$  is rewritten in a flat space version where  $g^{-1/4}(x) = 1$ ;

$$\underline{H} \equiv -\underline{F} = m^2 \underline{1} + \underline{\pi}_\mu \underline{\pi}^\mu, \quad (105)$$

In Eq.(105),  $\underline{\pi}_\mu \equiv (\nabla_\mu/i - eA_\mu)$  is the conjugate momentum derivative operator. The position operator in this proper time space is given by  $\underline{x}^\mu$ . The commutation relations of  $\underline{\pi}_\mu$  and  $\underline{x}_\mu$  are given by

$$[\underline{x}^\mu, \underline{x}^\nu] = 0, \quad [\underline{x}^\mu, \underline{\pi}^\nu] = i\delta^{\mu\nu} \underline{1}, \quad \text{and} \quad [\underline{\pi}^\mu, \underline{\pi}^\nu] = ieF^{\mu\nu} \underline{1}, \quad (106)$$

where  $F^{\mu\nu} \equiv (\partial^\mu A^\nu - \partial^\nu A^\mu)$  is the familiar electromagnetic field tensor. Using these commutation relations, the equations of motion for the operators  $\underline{x}^\mu$  and  $\underline{\pi}^\mu$  are

$$\frac{d\underline{x}^\mu}{ds} = i[\underline{H}, \underline{x}^\mu] = 2\underline{\pi}^\mu, \quad (107)$$

and

$$\frac{d\underline{\pi}^\mu}{ds} = i[\underline{H}, \underline{\pi}^\mu] = 2eF^{\mu\nu} \underline{\pi}_\nu - ie\partial_\nu F^{\mu\nu} \underline{1}. \quad (108)$$

Now consider the case of zero electromagnetic fields, or  $F^{\mu\nu} = 0$ . The equations of motion become

$$\frac{d\underline{\pi}^\mu}{ds} = 0 \Rightarrow \underline{\pi}^\mu(s) = \underline{\pi}^\mu(0). \quad (109)$$

and

$$\frac{d\underline{x}^\mu}{ds} = 2\underline{\pi}^\mu(0) = \frac{\underline{x}^\mu(s) - \underline{x}^\mu(0)}{s - 0}. \quad (110)$$

Taking the matrix element  $\langle x, s | \underline{H} | x', 0 \rangle$  requires all  $\underline{x}^\mu(s)$  be moved to the left of the  $\underline{x}^\mu(0)$ :

$$\begin{aligned} \underline{H} &= m^2 \underline{1} + \frac{(\underline{x}^\mu(s) - \underline{x}^\mu(0))(\underline{x}_\mu(s) - \underline{x}_\mu(0))}{4s^2} \\ &= m^2 \underline{1} + (4s^2)^{-1} (\underline{x}^\mu(s)\underline{x}_\mu(s) - 2\underline{x}^\mu(s)\underline{x}_\mu(0) + \underline{x}^\mu(0)\underline{x}_\mu(0)) + \\ &\quad (4s^2)^{-1} [\underline{x}^\mu(s), \underline{x}_\mu(0)]. \end{aligned} \quad (111)$$

Using

$$[\underline{x}^\mu(s), \underline{x}_\mu(0)] = [2s\underline{\pi}^\mu(0) + \underline{x}^\mu(0), \underline{x}_\mu(0)] = 2s(-i\delta^\mu{}_\mu) = -8is, \quad (112)$$

and taking matrix elements of Eq.(111) yields

$$\langle x, s | \underline{H} | x', 0 \rangle = \left[ m^2 + (4s^2)^{-1} (x^\mu(s) - x^\mu(0))^2 - \frac{2i}{s} \right] \langle x, s | x', 0 \rangle. \quad (113)$$

With  $x^\mu(s)$  and  $x^\mu(0)$  associated with the spacetime points  $x$  and  $x'$ , respectively, the coordinate space Schrödinger-like equation (103) now reads

$$i \frac{\partial}{\partial s} \langle x, s | x', 0 \rangle = \left[ m^2 + (4s^2)^{-1} (x^\mu - x'^\mu)^2 - \frac{2i}{s} \right] \langle x, s | x', 0 \rangle, \quad (114)$$

which is readily integrated to give

$$\langle x, s | x', 0 \rangle = C \Phi(x, x') s^{-2} e^{-i \left[ m^2 s - \frac{(x-x')^2}{4s} \right]}. \quad (115)$$

Eq.(115) must behave as a delta function as  $s \rightarrow 0$  according to Eq(104). This will be true so long as

$$\lim_{x' \rightarrow x} \Phi(x, x') = 1, \quad (116)$$

and

$$C s^{-2} \int e^{\left[ \frac{ix^2}{4s} \right]} d^4 x = 1. \quad (117)$$

Eq.(116) is required to be true by definition. Eq.(117) then shows that  $C = -i(4\pi)^{-2}$ .

DeWitt used this flat space example, culminating in Eq.(115), to make an ansatz for the solution of Eq.(103), namely,

$$\langle x, s | x', 0 \rangle = -\frac{i}{(4\pi)^2} \frac{D^{\frac{1}{2}}(x, x')}{s^2} \exp \left[ i \frac{\sigma(x, x')}{2s} - im^2 s \right] \Omega(x, x', s), \quad (118)$$

where  $D(x, x')$  is the Van Vleck-Morette determinant. The unknown function  $\Omega(x, x', s)$  is yet to be determined, yet DeWitt [38] proposed the flat space solution suggests the boundary condition

$$\lim_{x' \rightarrow x} \Omega(x, x', s) = 1, \quad (119)$$

in agreement with Eq.(116).

Geometric quantities are unaffected by the presence of the gauge field so no gauge field information can arise in the point-splitting procedure through the quantities  $\sigma(x, x')$  and  $D^{\frac{1}{2}}(x, x')$  in Eq.(118). The only way the gauge field can explicitly appear is through the unknown function  $\Omega(x, x', s)$ , which carries information about the gauge field. Whenever derivatives of this function are taken, they must be gauge covariant derivatives which obey the commutation relation

$$f(x, x')|_{\mu\nu} - f(x, x')|_{\nu\mu} = ieF_{\mu\nu}f(x, x'), \quad (120)$$

where  $f(x, x')$  is a function that explicitly carries information about the gauge field.

Substituting Eq.(118) into Eq.(103), and using Eq.(57), yields a differential equation for  $\Omega(x, x')$

$$i \frac{\partial \Omega}{\partial s} + \frac{i}{s} \Omega|_{\mu} \sigma^{\mu} = -D^{-1/2} (D^{1/2} \Omega)|_{\mu}{}^{\mu} + \xi R \Omega. \quad (121)$$

The function  $\Omega$  is given the *asymptotic* series representation [2],

$$\Omega(x, x') = \sum_0^{\infty} a_n(x, x') (is)^n. \quad (122)$$

This may be done, so long as the gravitational and electromagnetic fields, upon which the coefficients  $a_n(x, x')$  depend, are slowly varying over the infinitesimal distance between the points  $x$  and  $x'$ . Substituting Eq.(122) into Eq.(121), defining

$$\Delta(x, x') \equiv g^{-1/2}(x) D(x, x') g^{-1/2}(x'), \quad (123)$$

and equating like powers of  $s$  in the infinite series that result, yields the recursion relations which will be used to determine the  $a_n$ :

$$\sigma^{|\mu} a_{0|\mu} = 0, \quad (124)$$

and

$$\sigma^{|\mu} a_{n+1|\mu} + (n+1)a_{n+1} = \Delta^{-1/2}(\Delta^{1/2} a_n)_{|\mu}{}^\mu - \xi R a_n. \quad (125)$$

These relations are of the same form as those derived by Christensen [30]. The new feature here is that the presence of the gauge field requires that all derivatives be gauge covariant when appropriate. As mentioned previously, objects such as  $\sigma^{i\mu\nu}$ ,  $\Delta^{1/2; \mu\nu}$ ,  $R^{i\mu\nu\rho}$ , etc., will only require partial derivatives and the Christoffel connections in their covariant derivatives. Only the coefficients  $a_n(x, x')$ , which carry information about the gauge field, will require the gauge connection,  $A^\mu$ , in addition to partial derivatives and the christoffel connections, in their derivatives are gauge covariant.

Substituting Eq.(122) and Eq.(118) into Eq.(101) yields

$$G_F(x, x') = \frac{\Delta^{1/2}}{(4\pi)^2} \int_0^\infty s^{-2} e^{-i[m^2 s - \frac{\sigma}{2s}]} \sum_{n=0}^\infty a_n(x, x') (is)^n ds. \quad (126)$$

Note how the two factors  $g^{-1/4}(x)$  and  $g^{-1/4}(x')$  have been grouped with the factor  $D^{1/2}(x, x')$  to form  $\Delta^{1/2}(x, x')$  defined in Eq.(123). The summation and integration may be exchanged, and Eq.(126) may be rewritten as

$$G_F(x, x') = \frac{\Delta^{1/2}}{(4\pi)^2} \sum_{n=0}^\infty a_n(x, x') \left( -\frac{\partial}{\partial m^2} \right)^n \int_0^\infty s^{-2} e^{-i[m^2 s - \frac{\sigma}{2s}]} ds. \quad (127)$$

Using the identity [39]

$$\frac{1}{(4\pi)^2} \int_0^\infty s^{-2} e^{-i[m^2 s - \frac{\sigma}{2s}]} ds = -\frac{m^2 H_1^{(2)} [(-2m^2\sigma)^{1/2}]}{8\pi (-2m^2\sigma)^{1/2}}, \quad (128)$$

yields

$$G_F(x, x') = -\frac{\Delta^{1/2}}{8\pi} \sum_{n=0}^\infty a_n \left( -\frac{\partial}{\partial m^2} \right)^n \frac{m^2 H_1^{(2)} [(-2m^2\sigma)^{1/2}]}{(-2m^2\sigma)^{1/2}}, \quad (129)$$

where  $H_1^{(2)}(x)$  is the Hankel function of the second kind of order one;

$$H_1^{(2)}(x) = J_1(x) - iY_1(x). \quad (130)$$

The Hankel function  $H_1^{(2)}$  may be expanded in an asymptotic series in its small argument  $(-2m^2\sigma)^{1/2}$ ,

$$\begin{aligned} \frac{m^2 H_1^{(2)} [(-2m^2\sigma)^{1/2}]}{(-2m^2\sigma)^{1/2}} &= \frac{1}{i\pi} \left( \frac{1}{\sigma + i\epsilon} + \right. \\ &2m^2 \left\{ \left[ \gamma + \frac{1}{2} \ln\left(\frac{1}{2}m^2\right) + \frac{1}{2} \ln(\sigma + i\epsilon) \right] \left[ \frac{1}{2} + \frac{2m^2\sigma}{2^2 \times 4} + \frac{(2m^2\sigma)^2}{2^2 \times 4^2 \times 6} + \dots \right] \right. \\ &\left. \left. - \frac{1}{4} - \frac{2m^2\sigma}{2^2 \times 4} \left(1 + \frac{1}{4}\right) - \frac{(2m^2\sigma)^2}{2^2 \times 4^2 \times 6} \left(1 + \frac{1}{2} + \frac{1}{6}\right) - \dots \right\} - \dots \right), \end{aligned} \quad (131)$$

where  $\gamma$  is Euler's constant and the infinitesimal  $+i\epsilon$  has been explicitly included.

The differentiations and summations of Eq.(129) may be performed on this series.

Using the identities,

$$\frac{1}{\sigma + i\epsilon} = \frac{1}{\sigma} - i\pi\delta(\sigma) \quad , \quad \ln(\sigma + i\epsilon) = \ln|\sigma| + i\pi\Theta(-\sigma), \quad (132)$$

allows the identification of the real and imaginary parts of the Feynman Green function  $G_F(x, x')$ . Using Eq.(89), the DeWitt-Schwinger point-splitting expansion for the biscalar  $G^{(1)}(x, x')$  is obtained:

$$\begin{aligned} G^{(1)}(x, x') &= \frac{\Delta^{1/2}(x, x')}{4\pi^2} \times \\ &\left\{ a_0(x, x') \left[ \frac{1}{\sigma(x, x')} + m^2 \left( \gamma + \frac{1}{2} \ln \left| \frac{1}{2} m^2 \sigma(x, x') \right| \right) \left( 1 + \frac{1}{4} m^2 \sigma(x, x') + \dots \right) \right. \right. \\ &\quad \left. \left. - \frac{1}{2} m^2 - \frac{5}{16} m^2 \sigma(x, x') + \dots \right] \right. \\ &- a_1(x, x') \left[ \left( \gamma + \frac{1}{2} \ln \left| \frac{1}{2} m^2 \sigma(x, x') \right| \right) \left( 1 + \frac{1}{2} m^2 \sigma(x, x') + \dots \right) - \frac{1}{2} m^2 \sigma(x, x') - \dots \right] \\ &+ a_2(x, x') \sigma(x, x') \left[ \left( \gamma + \frac{1}{2} \ln \left| \frac{1}{2} m^2 \sigma(x, x') \right| \right) \left( \frac{1}{2} + \frac{1}{8} m^2 \sigma(x, x') + \dots \right) - \frac{1}{4} - \dots \right] \\ &\left. + \frac{1}{2m^2} [a_2(x, x') + \dots] + \frac{1}{2m^4} [a_3(x, x') + \dots] + \dots \right\}. \end{aligned} \quad (133)$$

In this form,  $G^{(1)}$  appears to have at least a quadratic divergence in the infinitesimal separation  $\sigma^\mu$  due to the presence of the term proportional to  $1/\sigma = 2/(\sigma_\mu \sigma^\mu)$ .

This is in agreement with the discussion of the degree of divergence of scalar QED. The task now is to determine the true form of the biscalars  $\Delta^{1/2}(x, x')$ ,  $\sigma(x, x')$  and  $a_n(x, x')$  in Eq.(133). With  $\langle j^\mu \rangle$  and  $\langle T_{\mu\nu} \rangle$  being constructed from  $G^{(1)}$  and its derivatives, it will also be necessary to determine the form of derivatives of these biscalars.

As discussed in the next chapter, it will not be possible to exactly determine either the biscalars appearing on the right hand side of Eq.(133), nor to exactly determine the derivatives of these biscalars. Yet we may construct approximations to the biscalars and their derivatives, or any bitensor, which will satisfy the requirements of the point-splitting procedure. As explained more fully at the beginning of the next chapter, each bitensor is expanded in a Taylor series about the stationary point  $x$  in terms of the infinitesimal separation vector  $\sigma^\mu$ . This expansion is given by

$$a^{\mu\nu\dots}(x, x') = a0^{\mu\nu\dots}(x) + a1^{\mu\nu\dots}{}_\alpha(x)\sigma^\alpha + \frac{1}{2!}a2^{\mu\nu\dots}{}_{\alpha\beta}(x)\sigma^\alpha\sigma^\beta + \dots, \quad (134)$$

where the coefficients  $a0^{\mu\nu\dots}(x)$ ,  $a1^{\mu\nu\dots}{}_\alpha(x)$ ,  $\dots$ , are functions of the stationary point  $x$  only. The next chapter is devoted to evaluating these coefficients and construction of the expansions of all of the bitensors required by Eq.(133) and its derivatives.

## CHAPTER 5

## Coincidence Limits and Covariant Expansions

The four coupled differential equations, Eqs.(44), (124), (125), and, (57), define the biscalars  $\sigma(x, x')$ ,  $a_0(x, x')$ ,  $a_n(x, x')$ , and  $D^{1/2}(x, x')$ , respectively. If these differential equations could be solved directly, then the DeWitt-Schwinger expansion, Eq.(133), for  $G^{(1)}(x, x')$  would be straightforward to compute. It would then be possible to take derivatives of the exact expression for  $G^{(1)}$  and construct  $\langle j^\mu \rangle$  and  $\langle T_{\mu\nu} \rangle$ . Unfortunately, these coupled, non-linear differential equations have not been solved exactly and another method for determining all of the bitensors required to construct  $G^{(1)}$  and its derivatives must be employed. The method employed here is due to DeWitt [2] and was used by Christensen in his work to regularize the real scalar field [30]. This chapter follows these two authors closely, extending their work to include the terms in the expansions of the biscalars due to the electromagnetic field.

The approach used by DeWitt and Christensen for constructing the bitensors needed for point-splitting regularization assumes the bitensors may all be written as a power series expansion using the geodesic separation vector  $\sigma^\mu$  [30, 2],

$$a^{\mu\nu\dots}(x, x') = a_0^{\mu\nu\dots}(x) + a_1^{\mu\nu\dots}{}_\alpha(x)\sigma^\alpha + \frac{1}{2!}a_2^{\mu\nu\dots}{}_{\alpha\beta}(x)\sigma^\alpha\sigma^\beta + \dots, \quad (135)$$

as mentioned at the end of Chapter 4. This assumption for the form of the bitensors is reasonable in that they are functions of the infinitesimally separated points  $x$  and  $x'$ . The power series expansion may be continued to arbitrarily high powers of  $\sigma^\alpha$  in order to achieve arbitrarily high precision when calculating the bitensors. Since all bitensors are known to be functions of the split points  $x$  and  $x'$ , the explicit inclusion of the argument  $(x, x')$  will be dropped hereafter to simplify the long expressions

to follow. Since all coefficients on the right hand side of Eq.(135) are known to be functions of the stationary point  $x$ , the explicit inclusion of the argument ( $x$ ) will also be dropped.

Evaluating the coefficients of the power series expansion for the bitensors involves the idea of *coincidence limits*, defined by taking the limit  $x' \rightarrow x$  of bitensors. Synge's bracket notation [32]

$$[a(x, x')] \equiv \lim_{x' \rightarrow x} a(x, x'), \quad (136)$$

will be used to simplify writing of the many limits needed. As the nearby point  $x'$  is brought together with the stationary point  $x$ , the geodesic distance between  $x$  and  $x'$  goes to zero. For example, since the vector  $\sigma^\mu \equiv \sigma^{i\mu}$  has length is equal to the geodesic distance between the two points,

$$[\sigma^{i\mu}] = 0 = [\sigma_{; \mu}]. \quad (137)$$

Now consider the expansion for the second rank tensor  $a^{\mu\nu}$  given by

$$a^{\mu\nu} = a0^{\mu\nu} + a1^{\mu\nu}{}_{\alpha} \sigma^{\alpha} + \frac{1}{2!} a2^{\mu\nu}{}_{\alpha\beta} \sigma^{\alpha} \sigma^{\beta} + \dots \quad (138)$$

Taking the coincidence limit of Eq.(138) and using Eq.(137) will yield the first coefficient of the expansion of  $a^{\mu\nu}$ ,

$$[a^{\mu\nu}] = a0^{\mu\nu} \Rightarrow a0^{\mu\nu} = [a^{\mu\nu}], \quad (139)$$

where no coincidence limit brackets [ ] are used with  $a0^{\mu\nu}$  since it is known to be an explicit function of the stationary point  $x$ . This indicates that, knowing the first coefficient in the series for  $a^{\mu\nu}$  requires knowing the coincidence limit of the bitensor  $[a^{\mu\nu}]$ . This may seem like a circular line of reasoning, since it is necessary to know the coincidence limit of the bitensor in order to know the zeroth order (in  $\sigma^\mu$ ) approximation for that very bitensor! Fortunately, the four differential equations

for  $\sigma(x, x')$ ,  $a_0(x, x')$ ,  $a_n(x, x')$ , and  $D^{1/2}(x, x')$  provide a way to determine all of the coincidence limits required to construct a series as in Eq.(135) for every bitensor needed for point-splitting.

To determine the required coincidence limits, recall the function

$$\Delta^{1/2}(x, x') = g^{-1/4}(x)D^{1/2}(x, x')g^{-1/4}(x'). \quad (140)$$

The differential equation for  $\Delta^{1/2}$ , Eq.(57), may be rewritten in the more convenient form

$$4\Delta^{1/2} = 2\Delta^{1/2}{}_{;\mu}\sigma^{i\mu} + \Delta^{1/2}\sigma_{;\mu}{}^\mu. \quad (141)$$

The other three may remain in their original form,

$$\sigma(x, x') = \frac{1}{2}\sigma^{i\mu}\sigma_{;\mu}, \quad (142)$$

$$\sigma^{i\mu}a_{0|\mu} = 0, \quad (143)$$

and

$$\sigma^{i\mu}a_{n+1|\mu} + (n+1)a_{n+1} = \Delta^{-1/2}(\Delta^{1/2}a_n)_{|\mu}{}^\mu - \xi Ra_n. \quad (144)$$

These differential equations may be solved iteratively for all needed coincidence limits. As an example of this procedure, consider Eq.(142) for  $\sigma$ . Since  $\sigma$  is one-half the square of the geodesic distance between the points  $x$  and  $x'$ , it is evident that

$$[\sigma] = 0. \quad (145)$$

One covariant derivative of Eq.(142) yields

$$\sigma^{i\alpha} = \sigma^{i\mu\alpha}\sigma_{;\mu} \quad (146)$$

whose coincidence limit yields the trivial equation

$$[\sigma^{i\alpha}] = [\sigma^{i\mu\alpha}][\sigma_{;\mu}] \rightarrow 0 = 0. \quad (147)$$

Two covariant derivatives of Eq.(142) gives

$$\sigma^{;\alpha\beta} = \sigma^{;\mu\alpha\beta}\sigma_{\mu} + \sigma^{;\mu\alpha}\sigma_{;\mu}{}^{\beta}, \quad (148)$$

whose coincidence limit yields the more useful equation

$$[\sigma^{;\alpha\beta}] = 0 + [\sigma^{;\mu\alpha}][\sigma_{;\mu}{}^{\beta}]. \quad (149)$$

Eq.(149) implies that

$$[\sigma_{;\mu}{}^{\beta}] = g_{\mu}{}^{\beta}. \quad (150)$$

The coincidence limit of three covariant derivatives of Eq.(142) yields

$$\begin{aligned} [\sigma^{;\alpha\beta\gamma}] &= [\sigma^{;\gamma\alpha\beta}] + [\sigma^{;\beta\alpha\gamma}] + [\sigma^{;\alpha\beta\gamma}] \\ &= [\sigma^{;\alpha\gamma\beta}] + 2[\sigma^{;\alpha\beta\gamma}] \\ &= [\sigma^{;\alpha\beta\gamma} + R^{\beta\gamma\alpha}{}_{\mu}\sigma^{;\mu}] + 2[\sigma^{;\alpha\beta\gamma}] \\ &= 3[\sigma^{;\alpha\beta\gamma}], \end{aligned} \quad (151)$$

and so

$$[\sigma^{;\alpha\beta\gamma}] = 0. \quad (152)$$

The Riemann tensor  $R^{\beta\gamma\alpha}{}_{\mu}$  arises in Eq.(151) through the commutator of two covariant derivatives of purely geometric quantities;

$$\sigma^{;\alpha\gamma\beta} = \sigma^{;\alpha\beta\gamma} + R^{\beta\gamma\alpha}{}_{\mu}\sigma^{;\mu}. \quad (153)$$

Using this same technique repeatedly gives [30, 31, 27]

$$[\sigma^{i\mu}] = 0, \quad (154)$$

$$[\sigma^{i\mu}{}_{\nu}] = g^{\mu}{}_{\nu}, \quad (155)$$

$$[\sigma^{i\mu}{}_{\nu\sigma}] = 0, \quad (156)$$

$$[\sigma^{i\mu}{}_{\nu\rho\tau}] = S^{\mu}{}_{\nu\rho\tau} \equiv -\frac{1}{3}(R^{\mu}{}_{\rho\nu\tau} + R^{\mu}{}_{\tau\nu\rho}), \quad (157)$$

$$[\sigma^{i\mu}{}_{\nu\alpha\tau\rho}] = S5^\mu{}_{\nu\alpha\tau\rho} \equiv \frac{3}{4}(S^\mu{}_{\nu\alpha\tau;\rho} + S^\mu{}_{\nu\tau\rho;\alpha} + S^\mu{}_{\nu\rho\alpha;\tau}), \quad (158)$$

$$[\sigma_{;\mu\nu\rho\tau\kappa\lambda}] = S6_{\mu\nu\rho\tau\kappa\lambda} \equiv$$

$$\frac{R_{\mu\rho\nu\tau;\kappa\lambda}}{5} - \frac{R_{\mu\rho\nu\kappa;\tau\lambda}}{5} - \frac{R_{\mu\rho\nu\lambda;\tau\kappa}}{5} - \frac{R_{\mu\tau\nu\rho;\kappa\lambda}}{5} - \frac{R_{\mu\tau\nu\kappa;\rho\lambda}}{5} - \frac{R_{\mu\tau\nu\lambda;\rho\kappa}}{5}$$

$$\frac{R_{\mu\kappa\nu\rho;\tau\lambda}}{5} - \frac{R_{\mu\kappa\nu\tau;\rho\lambda}}{5} - \frac{R_{\mu\kappa\nu\lambda;\rho\tau}}{5} - \frac{R_{\mu\lambda\nu\rho;\tau\kappa}}{5} - \frac{R_{\mu\lambda\nu\tau;\rho\kappa}}{5} - \frac{R_{\mu\lambda\nu\kappa;\rho\tau}}{5}$$

$$\frac{S_{\alpha\mu\kappa\lambda}S_{\nu}{}^{\alpha}{}_{\rho\tau}}{5} - \frac{S_{\alpha\mu\tau\lambda}S_{\nu}{}^{\alpha}{}_{\rho\kappa}}{5} - \frac{S_{\alpha\mu\tau\kappa}S_{\nu}{}^{\alpha}{}_{\rho\lambda}}{5} - \frac{S_{\alpha\mu\rho\lambda}S_{\nu}{}^{\alpha}{}_{\tau\kappa}}{5}$$

$$\frac{S_{\alpha\mu\rho\kappa}S_{\nu}{}^{\alpha}{}_{\tau\lambda}}{5} - \frac{S_{\alpha\mu\rho\tau}S_{\nu}{}^{\alpha}{}_{\kappa\lambda}}{5} - \frac{S_{\alpha\mu\nu\lambda}S_{\rho}{}^{\alpha}{}_{\tau\kappa}}{5} - \frac{S_{\alpha\mu\nu\kappa}S_{\rho}{}^{\alpha}{}_{\tau\lambda}}{5}$$

$$\frac{S_{\alpha\mu\nu\tau}S_{\rho}{}^{\alpha}{}_{\kappa\lambda}}{5} - \frac{S_{\alpha\mu\nu\rho}S_{\tau}{}^{\alpha}{}_{\kappa\lambda}}{5} - \frac{S_{\alpha\tau\kappa\lambda}R_{\mu}{}^{\alpha}{}_{\nu\rho}}{5} - \frac{S_{\alpha\rho\kappa\lambda}R_{\mu}{}^{\alpha}{}_{\nu\tau}}{5}$$

$$\frac{S_{\alpha\rho\tau\lambda}R_{\mu}{}^{\alpha}{}_{\nu\kappa}}{5} - \frac{S_{\alpha\rho\tau\kappa}R_{\mu}{}^{\alpha}{}_{\nu\lambda}}{5} - \frac{S_{\alpha\nu\kappa\lambda}R_{\mu}{}^{\alpha}{}_{\rho\tau}}{5} - \frac{S_{\alpha\nu\tau\lambda}R_{\mu}{}^{\alpha}{}_{\rho\kappa}}{5}$$

$$\frac{S_{\alpha\nu\tau\kappa}R_{\mu}{}^{\alpha}{}_{\rho\lambda}}{5} - \frac{S_{\alpha\nu\rho\lambda}R_{\mu}{}^{\alpha}{}_{\tau\kappa}}{5} - \frac{S_{\alpha\nu\rho\kappa}R_{\mu}{}^{\alpha}{}_{\tau\lambda}}{5} - \frac{S_{\alpha\nu\rho\tau}R_{\mu}{}^{\alpha}{}_{\kappa\lambda}}{5}$$

$$\frac{S_{\alpha\mu\kappa\lambda}R_{\nu}{}^{\alpha}{}_{\rho\tau}}{5} - \frac{S_{\alpha\mu\tau\lambda}R_{\nu}{}^{\alpha}{}_{\rho\kappa}}{5} - \frac{S_{\alpha\mu\tau\kappa}R_{\nu}{}^{\alpha}{}_{\rho\lambda}}{5} - \frac{S_{\alpha\mu\rho\lambda}R_{\nu}{}^{\alpha}{}_{\tau\kappa}}{5}$$

$$\frac{S_{\alpha\mu\rho\kappa}R_{\nu}{}^{\alpha}{}_{\tau\lambda}}{5} - \frac{S_{\alpha\mu\rho\tau}R_{\nu}{}^{\alpha}{}_{\kappa\lambda}}{5} - \frac{S_{\mu\nu\alpha\lambda}R_{\rho}{}^{\alpha}{}_{\tau\kappa}}{5} - \frac{S_{\mu\nu\alpha\kappa}R_{\rho}{}^{\alpha}{}_{\tau\lambda}}{5}$$

$$\frac{S_{\mu\nu\alpha\tau}R_{\rho}{}^{\alpha}{}_{\kappa\lambda}}{5} - \frac{S_{\mu\nu\rho\alpha}R_{\tau}{}^{\alpha}{}_{\kappa\lambda}}{5} \quad (159)$$

and so forth. Note that a semicolon has been used in Eqs.(154)–(159) to show that the derivatives are covariant derivatives and contain only Christoffel connections. While this procedure could continue indefinitely, it need continue in the present work only to the coincidence limit of eight derivatives of  $\sigma$ . The limits  $[\sigma_{;\mu\nu\rho\tau\kappa\lambda}] \equiv S7_{\mu\nu\rho\tau\kappa\lambda}$  and  $[\sigma_{;\mu\nu\rho\tau\kappa\lambda\eta}] \equiv S8_{\mu\nu\rho\tau\kappa\lambda\eta}$  are listed in the Appendix to save space here. (It is also interesting to note that, after taking the covariant derivatives and subsequent coincidence limits for  $\sigma$ , each new coincidence limit depends on knowing the limit *two* iterations before in the sequence. Thus, if one were inclined to calculate, say, the tenth limit, then it could be done without knowing the ninth!)

Evaluating the limits of Eq.(142) and its derivatives for  $[\sigma_{;\mu\nu}], [\sigma_{;\mu\nu\rho}], \dots$  is not simply a good illustration of the iterative procedure. Each new iteration depends on previous iterations: Taking the limits of Eq.(141) and its derivatives requires

the limits of  $\sigma$  and its derivatives; taking the limits of Eq.(143) and its derivatives requires the limits of  $\sigma$ ,  $\Delta^{1/2}$ , and their derivatives; taking the limits of Eq.(144) and its derivatives requires the limits of  $\sigma$ ,  $\Delta^{1/2}$ ,  $a_0$ , and their derivatives. As discussed later, it will be necessary to have the coincidence limits of eight derivatives of  $\sigma$ , six derivatives of  $\Delta^{1/2}$ , six derivatives of  $a_0$ , four derivatives of  $a_1$ , two derivatives of  $a_2$ , and the simple coincidence limit  $a_3$ . When  $[a_3]$  is evaluated, the extent in number of each set of iterations will be discussed.

In order to proceed as above with the differential equation defining  $\Delta^{1/2}$ , Eq.(141), note that obtaining coincidence limits up to six derivatives of  $\Delta^{1/2}$  will now involve the use of Eqs.(154)–(271). Consider the differential equation for  $D_{\alpha\beta'}$ , Eq.(52),

$$D_{\alpha\beta'} = D_{\alpha\beta';\mu}\sigma^{;\mu} + D_{\mu\beta'}\sigma_{;\alpha}{}^{\mu}. \quad (160)$$

Taking the coincidence limit gives

$$\begin{aligned} [D_{\alpha\beta'}] &= [D_{\mu\beta'}][\sigma_{;\alpha}{}^{\mu}] \\ &= [\sigma_{;\mu\beta'}][\sigma_{;\alpha}{}^{\mu}] \\ &= g_{\mu\beta}g_{\alpha}{}^{\mu} \\ &= g_{\alpha\beta}. \end{aligned} \quad (161)$$

Taking the coincidence limit of  $D(x, x')$  gives

$$[D(x, x')] = [\det(D_{\alpha\beta'})] = \det(g_{\alpha\beta}) = g, \quad (162)$$

while taking the coincidence limit of  $\Delta^{1/2}(x, x')$  gives

$$[\Delta^{1/2}(x, x')] = [g^{-1/4}(x)D^{1/2}(x, x')g^{-1/4}(x')] = [g^{-1/4}(x)][D^{1/2}(x, x')][g^{-1/4}(x')] = 1. \quad (163)$$

The coincidence limits of up to six derivatives of Eq.(141) yields

$$[\Delta^{1/2}{}_{;\mu}] = 0, \quad (164)$$

$$[\Delta^{1/2}{}_{;\mu\nu}] = \frac{1}{6}R_{\mu\nu}, \quad (165)$$

$$[\Delta^{1/2}{}_{;\mu\nu\rho}] = \frac{1}{12}(R_{\mu\nu;\rho} + R_{\nu\rho;\mu} + R_{\rho\mu;\nu}), \quad (166)$$

$$[\Delta^{1/2}{}_{;\mu\nu\rho\tau}] =$$

$$\begin{aligned} & -\frac{S6\alpha^{\mu\nu\rho\tau}}{8} + \frac{R_{\mu\tau}R_{\nu\rho}}{36} + \frac{R_{\mu\rho}R_{\nu\tau}}{36} + \frac{R_{\mu\nu}R_{\rho\tau}}{36} - \frac{R_{\alpha\tau}R_{\mu\nu\rho}{}^\alpha}{72} - \frac{R_{\alpha\rho}R_{\mu\nu\tau}{}^\alpha}{72} - \\ & \frac{R_{\alpha\tau}R_{\mu\rho\nu}{}^\alpha}{72} - \frac{R_{\alpha\nu}R_{\mu\rho\tau}{}^\alpha}{72} - \frac{R_{\alpha\rho}R_{\mu\tau\nu}{}^\alpha}{72} - \frac{R_{\alpha\nu}R_{\mu\tau\rho}{}^\alpha}{72} - \frac{R_{\alpha\tau}R_{\mu}{}^\alpha{}_{\nu\rho}}{24} - \\ & \frac{R_{\alpha\rho}R_{\mu}{}^\alpha{}_{\nu\tau}}{24} - \frac{R_{\alpha\nu}R_{\mu}{}^\alpha{}_{\rho\tau}}{24} - \frac{R_{\alpha\mu}R_{\nu\rho\tau}{}^\alpha}{72} - \frac{R_{\alpha\mu}R_{\nu\tau\rho}{}^\alpha}{72} - \frac{R_{\alpha\mu}R_{\nu}{}^\alpha{}_{\rho\tau}}{24} \end{aligned} \quad (167)$$

and the five- and six-derivative limits  $[\Delta^{1/2}{}_{;\mu\nu\rho\tau\kappa}]$  and  $[\Delta^{1/2}{}_{;\mu\nu\rho\tau\kappa\lambda}]$ , which are listed in Appendix B to save space here. Note how the four-, five- and six-derivative limits of  $\Delta^{1/2}$  depend on the six-, seven-, and eight-derivative limits of  $\sigma$ , illustrating how the  $n^{\text{th}}$ -derivative limits of  $\Delta^{1/2}$  depends on the  $(n+2)^{\text{th}}$ -derivative limits of  $\sigma$ .

For the limits of  $\Delta^{-1/2}$  and its derivatives, first note that Eq.(163) may be squared to give

$$[\Delta] = 1, \quad (168)$$

indicating

$$[\Delta^{\pm 1/2}] = [\Delta^{\pm 1}] = [\Delta^{\pm 3/2}] = \dots = 1. \quad (169)$$

The relation

$$\Delta^{1/2}{}_{;\mu} = \frac{1}{2}\Delta^{-1/2}\Delta_{;\mu}, \quad (170)$$

along with its derivatives, indicates

$$[\Delta_{;\mu}] = [\Delta^{1/2}{}_{;\mu}] = 0, \quad (171)$$

$$[\Delta_{;\mu\nu}] = 2[\Delta^{1/2}{}_{;\mu\nu}], \quad (172)$$

$$[\Delta_{;\mu\nu\rho}] = 2[\Delta^{1/2}{}_{;\mu\nu\rho}], \quad (173)$$

$$\begin{aligned} [\Delta_{;\mu\nu\rho\tau}] &= 2[\Delta^{1/2}{}_{;\mu\nu\rho\tau}] + 2[\Delta^{1/2}{}_{;\mu\nu}][\Delta^{1/2}{}_{;\rho\tau}] + \\ & 2[\Delta^{1/2}{}_{;\mu\rho}][\Delta^{1/2}{}_{;\nu\tau}] + 2[\Delta^{1/2}{}_{;\mu\tau}][\Delta^{1/2}{}_{;\nu\rho}]. \end{aligned} \quad (174)$$

The relation

$$\Delta^{-1/2}{}_{;\mu} = -\frac{1}{2}\Delta^{-3/2}\Delta_{;\mu}, \quad (175)$$

and its derivatives yield the coincidence limits

$$[\Delta^{-1/2}{}_{;\mu}] = 0, \quad (176)$$

$$[\Delta^{-1/2}{}_{;\mu\nu}] = -[\Delta^{1/2}{}_{;\mu\nu}], \quad (177)$$

$$[\Delta^{-1/2}{}_{;\mu\nu\rho}] = -[\Delta^{1/2}{}_{;\mu\nu\rho}], \quad (178)$$

$$[\Delta^{-1/2}{}_{;\mu\nu\rho\tau}] = -[\Delta^{1/2}{}_{;\mu\nu\rho\tau}] + 2[\Delta^{1/2}{}_{;\mu\nu}][\Delta^{1/2}{}_{;\rho\tau}] + 2[\Delta^{1/2}{}_{;\mu\rho}][\Delta^{1/2}{}_{;\nu\tau}] + 2[\Delta^{1/2}{}_{;\mu\tau}][\Delta^{1/2}{}_{;\nu\rho}]. \quad (179)$$

Next, Eqs.(143)–(144) may be solved iteratively. The Riemann tensor appeared in Eq.(151) due to the commutator of the covariant derivative of purely geometric quantities. The iterative technique for quantities that explicitly depend upon the gauge field is conceptually the same. The new feature now requires careful allowance for the commutator of the *gauge* covariant derivative that must be used with derivatives of  $a_0$  and the other  $a_n$ . This gauge commutation relation is defined by

$$a_n|_{\mu\nu} - a_n|_{\nu\mu} = ieF_{\mu\nu}a_n. \quad (180)$$

Differentiating Eq.(143) repeatedly gives

$$\sigma^{i\mu}a_{0|\mu} = 0, \quad (181)$$

$$\sigma^{i\mu\nu}a_{0|\mu} + \sigma^{i\mu}a_{0|\mu}{}^\nu = 0, \quad (182)$$

$$\sigma^{i\mu\nu\rho}a_{0|\mu} + \sigma^{i\mu\nu}a_{0|\mu}{}^\rho + \sigma^{i\mu\rho}a_{0|\mu}{}^\nu + \sigma^{i\mu}a_{0|\mu}{}^{\nu\rho} = 0, \quad (183)$$

and so forth, where the slash is used to indicate the gauge covariant derivative being applied to  $a_0$ . Now recall the boundary condition

$$\lim_{x' \rightarrow x} \Omega(x, x', s) = 1. \quad (184)$$

In the limit  $x' \rightarrow x$ ,  $s \rightarrow 0$  and  $\Omega(x, x', s) \rightarrow 1$ . For these limits to hold, it must be true that

$$[a_0] = 1. \quad (185)$$

Taking the coincidence limit of Eqs.(181)–(183), and using Eqs. (154)–(158) along with the commutation relation of the gauge covariant derivative, Eq.(180), yields (see Ref.[27] for the ones listed below),

$$[a_0|_{\mu}] = 0, \quad (186)$$

$$[a_0|_{\mu\nu}] = \frac{1}{2}ieF_{\mu\nu}, \quad (187)$$

$$[a_0|_{\mu\nu\rho}] = \frac{1}{3}ie(F_{\mu\nu;\rho} + F_{\mu\rho;\nu}), \quad (188)$$

and

$$\begin{aligned} [a_0|_{\mu\nu\rho\tau}] &= \frac{ie}{4}[F_{\mu\nu;\rho\tau} + F_{\mu\rho;\nu\tau} + F_{\mu\tau;\nu\rho}] - \frac{e^2}{4}[F_{\mu\nu}F_{\rho\tau} + F_{\mu\rho}F_{\nu\tau} + F_{\mu\tau}F_{\nu\rho}] \\ &+ \frac{ie}{4}[F_{\alpha\mu}S^{\alpha}_{\nu\rho\tau} + F_{\alpha\nu}S^{\alpha}_{\mu\tau\rho} + F_{\alpha\rho}S^{\alpha}_{\mu\tau\nu} + F_{\alpha\tau}S^{\alpha}_{\mu\rho\nu}]. \end{aligned} \quad (189)$$

The limits  $[a_0|_{\mu\nu\rho\tau\kappa}]$  and  $[a_0|_{\mu\nu\rho\tau\kappa\lambda}]$  are listed in Appendix C.

Now that the various coincidence limits involving  $a_0$  have been obtained, it is possible to indicate how to know how far to carry each iteration. To show this requires knowing the highest  $n$  value desired for the  $a_n$ . For example, in this thesis, the coincidence limit  $[a_3]$  will be the highest term needed. Consider the limit of Eq.(144). This has an immediate coincidence limit

$$\begin{aligned} [a_{n+1}] &= (n+1)^{-1} \left\{ [(\Delta^{1/2}a_n)|_{\mu}{}^{\mu}] - \xi R[a_n] \right\} \\ &= (n+1)^{-1} \left\{ [\Delta^{1/2}_{;\mu}{}^{\mu}] [a_n] + 2[\Delta^{1/2}_{;\mu}] [a_n^{\mu}] + [\Delta^{1/2}] [a_n]_{\mu}{}^{\mu} - \xi R[a_n] \right\} \\ &= (n+1)^{-1} \left\{ \left( \frac{1}{6} - \xi \right) R[a_n] + [a_n]_{\mu}{}^{\mu} \right\}. \end{aligned} \quad (190)$$

Here, the limits of up to two derivatives of  $\Delta^{1/2}$ ,  $[\Delta^{1/2}_{;\mu\nu}]$ , and two derivatives of  $a_n$  were needed. As stated above, the two-derivative limit of  $\Delta^{1/2}$  will require knowing

the four-derivative limit of  $\sigma$ . All of these coincidence limits will require knowing many other coincidence limits. As a further example, the coincidence limit of one derivative of Eq.(144) is

$$[a_{n+1|\alpha}] = (n+2)^{-1} \left\{ \left( \frac{1}{6} - \xi \right) (R_{;\alpha}[a_n] + R[a_n|\alpha]) + \frac{1}{3}[a_n^{|\mu}]R_{\mu\alpha} + [a_n|_{\mu}{}^{\mu}{}_{\alpha}] \right\}. \quad (191)$$

Here, the limits of up to three derivatives of  $\Delta^{1/2}$  and three derivatives of  $a_n$  were needed. Again, while this could continue endlessly, a pattern is emerging. It will be left as an exercise for the studious reader to verify the following statements are true with regard to the extent of number of each set of iterations of derivatives and subsequent coincidence limits:

- The limit  $[a_3]$  requires knowing the limits of two derivatives of  $a_2$  and two derivatives of  $\Delta^{1/2}$ .
- The limit of two derivatives of  $[a_2]$  requires knowing the limits of four derivatives of  $a_1$  and four derivatives of  $\Delta^{1/2}$ .
- The limit of four derivatives of  $[a_1]$  requires knowing the limits of six derivatives of  $a_0$  and six derivatives of  $\Delta^{1/2}$ .
- The limit of six derivatives of  $\Delta^{1/2}$  requires knowing the limits of eight derivatives of  $\sigma$ .

Herein lies the source of the excruciating algebraic difficulty of the point-splitting procedure!

The iteration procedure continues by determining  $[a_1]$ , which is obtained by setting  $n = 0$  in the recursion relation Eq.(144). This gives

$$\sigma^{;\mu}a_{1|\mu} + a_1 = \Delta^{-1/2}(\Delta^{1/2}a_0)_{|\mu}{}^{\mu} - \xi R a_0, \quad (192)$$

with an immediate coincidence limit

$$[a_1] = \left( \frac{1}{6} - \xi \right) R. \quad (193)$$

The limits of up to four derivatives of Eq.(144) with  $n = 0$  gives

$$[a_{1|\mu}] = \frac{1}{2} \left( \frac{1}{6} - \xi \right) R_{;\mu} + \frac{ie}{6} F_{\mu\alpha}{}^{;\alpha}, \quad (194)$$

$$[a_{1|\mu\nu}] = \left( \frac{1}{20} - \frac{1}{3} \xi \right) R_{;\mu\nu} + \frac{1}{60} R_{\mu\nu;\alpha}{}^\alpha + \frac{1}{90} R^{\alpha\beta} R_{\alpha\nu\beta\mu} - \frac{1}{45} R_{\mu\alpha} R^\alpha{}_\nu \\ + \frac{1}{90} R_{\alpha\beta\gamma\mu} R^{\alpha\beta\gamma}{}_\nu - \frac{e^2}{6} F_{\mu\alpha} F_\nu{}^\alpha - \frac{ie}{12} (F_{\mu\alpha}{}^{;\nu} + F_{\nu\alpha}{}^{;\mu}) + \frac{ie}{2} \left( \frac{1}{6} - \xi \right) R F_{\mu\nu}, \quad (195)$$

and the limits  $[a_{1|\mu\nu\rho}]$  and  $[a_{1|\mu\nu\rho\tau}]$  listed in the Appendix. The coincidence limits involving  $a_2$  are

$$[a_2] = -\frac{1}{180} R^{\alpha\beta} R_{\alpha\beta} + \frac{1}{180} R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} + \frac{1}{6} \left( \frac{1}{5} - \xi \right) R_{;\alpha}{}^\alpha \\ + \frac{1}{2} \left( \frac{1}{6} - \xi \right)^2 R^2 - \frac{e^2}{12} F^{\alpha\beta} F_{\alpha\beta}, \quad (196)$$

with  $[a_{2|\mu}]$  and  $[a_{2|\mu\nu}]$  listed in the Appendix. The final limit needed is  $[a_3]$ , given by [40]

$$[a_3] = \frac{R^3}{1296} - \frac{R^3 \xi}{72} + \frac{R^3 \xi^2}{12} - \frac{R^3 \xi^3}{6} + \frac{11 R_{;\alpha} R^{;\alpha}}{5040} - \\ \frac{\xi R_{;\alpha} R^{;\alpha}}{30} + \frac{\xi^2 R_{;\alpha} R^{;\alpha}}{12} - \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F_\gamma{}^{\beta;\gamma}}{180} - \frac{2e^2 F_{\alpha\beta;\gamma} F^{\alpha\beta;\gamma}}{135} - \\ \frac{2e^2 F_{\alpha\beta;\gamma} F^{\alpha\gamma;\beta}}{135} - \frac{R_{\alpha\beta;\gamma} R^{\alpha\beta;\gamma}}{1440} - \frac{R_{\alpha\beta;\gamma} R^{\alpha\gamma;\beta}}{480} - \frac{R_{\alpha\beta;\gamma} R_\delta{}^{\alpha\beta\gamma;\delta}}{560} + \\ \frac{11 R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_\epsilon{}^{\beta\gamma\delta;\epsilon}}{10080} + \frac{11 R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_\epsilon{}^{\gamma\beta\delta;\epsilon}}{10080} + \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon}}{1680} + \\ \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\epsilon;\delta}}{840} + \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\gamma\beta\delta;\epsilon}}{1680} + \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\gamma\beta\epsilon;\delta}}{840} + \frac{R R_{;\alpha}{}^\alpha}{180} - \\ \frac{11 R \xi R_{;\alpha}{}^\alpha}{180} + \frac{R \xi^2 R_{;\alpha}{}^\alpha}{6} + \frac{R_{;\alpha\beta}{}^{\beta\alpha}}{1260} + \frac{R_{;\alpha}{}^\alpha{}_\beta{}^\beta}{504} - \\ \frac{\xi R_{;\alpha}{}^\alpha{}_\beta{}^\beta}{60} + \frac{R_{\alpha\beta;\gamma}{}^{\gamma\alpha\beta}}{630} - \frac{e^2 F_{\alpha\beta;\gamma}{}^\gamma F^{\alpha\beta}}{60} - \frac{e^2 R F_{\alpha\beta} F^{\alpha\beta}}{72} + \\ \frac{e^2 R \xi F_{\alpha\beta} F^{\alpha\beta}}{12} - \frac{e^2 F_{\alpha\beta;\gamma}{}^\beta F^{\alpha\gamma}}{60} - \frac{e^2 F_{\alpha\beta}{}^{;\beta}{}_\gamma F^{\alpha\gamma}}{60} + \frac{R_{;\alpha\beta} R^{\alpha\beta}}{151200} - \\ \frac{\xi R_{;\alpha\beta} R^{\alpha\beta}}{90} - \frac{R_{\alpha\beta;\gamma}{}^\gamma R^{\alpha\beta}}{700} - \frac{R R_{\alpha\beta} R^{\alpha\beta}}{1080} + \frac{R \xi R_{\alpha\beta} R^{\alpha\beta}}{180} - \\ \frac{19 R_{\alpha\beta;\gamma}{}^\beta R^{\alpha\gamma}}{10800} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\beta\delta} R^{\alpha\gamma}}{700} + \frac{e^2 F_{\alpha\beta} F_\gamma{}^\alpha R^{\beta\gamma}}{180} + \frac{43 R_{\alpha\beta} R_\gamma{}^\alpha R^{\beta\gamma}}{226800} +$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta}}{1575} - \frac{e^2 F_{\alpha\beta} F_{\gamma\delta} R^{\alpha\beta\gamma\delta}}{180} + \frac{R R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}}{1296} - \frac{R\xi R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}}{216} + \\
& \frac{2R_{\alpha\beta\gamma\delta;\epsilon}{}^\delta R^{\alpha\beta\gamma\epsilon}}{1575} + \frac{2R_{\alpha\beta\gamma\delta;\delta}{}^\epsilon R^{\alpha\beta\gamma\epsilon}}{1575} + \frac{2R_{\alpha\beta;\gamma\delta} R^{\alpha\gamma\beta\delta}}{1575} + \frac{R_{\alpha\beta\gamma\delta;\epsilon}{}^\epsilon R^{\alpha\gamma\beta\delta}}{1575} - \\
& \frac{e^2 F_{\alpha\beta} F_{\gamma\delta} R^{\alpha\gamma\beta\delta}}{180} - \frac{41R_{\alpha\beta} R_{\gamma\delta} R^{\alpha\gamma\beta\delta}}{75600} + \frac{R R_{\alpha\beta\gamma\delta} R^{\alpha\gamma\beta\delta}}{3240} - \frac{R\xi R_{\alpha\beta\gamma\delta} R^{\alpha\gamma\beta\delta}}{540} + \\
& \frac{2R_{\alpha\beta\gamma\delta;\epsilon}{}^\delta R^{\alpha\gamma\beta\epsilon}}{1575} + \frac{2R_{\alpha\beta\gamma\delta;\delta}{}^\epsilon R^{\alpha\gamma\beta\epsilon}}{1575} - \frac{R_{\alpha\beta} R_{\gamma\delta\epsilon}{}^\alpha R^{\beta\gamma\delta\epsilon}}{1890} + \\
& \frac{4R_{\alpha\beta\gamma\delta} R_{\epsilon\zeta}{}^{\alpha\gamma} R^{\beta\delta\epsilon\zeta}}{14175} + \frac{2R_{\alpha\beta\gamma\delta} R_{\epsilon\zeta}{}^{\alpha\gamma} R^{\beta\delta\epsilon\zeta}}{14175} - \frac{R_{\alpha\beta} R_{\gamma\delta\epsilon}{}^\alpha R^{\beta\epsilon\gamma\delta}}{6300} - \\
& \frac{2R_{\alpha\beta\gamma\delta} R_{\epsilon\zeta}{}^{\alpha\gamma} R^{\beta\zeta\delta\epsilon}}{14175} + \frac{4R_{\alpha\beta\gamma\delta} R_{\epsilon\zeta}{}^{\alpha\beta} R^{\gamma\delta\epsilon\zeta}}{14175}
\end{aligned} \tag{197}$$

Having obtained the coincidence limits of the many biscalars and their derivatives that arise in the point-splitting procedure, it is possible to construct the bitensor series expansion of Eq.(135) for each bitensor in the point-splitting expansion of  $\langle\phi^2\rangle$ ,  $\langle j^\mu\rangle$  and  $\langle T_{\mu\nu}\rangle$ . Evaluating the coefficients  $a0^{\mu\nu\dots}$ ,  $a1^{\mu\nu\dots}{}_\alpha$ ,  $\dots$  of the bitensor power series expansion requires taking the coincidence limit  $x' \rightarrow x$  of Eq.(135) and its derivatives. For example, for the second rank bitensor  $a^{\mu\nu}(x, x')$ , this yields,

$$a0^{\mu\nu} = [a^{\mu\nu}], \tag{198}$$

$$a1^{\mu\nu}{}_\alpha = [a^{\mu\nu}{}_{;\alpha}] - a0^{\mu\nu}{}_{;\alpha}, \tag{199}$$

$$a2^{\mu\nu}{}_{\alpha\beta} = [a^{\mu\nu}{}_{;\alpha\beta}] - 2a1^{\mu\nu}{}_{\alpha;\beta} - a0^{\mu\nu}{}_{;\alpha\beta}, \tag{200}$$

$$a3^{\mu\nu}{}_{\alpha\beta\gamma} = [a^{\mu\nu}{}_{;\alpha\beta\gamma}] - 3a2^{\mu\nu}{}_{\alpha\beta;\gamma} - 3a1^{\mu\nu}{}_{\alpha;\beta\gamma} - a0^{\mu\nu}{}_{;\alpha\beta\gamma}, \tag{201}$$

and so forth. The numeric factors on the right-hand side of Eqs.(198-201) arise due to symmetrization on the dummy indices  $\alpha, \beta, \dots$ , in Eq.(135). Terms such as  $a1^{\mu\nu}{}_\rho S^\rho{}_{\alpha\beta\gamma} \sigma^\alpha \sigma^\beta \sigma^\gamma$  do not contribute due to this same symmetrization. Note that  $a^{\mu\nu}$  could involve summations over primed indices such as  $a^{\mu\nu} = g^{\mu\alpha'} g^{\nu\beta'} a_2^{|\alpha'\beta'}$ , whose evaluation will be discussed next.

The expansions will have bitensors constructed by taking primed derivatives of the biscalars in  $G^{(1)}$ . As discussed at the end of Chapter 3, these bitensors must

be parallel transported back to the stationary point  $x$  (see the form of Eq.(81) and Eq.(82)) in order to perform the expansion in Eq.(135). For example,

$$g^\nu_{\rho'} a^{i\mu\rho'} = a0^{\mu\nu} + a1^{\mu\nu}{}_\alpha \sigma^\alpha + \dots \quad (202)$$

In order to evaluate the coincidence limits in Eqs.(198)–(201) for these primed derivatives, a generalization by Christensen [30] of a theorem proved by Synge is used:

$$[T_{\alpha_1 \dots \alpha_n \beta'_1 \dots \beta'_m; \mu'}] = -[T_{\alpha_1 \dots \alpha_n \beta'_1 \dots \beta'_m; \mu}] + [T_{\alpha_1 \dots \alpha_n \beta'_1 \dots \beta'_m}]_{; \mu}, \quad (203)$$

where  $T_{\alpha_1 \dots \alpha_n \beta_1 \dots \beta_m}$  is any bitensor with equal weight at both  $x$  and  $x'$  and whose coincidence limit and derivative coincidence limits exist. The relation

$$[g^\mu_{\nu'; \alpha \beta \dots}] \sigma^\alpha \sigma^\beta \dots = 0, \quad (204)$$

which follows immediately from taking derivatives of Eq.(69), will also be used. As an example, consider  $g^\mu_{\tau'} g^\nu_{\rho'} a^{|\tau'\rho'}$ . Quantities such as the second rank bi-tensor  $a^{\mu\nu}$  appear in the stress-energy tensor whenever two derivatives of  $G^{(1)}$  are taken;

$$a^{\mu\nu} \equiv g^\mu_{\tau'} g^\nu_{\rho'} a^{|\tau'\rho'} = a0^{\mu\nu} + a1^{\mu\nu}{}_\alpha \sigma^\alpha + \dots \quad (205)$$

The coincidence limit of Eq.(205) gives

$$[g^\mu_{\tau'} g^\nu_{\rho'} a^{|\tau'\rho'}] = a0^{\mu\nu}, \quad (206)$$

and thus the first coefficient of the series,  $a0^{\mu\nu}$ , is given by

$$a0^{\mu\nu} = [a^{|\mu\nu}]. \quad (207)$$

The coincidence limit of one derivative of Eq.(205) gives the next coefficient in the series for  $g^\mu_{\tau'} g^\nu_{\rho'} a^{|\tau'\rho'}$ ;

$$\begin{aligned} a1^{\mu\nu}{}_\alpha &= [(g^\mu_{\tau'} g^\nu_{\rho'} a^{|\tau'\rho'})_{|\alpha}] - a0^{\mu\nu}{}_{;\alpha} \\ &= [a_{|\alpha}{}^{\mu'\nu'}] - a0^{\mu\nu}{}_{;\alpha} + [g^\mu{}_{\sigma'}{}_{;\alpha\beta}] \text{ terms,} \\ &= -[a_{|\alpha}{}^{\nu\mu'}] + [a_{|\alpha}{}^{\mu'}]{}_{;\nu} - a0^{\mu\nu}{}_{;\alpha} + [g^\mu{}_{\sigma'}{}_{;\alpha\beta}] \text{ terms,} \\ &= [a_{|\alpha}{}^{\nu\mu}] - [a_{|\alpha}{}^{\nu}]{}_{;\mu} - [a_{|\alpha}{}^{\mu}]{}_{;\nu} + [a_{|\alpha}]{}^{\mu\nu} - a0^{\mu\nu}{}_{;\alpha} + [g^\mu{}_{\sigma'}{}_{;\alpha\beta}] \text{ terms,} \end{aligned} \quad (208)$$

where the fact that primed derivatives commute with unprimed derivatives has been used, and terms containing objects such as  $[g^\mu_{\sigma';\alpha}]$ , etc., have been grouped together since they contribute nothing to the expansions due to (204). Note that derivatives of coincidence limits are always covariant since the coincidence limits of any quantity are constructed from local curvature and electromagnetic tensors, and such tensors are gauge invariant. Continuing this procedure gives

$$\begin{aligned}
a2^{\mu\nu}_{\alpha\beta} &= [(g^\mu_{\tau'} g^\nu_{\rho'} a^{|\tau'\rho'})_{|\alpha\beta}] - a0^{\mu\nu}_{;\alpha\beta} - a1^{\mu\nu}_{\alpha;\beta} - a1^{\mu\nu}_{\beta;\alpha}, \\
&= [a_{\alpha\beta}{}^{i\mu\nu}] - [a_{\alpha\beta}{}^{i\nu}]{}^{i\mu} - [a_{\alpha\beta}{}^{i\mu}]{}^{i\nu} + [a_{\alpha\beta}]{}^{i\mu\nu} - a0^{\mu\nu}_{;\alpha\beta} \\
&\quad - a1^{\mu\nu}_{\alpha;\beta} - a1^{\mu\nu}_{\beta;\alpha} + [g^\mu_{\sigma';\alpha\beta}] \text{ terms.}
\end{aligned} \tag{209}$$

As a specific example of Eq.(209), consider the expansion of  $g^\nu_{\rho'} \sigma^{i\mu\rho'}$ :

$$\begin{aligned}
g^\nu_{\rho'} \sigma^{i\mu\rho'} &= [g^\nu_{\rho'} \sigma^{i\mu\rho'}] + ([g^\nu_{\rho'} \sigma^{i\mu\rho'}]_{\alpha} - [g^\nu_{\rho'} \sigma^{i\mu\rho'}]_{;\alpha}) \sigma^\alpha + \dots \\
&= [g^\nu_{\rho'}] [\sigma^{i\mu\rho'}] + ([g^\nu_{\rho'}] [\sigma^{i\mu\rho'}]_{\alpha} - [g^\nu_{\rho'}]_{;\alpha} [\sigma^{i\mu\rho'}] - [g^\nu_{\rho'}] [\sigma^{i\mu\rho'}]_{;\alpha}) \sigma^\alpha + \dots \\
&= g^\nu_{\rho} \{ -[\sigma^{i\mu\rho}] + [\sigma^{i\mu}]{}^{i\rho} \} + \\
&\quad (g^\nu_{\rho} \{ -[\sigma^{i\mu}{}_{\alpha}{}^{\rho}] + [\sigma^{i\mu}{}_{\alpha}]{}^{i\rho} \} - g^\nu_{\rho;\alpha} [\sigma^{i\mu\rho'}] - g^\nu_{\rho} \{ [\sigma^{i\mu\rho}] + [\sigma^{i\mu}]{}^{i\rho} \}_{;\alpha}) \sigma^\alpha \\
&\quad + \dots \\
&= -g^{\mu\nu} + (0)\sigma^\alpha + \dots
\end{aligned} \tag{210}$$

The last issue to discuss before listing the bitensor expansions is the order to which the power series expansions must be carried in the separation vector  $\sigma^\mu$ . For example, if we were only constructing Eq.(133) for  $G^{(1)}$ , we note that the biscalars  $\Delta^{1/2}$  and  $a_0$  multiply the  $1/\sigma = 2/(\sigma^\mu \sigma_\mu)$  term. This term diverges quadratically as  $\sigma^\mu \rightarrow 0$  and thus the expansions for  $\Delta^{1/2}$  and  $a_0$  must be carried out to second order in  $\sigma^\mu$ . The rest of the  $a_n$  in  $G^{(1)}$  need not proceed beyond the zeroth order term in the power series expansion Eq.(135) since they multiply terms that either diverge logarithmically or remain finite as the points  $x$  and  $x'$  are brought together.

Constructing the DS point-splitting expansion for the current requires taking the derivatives of  $G^{(1)}$  indicated by Eq.(81). We may use

$$g^\mu_{\tau'} \sigma^{\tau'} = -\sigma^\mu, \quad (211)$$

which is indicated directly from the construction of Figure 2. In Figure 2, the bivector of parallel transport  $g^\mu_{\tau'}$  parallel transports the vector  $\sigma^{\mu'}$  from the point  $x'(\lambda')$  to the point  $x(\lambda)$  along the geodesic connecting the two points. When at  $x$ , it remains a tangent vector with length equal to the geodesic distance between  $x$  and  $x'$ . Yet at  $x$ ,  $\sigma^{\mu'}$  is still pointed in the direction  $x' \rightarrow x$  while the original vector at  $x$ ,  $\sigma^\mu$ , is pointed in the direction  $x \rightarrow x'$ .

Using Eq.(211), Eq.(81) becomes

$$\begin{aligned} 4\pi^2 \langle j^\mu(x) \rangle &= 4\pi^2 \lim_{x' \rightarrow x} \frac{ie}{4} \left\{ \left( G^{(1)|\mu} + g^\mu_{\tau'} G^{(1)|\tau'} \right) - \left( G^{(1)|\mu} + g^\mu_{\tau'} G^{(1)|\tau'} \right)^* \right\} \\ &= 4\pi^2 \lim_{x' \rightarrow x} \frac{ie}{4} \left\{ \left[ \frac{1}{\sigma} + m^2 \left( \gamma + \frac{1}{2} \ln \left| \frac{1}{2} m^2 \sigma \right| \right) - \frac{1}{2} m^2 \right] \times \right. \\ &\quad \left[ \Delta^{1/2} (a_0^{|\mu} + g^\mu_{\tau'} a_0^{|\tau'}) + a_0 (\Delta^{1/2; \mu} + g^\mu_{\tau'} \Delta^{1/2; \tau'}) \right] \\ &\quad - \left( \gamma + \frac{1}{2} \ln \left| \frac{1}{2} m^2 \sigma \right| \right) \times \\ &\quad \left[ \Delta^{1/2} (a_1^{|\mu} + g^\mu_{\tau'} a_1^{|\tau'}) + a_1 (\Delta^{1/2; \mu} + g^\mu_{\tau'} \Delta^{1/2; \tau'}) \right] \\ &\quad + \frac{1}{2m^2} \left[ \Delta^{1/2} (a_2^{|\mu} + g^\mu_{\tau'} a_2^{|\tau'}) + a_2 (\Delta^{1/2; \mu} + g^\mu_{\tau'} \Delta^{1/2; \tau'}) \right] \\ &\quad \left. + \frac{1}{2m^4} \left[ \Delta^{1/2} (a_3^{|\mu} + g^\mu_{\tau'} a_3^{|\tau'}) + a_3 (\Delta^{1/2; \mu} + g^\mu_{\tau'} \Delta^{1/2; \tau'}) \right] + \dots \right\} \quad (212) \end{aligned}$$

where terms such as  $\Delta^{1/2; \mu} a_0 \sigma$  have been dropped since they vanish in the limit  $x' \rightarrow x$ , and the semicolon and vertical bar have been used to indicate covariant and gauge covariant differentiation, respectively. Eq.(212) shows that the power series expansions for  $\Delta^{1/2}$ ,  $a_0$ , and their first derivatives must be carried to second order in  $\sigma^\mu$  due to the  $1/\sigma$  term, while the expansions for  $a_1, a_2, \dots$  and their first derivatives need go no further than zeroth order. As discussed in Chapter 2, one derivative of  $G^{(1)}$  would potentially have a cubic divergence. In the expansion of Eq.(212), the cubic

terms that do arise after performing the differentiations of Eq.(81) immediately cancel due to Eq.(211). There are still terms that are potentially quadratically divergent, but, as will be shown in the next chapter, these, too, will cancel, leaving  $\langle j^\mu \rangle$  with only a linearly divergent term.

It is not necessary to list the full expression for the stress-energy tensor to see how far it requires the power series expansions to be carried out. It is only useful to examine the single term

$$\left(\frac{\Delta^{1/2}a_0}{\sigma}\right)|_{\mu\nu} = \frac{2\Delta^{1/2}a_0\sigma^{i\mu}\sigma^{i\nu}}{\sigma^3} - \frac{\Delta^{1/2}a_0\sigma^{i\mu\nu}}{\sigma^2} - \frac{\Delta^{1/2}a_0|^\nu\sigma^{i\mu}}{\sigma^2} - \frac{\Delta^{1/2}{}_{;\nu}a_0\sigma^{i\mu}}{\sigma^2} + \dots, \quad (213)$$

where

$$(\sigma^{-1})|_{\mu\nu} = \frac{2\sigma^{i\mu}\sigma^{i\nu}}{\sigma^3} - \frac{\sigma^{i\mu\nu}}{\sigma^2}. \quad (214)$$

has been used. Eq.(213) shows that, to construct  $\langle T_{\mu\nu} \rangle$ , it would be necessary to carry the expansions of  $\Delta^{1/2}$ ,  $a_0$  and  $\sigma^{i\mu\nu}$  to fourth order in  $\sigma^\mu$ ;  $\Delta^{1/2}{}_{;\mu}$  and  $a_0|^\mu$  must be carried to third order; and  $\Delta^{1/2}{}_{;\mu\nu}$  and  $a_0|_{\mu\nu}$  must be carried to second order. In fact, these fourth order power series expansions are the lengthiest expansions that will be needed.

Without going through more analysis like the above, it is useful to simply list the expansions that will be needed for regularization of the expressions for  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$ . These expansions are

$$g^\mu{}_{\tau'}\sigma^{i\tau'} = -\sigma^\mu, \quad (215)$$

$$\begin{aligned} \sigma^{i\mu\nu} = & g^{\mu\nu} - \frac{1}{3}R^\mu{}_\alpha{}^\nu{}_\beta\sigma^\alpha\sigma^\beta + \frac{1}{12}R^\mu{}_\alpha{}^\nu{}_{\beta;\gamma}\sigma^\alpha\sigma^\beta\sigma^\gamma \\ & - \left(\frac{1}{60}R^\mu{}_\alpha{}^\nu{}_{\beta;\gamma\delta} + \frac{1}{45}R^\rho{}_\alpha{}^\mu{}_\beta R_{\rho\gamma}{}^\nu{}_\delta\right)\sigma^\alpha\sigma^\beta\sigma^\gamma\sigma^\delta + \dots, \end{aligned} \quad (216)$$

$$\begin{aligned} g^\nu{}_{\rho'}\sigma^{i\mu\rho'} = & -g^{\mu\nu} - \frac{1}{6}R^\mu{}_\alpha{}^\nu{}_\beta\sigma^\alpha\sigma^\beta + \frac{1}{12}R^\mu{}_\alpha{}^\nu{}_{\beta;\gamma}\sigma^\alpha\sigma^\beta\sigma^\gamma \\ & - \left(\frac{1}{40}R^\mu{}_\alpha{}^\nu{}_{\beta;\gamma\delta} + \frac{7}{360}R^\rho{}_\alpha{}^\mu{}_\beta R_{\rho\gamma}{}^\nu{}_\delta\right)\sigma^\alpha\sigma^\beta\sigma^\gamma\sigma^\delta + \dots, \end{aligned} \quad (217)$$

$$g^\mu{}_{\tau'}g^\nu{}_{\rho'}\sigma^{i\tau'\rho'} = g^{\mu\nu} - \frac{1}{3}R^\mu{}_\alpha{}^\nu{}_\beta\sigma^\alpha\sigma^\beta + \frac{1}{4}R^\mu{}_\alpha{}^\nu{}_{\beta;\gamma}\sigma^\alpha\sigma^\beta\sigma^\gamma$$

$$-\left(\frac{1}{10}R^\mu{}^\nu{}_{\beta;\gamma\delta} + \frac{1}{45}R^\rho{}^\mu{}_\beta R_{\rho\gamma}{}^\nu{}_\delta\right)\sigma^\alpha\sigma^\beta\sigma^\gamma\sigma^\delta + \dots, \quad (218)$$

$$\begin{aligned} \Delta^{\frac{1}{2}} &= 1 + \frac{1}{12}R_{\alpha\beta}\sigma^\alpha\sigma^\beta - \frac{1}{24}R_{\alpha\beta;\gamma}\sigma^\alpha\sigma^\beta\sigma^\gamma \\ &+ \left(\frac{1}{288}R_{\alpha\beta}R_{\gamma\delta} + \frac{1}{360}R^\rho{}_\alpha{}^\tau{}_\beta R_{\rho\gamma\tau\delta} + \frac{1}{80}R_{\alpha\beta;\gamma\delta}\right)\sigma^\alpha\sigma^\beta\sigma^\gamma\sigma^\delta + \dots, \end{aligned} \quad (219)$$

$$\begin{aligned} \Delta^{\frac{1}{2};\mu} &= \frac{1}{6}R^\mu{}_\alpha\sigma^\alpha - \frac{1}{24}(2R^\mu{}_{\alpha;\beta} - R_{\alpha\beta}{}^{;\mu})\sigma^\alpha\sigma^\beta + \left(\frac{1}{40}R^\mu{}_{\alpha;\beta\gamma} - \frac{1}{60}R_{\alpha\beta}{}^{;\mu}{}_\gamma\right. \\ &+ \left.\frac{1}{90}R^{\rho\mu\tau}{}_\alpha R_{\rho\beta\tau\gamma} + \frac{1}{72}R^\mu{}_\alpha R_{\beta\gamma} + \frac{1}{360}R_{\alpha\rho}{}^\rho R_{\rho\beta}{}^\mu{}_\gamma\right)\sigma^\alpha\sigma^\beta\sigma^\gamma + \dots, \end{aligned} \quad (220)$$

$$\begin{aligned} g^\mu{}_{\tau'}\Delta^{\frac{1}{2};\tau'} &= -\frac{1}{6}R^\mu{}_\alpha\sigma^\alpha + \frac{1}{24}(2R^\mu{}_{\alpha;\beta} + R_{\alpha\beta}{}^{;\mu})\sigma^\alpha\sigma^\beta + \left(-\frac{1}{40}R^\mu{}_{\alpha;\beta\gamma}\right. \\ &+ \left.\frac{1}{60}R_{\alpha\beta}{}^{;\mu}{}_\gamma - \frac{1}{90}R^{\rho\mu\tau}{}_\alpha R_{\rho\beta\tau\gamma} - \frac{1}{72}R^\mu{}_\alpha R_{\beta\gamma} + \frac{11}{360}R_{\alpha\rho}{}^\rho R_{\rho\beta}{}^\mu{}_\gamma\right. \\ &+ \left.\frac{1}{270}R^{\rho\tau\mu}{}_\alpha R_{\rho\beta\tau\gamma}\right)\sigma^\alpha\sigma^\beta\sigma^\gamma + \dots, \end{aligned} \quad (221)$$

$$\begin{aligned} \Delta^{\frac{1}{2};\mu\nu} &= \frac{1}{6}R^{\mu\nu} + \frac{1}{12}(R^\mu{}_\alpha{}^{;\nu} + R^\nu{}_\alpha{}^{;\mu} - R^{\mu\nu}{}_{;\alpha})\sigma^\alpha \\ &+ \left[\frac{1}{40}R^{\mu\nu}{}_{;\alpha\beta} + \frac{1}{80}(R_{\alpha\beta}{}^{;\mu\nu} + R_{\alpha\beta}{}^{;\nu\mu}) - \frac{1}{30}(R^\mu{}_\alpha{}^{;\nu}{}_\beta + R^\nu{}_\alpha{}^{;\mu}{}_\beta) + \frac{1}{72}R_{\alpha\beta}R^{\mu\nu}\right. \\ &+ \frac{1}{36}R^\mu{}_\alpha R^\nu{}_\beta + \frac{1}{360}(R^\mu{}_\rho R^\nu{}_\alpha{}^\rho{}_\beta + R^\nu{}_\rho R^\mu{}_\alpha{}^\rho{}_\beta) + \frac{1}{90}R^{\mu\rho\nu\tau}R_{\rho\alpha\tau\beta} \\ &+ \frac{1}{180}(R^\rho{}_\alpha{}^\tau{}_\mu R_{\rho}{}^\nu{}_\tau{}_\beta + R^\rho{}_\alpha{}^\tau{}_\nu R_{\rho}{}^\mu{}_\tau{}_\beta) + \frac{1}{180}(R^\rho{}_\alpha{}^\tau{}_\mu R_{\tau}{}^\nu{}_\rho{}_\beta + R^\rho{}_\alpha{}^\tau{}_\nu R_{\tau}{}^\mu{}_\rho{}_\beta) \\ &+ \left.\frac{11}{360}R_{\alpha\rho}(R^{\rho\mu\nu}{}_\beta + R^{\rho\nu\mu}{}_\beta)\right]\sigma^\alpha\sigma^\beta + \dots, \end{aligned} \quad (222)$$

$$\begin{aligned} g^\nu{}_{\rho'}\Delta^{\frac{1}{2};\mu\rho'} &= -\frac{1}{6}R^{\mu\nu} + \frac{1}{12}(R^\mu{}_\alpha{}^{;\nu} - R^\nu{}_\alpha{}^{;\mu} - R^{\mu\nu}{}_{;\alpha})\sigma^\alpha + \left[-\frac{1}{40}R^{\mu\nu}{}_{;\alpha\beta}\right. \\ &+ \frac{1}{60}R_{\alpha\beta}{}^{;\mu\nu} + \frac{1}{30}(2R^\mu{}_{\alpha;\beta}{}^\nu - 5R^\mu{}_\alpha{}^{;\nu}{}_\beta + 2R^\nu{}_\alpha{}^{;\mu}{}_\beta) - \frac{1}{72}R_{\alpha\beta}R^{\mu\nu} \\ &- \frac{1}{36}R^\mu{}_\alpha R^\nu{}_\beta - \frac{1}{180}(R^\nu{}_\rho R^\mu{}_\alpha{}^\rho{}_\beta - 11R^\mu{}_\rho R^\nu{}_\alpha{}^\rho{}_\beta) - \frac{1}{90}R^{\mu\rho\nu\tau}R_{\rho\alpha\tau\beta} \\ &- \frac{1}{90}R^\rho{}_\alpha{}^\tau{}_\mu(R_{\tau}{}^\nu{}_\rho{}_\beta + R_{\rho}{}^\nu{}_\tau{}_\beta) - \frac{1}{180}R_{\alpha\rho}(R^\rho{}_\beta{}^{\mu\nu} + 10R^{\rho\mu\nu}{}_\beta)\left.\right]\sigma^\alpha\sigma^\beta + \dots, \end{aligned} \quad (223)$$

$$\begin{aligned} g^\mu{}_{\tau'}g^\nu{}_{\rho'}\Delta^{\frac{1}{2};\tau'\rho'} &= \frac{1}{6}R^{\mu\nu} - \frac{1}{12}(R^\mu{}_\alpha{}^{;\nu} + R^\nu{}_\alpha{}^{;\mu} + R^{\mu\nu}{}_{;\alpha})\sigma^\alpha \\ &+ \left[\frac{1}{40}R^{\mu\nu}{}_{;\alpha\beta} + \frac{1}{80}(R_{\alpha\beta}{}^{;\mu\nu} + R_{\alpha\beta}{}^{;\nu\mu}) - \frac{1}{30}(R^\mu{}_{\alpha;\beta}{}^\nu + R^\nu{}_{\alpha;\beta}{}^\mu)\right. \\ &+ \frac{1}{12}(R^\mu{}_\alpha{}^{;\nu}{}_\beta + R^\nu{}_\alpha{}^{;\mu}{}_\beta) + \frac{1}{72}R_{\alpha\beta}R^{\mu\nu} + \frac{1}{36}R^\mu{}_\alpha R^\nu{}_\beta \\ &- \frac{11}{360}(R^\mu{}_\rho R^\nu{}_\alpha{}^\rho{}_\beta + R^\nu{}_\rho R^\mu{}_\alpha{}^\rho{}_\beta) + \frac{1}{180}R_{\rho\alpha\tau\beta}(R^{\mu\rho\nu\tau} + R^{\nu\rho\mu\tau}) \\ &+ \left.\frac{1}{180}(R^\rho{}_\alpha{}^\tau{}_\mu R_{\rho}{}^\nu{}_\tau{}_\beta + R^\rho{}_\alpha{}^\tau{}_\nu R_{\rho}{}^\mu{}_\tau{}_\beta) + \frac{1}{180}(R^\rho{}_\alpha{}^\tau{}_\mu R_{\tau}{}^\nu{}_\rho{}_\beta + R^\rho{}_\alpha{}^\tau{}_\nu R_{\tau}{}^\mu{}_\rho{}_\beta)\right] \end{aligned}$$

$$-\frac{1}{360}R_{\alpha\rho}(R^{\rho\mu\nu}{}_{\beta} + R^{\rho\nu\mu}{}_{\beta})\sigma^{\alpha}\sigma^{\beta} + \dots, \quad (224)$$

$$a_0 = 1, \quad (225)$$

$$a_0^{|\mu} = \frac{ie}{2}F^{\mu}{}_{\alpha}\sigma^{\alpha} - \frac{ie}{6}F^{\mu}{}_{\alpha;\beta}\sigma^{\alpha}\sigma^{\beta} + \frac{ie}{24}(F^{\mu}{}_{\alpha;\beta\gamma} + F_{\rho\alpha}R^{\mu}{}_{\beta}{}^{\rho}{}_{\gamma})\sigma^{\alpha}\sigma^{\beta}\sigma^{\gamma} + \dots, \quad (226)$$

$$g^{\mu}{}_{\tau'}a_0^{|\tau'} = \frac{ie}{2}F^{\mu}{}_{\alpha}\sigma^{\alpha} - \frac{ie}{3}F^{\mu}{}_{\alpha;\beta}\sigma^{\alpha}\sigma^{\beta} + \frac{ie}{24}(3F^{\mu}{}_{\alpha;\beta\gamma} + F_{\rho\alpha}R^{\mu}{}_{\beta}{}^{\rho}{}_{\gamma})\sigma^{\alpha}\sigma^{\beta}\sigma^{\gamma} + \dots, \quad (227)$$

$$a_0^{|\mu\nu} = \frac{ie}{2}F^{\mu\nu} + \frac{ie}{6}(F^{\mu}{}_{\alpha}{}^{;\nu} + F^{\nu}{}_{\alpha}{}^{;\mu})\sigma^{\alpha} + \left[\frac{ie}{24}(F_{\alpha}{}^{\mu}{}_{;\beta}{}^{\nu} + F_{\alpha}{}^{\nu}{}_{;\beta}{}^{\mu}) - \frac{e^2}{4}F_{\alpha}{}^{\mu}F_{\beta}{}^{\nu} + \frac{ie}{12}(F_{\rho}{}^{\mu}R^{\nu}{}_{\alpha}{}^{\rho}{}_{\beta} + F_{\rho}{}^{\nu}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}) - \frac{ie}{24}F_{\rho\alpha}(R^{\mu\rho\nu}{}_{\beta} + R^{\nu\rho\mu}{}_{\beta})\right]\sigma^{\alpha}\sigma^{\beta} + \dots, \quad (228)$$

$$g^{\nu}{}_{\rho'}a_0^{|\mu\rho'} = -\frac{ie}{2}F^{\mu\nu} + \frac{ie}{6}(F^{\mu\nu}{}_{;\alpha} + F^{\mu}{}_{\alpha}{}^{;\nu})\sigma^{\alpha} + \left(-\frac{ie}{24}F^{\mu\nu}{}_{;\alpha\beta} + \frac{ie}{12}F_{\alpha}{}^{\mu}{}_{;\beta}{}^{\nu} - \frac{e^2}{4}F_{\alpha}{}^{\mu}F_{\beta}{}^{\nu} + \frac{ie}{24}(3F_{\rho}{}^{\mu}R^{\nu}{}_{\alpha}{}^{\rho}{}_{\beta} - F_{\rho}{}^{\nu}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}) + \frac{ie}{12}F_{\rho\alpha}R^{\nu\rho\mu}{}_{\beta}\right)\sigma^{\alpha}\sigma^{\beta} + \dots, \quad (229)$$

$$g^{\mu}{}_{\tau'}g^{\nu}{}_{\rho'}a_0^{|\tau'\rho'} = \frac{ie}{2}F^{\mu\nu} + \frac{ie}{6}(F^{\mu}{}_{\alpha}{}^{;\nu} + F^{\nu}{}_{\alpha}{}^{;\mu})\sigma^{\alpha} + \left[\frac{3ie}{8}(F_{\alpha}{}^{\mu}{}_{;\beta}{}^{\nu} + F_{\alpha}{}^{\nu}{}_{;\beta}{}^{\mu}) - \frac{e^2}{4}F_{\alpha}{}^{\mu}F_{\beta}{}^{\nu} + \frac{ie}{4}(F_{\rho}{}^{\mu}R^{\nu}{}_{\alpha}{}^{\rho}{}_{\beta} + F_{\rho}{}^{\nu}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}) + \frac{3ie}{8}F_{\rho\alpha}(R^{\mu\rho\nu}{}_{\beta} + R^{\nu\rho\mu}{}_{\beta})\right]\sigma^{\alpha}\sigma^{\beta} + \dots, \quad (230)$$

$$a_1 = \left(\frac{1}{6} - \xi\right)R + \left(-\frac{1}{2}\left(\frac{1}{6} - \xi\right)R_{;\alpha} + \frac{ie}{6}F^{\rho}{}_{\alpha;\rho}\right)\sigma^{\alpha} + \left(-\frac{1}{90}R_{\alpha\rho}R^{\rho}{}_{\beta} + \frac{1}{180}R^{\rho\tau}R_{\rho\alpha\tau\beta} + \frac{1}{120}R_{\alpha\beta;\rho}{}^{\rho} + \frac{1}{180}R_{\rho\tau\kappa\alpha}R^{\rho\tau\kappa}{}_{\beta} + \left(\frac{1}{40} - \frac{1}{6}\xi\right)R_{;\alpha\beta} - \frac{ie}{12}F^{\rho}{}_{\alpha;\rho\beta} - \frac{e^2}{12}F^{\rho}{}_{\alpha}F_{\rho\beta}\right)\sigma^{\alpha}\sigma^{\beta} + \dots, \quad (231)$$

$$a_1^{|\mu} = \frac{1}{2}\left(\frac{1}{6} - \xi\right)R^{;\mu} + \frac{ie}{6}F^{\rho\mu}{}_{;\rho} + \left(-\frac{1}{45}R^{\mu}{}_{\rho}R^{\rho}{}_{\alpha} + \frac{1}{90}R^{\rho\tau}R^{\mu}{}_{\rho\alpha\tau} + \frac{1}{90}R_{\rho\tau\kappa}{}^{\mu}R^{\rho\tau\kappa}{}_{\alpha} + \frac{1}{60}R^{\mu}{}_{\alpha;\rho}{}^{\rho} + \frac{1}{6}\left(\xi - \frac{1}{5}\right)R^{;\mu}{}_{\alpha} + \frac{ie}{12}(F_{\rho\alpha}{}^{;\rho\mu} - F^{\rho\mu}{}_{;\rho\alpha}) + \frac{ie}{2}\left(\frac{1}{6} - \xi\right)RF^{\mu}{}_{\alpha}\right)\sigma^{\alpha} + \dots, \quad (232)$$

$$g^{\mu}{}_{\tau'}a_1^{|\tau'} = \frac{1}{2}\left(\frac{1}{6} - \xi\right)R^{;\mu} - \frac{ie}{6}F^{\rho\mu}{}_{;\rho} + \left(\frac{1}{45}R^{\mu}{}_{\rho}R^{\rho}{}_{\alpha} - \frac{1}{90}R^{\rho\tau}R^{\mu}{}_{\rho\alpha\tau} - \frac{1}{90}R_{\rho\tau\kappa}{}^{\mu}R^{\rho\tau\kappa}{}_{\alpha} - \frac{1}{60}R^{\mu}{}_{\alpha;\rho}{}^{\rho} + \frac{1}{3}\left(\xi - \frac{3}{20}\right)R^{;\mu}{}_{\alpha} + \frac{ie}{12}(F_{\rho\alpha}{}^{;\rho\mu} - F^{\rho\mu}{}_{;\rho\alpha}) + \frac{ie}{2}\left(\frac{1}{6} - \xi\right)RF^{\mu}{}_{\alpha}\right)\sigma^{\alpha} + \dots, \quad (233)$$

$$a_1^{|\mu\nu} = -\frac{1}{45}R^{\mu}{}_{\rho}R^{\nu\rho} + \frac{1}{90}R^{\rho\tau}R^{\mu}{}_{\rho}{}^{\nu}{}_{\tau} + \frac{1}{90}R_{\rho\tau\kappa}{}^{\mu}R^{\rho\tau\kappa\nu} + \frac{1}{60}R^{\mu\nu}{}_{;\rho}{}^{\rho} + \left(\frac{1}{20} - \frac{1}{3}\xi\right)R^{;\mu\nu} + \frac{ie}{12}(F_{\rho}{}^{\mu;\rho\nu} + F_{\rho}{}^{\nu;\rho\mu}) + \frac{ie}{2}\left(\frac{1}{6} - \xi\right)RF^{\mu\nu}$$

$$-\frac{e^2}{6}F_\rho^\mu F^{\rho\nu} + \dots, \quad (234)$$

$$g^\nu_{\rho'} a_1^{|\mu\rho'} = \frac{1}{45}R^\mu_{\rho} R^{\nu\rho} - \frac{1}{90}R^{\rho\tau} R^\mu_{\rho}{}^\nu{}_\tau - \frac{1}{90}R_{\rho\tau\kappa}{}^\mu R^{\rho\tau\kappa\nu} - \frac{1}{60}R^{\mu\nu}{}_{;\rho}{}^\rho + \frac{1}{6}\left(\frac{1}{5} - \xi\right)R^{;\mu\nu}$$

$$+ \frac{ie}{12}(F_\rho^{\mu;\rho\nu} - F_\rho^{\nu;\rho\mu}) - \frac{ie}{2}\left(\frac{1}{6} - \xi\right)RF^{\mu\nu} + \frac{e^2}{6}F_\rho^\mu F^{\rho\nu} + \dots, \quad (235)$$

$$g^\mu_{\tau'} g^\nu_{\rho'} a_1^{|\tau'\rho'} = -\frac{1}{45}R^\mu_{\rho} R^{\nu\rho} + \frac{1}{90}R^{\rho\tau} R^\mu_{\rho}{}^\nu{}_\tau + \frac{1}{90}R_{\rho\tau\kappa}{}^\mu R^{\rho\tau\kappa\nu} + \frac{1}{60}R^{\mu\nu}{}_{;\rho}{}^\rho$$

$$+ \left(\frac{1}{20} - \frac{1}{3}\xi\right)R^{;\mu\nu} - \frac{ie}{12}(F_\rho^{\mu;\rho\nu} + F_\rho^{\nu;\rho\mu}) + \frac{ie}{2}\left(\frac{1}{6} - \xi\right)RF^{\mu\nu}$$

$$- \frac{e^2}{6}F_\rho^\mu F^{\rho\nu} + \dots, \quad (236)$$

and

$$a_2 = \frac{1}{180}(R^{\rho\tau\kappa\lambda} R_{\rho\tau\kappa\lambda} - R^{\rho\tau} R_{\rho\tau}) + \frac{1}{6}\left(\frac{1}{5} - \xi\right)R_{;\rho}{}^\rho + \frac{1}{2}\left(\frac{1}{6} - \xi\right)^2 R^2$$

$$- \frac{e^2}{12}F_{\rho\tau} F^{\rho\tau} + \dots. \quad (237)$$

A semicolon has been used for all of the derivatives on the right hand sides of Eqs.(215)–(237) since all of these terms are gauge invariant and thus do not need the connection  $A^\mu$  in their derivatives. The expansions with either two unprimed or two primed derivatives have been symmetrized according to,

$$a^{|\mu\nu} = \frac{1}{2}(a^{|\mu\nu} + a^{|\nu\mu} + \chi^{\mu\nu} a), \quad (238)$$

where  $\chi^{\mu\nu} = ieF^{\mu\nu}$  for the  $a_n$  and  $\chi^{\mu\nu} = 0$  for the other biscalars. This ensures the expansions of Eqs.(215)–(237) obey the commutation relations of both the covariant and gauge covariant derivatives. Note that quantities such as  $g^\nu_{\rho'}(\sigma^{-1})^{;\mu\rho'}$  may only have their numerators expanded using Eqs.(215)–(237) since they diverge in the coincidence limit necessary for applying the expansion iterations in Eqs.(198)–(201):

$$g^\nu_{\rho'}(\sigma^{-1})^{;\mu\rho'} = g^\nu_{\rho'}(2\sigma^{-3}\sigma^{;\mu}\sigma^{;\rho'} - \sigma^{-2}\sigma^{;\mu\rho'})$$

$$= -2\sigma^{-3}\sigma^{;\mu}\sigma^{;\nu} - \sigma^{-2}(-g^{\mu\nu} - \dots). \quad (239)$$

These expansions will be used in the next chapter to regularize  $\langle\phi^2\rangle$ ,  $\langle j^\mu\rangle$ , and  $\langle T_{\mu\nu}\rangle$ .

## CHAPTER 6

## DeWitt-Schwinger Point-Splitting Expansions of $G^{(1)}$ , $\langle j^\mu \rangle$ , and $\langle T_{\mu\nu} \rangle$

The DeWitt-Schwinger point-splitting procedure is able to completely regularize the expressions for  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$  as mentioned in Chapter 1. This has been proven in more general terms elsewhere using an analysis of the effective action of gravitational theory [8]. In this chapter, however, the divergences of each expression will be identified explicitly by identifying terms that diverge as the distance  $|x - x'|$  goes to zero. The divergent terms will be shown to be proportional to non-negative powers of  $m$  in the DS point-splitting expansions for  $\langle \phi^2 \rangle$ ,  $\langle j^\mu \rangle$ , and  $\langle T_{\mu\nu} \rangle$ , where  $m$  is the mass of the quantized field. This chapter also details how DeWitt-Schwinger approximations for  $\langle \phi^2 \rangle$  and  $\langle T_{\mu\nu} \rangle$  are obtained. The point-splitting procedure will be shown incapable of yielding a DeWitt-Schwinger expansion for  $\langle j^\mu \rangle$ .

The vacuum polarization as given by Eq.(75) is

$$\langle \phi^2 \rangle = \lim_{x' \rightarrow x} \frac{1}{2} G^{(1)}(x, x') \quad (240)$$

and is thus regularized along with  $G^{(1)}$ . Substituting the expansions in Eqs.(215)–(237) for the biscalars into Eq.(133) and collecting terms of like powers of  $\sigma^\mu$  yields,

$$4\pi^2 G^{(1)}(x, x') = \frac{2}{(\sigma^\rho \sigma_\rho)} + [m^2 - (\frac{1}{6} - \xi)R][\gamma + \frac{1}{2} \ln |\frac{1}{4} m^2 (\sigma^\rho \sigma_\rho)|] - \frac{1}{2} m^2 + \frac{1}{6} R_{\alpha\beta} \frac{\sigma^\alpha \sigma^\beta}{(\sigma^\rho \sigma_\rho)} + \frac{[a_2]}{2m^2} + \frac{[a_3]}{2m^4} + \vartheta \left( \frac{1}{m^6} \right), \quad (241)$$

where the explicit expressions for  $[a_2]$  and  $[a_3]$  have not been included here for brevity, and the points  $x$  and  $x'$  are not coincident. The terms of  $\vartheta(m^2)$  and  $\vartheta(m^0)$  are the same

as those derived by Christensen for a real scalar field. Eq.(241) explicitly shows terms that diverge quadratically and logarithmically as  $\sigma^\mu \rightarrow 0$ . The quadratic divergence was predicted in the discussion of the degree of divergence in Chapter 2. Eq.(241) also contains two types of finite terms; one,  $\frac{1}{6}R_{\alpha\beta}\frac{\sigma^\alpha\sigma^\beta}{(\sigma^\rho\sigma_\rho)}$  depending on the direction of the point-splitting vector  $\sigma^\mu$ , and another,  $-\frac{1}{2}m^2$ , independent of the direction of  $\sigma^\mu$ . The terms proportional to non-negative powers of  $m$  come from the power series expansions of  $[a_0]$  and  $[a_1]$  and have been shown to be sufficient to regularize  $G^{(1)}$  [8, 2, 4].

The agreement of the regularization terms in Eq.(241) with Christensen's original result is to be expected. The complex scalar field is constructed from two real fields according to  $\phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$ . From the form of the Hadamard function Eq.(74), it is evident that  $G^{(1)}_{divergent}$  for the complex scalar field will be the sum of  $G^{(1)}_{divergent}$  for two real scalar fields. Thus both contain the same quadratic and logarithmic divergences, and the same direction-dependent ( $\sigma^\mu$ -dependent) and direction-independent terms.

The terms in Eq.(241) proportional to  $m^{-2}, m^{-4}, \dots$  remain finite as  $\sigma^\mu \rightarrow 0$ . The DeWitt-Schwinger point-splitting expansion contains both divergent and finite term. The renormalized DS approximation,  $G^{(1)}_{DS,ren}$ , is formed by subtracting terms proportional to non-negative powers of  $m$  in Eq.(241). Eq.(241) shows that the first few terms, or even the first term, in this series may be taken as a DS approximation to the exact value for  $G^{(1)}$  so long as the mass  $m$  of the quantized field is large when compared to the curvature terms contained in  $[a_2], [a_3]$ , etc. Thus, the DS approximation to  $G^{(1)}$  to  $\vartheta\left(\frac{1}{m^4}\right)$  is given by

$$\begin{aligned} 4\pi^2 G^{(1)}_{DS,ren} &\approx \frac{[a_2]}{2m^2} + \frac{[a_3]}{2m^4} \\ &= \frac{1}{2m^2} \left[ -\frac{1}{180} R^{\alpha\beta} R_{\alpha\beta} + \frac{1}{180} R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} + \frac{1}{6} \left( \frac{1}{5} - \xi \right) R_{;\alpha}{}^\alpha \right] \end{aligned}$$

$$+\frac{1}{2}\left(\frac{1}{6}-\xi\right)^2 R^2 - \frac{e^2}{12} F^{\alpha\beta} F_{\alpha\beta} \Big] + \frac{[a_3]}{2m^4} \quad (242)$$

where  $[a_3]$  is given by Eq.(197).

The gauge field first arises in the DS expansion of  $G^{(1)}$  through the  $[a_2]$  term. This may have been predicted for several reasons. Moving briefly to units where  $c = \hbar = 1$  but  $G \neq 1$ ,  $G^{(1)}$  is a second order quantity with units  $(length)^{-2}$ . The DS point-splitting procedure is an asymptotic expansion in inverse powers of  $m^2$  and the scalar wave equation, Eq.(71), shows  $m^2$  to have units  $(length)^{-2}$ . Thus, the coefficient  $A_{+2}$  in the DS expansion for  $G^{(1)}$ ,

$$G^{(1)}(x, x') = A_{+2}(x, x')m^2 + A_0(x, x')m^0 + A_{-2}(x, x')m^{-2} + \dots, \quad (243)$$

must have units  $(length)^0$ ,  $A_{+0}$  must have units  $(length)^{-2}$ ,  $A_{-2}$  must have units  $(length)^{-4}$ , and so forth.

The gauge field arises in the point-splitting procedure through the gauge commutation relation

$$a_n|_{\mu\nu} - a_n|_{\nu\mu} = ieF_{\mu\nu}a_n. \quad (244)$$

The sign of the charge  $e$  of the field can not change the physics of  $G^{(1)}$ , so no odd power of  $e$  may appear in any VEVs. This indicates the VEVs (returning to units where  $G = 1$ ) may only contain  $(eF)^2, (eF)^4, \dots$ , which have units  $(length)^{-4}, (length)^{-8}, \dots$ . Thus,  $(eF)^2$  may only appear in the coefficient of  $m^{-2}$  of  $G^{(1)}$ , or in  $[a_2]$ ;  $(eF)^4$  may only appear in the coefficient of  $m^{-6}$ , or  $[a_4]$ ; and so forth. The  $(eF)^2$  term in  $[a_2]$  is in agreement with the earlier results of DeWitt [38].

We may also count units to see where to expect cross terms involving the electromagnetic and gravitational fields to appear. For example,  $R(eF)^2$  has units  $(length)^{-6}$  and would first appear in the coefficient of  $m^{-4}$ , or in  $[a_3]$ . Eq.(197) shows this to be true. In fact, each of the terms in the DS point-splitting expansion for  $G^{(1)}$  may have been predicted from power counting. It must be left for point-splitting or some

other procedure to supply the numerical coefficients (which may be 0) for each of these terms. Davies, *et al* [18], constructed the stress-energy tensor of a conformally invariant scalar field in a conformally invariant spacetime using such power counting together with known constraints on the components.

Turning now to the current, the appropriate expansions from Eqs.(215)–(237) are substituted into Eq.(212) and like powers of  $\sigma^\mu$  are collected. This isolates the terms in  $\langle j^\mu \rangle$  which will diverge linearly and those which will remain finite as the points are brought together:

$$\langle j^\mu(x, x') \rangle_{linear} = \frac{1}{4\pi^2} \frac{e^2 \sigma^\alpha F_\alpha^\mu}{(\sigma^\rho \sigma_\rho)}, \quad (245)$$

and

$$\langle j^\mu(x, x') \rangle_{finite} = \frac{1}{4\pi^2} \frac{e^2 \sigma^\alpha \sigma^\beta F_{\alpha\beta}^\mu}{(2\sigma^\rho \sigma_\rho)}. \quad (246)$$

Eqs.(245)–(246) are proportional to  $m^0$ , a non-negative power of  $m$ , and were constructed from the coincidence limits of  $a_0$  and  $a_1$  and their derivatives. The linear term agrees with the prediction of Chapter 2 when the degree of divergence of the scalar field current was discussed. These two terms suffice to regularize  $\langle j^\mu \rangle$ .

The current is independent of the sign of the charge carrier having electromagnetic terms proportional to  $e^2$ . One factor of  $e$  in the current is explicit in Eq.(81). The other factor comes from the derivation of the expansions of  $G(1)^\mu$ , etc. This originates with the gauge commutation relation, Eq.(180), and is seen to carry one factor of  $eF$  through to the first derivative expansions of the  $a_n$  in Eqs.(215)–(237). The current is a third order quantity with units of  $(length)^{-3}$ . By counting powers, the linearly diverging term would thus require one factor of the second order  $eF$ , while the finite term would require the third order derivative of  $eF$ . Eqs. (245) and (246) are thus in the only possible form and, aside from numerical factors, could have been deduced immediately from simple power counting.

The divergence of Eqs.(245) and (246) must be taken in order to verify that the

point-splitting procedure does not violate the conservation of current. Naïvely taking the divergence of the current with respect to only  $x^\alpha$  and not  $x'^\alpha$  would give the wrong result since the separation vector  $\sigma^\mu(x, x')$ , which depends on both  $x^\alpha$  and  $x'^\alpha$ , is contained within the regularization counterterms of  $\langle j^\mu \rangle$ . The correct procedure for verifying current conservation requires evaluating the divergence of the classical current within the point-splitting regime whereby the derivatives with respect to the two spacetime points are symmetrized. Starting from the definition of the VEV of the current

$$\langle j^\mu \rangle = \lim_{x' \rightarrow x} \frac{ie}{4} \left[ \left( G^{(1)|m\mu} + g^\mu{}_{\tau'} G^{(1)|\tau'} \right) - c.c. \right], \quad (247)$$

the two derivatives are taken and the result is symmetrized, giving

$$\begin{aligned} \langle j^\mu \rangle_{;\mu} &= \frac{1}{2} \left[ \langle j^\mu \rangle_{;\mu} + \langle j^\mu \rangle_{\nu'} g_{\mu}{}^{\nu'} \right] \\ &= \lim_{x' \rightarrow x} \frac{ie}{8} \left[ G^{(1)|\mu}{}_{\mu} + 2g^{\mu\tau'} G^{(1)|\mu\tau'} + g^\mu{}_{\tau'} g_{\mu\nu'} G^{(1)|\tau'\nu'} - c.c. \right]. \end{aligned} \quad (248)$$

This may be seen to vanish to all orders by using the expansions in Eqs.(215)–(237), ensuring that the current is a conserved quantity.

It is also interesting to examine the divergence of the classical current,

$$j^\mu{}_{;\mu} = ie[(D^\mu \phi)\phi^* - \phi(D^\mu \phi)^*]_{;\mu}, \quad (249)$$

within the point-splitting regime. Eq.(249) may be written as,

$$j^\mu{}_{;\mu} = \frac{ie}{2} [\{D_\mu D^\mu \phi, \phi^*\} - \{D_\mu D^\mu \phi, \phi^*\}^*] = \frac{ie}{2} [\{\phi|^\mu{}_\mu, \phi^*\} - \{\phi|^\mu{}_\mu, \phi^*\}^*]. \quad (250)$$

Making the transition from classical to quantum fields and applying the point-splitting procedure to the right-hand-side of Eq.(250) yields

$$\begin{aligned} \langle \underline{j}^\mu{}_{;\mu}(x) \rangle &= \lim_{x' \rightarrow x} \frac{ie}{4} \left[ \left( G^{(1)|\mu}{}_{\mu} + g^\mu{}_{\tau'} g_{\mu\rho'} G^{(1)|\tau'\rho'} \right) - \right. \\ &\quad \left. \left( G^{(1)|\mu}{}_{\mu} + g^\mu{}_{\tau'} g_{\mu\rho'} G^{(1)|\tau'\rho'} \right)^* \right]. \end{aligned} \quad (251)$$

This, too, may be seen to vanish to all orders by using the expansions in Eqs.(215)–(237).

The point-splitting procedure can not yield a DS approximation for  $\langle j^\mu \rangle$ . This may be seen by examining the terms proportional to negative powers of  $m$  in Eq.(212);

$$\langle j^\mu \rangle_{DS,ren} \approx \frac{ie}{4} \left\{ \frac{1}{2m^2} \left[ \Delta^{1/2}(a_2^{|\mu} + g^\mu{}_{\tau'} a_2^{|\tau'}) + a_2(\Delta^{1/2;\mu} + g^\mu{}_{\tau'} \Delta^{1/2;\tau'}) \right] + \frac{1}{2m^4} \left[ \Delta^{1/2}(a_3^{|\mu} + g^\mu{}_{\tau'} a_3^{|\tau'}) + a_3(\Delta^{1/2;\mu} + g^\mu{}_{\tau'} \Delta^{1/2;\tau'}) \right] + \dots \right\}. \quad (252)$$

Since  $\langle j^\mu \rangle$  is explicitly proportional to  $\frac{ie}{4}$ , the only way for Eq.(212) to yield a DS approximation is for complex terms to be present inside the curly brackets of Eq.(252). This does not happen for several reasons. First, any complex terms in the bitensor expansion of Eq.(135) must arise due to the commutation relation of the gauge covariant derivative. The purely geometric quantities  $\Delta^{1/2;\mu}$  and  $\Delta^{1/2;\mu'}$  will not require gauge covariant derivatives and thus will have power series expansions that are purely real. This is seen in Eqs.(220)–(221). Second, the zeroth order terms in the power series expansions for  $a_2$  and  $a_3$  are the coincidence limits  $[a_2]$  and  $[a_3]$ , as indicated by Eq.(198). Eqs.(196)–(197) show that these limits are real. Finally, applying Christensen's generalization of Synge's Theorem to the zeroth order power series expansions for the combination  $a_n^{|\mu} + g^\mu{}_{\tau'} a_n^{|\tau'}$  gives

$$\begin{aligned} a_n^{|\mu} + g^\mu{}_{\tau'} a_n^{|\tau'} &= [a_n^{|\mu}] + (-[a_n^{|\mu}] + [a_n]^{|\mu}) + \vartheta(\sigma^\mu) \\ &= [a_n]^{|\mu} + \vartheta(\sigma^\mu). \end{aligned} \quad (253)$$

Since the coincidence limits of the  $a_n$  are real, this, too, is real. Thus, the real part of the right hand side of Eq.(252) is zero, and point-splitting can not yield a DS approximation for the current associated with a charged scalar field.

Now consider the stress-energy tensor. The second derivatives  $G^{(1)|\mu\nu}$ ,  $G^{(1)|\mu'\nu}$ ,  $G^{(1)|\mu\nu'}$ , and  $G^{(1)|\mu'\nu'}$  may be formed by differentiating Eq.(133). These are then used to form

the components of the stress-energy tensor of Eq.(82). Substituting the expansions from Eqs.(215)–(237), and collecting like powers of  $\sigma^\mu$ , yields the following expressions, with the points split,

$$\langle T^{\mu\nu} \rangle_{quartic} = \frac{1}{2\pi^2} \frac{1}{(\sigma^\rho \sigma_\rho)^2} \left[ g^{\mu\nu} - 4 \frac{\sigma^\mu \sigma^\nu}{(\sigma^\rho \sigma_\rho)} \right], \quad (254)$$

$$\begin{aligned} \langle T^{\mu\nu} \rangle_{quadratic} = & \frac{1}{4\pi^2} \frac{1}{(\sigma^\rho \sigma_\rho)} \times \left[ \frac{1}{3} (R^\mu{}_\alpha \sigma^\nu + R^\nu{}_\alpha \sigma^\mu) \frac{\sigma^\alpha}{(\sigma^\rho \sigma_\rho)} \right. \\ & - \left( \frac{1}{6} - \xi \right) \left\{ R^{\mu\nu} - \frac{1}{2} R \left[ g^{\mu\nu} - 2 \frac{\sigma^\mu \sigma^\nu}{(\sigma^\rho \sigma_\rho)} \right] \right. \\ & \quad \left. \left. + 2 (g^{\mu\nu} R_{\alpha\beta} - R^\mu{}_\alpha{}^\nu{}_\beta) \frac{\sigma^\alpha \sigma^\beta}{(\sigma^\rho \sigma_\rho)} \right\} \right. \\ & \left. - \frac{2}{3} R_{\alpha\beta} \frac{\sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu}{(\sigma^\rho \sigma_\rho)^2} - \frac{1}{2} m^2 \left[ g^{\mu\nu} - 2 \frac{\sigma^\mu \sigma^\nu}{(\sigma^\rho \sigma_\rho)} \right] \right], \quad (255) \end{aligned}$$

$$\begin{aligned} \langle T^{\mu\nu} \rangle_{linear} = & \frac{1}{4\pi^2} \left[ -\frac{1}{6} (R^\mu{}_\alpha g^\nu{}_\beta + R^\nu{}_\alpha g^\mu{}_\beta - \frac{1}{4} R_{\alpha\beta} g^{\mu\nu})_{;\gamma} \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma}{(\sigma^\rho \sigma_\rho)^2} \right. \\ & \frac{1}{12} (R^{\mu\nu} - \frac{1}{4} R g^{\mu\nu})_{;\alpha} \frac{\sigma^\alpha}{(\sigma^\rho \sigma_\rho)} \\ & - \left( \frac{1}{6} - \xi \right) \left\{ (R^\mu{}_\alpha{}^\nu{}_\beta - R_{\alpha\beta} g^{\mu\nu})_{;\gamma} \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma}{(\sigma^\rho \sigma_\rho)^2} \right. \\ & \left. + \frac{1}{4} R_{;\alpha} \frac{\sigma^\alpha}{(\sigma^\rho \sigma_\rho)} \left[ g^{\mu\nu} - 2 \frac{\sigma^\mu \sigma^\nu}{(\sigma^\rho \sigma_\rho)} \right] \right\} + \left( \frac{1}{6} - \xi \right)^2 \left[ \frac{3}{4} R_{;\alpha} \frac{\sigma^\alpha}{(\sigma^\rho \sigma_\rho)} g^{\mu\nu} \right] \\ & \left. - \frac{1}{12} R_{\alpha\beta;\gamma} \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma}{(\sigma^\rho \sigma_\rho)^2} \left[ g^{\mu\nu} - 4 \frac{\sigma^\mu \sigma^\nu}{(\sigma^\rho \sigma_\rho)} \right] \right], \quad (256) \end{aligned}$$

$$\begin{aligned} \langle T^{\mu\nu} \rangle_{logarithmic} = & \frac{1}{4\pi^2} \left[ \gamma + \frac{1}{2} \ln \left| \frac{1}{4} m^2 (\sigma^\rho \sigma_\rho) \right| \right] \times \\ & \left[ \frac{1}{60} (R^{\rho\mu\tau\nu} R_{\rho\tau} - \frac{1}{4} R^{\rho\tau} R_{\rho\tau} g^{\mu\nu}) - \frac{1}{2} \left( \frac{1}{6} - \xi \right) \left[ m^2 (R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}) \right] \right. \\ & + \frac{1}{120} R^{\mu\nu}{}_{;\rho}{}^\rho - \frac{1}{180} R (R^{\mu\nu} - \frac{1}{4} R g^{\mu\nu}) - \frac{1}{360} R^{;\mu\nu} - \frac{1}{720} R_{;\rho}{}^\rho g^{\mu\nu} \\ & - \frac{1}{2} \left( \frac{1}{6} - \xi \right)^2 \left[ R^{;\mu\nu} - R (R^{\mu\nu} - \frac{1}{4} R g^{\mu\nu}) - R_{;\rho}{}^\rho g^{\mu\nu} \right] \\ & \left. - \frac{1}{8} m^4 g^{\mu\nu} - \frac{e^2}{12} F^{\rho\mu} F_\rho{}^\nu + \frac{e^2}{48} g^{\mu\nu} F^{\rho\tau} F_{\rho\tau} \right], \quad (257) \end{aligned}$$

and

$$\begin{aligned}
\langle T^{\mu\nu} \rangle_{finite} = & \langle T^{\mu\nu} \rangle_{finite} + \frac{e^2}{4\pi^2} \left[ \frac{\sigma^\mu \sigma^\nu}{12(\sigma^\rho \sigma_\rho)^2} \sigma^\alpha \sigma^\beta F_\alpha^\gamma F_{\beta\gamma} \right. \\
& + \frac{1}{12(\sigma^\rho \sigma_\rho)} (\sigma^\alpha \sigma^\mu F_\alpha^\beta F_{\beta\nu} + \sigma^\alpha \sigma^\nu F_\alpha^\beta F_{\beta\mu}) \\
& + \frac{1}{(\sigma^\rho \sigma_\rho)} \left( \xi - \frac{1}{4} \right) \sigma^\alpha \sigma^\beta F_\alpha^\mu F_{\beta\nu} + \frac{1}{(\sigma^\rho \sigma_\rho)} \left( \xi - \frac{5}{24} \right) g^{\mu\nu} \sigma^\alpha \sigma^\beta F_\alpha^\gamma F_{\beta\gamma} \\
& \left. - \frac{1}{96} \left( g^{\mu\nu} - 2 \frac{\sigma^\mu \sigma^\nu}{(\sigma^\rho \sigma_\rho)} \right) F^{\alpha\beta} F_{\alpha\beta} + \vartheta \left( \frac{1}{m^2} \right) \right]. \quad (258)
\end{aligned}$$

For brevity, only the terms due to the electromagnetic field are included in Eq.(258). The terms  $\langle T^{\mu\nu} \rangle_{finite}$  have been previously calculated by Christensen and may be found in Ref.[30]. Also,  $\langle T^{\mu\nu} \rangle_{finite}$  is listed in a less compact form than that used by Christensen in Appendix F. Comparing these two expressions for  $\langle T^{\mu\nu} \rangle_{finite}$  shows how much work is involved in simplifying tremendously long tensor expressions. However, both expressions are entirely equivalent in the physical information they contain. Eqs.(254)–(258) are constructed from the coincidence limits of  $a_0, a_1$  and  $a_2$  and their derivatives and are sufficient to completely regularize  $\langle T^{\mu\nu} \rangle$  [8].

The structure of the electromagnetic contribution to the regularization terms for the stress-energy tensor may be deduced in the manner as for  $G^{(1)}$  and  $\langle j^\mu \rangle$ . They must be independent of the sign of the charge and thus proportional to  $e^2$ . There are no explicit factors of  $e$  in Eq.(82), so they must arise in the derivations of the expansions of  $G(1)^{\mu\nu}$ , etc., by virtue of the gauge commutator. The simplest term expected is the fourth order  $(eF)^2$ , which has units  $(length)^{-4}$  when  $c = \hbar = 1$  but  $G \neq 1$ . The fourth order stress-energy tensor would only allow such a term to be present in the logarithmic regularization term. This is in agreement with DeWitt's result for the generalized Yang-Mills field [4, 38]. The two possible fourth order combinations of the gravitational and electromagnetic field tensors are  $g^{\mu\nu} e^2 F^{\rho\tau} F_{\rho\tau}$  and  $e^2 F^{\rho\mu} F_\rho^\nu$ . The point-splitting procedure yields the numerical factors for these terms.

The point-splitting procedure yields a DS approximation for the stress-energy tensor and is given symbolically by

$$4\pi^2 \langle T^{\mu\nu} \rangle \approx \frac{1}{4m^2} \left[ \left( \xi - \frac{1}{2} \right) \left( g^{\mu\nu} g_{\alpha\beta'} a_2^{|\alpha\beta'} - g^\nu_{\beta'} a_2^{|\mu\beta'} - g^\mu_{\alpha'} a_2^{|\alpha'\nu} \right) \right. \\ \left. - \xi \left( a_2^{|\mu\nu} + g^\mu_{\alpha'} g^\nu_{\beta'} a_2^{|\alpha'\beta'} \right) + \xi g^{\mu\nu} \left( a_2|_{\alpha}{}^{\alpha} + g^\rho_{\alpha'} g_{\rho\beta'} a_2^{\alpha'\beta'} \right) \right. \\ \left. a_2 \left( \xi - \frac{1}{8} \right) \left( R^{\mu\nu} - \frac{3}{2} g^{\mu\nu} R \right) \right] + \vartheta \left( \frac{1}{m^4} \right), \quad (259)$$

where terms such as  $\Delta^{1/2;\mu}$  and  $\Delta^{1/2;\mu\nu}$  have been substituted for using Eqs.(215)–(237), and where Newton's constant  $G = 1$  once more. Many terms that could possibly arise from differentiation of  $G^{(1)}$  have power series expansions that are of order  $\sigma^\mu$  or greater, so they contribute nothing to  $\langle T^{\mu\nu} \rangle$ . Eq.(259) has been written to show how  $\langle T^{\mu\nu} \rangle$  will depend on the coincidence limits of  $a_2$  and its derivatives. Consider the power series expansion for the bitensors

$$a^{\mu\nu\dots}(x, x') = a0^{\mu\nu\dots}(x) + a1^{\mu\nu\dots}{}_{\alpha}(x) \sigma^\alpha + \frac{1}{2!} a2^{\mu\nu\dots}{}_{\alpha\beta}(x) \sigma^\alpha \sigma^\beta + \dots \quad (260)$$

There are no negative powers of  $\sigma^\mu$  Eq.(259), so only the first term in the series of Eq.(260),  $a0^{\mu\nu\dots}(x)$ , is needed. This term is given by the coincidence limit of the bitensor of interest. For example, the expansion for  $a_2^{|\mu\nu}$  contains only one term and is given by

$$a_2^{|\mu\nu} = [a_2^{|\mu\nu}]. \quad (261)$$

The expansion for  $g^\mu_{\alpha'} g^\nu_{\beta'} a_2^{|\alpha'\beta'}$  is more complicated but also is built from one coincidence limit involving primed derivatives;

$$g^\mu_{\alpha'} g^\nu_{\beta'} a_2^{|\alpha'\beta'} = [g^\mu_{\alpha'} g^\nu_{\beta'} a_2^{|\alpha'\beta'}] \\ = -[a_2^{|\nu\mu'}] + [a_2^{|\mu'}]{}^\nu \\ = [a_2^{|\nu\mu}] - [a_2^{|\nu}]{}^{\mu} - [a_2^{|\mu}]{}^\nu + [a_2]{}^{;\mu\nu}. \quad (262)$$

As discussed previously, both of these expressions must be symmetrized in their indices according to Eq.(238) before substitution into Eq.(259) for the stress-energy

tensor. Finally, using Eqs.(260), (198), and (203) to construct the zeroth order terms in the expansions of  $a_2$  and its derivatives yields the DS approximation to the stress-energy tensor  $\langle T^{\mu\nu} \rangle_{DS,ren}$ ,

$$4\pi^2 \langle T^{\mu\nu} \rangle_{DS,ren} \approx 4\pi^2 \langle T^{\mu\nu} \rangle_{DS,EM} + 4\pi^2 \langle T^{\mu\nu} \rangle_{DS,purely\ geometric}. \quad (263)$$

In Eq.(263),  $4\pi^2 \langle T^{\mu\nu} \rangle_{DS,EM}$  are the terms which depend on the electromagnetic field and are given by

$$\begin{aligned}
4\pi^2 \langle T^{\mu\nu} \rangle_{DS,EM} = & \\
& \frac{e^2 F_{\alpha}{}^{\mu;\alpha} F_{\beta}{}^{\nu;\beta}}{144m^2} - \frac{e^2 F_{\alpha\beta}{}^{;\mu} F^{\alpha\beta;\nu}}{120m^2} + \frac{e^2 \xi F_{\alpha\beta}{}^{;\mu} F^{\alpha\beta;\nu}}{24m^2} - \frac{e^2 F_{\alpha\beta}{}^{;\nu} F^{\alpha\mu;\beta}}{720m^2} - \\
& \frac{e^2 F_{\alpha\beta}{}^{;\mu} F^{\alpha\nu;\beta}}{720m^2} + \frac{e^2 F_{\alpha}{}^{\mu}{}_{;\beta} F^{\alpha\nu;\beta}}{180m^2} + \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F^{\beta\mu;\nu}}{720m^2} + \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F^{\beta\nu;\mu}}{720m^2} - \\
& \frac{11e^2 F_{\alpha\beta}{}^{;\mu\nu} F^{\alpha\beta}}{2880m^2} + \frac{e^2 \xi F_{\alpha\beta}{}^{;\mu\nu} F^{\alpha\beta}}{48m^2} - \frac{11e^2 F_{\alpha\beta}{}^{;\nu\mu} F^{\alpha\beta}}{2880m^2} + \frac{e^2 \xi F_{\alpha\beta}{}^{;\nu\mu} F^{\alpha\beta}}{48m^2} - \\
& \frac{e^2 F_{\alpha}{}^{\mu}{}_{;\beta}{}^{\nu} F^{\alpha\beta}}{1440m^2} - \frac{e^2 F_{\alpha}{}^{\nu}{}_{;\beta}{}^{\mu} F^{\alpha\beta}}{1440m^2} + \frac{e^2 F_{\alpha}{}^{\nu}{}_{;\beta}{}^{\beta} F^{\alpha\mu}}{320m^2} + \frac{e^2 F_{\alpha}{}^{\mu}{}_{;\beta}{}^{\beta} F^{\alpha\nu}}{320m^2} + \\
& \frac{e^2 R F_{\alpha}{}^{\mu} F^{\alpha\nu}}{144m^2} - \frac{e^2 R \xi F_{\alpha}{}^{\mu} F^{\alpha\nu}}{24m^2} - \frac{e^2 F_{\alpha\beta}{}^{;\alpha\nu} F^{\beta\mu}}{960m^2} + \frac{e^2 F_{\alpha}{}^{\nu;\alpha} F^{\beta\mu}}{192m^2} - \\
& \frac{e^2 F_{\alpha\beta}{}^{;\alpha\mu} F^{\beta\nu}}{960m^2} + \frac{e^2 F_{\alpha}{}^{\mu;\alpha} F^{\beta\nu}}{192m^2} - \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F_{\gamma}{}^{\beta;\gamma} g^{\mu\nu}}{480m^2} + \frac{e^2 \xi F_{\alpha\beta}{}^{;\alpha} F_{\gamma}{}^{\beta;\gamma} g^{\mu\nu}}{240m^2} + \\
& \frac{e^2 F_{\alpha\beta;\gamma} F^{\alpha\beta;\gamma} g^{\mu\nu}}{720m^2} - \frac{17e^2 \xi F_{\alpha\beta;\gamma} F^{\alpha\beta;\gamma} g^{\mu\nu}}{720m^2} + \frac{e^2 F_{\alpha\beta;\gamma} F^{\alpha\gamma;\beta} g^{\mu\nu}}{720m^2} - \frac{e^2 \xi F_{\alpha\beta;\gamma} F^{\alpha\gamma;\beta} g^{\mu\nu}}{360m^2} + \\
& \frac{e^2 F_{\alpha\beta;\gamma}{}^{\gamma} F^{\alpha\beta} g^{\mu\nu}}{1440m^2} - \frac{e^2 \xi F_{\alpha\beta;\gamma}{}^{\gamma} F^{\alpha\beta} g^{\mu\nu}}{45m^2} - \frac{e^2 R F_{\alpha\beta} F^{\alpha\beta} g^{\mu\nu}}{192m^2} + \frac{e^2 R \xi F_{\alpha\beta} F^{\alpha\beta} g^{\mu\nu}}{24m^2} - \\
& \frac{e^2 R \xi^2 F_{\alpha\beta} F^{\alpha\beta} g^{\mu\nu}}{24m^2} - \frac{e^2 F_{\alpha\beta;\gamma}{}^{\alpha} F^{\beta\gamma} g^{\mu\nu}}{1440m^2} + \frac{e^2 \xi F_{\alpha\beta;\gamma}{}^{\alpha} F^{\beta\gamma} g^{\mu\nu}}{720m^2} + \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F_{\gamma}{}^{\beta\gamma} g^{\mu\nu}}{160m^2} - \\
& \frac{e^2 \xi F_{\alpha\beta}{}^{;\alpha} F_{\gamma}{}^{\beta\gamma} g^{\mu\nu}}{80m^2} + \frac{e^2 F_{\alpha}{}^{\mu} F_{\beta}{}^{\nu} R^{\alpha\beta}}{480m^2} + \frac{e^2 F_{\alpha\beta} F_{\gamma}{}^{\alpha} g^{\mu\nu} R^{\beta\gamma}}{480m^2} - \frac{e^2 \xi F_{\alpha\beta} F_{\gamma}{}^{\alpha} g^{\mu\nu} R^{\beta\gamma}}{240m^2} - \\
& \frac{e^2 F_{\alpha\beta} F^{\alpha\nu} R^{\beta\mu}}{2880m^2} - \frac{e^2 F_{\alpha\beta} F^{\alpha\mu} R^{\beta\nu}}{2880m^2} + \frac{e^2 F_{\alpha\beta} F^{\alpha\beta} R^{\mu\nu}}{288m^2} - \frac{e^2 \xi F_{\alpha\beta} F^{\alpha\beta} R^{\mu\nu}}{48m^2} - \\
& \frac{e^2 F_{\alpha\beta} F_{\gamma\delta} g^{\mu\nu} R^{\alpha\beta\gamma\delta}}{480m^2} + \frac{e^2 \xi F_{\alpha\beta} F_{\gamma\delta} g^{\mu\nu} R^{\alpha\beta\gamma\delta}}{240m^2} + \frac{e^2 F_{\alpha\beta} F_{\gamma}{}^{\nu} R^{\alpha\beta\gamma\mu}}{480m^2} + \frac{e^2 F_{\alpha\beta} F_{\gamma}{}^{\mu} R^{\alpha\beta\gamma\nu}}{480m^2} - \\
& \frac{e^2 F_{\alpha\beta} F_{\gamma\delta} g^{\mu\nu} R^{\alpha\gamma\beta\delta}}{480m^2} + \frac{e^2 \xi F_{\alpha\beta} F_{\gamma\delta} g^{\mu\nu} R^{\alpha\gamma\beta\delta}}{240m^2} + \frac{e^2 F_{\alpha\beta} F_{\gamma}{}^{\nu} R^{\alpha\gamma\beta\mu}}{480m^2} + \frac{e^2 F_{\alpha\beta} F_{\gamma}{}^{\mu} R^{\alpha\gamma\beta\nu}}{480m^2} - \\
& \frac{e^2 F_{\alpha\beta} F_{\gamma}{}^{\alpha} R^{\beta\mu\gamma\nu}}{360m^2}
\end{aligned} \quad (264)$$

The expression  $4\pi^2 \langle T^{\mu\nu} \rangle_{DS,purely\ geometric}$  represents the 361 purely geometric terms

of  $4\pi^2 \langle T^{\mu\nu} \rangle_{DS,ren}$  which are listed in Appendix G. Eq.(264) includes terms which are purely electromagnetic and terms which have both electromagnetic and geometric parts. The purely electromagnetic terms will not disappear in flat space and thus may be used for studies of quantum field effects in flat space. It should be emphasized here that this result is for a general four-dimensional spacetime with a general electromagnetic field possessing  $U(1)$  symmetry.

The DeWitt-Schwinger regularization counterterms of Eqs.(241), (245)–(246), and (254)–(258) are the first set of major results of this thesis. The DS approximations  $G_{DS,ren}^{(1)}$  (or, equivalently,  $\langle \phi^2 \rangle_{DS,ren}$ ) and  $\langle T^{\mu\nu} \rangle_{DS,ren}$  of Eqs.(242) and (264) constitute the second group of major results of this thesis.

## CHAPTER 7

## Concluding Remarks and Future Work

One of the two major results of this thesis was the calculation of the DeWitt-Schwinger regularization counterterms for the three VEVs  $\langle\phi^2\rangle$ ,  $\langle j^\mu\rangle$ , and  $\langle T_{\mu\nu}\rangle$  for a charged scalar field coupled to the electromagnetic field of general four-dimensional spacetimes. These results are found in Eqs.(241), (245)–(246), and (254)–(258). These terms are sufficient to completely regularize the expressions for  $\langle\phi^2\rangle$ ,  $\langle j^\mu\rangle$ , and  $\langle T_{\mu\nu}\rangle$ . The DeWitt-Schwinger approximations for  $G_{DS,ren}^{(1)}$  (or, equivalently,  $\langle\phi^2\rangle_{DS,ren}$ ) and  $\langle T^{\mu\nu}\rangle_{DS,ren}$  are found in Eqs.(242) and (264) and constitute the second group of major results of this thesis. The applications of these two sets of results are discussed briefly in this chapter.

The second set of major results have more immediate applications than the regularization counterterms. Presented here for the first time is an analytic expression for an approximation to  $\langle T^{\mu\nu}\rangle$  for a quantized, charged massive scalar field. This approximation was derived for a general four-dimensional spacetime with a general electromagnetic field possessing  $U(1)$  symmetry. This arose from carrying the DeWitt-Schwinger geodesic point-splitting expansions for the asymptotic series for  $\langle T^{\mu\nu}\rangle$  out to order  $m^{-2}$ . The completely general expression for  $\langle T^{\mu\nu}\rangle$  does not have to be restricted to a spacetime having any particular symmetries.

The analytic approximation for  $\langle T^{\mu\nu}\rangle$  of Frolov and Zel'nikov [26] was restricted to the case of Ricci-flat spacetimes. Brown, Ottewill, and Page [28] derived an analytic approximation for  $\langle T^{\mu\nu}\rangle$  for conformally invariant massless scalar, spinor, and vector fields on static Einstein spacetimes. In neither of these works was there an electromagnetic field present. This new expression is a generalized version of this

prior work, with potential applications too numerous to fully mention here. However, an immediate application of concern to this author is described next.

The first set of results of this thesis will serve to renormalize the dynamical equations that will allow a full quantum electrodynamic (QED) study of the evolution of realistic initial conditions in charged black hole spacetimes. The evolution of these initial conditions will help answer the question of what is true interior structure of charged black hole spacetimes. Consider the Reissner-Nordström spacetime, with a line element given by

$$ds^2 = - \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2(\theta)d\phi^2). \quad (265)$$

In this line element,  $M$  is the mass as determined by Keplerian orbits in the large  $r$ , or nearly Newtonian limit, and  $Q$  is the charge enclosed by a Gaussian sphere surrounding the black hole. The analytic extension of the Reissner-Nordström solution extends across two horizons located at the two radii  $r_{\pm}$  given by the solution to  $g_{tt} = 0$ :

$$r_{\pm} = M \pm M \sqrt{1 - \frac{Q^2}{M^2}}. \quad (266)$$

The Reissner-Nordström solution has a Penrose spacetime diagram shown in Figure 4. This diagram shows features in the Reissner-Nordström spacetime quite different from those in the Schwarzschild spacetime, including the presence of the two horizons, timelike singularities, and an apparent tunnel to other asymptotically flat spacetimes.

Surfaces of constant  $(t, \theta, \phi)$  in the Reissner-Nordström spacetime have the line element

$$ds^2 = \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2. \quad (267)$$

With  $r > r_+$  or  $r < r_-$  in Regions I and III, respectively,  $ds^2 > 0$  and  $r$  is a spatial coordinate. Between the horizons in Region II,  $r_- < r < r_+$ ,  $ds^2 < 0$ , and  $r$  is

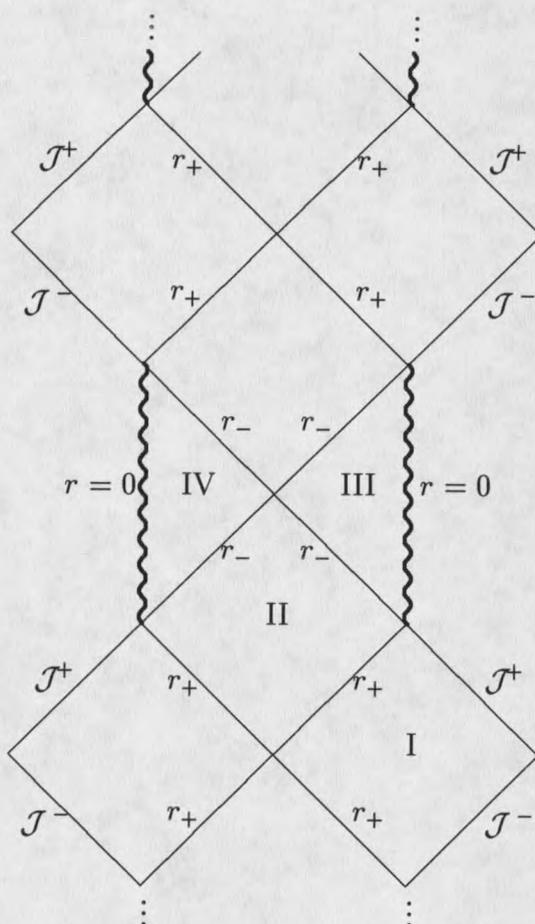


Figure 4: Penrose diagram for the Reissner-Nordström analytic extension

a temporal coordinate. In this region, dynamical evolution equations may assume a fortuitous form for two reasons. First, the future for all particles is towards decreasing  $r$ . Second, the spherical symmetry of the entire spacetime, and the fact that  $(\partial/\partial t)$  is a Killing vector, ensure that every point on each spacelike slice of constant  $r$  is equivalent with respect to the coordinates  $(t, \theta, \phi)$ . Thus, dynamical evolution equations in Region II assume the form of *ordinary* differential equations in  $r$  when these equations are restricted to the case of spherically symmetric solutions.

The inner horizons at  $r_-$  are Cauchy horizons for Region II. To the future of  $r_-$ , initial conditions defined on a spacetime slice may be propagated into the future.

However, the presence of the timelike singularities prevents the information propagated from these initial conditions into the future from completely determining the causal structure of the spacetime. For example, a spacelike slice may be chosen that connects Region IV to Region III, terminating on the two singularities. Initial conditions on this slice may be evolved some distance into the future, resulting in a new set of conditions determined on a new spacetime slice. However, without knowing the boundary conditions at, say, the right hand singularity, then anything could possibly come out of that singularity. This new information would make a contribution to the spacetime structure on the new spacetime slice, and this contribution would be distinct from the spacetime structure that evolved from the initial conditions.

The right hand Cauchy horizon in the Reissner-Nordström interior has been shown to be both classically [42, 43, 44, 45, 46, 47] and quantum mechanically unstable [48, 49, 50]. Uncharged fields have been defined on in Region I of the Reissner-Nordström spacetime and evolved into the future beyond the Cauchy horizon. As the fields evolve across  $r_-$ , the stress energy tensor diverges and thus the gravitational backreaction diverges. This divergence leads to the Cauchy horizon becoming a curvature singularity, via the so-called “mass inflation” process [51, 52]. It is, however, somewhat unclear whether this singularity is sufficiently strong to act as a true “edge” to the spacetime, enforcing strong cosmic censorship [53, 54].

The studies of Cauchy horizon instability have been performed in great detail, yet they have not dealt with the underlying electromagnetic field which is ultimately responsible for the formation of the exotic interior structures. Uncharged quantized fields may cause a gravitational backreaction, but they can not cause an electromagnetic backreaction since they do not possess their own electromagnetic field which will superpose on the electromagnetic field of the charged black hole spacetime. Only by evolving charged fields can the classical electromagnetic field of the Reissner-

Nordström black hole spacetime or general charged black hole spacetimes be changed.

The electric field of the Reissner-Nordström spacetime can serve as a source of charged pair creation and vacuum polarization. These effects will introduce a new, *dynamical* element to charged black hole spacetimes. The goal of this thesis has been to build the theoretical foundation for treating quantized charged scalar fields in a general curved spacetime containing an arbitrary classical electromagnetic field. The regularization counterterms for  $\langle j^\mu \rangle$  and  $\langle T_{\mu\nu} \rangle$  were derived in order to renormalize realistic source terms for the full semiclassical Einstein-Maxwell field equations,

$$G_{\mu\nu} = 8\pi\langle T_{\mu\nu} \rangle + 8\pi T_{\mu\nu}^{EM}, \quad (268)$$

and

$$F^{\mu\nu}{}_{;\nu} = 4\pi\langle j^\mu \rangle. \quad (269)$$

Eqs.(268)–(269) will be used by the author to study the causal structure of charged black hole spacetimes. These are the dynamical evolution equations which will allow initial conditions for realistic black holes to be set on some spacetime slice and evolved into the future. Using these equations, initial conditions may be evolved from an initial spacelike hypersurface into the future of the spacetime. This will be the foundation for determining the causal structure of the interiors of physically realistic charged black holes. Determining this causal structure will address the question of whether or not these black hole interiors contain exotic structures such as those in the Reissner-Nordström analytic extension.

As discussed above, the evolution equations for the region between the horizons of the Reissner-Nordström spacetime may assume the form of ordinary differential equations when restricting to the case of spherically symmetric solutions. These spherically symmetric solutions, along with the fact that this interior region is an anisotropic cosmology, will allow the method described by Anderson, Hiscock, and

Samuel for computing VEVs in static spherically symmetric spacetimes to be applied to dynamical studies of this region. Future work by the author will include applying their method to construct the current,  $\langle j^\mu \rangle$ , associated with the charged scalar field for these static spherically symmetric spacetimes. This current will serve as a source for the electromagnetic field in Eq.(269). Using the analytic expression for  $\langle T^{\mu\nu} \rangle$  as a source in Eq.(268) will provide a means to address the question of the true interior structure of charged black hole spacetimes.

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## APPENDIX A

Coincidence limits  $[\sigma_{;\mu\nu\rho\tau\kappa\lambda\iota}]$  and  $[\sigma_{;\mu\nu\rho\tau\kappa\lambda\iota\eta}]$

$$[\sigma; \mu\nu\rho\tau\kappa\lambda] =$$

$$\begin{aligned} & \frac{R_{\mu\rho\nu\tau;\kappa\lambda}}{6} - \frac{R_{\mu\rho\nu\kappa;\tau\lambda}}{6} - \frac{R_{\mu\rho\nu\lambda;\tau\kappa}}{6} - \frac{R_{\mu\rho\nu\iota;\tau\kappa\lambda}}{6} - \frac{R_{\mu\rho\tau\nu;\kappa\lambda}}{6} \\ & \frac{R_{\mu\tau\nu\kappa;\rho\lambda}}{6} - \frac{R_{\mu\tau\nu\lambda;\rho\kappa}}{6} - \frac{R_{\mu\tau\nu\iota;\rho\kappa\lambda}}{6} - \frac{R_{\mu\kappa\nu\rho;\tau\lambda}}{6} - \frac{R_{\mu\kappa\nu\tau;\rho\lambda}}{6} \\ & \frac{R_{\mu\kappa\nu\lambda;\rho\tau\iota}}{6} - \frac{R_{\mu\kappa\nu\iota;\rho\tau\lambda}}{6} - \frac{R_{\mu\lambda\nu\rho;\tau\kappa\iota}}{6} - \frac{R_{\mu\lambda\nu\tau;\rho\kappa\iota}}{6} - \frac{R_{\mu\lambda\nu\kappa;\rho\tau\iota}}{6} \\ & \frac{R_{\mu\lambda\nu\iota;\rho\tau\kappa}}{6} - \frac{R_{\mu\nu\rho;\tau\kappa\lambda}}{6} - \frac{R_{\mu\nu\tau;\rho\kappa\lambda}}{6} - \frac{R_{\mu\nu\kappa;\rho\tau\lambda}}{6} - \frac{R_{\mu\nu\lambda;\rho\tau\kappa}}{6} + \\ & \frac{R_{\alpha\tau\kappa\lambda;\iota}S_{\mu\nu\rho}^\alpha}{6} + \frac{R_{\alpha\tau\kappa\iota;\lambda}S_{\mu\nu\rho}^\alpha}{6} + \frac{R_{\alpha\rho\kappa\lambda;\iota}S_{\mu\nu}^\alpha\tau}{6} + \frac{R_{\alpha\rho\kappa\iota;\lambda}S_{\mu\nu}^\alpha\tau}{6} + \\ & \frac{R_{\alpha\rho\tau\lambda;\iota}S_{\mu\nu}^\alpha\kappa}{6} + \frac{R_{\alpha\rho\tau\iota;\lambda}S_{\mu\nu}^\alpha\kappa}{6} + \frac{R_{\alpha\rho\tau\kappa;\iota}S_{\mu\nu}^\alpha\lambda}{6} + \frac{R_{\alpha\rho\tau\iota;\kappa}S_{\mu\nu}^\alpha\lambda}{6} + \\ & \frac{R_{\alpha\rho\tau\kappa;\lambda}S_{\mu\nu}^\alpha\iota}{6} + \frac{R_{\alpha\rho\tau\lambda;\kappa}S_{\mu\nu}^\alpha\iota}{6} + \frac{R_{\alpha\nu\kappa\lambda;\iota}S_{\mu}^\alpha\rho\tau}{6} + \frac{R_{\alpha\nu\kappa\iota;\lambda}S_{\mu}^\alpha\rho\tau}{6} + \\ & \frac{R_{\alpha\nu\tau\lambda;\iota}S_{\mu}^\alpha\rho\kappa}{6} + \frac{R_{\alpha\nu\tau\iota;\lambda}S_{\mu}^\alpha\rho\kappa}{6} + \frac{R_{\alpha\nu\tau\kappa;\iota}S_{\mu}^\alpha\rho\lambda}{6} + \frac{R_{\alpha\nu\tau\iota;\kappa}S_{\mu}^\alpha\rho\lambda}{6} + \\ & \frac{R_{\alpha\nu\tau\kappa;\lambda}S_{\mu}^\alpha\rho\iota}{6} + \frac{R_{\alpha\nu\tau\lambda;\kappa}S_{\mu}^\alpha\rho\iota}{6} + \frac{R_{\alpha\nu\rho\lambda;\iota}S_{\mu}^\alpha\tau\kappa}{6} + \frac{R_{\alpha\nu\rho\iota;\lambda}S_{\mu}^\alpha\tau\kappa}{6} + \\ & \frac{R_{\alpha\nu\rho\kappa;\iota}S_{\mu}^\alpha\tau\lambda}{6} + \frac{R_{\alpha\nu\rho\iota;\kappa}S_{\mu}^\alpha\tau\lambda}{6} + \frac{R_{\alpha\nu\rho\kappa;\lambda}S_{\mu}^\alpha\tau\iota}{6} + \frac{R_{\alpha\nu\rho\lambda;\kappa}S_{\mu}^\alpha\tau\iota}{6} + \\ & \frac{R_{\alpha\nu\rho\tau;\iota}S_{\mu}^\alpha\kappa\lambda}{6} + \frac{R_{\alpha\nu\rho\iota;\tau}S_{\mu}^\alpha\kappa\lambda}{6} + \frac{R_{\alpha\nu\rho\tau;\lambda}S_{\mu}^\alpha\kappa\iota}{6} + \frac{R_{\alpha\nu\rho\lambda;\tau}S_{\mu}^\alpha\kappa\iota}{6} + \\ & 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\frac{R_{\alpha\mu\rho\kappa;\lambda}S_{\nu}^\alpha\tau\iota}{6} + \frac{R_{\alpha\mu\rho\lambda;\kappa}S_{\nu}^\alpha\tau\iota}{6} + \\ & \frac{R_{\alpha\mu\rho\tau;\iota}S_{\nu}^\alpha\kappa\lambda}{6} + \frac{R_{\alpha\mu\rho\iota;\tau}S_{\nu}^\alpha\kappa\lambda}{6} + \frac{R_{\alpha\mu\rho\tau;\lambda}S_{\nu}^\alpha\kappa\iota}{6} + \frac{R_{\alpha\mu\rho\lambda;\tau}S_{\nu}^\alpha\kappa\iota}{6} + \\ & \frac{R_{\alpha\mu\rho\tau;\kappa}S_{\nu}^\alpha\lambda\iota}{6} + \frac{R_{\alpha\mu\rho\kappa;\tau}S_{\nu}^\alpha\lambda\iota}{6} + \frac{R_{\alpha\mu\nu\lambda;\iota}S_{\rho}^\alpha\tau\kappa}{6} + \frac{R_{\alpha\mu\nu\iota;\lambda}S_{\rho}^\alpha\tau\kappa}{6} + \\ & \frac{R_{\alpha\mu\nu\kappa;\iota}S_{\rho}^\alpha\tau\lambda}{6} + \frac{R_{\alpha\mu\nu\iota;\kappa}S_{\rho}^\alpha\tau\lambda}{6} + \frac{R_{\alpha\mu\nu\kappa;\lambda}S_{\rho}^\alpha\tau\iota}{6} + \frac{R_{\alpha\mu\nu\lambda;\kappa}S_{\rho}^\alpha\tau\iota}{6} + \\ & \frac{R_{\alpha\mu\nu\tau;\iota}S_{\rho}^\alpha\kappa\lambda}{6} + \frac{R_{\alpha\mu\nu\iota;\tau}S_{\rho}^\alpha\kappa\lambda}{6} + \frac{R_{\alpha\mu\nu\tau;\lambda}S_{\rho}^\alpha\kappa\iota}{6} + \frac{R_{\alpha\mu\nu\lambda;\tau}S_{\rho}^\alpha\kappa\iota}{6} + \\ & \frac{R_{\alpha\mu\nu\tau;\kappa}S_{\rho}^\alpha\lambda\iota}{6} + \frac{R_{\alpha\mu\nu\kappa;\tau}S_{\rho}^\alpha\lambda\iota}{6} + \frac{R_{\alpha\mu\nu\rho;\iota}S_{\tau}^\alpha\kappa\lambda}{6} + \frac{R_{\alpha\mu\nu\iota;\rho}S_{\tau}^\alpha\kappa\lambda}{6} + \end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\mu\nu\rho;\lambda}S_{\tau}^{\alpha\kappa\iota}}{6} + \frac{R_{\alpha\mu\nu\lambda;\rho}S_{\tau}^{\alpha\kappa\iota}}{6} + \frac{R_{\alpha\mu\nu\rho;\kappa}S_{\tau}^{\alpha\lambda\iota}}{6} + \frac{R_{\alpha\mu\nu\kappa;\rho}S_{\tau}^{\alpha\lambda\iota}}{6} + \\
& \frac{R_{\alpha\mu\nu\rho;\tau}S_{\kappa}^{\alpha\lambda\iota}}{6} + \frac{R_{\alpha\mu\nu\tau;\rho}S_{\kappa}^{\alpha\lambda\iota}}{6} - \frac{S_{\alpha\kappa\lambda\iota}S_{5\mu}^{\alpha\nu\rho\tau}}{6} - \frac{S_{\alpha\tau\lambda\iota}S_{5\mu}^{\alpha\nu\rho\kappa}}{6} - \\
& \frac{S_{\alpha\tau\kappa\iota}S_{5\mu}^{\alpha\nu\rho\lambda}}{6} - \frac{S_{\alpha\tau\kappa\lambda}S_{5\mu}^{\alpha\nu\rho\iota}}{6} - \frac{S_{\alpha\rho\lambda\iota}S_{5\mu}^{\alpha\nu\tau\kappa}}{6} - \frac{S_{\alpha\rho\kappa\iota}S_{5\mu}^{\alpha\nu\tau\lambda}}{6} - \\
& \frac{S_{\alpha\rho\kappa\lambda}S_{5\mu}^{\alpha\nu\tau\iota}}{6} - \frac{S_{\alpha\rho\tau\iota}S_{5\mu}^{\alpha\nu\kappa\lambda}}{6} - \frac{S_{\alpha\rho\tau\lambda}S_{5\mu}^{\alpha\nu\kappa\iota}}{6} - \frac{S_{\alpha\rho\tau\kappa}S_{5\mu}^{\alpha\nu\lambda\iota}}{6} - \\
& \frac{S_{\alpha\nu\lambda\iota}S_{5\mu}^{\alpha\rho\tau\kappa}}{6} - \frac{S_{\alpha\nu\kappa\iota}S_{5\mu}^{\alpha\rho\tau\lambda}}{6} - \frac{S_{\alpha\nu\kappa\lambda}S_{5\mu}^{\alpha\rho\tau\iota}}{6} - \frac{S_{\alpha\nu\tau\iota}S_{5\mu}^{\alpha\rho\kappa\lambda}}{6} - \\
& \frac{S_{\alpha\nu\tau\lambda}S_{5\mu}^{\alpha\rho\kappa\iota}}{6} - \frac{S_{\alpha\nu\tau\kappa}S_{5\mu}^{\alpha\rho\lambda\iota}}{6} - \frac{S_{\alpha\nu\rho\iota}S_{5\mu}^{\alpha\tau\kappa\lambda}}{6} - \frac{S_{\alpha\nu\rho\lambda}S_{5\mu}^{\alpha\tau\kappa\iota}}{6} - \\
& \frac{S_{\alpha\nu\rho\kappa}S_{5\mu}^{\alpha\tau\lambda\iota}}{6} - \frac{S_{\alpha\nu\rho\tau}S_{5\mu}^{\alpha\kappa\lambda\iota}}{6} - \frac{S_{\alpha\mu\lambda\iota}S_{5\nu}^{\alpha\rho\tau\kappa}}{6} - \frac{S_{\alpha\mu\kappa\iota}S_{5\nu}^{\alpha\rho\tau\lambda}}{6} - \\
& \frac{S_{\alpha\mu\kappa\lambda}S_{5\nu}^{\alpha\rho\tau\iota}}{6} - \frac{S_{\alpha\mu\tau\iota}S_{5\nu}^{\alpha\rho\kappa\lambda}}{6} - \frac{S_{\alpha\mu\tau\lambda}S_{5\nu}^{\alpha\rho\kappa\iota}}{6} - \frac{S_{\alpha\mu\tau\kappa}S_{5\nu}^{\alpha\rho\lambda\iota}}{6} - \\
& \frac{S_{\alpha\mu\rho\iota}S_{5\nu}^{\alpha\tau\kappa\lambda}}{6} - \frac{S_{\alpha\mu\rho\lambda}S_{5\nu}^{\alpha\tau\kappa\iota}}{6} - \frac{S_{\alpha\mu\rho\kappa}S_{5\nu}^{\alpha\tau\lambda\iota}}{6} - \frac{S_{\alpha\mu\rho\tau}S_{5\nu}^{\alpha\kappa\lambda\iota}}{6} - \\
& \frac{S_{\alpha\mu\nu\iota}S_{5\rho}^{\alpha\tau\kappa\lambda}}{6} - \frac{S_{\alpha\mu\nu\lambda}S_{5\rho}^{\alpha\tau\kappa\iota}}{6} - \frac{S_{\alpha\mu\nu\kappa}S_{5\rho}^{\alpha\tau\lambda\iota}}{6} - \frac{S_{\alpha\mu\nu\tau}S_{5\rho}^{\alpha\kappa\lambda\iota}}{6} - \\
& \frac{S_{\alpha\mu\nu\rho}S_{5\tau}^{\alpha\kappa\lambda\iota}}{6} - \frac{S_{5\alpha\tau\kappa\lambda\iota}R_{\mu}^{\alpha\nu\rho}}{6} - \frac{S_{5\alpha\rho\kappa\lambda\iota}R_{\mu}^{\alpha\nu\tau}}{6} - \frac{S_{5\alpha\rho\tau\lambda\iota}R_{\mu}^{\alpha\nu\kappa}}{6} - \\
& \frac{S_{5\alpha\rho\tau\kappa\lambda}R_{\mu}^{\alpha\nu\lambda}}{6} - \frac{S_{5\alpha\rho\tau\kappa\lambda}R_{\mu}^{\alpha\nu\iota}}{6} - \frac{S_{5\alpha\nu\kappa\lambda\iota}R_{\mu}^{\alpha\rho\tau}}{6} - \frac{S_{5\alpha\nu\tau\lambda\iota}R_{\mu}^{\alpha\rho\kappa}}{6} - \\
& \frac{S_{5\alpha\nu\tau\kappa\lambda}R_{\mu}^{\alpha\rho\lambda}}{6} - \frac{S_{5\alpha\nu\tau\kappa\lambda}R_{\mu}^{\alpha\rho\iota}}{6} - \frac{S_{5\alpha\nu\rho\lambda\iota}R_{\mu}^{\alpha\tau\kappa}}{6} - \frac{S_{5\alpha\nu\rho\kappa\iota}R_{\mu}^{\alpha\tau\lambda}}{6} - \\
& \frac{S_{5\alpha\nu\rho\kappa\lambda}R_{\mu}^{\alpha\tau\iota}}{6} - \frac{S_{5\alpha\nu\rho\tau\iota}R_{\mu}^{\alpha\kappa\lambda}}{6} - \frac{S_{5\alpha\nu\rho\tau\lambda}R_{\mu}^{\alpha\kappa\iota}}{6} - \frac{S_{5\alpha\nu\rho\tau\kappa}R_{\mu}^{\alpha\lambda\iota}}{6} - \\
& \frac{S_{5\alpha\mu\kappa\lambda\iota}R_{\nu}^{\alpha\rho\tau}}{6} - \frac{S_{5\alpha\mu\tau\lambda\iota}R_{\nu}^{\alpha\rho\kappa}}{6} - \frac{S_{5\alpha\mu\tau\kappa\lambda}R_{\nu}^{\alpha\rho\lambda}}{6} - \frac{S_{5\alpha\mu\tau\kappa\lambda}R_{\nu}^{\alpha\rho\iota}}{6} - \\
& \frac{S_{5\alpha\mu\rho\lambda\iota}R_{\nu}^{\alpha\tau\kappa}}{6} - \frac{S_{5\alpha\mu\rho\kappa\lambda}R_{\nu}^{\alpha\tau\lambda}}{6} - \frac{S_{5\alpha\mu\rho\kappa\lambda}R_{\nu}^{\alpha\tau\iota}}{6} - \frac{S_{5\alpha\mu\rho\tau\iota}R_{\nu}^{\alpha\kappa\lambda}}{6} - \\
& \frac{S_{5\alpha\mu\rho\tau\lambda}R_{\nu}^{\alpha\kappa\iota}}{6} - \frac{S_{5\alpha\mu\rho\tau\kappa}R_{\nu}^{\alpha\lambda\iota}}{6} - \frac{S_{5\mu\nu\alpha\lambda\iota}R_{\rho}^{\alpha\tau\kappa}}{6} - \frac{S_{5\mu\nu\alpha\kappa\iota}R_{\rho}^{\alpha\tau\lambda}}{6} - \\
& \frac{S_{5\mu\nu\alpha\kappa\lambda}R_{\rho}^{\alpha\tau\iota}}{6} - \frac{S_{5\mu\nu\alpha\tau\iota}R_{\rho}^{\alpha\kappa\lambda}}{6} - \frac{S_{5\mu\nu\alpha\tau\lambda}R_{\rho}^{\alpha\kappa\iota}}{6} - \frac{S_{5\mu\nu\alpha\tau\kappa}R_{\rho}^{\alpha\lambda\iota}}{6} - \\
& \frac{S_{5\mu\nu\rho\alpha\lambda}R_{\tau}^{\alpha\kappa\lambda}}{6} - \frac{S_{5\mu\nu\rho\alpha\lambda}R_{\tau}^{\alpha\kappa\iota}}{6} - \frac{S_{5\mu\nu\rho\alpha\kappa}R_{\tau}^{\alpha\lambda\iota}}{6} - \frac{S_{5\mu\nu\rho\tau\alpha}R_{\kappa}^{\alpha\lambda\iota}}{6}
\end{aligned}$$

(270)

 $[\sigma; \mu\nu\rho\tau\kappa\lambda\iota\eta] =$ 

$$\frac{R_{\nu}^{\alpha\kappa\lambda;\iota\eta}S_{\alpha\mu\rho\tau}}{7} - \frac{R_{\nu}^{\alpha\tau\lambda;\iota\eta}S_{\alpha\mu\rho\kappa}}{7} - \frac{R_{\nu}^{\alpha\tau\kappa;\iota\eta}S_{\alpha\mu\rho\lambda}}{7} - \frac{R_{\nu}^{\alpha\tau\kappa;\lambda\eta}S_{\alpha\mu\rho\iota}}{7}$$

$$\begin{array}{cccc}
\frac{R_{\nu}^{\alpha} \tau \kappa; \lambda \iota S_{\alpha \mu \rho \eta}}{7} - \frac{R_{\nu}^{\alpha} \rho \lambda; \iota \eta S_{\alpha \mu \tau \kappa}}{7} - \frac{R_{\nu}^{\alpha} \rho \kappa; \iota \eta S_{\alpha \mu \tau \lambda}}{7} - \frac{R_{\nu}^{\alpha} \rho \kappa; \lambda \eta S_{\alpha \mu \tau \iota}}{7} \\
\frac{R_{\nu}^{\alpha} \rho \kappa; \lambda \iota S_{\alpha \mu \tau \eta}}{7} - \frac{R_{\nu}^{\alpha} \rho \tau; \iota \eta S_{\alpha \mu \kappa \lambda}}{7} - \frac{R_{\nu}^{\alpha} \rho \tau; \lambda \eta S_{\alpha \mu \kappa \iota}}{7} - \frac{R_{\nu}^{\alpha} \rho \tau; \lambda \iota S_{\alpha \mu \kappa \eta}}{7} \\
\frac{R_{\nu}^{\alpha} \rho \tau; \kappa \eta S_{\alpha \mu \lambda \iota}}{7} - \frac{R_{\nu}^{\alpha} \rho \tau; \kappa \iota S_{\alpha \mu \lambda \eta}}{7} - \frac{R_{\nu}^{\alpha} \rho \tau; \kappa \lambda S_{\alpha \mu \iota \eta}}{7} - \frac{R_{\nu}^{\alpha} \rho \lambda; \tau \kappa S_{\alpha \mu \iota \eta}}{7} \\
\frac{R_{\mu}^{\alpha} \kappa \lambda; \iota \eta S_{\alpha \nu \rho \tau}}{7} - \frac{R_{\mu}^{\alpha} \tau \lambda; \iota \eta S_{\alpha \nu \rho \kappa}}{7} - \frac{R_{\mu}^{\alpha} \tau \kappa; \iota \eta S_{\alpha \nu \rho \lambda}}{7} - \frac{R_{\mu}^{\alpha} \tau \kappa; \lambda \eta S_{\alpha \nu \rho \iota}}{7} \\
\frac{R_{\mu}^{\alpha} \tau \kappa; \lambda \iota S_{\alpha \nu \rho \eta}}{7} - \frac{R_{\mu}^{\alpha} \rho \lambda; \iota \eta S_{\alpha \nu \tau \kappa}}{7} - \frac{R_{\mu}^{\alpha} \rho \kappa; \iota \eta S_{\alpha \nu \tau \lambda}}{7} - \frac{R_{\mu}^{\alpha} \rho \kappa; \lambda \eta S_{\alpha \nu \tau \iota}}{7} \\
\frac{R_{\mu}^{\alpha} \rho \kappa; \lambda \iota S_{\alpha \nu \tau \eta}}{7} - \frac{R_{\mu}^{\alpha} \rho \tau; \iota \eta S_{\alpha \nu \kappa \lambda}}{7} - \frac{R_{\mu}^{\alpha} \rho \tau; \lambda \eta S_{\alpha \nu \kappa \iota}}{7} - \frac{R_{\mu}^{\alpha} \rho \tau; \lambda \iota S_{\alpha \nu \kappa \eta}}{7} \\
\frac{R_{\mu}^{\alpha} \rho \tau; \kappa \eta S_{\alpha \nu \lambda \iota}}{7} - \frac{R_{\mu}^{\alpha} \rho \tau; \kappa \iota S_{\alpha \nu \lambda \eta}}{7} - \frac{R_{\mu}^{\alpha} \rho \tau; \kappa \lambda S_{\alpha \nu \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \rho \lambda; \tau \kappa S_{\alpha \nu \iota \eta}}{7} \\
\frac{R_{\mu}^{\alpha} \nu \lambda; \iota \eta S_{\alpha \rho \tau \kappa}}{7} - \frac{R_{\mu}^{\alpha} \nu \kappa; \iota \eta S_{\alpha \rho \tau \lambda}}{7} - \frac{R_{\mu}^{\alpha} \nu \kappa; \lambda \eta S_{\alpha \rho \tau \iota}}{7} - \frac{R_{\mu}^{\alpha} \nu \kappa; \lambda \iota S_{\alpha \rho \tau \eta}}{7} \\
\frac{R_{\mu}^{\alpha} \nu \tau; \iota \eta S_{\alpha \rho \kappa \lambda}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \lambda \eta S_{\alpha \rho \kappa \iota}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \lambda \iota S_{\alpha \rho \kappa \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \kappa \eta S_{\alpha \rho \lambda \iota}}{7} \\
\frac{R_{\mu}^{\alpha} \nu \tau; \kappa \iota S_{\alpha \rho \lambda \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \kappa \lambda S_{\alpha \rho \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \lambda; \tau \kappa S_{\alpha \rho \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \rho; \iota \eta S_{\alpha \tau \kappa \lambda}}{7} \\
\frac{R_{\mu}^{\alpha} \nu \rho; \lambda \eta S_{\alpha \tau \kappa \iota}}{7} - \frac{R_{\mu}^{\alpha} \nu \rho; \lambda \iota S_{\alpha \tau \kappa \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \rho; \kappa \eta S_{\alpha \tau \lambda \iota}}{7} - \frac{R_{\mu}^{\alpha} \nu \rho; \kappa \iota S_{\alpha \tau \lambda \eta}}{7} \\
\frac{R_{\mu}^{\alpha} \nu \rho; \kappa \lambda S_{\alpha \tau \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \lambda; \rho \kappa S_{\alpha \tau \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \rho; \tau \eta S_{\alpha \kappa \lambda \iota}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \rho \eta S_{\alpha \kappa \lambda \iota}}{7} \\
\frac{R_{\mu}^{\alpha} \nu \rho; \tau \iota S_{\alpha \kappa \lambda \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \rho \iota S_{\alpha \kappa \lambda \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \rho; \tau \lambda S_{\alpha \kappa \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \rho \lambda S_{\alpha \kappa \iota \eta}}{7} \\
\frac{R_{\mu}^{\alpha} \nu \lambda; \rho \tau S_{\alpha \kappa \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \rho; \tau \kappa S_{\alpha \lambda \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \tau; \rho \kappa S_{\alpha \lambda \iota \eta}}{7} - \frac{R_{\mu}^{\alpha} \nu \kappa; \rho \tau S_{\alpha \lambda \iota \eta}}{7} \\
\frac{R_{\rho}^{\alpha} \kappa \lambda; \iota \eta S_{\mu \nu \alpha \tau}}{7} - \frac{R_{\rho}^{\alpha} \tau \lambda; \iota \eta S_{\mu \nu \alpha \kappa}}{7} - \frac{R_{\rho}^{\alpha} \tau \kappa; \iota \eta S_{\mu \nu \alpha \lambda}}{7} - \frac{R_{\rho}^{\alpha} \tau \kappa; \lambda \eta S_{\mu \nu \alpha \iota}}{7} \\
\frac{R_{\rho}^{\alpha} \tau \kappa; \lambda \iota S_{\mu \nu \alpha \eta}}{7} - \frac{R_{\tau}^{\alpha} \kappa \lambda; \iota \eta S_{\mu \nu \rho \alpha}}{7} + \frac{R_{\alpha \tau \kappa \iota; \lambda \eta S_{\mu \nu \rho}^{\alpha}}}{7} + \frac{R_{\alpha \tau \kappa \eta; \lambda \iota S_{\mu \nu \rho}^{\alpha}}}{7} + \\
\frac{R_{\alpha \rho \kappa \iota; \lambda \eta S_{\mu \nu}^{\alpha} \tau}}{7} + \frac{R_{\alpha \rho \kappa \eta; \lambda \iota S_{\mu \nu}^{\alpha} \tau}}{7} + \frac{R_{\alpha \rho \tau \iota; \lambda \eta S_{\mu \nu}^{\alpha} \kappa}}{7} + \frac{R_{\alpha \rho \tau \eta; \lambda \iota S_{\mu \nu}^{\alpha} \kappa}}{7} + \\
\frac{R_{\alpha \rho \tau \iota; \kappa \eta S_{\mu \nu}^{\alpha} \lambda}}{7} + \frac{R_{\alpha \rho \tau \eta; \kappa \iota S_{\mu \nu}^{\alpha} \lambda}}{7} + \frac{R_{\alpha \rho \tau \lambda; \kappa \eta S_{\mu \nu}^{\alpha} \iota}}{7} + \frac{R_{\alpha \rho \tau \eta; \kappa \lambda S_{\mu \nu}^{\alpha} \iota}}{7} + \\
\frac{R_{\alpha \rho \tau \lambda; \kappa \iota S_{\mu \nu}^{\alpha} \eta}}{7} + \frac{R_{\alpha \rho \tau \iota; \kappa \lambda S_{\mu \nu}^{\alpha} \eta}}{7} + \frac{R_{\alpha \nu \kappa \iota; \lambda \eta S_{\mu}^{\alpha} \rho \tau}}{7} + \frac{R_{\alpha \nu \kappa \eta; \lambda \iota S_{\mu}^{\alpha} \rho \tau}}{7} + \\
\frac{R_{\alpha \nu \tau \iota; \lambda \eta S_{\mu}^{\alpha} \rho \kappa}}{7} + \frac{R_{\alpha \nu \tau \eta; \lambda \iota S_{\mu}^{\alpha} \rho \kappa}}{7} + \frac{R_{\alpha \nu \tau \iota; \kappa \eta S_{\mu}^{\alpha} \rho \lambda}}{7} + \frac{R_{\alpha \nu \tau \eta; \kappa \iota S_{\mu}^{\alpha} \rho \lambda}}{7} + \\
\frac{R_{\alpha \nu \tau \lambda; \kappa \eta S_{\mu}^{\alpha} \rho \iota}}{7} + \frac{R_{\alpha \nu \tau \eta; \kappa \lambda S_{\mu}^{\alpha} \rho \iota}}{7} + \frac{R_{\alpha \nu \tau \lambda; \kappa \iota S_{\mu}^{\alpha} \rho \eta}}{7} + \frac{R_{\alpha \nu \tau \iota; \kappa \lambda S_{\mu}^{\alpha} \rho \eta}}{7} + \\
\frac{R_{\alpha \nu \rho \iota; \lambda \eta S_{\mu}^{\alpha} \tau \kappa}}{7} + \frac{R_{\alpha \nu \rho \eta; \lambda \iota S_{\mu}^{\alpha} \tau \kappa}}{7} + \frac{R_{\alpha \nu \rho \iota; \kappa \eta S_{\mu}^{\alpha} \tau \lambda}}{7} + \frac{R_{\alpha \nu \rho \eta; \kappa \iota S_{\mu}^{\alpha} \tau \lambda}}{7} +
\end{array}$$

$$\begin{aligned}
& \frac{R_{\alpha\nu\rho\lambda;\kappa\eta}S_{\mu}^{\alpha}\tau\iota}{7} + \frac{R_{\alpha\nu\rho\eta;\kappa\lambda}S_{\mu}^{\alpha}\tau\iota}{7} + \frac{R_{\alpha\nu\rho\lambda;\kappa\iota}S_{\mu}^{\alpha}\tau\eta}{7} + \frac{R_{\alpha\nu\rho\iota;\kappa\lambda}S_{\mu}^{\alpha}\tau\eta}{7} + \\
& \frac{R_{\alpha\nu\rho\iota;\tau\eta}S_{\mu}^{\alpha}\kappa\lambda}{7} + \frac{R_{\alpha\nu\rho\eta;\tau\iota}S_{\mu}^{\alpha}\kappa\lambda}{7} + \frac{R_{\alpha\nu\rho\lambda;\tau\eta}S_{\mu}^{\alpha}\kappa\iota}{7} + \frac{R_{\alpha\nu\rho\eta;\tau\lambda}S_{\mu}^{\alpha}\kappa\iota}{7} + \\
& \frac{R_{\alpha\nu\rho\lambda;\tau\iota}S_{\mu}^{\alpha}\kappa\eta}{7} + \frac{R_{\alpha\nu\rho\iota;\tau\lambda}S_{\mu}^{\alpha}\kappa\eta}{7} + \frac{R_{\alpha\nu\rho\kappa;\tau\eta}S_{\mu}^{\alpha}\lambda\iota}{7} + \frac{R_{\alpha\nu\rho\eta;\tau\kappa}S_{\mu}^{\alpha}\lambda\iota}{7} + \\
& \frac{R_{\alpha\nu\rho\kappa;\tau\iota}S_{\mu}^{\alpha}\lambda\eta}{7} + \frac{R_{\alpha\nu\rho\iota;\tau\kappa}S_{\mu}^{\alpha}\lambda\eta}{7} + \frac{R_{\alpha\nu\rho\kappa;\tau\lambda}S_{\mu}^{\alpha}\iota\eta}{7} + \frac{R_{\alpha\mu\kappa\iota;\lambda\eta}S_{\nu}^{\alpha}\rho\tau}{7} + \\
& \frac{R_{\alpha\mu\kappa\eta;\lambda\iota}S_{\nu}^{\alpha}\rho\tau}{7} + \frac{R_{\alpha\mu\tau\iota;\lambda\eta}S_{\nu}^{\alpha}\rho\kappa}{7} + \frac{R_{\alpha\mu\tau\eta;\lambda\iota}S_{\nu}^{\alpha}\rho\kappa}{7} + \frac{R_{\alpha\mu\tau\iota;\kappa\eta}S_{\nu}^{\alpha}\rho\lambda}{7} + \\
& \frac{R_{\alpha\mu\tau\eta;\kappa\iota}S_{\nu}^{\alpha}\rho\lambda}{7} + \frac{R_{\alpha\mu\tau\lambda;\kappa\eta}S_{\nu}^{\alpha}\rho\iota}{7} + \frac{R_{\alpha\mu\tau\eta;\kappa\lambda}S_{\nu}^{\alpha}\rho\iota}{7} + \frac{R_{\alpha\mu\tau\lambda;\kappa\iota}S_{\nu}^{\alpha}\rho\eta}{7} + \\
& \frac{R_{\alpha\mu\tau\iota;\kappa\lambda}S_{\nu}^{\alpha}\rho\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\lambda\eta}S_{\nu}^{\alpha}\tau\kappa}{7} + \frac{R_{\alpha\mu\rho\eta;\lambda\iota}S_{\nu}^{\alpha}\tau\kappa}{7} + \frac{R_{\alpha\mu\rho\iota;\kappa\eta}S_{\nu}^{\alpha}\tau\lambda}{7} + \\
& \frac{R_{\alpha\mu\rho\eta;\kappa\iota}S_{\nu}^{\alpha}\tau\lambda}{7} + \frac{R_{\alpha\mu\rho\lambda;\kappa\eta}S_{\nu}^{\alpha}\tau\iota}{7} + \frac{R_{\alpha\mu\rho\eta;\kappa\lambda}S_{\nu}^{\alpha}\tau\iota}{7} + \frac{R_{\alpha\mu\rho\lambda;\kappa\iota}S_{\nu}^{\alpha}\tau\eta}{7} + \\
& \frac{R_{\alpha\mu\rho\iota;\kappa\lambda}S_{\nu}^{\alpha}\tau\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\tau\eta}S_{\nu}^{\alpha}\kappa\lambda}{7} + \frac{R_{\alpha\mu\rho\eta;\tau\iota}S_{\nu}^{\alpha}\kappa\lambda}{7} + \frac{R_{\alpha\mu\rho\lambda;\tau\eta}S_{\nu}^{\alpha}\kappa\iota}{7} + \\
& \frac{R_{\alpha\mu\rho\eta;\tau\lambda}S_{\nu}^{\alpha}\kappa\iota}{7} + \frac{R_{\alpha\mu\rho\lambda;\tau\iota}S_{\nu}^{\alpha}\kappa\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\tau\lambda}S_{\nu}^{\alpha}\kappa\eta}{7} + \frac{R_{\alpha\mu\rho\kappa;\tau\eta}S_{\nu}^{\alpha}\lambda\iota}{7} + \\
& \frac{R_{\alpha\mu\rho\eta;\tau\kappa}S_{\nu}^{\alpha}\lambda\iota}{7} + \frac{R_{\alpha\mu\rho\kappa;\tau\iota}S_{\nu}^{\alpha}\lambda\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\tau\kappa}S_{\nu}^{\alpha}\lambda\eta}{7} + \frac{R_{\alpha\mu\rho\kappa;\tau\lambda}S_{\nu}^{\alpha}\iota\eta}{7} + \\
& \frac{R_{\alpha\mu\rho\iota;\lambda\eta}S_{\rho}^{\alpha}\tau\kappa}{7} + \frac{R_{\alpha\mu\rho\eta;\lambda\iota}S_{\rho}^{\alpha}\tau\kappa}{7} + \frac{R_{\alpha\mu\rho\iota;\kappa\eta}S_{\rho}^{\alpha}\tau\lambda}{7} + \frac{R_{\alpha\mu\rho\eta;\kappa\iota}S_{\rho}^{\alpha}\tau\lambda}{7} + \\
& \frac{R_{\alpha\mu\rho\lambda;\kappa\eta}S_{\rho}^{\alpha}\tau\iota}{7} + \frac{R_{\alpha\mu\rho\eta;\kappa\lambda}S_{\rho}^{\alpha}\tau\iota}{7} + \frac{R_{\alpha\mu\rho\lambda;\kappa\iota}S_{\rho}^{\alpha}\tau\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\kappa\lambda}S_{\rho}^{\alpha}\tau\eta}{7} + \\
& \frac{R_{\alpha\mu\rho\iota;\tau\eta}S_{\rho}^{\alpha}\kappa\lambda}{7} + \frac{R_{\alpha\mu\rho\eta;\tau\iota}S_{\rho}^{\alpha}\kappa\lambda}{7} + \frac{R_{\alpha\mu\rho\lambda;\tau\eta}S_{\rho}^{\alpha}\kappa\iota}{7} + \frac{R_{\alpha\mu\rho\eta;\tau\lambda}S_{\rho}^{\alpha}\kappa\iota}{7} + \\
& \frac{R_{\alpha\mu\rho\lambda;\tau\iota}S_{\rho}^{\alpha}\kappa\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\tau\lambda}S_{\rho}^{\alpha}\kappa\eta}{7} + \frac{R_{\alpha\mu\rho\kappa;\tau\eta}S_{\rho}^{\alpha}\lambda\iota}{7} + \frac{R_{\alpha\mu\rho\eta;\tau\kappa}S_{\rho}^{\alpha}\lambda\iota}{7} + \\
& \frac{R_{\alpha\mu\rho\kappa;\tau\iota}S_{\rho}^{\alpha}\lambda\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\tau\kappa}S_{\rho}^{\alpha}\lambda\eta}{7} + \frac{R_{\alpha\mu\rho\kappa;\tau\lambda}S_{\rho}^{\alpha}\iota\eta}{7} + \frac{R_{\alpha\mu\rho\iota;\rho\eta}S_{\tau}^{\alpha}\kappa\lambda}{7} + \\
& \frac{R_{\alpha\mu\rho\eta;\rho\iota}S_{\tau}^{\alpha}\kappa\lambda}{7} + \frac{R_{\alpha\mu\rho\lambda;\rho\eta}S_{\tau}^{\alpha}\kappa\iota}{7} + \frac{R_{\alpha\mu\rho\eta;\rho\lambda}S_{\tau}^{\alpha}\kappa\iota}{7} + \frac{R_{\alpha\mu\rho\lambda;\rho\iota}S_{\tau}^{\alpha}\kappa\eta}{7} + \\
& \frac{R_{\alpha\mu\rho\iota;\rho\lambda}S_{\tau}^{\alpha}\kappa\eta}{7} + \frac{R_{\alpha\mu\rho\kappa;\rho\eta}S_{\tau}^{\alpha}\lambda\iota}{7} + \frac{R_{\alpha\mu\rho\eta;\rho\kappa}S_{\tau}^{\alpha}\lambda\iota}{7} + \frac{R_{\alpha\mu\rho\kappa;\rho\iota}S_{\tau}^{\alpha}\lambda\eta}{7} + \\
& \frac{R_{\alpha\mu\rho\iota;\rho\kappa}S_{\tau}^{\alpha}\lambda\eta}{7} + \frac{R_{\alpha\mu\rho\kappa;\rho\lambda}S_{\tau}^{\alpha}\iota\eta}{7} + \frac{R_{\alpha\mu\rho\eta;\rho\tau}S_{\kappa}^{\alpha}\lambda\iota}{7} + \frac{R_{\alpha\mu\rho\iota;\rho\tau}S_{\kappa}^{\alpha}\lambda\eta}{7} - \\
& \frac{R_{\mu\rho\tau\iota;\kappa\lambda\iota\eta}}{7} - \frac{R_{\mu\rho\tau\kappa;\tau\lambda\iota\eta}}{7} - \frac{R_{\mu\rho\tau\lambda;\tau\kappa\iota\eta}}{7} - \frac{R_{\mu\rho\tau\iota;\tau\kappa\lambda\eta}}{7} - \\
& \frac{R_{\mu\rho\tau\eta;\tau\kappa\lambda\iota}}{7} - \frac{R_{\mu\tau\nu\rho;\kappa\lambda\iota\eta}}{7} - \frac{R_{\mu\tau\nu\kappa;\rho\lambda\iota\eta}}{7} - \frac{R_{\mu\tau\nu\lambda;\rho\kappa\iota\eta}}{7} - \\
& \frac{R_{\mu\tau\nu\iota;\rho\kappa\lambda\eta}}{7} - \frac{R_{\mu\tau\nu\eta;\rho\kappa\lambda\iota}}{7} - \frac{R_{\mu\kappa\nu\rho;\tau\lambda\iota\eta}}{7} - \frac{R_{\mu\kappa\nu\tau;\rho\lambda\iota\eta}}{7} -
\end{aligned}$$

|   |   |   |   |
|---|---|---|---|
| $\frac{R_{\mu\kappa\nu\lambda;\rho\tau\eta}}{7}$                      | $\frac{R_{\mu\kappa\nu\iota;\rho\tau\lambda\eta}}{7}$                 | $\frac{R_{\mu\kappa\nu\eta;\rho\tau\lambda\iota}}{7}$                 | $\frac{R_{\mu\lambda\nu\rho;\tau\kappa\iota\eta}}{7}$                 |
| $\frac{R_{\mu\lambda\nu\tau;\rho\kappa\iota\eta}}{7}$                 | $\frac{R_{\mu\lambda\nu\kappa;\rho\tau\iota\eta}}{7}$                 | $\frac{R_{\mu\lambda\nu\iota;\rho\tau\kappa\eta}}{7}$                 | $\frac{R_{\mu\lambda\nu\eta;\rho\tau\kappa\iota}}{7}$                 |
| $\frac{R_{\mu\iota\nu\rho;\tau\kappa\lambda\eta}}{7}$                 | $\frac{R_{\mu\iota\nu\tau;\rho\kappa\lambda\eta}}{7}$                 | $\frac{R_{\mu\iota\nu\kappa;\rho\tau\lambda\eta}}{7}$                 | $\frac{R_{\mu\iota\nu\lambda;\rho\tau\kappa\eta}}{7}$                 |
| $\frac{R_{\mu\iota\nu\eta;\rho\tau\kappa\lambda}}{7}$                 | $\frac{R_{\mu\eta\nu\rho;\tau\kappa\lambda\iota}}{7}$                 | $\frac{R_{\mu\eta\nu\tau;\rho\kappa\lambda\iota}}{7}$                 | $\frac{R_{\mu\eta\nu\kappa;\rho\tau\lambda\iota}}{7}$                 |
| $\frac{R_{\mu\eta\nu\lambda;\rho\tau\kappa\iota}}{7}$                 | $\frac{R_{\mu\eta\nu\iota;\rho\tau\kappa\lambda}}{7}$                 | $\frac{R_{\nu^\alpha\lambda\iota;\eta S5\alpha\mu\rho\tau\kappa}}{7}$ | $\frac{R_{\nu^\alpha\kappa\iota;\eta S5\alpha\mu\rho\tau\lambda}}{7}$ |
| $\frac{R_{\nu^\alpha\kappa\lambda;\eta S5\alpha\mu\rho\tau\iota}}{7}$ | $\frac{R_{\nu^\alpha\kappa\lambda;\iota S5\alpha\mu\rho\tau\eta}}{7}$ | $\frac{R_{\nu^\alpha\tau\iota;\eta S5\alpha\mu\rho\kappa\lambda}}{7}$ | $\frac{R_{\nu^\alpha\tau\lambda;\eta S5\alpha\mu\rho\kappa\iota}}{7}$ |
| $\frac{R_{\nu^\alpha\tau\lambda;\iota S5\alpha\mu\rho\kappa\eta}}{7}$ | $\frac{R_{\nu^\alpha\tau\kappa;\eta S5\alpha\mu\rho\lambda\iota}}{7}$ | $\frac{R_{\nu^\alpha\tau\kappa;\iota S5\alpha\mu\rho\lambda\eta}}{7}$ | $\frac{R_{\nu^\alpha\tau\kappa;\lambda S5\alpha\mu\rho\iota\eta}}{7}$ |
| $\frac{R_{\nu^\alpha\rho\iota;\eta S5\alpha\mu\tau\kappa\lambda}}{7}$ | $\frac{R_{\nu^\alpha\rho\lambda;\eta S5\alpha\mu\tau\kappa\iota}}{7}$ | $\frac{R_{\nu^\alpha\rho\lambda;\iota S5\alpha\mu\tau\kappa\eta}}{7}$ | $\frac{R_{\nu^\alpha\rho\kappa;\eta S5\alpha\mu\tau\lambda\iota}}{7}$ |
| $\frac{R_{\nu^\alpha\rho\kappa;\iota S5\alpha\mu\tau\lambda\eta}}{7}$ | $\frac{R_{\nu^\alpha\rho\kappa;\lambda S5\alpha\mu\tau\iota\eta}}{7}$ | $\frac{R_{\nu^\alpha\rho\tau;\eta S5\alpha\mu\kappa\lambda\iota}}{7}$ | $\frac{R_{\nu^\alpha\rho\tau;\iota S5\alpha\mu\kappa\lambda\eta}}{7}$ |
| $\frac{R_{\nu^\alpha\rho\tau;\iota S5\alpha\mu\kappa\iota\eta}}{7}$   | $\frac{R_{\nu^\alpha\rho\tau;\kappa S5\alpha\mu\lambda\iota\eta}}{7}$ | $\frac{R_{\mu^\alpha\lambda\iota;\eta S5\alpha\nu\rho\tau\kappa}}{7}$ | $\frac{R_{\mu^\alpha\kappa\iota;\eta S5\alpha\nu\rho\tau\lambda}}{7}$ |
| $\frac{R_{\mu^\alpha\kappa\lambda;\eta S5\alpha\nu\rho\tau\iota}}{7}$ | $\frac{R_{\mu^\alpha\kappa\lambda;\iota S5\alpha\nu\rho\tau\eta}}{7}$ | $\frac{R_{\mu^\alpha\tau\iota;\eta S5\alpha\nu\rho\kappa\lambda}}{7}$ | $\frac{R_{\mu^\alpha\tau\lambda;\eta S5\alpha\nu\rho\kappa\iota}}{7}$ |
| $\frac{R_{\mu^\alpha\tau\lambda;\iota S5\alpha\nu\rho\kappa\eta}}{7}$ | $\frac{R_{\mu^\alpha\tau\kappa;\eta S5\alpha\nu\rho\lambda\iota}}{7}$ | $\frac{R_{\mu^\alpha\tau\kappa;\iota S5\alpha\nu\rho\lambda\eta}}{7}$ | $\frac{R_{\mu^\alpha\tau\kappa;\lambda S5\alpha\nu\rho\iota\eta}}{7}$ |
| $\frac{R_{\mu^\alpha\rho\iota;\eta S5\alpha\nu\tau\kappa\lambda}}{7}$ | $\frac{R_{\mu^\alpha\rho\lambda;\eta S5\alpha\nu\tau\kappa\iota}}{7}$ | $\frac{R_{\mu^\alpha\rho\lambda;\iota S5\alpha\nu\tau\kappa\eta}}{7}$ | $\frac{R_{\mu^\alpha\rho\kappa;\eta S5\alpha\nu\tau\lambda\iota}}{7}$ |
| $\frac{R_{\mu^\alpha\rho\kappa;\iota S5\alpha\nu\tau\lambda\eta}}{7}$ | $\frac{R_{\mu^\alpha\rho\kappa;\lambda S5\alpha\nu\tau\iota\eta}}{7}$ | $\frac{R_{\mu^\alpha\rho\tau;\eta S5\alpha\nu\kappa\lambda\iota}}{7}$ | $\frac{R_{\mu^\alpha\rho\tau;\iota S5\alpha\nu\kappa\lambda\eta}}{7}$ |
| $\frac{R_{\mu^\alpha\rho\tau;\lambda S5\alpha\nu\kappa\iota\eta}}{7}$ | $\frac{R_{\mu^\alpha\rho\tau;\kappa S5\alpha\nu\lambda\iota\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\iota;\eta S5\alpha\rho\tau\kappa\lambda}}{7}$ | $\frac{R_{\mu^\alpha\nu\lambda;\eta S5\alpha\rho\tau\kappa\iota}}{7}$ |
| $\frac{R_{\mu^\alpha\nu\lambda;\iota S5\alpha\rho\tau\kappa\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\kappa;\eta S5\alpha\rho\tau\lambda\iota}}{7}$ | $\frac{R_{\mu^\alpha\nu\kappa;\iota S5\alpha\rho\tau\lambda\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\kappa;\lambda S5\alpha\rho\tau\iota\eta}}{7}$ |
| $\frac{R_{\mu^\alpha\nu\tau;\eta S5\alpha\rho\kappa\lambda\iota}}{7}$ | $\frac{R_{\mu^\alpha\nu\tau;\iota S5\alpha\rho\kappa\lambda\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\tau;\lambda S5\alpha\rho\kappa\iota\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\tau;\kappa S5\alpha\rho\lambda\iota\eta}}{7}$ |
| $\frac{R_{\mu^\alpha\nu\rho;\eta S5\alpha\tau\kappa\lambda\iota}}{7}$ | $\frac{R_{\mu^\alpha\nu\rho;\iota S5\alpha\tau\kappa\lambda\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\rho;\lambda S5\alpha\tau\kappa\iota\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\rho;\kappa S5\alpha\tau\lambda\iota\eta}}{7}$ |
| $\frac{R_{\mu^\alpha\nu\rho;\tau S5\alpha\kappa\lambda\iota\eta}}{7}$ | $\frac{R_{\mu^\alpha\nu\tau;\rho S5\alpha\kappa\lambda\iota\eta}}{7}$ | $\frac{R_{\rho^\alpha\lambda\iota;\eta S5\mu\nu\alpha\tau\kappa}}{7}$ | $\frac{R_{\rho^\alpha\kappa\iota;\eta S5\mu\nu\alpha\tau\lambda}}{7}$ |
| $\frac{R_{\rho^\alpha\kappa\lambda;\eta S5\mu\nu\alpha\tau\iota}}{7}$ | $\frac{R_{\rho^\alpha\kappa\lambda;\iota S5\mu\nu\alpha\tau\eta}}{7}$ | $\frac{R_{\rho^\alpha\tau\iota;\eta S5\mu\nu\alpha\kappa\lambda}}{7}$ | $\frac{R_{\rho^\alpha\tau\lambda;\eta S5\mu\nu\alpha\kappa\iota}}{7}$ |
| $\frac{R_{\rho^\alpha\tau\lambda;\iota S5\mu\nu\alpha\kappa\eta}}{7}$ | $\frac{R_{\rho^\alpha\tau\kappa;\eta S5\mu\nu\alpha\lambda\iota}}{7}$ | $\frac{R_{\rho^\alpha\tau\kappa;\iota S5\mu\nu\alpha\lambda\eta}}{7}$ | $\frac{R_{\rho^\alpha\tau\kappa;\lambda S5\mu\nu\alpha\iota\eta}}{7}$ |
| $\frac{R_{\tau^\alpha\lambda\iota;\eta S5\mu\nu\rho\alpha\kappa}}{7}$ | $\frac{R_{\tau^\alpha\kappa\iota;\eta S5\mu\nu\rho\alpha\lambda}}{7}$ | $\frac{R_{\tau^\alpha\kappa\lambda;\eta S5\mu\nu\rho\alpha\iota}}{7}$ | $\frac{R_{\tau^\alpha\kappa\lambda;\iota S5\mu\nu\rho\alpha\eta}}{7}$ |



$$\begin{array}{r}
\frac{R_{\alpha\mu\nu\eta;\tau}S5_{\rho}^{\alpha}κλι}{7} - \frac{S5_{\alpha\mu\nu\tau\eta}S5_{\rho}^{\alpha}κλι}{7} + \frac{R_{\alpha\mu\nu;\tau}S5_{\rho}^{\alpha}κλη}{7} - \frac{S5_{\alpha\mu\nu\tau\iota}S5_{\rho}^{\alpha}κλη}{7} + \\
\frac{R_{\alpha\mu\nu\lambda;\tau}S5_{\rho}^{\alpha}κιη}{7} - \frac{S5_{\alpha\mu\nu\tau\lambda}S5_{\rho}^{\alpha}κιη}{7} + \frac{R_{\alpha\mu\nu\kappa;\tau}S5_{\rho}^{\alpha}λιη}{7} - \frac{S5_{\alpha\mu\nu\tau\kappa}S5_{\rho}^{\alpha}λιη}{7} + \\
\frac{R_{\alpha\mu\nu\eta;\rho}S5_{\tau}^{\alpha}κλι}{7} - \frac{S5_{\alpha\mu\nu\rho\eta}S5_{\tau}^{\alpha}κλι}{7} + \frac{R_{\alpha\mu\nu;\rho}S5_{\tau}^{\alpha}κλη}{7} - \frac{S5_{\alpha\mu\nu\rho\iota}S5_{\tau}^{\alpha}κλη}{7} + \\
\frac{R_{\alpha\mu\nu\lambda;\rho}S5_{\tau}^{\alpha}κιη}{7} - \frac{S5_{\alpha\mu\nu\rho\lambda}S5_{\tau}^{\alpha}κιη}{7} + \frac{R_{\alpha\mu\nu\kappa;\rho}S5_{\tau}^{\alpha}λιη}{7} - \frac{S5_{\alpha\mu\nu\rho\kappa}S5_{\tau}^{\alpha}λιη}{7} - \\
\frac{S5_{\alpha\mu\nu\rho\tau}S5_{\kappa}^{\alpha}λιη}{7} - \frac{S_{\alpha\lambda\iota\eta}S6_{\mu}^{\alpha}\nu\rho\tau\kappa}{7} - \frac{S_{\alpha\kappa\iota\eta}S6_{\mu}^{\alpha}\nu\rho\tau\lambda}{7} - \frac{S_{\alpha\kappa\lambda\eta}S6_{\mu}^{\alpha}\nu\rho\tau\iota}{7} - \\
\frac{S_{\alpha\kappa\lambda\iota}S6_{\mu}^{\alpha}\nu\rho\tau\eta}{7} - \frac{S_{\alpha\tau\iota\eta}S6_{\mu}^{\alpha}\nu\rho\kappa\lambda}{7} - \frac{S_{\alpha\tau\lambda\eta}S6_{\mu}^{\alpha}\nu\rho\kappa\iota}{7} - \frac{S_{\alpha\tau\lambda\iota}S6_{\mu}^{\alpha}\nu\rho\kappa\eta}{7} - \\
\frac{S_{\alpha\tau\kappa\eta}S6_{\mu}^{\alpha}\nu\rho\lambda\iota}{7} - \frac{S_{\alpha\tau\kappa\iota}S6_{\mu}^{\alpha}\nu\rho\lambda\eta}{7} - \frac{S_{\alpha\tau\kappa\lambda}S6_{\mu}^{\alpha}\nu\rho\lambda\iota}{7} - \frac{S_{\alpha\tau\kappa\lambda\iota}S6_{\mu}^{\alpha}\nu\rho\lambda\eta}{7} - \\
\frac{S_{\alpha\rho\lambda\eta}S6_{\mu}^{\alpha}\nu\tau\kappa\iota}{7} - \frac{S_{\alpha\rho\lambda\iota}S6_{\mu}^{\alpha}\nu\tau\kappa\eta}{7} - \frac{S_{\alpha\rho\kappa\eta}S6_{\mu}^{\alpha}\nu\tau\lambda\iota}{7} - \frac{S_{\alpha\rho\kappa\iota}S6_{\mu}^{\alpha}\nu\tau\lambda\eta}{7} - \\
\frac{S_{\alpha\rho\kappa\lambda}S6_{\mu}^{\alpha}\nu\tau\iota\eta}{7} - \frac{S_{\alpha\rho\tau\eta}S6_{\mu}^{\alpha}\nu\kappa\lambda\iota}{7} - \frac{S_{\alpha\rho\tau\iota}S6_{\mu}^{\alpha}\nu\kappa\lambda\eta}{7} - \frac{S_{\alpha\rho\tau\lambda}S6_{\mu}^{\alpha}\nu\kappa\lambda\iota}{7} - \\
\frac{S_{\alpha\rho\tau\kappa}S6_{\mu}^{\alpha}\nu\lambda\iota\eta}{7} - \frac{S_{\alpha\nu\iota\eta}S6_{\mu}^{\alpha}\rho\tau\kappa\lambda}{7} - \frac{S_{\alpha\nu\lambda\eta}S6_{\mu}^{\alpha}\rho\tau\kappa\iota}{7} - \frac{S_{\alpha\nu\lambda\iota}S6_{\mu}^{\alpha}\rho\tau\kappa\eta}{7} - \\
\frac{S_{\alpha\nu\kappa\eta}S6_{\mu}^{\alpha}\rho\tau\lambda\iota}{7} - \frac{S_{\alpha\nu\kappa\iota}S6_{\mu}^{\alpha}\rho\tau\lambda\eta}{7} - \frac{S_{\alpha\nu\kappa\lambda}S6_{\mu}^{\alpha}\rho\tau\lambda\iota}{7} - \frac{S_{\alpha\nu\tau\eta}S6_{\mu}^{\alpha}\rho\kappa\lambda\iota}{7} - \\
\frac{S_{\alpha\nu\tau\iota}S6_{\mu}^{\alpha}\rho\kappa\lambda\eta}{7} - \frac{S_{\alpha\nu\tau\lambda}S6_{\mu}^{\alpha}\rho\kappa\lambda\iota}{7} - \frac{S_{\alpha\nu\tau\kappa}S6_{\mu}^{\alpha}\rho\lambda\iota\eta}{7} - \frac{S_{\alpha\nu\rho\eta}S6_{\mu}^{\alpha}\tau\kappa\lambda\iota}{7} - \\
\frac{S_{\alpha\nu\rho\iota}S6_{\mu}^{\alpha}\tau\kappa\lambda\eta}{7} - \frac{S_{\alpha\nu\rho\lambda}S6_{\mu}^{\alpha}\tau\kappa\lambda\iota}{7} - \frac{S_{\alpha\nu\rho\kappa}S6_{\mu}^{\alpha}\tau\lambda\iota\eta}{7} - \frac{S_{\alpha\nu\rho\tau}S6_{\mu}^{\alpha}\kappa\lambda\iota\eta}{7} - \\
\frac{S_{\alpha\mu\iota\eta}S6_{\nu}^{\alpha}\rho\tau\kappa\lambda}{7} - \frac{S_{\alpha\mu\lambda\eta}S6_{\nu}^{\alpha}\rho\tau\kappa\iota}{7} - \frac{S_{\alpha\mu\lambda\iota}S6_{\nu}^{\alpha}\rho\tau\kappa\eta}{7} - \frac{S_{\alpha\mu\kappa\eta}S6_{\nu}^{\alpha}\rho\tau\lambda\iota}{7} - \\
\frac{S_{\alpha\mu\kappa\iota}S6_{\nu}^{\alpha}\rho\tau\lambda\eta}{7} - \frac{S_{\alpha\mu\kappa\lambda}S6_{\nu}^{\alpha}\rho\tau\lambda\iota}{7} - \frac{S_{\alpha\mu\tau\eta}S6_{\nu}^{\alpha}\rho\kappa\lambda\iota}{7} - \frac{S_{\alpha\mu\tau\iota}S6_{\nu}^{\alpha}\rho\kappa\lambda\eta}{7} - \\
\frac{S_{\alpha\mu\tau\lambda}S6_{\nu}^{\alpha}\rho\kappa\lambda\iota}{7} - \frac{S_{\alpha\mu\tau\kappa}S6_{\nu}^{\alpha}\rho\lambda\iota\eta}{7} - \frac{S_{\alpha\mu\rho\eta}S6_{\nu}^{\alpha}\tau\kappa\lambda\iota}{7} - \frac{S_{\alpha\mu\rho\iota}S6_{\nu}^{\alpha}\tau\kappa\lambda\eta}{7} - \\
\frac{S_{\alpha\mu\rho\lambda}S6_{\nu}^{\alpha}\tau\kappa\lambda\iota}{7} - \frac{S_{\alpha\mu\rho\kappa}S6_{\nu}^{\alpha}\tau\lambda\iota\eta}{7} - \frac{S_{\alpha\mu\rho\tau}S6_{\nu}^{\alpha}\kappa\lambda\iota\eta}{7} - \frac{S_{\alpha\mu\nu\eta}S6_{\rho}^{\alpha}\tau\kappa\lambda\iota}{7} - \\
\frac{S_{\alpha\mu\nu\iota}S6_{\rho}^{\alpha}\tau\kappa\lambda\eta}{7} - \frac{S_{\alpha\mu\nu\lambda}S6_{\rho}^{\alpha}\tau\kappa\lambda\iota}{7} - \frac{S_{\alpha\mu\nu\kappa}S6_{\rho}^{\alpha}\tau\lambda\iota\eta}{7} - \frac{S_{\alpha\mu\nu\tau}S6_{\rho}^{\alpha}\kappa\lambda\iota\eta}{7} - \\
\frac{S_{\alpha\mu\nu\rho}S6_{\tau}^{\alpha}\kappa\lambda\iota\eta}{7} - \frac{S6_{\alpha\tau\kappa\lambda\iota\eta}R_{\mu}^{\alpha}\nu\rho}{7} - \frac{S6_{\alpha\rho\kappa\lambda\iota\eta}R_{\mu}^{\alpha}\nu\tau}{7} - \frac{S6_{\alpha\rho\tau\lambda\iota\eta}R_{\mu}^{\alpha}\nu\kappa}{7} - \\
\frac{S6_{\alpha\rho\tau\kappa\lambda\iota}R_{\mu}^{\alpha}\nu\lambda}{7} - \frac{S6_{\alpha\rho\tau\kappa\lambda\eta}R_{\mu}^{\alpha}\nu\iota}{7} - \frac{S6_{\alpha\rho\tau\kappa\lambda\iota}R_{\mu}^{\alpha}\nu\eta}{7} - \frac{S6_{\alpha\nu\kappa\lambda\iota\eta}R_{\mu}^{\alpha}\rho\tau}{7} - \\
\frac{S6_{\alpha\nu\tau\lambda\iota\eta}R_{\mu}^{\alpha}\rho\kappa}{7} - \frac{S6_{\alpha\nu\tau\kappa\lambda\iota\eta}R_{\mu}^{\alpha}\rho\lambda}{7} - \frac{S6_{\alpha\nu\tau\kappa\lambda\eta}R_{\mu}^{\alpha}\rho\iota}{7} - \frac{S6_{\alpha\nu\tau\kappa\lambda\iota}R_{\mu}^{\alpha}\rho\eta}{7} - \\
\frac{S6_{\alpha\nu\rho\lambda\iota\eta}R_{\mu}^{\alpha}\tau\kappa}{7} - \frac{S6_{\alpha\nu\rho\kappa\lambda\iota\eta}R_{\mu}^{\alpha}\tau\lambda}{7} - \frac{S6_{\alpha\nu\rho\kappa\lambda\eta}R_{\mu}^{\alpha}\tau\iota}{7} - \frac{S6_{\alpha\nu\rho\kappa\lambda\iota}R_{\mu}^{\alpha}\tau\eta}{7} -
\end{array}$$

$$\begin{array}{cccc}
\frac{S6_{\alpha\nu\rho\tau\iota\eta}R_{\mu}^{\alpha}\kappa\lambda}{7} & \frac{S6_{\alpha\nu\rho\tau\lambda\eta}R_{\mu}^{\alpha}\kappa\iota}{7} & \frac{S6_{\alpha\nu\rho\tau\lambda\iota}R_{\mu}^{\alpha}\kappa\eta}{7} & \frac{S6_{\alpha\nu\rho\tau\kappa\eta}R_{\mu}^{\alpha}\lambda\iota}{7} \\
\frac{S6_{\alpha\nu\rho\tau\kappa\iota}R_{\mu}^{\alpha}\lambda\eta}{7} & \frac{S6_{\alpha\nu\rho\tau\kappa\lambda}R_{\mu}^{\alpha}\iota\eta}{7} & \frac{S6_{\alpha\mu\kappa\lambda\iota\eta}R_{\nu}^{\alpha}\rho\tau}{7} & \frac{S6_{\alpha\mu\tau\lambda\iota\eta}R_{\nu}^{\alpha}\rho\kappa}{7} \\
\frac{S6_{\alpha\mu\tau\kappa\iota\eta}R_{\nu}^{\alpha}\rho\lambda}{7} & \frac{S6_{\alpha\mu\tau\kappa\lambda\eta}R_{\nu}^{\alpha}\rho\iota}{7} & \frac{S6_{\alpha\mu\tau\kappa\lambda\iota}R_{\nu}^{\alpha}\rho\eta}{7} & \frac{S6_{\alpha\mu\rho\lambda\iota\eta}R_{\nu}^{\alpha}\tau\kappa}{7} \\
\frac{S6_{\alpha\mu\rho\kappa\iota\eta}R_{\nu}^{\alpha}\tau\lambda}{7} & \frac{S6_{\alpha\mu\rho\kappa\lambda\eta}R_{\nu}^{\alpha}\tau\iota}{7} & \frac{S6_{\alpha\mu\rho\kappa\lambda\iota}R_{\nu}^{\alpha}\tau\eta}{7} & \frac{S6_{\alpha\mu\rho\tau\iota\eta}R_{\nu}^{\alpha}\kappa\lambda}{7} \\
\frac{S6_{\alpha\mu\rho\tau\lambda\eta}R_{\nu}^{\alpha}\kappa\iota}{7} & \frac{S6_{\alpha\mu\rho\tau\lambda\iota}R_{\nu}^{\alpha}\kappa\eta}{7} & \frac{S6_{\alpha\mu\rho\tau\kappa\eta}R_{\nu}^{\alpha}\lambda\iota}{7} & \frac{S6_{\alpha\mu\rho\tau\kappa\iota}R_{\nu}^{\alpha}\lambda\eta}{7} \\
\frac{S6_{\alpha\mu\rho\tau\kappa\lambda}R_{\nu}^{\alpha}\iota\eta}{7} & \frac{S6_{\mu\nu\alpha\lambda\iota\eta}R_{\rho}^{\alpha}\tau\kappa}{7} & \frac{S6_{\mu\nu\alpha\kappa\iota\eta}R_{\rho}^{\alpha}\tau\lambda}{7} & \frac{S6_{\mu\nu\alpha\kappa\lambda\eta}R_{\rho}^{\alpha}\tau\iota}{7} \\
\frac{S6_{\mu\nu\alpha\kappa\lambda\iota}R_{\rho}^{\alpha}\tau\eta}{7} & \frac{S6_{\mu\nu\alpha\tau\iota\eta}R_{\rho}^{\alpha}\kappa\lambda}{7} & \frac{S6_{\mu\nu\alpha\tau\lambda\eta}R_{\rho}^{\alpha}\kappa\iota}{7} & \frac{S6_{\mu\nu\alpha\tau\lambda\iota}R_{\rho}^{\alpha}\kappa\eta}{7} \\
\frac{S6_{\mu\nu\alpha\tau\kappa\eta}R_{\rho}^{\alpha}\lambda\iota}{7} & \frac{S6_{\mu\nu\alpha\tau\kappa\iota}R_{\rho}^{\alpha}\lambda\eta}{7} & \frac{S6_{\mu\nu\alpha\tau\kappa\lambda}R_{\rho}^{\alpha}\iota\eta}{7} & \frac{S6_{\mu\nu\rho\alpha\iota\eta}R_{\tau}^{\alpha}\kappa\lambda}{7} \\
\frac{S6_{\mu\nu\rho\alpha\lambda\eta}R_{\tau}^{\alpha}\kappa\iota}{7} & \frac{S6_{\mu\nu\rho\alpha\lambda\iota}R_{\tau}^{\alpha}\kappa\eta}{7} & \frac{S6_{\mu\nu\rho\alpha\kappa\eta}R_{\tau}^{\alpha}\lambda\iota}{7} & \frac{S6_{\mu\nu\rho\alpha\kappa\iota}R_{\tau}^{\alpha}\lambda\eta}{7} \\
\frac{S6_{\mu\nu\rho\alpha\kappa\lambda}R_{\tau}^{\alpha}\iota\eta}{7} & \frac{S6_{\mu\nu\rho\tau\alpha\eta}R_{\kappa}^{\alpha}\lambda\iota}{7} & \frac{S6_{\mu\nu\rho\tau\alpha\iota}R_{\kappa}^{\alpha}\lambda\eta}{7} & \frac{S6_{\mu\nu\rho\tau\alpha\lambda}R_{\kappa}^{\alpha}\iota\eta}{7} \\
\frac{S6_{\mu\nu\rho\tau\kappa\alpha}R_{\lambda}^{\alpha}\iota\eta}{7} & & & 
\end{array}$$

(271)

## APPENDIX B

Coincidence limits  $[\Delta^{1/2}_{;\mu\nu\rho\tau\kappa}]$  and  $[\Delta^{1/2}_{;\mu\nu\rho\tau\kappa\lambda}]$

$$[\Delta^{1/2}_{;\mu\nu\rho\tau\kappa}] =$$

|   |  |   |   |   |
|---|--|---|---|---|
| $\frac{S^7 \alpha^{\mu\nu\rho\tau\kappa}}{10}$                  | $\frac{R_{\nu\rho\tau}{}^\alpha R_{\alpha\mu}}{120}$             | $\frac{R_{\nu\rho\kappa}{}^\alpha{}_\tau R_{\alpha\mu}}{120}$   | $\frac{R_{\nu\tau\rho}{}^\alpha{}_\kappa R_{\alpha\mu}}{120}$   | $\frac{R_{\nu\tau\kappa}{}^\alpha{}_\rho R_{\alpha\mu}}{120}$   |
| $\frac{R_{\nu\kappa\rho}{}^\alpha{}_\tau R_{\alpha\mu}}{120}$   | $\frac{R_{\nu\kappa\tau}{}^\alpha{}_\rho R_{\alpha\mu}}{120}$    | $\frac{R_{\nu}{}^\alpha{}_\rho\tau{}_\kappa R_{\alpha\mu}}{30}$ | $\frac{R_{\mu\rho\tau}{}^\alpha{}_\kappa R_{\alpha\nu}}{120}$   | $\frac{R_{\mu\rho\kappa}{}^\alpha{}_\tau R_{\alpha\nu}}{120}$   |
| $\frac{R_{\mu\tau\rho}{}^\alpha{}_\kappa R_{\alpha\nu}}{120}$   | $\frac{R_{\mu\tau\kappa}{}^\alpha{}_\rho R_{\alpha\nu}}{120}$    | $\frac{R_{\mu\kappa\rho}{}^\alpha{}_\tau R_{\alpha\nu}}{120}$   | $\frac{R_{\mu\kappa\tau}{}^\alpha{}_\rho R_{\alpha\nu}}{120}$   | $\frac{R_{\mu}{}^\alpha{}_\rho\tau{}_\kappa R_{\alpha\nu}}{30}$ |
| $\frac{R_{\mu\nu\tau}{}^\alpha{}_\kappa R_{\alpha\rho}}{120}$   | $\frac{R_{\mu\nu\kappa}{}^\alpha{}_\tau R_{\alpha\rho}}{120}$    | $\frac{R_{\mu\tau\nu}{}^\alpha{}_\kappa R_{\alpha\rho}}{120}$   | $\frac{R_{\mu\tau\kappa}{}^\alpha{}_\nu R_{\alpha\rho}}{120}$   | $\frac{R_{\mu\kappa\nu}{}^\alpha{}_\tau R_{\alpha\rho}}{120}$   |
| $\frac{R_{\mu\kappa\tau}{}^\alpha{}_\nu R_{\alpha\rho}}{120}$   | $\frac{R_{\mu}{}^\alpha{}_\nu\tau{}_\kappa R_{\alpha\rho}}{120}$ | $\frac{R_{\mu\nu\rho}{}^\alpha{}_\kappa R_{\alpha\tau}}{120}$   | $\frac{R_{\mu\nu\kappa}{}^\alpha{}_\rho R_{\alpha\tau}}{120}$   | $\frac{R_{\mu\rho\nu}{}^\alpha{}_\kappa R_{\alpha\tau}}{120}$   |
| $\frac{R_{\mu\rho\kappa}{}^\alpha{}_\nu R_{\alpha\tau}}{120}$   | $\frac{R_{\mu\kappa\nu}{}^\alpha{}_\rho R_{\alpha\tau}}{120}$    | $\frac{R_{\mu\kappa\rho}{}^\alpha{}_\nu R_{\alpha\tau}}{120}$   | $\frac{R_{\mu}{}^\alpha{}_\nu\rho{}_\kappa R_{\alpha\tau}}{30}$ | $\frac{R_{\mu\nu\rho}{}^\alpha{}_\tau R_{\alpha\kappa}}{120}$   |
| $\frac{R_{\mu\nu\tau}{}^\alpha{}_\rho R_{\alpha\kappa}}{120}$   | $\frac{R_{\mu\rho\nu}{}^\alpha{}_\tau R_{\alpha\kappa}}{120}$    | $\frac{R_{\mu\rho\tau}{}^\alpha{}_\nu R_{\alpha\kappa}}{120}$   | $\frac{R_{\mu\tau\nu}{}^\alpha{}_\rho R_{\alpha\kappa}}{120}$   | $\frac{R_{\mu\tau\rho}{}^\alpha{}_\nu R_{\alpha\kappa}}{120}$   |
| $\frac{R_{\mu}{}^\alpha{}_\nu\rho{}_\tau R_{\alpha\kappa}}{30}$ | $\frac{R_{\mu}{}^\alpha{}_\nu\tau{}_\rho R_{\alpha\kappa}}{30}$  | $\frac{R_{\rho\tau}{}_\kappa R_{\mu\nu}}{72}$                   | $\frac{R_{\rho\kappa}{}_\tau R_{\mu\nu}}{72}$                   | $\frac{R_{\tau\kappa}{}_\rho R_{\mu\nu}}{72}$                   |
| $\frac{R_{\nu\tau}{}_\kappa R_{\mu\rho}}{72}$                   | $\frac{R_{\nu\kappa}{}_\tau R_{\mu\rho}}{72}$                    | $\frac{R_{\tau\kappa}{}_\nu R_{\mu\rho}}{72}$                   | $\frac{R_{\nu\rho}{}_\kappa R_{\mu\tau}}{72}$                   | $\frac{R_{\nu\kappa}{}_\rho R_{\mu\tau}}{72}$                   |
| $\frac{R_{\rho\kappa}{}_\nu R_{\mu\tau}}{72}$                   | $\frac{R_{\nu\rho}{}_\tau R_{\mu\kappa}}{72}$                    | $\frac{R_{\nu\tau}{}_\rho R_{\mu\kappa}}{72}$                   | $\frac{R_{\rho\tau}{}_\nu R_{\mu\kappa}}{72}$                   | $\frac{R_{\alpha\nu\rho\kappa}{}_\tau R_{\mu}{}^\alpha}{30}$    |
| $\frac{R_{\mu\tau}{}_\kappa R_{\nu\rho}}{72}$                   | $\frac{R_{\mu\kappa}{}_\tau R_{\nu\rho}}{72}$                    | $\frac{R_{\tau\kappa}{}_\mu R_{\nu\rho}}{72}$                   | $\frac{R_{\mu\rho}{}_\kappa R_{\nu\tau}}{72}$                   | $\frac{R_{\mu\kappa}{}_\rho R_{\nu\tau}}{72}$                   |
| $\frac{R_{\rho\kappa}{}_\mu R_{\nu\tau}}{72}$                   | $\frac{R_{\mu\rho}{}_\tau R_{\nu\kappa}}{72}$                    | $\frac{R_{\mu\tau}{}_\rho R_{\nu\kappa}}{72}$                   | $\frac{R_{\rho\tau}{}_\mu R_{\nu\kappa}}{72}$                   | $\frac{R_{\alpha\mu\rho\kappa}{}_\tau R_{\nu}{}^\alpha}{30}$    |
| $\frac{R_{\mu\nu}{}_\kappa R_{\rho\tau}}{72}$                   | $\frac{R_{\mu\kappa}{}_\nu R_{\rho\tau}}{72}$                    | $\frac{R_{\nu\kappa}{}_\mu R_{\rho\tau}}{72}$                   | $\frac{R_{\mu\nu}{}_\tau R_{\rho\kappa}}{72}$                   | $\frac{R_{\mu\tau}{}_\nu R_{\rho\kappa}}{72}$                   |
| $\frac{R_{\nu\tau}{}_\mu R_{\rho\kappa}}{72}$                   | $\frac{R_{\alpha\mu\nu\kappa}{}_\tau R_{\rho}{}^\alpha}{30}$     | $\frac{R_{\mu\nu}{}_\rho R_{\tau\kappa}}{72}$                   | $\frac{R_{\mu\rho}{}_\nu R_{\tau\kappa}}{72}$                   | $\frac{R_{\nu\rho}{}_\mu R_{\tau\kappa}}{72}$                   |
| $\frac{R_{\alpha\mu\nu\kappa}{}_\rho R_{\tau}{}^\alpha}{30}$    | $\frac{R_{\alpha\tau}{}_\kappa R_{\mu\nu\rho}{}^\alpha}{180}$    | $\frac{R_{\alpha\kappa}{}_\tau R_{\mu\nu\rho}{}^\alpha}{180}$   | $\frac{R_{\tau\kappa}{}_\alpha R_{\mu\nu\rho}{}^\alpha}{180}$   | $\frac{R_{\alpha\rho}{}_\kappa R_{\mu\nu\tau}{}^\alpha}{180}$   |
| $\frac{R_{\alpha\kappa}{}_\rho R_{\mu\nu\tau}{}^\alpha}{180}$   | $\frac{R_{\rho\kappa}{}_\alpha R_{\mu\nu\tau}{}^\alpha}{180}$    | $\frac{R_{\alpha\rho}{}_\tau R_{\mu\nu\kappa}{}^\alpha}{180}$   | $\frac{R_{\alpha\tau}{}_\rho R_{\mu\nu\kappa}{}^\alpha}{180}$   | $\frac{R_{\rho\tau}{}_\alpha R_{\mu\nu\kappa}{}^\alpha}{180}$   |
| $\frac{R_{\alpha\tau}{}_\kappa R_{\mu\rho\nu}{}^\alpha}{180}$   | $\frac{R_{\alpha\kappa}{}_\tau R_{\mu\rho\nu}{}^\alpha}{180}$    | $\frac{R_{\tau\kappa}{}_\alpha R_{\mu\rho\nu}{}^\alpha}{180}$   | $\frac{R_{\alpha\nu}{}_\kappa R_{\mu\rho\tau}{}^\alpha}{180}$   | $\frac{R_{\alpha\kappa}{}_\nu R_{\mu\rho\tau}{}^\alpha}{180}$   |
| $\frac{R_{\nu\kappa}{}_\alpha R_{\mu\rho\tau}{}^\alpha}{180}$   | $\frac{R_{\alpha\nu}{}_\tau R_{\mu\rho\kappa}{}^\alpha}{180}$    | $\frac{R_{\alpha\tau}{}_\nu R_{\mu\rho\kappa}{}^\alpha}{180}$   | $\frac{R_{\nu\tau}{}_\alpha R_{\mu\rho\kappa}{}^\alpha}{180}$   | $\frac{R_{\alpha\rho}{}_\kappa R_{\mu\tau\nu}{}^\alpha}{180}$   |
| $\frac{R_{\alpha\kappa}{}_\rho R_{\mu\tau\nu}{}^\alpha}{180}$   | $\frac{R_{\rho\kappa}{}_\alpha R_{\mu\tau\nu}{}^\alpha}{180}$    | $\frac{R_{\alpha\nu}{}_\kappa R_{\mu\tau\rho}{}^\alpha}{180}$   | $\frac{R_{\alpha\kappa}{}_\nu R_{\mu\tau\rho}{}^\alpha}{180}$   | $\frac{R_{\nu\kappa}{}_\alpha R_{\mu\tau\rho}{}^\alpha}{180}$   |
| $\frac{R_{\alpha\nu}{}_\rho R_{\mu\tau\kappa}{}^\alpha}{180}$   | $\frac{R_{\alpha\rho}{}_\nu R_{\mu\tau\kappa}{}^\alpha}{180}$    | $\frac{R_{\nu\rho}{}_\alpha R_{\mu\tau\kappa}{}^\alpha}{180}$   | $\frac{R_{\alpha\rho}{}_\tau R_{\mu\kappa\nu}{}^\alpha}{180}$   | $\frac{R_{\alpha\tau}{}_\rho R_{\mu\kappa\nu}{}^\alpha}{180}$   |

|  |  |  |  |  |
|--|--|--|--|--|
| $\frac{R_{\rho\tau;\alpha}R_{\mu\kappa\nu}{}^\alpha}{180}$     | $\frac{R_{\alpha\nu;\tau}R_{\mu\kappa\rho}{}^\alpha}{180}$     | $\frac{R_{\alpha\tau;\nu}R_{\mu\kappa\rho}{}^\alpha}{180}$     | $\frac{R_{\nu\tau;\alpha}R_{\mu\kappa\rho}{}^\alpha}{180}$     | $\frac{R_{\alpha\nu;\rho}R_{\mu\kappa\tau}{}^\alpha}{180}$     |
| $\frac{R_{\alpha\rho;\nu}R_{\mu\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\nu\rho;\alpha}R_{\mu\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\alpha\tau;\kappa}R_{\mu}{}^\alpha{}_{\nu\rho}}{60}$ | $\frac{R_{\alpha\kappa;\tau}R_{\mu}{}^\alpha{}_{\nu\rho}}{60}$ | $\frac{R_{\tau\kappa;\alpha}R_{\mu}{}^\alpha{}_{\nu\rho}}{60}$ |
| $\frac{R_{\alpha\rho;\kappa}R_{\mu}{}^\alpha{}_{\nu\tau}}{60}$ | $\frac{R_{\alpha\kappa;\rho}R_{\mu}{}^\alpha{}_{\nu\tau}}{60}$ | $\frac{R_{\rho\kappa;\alpha}R_{\mu}{}^\alpha{}_{\nu\tau}}{60}$ | $\frac{R_{\alpha\rho;\tau}R_{\mu}{}^\alpha{}_{\nu\kappa}}{60}$ | $\frac{R_{\alpha\tau;\rho}R_{\mu}{}^\alpha{}_{\nu\kappa}}{60}$ |
| $\frac{R_{\rho\tau;\alpha}R_{\mu}{}^\alpha{}_{\nu\kappa}}{60}$ | $\frac{R_{\alpha\nu;\kappa}R_{\mu}{}^\alpha{}_{\rho\tau}}{60}$ | $\frac{R_{\alpha\kappa;\nu}R_{\mu}{}^\alpha{}_{\rho\tau}}{60}$ | $\frac{R_{\nu\kappa;\alpha}R_{\mu}{}^\alpha{}_{\rho\tau}}{60}$ | $\frac{R_{\alpha\nu;\tau}R_{\mu}{}^\alpha{}_{\rho\kappa}}{60}$ |
| $\frac{R_{\alpha\tau;\nu}R_{\mu}{}^\alpha{}_{\rho\kappa}}{60}$ | $\frac{R_{\nu\tau;\alpha}R_{\mu}{}^\alpha{}_{\rho\kappa}}{60}$ | $\frac{R_{\alpha\nu;\rho}R_{\mu}{}^\alpha{}_{\tau\kappa}}{60}$ | $\frac{R_{\alpha\rho;\nu}R_{\mu}{}^\alpha{}_{\tau\kappa}}{60}$ | $\frac{R_{\nu\rho;\alpha}R_{\mu}{}^\alpha{}_{\tau\kappa}}{60}$ |
| $\frac{R_{\alpha\mu;\kappa}R_{\nu\rho\tau}{}^\alpha}{180}$     | $\frac{R_{\alpha\kappa;\mu}R_{\nu\rho\tau}{}^\alpha}{180}$     | $\frac{R_{\mu\kappa;\alpha}R_{\nu\rho\tau}{}^\alpha}{180}$     | $\frac{R_{\alpha\mu;\tau}R_{\nu\rho\kappa}{}^\alpha}{180}$     | $\frac{R_{\alpha\tau;\mu}R_{\nu\rho\kappa}{}^\alpha}{180}$     |
| $\frac{R_{\mu\tau;\alpha}R_{\nu\rho\kappa}{}^\alpha}{180}$     | $\frac{R_{\alpha\mu;\kappa}R_{\nu\tau\rho}{}^\alpha}{180}$     | $\frac{R_{\alpha\kappa;\mu}R_{\nu\tau\rho}{}^\alpha}{180}$     | $\frac{R_{\mu\kappa;\alpha}R_{\nu\tau\rho}{}^\alpha}{180}$     | $\frac{R_{\alpha\mu;\rho}R_{\nu\tau\kappa}{}^\alpha}{180}$     |
| $\frac{R_{\alpha\rho;\mu}R_{\nu\tau\kappa}{}^\alpha}{180}$     | $\frac{R_{\mu\rho;\alpha}R_{\nu\tau\kappa}{}^\alpha}{180}$     | $\frac{R_{\alpha\mu;\tau}R_{\nu\kappa\rho}{}^\alpha}{180}$     | $\frac{R_{\alpha\tau;\mu}R_{\nu\kappa\rho}{}^\alpha}{180}$     | $\frac{R_{\mu\tau;\alpha}R_{\nu\kappa\rho}{}^\alpha}{180}$     |
| $\frac{R_{\alpha\mu;\rho}R_{\nu\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\alpha\rho;\mu}R_{\nu\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\mu\rho;\alpha}R_{\nu\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\alpha\mu;\kappa}R_{\nu}{}^\alpha{}_{\rho\tau}}{60}$ | $\frac{R_{\alpha\kappa;\mu}R_{\nu}{}^\alpha{}_{\rho\tau}}{60}$ |
| $\frac{R_{\mu\kappa;\alpha}R_{\nu}{}^\alpha{}_{\rho\tau}}{60}$ | $\frac{R_{\alpha\mu;\tau}R_{\nu}{}^\alpha{}_{\rho\kappa}}{60}$ | $\frac{R_{\alpha\tau;\mu}R_{\nu}{}^\alpha{}_{\rho\kappa}}{60}$ | $\frac{R_{\mu\tau;\alpha}R_{\nu}{}^\alpha{}_{\rho\kappa}}{60}$ | $\frac{R_{\alpha\mu;\rho}R_{\nu}{}^\alpha{}_{\tau\kappa}}{60}$ |
| $\frac{R_{\alpha\rho;\mu}R_{\nu}{}^\alpha{}_{\tau\kappa}}{60}$ | $\frac{R_{\mu\rho;\alpha}R_{\nu}{}^\alpha{}_{\tau\kappa}}{60}$ | $\frac{R_{\alpha\mu;\nu}R_{\rho\tau\kappa}{}^\alpha}{180}$     | $\frac{R_{\alpha\nu;\mu}R_{\rho\tau\kappa}{}^\alpha}{180}$     | $\frac{R_{\mu\nu;\alpha}R_{\rho\tau\kappa}{}^\alpha}{180}$     |
| $\frac{R_{\alpha\mu;\nu}R_{\rho\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\alpha\nu;\mu}R_{\rho\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\mu\nu;\alpha}R_{\rho\kappa\tau}{}^\alpha}{180}$     | $\frac{R_{\alpha\mu;\nu}R_{\rho}{}^\alpha{}_{\tau\kappa}}{60}$ | $\frac{R_{\alpha\nu;\mu}R_{\rho}{}^\alpha{}_{\tau\kappa}}{60}$ |
| $\frac{R_{\mu\nu;\alpha}R_{\rho}{}^\alpha{}_{\tau\kappa}}{60}$ |  |  |  |  |

(272)

$$[\Delta^{1/2}{}_{;\alpha}{}^\alpha{}_{\beta}{}^\beta{}_{\mu\nu}] =$$

$$\begin{aligned} & \frac{R_{;\mu}R_{;\nu}}{18} + \frac{R_{;\alpha}R_{\mu\nu}{}^\alpha}{28} - \frac{13R_{;\alpha}R_{\mu}{}^\alpha{}_{;\nu}}{252} - \frac{19R_{\alpha\beta;\nu}R_{\mu}{}^{\alpha\beta}}{336} - \\ & \frac{19R_{\alpha\nu;\beta}R_{\mu}{}^{\beta\alpha}}{1008} - \frac{13R_{;\alpha}R_{\nu}{}^\alpha{}_{;\mu}}{252} - \frac{19R_{\alpha\beta;\mu}R_{\nu}{}^{\alpha\beta}}{336} - \frac{23R_{\alpha\mu;\beta}R_{\nu}{}^{\alpha\beta}}{1008} - \\ & \frac{19R_{\alpha\mu;\beta}R_{\nu}{}^{\beta\alpha}}{1008} - \frac{41R_{\alpha\beta;\mu}R_{\nu}{}^{\alpha\beta}}{1008} - \frac{R_{;\alpha}R_{\beta\mu\nu}{}^{\alpha\beta}}{126} - \frac{R_{;\alpha}R_{\beta\nu\mu}{}^{\alpha\beta}}{126} + \\ & \frac{25R_{\alpha\beta;\nu}R_{\gamma\mu}{}^{\alpha\beta\gamma}}{1008} - \frac{R_{\alpha\nu;\beta}R_{\gamma\mu}{}^{\alpha\beta\gamma}}{112} + \frac{25R_{\alpha\beta;\mu}R_{\gamma\nu}{}^{\alpha\beta\gamma}}{1008} - \frac{R_{\alpha\mu;\beta}R_{\gamma\nu}{}^{\alpha\beta\gamma}}{112} + \\ & \frac{R_{\alpha\beta;\nu}R_{\gamma}{}^\alpha{}_{\mu}{}^{\beta\gamma}}{48} + \frac{R_{\alpha\nu;\beta}R_{\gamma}{}^\alpha{}_{\mu}{}^{\beta\gamma}}{168} + \frac{R_{\alpha\beta;\mu}R_{\gamma}{}^\alpha{}_{\nu}{}^{\beta\gamma}}{48} + \frac{R_{\alpha\mu;\beta}R_{\gamma}{}^\alpha{}_{\nu}{}^{\beta\gamma}}{168} + \\ & \frac{5R_{\alpha\nu;\beta}R_{\gamma}{}^\beta{}_{\mu}{}^{\alpha\gamma}}{336} + \frac{5R_{\alpha\mu;\beta}R_{\gamma}{}^\beta{}_{\nu}{}^{\alpha\gamma}}{336} + \frac{R_{\alpha\beta\gamma\mu}{}^{\gamma}R_{\delta\nu}{}^{\alpha\beta\delta}}{336} - \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha}R_{\delta\nu}{}^{\beta\gamma\delta}}{336} + \end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\mu}{}^{;\gamma}R_{\delta}{}^{\alpha}{}_{\nu}{}^{\beta;\delta}}{336} - \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha}R_{\delta}{}^{\beta}{}_{\nu}{}^{\gamma;\delta}}{48} - \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha}R_{\delta}{}^{\gamma}{}_{\nu}{}^{\beta;\delta}}{56} + \frac{11R_{\alpha\beta;\gamma}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta;\gamma}}{504} + \\
& \frac{13R_{\alpha\beta;\gamma}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\gamma;\beta}}{1008} - \frac{R_{\alpha\beta;\gamma}R_{\mu}{}^{\alpha\beta\gamma}{}_{;\nu}}{112} + \frac{R_{\alpha\beta\gamma\delta;\nu}R_{\mu}{}^{\alpha\beta\gamma;\delta}}{84} + \frac{R_{\alpha\beta\gamma\nu;\delta}R_{\mu}{}^{\alpha\beta\gamma;\delta}}{84} - \\
& \frac{R_{\alpha\beta\gamma\delta;\nu}R_{\mu}{}^{\alpha\gamma\delta;\beta}}{42} - \frac{R_{\alpha\beta\gamma\nu;\delta}R_{\mu}{}^{\alpha\gamma\delta;\beta}}{84} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha}R_{\mu}{}^{\beta}{}_{\nu}{}^{\gamma;\delta}}{48} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha}R_{\mu}{}^{\beta\gamma\delta}{}_{;\nu}}{48} + \\
& \frac{13R_{\alpha\beta;\gamma}R_{\mu}{}^{\gamma}{}_{\nu}{}^{\alpha;\beta}}{1008} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha}R_{\mu}{}^{\gamma}{}_{\nu}{}^{\beta;\delta}}{48} - \frac{23R_{\alpha\beta;\gamma}R_{\mu}{}^{\gamma\alpha\beta}{}_{;\nu}}{336} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha}R_{\mu}{}^{\gamma\beta\delta}{}_{;\nu}}{48} - \\
& \frac{R_{\alpha\beta;\gamma}R_{\nu}{}^{\alpha\beta\gamma}{}_{;\mu}}{112} + \frac{R_{\alpha\beta\gamma\delta;\mu}R_{\nu}{}^{\alpha\beta\gamma;\delta}}{42} + \frac{R_{\alpha\beta\gamma\mu;\delta}R_{\nu}{}^{\alpha\beta\gamma;\delta}}{84} - \frac{R_{\alpha\beta\gamma\delta;\mu}R_{\nu}{}^{\alpha\gamma\delta;\beta}}{42} - \\
& \frac{R_{\alpha\beta\gamma\mu;\delta}R_{\nu}{}^{\alpha\gamma\delta;\beta}}{84} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha}R_{\nu}{}^{\beta\gamma\delta}{}_{;\mu}}{48} - \frac{61R_{\alpha\beta;\gamma}R_{\nu}{}^{\gamma\alpha\beta}{}_{;\mu}}{1008} - \frac{R_{\alpha\beta\gamma\mu;\delta}R_{\nu}{}^{\gamma\alpha\beta;\delta}}{42} - \\
& \frac{R_{\alpha\beta\gamma\mu;\delta}R_{\nu}{}^{\gamma\alpha\delta;\beta}}{42} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha}R_{\nu}{}^{\gamma\beta\delta}{}_{;\mu}}{48} + \frac{R_{\alpha\beta\gamma\delta;\mu}R_{\nu}{}^{\alpha\beta\gamma\delta}{}_{;\nu}}{84} + \frac{R_{\alpha\beta\gamma\delta;\mu}R_{\nu}{}^{\alpha\gamma\beta\delta}{}_{;\nu}}{84} + \\
& \frac{R_{\alpha\beta\gamma\mu;\delta}R^{\alpha\gamma\beta\delta}{}_{;\nu}}{84} + \frac{RR_{;\mu\nu}}{60} + \frac{RR_{\alpha\mu}{}^{;\alpha}{}_{\nu}}{30} + \frac{RR_{\alpha\nu}{}^{;\alpha}{}_{\mu}}{30} + \\
& \frac{RR_{\mu\nu; \alpha}{}^{\alpha}}{60} + \frac{R_{;\alpha}{}^{\alpha}{}_{\mu\nu}}{21} + \frac{R_{\alpha\beta}{}^{;\alpha\beta}{}_{\mu\nu}}{14} + \frac{R_{\alpha\beta}{}^{;\beta\alpha}{}_{\mu\nu}}{42} + \\
& \frac{R_{\alpha\mu; \beta}{}^{\beta\alpha}{}_{\nu}}{21} + \frac{R_{\alpha\mu}{}^{;\alpha}{}_{\beta}{}^{\beta}{}_{\nu}}{21} + \frac{R_{\alpha\nu; \beta}{}^{\beta\alpha}{}_{\mu}}{21} + \frac{R_{\alpha\nu}{}^{;\alpha}{}_{\beta}{}^{\beta}{}_{\mu}}{21} + \\
& \frac{R_{\mu\nu; \alpha}{}^{\alpha}{}_{\beta}{}^{\beta}}{42} + \frac{R^2R_{\mu\nu}}{216} + \frac{R_{;\alpha}{}^{\alpha}{}_{R_{\mu\nu}}}{60} + \frac{R_{\alpha\beta}{}^{;\alpha\beta}{}_{R_{\mu\nu}}}{40} + \\
& \frac{R_{\alpha\beta}{}^{;\beta\alpha}{}_{R_{\mu\nu}}}{120} + \frac{R_{;\alpha\nu}R_{\mu}{}^{\alpha}}{630} - \frac{47R_{\alpha\beta}{}^{;\beta}{}_{\nu}R_{\mu}{}^{\alpha}}{1260} - \frac{17R_{\alpha\nu; \beta}{}^{\beta}{}_{R_{\mu}{}^{\alpha}}}{630} - \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma\beta}{}_{R_{\mu}{}^{\alpha}}}{210} - \frac{R_{\alpha\nu\beta\gamma}{}^{;\beta\gamma}{}_{R_{\mu}{}^{\alpha}}}{210} - \frac{19R_{\alpha\beta}{}^{;\alpha}{}_{\nu}R_{\mu}{}^{\beta}}{1260} - \frac{17R_{\alpha\nu; \beta}{}^{\alpha}{}_{R_{\mu}{}^{\beta}}}{2520} + \\
& \frac{5R_{\alpha\nu}{}^{;\alpha}{}_{\beta}R_{\mu}{}^{\beta}}{504} + \frac{2R_{\alpha\nu\beta\gamma}{}^{;\alpha\gamma}{}_{R_{\mu}{}^{\beta}}}{315} + \frac{2R_{\alpha\beta\gamma\nu}{}^{;\alpha\beta}{}_{R_{\mu}{}^{\gamma}}}{315} + \frac{R_{;\alpha\mu}R_{\nu}{}^{\alpha}}{630} - \\
& \frac{47R_{\alpha\beta}{}^{;\beta}{}_{\mu}R_{\nu}{}^{\alpha}}{1260} - \frac{17R_{\alpha\mu; \beta}{}^{\beta}{}_{R_{\nu}{}^{\alpha}}}{630} - \frac{R_{\alpha\beta\gamma\mu}{}^{;\gamma\beta}{}_{R_{\nu}{}^{\alpha}}}{210} - \frac{R_{\alpha\mu\beta\gamma}{}^{;\beta\gamma}{}_{R_{\nu}{}^{\alpha}}}{210} - \\
& \frac{RR_{\alpha\mu}R_{\nu}{}^{\alpha}}{45} - \frac{19R_{\alpha\beta}{}^{;\alpha}{}_{\mu}R_{\nu}{}^{\beta}}{1260} - \frac{17R_{\alpha\mu; \beta}{}^{\alpha}{}_{R_{\nu}{}^{\beta}}}{2520} + \frac{5R_{\alpha\mu}{}^{;\alpha}{}_{\beta}R_{\nu}{}^{\beta}}{504} + \\
& \frac{2R_{\alpha\mu\beta\gamma}{}^{;\alpha\gamma}{}_{R_{\nu}{}^{\beta}}}{315} + \frac{R_{\alpha\beta}R_{\mu}{}^{\alpha}R_{\nu}{}^{\beta}}{540} + \frac{2R_{\alpha\beta\gamma\mu}{}^{;\alpha\beta}{}_{R_{\nu}{}^{\gamma}}}{315} - \frac{16R_{\alpha\beta; \mu\nu}R^{\alpha\beta}}{315} - \\
& \frac{19R_{\alpha\mu; \beta\nu}R^{\alpha\beta}}{315} - \frac{19R_{\alpha\nu; \beta\mu}R^{\alpha\beta}}{315} + \frac{4R_{\mu\nu; \alpha\beta}R^{\alpha\beta}}{315} + \frac{26R_{\alpha\beta\gamma\mu}{}^{;\gamma}{}_{\nu}R^{\alpha\beta}}{315} - \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma}{}_{\mu}R^{\alpha\beta}}{1260} + \frac{R_{\alpha\mu\beta\gamma}{}^{;\gamma}{}_{\nu}R^{\alpha\beta}}{45} + \frac{R_{\alpha\mu\beta\nu; \gamma}R^{\alpha\beta}}{90} + \frac{R_{\alpha\nu\beta\gamma}{}^{;\gamma}{}_{\mu}R^{\alpha\beta}}{45} + \\
& \frac{R_{\alpha\nu\beta\mu; \gamma}R^{\alpha\beta}}{90} - \frac{R_{\alpha\beta}R_{\mu\nu}R^{\alpha\beta}}{180} + \frac{R_{\alpha\beta\gamma\mu}{}^{;\beta}{}_{\nu}R^{\alpha\gamma}}{45} + \frac{R_{\alpha\beta\gamma\nu}{}^{;\beta}{}_{\mu}R^{\alpha\gamma}}{45} - \\
& \frac{R_{\alpha\mu\beta\gamma}{}^{;\beta}{}_{\nu}R^{\alpha\gamma}}{210} - \frac{R_{\alpha\mu\beta\nu}{}^{;\beta}{}_{\gamma}R^{\alpha\gamma}}{210} - \frac{R_{\alpha\nu\beta\gamma}{}^{;\beta}{}_{\mu}R^{\alpha\gamma}}{210} - \frac{R_{\alpha\nu\beta\mu}{}^{;\beta}{}_{\gamma}R^{\alpha\gamma}}{210} +
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha\nu}R^{\beta\gamma}}{90} + \frac{R_{\alpha\beta\gamma\nu}{}^{\alpha\mu}R^{\beta\gamma}}{90} + \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha\nu}R^{\beta\gamma}}{90} + \frac{R_{\alpha\mu\beta\nu}{}^{\alpha\gamma}R^{\beta\gamma}}{90} + \\
& \frac{7R_{\alpha\nu\beta\gamma}{}^{\alpha\mu}R^{\beta\gamma}}{180} + \frac{R_{\alpha\nu\beta\mu}{}^{\alpha\gamma}R^{\beta\gamma}}{90} + \frac{R_{\gamma\alpha\beta}R_{\mu\nu}{}^{\alpha\beta}}{60} + \frac{R_{\alpha\beta;\gamma}R_{\mu\nu}{}^{\alpha\beta}}{28} + \\
& \frac{RR_{\alpha\beta}R_{\mu\nu}{}^{\alpha\beta}}{54} + \frac{2R_{\alpha\beta\gamma\delta}{}^{;\beta\delta}R_{\mu\nu}{}^{\alpha\gamma}}{105} + \frac{2R_{\alpha\beta\gamma\delta}{}^{;\delta\beta}R_{\mu\nu}{}^{\alpha\gamma}}{105} - \frac{37R_{\alpha\beta}R_{\gamma}{}^{\alpha}R_{\mu\nu}{}^{\beta\gamma}}{2520} - \\
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\gamma\delta}R_{\mu\nu}{}^{\beta\epsilon}}{945} + \frac{17R_{\alpha\beta}R_{\gamma\delta}{}^{\alpha\beta}R_{\mu\nu}{}^{\gamma\delta}}{3780} + \frac{R_{\alpha\beta}R_{\gamma}{}^{\alpha\beta}R_{\mu\nu}{}^{\gamma\delta}}{180} + \\
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\beta\gamma}R_{\mu\nu}{}^{\delta\epsilon}}{945} + \frac{R_{\gamma\alpha\beta}R_{\mu}{}^{\alpha\nu\beta}}{90} - \frac{R_{\alpha\beta;\gamma}R_{\mu}{}^{\alpha\nu\beta}}{630} + \\
& \frac{RR_{\alpha\beta}R_{\mu}{}^{\alpha\nu\beta}}{180} + \frac{31R_{\alpha\beta;\gamma}{}^{\beta}R_{\mu}{}^{\alpha\nu\gamma}}{1260} + \frac{R_{\alpha\beta}{}^{;\beta}R_{\mu}{}^{\alpha\nu\gamma}}{126} + \frac{2R_{\alpha\beta\gamma\delta}{}^{;\beta\delta}R_{\mu}{}^{\alpha\nu\gamma}}{315} + \\
& \frac{2R_{\alpha\beta\gamma\delta}{}^{;\delta\beta}R_{\mu}{}^{\alpha\nu\gamma}}{315} - \frac{R_{\alpha\beta;\gamma\nu}R_{\mu}{}^{\alpha\beta\gamma}}{630} - \frac{23R_{\alpha\nu;\beta\gamma}R_{\mu}{}^{\alpha\beta\gamma}}{630} + \frac{4R_{\alpha\beta\gamma\delta}{}^{;\delta\nu}R_{\mu}{}^{\alpha\beta\gamma}}{315} - \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{;\delta}R_{\mu}{}^{\alpha\beta\gamma}}{105} - \frac{2R_{\alpha\nu\beta\gamma}{}^{;\delta}R_{\mu}{}^{\alpha\beta\gamma}}{315} + \frac{R_{\alpha\beta}R_{\gamma\nu}R_{\mu}{}^{\alpha\beta\gamma}}{56} + \frac{RR_{\alpha\beta\gamma\nu}R_{\mu}{}^{\alpha\beta\gamma}}{270} - \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma}R_{\mu}{}^{\alpha\beta\delta}}{63} - \frac{2R_{\alpha\nu\beta\gamma}{}^{;\delta}R_{\mu}{}^{\alpha\beta\delta}}{315} - \frac{2R_{\alpha\nu\beta\gamma}{}^{;\gamma}R_{\mu}{}^{\alpha\beta\delta}}{315} - \frac{4R_{\alpha\beta\gamma\delta}{}^{;\beta\nu}R_{\mu}{}^{\alpha\gamma\delta}}{315} - \\
& \frac{2R_{\alpha\beta\gamma\nu}{}^{;\delta}R_{\mu}{}^{\alpha\gamma\delta}}{315} - \frac{2R_{\alpha\beta\gamma\nu}{}^{;\beta\delta}R_{\mu}{}^{\alpha\gamma\delta}}{315} + \frac{R_{\gamma\alpha\beta}R_{\mu}{}^{\beta\nu\alpha}}{90} - \frac{R_{\alpha\beta;\gamma}R_{\mu}{}^{\beta\nu\alpha}}{630} + \\
& \frac{RR_{\alpha\beta}R_{\mu}{}^{\beta\nu\alpha}}{180} - \frac{13R_{\alpha\beta;\gamma}{}^{\alpha}R_{\mu}{}^{\beta\nu\gamma}}{1260} - \frac{R_{\alpha\beta}{}^{;\alpha}R_{\mu}{}^{\beta\nu\gamma}}{105} + \frac{2R_{\alpha\beta\gamma\delta}{}^{;\alpha\delta}R_{\mu}{}^{\beta\nu\gamma}}{315} + \\
& \frac{R_{\alpha\beta}R_{\gamma}{}^{\alpha}R_{\mu}{}^{\beta\nu\gamma}}{210} - \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\gamma\delta}R_{\mu}{}^{\beta\nu\epsilon}}{945} - \frac{R_{\alpha\beta;\gamma\nu}R_{\mu}{}^{\beta\alpha\gamma}}{210} + \frac{R_{\alpha\nu;\beta\gamma}R_{\mu}{}^{\beta\alpha\gamma}}{126} - \\
& \frac{4R_{\alpha\beta\gamma\delta}{}^{;\delta\nu}R_{\mu}{}^{\beta\alpha\gamma}}{105} - \frac{11R_{\alpha\beta\gamma\nu}{}^{;\delta}R_{\mu}{}^{\beta\alpha\gamma}}{315} - \frac{2R_{\alpha\nu\beta\gamma}{}^{;\delta}R_{\mu}{}^{\beta\alpha\gamma}}{315} - \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma}R_{\mu}{}^{\beta\alpha\delta}}{63} - \\
& \frac{2R_{\alpha\nu\beta\gamma}{}^{;\delta}R_{\mu}{}^{\beta\alpha\delta}}{315} - \frac{2R_{\alpha\nu\beta\gamma}{}^{;\gamma}R_{\mu}{}^{\beta\alpha\delta}}{315} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha\nu}R_{\mu}{}^{\beta\gamma\delta}}{315} - \frac{R_{\alpha\beta\gamma\nu}{}^{;\alpha}R_{\mu}{}^{\beta\gamma\delta}}{315} + \\
& \frac{193R_{\alpha\nu}R_{\beta\gamma\delta}{}^{\alpha}R_{\mu}{}^{\beta\gamma\delta}}{5670} + \frac{11R_{\alpha\beta}R_{\gamma\delta\nu}{}^{\alpha}R_{\mu}{}^{\beta\gamma\delta}}{1260} - \frac{23R_{\alpha\beta}R_{\gamma\nu\delta}{}^{\alpha}R_{\mu}{}^{\beta\gamma\delta}}{756} + \\
& \frac{32R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\delta}R_{\mu}{}^{\beta\gamma\epsilon}}{2835} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_{\mu}{}^{\beta\delta\epsilon}}{315} + \frac{44R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\nu\gamma}R_{\mu}{}^{\beta\delta\epsilon}}{2835} + \\
& \frac{31R_{\alpha\beta;\gamma}{}^{\beta}R_{\mu}{}^{\gamma\nu\alpha}}{1260} + \frac{R_{\alpha\beta}{}^{;\beta}R_{\mu}{}^{\gamma\nu\alpha}}{126} + \frac{2R_{\alpha\beta\gamma\delta}{}^{;\beta\delta}R_{\mu}{}^{\gamma\nu\alpha}}{315} + \frac{2R_{\alpha\beta\gamma\delta}{}^{;\delta\beta}R_{\mu}{}^{\gamma\nu\alpha}}{315} - \\
& \frac{13R_{\alpha\beta;\gamma}{}^{\alpha}R_{\mu}{}^{\gamma\nu\beta}}{1260} - \frac{R_{\alpha\beta}{}^{;\alpha}R_{\mu}{}^{\gamma\nu\beta}}{105} + \frac{2R_{\alpha\beta\gamma\delta}{}^{;\alpha\delta}R_{\mu}{}^{\gamma\nu\beta}}{315} + \frac{R_{\alpha\beta}R_{\gamma}{}^{\alpha}R_{\mu}{}^{\gamma\nu\beta}}{210} + \\
& \frac{2R_{\alpha\beta\gamma\delta}{}^{;\alpha\beta}R_{\mu}{}^{\gamma\nu\delta}}{315} - \frac{13R_{\alpha\beta}R_{\gamma\delta}{}^{\alpha\beta}R_{\mu}{}^{\gamma\nu\delta}}{2835} + \frac{44R_{\alpha\beta}R_{\gamma}{}^{\alpha\beta}R_{\mu}{}^{\gamma\nu\delta}}{2835} - \frac{47R_{\alpha\beta;\gamma\nu}R_{\mu}{}^{\gamma\alpha\beta}}{630} + \\
& \frac{R_{\alpha\nu;\beta\gamma}R_{\mu}{}^{\gamma\alpha\beta}}{90} - \frac{4R_{\alpha\beta\gamma\delta}{}^{;\delta\nu}R_{\mu}{}^{\gamma\alpha\beta}}{315} - \frac{2R_{\alpha\beta\gamma\nu}{}^{;\delta}R_{\mu}{}^{\gamma\alpha\beta}}{315} - \frac{2R_{\alpha\nu\beta\gamma}{}^{;\delta}R_{\mu}{}^{\gamma\alpha\beta}}{105} - \\
& \frac{317R_{\alpha\beta}R_{\gamma\nu}R_{\mu}{}^{\gamma\alpha\beta}}{11340} - \frac{4R_{\alpha\beta\gamma\delta}{}^{;\beta\nu}R_{\mu}{}^{\gamma\alpha\delta}}{315} - \frac{2R_{\alpha\beta\gamma\nu}{}^{;\beta}R_{\mu}{}^{\gamma\alpha\delta}}{315} - \frac{2R_{\alpha\beta\gamma\nu}{}^{;\beta\delta}R_{\mu}{}^{\gamma\alpha\delta}}{315} -
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\gamma\beta\delta}}{63} - \frac{R_{\alpha\nu\beta\gamma}{}^i{}_\delta R_\mu{}^{\gamma\beta\delta}}{315} - \frac{11R_{\alpha\beta}R_{\gamma\delta\nu}{}^\alpha R_\mu{}^{\gamma\beta\delta}}{756} - \frac{R_{\alpha\beta}R_{\gamma\nu\delta}{}^\alpha R_\mu{}^{\gamma\beta\delta}}{315} + \\
& \frac{26R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\gamma\delta\epsilon}}{2835} + \frac{83R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\gamma\delta\epsilon}}{2835} + \frac{2R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\delta\alpha\beta}}{315} + \\
& \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{189} - \frac{2R_{\alpha\nu\beta\gamma}{}^i{}_\delta R_\mu{}^{\delta\alpha\beta}}{105} - \frac{2R_{\alpha\nu\beta\gamma}{}^i{}_\delta R_\mu{}^{\delta\alpha\beta}}{105} - \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\delta\alpha\beta}}{105} - \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\delta\alpha\beta}}{105} - \\
& \frac{2R_{\alpha\beta\gamma\nu}{}^i{}_\delta R_\mu{}^{\delta\alpha\beta}}{105} - \frac{2R_{\alpha\beta\gamma\nu}{}^i{}_\delta R_\mu{}^{\delta\alpha\beta}}{105} - \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\delta\alpha\beta}}{315} + \frac{R_{\alpha\beta\gamma\nu}{}^i{}_\delta R_\mu{}^{\delta\alpha\beta}}{315} - \\
& \frac{R_{\alpha\nu\beta\gamma}{}^i{}_\delta R_\mu{}^{\delta\alpha\beta}}{315} - \frac{17R_{\alpha\nu}R_{\beta\gamma\delta}{}^\alpha R_\mu{}^{\delta\beta\gamma}}{11340} + \frac{8R_{\alpha\beta}R_{\gamma\nu\delta}{}^\alpha R_\mu{}^{\delta\beta\gamma}}{945} - \\
& \frac{11R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_\mu{}^{\delta\beta\epsilon}}{2835} - \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_\mu{}^{\delta\beta\epsilon}}{945} - \frac{97R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{11340} + \\
& \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{81} - \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{945} + \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{189} + \\
& \frac{8R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{2835} + \frac{8R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{2835} - \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{2835} - \\
& \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{567} - \frac{23R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_\mu{}^{\delta\gamma\epsilon}}{11340} - \frac{R_{\alpha\beta;\gamma\mu}R_\nu{}^{\alpha\beta\gamma}}{630} - \frac{23R_{\alpha\mu;\beta\gamma}R_\nu{}^{\alpha\beta\gamma}}{630} + \\
& \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\alpha\beta\gamma}}{315} - \frac{R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\alpha\beta\gamma}}{105} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\alpha\beta\gamma}}{315} + \frac{R_{\alpha\beta}R_{\gamma\mu}R_\nu{}^{\alpha\beta\gamma}}{56} + \\
& \frac{RR_{\alpha\beta\gamma\mu}R_\nu{}^{\alpha\beta\gamma}}{270} - \frac{R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\alpha\beta\delta}}{63} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\alpha\beta\delta}}{315} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\alpha\beta\delta}}{315} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\alpha\beta\delta}}{315} - \\
& \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\alpha\beta\gamma}}{315} - \frac{2R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\alpha\beta\gamma}}{315} - \frac{2R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\alpha\beta\gamma}}{315} - \frac{R_{\alpha\beta;\gamma\mu}R_\nu{}^{\beta\alpha\gamma}}{210} + \\
& \frac{R_{\alpha\mu;\beta\gamma}R_\nu{}^{\beta\alpha\gamma}}{126} - \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\beta\alpha\gamma}}{105} - \frac{11R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\beta\alpha\gamma}}{315} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\beta\alpha\gamma}}{315} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\beta\alpha\delta}}{63} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\beta\alpha\delta}}{315} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\beta\alpha\delta}}{315} - \frac{R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\beta\gamma\delta}}{315} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\beta\gamma\delta}}{315} + \frac{193R_{\alpha\mu}R_{\beta\gamma\delta}{}^\alpha R_\nu{}^{\beta\gamma\delta}}{5670} + \frac{11R_{\alpha\beta}R_{\gamma\delta\mu}{}^\alpha R_\nu{}^{\beta\gamma\delta}}{1260} - \\
& \frac{23R_{\alpha\beta}R_{\gamma\mu\delta}{}^\alpha R_\nu{}^{\beta\gamma\delta}}{756} + \frac{32R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\delta}R_\mu{}^{\beta\gamma\epsilon}}{2835} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\gamma}R_\nu{}^{\beta\delta\epsilon}}{315} + \\
& \frac{44R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\gamma}R_\nu{}^{\beta\delta\epsilon}}{2835} - \frac{47R_{\alpha\beta;\gamma\mu}R_\nu{}^{\gamma\alpha\beta}}{630} + \frac{R_{\alpha\mu;\beta\gamma}R_\nu{}^{\gamma\alpha\beta}}{90} - \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\gamma\alpha\beta}}{315} - \\
& \frac{2R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\gamma\alpha\beta}}{315} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\gamma\alpha\beta}}{105} - \frac{317R_{\alpha\beta}R_{\gamma\mu}R_\nu{}^{\gamma\alpha\beta}}{11340} - \frac{RR_{\alpha\beta\gamma\mu}R_\nu{}^{\gamma\alpha\beta}}{135} - \\
& \frac{4R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\gamma\alpha\delta}}{315} - \frac{2R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\gamma\alpha\delta}}{315} - \frac{2R_{\alpha\beta\gamma\mu}{}^i{}_\delta R_\nu{}^{\gamma\alpha\delta}}{315} - \frac{R_{\alpha\beta\gamma\delta}{}^i{}_\nu R_\mu{}^{\gamma\beta\delta}}{63} - \\
& \frac{R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\gamma\beta\delta}}{315} - \frac{11R_{\alpha\beta}R_{\gamma\delta\mu}{}^\alpha R_\nu{}^{\gamma\beta\delta}}{756} - \frac{R_{\alpha\beta}R_{\gamma\mu\delta}{}^\alpha R_\nu{}^{\gamma\beta\delta}}{315} + \\
& \frac{26R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\beta}R_\nu{}^{\gamma\delta\epsilon}}{2835} + \frac{83R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\beta}R_\nu{}^{\gamma\delta\epsilon}}{2835} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\delta\alpha\beta}}{105} - \frac{2R_{\alpha\mu\beta\gamma}{}^i{}_\delta R_\nu{}^{\delta\alpha\beta}}{105} -
\end{aligned}$$

$$\begin{aligned}
& \frac{4R_{\alpha\beta\gamma\delta}{}^{\beta}{}_{\mu}R_{\nu}{}^{\delta\alpha\gamma}}{105} - \frac{2R_{\alpha\beta\gamma\mu}{}^{\beta}{}_{\delta}R_{\nu}{}^{\delta\alpha\gamma}}{105} - \frac{2R_{\alpha\beta\gamma\mu}{}^{\beta}{}_{\delta}R_{\nu}{}^{\delta\alpha\gamma}}{105} - \frac{4R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\mu}R_{\nu}{}^{\delta\beta\gamma}}{315} + \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha}{}_{\delta}R_{\nu}{}^{\delta\beta\gamma}}{315} - \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha}{}_{\delta}R_{\nu}{}^{\delta\beta\gamma}}{315} - \frac{17R_{\alpha\mu}R_{\beta\gamma\delta}{}^{\alpha}R_{\nu}{}^{\delta\beta\gamma}}{11340} + \frac{8R_{\alpha\beta}R_{\gamma\mu\delta}{}^{\alpha}R_{\nu}{}^{\delta\beta\gamma}}{945} - \\
& \frac{11R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\gamma}R_{\nu}{}^{\delta\beta\epsilon}}{2835} - \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma}R_{\nu}{}^{\delta\beta\epsilon}}{945} - \frac{97R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\beta}R_{\nu}{}^{\delta\gamma\epsilon}}{11340} + \\
& \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\beta}R_{\nu}{}^{\delta\gamma\epsilon}}{81} + \frac{8R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\delta}R_{\nu}{}^{\epsilon\beta\gamma}}{2835} - \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma}R_{\nu}{}^{\epsilon\beta\delta}}{2835} - \\
& \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\beta}R_{\nu}{}^{\epsilon\gamma\delta}}{315} - \frac{23R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\beta}R_{\nu}{}^{\epsilon\gamma\delta}}{11340} + \frac{4R_{\alpha\beta\gamma\delta}{}_{;\mu\nu}R^{\alpha\beta\gamma\delta}}{315} + \frac{4R_{\alpha\beta\gamma\mu}{}_{;\delta\nu}R^{\alpha\beta\gamma\delta}}{315} + \\
& \frac{4R_{\alpha\beta\gamma\nu}{}_{;\delta\mu}R^{\alpha\beta\gamma\delta}}{315} + \frac{2R_{\alpha\mu\beta\nu}{}_{;\gamma\delta}R^{\alpha\beta\gamma\delta}}{105} + \frac{2R_{\alpha\nu\beta\mu}{}_{;\gamma\delta}R^{\alpha\beta\gamma\delta}}{105} + \frac{R_{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}{270} - \\
& \frac{R_{\alpha\nu}R_{\beta\gamma\delta\mu}R^{\alpha\beta\gamma\delta}}{81} - \frac{R_{\alpha\mu}R_{\beta\gamma\delta\nu}R^{\alpha\beta\gamma\delta}}{81} - \frac{4R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\gamma}R^{\alpha\beta\delta\epsilon}}{567} + \\
& \frac{32R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\gamma}R^{\alpha\beta\delta\epsilon}}{2835} - \frac{R_{\alpha\beta\gamma\delta}{}_{;\mu\nu}R^{\alpha\gamma\beta\delta}}{63} + \frac{2R_{\alpha\beta\gamma\mu}{}_{;\delta\nu}R^{\alpha\gamma\beta\delta}}{63} + \\
& \frac{2R_{\alpha\beta\gamma\nu}{}_{;\delta\mu}R^{\alpha\gamma\beta\delta}}{63} + \frac{4R_{\alpha\mu\beta\gamma}{}_{;\delta\nu}R^{\alpha\gamma\beta\delta}}{315} + \frac{2R_{\alpha\mu\beta\nu}{}_{;\gamma\delta}R^{\alpha\gamma\beta\delta}}{315} + \frac{4R_{\alpha\nu\beta\gamma}{}_{;\delta\mu}R^{\alpha\gamma\beta\delta}}{315} + \\
& \frac{2R_{\alpha\nu\beta\mu}{}_{;\gamma\delta}R^{\alpha\gamma\beta\delta}}{315} + \frac{R_{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\gamma\beta\delta}}{270} - \frac{R_{\alpha\beta}R_{\gamma\mu\delta\nu}R^{\alpha\gamma\beta\delta}}{81} + \\
& \frac{16R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\beta}R^{\alpha\gamma\delta\epsilon}}{945} - \frac{R_{\alpha\beta\gamma\delta}{}_{;\mu\nu}R^{\alpha\delta\beta\gamma}}{35} + \frac{2R_{\alpha\beta\gamma\mu}{}_{;\delta\nu}R^{\alpha\delta\beta\gamma}}{105} + \frac{2R_{\alpha\beta\gamma\nu}{}_{;\delta\mu}R^{\alpha\delta\beta\gamma}}{105} + \\
& \frac{4R_{\alpha\mu\beta\gamma}{}_{;\delta\nu}R^{\alpha\delta\beta\gamma}}{315} + \frac{2R_{\alpha\mu\beta\nu}{}_{;\gamma\delta}R^{\alpha\delta\beta\gamma}}{315} + \frac{4R_{\alpha\nu\beta\gamma}{}_{;\delta\mu}R^{\alpha\delta\beta\gamma}}{315} + \frac{2R_{\alpha\nu\beta\mu}{}_{;\gamma\delta}R^{\alpha\delta\beta\gamma}}{315} + \\
& \frac{R_{\alpha\nu}R_{\beta\gamma\delta\mu}R^{\alpha\delta\beta\gamma}}{81} + \frac{R_{\alpha\mu}R_{\beta\gamma\delta\nu}R^{\alpha\delta\beta\gamma}}{81} - \frac{8R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\gamma}R^{\alpha\delta\beta\epsilon}}{2835} + \\
& \frac{8R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\gamma}R^{\alpha\delta\beta\epsilon}}{2835} - \frac{4R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\gamma}R^{\alpha\epsilon\beta\delta}}{945} - \frac{4R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\beta}R^{\alpha\epsilon\gamma\delta}}{945} - \\
& \frac{32R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{2835} - \frac{16R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{945} + \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon\mu\nu}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{945} + \\
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon\nu\mu}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{945} - \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu\nu}R^{\beta\gamma\delta\epsilon}}{945} - \frac{22R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\alpha}R^{\beta\delta\gamma\epsilon}}{2835} - \\
& \frac{2R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\alpha}R^{\beta\epsilon\gamma\delta}}{2835} + \frac{4R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\alpha}R^{\beta\epsilon\gamma\delta}}{945} - \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\mu\nu}{}^{\alpha}R^{\beta\epsilon\gamma\delta}}{189} - \\
& \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu\mu}{}^{\alpha}R^{\beta\epsilon\gamma\delta}}{189} + \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu\nu}R^{\beta\epsilon\gamma\delta}}{945}
\end{aligned}$$

## APPENDIX C

Coincidence limits  $[a_{0|\mu\nu\rho\tau\kappa}]$  and  $[a_{0|\mu\nu\rho\tau\kappa\lambda}]$

$$\begin{aligned}
[a_{0|\mu\nu\rho\tau\kappa}] = & \frac{ie}{5}F_{\mu\nu;\rho\tau\kappa} + \frac{ie}{5}F_{\mu\rho;\nu\tau\kappa} + \frac{ie}{5}F_{\mu\tau;\nu\rho\kappa} + \frac{ie}{5}F_{\mu\kappa;\nu\rho\tau} - \\
& \frac{e^2F_{\rho\tau;\kappa}F_{\mu\nu}}{6} - \frac{e^2F_{\rho\kappa;\tau}F_{\mu\nu}}{6} - \frac{e^2F_{\nu\tau;\kappa}F_{\mu\rho}}{6} - \frac{e^2F_{\nu\kappa;\tau}F_{\mu\rho}}{6} - \\
& \frac{e^2F_{\nu\rho;\kappa}F_{\mu\tau}}{6} - \frac{e^2F_{\nu\kappa;\rho}F_{\mu\tau}}{6} - \frac{e^2F_{\nu\rho;\tau}F_{\mu\kappa}}{6} - \frac{e^2F_{\nu\tau;\rho}F_{\mu\kappa}}{6} + \\
& \frac{ie}{10}R_{\alpha\nu\rho\tau;\kappa}F_{\mu}^{\alpha} + \frac{ie}{10}R_{\alpha\nu\rho\kappa;\tau}F_{\mu}^{\alpha} - \frac{ie}{40}R_{\alpha\rho\nu\tau;\kappa}F_{\mu}^{\alpha} - \frac{ie}{40}R_{\alpha\rho\nu\kappa;\tau}F_{\mu}^{\alpha} - \\
& \frac{ie}{40}R_{\alpha\tau\nu\rho;\kappa}F_{\mu}^{\alpha} - \frac{ie}{40}R_{\alpha\tau\nu\kappa;\rho}F_{\mu}^{\alpha} - \frac{ie}{40}R_{\alpha\kappa\nu\rho;\tau}F_{\mu}^{\alpha} - \frac{ie}{40}R_{\alpha\kappa\nu\tau;\rho}F_{\mu}^{\alpha} - \\
& \frac{e^2F_{\mu\tau;\kappa}F_{\nu\rho}}{6} - \frac{e^2F_{\mu\kappa;\tau}F_{\nu\rho}}{6} - \frac{e^2F_{\mu\rho;\kappa}F_{\nu\tau}}{6} - \frac{e^2F_{\mu\kappa;\rho}F_{\nu\tau}}{6} - \\
& \frac{e^2F_{\mu\rho;\tau}F_{\nu\kappa}}{6} - \frac{e^2F_{\mu\tau;\rho}F_{\nu\kappa}}{6} - \frac{ie}{10}R_{\alpha\mu\rho\tau;\kappa}F_{\nu}^{\alpha} - \frac{ie}{10}R_{\alpha\mu\rho\kappa;\tau}F_{\nu}^{\alpha} - \\
& \frac{ie}{40}R_{\alpha\rho\mu\tau;\kappa}F_{\nu}^{\alpha} - \frac{ie}{40}R_{\alpha\rho\mu\kappa;\tau}F_{\nu}^{\alpha} - \frac{ie}{40}R_{\alpha\tau\mu\rho;\kappa}F_{\nu}^{\alpha} - \frac{ie}{40}R_{\alpha\tau\mu\kappa;\rho}F_{\nu}^{\alpha} - \\
& \frac{ie}{40}R_{\alpha\kappa\mu\rho;\tau}F_{\nu}^{\alpha} - \frac{ie}{40}R_{\alpha\kappa\mu\tau;\rho}F_{\nu}^{\alpha} - \frac{e^2F_{\mu\nu;\kappa}F_{\rho\tau}}{6} - \frac{e^2F_{\mu\kappa;\nu}F_{\rho\tau}}{6} - \\
& \frac{e^2F_{\mu\nu;\tau}F_{\rho\kappa}}{6} - \frac{e^2F_{\mu\tau;\nu}F_{\rho\kappa}}{6} - \frac{ie}{10}R_{\alpha\mu\nu\tau;\kappa}F_{\rho}^{\alpha} - \frac{ie}{10}R_{\alpha\mu\nu\kappa;\tau}F_{\rho}^{\alpha} - \\
& \frac{ie}{40}R_{\alpha\nu\mu\tau;\kappa}F_{\rho}^{\alpha} - \frac{ie}{40}R_{\alpha\nu\mu\kappa;\tau}F_{\rho}^{\alpha} - \frac{ie}{40}R_{\alpha\tau\mu\nu;\kappa}F_{\rho}^{\alpha} - \frac{ie}{40}R_{\alpha\tau\mu\kappa;\nu}F_{\rho}^{\alpha} - \\
& \frac{ie}{40}R_{\alpha\kappa\mu\nu;\tau}F_{\rho}^{\alpha} - \frac{ie}{40}R_{\alpha\kappa\mu\tau;\nu}F_{\rho}^{\alpha} - \frac{e^2F_{\mu\nu;\rho}F_{\tau\kappa}}{6} - \frac{e^2F_{\mu\kappa;\nu}F_{\tau\kappa}}{6} - \\
& \frac{ie}{10}R_{\alpha\mu\nu\rho;\kappa}F_{\tau}^{\alpha} - \frac{ie}{10}R_{\alpha\mu\nu\kappa;\rho}F_{\tau}^{\alpha} - \frac{ie}{40}R_{\alpha\nu\mu\rho;\kappa}F_{\tau}^{\alpha} - \frac{ie}{40}R_{\alpha\nu\mu\kappa;\rho}F_{\tau}^{\alpha} - \\
& \frac{ie}{40}R_{\alpha\rho\mu\nu;\kappa}F_{\tau}^{\alpha} - \frac{ie}{40}R_{\alpha\rho\mu\kappa;\nu}F_{\tau}^{\alpha} - \frac{ie}{40}R_{\alpha\kappa\mu\nu;\rho}F_{\tau}^{\alpha} - \frac{ie}{40}R_{\alpha\kappa\mu\rho;\nu}F_{\tau}^{\alpha} - \\
& \frac{ie}{10}R_{\alpha\mu\nu\rho;\tau}F_{\kappa}^{\alpha} - \frac{ie}{10}R_{\alpha\mu\nu\tau;\rho}F_{\kappa}^{\alpha} - \frac{ie}{40}R_{\alpha\nu\mu\rho;\tau}F_{\kappa}^{\alpha} - \frac{ie}{40}R_{\alpha\nu\mu\tau;\rho}F_{\kappa}^{\alpha} - \\
& \frac{ie}{40}R_{\alpha\rho\mu\nu;\tau}F_{\kappa}^{\alpha} - \frac{ie}{40}R_{\alpha\rho\mu\tau;\nu}F_{\kappa}^{\alpha} - \frac{ie}{40}R_{\alpha\tau\mu\nu;\rho}F_{\kappa}^{\alpha} - \frac{ie}{40}R_{\alpha\tau\mu\rho;\nu}F_{\kappa}^{\alpha} - \\
& \frac{ie}{45}F_{\alpha\tau;\kappa}R_{\mu\nu\rho}^{\alpha} - \frac{ie}{45}F_{\alpha\kappa;\tau}R_{\mu\nu\rho}^{\alpha} - \frac{ie}{45}F_{\alpha\rho;\kappa}R_{\mu\nu\tau}^{\alpha} - \frac{ie}{45}F_{\alpha\kappa;\rho}R_{\mu\nu\tau}^{\alpha} - \\
& \frac{ie}{45}F_{\alpha\rho;\tau}R_{\mu\nu\kappa}^{\alpha} - \frac{ie}{45}F_{\alpha\tau;\rho}R_{\mu\nu\kappa}^{\alpha} - \frac{ie}{45}F_{\alpha\tau;\kappa}R_{\mu\rho\nu}^{\alpha} - \frac{ie}{45}F_{\alpha\kappa;\tau}R_{\mu\rho\nu}^{\alpha} - \\
& \frac{ie}{45}F_{\alpha\nu;\kappa}R_{\mu\rho\tau}^{\alpha} - \frac{ie}{45}F_{\alpha\kappa;\nu}R_{\mu\rho\tau}^{\alpha} - \frac{ie}{45}F_{\alpha\nu;\tau}R_{\mu\rho\kappa}^{\alpha} - \frac{ie}{45}F_{\alpha\tau;\nu}R_{\mu\rho\kappa}^{\alpha} - \\
& \frac{ie}{45}F_{\alpha\rho;\kappa}R_{\mu\tau\nu}^{\alpha} - \frac{ie}{45}F_{\alpha\kappa;\rho}R_{\mu\tau\nu}^{\alpha} - \frac{ie}{45}F_{\alpha\nu;\kappa}R_{\mu\tau\rho}^{\alpha} - \frac{ie}{45}F_{\alpha\kappa;\nu}R_{\mu\tau\rho}^{\alpha} - \\
& \frac{ie}{45}F_{\alpha\nu;\rho}R_{\mu\tau\kappa}^{\alpha} - \frac{ie}{45}F_{\alpha\rho;\nu}R_{\mu\tau\kappa}^{\alpha} - \frac{ie}{45}F_{\alpha\rho;\tau}R_{\mu\kappa\nu}^{\alpha} - \frac{ie}{45}F_{\alpha\tau;\rho}R_{\mu\kappa\nu}^{\alpha} -
\end{aligned}$$

$$\begin{aligned}
& \frac{ie}{45} F_{\alpha\nu;\tau} R_{\mu\kappa\rho}{}^\alpha - \frac{ie}{45} F_{\alpha\tau;\nu} R_{\mu\kappa\rho}{}^\alpha - \frac{ie}{45} F_{\alpha\nu;\rho} R_{\mu\kappa\tau}{}^\alpha - \frac{ie}{45} F_{\alpha\rho;\nu} R_{\mu\kappa\tau}{}^\alpha - \\
& \frac{ie}{15} F_{\alpha\tau;\kappa} R_{\mu}{}^\alpha{}_{\nu\rho} - \frac{ie}{15} F_{\alpha\kappa;\tau} R_{\mu}{}^\alpha{}_{\nu\rho} - \frac{ie}{15} F_{\alpha\rho;\kappa} R_{\mu}{}^\alpha{}_{\nu\tau} - \frac{ie}{15} F_{\alpha\kappa;\rho} R_{\mu}{}^\alpha{}_{\nu\tau} - \\
& \frac{ie}{15} F_{\alpha\rho;\tau} R_{\mu}{}^\alpha{}_{\nu\kappa} - \frac{ie}{15} F_{\alpha\tau;\rho} R_{\mu}{}^\alpha{}_{\nu\kappa} - \frac{ie}{15} F_{\alpha\nu;\kappa} R_{\mu}{}^\alpha{}_{\rho\tau} - \frac{ie}{15} F_{\alpha\kappa;\nu} R_{\mu}{}^\alpha{}_{\rho\tau} - \\
& \frac{ie}{15} F_{\alpha\nu;\tau} R_{\mu}{}^\alpha{}_{\rho\kappa} - \frac{ie}{15} F_{\alpha\tau;\nu} R_{\mu}{}^\alpha{}_{\rho\kappa} - \frac{ie}{15} F_{\alpha\nu;\rho} R_{\mu}{}^\alpha{}_{\tau\kappa} - \frac{ie}{15} F_{\alpha\rho;\nu} R_{\mu}{}^\alpha{}_{\tau\kappa} - \\
& \frac{ie}{45} F_{\alpha\mu;\kappa} R_{\nu\rho\tau}{}^\alpha - \frac{ie}{45} F_{\alpha\kappa;\mu} R_{\nu\rho\tau}{}^\alpha - \frac{ie}{45} F_{\alpha\mu;\tau} R_{\nu\rho\kappa}{}^\alpha - \frac{ie}{45} F_{\alpha\tau;\mu} R_{\nu\rho\kappa}{}^\alpha - \\
& \frac{ie}{45} F_{\alpha\mu;\kappa} R_{\nu\tau\rho}{}^\alpha - \frac{ie}{45} F_{\alpha\kappa;\mu} R_{\nu\tau\rho}{}^\alpha - \frac{ie}{45} F_{\alpha\mu;\rho} R_{\nu\tau\kappa}{}^\alpha - \frac{ie}{45} F_{\alpha\rho;\mu} R_{\nu\tau\kappa}{}^\alpha - \\
& \frac{ie}{45} F_{\alpha\mu;\tau} R_{\nu\kappa\rho}{}^\alpha - \frac{ie}{45} F_{\alpha\tau;\mu} R_{\nu\kappa\rho}{}^\alpha - \frac{ie}{45} F_{\alpha\mu;\rho} R_{\nu\kappa\tau}{}^\alpha - \frac{ie}{45} F_{\alpha\rho;\mu} R_{\nu\kappa\tau}{}^\alpha + \\
& \frac{ie}{15} F_{\alpha\mu;\kappa} R_{\nu}{}^\alpha{}_{\rho\tau} - \frac{ie}{15} F_{\mu\kappa;\alpha} R_{\nu}{}^\alpha{}_{\rho\tau} + \frac{ie}{15} F_{\alpha\mu;\tau} R_{\nu}{}^\alpha{}_{\rho\kappa} - \frac{ie}{15} F_{\mu\tau;\alpha} R_{\nu}{}^\alpha{}_{\rho\kappa} + \\
& \frac{ie}{15} F_{\alpha\mu;\rho} R_{\nu}{}^\alpha{}_{\tau\kappa} - \frac{ie}{15} F_{\mu\rho;\alpha} R_{\nu}{}^\alpha{}_{\tau\kappa} - \frac{ie}{45} F_{\alpha\mu;\nu} R_{\rho\tau\kappa}{}^\alpha - \frac{ie}{45} F_{\alpha\nu;\mu} R_{\rho\tau\kappa}{}^\alpha - \\
& \frac{ie}{45} F_{\alpha\mu;\nu} R_{\rho\kappa\tau}{}^\alpha - \frac{ie}{45} F_{\alpha\nu;\mu} R_{\rho\kappa\tau}{}^\alpha + \frac{ie}{15} F_{\alpha\mu;\nu} R_{\rho}{}^\alpha{}_{\tau\kappa} - \frac{ie}{15} F_{\mu\nu;\alpha} R_{\rho}{}^\alpha{}_{\tau\kappa} \quad (274)
\end{aligned}$$

$$[a_0]_{\mu\nu\rho\tau\kappa\lambda} =$$

$$\begin{array}{cccc}
\frac{e^2 F_{\mu\kappa;\lambda} F_{\nu\rho;\tau}}{9} & \frac{e^2 F_{\mu\lambda;\kappa} F_{\nu\rho;\tau}}{9} & \frac{e^2 F_{\mu\tau;\lambda} F_{\nu\rho;\kappa}}{9} & \frac{e^2 F_{\mu\lambda;\tau} F_{\nu\rho;\kappa}}{9} \\
\frac{e^2 F_{\mu\tau;\kappa} F_{\nu\rho;\lambda}}{9} & \frac{e^2 F_{\mu\kappa;\tau} F_{\nu\rho;\lambda}}{9} & \frac{e^2 F_{\mu\kappa;\lambda} F_{\nu\tau;\rho}}{9} & \frac{e^2 F_{\mu\lambda;\kappa} F_{\nu\tau;\rho}}{9} \\
\frac{e^2 F_{\mu\rho;\lambda} F_{\nu\tau;\kappa}}{9} & \frac{e^2 F_{\mu\lambda;\rho} F_{\nu\tau;\kappa}}{9} & \frac{e^2 F_{\mu\rho;\kappa} F_{\nu\tau;\lambda}}{9} & \frac{e^2 F_{\mu\kappa;\rho} F_{\nu\tau;\lambda}}{9} \\
\frac{e^2 F_{\mu\tau;\lambda} F_{\nu\kappa;\rho}}{9} & \frac{e^2 F_{\mu\lambda;\tau} F_{\nu\kappa;\rho}}{9} & \frac{e^2 F_{\mu\rho;\lambda} F_{\nu\kappa;\tau}}{9} & \frac{e^2 F_{\mu\lambda;\rho} F_{\nu\kappa;\tau}}{9} \\
\frac{e^2 F_{\mu\rho;\tau} F_{\nu\kappa;\lambda}}{9} & \frac{e^2 F_{\mu\tau;\rho} F_{\nu\kappa;\lambda}}{9} & \frac{e^2 F_{\mu\tau;\kappa} F_{\nu\lambda;\rho}}{9} & \frac{e^2 F_{\mu\kappa;\tau} F_{\nu\lambda;\rho}}{9} \\
\frac{e^2 F_{\mu\rho;\kappa} F_{\nu\lambda;\tau}}{9} & \frac{e^2 F_{\mu\kappa;\rho} F_{\nu\lambda;\tau}}{9} & \frac{e^2 F_{\mu\rho;\tau} F_{\nu\lambda;\kappa}}{9} & \frac{e^2 F_{\mu\tau;\rho} F_{\nu\lambda;\kappa}}{9} \\
\frac{e^2 F_{\mu\nu;\lambda} F_{\rho\tau;\kappa}}{9} & \frac{e^2 F_{\mu\lambda;\nu} F_{\rho\tau;\kappa}}{9} & \frac{e^2 F_{\mu\nu;\kappa} F_{\rho\tau;\lambda}}{9} & \frac{e^2 F_{\mu\kappa;\nu} F_{\rho\tau;\lambda}}{9} \\
\frac{e^2 F_{\mu\nu;\kappa} F_{\rho\lambda;\tau}}{9} & \frac{e^2 F_{\mu\kappa;\nu} F_{\rho\lambda;\tau}}{9} & \frac{e^2 F_{\mu\nu;\tau} F_{\rho\kappa;\lambda}}{9} & \frac{e^2 F_{\mu\tau;\nu} F_{\rho\kappa;\lambda}}{9} \\
\frac{e^2 F_{\mu\nu;\rho} F_{\tau\kappa;\lambda}}{9} & \frac{e^2 F_{\mu\rho;\nu} F_{\tau\kappa;\lambda}}{9} & \frac{e^2 F_{\mu\nu;\rho} F_{\tau\lambda;\kappa}}{9} & \frac{e^2 F_{\mu\rho;\nu} F_{\tau\lambda;\kappa}}{9}
\end{array}$$

$$\begin{aligned}
& \frac{ie}{18} F_{\alpha\kappa;\lambda} R_{\mu}^{\alpha}{}_{\nu\rho;\tau} - \frac{ie}{18} F_{\alpha\lambda;\kappa} R_{\mu}^{\alpha}{}_{\nu\rho;\tau} - \frac{ie}{18} F_{\alpha\tau;\lambda} R_{\mu}^{\alpha}{}_{\nu\rho;\kappa} - \frac{ie}{18} F_{\alpha\lambda;\tau} R_{\mu}^{\alpha}{}_{\nu\rho;\kappa} - \\
& \frac{ie}{18} F_{\alpha\tau;\kappa} R_{\mu}^{\alpha}{}_{\nu\rho;\lambda} - \frac{ie}{18} F_{\alpha\kappa;\tau} R_{\mu}^{\alpha}{}_{\nu\rho;\lambda} - \frac{ie}{18} F_{\alpha\kappa;\lambda} R_{\mu}^{\alpha}{}_{\nu\tau;\rho} - \frac{ie}{18} F_{\alpha\lambda;\kappa} R_{\mu}^{\alpha}{}_{\nu\tau;\rho} - \\
& \frac{ie}{18} F_{\alpha\rho;\lambda} R_{\mu}^{\alpha}{}_{\nu\tau;\kappa} - \frac{ie}{18} F_{\alpha\lambda;\rho} R_{\mu}^{\alpha}{}_{\nu\tau;\kappa} - \frac{ie}{18} F_{\alpha\rho;\kappa} R_{\mu}^{\alpha}{}_{\nu\tau;\lambda} - \frac{ie}{18} F_{\alpha\kappa;\rho} R_{\mu}^{\alpha}{}_{\nu\tau;\lambda} - \\
& \frac{ie}{18} F_{\alpha\tau;\lambda} R_{\mu}^{\alpha}{}_{\nu\kappa;\rho} - \frac{ie}{18} F_{\alpha\lambda;\tau} R_{\mu}^{\alpha}{}_{\nu\kappa;\rho} - \frac{ie}{18} F_{\alpha\rho;\lambda} R_{\mu}^{\alpha}{}_{\nu\kappa;\tau} - \frac{ie}{18} F_{\alpha\lambda;\rho} R_{\mu}^{\alpha}{}_{\nu\kappa;\tau} - \\
& \frac{ie}{18} F_{\alpha\rho;\tau} R_{\mu}^{\alpha}{}_{\nu\kappa;\lambda} - \frac{ie}{18} F_{\alpha\tau;\rho} R_{\mu}^{\alpha}{}_{\nu\kappa;\lambda} - \frac{ie}{18} F_{\alpha\tau;\kappa} R_{\mu}^{\alpha}{}_{\nu\lambda;\rho} - \frac{ie}{18} F_{\alpha\kappa;\tau} R_{\mu}^{\alpha}{}_{\nu\lambda;\rho} - \\
& \frac{ie}{18} F_{\alpha\rho;\kappa} R_{\mu}^{\alpha}{}_{\nu\lambda;\tau} - \frac{ie}{18} F_{\alpha\kappa;\rho} R_{\mu}^{\alpha}{}_{\nu\lambda;\tau} - \frac{ie}{18} F_{\alpha\rho;\tau} R_{\mu}^{\alpha}{}_{\nu\lambda;\kappa} - \frac{ie}{18} F_{\alpha\tau;\rho} R_{\mu}^{\alpha}{}_{\nu\lambda;\kappa} - \\
& \frac{ie}{18} F_{\alpha\nu;\lambda} R_{\mu}^{\alpha}{}_{\rho\tau;\kappa} - \frac{ie}{18} F_{\alpha\lambda;\nu} R_{\mu}^{\alpha}{}_{\rho\tau;\kappa} - \frac{ie}{18} F_{\alpha\nu;\kappa} R_{\mu}^{\alpha}{}_{\rho\tau;\lambda} - \frac{ie}{18} F_{\alpha\kappa;\nu} R_{\mu}^{\alpha}{}_{\rho\tau;\lambda} - \\
& \frac{ie}{18} F_{\alpha\nu;\lambda} R_{\mu}^{\alpha}{}_{\rho\kappa;\tau} - \frac{ie}{18} F_{\alpha\lambda;\nu} R_{\mu}^{\alpha}{}_{\rho\kappa;\tau} - \frac{ie}{18} F_{\alpha\nu;\tau} R_{\mu}^{\alpha}{}_{\rho\kappa;\lambda} - \frac{ie}{18} F_{\alpha\tau;\nu} R_{\mu}^{\alpha}{}_{\rho\kappa;\lambda} - \\
& \frac{ie}{18} F_{\alpha\nu;\kappa} R_{\mu}^{\alpha}{}_{\rho\lambda;\tau} - \frac{ie}{18} F_{\alpha\kappa;\nu} R_{\mu}^{\alpha}{}_{\rho\lambda;\tau} - \frac{ie}{18} F_{\alpha\nu;\tau} R_{\mu}^{\alpha}{}_{\rho\lambda;\kappa} - \frac{ie}{18} F_{\alpha\tau;\nu} R_{\mu}^{\alpha}{}_{\rho\lambda;\kappa} - \\
& \frac{ie}{18} F_{\alpha\nu;\rho} R_{\mu}^{\alpha}{}_{\tau\kappa;\lambda} - \frac{ie}{18} F_{\alpha\rho;\nu} R_{\mu}^{\alpha}{}_{\tau\kappa;\lambda} - \frac{ie}{18} F_{\alpha\nu;\rho} R_{\mu}^{\alpha}{}_{\tau\lambda;\kappa} - \frac{ie}{18} F_{\alpha\rho;\nu} R_{\mu}^{\alpha}{}_{\tau\lambda;\kappa} + \\
& \frac{ie}{18} F_{\alpha\mu;\lambda} R_{\nu}^{\alpha}{}_{\rho\tau;\kappa} - \frac{ie}{18} F_{\mu\lambda;\alpha} R_{\nu}^{\alpha}{}_{\rho\tau;\kappa} + \frac{ie}{18} F_{\alpha\mu;\kappa} R_{\nu}^{\alpha}{}_{\rho\tau;\lambda} - \frac{ie}{18} F_{\mu\kappa;\alpha} R_{\nu}^{\alpha}{}_{\rho\tau;\lambda} + \\
& \frac{ie}{18} F_{\alpha\mu;\lambda} R_{\nu}^{\alpha}{}_{\rho\kappa;\tau} - \frac{ie}{18} F_{\mu\lambda;\alpha} R_{\nu}^{\alpha}{}_{\rho\kappa;\tau} + \frac{ie}{18} F_{\alpha\mu;\tau} R_{\nu}^{\alpha}{}_{\rho\kappa;\lambda} - \frac{ie}{18} F_{\mu\tau;\alpha} R_{\nu}^{\alpha}{}_{\rho\kappa;\lambda} + \\
& \frac{ie}{18} F_{\alpha\mu;\kappa} R_{\nu}^{\alpha}{}_{\rho\lambda;\tau} - \frac{ie}{18} F_{\mu\kappa;\alpha} R_{\nu}^{\alpha}{}_{\rho\lambda;\tau} + \frac{ie}{18} F_{\alpha\mu;\tau} R_{\nu}^{\alpha}{}_{\rho\lambda;\kappa} - \frac{ie}{18} F_{\mu\tau;\alpha} R_{\nu}^{\alpha}{}_{\rho\lambda;\kappa} + \\
& \frac{ie}{18} F_{\alpha\mu;\rho} R_{\nu}^{\alpha}{}_{\tau\kappa;\lambda} - \frac{ie}{18} F_{\mu\rho;\alpha} R_{\nu}^{\alpha}{}_{\tau\kappa;\lambda} + \frac{ie}{18} F_{\alpha\mu;\rho} R_{\nu}^{\alpha}{}_{\tau\lambda;\kappa} - \frac{ie}{18} F_{\mu\rho;\alpha} R_{\nu}^{\alpha}{}_{\tau\lambda;\kappa} + \\
& \frac{ie}{18} F_{\alpha\mu;\nu} R_{\rho}^{\alpha}{}_{\tau\kappa;\lambda} - \frac{ie}{18} F_{\mu\nu;\alpha} R_{\rho}^{\alpha}{}_{\tau\kappa;\lambda} + \frac{ie}{18} F_{\alpha\mu;\nu} R_{\rho}^{\alpha}{}_{\tau\lambda;\kappa} - \frac{ie}{18} F_{\mu\nu;\alpha} R_{\rho}^{\alpha}{}_{\tau\lambda;\kappa} + \\
& \frac{ie}{6} F_{\mu\nu;\rho\tau\kappa\lambda} + \frac{ie}{6} F_{\mu\rho;\nu\tau\kappa\lambda} + \frac{ie}{6} F_{\mu\tau;\nu\rho\kappa\lambda} + \frac{ie}{6} F_{\mu\kappa;\nu\rho\tau\lambda} + \\
& \frac{ie}{6} F_{\mu\lambda;\nu\rho\tau\kappa} - \frac{e^2 F_{\rho\tau;\kappa\lambda} F_{\mu\nu}}{8} - \frac{e^2 F_{\rho\kappa;\tau\lambda} F_{\mu\nu}}{8} - \frac{e^2 F_{\rho\lambda;\tau\kappa} F_{\mu\nu}}{8} - \\
& \frac{e^2 F_{\nu\tau;\kappa\lambda} F_{\mu\rho}}{8} - \frac{e^2 F_{\nu\kappa;\tau\lambda} F_{\mu\rho}}{8} - \frac{e^2 F_{\nu\lambda;\tau\kappa} F_{\mu\rho}}{8} - \frac{e^2 F_{\nu\rho;\kappa\lambda} F_{\mu\tau}}{8} - \\
& \frac{e^2 F_{\nu\kappa;\rho\lambda} F_{\mu\tau}}{8} - \frac{e^2 F_{\nu\lambda;\rho\kappa} F_{\mu\tau}}{8} - \frac{e^2 F_{\nu\rho;\tau\lambda} F_{\mu\kappa}}{8} - \frac{e^2 F_{\nu\tau;\rho\lambda} F_{\mu\kappa}}{8} - \\
& \frac{e^2 F_{\nu\lambda;\rho\tau} F_{\mu\kappa}}{8} - \frac{e^2 F_{\nu\rho;\tau\kappa} F_{\mu\lambda}}{8} - \frac{e^2 F_{\nu\tau;\rho\kappa} F_{\mu\lambda}}{8} - \frac{e^2 F_{\nu\kappa;\rho\tau} F_{\mu\lambda}}{8} + \\
& \frac{ie}{12} R_{\alpha\nu\rho\tau;\kappa\lambda} F_{\mu}^{\alpha} + \frac{ie}{12} R_{\alpha\nu\rho\tau;\kappa\lambda} F_{\mu}^{\alpha} + \frac{ie}{12} R_{\alpha\nu\rho\kappa;\tau\lambda} F_{\mu}^{\alpha} + \frac{ie}{12} R_{\alpha\nu\rho\lambda;\tau\kappa} F_{\mu}^{\alpha} - \\
& \frac{e^2 F_{\mu\tau;\kappa\lambda} F_{\nu\rho}}{8} - \frac{e^2 F_{\mu\kappa;\tau\lambda} F_{\nu\rho}}{8} - \frac{e^2 F_{\mu\lambda;\tau\kappa} F_{\nu\rho}}{8} - \frac{e^2 F_{\mu\rho;\kappa\lambda} F_{\nu\tau}}{8} -
\end{aligned}$$

$$\begin{aligned}
& \frac{e^2 F_{\mu\kappa;\rho\lambda} F_{\nu\tau}}{8} - \frac{e^2 F_{\mu\lambda;\rho\kappa} F_{\nu\tau}}{8} - \frac{e^2 F_{\mu\rho;\tau\lambda} F_{\nu\kappa}}{8} - \frac{e^2 F_{\mu\tau;\rho\lambda} F_{\nu\kappa}}{8} - \\
& \frac{e^2 F_{\mu\lambda;\rho\tau} F_{\nu\kappa}}{8} - \frac{e^2 F_{\mu\rho;\tau\kappa} F_{\nu\lambda}}{8} - \frac{e^2 F_{\mu\tau;\rho\kappa} F_{\nu\lambda}}{8} - \frac{e^2 F_{\mu\kappa;\rho\tau} F_{\nu\lambda}}{8} - \\
& \frac{ie}{12} R_{\alpha\mu\rho\tau;\kappa\lambda} F_{\nu}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\rho\kappa;\tau\lambda} F_{\nu}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\rho\lambda;\tau\kappa} F_{\nu}^{\alpha} - \frac{e^2 F_{\mu\nu;\kappa\lambda} F_{\rho\tau}}{8} - \\
& \frac{e^2 F_{\mu\kappa;\nu\lambda} F_{\rho\tau}}{8} - \frac{e^2 F_{\mu\lambda;\nu\kappa} F_{\rho\tau}}{8} - \frac{ie^3}{8} F_{\mu\lambda} F_{\nu\kappa} F_{\rho\tau} - \frac{ie^3}{8} F_{\mu\kappa} F_{\nu\lambda} F_{\rho\tau} - \\
& \frac{e^2 F_{\mu\nu;\tau\lambda} F_{\rho\kappa}}{8} - \frac{e^2 F_{\mu\tau;\nu\lambda} F_{\rho\kappa}}{8} - \frac{e^2 F_{\mu\lambda;\nu\tau} F_{\rho\kappa}}{8} - \frac{ie^3}{8} F_{\mu\lambda} F_{\nu\tau} F_{\rho\kappa} - \\
& \frac{ie^3}{8} F_{\mu\tau} F_{\nu\lambda} F_{\rho\kappa} - \frac{e^2 F_{\mu\nu;\tau\kappa} F_{\rho\lambda}}{8} - \frac{e^2 F_{\mu\tau;\nu\kappa} F_{\rho\lambda}}{8} - \frac{e^2 F_{\mu\kappa;\nu\tau} F_{\rho\lambda}}{8} - \\
& \frac{ie^3}{8} F_{\mu\kappa} F_{\nu\tau} F_{\rho\lambda} - \frac{ie^3}{8} F_{\mu\tau} F_{\nu\kappa} F_{\rho\lambda} - \frac{ie}{12} R_{\alpha\mu\nu\tau;\kappa\lambda} F_{\rho}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\nu\kappa;\tau\lambda} F_{\rho}^{\alpha} - \\
& \frac{ie}{12} R_{\alpha\mu\nu\lambda;\tau\kappa} F_{\rho}^{\alpha} - \frac{e^2 F_{\mu\nu;\rho\lambda} F_{\tau\kappa}}{8} - \frac{e^2 F_{\mu\rho;\nu\lambda} F_{\tau\kappa}}{8} - \frac{e^2 F_{\mu\lambda;\nu\rho} F_{\tau\kappa}}{8} - \\
& \frac{ie^3}{8} F_{\mu\lambda} F_{\nu\rho} F_{\tau\kappa} - \frac{ie^3}{8} F_{\mu\rho} F_{\nu\lambda} F_{\tau\kappa} - \frac{ie^3}{8} F_{\mu\nu} F_{\rho\lambda} F_{\tau\kappa} - \frac{e^2 F_{\mu\nu;\rho\kappa} F_{\tau\lambda}}{8} - \\
& \frac{e^2 F_{\mu\rho;\nu\kappa} F_{\tau\lambda}}{8} - \frac{e^2 F_{\mu\kappa;\nu\rho} F_{\tau\lambda}}{8} - \frac{ie^3}{8} F_{\mu\kappa} F_{\nu\rho} F_{\tau\lambda} - \frac{ie^3}{8} F_{\mu\rho} F_{\nu\kappa} F_{\tau\lambda} - \\
& \frac{ie^3}{8} F_{\mu\nu} F_{\rho\kappa} F_{\tau\lambda} - \frac{ie}{12} R_{\alpha\mu\nu\rho;\kappa\lambda} F_{\tau}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\nu\kappa;\rho\lambda} F_{\tau}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\nu\lambda;\rho\kappa} F_{\tau}^{\alpha} - \\
& \frac{e^2 F_{\mu\nu;\rho\tau} F_{\kappa\lambda}}{8} - \frac{e^2 F_{\mu\rho;\nu\tau} F_{\kappa\lambda}}{8} - \frac{e^2 F_{\mu\tau;\nu\rho} F_{\kappa\lambda}}{8} - \frac{ie^3}{8} F_{\mu\tau} F_{\nu\rho} F_{\kappa\lambda} - \\
& \frac{ie^3}{8} F_{\mu\rho} F_{\nu\tau} F_{\kappa\lambda} - \frac{ie^3}{8} F_{\mu\nu} F_{\rho\tau} F_{\kappa\lambda} - \frac{ie}{12} R_{\alpha\mu\nu\rho;\tau\lambda} F_{\kappa}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\nu\tau;\rho\lambda} F_{\kappa}^{\alpha} - \\
& \frac{ie}{12} R_{\alpha\mu\nu\lambda;\rho\tau} F_{\kappa}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\nu\rho;\tau\kappa} F_{\lambda}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\nu\tau;\rho\kappa} F_{\lambda}^{\alpha} - \frac{ie}{12} R_{\alpha\mu\nu\kappa;\rho\tau} F_{\lambda}^{\alpha} - \\
& \frac{ie}{24} F_{\alpha\tau;\kappa\lambda} R_{\mu}^{\alpha}{}_{\nu\rho} - \frac{ie}{24} F_{\alpha\kappa;\tau\lambda} R_{\mu}^{\alpha}{}_{\nu\rho} - \frac{ie}{24} F_{\alpha\lambda;\tau\kappa} R_{\mu}^{\alpha}{}_{\nu\rho} + \frac{e^2 F_{\alpha\lambda} F_{\tau\kappa} R_{\mu}^{\alpha}{}_{\nu\rho}}{24} + \\
& \frac{e^2 F_{\alpha\kappa} F_{\tau\lambda} R_{\mu}^{\alpha}{}_{\nu\rho}}{24} + \frac{e^2 F_{\alpha\tau} F_{\kappa\lambda} R_{\mu}^{\alpha}{}_{\nu\rho}}{24} - \frac{ie}{24} F_{\alpha\rho;\kappa\lambda} R_{\mu}^{\alpha}{}_{\nu\tau} - \frac{ie}{24} F_{\alpha\kappa;\rho\lambda} R_{\mu}^{\alpha}{}_{\nu\tau} - \\
& \frac{ie}{24} F_{\alpha\lambda;\rho\kappa} R_{\mu}^{\alpha}{}_{\nu\tau} + \frac{e^2 F_{\alpha\lambda} F_{\rho\kappa} R_{\mu}^{\alpha}{}_{\nu\tau}}{24} + \frac{e^2 F_{\alpha\kappa} F_{\rho\lambda} R_{\mu}^{\alpha}{}_{\nu\tau}}{24} + \frac{e^2 F_{\alpha\rho} F_{\kappa\lambda} R_{\mu}^{\alpha}{}_{\nu\tau}}{24} - \\
& \frac{ie}{24} F_{\alpha\rho;\tau\lambda} R_{\mu}^{\alpha}{}_{\nu\kappa} - \frac{ie}{24} F_{\alpha\tau;\rho\lambda} R_{\mu}^{\alpha}{}_{\nu\kappa} - \frac{ie}{24} F_{\alpha\lambda;\rho\tau} R_{\mu}^{\alpha}{}_{\nu\kappa} + \frac{e^2 F_{\alpha\lambda} F_{\rho\tau} R_{\mu}^{\alpha}{}_{\nu\kappa}}{24} + \\
& \frac{e^2 F_{\alpha\tau} F_{\rho\lambda} R_{\mu}^{\alpha}{}_{\nu\kappa}}{24} + \frac{e^2 F_{\alpha\rho} F_{\tau\lambda} R_{\mu}^{\alpha}{}_{\nu\kappa}}{24} - \frac{ie}{24} F_{\alpha\rho;\tau\kappa} R_{\mu}^{\alpha}{}_{\nu\lambda} - \frac{ie}{24} F_{\alpha\tau;\rho\kappa} R_{\mu}^{\alpha}{}_{\nu\lambda} - \\
& \frac{ie}{24} F_{\alpha\kappa;\rho\tau} R_{\mu}^{\alpha}{}_{\nu\lambda} + \frac{e^2 F_{\alpha\kappa} F_{\rho\tau} R_{\mu}^{\alpha}{}_{\nu\lambda}}{24} + \frac{e^2 F_{\alpha\tau} F_{\rho\kappa} R_{\mu}^{\alpha}{}_{\nu\lambda}}{24} + \frac{e^2 F_{\alpha\rho} F_{\tau\kappa} R_{\mu}^{\alpha}{}_{\nu\lambda}}{24} - \\
& \frac{ie}{24} F_{\alpha\nu;\kappa\lambda} R_{\mu}^{\alpha}{}_{\rho\tau} - \frac{ie}{24} F_{\alpha\kappa;\nu\lambda} R_{\mu}^{\alpha}{}_{\rho\tau} - \frac{ie}{24} F_{\alpha\lambda;\nu\kappa} R_{\mu}^{\alpha}{}_{\rho\tau} + \frac{e^2 F_{\alpha\lambda} F_{\nu\kappa} R_{\mu}^{\alpha}{}_{\rho\tau}}{24} +
\end{aligned}$$



$$\begin{aligned}
& \frac{e^2 F_{\alpha\nu} F_{\mu\tau} R_{\rho}^{\alpha\kappa\lambda}}{24} - \frac{e^2 F_{\alpha\mu} F_{\nu\tau} R_{\rho}^{\alpha\kappa\lambda}}{24} + \frac{ie}{24} F_{\alpha\mu;\nu\rho} R_{\tau}^{\alpha\kappa\lambda} - \frac{ie}{24} F_{\mu\nu;\rho\alpha} R_{\tau}^{\alpha\kappa\lambda} - \\
& \frac{ie}{24} F_{\mu\rho;\nu\alpha} R_{\tau}^{\alpha\kappa\lambda} - \frac{e^2 F_{\alpha\rho} F_{\mu\nu} R_{\tau}^{\alpha\kappa\lambda}}{24} - \frac{e^2 F_{\alpha\nu} F_{\mu\rho} R_{\tau}^{\alpha\kappa\lambda}}{24} - \frac{e^2 F_{\alpha\mu} F_{\nu\rho} R_{\tau}^{\alpha\kappa\lambda}}{24} - \\
& \frac{e^2 F_{\tau}^{\alpha} F_{\kappa\lambda} S_{\alpha\mu\nu\rho}}{24} - \frac{e^2 F_{\tau\lambda} F_{\kappa}^{\alpha} S_{\alpha\mu\nu\rho}}{24} - \frac{e^2 F_{\tau\kappa} F_{\lambda}^{\alpha} S_{\alpha\mu\nu\rho}}{24} - \frac{e^2 F_{\rho}^{\alpha} F_{\kappa\lambda} S_{\alpha\mu\nu\tau}}{24} - \\
& \frac{e^2 F_{\rho\lambda} F_{\kappa}^{\alpha} S_{\alpha\mu\nu\tau}}{24} - \frac{e^2 F_{\rho\kappa} F_{\lambda}^{\alpha} S_{\alpha\mu\nu\tau}}{24} - \frac{e^2 F_{\rho}^{\alpha} F_{\tau\lambda} S_{\alpha\mu\nu\kappa}}{24} - \frac{e^2 F_{\rho\lambda} F_{\tau}^{\alpha} S_{\alpha\mu\nu\kappa}}{24} - \\
& \frac{e^2 F_{\rho\tau} F_{\lambda}^{\alpha} S_{\alpha\mu\nu\kappa}}{24} - \frac{e^2 F_{\rho}^{\alpha} F_{\tau\kappa} S_{\alpha\mu\nu\lambda}}{24} - \frac{e^2 F_{\rho\kappa} F_{\tau}^{\alpha} S_{\alpha\mu\nu\lambda}}{24} - \frac{e^2 F_{\rho\tau} F_{\kappa}^{\alpha} S_{\alpha\mu\nu\lambda}}{24} - \\
& \frac{e^2 F_{\nu}^{\alpha} F_{\kappa\lambda} S_{\alpha\mu\rho\tau}}{24} - \frac{e^2 F_{\nu\lambda} F_{\kappa}^{\alpha} S_{\alpha\mu\rho\tau}}{24} - \frac{e^2 F_{\nu\kappa} F_{\lambda}^{\alpha} S_{\alpha\mu\rho\tau}}{24} - \frac{e^2 F_{\nu}^{\alpha} F_{\tau\lambda} S_{\alpha\mu\rho\kappa}}{24} - \\
& \frac{e^2 F_{\nu\lambda} F_{\tau}^{\alpha} S_{\alpha\mu\rho\kappa}}{24} - \frac{e^2 F_{\nu\tau} F_{\lambda}^{\alpha} S_{\alpha\mu\rho\kappa}}{24} - \frac{e^2 F_{\nu}^{\alpha} F_{\tau\kappa} S_{\alpha\mu\rho\lambda}}{24} - \frac{e^2 F_{\nu\kappa} F_{\tau}^{\alpha} S_{\alpha\mu\rho\lambda}}{24} - \\
& \frac{e^2 F_{\nu\tau} F_{\kappa}^{\alpha} S_{\alpha\mu\rho\lambda}}{24} - \frac{e^2 F_{\nu}^{\alpha} F_{\rho\lambda} S_{\alpha\mu\tau\kappa}}{24} - \frac{e^2 F_{\nu\lambda} F_{\rho}^{\alpha} S_{\alpha\mu\tau\kappa}}{24} - \frac{e^2 F_{\nu\rho} F_{\lambda}^{\alpha} S_{\alpha\mu\tau\kappa}}{24} - \\
& \frac{e^2 F_{\nu}^{\alpha} F_{\rho\kappa} S_{\alpha\mu\tau\lambda}}{24} - \frac{e^2 F_{\nu\kappa} F_{\rho}^{\alpha} S_{\alpha\mu\tau\lambda}}{24} - \frac{e^2 F_{\nu\rho} F_{\kappa}^{\alpha} S_{\alpha\mu\tau\lambda}}{24} - \frac{e^2 F_{\nu}^{\alpha} F_{\rho\tau} S_{\alpha\mu\kappa\lambda}}{24} - \\
& \frac{e^2 F_{\nu\tau} F_{\rho}^{\alpha} S_{\alpha\mu\kappa\lambda}}{24} - \frac{e^2 F_{\nu\rho} F_{\tau}^{\alpha} S_{\alpha\mu\kappa\lambda}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\kappa\lambda} S_{\alpha\nu\rho\tau}}{24} - \frac{e^2 F_{\mu\lambda} F_{\kappa}^{\alpha} S_{\alpha\nu\rho\tau}}{24} - \\
& \frac{e^2 F_{\mu\kappa} F_{\lambda}^{\alpha} S_{\alpha\nu\rho\tau}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\tau\lambda} S_{\alpha\nu\rho\kappa}}{24} - \frac{e^2 F_{\mu\lambda} F_{\tau}^{\alpha} S_{\alpha\nu\rho\kappa}}{24} - \frac{e^2 F_{\mu\tau} F_{\lambda}^{\alpha} S_{\alpha\nu\rho\kappa}}{24} - \\
& \frac{e^2 F_{\mu}^{\alpha} F_{\tau\kappa} S_{\alpha\nu\rho\lambda}}{24} - \frac{e^2 F_{\mu\kappa} F_{\tau}^{\alpha} S_{\alpha\nu\rho\lambda}}{24} - \frac{e^2 F_{\mu\tau} F_{\kappa}^{\alpha} S_{\alpha\nu\rho\lambda}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\rho\lambda} S_{\alpha\nu\tau\kappa}}{24} - \\
& \frac{e^2 F_{\mu\lambda} F_{\rho}^{\alpha} S_{\alpha\nu\tau\kappa}}{24} - \frac{e^2 F_{\mu\rho} F_{\lambda}^{\alpha} S_{\alpha\nu\tau\kappa}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\rho\kappa} S_{\alpha\nu\tau\lambda}}{24} - \frac{e^2 F_{\mu\kappa} F_{\rho}^{\alpha} S_{\alpha\nu\tau\lambda}}{24} - \\
& \frac{e^2 F_{\mu\rho} F_{\kappa}^{\alpha} S_{\alpha\nu\tau\lambda}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\rho\tau} S_{\alpha\nu\kappa\lambda}}{24} - \frac{e^2 F_{\mu\tau} F_{\rho}^{\alpha} S_{\alpha\nu\kappa\lambda}}{24} - \frac{e^2 F_{\mu\rho} F_{\tau}^{\alpha} S_{\alpha\nu\kappa\lambda}}{24} - \\
& \frac{e^2 F_{\mu}^{\alpha} F_{\nu\lambda} S_{\alpha\rho\tau\kappa}}{24} - \frac{e^2 F_{\mu\lambda} F_{\nu}^{\alpha} S_{\alpha\rho\tau\kappa}}{24} - \frac{e^2 F_{\mu\nu} F_{\lambda}^{\alpha} S_{\alpha\rho\tau\kappa}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\nu\kappa} S_{\alpha\rho\tau\lambda}}{24} - \\
& \frac{e^2 F_{\mu\kappa} F_{\nu}^{\alpha} S_{\alpha\rho\tau\lambda}}{24} - \frac{e^2 F_{\mu\nu} F_{\kappa}^{\alpha} S_{\alpha\rho\tau\lambda}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\nu\tau} S_{\alpha\rho\kappa\lambda}}{24} - \frac{e^2 F_{\mu\tau} F_{\nu}^{\alpha} S_{\alpha\rho\kappa\lambda}}{24} - \\
& \frac{e^2 F_{\mu\nu} F_{\tau}^{\alpha} S_{\alpha\rho\kappa\lambda}}{24} - \frac{e^2 F_{\mu}^{\alpha} F_{\nu\rho} S_{\alpha\tau\kappa\lambda}}{24} - \frac{e^2 F_{\mu\rho} F_{\nu}^{\alpha} S_{\alpha\tau\kappa\lambda}}{24} - \frac{e^2 F_{\mu\nu} F_{\rho}^{\alpha} S_{\alpha\tau\kappa\lambda}}{24} + \\
& \frac{ie}{24} F_{\lambda}^{\alpha} R_{\nu}^{\beta}{}_{\tau\kappa} S_{\beta\mu\rho\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\nu}^{\beta}{}_{\tau\lambda} S_{\beta\mu\rho\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\nu}^{\beta}{}_{\kappa\lambda} S_{\beta\mu\rho\alpha} + \\
& \frac{ie}{24} F_{\lambda}^{\alpha} R_{\nu}^{\beta}{}_{\rho\kappa} S_{\beta\mu\tau\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\nu}^{\beta}{}_{\rho\lambda} S_{\beta\mu\tau\alpha} + \frac{ie}{24} F_{\rho}^{\alpha} R_{\nu}^{\beta}{}_{\kappa\lambda} S_{\beta\mu\tau\alpha} + \\
& \frac{ie}{24} F_{\nu}^{\alpha} R_{\rho}^{\beta}{}_{\kappa\lambda} S_{\beta\mu\tau\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} R_{\nu}^{\beta}{}_{\rho\tau} S_{\beta\mu\kappa\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\nu}^{\beta}{}_{\rho\lambda} S_{\beta\mu\kappa\alpha} + \\
& \frac{ie}{24} F_{\rho}^{\alpha} R_{\nu}^{\beta}{}_{\tau\lambda} S_{\beta\mu\kappa\alpha} + \frac{ie}{24} F_{\nu}^{\alpha} R_{\rho}^{\beta}{}_{\tau\lambda} S_{\beta\mu\kappa\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\nu}^{\beta}{}_{\rho\tau} S_{\beta\mu\lambda\alpha} +
\end{aligned}$$

$$\begin{aligned}
& \frac{ie}{24} F_{\tau}^{\alpha} R_{\nu}^{\beta}{}_{\rho\kappa} S_{\beta\mu\lambda\alpha} + \frac{ie}{24} F_{\rho}^{\alpha} R_{\nu}^{\beta}{}_{\tau\kappa} S_{\beta\mu\lambda\alpha} + \frac{ie}{24} F_{\nu}^{\alpha} R_{\rho}^{\beta}{}_{\tau\kappa} S_{\beta\mu\lambda\alpha} + \\
& \frac{ie}{24} F_{\lambda}^{\alpha} R_{\mu}^{\beta}{}_{\tau\kappa} S_{\beta\nu\rho\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\mu}^{\beta}{}_{\tau\lambda} S_{\beta\nu\rho\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\mu}^{\beta}{}_{\kappa\lambda} S_{\beta\nu\rho\alpha} + \\
& \frac{ie}{24} F_{\mu}^{\alpha} R_{\tau}^{\beta}{}_{\kappa\lambda} S_{\beta\nu\rho\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} R_{\mu}^{\beta}{}_{\rho\kappa} S_{\beta\nu\tau\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\mu}^{\beta}{}_{\rho\lambda} S_{\beta\nu\tau\alpha} + \\
& \frac{ie}{24} F_{\rho}^{\alpha} R_{\mu}^{\beta}{}_{\kappa\lambda} S_{\beta\nu\tau\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} R_{\mu}^{\beta}{}_{\rho\tau} S_{\beta\nu\kappa\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\mu}^{\beta}{}_{\rho\lambda} S_{\beta\nu\kappa\alpha} + \\
& \frac{ie}{24} F_{\rho}^{\alpha} R_{\mu}^{\beta}{}_{\tau\lambda} S_{\beta\nu\kappa\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\mu}^{\beta}{}_{\rho\tau} S_{\beta\nu\lambda\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\mu}^{\beta}{}_{\rho\kappa} S_{\beta\nu\lambda\alpha} + \\
& \frac{ie}{24} F_{\rho}^{\alpha} R_{\mu}^{\beta}{}_{\tau\kappa} S_{\beta\nu\lambda\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} R_{\mu}^{\beta}{}_{\nu\kappa} S_{\beta\rho\tau\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\mu}^{\beta}{}_{\nu\lambda} S_{\beta\rho\tau\alpha} + \\
& \frac{ie}{24} F_{\nu}^{\alpha} R_{\mu}^{\beta}{}_{\kappa\lambda} S_{\beta\rho\tau\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} R_{\mu}^{\beta}{}_{\nu\tau} S_{\beta\rho\kappa\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\mu}^{\beta}{}_{\nu\lambda} S_{\beta\rho\kappa\alpha} + \\
& \frac{ie}{24} F_{\nu}^{\alpha} R_{\mu}^{\beta}{}_{\tau\lambda} S_{\beta\rho\kappa\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\mu}^{\beta}{}_{\nu\tau} S_{\beta\rho\lambda\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\mu}^{\beta}{}_{\nu\kappa} S_{\beta\rho\lambda\alpha} + \\
& \frac{ie}{24} F_{\nu}^{\alpha} R_{\mu}^{\beta}{}_{\tau\kappa} S_{\beta\rho\lambda\alpha} + \frac{ie}{24} F_{\mu}^{\alpha} R_{\nu}^{\beta}{}_{\kappa\lambda} S_{\beta\tau\rho\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} R_{\mu}^{\beta}{}_{\nu\rho} S_{\beta\tau\kappa\alpha} + \\
& \frac{ie}{24} F_{\rho}^{\alpha} R_{\mu}^{\beta}{}_{\nu\lambda} S_{\beta\tau\kappa\alpha} + \frac{ie}{24} F_{\nu}^{\alpha} R_{\mu}^{\beta}{}_{\rho\lambda} S_{\beta\tau\kappa\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\mu}^{\beta}{}_{\nu\rho} S_{\beta\tau\lambda\alpha} + \\
& \frac{ie}{24} F_{\rho}^{\alpha} R_{\mu}^{\beta}{}_{\nu\kappa} S_{\beta\tau\lambda\alpha} + \frac{ie}{24} F_{\nu}^{\alpha} R_{\mu}^{\beta}{}_{\rho\kappa} S_{\beta\tau\lambda\alpha} + \frac{ie}{24} F_{\mu}^{\alpha} R_{\nu}^{\beta}{}_{\tau\lambda} S_{\beta\kappa\rho\alpha} + \\
& \frac{ie}{24} F_{\mu}^{\alpha} R_{\nu}^{\beta}{}_{\rho\lambda} S_{\beta\kappa\tau\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} R_{\mu}^{\beta}{}_{\nu\rho} S_{\beta\kappa\lambda\alpha} + \frac{ie}{24} F_{\rho}^{\alpha} R_{\mu}^{\beta}{}_{\nu\tau} S_{\beta\kappa\lambda\alpha} + \\
& \frac{ie}{24} F_{\nu}^{\alpha} R_{\mu}^{\beta}{}_{\rho\tau} S_{\beta\kappa\lambda\alpha} + \frac{ie}{24} F_{\mu}^{\alpha} R_{\nu}^{\beta}{}_{\tau\kappa} S_{\beta\lambda\rho\alpha} + \frac{ie}{24} F_{\mu}^{\alpha} R_{\nu}^{\beta}{}_{\rho\kappa} S_{\beta\lambda\tau\alpha} + \\
& \frac{ie}{24} F_{\mu}^{\alpha} R_{\nu}^{\beta}{}_{\rho\tau} S_{\beta\lambda\kappa\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} R_{\rho}^{\beta}{}_{\tau\kappa} S_{\mu\nu\beta\alpha} + \frac{ie}{24} F_{\kappa}^{\alpha} R_{\rho}^{\beta}{}_{\tau\lambda} S_{\mu\nu\beta\alpha} + \\
& \frac{ie}{24} F_{\tau}^{\alpha} R_{\rho}^{\beta}{}_{\kappa\lambda} S_{\mu\nu\beta\alpha} + \frac{ie}{24} F_{\rho}^{\alpha} R_{\tau}^{\beta}{}_{\kappa\lambda} S_{\mu\nu\beta\alpha} + \frac{e^2 F_{\tau}^{\alpha} F_{\kappa\lambda} S_{\mu\nu\rho\alpha}}{24} + \\
& \frac{e^2 F_{\tau\lambda} F_{\kappa}^{\alpha} S_{\mu\nu\rho\alpha}}{24} + \frac{e^2 F_{\tau\kappa} F_{\lambda}^{\alpha} S_{\mu\nu\rho\alpha}}{24} + \frac{ie}{24} F_{\alpha}^{\beta} R_{\tau}^{\alpha}{}_{\kappa\lambda} S_{\mu\nu\rho\beta} + \\
& \frac{e^2 F_{\rho}^{\alpha} F_{\kappa\lambda} S_{\mu\nu\tau\alpha}}{24} + \frac{e^2 F_{\rho\lambda} F_{\kappa}^{\alpha} S_{\mu\nu\tau\alpha}}{24} + \frac{e^2 F_{\rho\kappa} F_{\lambda}^{\alpha} S_{\mu\nu\tau\alpha}}{24} + \\
& \frac{ie}{24} F_{\alpha}^{\beta} R_{\rho}^{\alpha}{}_{\kappa\lambda} S_{\mu\nu\tau\beta} + \frac{e^2 F_{\rho}^{\alpha} F_{\tau\lambda} S_{\mu\nu\kappa\alpha}}{24} + \frac{e^2 F_{\rho\lambda} F_{\tau}^{\alpha} S_{\mu\nu\kappa\alpha}}{24} + \\
& \frac{e^2 F_{\rho\tau} F_{\lambda}^{\alpha} S_{\mu\nu\kappa\alpha}}{24} + \frac{ie}{24} F_{\alpha}^{\beta} R_{\rho}^{\alpha}{}_{\tau\lambda} S_{\mu\nu\kappa\beta} + \frac{e^2 F_{\rho}^{\alpha} F_{\tau\kappa} S_{\mu\nu\lambda\alpha}}{24} + \\
& \frac{e^2 F_{\rho\kappa} F_{\tau}^{\alpha} S_{\mu\nu\lambda\alpha}}{24} + \frac{e^2 F_{\rho\tau} F_{\kappa}^{\alpha} S_{\mu\nu\lambda\alpha}}{24} + \frac{ie}{24} F_{\alpha}^{\beta} R_{\rho}^{\alpha}{}_{\tau\kappa} S_{\mu\nu\lambda\beta} + \\
& \frac{ie}{24} F_{\nu}^{\alpha} R_{\tau}^{\beta}{}_{\kappa\lambda} S_{\mu\rho\beta\alpha} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\tau\kappa\lambda} S_{\mu\rho\nu\beta} + \frac{e^2 F_{\nu}^{\alpha} F_{\kappa\lambda} S_{\mu\rho\tau\alpha}}{24} + \\
& \frac{e^2 F_{\nu\lambda} F_{\kappa}^{\alpha} S_{\mu\rho\tau\alpha}}{24} + \frac{e^2 F_{\nu\kappa} F_{\lambda}^{\alpha} S_{\mu\rho\tau\alpha}}{24} + \frac{ie}{24} F_{\alpha}^{\beta} R_{\nu}^{\alpha}{}_{\kappa\lambda} S_{\mu\rho\tau\beta} +
\end{aligned}$$

$$\begin{aligned}
& \frac{e^2 F_\nu^\alpha F_{\tau\lambda} S_{\mu\rho\kappa\alpha}}{24} + \frac{e^2 F_{\nu\lambda} F_\tau^\alpha S_{\mu\rho\kappa\alpha}}{24} + \frac{e^2 F_{\nu\tau} F_\lambda^\alpha S_{\mu\rho\kappa\alpha}}{24} + \\
& \frac{ie}{24} F_\alpha^\beta R_\nu^\alpha{}_{\tau\lambda} S_{\mu\rho\kappa\beta} + \frac{e^2 F_\nu^\alpha F_{\tau\kappa} S_{\mu\rho\lambda\alpha}}{24} + \frac{e^2 F_{\nu\kappa} F_\tau^\alpha S_{\mu\rho\lambda\alpha}}{24} + \\
& \frac{e^2 F_{\nu\tau} F_\kappa^\alpha S_{\mu\rho\lambda\alpha}}{24} + \frac{ie}{24} F_\alpha^\beta R_\nu^\alpha{}_{\tau\kappa} S_{\mu\rho\lambda\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\rho\kappa\lambda} S_{\mu\tau\nu\beta} + \\
& \frac{ie}{24} F^{\alpha\beta} S_{\alpha\nu\kappa\lambda} S_{\mu\tau\rho\beta} + \frac{e^2 F_\nu^\alpha F_{\rho\lambda} S_{\mu\tau\kappa\alpha}}{24} + \frac{e^2 F_{\nu\lambda} F_\rho^\alpha S_{\mu\tau\kappa\alpha}}{24} + \\
& \frac{e^2 F_{\nu\rho} F_\lambda^\alpha S_{\mu\tau\kappa\alpha}}{24} + \frac{ie}{24} F_\alpha^\beta R_\nu^\alpha{}_{\rho\lambda} S_{\mu\tau\kappa\beta} + \frac{e^2 F_\nu^\alpha F_{\rho\kappa} S_{\mu\tau\lambda\alpha}}{24} + \\
& \frac{e^2 F_{\nu\kappa} F_\rho^\alpha S_{\mu\tau\lambda\alpha}}{24} + \frac{e^2 F_{\nu\rho} F_\kappa^\alpha S_{\mu\tau\lambda\alpha}}{24} + \frac{ie}{24} F_\alpha^\beta R_\nu^\alpha{}_{\rho\kappa} S_{\mu\tau\lambda\beta} + \\
& \frac{ie}{24} F^{\alpha\beta} S_{\alpha\rho\tau\lambda} S_{\mu\kappa\nu\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\nu\tau\lambda} S_{\mu\kappa\rho\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\nu\rho\lambda} S_{\mu\kappa\tau\beta} + \\
& \frac{e^2 F_\nu^\alpha F_{\rho\tau} S_{\mu\kappa\lambda\alpha}}{24} + \frac{e^2 F_{\nu\tau} F_\rho^\alpha S_{\mu\kappa\lambda\alpha}}{24} + \frac{e^2 F_{\nu\rho} F_\tau^\alpha S_{\mu\kappa\lambda\alpha}}{24} + \\
& \frac{ie}{24} F_\alpha^\beta R_\nu^\alpha{}_{\rho\tau} S_{\mu\kappa\lambda\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\rho\tau\kappa} S_{\mu\lambda\nu\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\nu\tau\kappa} S_{\mu\lambda\rho\beta} + \\
& \frac{ie}{24} F^{\alpha\beta} S_{\alpha\nu\rho\kappa} S_{\mu\lambda\tau\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\nu\rho\tau} S_{\mu\lambda\kappa\beta} - \frac{ie}{24} F_{\alpha\tau;\kappa\lambda} S_\mu^\alpha{}_{\nu\rho} - \\
& \frac{ie}{24} F_{\alpha\kappa;\tau\lambda} S_\mu^\alpha{}_{\nu\rho} - \frac{ie}{24} F_{\alpha\lambda;\tau\kappa} S_\mu^\alpha{}_{\nu\rho} - \frac{ie}{24} F_{\alpha\rho;\kappa\lambda} S_\mu^\alpha{}_{\nu\tau} - \\
& \frac{ie}{24} F_{\alpha\kappa;\rho\lambda} S_\mu^\alpha{}_{\nu\tau} - \frac{ie}{24} F_{\alpha\lambda;\rho\kappa} S_\mu^\alpha{}_{\nu\tau} - \frac{ie}{24} F_{\alpha\rho;\tau\lambda} S_\mu^\alpha{}_{\nu\kappa} - \\
& \frac{ie}{24} F_{\alpha\tau;\rho\lambda} S_\mu^\alpha{}_{\nu\kappa} - \frac{ie}{24} F_{\alpha\lambda;\rho\tau} S_\mu^\alpha{}_{\nu\kappa} - \frac{ie}{24} F_{\alpha\rho;\tau\kappa} S_\mu^\alpha{}_{\nu\lambda} - \\
& \frac{ie}{24} F_{\alpha\tau;\rho\kappa} S_\mu^\alpha{}_{\nu\lambda} - \frac{ie}{24} F_{\alpha\kappa;\rho\tau} S_\mu^\alpha{}_{\nu\lambda} - \frac{ie}{24} F_{\alpha\nu;\kappa\lambda} S_\mu^\alpha{}_{\rho\tau} - \\
& \frac{ie}{24} F_{\alpha\kappa;\nu\lambda} S_\mu^\alpha{}_{\rho\tau} - \frac{ie}{24} F_{\alpha\lambda;\nu\kappa} S_\mu^\alpha{}_{\rho\tau} - \frac{ie}{24} F_{\alpha\nu;\tau\lambda} S_\mu^\alpha{}_{\rho\kappa} - \\
& \frac{ie}{24} F_{\alpha\tau;\nu\lambda} S_\mu^\alpha{}_{\rho\kappa} - \frac{ie}{24} F_{\alpha\lambda;\nu\tau} S_\mu^\alpha{}_{\rho\kappa} - \frac{ie}{24} F_{\alpha\nu;\tau\kappa} S_\mu^\alpha{}_{\rho\lambda} - \\
& \frac{ie}{24} F_{\alpha\tau;\nu\kappa} S_\mu^\alpha{}_{\rho\lambda} - \frac{ie}{24} F_{\alpha\kappa;\nu\tau} S_\mu^\alpha{}_{\rho\lambda} - \frac{ie}{24} F_{\alpha\nu;\rho\lambda} S_\mu^\alpha{}_{\tau\kappa} - \\
& \frac{ie}{24} F_{\alpha\rho;\nu\lambda} S_\mu^\alpha{}_{\tau\kappa} - \frac{ie}{24} F_{\alpha\lambda;\nu\rho} S_\mu^\alpha{}_{\tau\kappa} - \frac{ie}{24} F_{\alpha\nu;\rho\kappa} S_\mu^\alpha{}_{\tau\lambda} - \\
& \frac{ie}{24} F_{\alpha\rho;\nu\kappa} S_\mu^\alpha{}_{\tau\lambda} - \frac{ie}{24} F_{\alpha\kappa;\nu\rho} S_\mu^\alpha{}_{\tau\lambda} - \frac{ie}{24} F_{\alpha\nu;\rho\tau} S_\mu^\alpha{}_{\kappa\lambda} - \\
& \frac{ie}{24} F_{\alpha\rho;\nu\tau} S_\mu^\alpha{}_{\kappa\lambda} - \frac{ie}{24} F_{\alpha\tau;\nu\rho} S_\mu^\alpha{}_{\kappa\lambda} + \frac{e^2 F_{\mu\lambda} F_\kappa^\alpha S_{\nu\rho\tau\alpha}}{24} + \\
& \frac{e^2 F_{\mu\kappa} F_\lambda^\alpha S_{\nu\rho\tau\alpha}}{24} + \frac{e^2 F_{\mu\lambda} F_\tau^\alpha S_{\nu\rho\kappa\alpha}}{24} + \frac{e^2 F_{\mu\tau} F_\lambda^\alpha S_{\nu\rho\kappa\alpha}}{24} +
\end{aligned}$$

$$\begin{aligned}
& \frac{e^2 F_{\mu\kappa} F_{\tau}^{\alpha} S_{\nu\rho\lambda\alpha}}{24} + \frac{e^2 F_{\mu\tau} F_{\kappa}^{\alpha} S_{\nu\rho\lambda\alpha}}{24} + \frac{ie}{24} F_{\mu}^{\alpha} R_{\rho}^{\beta}{}_{\kappa\lambda} S_{\nu\tau\beta\alpha} + \\
& \frac{e^2 F_{\mu}^{\alpha} F_{\kappa\lambda} S_{\nu\tau\rho\alpha}}{24} + \frac{ie}{24} F_{\alpha}^{\beta} R_{\mu}^{\alpha}{}_{\kappa\lambda} S_{\nu\tau\rho\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\kappa\lambda} S_{\nu\tau\rho\beta} + \\
& \frac{e^2 F_{\mu\lambda} F_{\rho}^{\alpha} S_{\nu\tau\kappa\alpha}}{24} + \frac{e^2 F_{\mu\rho} F_{\lambda}^{\alpha} S_{\nu\tau\kappa\alpha}}{24} + \frac{e^2 F_{\mu\kappa} F_{\rho}^{\alpha} S_{\nu\tau\lambda\alpha}}{24} + \\
& \frac{e^2 F_{\mu\rho} F_{\kappa}^{\alpha} S_{\nu\tau\lambda\alpha}}{24} + \frac{ie}{24} F_{\mu}^{\alpha} R_{\rho}^{\beta}{}_{\tau\lambda} S_{\nu\kappa\beta\alpha} + \frac{e^2 F_{\mu}^{\alpha} F_{\tau\lambda} S_{\nu\kappa\rho\alpha}}{24} + \\
& \frac{ie}{24} F_{\alpha}^{\beta} R_{\mu}^{\alpha}{}_{\tau\lambda} S_{\nu\kappa\rho\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\tau\lambda} S_{\nu\kappa\rho\beta} + \frac{e^2 F_{\mu}^{\alpha} F_{\rho\lambda} S_{\nu\kappa\tau\alpha}}{24} + \\
& \frac{ie}{24} F_{\alpha}^{\beta} R_{\mu}^{\alpha}{}_{\rho\lambda} S_{\nu\kappa\tau\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\rho\lambda} S_{\nu\kappa\tau\beta} + \frac{e^2 F_{\mu\tau} F_{\rho}^{\alpha} S_{\nu\kappa\lambda\alpha}}{24} + \\
& \frac{e^2 F_{\mu\rho} F_{\tau}^{\alpha} S_{\nu\kappa\lambda\alpha}}{24} + \frac{ie}{24} F_{\mu}^{\alpha} R_{\rho}^{\beta}{}_{\tau\kappa} S_{\nu\lambda\beta\alpha} + \frac{e^2 F_{\mu}^{\alpha} F_{\tau\kappa} S_{\nu\lambda\rho\alpha}}{24} + \\
& \frac{ie}{24} F_{\alpha}^{\beta} R_{\mu}^{\alpha}{}_{\tau\kappa} S_{\nu\lambda\rho\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\tau\kappa} S_{\nu\lambda\rho\beta} + \frac{e^2 F_{\mu}^{\alpha} F_{\rho\kappa} S_{\nu\lambda\tau\alpha}}{24} + \\
& \frac{ie}{24} F_{\alpha}^{\beta} R_{\mu}^{\alpha}{}_{\rho\kappa} S_{\nu\lambda\tau\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\rho\kappa} S_{\nu\lambda\tau\beta} + \frac{e^2 F_{\mu}^{\alpha} F_{\rho\tau} S_{\nu\lambda\kappa\alpha}}{24} + \\
& \frac{ie}{24} F_{\alpha}^{\beta} R_{\mu}^{\alpha}{}_{\rho\tau} S_{\nu\lambda\kappa\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\rho\tau} S_{\nu\lambda\kappa\beta} - \frac{ie}{24} F_{\alpha\mu;\kappa\lambda} S_{\nu}^{\alpha}{}_{\rho\tau} - \\
& \frac{ie}{24} F_{\alpha\kappa;\mu\lambda} S_{\nu}^{\alpha}{}_{\rho\tau} - \frac{ie}{24} F_{\alpha\lambda;\mu\kappa} S_{\nu}^{\alpha}{}_{\rho\tau} - \frac{ie}{24} F_{\alpha\mu;\tau\lambda} S_{\nu}^{\alpha}{}_{\rho\kappa} - \\
& \frac{ie}{24} F_{\alpha\tau;\mu\lambda} S_{\nu}^{\alpha}{}_{\rho\kappa} - \frac{ie}{24} F_{\alpha\lambda;\mu\tau} S_{\nu}^{\alpha}{}_{\rho\kappa} - \frac{ie}{24} F_{\alpha\mu;\tau\kappa} S_{\nu}^{\alpha}{}_{\rho\lambda} - \\
& \frac{ie}{24} F_{\alpha\tau;\mu\kappa} S_{\nu}^{\alpha}{}_{\rho\lambda} - \frac{ie}{24} F_{\alpha\kappa;\mu\tau} S_{\nu}^{\alpha}{}_{\rho\lambda} - \frac{ie}{24} F_{\alpha\mu;\rho\lambda} S_{\nu}^{\alpha}{}_{\tau\kappa} - \\
& \frac{ie}{24} F_{\alpha\rho;\mu\lambda} S_{\nu}^{\alpha}{}_{\tau\kappa} - \frac{ie}{24} F_{\alpha\lambda;\mu\rho} S_{\nu}^{\alpha}{}_{\tau\kappa} - \frac{ie}{24} F_{\alpha\mu;\rho\kappa} S_{\nu}^{\alpha}{}_{\tau\lambda} - \\
& \frac{ie}{24} F_{\alpha\rho;\mu\kappa} S_{\nu}^{\alpha}{}_{\tau\lambda} - \frac{ie}{24} F_{\alpha\kappa;\mu\rho} S_{\nu}^{\alpha}{}_{\tau\lambda} - \frac{ie}{24} F_{\alpha\mu;\rho\tau} S_{\nu}^{\alpha}{}_{\kappa\lambda} - \\
& \frac{ie}{24} F_{\alpha\rho;\mu\tau} S_{\nu}^{\alpha}{}_{\kappa\lambda} - \frac{ie}{24} F_{\alpha\tau;\mu\rho} S_{\nu}^{\alpha}{}_{\kappa\lambda} + \frac{ie}{24} F_{\lambda}^{\alpha} S_{\beta\mu\tau\kappa} S_{\nu}^{\beta}{}_{\rho\alpha} + \\
& \frac{ie}{24} F_{\kappa}^{\alpha} S_{\beta\mu\tau\lambda} S_{\nu}^{\beta}{}_{\rho\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} S_{\beta\mu\kappa\lambda} S_{\nu}^{\beta}{}_{\rho\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} S_{\beta\mu\kappa\alpha} S_{\nu}^{\beta}{}_{\rho\tau} + \\
& \frac{ie}{24} F_{\kappa}^{\alpha} S_{\beta\mu\lambda\alpha} S_{\nu}^{\beta}{}_{\rho\tau} + \frac{ie}{24} F_{\lambda}^{\alpha} S_{\beta\mu\tau\alpha} S_{\nu}^{\beta}{}_{\rho\kappa} + \frac{ie}{24} F_{\tau}^{\alpha} S_{\beta\mu\lambda\alpha} S_{\nu}^{\beta}{}_{\rho\kappa} + \\
& \frac{ie}{24} F_{\kappa}^{\alpha} S_{\beta\mu\tau\alpha} S_{\nu}^{\beta}{}_{\rho\lambda} + \frac{ie}{24} F_{\tau}^{\alpha} S_{\beta\mu\kappa\alpha} S_{\nu}^{\beta}{}_{\rho\lambda} + \frac{ie}{24} F_{\lambda}^{\alpha} S_{\beta\mu\rho\kappa} S_{\nu}^{\beta}{}_{\tau\alpha} + \\
& \frac{ie}{24} F_{\kappa}^{\alpha} S_{\beta\mu\rho\lambda} S_{\nu}^{\beta}{}_{\tau\alpha} + \frac{ie}{24} F_{\rho}^{\alpha} S_{\beta\mu\kappa\lambda} S_{\nu}^{\beta}{}_{\tau\alpha} + \frac{ie}{24} F_{\lambda}^{\alpha} S_{\beta\mu\rho\alpha} S_{\nu}^{\beta}{}_{\tau\kappa} + \\
& \frac{ie}{24} F_{\rho}^{\alpha} S_{\beta\mu\lambda\alpha} S_{\nu}^{\beta}{}_{\tau\kappa} + \frac{ie}{24} F_{\kappa}^{\alpha} S_{\beta\mu\rho\alpha} S_{\nu}^{\beta}{}_{\tau\lambda} + \frac{ie}{24} F_{\rho}^{\alpha} S_{\beta\mu\kappa\alpha} S_{\nu}^{\beta}{}_{\tau\lambda} + \\
& \frac{ie}{24} F_{\lambda}^{\alpha} S_{\beta\mu\rho\tau} S_{\nu}^{\beta}{}_{\kappa\alpha} + \frac{ie}{24} F_{\tau}^{\alpha} S_{\beta\mu\rho\lambda} S_{\nu}^{\beta}{}_{\kappa\alpha} + \frac{ie}{24} F_{\rho}^{\alpha} S_{\beta\mu\tau\lambda} S_{\nu}^{\beta}{}_{\kappa\alpha} +
\end{aligned}$$

$$\begin{aligned}
& \frac{ie}{24} F_\tau^\alpha S_{\beta\mu\rho\alpha} S_\nu^\beta{}_{\kappa\lambda} + \frac{ie}{24} F_\rho^\alpha S_{\beta\mu\tau\alpha} S_\nu^\beta{}_{\kappa\lambda} + \frac{ie}{24} F_\kappa^\alpha S_{\beta\mu\rho\tau} S_\nu^\beta{}_{\lambda\alpha} + \\
& \frac{ie}{24} F_\tau^\alpha S_{\beta\mu\rho\kappa} S_\nu^\beta{}_{\lambda\alpha} + \frac{ie}{24} F_\rho^\alpha S_{\beta\mu\tau\kappa} S_\nu^\beta{}_{\lambda\alpha} + \frac{e^2 F_{\mu\nu} F_\lambda^\alpha S_{\rho\tau\kappa\alpha}}{24} + \\
& \frac{e^2 F_{\mu\nu} F_\kappa^\alpha S_{\rho\tau\lambda\alpha}}{24} + \frac{e^2 F_\mu^\alpha F_{\nu\lambda} S_{\rho\kappa\tau\alpha}}{24} + \frac{e^2 F_{\mu\lambda} F_\nu^\alpha S_{\rho\kappa\tau\alpha}}{24} + \\
& \frac{ie}{24} F_\alpha^\beta R_{\mu\nu\lambda}^\alpha S_{\rho\kappa\tau\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\nu\lambda} S_{\rho\kappa\tau\beta} + \frac{e^2 F_{\mu\nu} F_\tau^\alpha S_{\rho\kappa\lambda\alpha}}{24} + \\
& \frac{e^2 F_\mu^\alpha F_{\nu\kappa} S_{\rho\lambda\tau\alpha}}{24} + \frac{e^2 F_{\mu\kappa} F_\nu^\alpha S_{\rho\lambda\tau\alpha}}{24} + \frac{ie}{24} F_\alpha^\beta R_{\mu\nu\kappa}^\alpha S_{\rho\lambda\tau\beta} + \\
& \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\nu\kappa} S_{\rho\lambda\tau\beta} + \frac{e^2 F_\mu^\alpha F_{\nu\tau} S_{\rho\lambda\kappa\alpha}}{24} + \frac{e^2 F_{\mu\tau} F_\nu^\alpha S_{\rho\lambda\kappa\alpha}}{24} + \\
& \frac{ie}{24} F_\alpha^\beta R_{\mu\nu\tau}^\alpha S_{\rho\lambda\kappa\beta} + \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\nu\tau} S_{\rho\lambda\kappa\beta} - \frac{ie}{24} F_{\alpha\mu;\nu\lambda} S_\rho^\alpha{}_{\tau\kappa} - \\
& \frac{ie}{24} F_{\alpha\nu;\mu\lambda} S_\rho^\alpha{}_{\tau\kappa} - \frac{ie}{24} F_{\alpha\lambda;\mu\nu} S_\rho^\alpha{}_{\tau\kappa} - \frac{ie}{24} F_{\alpha\mu;\nu\kappa} S_\rho^\alpha{}_{\tau\lambda} - \\
& \frac{ie}{24} F_{\alpha\nu;\mu\kappa} S_\rho^\alpha{}_{\tau\lambda} - \frac{ie}{24} F_{\alpha\kappa;\mu\nu} S_\rho^\alpha{}_{\tau\lambda} - \frac{ie}{24} F_{\alpha\mu;\nu\tau} S_\rho^\alpha{}_{\kappa\lambda} - \\
& \frac{ie}{24} F_{\alpha\nu;\mu\tau} S_\rho^\alpha{}_{\kappa\lambda} - \frac{ie}{24} F_{\alpha\tau;\mu\nu} S_\rho^\alpha{}_{\kappa\lambda} + \frac{ie}{24} F_\lambda^\alpha S_{\beta\mu\nu\kappa} S_\rho^\beta{}_{\tau\alpha} + \\
& \frac{ie}{24} F_\kappa^\alpha S_{\beta\mu\nu\lambda} S_\rho^\beta{}_{\tau\alpha} + \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\kappa\lambda} S_\rho^\beta{}_{\tau\alpha} + \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\kappa\lambda} S_\rho^\beta{}_{\tau\alpha} + \\
& \frac{ie}{24} F_\lambda^\alpha S_{\beta\mu\nu\alpha} S_\rho^\beta{}_{\tau\kappa} + \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\lambda\alpha} S_\rho^\beta{}_{\tau\kappa} + \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\lambda\alpha} S_\rho^\beta{}_{\tau\kappa} + \\
& \frac{ie}{24} F_\kappa^\alpha S_{\beta\mu\nu\alpha} S_\rho^\beta{}_{\tau\lambda} + \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\kappa\alpha} S_\rho^\beta{}_{\tau\lambda} + \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\kappa\alpha} S_\rho^\beta{}_{\tau\lambda} + \\
& \frac{ie}{24} F_\lambda^\alpha S_{\beta\mu\nu\tau} S_\rho^\beta{}_{\kappa\alpha} + \frac{ie}{24} F_\tau^\alpha S_{\beta\mu\nu\lambda} S_\rho^\beta{}_{\kappa\alpha} + \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\tau\lambda} S_\rho^\beta{}_{\kappa\alpha} + \\
& \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\tau\lambda} S_\rho^\beta{}_{\kappa\alpha} + \frac{ie}{24} F_\tau^\alpha S_{\beta\mu\nu\alpha} S_\rho^\beta{}_{\kappa\lambda} + \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\tau\alpha} S_\rho^\beta{}_{\kappa\lambda} + \\
& \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\tau\alpha} S_\rho^\beta{}_{\kappa\lambda} + \frac{ie}{24} F_\kappa^\alpha S_{\beta\mu\nu\tau} S_\rho^\beta{}_{\lambda\alpha} + \frac{ie}{24} F_\tau^\alpha S_{\beta\mu\nu\kappa} S_\rho^\beta{}_{\lambda\alpha} + \\
& \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\tau\kappa} S_\rho^\beta{}_{\lambda\alpha} + \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\tau\kappa} S_\rho^\beta{}_{\lambda\alpha} + \frac{e^2 F_\mu^\alpha F_{\nu\rho} S_{\tau\lambda\kappa\alpha}}{24} + \\
& \frac{e^2 F_{\mu\rho} F_\nu^\alpha S_{\tau\lambda\kappa\alpha}}{24} + \frac{e^2 F_{\mu\nu} F_\rho^\alpha S_{\tau\lambda\kappa\alpha}}{24} + \frac{ie}{24} F_\alpha^\beta R_{\mu\nu\rho}^\alpha S_{\tau\lambda\kappa\beta} + \\
& \frac{ie}{24} F^{\alpha\beta} S_{\alpha\mu\nu\rho} S_{\tau\lambda\kappa\beta} - \frac{ie}{24} F_{\alpha\mu;\nu\rho} S_\tau^\alpha{}_{\kappa\lambda} - \frac{ie}{24} F_{\alpha\nu;\mu\rho} S_\tau^\alpha{}_{\kappa\lambda} - \\
& \frac{ie}{24} F_{\alpha\rho;\mu\nu} S_\tau^\alpha{}_{\kappa\lambda} + \frac{ie}{24} F_\lambda^\alpha S_{\beta\mu\nu\rho} S_\tau^\beta{}_{\kappa\alpha} + \frac{ie}{24} F_\rho^\alpha S_{\beta\mu\nu\lambda} S_\tau^\beta{}_{\kappa\alpha} + \\
& \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\rho\lambda} S_\tau^\beta{}_{\kappa\alpha} + \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\rho\lambda} S_\tau^\beta{}_{\kappa\alpha} + \frac{ie}{24} F_\rho^\alpha S_{\beta\mu\nu\alpha} S_\tau^\beta{}_{\kappa\lambda} + \\
& \frac{ie}{24} F_\nu^\alpha S_{\beta\mu\rho\alpha} S_\tau^\beta{}_{\kappa\lambda} + \frac{ie}{24} F_\mu^\alpha S_{\beta\nu\rho\alpha} S_\tau^\beta{}_{\kappa\lambda} + \frac{ie}{24} F_\kappa^\alpha S_{\beta\mu\nu\rho} S_\tau^\beta{}_{\lambda\alpha} +
\end{aligned}$$

$$\begin{aligned}
& \frac{ie}{24} F_{\rho}^{\alpha} S_{\beta\mu\nu\kappa} S_{\tau}^{\beta} \lambda_{\alpha} + \frac{ie}{24} F_{\nu}^{\alpha} S_{\beta\mu\rho\kappa} S_{\tau}^{\beta} \lambda_{\alpha} + \frac{ie}{24} F_{\mu}^{\alpha} S_{\beta\nu\rho\kappa} S_{\tau}^{\beta} \lambda_{\alpha} + \\
& \frac{ie}{24} F_{\tau}^{\alpha} S_{\beta\mu\nu\rho} S_{\kappa}^{\beta} \lambda_{\alpha} + \frac{ie}{24} F_{\rho}^{\alpha} S_{\beta\mu\nu\tau} S_{\kappa}^{\beta} \lambda_{\alpha} + \frac{ie}{24} F_{\nu}^{\alpha} S_{\beta\mu\rho\tau} S_{\kappa}^{\beta} \lambda_{\alpha} + \\
& \frac{ie}{24} F_{\mu}^{\alpha} S_{\beta\nu\rho\tau} S_{\kappa}^{\beta} \lambda_{\alpha} - \frac{ie}{18} F_{\alpha\kappa;\lambda} S_{5\mu}^{\alpha} \nu\rho\tau - \frac{ie}{18} F_{\alpha\lambda;\kappa} S_{5\mu}^{\alpha} \nu\rho\tau - \\
& \frac{ie}{18} F_{\alpha\tau;\lambda} S_{5\mu}^{\alpha} \nu\rho\kappa - \frac{ie}{18} F_{\alpha\lambda;\tau} S_{5\mu}^{\alpha} \nu\rho\kappa - \frac{ie}{18} F_{\alpha\tau;\kappa} S_{5\mu}^{\alpha} \nu\rho\lambda - \\
& \frac{ie}{18} F_{\alpha\kappa;\tau} S_{5\mu}^{\alpha} \nu\rho\lambda - \frac{ie}{18} F_{\alpha\rho;\lambda} S_{5\mu}^{\alpha} \nu\tau\kappa - \frac{ie}{18} F_{\alpha\lambda;\rho} S_{5\mu}^{\alpha} \nu\tau\kappa - \\
& \frac{ie}{18} F_{\alpha\rho;\kappa} S_{5\mu}^{\alpha} \nu\tau\lambda - \frac{ie}{18} F_{\alpha\kappa;\rho} S_{5\mu}^{\alpha} \nu\tau\lambda - \frac{ie}{18} F_{\alpha\rho;\tau} S_{5\mu}^{\alpha} \nu\kappa\lambda - \\
& \frac{ie}{18} F_{\alpha\tau;\rho} S_{5\mu}^{\alpha} \nu\kappa\lambda - \frac{ie}{18} F_{\alpha\nu;\lambda} S_{5\mu}^{\alpha} \rho\tau\kappa - \frac{ie}{18} F_{\alpha\lambda;\nu} S_{5\mu}^{\alpha} \rho\tau\kappa - \\
& \frac{ie}{18} F_{\alpha\nu;\kappa} S_{5\mu}^{\alpha} \rho\tau\lambda - \frac{ie}{18} F_{\alpha\kappa;\nu} S_{5\mu}^{\alpha} \rho\tau\lambda - \frac{ie}{18} F_{\alpha\nu;\tau} S_{5\mu}^{\alpha} \rho\kappa\lambda - \\
& \frac{ie}{18} F_{\alpha\tau;\nu} S_{5\mu}^{\alpha} \rho\kappa\lambda - \frac{ie}{18} F_{\alpha\nu;\rho} S_{5\mu}^{\alpha} \tau\kappa\lambda - \frac{ie}{18} F_{\alpha\rho;\nu} S_{5\mu}^{\alpha} \tau\kappa\lambda - \\
& \frac{ie}{18} F_{\alpha\mu;\lambda} S_{5\nu}^{\alpha} \rho\tau\kappa - \frac{ie}{18} F_{\alpha\lambda;\mu} S_{5\nu}^{\alpha} \rho\tau\kappa - \frac{ie}{18} F_{\alpha\mu;\kappa} S_{5\nu}^{\alpha} \rho\tau\lambda - \\
& \frac{ie}{18} F_{\alpha\kappa;\mu} S_{5\nu}^{\alpha} \rho\tau\lambda - \frac{ie}{18} F_{\alpha\mu;\tau} S_{5\nu}^{\alpha} \rho\kappa\lambda - \frac{ie}{18} F_{\alpha\tau;\mu} S_{5\nu}^{\alpha} \rho\kappa\lambda - \\
& \frac{ie}{18} F_{\alpha\mu;\rho} S_{5\nu}^{\alpha} \tau\kappa\lambda - \frac{ie}{18} F_{\alpha\rho;\mu} S_{5\nu}^{\alpha} \tau\kappa\lambda - \frac{ie}{18} F_{\alpha\mu;\nu} S_{5\rho}^{\alpha} \tau\kappa\lambda - \\
& \frac{ie}{18} F_{\alpha\nu;\mu} S_{5\rho}^{\alpha} \tau\kappa\lambda + \frac{ie}{12} F_{\lambda}^{\alpha} S_{6\alpha\mu\nu\rho\tau\kappa} + \frac{ie}{12} F_{\kappa}^{\alpha} S_{6\alpha\mu\nu\rho\tau\lambda} + \\
& \frac{ie}{12} F_{\tau}^{\alpha} S_{6\alpha\mu\nu\rho\kappa\lambda} + \frac{ie}{12} F_{\rho}^{\alpha} S_{6\alpha\mu\nu\tau\kappa\lambda} + \frac{ie}{12} F_{\nu}^{\alpha} S_{6\alpha\mu\rho\tau\kappa\lambda} + \\
& \frac{ie}{12} F_{\mu}^{\alpha} S_{6\alpha\nu\rho\tau\kappa\lambda}
\end{aligned}$$

(275)

## APPENDIX D

Coincidence limits  $[a_{1|\mu\nu\rho}]$  and  $[a_{1|\alpha}{}^\alpha{}_{\mu\nu}]$

$[a_1]_{\mu\nu\rho} =$ 

$$\begin{aligned}
& \frac{ie}{3} \left( \frac{1}{6} - \xi \right) RF_{\mu\nu;\rho} + \frac{ie}{3} \left( \frac{1}{6} - \xi \right) RF_{\mu\rho;\nu} + \left( \frac{1}{30} - \frac{\xi}{4} \right) R_{;\mu\nu\rho} + \frac{ie}{20} F_{\alpha\mu;\nu\rho} + \\
& \frac{ie}{20} F_{\alpha\nu;\mu\rho} + \frac{ie}{20} F_{\alpha\rho;\mu\nu} + \frac{1}{120} R_{\mu\nu;\alpha}{}^\alpha{}_\rho + \frac{1}{120} R_{\mu\rho;\alpha}{}^\alpha{}_\nu + \\
& \frac{1}{120} R_{\nu\rho;\alpha}{}^\alpha{}_\mu + \frac{ie}{4} \left( \frac{1}{6} - \xi \right) R_{;\rho} F_{\mu\nu} - \frac{e^2}{12} F_{\alpha\rho;\alpha} F_{\mu\nu} + \frac{ie}{4} \left( \frac{1}{6} - \xi \right) R_{;\nu} F_{\mu\rho} - \\
& \frac{e^2}{12} F_{\alpha\nu;\alpha} F_{\mu\rho} + \frac{e^2}{12} F_{\alpha\nu;\rho} F_{\mu}{}^\alpha + \frac{e^2}{12} F_{\alpha\rho;\nu} F_{\mu}{}^\alpha + \frac{ie}{96} R_{\alpha\nu;\rho} F_{\mu}{}^\alpha + \\
& \frac{ie}{96} R_{\alpha\rho;\nu} F_{\mu}{}^\alpha - \frac{ie}{48} R_{\nu\rho;\alpha} F_{\mu}{}^\alpha - \frac{ie}{160} R_{\alpha\nu\beta\rho;\alpha} F_{\mu}{}^\beta - \frac{ie}{160} R_{\alpha\rho\beta\nu;\alpha} F_{\mu}{}^\beta + \\
& \frac{ie}{4} \left( \frac{1}{6} - \xi \right) R_{;\mu} F_{\nu\rho} - \frac{e^2}{12} F_{\alpha\mu;\alpha} F_{\nu\rho} + \frac{e^2}{12} F_{\alpha\mu;\rho} F_{\nu}{}^\alpha + \frac{e^2}{12} F_{\alpha\rho;\mu} F_{\nu}{}^\alpha + \\
& \frac{ie}{96} R_{\alpha\mu;\rho} F_{\nu}{}^\alpha + \frac{ie}{96} R_{\alpha\rho;\mu} F_{\nu}{}^\alpha - \frac{ie}{48} R_{\mu\rho;\alpha} F_{\nu}{}^\alpha - \frac{ie}{160} R_{\alpha\mu\beta\rho;\alpha} F_{\nu}{}^\beta - \\
& \frac{ie}{160} R_{\alpha\rho\beta\mu;\alpha} F_{\nu}{}^\beta + \frac{e^2}{12} F_{\alpha\mu;\nu} F_{\rho}{}^\alpha + \frac{e^2}{12} F_{\alpha\nu;\mu} F_{\rho}{}^\alpha + \frac{ie}{96} R_{\alpha\mu;\nu} F_{\rho}{}^\alpha + \\
& \frac{ie}{96} R_{\alpha\nu;\mu} F_{\rho}{}^\alpha - \frac{ie}{48} R_{\mu\nu;\alpha} F_{\rho}{}^\alpha - \frac{ie}{160} R_{\alpha\mu\beta\nu;\alpha} F_{\rho}{}^\beta - \frac{ie}{160} R_{\alpha\nu\beta\mu;\alpha} F_{\rho}{}^\beta - \\
& \frac{ie}{160} R_{\alpha\mu\beta\nu;\rho} F^{\alpha\beta} - \frac{ie}{160} R_{\alpha\mu\beta\rho;\nu} F^{\alpha\beta} - \frac{ie}{160} R_{\alpha\nu\beta\mu;\rho} F^{\alpha\beta} - \frac{ie}{160} R_{\alpha\nu\beta\rho;\mu} F^{\alpha\beta} - \\
& \frac{ie}{160} R_{\alpha\rho\beta\mu;\nu} F^{\alpha\beta} - \frac{ie}{160} R_{\alpha\rho\beta\nu;\mu} F^{\alpha\beta} + \frac{ie}{180} F_{\alpha\nu;\rho} R_{\mu}{}^\alpha + \frac{ie}{180} F_{\alpha\rho;\nu} R_{\mu}{}^\alpha - \\
& \frac{31}{2880} R_{\alpha\nu;\rho} R_{\mu}{}^\alpha - \frac{31}{2880} R_{\alpha\rho;\nu} R_{\mu}{}^\alpha - \frac{1}{1440} R_{\nu\rho;\alpha} R_{\mu}{}^\alpha + \frac{1}{2880} R_{\alpha\nu\beta\rho;\alpha} R_{\mu}{}^\beta + \\
& \frac{1}{2880} R_{\alpha\rho\beta\nu;\alpha} R_{\mu}{}^\beta + \frac{ie}{180} F_{\alpha\mu;\rho} R_{\nu}{}^\alpha + \frac{ie}{180} F_{\alpha\rho;\mu} R_{\nu}{}^\alpha - \frac{31}{2880} R_{\alpha\mu;\rho} R_{\nu}{}^\alpha - \\
& \frac{31}{2880} R_{\alpha\rho;\mu} R_{\nu}{}^\alpha - \frac{1}{1440} R_{\mu\rho;\alpha} R_{\nu}{}^\alpha + \frac{1}{2880} R_{\alpha\mu\beta\rho;\alpha} R_{\nu}{}^\beta + \frac{1}{2880} R_{\alpha\rho\beta\mu;\alpha} R_{\nu}{}^\beta + \\
& \frac{ie}{180} F_{\alpha\mu;\nu} R_{\rho}{}^\alpha + \frac{ie}{180} F_{\alpha\nu;\mu} R_{\rho}{}^\alpha - \frac{31}{2880} R_{\alpha\mu;\nu} R_{\rho}{}^\alpha - \frac{31}{2880} R_{\alpha\nu;\mu} R_{\rho}{}^\alpha - \\
& \frac{1}{1440} R_{\mu\nu;\alpha} R_{\rho}{}^\alpha + \frac{1}{2880} R_{\alpha\mu\beta\nu;\alpha} R_{\rho}{}^\beta + \frac{1}{2880} R_{\alpha\nu\beta\mu;\alpha} R_{\rho}{}^\beta - \frac{1}{60} R_{\alpha\beta\mu\nu;\rho} R^{\alpha\beta} + \\
& \frac{1}{192} R_{\alpha\mu\beta\nu;\rho} R^{\alpha\beta} + \frac{1}{192} R_{\alpha\mu\beta\rho;\nu} R^{\alpha\beta} + \frac{1}{2880} R_{\alpha\nu\beta\mu;\rho} R^{\alpha\beta} + \frac{1}{192} R_{\alpha\nu\beta\rho;\mu} R^{\alpha\beta} + \\
& \frac{1}{2880} R_{\alpha\rho\beta\mu;\nu} R^{\alpha\beta} + \frac{1}{2880} R_{\alpha\rho\beta\nu;\mu} R^{\alpha\beta} - \frac{1}{120} R_{;\alpha} R_{\mu\nu\rho}{}^\alpha + \frac{1}{24} \xi R_{;\alpha} R_{\mu\nu\rho}{}^\alpha + \\
& \frac{ie}{120} F_{\alpha\beta;\beta} R_{\mu\nu\rho}{}^\alpha + \frac{ie}{90} F_{\alpha\beta;\alpha} R_{\mu\nu\rho}{}^\beta - \frac{ie}{180} F_{\alpha\rho;\beta} R_{\mu\nu}{}^{\alpha\beta} - \frac{1}{960} R_{\alpha\beta\delta\rho;\beta} R_{\mu\nu}{}^{\alpha\delta} + \\
& \frac{1}{960} R_{\alpha\rho\beta\delta;\beta} R_{\mu\nu}{}^{\alpha\delta} - \frac{1}{120} R_{;\alpha} R_{\mu\rho\nu}{}^\alpha + \frac{1}{24} \xi R_{;\alpha} R_{\mu\rho\nu}{}^\alpha + \frac{ie}{120} F_{\alpha\beta;\beta} R_{\mu\rho\nu}{}^\alpha -
\end{aligned}$$

$$\begin{aligned}
& \frac{ie}{180} F_{\alpha\nu;\beta} R_{\mu\rho}^{\alpha\beta} - \frac{1}{960} R_{\alpha\beta\delta\nu;\beta} R_{\mu\rho}^{\alpha\delta} + \frac{1}{960} R_{\alpha\nu\beta\delta;\beta} R_{\mu\rho}^{\alpha\delta} - \frac{1}{40} R_{;\alpha} R_{\mu}^{\alpha}{}_{\nu\rho} + \\
& \frac{1}{8} \xi R_{;\alpha} R_{\mu}^{\alpha}{}_{\nu\rho} + \frac{ie}{24} F_{\alpha\beta;\beta} R_{\mu}^{\alpha}{}_{\nu\rho} + \frac{ie}{60} F_{\alpha\rho;\beta} R_{\mu}^{\alpha}{}_{\nu\beta} + \frac{1}{1440} R_{\alpha\beta;\rho} R_{\mu}^{\alpha}{}_{\nu\beta} + \\
& \frac{1}{480} R_{\alpha\rho;\beta} R_{\mu}^{\alpha}{}_{\nu\beta} - \frac{1}{720} R_{\alpha\beta\delta\rho;\delta} R_{\mu}^{\alpha}{}_{\nu\beta} + \frac{1}{2880} R_{\alpha\rho\beta\delta;\delta} R_{\mu}^{\alpha}{}_{\nu\beta} + \frac{1}{1440} R_{\alpha\beta\delta\rho;\beta} R_{\mu}^{\alpha}{}_{\nu}{}^{\delta} - \\
& \frac{1}{960} R_{\alpha\rho\beta\delta;\beta} R_{\mu}^{\alpha}{}_{\nu}{}^{\delta} + \frac{ie}{60} F_{\alpha\nu;\beta} R_{\mu}^{\alpha}{}_{\rho\beta} + \frac{1}{1440} R_{\alpha\beta;\nu} R_{\mu}^{\alpha}{}_{\rho\beta} + \frac{1}{480} R_{\alpha\nu;\beta} R_{\mu}^{\alpha}{}_{\rho\beta} - \\
& \frac{1}{720} R_{\alpha\beta\delta\nu;\delta} R_{\mu}^{\alpha}{}_{\rho\beta} + \frac{1}{2880} R_{\alpha\nu\beta\delta;\delta} R_{\mu}^{\alpha}{}_{\rho\beta} + \frac{1}{1440} R_{\alpha\beta\delta\nu;\beta} R_{\mu}^{\alpha}{}_{\rho}{}^{\delta} - \frac{1}{960} R_{\alpha\nu\beta\delta;\beta} R_{\mu}^{\alpha}{}_{\rho}{}^{\delta} + \\
& \frac{1}{480} R_{\alpha\beta\delta\nu;\rho} R_{\mu}^{\alpha\beta\delta} + \frac{1}{480} R_{\alpha\beta\delta\rho;\nu} R_{\mu}^{\alpha\beta\delta} - \frac{1}{720} R_{\alpha\nu\beta\delta;\rho} R_{\mu}^{\alpha\beta\delta} - \frac{1}{480} R_{\alpha\nu\beta\rho;\delta} R_{\mu}^{\alpha\beta\delta} - \\
& \frac{1}{720} R_{\alpha\rho\beta\delta;\nu} R_{\mu}^{\alpha\beta\delta} - \frac{1}{1440} R_{\alpha\rho\beta\nu;\delta} R_{\mu}^{\alpha\beta\delta} - \frac{ie}{90} F_{\alpha\beta;\alpha} R_{\mu}^{\beta}{}_{\nu\rho} + \frac{ie}{180} F_{\alpha\rho;\beta} R_{\mu}^{\beta}{}_{\nu}{}^{\alpha} + \\
& \frac{1}{1440} R_{\alpha\beta;\rho} R_{\mu}^{\beta}{}_{\nu}{}^{\alpha} + \frac{1}{480} R_{\alpha\rho;\beta} R_{\mu}^{\beta}{}_{\nu}{}^{\alpha} - \frac{1}{720} R_{\alpha\beta\delta\rho;\delta} R_{\mu}^{\beta}{}_{\nu}{}^{\alpha} + \frac{1}{2880} R_{\alpha\rho\beta\delta;\delta} R_{\mu}^{\beta}{}_{\nu}{}^{\alpha} + \\
& \frac{1}{1440} R_{\alpha\rho\beta\delta;\alpha} R_{\mu}^{\beta}{}_{\nu}{}^{\delta} + \frac{ie}{180} F_{\alpha\nu;\beta} R_{\mu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{1440} R_{\alpha\beta;\nu} R_{\mu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{480} R_{\alpha\nu;\beta} R_{\mu}^{\beta}{}_{\rho}{}^{\alpha} - \\
& \frac{1}{720} R_{\alpha\beta\delta\nu;\delta} R_{\mu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{2880} R_{\alpha\nu\beta\delta;\delta} R_{\mu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{1440} R_{\alpha\nu\beta\delta;\alpha} R_{\mu}^{\beta}{}_{\rho}{}^{\delta} + \frac{1}{576} R_{\alpha\beta\delta\nu;\rho} R_{\mu}^{\beta\alpha\delta} + \\
& \frac{1}{576} R_{\alpha\beta\delta\rho;\nu} R_{\mu}^{\beta\alpha\delta} - \frac{1}{480} R_{\alpha\nu\beta\rho;\delta} R_{\mu}^{\beta\alpha\delta} - \frac{1}{1440} R_{\alpha\rho\beta\nu;\delta} R_{\mu}^{\beta\alpha\delta} + \frac{1}{1440} R_{\alpha\beta\delta\rho;\beta} R_{\mu}^{\delta}{}_{\nu}{}^{\alpha} - \\
& \frac{1}{960} R_{\alpha\rho\beta\delta;\beta} R_{\mu}^{\delta}{}_{\nu}{}^{\alpha} + \frac{1}{1440} R_{\alpha\rho\beta\delta;\alpha} R_{\mu}^{\delta}{}_{\nu}{}^{\beta} + \frac{1}{1440} R_{\alpha\beta\delta\nu;\beta} R_{\mu}^{\delta}{}_{\rho}{}^{\alpha} - \frac{1}{960} R_{\alpha\nu\beta\delta;\beta} R_{\mu}^{\delta}{}_{\rho}{}^{\alpha} + \\
& \frac{1}{1440} R_{\alpha\nu\beta\delta;\alpha} R_{\mu}^{\delta}{}_{\rho}{}^{\beta} - \frac{1}{720} R_{\alpha\beta\delta\nu;\rho} R_{\mu}^{\delta\alpha\beta} - \frac{1}{720} R_{\alpha\beta\delta\rho;\nu} R_{\mu}^{\delta\alpha\beta} + \frac{7}{2880} R_{\alpha\nu\beta\delta;\rho} R_{\mu}^{\delta\alpha\beta} + \\
& \frac{7}{2880} R_{\alpha\rho\beta\delta;\nu} R_{\mu}^{\delta\alpha\beta} - \frac{ie}{180} F_{\alpha\mu;\beta} R_{\nu\rho}^{\alpha\beta} - \frac{1}{960} R_{\alpha\beta\delta\mu;\beta} R_{\nu\rho}^{\alpha\delta} + \frac{1}{960} R_{\alpha\mu\beta\delta;\beta} R_{\nu\rho}^{\alpha\delta} + \\
& \frac{ie}{60} F_{\alpha\mu;\beta} R_{\nu}^{\alpha}{}_{\rho}{}^{\beta} + \frac{1}{1440} R_{\alpha\beta;\mu} R_{\nu}^{\alpha}{}_{\rho}{}^{\beta} + \frac{1}{480} R_{\alpha\mu;\beta} R_{\nu}^{\alpha}{}_{\rho}{}^{\beta} - \frac{1}{720} R_{\alpha\beta\delta\mu;\delta} R_{\nu}^{\alpha}{}_{\rho}{}^{\beta} + \\
& \frac{1}{2880} R_{\alpha\mu\beta\delta;\delta} R_{\nu}^{\alpha}{}_{\rho}{}^{\beta} + \frac{1}{1440} R_{\alpha\beta\delta\mu;\beta} R_{\nu}^{\alpha}{}_{\rho}{}^{\delta} - \frac{1}{960} R_{\alpha\mu\beta\delta;\beta} R_{\nu}^{\alpha}{}_{\rho}{}^{\delta} + \frac{1}{480} R_{\alpha\beta\delta\mu;\rho} R_{\nu}^{\alpha\beta\delta} + \\
& \frac{1}{480} R_{\alpha\beta\delta\rho;\mu} R_{\nu}^{\alpha\beta\delta} - \frac{1}{720} R_{\alpha\mu\beta\delta;\rho} R_{\nu}^{\alpha\beta\delta} - \frac{1}{480} R_{\alpha\mu\beta\rho;\delta} R_{\nu}^{\alpha\beta\delta} - \frac{1}{720} R_{\alpha\rho\beta\delta;\mu} R_{\nu}^{\alpha\beta\delta} - \\
& \frac{1}{1440} R_{\alpha\rho\beta\mu;\delta} R_{\nu}^{\alpha\beta\delta} + \frac{ie}{180} F_{\alpha\mu;\beta} R_{\nu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{1440} R_{\alpha\beta;\mu} R_{\nu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{480} R_{\alpha\mu;\beta} R_{\nu}^{\beta}{}_{\rho}{}^{\alpha} - \\
& \frac{1}{720} R_{\alpha\beta\delta\mu;\delta} R_{\nu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{2880} R_{\alpha\mu\beta\delta;\delta} R_{\nu}^{\beta}{}_{\rho}{}^{\alpha} + \frac{1}{1440} R_{\alpha\mu\beta\delta;\alpha} R_{\nu}^{\beta}{}_{\rho}{}^{\delta} + \frac{1}{576} R_{\alpha\beta\delta\mu;\rho} R_{\nu}^{\beta\alpha\delta} + \\
& \frac{1}{576} R_{\alpha\beta\delta\rho;\mu} R_{\nu}^{\beta\alpha\delta} - \frac{1}{480} R_{\alpha\mu\beta\rho;\delta} R_{\nu}^{\beta\alpha\delta} - \frac{1}{1440} R_{\alpha\rho\beta\mu;\delta} R_{\nu}^{\beta\alpha\delta} + \frac{1}{1440} R_{\alpha\beta\delta\mu;\beta} R_{\nu}^{\delta}{}_{\rho}{}^{\alpha} - \\
& \frac{1}{960} R_{\alpha\mu\beta\delta;\beta} R_{\nu}^{\delta}{}_{\rho}{}^{\alpha} + \frac{1}{1440} R_{\alpha\mu\beta\delta;\alpha} R_{\nu}^{\delta}{}_{\rho}{}^{\beta} - \frac{1}{720} R_{\alpha\beta\delta\mu;\rho} R_{\nu}^{\delta\alpha\beta} - \frac{1}{720} R_{\alpha\beta\delta\rho;\mu} R_{\nu}^{\delta\alpha\beta} + \\
& \frac{7}{2880} R_{\alpha\mu\beta\delta;\rho} R_{\nu}^{\delta\alpha\beta} + \frac{7}{2880} R_{\alpha\rho\beta\delta;\mu} R_{\nu}^{\delta\alpha\beta} + \frac{1}{480} R_{\alpha\beta\delta\mu;\nu} R_{\rho}^{\alpha\beta\delta} + \frac{1}{480} R_{\alpha\beta\delta\nu;\mu} R_{\rho}^{\alpha\beta\delta} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{720} R_{\alpha\mu\beta\delta;\nu} R_{\rho}^{\alpha\beta\delta} - \frac{1}{480} R_{\alpha\mu\beta\nu;\delta} R_{\rho}^{\alpha\beta\delta} - \frac{1}{720} R_{\alpha\nu\beta\delta;\mu} R_{\rho}^{\alpha\beta\delta} - \frac{1}{1440} R_{\alpha\nu\beta\mu;\delta} R_{\rho}^{\alpha\beta\delta} + \\
& \frac{1}{576} R_{\alpha\beta\delta\mu;\nu} R_{\rho}^{\beta\alpha\delta} + \frac{1}{576} R_{\alpha\beta\delta\nu;\mu} R_{\rho}^{\beta\alpha\delta} - \frac{1}{480} R_{\alpha\mu\beta\nu;\delta} R_{\rho}^{\beta\alpha\delta} - \frac{1}{1440} R_{\alpha\nu\beta\mu;\delta} R_{\rho}^{\beta\alpha\delta} - \\
& \frac{1}{720} R_{\alpha\beta\delta\mu;\nu} R_{\rho}^{\delta\alpha\beta} - \frac{1}{720} R_{\alpha\beta\delta\nu;\mu} R_{\rho}^{\delta\alpha\beta} + \frac{7}{2880} R_{\alpha\mu\beta\delta;\nu} R_{\rho}^{\delta\alpha\beta} + \frac{7}{2880} R_{\alpha\nu\beta\delta;\mu} R_{\rho}^{\delta\alpha\beta}
\end{aligned} \tag{276}$$

$$[a_1]_{\alpha}^{\mu\nu} =$$

$$\begin{aligned}
& \frac{R_{;\mu} R_{;\nu}}{120} + \frac{ie}{45} R_{;\nu} F_{\alpha\mu}{}^{;\alpha} - \frac{ie}{6} \xi R_{;\nu} F_{\alpha\mu}{}^{;\alpha} + \\
& \frac{ie}{45} R_{;\mu} F_{\alpha\nu}{}^{;\alpha} - \frac{ie}{6} \xi R_{;\mu} F_{\alpha\nu}{}^{;\alpha} - \frac{e^2 F_{\alpha\mu}{}^{;\alpha} F_{\beta\nu}{}^{;\beta}}{9} - \\
& \frac{ie}{18} R_{;\alpha} F_{\mu}{}^{\alpha}{}_{;\nu} + \frac{ie}{3} \xi R_{;\alpha} F_{\mu}{}^{\alpha}{}_{;\nu} + \frac{4e^2 F_{\alpha\beta;\nu} F_{\mu}{}^{\alpha;\beta}}{45} + \\
& \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F_{\mu}{}^{\beta}{}_{;\nu}}{45} - \frac{ie}{18} R_{;\alpha} F_{\nu}{}^{\alpha}{}_{;\mu} + \frac{ie}{3} \xi R_{;\alpha} F_{\nu}{}^{\alpha}{}_{;\mu} + \\
& \frac{4e^2 F_{\alpha\beta;\mu} F_{\nu}{}^{\alpha;\beta}}{45} + \frac{4e^2 F_{\alpha\mu;\beta} F_{\nu}{}^{\alpha;\beta}}{45} + \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F_{\nu}{}^{\beta}{}_{;\mu}}{45} - \\
& \frac{4e^2 F_{\alpha\beta;\mu} F_{\nu}{}^{\alpha\beta}}{45} - \frac{R_{;\nu} R_{\alpha\mu}{}^{;\alpha}}{180} - \frac{R_{;\mu} R_{\alpha\nu}{}^{;\alpha}}{180} + \\
& \frac{ie}{90} F_{\alpha\nu}{}^{;\alpha} R_{\beta\mu}{}^{;\beta} + \frac{ie}{90} F_{\alpha\mu}{}^{;\alpha} R_{\beta\nu}{}^{;\beta} - \frac{R_{\alpha\mu}{}^{;\alpha} R_{\beta\nu}{}^{;\beta}}{90} + \\
& \frac{ie}{45} F_{\alpha\mu;\nu} R_{\beta}{}^{\alpha;\beta} + \frac{ie}{45} F_{\alpha\nu;\mu} R_{\beta}{}^{\alpha;\beta} + \frac{11 R_{;\alpha} R_{\mu\nu}{}^{;\alpha}}{2520} - \\
& \frac{ie}{45} F_{\alpha\beta}{}^{;\alpha} R_{\mu\nu}{}^{;\beta} - \frac{R_{\alpha\beta}{}^{;\alpha} R_{\mu\nu}{}^{;\beta}}{180} - \frac{5 R_{;\alpha} R_{\mu}{}^{\alpha}{}_{;\nu}}{168} + \\
& \frac{\xi R_{;\alpha} R_{\mu}{}^{\alpha}{}_{;\nu}}{10} - \frac{ie}{45} F_{\alpha\beta;\nu} R_{\mu}{}^{\alpha;\beta} - \frac{19 R_{\alpha\beta;\nu} R_{\mu}{}^{\alpha;\beta}}{1680} - \\
& \frac{2ie}{45} F_{\alpha\beta}{}^{;\alpha} R_{\mu}{}^{\beta}{}_{;\nu} - \frac{R_{\alpha\beta}{}^{;\alpha} R_{\mu}{}^{\beta}{}_{;\nu}}{180} + \frac{ie}{45} F_{\alpha\nu;\beta} R_{\mu}{}^{\beta;\alpha} - \\
& \frac{19 R_{\alpha\nu;\beta} R_{\mu}{}^{\beta;\alpha}}{5040} - \frac{5 R_{;\alpha} R_{\nu}{}^{\alpha}{}_{;\mu}}{168} + \frac{\xi R_{;\alpha} R_{\nu}{}^{\alpha}{}_{;\mu}}{10} - \\
& \frac{ie}{45} F_{\alpha\beta;\mu} R_{\nu}{}^{\alpha;\beta} - \frac{19 R_{\alpha\beta;\mu} R_{\nu}{}^{\alpha;\beta}}{1680} - \frac{23 R_{\alpha\mu;\beta} R_{\nu}{}^{\alpha;\beta}}{5040} - \\
& \frac{2ie}{45} F_{\alpha\beta}{}^{;\alpha} R_{\nu}{}^{\beta}{}_{;\mu} - \frac{R_{\alpha\beta}{}^{;\alpha} R_{\nu}{}^{\beta}{}_{;\mu}}{180} + \frac{ie}{45} F_{\alpha\mu;\beta} R_{\nu}{}^{\beta;\alpha} - \\
& \frac{19 R_{\alpha\mu;\beta} R_{\nu}{}^{\beta;\alpha}}{5040} - \frac{41 R_{\alpha\beta;\mu} R_{\nu}{}^{\alpha\beta}}{5040} - \frac{R_{;\alpha} R_{\beta\mu\nu}{}^{\alpha;\beta}}{630}
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{;\alpha} R_{\beta\nu\mu}{}^{\alpha;\beta}}{630} + \frac{5R_{\alpha\beta;\nu} R_{\gamma\mu}{}^{\alpha\beta;\gamma}}{1008} - \frac{R_{\alpha\nu;\beta} R_{\gamma\mu}{}^{\alpha\beta;\gamma}}{560} + \\
& \frac{5R_{\alpha\beta;\mu} R_{\gamma\nu}{}^{\alpha\beta;\gamma}}{1008} - \frac{R_{\alpha\mu;\beta} R_{\gamma\nu}{}^{\alpha\beta;\gamma}}{560} + \frac{R_{\alpha\beta;\nu} R_{\gamma\mu}{}^{\alpha\beta;\gamma}}{240} + \\
& \frac{R_{\alpha\nu;\beta} R_{\gamma\mu}{}^{\alpha\beta;\gamma}}{840} + \frac{R_{\alpha\beta;\mu} R_{\gamma\nu}{}^{\alpha\beta;\gamma}}{240} + \frac{R_{\alpha\mu;\beta} R_{\gamma\nu}{}^{\alpha\beta;\gamma}}{840} + \\
& \frac{R_{\alpha\nu;\beta} R_{\gamma\mu}{}^{\beta\alpha;\gamma}}{336} + \frac{R_{\alpha\mu;\beta} R_{\gamma\nu}{}^{\beta\alpha;\gamma}}{336} + \frac{R_{\alpha\beta\gamma\mu}{}^{;\gamma} R_{\delta\nu}{}^{\alpha\beta;\delta}}{1680} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha} R_{\delta\nu}{}^{\beta\gamma;\delta}}{1680} + \frac{R_{\alpha\beta\gamma\mu}{}^{;\gamma} R_{\delta\nu}{}^{\alpha\beta;\delta}}{1680} - \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha} R_{\delta\nu}{}^{\beta\gamma;\delta}}{240} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha} R_{\delta\nu}{}^{\gamma\beta;\delta}}{280} + \frac{11R_{\alpha\beta;\gamma} R_{\mu\nu}{}^{\alpha\beta;\gamma}}{2520} + \frac{13R_{\alpha\beta;\gamma} R_{\mu\nu}{}^{\alpha\gamma;\beta}}{5040} - \\
& \frac{R_{\alpha\beta;\gamma} R_{\mu\nu}{}^{\alpha\beta\gamma}{}_{;\nu}}{560} + \frac{R_{\alpha\beta\gamma\delta;\nu} R_{\mu\nu}{}^{\alpha\beta\gamma;\delta}}{420} + \frac{R_{\alpha\beta\gamma\nu;\delta} R_{\mu\nu}{}^{\alpha\beta\gamma;\delta}}{420} - \\
& \frac{R_{\alpha\beta\gamma\delta;\nu} R_{\mu\nu}{}^{\alpha\gamma\delta;\beta}}{210} - \frac{R_{\alpha\beta\gamma\nu;\delta} R_{\mu\nu}{}^{\alpha\gamma\delta;\beta}}{420} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_{\mu\nu}{}^{\beta\gamma;\delta}}{240} + \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_{\mu\nu}{}^{\beta\gamma\delta}{}_{;\nu}}{240} + \frac{13R_{\alpha\beta;\gamma} R_{\mu\nu}{}^{\gamma\alpha;\beta}}{5040} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_{\mu\nu}{}^{\gamma\beta;\delta}}{240} - \\
& \frac{23R_{\alpha\beta;\gamma} R_{\mu\nu}{}^{\gamma\alpha\beta}{}_{;\nu}}{1680} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_{\mu\nu}{}^{\gamma\beta\delta}{}_{;\nu}}{240} - \frac{R_{\alpha\beta;\gamma} R_{\nu\mu}{}^{\alpha\beta\gamma}{}_{;\mu}}{560} + \\
& \frac{R_{\alpha\beta\gamma\delta}{}_{;\mu} R_{\nu\mu}{}^{\alpha\beta\gamma;\delta}}{210} + \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu\nu}{}^{\alpha\beta\gamma;\delta}}{420} - \frac{R_{\alpha\beta\gamma\delta}{}_{;\mu} R_{\nu\nu}{}^{\alpha\gamma\delta;\beta}}{210} - \\
& \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu\nu}{}^{\alpha\gamma\delta;\beta}}{420} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_{\nu\nu}{}^{\beta\gamma\delta}{}_{;\mu}}{240} - \frac{61R_{\alpha\beta;\gamma} R_{\nu\nu}{}^{\gamma\alpha\beta}{}_{;\mu}}{5040} - \\
& \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu\nu}{}^{\gamma\alpha\beta;\delta}}{210} - \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu\nu}{}^{\gamma\alpha\delta;\beta}}{210} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha} R_{\nu\nu}{}^{\gamma\beta\delta}{}_{;\mu}}{240} + \\
& \frac{R_{\alpha\beta\gamma\delta}{}_{;\mu} R^{\alpha\beta\gamma\delta}{}_{;\nu}}{420} + \frac{R_{\alpha\beta\gamma\delta}{}_{;\mu} R^{\alpha\gamma\beta\delta}{}_{;\nu}}{420} + \frac{R_{\alpha\beta\gamma\mu;\delta} R^{\alpha\gamma\beta\delta}{}_{;\nu}}{420} + \\
& \frac{RR_{;\mu\nu}}{300} + \frac{ie}{5} \left( \frac{1}{6} - \xi \right) RF_{\alpha\mu}{}^{;\alpha}{}_{\nu} + \frac{ie}{120} RF_{\alpha\mu}{}^{;\alpha}{}_{\nu} - \\
& \frac{ie}{20} R\xi F_{\alpha\mu}{}^{;\alpha}{}_{\nu} + \frac{ie}{5} \left( \frac{1}{6} - \xi \right) RF_{\alpha\nu}{}^{;\alpha}{}_{\mu} + \frac{ie}{120} RF_{\alpha\nu}{}^{;\alpha}{}_{\mu} - \\
& \frac{ie}{20} R\xi F_{\alpha\nu}{}^{;\alpha}{}_{\mu} + \frac{RR_{\alpha\mu}{}^{;\alpha}{}_{\nu}}{150} + \frac{RR_{\alpha\nu}{}^{;\alpha}{}_{\mu}}{150} + \\
& \frac{RR_{\mu\nu}{}^{;\alpha}{}_{\alpha}}{300} + \frac{R_{;\alpha}{}^{\alpha}{}_{\mu\nu}}{105} - \frac{\xi R_{;\alpha}{}^{\alpha}{}_{\mu\nu}}{5} + \\
& \frac{ie}{15} F^{\alpha\beta}{}_{\mu\nu} + \frac{ie}{30} F^{\alpha\mu}{}^{;\alpha}{}_{\beta\nu} + \frac{ie}{30} F^{\alpha\nu}{}^{;\alpha}{}_{\beta\mu} + \\
& \frac{R_{\alpha\beta}{}^{;\alpha\beta}{}_{\mu\nu}}{70} + \frac{R_{\alpha\beta}{}^{;\beta\alpha}{}_{\mu\nu}}{210} + \frac{R_{\alpha\mu;\beta}{}^{\beta\alpha}{}_{\nu}}{105} + \\
& \frac{R_{\alpha\mu}{}^{;\alpha}{}_{\beta\nu}}{105} + \frac{R_{\alpha\nu;\beta}{}^{\beta\alpha}{}_{\mu}}{105} + \frac{R_{\alpha\nu}{}^{;\alpha}{}_{\beta\mu}}{105} +
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\mu\nu;\alpha}{}^{\alpha}{}_{\beta}{}^{\beta}}{210} + \frac{ie}{75}R_{;\alpha}{}^{\alpha}F_{\mu\nu} - \frac{ie}{6}\xi R_{;\alpha}{}^{\alpha}F_{\mu\nu} - \\
& \frac{e^2F_{\alpha\beta}{}^{;\alpha\beta}F_{\mu\nu}}{15} + \frac{e^2F_{\alpha\beta}{}^{;\beta\alpha}F_{\mu\nu}}{60} - \frac{ie}{50}R_{;\alpha\nu}F_{\mu}{}^{\alpha} + \\
& \frac{ie}{3}\xi R_{;\alpha\nu}F_{\mu}{}^{\alpha} + \frac{e^2F_{\alpha\beta}{}^{;\beta}{}_{\nu}F_{\mu}{}^{\alpha}}{24} + \frac{e^2F_{\alpha\nu}{}^{;\beta}{}_{\mu}F_{\mu}{}^{\alpha}}{20} + \\
& \frac{ie}{30}R_{\alpha\beta}{}^{;\beta}{}_{\nu}F_{\mu}{}^{\alpha} + \frac{ie}{100}R_{\alpha\nu}{}^{;\beta}{}_{\mu}F_{\mu}{}^{\alpha} + \frac{e^2F_{\alpha\beta}{}^{;\alpha}{}_{\nu}F_{\mu}{}^{\beta}}{40} + \\
& \frac{e^2F_{\alpha\nu}{}^{;\alpha}{}_{\beta}F_{\mu}{}^{\beta}}{12} - \frac{ie}{50}R_{;\alpha\mu}F_{\nu}{}^{\alpha} + \frac{ie}{3}\xi R_{;\alpha\mu}F_{\nu}{}^{\alpha} + \\
& \frac{e^2F_{\alpha\beta}{}^{;\beta}{}_{\mu}F_{\nu}{}^{\alpha}}{24} + \frac{e^2F_{\alpha\mu}{}^{;\beta}{}_{\nu}F_{\nu}{}^{\alpha}}{20} + \frac{ie}{30}R_{\alpha\beta}{}^{;\beta}{}_{\mu}F_{\nu}{}^{\alpha} + \\
& \frac{ie}{100}R_{\alpha\mu}{}^{;\beta}{}_{\nu}F_{\nu}{}^{\alpha} + \frac{e^2RF_{\alpha\mu}F_{\nu}{}^{\alpha}}{12} - \frac{e^2R\xi F_{\alpha\mu}F_{\nu}{}^{\alpha}}{2} + \\
& \frac{e^2F_{\alpha\beta}{}^{;\alpha}{}_{\mu}F_{\nu}{}^{\beta}}{40} + \frac{e^2F_{\alpha\mu}{}^{;\alpha}{}_{\beta}F_{\nu}{}^{\beta}}{12} - \frac{e^2F_{\alpha\beta}{}^{;\mu\nu}F^{\alpha\beta}}{10} - \\
& \frac{e^2F_{\alpha\mu}{}^{;\beta\nu}F^{\alpha\beta}}{10} - \frac{e^2F_{\alpha\nu}{}^{;\beta\mu}F^{\alpha\beta}}{10} - \frac{ie}{30}R_{\alpha\beta}{}^{;\mu\nu}F^{\alpha\beta} - \\
& \frac{ie}{30}R_{\alpha\mu}{}^{;\beta\nu}F^{\alpha\beta} - \frac{ie}{30}R_{\alpha\nu}{}^{;\beta\mu}F^{\alpha\beta} - \frac{ie^3}{12}F_{\alpha\beta}F_{\mu\nu}F^{\alpha\beta} + \\
& \frac{R_{;\alpha}{}^{\alpha}R_{\mu\nu}}{300} + \frac{R_{\alpha\beta}{}^{;\alpha\beta}R_{\mu\nu}}{200} + \frac{R_{\alpha\beta}{}^{;\beta\alpha}R_{\mu\nu}}{600} - \\
& \frac{61R_{;\alpha\nu}R_{\mu}{}^{\alpha}}{6300} + \frac{\xi R_{;\alpha\nu}R_{\mu}{}^{\alpha}}{15} + \frac{ie}{24}F_{\alpha\beta}{}^{;\beta}{}_{\nu}R_{\mu}{}^{\alpha} + \\
& \frac{ie}{120}F_{\alpha\nu}{}^{;\beta}{}_{\mu}R_{\mu}{}^{\alpha} - \frac{47R_{\alpha\beta}{}^{;\beta}{}_{\nu}R_{\mu}{}^{\alpha}}{6300} - \frac{11R_{\alpha\nu}{}^{;\beta}{}_{\mu}R_{\mu}{}^{\alpha}}{1260} - \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma\beta}R_{\mu}{}^{\alpha}}{1050} - \frac{R_{\alpha\nu\beta\gamma}{}^{;\beta\gamma}R_{\mu}{}^{\alpha}}{1050} - \frac{ie}{60}RF_{\alpha\nu}R_{\mu}{}^{\alpha} + \\
& \frac{ie}{10}R\xi F_{\alpha\nu}R_{\mu}{}^{\alpha} - \frac{ie}{40}F_{\alpha\nu}{}^{;\alpha}{}_{\beta}R_{\mu}{}^{\beta} - \frac{19R_{\alpha\beta}{}^{;\alpha}{}_{\nu}R_{\mu}{}^{\beta}}{6300} - \\
& \frac{17R_{\alpha\nu}{}^{;\beta}{}_{\mu}R_{\mu}{}^{\beta}}{12600} + \frac{R_{\alpha\nu}{}^{;\alpha}{}_{\beta}R_{\mu}{}^{\beta}}{504} + \frac{2R_{\alpha\nu\beta\gamma}{}^{;\alpha\gamma}R_{\mu}{}^{\beta}}{1575} - \\
& \frac{e^2F_{\alpha\beta}F_{\nu}{}^{\alpha}R_{\mu}{}^{\beta}}{15} - \frac{23ie}{900}F_{\alpha\nu}R_{\beta}{}^{\alpha}R_{\mu}{}^{\beta} + \frac{2R_{\alpha\beta\gamma\nu}{}^{;\alpha\beta}R_{\mu}{}^{\gamma}}{1575} - \\
& \frac{61R_{;\alpha\mu}R_{\nu}{}^{\alpha}}{6300} + \frac{\xi R_{;\alpha\mu}R_{\nu}{}^{\alpha}}{15} + \frac{ie}{24}F_{\alpha\beta}{}^{;\beta}{}_{\mu}R_{\nu}{}^{\alpha} + \\
& \frac{ie}{120}F_{\alpha\mu}{}^{;\beta}{}_{\nu}R_{\nu}{}^{\alpha} - \frac{47R_{\alpha\beta}{}^{;\beta}{}_{\mu}R_{\nu}{}^{\alpha}}{6300} - \frac{11R_{\alpha\mu}{}^{;\beta}{}_{\nu}R_{\nu}{}^{\alpha}}{1260} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{;\gamma\beta}R_{\nu}{}^{\alpha}}{1050} - \frac{R_{\alpha\mu\beta\gamma}{}^{;\beta\gamma}R_{\nu}{}^{\alpha}}{1050} - \frac{ie}{60}RF_{\alpha\mu}R_{\nu}{}^{\alpha} + \\
& \frac{ie}{10}R\xi F_{\alpha\mu}R_{\nu}{}^{\alpha} - \frac{RR_{\alpha\mu}R_{\nu}{}^{\alpha}}{1350} - \frac{ie}{40}F_{\alpha\mu}{}^{;\alpha}{}_{\beta}R_{\nu}{}^{\beta} -
\end{aligned}$$

$$\begin{aligned}
& \frac{19R_{\alpha\beta}{}^{\alpha\mu}R_{\nu}{}^{\beta}}{6300} - \frac{17R_{\alpha\mu}{}^{\alpha\beta}R_{\nu}{}^{\beta}}{12600} + \frac{R_{\alpha\mu}{}^{\alpha\beta}R_{\nu}{}^{\beta}}{504} + \\
& \frac{2R_{\alpha\mu\beta\gamma}{}^{\alpha\gamma}R_{\nu}{}^{\beta}}{1575} - \frac{e^2F_{\alpha\beta}F_{\mu}{}^{\alpha}R_{\nu}{}^{\beta}}{15} - \frac{23ie}{900}F_{\alpha\mu}R_{\beta}{}^{\alpha}R_{\nu}{}^{\beta} + \\
& \frac{R_{\alpha\beta}R_{\mu}{}^{\alpha}R_{\nu}{}^{\beta}}{60} + \frac{2R_{\alpha\beta\gamma\mu}{}^{\alpha\beta}R_{\nu}{}^{\gamma}}{1575} + \frac{ie}{60}F_{\alpha\beta}{}^{\mu\nu}R^{\alpha\beta} + \\
& \frac{ie}{60}F_{\alpha\mu}{}^{\beta\nu}R^{\alpha\beta} + \frac{ie}{60}F_{\alpha\nu}{}^{\beta\mu}R^{\alpha\beta} - \frac{16R_{\alpha\beta}{}^{\mu\nu}R^{\alpha\beta}}{1575} - \\
& \frac{19R_{\alpha\mu}{}^{\beta\nu}R^{\alpha\beta}}{1575} - \frac{19R_{\alpha\nu}{}^{\beta\mu}R^{\alpha\beta}}{1575} + \frac{4R_{\mu\nu}{}^{\alpha\beta}R^{\alpha\beta}}{1575} + \\
& \frac{26R_{\alpha\beta\gamma\mu}{}^{\gamma\nu}R^{\alpha\beta}}{1575} - \frac{R_{\alpha\beta\gamma\nu}{}^{\gamma\mu}R^{\alpha\beta}}{6300} + \frac{R_{\alpha\mu\beta\gamma}{}^{\gamma\nu}R^{\alpha\beta}}{225} + \\
& \frac{R_{\alpha\mu\beta\nu}{}^{\gamma}R^{\alpha\beta}}{450} + \frac{R_{\alpha\nu\beta\gamma}{}^{\gamma\mu}R^{\alpha\beta}}{225} + \frac{R_{\alpha\nu\beta\mu}{}^{\gamma}R^{\alpha\beta}}{450} - \\
& \frac{e^2F_{\alpha\mu}F_{\beta\nu}R^{\alpha\beta}}{30} - \frac{e^2F_{\alpha\beta}F_{\mu\nu}R^{\alpha\beta}}{60} - \frac{ie}{450}F_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - \\
& \frac{R_{\alpha\beta}R_{\mu\nu}R^{\alpha\beta}}{900} + \frac{R_{\alpha\beta\gamma\mu}{}^{\beta\nu}R^{\alpha\gamma}}{225} + \frac{R_{\alpha\beta\gamma\nu}{}^{\beta\mu}R^{\alpha\gamma}}{225} - \\
& \frac{R_{\alpha\mu\beta\gamma}{}^{\beta\nu}R^{\alpha\gamma}}{1050} - \frac{R_{\alpha\mu\beta\nu}{}^{\beta\gamma}R^{\alpha\gamma}}{1050} - \frac{R_{\alpha\nu\beta\gamma}{}^{\beta\mu}R^{\alpha\gamma}}{1050} - \\
& \frac{R_{\alpha\nu\beta\mu}{}^{\beta\gamma}R^{\alpha\gamma}}{1050} + \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha\nu}R^{\beta\gamma}}{450} + \frac{R_{\alpha\beta\gamma\nu}{}^{\alpha\mu}R^{\beta\gamma}}{450} + \\
& \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha\nu}R^{\beta\gamma}}{450} + \frac{R_{\alpha\mu\beta\nu}{}^{\alpha\gamma}R^{\beta\gamma}}{450} + \frac{7R_{\alpha\nu\beta\gamma}{}^{\alpha\mu}R^{\beta\gamma}}{900} + \\
& \frac{R_{\alpha\nu\beta\mu}{}^{\alpha\gamma}R^{\beta\gamma}}{450} + \frac{R_{\alpha\beta}R_{\mu\nu}{}^{\alpha\beta}}{300} + \frac{R_{\alpha\beta}{}^{\gamma}R_{\mu\nu}{}^{\alpha\beta}}{140} + \\
& \frac{RR_{\alpha\beta}R_{\mu\nu}{}^{\alpha\beta}}{1080} - \frac{ie}{60}F_{\alpha\beta}{}^{\beta\gamma}R_{\mu\nu}{}^{\alpha\gamma} + \frac{2R_{\alpha\beta\gamma\delta}{}^{\beta\delta}R_{\mu\nu}{}^{\alpha\gamma}}{525} + \\
& \frac{2R_{\alpha\beta\gamma\delta}{}^{\delta\beta}R_{\mu\nu}{}^{\alpha\gamma}}{525} - \frac{ie}{60}F_{\alpha\beta}R_{\gamma}{}^{\beta}R_{\mu\nu}{}^{\alpha\gamma} - \frac{ie}{60}F_{\alpha\beta}{}^{\alpha\gamma}R_{\mu\nu}{}^{\beta\gamma} + \\
& \frac{e^2F_{\alpha\beta}F_{\gamma}{}^{\alpha}R_{\mu\nu}{}^{\beta\gamma}}{15} - \frac{ie}{60}F_{\alpha\beta}R_{\gamma}{}^{\alpha}R_{\mu\nu}{}^{\beta\gamma} - \frac{37R_{\alpha\beta}R_{\gamma}{}^{\alpha}R_{\mu\nu}{}^{\beta\gamma}}{12600} - \\
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\gamma\delta}R_{\mu\nu}{}^{\beta\epsilon}}{4725} + \frac{17R_{\alpha\beta}R_{\gamma\delta}{}^{\alpha\beta}R_{\mu\nu}{}^{\gamma\delta}}{18900} + \frac{R_{\alpha\beta}R_{\gamma}{}^{\alpha\delta}R_{\mu\nu}{}^{\gamma\delta}}{900} + \\
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\beta\gamma}R_{\mu\nu}{}^{\delta\epsilon}}{4725} + \frac{R_{\alpha\beta}R_{\mu}{}^{\alpha\nu}{}^{\beta}}{450} - \frac{R_{\alpha\beta}{}^{\gamma}R_{\mu}{}^{\alpha\nu}{}^{\beta}}{3150} - \\
& \frac{RR_{\alpha\beta}R_{\mu}{}^{\alpha\nu}{}^{\beta}}{1350} + \frac{31R_{\alpha\beta}{}^{\beta}R_{\mu}{}^{\alpha\nu}{}^{\gamma}}{6300} + \frac{R_{\alpha\beta}{}^{\beta\gamma}R_{\mu}{}^{\alpha\nu}{}^{\gamma}}{630} + \\
& \frac{2R_{\alpha\beta\gamma\delta}{}^{\beta\delta}R_{\mu}{}^{\alpha\nu}{}^{\gamma}}{1575} + \frac{2R_{\alpha\beta\gamma\delta}{}^{\delta\beta}R_{\mu}{}^{\alpha\nu}{}^{\gamma}}{1575} + \frac{ie}{180}F_{\alpha\beta}R_{\gamma}{}^{\beta}R_{\mu}{}^{\alpha\nu}{}^{\gamma} - \\
& \frac{R_{\alpha\beta}{}^{\gamma\nu}R_{\mu}{}^{\alpha\beta\gamma}}{3150} - \frac{23R_{\alpha\nu}{}^{\beta\gamma}R_{\mu}{}^{\alpha\beta\gamma}}{3150} + \frac{4R_{\alpha\beta\gamma\delta}{}^{\delta\nu}R_{\mu}{}^{\alpha\beta\gamma}}{1575} -
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\nu;\delta} R_{\mu}^{\alpha\beta\gamma}}{525} - \frac{2R_{\alpha\nu\beta\gamma;\delta} R_{\mu}^{\alpha\beta\gamma}}{1575} - \frac{ie}{90} F_{\alpha\nu} R_{\beta\gamma} R_{\mu}^{\alpha\beta\gamma} + \\
& \frac{ie}{60} F_{\alpha\beta} R_{\gamma\nu} R_{\mu}^{\alpha\beta\gamma} + \frac{359R_{\alpha\beta} R_{\gamma\nu} R_{\mu}^{\alpha\beta\gamma}}{37800} + \frac{RR_{\alpha\beta\gamma\nu} R_{\mu}^{\alpha\beta\gamma}}{1350} - \\
& \frac{R_{\alpha\beta\gamma\nu;\gamma} R_{\mu}^{\alpha\beta\delta}}{315} - \frac{2R_{\alpha\nu\beta\gamma;\delta} R_{\mu}^{\alpha\beta\delta}}{1575} - \frac{2R_{\alpha\nu\beta\gamma;\gamma} R_{\mu}^{\alpha\beta\delta}}{1575} - \\
& \frac{4R_{\alpha\beta\gamma\delta} R_{\mu}^{\alpha\gamma\delta}}{1575} - \frac{2R_{\alpha\beta\gamma\nu;\delta} R_{\mu}^{\alpha\gamma\delta}}{1575} - \frac{2R_{\alpha\beta\gamma\nu;\beta} R_{\mu}^{\alpha\gamma\delta}}{1575} + \\
& \frac{R_{\mu;\alpha\beta} R_{\nu}^{\beta\alpha}}{450} - \frac{R_{\alpha\beta;\gamma} R_{\mu}^{\beta\alpha}}{3150} - \frac{RR_{\alpha\beta} R_{\mu}^{\beta\alpha}}{1350} - \\
& \frac{13R_{\alpha\beta;\gamma} R_{\mu}^{\beta\gamma}}{6300} - \frac{R_{\alpha\beta;\alpha} R_{\mu}^{\beta\gamma}}{525} + \frac{2R_{\alpha\beta\gamma\delta} R_{\mu}^{\beta\gamma}}{1575} + \\
& \frac{ie}{180} F_{\alpha\beta} R_{\gamma}^{\alpha} R_{\mu}^{\beta\gamma} + \frac{R_{\alpha\beta} R_{\gamma}^{\alpha} R_{\mu}^{\beta\gamma}}{1050} - \frac{16R_{\alpha\beta\gamma\delta} R_{\mu}^{\beta\gamma}}{4725} - \\
& \frac{R_{\alpha\beta;\gamma\nu} R_{\mu}^{\beta\alpha\gamma}}{1050} + \frac{R_{\alpha\nu;\beta\gamma} R_{\mu}^{\beta\alpha\gamma}}{630} - \frac{4R_{\alpha\beta\gamma\delta} R_{\mu}^{\beta\alpha\gamma}}{525} - \\
& \frac{11R_{\alpha\beta\gamma\nu;\delta} R_{\mu}^{\beta\alpha\gamma}}{1575} - \frac{2R_{\alpha\nu\beta\gamma;\delta} R_{\mu}^{\beta\alpha\gamma}}{1575} + \frac{ie}{180} F_{\alpha\nu} R_{\beta\gamma} R_{\mu}^{\beta\alpha\gamma} + \\
& \frac{ie}{180} F_{\alpha\beta} R_{\gamma\nu} R_{\mu}^{\beta\alpha\gamma} - \frac{R_{\alpha\beta\gamma\nu;\gamma} R_{\mu}^{\beta\alpha\delta}}{315} - \frac{2R_{\alpha\nu\beta\gamma;\delta} R_{\mu}^{\beta\alpha\delta}}{1575} - \\
& \frac{2R_{\alpha\nu\beta\gamma;\gamma} R_{\mu}^{\beta\alpha\delta}}{1575} - \frac{R_{\alpha\beta\gamma\delta;\alpha} R_{\mu}^{\beta\gamma\delta}}{1575} - \frac{R_{\alpha\beta\gamma\nu;\alpha} R_{\mu}^{\beta\gamma\delta}}{1575} + \\
& \frac{193R_{\alpha\nu} R_{\beta\gamma\delta} R_{\mu}^{\beta\gamma\delta}}{28350} + \frac{11R_{\alpha\beta} R_{\gamma\delta\nu} R_{\mu}^{\beta\gamma\delta}}{6300} - \frac{23R_{\alpha\beta} R_{\gamma\nu\delta} R_{\mu}^{\beta\gamma\delta}}{3780} + \\
& \frac{R_{\alpha\beta\gamma\delta} R_{\mu}^{\beta\delta\epsilon}}{1575} + \frac{4R_{\alpha\beta\gamma\delta} R_{\mu}^{\beta\delta\epsilon}}{4725} + \frac{31R_{\alpha\beta;\gamma} R_{\mu}^{\beta\delta\epsilon}}{6300} + \\
& \frac{R_{\alpha\beta;\gamma} R_{\mu}^{\beta\delta\epsilon}}{630} + \frac{2R_{\alpha\beta\gamma\delta;\beta\delta} R_{\mu}^{\beta\delta\epsilon}}{1575} + \frac{2R_{\alpha\beta\gamma\delta;\delta\beta} R_{\mu}^{\beta\delta\epsilon}}{1575} + \\
& \frac{ie}{180} F_{\alpha\beta} R_{\gamma}^{\beta} R_{\mu}^{\beta\delta\epsilon} - \frac{13R_{\alpha\beta;\gamma} R_{\mu}^{\beta\delta\epsilon}}{6300} - \frac{R_{\alpha\beta;\alpha} R_{\mu}^{\beta\delta\epsilon}}{525} + \\
& \frac{2R_{\alpha\beta\gamma\delta;\alpha\delta} R_{\mu}^{\beta\delta\epsilon}}{1575} + \frac{ie}{180} F_{\alpha\beta} R_{\gamma}^{\alpha} R_{\mu}^{\beta\delta\epsilon} + \frac{R_{\alpha\beta} R_{\gamma}^{\alpha} R_{\mu}^{\beta\delta\epsilon}}{1050} + \\
& \frac{2R_{\alpha\beta\gamma\delta;\alpha\beta} R_{\mu}^{\beta\delta\epsilon}}{1575} - \frac{13R_{\alpha\beta} R_{\gamma\delta} R_{\mu}^{\beta\delta\epsilon}}{14175} + \frac{44R_{\alpha\beta} R_{\gamma}^{\alpha} R_{\mu}^{\beta\delta\epsilon}}{14175} - \\
& \frac{47R_{\alpha\beta;\gamma\nu} R_{\mu}^{\beta\delta\epsilon}}{3150} + \frac{R_{\alpha\nu;\beta\gamma} R_{\mu}^{\beta\delta\epsilon}}{450} - \frac{4R_{\alpha\beta\gamma\delta;\delta} R_{\mu}^{\beta\delta\epsilon}}{1575} - \\
& \frac{2R_{\alpha\beta\gamma\nu;\delta} R_{\mu}^{\beta\delta\epsilon}}{1575} - \frac{2R_{\alpha\nu\beta\gamma;\delta} R_{\mu}^{\beta\delta\epsilon}}{525} + \frac{19ie}{900} F_{\alpha\nu} R_{\beta\gamma} R_{\mu}^{\beta\delta\epsilon} + \\
& \frac{ie}{180} F_{\alpha\beta} R_{\gamma\nu} R_{\mu}^{\beta\delta\epsilon} - \frac{317R_{\alpha\beta} R_{\gamma\nu} R_{\mu}^{\beta\delta\epsilon}}{56700} - \frac{4R_{\alpha\beta\gamma\delta;\beta} R_{\mu}^{\beta\delta\epsilon}}{1575} - \\
& \frac{2R_{\alpha\beta\gamma\nu;\delta} R_{\mu}^{\beta\delta\epsilon}}{1575} - \frac{2R_{\alpha\beta\gamma\nu;\beta} R_{\mu}^{\beta\delta\epsilon}}{1575} - \frac{R_{\alpha\beta\gamma\delta;\alpha} R_{\mu}^{\beta\delta\epsilon}}{315} -
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\nu\beta\gamma}{}^{\alpha}{}_{\delta}R_{\mu}{}^{\gamma\beta\delta}}{1575} - \frac{11R_{\alpha\beta}R_{\gamma\delta\nu}{}^{\alpha}R_{\mu}{}^{\gamma\beta\delta}}{3780} - \frac{R_{\alpha\beta}R_{\gamma\nu\delta}{}^{\alpha}R_{\mu}{}^{\gamma\beta\delta}}{1575} + \\
& \frac{67R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_{\mu}{}^{\gamma\delta\epsilon}}{18900} + \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\nu}{}^{\beta}R_{\mu}{}^{\gamma\delta\epsilon}}{4725} + \frac{2R_{\alpha\beta\gamma\delta}{}^{\alpha\beta}R_{\mu}{}^{\delta}{}_{\nu}{}^{\gamma}}{1575} + \\
& \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\beta\gamma}R_{\mu}{}^{\delta}{}_{\nu}{}^{\epsilon}}{945} - \frac{2R_{\alpha\nu\beta\gamma}{}^{\gamma}R_{\mu}{}^{\delta\alpha\beta}}{525} - \frac{2R_{\alpha\nu\beta\gamma}{}^{\gamma}{}_{\delta}R_{\mu}{}^{\delta\alpha\beta}}{525} - \\
& \frac{4R_{\alpha\beta\gamma\delta}{}^{\beta}{}_{\nu}R_{\mu}{}^{\delta\alpha\gamma}}{525} - \frac{2R_{\alpha\beta\gamma\nu}{}^{\beta}R_{\mu}{}^{\delta\alpha\gamma}}{525} - \frac{2R_{\alpha\beta\gamma\nu}{}^{\beta}{}_{\delta}R_{\mu}{}^{\delta\alpha\gamma}}{525} - \\
& \frac{4R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\nu}R_{\mu}{}^{\delta\beta\gamma}}{1575} + \frac{R_{\alpha\beta\gamma\nu}{}^{\alpha}{}_{\delta}R_{\mu}{}^{\delta\beta\gamma}}{1575} - \frac{R_{\alpha\nu\beta\gamma}{}^{\alpha}{}_{\delta}R_{\mu}{}^{\delta\beta\gamma}}{1575} - \\
& \frac{ie}{225}F_{\alpha\nu}R_{\beta\gamma\delta}{}^{\alpha}R_{\mu}{}^{\delta\beta\gamma} + \frac{109R_{\alpha\nu}R_{\beta\gamma\delta}{}^{\alpha}R_{\mu}{}^{\delta\beta\gamma}}{56700} + \frac{8R_{\alpha\beta}R_{\gamma\nu\delta}{}^{\alpha}R_{\mu}{}^{\delta\beta\gamma}}{4725} - \\
& \frac{11R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_{\mu}{}^{\delta\beta\epsilon}}{14175} - \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\nu}{}^{\gamma}R_{\mu}{}^{\delta\beta\epsilon}}{4725} - \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\gamma\delta}R_{\mu}{}^{\epsilon}{}_{\nu}{}^{\beta}}{4725} + \\
& \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta\gamma}R_{\mu}{}^{\epsilon}{}_{\nu}{}^{\delta}}{945} + \frac{8R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_{\mu}{}^{\epsilon\beta\delta}}{14175} - \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\nu}{}^{\gamma}R_{\mu}{}^{\epsilon\beta\delta}}{4725} - \\
& \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_{\mu}{}^{\epsilon\gamma\delta}}{2835} - \frac{23R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\nu}{}^{\beta}R_{\mu}{}^{\epsilon\gamma\delta}}{56700} - \frac{R_{\alpha\beta;\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{3150} - \\
& \frac{23R_{\alpha\mu;\beta\gamma}R_{\nu}{}^{\alpha\beta\gamma}}{3150} + \frac{4R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\mu}R_{\nu}{}^{\alpha\beta\gamma}}{1575} - \frac{R_{\alpha\beta\gamma\mu}{}^{\delta}R_{\nu}{}^{\alpha\beta\gamma}}{525} - \\
& \frac{2R_{\alpha\mu\beta\gamma}{}^{\delta}R_{\nu}{}^{\alpha\beta\gamma}}{1575} - \frac{ie}{90}F_{\alpha\mu}R_{\beta\gamma}R_{\nu}{}^{\alpha\beta\gamma} + \frac{ie}{60}F_{\alpha\beta}R_{\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma} + \\
& \frac{359R_{\alpha\beta}R_{\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{37800} + \frac{RR_{\alpha\beta\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{1350} - \frac{R_{\alpha\beta\gamma\mu}{}^{\gamma}{}_{\delta}R_{\nu}{}^{\alpha\beta\delta}}{315} - \\
& \frac{2R_{\alpha\mu\beta\gamma}{}^{\gamma}R_{\nu}{}^{\alpha\beta\delta}}{1575} - \frac{2R_{\alpha\mu\beta\gamma}{}^{\gamma}{}_{\delta}R_{\nu}{}^{\alpha\beta\delta}}{1575} - \frac{4R_{\alpha\beta\gamma\delta}{}^{\beta}{}_{\mu}R_{\nu}{}^{\alpha\gamma\delta}}{1575} - \\
& \frac{2R_{\alpha\beta\gamma\mu}{}^{\beta}R_{\nu}{}^{\alpha\gamma\delta}}{1575} - \frac{2R_{\alpha\beta\gamma\mu}{}^{\beta}{}_{\delta}R_{\nu}{}^{\alpha\gamma\delta}}{1575} - \frac{R_{\alpha\beta;\gamma\mu}R_{\nu}{}^{\beta\alpha\gamma}}{1050} + \\
& \frac{R_{\alpha\mu;\beta\gamma}R_{\nu}{}^{\beta\alpha\gamma}}{630} - \frac{4R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\mu}R_{\nu}{}^{\beta\alpha\gamma}}{525} - \frac{11R_{\alpha\beta\gamma\mu}{}^{\delta}R_{\nu}{}^{\beta\alpha\gamma}}{1575} - \\
& \frac{2R_{\alpha\mu\beta\gamma}{}^{\delta}R_{\nu}{}^{\beta\alpha\gamma}}{1575} + \frac{ie}{180}F_{\alpha\mu}R_{\beta\gamma}R_{\nu}{}^{\beta\alpha\gamma} + \frac{ie}{180}F_{\alpha\beta}R_{\gamma\mu}R_{\nu}{}^{\beta\alpha\gamma} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{\gamma}{}_{\delta}R_{\nu}{}^{\beta\alpha\delta}}{315} - \frac{2R_{\alpha\mu\beta\gamma}{}^{\gamma}R_{\nu}{}^{\beta\alpha\delta}}{1575} - \frac{2R_{\alpha\mu\beta\gamma}{}^{\gamma}{}_{\delta}R_{\nu}{}^{\beta\alpha\delta}}{1575} - \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\mu}R_{\nu}{}^{\beta\gamma\delta}}{1575} - \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha}{}_{\delta}R_{\nu}{}^{\beta\gamma\delta}}{1575} + \frac{193R_{\alpha\mu}R_{\beta\gamma\delta}{}^{\alpha}R_{\nu}{}^{\beta\gamma\delta}}{28350} + \\
& \frac{11R_{\alpha\beta}R_{\gamma\delta\mu}{}^{\alpha}R_{\nu}{}^{\beta\gamma\delta}}{6300} - \frac{23R_{\alpha\beta}R_{\gamma\mu\delta}{}^{\alpha}R_{\nu}{}^{\beta\gamma\delta}}{3780} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\gamma}R_{\nu}{}^{\beta\delta\epsilon}}{1575} + \\
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma}R_{\nu}{}^{\beta\delta\epsilon}}{4725} - \frac{47R_{\alpha\beta;\gamma\mu}R_{\nu}{}^{\gamma\alpha\beta}}{3150} + \frac{R_{\alpha\mu;\beta\gamma}R_{\nu}{}^{\gamma\alpha\beta}}{450} - \\
& \frac{4R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\mu}R_{\nu}{}^{\gamma\alpha\beta}}{1575} - \frac{2R_{\alpha\beta\gamma\mu}{}^{\delta}R_{\nu}{}^{\gamma\alpha\beta}}{1575} - \frac{2R_{\alpha\mu\beta\gamma}{}^{\delta}R_{\nu}{}^{\gamma\alpha\beta}}{525} +
\end{aligned}$$

$$\begin{aligned}
& \frac{19ie}{900} F_{\alpha\mu} R_{\beta\gamma} R_{\nu}^{\gamma\alpha\beta} + \frac{ie}{180} F_{\alpha\beta} R_{\gamma\mu} R_{\nu}^{\gamma\alpha\beta} - \frac{317 R_{\alpha\beta} R_{\gamma\mu} R_{\nu}^{\gamma\alpha\beta}}{56700} - \\
& \frac{R R_{\alpha\beta\gamma\mu} R_{\nu}^{\gamma\alpha\beta}}{675} - \frac{4 R_{\alpha\beta\gamma\delta}{}^{\beta}{}_{\mu} R_{\nu}^{\gamma\alpha\delta}}{1575} - \frac{2 R_{\alpha\beta\gamma\mu}{}^{\beta} R_{\nu}^{\gamma\alpha\delta}}{1575} - \\
& \frac{2 R_{\alpha\beta\gamma\mu}{}^{\beta}{}_{\delta} R_{\nu}^{\gamma\alpha\delta}}{1575} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\mu} R_{\nu}^{\gamma\beta\delta}}{315} - \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha}{}_{\delta} R_{\nu}^{\gamma\beta\delta}}{1575} - \\
& \frac{11 R_{\alpha\beta} R_{\gamma\delta\mu}{}^{\alpha} R_{\nu}^{\gamma\beta\delta}}{3780} - \frac{R_{\alpha\beta} R_{\gamma\mu\delta}{}^{\alpha} R_{\nu}^{\gamma\beta\delta}}{1575} + \frac{67 R_{\alpha\beta\gamma\delta} R_{\epsilon\mu}{}^{\alpha\beta} R_{\nu}^{\gamma\delta\epsilon}}{18900} + \\
& \frac{16 R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\beta} R_{\nu}^{\gamma\delta\epsilon}}{4725} - \frac{2 R_{\alpha\mu\beta\gamma}{}^{\gamma} R_{\nu}^{\delta\alpha\beta}}{525} - \frac{2 R_{\alpha\mu\beta\gamma}{}^{\gamma}{}_{\delta} R_{\nu}^{\delta\alpha\beta}}{525} - \\
& \frac{4 R_{\alpha\beta\gamma\delta}{}^{\beta}{}_{\mu} R_{\nu}^{\delta\alpha\gamma}}{525} - \frac{2 R_{\alpha\beta\gamma\mu}{}^{\beta} R_{\nu}^{\delta\alpha\gamma}}{525} - \frac{2 R_{\alpha\beta\gamma\mu}{}^{\beta}{}_{\delta} R_{\nu}^{\delta\alpha\gamma}}{525} - \\
& \frac{4 R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\mu} R_{\nu}^{\delta\beta\gamma}}{1575} + \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha}{}_{\delta} R_{\nu}^{\delta\beta\gamma}}{1575} - \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha}{}_{\delta} R_{\nu}^{\delta\beta\gamma}}{1575} - \\
& \frac{ie}{225} F_{\alpha\mu} R_{\beta\gamma\delta}{}^{\alpha} R_{\nu}^{\delta\beta\gamma} + \frac{109 R_{\alpha\mu} R_{\beta\gamma\delta}{}^{\alpha} R_{\nu}^{\delta\beta\gamma}}{56700} + \frac{8 R_{\alpha\beta} R_{\gamma\mu\delta}{}^{\alpha} R_{\nu}^{\delta\beta\gamma}}{4725} - \\
& \frac{11 R_{\alpha\beta\gamma\delta} R_{\epsilon\mu}{}^{\alpha\gamma} R_{\nu}^{\delta\beta\epsilon}}{14175} - \frac{4 R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma} R_{\nu}^{\delta\beta\epsilon}}{4725} - \frac{4 R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma} R_{\nu}^{\epsilon\beta\delta}}{4725} - \\
& \frac{2 R_{\alpha\beta\gamma\delta} R_{\epsilon\mu}{}^{\alpha\beta} R_{\nu}^{\epsilon\gamma\delta}}{1575} - \frac{23 R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\beta} R_{\nu}^{\epsilon\gamma\delta}}{56700} + \frac{4 R_{\alpha\beta\gamma\delta}{}^{\mu\nu} R^{\alpha\beta\gamma\delta}}{1575} + \\
& \frac{4 R_{\alpha\beta\gamma\mu}{}^{\delta\nu} R^{\alpha\beta\gamma\delta}}{1575} + \frac{4 R_{\alpha\beta\gamma\nu}{}^{\delta\mu} R^{\alpha\beta\gamma\delta}}{1575} + \frac{2 R_{\alpha\mu\beta\nu}{}^{\gamma\delta} R^{\alpha\beta\gamma\delta}}{525} + \\
& \frac{2 R_{\alpha\nu\beta\mu}{}^{\gamma\delta} R^{\alpha\beta\gamma\delta}}{525} + \frac{ie}{450} F_{\mu\nu} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \frac{R_{\mu\nu} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}}{1350} - \\
& \frac{R_{\alpha\nu} R_{\beta\gamma\delta\mu} R^{\alpha\beta\gamma\delta}}{405} - \frac{R_{\alpha\mu} R_{\beta\gamma\delta\nu} R^{\alpha\beta\gamma\delta}}{405} - \frac{4 R_{\alpha\beta\gamma\mu} R_{\delta\epsilon\nu}{}^{\gamma} R^{\alpha\beta\delta\epsilon}}{2835} + \\
& \frac{32 R_{\alpha\beta\gamma\mu} R_{\delta\epsilon\nu}{}^{\gamma} R^{\alpha\beta\delta\epsilon}}{14175} - \frac{R_{\alpha\beta\gamma\delta}{}^{\mu\nu} R^{\alpha\gamma\beta\delta}}{315} + \frac{2 R_{\alpha\beta\gamma\mu}{}^{\delta\nu} R^{\alpha\gamma\beta\delta}}{315} + \\
& \frac{2 R_{\alpha\beta\gamma\nu}{}^{\delta\mu} R^{\alpha\gamma\beta\delta}}{315} + \frac{4 R_{\alpha\mu\beta\gamma}{}^{\delta\nu} R^{\alpha\gamma\beta\delta}}{1575} + \frac{2 R_{\alpha\mu\beta\nu}{}^{\gamma\delta} R^{\alpha\gamma\beta\delta}}{1575} + \\
& \frac{4 R_{\alpha\nu\beta\gamma}{}^{\delta\mu} R^{\alpha\gamma\beta\delta}}{1575} + \frac{2 R_{\alpha\nu\beta\mu}{}^{\gamma\delta} R^{\alpha\gamma\beta\delta}}{1575} + \frac{R_{\mu\nu} R_{\alpha\beta\gamma\delta} R^{\alpha\gamma\beta\delta}}{1350} - \\
& \frac{R_{\alpha\beta} R_{\gamma\mu\delta\nu} R^{\alpha\gamma\beta\delta}}{405} - \frac{R_{\alpha\beta\gamma\delta}{}^{\mu\nu} R^{\alpha\delta\beta\gamma}}{175} + \frac{2 R_{\alpha\beta\gamma\mu}{}^{\delta\nu} R^{\alpha\delta\beta\gamma}}{525} + \\
& \frac{2 R_{\alpha\beta\gamma\nu}{}^{\delta\mu} R^{\alpha\delta\beta\gamma}}{525} + \frac{4 R_{\alpha\mu\beta\gamma}{}^{\delta\nu} R^{\alpha\delta\beta\gamma}}{1575} + \frac{2 R_{\alpha\mu\beta\nu}{}^{\gamma\delta} R^{\alpha\delta\beta\gamma}}{1575} + \\
& \frac{4 R_{\alpha\nu\beta\gamma}{}^{\delta\mu} R^{\alpha\delta\beta\gamma}}{1575} + \frac{2 R_{\alpha\nu\beta\mu}{}^{\gamma\delta} R^{\alpha\delta\beta\gamma}}{1575} + \frac{R_{\alpha\nu} R_{\beta\gamma\delta\mu} R^{\alpha\delta\beta\gamma}}{405} + \\
& \frac{R_{\alpha\mu} R_{\beta\gamma\delta\nu} R^{\alpha\delta\beta\gamma}}{405} - \frac{8 R_{\alpha\beta\gamma\mu} R_{\delta\epsilon\nu}{}^{\gamma} R^{\alpha\delta\beta\epsilon}}{14175} + \frac{4 R_{\alpha\beta\gamma\mu} R_{\delta\epsilon\nu}{}^{\gamma} R^{\alpha\delta\beta\epsilon}}{2835} - \\
& \frac{32 R_{\alpha\beta\gamma\mu} R_{\delta\epsilon\nu}{}^{\alpha} R^{\beta\gamma\delta\epsilon}}{14175} - \frac{32 R_{\alpha\beta\gamma\mu} R_{\delta\epsilon\nu}{}^{\alpha} R^{\beta\gamma\delta\epsilon}}{4725} + \frac{4 R_{\alpha\beta\gamma\delta} R_{\epsilon\mu\nu}{}^{\alpha} R^{\beta\gamma\delta\epsilon}}{4725} +
\end{aligned}$$

$$\begin{aligned}
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon\nu\mu}{}^\alpha R^{\beta\gamma\delta\epsilon}}{4725} - \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^\alpha{}_{\mu\nu}R^{\beta\gamma\delta\epsilon}}{4725} - \frac{4R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^\alpha R^{\beta\delta\gamma\epsilon}}{2835} + \\
& \frac{8R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^\alpha R^{\beta\epsilon\gamma\delta}}{4725} - \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\mu\nu}{}^\alpha R^{\beta\epsilon\gamma\delta}}{945} - \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu\mu}{}^\alpha R^{\beta\epsilon\gamma\delta}}{945} + \\
& \frac{16R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^\alpha{}_{\mu\nu}R^{\beta\epsilon\gamma\delta}}{4725} - \frac{e^2 F_\nu{}^\alpha F^{\beta\gamma} S_{\alpha\beta\gamma\mu}}{15} - \frac{e^2 F_\mu{}^\alpha F^{\beta\gamma} S_{\alpha\beta\gamma\nu}}{15} + \\
& \frac{e^2 F_\mu{}^\alpha F_\nu{}^\beta S_{\alpha\beta\gamma}{}^\gamma}{30} + \frac{e^2 F_{\mu\nu} F^{\alpha\beta} S_{\alpha\beta\gamma}{}^\gamma}{60} - \frac{e^2 F_\nu{}^\alpha F^{\beta\gamma} S_{\alpha\beta\mu\gamma}}{20} - \\
& \frac{e^2 F_\mu{}^\alpha F^{\beta\gamma} S_{\alpha\beta\nu\gamma}}{20} + \frac{ie}{60} F^{\alpha\beta} R_{\mu\nu}{}^{\gamma\delta} S_{\alpha\gamma\delta\beta} + \frac{ie}{120} F^{\alpha\beta} R_\nu{}^\gamma S_{\alpha\mu\gamma\beta} - \\
& \frac{ie}{120} R F^{\alpha\beta} S_{\alpha\mu\nu\beta} + \frac{ie}{20} R\xi F^{\alpha\beta} S_{\alpha\mu\nu\beta} + \frac{ie}{120} F^{\alpha\beta} R_\mu{}^\gamma S_{\alpha\nu\gamma\beta} - \\
& \frac{ie}{120} R F^{\alpha\beta} S_{\alpha\nu\mu\beta} + \frac{ie}{20} R\xi F^{\alpha\beta} S_{\alpha\nu\mu\beta} + \frac{ie}{60} R F_\nu{}^\alpha S_{\beta\alpha}{}^\beta{}_\mu - \\
& \frac{ie}{10} R\xi F_\nu{}^\alpha S_{\beta\alpha}{}^\beta{}_\mu + \frac{ie}{60} R F_\mu{}^\alpha S_{\beta\alpha}{}^\beta{}_\nu - \frac{ie}{10} R\xi F_\mu{}^\alpha S_{\beta\alpha}{}^\beta{}_\nu - \\
& \frac{e^2 F_\nu{}^\alpha F^{\beta\gamma} S_{\beta\gamma\alpha\mu}}{30} - \frac{e^2 F_\mu{}^\alpha F^{\beta\gamma} S_{\beta\gamma\alpha\nu}}{30} + \frac{e^2 F_\nu{}^\alpha F^{\beta\gamma} S_{\beta\gamma\mu\alpha}}{60} - \\
& \frac{ie}{60} F_\nu{}^\alpha R^{\beta\gamma} S_{\beta\gamma\mu\alpha} + \frac{e^2 F_\alpha{}^\beta F^{\alpha\gamma} S_{\beta\gamma\mu\nu}}{15} + \frac{2R_\alpha{}^\beta R^{\alpha\gamma} S_{\beta\gamma\mu\nu}}{225} + \\
& \frac{e^2 F_\mu{}^\alpha F^{\beta\gamma} S_{\beta\gamma\nu\alpha}}{60} - \frac{ie}{60} F_\mu{}^\alpha R^{\beta\gamma} S_{\beta\gamma\nu\alpha} - \frac{ie}{120} F_\nu{}^\alpha R_\mu{}^\beta S_{\beta\gamma}{}^\gamma{}_\alpha - \\
& \frac{ie}{120} F_\mu{}^\alpha R_\nu{}^\beta S_{\beta\gamma}{}^\gamma{}_\alpha - \frac{e^2 F_\nu{}^\alpha F^{\beta\gamma} S_{\beta\mu\alpha\gamma}}{60} - \frac{e^2 F_\alpha{}^\beta F^{\alpha\gamma} S_{\beta\mu\nu\gamma}}{40} - \\
& \frac{e^2 F_\mu{}^\alpha F^{\beta\gamma} S_{\beta\nu\alpha\gamma}}{60} + \frac{e^2 F_\alpha{}^\beta F^{\alpha\gamma} S_{\beta\nu\mu\gamma}}{60} - \frac{R_{;\alpha\nu} S_{\beta}{}^{\alpha\beta}{}_\mu}{100} + \\
& \frac{\xi R_{;\alpha\nu} S_{\beta}{}^{\alpha\beta}{}_\mu}{15} - \frac{R_{;\alpha\mu} S_{\beta}{}^{\alpha\beta}{}_\nu}{100} + \frac{\xi R_{;\alpha\mu} S_{\beta}{}^{\alpha\beta}{}_\nu}{15} - \\
& \frac{ie}{120} R F_\nu{}^\alpha S_{\beta}{}^\beta{}_{\mu\alpha} + \frac{ie}{20} R\xi F_\nu{}^\alpha S_{\beta}{}^\beta{}_{\mu\alpha} - \frac{ie}{120} R F_\mu{}^\alpha S_{\beta}{}^\beta{}_{\nu\alpha} + \\
& \frac{ie}{20} R\xi F_\mu{}^\alpha S_{\beta}{}^\beta{}_{\nu\alpha} - \frac{ie}{60} F^{\alpha\beta} R_{\mu\nu}{}^{\gamma\delta} S_{\gamma\alpha\delta\beta} - \frac{ie}{120} F^{\alpha\beta} R_\nu{}^\gamma S_{\gamma\alpha\mu\beta} - \\
& \frac{ie}{120} F^{\alpha\beta} R_\mu{}^\gamma S_{\gamma\alpha\nu\beta} + \frac{e^2 F_{\mu\nu} F^{\alpha\beta} S_{\gamma\alpha}{}^\gamma{}_\beta}{30} - \frac{e^2 F_\alpha{}^\beta F_\nu{}^\alpha S_{\gamma\beta}{}^\gamma{}_\mu}{60} - \\
& \frac{e^2 F_{\alpha\nu} F^{\alpha\beta} S_{\gamma\beta}{}^\gamma{}_\mu}{60} + \frac{R_\alpha{}^\beta R_\nu{}^\alpha S_{\gamma\beta}{}^\gamma{}_\mu}{225} - \frac{e^2 F_\alpha{}^\beta F_\mu{}^\alpha S_{\gamma\beta}{}^\gamma{}_\nu}{60} - \\
& \frac{e^2 F_{\alpha\mu} F^{\alpha\beta} S_{\gamma\beta}{}^\gamma{}_\nu}{60} + \frac{R_\alpha{}^\beta R_\mu{}^\alpha S_{\gamma\beta}{}^\gamma{}_\nu}{225} - \frac{ie}{30} F^{\alpha\beta} R_{\mu\nu}{}^{\gamma\delta} S_{\gamma\delta\alpha\beta} + \\
& \frac{ie}{30} F_\nu{}^\alpha R_\mu{}^\beta{}^{\gamma\delta} S_{\gamma\delta\beta\alpha} + \frac{ie}{30} F_\mu{}^\alpha R_\nu{}^\beta{}^{\gamma\delta} S_{\gamma\delta\beta\alpha} + \frac{ie}{30} F_\alpha{}^\beta R_\nu{}^\alpha{}^{\gamma\delta} S_{\gamma\delta\mu\beta} - \\
& \frac{R_{\alpha\beta} R^{\alpha\gamma\beta\delta} S_{\gamma\delta\mu\nu}}{450} - \frac{R_{\alpha\beta} R^{\alpha\delta\beta\gamma} S_{\gamma\delta\mu\nu}}{450} + \frac{ie}{30} F_\alpha{}^\beta R_\mu{}^\alpha{}^{\gamma\delta} S_{\gamma\delta\nu\beta} -
\end{aligned}$$

$$\begin{aligned}
& \frac{e^2 F_\alpha^\beta F^{\alpha\gamma} S_{\gamma\mu\nu\beta}}{120} - \frac{ie}{60} F_\alpha^\beta R^{\alpha\gamma} S_{\gamma\mu\nu\beta} - \frac{e^2 F_\alpha^\beta F_\nu^\alpha S_{\gamma\mu}{}^\gamma{}_\beta}{120} + \\
& \frac{e^2 F_{\alpha\nu} F^{\alpha\beta} S_{\gamma\mu}{}^\gamma{}_\beta}{120} - \frac{ie}{120} F_\alpha^\beta R_\nu^\alpha S_{\gamma\mu}{}^\gamma{}_\beta + \frac{e^2 F_\alpha^\beta F^{\alpha\gamma} S_{\gamma\nu\mu\beta}}{60} - \\
& \frac{ie}{60} F_\alpha^\beta R^{\alpha\gamma} S_{\gamma\nu\mu\beta} - \frac{e^2 F_\alpha^\beta F_\mu^\alpha S_{\gamma\nu}{}^\gamma{}_\beta}{120} + \frac{e^2 F_{\alpha\mu} F^{\alpha\beta} S_{\gamma\nu}{}^\gamma{}_\beta}{120} - \\
& \frac{ie}{120} F_\alpha^\beta R_\mu^\alpha S_{\gamma\nu}{}^\gamma{}_\beta - \frac{ie}{60} F_{\alpha\nu;\beta} S_\gamma{}^{\alpha\gamma}{}_\mu - \frac{R_{\alpha\nu;\beta} S_\gamma{}^{\alpha\gamma}{}_\mu}{300} - \\
& \frac{ie}{60} F_{\alpha\mu;\beta} S_\gamma{}^{\alpha\gamma}{}_\nu - \frac{R_{\alpha\mu;\beta} S_\gamma{}^{\alpha\gamma}{}_\nu}{300} - \frac{ie}{60} F_{\alpha\beta;\mu\nu} S_\gamma{}^{\alpha\gamma\beta} - \\
& \frac{ie}{60} F_{\alpha\mu;\beta\nu} S_\gamma{}^{\alpha\gamma\beta} - \frac{ie}{60} F_{\alpha\nu;\beta\mu} S_\gamma{}^{\alpha\gamma\beta} + \frac{ie}{60} F_{\alpha\beta}{}^\alpha{}_\nu S_\gamma{}^{\beta\gamma}{}_\mu - \\
& \frac{ie}{60} F_{\alpha\nu}{}^\alpha{}_\beta S_\gamma{}^{\beta\gamma}{}_\mu + \frac{ie}{60} F_{\alpha\beta}{}^\alpha{}_\mu S_\gamma{}^{\beta\gamma}{}_\nu - \frac{ie}{60} F_{\alpha\mu}{}^\alpha{}_\beta S_\gamma{}^{\beta\gamma}{}_\nu - \\
& \frac{e^2 F_\mu^\alpha F_\nu^\beta S_\gamma{}^{\gamma}{}_{\alpha\beta}}{60} + \frac{e^2 F_{\mu\nu} F^{\alpha\beta} S_\gamma{}^{\gamma}{}_{\alpha\beta}}{60} - \frac{e^2 F_\mu^\alpha F_\nu^\beta S_\gamma{}^{\gamma}{}_{\beta\alpha}}{60} + \\
& \frac{ie}{120} F_\nu^\alpha R_\mu^\beta S_\gamma{}^{\gamma}{}_{\beta\alpha} + \frac{ie}{120} F_\mu^\alpha R_\nu^\beta S_\gamma{}^{\gamma}{}_{\beta\alpha} - \frac{e^2 F_\alpha^\beta F_\nu^\alpha S_\gamma{}^{\gamma}{}_{\mu\beta}}{120} + \\
& \frac{e^2 F_{\alpha\nu} F^{\alpha\beta} S_\gamma{}^{\gamma}{}_{\mu\beta}}{120} + \frac{ie}{120} F_\alpha^\beta R_\nu^\alpha S_\gamma{}^{\gamma}{}_{\mu\beta} - \frac{e^2 F_\alpha^\beta F_\mu^\alpha S_\gamma{}^{\gamma}{}_{\nu\beta}}{120} + \\
& \frac{e^2 F_{\alpha\mu} F^{\alpha\beta} S_\gamma{}^{\gamma}{}_{\nu\beta}}{120} + \frac{ie}{120} F_\alpha^\beta R_\mu^\alpha S_\gamma{}^{\gamma}{}_{\nu\beta} + \frac{R_{\alpha\beta} R_\nu{}^{\beta\alpha\gamma} S_{\delta\gamma}{}^\delta{}_\mu}{450} + \\
& \frac{R_{\alpha\beta} R_\mu{}^{\beta\alpha\gamma} S_{\delta\gamma}{}^\delta{}_\nu}{450} - \frac{R_{\alpha\beta\gamma}{}^\delta R^{\alpha\beta\gamma\epsilon} S_{\delta\epsilon\mu\nu}}{225} + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\alpha}{}^\gamma{}_\nu S_{\delta\mu}{}^\delta{}_\beta + \\
& \frac{ie}{60} F^{\alpha\beta} S_{\gamma\alpha}{}^\gamma{}_\mu S_{\delta\nu}{}^\delta{}_\beta + \frac{ie}{15} F^{\alpha\beta} S_{\gamma\alpha\mu\nu} S_{\delta}{}^{\gamma\delta}{}_\beta + \frac{ie}{60} F_\nu^\alpha S_{\beta\gamma}{}^\beta{}_\alpha S_{\delta}{}^{\gamma\delta}{}_\mu + \\
& \frac{ie}{30} F^{\alpha\beta} S_{\gamma\alpha\nu\beta} S_{\delta}{}^{\gamma\delta}{}_\mu + \frac{ie}{60} F_\mu^\alpha S_{\beta\gamma}{}^\beta{}_\alpha S_{\delta}{}^{\gamma\delta}{}_\nu + \frac{ie}{30} F^{\alpha\beta} S_{\gamma\alpha\mu\beta} S_{\delta}{}^{\gamma\delta}{}_\nu + \\
& \frac{R_{\alpha\beta\gamma}{}^\delta R_\nu{}^{\gamma\alpha\beta} S_{\epsilon\delta}{}^\epsilon{}_\mu}{450} + \frac{R_{\alpha\beta\gamma}{}^\delta R_\mu{}^{\gamma\alpha\beta} S_{\epsilon\delta}{}^\epsilon{}_\nu}{450} + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\alpha\delta}{}^\delta S_\mu{}^{\gamma}{}_{\nu\beta} + \\
& \frac{ie}{30} F^{\alpha\beta} S_{\gamma\alpha\delta\nu} S_\mu{}^{\gamma\delta}{}_\beta + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\delta}{}^\gamma{}_\alpha S_\mu{}^\delta{}_{\nu\beta} + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\alpha\delta}{}^\delta S_\nu{}^{\gamma}{}_{\mu\beta} + \\
& \frac{ie}{30} F^{\alpha\beta} S_{\gamma\alpha\delta\mu} S_\nu{}^{\gamma\delta}{}_\beta + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\alpha}{}^\gamma{}_\delta S_\nu{}^\delta{}_{\mu\beta} - \frac{ie}{60} F_{\alpha\beta;\mu\nu} S^{\alpha\beta}{}^\gamma{} - \\
& \frac{ie}{60} F_{\alpha\mu;\beta\nu} S^{\alpha\beta}{}^\gamma{} - \frac{ie}{60} F_{\alpha\nu;\beta\mu} S^{\alpha\beta}{}^\gamma{} - \frac{R_{;\alpha\beta} S^{\alpha\beta}{}_{\mu\nu}}{50} + \\
& \frac{2\xi R_{;\alpha\beta} S^{\alpha\beta}{}_{\mu\nu}}{15} - \frac{ie}{30} F_{\alpha\beta;\gamma} S^{\alpha\beta}{}_{\mu\nu} - \frac{R_{\alpha\beta;\gamma} S^{\alpha\beta}{}_{\mu\nu}}{150} - \\
& \frac{ie}{30} F_{\alpha\beta;\gamma\nu} S^{\alpha\beta\gamma}{}_\mu - \frac{ie}{30} F_{\alpha\nu;\beta\gamma} S^{\alpha\beta\gamma}{}_\mu - \frac{ie}{30} F_{\alpha\beta;\gamma\mu} S^{\alpha\beta\gamma}{}_\nu - \\
& \frac{ie}{30} F_{\alpha\mu;\beta\gamma} S^{\alpha\beta\gamma}{}_\nu - \frac{ie}{30} F_{\alpha\beta;\gamma\nu} S^{\alpha\gamma\beta}{}_\mu - \frac{ie}{30} F_{\alpha\beta;\gamma\mu} S^{\alpha\gamma\beta}{}_\nu +
\end{aligned}$$

$$\begin{aligned}
& \frac{ie}{120} F_\nu^\alpha S_{\beta\gamma\mu\alpha} S^{\beta\gamma\delta} + \frac{ie}{120} F_\mu^\alpha S_{\beta\gamma\nu\alpha} S^{\beta\gamma\delta} + \frac{ie}{120} F_\nu^\alpha S_{\beta\gamma\delta} S^{\beta\gamma}{}_{\mu\alpha} + \\
& \frac{ie}{30} F_{\alpha\beta;\gamma}{}^\alpha S^{\beta\gamma}{}_{\mu\nu} - \frac{ie}{30} F_{\alpha\beta}{}^\alpha{}_\gamma S^{\beta\gamma}{}_{\mu\nu} + \frac{ie}{120} F_\mu^\alpha S_{\beta\gamma\delta} S^{\beta\gamma}{}_{\nu\alpha} + \\
& \frac{ie}{60} F_\nu^\alpha S_{\beta\gamma\delta\mu} S^{\beta\gamma\delta}{}_\alpha + \frac{ie}{60} F_\mu^\alpha S_{\beta\gamma\delta\nu} S^{\beta\gamma\delta}{}_\alpha + \frac{ie}{60} F_\nu^\alpha S_{\beta\gamma\delta\alpha} S^{\beta\gamma\delta}{}_\mu + \\
& \frac{ie}{60} F_\mu^\alpha S_{\beta\gamma\delta\alpha} S^{\beta\gamma\delta}{}_\nu + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\delta\mu\nu} S^{\gamma\delta}{}_{\alpha\beta} - \frac{ie}{60} F^{\alpha\beta} S_{\gamma\delta\nu\alpha} S^{\gamma\delta}{}_{\beta\mu} - \\
& \frac{ie}{60} F^{\alpha\beta} S_{\gamma\delta\mu\alpha} S^{\gamma\delta}{}_{\beta\nu} + \frac{ie}{60} F_\nu^\alpha S_{\beta\gamma}{}^\beta{}_\delta S^{\gamma\delta}{}_{\mu\alpha} + \frac{ie}{30} F^{\alpha\beta} S_{\gamma\alpha\delta\nu} S^{\gamma\delta}{}_{\mu\beta} + \\
& \frac{ie}{60} F^{\alpha\beta} S_{\gamma\delta\alpha\nu} S^{\gamma\delta}{}_{\mu\beta} + \frac{ie}{30} F^{\alpha\beta} S_{\gamma\alpha\delta\beta} S^{\gamma\delta}{}_{\mu\nu} + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\delta\alpha\beta} S^{\gamma\delta}{}_{\mu\nu} + \\
& \frac{ie}{60} F_\mu^\alpha S_{\beta\gamma}{}^\beta{}_\delta S^{\gamma\delta}{}_{\nu\alpha} + \frac{ie}{30} F^{\alpha\beta} S_{\gamma\alpha\delta\mu} S^{\gamma\delta}{}_{\nu\beta} + \frac{ie}{60} F^{\alpha\beta} S_{\gamma\delta\alpha\mu} S^{\gamma\delta}{}_{\nu\beta} - \\
& \frac{ie}{90} F_{\alpha\mu;\nu} S 5^{\alpha\beta}{}_{\gamma} - \frac{ie}{90} F_{\alpha\nu;\mu} S 5^{\alpha\beta}{}_{\gamma} - \frac{R_{;\alpha} S 5^{\alpha\beta}{}_{\mu\nu}}{60} + \\
& \frac{\xi R_{;\alpha} S 5^{\alpha\beta}{}_{\mu\nu}}{10} - \frac{ie}{45} F_{\alpha\beta;\nu} S 5^{\alpha\gamma\beta}{}_\mu - \frac{ie}{45} F_{\alpha\nu;\beta} S 5^{\alpha\gamma\beta}{}_\mu - \\
& \frac{ie}{45} F_{\alpha\beta;\mu} S 5^{\alpha\gamma\beta}{}_\nu - \frac{ie}{45} F_{\alpha\mu;\beta} S 5^{\alpha\gamma\beta}{}_\nu + \frac{ie}{90} F_{\alpha\beta}{}^\alpha S 5^{\beta\gamma}{}_{\mu\nu} - \\
& \frac{ie}{45} F_{\alpha\beta;\nu} S 5^{\alpha\beta}{}_{\gamma\mu} - \frac{ie}{45} F_{\alpha\nu;\beta} S 5^{\alpha\beta}{}_{\gamma\mu} - \frac{ie}{45} F_{\alpha\beta;\mu} S 5^{\alpha\beta}{}_{\gamma\nu} - \\
& \frac{ie}{45} F_{\alpha\mu;\beta} S 5^{\alpha\beta}{}_{\gamma\nu} - \frac{2ie}{45} F_{\alpha\beta;\gamma} S 5^{\alpha\beta\gamma}{}_{\mu\nu} - \frac{2ie}{45} F_{\alpha\beta;\gamma} S 5^{\alpha\gamma\beta}{}_{\mu\nu} - \\
& \frac{ie}{30} F^{\alpha\beta} S 6_{\alpha\beta\gamma}{}^\gamma{}_{\mu\nu} + \frac{ie}{60} F_\nu^\alpha S 6_{\alpha\beta}{}^\beta{}_\gamma{}^\gamma{}_\mu + \frac{ie}{60} F_\mu^\alpha S 6_{\alpha\beta}{}^\beta{}_\gamma{}^\gamma{}_\nu - \\
& \frac{ie}{30} F^{\alpha\beta} S 6_{\alpha\gamma}{}^\gamma{}_{\beta\mu\nu} - \frac{ie}{80} F_{\mu\nu} S 6_{\alpha}{}^\alpha{}_{\beta}{}^\beta{}_\gamma{}^\gamma + \frac{R_{\mu\nu} S 6_{\alpha}{}^\alpha{}_{\beta}{}^\beta{}_\gamma{}^\gamma}{240} + \\
& \frac{RS 6_{\alpha}{}^\alpha{}_{\beta}{}^\beta{}_{\mu\nu}}{120} + \frac{ie}{40} F_\nu^\alpha S 6_{\beta}{}^\beta{}_{\gamma\alpha}{}^\gamma{}_\mu + \frac{ie}{40} F_\mu^\alpha S 6_{\beta}{}^\beta{}_{\gamma\alpha}{}^\gamma{}_\nu + \\
& \frac{ie}{40} F_\nu^\alpha S 6_{\beta}{}^\beta{}_{\gamma}{}^\gamma{}_{\alpha\mu} + \frac{R_\nu{}^\alpha S 6_{\beta}{}^\beta{}_{\gamma}{}^\gamma{}_{\alpha\mu}}{120} + \frac{ie}{40} F_\mu^\alpha S 6_{\beta}{}^\beta{}_{\gamma}{}^\gamma{}_{\alpha\nu} + \\
& \frac{R_\mu{}^\alpha S 6_{\beta}{}^\beta{}_{\gamma}{}^\gamma{}_{\alpha\nu}}{120} + \frac{ie}{20} F^{\alpha\beta} S 6_{\gamma}{}^\gamma{}_{\alpha\beta\mu\nu}
\end{aligned}$$

(277)

## APPENDIX E

Coincidence limits  $[a_{2|\mu}]$  and  $[a_{2|\mu\nu}]$ .

$$\begin{aligned}
[a_{2|\mu}] = & \frac{1}{2} \left( \frac{1}{6} - \xi \right)^2 RR_{;\mu} + \frac{ie}{6} \left( \frac{1}{6} - \xi \right) RF_{\alpha\mu}{}^{;\alpha} + \frac{1}{12} \left( \frac{1}{6} - \xi \right) R_{;\alpha}{}^{\alpha}{}_{\mu} + \\
& \frac{ie}{60} F_{\alpha\mu}{}^{;\alpha}{}_{\beta}{}^{\beta} + \frac{R_{\alpha\mu;\beta}{}^{\beta\alpha}}{180} - \frac{ie}{5} \left( \frac{1}{6} - \xi \right) R_{;\alpha} F_{\mu}{}^{\alpha} - \frac{e^2 F_{\alpha\beta;\mu} F^{\alpha\beta}}{18} - \\
& \frac{e^2 F_{\alpha\mu;\beta} F^{\alpha\beta}}{18} - \frac{ie}{48} R_{\alpha\mu;\beta} F^{\alpha\beta} + \frac{ie}{240} R_{\alpha\beta\gamma\mu}{}^{;\gamma} F^{\alpha\beta} + \frac{ie}{240} R_{\alpha\beta\gamma\mu}{}^{;\beta} F^{\alpha\gamma} - \\
& \frac{R_{;\alpha} R_{\mu}{}^{\alpha}}{180} - \frac{ie}{180} F_{\alpha\beta}{}^{;\alpha} R_{\mu}{}^{\beta} + \frac{ie}{90} F_{\alpha\mu;\beta} R^{\alpha\beta} - \frac{7R_{\alpha\beta;\mu} R^{\alpha\beta}}{1440} - \\
& \frac{R_{\alpha\mu;\beta} R^{\alpha\beta}}{160} + \frac{7R_{\alpha\mu\beta\gamma}{}^{;\gamma} R^{\alpha\beta}}{1440} - \frac{R_{\alpha\beta;\gamma} R_{\mu}{}^{\alpha\beta\gamma}}{720} + \frac{7R_{\alpha\beta\gamma\delta}{}^{;\delta} R_{\mu}{}^{\alpha\beta\gamma}}{2160} - \\
& \frac{7R_{\alpha\beta\gamma\delta}{}^{;\beta} R_{\mu}{}^{\alpha\gamma\delta}}{2160} - \frac{ie}{90} F_{\alpha\beta;\gamma} R_{\mu}{}^{\gamma\alpha\beta} + \frac{R_{\alpha\beta\gamma\delta;\mu} R^{\alpha\beta\gamma\delta}}{540} + \frac{R_{\alpha\beta\gamma\mu;\delta} R^{\alpha\beta\gamma\delta}}{270} - \\
& \frac{R_{\alpha\mu\beta\gamma;\delta} R^{\alpha\beta\gamma\delta}}{270} + \frac{R_{\alpha\beta\gamma\delta;\mu} R^{\alpha\gamma\beta\delta}}{540}
\end{aligned} \tag{278}$$

$$\begin{aligned}
[a_{2|\mu\nu}] = & \frac{R_{;\mu} R_{;\nu}}{144} - \frac{\xi R_{;\mu} R_{;\nu}}{12} + \frac{\xi^2 R_{;\mu} R_{;\nu}}{4} - \frac{e^2 F_{\alpha\mu}{}^{;\alpha} F_{\beta\nu}{}^{;\beta}}{36} + \\
& \frac{e^2 F_{\alpha\beta;\nu} F_{\mu}{}^{\alpha;\beta}}{45} + \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F_{\mu}{}^{\beta}{}_{;\nu}}{180} + \frac{e^2 F_{\alpha\beta;\mu} F_{\nu}{}^{\alpha;\beta}}{45} + \frac{e^2 F_{\alpha\mu;\beta} F_{\nu}{}^{\alpha;\beta}}{45} + \\
& \frac{e^2 F_{\alpha\beta}{}^{;\alpha} F_{\nu}{}^{\beta}{}_{;\mu}}{180} - \frac{e^2 F_{\alpha\beta;\mu} F^{\alpha\beta}{}_{;\nu}}{45} + \frac{13R_{;\alpha} R_{\mu\nu}{}^{;\alpha}}{3360} - \frac{\xi R_{;\alpha} R_{\mu\nu}{}^{;\alpha}}{48} - \\
& \frac{23R_{;\alpha} R_{\mu}{}^{\alpha}{}_{;\nu}}{4032} + \frac{\xi R_{;\alpha} R_{\mu}{}^{\alpha}{}_{;\nu}}{96} - \frac{19R_{\alpha\beta;\nu} R_{\mu}{}^{\alpha;\beta}}{6720} - \frac{19R_{\alpha\nu;\beta} R_{\mu}{}^{\beta;\alpha}}{20160} - \\
& \frac{23R_{;\alpha} R_{\nu}{}^{\alpha}{}_{;\mu}}{4032} + \frac{\xi R_{;\alpha} R_{\nu}{}^{\alpha}{}_{;\mu}}{96} - \frac{19R_{\alpha\beta;\mu} R_{\nu}{}^{\alpha;\beta}}{6720} - \frac{23R_{\alpha\mu;\beta} R_{\nu}{}^{\alpha;\beta}}{20160} - \\
& \frac{19R_{\alpha\mu;\beta} R_{\nu}{}^{\beta;\alpha}}{20160} - \frac{41R_{\alpha\beta;\mu} R^{\alpha\beta}{}_{;\nu}}{20160} - \frac{29R_{;\alpha} R_{\beta\mu\nu}{}^{\alpha;\beta}}{20160} + \frac{\xi R_{;\alpha} R_{\beta\mu\nu}{}^{\alpha;\beta}}{160} - \\
& \frac{29R_{;\alpha} R_{\beta\nu\mu}{}^{\alpha;\beta}}{20160} + \frac{\xi R_{;\alpha} R_{\beta\nu\mu}{}^{\alpha;\beta}}{160} + \frac{5R_{\alpha\beta;\nu} R_{\gamma\mu}{}^{\alpha\beta;\gamma}}{4032} - \frac{R_{\alpha\nu;\beta} R_{\gamma\mu}{}^{\alpha\beta;\gamma}}{2240} + \\
& \frac{5R_{\alpha\beta;\mu} R_{\gamma\nu}{}^{\alpha\beta;\gamma}}{4032} - \frac{R_{\alpha\mu;\beta} R_{\gamma\nu}{}^{\alpha\beta;\gamma}}{2240} + \frac{R_{\alpha\beta;\nu} R_{\gamma}{}^{\alpha}{}_{\mu}{}^{\beta;\gamma}}{960} + \frac{R_{\alpha\nu;\beta} R_{\gamma}{}^{\alpha}{}_{\mu}{}^{\beta;\gamma}}{3360} + \\
& \frac{R_{\alpha\beta;\mu} R_{\gamma}{}^{\alpha}{}_{\nu}{}^{\beta;\gamma}}{960} + \frac{R_{\alpha\mu;\beta} R_{\gamma}{}^{\alpha}{}_{\nu}{}^{\beta;\gamma}}{3360} + \frac{R_{\alpha\nu;\beta} R_{\gamma}{}^{\beta}{}_{\mu}{}^{\alpha;\gamma}}{1344} + \frac{R_{\alpha\mu;\beta} R_{\gamma}{}^{\beta}{}_{\nu}{}^{\alpha;\gamma}}{1344} + \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{;\gamma} R_{\delta\nu}{}^{\alpha\beta;\delta}}{6720} - \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha} R_{\delta\nu}{}^{\beta\gamma;\delta}}{6720} + \frac{R_{\alpha\beta\gamma\mu}{}^{;\gamma} R_{\delta}{}^{\alpha}{}_{\nu}{}^{\beta;\delta}}{6720} - \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha} R_{\delta}{}^{\beta}{}_{\nu}{}^{\gamma;\delta}}{960} -
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha} R_{\delta}{}^{\gamma}{}_{\nu}{}^{\beta;\delta}}{1120} + \frac{11R_{\alpha\beta;\gamma} R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta;\gamma}}{10080} + \frac{13R_{\alpha\beta;\gamma} R_{\mu}{}^{\alpha}{}_{\nu}{}^{\gamma;\beta}}{20160} - \frac{R_{\alpha\beta;\gamma} R_{\mu}{}^{\alpha\beta\gamma}{}_{;\nu}}{2240} + \\
& \frac{R_{\alpha\beta\gamma\delta;\nu} R_{\mu}{}^{\alpha\beta\gamma;\delta}}{1680} + \frac{R_{\alpha\beta\gamma\nu;\delta} R_{\mu}{}^{\alpha\beta\gamma;\delta}}{1680} - \frac{R_{\alpha\beta\gamma\delta;\nu} R_{\mu}{}^{\alpha\gamma\delta;\beta}}{840} - \frac{R_{\alpha\beta\gamma\nu;\delta} R_{\mu}{}^{\alpha\gamma\delta;\beta}}{1680} - \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\mu}{}^{\beta}{}_{\nu}{}^{\gamma;\delta}}{960} + \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\mu}{}^{\beta\gamma\delta}{}_{;\nu}}{960} + \frac{13R_{\alpha\beta;\gamma} R_{\mu}{}^{\gamma}{}_{\nu}{}^{\alpha;\beta}}{20160} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\mu}{}^{\gamma}{}_{\nu}{}^{\beta;\delta}}{960} - \\
& \frac{23R_{\alpha\beta;\gamma} R_{\mu}{}^{\gamma\alpha\beta}{}_{;\nu}}{6720} + \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\mu}{}^{\gamma\beta\delta}{}_{;\nu}}{960} - \frac{R_{\alpha\beta;\gamma} R_{\nu}{}^{\alpha\beta\gamma}{}_{;\mu}}{2240} + \frac{R_{\alpha\beta\gamma\delta;\mu} R_{\nu}{}^{\alpha\beta\gamma;\delta}}{840} + \\
& \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu}{}^{\alpha\beta\gamma;\delta}}{1680} - \frac{R_{\alpha\beta\gamma\delta;\mu} R_{\nu}{}^{\alpha\gamma\delta;\beta}}{840} - \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu}{}^{\alpha\gamma\delta;\beta}}{1680} + \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\nu}{}^{\beta\gamma\delta}{}_{;\mu}}{960} - \\
& \frac{61R_{\alpha\beta;\gamma} R_{\nu}{}^{\gamma\alpha\beta}{}_{;\mu}}{20160} - \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu}{}^{\gamma\alpha\beta;\delta}}{840} - \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu}{}^{\gamma\alpha\delta;\beta}}{840} + \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\nu}{}^{\gamma\beta\delta}{}_{;\mu}}{960} + \\
& \frac{R_{\alpha\beta\gamma\delta;\mu} R_{\nu}{}^{\alpha\beta\gamma\delta}{}_{;\nu}}{1680} + \frac{R_{\alpha\beta\gamma\delta;\mu} R_{\nu}{}^{\alpha\gamma\beta\delta}{}_{;\nu}}{1680} + \frac{R_{\alpha\beta\gamma\mu;\delta} R_{\nu}{}^{\alpha\gamma\beta\delta}{}_{;\nu}}{1680} + \frac{RR_{;\mu\nu}}{240} - \\
& \frac{29R\xi R_{;\mu\nu}}{360} + \frac{R\xi^2 R_{;\mu\nu}}{3} + \frac{RR_{\alpha\mu}{}^{\alpha}{}_{\nu}}{240} - \frac{R\xi R_{\alpha\mu}{}^{\alpha}{}_{\nu}}{40} + \\
& \frac{RR_{\alpha\nu}{}^{\alpha}{}_{\mu}}{240} - \frac{R\xi R_{\alpha\nu}{}^{\alpha}{}_{\mu}}{40} + \frac{RR_{\mu\nu;\alpha}{}^{\alpha}}{360} - \frac{R\xi R_{\mu\nu;\alpha}{}^{\alpha}}{60} + \\
& \frac{R_{;\alpha}{}^{\alpha}{}_{\mu\nu}}{420} - \frac{\xi R_{;\alpha}{}^{\alpha}{}_{\mu\nu}}{20} + \frac{R_{\alpha\beta}{}^{\alpha\beta}{}_{\mu\nu}}{280} + \frac{R_{\alpha\beta}{}^{\beta\alpha}{}_{\mu\nu}}{840} + \\
& \frac{R_{\alpha\mu;\beta}{}^{\beta\alpha}{}_{\nu}}{420} + \frac{R_{\alpha\mu}{}^{\alpha}{}_{\beta}{}^{\beta}{}_{\nu}}{420} + \frac{R_{\alpha\nu;\beta}{}^{\beta\alpha}{}_{\mu}}{420} + \frac{R_{\alpha\nu}{}^{\alpha}{}_{\beta}{}^{\beta}{}_{\mu}}{420} + \\
& \frac{R_{\mu\nu;\alpha}{}^{\alpha}{}_{\beta}{}^{\beta}}{840} - \frac{e^2 F_{\alpha\beta}{}^{\alpha\beta} F_{\mu\nu}}{60} + \frac{e^2 F_{\alpha\beta}{}^{\beta\alpha} F_{\mu\nu}}{240} + \frac{e^2 F_{\alpha\beta}{}^{\beta}{}_{\nu} F_{\mu}{}^{\alpha}}{96} + \\
& \frac{e^2 F_{\alpha\nu;\beta}{}^{\beta} F_{\mu}{}^{\alpha}}{80} + \frac{e^2 F_{\alpha\beta}{}^{\alpha}{}_{\nu} F_{\mu}{}^{\beta}}{160} + \frac{e^2 F_{\alpha\nu}{}^{\alpha}{}_{\beta} F_{\mu}{}^{\beta}}{48} + \frac{e^2 F_{\alpha\beta}{}^{\beta}{}_{\mu} F_{\nu}{}^{\alpha}}{96} + \\
& \frac{e^2 F_{\alpha\mu;\beta}{}^{\beta} F_{\nu}{}^{\alpha}}{80} + \frac{e^2 R F_{\alpha\mu} F_{\nu}{}^{\alpha}}{36} - \frac{e^2 R \xi F_{\alpha\mu} F_{\nu}{}^{\alpha}}{6} + \frac{e^2 F_{\alpha\beta}{}^{\alpha}{}_{\mu} F_{\nu}{}^{\beta}}{160} + \\
& \frac{e^2 F_{\alpha\mu}{}^{\alpha}{}_{\beta} F_{\nu}{}^{\beta}}{48} - \frac{e^2 F_{\alpha\beta;\mu\nu} F^{\alpha\beta}}{40} - \frac{e^2 F_{\alpha\mu;\beta\nu} F^{\alpha\beta}}{40} - \frac{e^2 F_{\alpha\nu;\beta\mu} F^{\alpha\beta}}{40} + \\
& \frac{R_{;\alpha\nu} R_{\mu}{}^{\alpha}}{12600} - \frac{\xi R_{;\alpha\nu} R_{\mu}{}^{\alpha}}{180} - \frac{17R_{\alpha\beta}{}^{\beta}{}_{\nu} R_{\mu}{}^{\alpha}}{6300} - \frac{R_{\alpha\nu;\beta}{}^{\beta} R_{\mu}{}^{\alpha}}{525} - \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{\gamma\beta} R_{\mu}{}^{\alpha}}{4200} - \frac{R_{\alpha\nu\beta\gamma}{}^{\beta\gamma} R_{\mu}{}^{\alpha}}{4200} - \frac{R_{\alpha\beta}{}^{\alpha}{}_{\nu} R_{\mu}{}^{\beta}}{630} - \frac{17R_{\alpha\nu;\beta}{}^{\alpha} R_{\mu}{}^{\beta}}{50400} - \\
& \frac{59R_{\alpha\nu}{}^{\alpha}{}_{\beta} R_{\mu}{}^{\beta}}{50400} + \frac{R_{\alpha\nu\beta\gamma}{}^{\alpha\gamma} R_{\mu}{}^{\beta}}{3150} - \frac{e^2 F_{\alpha\beta} F_{\nu}{}^{\alpha} R_{\mu}{}^{\beta}}{720} + \frac{R_{\alpha\beta\gamma\nu}{}^{\alpha\beta} R_{\mu}{}^{\gamma}}{3150} + \\
& \frac{R_{;\alpha\mu} R_{\nu}{}^{\alpha}}{12600} - \frac{\xi R_{;\alpha\mu} R_{\nu}{}^{\alpha}}{180} - \frac{17R_{\alpha\beta}{}^{\beta}{}_{\mu} R_{\nu}{}^{\alpha}}{6300} - \frac{R_{\alpha\mu;\beta}{}^{\beta} R_{\nu}{}^{\alpha}}{525} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{\gamma\beta} R_{\nu}{}^{\alpha}}{4200} - \frac{R_{\alpha\mu\beta\gamma}{}^{\beta\gamma} R_{\nu}{}^{\alpha}}{4200} - \frac{RR_{\alpha\mu} R_{\nu}{}^{\alpha}}{270} + \frac{R\xi R_{\alpha\mu} R_{\nu}{}^{\alpha}}{45} - \\
& \frac{R_{\alpha\beta}{}^{\alpha}{}_{\mu} R_{\nu}{}^{\beta}}{630} - \frac{17R_{\alpha\mu;\beta}{}^{\alpha} R_{\nu}{}^{\beta}}{50400} - \frac{59R_{\alpha\mu}{}^{\alpha}{}_{\beta} R_{\nu}{}^{\beta}}{50400} + \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha\gamma} R_{\nu}{}^{\beta}}{3150} -
\end{aligned}$$

$$\begin{aligned}
& \frac{e^2 F_{\alpha\beta} F_{\mu}^{\alpha} R_{\nu}^{\beta}}{720} + \frac{17 R_{\alpha\beta} R_{\mu}^{\alpha} R_{\nu}^{\beta}}{10800} + \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha\beta} R_{\nu}^{\gamma}}{3150} - \frac{4 R_{\alpha\beta;\mu\nu} R^{\alpha\beta}}{1575} - \\
& \frac{19 R_{\alpha\mu;\beta\nu} R^{\alpha\beta}}{6300} - \frac{19 R_{\alpha\nu;\beta\mu} R^{\alpha\beta}}{6300} + \frac{R_{\mu\nu;\alpha\beta} R^{\alpha\beta}}{1575} + \frac{13 R_{\alpha\beta\gamma\mu}{}^{;\gamma\nu} R^{\alpha\beta}}{3150} - \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma\mu} R^{\alpha\beta}}{25200} + \frac{R_{\alpha\mu\beta\gamma}{}^{;\gamma\nu} R^{\alpha\beta}}{900} + \frac{R_{\alpha\mu\beta\nu;\gamma}{}^{\gamma} R^{\alpha\beta}}{1800} + \frac{R_{\alpha\nu\beta\gamma}{}^{;\gamma\mu} R^{\alpha\beta}}{900} + \\
& \frac{R_{\alpha\nu\beta\mu;\gamma}{}^{\gamma} R^{\alpha\beta}}{1800} - \frac{e^2 F_{\alpha\mu} F_{\beta\nu} R^{\alpha\beta}}{120} - \frac{e^2 F_{\alpha\beta} F_{\mu\nu} R^{\alpha\beta}}{144} + \frac{R_{\alpha\beta\gamma\mu}{}^{;\beta\nu} R^{\alpha\gamma}}{900} + \\
& \frac{R_{\alpha\beta\gamma\nu}{}^{;\beta\mu} R^{\alpha\gamma}}{900} - \frac{R_{\alpha\mu\beta\gamma}{}^{;\beta\nu} R^{\alpha\gamma}}{4200} - \frac{R_{\alpha\mu\beta\nu;\gamma}{}^{\beta} R^{\alpha\gamma}}{4200} - \frac{R_{\alpha\nu\beta\gamma}{}^{;\beta\mu} R^{\alpha\gamma}}{4200} - \\
& \frac{R_{\alpha\nu\beta\mu}{}^{;\beta\gamma} R^{\alpha\gamma}}{4200} + \frac{R_{\alpha\beta\gamma\mu}{}^{;\alpha\nu} R^{\beta\gamma}}{1800} + \frac{R_{\alpha\beta\gamma\nu}{}^{;\alpha\mu} R^{\beta\gamma}}{1800} + \frac{R_{\alpha\mu\beta\gamma}{}^{;\alpha\nu} R^{\beta\gamma}}{1800} + \\
& \frac{R_{\alpha\mu\beta\nu}{}^{;\alpha\gamma} R^{\beta\gamma}}{1800} + \frac{7 R_{\alpha\nu\beta\gamma}{}^{;\alpha\mu} R^{\beta\gamma}}{3600} + \frac{R_{\alpha\nu\beta\mu}{}^{;\alpha\gamma} R^{\beta\gamma}}{1800} + \frac{R_{\gamma\alpha\beta} R_{\mu\nu}{}^{\alpha\beta}}{1200} + \\
& \frac{R_{\alpha\beta;\gamma}{}^{\gamma} R_{\mu\nu}{}^{\alpha\beta}}{560} + \frac{R R_{\alpha\beta} R_{\mu\nu}{}^{\alpha\beta}}{4320} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\beta\delta} R_{\mu\nu}{}^{\alpha\gamma}}{1050} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\delta\beta} R_{\mu\nu}{}^{\alpha\gamma}}{1050} + \\
& \frac{11 e^2 F_{\alpha\beta} F_{\gamma}{}^{\alpha} R_{\mu\nu}{}^{\beta\gamma}}{720} - \frac{37 R_{\alpha\beta} R_{\gamma}{}^{\alpha} R_{\mu\nu}{}^{\beta\gamma}}{50400} - \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma\delta} R_{\mu\nu}{}^{\beta\epsilon}}{4725} + \\
& \frac{17 R_{\alpha\beta} R_{\gamma\delta}{}^{\alpha\beta} R_{\mu\nu}{}^{\gamma\delta}}{75600} + \frac{R_{\alpha\beta} R_{\gamma}{}^{\alpha\beta} R_{\mu\nu}{}^{\gamma\delta}}{3600} + \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\beta\gamma} R_{\mu\nu}{}^{\delta\epsilon}}{4725} + \\
& \frac{R_{\gamma\alpha\beta} R_{\mu}{}^{\alpha\nu\beta}}{450} - \frac{\xi R_{\gamma\alpha\beta} R_{\mu}{}^{\alpha\nu\beta}}{90} + \frac{R_{\alpha\beta;\gamma}{}^{\gamma} R_{\mu}{}^{\alpha\nu\beta}}{2100} + \frac{R R_{\alpha\beta} R_{\mu}{}^{\alpha\nu\beta}}{864} - \\
& \frac{R \xi R_{\alpha\beta} R_{\mu}{}^{\alpha\nu\beta}}{144} + \frac{31 R_{\alpha\beta;\gamma}{}^{\beta} R_{\mu}{}^{\alpha\nu\gamma}}{25200} + \frac{R_{\alpha\beta}{}^{;\beta\gamma} R_{\mu}{}^{\alpha\nu\gamma}}{2520} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\beta\delta} R_{\mu}{}^{\alpha\nu\gamma}}{3150} + \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{;\delta\beta} R_{\mu}{}^{\alpha\nu\gamma}}{3150} - \frac{R_{\alpha\beta;\gamma\nu} R_{\mu}{}^{\alpha\beta\gamma}}{12600} - \frac{23 R_{\alpha\nu;\beta\gamma} R_{\mu}{}^{\alpha\beta\gamma}}{12600} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\delta\nu} R_{\mu}{}^{\alpha\beta\gamma}}{1575} - \\
& \frac{R_{\alpha\beta\gamma\nu;\delta}{}^{\delta} R_{\mu}{}^{\alpha\beta\gamma}}{2100} - \frac{R_{\alpha\nu\beta\gamma;\delta}{}^{\delta} R_{\mu}{}^{\alpha\beta\gamma}}{3150} - \frac{e^2 F_{\alpha\beta} F_{\gamma\nu} R_{\mu}{}^{\alpha\beta\gamma}}{120} + \frac{191 R_{\alpha\beta} R_{\gamma\nu} R_{\mu}{}^{\alpha\beta\gamma}}{151200} + \\
& \frac{R R_{\alpha\beta\gamma\nu} R_{\mu}{}^{\alpha\beta\gamma}}{2160} - \frac{R \xi R_{\alpha\beta\gamma\nu} R_{\mu}{}^{\alpha\beta\gamma}}{360} - \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma\delta} R_{\mu}{}^{\alpha\beta\delta}}{1260} - \frac{R_{\alpha\nu\beta\gamma\delta}{}^{\gamma} R_{\mu}{}^{\alpha\beta\delta}}{3150} - \\
& \frac{R_{\alpha\nu\beta\gamma}{}^{;\gamma\delta} R_{\mu}{}^{\alpha\beta\delta}}{3150} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\beta\nu} R_{\mu}{}^{\alpha\gamma\delta}}{1575} - \frac{R_{\alpha\beta\gamma\nu;\delta}{}^{\beta} R_{\mu}{}^{\alpha\gamma\delta}}{3150} - \frac{R_{\alpha\beta\gamma\nu}{}^{;\beta\delta} R_{\mu}{}^{\alpha\gamma\delta}}{3150} + \\
& \frac{R_{\gamma\alpha\beta} R_{\mu}{}^{\beta\nu\alpha}}{450} - \frac{\xi R_{\gamma\alpha\beta} R_{\mu}{}^{\beta\nu\alpha}}{90} + \frac{R_{\alpha\beta;\gamma}{}^{\gamma} R_{\mu}{}^{\beta\nu\alpha}}{2100} + \frac{R R_{\alpha\beta} R_{\mu}{}^{\beta\nu\alpha}}{1440} - \\
& \frac{R \xi R_{\alpha\beta} R_{\mu}{}^{\beta\nu\alpha}}{240} - \frac{13 R_{\alpha\beta;\gamma}{}^{\alpha} R_{\mu}{}^{\beta\nu\gamma}}{25200} - \frac{R_{\alpha\beta}{}^{;\alpha\gamma} R_{\mu}{}^{\beta\nu\gamma}}{2100} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha\delta} R_{\mu}{}^{\beta\nu\gamma}}{3150} + \\
& \frac{7 e^2 F_{\alpha\beta} F_{\gamma}{}^{\alpha} R_{\mu}{}^{\beta\nu\gamma}}{1440} - \frac{19 R_{\alpha\beta} R_{\gamma}{}^{\alpha} R_{\mu}{}^{\beta\nu\gamma}}{37800} - \frac{4 R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma\delta} R_{\mu}{}^{\beta\nu\epsilon}}{4725} - \\
& \frac{R_{\alpha\beta;\gamma\nu} R_{\mu}{}^{\beta\alpha\gamma}}{4200} + \frac{R_{\alpha\nu;\beta\gamma} R_{\mu}{}^{\beta\alpha\gamma}}{2520} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\delta\nu} R_{\mu}{}^{\beta\alpha\gamma}}{525} - \frac{11 R_{\alpha\beta\gamma\nu;\delta}{}^{\delta} R_{\mu}{}^{\beta\alpha\gamma}}{6300} - \\
& \frac{R_{\alpha\nu\beta\gamma;\delta}{}^{\delta} R_{\mu}{}^{\beta\alpha\gamma}}{3150} - \frac{R_{\alpha\beta\gamma\nu}{}^{;\gamma\delta} R_{\mu}{}^{\beta\alpha\delta}}{1260} - \frac{R_{\alpha\nu\beta\gamma;\delta}{}^{\gamma} R_{\mu}{}^{\beta\alpha\delta}}{3150} - \frac{R_{\alpha\nu\beta\gamma}{}^{;\gamma\delta} R_{\mu}{}^{\beta\alpha\delta}}{3150} -
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha\nu}R_{\mu}{}^{\beta\gamma\delta}}{6300} - \frac{R_{\alpha\beta\gamma\nu}{}^{\alpha\delta}R_{\mu}{}^{\beta\gamma\delta}}{6300} + \frac{151R_{\alpha\nu}R_{\beta\gamma\delta}{}^{\alpha}R_{\mu}{}^{\beta\gamma\delta}}{113400} + \frac{11R_{\alpha\beta}R_{\gamma\delta\nu}{}^{\alpha}R_{\mu}{}^{\beta\gamma\delta}}{25200} - \\
& \frac{23R_{\alpha\beta}R_{\gamma\nu\delta}{}^{\alpha}R_{\mu}{}^{\beta\gamma\delta}}{15120} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_{\mu}{}^{\beta\delta\epsilon}}{6300} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\nu}{}^{\gamma}R_{\mu}{}^{\beta\delta\epsilon}}{4725} + \\
& \frac{31R_{\alpha\beta;\gamma}{}^{\beta}R_{\mu}{}^{\gamma\nu}{}^{\alpha}}{25200} + \frac{R_{\alpha\beta}{}^{\beta\gamma}R_{\mu}{}^{\gamma\nu}{}^{\alpha}}{2520} + \frac{R_{\alpha\beta\gamma\delta}{}^{\beta\delta}R_{\mu}{}^{\gamma\nu}{}^{\alpha}}{3150} + \frac{R_{\alpha\beta\gamma\delta}{}^{\delta\beta}R_{\mu}{}^{\gamma\nu}{}^{\alpha}}{3150} - \\
& \frac{13R_{\alpha\beta;\gamma}{}^{\alpha}R_{\mu}{}^{\gamma\nu}{}^{\beta}}{25200} - \frac{R_{\alpha\beta}{}^{\alpha\gamma}R_{\mu}{}^{\gamma\nu}{}^{\beta}}{2100} + \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha\delta}R_{\mu}{}^{\gamma\nu}{}^{\beta}}{3150} + \frac{e^2F_{\alpha\beta}F_{\gamma}{}^{\alpha}R_{\mu}{}^{\gamma\nu}{}^{\beta}}{160} - \\
& \frac{19R_{\alpha\beta}R_{\gamma}{}^{\alpha}R_{\mu}{}^{\gamma\nu}{}^{\beta}}{37800} + \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha\beta}R_{\mu}{}^{\gamma\nu}{}^{\delta}}{3150} - \frac{13R_{\alpha\beta}R_{\gamma\delta}{}^{\alpha\beta}R_{\mu}{}^{\gamma\nu}{}^{\delta}}{56700} + \\
& \frac{11R_{\alpha\beta}R_{\gamma}{}^{\alpha\delta}{}^{\beta}R_{\mu}{}^{\gamma\nu}{}^{\delta}}{14175} - \frac{47R_{\alpha\beta;\gamma\nu}R_{\mu}{}^{\gamma\alpha\beta}}{12600} + \frac{R_{\alpha\nu;\beta\gamma}R_{\mu}{}^{\gamma\alpha\beta}}{1800} - \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{\delta\nu}R_{\mu}{}^{\gamma\alpha\beta}}{1575} - \frac{R_{\alpha\beta\gamma\nu;\delta}{}^{\delta}R_{\mu}{}^{\gamma\alpha\beta}}{3150} - \frac{R_{\alpha\nu\beta\gamma;\delta}{}^{\delta}R_{\mu}{}^{\gamma\alpha\beta}}{1050} + \frac{e^2F_{\alpha\beta}F_{\gamma\nu}R_{\mu}{}^{\gamma\alpha\beta}}{120} - \\
& \frac{317R_{\alpha\beta}R_{\gamma\nu}R_{\mu}{}^{\gamma\alpha\beta}}{226800} - \frac{R_{\alpha\beta\gamma\delta}{}^{\beta\nu}R_{\mu}{}^{\gamma\alpha\delta}}{1575} - \frac{R_{\alpha\beta\gamma\nu;\delta}{}^{\beta}R_{\mu}{}^{\gamma\alpha\delta}}{3150} - \\
& \frac{R_{\alpha\beta\gamma\nu;\delta}{}^{\beta}R_{\mu}{}^{\gamma\alpha\delta}}{3150} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha\nu}R_{\mu}{}^{\gamma\beta\delta}}{1260} - \frac{R_{\alpha\nu\beta\gamma;\delta}{}^{\alpha}R_{\mu}{}^{\gamma\beta\delta}}{6300} - \frac{11R_{\alpha\beta}R_{\gamma\delta\nu}{}^{\alpha}R_{\mu}{}^{\gamma\beta\delta}}{15120} - \\
& \frac{R_{\alpha\beta}R_{\gamma\nu\delta}{}^{\alpha}R_{\mu}{}^{\gamma\beta\delta}}{6300} + \frac{67R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_{\mu}{}^{\gamma\delta\epsilon}}{75600} + \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\nu}{}^{\beta}R_{\mu}{}^{\gamma\delta\epsilon}}{4725} + \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha\beta}R_{\mu}{}^{\delta\nu}{}^{\gamma}}{3150} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\beta\gamma}R_{\mu}{}^{\delta\nu}{}^{\epsilon}}{1890} - \frac{R_{\alpha\nu\beta\gamma;\delta}{}^{\gamma}R_{\mu}{}^{\delta\alpha\beta}}{1050} - \frac{R_{\alpha\nu\beta\gamma;\gamma\delta}R_{\mu}{}^{\delta\alpha\beta}}{1050} - \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{\beta\nu}R_{\mu}{}^{\delta\alpha\gamma}}{525} - \frac{R_{\alpha\beta\gamma\nu;\delta}{}^{\beta}R_{\mu}{}^{\delta\alpha\gamma}}{1050} - \frac{R_{\alpha\beta\gamma\nu;\delta}{}^{\beta}R_{\mu}{}^{\delta\alpha\gamma}}{1050} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha\nu}R_{\mu}{}^{\delta\beta\gamma}}{1575} + \\
& \frac{R_{\alpha\beta\gamma\nu;\alpha\delta}R_{\mu}{}^{\delta\beta\gamma}}{6300} - \frac{R_{\alpha\nu\beta\gamma;\alpha\delta}R_{\mu}{}^{\delta\beta\gamma}}{6300} + \frac{R_{\alpha\nu}R_{\beta\gamma\delta}{}^{\alpha}R_{\mu}{}^{\delta\beta\gamma}}{9072} + \frac{2R_{\alpha\beta}R_{\gamma\nu\delta}{}^{\alpha}R_{\mu}{}^{\delta\beta\gamma}}{4725} - \\
& \frac{11R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_{\mu}{}^{\delta\beta\epsilon}}{56700} - \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\nu}{}^{\gamma}R_{\mu}{}^{\delta\beta\epsilon}}{4725} - \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\gamma\delta}R_{\mu}{}^{\epsilon\nu}{}^{\beta}}{4725} + \\
& \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\beta\gamma}R_{\mu}{}^{\epsilon\nu}{}^{\delta}}{1890} + \frac{2R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\gamma}R_{\mu}{}^{\epsilon\beta\delta}}{14175} - \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\nu}{}^{\gamma}R_{\mu}{}^{\epsilon\beta\delta}}{4725} - \\
& \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\nu}{}^{\alpha\beta}R_{\mu}{}^{\epsilon\gamma\delta}}{5670} - \frac{23R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha\nu}{}^{\beta}R_{\mu}{}^{\epsilon\gamma\delta}}{226800} - \frac{R_{\alpha\beta;\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{12600} - \\
& \frac{23R_{\alpha\mu;\beta\gamma}R_{\nu}{}^{\alpha\beta\gamma}}{12600} + \frac{R_{\alpha\beta\gamma\delta}{}^{\delta\mu}R_{\nu}{}^{\alpha\beta\gamma}}{1575} - \frac{R_{\alpha\beta\gamma\mu;\delta}{}^{\delta}R_{\nu}{}^{\alpha\beta\gamma}}{2100} - \frac{R_{\alpha\mu\beta\gamma;\delta}{}^{\delta}R_{\nu}{}^{\alpha\beta\gamma}}{3150} - \\
& \frac{e^2F_{\alpha\beta}F_{\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{120} + \frac{191R_{\alpha\beta}R_{\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{151200} + \frac{RR_{\alpha\beta\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{2160} - \\
& \frac{R_{\xi}R_{\alpha\beta\gamma\mu}R_{\nu}{}^{\alpha\beta\gamma}}{360} - \frac{R_{\alpha\beta\gamma\mu;\gamma\delta}R_{\nu}{}^{\alpha\beta\delta}}{1260} - \frac{R_{\alpha\mu\beta\gamma;\delta}{}^{\gamma}R_{\nu}{}^{\alpha\beta\delta}}{3150} - \frac{R_{\alpha\mu\beta\gamma;\gamma\delta}R_{\nu}{}^{\alpha\beta\delta}}{3150} - \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{\beta\mu}R_{\nu}{}^{\alpha\gamma\delta}}{1575} - \frac{R_{\alpha\beta\gamma\mu;\delta}{}^{\beta}R_{\nu}{}^{\alpha\gamma\delta}}{3150} - \frac{R_{\alpha\beta\gamma\mu;\delta}{}^{\beta}R_{\nu}{}^{\alpha\gamma\delta}}{3150} - \frac{R_{\alpha\beta;\gamma\mu}R_{\nu}{}^{\beta\alpha\gamma}}{4200} + \\
& \frac{R_{\alpha\mu;\beta\gamma}R_{\nu}{}^{\beta\alpha\gamma}}{2520} - \frac{R_{\alpha\beta\gamma\delta}{}^{\delta\mu}R_{\nu}{}^{\beta\alpha\gamma}}{525} - \frac{11R_{\alpha\beta\gamma\mu;\delta}{}^{\delta}R_{\nu}{}^{\beta\alpha\gamma}}{6300} - \frac{R_{\alpha\mu\beta\gamma;\delta}{}^{\delta}R_{\nu}{}^{\beta\alpha\gamma}}{3150} -
\end{aligned}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta\gamma\mu}{}^{\gamma\delta}R_{\nu}{}^{\beta\alpha\delta}}{1260} - \frac{R_{\alpha\mu\beta\gamma;\delta}{}^{\gamma}R_{\nu}{}^{\beta\alpha\delta}}{3150} - \frac{R_{\alpha\mu\beta\gamma}{}^{\gamma\delta}R_{\nu}{}^{\beta\alpha\delta}}{3150} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\mu}R_{\nu}{}^{\beta\gamma\delta}}{6300} \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha}{}_{\delta}R_{\nu}{}^{\beta\gamma\delta}}{6300} + \frac{151R_{\alpha\mu}R_{\beta\gamma\delta}{}^{\alpha}R_{\nu}{}^{\beta\gamma\delta}}{113400} + \frac{11R_{\alpha\beta}R_{\gamma\delta\mu}{}^{\alpha}R_{\nu}{}^{\beta\gamma\delta}}{25200} - \\
& \frac{23R_{\alpha\beta}R_{\gamma\mu\delta}{}^{\alpha}R_{\nu}{}^{\beta\gamma\delta}}{15120} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\gamma}R_{\nu}{}^{\beta\delta\epsilon}}{6300} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma}R_{\nu}{}^{\beta\delta\epsilon}}{4725} - \\
& \frac{47R_{\alpha\beta;\gamma\mu}R_{\nu}{}^{\gamma\alpha\beta}}{12600} + \frac{R_{\alpha\mu;\beta\gamma}R_{\nu}{}^{\gamma\alpha\beta}}{1800} - \frac{R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\mu}R_{\nu}{}^{\gamma\alpha\beta}}{1575} - \frac{R_{\alpha\beta\gamma\mu;\delta}R_{\nu}{}^{\gamma\alpha\beta}}{3150} - \\
& \frac{R_{\alpha\mu\beta\gamma;\delta}R_{\nu}{}^{\gamma\alpha\beta}}{1050} + \frac{e^2F_{\alpha\beta}F_{\gamma\mu}R_{\nu}{}^{\gamma\alpha\beta}}{120} - \frac{317R_{\alpha\beta}R_{\gamma\mu}R_{\nu}{}^{\gamma\alpha\beta}}{226800} - \\
& \frac{RR_{\alpha\beta\gamma\mu}R_{\nu}{}^{\gamma\alpha\beta}}{720} + \frac{R\xi R_{\alpha\beta\gamma\mu}R_{\nu}{}^{\gamma\alpha\beta}}{120} - \frac{R_{\alpha\beta\gamma\delta}{}^{\beta}{}_{\mu}R_{\nu}{}^{\gamma\alpha\delta}}{1575} - \frac{R_{\alpha\beta\gamma\mu;\delta}{}^{\beta}R_{\nu}{}^{\gamma\alpha\delta}}{3150} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{\beta}{}_{\delta}R_{\nu}{}^{\gamma\alpha\delta}}{3150} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\mu}R_{\nu}{}^{\gamma\beta\delta}}{1260} - \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha}{}_{\delta}R_{\nu}{}^{\gamma\beta\delta}}{6300} - \frac{11R_{\alpha\beta}R_{\gamma\delta\mu}{}^{\alpha}R_{\nu}{}^{\gamma\beta\delta}}{15120} - \\
& \frac{R_{\alpha\beta}R_{\gamma\mu\delta}{}^{\alpha}R_{\nu}{}^{\gamma\beta\delta}}{6300} + \frac{67R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\beta}R_{\nu}{}^{\gamma\delta\epsilon}}{75600} + \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\beta}R_{\nu}{}^{\gamma\delta\epsilon}}{4725} - \\
& \frac{R_{\alpha\mu\beta\gamma;\delta}{}^{\gamma}R_{\nu}{}^{\delta\alpha\beta}}{1050} - \frac{R_{\alpha\mu\beta\gamma}{}^{\gamma\delta}R_{\nu}{}^{\delta\alpha\beta}}{1050} - \frac{R_{\alpha\beta\gamma\delta}{}^{\beta}{}_{\mu}R_{\nu}{}^{\delta\alpha\gamma}}{525} - \frac{R_{\alpha\beta\gamma\mu;\delta}{}^{\beta}R_{\nu}{}^{\delta\alpha\gamma}}{1050} - \\
& \frac{R_{\alpha\beta\gamma\mu}{}^{\beta}{}_{\delta}R_{\nu}{}^{\delta\alpha\gamma}}{1050} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha}{}_{\mu}R_{\nu}{}^{\delta\beta\gamma}}{1575} + \frac{R_{\alpha\beta\gamma\mu}{}^{\alpha}{}_{\delta}R_{\nu}{}^{\delta\beta\gamma}}{6300} - \frac{R_{\alpha\mu\beta\gamma}{}^{\alpha}{}_{\delta}R_{\nu}{}^{\delta\beta\gamma}}{6300} + \\
& \frac{R_{\alpha\mu}R_{\beta\gamma\delta}{}^{\alpha}R_{\nu}{}^{\delta\beta\gamma}}{9072} + \frac{2R_{\alpha\beta}R_{\gamma\mu\delta}{}^{\alpha}R_{\nu}{}^{\delta\beta\gamma}}{4725} - \frac{11R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\gamma}R_{\nu}{}^{\delta\beta\epsilon}}{56700} - \\
& \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma}R_{\nu}{}^{\delta\beta\epsilon}}{4725} - \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\gamma}R_{\nu}{}^{\epsilon\beta\delta}}{4725} - \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\mu}{}^{\alpha\beta}R_{\nu}{}^{\epsilon\gamma\delta}}{3150} - \\
& \frac{23R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu}{}^{\beta}R_{\nu}{}^{\epsilon\gamma\delta}}{226800} + \frac{R_{\alpha\beta\gamma\delta;\mu\nu}R^{\alpha\beta\gamma\delta}}{1575} + \frac{R_{\alpha\beta\gamma\mu;\delta\nu}R^{\alpha\beta\gamma\delta}}{1575} + \frac{R_{\alpha\beta\gamma\nu;\delta\mu}R^{\alpha\beta\gamma\delta}}{1575} + \\
& \frac{R_{\alpha\mu\beta\nu;\gamma\delta}R^{\alpha\beta\gamma\delta}}{1050} + \frac{R_{\alpha\nu\beta\mu;\gamma\delta}R^{\alpha\beta\gamma\delta}}{1050} - \frac{R_{\alpha\nu}R_{\beta\gamma\delta\mu}R^{\alpha\beta\gamma\delta}}{1620} - \\
& \frac{R_{\alpha\mu}R_{\beta\gamma\delta\nu}R^{\alpha\beta\gamma\delta}}{1620} - \frac{R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\gamma}R^{\alpha\beta\delta\epsilon}}{2835} + \frac{8R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\gamma}R^{\alpha\beta\delta\epsilon}}{14175} - \\
& \frac{R_{\alpha\beta\gamma\delta;\mu\nu}R^{\alpha\gamma\beta\delta}}{1260} + \frac{R_{\alpha\beta\gamma\mu;\delta\nu}R^{\alpha\gamma\beta\delta}}{630} + \frac{R_{\alpha\beta\gamma\nu;\delta\mu}R^{\alpha\gamma\beta\delta}}{630} + \frac{R_{\alpha\mu\beta\gamma;\delta\nu}R^{\alpha\gamma\beta\delta}}{1575} + \\
& \frac{R_{\alpha\mu\beta\nu;\gamma\delta}R^{\alpha\gamma\beta\delta}}{3150} + \frac{R_{\alpha\nu\beta\gamma;\delta\mu}R^{\alpha\gamma\beta\delta}}{1575} + \frac{R_{\alpha\nu\beta\mu;\gamma\delta}R^{\alpha\gamma\beta\delta}}{3150} + \\
& \frac{R_{\alpha\beta}R_{\gamma\mu\delta\nu}R^{\alpha\gamma\beta\delta}}{8100} - \frac{R_{\alpha\beta\gamma\delta;\mu\nu}R^{\alpha\delta\beta\gamma}}{700} + \frac{R_{\alpha\beta\gamma\mu;\delta\nu}R^{\alpha\delta\beta\gamma}}{1050} + \\
& \frac{R_{\alpha\beta\gamma\nu;\delta\mu}R^{\alpha\delta\beta\gamma}}{1050} + \frac{R_{\alpha\mu\beta\gamma;\delta\nu}R^{\alpha\delta\beta\gamma}}{1575} + \frac{R_{\alpha\mu\beta\nu;\gamma\delta}R^{\alpha\delta\beta\gamma}}{3150} + \frac{R_{\alpha\nu\beta\gamma;\delta\mu}R^{\alpha\delta\beta\gamma}}{1575} + \\
& \frac{R_{\alpha\nu\beta\mu;\gamma\delta}R^{\alpha\delta\beta\gamma}}{3150} + \frac{R_{\alpha\nu}R_{\beta\gamma\delta\mu}R^{\alpha\delta\beta\gamma}}{1620} + \frac{R_{\alpha\mu}R_{\beta\gamma\delta\nu}R^{\alpha\delta\beta\gamma}}{1620} - \\
& \frac{2R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\gamma}R^{\alpha\delta\beta\epsilon}}{14175} + \frac{R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\gamma}R^{\alpha\delta\beta\epsilon}}{2835} - \frac{8R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{14175} -
\end{aligned}$$

$$\begin{aligned}
& \frac{8R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{4725} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\mu\nu}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{4725} + \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\nu\mu}{}^{\alpha}R^{\beta\gamma\delta\epsilon}}{4725} - \\
& \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu\nu}R^{\beta\gamma\delta\epsilon}}{4725} - \frac{R_{\alpha\beta\gamma\mu}R_{\delta\epsilon\nu}{}^{\alpha}R^{\beta\delta\gamma\epsilon}}{2835} + \frac{2R_{\alpha\beta\gamma\mu}R_{\delta\nu\epsilon}{}^{\alpha}R^{\beta\epsilon\gamma\delta}}{4725} - \\
& \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\mu\nu}{}^{\alpha}R^{\beta\epsilon\gamma\delta}}{6300} - \frac{R_{\alpha\beta\gamma\delta}R_{\epsilon\nu\mu}{}^{\alpha}R^{\beta\epsilon\gamma\delta}}{6300} + \frac{4R_{\alpha\beta\gamma\delta}R_{\epsilon}{}^{\alpha}{}_{\mu\nu}R^{\beta\epsilon\gamma\delta}}{4725}
\end{aligned}$$

(279)

## APPENDIX F

$$4\pi^2 \langle T_{\mu\nu} \rangle_{finite}$$

$$\begin{aligned}
4\pi^2 \langle T^{\mu\nu} \rangle_{finite} = & \\
& - \frac{m^4 \sigma^\mu \sigma^\nu}{16\sigma} + \frac{m^2 R \sigma^\mu \sigma^\nu}{48\sigma} - \frac{R^2 \sigma^\mu \sigma^\nu}{576\sigma} - \frac{m^2 R \xi \sigma^\mu \sigma^\nu}{8\sigma} + \\
& \frac{R^2 \xi \sigma^\mu \sigma^\nu}{48\sigma} - \frac{R^2 \xi^2 \sigma^\mu \sigma^\nu}{16\sigma} - \frac{\sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{;\alpha\beta}}{160\sigma^2} + \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{;\alpha\beta}}{24\sigma^2} - \\
& \frac{\sigma^\mu \sigma^\nu R_{;\alpha}{}^\alpha}{240\sigma} + \frac{\xi \sigma^\mu \sigma^\nu R_{;\alpha}{}^\alpha}{48\sigma} + \frac{\sigma^\alpha \sigma^\nu R_{;\alpha}{}^\mu}{480\sigma} - \frac{\xi \sigma^\alpha \sigma^\nu R_{;\alpha}{}^\mu}{48\sigma} + \\
& \frac{\sigma^\alpha \sigma^\mu R_{;\alpha}{}^\nu}{480\sigma} - \frac{\xi \sigma^\alpha \sigma^\mu R_{;\alpha}{}^\nu}{48\sigma} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \sigma^\mu \sigma^\nu R_{\alpha\beta;\gamma\delta}}{80\sigma^3} - \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{\alpha\beta;\gamma}{}^\gamma}{480\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha\beta;\gamma}{}^\mu}{240\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha\beta;\gamma}{}^\nu}{240\sigma^2} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha\beta}{}^{i\mu}{}_\gamma}{160\sigma^2} + \frac{\sigma^\alpha \sigma^\beta R_{\alpha\beta}{}^{i\mu\nu}}{240\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\beta}{}^{i\mu\nu}}{48\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha\beta}{}^{i\nu}{}_\gamma}{160\sigma^2} + \frac{\sigma^\alpha \sigma^\beta R_{\alpha\beta}{}^{i\nu\mu}}{240\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\beta}{}^{i\nu\mu}}{48\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha}{}^\mu{}_{;\beta\gamma}}{80\sigma^2} + \frac{\sigma^\alpha \sigma^\nu R_{\alpha}{}^\mu{}_{;\beta}{}^\beta}{240\sigma} + \frac{\sigma^\alpha \sigma^\beta R_{\alpha}{}^\mu{}_{;\beta}{}^\nu}{120\sigma} - \\
& \frac{\sigma^\alpha \sigma^\beta R_{\alpha}{}^\mu{}_{;\nu}{}^\beta}{80\sigma} + \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha}{}^\nu{}_{;\beta\gamma}}{80\sigma^2} + \frac{\sigma^\alpha \sigma^\mu R_{\alpha}{}^\nu{}_{;\beta}{}^\beta}{240\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta R_{\alpha}{}^\nu{}_{;\beta}{}^\mu}{120\sigma} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha}{}^\nu{}_{;\mu}{}^\beta}{80\sigma} - \frac{\sigma^\alpha \sigma^\beta R^{\mu\nu}{}_{;\alpha\beta}}{80\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha}{}^\mu{}_{\beta}{}^\nu{}_{;\gamma\delta}}{80\sigma^2} - \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha}{}^\mu{}_{\beta}{}^\nu{}_{;\gamma\delta}}{12\sigma^2} - \frac{5m^2 g^{\mu\nu}}{32} + \\
& \frac{m^4 g^{\mu\nu}}{8} - \frac{m^2 R g^{\mu\nu}}{48} + \frac{R^2 g^{\mu\nu}}{576} - \frac{R^2 \xi g^{\mu\nu}}{48} + \\
& \frac{R^2 \xi^2 g^{\mu\nu}}{16} + \frac{\sigma^\alpha \sigma^\beta R_{;\alpha\beta} g^{\mu\nu}}{96\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_{;\alpha\beta} g^{\mu\nu}}{24\sigma} + \\
& \frac{\xi^2 \sigma^\alpha \sigma^\beta R_{;\alpha\beta} g^{\mu\nu}}{8\sigma} + \frac{R_{;\alpha}{}^\alpha g^{\mu\nu}}{240} - \frac{\xi R_{;\alpha}{}^\alpha g^{\mu\nu}}{48} - \\
& \frac{\sigma_\alpha \sigma^\beta R_{;\beta}{}^\alpha g^{\mu\nu}}{160\sigma} + \frac{\xi \sigma_\alpha \sigma^\beta R_{;\beta}{}^\alpha g^{\mu\nu}}{16\sigma} - \frac{\xi^2 \sigma_\alpha \sigma^\beta R_{;\beta}{}^\alpha g^{\mu\nu}}{8\sigma} + \\
& \frac{11 \xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\beta;\gamma\delta} g^{\mu\nu}}{120\sigma^2} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha\beta;\gamma}{}^\gamma g^{\mu\nu}}{160\sigma} + \frac{7 \xi \sigma^\alpha \sigma^\beta R_{\alpha\beta;\gamma}{}^\gamma g^{\mu\nu}}{240\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\beta;\delta\gamma} g^{\mu\nu}}{240\sigma^2} - \frac{7 \xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\beta;\delta\gamma} g^{\mu\nu}}{240\sigma^2} + \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\beta}{}^{i\gamma}{}_\gamma g^{\mu\nu}}{80\sigma} - \\
& \frac{\sigma^\alpha \sigma^\beta R_{\alpha\gamma;\beta}{}^\gamma g^{\mu\nu}}{120\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\gamma;\beta}{}^\gamma g^{\mu\nu}}{60\sigma} + \frac{\sigma^\alpha \sigma^\beta R_{\alpha\gamma}{}^{i\gamma}{}_\beta g^{\mu\nu}}{48\sigma} - \\
& \frac{7 \xi \sigma^\alpha \sigma^\beta R_{\alpha\gamma}{}^{i\gamma}{}_\beta g^{\mu\nu}}{120\sigma} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\delta;\beta\gamma} g^{\mu\nu}}{160\sigma^2} - \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha}{}^\gamma{}_{;\beta\gamma} g^{\mu\nu}}{60\sigma}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sigma^\alpha \sigma^\beta R_{\alpha\gamma}{}^\gamma{}_{;\gamma\beta} g^{\mu\nu}}{120\sigma} + \frac{7\xi \sigma^\alpha \sigma^\beta R_{\alpha\gamma}{}^\gamma{}_{;\gamma\beta} g^{\mu\nu}}{120\sigma} + \frac{\sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\beta\gamma;\delta}{}^\alpha g^{\mu\nu}}{240\sigma^2} - \\
& \frac{\xi \sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\beta\gamma;\delta}{}^\alpha g^{\mu\nu}}{120\sigma^2} - \frac{\sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\beta\gamma}{}^{;\alpha}{}_\delta g^{\mu\nu}}{96\sigma^2} + \frac{7\xi \sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\beta\gamma}{}^{;\alpha}{}_\delta g^{\mu\nu}}{240\sigma^2} - \\
& \frac{\sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\beta\alpha}{}^{;\gamma}{}_\delta g^{\mu\nu}}{160\sigma^2} - \frac{\sigma_\alpha \sigma^\beta R_{\beta\alpha}{}^{;\gamma}{}_\delta g^{\mu\nu}}{480\sigma} + \frac{m^2 \sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{\alpha\beta}}{48\sigma^2} - \\
& \frac{R \sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{\alpha\beta}}{288\sigma^2} + \frac{R \xi \sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{\alpha\beta}}{48\sigma^2} + \frac{m^2 \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\beta}}{48\sigma} - \\
& \frac{R \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\beta}}{288\sigma} - \frac{m^2 \xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\beta}}{4\sigma} + \frac{R \xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\beta}}{16\sigma} - \\
& \frac{R \xi^2 \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\beta}}{4\sigma} - \frac{m^2 \sigma^\alpha \sigma^\nu R_{\alpha}{}^\mu}{24\sigma} + \frac{R \sigma^\alpha \sigma^\nu R_{\alpha}{}^\mu}{144\sigma} - \\
& \frac{R \xi \sigma^\alpha \sigma^\nu R_{\alpha}{}^\mu}{24\sigma} - \frac{m^2 \sigma^\alpha \sigma^\mu R_{\alpha}{}^\nu}{24\sigma} + \frac{R \sigma^\alpha \sigma^\mu R_{\alpha}{}^\nu}{144\sigma} - \\
& \frac{R \xi \sigma^\alpha \sigma^\mu R_{\alpha}{}^\nu}{24\sigma} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \sigma^\mu \sigma^\nu R_{\alpha\delta} R_{\beta\gamma}}{288\sigma^3} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\delta} R_{\beta\gamma}}{576\sigma^2} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{\alpha}{}^\gamma R_{\beta\gamma}}{360\sigma^2} + \frac{7\sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}{}^\gamma R_{\beta\gamma}}{288\sigma} - \frac{77\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}{}^\gamma R_{\beta\gamma}}{720\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha}{}^\mu R_{\beta\gamma}}{144\sigma^2} + \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha}{}^\nu R_{\beta\gamma}}{144\sigma^2} + \frac{m^2 \sigma_\alpha \sigma^\beta g^{\mu\nu} R_{\beta}{}^\alpha}{48\sigma} - \\
& \frac{R \sigma_\alpha \sigma^\beta g^{\mu\nu} R_{\beta}{}^\alpha}{288\sigma} + \frac{R \xi \sigma_\alpha \sigma^\beta g^{\mu\nu} R_{\beta}{}^\alpha}{48\sigma} - \frac{\sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\gamma} R_{\beta}{}^\gamma}{1440\sigma} + \\
& \frac{17\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\gamma} R_{\beta}{}^\gamma}{720\sigma} - \frac{\sigma^\alpha \sigma^\nu R_{\alpha}{}^\beta R_{\beta}{}^\mu}{360\sigma} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha}{}^\nu R_{\beta}{}^\mu}{144\sigma} + \\
& \frac{\sigma^\alpha \sigma^\mu R_{\alpha}{}^\beta R_{\beta}{}^\nu}{360\sigma} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha}{}^\mu R_{\beta}{}^\nu}{144\sigma} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta}}{288\sigma^2} + \\
& \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta}}{24\sigma^2} - \frac{\sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta}{}^\alpha R_{\gamma\delta}}{288\sigma^2} + \frac{\xi \sigma_\alpha \sigma^\beta g^{\mu\nu} R_{\beta}{}^\gamma R_{\gamma}{}^\alpha}{180\sigma} + \\
& \frac{\sigma^\mu \sigma^\nu R_{\alpha\beta} R^{\alpha\beta}}{1440\sigma} - \frac{g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta}}{1440} + \frac{\sigma_\alpha \sigma^\beta g^{\mu\nu} R_{\beta\gamma} R^{\alpha\gamma}}{360\sigma} - \\
& \frac{\xi \sigma_\alpha \sigma^\beta g^{\mu\nu} R_{\beta\gamma} R^{\alpha\gamma}}{180\sigma} - \frac{\sigma^\alpha \sigma^\nu R_{\alpha\beta} R^{\beta\mu}}{360\sigma} - \frac{\sigma^\alpha \sigma^\mu R_{\alpha\beta} R^{\beta\nu}}{360\sigma} + \\
& \frac{m^2 R^{\mu\nu}}{24} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha\beta} R^{\mu\nu}}{144\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\beta} R^{\mu\nu}}{24\sigma} - \\
& \frac{\sigma^\alpha \sigma^\nu R^{\beta\gamma} R_{\alpha\beta\gamma}{}^\mu}{720\sigma} - \frac{\sigma^\alpha \sigma^\mu R^{\beta\gamma} R_{\alpha\beta\gamma}{}^\nu}{720\sigma} - \frac{\sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R^{\gamma\delta} R_{\alpha\gamma\beta\delta}}{720\sigma^2} - \\
& \frac{\sigma^\alpha \sigma^\beta g^{\mu\nu} R^{\gamma\delta} R_{\alpha\gamma\beta\delta}}{1440\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R^{\gamma\delta} R_{\alpha\gamma\beta\delta}}{72\sigma} - \frac{\sigma^\alpha \sigma^\nu R^{\beta\gamma} R_{\alpha\gamma\beta}{}^\mu}{720\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta R^{\gamma\nu} R_{\alpha\gamma\beta}{}^\mu}{144\sigma} - \frac{\sigma^\alpha \sigma^\mu R^{\beta\gamma} R_{\alpha\gamma\beta}{}^\nu}{720\sigma} + \frac{\sigma^\alpha \sigma^\beta R^{\gamma\mu} R_{\alpha\gamma\beta}{}^\nu}{144\sigma} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R^{\gamma\delta} R_{\alpha\delta\beta\gamma}}{360\sigma} - \frac{7\sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\gamma}{}^\delta R_{\alpha\delta\beta\gamma}}{1440\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\gamma}{}^\delta R_{\alpha\delta\beta\gamma}}{60\sigma} +
\end{aligned}$$

$$\begin{aligned}
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_\gamma^\delta R_{\alpha\delta\beta}^\mu}{720\sigma^2} + \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_\gamma^\delta R_{\alpha\delta\beta}^\nu}{720\sigma^2} - \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_\gamma^\epsilon R_{\alpha\epsilon\beta\delta}}{360\sigma^2} - \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\gamma\delta} R_{\alpha}^\delta R_{\beta}^\mu}{120\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\gamma\delta} R_{\alpha}^\delta R_{\beta}^\nu}{120\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\gamma\epsilon} R_{\alpha}^\epsilon R_{\beta\delta}}{1440\sigma^2} + \\
& \frac{7\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\gamma\epsilon} R_{\alpha}^\epsilon R_{\beta\delta}}{360\sigma^2} - \frac{m^2 \sigma^\alpha \sigma^\beta R_{\alpha}^\mu R_{\beta}^\nu}{24\sigma} + \frac{R \sigma^\alpha \sigma^\beta R_{\alpha}^\mu R_{\beta}^\nu}{144\sigma} + \\
& \frac{m^2 \xi \sigma^\alpha \sigma^\beta R_{\alpha}^\mu R_{\beta}^\nu}{4\sigma} - \frac{R \xi \sigma^\alpha \sigma^\beta R_{\alpha}^\mu R_{\beta}^\nu}{12\sigma} + \frac{R \xi^2 \sigma^\alpha \sigma^\beta R_{\alpha}^\mu R_{\beta}^\nu}{4\sigma} + \\
& \frac{\sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\mu R_{\gamma}^\nu}{1440\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\mu R_{\gamma}^\nu}{480\sigma} - \frac{7\sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\mu R_{\gamma}^\nu}{480\sigma} + \\
& \frac{7\xi \sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\mu R_{\gamma}^\nu}{160\sigma} + \frac{\sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\nu R_{\gamma}^\mu}{1440\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\nu R_{\gamma}^\mu}{480\sigma} - \\
& \frac{7\sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\nu R_{\gamma}^\mu}{480\sigma} + \frac{7\xi \sigma^\alpha \sigma^\beta R_\beta^\gamma R_{\alpha}^\nu R_{\gamma}^\mu}{160\sigma} + \frac{\xi \sigma_\alpha \sigma^\beta g^{\mu\nu} R^{\gamma\delta} R_{\beta\gamma\delta}^\alpha}{360\sigma} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\epsilon}^{\gamma\delta} R_{\beta\gamma\delta}^\epsilon}{360\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\gamma\delta} R_{\beta\gamma\delta}^\epsilon}{360\sigma} - \frac{\sigma^\alpha \sigma^\nu R_{\alpha}^{\delta\beta\gamma} R_{\beta\gamma\delta}^\mu}{720\sigma} - \\
& \frac{\sigma^\alpha \sigma^\mu R_{\alpha}^{\delta\beta\gamma} R_{\beta\gamma\delta}^\nu}{720\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\delta\epsilon}^{\gamma} R_{\beta\gamma}^{\delta\epsilon}}{360\sigma} + \frac{\sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\gamma\delta\epsilon} R_{\beta\gamma}^{\delta\epsilon}}{360\sigma} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\gamma\delta\epsilon} R_{\beta\gamma}^{\delta\epsilon}}{120\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\delta}^{\gamma\nu} R_{\beta\gamma}^{\delta\mu}}{360\sigma} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\nu} R_{\beta\gamma}^{\delta\mu}}{360\sigma} + \\
& \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\nu} R_{\beta\gamma}^{\delta\mu}}{360\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\delta}^{\gamma\mu} R_{\beta\gamma}^{\delta\nu}}{360\sigma} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\mu} R_{\beta\gamma}^{\delta\nu}}{360\sigma} + \\
& \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\mu} R_{\beta\gamma}^{\delta\nu}}{360\sigma} + \frac{\sigma_\alpha \sigma^\beta g^{\mu\nu} R^{\gamma\delta} R_{\beta\delta\gamma}^\alpha}{360\sigma} - \frac{\xi \sigma_\alpha \sigma^\beta g^{\mu\nu} R^{\gamma\delta} R_{\beta\delta\gamma}^\alpha}{360\sigma} - \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha\epsilon}^{\delta\mu} R_{\beta\delta\gamma}^\epsilon}{1080\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha\epsilon}^{\delta\nu} R_{\beta\delta\gamma}^\epsilon}{1080\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha}^{\delta\epsilon\mu} R_{\beta\delta\gamma}^\epsilon}{360\sigma^2} - \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha}^{\delta\epsilon\nu} R_{\beta\delta\gamma}^\epsilon}{360\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\mu \sigma^\nu R_{\alpha}^{\epsilon\gamma\delta} R_{\beta\epsilon\gamma\delta}}{720\sigma^2} - \frac{\sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\epsilon\gamma\delta} R_{\beta\epsilon\gamma\delta}}{720\sigma} + \\
& \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\epsilon\gamma\delta} R_{\beta\epsilon\gamma\delta}}{360\sigma} - \frac{\sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\delta}^\epsilon R_{\beta\epsilon\gamma}^\alpha}{720\sigma^2} + \frac{\xi \sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\delta}^\epsilon R_{\beta\epsilon\gamma}^\alpha}{720\sigma} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\eta\delta}^\epsilon R_{\beta\epsilon\gamma}^\eta}{540\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha}^{\epsilon\delta\eta} R_{\beta\epsilon\gamma}^\eta}{360\sigma^2} + \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha}^{\epsilon\delta\eta} R_{\beta\epsilon\gamma}^\eta}{180\sigma^2} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\delta}^{\gamma\nu} R_{\beta}^{\delta\gamma\mu}}{360\sigma} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\nu} R_{\beta}^{\delta\gamma\mu}}{360\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\nu} R_{\beta}^{\delta\gamma\mu}}{360\sigma} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\delta}^{\gamma\mu} R_{\beta}^{\delta\gamma\nu}}{360\sigma} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\mu} R_{\beta}^{\delta\gamma\nu}}{360\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha}^{\gamma\delta\mu} R_{\beta}^{\delta\gamma\nu}}{360\sigma} + \\
& \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\epsilon}^{\gamma\delta} R_{\beta}^{\epsilon\gamma\delta}}{360\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\gamma\delta} R_{\beta}^{\epsilon\gamma\delta}}{360\sigma} + \frac{13\sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\delta\epsilon} R_{\beta}^{\epsilon\gamma\alpha}}{1440\sigma^2} - \\
& \frac{7\xi \sigma_\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\delta\epsilon} R_{\beta}^{\epsilon\gamma\alpha}}{360\sigma^2} - \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\epsilon\delta}^{\gamma} R_{\beta}^{\epsilon\gamma\delta}}{360\sigma} - \frac{\sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\gamma\delta\epsilon} R_{\beta}^{\epsilon\gamma\delta}}{360\sigma} + \\
& \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha}^{\gamma\delta\epsilon} R_{\beta}^{\epsilon\gamma\delta}}{120\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\alpha\epsilon\gamma\delta} R_{\beta}^{\epsilon\gamma\delta}}{360\sigma} + \frac{\xi \sigma_\alpha \sigma^\beta g^{\mu\nu} R_{\beta}^{\epsilon\gamma\delta} R_{\gamma\delta\epsilon}^\alpha}{360\sigma} -
\end{aligned}$$

$$\begin{aligned}
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha\epsilon\beta}{}^\delta R_{\gamma\delta}{}^{\epsilon\mu}}{1080\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha\epsilon\beta}{}^\delta R_{\gamma\delta}{}^{\epsilon\nu}}{1080\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \sigma^\mu \sigma^\nu R_{\alpha\eta\beta}{}^\epsilon R_{\gamma\epsilon\delta}{}^\eta}{360\sigma^3} + \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\eta\beta}{}^\epsilon R_{\gamma\epsilon\delta}{}^\eta}{720\sigma^2} - \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\eta\beta}{}^\epsilon R_{\gamma\epsilon\delta}{}^\eta}{540\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta\eta}{}^{\alpha\epsilon} R_{\gamma\epsilon\delta}{}^\eta}{1080\sigma^2} + \\
& \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta\eta}{}^{\alpha\epsilon} R_{\gamma\epsilon\delta}{}^\eta}{540\sigma^2} + \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta}{}^{\epsilon\alpha} R_{\gamma\epsilon\delta}{}^\eta}{180\sigma^2} + \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\epsilon\beta}{}^\eta R_{\gamma}{}^{\epsilon\delta\eta}}{90\sigma^2} - \\
& \frac{7\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\epsilon\beta\eta} R_{\gamma}{}^{\epsilon\delta\eta}}{1440\sigma^2} + \frac{11\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\epsilon\beta\eta} R_{\gamma}{}^{\epsilon\delta\eta}}{360\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\nu R_{\alpha\epsilon\beta}{}^\delta R_{\gamma}{}^{\epsilon\delta\mu}}{1080\sigma^2} + \\
& \frac{7\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\epsilon\beta}{}^\nu R_{\gamma}{}^{\epsilon\delta\mu}}{720\sigma^2} - \frac{11\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\epsilon\beta}{}^\nu R_{\gamma}{}^{\epsilon\delta\mu}}{360\sigma^2} - \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\mu R_{\alpha\epsilon\beta}{}^\delta R_{\gamma}{}^{\epsilon\delta\nu}}{1080\sigma^2} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\epsilon\beta}{}^\mu R_{\gamma}{}^{\epsilon\delta\nu}}{90\sigma^2} - \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\alpha\eta\beta}{}^\epsilon R_{\gamma}{}^{\eta\delta\epsilon}}{540\sigma^2} + \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\beta} R_{\gamma}{}^{\mu\delta\nu}}{144\sigma^2} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta R_{\alpha\beta} R_{\gamma}{}^{\mu\delta\nu}}{24\sigma^2} - \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\delta\beta}{}^\gamma R_{\gamma}{}^{\mu\delta\nu}}{360\sigma} - \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\delta\beta}{}^\gamma R_{\gamma}{}^{\nu\delta\mu}}{360\sigma} - \\
& \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta\eta\gamma}{}^\epsilon R_{\delta\epsilon}{}^{\alpha\eta}}{1080\sigma^2} + \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta\eta\gamma}{}^\epsilon R_{\delta\epsilon}{}^{\alpha\eta}}{540\sigma^2} + \frac{\sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta\eta\gamma}{}^\epsilon R_{\delta}{}^{\eta\epsilon\alpha}}{1080\sigma^2} - \\
& \frac{\xi \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta g^{\mu\nu} R_{\beta\eta\gamma}{}^\epsilon R_{\delta}{}^{\eta\epsilon\alpha}}{540\sigma^2} - \frac{\sigma^\alpha \sigma^\beta R_{\alpha\gamma\beta}{}^\delta R_{\delta}{}^{\mu\gamma\nu}}{360\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\gamma\beta}{}^\delta R_{\delta}{}^{\mu\gamma\nu}}{360\sigma} - \\
& \frac{\sigma^\alpha \sigma^\beta R_{\alpha\gamma\beta}{}^\delta R_{\delta}{}^{\nu\gamma\mu}}{360\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta R_{\alpha\gamma\beta}{}^\delta R_{\delta}{}^{\nu\gamma\mu}}{360\sigma} - \frac{\sigma^\mu \sigma^\nu R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}}{1440\sigma} + \\
& \frac{g^{\mu\nu} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}}{1440} - \frac{\sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\beta\epsilon\gamma\delta} R^{\alpha\epsilon\gamma\delta}}{720\sigma} + \frac{\xi \sigma^\alpha \sigma^\beta g^{\mu\nu} R_{\beta\epsilon\gamma\delta} R^{\alpha\epsilon\gamma\delta}}{360\sigma} - \\
& \frac{\sigma^\alpha \sigma^\nu R_{\alpha\delta\beta\gamma} R^{\beta\gamma\delta\mu}}{720\sigma} - \frac{\sigma^\alpha \sigma^\mu R_{\alpha\delta\beta\gamma} R^{\beta\gamma\delta\nu}}{720\sigma}
\end{aligned}$$

(280)

## APPENDIX G

$$4\pi^2 \langle T_{\mu\nu} \rangle_{DS, \text{purely geometric}}$$

$$4\pi^2 \langle T^{\mu\nu} \rangle_{DS,ren,purely\ geometric} = \frac{1}{m^2} \times \left[ \begin{array}{l} \frac{R^{;\mu} R^{;\nu}}{576} - \frac{\xi R^{;\mu} R^{;\nu}}{36} + \frac{7\xi^2 R^{;\mu} R^{;\nu}}{48} - \frac{\xi^3 R^{;\mu} R^{;\nu}}{4} - \\ \frac{19R_{\alpha\beta}^{;\mu} R^{\alpha\beta;\nu}}{26880} + \frac{\xi R_{\alpha\beta}^{;\mu} R^{\alpha\beta;\nu}}{360} - \frac{R_{\alpha\beta}^{;\nu} R^{\alpha\mu;\beta}}{13440} + \frac{59R_{;\alpha} R^{\alpha\mu;\nu}}{80640} - \\ \frac{\xi R_{;\alpha} R^{\alpha\mu;\nu}}{384} - \frac{R_{\alpha\beta}^{;\mu} R^{\alpha\nu;\beta}}{13440} + \frac{23R_{\alpha}^{\mu}{}_{;\beta} R^{\alpha\nu;\beta}}{80640} + \frac{59R_{;\alpha} R^{\alpha\nu;\mu}}{80640} - \\ \frac{\xi R_{;\alpha} R^{\alpha\nu;\mu}}{384} + \frac{19R_{\alpha}^{\nu}{}_{;\beta} R^{\beta\mu;\alpha}}{40320} - \frac{13R_{;\alpha} R^{\mu\nu;\alpha}}{13440} + \frac{\xi R_{;\alpha} R^{\mu\nu;\alpha}}{192} - \\ \frac{29R_{;\alpha} R_{\beta}^{\mu\alpha\nu;\beta}}{80640} + \frac{\xi R_{;\alpha} R_{\beta}^{\mu\alpha\nu;\beta}}{640} - \frac{29R_{;\alpha} R_{\beta}^{\nu\alpha\mu;\beta}}{80640} + \frac{\xi R_{;\alpha} R_{\beta}^{\nu\alpha\mu;\beta}}{640} - \\ \frac{R_{\alpha\beta}^{;\nu} R_{\gamma}^{\alpha\beta\mu;\gamma}}{2880} + \frac{R_{\alpha}^{\nu}{}_{;\beta} R_{\gamma}^{\alpha\beta\mu;\gamma}}{13440} - \frac{R_{\alpha\beta}^{;\mu} R_{\gamma}^{\alpha\beta\nu;\gamma}}{2880} + \frac{R_{\alpha}^{\mu}{}_{;\beta} R_{\gamma}^{\alpha\beta\nu;\gamma}}{13440} + \\ \frac{R_{\alpha}^{\nu}{}_{;\beta} R_{\gamma}^{\beta\alpha\mu;\gamma}}{5376} + \frac{R_{\alpha}^{\mu}{}_{;\beta} R_{\gamma}^{\beta\alpha\nu;\gamma}}{5376} + \frac{R_{\alpha}^{\nu}{}_{;\beta} R_{\gamma}^{\mu\alpha\beta;\gamma}}{8960} + \frac{R_{\alpha}^{\mu}{}_{;\beta} R_{\gamma}^{\nu\alpha\beta;\gamma}}{8960} + \\ \frac{R_{\alpha\beta\gamma}^{\mu;\nu} R_{\delta}^{\alpha\beta\nu;\delta}}{26880} - \frac{R_{\alpha\beta\gamma}^{\mu;\alpha} R_{\delta}^{\beta\gamma\nu;\delta}}{3840} - \frac{R_{\alpha\beta\gamma}^{\mu;\alpha} R_{\delta}^{\gamma\beta\nu;\delta}}{4480} - \frac{R_{\alpha\beta\gamma}^{\nu;\gamma} R_{\delta}^{\mu\alpha\beta;\delta}}{26880} + \\ \frac{R_{\alpha\beta\gamma}^{\mu;\alpha} R_{\delta}^{\nu\beta\gamma;\delta}}{26880} + \frac{19R_{\alpha\beta\gamma\delta}^{\mu} R^{\alpha\beta\gamma\delta;\nu}}{60480} - \frac{\xi R_{\alpha\beta\gamma\delta}^{\mu} R^{\alpha\beta\gamma\delta;\nu}}{360} + \frac{R_{\alpha\beta\gamma\delta}^{\nu} R^{\alpha\beta\gamma\mu;\delta}}{6048} + \\ \frac{R_{\alpha\beta\gamma\delta}^{\mu} R^{\alpha\beta\gamma\nu;\delta}}{6048} - \frac{R_{\alpha\beta\gamma}^{\mu}{}_{;\delta} R^{\alpha\beta\gamma\nu;\delta}}{3360} + \frac{19R_{\alpha\beta\gamma\delta}^{\nu} R^{\alpha\gamma\beta\delta;\mu}}{60480} + \frac{R_{\alpha\beta\gamma}^{\nu}{}_{;\delta} R^{\alpha\gamma\beta\delta;\mu}}{6048} + \\ \frac{R_{\alpha\beta\gamma}^{\mu}{}_{;\delta} R^{\alpha\gamma\beta\delta;\nu}}{6048} - \frac{R_{\alpha\beta\gamma}^{\nu}{}_{;\delta} R^{\alpha\gamma\beta\mu;\delta}}{3360} + \frac{R_{\alpha\beta;\gamma} R^{\alpha\gamma\beta\mu;\nu}}{16128} + \frac{R_{\alpha\beta;\gamma} R^{\alpha\gamma\beta\nu;\mu}}{16128} - \\ \frac{R_{\alpha\beta\gamma}^{\nu}{}_{;\delta} R^{\alpha\delta\gamma\mu;\beta}}{3360} - \frac{11R_{\alpha\beta;\gamma} R^{\alpha\mu\beta\nu;\gamma}}{40320} - \frac{R_{\alpha\beta\gamma}^{\nu}{}_{;\delta} R^{\alpha\mu\gamma\delta;\beta}}{3360} - \frac{13R_{\alpha\beta;\gamma} R^{\alpha\mu\gamma\nu;\beta}}{80640} - \\ \frac{13R_{\alpha\beta;\gamma} R^{\alpha\nu\gamma\mu;\beta}}{80640} + \frac{R_{\alpha\beta\gamma\delta}^{\alpha} R^{\beta\gamma\delta\mu;\nu}}{6912} + \frac{R_{\alpha\beta\gamma\delta}^{\alpha} R^{\beta\gamma\delta\nu;\mu}}{6912} - \frac{R_{\alpha\beta\gamma\delta}^{\alpha} R^{\beta\mu\gamma\delta;\nu}}{6912} + \\ \frac{R_{\alpha\beta\gamma\delta}^{\alpha} R^{\beta\mu\gamma\nu;\delta}}{3840} - \frac{R_{\alpha\beta\gamma\delta}^{\alpha} R^{\beta\nu\gamma\delta;\mu}}{6912} + \frac{R_{\alpha\beta\gamma\delta}^{\alpha} R^{\beta\nu\gamma\mu;\delta}}{3840} + \frac{7RR^{\mu\nu}}{2880} - \\ \frac{41R\xi R^{;\mu\nu}}{1440} + \frac{R\xi^2 R^{;\mu\nu}}{8} - \frac{R\xi^3 R^{;\mu\nu}}{4} - \frac{RR_{\alpha}^{\mu;\alpha\nu}}{960} + \\ \frac{R\xi R_{\alpha}^{\mu;\alpha\nu}}{160} - \frac{RR_{\alpha}^{\nu;\alpha\mu}}{960} + \frac{R\xi R_{\alpha}^{\nu;\alpha\mu}}{160} - \frac{RR^{\mu\nu}{}_{;\alpha}{}^{\alpha}}{1440} + \\ \frac{R\xi R^{\mu\nu}{}_{;\alpha}{}^{\alpha}}{240} + \frac{29R_{;\alpha}{}^{\alpha\mu\nu}}{20160} - \frac{\xi R_{;\alpha}{}^{\alpha\mu\nu}}{120} + \frac{\xi^2 R_{;\alpha}{}^{\alpha\mu\nu}}{48} + \\ \frac{29R_{;\alpha}{}^{\alpha\nu\mu}}{20160} - \frac{\xi R_{;\alpha}{}^{\alpha\nu\mu}}{120} + \frac{\xi^2 R_{;\alpha}{}^{\alpha\nu\mu}}{48} - \frac{R_{\alpha\beta}{}^{\alpha\beta\mu\nu}}{1680} - \\ \frac{R_{\alpha\beta}{}^{\alpha\beta\nu\mu}}{1680} + \frac{R_{\alpha}^{\mu}{}_{;\beta}{}^{\beta\alpha\nu}}{10080} - \frac{R_{\alpha}^{\mu;\alpha}{}_{\beta}{}^{\beta\nu}}{1680} + \frac{R_{\alpha}^{\nu}{}_{;\beta}{}^{\beta\alpha\mu}}{10080} - \end{array} \right]$$

$$\begin{aligned}
& \frac{R_{\alpha}{}^{\nu;\alpha}{}_{\beta}{}^{\beta\mu}}{1680} - \frac{R^{\mu\nu}{}_{;\alpha}{}^{\alpha}{}_{\beta}{}^{\beta}}{3360} + \frac{\text{Scalar } R^3 g^{\mu\nu}}{3456} - \frac{5 \text{Scalar } R^3 \xi g^{\mu\nu}}{864} + \\
& \frac{11 \text{Scalar } R^3 \xi^2 g^{\mu\nu}}{288} - \frac{\text{Scalar } R^3 \xi^3 g^{\mu\nu}}{12} - \frac{31 R_{;\alpha} R^{\alpha} g^{\mu\nu}}{80640} + \frac{27 \xi R_{;\alpha} R^{\alpha} g^{\mu\nu}}{2240} - \\
& \frac{17 \xi^2 R_{;\alpha} R^{\alpha} g^{\mu\nu}}{192} + \frac{3 \xi^3 R_{;\alpha} R^{\alpha} g^{\mu\nu}}{16} - \frac{R_{;\alpha} R_{\beta}{}^{\alpha;\beta} g^{\mu\nu}}{2688} + \frac{\xi R_{;\alpha} R_{\beta}{}^{\alpha;\beta} g^{\mu\nu}}{560} - \\
& \frac{\xi^2 R_{;\alpha} R_{\beta}{}^{\alpha;\beta} g^{\mu\nu}}{480} + \frac{R_{\alpha\beta;\gamma} R^{\alpha\beta;\gamma} g^{\mu\nu}}{2880} - \frac{\xi R_{\alpha\beta;\gamma} R^{\alpha\beta;\gamma} g^{\mu\nu}}{480} + \frac{R_{\alpha\beta;\gamma} R_{\delta}{}^{\alpha\beta\gamma;\delta} g^{\mu\nu}}{8960} - \\
& \frac{\xi R_{\alpha\beta;\gamma} R_{\delta}{}^{\alpha\beta\gamma;\delta} g^{\mu\nu}}{4480} + \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\epsilon}{}^{\beta\gamma\delta;\epsilon} g^{\mu\nu}}{241920} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\epsilon}{}^{\beta\gamma\delta;\epsilon} g^{\mu\nu}}{120960} - \frac{\xi R_{\alpha\beta\gamma\delta}{}^{\alpha} R_{\epsilon}{}^{\beta\gamma\delta;\epsilon} g^{\mu\nu}}{120960} + \\
& \frac{17 \xi R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon} g^{\mu\nu}}{12096} - \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon} g^{\mu\nu}}{60480} + \frac{\xi R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon} g^{\mu\nu}}{30240} - \\
& \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon} g^{\mu\nu}}{120960} + \frac{\xi R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon} g^{\mu\nu}}{60480} - \frac{R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon} g^{\mu\nu}}{60480} + \\
& \frac{\xi R_{\alpha\beta\gamma\delta;\epsilon} R^{\alpha\beta\gamma\delta;\epsilon} g^{\mu\nu}}{30240} - \frac{R R_{;\alpha} g^{\mu\nu}}{5760} + \frac{7 R \xi R_{;\alpha} g^{\mu\nu}}{1440} - \\
& \frac{5 R \xi^2 R_{;\alpha} g^{\mu\nu}}{96} + \frac{R \xi^3 R_{;\alpha} g^{\mu\nu}}{6} + \frac{R R_{\alpha\beta}{}^{\alpha\beta} g^{\mu\nu}}{960} - \\
& \frac{R \xi R_{\alpha\beta}{}^{\alpha\beta} g^{\mu\nu}}{120} + \frac{R \xi^2 R_{\alpha\beta}{}^{\alpha\beta} g^{\mu\nu}}{80} - \frac{13 R_{;\alpha}{}^{\alpha}{}_{\beta}{}^{\beta} g^{\mu\nu}}{10080} + \\
& \frac{11 \xi R_{;\alpha}{}^{\alpha}{}_{\beta}{}^{\beta} g^{\mu\nu}}{1008} - \frac{7 \xi^2 R_{;\alpha}{}^{\alpha}{}_{\beta}{}^{\beta} g^{\mu\nu}}{240} - \frac{R_{\alpha\beta;\gamma}{}^{\gamma\alpha\beta} g^{\mu\nu}}{10080} + \\
& \frac{\xi R_{\alpha\beta;\gamma}{}^{\gamma\alpha\beta} g^{\mu\nu}}{5040} + \frac{R_{\alpha\beta}{}^{\alpha}{}_{\gamma}{}^{\gamma\beta} g^{\mu\nu}}{1680} - \frac{\xi R_{\alpha\beta}{}^{\alpha}{}_{\gamma}{}^{\gamma\beta} g^{\mu\nu}}{840} + \\
& \frac{R_{\alpha\beta}{}^{\alpha\beta}{}_{\gamma}{}^{\gamma} g^{\mu\nu}}{1680} - \frac{\xi R_{\alpha\beta}{}^{\alpha\beta}{}_{\gamma}{}^{\gamma} g^{\mu\nu}}{840} - \frac{13 R_{\alpha\beta}{}^{\mu\nu} R^{\alpha\beta}}{44800} + \\
& \frac{\xi R_{\alpha\beta}{}^{\mu\nu} R^{\alpha\beta}}{720} - \frac{13 R_{\alpha\beta}{}^{\nu\mu} R^{\alpha\beta}}{44800} + \frac{\xi R_{\alpha\beta}{}^{\nu\mu} R^{\alpha\beta}}{720} - \frac{11 R_{\alpha}{}^{\mu}{}_{;\beta}{}^{\nu} R^{\alpha\beta}}{720} - \\
& \frac{11 R_{\alpha}{}^{\nu}{}_{;\beta}{}^{\mu} R^{\alpha\beta}}{720} - \frac{R^{\mu\nu}{}_{;\alpha\beta} R^{\alpha\beta}}{6300} + \frac{7 R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}^{\gamma\nu} R^{\alpha\beta}}{172800} - \frac{R_{\alpha}{}^{\mu}{}_{\beta}{}^{\nu}{}_{;\gamma}{}^{\gamma} R^{\alpha\beta}}{3600} + \\
& \frac{7 R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}^{\gamma\mu} R^{\alpha\beta}}{172800} + \frac{17 R_{;\alpha\beta} g^{\mu\nu} R^{\alpha\beta}}{12600} - \frac{173 \xi R_{;\alpha\beta} g^{\mu\nu} R^{\alpha\beta}}{25200} + \\
& \frac{\xi^2 R_{;\alpha\beta} g^{\mu\nu} R^{\alpha\beta}}{120} + \frac{29 R_{\alpha\beta;\gamma}{}^{\gamma} g^{\mu\nu} R^{\alpha\beta}}{720} - \frac{103 \xi R_{\alpha\beta;\gamma}{}^{\gamma} g^{\mu\nu} R^{\alpha\beta}}{67200} - \\
& \frac{R g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta}}{2880} + \frac{R \xi g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta}}{360} - \frac{R \xi^2 g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta}}{360} + \\
& \frac{11 R_{\alpha\beta\gamma}{}^{\mu;\beta\nu} R^{\alpha\gamma}}{120960} + \frac{11 R_{\alpha\beta\gamma}{}^{\nu;\beta\mu} R^{\alpha\gamma}}{120960} - \frac{R_{\alpha}{}^{\mu}{}_{\beta}{}^{\nu;\beta}{}_{\gamma} R^{\alpha\gamma}}{12600} - \frac{R_{\alpha}{}^{\nu}{}_{\beta}{}^{\mu;\beta}{}_{\gamma} R^{\alpha\gamma}}{12600} + \\
& \frac{7 R_{\alpha\beta;\gamma}{}^{\beta} g^{\mu\nu} R^{\alpha\gamma}}{57600} - \frac{7 \xi R_{\alpha\beta;\gamma}{}^{\beta} g^{\mu\nu} R^{\alpha\gamma}}{28800} - \frac{263 R_{\alpha\beta}{}^{\beta}{}_{\gamma} g^{\mu\nu} R^{\alpha\gamma}}{201600} +
\end{aligned}$$

$$\begin{aligned}
& \frac{263\xi R_{\alpha\beta}{}^{;\beta}{}_{\gamma}g^{\mu\nu}R^{\alpha\gamma}}{100800} - \frac{29R_{\alpha\beta\gamma\delta}{}^{;\beta\delta}g^{\mu\nu}R^{\alpha\gamma}}{720} + \frac{29\xi R_{\alpha\beta\gamma\delta}{}^{;\beta\delta}g^{\mu\nu}R^{\alpha\gamma}}{201600} - \\
& \frac{R_{;\alpha}{}^{\nu}R^{\alpha\mu}}{1400} + \frac{\xi R_{;\alpha}{}^{\nu}R^{\alpha\mu}}{720} + \frac{3R_{\alpha\beta}{}^{;\beta\nu}R^{\alpha\mu}}{2800} + \frac{R_{\alpha}{}^{\nu}{}_{;\beta}{}^{\beta}R^{\alpha\mu}}{2100} - \\
& \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\gamma}{}^{\beta}R^{\alpha\mu}}{50400} - \frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}^{;\beta\gamma}R^{\alpha\mu}}{50400} - \frac{R_{;\alpha}{}^{\mu}R^{\alpha\nu}}{1400} + \frac{\xi R_{;\alpha}{}^{\mu}R^{\alpha\nu}}{720} + \\
& \frac{3R_{\alpha\beta}{}^{;\beta\mu}R^{\alpha\nu}}{2800} + \frac{R_{\alpha}{}^{\mu}{}_{;\beta}{}^{\beta}R^{\alpha\nu}}{2100} - \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\gamma}{}^{\beta}R^{\alpha\nu}}{50400} - \frac{R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}^{;\beta\gamma}R^{\alpha\nu}}{50400} + \\
& \frac{RR_{\alpha}{}^{\mu}R^{\alpha\nu}}{1080} - \frac{R\xi R_{\alpha}{}^{\mu}R^{\alpha\nu}}{180} + \frac{43g^{\mu\nu}R_{\alpha\beta}R_{\gamma}{}^{\alpha}R^{\beta\gamma}}{604800} - \\
& \frac{43\xi g^{\mu\nu}R_{\alpha\beta}R_{\gamma}{}^{\alpha}R^{\beta\gamma}}{302400} + \frac{17R_{\alpha}{}^{\nu}{}_{;\beta}{}^{\alpha}R^{\beta\mu}}{201600} + \frac{59R_{\alpha}{}^{\nu}{}_{;\alpha}{}^{\beta}R^{\beta\mu}}{201600} + \\
& \frac{17R_{\alpha}{}^{\mu}{}_{;\beta}{}^{\alpha}R^{\beta\nu}}{201600} + \frac{59R_{\alpha}{}^{\mu}{}_{;\alpha}{}^{\beta}R^{\beta\nu}}{201600} - \frac{17R_{\alpha\beta}R^{\alpha\mu}R^{\beta\nu}}{43200} - \\
& \frac{R^2R^{\mu\nu}}{1728} + \frac{R^2\xi R^{\mu\nu}}{96} - \frac{R^2\xi^2R^{\mu\nu}}{16} + \frac{R^2\xi^3R^{\mu\nu}}{8} - \\
& \frac{R_{;\alpha}{}^{\alpha}R^{\mu\nu}}{720} + \frac{11\xi R_{;\alpha}{}^{\alpha}R^{\mu\nu}}{720} - \frac{\xi^2 R_{;\alpha}{}^{\alpha}R^{\mu\nu}}{24} + \\
& \frac{R_{\alpha\beta}R^{\alpha\beta}R^{\mu\nu}}{4320} - \frac{\xi R_{\alpha\beta}R^{\alpha\beta}R^{\mu\nu}}{720} + \frac{23R_{\alpha\beta\gamma\delta}{}^{;\mu\nu}R^{\alpha\beta\gamma\delta}}{151200} - \\
& \frac{\xi R_{\alpha\beta\gamma\delta}{}^{;\mu\nu}R^{\alpha\beta\gamma\delta}}{720} + \frac{23R_{\alpha\beta\gamma\delta}{}^{;\nu\mu}R^{\alpha\beta\gamma\delta}}{151200} - \frac{\xi R_{\alpha\beta\gamma\delta}{}^{;\nu\mu}R^{\alpha\beta\gamma\delta}}{720} - \frac{13R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta}{}^{\nu}R^{\alpha\beta\gamma\delta}}{302400} - \\
& \frac{13R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}{}^{\mu}R^{\alpha\beta\gamma\delta}}{302400} - \frac{R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}_{;\delta}{}^{\nu}R^{\alpha\beta\gamma\delta}}{17280} - \frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}_{;\delta}{}^{\mu}R^{\alpha\beta\gamma\delta}}{17280} + \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{;\epsilon}g^{\mu\nu}R^{\alpha\beta\gamma\delta}}{151200} + \frac{13\xi R_{\alpha\beta\gamma\delta}{}^{;\epsilon}g^{\mu\nu}R^{\alpha\beta\gamma\delta}}{9450} + \frac{Rg^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}{3456} - \\
& \frac{R\xi g^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}{432} + \frac{R\xi^2 g^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}{480} - \frac{R^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}{4320} + \\
& \frac{\xi R^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}{720} + \frac{R_{\alpha}{}^{\nu}R_{\beta\gamma\delta}{}^{\mu}R^{\alpha\beta\gamma\delta}}{6480} + \frac{R_{\alpha}{}^{\mu}R_{\beta\gamma\delta}{}^{\nu}R^{\alpha\beta\gamma\delta}}{6480} + \\
& \frac{37R_{\alpha\beta\gamma\delta}{}^{;\delta}g^{\mu\nu}R^{\alpha\beta\gamma\epsilon}}{302400} - \frac{37\xi R_{\alpha\beta\gamma\delta}{}^{;\delta}g^{\mu\nu}R^{\alpha\beta\gamma\epsilon}}{151200} + \frac{37R_{\alpha\beta\gamma\delta}{}^{;\delta}g^{\mu\nu}R^{\alpha\beta\gamma\epsilon}}{302400} - \\
& \frac{37\xi R_{\alpha\beta\gamma\delta}{}^{;\delta}g^{\mu\nu}R^{\alpha\beta\gamma\epsilon}}{151200} + \frac{R_{\alpha}{}^{\nu}{}_{;\beta\gamma}R^{\alpha\beta\gamma\mu}}{7200} - \frac{13R_{\alpha\beta\gamma\delta}{}^{;\delta\nu}R^{\alpha\beta\gamma\mu}}{302400} - \\
& \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}{}^{\delta}R^{\alpha\beta\gamma\mu}}{12600} - \frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}_{;\delta}{}^{\delta}R^{\alpha\beta\gamma\mu}}{4200} + \frac{R_{\alpha}{}^{\mu}{}_{;\beta\gamma}R^{\alpha\beta\gamma\nu}}{7200} + \frac{53R_{\alpha\beta\gamma\delta}{}^{;\delta\mu}R^{\alpha\beta\gamma\nu}}{1209600} - \\
& \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta}{}^{\delta}R^{\alpha\beta\gamma\nu}}{12600} + \frac{R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}_{;\delta}{}^{\delta}R^{\alpha\beta\gamma\nu}}{3600} - \frac{RR_{\alpha\beta\gamma}{}^{\mu}R^{\alpha\beta\gamma\nu}}{2880} + \\
& \frac{R\xi R_{\alpha\beta\gamma}{}^{\mu}R^{\alpha\beta\gamma\nu}}{480} + \frac{R_{\alpha\beta\gamma\delta}{}^{;\gamma}g^{\mu\nu}R^{\alpha\beta\delta\epsilon}}{11520} - \frac{\xi R_{\alpha\beta\gamma\delta}{}^{;\gamma}g^{\mu\nu}R^{\alpha\beta\delta\epsilon}}{5760} - \\
& \frac{R_{\alpha\beta\gamma}{}^{\mu}R_{\delta}{}^{\epsilon}{}^{\nu}R^{\alpha\beta\delta\epsilon}}{11340} + \frac{2R_{\alpha\beta\gamma}{}^{\mu}R_{\delta}{}^{\gamma}{}^{\nu}R^{\alpha\beta\delta\epsilon}}{14175} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\gamma\nu}R^{\alpha\beta\delta\mu}}{11520} +
\end{aligned}$$

$$\begin{aligned}
& \frac{23R_{\alpha\beta\gamma\delta}{}^{\mu\nu}R^{\alpha\gamma\beta\delta}}{151200} + \frac{23R_{\alpha\beta\gamma\delta}{}^{\nu\mu}R^{\alpha\gamma\beta\delta}}{151200} + \frac{53R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta}{}^{\nu}R^{\alpha\gamma\beta\delta}}{604800} + \frac{53R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}{}^{\mu}R^{\alpha\gamma\beta\delta}}{604800} - \\
& \frac{R_{\alpha}{}^{\mu}{}_{\beta}{}^{\nu}{}_{;\gamma\delta}R^{\alpha\gamma\beta\delta}}{6300} - \frac{R_{\alpha}{}^{\nu}{}_{\beta}{}^{\mu}{}_{;\gamma\delta}R^{\alpha\gamma\beta\delta}}{6300} + \frac{61R_{\alpha\beta;\gamma\delta}g^{\mu\nu}R^{\alpha\gamma\beta\delta}}{201600} - \\
& \frac{61\xi R_{\alpha\beta;\gamma\delta}g^{\mu\nu}R^{\alpha\gamma\beta\delta}}{100800} + \frac{R_{\alpha\beta\gamma\delta}{}^{\epsilon}g^{\mu\nu}R^{\alpha\gamma\beta\delta}}{151200} - \frac{\xi R_{\alpha\beta\gamma\delta}{}^{\epsilon}g^{\mu\nu}R^{\alpha\gamma\beta\delta}}{75600} - \\
& \frac{41g^{\mu\nu}R_{\alpha\beta}R_{\gamma\delta}R^{\alpha\gamma\beta\delta}}{201600} + \frac{41\xi g^{\mu\nu}R_{\alpha\beta}R_{\gamma\delta}R^{\alpha\gamma\beta\delta}}{100800} + \frac{Rg^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\gamma\beta\delta}}{8640} - \\
& \frac{R\xi g^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\gamma\beta\delta}}{1080} + \frac{R\xi^2 g^{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\gamma\beta\delta}}{720} - \frac{R_{\alpha\beta}R_{\gamma}{}^{\mu}{}_{\delta}{}^{\nu}R^{\alpha\gamma\beta\delta}}{32400} + \\
& \frac{13R_{\alpha\beta\gamma\delta}{}^{\delta}g^{\mu\nu}R^{\alpha\gamma\beta\epsilon}}{75600} - \frac{13\xi R_{\alpha\beta\gamma\delta}{}^{\delta}g^{\mu\nu}R^{\alpha\gamma\beta\epsilon}}{37800} - \frac{197R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\epsilon}g^{\mu\nu}R^{\alpha\gamma\beta\epsilon}}{1209600} + \\
& \frac{197\xi R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\epsilon}g^{\mu\nu}R^{\alpha\gamma\beta\epsilon}}{604800} + \frac{11R_{\alpha\beta;\gamma}{}^{\nu}R^{\alpha\gamma\beta\mu}}{720} + \frac{R_{\alpha}{}^{\nu}{}_{;\beta\gamma}R^{\alpha\gamma\beta\mu}}{10080} + \\
& \frac{53R_{\alpha\beta\gamma\delta}{}^{\delta\nu}R^{\alpha\gamma\beta\mu}}{1209600} - \frac{13R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}R^{\alpha\gamma\beta\mu}}{25200} + \frac{191R_{\alpha\beta}R_{\gamma}{}^{\nu}R^{\alpha\gamma\beta\mu}}{604800} - \\
& \frac{R_{\alpha\beta\gamma}{}^{\nu}R^{\alpha\gamma\beta\mu}}{4320} + \frac{R\xi R_{\alpha\beta\gamma}{}^{\nu}R^{\alpha\gamma\beta\mu}}{720} + \frac{11R_{\alpha\beta;\gamma}{}^{\mu}R^{\alpha\gamma\beta\nu}}{720} + \\
& \frac{R_{\alpha}{}^{\mu}{}_{;\beta\gamma}R^{\alpha\gamma\beta\nu}}{10080} + \frac{7R_{\alpha\beta\gamma\delta}{}^{\delta\mu}R^{\alpha\gamma\beta\nu}}{34560} + \frac{191R_{\alpha\beta}R_{\gamma}{}^{\mu}R^{\alpha\gamma\beta\nu}}{604800} + \\
& \frac{19R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}^{\nu}R^{\alpha\delta\beta\gamma}}{100800} + \frac{19R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}^{\delta}R^{\alpha\delta\beta\gamma}}{100800} - \frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma\delta}{}^{\mu}R^{\alpha\delta\beta\gamma}}{6480} - \\
& \frac{R_{\alpha}{}^{\mu}R_{\beta\gamma\delta}{}^{\nu}R^{\alpha\delta\beta\gamma}}{6480} - \frac{R_{\alpha\beta\gamma}{}^{\mu}R_{\delta\epsilon}{}^{\gamma\nu}R^{\alpha\delta\beta\epsilon}}{28350} + \frac{R_{\alpha\beta\gamma}{}^{\mu}R_{\delta}{}^{\gamma}{}_{\epsilon}{}^{\nu}R^{\alpha\delta\beta\epsilon}}{11340} - \\
& \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}R^{\alpha\delta\beta\mu}}{5040} - \frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}^{\delta}R^{\alpha\delta\beta\mu}}{12600} - \frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}^{\gamma}{}_{\delta}R^{\alpha\delta\beta\mu}}{12600} - \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta}R^{\alpha\delta\beta\nu}}{5040} - \\
& \frac{R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}^{\delta}R^{\alpha\delta\beta\nu}}{12600} - \frac{R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}^{\gamma}{}_{\delta}R^{\alpha\delta\beta\nu}}{12600} - \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}R^{\alpha\delta\gamma\mu}}{6300} - \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\beta}{}_{\delta}R^{\alpha\delta\gamma\mu}}{6300} - \\
& \frac{13R_{\alpha\beta\gamma\delta}{}^{\beta\mu}R^{\alpha\delta\gamma\nu}}{172800} - \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta}R^{\alpha\delta\gamma\nu}}{6300} - \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\beta}{}_{\delta}R^{\alpha\delta\gamma\nu}}{6300} + \\
& \frac{R_{\alpha\beta\gamma\delta}{}^{\delta}g^{\mu\nu}R^{\alpha\epsilon\beta\gamma}}{6300} - \frac{\xi R_{\alpha\beta\gamma\delta}{}^{\delta}g^{\mu\nu}R^{\alpha\epsilon\beta\gamma}}{3150} - \frac{283R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\epsilon}g^{\mu\nu}R^{\alpha\epsilon\beta\gamma}}{1209600} + \\
& \frac{283\xi R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\epsilon}g^{\mu\nu}R^{\alpha\epsilon\beta\gamma}}{604800} - \frac{11R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\epsilon}g^{\mu\nu}R^{\alpha\epsilon\gamma\delta}}{100800} + \frac{11\xi R_{\alpha\beta\gamma\delta}{}^{\delta}{}_{\epsilon}g^{\mu\nu}R^{\alpha\epsilon\gamma\delta}}{50400} + \\
& \frac{43R_{\alpha\beta\gamma\delta}{}^{\beta}g^{\mu\nu}R^{\alpha\epsilon\gamma\delta}}{1209600} - \frac{43\xi R_{\alpha\beta\gamma\delta}{}^{\beta}g^{\mu\nu}R^{\alpha\epsilon\gamma\delta}}{604800} + \frac{3R_{\alpha\beta;\gamma}{}^{\nu}R^{\alpha\mu\beta\gamma}}{44800} - \\
& \frac{23R_{\alpha}{}^{\nu}{}_{;\beta\gamma}R^{\alpha\mu\beta\gamma}}{50400} - \frac{101R_{\alpha\beta\gamma\delta}{}^{\delta\nu}R^{\alpha\mu\beta\gamma}}{1209600} - \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}R^{\alpha\mu\beta\gamma}}{8400} - \frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}^{\delta}R^{\alpha\mu\beta\gamma}}{12600} - \\
& \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}R^{\alpha\mu\beta\delta}}{6300} - \frac{R_{\iota\alpha\beta}R^{\alpha\mu\beta\nu}}{1800} + \frac{\xi R_{\iota\alpha\beta}R^{\alpha\mu\beta\nu}}{360} - \frac{R_{\alpha\beta;\gamma}{}^{\gamma}R^{\alpha\mu\beta\nu}}{4200} - \\
& \frac{R_{\alpha\beta}R^{\alpha\mu\beta\nu}}{2160} + \frac{R\xi R_{\alpha\beta}R^{\alpha\mu\beta\nu}}{360} - \frac{29R_{\alpha\beta\gamma\delta}{}^{\beta\nu}R^{\alpha\mu\gamma\delta}}{720} -
\end{aligned}$$

$$\begin{array}{r}
\frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta}{}^{\beta} R^{\alpha\mu\gamma\delta}}{12600} - \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta} R^{\alpha\mu\gamma\delta}}{12600} - \frac{31R_{\alpha\beta;\gamma}{}^{\beta} R^{\alpha\mu\gamma\nu}}{100800} - \\
\frac{R_{\alpha\beta}{}^{;\beta}{}_{\gamma} R^{\alpha\mu\gamma\nu}}{10080} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\beta\delta} R^{\alpha\mu\gamma\nu}}{6300} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\delta\beta} R^{\alpha\mu\gamma\nu}}{6300} + \frac{3R_{\alpha\beta;\gamma}{}^{\mu} R^{\alpha\nu\beta\gamma}}{44800} - \\
\frac{23R_{\alpha}{}^{\mu}{}_{;\beta\gamma} R^{\alpha\nu\beta\gamma}}{50400} - \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta} R^{\alpha\nu\beta\gamma}}{8400} - \frac{R_{\alpha}{}^{\mu}{}_{\beta\gamma;\delta} R^{\alpha\nu\beta\gamma}}{12600} - \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta} R^{\alpha\nu\beta\delta}}{6300} - \\
\frac{R_{;\alpha\beta} R^{\alpha\nu\beta\mu}}{1800} + \frac{\xi R_{;\alpha\beta} R^{\alpha\nu\beta\mu}}{360} - \frac{29R_{\alpha\beta\gamma\delta}{}^{;\beta\mu} R^{\alpha\nu\gamma\delta}}{720} - \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta} R^{\alpha\nu\gamma\delta}}{12600} - \\
\frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta} R^{\alpha\nu\gamma\delta}}{12600} - \frac{31R_{\alpha\beta;\gamma}{}^{\beta} R^{\alpha\nu\gamma\mu}}{100800} - \frac{R_{\alpha\beta}{}^{;\beta}{}_{\gamma} R^{\alpha\nu\gamma\mu}}{10080} - \\
\frac{g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta\epsilon}{}^{\alpha} R^{\beta\gamma\delta\epsilon}}{5040} + \frac{\xi g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta\epsilon}{}^{\alpha} R^{\beta\gamma\delta\epsilon}}{2520} - \frac{2R_{\alpha\beta\gamma}{}^{\mu} R_{\delta\epsilon}{}^{\alpha\nu} R^{\beta\gamma\delta\epsilon}}{14175} - \\
\frac{2R_{\alpha\beta\gamma}{}^{\mu} R_{\delta\epsilon}{}^{\alpha\nu} R^{\beta\gamma\delta\epsilon}}{4725} + \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\mu\alpha\nu} R^{\beta\gamma\delta\epsilon}}{9450} + \frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta} R^{\beta\gamma\delta\mu}}{25200} - \\
\frac{R_{\alpha}{}^{\nu}{}_{\beta\gamma}{}^{;\alpha}{}_{\delta} R^{\beta\gamma\delta\mu}}{25200} + \frac{R_{\alpha}{}^{\nu} R_{\beta\gamma\delta}{}^{\alpha} R^{\beta\gamma\delta\mu}}{4725} - \frac{11R_{\alpha\beta} R_{\gamma\delta}{}^{\alpha\nu} R^{\beta\gamma\delta\mu}}{60480} - \\
\frac{R_{\alpha\beta} R_{\gamma}{}^{\alpha}{}_{\delta}{}^{\nu} R^{\beta\gamma\delta\mu}}{12600} + \frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta} R^{\beta\gamma\delta\nu}}{25200} - \frac{R_{\alpha}{}^{\mu}{}_{\beta\gamma}{}^{;\alpha}{}_{\delta} R^{\beta\gamma\delta\nu}}{25200} + \\
\frac{R_{\alpha}{}^{\mu} R_{\beta\gamma\delta}{}^{\alpha} R^{\beta\gamma\delta\nu}}{4725} - \frac{11R_{\alpha\beta} R_{\gamma\delta}{}^{\alpha\mu} R^{\beta\gamma\delta\nu}}{60480} - \frac{R_{\alpha\beta\gamma}{}^{\mu} R_{\delta\epsilon}{}^{\alpha\nu} R^{\beta\delta\gamma\epsilon}}{28350} - \\
\frac{R_{\alpha\beta\gamma}{}^{\mu} R_{\delta\epsilon}{}^{\alpha\nu} R^{\beta\delta\gamma\epsilon}}{9450} - \frac{151R_{\alpha}{}^{\nu} R_{\beta\gamma\delta}{}^{\alpha} R^{\beta\delta\gamma\mu}}{453600} + \frac{R_{\alpha\beta} R_{\gamma}{}^{\alpha}{}_{\delta}{}^{\nu} R^{\beta\delta\gamma\mu}}{4725} - \\
\frac{151R_{\alpha}{}^{\mu} R_{\beta\gamma\delta}{}^{\alpha} R^{\beta\delta\gamma\nu}}{453600} - \frac{121g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\gamma} R^{\beta\delta\epsilon\eta}}{907200} + \frac{121\xi g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\gamma} R^{\beta\delta\epsilon\eta}}{453600} - \\
\frac{g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma} R^{\beta\delta\epsilon\eta}}{6300} + \frac{\xi g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma} R^{\beta\delta\epsilon\eta}}{3150} + \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma\nu} R^{\beta\delta\epsilon\mu}}{18900} + \\
\frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\nu\alpha\gamma} R^{\beta\delta\epsilon\mu}}{28350} - \frac{g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta\epsilon}{}^{\alpha} R^{\beta\epsilon\gamma\delta}}{16800} + \frac{\xi g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta\epsilon}{}^{\alpha} R^{\beta\epsilon\gamma\delta}}{8400} + \\
\frac{13R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\mu\alpha\nu} R^{\beta\epsilon\gamma\delta}}{37800} + \frac{g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\gamma} R^{\beta\epsilon\delta\eta}}{4725} - \frac{2\xi g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\gamma} R^{\beta\epsilon\delta\eta}}{4725} + \\
\frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma\nu} R^{\beta\epsilon\delta\mu}}{9450} - \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\nu\alpha\gamma} R^{\beta\epsilon\delta\mu}}{11340} - \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\mu\alpha\gamma} R^{\beta\epsilon\delta\nu}}{11340} - \\
\frac{g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma} R^{\beta\eta\delta\epsilon}}{18900} + \frac{\xi g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma} R^{\beta\eta\delta\epsilon}}{9450} - \frac{47R_{\alpha\beta\gamma\delta}{}^{;\alpha\nu} R^{\beta\mu\gamma\delta}}{302400} - \\
\frac{R_{\alpha\beta\gamma}{}^{\nu}{}_{;\delta} R^{\beta\mu\gamma\delta}}{25200} - \frac{11R_{\alpha\beta} R_{\gamma\delta}{}^{\alpha\nu} R^{\beta\mu\gamma\delta}}{50400} + \frac{23R_{\alpha\beta} R_{\gamma}{}^{\alpha}{}_{\delta}{}^{\nu} R^{\beta\mu\gamma\delta}}{60480} + \\
\frac{13R_{\alpha\beta;\gamma}{}^{\alpha} R^{\beta\mu\gamma\nu}}{100800} + \frac{R_{\alpha\beta}{}^{;\alpha}{}_{\gamma} R^{\beta\mu\gamma\nu}}{8400} - \frac{R_{\alpha\beta\gamma\delta}{}^{;\alpha\delta} R^{\beta\mu\gamma\nu}}{12600} + \\
\frac{19R_{\alpha\beta} R_{\gamma}{}^{\alpha} R^{\beta\mu\gamma\nu}}{75600} - \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\gamma\nu} R^{\beta\mu\delta\epsilon}}{9450} - \frac{47R_{\alpha\beta\gamma\delta}{}^{;\alpha\mu} R^{\beta\nu\gamma\delta}}{302400} - \\
\frac{R_{\alpha\beta\gamma}{}^{\mu}{}_{;\delta} R^{\beta\nu\gamma\delta}}{25200} + \frac{23R_{\alpha\beta} R_{\gamma}{}^{\alpha}{}_{\delta}{}^{\mu} R^{\beta\nu\gamma\delta}}{60480} + \frac{13R_{\alpha\beta;\gamma}{}^{\alpha} R^{\beta\nu\gamma\mu}}{100800} +
\end{array}$$

$$\begin{aligned}
& \frac{R_{\alpha\beta}{}^{\gamma} R^{\beta\nu\gamma\mu}}{8400} - \frac{R_{\alpha\beta\gamma\delta}{}^{\alpha\delta} R^{\beta\nu\gamma\mu}}{12600} + \frac{g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\beta} R^{\gamma\delta\epsilon\eta}}{9450} - \\
& \frac{\xi g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\beta} R^{\gamma\delta\epsilon\eta}}{4725} + \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\beta\nu} R^{\gamma\delta\epsilon\mu}}{4050} - \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\nu\alpha\beta} R^{\gamma\delta\epsilon\mu}}{8100} + \\
& \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\beta\mu} R^{\gamma\delta\epsilon\nu}}{4050} + \frac{31 g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\beta} R^{\gamma\delta\epsilon\eta}}{129600} - \frac{31 \xi g^{\mu\nu} R_{\alpha\beta\gamma\delta} R_{\epsilon\eta}{}^{\alpha\beta} R^{\gamma\delta\epsilon\eta}}{64800} + \\
& \left. \frac{2 R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\beta\nu} R^{\gamma\delta\epsilon\mu}}{4725} - \frac{11 R_{\alpha\beta} R_{\gamma}{}^{\alpha\beta} R^{\gamma\mu\delta\nu}}{56700} - \frac{R_{\alpha\beta\gamma\delta} R_{\epsilon}{}^{\alpha\beta\gamma} R^{\delta\mu\epsilon\nu}}{3780} \right] \quad (281)
\end{aligned}$$

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