

THE EFFECT OF ASSESSMENT/INSTRUCTION IMPLEMENTING A “RULE OF
FOUR” ON THE MATHEMATICS ACHIEVEMENT OF ELEMENTARY
EDUCATION MAJORS

by

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ABSTRACT

The mathematics content knowledge of elementary education majors is well documented as being weak in most instances. Mathematics content courses, aimed at helping pre-service elementary teachers become more competent and confident in their knowledge of the mathematics they will teach, often provide inadequate time for practice and demonstration of mathematics knowledge and skills.

As a response to this issue, this study examined how implementation of assessments based on a “Rule of Four” might increase the mathematics content knowledge of pre-service elementary teachers. The “Rule of Four” performance tasks used in the study required the students to demonstrate their understanding of mathematics concepts relating to addition, subtraction, multiplication and division of whole numbers, integers, and fractions. Three of the four performance tasks devised for each concept aligned with the following developmental levels advocated by Jerome Bruner: enactive (use of manipulatives), iconic (drawing a picture or diagram), and symbolic (writing the concept in words). The fourth performance task required demonstration of an application or real world representation of the mathematical concept.

A quasi-experimental study was conducted using a control group taught and assessed using traditional methods and a treatment group taught traditionally and assessed using the “Rule of Four” performance assessments along with other traditional assessment tools. Pretests and posttests were given to both groups to determine whether there was a difference in the two groups’ mathematics achievement and when gender, age, or class (e.g., freshman, sophomore) was factored in.

Because the control and treatment groups were significantly different in their pretest scores (with the control group having higher scores), an analysis of covariance was used for data analysis. Results showed that there were no significant differences between the groups on their posttest scores, including when gender, age, or class was factored in.

The study’s results indicate that assessment by “Rule of Four” did not produce higher levels of mathematics achievement than instruction using traditional assessment methods. While there was no difference in performance on the posttest by the two groups, the treatment group made substantial gains from their pretest scores.

CHAPTER 1

DEVELOPMENT OF THE PROBLEM

Introduction

Institutions of higher education that are charged with providing preparation for prospective elementary and secondary teachers are being held accountable for the quality of their graduates in these programs. The reauthorization of public law PL 107-110, the Elementary and Secondary Education Act (ESEA) was signed into law on January 8, 2002, by President George W. Bush. It mandates a “highly qualified” teacher in every elementary and secondary classroom. The intent of this requirement is to raise academic achievement for all students and to hold schools and teachers accountable in the process of meeting that goal.

Context of the Problem

To be “highly qualified” according to the ESEA, also known as the No Child Left Behind Act of 2001 (NCLB), a teacher must hold majors in the core content areas in which that person teaches, or the teacher must demonstrate how the definition of “highly qualified” is met as determined by the state in which the teacher is licensed. In North Dakota the Education Standards and Practices Board (ESPB) is responsible for establishing the criteria teachers must meet to demonstrate that they meet the “highly qualified” definition.

Currently elementary teachers in North Dakota need only to have completed a major in elementary education to be “highly qualified.” However, July 1, 2006, all elementary education majors applying for licensure in North Dakota will need to have achieved a minimum score of 158 on the Praxis II grades 1-6 content test, Test Form 0011, and achieved a minimum score of 162 on the K-6 Principles of Learning and Teaching test, Test Form 0055, in order to meet the new definition. Additionally, those elementary education majors planning to teach middle school mathematics must also achieve a minimum score of 148 on the Praxis II test for middle school mathematics, Test Form 10069, in order to be licensed at that level.

Minot State University has more than 50 students graduating from the elementary education program each year. A concern of the university is that all of its graduates in elementary education will be able to pass the appropriate Praxis II exams in order to be licensed in North Dakota and in other states where they might seek employment. The types of questions asked on the Praxis II elementary content and middle school mathematics tests require the test takers to explain how they arrive at particular answers or describe a process used to determine a solution to a problem. It is unclear at this time whether elementary majors at Minot State University will be able to adequately respond to these types of questions.

Many elementary education majors have inadequate backgrounds in mathematics. As part of their undergraduate program of study at Minot State University, elementary education majors are required to complete a four-credit college algebra course, Math 103, and pass it with a grade of “D” or higher. The college algebra course is a requirement in

the general education component for most programs of study at Minot State University. Those students who wish to gain North Dakota licensure in the area of middle school mathematics must have a passing grade in college algebra on their transcripts. All elementary education majors are also required to complete Math 277 and Math 377, semester long three-credit and two-credit mathematics content courses, respectively, geared specifically for their program of study. Teacher/student contact time for Math 277 is approximately 48 clock hours each semester. For Math 377 teacher/student contact time is approximately 32 clock hours each semester. Both courses must be passed with grades of “C” or higher. (Syllabi for the Math 277 and Math 377 courses are in Appendices D and E.) Even when Minot State University elementary education majors meet the program requirements for mathematics, it is uncertain whether they understand and retain enough of the mathematics learned in order to effectively teach it to students.

There is not sufficient time allocated to the current courses to adequately address all of the topics that should be covered in each course. There is not sufficient class time for students to gain experience and confidence in use of manipulatives for teaching and learning mathematics concepts. As a result, course instructors are unsure of students’ abilities to use manipulatives effectively to teach relevant concepts. Often there is not enough time for the instructor to observe each student working with manipulatives or to determine whether they have misconceptions or misunderstandings about the content and how the manipulatives illustrate concepts.

Students’ understanding of mathematics concepts tends to be assessed primarily through paper and pencil testing. While there are some essay questions that require

students to explain concepts, mathematical processes, or reasoning, class sizes prohibit extensive use of such questions.

In order to raise the level of confidence in the mathematical understanding of the elementary education majors at Minot State University, a proposal to make programmatic changes has been suggested. The proposal would eliminate the college algebra course as a general education requirement for elementary education majors, except for those who wish to get the middle school mathematics endorsement for licensure. Elementary education majors who would not take the college algebra course would need to demonstrate that they have the algebraic skills at a level that would meet the Minot State University entrance requirements for the college algebra course. This can be accomplished by having an ACT mathematics score of 21 or higher, by attaining a minimum score of 66 on the ACT Compass exam, or by having a grade of “C” or higher in Math 102, an intermediate algebra course intended to prepare students for college algebra.

The four program credits for college algebra and the five credits for Math 277 and Math 377 would be reallocated in order to create three three-credit replacements: Math 277, Math 278, and Math 377. These courses would each meet three times a week for regular classes and one time each week for lab. Topics from the current Math 277 and Math 377 would be redistributed among the new courses. A few new topics, including algebraic reasoning, that are not included in the current Math 277 or Math 377 courses, would be added to the curriculum in the new Math 377 course. In the new curriculum

students would gain the opportunity to explore topics that have been given little or no emphasis in the current Math 277 and Math 377 courses due to time limitations.

The laboratory time for each course would allow for student exploration of concepts with manipulatives. It would also allow course instructors to have opportunities to interact with each student one-on-one. This would provide the instructors with greater knowledge of students' levels of understanding of mathematical concepts and algorithms, their ability to use mathematical terminology appropriately, and their ability to reason mathematically. Students would gain the opportunity to experience the types of questions they will encounter on the Praxis II content examination required for licensure.

Additional formal assessments of student learning would also take place during the proposed laboratory setting. These assessments would follow a "Rule of Four," developed for this study, in which students would be required to demonstrate understanding of particular mathematical concepts at four levels: 1) through use of manipulatives, 2) through use of a numeric representation of a property or operational algorithm, 3) through use of an appropriate algebraic, geometric, or real world model with applicable diagrams, charts, or tables, and 4) through use of the written word. Instruction in the current Math 277 and Math 377 addresses all four of these levels of understanding for most concepts. However, assessment of student understanding of a particular concept generally is performed addressing only one or two of the levels of understanding, largely due to time constraints.

The impetus for development of this study's "Rule of Four" came from knowledge of a "Rule of Four" that was developed in the 1980s for the teaching of what

was termed reform calculus. Deanin (2003) stated that when teaching in a manner that implements this reform calculus principle, “every topic should be presented numerically, graphically, symbolically, and verbally” (p. 1). This researcher believed that development and implementation of “Rule of Four” assessment and instruction would enhance elementary education majors’ conceptual understanding of mathematics topics taught in Math 277. This belief is supported by a study by Bookman and Friedman (1998) which showed that reform calculus students were better problem solvers than those calculus students taught using traditional methods, that reform calculus students “were better able to formulate mathematical interpretations of verbal problems and solve and interpret the results of some verbal problems that required calculus in their solution” (p. 118), and that these students “were less confident in their pencil and paper computational skills, but more confident that they understood how calculus and mathematics was used to solve real-world problems” (p. 119).

The “Rule of Four” utilized for instruction in reform calculus could be considered a horizontal model in that the four representations of any concept could be used in any order. None of the representations would be considered as being a prerequisite for understanding of any of the other representations for the same concept. In contrast, the “Rule of Four” used for instruction and assessment in this study would be considered a vertical model where order of presentation matters. The four representations used for the teaching of mathematics concepts in this model are hierarchical, with successive representations matching successively higher levels of students’ intellectual development. While all participants in this study would be taught concepts using the four elements of

the “Rule of Four,” assessment by “Rule of Four” would only be implemented in the sections of Math 277 that comprised the treatment group.

A study by Roddick (1995) found that when presented tasks that could be solved using either procedural or conceptual approaches, students taught using the reform calculus “Rule of Four” generally selected conceptual approaches, while students taught using traditional methods selected procedural approaches. Roddick noted the following difference:

Calculus & *Mathematica* students demonstrated a stronger ability to discuss all aspects of a problem, including both conceptual and procedural issues, while traditional students expressed more uncertainty in their work and were less comfortable in discussions as to how to use other knowledge to check their solutions. (p. 7)

A study by Garner and Garner (2001) found that students from reform calculus courses retain better conceptual knowledge than the students in traditional calculus courses, while traditional calculus students retain better procedural knowledge than reform calculus students. A further finding was that students in reform calculus courses understand concepts before they gain computational competence. This type of mathematical conceptual understanding, which is often missing in elementary education majors, is desirable for them to gain in addition to the procedural understanding which is often over-emphasized.

Results of a study of reform calculus students conducted by Ratay (1993) show that weaker students, those with lower grades, showed high levels of improvement. Those students who had higher grades showed either moderate improvement or no improvement. If similar results can be achieved using this study’s “Rule of Four,” those

students who struggle with mathematics or have weak backgrounds in mathematics should benefit from the multilevel assessments.

Statement of the Problem

Elementary education majors in the Math 277 Mathematics for Elementary Teachers course at Minot State University are required to learn about number systems, their properties, and how to perform operations with numbers within those systems. There is insufficient time allotted for class time to allow for adequate assessment of students' abilities in all of the required areas of study. As a result, insufficient knowledge is gained concerning student knowledge of content by the end of the course. It is unclear whether the students will have the knowledge needed to pass the mathematics portion of the North Dakota licensure tests which will be required starting in July of 2006.

Purpose of the Study

The purpose of this comparative study is to determine whether implementation of assessment utilizing a "Rule of Four" in Math 277 would significantly raise the level of mathematics achievement for Minot State University elementary education majors.

Significance of the Study

Having knowledgeable and highly qualified teachers in every classroom is a goal advocated by the United States, as prescribed by the No Child Left Behind legislation passed in 2002. If the results of this study show that assessment by "Rule of Four" will

significantly increase the mathematics content knowledge of elementary education majors, this type of assessment could be implemented at other universities to bolster the mathematics content knowledge of prospective elementary teachers. This could have a significant impact on mathematics instruction and student learning in the elementary grades. At Minot State University results of the study will inform the members of the Department of Mathematics & Computer Science as it considers requesting changes in the elementary education program.

Limitations

A limitation of the study was that students were not randomly assigned to Math 277 sections each semester. Students determined their own schedules with the assistance of their advisors. Thus student self-selection was used to determine the class make-up for each section of Math 277. Additionally, students do not all take Math 277 at the same point in their programs of study. Student enrollment placement in Math 277 was determined by a variety of factors. Those factors included but were not limited to scheduling conflicts with other courses required in their programs of study, preference for either morning or afternoon times, conflicts with job times, and student study mates wishing to be in the same section.

Another limitation of the study was the number of hours of teacher/student contact time in Math 277. Fitting the assessments utilized in the “Rule of Four” into the course while addressing all of the required content was difficult and created a burden on the instructor. The course instructor was limited to using the “Rule of Four” assessments

as an extra feature of the course and not as a factor for students' grades due to the design of the study.

A final limitation was that the study was conducted over two semesters of a single academic year. The total number of students in the four sections may have been too low to have enough students in each of the control and treatment groups' subgroups identified by gender, age, or class.

Delimitations

A delimitation of the study was that the variables that were used in this study (group membership, age, gender, class, GPA, grade in college algebra, and ACT math score or COMPASS score) may not be the only ones that affect mathematics achievement. These variables also may not be the most significant variables affecting mathematics achievement. A second delimitation was that the study was conducted during the fall and spring semesters of the 2005-2006 academic term at Minot State University in Minot, ND. A third delimitation was that the study was limited to students enrolled in Math 277—Mathematics for Elementary Teachers I at Minot State University during the fall 2005 and spring 2006 semesters.

Research Hypotheses

The study will test the following hypotheses:

1. There is no significant difference in the mathematics achievement of those students who participate in assessments using a “Rule of Four” and the mathematics achievement of those students not involved with “Rule of Four” assessments.
2. There is no significant difference in the mathematics achievement of males and females who participate in assessments using a “Rule of Four” and the mathematics achievement of males and females who are not involved with “Rule of Four” assessments.
3. There is no significant difference in the mathematics achievement of students 25 years of age or younger who participate in assessments using a “Rule of Four” and the mathematics achievement of those students 26 years of age or older who are not involved with “Rule of Four” assessments.
4. There is no significant difference in the mathematics achievement of underclass students (freshmen and sophomores) who participate in assessments using a “Rule of Four” and the mathematics achievement of upper class students (juniors and seniors) who are not involved with “Rule of Four” assessments.

Definition of Terms

For the purpose of this study, mathematics achievement will be defined as student’s scores on a pretest and a posttest.

Summary

This study will focus on the level of mathematics achievement of elementary education majors taking Math 277: Mathematics for Elementary Teachers I at Minot State University. It addresses the need of adequately assessing the mathematics content knowledge of these prospective elementary teachers. The researcher has developed a series of assessment tasks and rubrics using a “Rule of Four” similar to the “Rule of Four” implemented in the teaching of reform calculus. These assessment tasks will be used with students in some sections of Math 277 with traditional methods of assessment used in the other sections. Scores on a pretest and posttest for students in each group will be compared to determine whether assessment using a “Rule of Four” improves mathematics achievement.

CHAPTER 2

LITERATURE REVIEW

Introduction

The focus of this study was to determine whether instruction and assessment by a “Rule of Four” (using manipulatives, numeric representations of properties or operational algorithms, appropriate algebraic, geometric, or real world models with applicable diagrams, charts, or tables, and the written word) would significantly increase mathematics achievement of pre-service elementary teachers. The study assessed the NCTM standards-based number and operation knowledge of these prospective elementary teachers.

Criteria for Selecting the Literature

The literature reviewed provides insights on how students learn mathematics, factors that impact student learning of mathematics, and the best practices for facilitating the learning of mathematics. Literature pertaining to how students learn mathematics included examination of the learning theories of Jean Piaget and Jerome Bruner and their implications for the teaching of mathematics. Literature was included that informs best practices for facilitating mathematical learning, including the number and operation content standard and the communication, connections, and representation process standards of the National Council of Teachers of Mathematics. Additionally literature on

assessment of student learning and alignment of assessment with instruction was examined. Literature was included that relates to factors that impact student learning of mathematics. That literature addressed teachers' content knowledge, student attitudes toward mathematics, and demographic and instructional factors that may be predictors of mathematics achievement.

Development of Conceptual Understanding of Mathematics

The work of Jean Piaget in the area of child development and learning demonstrated a need for teachers of mathematics to use manipulatives as young children were developing the concepts of number, conservation, and one-to-one correspondence. Piaget believed that, "The child must be given opportunities for active exploration of materials in order to understand how certain phenomena take place. It is only through such manipulation of objects that the child can begin to understand the operation of his acts" (Singer & Revenson, 1996, p. 109).

Piaget's learning theory was that of a constructivist, where "the emphasis on innate potential, internal maturation and the interaction of the pupil with a passive environment is the start" (Sutherland, 1989, p.104). Sutherland described the teacher's role in Piagetan classrooms as one of providing a "stimulating but structured environment" (p. 103). Within this environment the student would be free to explore while implementing strategies brought from outside the school, the primary source of student learning for the constructivist.

The learning theory of Jerome Bruner at first glance looks very much like the learning theory of Piaget, in that both advocated for student learning through exploration rather than through rote learning of rules and algorithms (Sutherland, 1989). But Bruner did not agree with the notion that the learning environment is a passive domain for the teacher. Rather Bruner viewed the role of the teacher as that of an interventionist. Sutherland described the interventionist role as “the need for dynamic intervention by the teacher to facilitate that interaction by asking challenging questions and providing stimulating, didactic material which the child would not normally encounter” (p.104).

Another key difference between the theories of Piaget and Bruner is in the timeline for the intellectual development of children. Bruner (1963) stated that Piaget’s work suggests three distinct stages of intellectual development beyond developmental stage one, the sensory-motor stage from ages 0 to 2. Stage two, in which relationships between experience and action are established, ends at approximately age five or six. That stage is also known as the preoperational stage. In stage three, known also as the stage of concrete operations, the child takes information or data about the real world and then manipulates it. This manipulation may be in the form of organizing the data or selecting particular data and using it to solve problems. Children move from stage three to stage four, the stage of formal operations, between the ages of ten and fourteen. In the fourth stage children hypothesize about things they have not experienced and use deduction or tests to determine truth or validity of the hypotheses.

In contrast to the lengthy process of progressing through Piaget’s last three stages of intellectual development, Bruner (1963) hypothesized “that any subject can be taught

effectively in some intellectually honest form to any child at any stage of development” (p.33). Thus the focus of his learning theory was on how children think, how they learn, and how they can be helped to learn (Gallenstein, 2003).

Bruner identified three stages of intellectual development through which children progressed. Unlike Piaget, Bruner believed that learning in each stage was not limited to a particular range of ages (Gallenstein, 2003). The first of Bruner’s developmental stages is called enactive or concrete. Children in this stage of development take actions on objects, which in the learning of mathematics translates to using manipulatives. The second stage is known as iconic or pictorial. As stated by Gallenstein, in this stage of development children “can express their understandings through conversation or by creating a mental image or picture of their concrete understanding” (p. 17). The pictorial stage is a transitional stage connecting concepts learned through the actions on objects to symbolic representations of those concepts. The third stage is the symbolic stage where concepts are represented abstractly by symbols and words (Gallenstein, 2003; Tomic & Kingma, 1996; Bruner et al., 1966). Children were expected to develop to a point where they would primarily use symbolic representations to accomplish mathematical tasks. Bruner’s stages of intellectual development support the tasks designed to implement this study’s “Rule of Four.”

Bruner’s Process of Concept Attainment

The learning of a concept or the understanding of categorical distinctions relative to a concept has been termed as concept attainment. Bruner, Goodnow, and Austin (1999)

stated that the process of concept attainment includes the following tasks: achieving relevant information, retaining information gained from potentially relevant experiences so it may be of later use, and transforming retained information so that it can be used in the testing of future hypotheses. They went on to describe the process of concept attainment as a series of decisions in which the earlier decisions affect the degrees of freedom that are possible for later decisions. Patterns or regularities in these decisions were termed as strategies.

Bruner et al. (1999) identified the following objectives for strategies:

- a. To insure that the concept will be attained after the minimum number of encounters with relevant instances.
 - b. To insure that a concept will be attained with certainty, regardless of the number of instances one must test en route to attainment.
 - c. To minimize the amount of strain on inference and memory capacity while at the same time insuring that a concept will be attained.
 - d. To minimize the number of wrong categorizations prior to attaining a concept.
- (p. 104)

The strategies implemented by individuals vary based on circumstances and situations encountered during the decision making process. Factors that affect the decision making process, as stated by Bruner et al., included the definition of tasks, the nature or attributes of instances encountered, the nature of validation of instances as exemplars or non-exemplars, consequences of incorrect and correct categorizations, and the nature of imposed restrictions such as time or cost restraints (p. 105). The strategy objectives and the factors influencing the concept attainment decision making process have implications for mathematics instruction.

Use of Manipulatives for Learning and Instruction

Jerome Bruner's work gained prominence in the 1960s. The enactive developmental stage defined by Bruner required that children work first with objects that they could manipulate in order to gain understanding of mathematical concepts (Gallenstein, 2003). In the early 1970s Zoltan Dienes (1973) developed what were known as Dienes multibase arithmetic blocks. These blocks, among the first standardized mathematics manipulatives, were used to represent numbers in various bases, including base 10. The base 10 blocks that are used today use his design, where small cubes, rods, flats, and large cubes are each used to represent the one's place, and the first, second, and third place to the left of the one's place, respectively. Each of these pieces was scored or marked to show how they would be made using 1s. Because these small cube representations did not come apart, students using them to do addition and subtraction had to make exchanges in order to show what has traditionally been called "carrying" or "borrowing."

Dienes (1973) also suggested using shapes with various qualities to help students at various levels, elementary through secondary, learn about sets and operations on sets. He suggested creating sets of pieces that represented all combinations of three shapes, three sizes, and three colors. These sets and the accompanying games he created are described in detail in the article. The pieces he described are included in larger sets of pieces, now known as attribute blocks. The games that Dienes described require game players to manipulate the objects by either keeping or giving away or addition of pieces

that have particular attributes or qualities, with the actions representing set operations such as union, intersection, and complement or negation. Dienes felt that only through the use of such physical manipulation of pieces with easily identified properties could young children and adolescents develop an understanding of sets and set operations.

Meira (1998) and Hall (1998) both addressed the issue of transparency of mathematics manipulatives. Transparency in Meira's words is "an index of learners' access to mathematical knowledge and activities" through use of the manipulative (p. 122). Hall described transparent manipulatives as those that "show both the teacher and the student what the student is thinking" (pp. 38-39). In other words, transparent manipulatives are those that make it easy for the student to connect the process of manipulating materials to the underlying mathematical concept or process. The point was made that planning is required to find appropriate manipulatives to be used in learning particular concepts so that the level of transparency is as high as possible. Waite-Stupiansky and Stupiansky (1998) and Stein and Bovalino (2001) also stressed that having a well-defined task to be accomplished through use of manipulatives is essential for student learning. They also pointed out that students must have time to explore with the manipulatives without having the teacher trying to give too many directions. This permits students to come up with their own strategies and alternate methods for completing the task, methods that the teacher may not have even considered. Both studies also indicate that students need time to reflect on their manipulations and their strategies in order to foster the connections to the underlying mathematical concepts. In addition, students need to have opportunities to communicate orally and in writing about their

manipulations and efforts with other students and with the teacher, as it also helps students to make connections.

Most pre-service teachers have not experienced learning mathematics with manipulatives, yet they are expected to be able to use them to teach mathematics. This was the focus of several studies in which attitudes toward manipulative use and the learning of mathematical concepts via activities involving manipulatives of preservice teachers were examined. Spungin (1996), Steele and Widman (1997), Quinn (1998), and Thatcher (2001) all found that preservice teachers' mathematical knowledge and understanding grew when they were taught mathematics with methods, including use of manipulatives, consistent with the NCTM standards. They also found that the preservice teachers gained an understanding of the need for providing processing and reflection time for their students, as it was an important time in their own learning during which connections to prior mathematical knowledge were made.

Sawada (1996) examined the use of manipulatives in Japanese elementary schools in an effort to determine how students arrive at connections to prior knowledge of mathematics. He determined that one factor influencing the formation of connections occurred through what he termed manipulative fluency. Sawada noted that teachers in the United States teach with a wider variety of manipulatives than do teachers in Japan, who use one basic set of manipulatives in many ways. Citing and agreeing with Dienes' Multiple Embodiment Principle, which espouses value in variety of mathematical representations, Sawada suggested that too much variety might be problematic. Sawada (1996) suggested the following:

The frequent use of a basic set of concrete manipulatives within a definitely constrained variety was important in becoming fluent in mathematics. The type of fluency under development is not unlike the kind of fluency that is important in acquiring a language. This kind of fluency is the establishment of connections that become so well known they are felt “in the bones.” (p. 260)

Salend and Hofstetter (1996) found in their study that using manipulatives in the context of a problem solving approach is beneficial for students who have mild learning disabilities. This approach helps students learn how to reason and communicate mathematically. It enables the students to learn math concepts because the method is non-threatening and “makes the connections between mathematics and their everyday lives” (Salend & Hofstetter, 1996, p. 211).

Virtual manipulatives were the subject of a few studies. Hennessy et al. (1988), Ippel (1992), and Moyer, Bolyard, and Spikell (2002) all promoted the use of virtual manipulatives, with the Moyer study defining them as being “dynamic visual representations of concrete manipulatives ... essentially ‘objects’” rather than “static visual representations” (Moyer et al., 2002, p. 372). They are true objects in that they can be moved and adjusted and manipulated through a mouse or keyboard commands. Such objects include interactive applets that demonstrate mathematical properties or concepts and computer generated models of multibase blocks. Dorward (2002) studied three groups of students who were each taught using one of three methods: using virtual manipulatives, using physical manipulatives, or using no manipulatives. In that study, no significant differences were found among the three groups. It was noted in the results, however, that scores were influenced by several other factors.

A recent study also provides support for Bruner's belief that students should work first with physical objects as they learn concepts. The study, sponsored by Hobby Industry Association (2002), examined the impact of hands-on craft projects on student learning when the projects are implemented as an instructional method within the academic core's curriculum. The academic core was identified as mathematics, science, language arts, and social studies.

The Hobby Industry Association (2002) study utilized teacher surveys, student surveys, and assessments of student learning on knowledge application tasks. The knowledge application tasks required students "to apply knowledge of what they had been studying to a new, not previously studied, situation. The main research purpose of these tasks was to obtain a measure of students' ability to apply or transfer knowledge" (p. 3). Seventy-six teachers in grades kindergarten through grade six and their 1600 students from urban, suburban, and rural schools in several states across the country participated in the study. They were not randomly selected.

Results of the Hobby Industry Association (2002) study included the following conclusions:

- Student learning improves when classroom lessons incorporate hands-on craft projects.
- Teachers regularly use hands-on craft projects to teach the core subjects and link the projects to state and national curriculum standards.
- Students develop greater curiosity about the subject matter when hands-on craft projects are included.
- Teachers say learning through hands-on craft projects accommodates students with different learning styles.
- Student behavior and socialization skill improve when hands-on craft projects are undertaken. (pp. ii-iii)

A conclusion drawn in the study was that “hands-on projects appear to function as learning anchors that organize and integrate various classroom learning activities” (p. ii). The study reported significant differences in students’ ability to transfer previously learned skills and knowledge to new contexts between students who took part in hands-on projects and those who did not, with the hands-on projects students demonstrating higher levels of transfer. DeGeorge and Santoro (2004) cited the Hobby Industry Association study as confirming the following commonly held belief of teachers and principals:

Active learning experiences using manipulatives appear to function as anchors that organize and integrate classroom learning, helping make aspects of what students need to learn more visible than abstract, conceptual instruction. (p. 28)

Features of Brunerian Instruction

Gallenstein (2003) stated that all three of Bruner’s learning modes are found in today’s mathematics classrooms where students work with manipulatives, use memories of visual, auditory, and kinesthetic mathematics experiences to facilitate mental mathematics, and use mathematical symbols with meaning. According to Gallenstein, students “take ownership of their knowledge by re-creating, reinventing, reconstructing, and redefining concepts on their own” when they move through the three stages of learning (p. 18). In classes that operate utilizing Brunerian instruction, students would be provided opportunities to learn through discovery learning and through problem solving situations. The focus in such instruction, according to Gallenstein, would be for the students to seek intrinsic rather than extrinsic rewards when learning new concepts.

Hopkins (1981) described Brunerian instruction as having its emphasis “on the development of the intellect for independent inquiry” (p. 275). He went on to state that while educational experiences would start with some basic knowledge and an area of interest to the student, it should rapidly move toward thinking about related concepts abstractly with an emphasis on analysis and synthesis. Gallenstein indicated that the most important goal of instruction in the Brunerian philosophy is learning how to learn. This goal would be supported by focusing on the structure of knowledge and the methodologies needed to create that knowledge. Teachers who embraced this philosophy would amply reward students for implementing mathematical processes correctly and give only small penalties for simple computational errors.

A list of requirements needed for teachers who implement Brunerian instruction was provided by Ediger (1999), with the following items included on the list:

1. The teacher needs to have an excellent knowledge of the structure of knowledge since these key ideas become objectives for learner attainment.
2. To achieve objectives on the pupils’ part, the teacher needs to sequence learning opportunities in that individuals experience the enactive, the iconic, and the symbolic in that order.
3. The teacher must appraise pupils to ascertain how many of structural knowledge objectives are being attained by pupils in a spiral curriculum. With a spiral curriculum, pupils meet up again again [sic] in increasing levels of complexity the structural ideas which serve as objectives of instruction.
4. The teacher needs to become a quality asker of questions involving the ongoing mathematics lesson so that pupils can truly learn in an inductive manner. Inductive teaching then assists pupils to achieve the structural ideas.
5. Inductive teaching in mathematics must be used together with the enactive, iconic, and symbolic materials of instruction. (p. 12)

Several studies, which are described later in this chapter (e. g., Allen, 2003; Goldhaber & Brewer, 1996; Haycock, 1998), identified teacher knowledge as a crucial factor in students’ mathematics achievement. These studies support the first teacher requirement in

Ediger's list. Bruner's (1963) support of the second teacher requirement listed above is shown in his statement that "what is most important for teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization of more conceptually adequate modes of thought" (p. 38).

Tomic and Kingma (1996) cited Bruner's belief that teachers should present students with problem solving situations that motivate and stimulate students to discover subject structure or fundamental concepts on their own. They also stated that teachers should design educational experiences that would facilitate students' learning through inductive reasoning, defined as arriving at a general principle from observing or encountering multiple examples and details. Lawton, Saunders, and Muhs (1980) stated the following implications for education when the discovery learning advocated by Bruner is implemented:

a) the underlying principles that give structure to subject matter should be used to determine curricula, b) concepts should be developed and redeveloped in a "spiraling" sequence towards greater levels of abstraction to facilitate the acquisition of generic codes, and c) efforts should be directed towards improving the individual's ability to recognize the plausibility of guesses. (p. 133)

Lawton et al. went on to state that acquisition of generic codes was the most important factor in real learning and that these generic codes lead to "the highest degree of transfer and to a lasting retention of information" (p. 133).

Bruner (1986) also advocated for multiple models of the learner based on his view that there is more than one kind of learning. Bruner went on to state that the models of the learner are to be treated as stipulative, that is, that each model should be used under conditions where its view of the learner "might be effective or useful or comforting" (p.

200). Samples (1992) agreed with Bruner and based his definitions of five learning modalities on Bruner's iconic, enactive, and symbolic classes of knowing and were identified in the following table on the next page.

Table 1: Learning Modalities by Bob Samples.

Modality	Definition	Adapted from Bruner	Short Form
Symbolic Abstract	The coding and processing of experience represented in symbols (for example, reading writing, and ciphering)	Symbolic	3Rs-Based
Visual-Spatial	Sensing, reasoning, and expressing experience through the visual and spatial media	Iconic	Arts-Based
Kinesthetic	Sensing, reasoning, and expressing experience through the visual and spatial media	Enactive	Movement-Based
Auditory	Sensing, reasoning, and expressing experience through patterned sound and music	Complex	Sound-Based
Synergic	More complex combinations of the previous modes (for example, visual-spatial and auditory)	Complex	Combination

(Samples, 1992, p. 64)

Samples also cited the work of Gregory Bateson and Howard Gardner's multiple intelligences as supporting multiple learning modalities. According to Samples, Bateson viewed learning as requiring a blending of both knowledge and experience, a view contrary to the commonly held belief that knowledge and experience were discrete entities.

Deng (2004) challenged the key premise of Bruner's learning theory which he presents as follows:

The structure of an academic discipline can be transformed via various modes of representation. Therefore, the subject matter of the scholar could be adapted into

different stages of learning readiness instead of waiting for the learner to exhibit readiness. (p. 152)

Deng claimed that the curriculum reform movement of the 1960s, based on Bruner's theories, has not met expectations. He cites several reasons for the failure of the reform movement, each addressing a major component of Brunerian instruction. Those reasons included not taking into account students' interests, experience, and their readiness for learning; the difficulties encountered in teaching and learning with discovery methods, and inadequate preparation of teachers in discovery learning methods.

Research Findings Concerning Brunerian Instruction

A study in Vermont utilized a practicum experience to explore the connection of critical thinking to Bruner's discovery learning methodology. In that study Forest (1978) cited four positive student effects of discovery learning identified by Bruner. Those four effects included expansion of students' intellectual possibilities, a change from external to internal motivation, development of a learning system allowing for further discoveries, and improved retention and recall of previously learned knowledge. Hence discovery learning was viewed as a desirable mode of learning. The development of students' critical thinking skills was viewed as an important component of discovery learning that needed to be specifically addressed and facilitated by teachers. Therefore a critical thinking unit was developed by participants as part of the practicum.

Results of the study showed that "teaching a unit on critical thinking skills to education majors increased their specific knowledge of the [reasoning] competencies and also increased the comfort level of these future teachers to deal with reasoning skills"

(Forest, 1978, p. 10). While an inquiry approach to learning was seen as valid and logical by study participants, the study found that practicum participants deemed that Bruner's book, *The Process of Education*, did not provide sufficient detail on inquiry learning methods.

Winer (1981) conducted a study that tested whether Brunerian learning theory could provide a teacher with a framework for the development of effective learning materials. Results of the study showed that the group taught using materials implementing Brunerian learning theory scored significantly higher on posttests than students in the control group who were taught using a traditional textbook learning approach. Additionally a significant difference in long-term learning and retention was found between the low spatial ability students in the two groups, with the treatment group performing at higher levels than the control group.

Smith's (1979) study examined Bruner's premise that every subject can be successfully learned by a child at any stage of development if it is presented in a form that is recognizable and simple enough to understand. In Smith's study authors of junior high school history texts were asked to lower the reading levels of the materials while still presenting "a detailed, analytical, and understandable picture of some of the important events in American history" (p. 385). The results of the study were mixed, with only one author in the first group succeeding in the task. Most of the texts in that group were not able to depict or provide critical analysis of historic events at the level of the higher reading level textbooks. Thus Bruner's premise was not fully actualized. A somewhat surprising finding of Smith's study was that the lower reading level textbooks

were more successful in showing causes of the Great Depression than were the higher level reading texts, which suggests that it is possible, with concerted effort, to lower reading level and still provide sufficient analysis and detail.

NCTM Content Standard for Number and Operation

In 2000 the National Council of Teachers of Mathematics published *Principles and Standards for School Mathematics*, which identified six principles and ten standards for the teaching of mathematics to students in prekindergarten through grade 12. Five of the standards focus on the mathematics content that students should learn and know. The five content standards address mathematical strands as follows:

- Standard 1: Number and Operations,
- Standard 2: Algebra,
- Standard 3: Geometry,
- Standard 4: Measurement, and
- Standard 5: Data Analysis and Probability.

The remaining five standards focus on the processes through which students acquire and apply that mathematical content. They address the learning processes as shown below:

- Standard 6: Problem Solving,
- Standard 7: Reasoning and Proof,
- Standard 8: Communication,
- Standard 9: Connections, and
- Standard 10: Representation.

Each of the ten standards is addressed in four grade bands. The grade bands used are Pre-K-2, Grades 3-5, Grades 6-8, and Grades 9-12. For each of the content standards, expectations for student performance within each grade band are identified. The content

and performance standards provide a framework for what is considered to be “best practice” for the teaching of mathematics.

The mathematics content knowledge of elementary education majors that is being assessed in this study has number and operation as its focus. It is also the major content focus for the Math 277 course being taken by the study participants. Since graduates of elementary education programs become licensed to teach students in grades 1-8 in North Dakota, the expectations concerning number and operation for grades 9-12 are not examined here.

Each of the three parts of the standard concerning number and operations remains the same for all four grade bands, with only the expectations changing as one progresses through the grade bands. Expectations for student performance concerning number and operation at each of the three younger grade bands are shown in the following tables. Each bulleted item represents a particular skill that students are expected to effectively demonstrate by the time they reach the end of the highest grade in the band. The first component and related expectations of the NCTM Number and Operations Standard addresses students’ development of the concept of number, their understanding of how numbers can be represented, their knowledge of relationships between numbers of various types, and their understanding and use of a variety of number systems. The second component of the standard addresses students’ understanding and use of mathematical operations and how they relate to other operations. The final component of the number and operations standard addresses the development of students’ computational fluency and their competence in making and using reasonable estimates.

Table 2 contains expectations for students in prekindergarten through grade 2.

Table 2. Number and Operations Standard for Grades Pre-K-2.

Instructional programs from prekindergarten through grade 12 should enable all students to—	Grades Pre-K-2 Expectations
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	<p data-bbox="657 447 1383 489">In prekindergarten through grade 2 all students should—</p> <ul style="list-style-type: none"> <li data-bbox="657 527 1383 600">• Count with understanding and recognize “how many” in sets of objects; <li data-bbox="657 604 1383 678">• Use multiple models to develop initial understandings of place value and the base-ten number system; <li data-bbox="657 682 1383 783">• Develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections; <li data-bbox="657 787 1383 888">• Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers; <li data-bbox="657 892 1383 993">• Connect number words and numerals to the quantities they represent, using various physical models and representations; <li data-bbox="657 997 1383 1087">• Understand and represent commonly used fractions, such as $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$.
Understand meanings of operations and how they relate to one another	<ul style="list-style-type: none"> <li data-bbox="657 1094 1427 1194">• Understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations; <li data-bbox="657 1199 1427 1272">• Understand the effects of adding and subtracting whole numbers; <li data-bbox="657 1276 1427 1381">• Understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.
Compute fluently and make reasonable estimates	<ul style="list-style-type: none"> <li data-bbox="657 1388 1427 1461">• Develop and use strategies for whole-number computations, with a focus on addition and subtraction; <li data-bbox="657 1465 1427 1539">• Develop fluency with basic number combinations for addition and subtraction; <li data-bbox="657 1543 1427 1648">• Use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.

(NCTM, 2000, p. 78).

Table 3 shows the expectations for students in grades 3-5 concerning the standard for number and operation.

Table 3. Number and Operations Standard for Grades 3-5.

Instructional programs from prekindergarten through grade 12 should enable all students to—	Grades 3-5 Expectations
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	<p data-bbox="656 373 1101 405">In grades 3-5 all students should—</p> <ul style="list-style-type: none"> <li data-bbox="656 451 1382 558">• Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals; <li data-bbox="656 564 1349 669">• Recognize equivalent representations for the same number and generate them by decomposing and composing numbers; <li data-bbox="656 676 1414 781">• Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers; <li data-bbox="656 787 1344 856">• Use models, benchmarks, and equivalent forms to judge the size of fractions; <li data-bbox="656 863 1414 932">• Recognize and generate equivalent forms of commonly used fractions, decimals, and percents; <li data-bbox="656 938 1393 1008">• Explore numbers less than 0 by extending the number line and through familiar applications; <li data-bbox="656 1014 1235 1083">• Describe classes of numbers according to characteristics such as the nature of their factors.
Understand meanings of operations and how they relate to one another	<ul style="list-style-type: none"> <li data-bbox="656 1089 1360 1159">• Understand various meanings of multiplication and division; <li data-bbox="656 1165 1354 1234">• Understand the effects of multiplying and dividing whole numbers; <li data-bbox="656 1241 1414 1346">• Identify and use relationships between operations, such as divisions as the inverse of multiplication, to solve problems; <li data-bbox="656 1352 1377 1425">• Understand and use properties of operations, such as the distributivity of multiplication over addition.
Compute fluently and make reasonable estimates	<ul style="list-style-type: none"> <li data-bbox="656 1432 1414 1537">• Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 X 50; <li data-bbox="656 1543 1377 1612">• Develop fluency in adding, subtracting, multiplying, and dividing whole numbers; <li data-bbox="656 1619 1370 1724">• Develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results; <li data-bbox="656 1730 1398 1835">• Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience; <li data-bbox="656 1841 1393 1875">• Use visual models, benchmarks, and equivalent forms

to add and subtract commonly used fractions and decimals;

- Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tool.
-

(NCTM, 2000, p. 148).

Table 4 shows the expectations for students in grades 6-8 concerning the standard for number and operation.

Table 4. Number and Operations Standard for Grades 6-8.

Instructional programs from prekindergarten through grade 12 should enable all students to—	Grades 6-8 Expectations
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	<p>In grades 6-8 all students should—</p> <ul style="list-style-type: none"> • Work flexibly with fractions, decimals, and percents to solve problems; • Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line; • Develop meaning for percents greater than 100 and less than 1; • Understand and use ratios and proportions to represent quantitative relationships; • Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; • Use factors, multiples, prime factorization, and relatively prime numbers to solve problems; • Develop meaning form integers and represent and compare quantities with them.
Understand meanings of operations and how they relate to one another	<ul style="list-style-type: none"> • Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers; • Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals; • Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.

Compute fluently and make reasonable estimates

- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;
- Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;
- Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;
- Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

(NCTM, 2000, p. 214).

NCTM Process Standards: Communication, Connections, and Representation

The five NCTM process standards identify ways in which students acquire and use mathematics content knowledge. Three of the process standards, communication, connections, and representation, have direct bearing on the “Rule of Four” assessment tasks and instruction being used in this study.

The communication, connections, and representation standards provided direct support the “Rule of four” performance tasks and instruction being implemented in this study. Multiple representations of a concept are required through the four tasks, with the concept represented with manipulatives, with numbers and operation symbols, with an algebraic, geometric, or real-world model, and with written words. Connections of the representations to each other and connections to algebra, geometry, or a real-world context are required of each student. Communication of the concepts addressed is

required orally, as the student explains the concept while demonstrating with manipulatives, and in written form.

The communication, connections, and representation standards are stated in Table 5 below.

Table 5. Selected NCTM Process Standards.

Standard	Instructional programs from prekindergarten through grade 12 should enable all students to—
Communication	<ul style="list-style-type: none"> • Organize and consolidate their mathematical thinking through communications; • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • Analyze and evaluate the mathematical thinking and strategies of others; • Use the language of mathematics to express mathematical ideas precisely.
Connections	<ul style="list-style-type: none"> • Recognize and use connections among mathematical ideas; • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • Recognize and apply mathematics in contexts outside of mathematics.
Representation	<ul style="list-style-type: none"> • Create and use representations to organize, record, and communicate mathematical ideas; • Select, apply, and translate among mathematical representations to solve problems; • Use representations to model and interpret physical, social, and mathematical phenomena.

(NCTM, 2000, p. 402).

Support for the NCTM Communication Standard

The NCTM (2000) adopted a communications standard because of its belief that “communication is an essential part of mathematics and mathematics education” (p. 60). It advocated that teachers provide frequent opportunities for students to engage in activities that challenge them “to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing” (p. 60). The NCTM has continued to hold the belief that when students have the opportunity to discuss the mathematics they are learning, they gain insights into their thinking concerning methods of solving problems, justifying their reasoning, and formulating questions about things that puzzle them. With regard to writing, the NCTM has stated that inclusion of writing in mathematics helps students to “consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson” (p. 61).

Communication was cited by Pritchard (1994) as an important aspect of instruction that uses Bruner’s concept-attainment approach. Pritchard stated that teachers using this model must “engage with their students in intellectual discourse” (p. 18). The purpose of this discourse was to help students to articulate their views, to clearly identify and describe phenomenon observed, and to present and use evidence to support their hypotheses and reasoning. Bruner (1971), in a revisit of his primary work *The Process of Education*, maintained that a key to instruction is activating the learner, that is, first getting people to want to learn and then making the experience both compelling and

sustained. He suggested that schools might consider having three days each week focus on standard curriculum topics. The remaining days each week would be used by students to prepare what Bruner called “briefs” that expressed their views and rationales for issues that were compelling and of importance to them.

Bruner (1971) also suggested that schools should be learning communities where everyone in the class is responsible for the learning of every student in that class. He also cited cross-age tutoring as a positive learning experience, not only for the person being tutored, but also for the tutor. As Bruner stated, when a student teaches, it makes the student a better learner. The teaching of ideas or concepts to another would require the person teaching to utilize appropriate language, identify relationships, and present logical arguments or rationales. Pritchard (1994) stated that learning environments that foster this type of discourse do not develop in short time frames. They must be nurtured over long periods of time so that meaningful discourse becomes habitual.

Research on the effectiveness of written and oral communication in mathematics instruction and learning has given mixed results. Studies varied greatly in the grade levels of participants, the types of communication activities implemented, and the hypotheses examined. Several studies focused on the impact of writing on student achievement in mathematics, while others focused on the impact of writing on student attitudes toward mathematics, and some focused on both.

Of the studies that addressed the impact of writing on mathematics achievement, Hirsch and King (1983), Kasperek (1996), and Riegler (2005) found no significant differences in mathematics achievement between students in classes that used writing

activities and students in classes that did not include writing activities. DeVaney (1996) found a significant negative association between frequent writing about problem solving and student achievement in geometry. Gladstone (1987), Sharp (1998), and Card (1998) all found significant positive impacts of writing on student mathematics achievement.

Hirsch and King (1983) required teachers, who had students provide written responses for mathematics questions dealing with conceptual knowledge, to be non-directive in providing feedback on the written responses to students. While Hirsch and King did not find significant differences in mathematics achievement between students who used the writing assignments and those who did not, they did make a recommendation that “If teachers are going to use writing assignments, they must integrate them into the math class” (p. 14). Card’s (1998) study examined second-grade students’ mathematics achievement and metacognition through use of interviews and observation of student work which included daily writing activities. It found that both students’ mathematics achievement and metacognition increased. Sharp’s (1998) study showed significant differences between pretest and posttest scores on tests of fraction concepts for elementary students who did journal writing about fractional situations. Students who did not do this journal writing had no significant difference between pretest and posttest scores.

Gladstone (1987) examined the effect that writing instruction had on 9th grade students’ mathematics achievement as measured on the New York State Regency Competency Test in Mathematics. The study also looked at knowledge transfer from one discipline to another. In this case, the transfer examined was from instruction in writing

and language arts to mathematics. To facilitate this Gladstone had students write a guidebook for passing the Regency Competency Test in Mathematics. Results of this study also showed a positive impact of the writing enrichment activities on students' class attendance.

Sierpiska (1997) studied the interaction of textbooks, tutors, and students of a linear algebra course. Sierpiska hypothesized that "the mathematics that is learned at school depends very much on the characteristics and the modes of communication developed in the classroom" (p. 3). Thus all of the interactions that facilitate this communication would become important items to examine. Results of the study, which used discourse analysis methodology, showed positive effects on learning when student and tutor discourse styles and textbook discourse formats matched well. When discourse styles did not match well and did not support student engagement or motivation, learning was negatively impacted.

Studies that examined the effect of writing on student attitudes toward mathematics included studies by Klein, Pfloderer, and Truckenmiller (1998), Kasparek (1996), Riegler (2005), Miller (1992). In Miller's study students were asked to complete impromptu timed writing assignments in which they were to explain a concept, skill or generalization. Key findings included that teachers often incorrectly measured students' levels of understanding and that students have limited understanding of and ability to use mathematics vocabulary.

Klein et al. (1998) used journal writing and cooperative learning experiences designed around Gardner's multiple intelligences in an intervention program intended to

foster and raise student motivation. Students communicated both orally and in writing about the mathematics being learned and about the solving of real-life application problems. Results of the study showed that student participation and engagement increased. These results may have been due in part to greater variety in class routine than there had been prior to the study.

Riegler (2005) examined students' perceptions of their mathematical writing ability and their attitudes toward mathematics by means of a survey given prior to instruction implementing writing assignments and the same survey given after such instruction. Survey results showed no significant change in students' perceptions of mathematical writing ability or in their attitudes toward mathematics.

A study aimed at determining the response of students to reform efforts in mathematics was undertaken by Wilson (1995). A survey was given to eighth grade mathematics students to determine their starting views about what it means to do mathematics. Results of that survey showed that students held traditional views about mathematical tasks and instruction centered about those tasks. Nine months later they were surveyed again after having experienced an academic year of reform-oriented mathematics instruction that included the use of journal writing. Results of the second survey showed that students had negative opinions about journal writing in mathematics. Students also continued to view having the teacher explain concepts to students as a model for mathematics instruction, even though they were receptive to and accepting of mathematical tasks that were non-traditional and exploratory in nature.

Exploration of the use of talk as a vehicle for mathematical learning of second-graders was conducted by Shand (2002). In that qualitative study, students engaged in mathematical experiences where they talked about the mathematics they were learning. Purposes of the talking were to provide opportunities for students to clarify their thinking about mathematical concepts, to compare and negotiate ideas with others, and to answer questions or respond to statements posed by the teacher that would assist students in initiating mathematical learning. Results of the study showed that student learning for study participants was enhanced and enriched by the inclusion of the talk experiences.

A comparative study was conducted by Stephenson (2002) to examine how language was used for social interaction, representation of experience, and socialization as well as how language was used in students' mathematical experiences. Participants for the study were from two fifth grade classes in which the teachers used contrasting methods for teaching mathematics. In what would be termed the "traditional" classroom students used language primarily through teacher-directed Interaction-Response-Feedback sequences. The interactions and responses centered on tasks that required only low levels of cognition of the students. Students in this class had limited opportunities for giving narrative responses or for choosing topics of interactions or methods of response.

The second, contrasting classroom would be termed an "open" classroom where students would have ample opportunities to select from a variety of activities or interactions that required high levels of cognition. The students in this class would also have opportunities to work with a variety of partners and tools to explore concepts and ideas.

Results of the Stephenson (2002) study indicated that students in both groups performed equally well on traditional mathematical tasks. However, students in the “open” classroom performed better than their counterparts in the “traditional” classroom on tasks that were more open-ended, more conceptual in nature, and that required socialization skills. The “open” classroom students also used more conceptual problem solving skills than the students in the “traditional” classroom. Another finding was that students in the “open” classroom had more diverse and connected views of mathematics than students in the “traditional” classroom. Connections that were supported by the study were between mathematics and the students themselves, between mathematics and other disciplines, and between mathematics and student life outside of class.

Support for the NCTM Connections Standard

The belief that students’ understanding of mathematical concepts is deeper and longer lasting if they can connect mathematical ideas was the basis for the NCTM’s (2000) development of a connections standard. The NCTM stated the importance that students “can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience” (p. 64). The NCTM has advocated that in order for teachers to facilitate students’ development of mathematical connections, they must first “know the needs of their students as well as the mathematics that the students studied in the preceding grades and what they will study in the following grades” (p. 64).

Teachers' beliefs about mathematics affect their ability to facilitate students' making of mathematical connections. Stigler and Perry, as cited in Lappan (2000), stated the following:

In order for teachers to help students obtain more authentic and productive notions about mathematics, teachers themselves need to believe that mathematics is more than just memorizing rules. (p. 323)

Yet frequently teachers in the United States have given emphasis to rules and procedures rather than fostering the making of connections (Lappan, 2000). Because of this practice, Lappan has issued the following challenge:

The challenge is to help both mathematics teacher educators and preservice teachers build a strong sense of efficacy around a vision of mathematics, in which solving problems and connecting what one learns are the central activities. This is in contrast to a vision of mathematics that is limited to filling a toolbox with a set of discrete skills and algorithms. (p. 323)

Emphasis in the teaching of mathematics at the elementary level in the United States has contrasted dramatically with mathematics instruction in Japan, where students consistently do well on international tests. Sawada (1996) studied the instructional methods used to teach mathematics in Japanese elementary schools. Results of that study indicated that mathematics was taught as a connection making activity that incorporated use of graphing with a belt graph, implemented multiple solutions and modes of presenting problems, built manipulative fluency with a basic set of concrete manipulatives, had students remain with the same teacher for two or three years, drew on students' experiences, and integrated mathematics across other disciplines.

Gladstone (1987) examined the effect on mathematics achievement and student motivation of writing that was connected to other disciplines. The mathematics writing

done by students in this study took place in an English class. Among the reported results were the following statements:

They [sic] youngsters saw how connections with what they did from class to class applied not only by calling up files from diskette on a daily basis, but also building on the previous day's concepts in basic skills and problem solving. Finally, and perhaps most importantly, they became aware that school has a connection to life. (p. 6)

Helping students to make and value such connections has been a focus of the NCTM connections standard.

Support for the NCTM Representation Standard

Representation, as defined in the NCTM representation standard, “refers to process and product—in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself” (NCTM, 2000, p. 67). While symbolic expressions, numeric expressions, diagrams, and graphs have been frequently used in mathematics instruction, they have often been taught and learned as isolated and unrelated skills, without the recognition that all the forms could be used to represent the same concept. The NCTM (2000) has made the following statement about the role of representation:

Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling. New forms of representation associated with electronic technology create a need for even greater instructional attention to representation. (p. 67)

Representation was also an important element of Bruner's work. Bruner et al. (1966) presented three stages of development in which different forms of representation are featured. Bruner stated that the learner would move through the three forms—enactive, iconic, and symbolic—in that order. Tomic and Kingma (1996) articulately described Bruner's forms of representation. They stated that in the enactive mode, representations of objects or events are created through actions that have been performed with them. Tomic and Kingma also stated that “according to Bruner, the coordination of the various behaviors requires a form of representation: the mental schema originates from the action and the sensory feedback” (p. 14).

With respect to iconic representations Tomic and Kingma (1996) stated that one of its advantages is that representations of this type (accession of mental images) are relatively independent of actions with objects or events. They went on to describe symbolic representation as consisting of codes with linguistic or abstract bases that do not require physical resemblance to reality. An important tenet of this type of representational system was that each form of representation would continue to be available for the learner as the learner moved through the three developmental stages.

Samples (1992) stated that “students engage in ways of knowing other than that which is 3Rs-based. Moreover, these ways of knowing are vital to the mental well-being of students” (p. 63). These ways of knowing included representations that were based on sensory experiences including movement, sound, touch, or smell.

Assessment of Student Learning

In order to determine students' levels of conceptual knowledge and understanding and to ascertain their reasoning and the strategies they use, teachers must design and implement meaningful performance assessments. Stiggins (1997) described performance assessment as "a complex way to assess. It requires that users prepare and conduct their assessments in a thoughtful and rigorous manner" (p. 178). He went on to state that performance assessment cannot rely on teachers' feelings about student knowledge or achievement. Rather they must collect credible evidence that demonstrates students' knowledge or skill relative to particular tasks.

The term "authentic assessment" has become a buzzword in education. Wiggins (1990) defines authentic assessment and contrasts it with traditional assessment as follows:

Assessment is authentic when we directly examine student performance on worthy intellectual tasks. Traditional assessment, by contrast, relies on indirect or proxy 'items'—efficient, simplistic substitutes from which we think valid inferences can be made about the student's performance at those valued challenges. (p. 1)

Wiggins went on to describe authentic assessments as requiring students to effectively use acquired knowledge rather than to merely recognize, recall, or "plug in" knowledge out of context. He also stated that authentic assessments must present the student with a variety of tasks that mirror the tasks undertaken in learning experiences where students use higher order thinking skills and effectively communicate their ideas and knowledge to others.

Wiggins (1993) addressed conventional test design and its inherent flaws. He suggested that good tests must be congruent with reality and not place arbitrary constraints on students in testing situations. Those constraints might include limiting time for completion of testing tasks or restricting methods or resources that can be used to complete the tasks. Wiggins stated that testing needs to be authentic and defined that authenticity as follows:

Authenticity in testing, then, might well be thought of as an obligation to make the student experience questions and tasks under constraints as they typically and “naturally” occur, with access to the tools that are usually available for solving such problems. (p. 208)

Wiggins indicated that congruence of a test with reality was of utmost importance, while coherence of a test with other tests is of secondary importance. Wiggins’ views would indicate that the standardized tests given in educational settings today are not authentic tests.

Wiggins (1990) suggested that both teaching and learning can benefit from assessment when he stated the following:

A move toward more authentic tasks and outcomes thus improves teaching and learning: students have greater clarity about their obligations (and are asked to master more engaging tasks), and teachers can come to believe that assessment results are both meaningful and useful for improving instruction. (p. 2)

In line with this view, assessment is thought by some to be a tool that should be used to facilitate learning in addition to evaluating it.

Arter and Stiggins (1992) suggested that performance assessment can be a powerful tool in instruction because it helps teachers communicate to students and others what is valued in learning experiences. Traditional tests typically do not assess many of

the skills and performances that teachers claim to value, such as writing, critical thinking, and problem solving. In order to assess such skills, teachers must first clearly identify essential learning goals associated with those skills and build learning experiences that facilitate students achieving those goals. However, Arter and Stiggins stated that many teachers are not trained in performance assessment methodology. They also indicated that teachers are often constrained to using traditional curriculum and testing methodologies because of high-stakes testing.

Cross (1986) advocated using assessment to improve instruction at the college level. Cross suggested that “assessment designed for the improvement of teaching should be situation specific, and it should provide immediate and useful feedback on what students are learning” (p. 67). She also suggested that college faculty should be reflective practitioners who can gain insights about their teaching by reflecting on that teaching and articulating what they and their students experience. According to Cross, such reflection when built into classroom assessment as part of a larger institutional assessment program could improve faculty morale through intellectual stimulation relevant to teaching.

The need to address student learning at the undergraduate level was advocated by Cross (1987) as well. Cross identified three major findings of research on teacher effectiveness that could each have positive effects on student learning in the college setting if implemented. One of those findings was that students learn more when they are actively involved in learning situations than they do when they are passive recipients of instruction. Continuation of lecturing on college campuses as the primary method of instruction would be contrary to this finding. Another finding of research on teacher

effectiveness was that students generally learn what they practice. Thus, practice time should be built into teaching and learning experiences to facilitate such practice when related to desired instructional outcomes. Cross indicated that the third research finding was that students would rise to meet expectations if teachers set high but attainable learning goals. All of these findings could be addressed by incorporation of authentic assessments in instruction as described by Wiggins (1990).

Stiggins (1997) identified three components necessary for creation of meaningful performance assessments: clarification of the performance to be evaluated, preparation of the performance exercises, and the devising of systems for scoring performance and recording results. According to Stiggins, the performances to be assessed may include products created or demonstrations of skill. Each product or skill would have to be defined well enough to provide students with instructions that help them to understand expectations of performance. Stiggins went on to state that assessments of those items must be designed so that they address all key elements of quality performance expected in the product or skill. Identification of the key elements of quality would then lead to development of an appropriate scoring system, plan, or rubric to assess the product or skill demonstrated.

Stiggins (1997) also stated that the targets of performance assessments must have “the strongest possible basis in the collective academic wisdom of experts in the discipline within which you assess” (p. 188). In mathematics instruction the collective wisdom of experts would be found in the standards and principles advocated by the National Council of Teachers of Mathematics.

Stiggins (1997) cited the work of Baron who provided a list of questions to ask when evaluating performance exercises or tasks. Among the questions given by Baron (1991) were the following:

- Are some of my tasks designed to have students make connections and forge relationships among various aspects of the curriculum?
- Do my tasks encourage students to access prior knowledge and problem solving skills to work well?
- Do some exercises ask students to work collaboratively to solve complex problems?
- Are my students sometimes expected to sustain their efforts over a period of time to succeed?
- Do my exercises expect self-assessment and reflection on the part of my students?
- Do some of my exercises present problems that are situated in real-world contexts? (p. 290)

These questions would suggest that making connections, incorporation of writing and reflection, and establishing relevance of learning to the real world are all important aspects of student learning and assessment.

Kitchen and Wilson (2004) used released items from the 1996 National Assessment of Educational Progress (NAEP) first to analyze the alignment between student performance tasks and particular benchmarks and standards and second to determine whether a particular performance task caused students to think in ways that matched the quality of the learning goals for that task. The items used for the study were well-aligned with the NCTM performance expectations for the geometry standard in the grades three through five grade band. Results of the analysis indicated that while the task was well aligned with the standard, the instructions provided to the student were ambiguous and did not clearly define expectations for student performance. Thus Kitchen and Wilson (2004) arrived at the following conclusion:

Alignment of a test or test item with a set of learning goals is necessary but not sufficient to ensure that the test or test item is of high quality. For a test or test item to be of high quality, it not only must be aligned with a set of high-quality learning goals but also must elicit student thinking that is aligned with the learning goals. (p. 397)

As a result of the ambiguity of the instructions in the task analyzed for this study, student thinking did not align well with the learning goal that best aligned with the task. The ambiguity then led to student responses that were not of high-quality as defined in the scoring rubric used for analysis.

Alignment of Instruction and Assessment

The assessment standards adopted by the NCTM in 1995 suggested that evaluation (the assigning of value to or determining the worth of) is not the only purpose for assessment. Rather “assessment should reflect the mathematics that all students need to know and be able to do” (p. 11), “should enhance mathematics learning” (p. 13), “should promote equity” (p. 15), “should be an open process” (p. 17), “should promote valid inference about mathematics learning” (p. 19), and “should be a coherent process” (p. 21). The NCTM further recommended that many assessment methods should be used, some formal and some informal, and that assessment should be aligned with instruction. Assessment needed to move away from the traditional testing of only factual information in the mathematics classroom to a more comprehensive assessment of students’ understanding of mathematical concepts.

Two studies revealed consequences when assessments are not aligned with instructional practices. Wilson (1993) conducted a case study in an Algebra 2 class where

“there were several aspects of assessment that were non-traditional, and were aligned with some of the current rhetoric of authentic assessment” (p. 2). Among these assessments were student writings about their learning. She reported that students basically ignored these assessments and viewed them as not being valued by the teacher because they did not count toward the students’ grades. While the teacher used the assessments to determine student knowledge and understanding, learning aspects valuable to the teacher, the students did not view these assessments as part of the evaluation of their learning. Since the evaluation aspect of assessment was what was valued by the students, they viewed the writing assignments as unimportant or irrelevant.

In the second study Smith (2000) observed the teaching and assessment strategies implemented by an eighth grade mathematics teacher in a pre-algebra class and an algebra class. In the pre-algebra class the teacher began using curriculum materials that implemented non-traditional learning strategies being recommended by the National Council of Teachers of Mathematics in 1989. In the algebra class the teacher continued to use traditional curriculum materials. Although the students in the pre-algebra course performed learning tasks that required deeper conceptual knowledge and required students to write explanations about concepts and processes, the teacher used only traditional methods of assessment and evaluation in both courses. Thus there was incoherence between instruction and the assessment and evaluation of student work. The focus of learning then became completion of work rather than production of higher-quality responses.

Factors That Impact or Predict Student Learning of Mathematics

A variety of factors have the potential to impact students' learning of mathematics. Among those factors are gender, attitude, math anxiety, high school grade point average, American College Test (ACT) or Scholastic Aptitude Test (SAT) scores, other standardized test scores, socioeconomic status, and highest level of previous mathematics coursework.

Gender has been the focus of many studies on student mathematics achievement. Results of those studies have produced conflicting views of the effect of gender on mathematics achievement. However, there has been a well documented closing of a perceived difference in the performance of males and females in mathematics world-wide (Baker & LeTendre, 2005). Baker and LeTendre attribute the narrowing of this gap to the rise of gender equality around the world and the increased access over the past 30 years for females to all levels of schooling. Catsambis (2005) suggests that the narrowing of the gender gap could be attributed to the strengthening of academic requirements for high school students that require all students to take more mathematics courses. Yet according to Catsambis, significant differences persist when choice of coursework is a factor, such as at the college level. At that level significantly fewer women than men major in mathematics or science or continue on to do doctoral work in mathematics.

Baker and LeTendre (2005) examined data from the First International Mathematics Study (FIMS) done in 1964, the Second International Mathematics Study (SIMS) done in 1982, and the Third International Mathematics and Science Study

(TIMSS) done in 1995. They found that the proportion of participating nations with statistically significant male-dominated gender differences in math scores dropped from 33% to 9% between 1967 and 1994 and from 35% to 18% from 1981 to 1994. Baker and LeTendre cited a study of NAEP trends by Campbell, Hambro, and Mazzeo (2000) that showed no gender differences in eighth grade mathematics in 1990, 1992, and 1996. That study also showed that male dominated differences among American students in grade 12 mathematics in the early 1990s declined significantly by 1996. They noted, however, that gender differences do persist in terms of mathematics enrollment and achievement.

Fox and Soller (2001) examined NAEP data for the years 1978-1996. They noted that from 1978-1996 there were relatively small, male dominated gender differences in mathematics achievement. The differences decreased in magnitude as grade levels increased from grade 4 to grade 8 and from grade 8 to grade 12. Fox and Soller also found that the gender gap on the SAT-Math scores had narrowed. Data from 1975-1976 showed that 50% of males had scores of 500 or better on the SAT-Math test while only 33% of females had scores at that level, a 17% differential. Data from 1994-1995 showed that the mean score difference between males and females had dropped approximately 10 points, and the difference in percentages of males and females scoring over 500 points on the SAT-Math test had fallen to 14%.

Bridgeman and Wendler (1991) conducted a study examining the relationship between SAT-Math scores and college mathematics course grades by gender. The study participants were students enrolled in algebra, precalculus, and calculus courses at several institutions. Results of the study showed that average SAT-Math scores of females were

at least one-third of a standard deviation below that of males. However, college mathematics course grades of females were found to be equal to or higher than those of males in the same course. Results were consistent across all campuses.

Educational Testing Service (ETS) prepared a summary and synthesis of several decades of research that suggests that the gender gap in mathematics achievement is narrowing. ETS also found that at the top levels of mathematics, women do not perform as well as men in some aspects of spatial ability and achievement. However, in a study applying Brunerian theory to instruction, Winer (1981) found no significant difference in spatial ability between male and female participants, all of whom were enrolled in an educational technology graduate program.

A study examining gender-related differences in mathematics achievement of 10th grade students was conducted by Randhawa and Randhawa (1989), using the Canadian Tests of Basic Skills reading comprehension and mathematics tests. For analysis test items were grouped into content by skill micro-components. The researchers were then able to examine gender-related differences for each process, content, or micro-component aspect tested. Results of the analysis showed that males scored significantly higher than females on all three process components, on five of the seven content components, and on five of the micro-components.

Wilson and Zhang (1998) used data from the mathematics portion of a statewide assessment to determine whether test item response types (multiple choice vs. constructed response) made a difference in achievement by gender groups. The Iowa Test of Basic Skills (ITBS) was used as the test with multiple choice responses, and the Delaware

“Interim Assessment” test was used as the constructed response test. Both tests were given to students in grades 3, 5, 8, and 10. Results of this study contradict findings of other studies that have shown a narrowing of the gender gap, as male participants in this study outperformed females in grades 5, 8, and 10 on the Interim Assessment and in grades 3 and 8 on the ITBS. Females outperformed males only in grade 3 on the Interim Assessment while there was no difference between gender groups at grades 5 or 10 on the ITBS. Wilson and Zhang pointed out that size of the gender gaps in mathematics achievement found in this study increased by grade level.

In a follow up to the study just cited, Zhang, Wilson, and Manon (1999) used two extended constructed response questions at the 3rd grade level to determine whether there were gender differences in problem solving strategies. Results of the study showed that more males than females utilized the most sophisticated problem solving approach. The study also revealed that more females were able to successfully accomplish tasks. Females in the study used visual or concrete approaches more often than males, while males more often were able to give adequate explanations for the strategies they used than were females.

In a study by Dees (1982) there was no significant difference found in mathematics achievement between male and female geometry students when entering knowledge was factored in. At entry to the course, male students had greater levels of content knowledge than did females. When scores were adjusted to control for this initial difference, females scored as well as the males on test items. An additional result of the

study was that females demonstrated higher cognitive reasoning, needed for the writing of proofs, than did males.

Tartre and Fennema (1995) conducted a study of 60 randomly selected students as they progressed through grades 6, 8, 10, and 12. Gender was among the variables examined in the study. Results indicated that there was no significant difference between mathematics achievement of males and that of females. In a study of 4th and 7th graders' comprehension of mathematical relationships expressed in graphs, Curcio (1987) found no significant differences between the graph comprehension of males and that of females.

Determining factors that can best predict mathematics achievement was the purpose of the following studies. The first of those examined whether various levels of mathematics anxiety or student attitudes and values relating to the learning of mathematics would be strong predictors of mathematics achievement.

Boe, May, Barkanic, and Boruch (2004) found that there was a significant correlation between students beliefs with regard to the variable "usually do well in math" and mathematics achievement. They also found a significant inverse relationship between mathematics achievement and the variable "mother encourages sports," tied to a student belief that parents value sports more than academics, and another significant inverse relationship between mathematics achievement and the variable representing a student belief that one needs luck to do well in math.

A study by Hong, O'Neil, and Feldon (2005) involving 11th grade students in South Korea examined the mediating roles of self-regulation and test anxiety when paired with gender, with self-regulation taking into account metacognition and motivation.

Metacognition values were based on students' ability to plan and self-check, while motivation values were based on students' feelings of self-efficacy and the levels of effort. Study results showed that test anxiety did not have a significant relationship with motivation. Two models for variable interactions were examined in the study. Gender effects on test anxiety and self-regulation were not statistically significant in either model. In the second model an additional interaction arrow was added to indicate interaction between gender and mathematics achievement. In this model test anxiety and self-regulation were shown to have significant effects on mathematics achievement.

Summarizing their findings, Hong et al. made the following statement:

Regardless of gender, students who plan, monitor, expend effort, and have self-efficacy tend to have high achievement scores in mathematics. However, students who worry in general or while taking tests tend to have low mathematics performance scores. (p. 283)

The extent to which students believed that mathematics was a male domain was one factor examined in studies by Tocci and Engelhard (1991) and Tartre and Fennema (1995). Tocci and Engelhard found that "females believe more strongly than males do that studying mathematics is as appropriate for them as it is for their male peers" (p. 284). The same conclusion was reached in the Tartre and Fennema study. Other findings of Tocci and Engelhard were that gender, mathematics achievement, and parental support were all significant predictors of student attitudes toward mathematics, with higher levels of achievement and parental support highly correlated with positive attitudes towards mathematics. An additional finding of Tartre and Fennema was that there was a strong positive correlation between the level of spatial skills and mathematics achievement for

females but not for males, thus making spatial skills a strong predictor of mathematics achievement for females.

A study by Pedersen, Elmore, and Bleyer (1986) found that parent attitudes and student career interests were significant predictors of mathematics achievement, while spatial visualization ability, student attitudes, and gender were not. The student attitudes variable in this study included attitudes concerning students' confidence in learning mathematics, their parents, their teachers, the usefulness of mathematics, anxiety, motivation, attitude toward success in mathematics, and math as a male domain.

Meece, Wigfield, and Eccles (1990) examined factors that influence the levels of math anxiety of a sample of students in grades 7 through 9. They found that "students' current performance expectancies in math and, to a somewhat lesser extent, the perceived importance of mathematics have the strongest direct effects on their anxiety" (p. 68). Meece et al. also found that students who put more importance on the belief that achievement in mathematics is important had lower levels of mathematics anxiety. Gender was found not to be a significant predictor of math anxiety.

Contrary to the studies by Pedersen et al. and Meece et al., a study conducted by Bretscher, Dwinell, Heyl, and Higbee (1989) found that there were significant differences between males and females with regard to math anxiety and attitude toward success in mathematics. The participants in this study were high risk university students enrolled in a developmental studies program. Another finding of the study was that "students who are more involved in learning for learning's sake performed significantly better during their first quarter during their first quarter in Developmental Studies mathematics" (p. 6).

Grade point averages were found to be significant predictors of mathematics achievement in studies by Gliner (1987), Edge and Friedberg (1984), and Sue and Abe (1988), while ACT and/or SAT scores were found to be significant predictors of mathematics achievement in studies by House (1995) and Sue and Abe (1988). Several studies (Gliner, 1987; House, 1995; Sue & Abe, 1988; Nuttall & Hell, 2001) found gender not to be a predictor of mathematics achievement. Gliner also determined that age is not a good predictor of mathematics achievement.

Level of spatial skills was found to be a predictor of mathematics achievement for females only in a study by Tartre and Fennema (1995). They also found that level of verbal skills was a significant predictor of mathematics achievement only for males. These findings contradict generally held beliefs about verbal and spatial skill levels as they relate to gender.

Student confidence level, parents' educational levels, student attitudes toward mathematics, and home environment were all found to be significant predictors of mathematics achievement in a study by Erickam, McCreith, and Lapointe (2005). This contrasts with the findings of Curcio (1987) who found that previous levels of mathematics achievement, prior knowledge of the mathematics topic being learned, math content, and form of graph were all significant predictors of mathematics achievement.

Riggs and Nelson (1976) examined the effect of verbal vs. nonverbal tests on the mathematics achievement of 1st grade students. The topic for both tests was conservation of length. Riggs and Nelson found that "knowledge of conservation scores on a verbal conservation test significantly improves the ability to predict achievement scores made

on the basis of I. Q. scores” (pp. 318-319). They also found that the same type of knowledge on a nonverbal test did not adequately predict mathematics achievement scores.

A study by Webb, Troper, and Fall (1995) found the best predictors of mathematics learning for students working in small, peer-directed groups. They identified the extent to which students received explanations and the extent to which they carried out constructive activity as the strongest predictors of mathematics learning. Verbalization was identified as an important component in both of those factors. This aligned well with Bradley’s (1988) findings that language facility and procedural knowledge are significant predictors of mathematics achievement with regard to conceptual knowledge.

Alignment of curriculum with what is to be tested was found to be a significant predictor of mathematics achievement for third grade students who were matched according to socioeconomic status in a study by Mitchell (1999). The results of this study weakened the predictability of mathematics achievement based on gender, race, school size, or socioeconomic status.

Hill (1989) found that gender, the school district’s test of mathematics achievement, and recommendation of the students’ 6th grade teacher were all significant predictors of mathematics achievement for 7th grade students. Meynsse and Tashakkori (1994) found significant predictors of mathematics achievement to be socioeconomic status and gender. This contradicts results of a study of 10th grade students by Nuttall and Hell (2001) that found socioeconomic status and gender not to be significant predictors of

mathematics achievement. Nutall and Hell did find that previous coursework in mathematics and science and achievement level on the state test's science and mathematics scores were significant predictors of mathematics achievement.

Effect of Teachers' Content Knowledge on Student Math Achievement

Many studies have been conducted to determine the effect of various factors on student learning. Teacher content knowledge is one of those factors. Director Michael Allen prepared a summary of the findings of an Education Commission of the States sponsored study examining educational research on teacher preparation. In response to the question of the extent to which subject knowledge contributes to the effectiveness of a teacher, Allen (2003) stated the following:

Although the research on this topic is spotty and focuses largely on the teaching of mathematics, it provides moderate support for the importance of solid subject-matter knowledge. The research generally is not fine-grained enough, however, to make it clear how much subject-matter knowledge is important for teaching specific courses and grade levels. (p. 1)

He went on to say that while a critical number of content courses is helpful, there is insufficient evidence to conclude that holding a major in the content area is necessary and “limited support for the conclusion that in addition to a strong grasp of the subject itself, knowledge of how to teach a particular subject is important” (Allen, 2003, p. 1).

Goldhaber and Brewer (1996) studied data from the National Educational Longitudinal Study of 1988 (NELS: 88), which allowed students to be linked to particular teachers. The NELS data came from a nationally representative survey of approximately 24,000 eighth grade students, with approximately 18,000 of those students resurveyed

two years later. At the time of each survey the students also took one or more subject based tests in mathematics, science, English, and history. The ability to link students to specific classes and teachers allowed the researchers to determine the effect of subject-specific degree levels on student achievement, something that was not possible in other studies where data was aggregated to school levels. Results of the study found significant differences in student achievement when the teachers had subject specific degrees or certification in mathematics or science. Goldhaber and Brewer (1996) drew the following conclusion:

Teachers who are certified in mathematics, and those with Bachelor or Masters degrees in math and science, are associated with higher test scores. Because math and science degrees were not found to influence student outcomes in English and history, we believe that these results suggest that it is the subject-specific training rather than teacher ability that leads to these findings. This is important because it suggests that student achievement in technical subjects can be improved by requiring in subject teaching. (p. 15)

Haycock (1998) examined the results of large research studies in Tennessee, Texas, and Massachusetts to determine what makes an effective teacher. The Tennessee study focused on state collected data which, unlike data collected by most states, ties teachers to the achievement of their students. In that study William L. Sanders, director of the Value-Added Research and Assessment Center at the University of Tennessee, Knoxville, grouped teachers into quintiles by using the learning gain scores of their students. The teachers in the lowest quintile produced gains of about 14 percentile points for low-achieving students during the school year, while teachers in the highest quintile produced gains for low-achieving students averaging 53 percentile points. Gains for middle-achieving students averaged 10 points for lowest quintile teachers and mid-30s

with highest quintile teachers. High-achieving students gained an average of 2 points with lowest quintile teachers and 25 points with highest quintile teachers. Evidence from the study also indicates that the effects of teachers on student achievement, both positive and negative, are long-lived and affect student performance two years later.

A 1997 study of teacher effects in the Dallas Independent School District in Texas found results similar to those of the Tennessee study. Haycock (1998) reported the following example:

The average reading scores of a group of Dallas fourth graders who were assigned to three highly effective teachers in a row rose from the 59th percentile in fourth grade to the 76th percentile by the conclusion of sixth grade. A fairly similar (but slightly higher achieving) group of students was assigned three consecutive ineffective teachers and fell from the 60th percentile in fourth grade to the 42nd percentile by the end of sixth grade. (p. 4)

Haycock provided another example concerning mathematics achievement by a similar group of third grade students. He found that those students assigned to three highly effective teachers in a row moved from averaging around the 55th percentile at the start of third grade to the 76th percentile at the end of fifth grade. This contrasted with students, starting at a 57th percentile average, who were assigned to three of the least effective teachers. At the end of fifth grade the average of these students had fallen to the 27th percentile.

Bain and Company conducted a study for the Boston Public Schools, with results published in 1998. In that study classes of tenth grade students with approximately equal average starting scores were examined. Progress through the year was charted for each teacher. Average gains by the classes of each teacher were determined and placed in order. The teachers were then split into thirds according to the ranking of average gains.

While differences were found in reading, more striking differences were found in mathematics where “the top third teachers produced gains on average that exceeded the national median [for growth] (14.6 to 11.0 nationally), whereas the bottom third again showed virtually no growth (-0.6)” (Haycock, 1998, p. 5).

None of the three studies cited by Haycock identified the qualities that make a teacher effective. She cites the work of Goldhaber and Brewer (1996) as reason to believe that deep content knowledge on the part of the teacher is a crucial element for effective teaching and is a significant factor in student learning and achievement.

Approximately 350 mathematics and science teachers from 25 randomly selected middle schools in each of Texas Education Regions 1, 2, 3, and 20 were surveyed to “identify current educational reform issues that would increase student achievement in mathematics and science in South Texas middle schools, specifically grades 4 – 8” (Adams, Brower, Hill, & Marshall, 2000, p. 1). Information requested included the teachers’ knowledge of state and national standards, use of technology, professional development, effective teaching practices, and teaching assignments (grades and subjects taught).

One finding of the study was that there was tremendous discrepancy in teachers’ content knowledge. Survey results indicated that in order to have professional development of teachers lead to effective teaching, one aspect of that professional development had to help teachers expand their content knowledge. Additionally, that professional development would need to incorporate the notion that teachers must be the key to student learning (Adams et al., 2000).

Kim (1993) conducted a school-level analysis of the 1990 National Assessment of Educational Progress (NAEP) Mathematics Trial State Assessment (TSA) in order to determine factors that affect student learning outcomes. The NAEP TSA data was collected from more than 100,000 eighth-grade students in about 3700 schools distributed equally among the 37 participating states. The TSA consisted of both multiple choice and open-ended test questions, a demographics questionnaire, and a mathematics background questionnaire, all completed by the students; questionnaires completed by the mathematics teachers of the assessed students; and school characteristics and policy questionnaires completed by each participating school.

Eight predictor variables and one dependent variable were identified for data analysis. The predictor variables measured five predictor constructs—student characteristics, school conditions, teacher characteristics, teacher behavior, and student behavior, and the dependent variable represented student achievement. A structural model using directional pathways was developed and tested for effects on educational productivity. Kim (1993) determined that the number and level of math courses taken by teachers had a statistically significant effect on the amount of teacher emphasis on higher math skills. Kim found that all eight predictors were found to have statistically significant relationships to student achievement. She also found that much of the variance in student achievement was accounted for by the eight variables.

Results from Kim's study indicated that student behavior can be influenced by both teacher behavior and school conditions. Since both of these factors can be changed or controlled by schools, Kim stated that schools should modify policies and teaching

strategies to provide the greatest possible positive effects on student achievement. Kim (1993) made the following statement:

In summary, the study findings seem to shed light on efforts to search for higher education productivity. The results of the study indicate that, as was found in new research findings since the mid-1980s, teacher content knowledge and academic emphasis were important factors through which schools can make a difference in student achievement....The study findings suggest that student mathematics achievement can be positively impacted by taking academic courses and teachers' placing academic emphasis on higher math skills in classrooms. Schools needs [*sic*] to design the curriculum organization to encourage higher student enrollment in academic core courses and teachers must possess content knowledge to be able to emphasize academic skills in classrooms. (p.14)

Mullens, Murnane, and Willett (1996) conducted a longitudinal study to determine whether the academic accomplishments, pedagogical training and knowledge of mathematics of primary school teachers in Belize could predict how effective they would be in helping students learn mathematics. Data were collected during the 1990-91 school year from a stratified random sample of approximately 1000 third grade students from 72 of the 215 third grade classrooms in Belize. One factor used for stratification was classroom accessibility, a government developed measure that classified schools as urban, rural, remote, or most remote. The other factor was teacher qualification that was based on the six-level indicator used by the government to define wage scales. Student achievement levels were measured using a pretest given in October 1990 and given in May 1991.

Because Belize was a developing country at the time of the study, only 50% of the population continued schooling past the eighth grade. Problems identified relative to the primary (through eighth grade) school system included low student achievement, especially in rural schools, and a shortage of qualified teachers nationwide. Teachers

sampled for the study reflected the lack of qualifications experienced by the country.

Mullens et al. (1996) indicated the following levels of qualification:

Thirty-two percent of the sampled teachers had completed teacher training, consisting of 2 years of instruction in academics (including 180 hours of mathematics) and teaching methodology at Belize Teachers' College and a third year of supervised classroom internship. Fifty-eight percent had completed 4 years of high school, including 4 years of mathematics, while the rest were simply primary school graduates. Of the 49 teachers for whom we have a grade on the mathematics portion of the BNSE [Belize National Selection Examination, the exit examination for primary schools] (taken at the conclusion of the teacher's own eighth school year), two-thirds received a grade of C, 12 percent an A or B, and 20 percent a D or E. (p. 150)

Mullens et al. (1996) found a statistically significant relationship between the mathematical ability of teachers and their students' mathematics achievement. Students whose teachers demonstrated high levels of mathematical ability during their own schooling were able to learn advanced mathematics concepts more quickly than students whose teachers possessed lower levels of mathematical ability. For the teachers for whom no BNSE scores were available, completion of high school was a statistically significant indicator of mathematical ability. Because all high school students in Belize must take four years of mathematics, they have more experience with a broader range of mathematical concepts and at greater levels of detail than those students who only complete primary school. One would then expect that teachers who have completed high school would have higher levels of mathematics ability than teachers who have only completed primary school. Their conclusion was "that a teacher's knowledge of mathematics is important to student learning and that completion of high school may be a reliable secondary indicator of subject matter competence" (Mullens et al., 1996, p. 155).

Collias, Pajak, and Rigden (2000) established an initiative for the Council of Basic Education and the American Association of Colleges for Teacher Education that was based on previous research findings. Those findings indicate that teachers' deep content knowledge leads to greater student achievement and that graduates of teacher education programs often have insufficient content knowledge in the subjects they teach, often because schools of education in university settings do not believe it is their responsibility to provide instruction in content areas. They proposed the Standards-based Teacher Education Project (STEP) with the purpose of coordinating the collaboration of education and arts and sciences faculty to jointly design and monitor teacher education programs. The STEP initiative is a three year program that according to Collias et al. does the following:

STEP guides campuses through a four-stage process: establish a campus task force, conduct an institutional analysis, create a workplan, and create (or refine) an accountability system to ascertain whether new graduates have the knowledge and skills they need to teach in a standards-based school environment. (pp. 1-2)

Several institutions in Georgia, Indiana, Kentucky, and Maryland have implemented the STEP initiative on their campuses. Results of those implementations "suggest a new focus on teacher preparation [should be adopted] that provides a basic liberal education and a thorough grounding in specific subject areas, complemented by an overarching emphasis on the art of teaching" (Collias et al., 2000, p. 8). STEP has proposed to jointly design with the American Council of Learned Societies a teacher core curriculum that will provide prospective teachers with a basic liberal education that would be completed during their first two years of university study. This liberal education would then be augmented by specialization in a content area of the student's choice.

Suh and Fore (2002) suggested that an additional focus must be made in the discussion of teacher quality. The focus suggested is to look at teacher preparation in terms of teacher quality rather than teacher certification. Suh and Fore gave the following rationale:

Based on studies that examine individual student learning, we know that the quality of a student's teacher is the single-most important factor in a student's education. Improving teacher preparation may, therefore, require going beyond the realm of current state certification, and focusing on actual student understanding and achievement needs. (p. 2)

Suh and Fore (2002) cited a 1999 study by Goldhaber and Brewer which indicates "that teachers who major in the subject-area taught have a more positive impact on student achievement than teachers majoring in an out-of-field discipline, including those who major in education" (p. 1). This finding, coupled with the perceived need to focus on student need for quality teaching, presses the education community to re-examine and revise teacher licensing and certification requirements so that content knowledge of new teachers is assured.

The Missouri K-16 Task Force on Achievement Gap Elimination (K-16 TAGE) (2002) examined Missouri Assessment Program (MAP) test data for 1999-2000, ACT participation data and scores, high school graduation rates, college attendance rates, and data on college freshmen in need of remediation in order to determine where student achievement gaps occurred in Missouri schools and to determine the causes of those gaps. ACT scores of new teachers and the distribution of new teachers in Missouri schools by ACT score were also examined. The K-16 TAGE also recommended actions that should be taken by schools and the state in order to eliminate the achievement gaps.

Throughout grades K-12 Missouri students take three MAP tests in each of the areas mathematics, science, social studies, and communication arts. Achievement gaps have been identified in MAP scores based on student ethnicity and school location in an urban or rural area. At the elementary level schools that ranked in the lowest quartile on MAP tests had significantly higher percentages of minority students than schools in the other quartiles. Data showed that this gap continued and widened as students progressed through the educational system. While urban school districts studied had significantly high percentages of minority students, some rural areas have similar percentages of such students. A high percentage of these urban and rural schools ranked in the lowest quartile on MAP tests.

Data showed that schools with high percentages of minority students or high percentages of students in poverty had a disproportionate share of new teachers with below average ACT scores and lower than average percentages of new teachers with above average ACT scores. These data suggested that “student achievement in low-performing schools is strongly influenced by teacher quality rather than by student characteristics or school location” (K-16 TAGE, 2002, pp. 17-18).

While the K-16 TAGE recognized that many factors that contribute to low student achievement are beyond the control of school districts, it advocated that teacher quality was a significant factor in student achievement that can be controlled through the hiring, mentoring, and professional development practices of the districts. The task force believed “that improving teacher quality is the single most important factor in eliminating the achievement gaps among Missouri students” (K-16 TAGE, 2002, p. 7). One of the

primary recommendations of the K-16 TAGE was tied to the belief that quality teaching requires deep content knowledge. That recommendation is to “Assess the content knowledge of teachers in low-performing schools and provide content-based professional development for those with deficiencies” (K-16 TAGE, 2002, p. 7).

The Southern Regional Education Board (SREB) (1998) looked at qualifications of middle school teachers in SREB states and tied those qualifications to student achievement as measured in the 1996 National Assessment of Educational Progress (NAEP). The purpose of the study was to identify ways to improve teaching in the middle grades and to make recommendations that would facilitate such improvement.

A description of a perceived problem with current middle school teacher preparation was identified by SREB (1998) as follows:

Because of practices in teacher preparation, licensure, and assignment to classrooms, too many teachers in the middle grades have too little knowledge of the subjects they teach. Teachers who never have taken advanced English courses, physics, chemistry, or college algebra can teach seventh- and eighth- grade pre-algebra, algebra, physical science and English in most SREB states. In SREB states, those who teach eighth-grade mathematics and science are less likely than their peers nationwide to have had college courses in their content area during the last two years. The results are predictable: lagging student achievement in the middle grades in mathematics, science, and language arts. (p. 3)

The Board went on to state that in SREB states at least one third of the teachers in grades 6-8 are licensed as elementary teachers. NAEP results from 1996 indicated that eighth grade students in rural areas of SREB states “trail the nation in student achievement by a larger margin than do students in urban and suburban areas of the region” (SREB, 1998, p. 5). It was reported that in those rural areas 29% of eighth-grade mathematics teachers have elementary education majors rather than content area majors as opposed to the rest

of the nation where only 16% of eighth-grade mathematics teachers have elementary education majors rather than content area majors.

SREB (1998) made several recommendations to improve teaching in the middle grades. One of those recommendations addressed the need for greater content knowledge of middle school teachers. The Board recommended that states require prospective middle school teachers to have at least a minor in the content areas they would teach. They also recommended that states should require all current middle school teachers who do not have a content major or minor in the subject areas they teach to acquire within five years a minor equivalent in those content areas.

Goldhaber and Anthony (2003) reviewed research on characteristics of highly qualified teachers and connections between those characteristics and student achievement. They also identified recommendations for changes in public policy to increase teacher quality.

The importance of teacher quality was identified in the study overview.

Goldhaber and Anthony (2003) stated it as follows:

Teachers clearly play an important role in shaping the future of individuals as well as of entire generations. In recent years, new research has demonstrated the dramatic effect that teachers can have on the outcomes of students from all academic and social backgrounds. In fact, studies have shown that teacher quality is the most important educational input predicting student achievement. (p.1)

They went on to state that despite the importance of teacher quality, a large part of the teaching workforce are less academically skilled than college graduates in other professions. The lower degree of academic skill has been a concern addressed by task forces and commissions for two decades in an effort to produce higher quality teaching.

In their study Goldhaber and Anthony (2003) examined research that connects various teacher attributes to student learning to determine which teacher attributes best predict teacher effectiveness and hence, teacher quality. They stated that results of the 1986 Greenwald, Hedges, and Laine meta-analysis of 106 studies concerning the effect of teacher degree level to student achievement were not definitive, with those studies showing statistically significant relationships evenly split between having a positive or negative effect. Goldhaber and Anthony's review of a large, 1997 study done by Goldhaber and Brewer indicated that increases in student learning in grades eight through ten were not attributable to teachers holding advanced degrees. An exception was noted in that mathematics and science teachers holding advanced degrees in the subjects they taught had a positive influence on student learning. The authors cited a problem concerning both of the large reviews in that they lacked specificity concerning some of the teacher variables used. Advanced degree information was often given by level of the degree only, with the subject of the degree not given. This lack of specificity may have contributed to the inconclusiveness of results.

Goldhaber and Anthony (2003) also reviewed studies that examined the effect of teacher content knowledge on student achievement. Goldhaber and Anthony stated that findings from a 1994 study of eighth grade students done by Monk and King-Rice produced the following conclusions:

Even in subjects where training appears to make a difference (e.g., math), the impact of subject-specific training depends on the context of the classes taught. They found that the number of math courses taken by teachers in college had an impact on high school students' achievement in math, but that additional teacher coursework beyond that only mattered if the teacher was teaching a more

advanced course. Furthermore, they showed that after some point, there were diminishing returns to additional teacher coursework. (p. 13)

Results of an Eberts and Stone study from 1984 provided contrasting results concerning math teachers at the elementary level. Goldhaber and Anthony (2003) reported those findings as follows:

They did not find a positive relationship between the number of math courses taken by teachers and their fourth grade students' achievement in math. Subject matter training, proxied by advanced degrees, made a difference to student outcomes in some contexts but not all. (pp. 13-14)

Effect of Beliefs and Attitudes on Mathematics Achievement

Kloosterman (1996) identified several beliefs held by most students concerning mathematics. Among them are that mathematics is computational and that mathematics is useful. These beliefs, along with others having to do with the learning of mathematics, were incorporated into Kloosterman's model of the belief/motivation/achievement process in mathematics. In the model, self-confidence on the part of the learner is identified as an important factor in the learning of mathematics. McLeod (1992) also identified confidence as an important factor in students' and teachers' beliefs about themselves and their relationship to mathematics.

Discourse is another important belief factor for the learning of mathematics in Kloosterman's model. Often discourse is an element in the learning of mathematics during group learning situations. Grouws and Lembke (1996) offered insight into how the culture of the classroom impacts such group learning activities. They stated that for learners in situations where group work is done in non-competitive settings, students can

offer each other support through their discourse. However, these students may have felt they have less responsibility for the learning and may thus be less motivated to learn the mathematics. Additionally, for learners in situations where group work is done in competitive settings, Grouws and Lembke (1996) believed that students may attribute their success or failure to ability and/or luck and may not be motivated to put forth much effort.

The culture of the classroom has had a great deal of impact on student learning of mathematics. One way in which it has manifested itself is in how students view the role of the teacher. In Kloosterman's (1996) model, students often saw the teacher as a transmitter of knowledge and a source of answers. Stage and Kloosterman (1995) saw the role of the teacher in remedial mathematics courses as often being that of teaching rules and procedures. Should a teacher have decided to alter these roles and adopt a different type of instructional model such as inquiry learning, in which students construct their own knowledge, there would be a problem with class culture unless the students also decided to alter their roles as receivers of information.

According to Uttal (1996), another factor which may play a key role in the learning of mathematics is parent and student beliefs concerning whether innate ability or student effort is the cause of success in mathematics. Utal's research indicated that American parents tend to believe that innate ability is more important than effort, largely due to the popularity of the theory of genetic determinism. Asian parents, on the other hand, tend to view effort as a more important factor in the learning of mathematics than innate ability.

The role of emotion in the learning of mathematics has also been studied. Barbara McDonald (1992) sees cognition and emotion as being intertwined in the learning process, with one not being able to be examined without the other. She raised two issues as follows: (1) “What is the nature of the process by which emotions interact with information processing in the learning of mathematics?” and (2) there are “belief systems and attitudes that predispose students to differentially respond to mathematics learning” (McDonald, 1992, p. 220). The emotions and attitudes addressed here include what has been called math anxiety and also mathematical confidence. McLeod (1992) suggests that one should “think of attitudes as the end result of emotional reactions that have become automatized” (p. 249).

A gap exists in the research presented as it has not answered how the beliefs held concerning mathematics and the learning of mathematics are formed. Additionally, there are limitations to some of the research. Grouws and Lembke (1996) indicated one limitation that arises from getting measures of student motivation by measuring student achievement. It is not clear that these two measures should be equated. They indicated that a second limitation exists due to much of the research on student motivation being done in free-choice situations while students are not in free-choice situations in classrooms.

Summary

Literature reviewed has shown that student learning of mathematics is enhanced when multiple representations of concepts are explored and presented; when students

make connections between mathematical concepts, connections to other disciplines, and connections to their own experiences and past learning; and when students have opportunities to communicate orally and in writing about the mathematics they are learning. Several research studies have shown that the teacher and the teacher's level of content knowledge are among the most important factors that affect student learning. Studies reviewed indicated that deep content knowledge depends on building conceptual understanding along with procedural understanding. The work of Jerome Bruner has provided a context in which meaningful mathematical explorations can take place. His enactive, iconic, and symbolic stages of development align well with the major tenets of the "Rule of Four" assessments and instruction used in this study.

Since deeper conceptual understanding of mathematics topics for elementary education majors has been the underlying goal of this study, implementation in the Math 277 course of a "Rule of Four" assessment system supported by the findings from the literature would seem to be appropriate. Alignment of the "Rule of Four" assessments and instruction with the NCTM content standard for number and operations and the process standards for communication, connections, and representation would indicate that such assessments would be considered best practice.

Studies informing the use of variables being tested in this study provided mixed results. The effect of gender on mathematics achievement was found to be significant in some studies and not in others. Few studies examined effects of age and class on mathematics achievement.

CHAPTER 3

METHODOLOGY

Introduction

This chapter includes a discussion of the participants, research design, instruments, data collection, data analysis strategies, and procedures used in this study.

Participants

Elementary Education Program

The elementary education program at Minot State University graduates between 50 and 60 students each year. There are between 200 and 300 students at various stages in the program during any academic term. These students are required to take content courses in several disciplines including mathematics. College algebra is currently a prerequisite for the two mathematics courses required of all elementary education majors at Minot State. Over 90% of the candidates take their college algebra course at Minot State, with the remaining candidates taking an equivalent course at another institution and transferring those credits to Minot State (Johnson, 2004).

Math 277 – Mathematics for Elementary Teachers I and Math 377 – Mathematics for Elementary Teachers II are the two required mathematics content courses in the elementary education program. The three semester hour Math 277 course focuses on the following topics: problem solving, number systems (including the real number system

and its subsets), number theory, binary operations, and proportional reasoning. The two semester hour Math 377 course addresses the following topics: statistics, probability, geometry, and measurement. These courses may be taken sequentially or concurrently.

Enrollment in Math 277 – Mathematics for Elementary Teachers I and Math 377 – Mathematics for Elementary Teachers II does not require admission to the teacher education program. As a result, many students in these courses have not even applied for admission to the teacher education program. Often they wait to apply until after they have completed both courses with a grade of “C” or higher in each.

Students in Math 277 and Math 377 are diverse in age. A substantial number of these students are older than the traditional college student who starts college immediately following high school. Several of these older students are mothers who have waited to start their college educations until their children are in school. Others have found themselves facing economic hardships due to lack of education and so have decided to get a bachelor’s degree. Still others have found a calling to the teaching profession after having worked with children in daycare or having worked in school settings in capacities other than as teachers. Females outnumber males by a wide margin in the two courses, although there has been a steady increase in the number of males in the elementary education program.

Participants

The participants in the study consisted of all elementary education majors enrolled at Minot State University during the course of the study. Participants in the study were pre-service elementary education majors in four sections of Math 277, two sections

in each of the fall and spring semesters of the 2005-2006 academic year at Minot State University.

Content Focus

For the purpose of this study research questions were limited to those that showed student performance concerning content addressed in NCTM Standard 1: Number and Operations. Because the content concerning rational numbers expressed as decimals comes late in the Math 277 course and often gets less emphasis than other subsets of the real numbers studied in the course, this study did not address students' understanding of numbers in decimal form or their ability to perform binary operations using such numbers. Questions on the pretest- instrument required students to work with whole numbers, integers, and fractions, use the four basic binary operations of addition, subtraction, multiplication, and division, and demonstrate understanding of several field properties of the real numbers.

Course Descriptions

Math 277 and Math 377 are the two required mathematics content courses in the elementary education program. The Math 277 course focuses on the following topics: problem solving, number systems (including the real number system and its subsets), number theory, binary operations, and proportional reasoning. The Math 377 course addresses the following topics: statistics, probability, geometry, and measurement. In both of these courses, students are provided experiences in which they learn about and use manipulatives that they will later use in their teaching of various mathematical

concepts. The use of these manipulatives is then connected to algorithms and properties that are generally already known by the students. Students are sometimes asked to write explanations of concepts and about connections between manipulatives and algorithms. In their writing, students are to use mathematical vocabulary and notation appropriately.

Most of the content in the Math 277 course is intended to help students gain knowledge about subsets of the real numbers, binary operations performed using those subsets, and field properties of the real numbers. Subsets of the real numbers emphasized in the course include whole numbers, integers, rational numbers expressed in fraction form, and rational numbers expressed in decimal form, although the work with decimals is limited due to time constraints. The binary operations given emphasis in Math 277 are addition, subtraction, multiplication, and division. Field properties studied include the commutative and associative properties for addition and multiplication, the distributive property of multiplication over addition, and the identity and inverse properties for addition and multiplication. The Math 277 course content is supported by the Number and Operation Standard of the NCTM.

Research Design

This study employed a quasi-experimental research design using a pretest and posttest. The purpose of the research was to compare mathematics achievement of students who participated in instruction and assessments implementing a “Rule of Four” to the mathematics achievement of students not exposed to “Rule of Four” assessment procedures, but provided instruction that included all four representational forms (using

manipulatives, numeric representations of properties or operational algorithms, appropriate algebraic, geometric, or real world models with applicable diagrams, charts, or tables, and the written word). Participants were students in four sections of the Math 277—Mathematics for Elementary Teachers I course at Minot State University during fall and spring semesters of the 2005-2006 academic year. Enrollment for those courses was done by student self-selection. All four sections were taught by the same instructor, Dr. Warren Gamas.

Each semester one section of Math 277 employed “Rule of Four” assessment tasks pertaining to the following five content strands: addition, subtraction, multiplication, division, field properties of the real numbers. The “Rule of Four” tasks were used in addition to the methods of assessment traditionally used to determine student achievement in the Math 277 course. The traditional assessment methods were the only types of assessment activities used in the other two sections, one each semester. The course instructor determined which section of Math 277 used the “Rule of Four” assessment tasks each semester.

All sections of Math 277 were taught using the same general methods and curriculum as in previous semesters. That curriculum included the four types of representation utilized in the “Rule of Four.” A pretest developed by the researcher was given to all enrolled students near the beginning of the Math 277 course in order to determine the mathematics achievement level of each student at that point. They took the same test as a posttest at the end of the Math 277 course in order to determine the levels of mathematics achievement after instruction. Since approximately four months had

elapsed between the pretest and posttest dates, there was little likelihood that students would recall the wording or specific numbers used in the pretest questions. Data including participants' gender, age, class, cumulative GPA, College Algebra grade, and ACT math score were obtained using the university system database with access granted through the registrar's office at Minot State University.

Pretests were coded by the course instructor, who then removed the student names from the pretest forms before giving the pretests to the researcher for scoring. The instructor maintained the coded list of students in order to pair the posttest forms with the same student codes used for the pretests. Student names were removed from coded posttest forms by the course instructor prior to giving the posttests to the researcher for scoring.

After a pilot study was conducted, it was determined that two neutral parties should be trained to use the scoring rubric developed by the researcher and that they should do the scoring of the pretests and posttests. The researcher collected and maintained the demographic data, pretest scores, and posttest scores in an SPSS database.

Since all Math 277 students were taking the pretest and posttest for the study, prior to the pretest they were asked to sign informed consent forms indicating that they agreed to participate in the study (see Appendix C). The course instructor distributed the informed consent forms to students to obtain their signatures. Signed forms were collected and kept in a sealed envelope by the course instructor until the study was completed. This allowed the participants to remain completely anonymous to the researcher.

Independent Variables

For this study one independent variable was the experimental group to which each participant belongs. There were two groups for the study. The treatment group consisted of the study participants in the sections of Math 277—Mathematics for Elementary Teachers I that were using the “Rule of Four” assessment tasks. The control group was made up of the participants in the sections of Math 277 that were not using the “Rule of Four” assessment tasks. There was one section of Math 277 for the control group and one section for the treatment group in each fall or spring semester of the 2005-2006 academic year. A second independent variable was gender. Age (25 or under vs. 26 or older) was the third independent variable. Class was the final independent variable, with two groups established for the variable. One group was composed of underclass students (freshmen and sophomores), while the other included upper class students (juniors and seniors). Pretest score, Math 103 – College Algebra grade, ACT Math score, and cumulative grade point average (GPA) were variables used as covariates.

Dependent Variables

The dependent variable for the hypotheses testing was mathematics achievement after instruction as measured by the posttest. Pretest scores were used as a dependent variable in establishing equality of experimental groups.

Instruments

“Rule of Four” Assessment Instruments and Rubrics

The “Rule of Four” performance tasks and rubrics used for assessing student knowledge were developed by this researcher. The performance tasks were designed to demonstrate student knowledge of whole numbers, integers, and rational numbers expressed as fractions as they are used in the binary operations of addition, subtraction, multiplication, and division. The tasks also addressed several field properties of the real numbers, namely the distributive property of multiplication over addition and the associative and commutative properties of addition and multiplication. To make a grid for scoring student performance, the tasks were organized and numbered as follows:

1. Addition;
2. Subtraction;
3. Multiplication;
4. Division;
5. Field properties.

Rubrics for assessing student responses for each performance task were developed for each part or level of the “Rule of Four.” The four levels were identified as follows:

- Level A: Using an appropriate manipulative to demonstrate a concept.
- Level B: Using an appropriate algorithm involving a binary operation or providing a numeric representation of a field property.
- Level C: Providing an appropriate algebraic, geometric, or real world application of a concept.
- Level D: Writing the concept in words using appropriate terminology.

A five-point scale ranging from zero to four was developed to score student performance on the tasks. The meaning for each score is as follows:

- 4—Proficient (Exceptional);
- 3—Target (Acceptable);
- 2—Limited Knowledge (Not Acceptable);
- 1—Unsuccessful (Not Acceptable);
- 0—No Knowledge (Not Acceptable).

Criteria for each of the performance scores identified in the scale were developed for each level of the “Rule of Four” (e.g., Level A, Level B). Except for Level B, the criteria for each level were the same for performance tasks involving operations and field properties. For Level B a set of criteria was developed for performance tasks relating to binary operations, and a different set of criteria was developed for performance tasks pertaining to field properties. The assessment rubrics can be seen in Appendix G. As the instruction implementing the “Rule of Four” was undertaken in the study, it was determined that the concepts addressed in some of the performance tasks did not readily lend themselves to Level C of the “Rule of Four,” where students were to provide an appropriate algebraic, geometric, or real world application of the concept. Examination of all “Rule of Four” levels and tasks was then undertaken to determine appropriateness of each of the four levels to each performance task. A chart showing the findings of that review is located in Appendix H.

Pretest and Posttest Instrument

The pretest and posttest utilized the same test instrument. Since four months had elapsed between when the pretest and posttest were taken, there was little likelihood that students would recall the wording or specific numbers used in the pretest questions. The paper and pencil instrument was written by the researcher and consisted of ten questions that were representative of the content for the Math 277 course.

The paper and pencil rubric used to score the pretest and posttest was also developed by the researcher. Two scorers, each using this rubric, assigned scores ranging from four to zero on each question of the pretest and posttest, with the highest score given for successful completion of the task. Scores of two and one represented low levels of success in completing tasks. A score of zero was given for not attempting the task (see Appendix B to view the scoring rubric).

Validation

After the questions were written, Dr. Laurie Geller and Dr. Warren Gamas, faculty members of the Department of Mathematics & Computer Science at Minot State University, examined the questions to determine content validity of the pretest/posttest instrument. Both of these faculty members have taught the Math 277 course at Minot State University and were therefore familiar with the content of the course and expectations for student work. They reviewed pretest/posttest questions to assess how well they aligned with content and focus of questions posed to students their previous sections of Math 277. They also examined the questions to determine consistency with the types of questions posed in the textbook used in the Math 277 course. After this review, two questions were changed slightly for clarity of instructions to the student (see Appendix A to view pretest/posttest questions). These faculty members determined that all ten questions were consistent with content covered in the course and were in alignment with questions typically posed to students on tests and homework assignments Math 277.

Reliability

A group of six volunteers who were just completing Math 277 in the spring of 2002 piloted the pretest/posttest so their scores could be used to measure the internal and scoring reliability of the test. The method of split halves was used to determine internal reliability of the test. For this test, the five odd-numbered questions were scored separately from the five even-numbered questions. Each question was scored using the rubric in Appendix B. Total scores for the odd-numbered questions and even-numbered questions are shown for each student in Table 6 below.

Table 6. Test Scores for Determining Reliability of the Test

Test Number	Evens	Odds
1	17	13
2	16	13
3	17	12
4	11	7
5	14	12
6	15	10
Total N	6	6

A correlation coefficient or Pearson's "r" value of +0.87 was determined using linear regression. This value indicated that the one-half of the test measured nearly the same ability level as did the other half. The measurements between halves were consistent. The Spearman-Brown formula was applied to the correlation coefficient to estimate the internal reliability of the whole test, rather than half of the test as was measured by "r." The formula was as follows: $2r/(1+r)$. The results of this formula, using $r = 0.87$, indicated a split-half reliability of 0.93 for the whole test. This was a very high number

for the reliability factor and indicated that the test instrument was internally consistent or homogenous.

Initially it was thought that the researcher would be the only scorer for the pretest and posttest, so scoring reliability was determined through a correlation of scores for the same set of completed tests by that single scorer over time. More than two years elapsed between scorings by the researcher. Without observing the original scoring or student work, the researcher gave another faculty member in the MSU Department of Mathematics & Computer Science a file containing the original, scored tests. That faculty member then made copies of the original tests in their scored format. The scoring markings were then eliminated on the copies. New copies of these revised copies were then made to mask “cleaning” markings that might provide scoring clues from the previous scoring. The new, clean copies of the students’ tests were returned to the researcher to be scored. The original scored tests were kept by the researcher’s colleague until the new copies of the tests were scored by the researcher. Scores for the ten questions on all six original tests and all six copies were then entered as paired data in an SPSS file.

Table 7 shows the Pearson correlation coefficient for the paired scores. The correlation was significant at the 0.01 level, using a two-tailed test, showing that there was a high level of scoring reliability when one scorer used the rubric in Appendix B. Having arrived at such a high correlation coefficient with discrete data values provides confidence in the use of the rubric for scoring on the pretests and posttests.

Table 7. Correlation Coefficients for Scoring Reliability for One Scorer.

		First	Second
First	Pearson Correlation	1	.831**
	Sig. (2-tailed)	.	.000
	N	60	60
Second	Pearson Correlation	.831**	1
	Sig. (2-tailed)	.000	.
	N	60	60

** . Correlation is significant at the 0.01 level (2-tailed).

Revision of Pretest and Posttest Instrument

On each of three questions in the pretest/posttest instrument, the work of one student was scored quite differently, with a rubric score of 3 given for one scoring and a rubric score of 1 given for the other. Upon examination of their work and the rubric used to score two of those questions, it was determined by the researcher that an additional descriptor needed to be added to the level three scoring classification. On questions 3 and 4 the rubric was changed from “Correctly demonstrates but cannot adequately explain addition/subtraction of the signed numbers,” to “Correctly demonstrates or explains but cannot adequately do the other for addition/subtraction of the signed numbers.” For question number 7, no changes were made to the rubric as it was determined that the rubric had been incorrectly applied in one of the disparate scorings. The final pretest/posttest instrument is in Appendix A.

Curriculum

“Rule of Four” Instruction and Assessment

Participants in the classes where “Rule of Four” assessments are implemented were taught using methods similar to those used in the classes in which “Rule of Four” assessments were not used. The “Rule of Four” assessments took place during class time. For each of the concepts the participants were asked to demonstrate knowledge of the concept via tasks that adhered to the “Rule of Four.” These assessment tasks are listed in Appendix F. In the case of the level A assessments, where the student was to demonstrate a concept using manipulatives, the instructor had each student individually demonstrate the concept and verbally explain how to use the manipulative and how it represented the concept while others were doing practice exercises with the same type of manipulative.

The level B tasks required the student to provide a numeric and symbolic representation to be used with an algorithm to demonstrate a mathematical concept. For the level C tasks the students had to use an appropriate algebraic, geometric, or real world model to represent a mathematical concept. Level D tasks required the student to write a paragraph that explained a mathematical concept. The student was to use mathematical terminology appropriately and to clearly articulate mathematical procedures and processes. In the cases of the assessments for levels B, C, and D, the students were asked to give written performances that were submitted to the instructor during class time. The instructor scored these written responses outside of class time.

The course instructor evaluated student performance using the rubric in Appendix G and tracked student performance on each concept on grids, also in Appendix G. For the grids having only columns a-c, the goal was to have students receive scores of three or four in all three of the tasks in each column and in at least three of the rows. For grids having columns a-e, students were expected to achieve scores of three or four in at least three of the tasks in each column and in at least four of the tasks in each of the four rows.

Procedures

Institutional Review

Since the study used human participants, review by the Institutional Review Boards (IRB) at both Minot State University and Montana State University was undertaken. The IRB at Minot State University reviewed the proposal for the pilot study conducted during spring semester of 2005 and for the full study conducted during fall semester of 2005 and spring semester of 2006. Approval for both the pilot study and the full study were given in January of 2005. A copy of each approval letter can be seen in Appendix I.

All study participants were provided information about the study prior to their participation. Each participant signed an informed consent letter explaining the study's purpose and procedures and participant confidentiality protections. The informed consent letter (see Appendix C) was approved by the Minot State University IRB.

Training of Instructor in Use of the “Rule of Four”

Dr. Warren Gamas taught all of the sections of Math 277 that were part of this study. He also taught the two sections of Math 277 that took part in a pilot study conducted in the spring semester of 2005. Prior to the pilot study this researcher met with Dr. Gamas to acquaint him with the procedures and instruments that would be used in the study. Instruction on using the “Rule of Four” assessment rubric was given.

Periodically throughout the pilot study, Dr. Gamas and this researcher met to discuss issues that arose concerning implementation of study procedures and to discuss how the pilot study was going in general. They also met after the pilot study was completed to determine what changes needed to be made in instruments and procedures so that the full study would run smoothly.

Selection of Participants

Participants for the study were all of the students enrolled in Math 277 at Minot State University in the fall 2005 and spring 2006 semesters. Students self-selected the sections in which they enrolled based on availability of sections and how the time slots for these sections fit with the rest of the courses in which they enrolled.

Arrangement of Classes into Control and Treatment Groups

Selection of Math 277 sections as treatment groups was determined after conversations between the researcher and the course instructor. Because the “Rule of Four” assessments would take a great deal of class time, the researcher and the course

instructor decided that the section with the smaller number of students each semester would be designated as the treatment group. The larger section each semester would then be designated as the control group.

Comparison of Instruction in Control and Treatment Classes

During the study the researcher observed instruction in both control and treatment classes on three consecutive class days. This was done to see how the “Rule of Four” assessment tasks were being implemented in the Treatment class and to more clearly identify differences and similarities in instructional practices for the classes.

On the first observation day some differences in expectations for student work in the two classes became apparent. This was day two of instruction relating to the instructional algorithm for multiplication. For each of the classes the instructor had developed a problem sheet where students would use the instructional model for multiplication and tie it to drawings of blocks in base 10 or some other number base. The problems on the sheets for the two sections were nearly identical, but they did not have the same expectations for student work. The heading and directions for the first two problems on the single version of the control class problem sheet read as follows:

Name _____

In this in-class activity, you are expected to show geometrically, numerically, algebraically, and verbally how to multiply two and three digit numbers.

- a) Solve the problem numerically using the final algorithm.
- b) Use the Instructional algorithm to show partial products for the same multiplication problem.
- c) Use the base 10 blocks at your table to represent the problem. Draw the base 10 block representation to the right of the problem (below).

In contrast, the heading and directions for the first two problems on the two versions of the problem sheet for the treatment section read as follows:

Version 1

Name _____

Partner _____

In this in-class activity, you are expected to show geometrically, numerically, algebraically, and verbally how to multiply two and three digit numbers.

- a) Solve the problem numerically using the final algorithm.
- b) Use the Instructional algorithm to show partial products for the same multiplication problem.
- c) Use the base 10 blocks at your table to represent the problem. Show the problem to your partner and have them “check” your representation before you draw the problem in the section to the right.
- d) Explain to your partner how your base 10 block representation (either the blocks on your table or your representation drawn here) demonstrate the partial products.

On both sets of problem sheets similar directions were given for two additional problems containing the same digits to be completed in another base. The problems were worked at students’ tables during class in both sections.

In the control group students identified and discussed expectations for in-class work and also talked about “how to” proceed in working the assigned problems. As the students worked at their tables the instructor walked though the room observing student work and stopped to help those that needed assistance or who had questions. Although base 10 blocks were available at student tables for use in working the problems, students were not required to model the problem with them or to use them as a basis for their drawings. Students were not required to verbalize, orally or in writing, any concept or mathematical process with another student or with the instructor.

Expectations for student interaction were different in the treatment section. Directions for completing the problem sheet required students to have much more

interaction with each other than in the control section. In the treatment section, two versions of the problem sheet were used so that each of the partners had different but similar problems, thus requiring each student to be responsible for working a problem not previously seen or explained. As the instructor moved about the room, he monitored student progress, observed models, and listened to student explanations. He reminded students that they needed to explain to their partners what the drawing in the second problem represented and where partial products were used in that problem.

Differences in performance expectations for the two groups were also apparent on the second day of observation. At the start of class the instructor reviewed models for division in both class sections. In the control section the instructor then guided students through a base 10 division problem using the partition model shown only by drawings. Each student had a handout on which they recorded the process. Connections between parts of the partition model and parts of the instructional algorithm were made by the instructor. Another division problem in base 5 was done following the same format, with the instructor and the students making drawings that would represent mats, flats, and units in base 5. The division in base 5 was connected to the missing factor model for division with the third division problem. Throughout the class period, base 10 blocks were present on all tables, but no students used them. There was no partnering with other students to provide explanations and no requirement for written explanations.

After the initial review by the instructor in the treatment section, students were asked to use the base 10 blocks to model the dividend of the first partition model problem on a handout. The handout was nearly identical to the one used in the control section.

Although the dividend was modeled with the blocks, the partitioning was done only through drawings. Students were asked to check with a partner on how to complete charts and to compare their work with that of a partner. In the treatment section the instructor was better able than in the control section to observe student work. He made it a point to do so. Students worked together and would raise their hands to get assistance or to get feedback on whether they were using the right approach. As with the control section, the instructor tied the final example in base 5 to the missing factor model of division.

The main focus of instruction on the third observation day was the introduction of new material. The instructor primarily used a lecture format for delivery of instruction on alternative algorithms for multiplication in both the control and treatment sections. In both sections the instructor asked students to make connections between multiplication models and to do problems utilizing each model. Identical problems were used in both sections. A difference in student performance expectations became apparent when the instructor asked the treatment section to partner with another student after completing the final three problems to check their work against that of the partner and to verbalize what was done in each problem. A written description was required as part of their homework assignment. This description was to be similar in content to the oral verbalization done in class with a partner. The writing was not part of the homework for the control group. As students worked with their partners, the instructor observed and monitored the verbalization content and process. He also assessed student use of drawings and algorithms. During and after each of the treatment class sessions, the instructor filled in the grids for student performance on the tasks relevant to the day's activities.

Administration of Pretests and Posttests

Pretests were given to all participants within the first three weeks of the Math 277 course. A class day was taken for administration of the pretests. The posttest questions were administered as part of the final examination for the course.

Collection of Demographic Data

Demographic and education data for participants was collected through the Records Office at Minot State University. Data collected included age, class, gender, college algebra grade, ACT-Math score, and cumulative grade point average at the start of the semester in which the participant was enrolled in Math 277. The data was then coded by the course instructor and names removed so that pretest and posttest scores could be paired with the demographic and educational data.

Selection and Training of Scorers

Two neutral parties were selected to score the pretests and posttests. Dr. Laurie Geller and Dr. Susan Jensen, both faculty members in the Department of Mathematics, agreed to be scorers. The researcher trained both individuals in use of the rubric. Each of these individuals then scored the set of posttests from the pilot study to determine the consistency with which they applied the rubric.

Scores from each of the scorers were paired by posttest question. A correlation coefficient was determined for each posttest question. The correlation coefficients on seven of the ten questions was .80 or higher, indicating consistent application of the rubric by both scorers on those questions. The remaining three correlation coefficients

were below .80 with one being less than .50. The questions on which correlation was low were all questions that were changed after the pilot study because their wording did not require students to do things that would be needed in order to meet the highest level of the scoring rubric. As the scorers had begun their work on the pilot study posttests, they asked about the lack of correlation between one part of the rubric and the test questions and how they should score those questions. Their interpretation of my response was differed which resulted in applying the rubric quite differently. This ultimately caused the low correlation coefficients.

The researcher went through the rubric with both scorers prior to their scoring of the study pretests and posttests. One of the scorers asked for clarification of a rubric category as she started the scoring of the study's pretests and posttests. After getting the clarification, she was able to complete the scoring.

Pilot Study

A pilot study was conducted in spring 2005. Participants in the pilot study were from the two sections of Math 277—Mathematics for Elementary Teachers I offered at Minot State University that semester. Both sections were taught by Dr. Warren Gamas, who was the instructor for all of the sections that will participate in the full study.

Effectiveness of Pilot Study Procedures

Periodically throughout the pilot study, Dr. Gamas and this researcher met to clarify study procedures and instruments and their effectiveness. During these meetings other issues relating to time constraints were also discussed.

The two sections of Math 277 that took part in the pilot study were quite different in size. One section had only seven students, while there were more than twenty in the other. It was determined that it would be easier to implement the “Rule of Four” assessments in the smaller section because of the class time required for the level A assessments where students used manipulatives to demonstrate and explain a concept.

At the outset of the pilot study the course instructor was concerned that the “Rule of Four” assessments would take too much class time. As the semester progressed, those concerns were alleviated as the instructor became more comfortable with the manner in which each assessment took place.

The pretest was given on a class day early in the semester. It was given after the last add date, but prior to any instruction on the concepts being tested in the pretest. Several students were absent that day for a variety of reasons. Students who missed the pretest were asked to make arrangements to get together with the course instructor to take the pretest at a later date. Some did this within a couple of weeks, but there were others who did not. Those who had not taken the pretest by the time instruction was given on pretest topics did not take the pretest at all. As a result the number of pretest/posttest pairs was lower than expected.

A concern was raised about taking a second instruction day to give the posttest. The instructor requested that the posttest questions be incorporated into his comprehensive final examination with modification of some questions so that they specified a particular method or manipulative be shown. It was decided to make this change. Making the posttest part of the final examination meant that the instructor had to

photocopy each student's final exam, code the photocopies, and remove the student names from the photocopies. The coded photocopies were then given to this researcher to score. Final examination questions that were not on the original posttest were not scored by the researcher. All of the original work of the students on the final examinations was scored by the course instructor using his own scoring rubric for the purpose of giving the students grades. The examinations graded by the course instructor were not seen by the researcher. The researcher's scores were not used for grades and were not shared with the course instructor.

The course instructor revised some posttest questions when he included them in his final examination. The expectation was that changes would relate to requiring use of a particular method or manipulative on a problem rather than leaving the choice to the student. This researcher did not have the opportunity to see the changes prior to the instructor giving the test. While the revisions generally did what was expected, in two instances, the student was asked only to demonstrate a concept on the final exam when the posttest question had asked for a demonstration and an explanation. On another question the word "and" used on the posttest was changed to "or" on the final examination. These changes resulted in the researcher's scoring rubric not fitting the question well. Question revision that was in line with the researcher's expectations made student responses more consistent and thus made it easier for the researcher to be consistent in applying the scoring rubric.

"Rule of Four" instruction and assessment tasks concerning use of decimal numbers were not used during the pilot study. It was determined that there was

insufficient time to adequately include those tasks during the course of the semester. Less emphasis was given to completing those tasks during the pilot study since none of the pretest/posttest questions involved working with decimal values.

Changes Made Due to Pilot Study

Since the incorporation of the posttest into the final examination for the course worked well procedurally in the pilot study, it was decided to do so in the full study. The researcher met with the course instructor to discuss the revised posttest questions. Wording acceptable to both individuals was adopted and was used for both pretest and posttest questions in the full study.

It was determined that a cutoff date would be established for taking the pretest each semester. Students who were not in class on the date that the pretest was given each semester would be required to make arrangements to take the pretest by the cutoff date. Students who for some reason could not take the pretest prior to the cutoff date would not have their posttest scores used in the study.

A decision was made to eliminate during the full study those “Rule of Four” assessment tasks relating to operations with decimal values. Operations with decimal values were not tested on the pretest or posttest, so leaving the decimal assessment tasks did not adversely affect the study.

As the pilot study progressed, the instructor became more aware of the intent and scope of the “Rule of Four” assessment model. His emphasis in class on that assessment model increased in the later part of the semester. To have that higher level of emphasis and to give students greater awareness of expectations, the instructor developed a set of

“Rule of Four” expectations to give to each participant in the treatment group at the start of the semester. The instructor worked to keep students better informed of their progress in meeting the “Rule of Four” expectations as they monitored the scoring grid used when the assessment task rubrics were implemented.

Data Analysis

Data Analysis Strategy

It was not expected that participants would have a great deal of prior knowledge of instructional models and multiple representations of the mathematical concepts studied in Math 277—Mathematics for Elementary Teachers I. Therefore the mean level of mathematics achievement for the treatment group was not expected to be significantly different from the mean level of mathematics achievement for the control group. This was tested using a *t*-test of independent samples. If results of a *t*-test showed that the mean pretest scores for the control group and the treatment group were not significantly different, then the treatment and control groups could be assumed to be of equal ability at the start of the study. This allowed the researcher to use a *t*-test of independent samples using the posttest scores to determine if there was a significant difference in mathematics achievement between the treatment and control groups. Since the “Rule of Four” assessment procedures were the only difference in how the sections were taught and assessed, a significant difference in the mean of the posttest scores should be attributable to the implementation of the “Rule of Four” procedures.

If a *t*-test of independent samples for the pretest scores indicated that the mean scores of the groups were significantly different from each other, a one-way analysis of covariance (ANCOVA) would be performed to determine whether there was a significant difference between the mathematics achievement of the treatment and control groups. For the One-Way ANCOVA the posttest score was the dependent variable. The independent variable was the experimental group (treatment or control), and the covariates were cumulative GPA, ACT Math score, and grade in Math 103-College Algebra.

Two-way analyses of covariance (Two-Way ANCOVA) were conducted on posttest scores to determine if there were differences with respect to “Rule of Four” usage in the mathematics achievement of experimental groups for various demographic groups. The demographic groups to be studied were males vs. females, age 25 or under vs. age 26 or over, and class status groups of underclass students vs. upper class students.

Alpha Level

An alpha level of 0.05 was used for this study. This level will be used rather than a more conservative value of 0.01 since it was difficult to imagine that harm could be done to students by advocating additional levels of assessment concerning their knowledge of real numbers and binary operations using those numbers, even if no measurable advantages exist. Thus a Type I error (rejecting the null hypothesis even though it is true) would not have serious negative effects. The possibility of making a Type II error (failing to reject the null hypothesis even though it is false) was of concern to this researcher. Making such an error would mean that the university would miss an opportunity to implement “Rule of Four” assessments in the Math 277 and Math 377

courses and thus help elementary education majors to be better prepared to teach mathematics.

Assumptions

Instruction for all four Math 277 sections in the fall 2005 and spring 2006 semesters at Minot State University would be taught by the same instructor. An assumption was made that delivery of instruction on each topic addressed in the study will be consistent for all four sections. An additional assumption was that “Rule of Four” assessment procedures will be implemented in the same manner each semester.

Summary

The purpose of this study was to determine if students enrolled in Math 277—Mathematics for Elementary Teachers I acquire more content knowledge concerning number and operations when assessed using a “Rule of Four” than those students not assessed using a “Rule of Four.” Data from pretests and posttests was gathered from students in four sections of Math 277 at Minot State University during the 2005-2006 school year. Data was analyzed using *t*-tests of independent samples, a One-Way ANCOVA, and Two-Way ANCOVAs to determine if significant differences in mathematics achievement existed between experimental groups and to determine if demographics produced significant differences in mathematics achievement between experimental groups.

CHAPTER 4

RESULTS

Introduction

This chapter shows data gathered and results of statistical analyses of pretest and posttest scoring for this study concerning assessment and instruction implementing a “Rule of Four” using manipulatives, numeric representations of a properties or operational algorithms, appropriate algebraic, geometric, or real world models with applicable diagrams, charts, or tables, and the written word. Data analysis addressed the following research hypotheses:

1. There is no significant difference in the mathematics achievement of those students who participate in assessments using a “Rule of Four” and the mathematics achievement of those students not involved with “Rule of Four” assessments.
2. There is no significant difference in the mathematics achievement of males and females who participate in assessments using a “Rule of Four” and the mathematics achievement of males and females who are not involved with “Rule of Four” assessments.
3. There is no significant difference in the mathematics achievement of students 25 years of age or younger who participate in assessments using a “Rule of Four” and the mathematics achievement of those students 26 years of age or older who are not involved with “Rule of Four” assessments.

4. There is no significant difference in the mathematics achievement of underclass students (freshmen and sophomores) who participate in assessments using a “Rule of Four” and the mathematics achievement of upper class students (juniors and seniors) who are not involved with “Rule of Four” assessments.

Hypothesis one was tested with a One-Way ANCOVA. Hypotheses two through four were tested with a Two-Way ANCOVA.

Reliability Analysis of Pretest and Posttest Scoring by Two Scorers

The study included data only for participants who took both the pretest and posttest. Two individuals independently used the scoring rubric to score pretests and posttests. Single measure intraclass correlation coefficients were determined using the one-way random effect model for each question and the total score on the pretests and posttests. Intraclass correlation coefficients for the pretest are shown in Table 8 below.

Table 8. Single Measure Intraclass Correlation Coefficients for Pretest Questions 1-10 Using the One-way Random Effect Model.

Question Number	Intraclass Correlation
1	.900
2	.895
3	.744
4	.751
5	.875
6	.778
7	.697
8	.873
9	.886
10	.756
Total	.928
N of Cases = 68 N of Items Compared = 2	

The intraclass correlation coefficients for the pretest and posttest data show high levels of correlation between scorers on each question and on score totals. Question 5 on the posttest had a lower, but acceptable level of correlation between scorers.

Single sets of pretest scores and posttest scores were created for analysis. This was accomplished first by averaging the scores on questions where the scorers' values differed by two. For questions on which the scorers' values differed by three, the higher of the two rubric values between their values was used.

Posttest reliability analysis for the two scorers is summarized in Table 9.

Table 9. Single Measure Intraclass Correlation Coefficients for Posttest Questions 1-10 Using the One-way Random Effect Model.

Question Number	Intraclass Correlation
1	.847
2	.687
3	.934
4	.912
5	.567
6	.729
7	.840
8	.668
9	.818
10	.789
Total	.909

N of Cases = 68 N of Items Compared = 2

t-Tests of Independent Samples for Pretests

Pretest total scores for the control group and treatment group were compared using an independent-samples *t* test. The mean of the control group ($M = 9.40$, $SD = 5.31$) was found to be significantly higher than that of the treatment (Rule of Four) group

($M = 5.40$, $SD = 4.56$), $t(66) = 3.15$, $p = .002$ (two-tailed). Figure 1 below shows the distributions of the two groups.

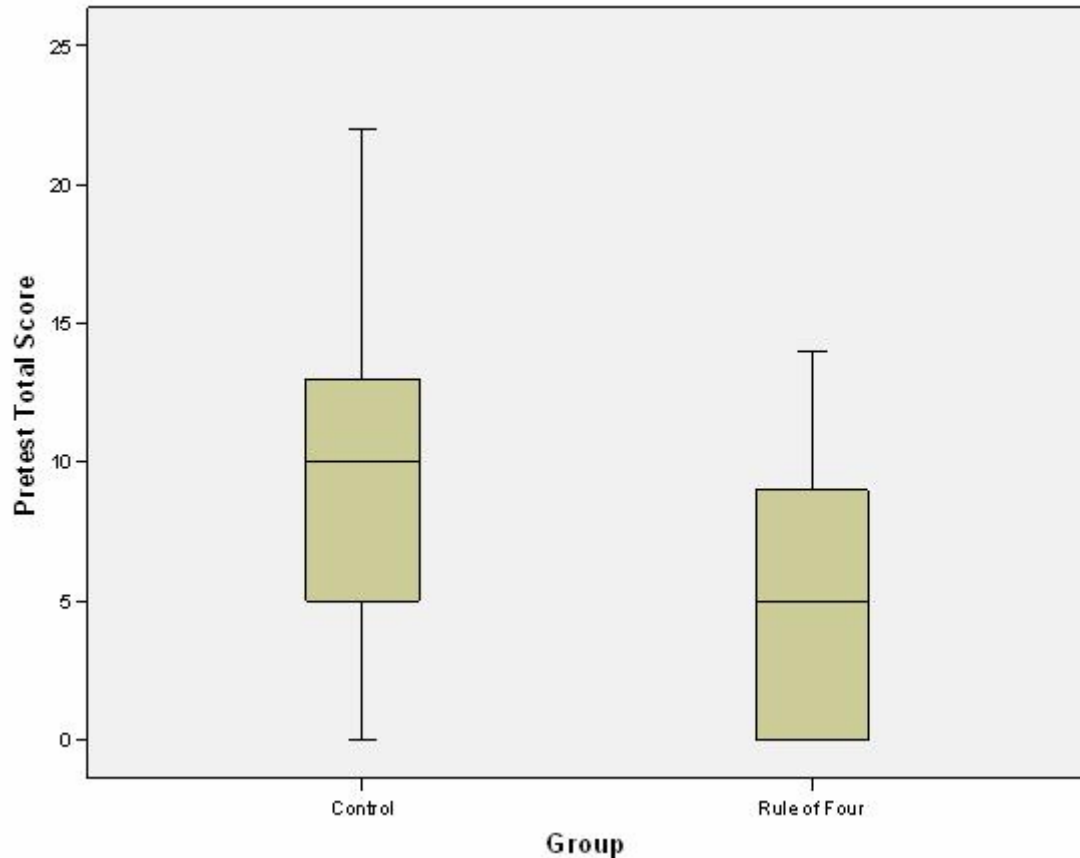


Figure 1. Distribution of pretest total scores for control group and treatment (Rule of Four) group.

Data Issues

Since there was a statistically significant difference between pretest scores of the two experimental groups, One-Way ANCOVA and Two-Way ANCOVA tests were used to test the four research hypotheses. In order to control for differences in the groups, ACT-Math scores, cumulative GPAs, and college algebra grades were used as covariates. While cumulative GPAs were available for all participants, college algebra grades were

not available for two participants, and ACT-Math scores were not available for 30 of the 68 participants. In order to maintain an n of 68 for the data analysis, scores for all three covariates had to be in place for each participant. To accomplish this, the average college algebra grade of the participants for which they were available was used for the two participants who did not have reported college algebra grades. Similarly, the average of the reported ACT-Math scores was used for each of the participants that did not have a reported ACT-Math score.

Prior to conducting the One-Way and Two-Way ANCOVAs, a preliminary analysis was performed to evaluate the homogeneity-of-slopes for the covariate and dependent variable. Results of this analysis indicated that the relationship between the covariate, the pretest total score, and the dependent variable, posttest total score, did not differ significantly as a function of the pretest total score, $F(1, 64) = .509$, $MSE = 26.66$, $p = .478$, partial $\eta^2 = .01$.

One-Way ANCOVA Analysis

A one-way analysis of covariance (One-Way ANCOVA) was conducted to test research hypothesis one. Hypothesis 1 was stated as follows: There is no significant difference in the mathematics achievement of those students who participate in assessments using a “Rule of Four” and the mathematics achievement of those students not involved with “Rule of Four” assessments. The One-Way ANCOVA showed no significant difference between the posttest scores of the treatment and control groups when controlling for differences in pretest scores, cumulative GPAs, college algebra

grades, and ACT-Math scores, $F(1, 62) = 0.402$, $MSE = 8.367$, $p = .528$. Therefore Hypothesis 1 was retained.

Two-Way ANCOVA Analyses

A two-way analysis of covariance (Two-Way ANCOVA) was used to test hypotheses two through 4. Hypothesis 2 was stated as follows: There is no significant difference in the mathematics achievement of males and females who participate in assessments using a “Rule of Four” and the mathematics achievement of males and females who are not involved with “Rule of Four” assessments. Results of the analysis showed no statistically significant difference between the posttest scores of males and females in the two experimental groups when controlling for differences in pretest scores, cumulative GPAs, college algebra grades, and ACT-Math scores, $F(1, 60) = 1.504$, $MSE = 30.249$, $p = .225$. Results from a 2 (gender) X 2 (experimental group) ANCOVA showed no statistically significant differences between groups. Thus, Hypothesis 2 was retained.

Hypothesis 3 was stated as follows: There is no significant difference in the mathematics achievement of students 25 years of age or younger who participate in assessments using a “Rule of Four” and the mathematics achievement of those students 26 years of age or older who are not involved with “Rule of Four” assessments. Results of the analysis showed no statistically significant difference between the posttest scores of students ages 25 and under and those 26 and over in the two experimental groups when controlling for differences in pretest scores, cumulative GPAs, college algebra grades,

and ACT-Math scores, $F(1, 60) = 0.407$, $MSE = 8.432$, $p = .526$. Results from a 2 (age) X 2 (experimental group) ANCOVA found no statistically significant difference between groups. Therefore, Hypothesis 3 was retained.

Hypothesis 4 was stated as follows: There is no significant difference in the mathematics achievement of underclass students (freshmen and sophomores) who participate in assessments using a “Rule of Four” and the mathematics achievement of upper class students (juniors and seniors) who are not involved with “Rule of Four” assessments. Results of the analysis showed no statistically significant difference between the posttest scores of the underclass students compared to the upper class students in the two experimental groups, when controlling for differences in pretest scores, cumulative GPAs, college algebra grades, and ACT-Math scores, $F(1, 60) = 0.274$, $MSE = 5.515$, $p = .603$. Results from a 2 (class) X 2 (experimental group) ANCOVA found no statistically significant difference between groups. Thus, Hypothesis 4 was retained.

Summary

This research study used a one-way analysis of covariance and two-way analyses of covariance to test four null hypotheses to examine the effects of “Rule of Four” assessments, gender, age, and class on mathematics achievement of elementary education majors. All four null hypotheses were retained.

CHAPTER 5

SUMMARY, IMPLICATIONS, AND RECOMMENDATIONS

Summary of the Study

The purpose of this study was to determine whether implementation of instruction and assessment utilizing a “Rule of Four” (using manipulatives, numeric representations of a properties or operational algorithms, appropriate algebraic, geometric, or real world models with applicable diagrams, charts, or tables, and the written word) in Math 277 would significantly raise the level of mathematics achievement for Minot State University elementary education majors. In particular this study focused on student knowledge and achievement in the content area of number and operation. “Rule of Four” instruction and assessments were aligned with performance expectations in the content standard for number and operation and the communication, representation, and connections process standards identified by the National Council of Teachers of Mathematics in *Principles and Standards for School Mathematics* (2000).

“Rule of Four” assessment tasks and a scoring rubric were developed for use in the study. A pretest-posttest instrument was developed that contained ten questions that aligned with the content addressed in the “Rule of Four” assessment tasks. Training was conducted for the teacher who would teach using the “Rule of Four” and implement the related assessment tasks. A pilot study was conducted during the spring semester of 2005

to assess effectiveness of study procedures and instruments. Some changes in procedures and pretest-posttests questions were made as a result of the pilot study.

Data collection took place during the fall and spring semesters of the 2005-2006 academic year at Minot State University in the four sections, two each semester, of Math 277 – Mathematics for Elementary Teachers I. Each semester both sections were taught using “Rule of Four” representations. Only one section each semester included the “Rule of Four” performance assessment tasks designed for this study. The data obtained was analyzed using the SPSS statistical analysis program to test the following research hypotheses:

1. There is no significant difference in the mathematics achievement of those students who participate in assessments using a “Rule of Four” and the mathematics achievement of those students not involved with “Rule of Four” assessments.
2. There is no significant difference in the mathematics achievement of males and females who participate in assessments using a “Rule of Four” and the mathematics achievement of males and females who are not involved with “Rule of Four” assessments.
3. There is no significant difference in the mathematics achievement of students 25 years of age or younger who participate in assessments using a “Rule of Four” and the mathematics achievement of those students 26 years of age or older who are not involved with “Rule of Four” assessments.
4. There is no significant difference in the mathematics achievement of underclass students (freshmen and sophomores) who participate in assessments using a “Rule of

Four” and the mathematics achievement of upper class students (juniors and seniors) who are not involved with “Rule of Four” assessments.

Data for the study was compiled in an SPSS spreadsheet. Data analysis for hypothesis 1 was completed using a One-Way ANCOVA and for hypotheses 2 through 4 was completed using Two-Way ANCOVAs.

Conclusions

This study sought to determine whether implementation of assessment utilizing a “Rule of Four” in Math 277 would significantly raise the level of mathematics achievement for Minot State University elementary education majors. Four null hypotheses were tested, and all four null hypotheses were retained. Those results would suggest that use of the “Rule of Four” assessments does not significantly increase overall mathematics achievement of elementary education majors relative to the ten topics assessed, regardless of gender, age, or class.

It had been expected that use of “Rule of Four” assessments would cause an increase in students’ mathematics achievement. The performance tasks used as “Rule of Four” assessments were designed following the tenets of Wiggins’ (1990) definition of authentic assessment. The “Rule of Four” performance tasks were tools used for facilitating learning as well as for evaluating it. They provided students in the experimental group with clarification of learning obligations and concepts that would be learned. In addition, as recommended by Stiggins (1997), the learning targets assessed

via the “Rule of Four” performance tasks were based on the collective academic wisdom of experts in mathematics, namely the National Council of Teachers of Mathematics.

The “Rule of Four” tasks also helped to communicate to students what was being valued in Math 277 learning experiences for the experimental group, which aligned with the recommendation of assessment by Arter and Stiggins (1992). The “Rule of Four” performance tasks required students in the experimental group to be actively engaged in learning experiences rather than being passive recipients of knowledge and provided opportunities for practicing skills. According to Cross (1987), students learn more when learning experiences have those characteristics. Thus, the experimental group was expected to have higher levels of mathematics achievement than the control group.

There are several possible reasons that no difference was found in mathematics achievement levels of the control and experimental groups. First, the concepts being studied may have been at too low a level to find stratification of achievement levels regardless how they were taught or assessed. The concepts being learned in Math 277 address numbers and operations at an extremely fundamental level. Greater complexity of concepts would allow for more variety in approaches for teaching and learning and would possibly lead to a greater range of success in learning the concepts.

Another possible reason that the experimental group did not reach significantly higher levels of achievement than the control group could be that students in the experimental group may not have looked on some of the performance tasks as assessments of their knowledge and skill. They may have looked on them as just another part of the learning experience having no more significance than other tasks asked of

them by the instructor. This may have been due to the instructor not consistently recording student results on the “Rule of Four” performance tasks as they were observed or completed.

A third reason that the experimental group did not meet expectations for higher levels of achievement than the control group may be a reflection of a phenomenon reported by Wilson (1993). Wilson found that students value what is graded and tend not to value what is not graded. In this study, the participants were told in the informed consent form that the “Rule of Four” assessments would not be included as part of the grading scheme for Math 277. Thus participants in the experimental sections of Math 277 may not have viewed the “Rule of Four” performance tasks as being worth the effort. Alternatively, they may have viewed them as valuable at the moment they were done, but not relevant or of enough importance when it came time to prepare for taking the posttest.

One more possible reason that performance expectations were not met for the experimental group is related to the finding by Smith (2000) that students do not perform as well when learning experiences are non-traditional and evaluation tools and methods are traditional. In this study the “Rule of Four” performance tasks provided a non-traditional approach to demonstrating learning through four distinct representations of a concept. The tests that were given to measure student achievement employed more traditional types of questions that asked students to demonstrate one particular representation that may look quite different from the performance tasks in the “Rule of Four” assessments.

It was unclear at the onset of the study whether gender would have a significant impact on student achievement with implementation of “Rule of Four” assessments. With several studies (Baker & LeTendre, 2005; Campbell et al., 2000; Catsambis, 2005; Fox & Soller, 2001) showing a narrowing of the gender gap in mathematics achievement, significant differences would be less likely than they might have been 30 years ago. Results of this study showed that there was no significant difference by gender group between mathematics achievement of the experimental group and that of the control group. Those results concur with several studies that found gender not make a difference in mathematic achievement (Dees, 1982; Tartre & Fennema, 1995; Curcio, 1987) or did not find gender to be a predictor of mathematics achievement (Meece et al., 1990; Pedersen et al., 1986; Gliner, 1987; House, 1995; Sue & Abe, 1988; Nuttall & Hell, 2001). The findings contradict those of several other studies (Randhawa & Randhawa, 1989; Wilson & Zhang, 1998; Hill, 1989) that found significant differences in mathematics achievement between males and females. Results of this study may be somewhat unreliable due to the low number of male participants. There were only 5 males in the control group and 4 males in the treatment group. Therefore the samples for these groups may not be representative of those segments of the population being studied.

Results of data analysis for the effect of age on mathematics achievement of students in the control and treatment groups showed no significant differences in achievement among groups, with age groups representing students 25 years of age or younger and students 26 years of age or older. This finding agrees with that of Gliner (1987), whose study found that age is not a predictor of mathematics achievement.

However, results of this study regarding the effect of age may be somewhat unreliable due to small numbers of participants in two of the subgroups. There were only 3 participants 26 years of age or older in the control group and 4 participants 25 years of age or younger in the treatment group. Therefore, the samples for these groups are more likely not to be representative of those segments of the population being studied.

This study also examined the relationship between mathematics achievement and class, where participants were grouped as being underclassmen (freshmen and sophomores) or upper classmen (juniors and seniors). No difference was found in the mathematics achievement of experimental subgroups formed by class groupings. This aligns with the findings for mathematics achievement of experimental subgroups formed by age groupings. This might have been expected as age and class have a strong correlation. There are not many studies that use class as an independent variable, likely due to the high correlation between age and class. In most of the research studies reviewed, study participants were members of classes at either the elementary or secondary level, where students would be very close to the same age. Very few studies involved participants from more than one class (e.g., freshman, sophomore, junior, senior).

Implications for Teacher Education

The purpose of this study was to determine whether implementation of assessment utilizing a “Rule of Four” in Math 277 would significantly raise the level of mathematics achievement for Minot State University elementary education majors. Study

results were to inform the Department of Mathematics and Computer Science at Minot State University about whether it should request changes in the elementary education program that would allow more time in Math 277-Mathematics for Elementary Teachers I and Math 377-Mathematics for Elementary Teachers II for assessing student performance through use of “Rule of Four” assessments. Results of the study showed that the “Rule of Four” assessments did not significantly raise participants’ mathematics achievement levels.

While the treatment group did not have significantly higher levels of achievement than the control group, they did have greater levels of engagement in their learning as was evident in the student-student and teacher-student interaction that was observed by the researcher. The increasing students’ level of engagement is supported by Bruner (1971) who maintained that a key to instruction is activating the learner, that is, first getting people to want to learn and then making the experience both compelling and sustained. Such increased engagement would be a good reason for the Department of Mathematics and Computer Science to request additional credit and teacher/student contact hours for the Math 277 and Math 377 courses.

Students in the treatment sections showed strong gains in math achievement over the course of the semester. These students demonstrated significantly lower levels of content knowledge at the start of the semester than the students in the control sections. Although they had large gains, on average their marginal mean posttest score was slightly lower than that of the students in the control group.

The “Rule of Four” instruction and assessments did not hinder the learning of the students in those sections and may have been the cause for the strong gains in achievement. It is possible, however, that these students would have reached the same levels of achievement through implementation of traditional methods of assessment. An emphasis on accountability for knowing each concept on multiple levels may have positively impacted the learning of “Rule of Four” participants.

Certainly ongoing assessment enables a teacher to learn the thinking, reasoning, and level of understanding of students in their classes. Ensuring that the assessment uses multiple measures at varying levels of complexity is strongly recommended by the National Council of Teachers of Mathematics. Instruction and assessment by “Rule of Four” aligns with this recommendation. Since it does not harm students and may help teachers to better their students’ knowledge and understanding of concepts, “Rule of Four” performance tasks should be looked upon as a desirable assessment option.

Recommendations for Further Research

This study limited its scope to examining mathematics achievement of elementary education majors in the area of number and operation. It may be of value to conduct a similar study to examine the mathematics achievement of elementary education majors in the areas of measurement, geometry, probability, and statistics. The development of assessment activities that can be built into instruction in those areas may have a positive effect on mathematics instruction at the elementary level.

It may be worthwhile to replicate this study and conduct an attitude survey to determine whether use of “Rule of Four” instruction and assessments would affect the attitudes and beliefs of participants with regard to mathematical confidence and the value of mathematics. Several studies (Boe et al., 2004; Hong et al., 2005; Tocci & Engelhard, 1991; Tartre & Fennema, 1995) indicated that student attitudes can have a significant impact on mathematics achievement.

Should this study be replicated, it may be helpful to get more frequent feedback from the course instructor on implementation of assessment tasks and the recording of successful completion of the tasks. It may also be a good idea to increase the number of items on the pretest and posttest so that multiple questions address the same content. This would allow for a broader view of student understanding. Another consideration for replication of the study would be to increase the number of semesters during which data would be collected to achieve a higher number of participants. This would eliminate the need to collapse data into broader ranges in the areas of age and class as they were in this study.

Future studies based on this model should align pretest and posttest questions and performance expectations more closely with those of the performance tasks used in the “Rule of Four” assessments. This would be supported by Smith (2000) who found that students do not perform as well when learning experiences are non-traditional and evaluation tools and methods are traditional. The “Rule of Four” performance tasks should also be included as part of the grading scheme for Math 277 so that students value

them. This is supported by the findings of Wilson (1993) who found that students value what is graded and tend not to value what is not graded.

Now that passing the PRAXIS II elementary education content test is being required as a licensure and graduation requirement for all elementary education program completers at Minot State University, it may be worthwhile to do a follow-up study of this study's participants and their success rate in passing the PRAXIS II exam. Most of the study's participants are just getting to the point where they will take the PRAXIS II exam, as they generally take this test during their senior year, often during student teaching. The focus of that study would be to determine whether participants in the treatment group have greater success rates in passing the PRAXIS II exam than those in the control group. The results of such a study would help to inform the teacher educators at Minot State University on how they might better prepare elementary education majors to pass the PRAXIS II exam by providing better training in mathematics.

A second follow-up study could be done in four or five years after the study's participants have entered the teaching profession and have taught for 2 or more years. The purpose of such a study would be to determine whether instruction and assessment by "Rule of Four" made a difference in participants' retention of mathematics knowledge and their ability to utilize that knowledge to create meaningful mathematics learning experiences for their students.

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APPENDICES

APPENDIX A

PRETEST/ INSTRUMENT

Appendix A
Pre-Test and Posttest

Demographic information: Please place an X in the appropriate blank.

A. Gender: Female _____ Male _____

B. Age group: 25 or under _____ 26 or older _____

C. Class status: Fr. _____ Soph. _____ Jr. _____ Sr. _____ UGSpecial _____

1. Demonstrate using drawings how to add the following numbers using regrouping:

$$\begin{array}{r} 384 \\ +537 \\ \hline \end{array}$$

2. Demonstrate using drawings how to subtract the following numbers using regrouping:

$$\begin{array}{r} 631 \\ -265 \\ \hline \end{array}$$

3. Explain and demonstrate addition of the following signed numbers:

$$3 + (-8) = ?$$

4. Explain and demonstrate subtraction of the indicated signed numbers:

$$7 - (-4) = ?$$

5. Illustrate with drawings of appropriate manipulatives, how to determine which fraction is larger:

$$\frac{3}{8} \quad \frac{1}{3}$$

6. Show how to connect the use of base 10 units, rods, and mats to the division algorithm by demonstrating (on paper with drawings and the algorithm) how to divide 675 by 4.

$$4 \overline{)675}$$

7. Show how to use linker cubes and the area model to demonstrate the commutative property of multiplication when 3 and 4 are multiplied together.

8. Show how to use Cuisenaire rods to demonstrate the associative property of addition using the numbers 2, 3, and 5.
9. Demonstrate how to multiply 376 by 247 using partial products.

$$\begin{array}{r} 376 \\ \times 247 \\ \hline \end{array}$$

10. Demonstrate how to use manipulatives to find the answer to $1\frac{3}{4} \div \frac{3}{8}$, and then explain your work.

$$1\frac{3}{4} \div \frac{3}{8} = ?$$

Revised Pretest/ Questions

1. Demonstrate using drawings how to add the following numbers using regrouping:

$$\begin{array}{r} 384 \\ +537 \\ \hline \end{array}$$

2. Demonstrate using drawings how to subtract the following numbers using regrouping:

$$\begin{array}{r} 631 \\ -265 \\ \hline \end{array}$$

3. Explain and demonstrate addition of the following signed numbers using red and black chips:

$$3 + (-8) = ?$$

4. Explain and demonstrate subtraction of the indicated signed numbers using red and black chips:

$$7 - (-4) = ?$$

5. Illustrate with drawings of appropriate manipulatives, how to determine which fraction is larger and then tell which is larger:

$$\frac{3}{8} \quad \frac{1}{3}$$

6. Show how to connect the partition model with base 10 units, rods, and mats to the division algorithm by demonstrating (on paper with drawings and the algorithm) how to divide 675 by 4. Be sure to show how they are connected.

$$4 \overline{)675}$$

Place Value Table

Partition Table

--	--	--	--

7. Show how to use the area model to demonstrate the commutative property of multiplication when 3 and 4 are multiplied together.
8. Show how to use Cuisenaire rods to demonstrate the associative property of addition using the numbers 2, 3, and 5.
9. Do the following multiplication problem using the instructional algorithm. To the right of each partial product, indicate what you are actually multiplying

$$\begin{array}{r} 376 \\ \times \underline{247} \end{array}$$

10. Demonstrate how to use manipulatives to find the answer to $1\frac{3}{4} \div \frac{3}{8}$, and then explain your work.

$$1\frac{3}{4} \div \frac{3}{8} = ?$$

APPENDIX B

PRESTEST/ SCORING RUBRIC

Appendix B

Scoring Rubric for Pre-Test and Posttest

1. 4 - Arrives at correct answer and provides accurate drawings.
3 - Arrives at correct answer but leaves out one or two steps in drawings.
2 - Arrives at incorrect answer or leaves out several steps in drawings.
1 - Does not arrive at an answer or does not do any drawings.
0 - Does not attempt the task.

2. 4 - Arrives at correct answer and provides accurate drawings.
3 - Arrives at correct answer but leaves out one or two steps in drawings.
2 - Arrives at incorrect answer or leaves out several steps in drawings.
1 - Does not arrive at an answer or does not do any drawings.
0 - Does not attempt the task.

3. 4 - Correctly demonstrates and explains addition of the signed numbers.
3 - Correctly demonstrates or explains but cannot adequately do the other for addition of the signed numbers.
2 - Incorrectly demonstrates and cannot adequately explain addition of the signed numbers.
1 - Cannot correctly demonstrate or explain addition of the signed numbers or does only one of the two tasks.
0 - Does not attempt the task.

4. 4 - Correctly demonstrates and explains subtraction of signed numbers.
3 - Correctly demonstrates or explains but cannot adequately do the other for subtraction of signed numbers.
2 - Incorrectly demonstrates and cannot adequately explain subtraction of signed numbers.
1 - Cannot correctly demonstrate or explain subtraction of the signed numbers or does only one of the two tasks.
0 - Does not attempt the task.

5. 4 - Represents fractions appropriately and correctly determines which is larger.
3 - Does not represent fractions appropriately but correctly determines which is larger.
2 - Does not represent fractions appropriately and incorrectly determines which is larger.
1 - Cannot complete the task.
0 - Does not attempt the task.

6. 4 - Divides correctly and connects drawings correctly to the division algorithm.
3 - Divides correctly but does not make appropriate drawings and connect them to the algorithm, or doesn't show algorithm but makes appropriate drawings.
2 - Divides incorrectly and cannot complete drawings or does not connect drawings to the algorithm.
1 - Cannot complete the division or does not do any drawings.
0 - Does not attempt the task.

7. 4 - Correctly demonstrates commutative property of multiplication using the area model.
3 - Correctly demonstrates commutative property of multiplication but uses a different model or uses area model incorrectly.
2 - Demonstrates the wrong property but uses the area model appropriately.
1 - Cannot complete the task.
0 - Does not attempt the task.

8. 4 - Correctly demonstrates associative property of addition using rods.
3 - Correctly demonstrates associative property of addition but uses rods incorrectly.
2 - Demonstrates the wrong property but uses the rods appropriately.
1 - Cannot complete the task.
0 - Does not attempt the task.

9. 4 - Multiplies correctly and shows partial products correctly.
3 - Multiplies correctly but does not use method of partial products.
2 - Multiplies incorrectly and does not correctly show partial products.
1 - Cannot complete the multiplication problem.
0 - Does not attempt the task.

10. 4 - Divides correctly, shows appropriate manipulatives, and adequately explains the work.
3 - Divides correctly, but uses the algorithm and explains the algorithm adequately.
2 - Divides incorrectly and cannot adequately explain the work, or divides using the algorithm and does not explain any of the work.
1 - Cannot complete the division problem.
0 - Does not attempt the task.

APPENDIX C

INFORMED CONSENT FORM

Appendix C

Informed Consent for Student Participants in the Study
Concerning Elementary Education Majors' Mathematics Achievement
Using an Assessment Model Based on a "Rule of Four"

Date: _____

Student's Name: _____

I am a faculty member in the Department of Mathematics and Computer Science at Minot State University. As part of my coursework for my doctoral program, I am doing a research study concerning assessment of mathematics achievement of elementary education majors.

As someone who is as an elementary major enrolled in Math 277, I am inviting you to participate in this study. The purpose of the study is to determine whether assessment based on a "Rule of Four," as implemented in Math 277, will increase mathematics achievement levels for elementary education majors. All members of the class will take a pretest sometime during the first two weeks of the Math 277 course and a posttest at the end of the course. One section of Math 277 will have assessments of your learning done as in previous semesters. Students in the other section will have additional "Rule of Four" assessments done during class time as a part of the course. Scores of the participants from the "Rule of Four" assessment section will be compared to the scores of those Students in the other section who did not participate in the "Rule of Four" assessments. As a result of participating in this study, perhaps the student will gain a higher level of mathematics content knowledge, which ultimately may help that candidate become a more effective teacher of mathematics. Pretest and posttest scores are not factors in the computing of Math 277 grades.

Your name will be used only to pair up the pretest with the posttest in the research study. The course instructor will record the test form number used by each participant on the pretest. The corner of the first test page where the test-taker's name is written will be cut off after the test number is recorded. The pretests, identified now only by number, will then be given to the researcher to score. When the posttest is taken, students will write their names in the upper right hand corner. The course instructor will write the pretest number for the students below their names and will then cut off the name. This is done for the purpose of pairing pretest and scores for each individual. The list of names paired with test form numbers will be seen only by the instructor and will be kept by the instructor until all posttests are taken. The instructor will then destroy the list via a shredding process. The researcher will never see the list of names paired with test form numbers.

Three pieces of demographic data will be gathered as part of the study. They are gender, age group (25 or under vs. 26 or older), and class status (freshman, sophomore, junior, senior, or undergraduate special). That demographic information will be collected on the pretest question sheet. No student names will be used in the study report. The informed consent forms will be kept in a sealed envelope in a filing cabinet in the instructor's office until the study is completed. At that time the sealed envelope will be given to the researcher, who will keep it in a filing cabinet in her office in Model Hall until the research study and thesis are approved.

If you have any concerns about the study or would like to report any abuses you suspect, please notify Dr. David McCormack, Chair, Dept. of Mathematics & Computer Science (858-3281) or Dr. Margi Coxwell, Chair, MSU Institutional Review Board (858-3125).

Your participation in this study is voluntary. If at any time you decide not to continue participating in the study, your pretest and posttest scores will be eliminated and the pretest and posttest papers destroyed. Should you decide not to participate in the study, your Math 277 grade and your relationship with the Mathematics & Computer Science Department, Minot State University, your instructor, or the researcher will not be affected. Your signature below signifies your intent to participate. Thank you very much!

Sincerely,

Cheryl Nilsen

Student's Signature

APPENDIX D

MATH 277 SYLLABUS

Appendix D



Mathematics for Elementary Teachers I Spring 2004

Course:

Mathematics for Elementary Teachers I, MATH 277, 3 credits

Prerequisite: Math 103

Course Description:

This is a course for elementary education majors. Topics include problem solving, sets of numbers (natural numbers through the reals), number theory, and proportional reasoning. Students will gain an understanding of the mathematics taught at the elementary level and will learn how to communicate, explain, and demonstrate the mathematics using various physical models, conceptual models, and manipulatives. Prerequisite: MATH 103

Course Materials:

1. Long, C. T. & DeTemple, D. W. (2000). *Mathematical Reasoning for Elementary Teachers* (2nd Ed.). Reading, MA: Addison Wesley Education Publishers, Inc.
2. (Optional) Bennett, A. B., Jr. & Nelson, L. T. (2001). *Mathematics for Elementary Teachers: An Activity Approach* (5th Ed.). New York, NY: McGraw-Hill Companies, Inc.
3. (Optional) Manipulative kit to accompany text 2.
4. A three-ring binder to keep all your handouts. You are going to need it. I will hole-punch everything for you.
5. The National Library of Virtual Manipulative for Interactive Mathematics Web site: <http://matti.usu.edu/nlvm/nav/index.html>

Course Overview and Purpose:

This course explores the fundamental topics in the elementary curriculum: problem solving, sets, number systems and operations on these systems. Each of these topics is introduced in the early elementary grades and developed in greater and greater detail as students progress. ***My main goals this semester are to help you better understand the mathematical concepts and skills taught in elementary school, as well as provide you with methods for teaching these concepts and skills to elementary students.***

This course will assess your ability in two areas: (1) your understanding of the mathematics, and (2) your ability to communicate your understanding of the mathematics. The latter goal, your ability to communicate your understanding, is especially relevant to your future as an elementary teacher. During this course you will also be asked to demonstrate and to explain the mathematical concepts taught in the elementary grades. You will often be asked to do this using manipulatives or physical models. Thus, you will need to explain the what, how, and why all the time.

I hope you emerge from MATH 277 with a better understanding of the mathematics that you will teach and with multiple ways to teach it. I also hope that you will come to view mathematics in a positive way. The attitude that you bring into your mathematics classroom will impact the attitude your students have about mathematics. So bring a positive attitude and get fired up about math! Enthusiasm is contagious!

Course Goals: The goals of MATH 277 are:

1. To develop students' understanding of the mathematical concepts taught at the elementary level (knowledge and reflection).
2. To improve students' abilities to explain and demonstrate, thus communicate and teach, the mathematical concepts taught at the elementary level (action & knowledge).
3. To teach students how to use manipulatives and models to demonstrate and explain the mathematics taught at the elementary level (action, knowledge, & reflection).
4. To develop students' problem solving skills (action & knowledge).
5. To improve students' attitudes about mathematics (reflection).

Student Outcomes: Students who complete MATH 277 will be able to:

1. Understand, demonstrate, and explain the operations of addition, subtraction, multiplication, and division on each set of numbers (natural numbers through the real numbers) using physical models, conceptual models, algorithms, and manipulatives (action, knowledge, & reflection).
2. Understand and use mathematical notation and language correctly (action & knowledge).
3. Understand, demonstrate, and explain place value in base 10 and in other bases (action, knowledge, & reflection).
4. Understand, demonstrate and explain various number theory concepts numerically and using physical models (action, knowledge, & reflection).
5. Demonstrate and explain the properties of each operation in each set of numbers (action & knowledge).
6. Demonstrate and explain sets, operations on sets, and relationships between sets (action & knowledge).

Assessment: Students will be assessed using the following:

1. Participation in classroom discussions and activities (action, knowledge, & reflection).
2. Assignments that require explanations and demonstrations of key concepts and models (action, knowledge, & reflection).
3. Assigned readings and homework problems (action, knowledge & reflection).
4. A presentation (action, knowledge, & reflection).
5. Reading of journal articles and corresponding writing assignments (knowledge & reflection).
6. Tests and a final exam (knowledge & reflection).

Course Structure and Format:

This course will be taught using a combination of lecture and hands-on activities done individually or in groups. I want you to be an active participant in your learning while I try to facilitate your learning. I do not expect you to open your head and have me fill it with knowledge. Of course, I may show you new things you have never seen before or have never thought of in a certain way, but I want you to interact with me and with the material as you learn.

Assignments:

Homework: Homework is an important part of this course. The amount of time and effort you put into **truly understanding** the homework will not only help you in this course, but also when you teach your own class. Math is **NOT** about memorizing formulas and solving problems with a “shortcut.” Rather, math is about problem solving, understanding concepts, critical thinking, and yes, often persisting through frustration (even when you want to throw your math book against the wall). Understanding concepts, and then being able to explain and demonstrate these concepts, will be most important in this course! You need to be able to do more than just show students how to perform calculations. You need to be able to explain to them how and why mathematics works.

You will be assigned two kinds of homework problems, and it is essential that you do both kinds. The first kind is problems in the text. These problems are in the course schedule. You should complete these problems after each lesson. The second kind of homework problems are ones that I create and hand out for you to do. These problems focus on your ability to explain and demonstrate crucial concepts. I will collect and grade one or both of these kinds of homework! I will grade for accuracy, thoroughness, and logical and complete explanations. I take your homework assignments very seriously, and you should too. They will be your biggest help in preparing for exams.

Readings and Writings: You are required to read 3 articles or other readings over the course of the semester. After reading them you are required to write a 2-page paper on each of these articles. The articles deal with teaching mathematics in the elementary grades or teaching and education in general. I hope these articles will make you think, question, and provide you with new ideas. The details of this assignment are provided at the end of the syllabus on a separate sheet, and the due dates are in the course schedule.

Web and Presentation Assignment: I want you to find: (a) 3 or more excellent Web sources of relevant elementary teaching information, and (b) one lesson plan on the topic of your choice—please be creative. You will then present and explain the Web sites and lesson plan to the class. The lesson plan can be one that you created, one that you found on a Website, or one that you adapted that was authored by someone else. If you use someone else’s work you **MUST** reference this!! Each person will bring copies of the Web addresses and the lesson plan for each student in the class. I hope that this will provide each of you with valuable resources that you might use in the classroom. We will discuss the format and grading of this in more detail later.

Other assignments: Other assignments may be assigned during the semester. For example, sometimes you will receive points for an in-class individual or group activity or assignment, so it is imperative that you attend each day.

Tests and Quizzes:

Chapter tests and a comprehensive final examination will be administered during the semester. The tentative dates are given in the course schedule. If for some reason you have to miss an exam, you must get permission from me **prior** to the exam to schedule a make-up (or as soon thereafter as possible for an unforeseen occurrence).

Attendance and Class Participation:

Daily attendance is expected and absences will be noted. This course will have in-class activities that count toward your grade. Thus, it is important for you to attend. Learning takes place through participation in discussion and group activities. Thus, it is essential that you attend

class. If, by chance, an emergency requires that you miss a substantial amount of class, please let me know so that we can make arrangements for your absence. Attendance and participation will be considered when computing your final grade in this course.

Grading:	Percent of your Final Grade
Exams	50%
Final	20%
In-class work, readings, presentation	20%
Homework	10%

Lowest A – 92% Lowest B – 83% Lowest C – 74% Lowest D – 65%

APPENDIX E

MATH 377 SYLLABUS

Appendix E

Math 377 Mathematics for Elementary Teachers

2 Semester Hours

Fall Semester 2004



1. **Title:** Mathematics for Elementary Teachers II
Prerequisite: Math 103
2. **Offering:** This course is offered every fall and spring semester.

Chapter 11 **Course Description:** Minot State University's organizing theme for teacher education is Teacher as Reflective Decision Maker. To develop reflective decision makers, MSU has implemented the conceptual model ARK representing action, reflection, and knowledge. Math 377 addresses the knowledge component of the model. It is a required mathematics content course intended to ensure that all Minot State University elementary education majors have the content knowledge necessary for them to be effective teachers of mathematics in the elementary and middle school grades.

The following course objectives contribute to this necessity. Each objective of Math 377 is labeled (A, R, and/or K) to indicate the dominant component of the ARK model.

- A. To acquaint students with modern developments in the field of elementary and middle school mathematics, including the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*. (RK)
- B. To develop coherence and organization in students' understanding of mathematical concepts, structures, and procedures in the area of geometry. (K)
- C. To develop coherence and organization in students' understanding of mathematical concepts, structures, and procedures in the area of measurement. (K)
- D. To develop coherence and organization in students' understanding of mathematical concepts, structures, and procedures in the area of statistics. (K)
- E. To develop coherence and organization in students' understanding of mathematical concepts, structures, and procedures in the area of probability. (K)

- F. To identify and observe effective methods for teaching mathematical concepts to elementary and middle school students, identify common student misconceptions and error patterns, and determine how the misconceptions and error patterns might be addressed. (AR)
- G. To become acquainted with current literature in the field, for professional reading and for enrichment for both teacher and student. (RK)
- H. To become acquainted with the use of computers, calculators and manipulatives in teaching elementary and middle school mathematics. (ARK)

4. **Textbook:** *Required:* Long & DeTemple, (2000), Mathematical Reasoning for Elementary Teachers, (2nd ed.), Addison-Wesley, Reading, MA
Optional: Bennett & Nelson, (2001), Mathematics for Elementary Teachers: An Activity Approach, (5th ed.), McGraw-Hill, New York, NY

5. **Work Requirements:** Following is a listing of activities that will be completed by candidates in Math 277:

- A. Candidates will be given problem sets and exercises to do, some as in class activities, and some as homework. Most of these will be graded.
- B. Candidates will work with manipulatives to do mathematical investigations both in and out of class.
- C. Candidates will read journal articles on topics related to concepts being studied and will write summaries and reflections concerning how they, in their future classrooms, might implement teaching strategies or activities described in the articles.
- D. Candidates will demonstrate proficiency in use of tools such as compasses, protractors, rulers, reflecting devices, and other manipulatives for learning and teaching mathematical concepts.
- E. Candidates will find interactive mathematical applets in the areas of geometry, measurement, probability, and statistics that are available through the Internet and reflect on how they can be used to help students understand mathematical concepts.

Chapter 11 **Grading:** Grades for Math 377 will be determined based on the following weighing factors and using the grading scale below:

	<u>Percent of Grade</u>
Exams and Final Exam	60%
Projects and Papers	20%
Homework	20%

A – 90-100% B – 80-89% C – 70-79% D – 60-69% F – Below 60%

Homework will regularly be assigned, collected, and graded. Many activities that are parts of assignments will be started and/or completed during class time. Make up work for excused absences will be allowed. You will have as many days as you missed to make up homework.

The following sections of the textbook will be covered in the order listed below:

Chapter 8	8.1 & 8.2
Chapter 9	Parts of sections 9.1 – 9.3

Chapter 10 10.1 – 10.3

Chapter 11 All sections 11.1 – 11.4

The final exam will be comprehensive.

You are responsible for all of the material covered in lecture, the textbook, and class handouts. Attendance is expected and will be considered when computing your grade for this course.

APPENDIX F

“RULE OF FOUR” PERFORMANCE TASKS

Appendix F

Performance Tasks:

- 1A) The student will select appropriate manipulatives (from those displayed at the assessment site) and demonstrate addition of
- the whole numbers 365 and 238.
 - the integers -13 and 7.
 - the fractions $\frac{2}{3}$ and $\frac{3}{4}$.
- 1B) The student will make a pictorial representation, chart, or diagram showing the addition of
- the whole numbers 365 and 238.
 - the integers -13 and 7.
 - the fractions $\frac{2}{3}$ and $\frac{3}{4}$.
- 1C) The student will use an appropriate algebraic, geometric, or real world model to represent the addition of
- two whole numbers larger than 10.
 - two integers with different signs.
 - two fractions with different denominators.
- 1D) The student will write a paragraph explaining how to add
- the whole numbers 365 and 238.
 - the integers -13 and 7.
 - the fractions $\frac{2}{3}$ and $\frac{3}{4}$.
- 2A) The student will select appropriate manipulatives (from those displayed at the assessment site) and demonstrate subtraction of
- the whole numbers 365 minus 288.
 - the integers -12 minus 8.
 - the fractions $\frac{5}{6}$ minus $\frac{1}{4}$.
- 2B) The student will make a pictorial representation, chart, or diagram showing the subtraction of
- the whole numbers 365 minus 288.
 - the integers -12 minus 8.

- c.) the fractions $\frac{5}{6}$ minus $\frac{1}{4}$.
- 2C) The student will use an appropriate algebraic, geometric, or real world model to represent the subtraction of
- two whole numbers larger than 10.
 - two integers with different signs.
 - two fractions with different denominators.
- 2D) The student will write a paragraph explaining how to subtract
- the whole numbers 365 minus 288.
 - the integers -12 minus 8.
 - the fractions $\frac{5}{6}$ minus $\frac{1}{4}$.
- 3A) The student will select appropriate manipulatives (from those displayed at the assessment site) and demonstrate multiplication of
- the whole numbers 407 and 26.
 - the integers -15 and 12.
 - the fractions $\frac{3}{4}$ and $\frac{5}{8}$.
- 3B) The student will make a pictorial representation, chart, or diagram showing the multiplication of
- the whole numbers 407 and 26.
 - the integers -15 and 12.
 - the fractions $\frac{3}{4}$ and $\frac{5}{8}$.
- 3C) The student will use an appropriate algebraic, geometric, or real world model to represent the multiplication of
- two whole numbers larger than 10.
 - two integers with different signs.
 - two fractions with different denominators.
- 3D) The student will write a paragraph explaining how to multiply
- the whole numbers 407 and 26.
 - the integers -15 and 12.
 - the fractions $\frac{3}{4}$ and $\frac{5}{8}$.
- 4A) The student will select appropriate manipulatives (from those displayed at the assessment site) and demonstrate division of

- a.) the whole numbers 34 by 5.
 - b.) the integers -27 by 4.
 - c.) the fractions $\frac{5}{2}$ by $\frac{7}{8}$.
- 4B) The student will make a pictorial representation, chart, or diagram showing the division of
- a.) the whole numbers 34 by 5.
 - b.) the integers -27 by 4.
 - c.) the fractions $\frac{5}{2}$ by $\frac{7}{8}$.
- 4C) The student will use an appropriate algebraic, geometric, or real world model to represent the division of
- a.) two whole numbers larger than 10.
 - b.) two integers with different signs.
 - c.) two fractions with different denominators.
- 4D) The student will write a paragraph explaining how to divide
- a.) the whole numbers 34 by 5.
 - b.) the integers -27 by 4.
 - c.) the fractions $\frac{5}{2}$ by $\frac{7}{8}$.
- 5A) The student will select appropriate manipulatives (from those displayed at the assessment site) and demonstrate
- a.) the commutative property of addition, using the numbers 5 and 9.
 - b.) the commutative property of multiplication, using the numbers 4 and 7.
 - c.) the associative property of addition, using the numbers 2, 5, and 7.
 - d.) the associative property of multiplication, using the numbers 2, 3, and 4.
 - e.) the distributive property of multiplication over addition, using the numbers 3, 4, and 5.
- 5B) The student will make a pictorial representation, chart, or diagram illustrating
- a.) the commutative property of addition, using the numbers 5 and 9.
 - b.) the commutative property of multiplication, using the numbers 4 and 7.
 - c.) the associative property of addition, using the numbers 2, 5, and 7.
 - d.) the associative property of multiplication, using the numbers 2, 3, and 4.
 - e.) the distributive property of multiplication over addition, using the numbers 3, 4, and 5.

- 5C) The student will use an appropriate algebraic, geometric, or real world model to represent
- a.) the commutative property of addition, using two numbers or variables.
 - b.) the commutative property of multiplication, using two numbers or variables.
 - c.) the associative property of addition, using three numbers or variables.
 - d.) the associative property of multiplication, using three numbers or variables.
 - e.) the distributive property of multiplication over addition, using three numbers or variables.
- 5D) The student will write a paragraph explaining
- a.) the commutative property of addition, using the numbers 5 and 9.
 - b.) the commutative property of multiplication, using the numbers 4 and 7.
 - c.) the associative property of addition, using the numbers 2, 5, and 7.
 - d.) the associative property of multiplication, using the numbers 2, 3, and 4.
 - e.) the distributive property of multiplication over addition, using the numbers 3, 4, and 5.

APPENDIX G

ASSESSMENT RUBRICS FOR “RULE OF FOUR”
PERFORMANCE TASKS

Appendix G

Assessment Rubrics:

Performance Tasks 1A-5D

For each performance task at level A (e.g., 1Aa, 3Ac, 4Ab) the following rubric will be used. For levels B, C, and D, the subsequent rubrics will be used respectively.

Score	Meaning	Criterion Level A
4	Proficient (Exceptional)	The student selects an appropriate manipulative, demonstrates the task correctly, and provides adequate explanation without prompts.
3	Target (Acceptable)	The student selects an appropriate manipulative, demonstrates the task correctly, and provides adequate explanation with one or two prompts.
2	Limited Knowledge (Not Acceptable)	The student selects an appropriate manipulative, demonstrates the task correctly, and provides some explanation with more than two prompts.
1	Unsuccessful (Not Acceptable)	The student selects an appropriate manipulative but cannot demonstrate the task correctly or provides adequate explanation even with prompting.
0	No Knowledge (Not Acceptable)	The student selects an inappropriate manipulative and cannot complete the task, or the student does not attempt the task.

Score	Meaning	Criterion Level B - Operations
4	Proficient (Exceptional)	The student selects an appropriate algorithm, correctly names the algorithm (when applicable) and uses it to get a correct value. The student shows all steps of the algorithm and provides explanatory comments.
3	Target (Acceptable)	The student selects an appropriate algorithm, correctly names the algorithm (when applicable) and uses it to get a correct value. The student shows all steps of the algorithm.
2	Limited Knowledge (Not Acceptable)	The student selects an appropriate algorithm, correctly names the algorithm (when applicable) and uses it, but gets an incorrect value. The student shows most of the steps of the algorithm.
1	Unsuccessful (Not Acceptable)	The student selects an appropriate algorithm but cannot name the algorithm (when applicable) and can identify a limited number of steps in the algorithm but arrives at a correct or incorrect value, or the student identifies steps but cannot get through them to find a value.

0	No Knowledge (Not Acceptable)	The student selects an inappropriate algorithm and cannot complete the task, or the student does not attempt the task.
Score	Meaning	Criterion Level B - Properties
4	Proficient (Exceptional)	The student demonstrates the correct property and uses an appropriate numerical and symbolic representation and provides explanatory comments.
3	Target (Acceptable)	The student demonstrates the correct property and uses an appropriate numerical and symbolic representation.
2	Limited Knowledge (Not Acceptable)	The student demonstrates the correct property but provides an incorrect numerical and symbolic representation.
1	Unsuccessful (Not Acceptable)	The student demonstrates the incorrect property and provides a correct numeric and symbolic representation for a different property.
0	No Knowledge (Not Acceptable)	The student demonstrates the incorrect property and provides an incorrect numeric and symbolic representation for a different property or the student does not attempt the task.

Score	Meaning	Criterion Level C
4	Proficient (Exceptional)	The student articulates an appropriate algebraic, geometric, or real world model which fits the task and includes all of the conditions needed to complete the task. The student provides information on how the model fits the task. The student includes appropriate figures, diagrams, or tables with labeling that shows the task's conditions (e.g., a geometric figure with its parts labeled to show dimensions or relationships to other parts).
3	Target (Acceptable)	The student articulates an appropriate algebraic, geometric, or real world model which fits the task and includes all of the conditions needed to complete the task. The student includes appropriate figures, diagrams, or tables with labeling that shows the task's conditions (e.g., a geometric figure with its parts labeled to show dimensions or relationships to other parts).
2	Limited Knowledge (Not Acceptable)	The student articulates an appropriate algebraic, geometric, or real world model which fits the task, but does not address all of the conditions needed to complete the task. The student includes figures, diagrams, or tables but does not include labeling that shows the task's conditions (e.g., a geometric figure with its parts labeled to show dimensions or relationships to other parts).

1	Unsuccessful (Not Acceptable)	The student articulates an algebraic, geometric, or real world model which does not fit the task and may or may not address all of the conditions needed to complete the task. The student includes inappropriate figures, diagrams, or tables.
0	No Knowledge (Not Acceptable)	The student articulates an algebraic, geometric, or real world model which does not fit the task and does not address any of the conditions needed to complete the task, or the student does not attempt the task.

Score	Meaning	Criterion Level D
4	Proficient (Exceptional)	The student's writing correctly identifies all steps to complete the task, puts them in a correct order, uses appropriate mathematical terminology, and does not add extraneous information. The student may also provide a second correct method for completing the task.
3	Target (Acceptable)	The student's writing correctly identifies all steps to complete the task and uses correct mathematical terminology. The student either adds extraneous information or puts steps out of sequence, but tells how they are out of sequence.
2	Limited Knowledge (Not Acceptable)	The student's writing leaves out one necessary step to complete the task and uses some mathematical terminology appropriately and the student may include large amounts of information unrelated to the task.
1	Unsuccessful (Not Acceptable)	The student's writing leaves out two or more necessary steps to complete the task or does not use mathematical terminology related to the task.
0	No Knowledge (Not Acceptable)	The student's writing does not identify any of the necessary steps to complete the task and does not use mathematical terminology related to the task, or the student does not attempt the task.

Scores obtained for each task will be placed in a grid as shown below.

Task	a	b	c
1A			
1B			
1C			
1D			

Task	a	b	c
2A			
2B			
2C			
2D			

Task	a	b	c
3A			
3B			
3C			
3D			

Task	a	b	c
4A			
4B			
4C			
4D			

Task	a	b	c	d	e
5A					
5B					
5C					
5D					

Students will need to score 3 or 4 in all 3 of the tasks in each column and in at least 3 of the rows for those grids having parts a-c only. For grids having parts a-e, students would need to score 3 or 4 in at least 3 of the tasks in each column and in at least 4 of the tasks in each of the 4 rows.

APPENDIX H

GRID SHOWING APPROPRIATENESS OF PERFORMANCE TASKS

Appendix H

Grid showing for which performance tasks a particular assessment level is not a good fit. An X indicates that a task is not a good fit.

Task	a	b	c
1A			
1B			
1C			
1D			

Task	a	b	c
2A			
2B			
2C			
2D			

Task	a	b	c
3A			
3B			
3C	X	X	X
3D			

Task	a	b	c
4A			
4B			
4C	X	X	X
4D			

Task	a	b	c	d	e
5A					
5B					
5C					
5D					

APPENDIX I

INSTITUTIONAL REVIEW APPROVAL LETTERS

Appendix I



Institutional Review Board

MINOT STATE UNIVERSITY

500 University Avenue West • Minot, North Dakota 58707 • (701) 858-3125 • 1-800-777-0750 • FAX (701) 858-4286

NOTICE OF IRB APPROVAL

Name of Principal Investigator: Cheryl Nelson

University Address: Model Hall

Title of Project: The Effect of Assessment Implementing
a "Rule of Four"

Date 1/28/05

The above project has been reviewed and approved by the IRB under the provisions of Federal Regulations 45 CFR 46.

This approval is based on the following conditions:

1. The materials you submitted to the IRB (through the Office of Research & Sponsored Programs) provide a complete and accurate account of how human subjects are involved in your project.
2. You will carry on your research strictly according to the procedures as described in materials presented to the IRB.
3. You will report to the Office of Research and Sponsored Programs any changes in procedures that may have a bearing on this approval and require another IRB review.
4. If any changes are made, you will submit the modified project for IRB review.
5. You will immediately report to the Office of Research & Sponsored Programs any problem(s) that you encounter while using human subjects.

Signed

Margi Coxwell

Dr. Margi Coxwell
Chair, Minot State University IRB



INSTITUTIONAL REVIEW BOARD
For the Protection of Human Subjects
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Chair: Mark Quinn
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MEMORANDUM

TO: Cheryl Nilsen

FROM: Mark Quinn, Ph.D. Chair *Mark Quinn*
 Institutional Review Board for the Protection of Human Subjects

DATE: September 21, 2007

SUBJECT: *The Effect of Assessment Implementing a "Rule of Four" on the Mathematics Achievement of Elementary Education Majors*

The above research, described in your submission of September 21, 2007, is exempt from the requirement of review by the Institutional Review Board in accordance with the Code of Federal Regulations, Part 46, section 101. The specific paragraph which applies to your research is:

- (b)(1) Research conducted in established or commonly accepted educational settings, involving normal educational practices such as (i) research on regular and special education instructional strategies, or (ii) research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.
- (b)(2) Research involving the use of educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures or observation of public behavior, unless: (i) information obtained is recorded in such a manner that human subjects can be identified, directly or through identifiers linked to the subjects; and (ii) any disclosure of the human subjects' responses outside the research could reasonably place the subjects at risk of criminal or civil liability, or be damaging to the subjects' financial standing, employability, or reputation.
- (b)(3) Research involving the use of educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures, or observation of public behavior that is not exempt under paragraph (b)(2) of this section, if: (i) the human subjects are elected or appointed public officials or candidates for public office; or (ii) federal statute(s) without exception that the confidentiality of the personally identifiable information will be maintained throughout the research and thereafter.
- (b)(4) Research involving the collection or study of existing data, documents, records, pathological specimens, or diagnostic specimens, if these sources are publicly available, or if the information is recorded by the investigator in such a manner that the subjects cannot be identified, directly or through identifiers linked to the subjects.

The MSU IRB will defer oversight of the research to Minot State University's IRB.