



An economic evaluation of nitrogen fertilization of Montana winter wheat
by Bradley Eugene Garnick

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
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Abstract:

Relationships between yield and protein response of winter wheat to nitrogen fertilizer and various soil and climatic variables were determined using data from 43 fertility experiments conducted in Montana during the years 1967, 1968, 1970, and 1971. Relationships were estimated using a generalized nonlinear least squares algorithm.

Additive and multiplicative error models were examined. Explanatory variables were applied nitrogen, applied phosphorus, April through July precipitation, NO₃-N to 4 feet in early spring, and soil water to 4 feet in early spring.

Variables important in explaining yield response were first determined using an additive error model. Precipitation, NO₃-N, and applied nitrogen were important in explaining yield response. A multiplicative error model was then estimated and additive and multiplicative models were compared. Multiplicative models were chosen for estimating yield and protein response based on the properties associated with the error term. Important variables for explaining protein response were precipitation, NO₃-N, applied nitrogen, and applied phosphorus.

Estimated yield and protein response equations were used to determine optimal rates of nitrogen application under varying nitrogen-wheat price conditions. Optimal rates were first determined without a protein premium structure. Protein response and a protein premium structure were then included to illustrate the magnitude of the effect protein premiums may have on optimal nitrogen fertilization practices. High levels of soil nitrate did not prevent economic application of nitrogen fertilizer. As expected, protein premiums increased optimal rates of nitrogen application under most conditions.

In the final section, a 25-percent marginal rate of return was specified and used to derive optimal nitrogen rates without a protein premium. Specification of this marginal rate of return reduced optimal nitrogen rates.

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Date

July 18, 1979

AN ECONOMIC EVALUATION OF NITROGEN
FERTILIZATION OF MONTANA WINTER WHEAT

by

BRADLEY EUGENE GARNICK

A thesis submitted in partial fulfillment
of the requirements for the degree


of

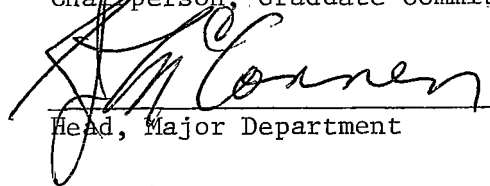
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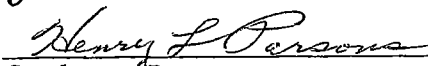
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TABLE OF CONTENTS

	<u>Page</u>
VITA	ii
ACKNOWLEDGMENTS.	iii
LIST OF TABLES AND FIGURES	vi
ABSTRACT	viii
CHAPTER I: INTRODUCTION	1
Statement of the Problem.	1
Purpose of the Study.	2
CHAPTER II: LITERATURE REVIEW	4
CHAPTER III: ESTIMATION OF YIELD AND PROTEIN RESPONSE	10
Source of Data.	10
Algebraic Specification of the Response Function.	11
Additive Error Model.	13
Multiplicative Error Model.	18
Estimation Procedures	20
Statistical Tests	22
Estimation Procedures	23
Determining Relevant Variables in the Additive Error Model	26
Comparison of Additive and Multiplicative Error Models.	29
Constrained Multiplicative Error Model.	35
Estimated Conditional Mean Yields	39
Assumption Concerning Soil Water.	44
Protein Estimation.	45
CHAPTER IV: OPTIMAL FERTILIZER POLICIES	51
Unlimited Capital	51
Limited Capital	53
Nitrogen Fertilizer Recommendations Without Protein Response.	54
Introduction of Protein Premium	58
Nitrogen Fertilizer Recommendations With Protein Premium.	62
Risk and Uncertainty.	66

TABLE OF CONTENTS (cont'd.)

	<u>Page</u>
CHAPTER V: SUMMARY AND CONCLUSIONS.	71
APPENDIX	80
LITERATURE CITED	96

LIST OF TABLES AND FIGURES

<u>Table</u>		<u>Page</u>
1	VARIABLES USED IN DEVELOPING PREDICTIVE EQUATIONS FOR YIELD AND PERCENT PROTEIN OF WINTER WHEAT.	24
2	EQUATIONS USED TO TEST THE SIGNIFICANCE OF SOIL WATER, APPLIED PHOSPHORUS, PRECIPITATION, AND SOIL NITRATE IN EXPLAINING YIELD RESPONSE.	25
3	REGRESSION COEFFICIENTS FOR MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.14).	30
4	REGRESSION COEFFICIENTS FOR THE ADDITIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.11).	31
5	WINTER WHEAT YIELDS PREDICTED BY THE MULTIPLICATIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN ($\text{NO}_3\text{-N}$ to 4' = 90 lb/A)	32
6	WINTER WHEAT YIELDS PREDICTED BY THE ADDITIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN ($\text{NO}_3\text{-N}$ to 4' = 90 lb/A).	33
7	REGRESSION COEFFICIENTS FOR THE CONSTRAINED MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.15)	36
8	WINTER WHEAT YIELDS PREDICTED BY THE CONSTRAINED MULTIPLICATIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN ($\text{NO}_3\text{-N}$ to 4' = 90 lb/A)	38
9	95-PERCENT CONFIDENCE INTERVALS FOR YIELD WHEN EXPECTED PRECIPITATION EQUALS 7.47 INCHES AND EXPECTED PRECIPITATION SQUARED EQUALS 62.38 INCHES	41
10	YIELD EQUATIONS WHEN PRECIPITATION EQUALS 7.47 INCHES, PRECIPITATION SQUARED EQUALS 62.38 INCHES, AND SOIL NITRATE IS SET AT THE VALUES GIVEN	43
11	REGRESSION COEFFICIENTS FOR THE MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT PROTEIN CONTENT (EQ. 3.16)	46

LIST OF TABLES AND FIGURES (cont'd.)

<u>Table</u>	<u>Page</u>
12 PERCENT PROTEIN OF WINTER WHEAT PREDICTED BY THE MULTIPLICATIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN ($\text{NO}_3\text{-N}$ TO 4' = 90 lb/A and Applied Phosphorus = 25 lb/A)	47
13 OPTIMAL RATES OF N APPLICATION BASED ON $\text{NO}_3\text{-N}$ TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 (No Protein Premium Structure)	55
14 OPTIMAL RATE OF NITROGEN BASED ON THE INPUT-OUTPUT PRICE RATIO AND TOTAL EXPECTED WATER WHEN $\text{NO}_3\text{-N}$ TO 4' = 90 lb/A (No Protein Premium)	57
15 OPTIMAL RATES OF N APPLICATION BASED ON $\text{NO}_3\text{-N}$ TO 4' AND TOAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 (With Protein Premium Structure and Applied Phosphorus = 25 lb/A)	63
16 OPTIMAL RATES OF N APPLICATION BASED ON $\text{NO}_3\text{-N}$ TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 (With Protein Premium Structure and Applied Phosphorus = 25 lb/A)	65
17 OPTIMAL RATES OF N APPLICATION BASED ON $\text{NO}_3\text{-N}$ TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 AND THE MARGINAL RATE OF RETURN EQUALS .25 (No Protein Premium Structure)	69
 <u>Figure</u>	
1 Recommended Rates of Nitrogen Application Plotted Against Varying Nitrogen-Wheat Price Ratios When No Protein Premium Exists (Total $\text{NO}_3\text{-N}$ to 4' = 90 lb/A and Total Expected Water = 12 inches)	59

ABSTRACT

Relationships between yield and protein response of winter wheat to nitrogen fertilizer and various soil and climatic variables were determined using data from 43 fertility experiments conducted in Montana during the years 1967, 1968, 1970, and 1971. Relationships were estimated using a generalized nonlinear least squares algorithm. Additive and multiplicative error models were examined. Explanatory variables were applied nitrogen, applied phosphorus, April through July precipitation, $\text{NO}_3\text{-N}$ to 4 feet in early spring, and soil water to 4 feet in early spring.

Variables important in explaining yield response were first determined using an additive error model. Precipitation, $\text{NO}_3\text{-N}$, and applied nitrogen were important in explaining yield response. A multiplicative error model was then estimated and additive and multiplicative models were compared. Multiplicative models were chosen for estimating yield and protein response based on the properties associated with the error term. Important variables for explaining protein response were precipitation, $\text{NO}_3\text{-N}$, applied nitrogen, and applied phosphorus.

Estimated yield and protein response equations were used to determine optimal rates of nitrogen application under varying nitrogen-wheat price conditions. Optimal rates were first determined without a protein premium structure. Protein response and a protein premium structure were then included to illustrate the magnitude of the effect protein premiums may have on optimal nitrogen fertilization practices. High levels of soil nitrate did not prevent economic application of nitrogen fertilizer. As expected, protein premiums increased optimal rates of nitrogen application under most conditions.

In the final section, a 25-percent marginal rate of return was specified and used to derive optimal nitrogen rates without a protein premium. Specification of this marginal rate of return reduced optimal nitrogen rates.

CHAPTER I

INTRODUCTION

Statement of the Problem

Nitrogen fertilizer usage in Montana has increased from 10,932 tons in 1965 to over 52,784 tons in 1976. Nitrogen fertilizers have increased yields and profitability of winter wheat production on many soils. Montana winter wheat producers need precise information concerning the rates of nitrogen fertilizer application best suited for their situation. Rates of nitrogen fertilizer application that produce the highest yields seldom result in the highest profits.

The costs of the inputs necessary to produce a winter wheat crop have risen considerably in the 1970's. During this same time period, extremely volatile price levels have prevailed in winter wheat markets. Winter 1977 prices received by Montana wheat producers are approximately 50 percent of 1973-74 price levels. Nitrogen fertilizer prices have also fluctuated widely from a level of 10 cents per pound of actual N in 1970 to over 30 cents per pound in 1974. Winter 1977 nitrogen price is 20 cents per pound of N. With rapidly changing input and output prices, it is imperative that producers strive to determine profit-maximizing levels of nitrogen fertilizer use. An uneconomic fertilizer application could result in a considerable loss of profits to wheat producers.

The criteria used by farmers for making fertilizer decisions vary considerably. Fertilizer applications are often made with little regard for changing price structures. In addition, differences in response to nitrogen arising from varying soil and/or moisture conditions are not well defined. If wheat farmers are to maximize profits with respect to fertilizer costs, it is necessary for them to have a decision criterion for nitrogen fertilizer application which is applicable under variable moisture and soil fertility conditions.

In order to make economic decisions regarding the level of nitrogen to apply to winter wheat, two types of information are needed. First, the decision maker needs information concerning the physical response of wheat to nitrogen. Two types of response must be quantified: (a) incremental yields forthcoming from different levels of nitrogen application, and (b) incremental changes in protein percentage associated with different levels of nitrogen. Second, the decision maker must have input cost information for nitrogen and product price information for wheat, including protein premium structures. If this information is available, basic economic logic can be used to determine optimum levels of nitrogen application (6).

Purpose of the Study

The general purpose of the study is to develop and present a profit maximizing decision criterion based on the best information

available which will be of use to Montana winter wheat producers.

In order to accomplish this end, the specific objectives of the study are to:

- 1) Estimate yield and protein response of winter wheat to applied nitrogen fertilizer and important soil and climatic variables.
- 2) Determine the optimal levels of nitrogen application given specified protein premiums, wheat prices, and nitrogen fertilizer costs.
- 3) Assemble and present the derived data and information in a form which can be used by Montana wheat producers to make economically rational decisions concerning nitrogen fertilizer use.

The chapters which follow discuss the means used to complete these steps. More precisely, Chapter II presents a review of previous work on wheat response to nitrogen fertilization. Chapter III discusses the specification and statistical estimation of yield and protein response to nitrogen fertilizer. In Chapter IV, input and output prices are introduced and economic logic is used to derive optimal application rates. Chapter V offers concluding remarks and suggestions for future work.

CHAPTER II

LITERATURE REVIEW

The ability to predict wheat yield and yield response is necessary if decision makers are to determine optimal levels of fertilizer application. A considerable amount of research has been completed which has studied the effects of measurable soil and climatic variables on the response of wheat to applied fertilizer. Significant increases in yields and protein content of the grain have been attributed to nitrogen fertilizers (9, 14, 18, 19). Results from this and other research suggest that a number of factors influence yield-protein relations. Variation in crop management practices can have a substantial effect on grain response. Seeding rates, row spacing, varieties, tillage practices, and land-use systems are all thought to influence wheat yields (12, 21, 26).

Studies which have attempted to explain wheat yield relations have been quite varied in terms of the explanatory variables included and the geographic areas studied. Early work by Fisher (10) utilized linear regression techniques to examine the effects of rainfall on wheat yields. Response curves were estimated giving the expected change in yield for an additional inch of precipitation falling above the average at any time of the year. Similar techniques were used by researchers in India (11). Results from their study indicated that rainfall distribution accounted for 75 percent of the total variation in yields on unfertilized lands.

Robertson (23) used a factorial yield-weather model to analyze 50 years of spring wheat yield and weather data from southwestern Saskatchewan. Precipitation for the summer-fallow period and for May, June and August; global radiation for May; and maximum temperature for June, and July proved to be the most important variables in explaining wheat yield variation. Later research in southwestern Saskatchewan by Read and Warder (22) found growing season precipitation to be more important than stored soil moisture in explaining yield and protein variability on unfertilized plots. On fertilized plots, stored soil moisture exerted a greater influence on yield and protein content of the grain than did growing season precipitation. They concluded variables which could be measured before seeding had the greatest influence on spring wheat response to nitrogen fertilizer.

Bauer et al. (3) correlated rainfall and stored available moisture with barley and spring wheat response. Total moisture accounted for 40.3 percent of the yield response to nitrogen fertilizer. Bair and Robertson (2) used maximum and minimum temperature, rainfall, and stored soil moisture to predict wheat yields. In western Oklahoma, positive and significant correlations between wheat yields and soil moisture at seeding, growing season precipitation, temperature during the ripening period, and soil moisture in the spring were reported by Eck and Tucker (8). The relationships, however, did not yield satisfactory prediction equations.

Recently Black (4) published results indicating that, under Montana conditions, successful annual cropping of winter grains requires 8 to 10 inches of water from stored soil moisture and growing season precipitation. In Texas (1), October through June rainfall affected wheat yields under summer fallow conditions. Approximately 5 inches of rainfall were required during this time period before any measurable amount of grain was produced. Each additional inch received during this period increased production by about 2 bushels per acre.

Other studies have related grain protein to moisture components in the environment. Under irrigated conditions in Mexico (9), grain protein increased as the available moisture percentage at time of irrigation decreased. Protein content of the grain was decreased by small applications of nitrogen but was increased by large application rates. Brengle (5) correlated available soil moisture with grain protein in Colorado. Significant negative correlations existed between the two variables. In the Great Plains (26), increases in rainfall and soil moisture decreased grain protein.

Levels of soil fertility have also been shown to have a marked effect on yields and protein content. Significant positive relationships between these response components and soil nitrate nitrogen have been documented in the literature (20, 28). In Nebraska, Terman et al. (29) reported grain yield-protein relationships to be tempered by soil nitrate levels. Under adequate moisture conditions and low

available soil nitrate levels, response to applied nitrogen was high and protein levels were low. At higher soil nitrate levels, yield response to applied nitrogen was low and the chief effect of applied nitrogen was to increase grain protein. Young et al. (32) included soil nitrate at seeding, total nitrogen content, and organic matter in an analysis of spring wheat response. Soil nitrate at seeding was significant in the model at the .01 level.

Taylor et al. (28) indicated the effect of soil nitrate nitrogen on yields was most beneficial when growing season precipitation was high. Results published by Smika et al. (27) suggest soil moisture at seeding has an influence on soil nitrate-yield relationships. When soil moisture at seeding was included in the analysis, the largest grain yields were obtained where soil nitrate levels were lowest. With the exclusion of soil moisture at seeding, no relationships existed between soil nitrate and grain yields either with or without nitrogen fertilizer. Researchers in Montana (7) attributed the lack of response to nitrogen fertilizer to pre-existing high soil nitrate levels.

Recent efforts to quantify yield and protein responses under Montana conditions were made by Jackson (15). A stepwise multiple regression program was used to analyze winter wheat data from 47 summer fallow locations. Prediction equations were generated for potential grain yields. Predicted potential yields were then used along with soil and climatic variables to formulate models to estimate

the nitrogen fertilizer rates required to achieve the predicted potential yields. Finally, grain protein models were developed using potential yields, nitrogen fertilizer rates, and soil and climatic factors. Only one observation from each available set of data was picked for use in formulating these models. Growing season precipitation, soil organic matter, and evaporation rates during the first half of the growing season were important in explaining potential yields. Soil moisture measurements proved to be nonsignificant in these models. When nitrogen fertilizer requirements were estimated, soil nitrate nitrogen, potential yield, available soil water, and evaporation rates during the first half of the growing season were the most important variables. Important variables for estimating protein content were potential yield, soil nitrate nitrogen, nitrogen fertilizer requirements, organic matter, and growing season rainfall. Positive relationships were reported between protein content and soil nitrate nitrogen, soil organic matter, and nitrogen fertilizer requirements. Multiple correlation coefficients for potential yield, nitrogen fertilizer requirement, and grain protein equations were 41 percent, 58 percent, and 41 percent, respectively. Models were also generated to predict yield and percent protein on unfertilized plots and residual soil nitrate after harvest.

Results from these studies suggest that a number of soil and climatic variables and variable interactions influence grain response.

In these experiments, soil nitrate and applied nitrogen were positively correlated with grain yield and protein content. Soil water and precipitation were also positively correlated with grain yield but negatively correlated with protein content. Variability in the success obtained using these factors along with other soil and climatic variables to predict yield and/or protein response may be explained by variation in individual soil characteristics, the distribution of soil water and soil nitrate within the top 4 feet of the soil profile, management practices, and many other factors. It is hoped that the following research will contribute to a better understanding of these relationships.

CHAPTER III

ESTIMATION OF YIELD AND PROTEIN RESPONSE

Source of Data

The data used for the analysis were collected from a series of nitrogen top-dressing experiments conducted by the Montana Cooperative Extension Service and the Montana Agricultural Experiment Station. Forty-three sets of data representing 38 locations were included in the final statistical analysis. Site locations and investigators are listed in the Appendix.

Data were selected according to their ability to meet a predetermined set of criteria. Initial selection was limited to experimental plots located on summer fallow land with good stands of a recommended variety of hard red winter wheat. Experimental sites characterized by hail damage were excluded from the analysis.

Observations on a predetermined set of explanatory variables to be used in the estimation of yield and protein response were also required. These explanatory variables included available soil nitrate measured to a depth of 4 feet in early spring, April through July precipitation, and available soil water measured to 4 feet in early spring. In many cases moisture content in the soil was not expressed as inches of available soil water. As a result, it was necessary to convert these measurements to inches of available water using the best information available. In addition, dates for the installation of rain gauges varied between experiments and were not available for

a limited number of locations. Hence, precipitation records from nearby weather stations were used to replace missing rainfall data. A standard soil analysis with measurements on organic matter and phosphorus and potassium content present in the top 6 inches of soil was desired, but was not available for all locations.

Individual experiments were conducted using a randomized complete block design with three replications. Rates of broadcast nitrogen common to all locations were 0, 20, 40, 60, and 80 pounds per acre of spring-applied nitrogen in the form of ammonium nitrate. Potassium and phosphorus were also applied in the spring at rates of 25 and 20 pounds per acre respectively for 1967 and 1968, and at rates of 25 and 40 pounds per acre for 1970 and 1971. Rates represented actual pounds of elemental phosphorus and potassium. In cases where starter fertilizer was applied in the fall, total amounts of applied phosphorus and nitrogen were somewhat larger.

Varieties of winter wheat represented in the data were Winalta, Cheyenne, and Warrior.

Algebraic Specification of the Response Function

The selection of an algebraic equation to use when fitting crop response functions warrants careful consideration. In the selection procedure, it is important to take into account the nature of the phenomenon under investigation and any limitations or assumptions

which may be imposed by a particular model. The functional form of the response function is important; of equal importance are the assumptions made concerning the specifications of the error term. The specification of the regression disturbance or error term will be discussed in detail later in this section.

For the purpose of this study, polynomial equations were selected for estimating yield and protein response. The properties associated with this type of equation are well suited to the data under consideration. This can be readily observed if the behavior of a second degree polynomial with one explanatory variable is examined. The equation is expressed in the following form:

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_1^2 \quad (3.1)$$

where: Y = total output;

X_1 = units of the variable resource;

α = level of output when X_1 equals zero; and

β_1 and β_2 = parameters of the equation.

If we expect total product to reach a maximum and then decline, we would hypothesize β_2 to be negative. Thus the equation lends itself to situations where both positive and negative marginal products exist. The marginal product curve is linear for second degree polynomials, although this restriction is overcome when cubic terms or higher order interactions are included. Extensions and transformations of equation

(3.1) are easily obtained and allow considerable flexibility in the specification of crop response functions.

Attempts to use polynomial equations to fit crop-yield data have generally been quite successful (13). Research workers investigating yield relationships have demonstrated that the influence of explanatory variables is often dependent on the levels of other variables present. Consequently, effects are not additive and interactions occur between variables. The models for this analysis were formulated under the hypothesis that interactions of this nature do occur. More precisely, two models were specified and used to arrive at parameter estimates for relevant explanatory variables. Models differed in the specification of the regression disturbance and in the assumptions made regarding nonlinear characteristics of the equations.

Additive Error Model

The first model employed in the estimation of yield and protein response is represented by (3.2).

$$\begin{aligned}
 Y = & b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 \\
 & + b_7X_7 + b_8X_8 + b_9X_9 + b_{10}X_{10} + b_{11}X_{11} + b_{12}X_{12} \\
 & + b_{13}X_{13} + b_{14}X_{14} + b_{15}X_{15} + b_{16}X_{16} + b_{17}X_{17} \\
 & + b_{18}X_{18} + \epsilon_{iL} + U_L
 \end{aligned}
 \tag{3.2}$$

where: Y = estimated yield;

b_0 = intercept;

X_i 's = independent variables designated in Table 1 on page 24;

b_i 's = estimated regression coefficients;

ϵ_{iL} = random variation associated with the i^{th} treatment at location L ; and

U_L = random variation due to location L .

By definition (3.2) is an intrinsically linear third degree polynomial. That is, the equation is nonlinear with respect to the variables but linear with respect to the parameters to be estimated. The model can be converted to a form which is more obviously linear in the parameters by redefining the variables. In this case, redefinition of second and third degree polynomial terms as additional variables in a linear form will convert the equation to an obviously linear relationship in the parameters.

The regression disturbance in (3.2) is additive and composed of the terms ϵ_{iL} and U_L . Although the separation of these components is not possible in the actual estimation, the logic and method of incorporation of these error terms are extremely important to the study and are thus outlined.

The first source of variation in the model is represented by ϵ_{iL} . Specifically, ϵ_{iL} is defined as a random disturbance which is independently and identically distributed across treatments, i , and

locations, L . The effects of random variation in soil and drainage characteristics within experimental plots are examples of factors inherent in this term. The usual properties (16) are associated with ϵ_{iL} and are outlined below.

ϵ_{iL} is normally distributed

$$E(\epsilon_{iL}) = 0$$

$$E(\epsilon_{iL}^2) = \sigma_\epsilon^2$$

$$E(\epsilon_{iL} \epsilon_{jL}) = 0 \text{ for } i \neq j$$

$$E(\epsilon_{iL} \epsilon_{iL'}) = 0 \text{ for } L \neq L'$$

A second disturbance is introduced into the additive model when observations from heterogeneous locations are used to estimate yield response. Obviously, an effect due to location exists between experiments and therefore introduces a major source of variation into the model. This location effect is represented by U_L in (3.2) and is defined as a random disturbance common to all treatments at a particular location. More specifically, U_L is added to all ϵ_{iL} for $i = 1, \dots, n$, where U_L is independent and identically distributed across locations with $E(U_L) = 0$. If we let $V_{iL} = \epsilon_{iL} + U_L$ and further assume that ϵ_{iL} and U_L are independent across locations and treatments, then

$$\begin{aligned} \text{Cov}(V_{iL}, V_{jL}) &= E(V_{iL} V_{jL}) = E(\epsilon_{iL} + U_L)(\epsilon_{jL} + U_L) \\ &= E(U_L^2) = \sigma_u^2, \text{ for } i \neq j. \end{aligned}$$

When ρ_{ij} denotes the correlation between treatments i and j at a given location then

$$\rho_{ij} = \frac{\text{Cov}(V_{iL}, V_{jL})}{\sigma^2} = \frac{\sigma_u^2}{\sigma^2} = \rho$$

where: $\sigma_u^2 = \text{Var}(U_L)$ and $\sigma^2 = \text{Var}(V_{jL})$.

Since we assumed ϵ_{iL} was identically distributed across treatments and locations and that ϵ_{iL} and U_L were independently distributed, it follows that $\text{Var}(V_{iL})$ is constant across locations and treatments as shown below,

$$\begin{aligned} \text{Var}(V_{iL}) &= E(V_{iL}^2) = E(\epsilon_{iL} + U_L)^2 = E(\epsilon_{iL}^2) + E(U_L^2) \\ &= \sigma_\epsilon^2 + \sigma_u^2 = \sigma^2 \end{aligned}$$

Inherent in this property is the assumption that the effect of U_L is identical across all treatment levels of nitrogen at a specific location. However, this assumption may not be valid in all cases, as we might expect that the effect of U_L would differ as nitrogen increases across treatments. As a result, an alternative definition of V_{iL} is

$$V_{iL} = \alpha_i(\epsilon_{iL} + U_L) \quad (3.3)$$

where α_i is a proportional factor which reflects a treatment-specific effect on both components of the regression disturbance. Intuitively,

differential variability exists over treatments in the error term.

Therefore:

$$\begin{aligned} \text{Cov } (V_{iL}, V_{jL}) &= E(V_{iL} V_{jL}) = E[\alpha_i (\epsilon_{iL} + U_L) \alpha_j (\epsilon_{jL} + U_L)] \\ &= E[\alpha_i \alpha_j (\epsilon_{iL} + U_L) (\epsilon_{jL} + U_L)] = \alpha_i \alpha_j E(U_L^2) \\ &= \alpha_i \alpha_j \sigma_u^2 \end{aligned}$$

and

$$\begin{aligned} \text{Var } (V_{jL}) &= E(V_{jL}^2) = E[\alpha_j (\epsilon_{jL} + U_L) \alpha_j (\epsilon_{jL} + U_L)] \\ &= \alpha_j^2 (\sigma_\epsilon^2 + \sigma_u^2) = \alpha_j^2 \sigma^2 \end{aligned}$$

This latter derivation shows the nature of the heteroskedastic variances between treatment residuals. If the correlations between treatment residuals are examined, then

$$\rho_{ij} = \frac{\text{Cov } (V_{iL}, V_{jL})}{\alpha_i \alpha_j \sigma^2} = \frac{\alpha_i \alpha_j \sigma_u^2}{\alpha_i \alpha_j \sigma^2} = \frac{\sigma_u^2}{\sigma^2} = \rho$$

Correlations between treatment residuals are therefore constant and equal to ρ but variances are unequal across treatment levels. The correlation ρ is equal to the ratio of the variance of the location specific effect to the total variance. This latter specification was used in the estimation of yield and protein response.

Multiplicative Error Model

A second possible specification of this response function is a multiplicative error model. This model can be expressed in natural numbers as

$$\begin{aligned}
 Y = & (\alpha + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_6 X_6 & (3.4) \\
 & + b_7 X_7 + b_8 X_8 + b_9 X_9 + b_{10} X_{10} + b_{11} X_{11} + b_{12} X_{12} \\
 & + b_{13} X_{13} + b_{14} X_{14} + b_{15} X_{15} + b_{16} X_{16} + b_{17} X_{17} \\
 & + b_{18} X_{18}) \epsilon_{iL}^U
 \end{aligned}$$

where once again the explanatory variables maintain the same definitions as those in (3.2). In this equation, the error components are multiplied by the mean response function as opposed to the additive disturbance previously described.

The parameters of (3.4) are estimated by taking the natural logarithm of both sides of the equation. Performing this transformation gives

$$\begin{aligned}
 \log_e Y = & \log_e (\alpha + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 & (3.5) \\
 & + b_6 X_6 + b_7 X_7 + b_8 X_8 + b_9 X_9 + b_{10} X_{10} + b_{11} X_{11} \\
 & + b_{12} X_{12} + b_{13} X_{13} + b_{14} X_{14} + b_{15} X_{15} + b_{16} X_{16} \\
 & + b_{17} X_{17} + b_{18} X_{18}) + \log_e \epsilon_{iL} + \log_e U_L
 \end{aligned}$$

which is an intrinsically nonlinear model. That is, (3.5) is nonlinear with respect to the parameters as well as being nonlinear with respect to the variables.

The two components of the multiplicative regression disturbances in (3.4) are defined in the same manner as ϵ_{iL} and U_L in the additive model. However, two additional assumptions regarding the expected values of ϵ_{iL} and U_L are necessary in the multiplicative framework. The assumptions are outlined below.

$$E(\epsilon_{iL}) = 1$$

$$E(U_L) = 1$$

Taking the expected value of (3.4) with these assumptions results in (3.6),

$$\begin{aligned} E(Y) = & \alpha + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_6 X_6 & (3.6) \\ & + b_7 X_7 + b_8 X_8 + b_9 X_9 + b_{10} X_{10} + b_{11} X_{11} + b_{12} X_{12} \\ & + b_{13} X_{13} + b_{14} X_{14} + b_{15} X_{15} + b_{16} X_{16} + b_{17} X_{17} \\ & + b_{18} X_{18} \end{aligned}$$

Again, it is important to recognize that the separate components of V_{iL} , defined as ϵ_{iL} and U_L , are not actually in the calculations. However, an argument analogous to that presented for the additive model can be applied to (3.5) to show the existence of nonzero covariances

between treatments at specific locations. Similarly, heteroskedastic variances and constant correlations also exist when differential variability occurs across treatments in the error term.

Estimation Procedures

Parameters for these models were estimated using a generalized nonlinear program written by O. Burt, Montana State University. The interested reader is referred to Malinvaud, Chapter 9 (17) for a discussion of the statistical theory underlying the development of the program. The assumptions previously outlined concerning the regression disturbances were specified in the program.

The technique of maximum likelihood estimation is used to arrive at parameter estimates for nonlinear models. Although the concept of maximum likelihood estimation is not new, the technique is one which may be unfamiliar to the reader. Therefore, a brief discussion of the general principles associated with this form of estimation is presented.

Intuitively, the method of maximum likelihood estimation is based on the idea that certain populations are more likely to generate a given sample than other populations. When the nature of a population is estimated from a random sample of data, maximum likelihood estimates can be defined in the following manner:

If a random variable X has a probability distribution $f(x)$ characterized by parameters $\theta_1, \theta_2, \dots, \theta_k$ and if we observe a sample X_1, X_2, \dots, X_n , then the maximum likelihood estimators of $\theta_1, \theta_2, \dots, \theta_k$ are those values of these parameters that would generate the observed sample most often (16).

The problem thus becomes one of finding the values of $\theta_1, \theta_2, \dots, \theta_k$ which maximize the probability of the observed sample values. The objective of finding these values can be accomplished through the formulation of a likelihood function. If the sample observations are independent, the likelihood function takes on the formula of the joint probability distribution of the sample represented in equation (3.7).

$$L = g(X_1, X_2, \dots, X_n) = f(X_1) f(X_2) \dots f(X_n) \quad (3.7)$$

We can now maximize (3.7) with respect to $\theta_1, \theta_2, \dots, \theta_k$ and obtain estimators $(\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_k)$ of the unknown parameters. More specifically, $\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_k$ are estimators obtained by the method of maximum likelihood estimation if the following condition is satisfied:

$$L(X_1, X_2, \dots, X_n; \hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_k) > L(X_1, X_2, \dots, X_n; \tilde{\theta}_1, \dots, \tilde{\theta}_k)$$

where $\tilde{\theta}_1, \dots, \tilde{\theta}_k$ are any other estimators of $\theta_1, \theta_2, \dots, \theta_n$. Since the logarithm of L is a monotonic transformation of L , the maximization procedure can be accomplished by maximizing the function

$$L = \log_e L$$

Solutions are then obtained by setting the partial derivatives of L with respect to each parameter equal to zero and solving simultaneously.

Statistical Tests

Ratios of the likelihood functions generated in the estimation procedure described can be used to evaluate the significance of explanatory variables in the models. The test is based on the property that the negative of twice the logarithm of a likelihood function ratio is asymptotically distributed as χ^2 (30). The chi-square test statistic is given by (3.8)

$$T \log_e \left[\frac{D_r}{D_g} \right] \sim \chi_q^2 \quad (3.8)$$

where: T = number of experiments (43);

D = determinant of the residual covariance matrix;

g = general model with fewer restrictions;

r = restricted model; and

q = difference in the number of parameters between the two models.

This statistical test is particularly useful in evaluating the relevance of an explanatory variable which is a component of more than one term in the response equation. Such is the case with soil moisture, soil nitrate, precipitation, and applied nitrogen. The test is also appropriate for models with linear constraints on parameter values.

T-ratios for individual coefficients were also used to evaluate and compare models. Standard errors for linear combinations of variables were examined to determine the forecasting ability of the model.

Estimation Procedures

Initial estimates of yield response relationships were made using additive regression disturbances. Table 1 contains a listing of variable descriptions and associated means, standard deviations, and coefficients of variation. Parameters for each of these variables were estimated and the nature of the response function was evaluated. Equations were also generated to test the significance of soil nitrate, precipitation, soil moisture, and applied phosphorus in explaining yield response to nitrogen fertilizer. This set of equations is presented in Table 2. Chi-square tests were used to determine variables important in explaining yield response in this set of equations.

Computational costs involved in the estimation of multiplicative models did not allow duplication of the procedures outlined above. Therefore, significant variables determined in the previous section were used when multiplicative models were generated. A comparison was then made between multiplicative and additive models to determine the response function best suited for use in developing fertilizer recommendations. T-ratios for identical terms in the two models were initially used as a criterion for comparing models in this section of the analysis.

TABLE 1. VARIABLES USED IN DEVELOPING PREDICTIVE EQUATIONS FOR YIELD AND PERCENT PROTEIN OF WINTER WHEAT.

Variable Designation	Variable Description	Units	Mean	Standard Deviation	Adjusted Coefficient of Variation
Y	Grain yield	bu/A	38.00	8.74	0.54
X ₁	Total applied phosphorus	lb/A	44.49	15.99	0.63
X ₂	Binary variable (Warrior)		0.07	0.26	3.66
X ₃	Binary variable (Cheyenne)		0.12	0.32	2.76
X ₄	Total applied nitrogen (N)	lb/A	43.47	28.62	0.66
X ₅	N ²	lb/A	2704.30	2857.40	0.96
X ₆	Soil NO ₃ -N in 4' of soil (SN)	lb/A	105.41	105.34	1.12
X ₇	(SN) ²	lb/A	22157.00	71255.00	3.24
X ₈	Avail. soil water in 4' of soil (SW)	in.	4.84	1.72	0.64
X ₉	(SW) ²	in.	26.33	17.40	0.80
X ₁₀	Total April-July Precipitation (PREC)	in.	6.76	2.01	0.63
X ₁₁	(PREC) ²	in.	49.73	29.83	0.81
X ₁₂	N x PREC		297.23	224.67	0.76
X ₁₃	SN x N		4471.70	5943.70	1.33
X ₁₄	SN x SW		625.81	569.37	1.12
X ₁₅	N x SW		282.33	247.71	0.88
X ₁₆	SN x PREC		697.01	654.99	1.03
X ₁₇	SW x PREC		43.52	26.28	0.81
X ₁₈	N ² x PREC		18581.00	19684.00	1.06

2/4

TABLE 2. EQUATIONS USED TO TEST THE SIGNIFICANCE OF SOIL WATER, APPLIED PHOSPHORUS, PRECIPITATION, AND SOIL NITRATE IN EXPLAINING YIELD RESPONSE.

Eq. No.	Equation
3.9	$Y = 20.27 + 0.0947X_1 + 9.292X_2 + 3.009X_3 - 0.04X_4 + 0.000338X_5 - 0.127X_6 + 0.000056X_7$ $- 6.154X_8 + 0.577X_9 + 7.67X_{10} - 0.468X_{11} + 0.0297X_{12} - 0.000162X_{13} + 0.0117X_{14}$ $- 0.00303X_{15} + 0.0103X_{16} - 0.167X_{17} - 0.000163X_{18}$
3.10	$Y = 1.421 + 0.0648X_1 + 10.97X_2 + 3.609X_3 - 0.066X_4 + 0.000373X_5 + 0.00339X_6$ $- 0.0000316X_7 + 7.787X_{10} - 0.57X_{11} + 0.0306X_{12} - 0.000147X_{13} + 0.00727X_{16}$ $- 0.000171X_{18}$
3.11	$Y = 2.877 + 11.229X_2 + 2.657X_3 - 0.066X_4 + 0.000383X_5 + 0.012X_6 - 0.00003X_7 + 8.088X_{10}$ $- 0.568X_{11} + 0.0308X_{12} - 0.000148X_{13} + 0.00508X_{16} - 0.000174X_{18}$
3.12	$Y = 29.40 + 12.875X_2 + 4.609X_3 + 0.135X_4 - 0.000691X_5 + 0.0464X_6 - 0.0000389X_7$ $- 0.000167X_{13}$
3.13	$Y = 4.634 + 12.515X_2 + 1.666X_3 - 0.0939X_4 + 0.000486X_5 + 8.422X_{10} - 0.561X_{11}$ $+ 0.0312X_{12} - 0.000173X_{18}$

Determining Relevant Variables in the Additive Error Model

Behavior of estimated response functions varied considerably with the inclusion and exclusion of soil water variables. Results consistent with conventional theory were obtained only when soil water variables were excluded from the statistical analysis. A comparison of the behavior of equations (3.9) and (3.10) in Table 2 illustrates this point. Estimated parameters for (3.9) showed that increasing rather than decreasing marginal response existed for soil nitrate and soil moisture over all ranges of the data, with incremental response being negative at lower levels of these variables. In this model diminishing marginal response existed only for applied nitrogen and precipitation. However, when soil water variables were eliminated in (3.10), the estimated response function exhibited diminishing marginal response for each of the resources under consideration.

This result is difficult to understand. The significant change in the nature of the response function raises the question of the validity of including the soil water variables as measured in the data in the model. Soil water measurements at 15 locations were reported as inches of available soil water in the data source but actually represented some other measurement. As a result, it is possible that difficulties encountered in converting these soil water

measurements to inches of available soil water together with other sources of measurement error severely limited the accuracy of soil water variables. In order to determine the importance of soil water variables in the model, a chi-square test statistic was calculated using the residual covariance matrices of (3.9) and (3.10). The null hypothesis tested was expressed as

$$H_0: b_8 = b_9 = b_{14} = b_{15} = b_{17} = 0$$

where the b_i 's refer to estimated parameters in (3.9). Each of these parameters is associated with a variable which includes soil water. Results from this test indicated that the null hypothesis could not be rejected at the .10 level of significance. Conclusions from previous studies cited in the literature review suggest that soil water is very important in explaining yield response. However, in this analysis soil water appears to only confound the behavior of estimated response functions. Based on these results, soil water was eliminated as an explanatory variable in the analysis which means that the net effects of soil water variation are moved to the error term of the model.

The importance of applied phosphorus was also examined by applying similar techniques to equations (3.10) and (3.11) in Table 2. The chi-square test statistic calculated from these regression equations showed that applied phosphorus was of little consequence in

explaining yield variability. The coefficient for linear applied phosphorus was not significantly different from zero at the .10 level. Therefore, it too was excluded from the analysis.

With the exclusion of applied phosphorus and soil water, the model predicting yield can be expressed in the form of equation (3.11). Investigation of the statistical results for this model indicates that seasonal precipitation and soil nitrate are very important in explaining yield response to nitrogen fertilizer. The null hypothesis tested for precipitation can be expressed as

$$H_0: b_{10} = b_{11} = b_{12} = b_{16} = b_{18} = 0$$

where the b_i 's refer to estimated coefficients for terms in (3.11). Similarly, the null hypothesis used to test the significance of soil nitrate was expressed as

$$H_0: b_6 = b_7 = b_{13} = b_{16} = 0$$

where, once again, the b_i 's symbolize coefficients in equation (3.11) in Table 2. Parameters for terms which included precipitation as a variable were significantly different from zero at the .001 level. Thus, we can be 99.9 percent confident that precipitation contributes to the explanation of yield variation in this set of experiments. Similarly, coefficients for soil nitrate terms were significantly different from zero at the .01 level. Comparison of these confidence

levels suggests that precipitation is the more important of the two variables.

Comparison of Additive and Multiplicative Error Models

Important variables from the additive analysis were next used to generate a model with a multiplicative disturbance term. Regression results for comparable multiplicative and additive models are presented in Tables 3 and 4 respectively. Models differ only in the specification of the disturbance term and in associated properties discussed earlier.

In both instances, estimated response functions are characterized by maximum yields and declining marginal productivity to the resources under consideration. Similarly, signs of estimated coefficients do not vary between models. A comparison of yields predicted by the two models for specified levels of precipitation and soil nitrate is presented in Tables 5 and 6. Predicted yields are similar over most ranges of the data.

Examination of the statistical significance of estimated parameters reveals that t-ratios for the multiplicative model are higher for all but four terms. The significance of linear soil nitrate, applied nitrogen x soil nitrate, and the two binary variables were reduced with the use of the multiplicative disturbance. However, differences between corresponding significance levels in the two

TABLE 3. REGRESSION COEFFICIENTS FOR MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.14).

Variable Description	Regression Coefficient	t-value
Intercept	3.925300	0.380
Variety 2 (Warrior)	11.248000 ***	2.687
Variety 3 (Cheyenne)	1.108200	0.410
N	-0.096910 *	-1.532
N ²	0.000654	0.894
SN	0.001654	0.037
SN ²	-0.000035	-0.925
PREC	7.834700 ***	2.856
PREC ²	-0.580110 ***	-3.443
N x PREC	0.035205 ***	3.768
N x SN	-0.000119 **	-1.816
PREC x SN	0.008295 *	1.511
N ² x PREC	-0.000223 *	-2.106
Multiple R ² = 36.8%		
* p = .10 ** p = .05 *** p = .005		

N = Total applied nitrogen (lb/A).

SN = Total NO₃-N to 4' in early spring (lb/A).

PREC = Total April-July precipitation (in.).

TABLE 4. REGRESSION COEFFICIENTS FOR THE ADDITIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.11).

Variable Description	Regression Coefficient	t-value
Intercept	2.876600	0.249
Variety 2 Binary (Warrior)	11.229000 ***	3.431
Variety 3 Binary (Cheyenne)	2.656600	0.991
N	-0.066648	-0.924
N ²	0.000383	0.453
SN	0.011962	0.250
SN ²	-0.000030	-0.803
PREC	8.088100 ***	2.682
PREC ²	-0.568450 ***	-2.959
N x PREC	0.030841 ***	2.985
N x SN	-0.000148 ***	-2.191
PREC x SN	0.005084	0.926
N ² x PREC	-0.000174 *	-1.453
Multiple R ² = 39.1%		
* p = .10 ** p = .05 *** p = .005		

N = Total applied nitrogen (lb/A).

SN = Total NO₃-N to 4' in early spring (lb/A).

PREC = Total April-July precipitation (in.)

TABLE 5. WINTER WHEAT YIELDS PREDICTED BY THE MULTIPLICATIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN ($\text{NO}_3\text{-N}$ to 4' = 90 lb/A).

Total Applied Nitrogen	Total April-July Precipitation (in.)									
	3	4	5	6	7	8	9	10	11	12
<u>lb N/A</u>	<u>Yield (bu/A)</u>									
0	24.31	28.83	32.19	34.39	35.43	35.31	34.03	31.59	27.99	23.23
20	24.26	29.40	33.38	36.19	37.05	38.34	37.67	35.85	32.86	27.72
40	24.21	29.78	34.19	37.44	39.54	40.47	40.24	38.85	36.30	32.59
60	24.14	29.97	34.64	38.15	40.50	41.69	41.72	40.59	38.30	34.85
80	24.06	29.97	34.72	38.31	40.74	42.01	42.12	41.07	38.86	35.49
100	23.97	29.78	34.43	37.93	40.26	41.43	41.45	40.30	37.99	34.52

TABLE 6. WINTER WHEAT YIELDS PREDICTED BY THE ADDITIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN (NO₃-N to 4' = 90 lb/A).

Total Applied Nitrogen	Total April-July Precipitation (in.)									
	3	4	5	6	7	8	9	10	11	12
<u>lb N/A</u>	<u>Yield (bu/A)</u>									
0	24.23	28.80	32.23	34.52	35.68	35.69	34.58	32.32	28.93	24.40
20	24.43	29.54	33.52	36.36	38.06	38.63	38.06	36.35	33.50	29.52
40	24.51	30.03	34.42	37.67	39.78	40.75	40.59	39.29	36.86	33.28
60	24.49	30.28	34.93	38.45	40.83	42.08	42.18	41.14	38.99	35.68
80	24.35	30.27	35.06	38.71	41.22	42.59	42.83	41.93	39.89	36.72
100	24.10	30.02	34.79	38.43	40.94	42.30	42.53	41.62	39.58	36.40

models are small. As a result, the selection of a model for use in developing fertilizer recommendations was based on properties associated with the two error terms. In the multiplicative framework, standard errors of the estimate are directly proportional to the mean. Hence, for agronomic data this form of specification would appear to make sense intuitively. Based on this intuitive appeal, the multiplicative model was chosen to estimate yield.

Careful evaluation of the multiplicative model reveals that yields are depressed at high levels of precipitation. This phenomenon is rather difficult to explain, particularly when one considers that the experiments were conducted under dryland conditions. Under semi-arid conditions, moisture has often been thought to be the factor most limiting grain yields. Hence, this result does not seem plausible in lieu of a priori information.

One possible explanation for this occurrence may be found in the distribution of these high levels of precipitation. If large amounts of rainfall were received during a short time period, adverse effects on yield could result from increased weed infestation and disease. Methods analyzed to control weeds were not recorded for this group of experiments. As a result, competition with weeds for available nutrients and moisture may have confounded response results. It is the opinion of this researcher that the yields predicted by the model at these high levels of precipitation do not reflect normal.

yield relationships. After consultation with other researchers, a decision was reached to place a constraint on the net effects of precipitation in the model. The constraint involved the marginal product of precipitation. Values were specified for precipitation, applied nitrogen, and soil nitrate in order to force the first derivative of yield with respect to precipitation (marginal product of precipitation) equal to zero at a specified level. In this case, the first derivative was forced to equal zero at a level of 100 pounds of applied nitrogen, 13 inches of seasonal precipitation, and 10 pounds of available soil nitrate. Specifically, the constraint can be expressed as

$$b_{10} + 26b_{11} + 100b_{12} + 10b_{16} + 10,000b_{18} = 0$$

where b_i 's refer to regression parameters in the multiplicative model. Parameters were then estimated with this constraint on the model.

Constrained Multiplicative Error Model

Table 7 contains regression results for the constrained model. T-ratios for terms which included precipitation as a variable were generally reduced by the imposition of the constraint. The greatest decline in significance occurred in the linear and quadratic precipitation terms which is not surprising since the constraint forces less curvature on the quadratic relationship. Both the linear and quad-

TABLE 7. REGRESSION COEFFICIENTS FOR THE CONSTRAINED MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.15).

Variable Description	Regression Coefficient	t-value
Intercept	25.902000 ***	3.091
Variety 2 Binary (Warrior)	12.806000 ***	2.864
Variety 3 Binary (Cheyenne)	3.266100	1.166
N	-0.084284	-1.240
N ²	0.000275	0.354
SN	-0.006196	-0.122
SN ²	-0.000038	-0.914
PREC	0.959140	0.554
PREC ²	-0.105220 *	-1.414
N x PREC	0.033287 ***	3.287
N x SN	-0.000114 **	-1.751
PREC x SN	0.010248 *	1.623
N ² x PREC	-0.000165 *	-1.452
Multiple R ² = 32.5%		
* p = .10 ** p = .05 *** p = .005		

N = Total applied nitrogen (lb/A).

SN = Total NO₃-N to 4' in early spring (lb/A).

PREC = Total April-July precipitation (in.).

ratic terms dropped below the .05 level of significance. Comparison of Tables 5 and 8 shows that the constrained function is much flatter over the range of precipitation values than the unconstrained equation.

As with the other models, care must be exercised in evaluating the signs of estimated parameters. Ordinarily, a positive linear term and a negative quadratic term indicate diminishing returns to the variable input under consideration. However, in an analysis of this type, the effects of interaction terms must be taken into account before the nature of the response function can be determined. Second degree interaction terms are of the hypothesized sign. For instance, a positive coefficient for b_{12} indicates that the effect of applied nitrogen is most beneficial when precipitation levels are high. Similarly, the coefficient for the interaction between soil nitrate and precipitation is also positive. A negative coefficient for b_{13} implies that soil nitrate and applied nitrogen are substitutes by nature. This result is in agreement with previous research (20). In the case of the third degree interaction between applied nitrogen squared and precipitation, the expected sign is not clear, as a priori information was not available for this term.

Interactions appear to be very important in explaining yield variation in the model. The t-ratio for the interaction between applied nitrogen and precipitation suggests that this term exerts

