An analytical model of the patello-femoral joint
by Jack Stephen Hagelin

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE * in Mechanical Engineering
Montana State University

Abstract:
The intent of this research was to develop an analytical model of the human knee joint for the purpose of aiding orthopedic surgeons in determining the optimum corrective operation to alleviate the condition of recurrent patellar dislocation.

A kinematic model was developed defining the relative positions of the femur, tibia, and patella for all angles of flexion. The patellar - surface was approximated by a cosine curve rotated through a specified arc length and the patella was modeled as a cylinder. The assumption was made that the quadriceps muscle group could be approximated by a single muscle in which the resultant force acting on the patella was equivalent to the total muscle group. The accuracy of this model is contingent upon the accurate determination of the proximal attachment for the effective quadriceps muscle.

Nineteen nonlinear simultaneous equations were developed to relate the nineteen unknown kinematic parameters and were solved using Newton-Raphson's Method on the computer. The computer output is in the form of a video display indicating the position of the patella in the patellar groove. Surgeons may simulate operations on the computer model and determine visually the path of the Datella throughout flexion. Trial operations are simulated until the desired patellar path is reached.

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ABSTRACT

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CHAPTER 1

INTRODUCTION

The intent of this research was to develop an analytical model of the human knee joint for the purpose of aiding orthopedic surgeons in determining the optimum remedial operation for recurrent patellar subluxation.

Recurrent dislocation of the patella is primarily due to an irregular geometrical configuration in the leg anatomy. Since there are a number of different abnormalities possible, there are also a number of operative techniques available to surgeons. This results in a degree of uncertainty in their decisions, which in turn results in an undesirable failure rate.

To reduce this failure rate, a model was desired which could predict the effect of each of the different operations performed. Surgeons could then select the operation which would result in a desirable force pattern and acceptable patellar path.

To accomplish this, a kinematic model was developed which included parameters for ligament directions and a description of the relative positions of the femur, tibia, and patella. The assumption was made that at any given angle of flexion, there is only one point on the patella which is in contact with the femur. The result is a computer model which outputs a video display indicating the path of the patella in the patellar groove through flexion.

CHAPTER 2

ANALYTICAL MODEL

2.1 Introduction

The purpose of this research was to develop an analytical model of the patello-femoral joint, accurately defining the position of the patella in the joint for all angles of flexion.

The kinematic model developed incorporates the use of vectors in defining all bone and ligament positions. A force analysis is included requiring the patella to be in static equilibrium throughout flexion.

2.2 Definition of Kinematic Model

Figure 1 depicts the model used in this study. Listed below is a description of the coordinate system and all points in the model.

A rectangular coordinate system is used in this model as defined in Figure 1-a. Both the left and right knees incorporate a left-handed coordinate system. The Y-axis is the axis of rotation of the tibia about the femur. The X-axis is parallel to the femoral shaft and is located laterally in the same plane as the tibial tuberosity. This axis is positive in the distal direction. The Z-axis is positive anteriorly.

Point '0' is the origin of the coordinate system. Point '1' is the point of contact between the patella and the patellar surface. (The gap shown in Figure 1-b is due to the fact that there is a layer of cartiledge between the femur and the patella.) The patellar surface will hereafter be referred to as the cam. Point '2' is the tibial tuberosity,

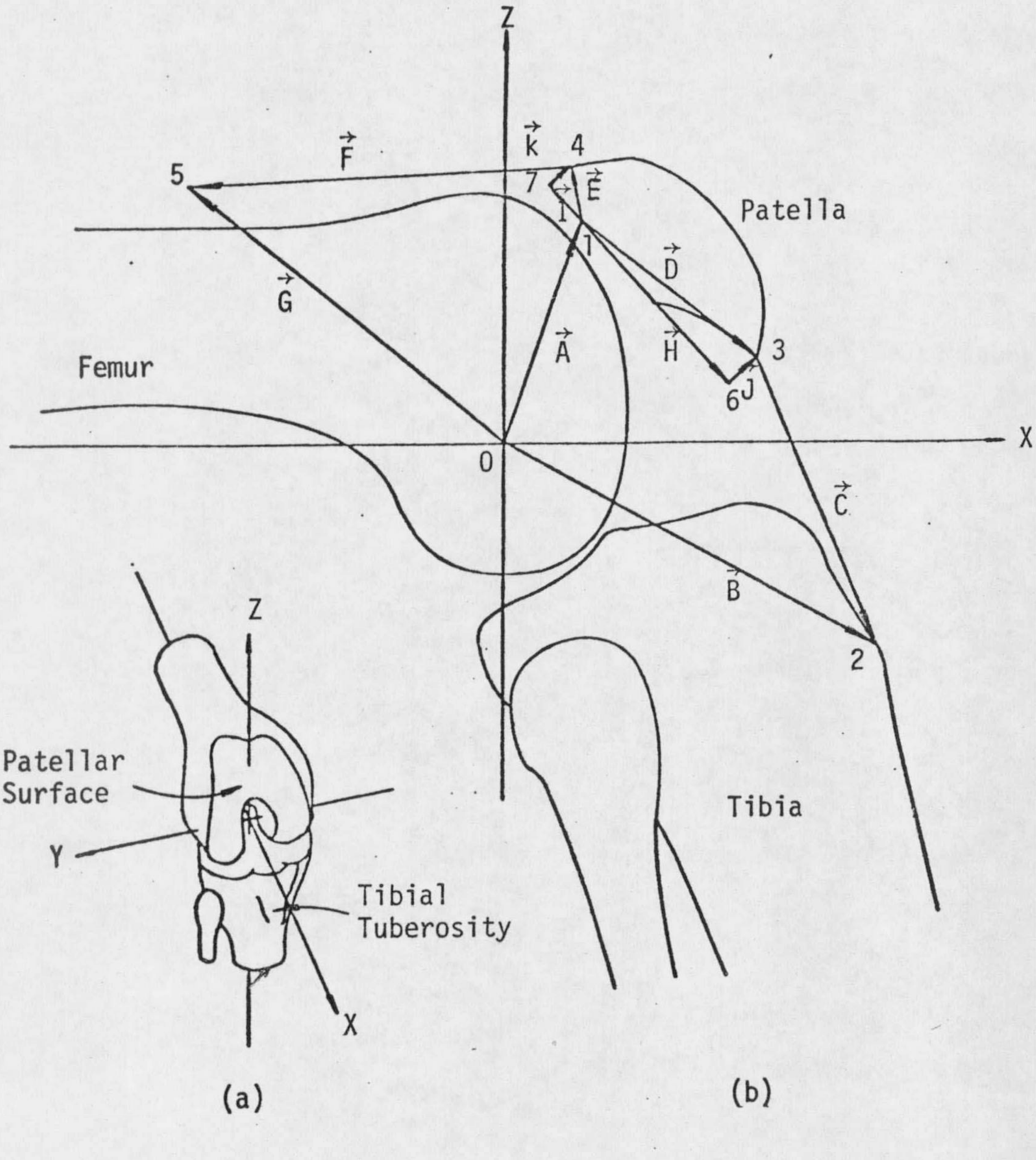


Figure 1.--Analytical model parameter definition

the point of insertion of the Ligament Patellae; and point '3' is the point of origin for this ligament, located on the distal end of the patella. Point '4' is the point of insertion of the quadriceps group, located on the proximal end of the patella; and point '5' is the effective point of origin for this muscle group, which is to be determined by the surgeon. Points '6' and '7' represent the extreme points on the inferior surface of the patella and are colinear with point '1'.

All vectors are defined in terms of the points listed above. Note that vectors $\vec{C}$ and $\vec{F}$ represent the directions of the patellar ligament and the quadriceps group respectively.

A number of assumptions are inherent in this model and will be discussed in the following section.

2.3 Assumptions

The quadriceps group is made up of four muscles: the rectus femoris, the vastus lateralis, the vastus intermedius, and the vastus medialis (see Figure 2-a). All of these have as their insertion the proximal end of the patella, but their origins differ greatly, ranging from the anterior inferior iliac spine for the rectus femoris, to points all along the shaft of the femur for the vastus group. Figures 3-a through 3-c illustrate these proximal points of attachment. The assumption was made that the quadriceps group could be approximated by a single muscle with its proximal attachment at the appropriate position, as shown in

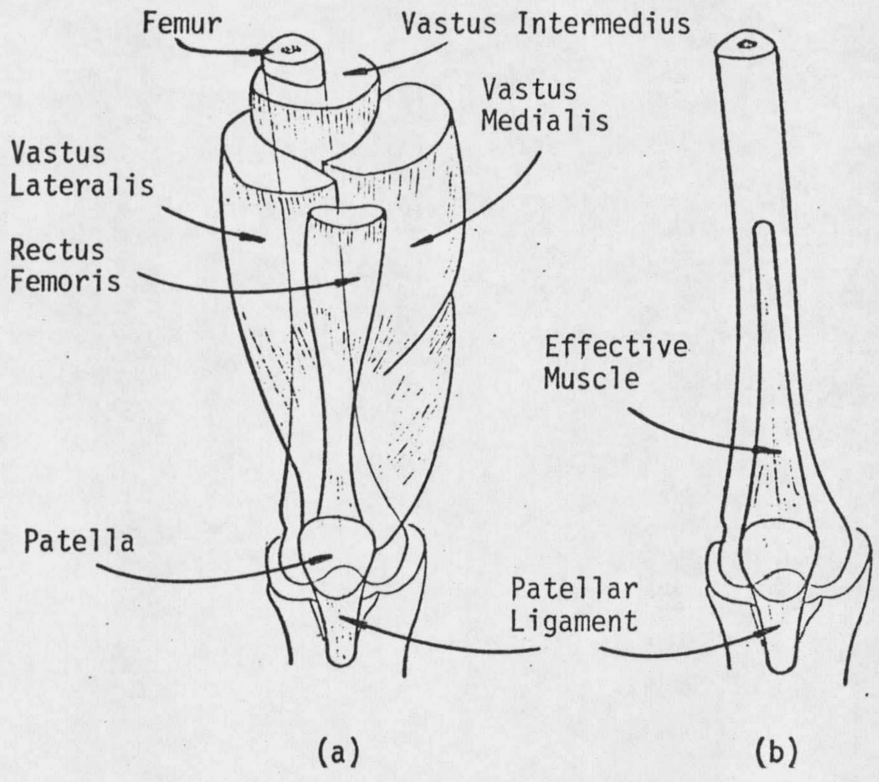


Figure 2.--Quadriceps muscle group
(a) actual
(b) modelled

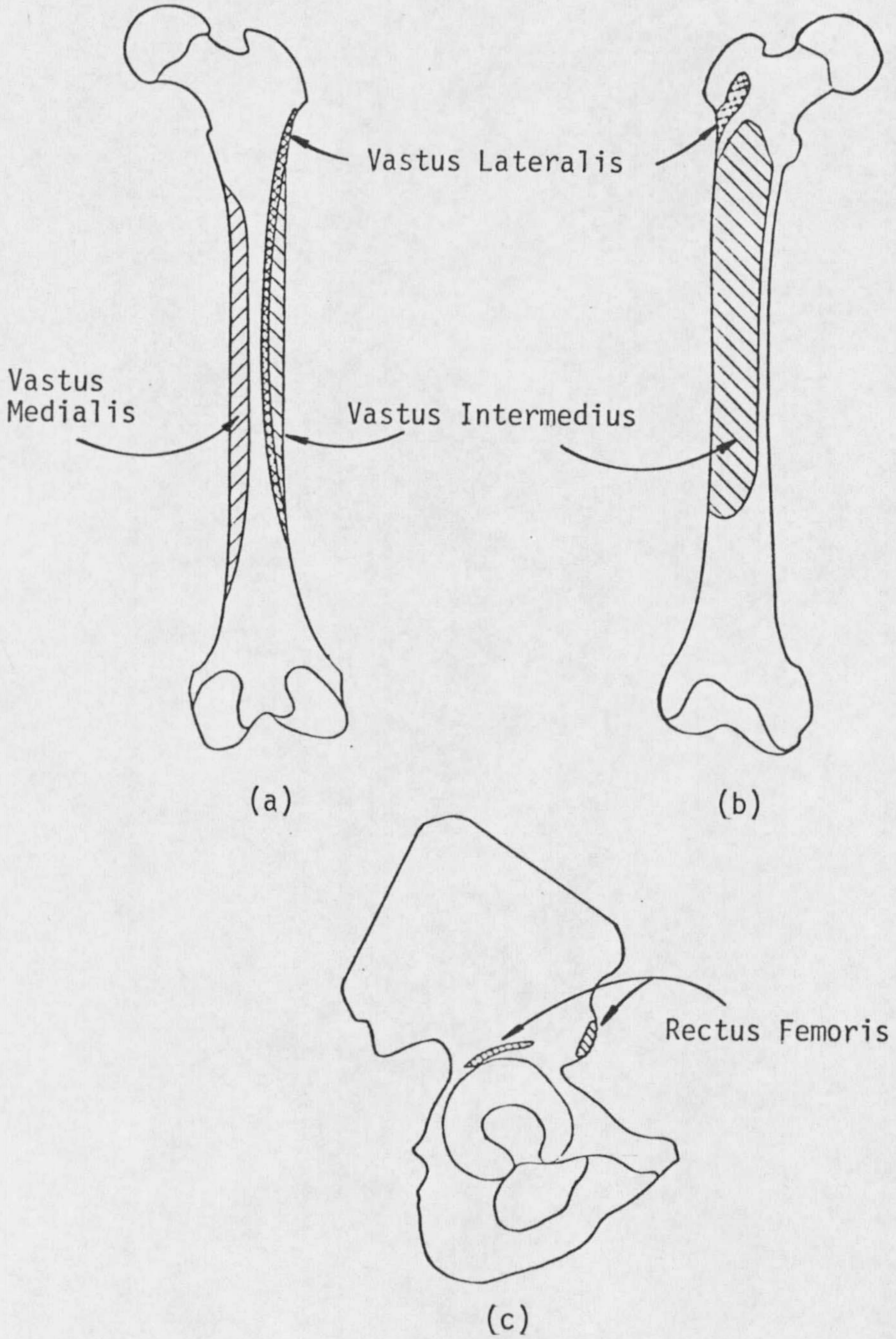


Figure 3.--Quadriceps proximal muscle attachments. (a) femur - posterior view, (b) femur - anterior view, and (c) right ilium, ischium, and pubis - lateral surface.

Figure 2-b. The effective proximal point of attachment is left as a variable to be determined by the surgeon for the patient under consideration. For the purposes of this study, the point of attachment was chosen on the anterior side of the femoral shaft, located approximately one-third the length of the femur from the distal end and slightly lateral. This point corresponds to point '5' in the model. It was also assumed that the center of rotation of the joint does not change with the angle of flexion. It is left to the surgeon to accurately determine this point for the patient in question.

The patella was modelled as a cylinder with a radius equal to the radius of curvature of the posterior side of the patella. This can be obtained easily from a 'sunrise-view' x-ray.

2.4 Solution

In general, the problem consists of specifying an angle of flexion and determining the position of the patella relative to the femur. The components of all seven points in the model are unknown with the exception of point '5', the y-component of point '2', and the x-component of point '1'. Point '5' is the effective point of origin for the quadriceps group, which is determined by the surgeon. The value of y_2 corresponds to the lateral displacement of the insertion point for the patellar ligament and is therefore, a control variable in the operation simulation. There is a one-to-one correspondence between x_1 and the angle of flexion; therefore, specifying x_1 is equivalent to specifying the angle of flexion. This leaves a total of sixteen unknowns as shown in Table 1.

Table 1.--Variables

pt.	unknown	specified
1	y_1 z_1	x_1
2	x_2 z_2	y_2
3	x_3 y_3 z_3	
4	x_4 y_4 z_4	
5		x_5 y_5 z_5
6	x_6 y_6 z_6	
7	x_7 y_7 z_7	

The force analysis, to be discussed in more detail later, includes the two forces exerted on the patella by the ligaments, and also a resultant force acting on the patella at the point of contact. The direction of this resultant force is normal to the cam at the point of contact. (It is assumed that there are no tangential components of the resultant force due to the fact that there is essentially no friction between the patella and the cam.) Since point '1' is unknown, the direction cosines of the normal force are also unknown, adding three more unknown to the problem: p_x, p_y, and p_z. There are now nineteen unknowns and hence, nineteen equations are necessary to obtain a solution.

Figures 4-a and 4-b illustrate the relationship between the three components of point '1' to describe the shape of the cam. The two-dimensional curve in the X-Z plane, S_0, is approximated by the relationship:

$$R_0 = a\theta_0 + b$$

where: R_0 is the distance from the origin to the curve S_0, and θ_0 is the angle of the R-axis with the X-axis, positive counter-clockwise. The constants a and b are obtained easily from x-rays.

To approximate the shape of the groove in the cam, a cosine curve is used as shown in Figure 4-b. The following relationship results:

$$R = R_0 + .5D[1-\cos(\pi y_1/P)]$$

where: R is the distance from the Y-axis to the cam and y_1 is the y-component of point '1' on the cam. The constant D is the average depth of the patellar groove, and P is one-half the width of the groove, (one-half the period of the cosine curve).

Noting that $R^2 = x_1^2 + z_1^2$, and $\cos\theta_0 = x_1/(x_1^2 + z_1^2)^{1/2}$, the surface equation reduces to:

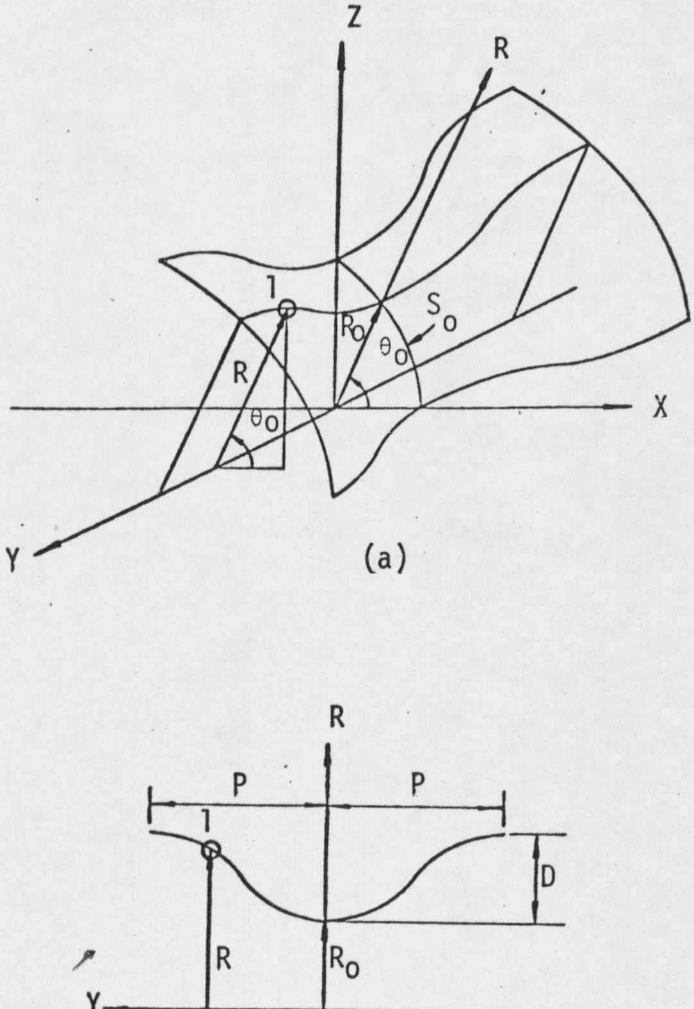


Figure 4.--Patellar surface model

$$(x_1^2+z_1^2)^{1/2} = (a \cos^{-1}[x_1/(x_1^2+z_1^2)^{1/2}] + b] + .5D[1-\cos(\pi y_1/P)].$$

It will be necessary, for the purpose of operation simulation, to introduce an angle of rotation about the X-axis into this equation. This is accomplished using a coordinate transformation with the angle of rotation being positive in the counter-clockwise direction. This results in the following equation:

$$S_1(x_1,y_1,z_1) = [x_1^2+(z_1\cos\theta + y_1\sin\theta)^2]^{1/2} -$$

$$\alpha[\rho+\cos^{-1}[x_1/(x_1^2+(z_1\cos\theta + y_1\sin\theta)^2)^{1/2}]]-$$

$$.5D[1-\cos[\pi(y_1\cos\theta-z_1\sin\theta)/P]] = 0 \qquad (1)$$

where: θ is the angle of rotation, $\alpha = a$, and $\rho = b/a$.

A second relationship is that the normal vector to the cam at the point of contact is the gradient of the surface function, S_1, at that point. This results in the following three equations:

$$p_x = \partial S_1/\partial x_1 \qquad (2)$$

$$p_y = \partial S_1/\partial y_1 \qquad (3)$$

$$p_z = \partial S_1/\partial z_1 \qquad (4)$$

In the computer program, these derivatives are arrived at numerically.

It was assumed that the patellar ligament does not stretch, and hence, the magnitude of vector $\vec{C}$ is a constant. This may be written as:

$$(x_2-x_3)^2 + (y_2-y_3)^2 + (z_2-z_3)^2 = r_3^2 \qquad (5)$$

where: r_3 is the length of the ligament, determined from x-rays.

Vectors $\vec{J}$ and $\vec{K}$ have constant magnitudes throughout flexion. This magnitude is approximated as one-half the radius of the patella, hence:

$$(x_3-x_6)^2 + (y_3-y_6)^2 + (z_3-z_6)^2 = (r/2)^2 \qquad (6)$$

$$(x_4-x_7)^2 + (y_4-y_7)^2 + (z_4-z_7)^2 = (r/2)^2 \qquad (7)$$

where 4 is the radius of the patella.

The sum of the magnitudes of vectors $\vec{H}$ and $\vec{I}$ is a constant equaling the length of the patella, C_p. Therefore:

$$(x_6-x_7)^2 + (y_6-y_7)^2 + (z_6-z_7)^2 = C_p{}^2 \qquad (8)$$

Based on the assumption that the center of rotation of the tibia (point 'O') remains stationary, it can be stated that the distance from the Y-axis to the point of insertion of the patellar ligament also is a fixed constant throughout flexion. This may be written as:

$$x_2{}^2 + z_2{}^2 = r_2{}^2. \qquad (9)$$

The value of r_2, the distance from the Y-axis to the point of insertion of the ligament, is obtained from the following relationship:

$$r_2 = (r_{2_o}{}^2 + \delta^2 + 2r_{2_o}\delta\cos\omega)^{1/2},$$

where r_{2_o} is the distance from the Y-axis to the tibial tuberosity, which can be obtained from x-rays; δ is the longitudinal displacement of the patellar ligament along the tibial shaft, measured positively in the distal direction; and ω is the angle between the vector from the origin to the tibial tuberosity and the vector parallel to the tibial shaft passing through the origin. The variable δ can be adjusted to simulate the operative technique of displacing the insertion of the patellar ligament longitudinally.

Four additional equations are derived from the assumption that vectors $\vec{J}$ and $\vec{K}$ are parallel to the normal vector at the point of contact. Thus,

$$\frac{x_3 - x_6}{z_3 - z_6} = \frac{p_x}{p_z} \qquad (10)$$

$$\frac{y_3 - y_6}{z_3 - z_6} = \frac{p_y}{p_z} \qquad (11)$$

$$\frac{x_4 - x_7}{z_4 - z_7} = \frac{p_x}{p_z} \qquad (12)$$

$$\frac{y_4 - y_7}{z_4 - z_7} = \frac{p_y}{p_z} \qquad (13)$$

Vectors $\vec{H}$ and $\vec{I}$ are required to be colinear resulting in the following two equations:

$$\frac{x_7 - x_1}{z_y - z_1} = \frac{x_1 - x_6}{z_1 - z_6} \qquad (14)$$

$$\frac{y_7 - y_1}{z_7 - z_1} = \frac{y_1 - y_6}{z_1 - z_6} \qquad (15)$$

It is required that the line 6-1-7 lie in the plane tangent to the cam at the point of contact. This may be stated in the following equation:

$$(x_7 - x_1)p_x + (y_7 - y_1)p_y + (z_7 - z_1)p_z = 0 \qquad (16)$$

The final three equations result from the force balance on the patella. Figure 5 depicts the forces acting on the patella, $\vec{F}_1$ and $\vec{F}_2$ represent the tensile forces exerted on the patella by the quadriceps group and the patellar ligament respectively. The normal force $\vec{N}$ is the resultant force between the patella and the femur at the point of contact. These forces are written in vector notation as follows:

$$\vec{F}_1 = F_1[(x_5-x_4)\vec{i} + (y_5-y_4)\vec{j} + (z_5-z_4)\vec{k}]$$

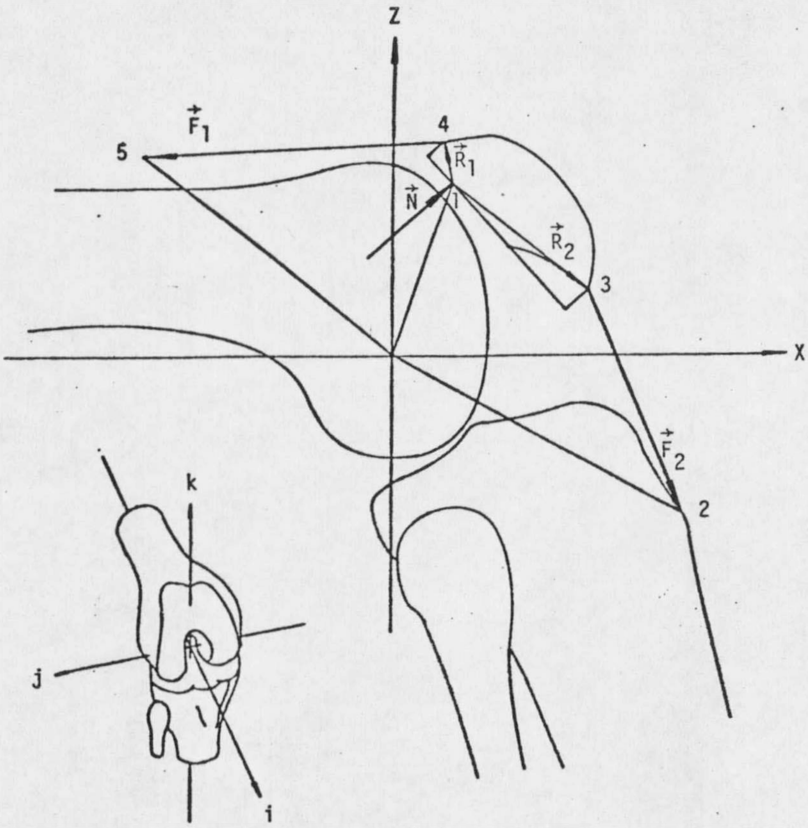


Figure 5.--Free body diagram of patella

$$\vec{F}_2 = F_2[(x_2-x_3)\vec{i} + (y_2-y_3)\vec{j} + (z_2-z_3)\vec{k}]$$

$$\vec{N} = N[p_x\vec{i} + p_y\vec{j} + p_z\vec{k}]$$

where F_1, F_2, and N are re-normalized force magnitudes.

The displacement vectors $\vec{R}_1$ and $\vec{R}_2$ shown in Figure 5 may be written as:

$$\vec{R}_1 = (x_4-x_1)\vec{i} + (y_4-y_1)\vec{j} + (z_4-z_1)\vec{k}$$

$$\vec{R}_2 = (x_3-x_1)\vec{i} + (y_3-y_1)\vec{j} + (z_3-z_1)\vec{k}$$

It is required that the patella be in static equilibrium throughout flexion, therefore, the sum of the forces in all three directions and the sum of the moments about all three axes must be zero. Balancing the forces yields the following matrix equation:

$$\begin{bmatrix} x_5-x_4 & x_2-x_3 & p_x \\ y_5-y_4 & y_2-y_3 & p_y \\ z_5-z_4 & z_2-z_3 & p_z \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

For a solution to exist, the determinate of the coefficient matrix must equal zero, hence:

$$\begin{aligned} &p_x[(z_2-z_3)(y_5-y_4)-(y_2-y_3)(z_5-z_4)] + \\ &p_y[(x_2-x_3)(z_5-z_4)-(z_2-z_3)(x_5-x_4)] + \\ &p_z[(y_2-y_3)(x_5-x_4)-(x_2-x_3)(y_5-y_4)] = 0 \end{aligned} \tag{17}$$

Balancing the moments about all three axes yields the following:

$$\begin{aligned} &F_1[(y_4-y_1)(x_5-x_4)-(x_4-x_1)(y_5-y_4)] + \\ &F_2[(y_3-y_1)(x_2-x_3)-(x_3-x_1)(y_2-y_4)] = 0 \end{aligned}$$

$$\begin{aligned} &F_1[(x_4-x_1)(z_5-z_4)-(z_4-z_1)(x_5-x_4)] + \\ &F_2[(x_3-x_1)(z_2-z_3)-(z_3-z_1)(x_2-x_3)] = 0 \end{aligned}$$

$$F_1[(z_4-z_1)(y_5-y_4)-(y_4-y_1)(z_5-z_4)] +$$
$$F_2[(z_3-z_1)(y_2-y_3)-(y_3-y_1)(z_2-z_3)] = 0$$

Combining the first two of these equations results in the following:

$$[(y_4-y_1)(x_5-x_4)-(x_4-x_1)(y_5-y_4)][(x_3-x_1)(z_2-z_3)-(z_3-z_1)(x_2-x_3)] -$$
$$[(x_4-x_1)(z_5-z_4)-(z_4-z_1)(x_5-x_4)][(y_3-y_1)(x_2-x_3)-(x_3-x_1)(y_2-y_3)] = 0 \quad (18)$$

Finally, combining the force and moment equations yields:

$$[p_z(x_5-x_4)-p_x(z_5-z_4)][(x_3-x_1)(z_2-z_3)-(z_3-z_1)(x_2-x_3)] -$$
$$[p_z(x_2-x_3)-p_x(z_2-z_3)][(x_4-x_1)(z_5-z_4)-(z_4-z_1)(x_5-x_4)] = 0 \quad (19)$$

This set of nineteen nonlinear simultaneous equations was solved using Newton-Raphson's Method on the computer.

2.5 Operation Simulation

To be useful to surgeons, this model must incorporate the ability to simulate a spectrum of the most common corrective operations intended to alleviate the condition of recurrent patellar subluxation. Two operations are simulated in this study. The first is the movement in the lateral and longitudinal directions of the insertion point for the patellar ligament. These movements are accomplished by adjusting the variables y_2 and δ respectively. This operation is the most common as it is the easiest to perform, and has the shortest recovery time. The second operation is the axial rotation of the distal end of the femur.

This has a much greater effect than the previous operation, but has a much longer recovery rate. This operation is accomplished by adjusting the variable θ, which is the angle of rotation about the X-axis.

CHAPTER 3

RESULTS

3.1 Introduction

The formulation of this model results in a computer model of the Patello-Femoral joint. The output is in the form of a video display indicating the position of the patella in the X-Z and Y-Z planes. Figure 6 is a sample of the output from a graphics terminal.

The figure at the top is a motion view of the joint in the X-Z plane illustrating the X and Z components of the kinematic configuration for the system. Its main purpose is to indicate to surgeons how high the patella is riding in the cam. The figure on the right is a view of the cam in the Y-Z plane indicating the path of the patella in the groove of the cam.

3.2 Compatibility of Model with X-ray

Figure 7 is a full scale view demonstrating the accuracy of the model in the X-Z plane. The solid lines represent actual x-ray data, and the dashed lines represent the model incorporating data from the same x-ray. The position of the modelled patella is the critical factor in this view, and as shown, matches closely the actual position.

3.3 Sensitivity of Model to Effective Muscle Position

More important than the position of the patella in the X-Z plane is its position in the Y-Z plane. All of the input data for this model is

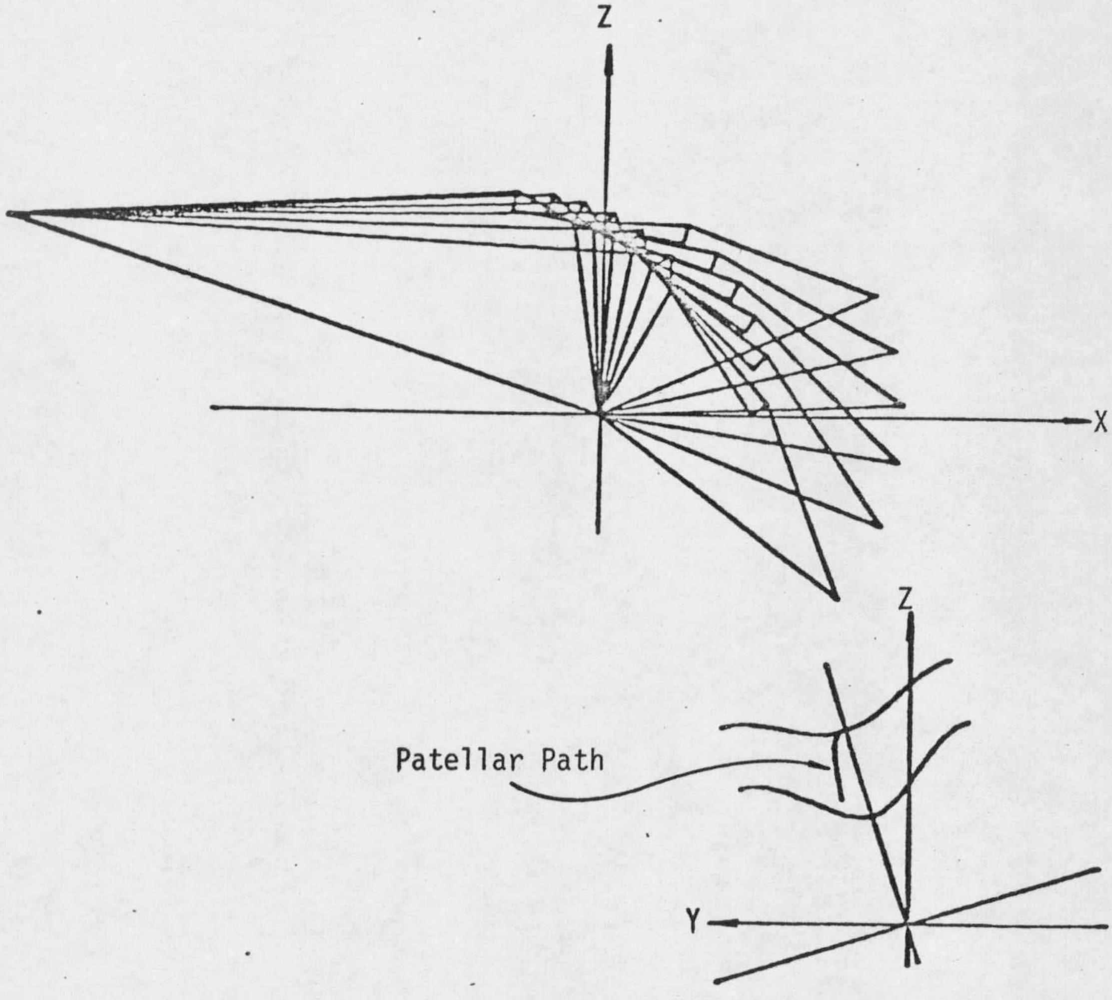


Figure 6.--Actual video display output

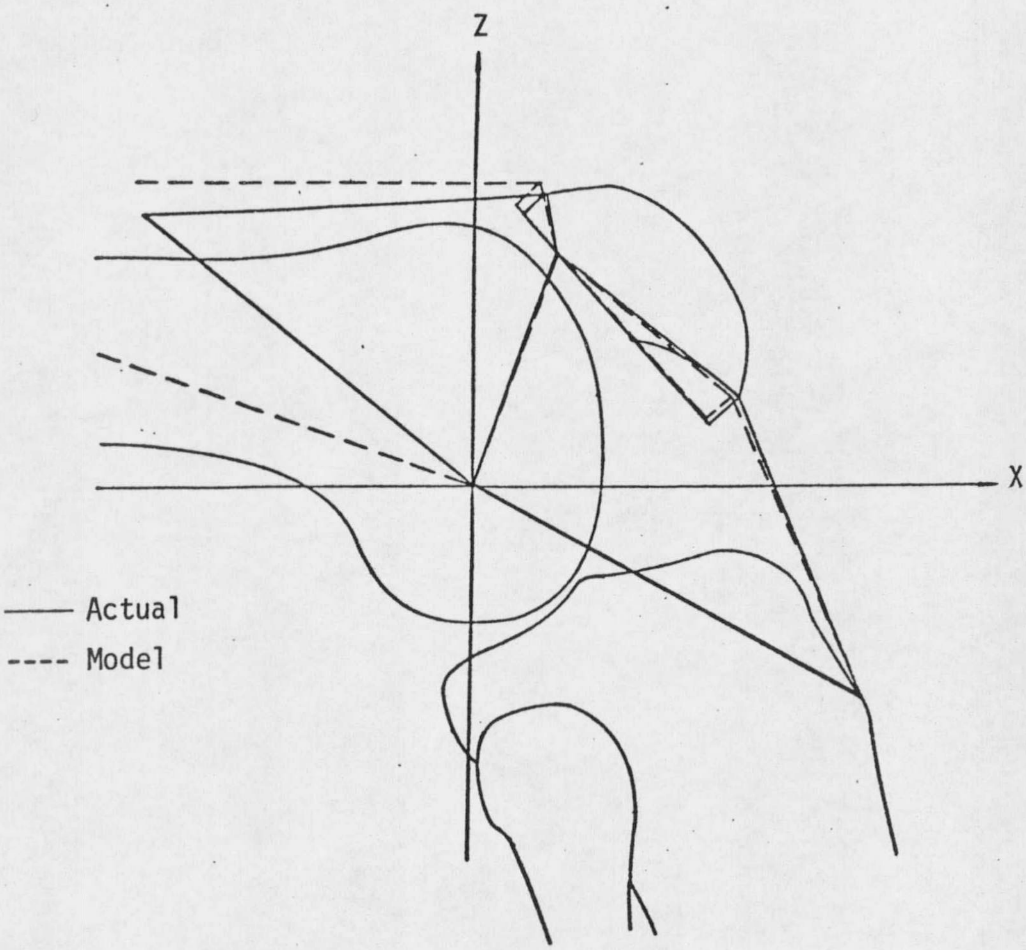


Figure 7.--Comparison of model with actual x-ray data

fairly straightforward except for the effective quadriceps group point of origin (x_5,y_5,z_5). Figures 8-a through 8-c indicate the sensitivity of the patellar path to this point.

Figure 8-a demonstrates the effect of altering the x-component of point '5'. There is a slight shift inward as x_5 is increased. The general shape and tendency of the path does not change as x_5 is changed, and hence, the selection of x_5 is not a critical factor if it is chosen in the correct range ($\pm$ 2 centimeters).

The y-component of point '5' is varied in Figure 8-b. It is evident that this value is critically important as the whole path can be radically affected in a fifteen centimeter range. It is therefore necessary that a very accurate value be used for y_5. It is recommended that it be to the nearest centimeter.

The change in the patellar path with a four centimeter change in z_5 is shown in Figure 8-c. This value is not as critical as y_5, but should be found to the nearest 2 centimeters.

3.4 Operation Simulation

Figures 9-a through 9-f demonstrate the effect of a series of operations performed on the model as an example of the procedure to be used by the surgeon and does not represent an actual patient. In this case, point five is (-15,7.5,5.) in the units of centimeters. This represents a right leg, therefore, the left side of the groove in the

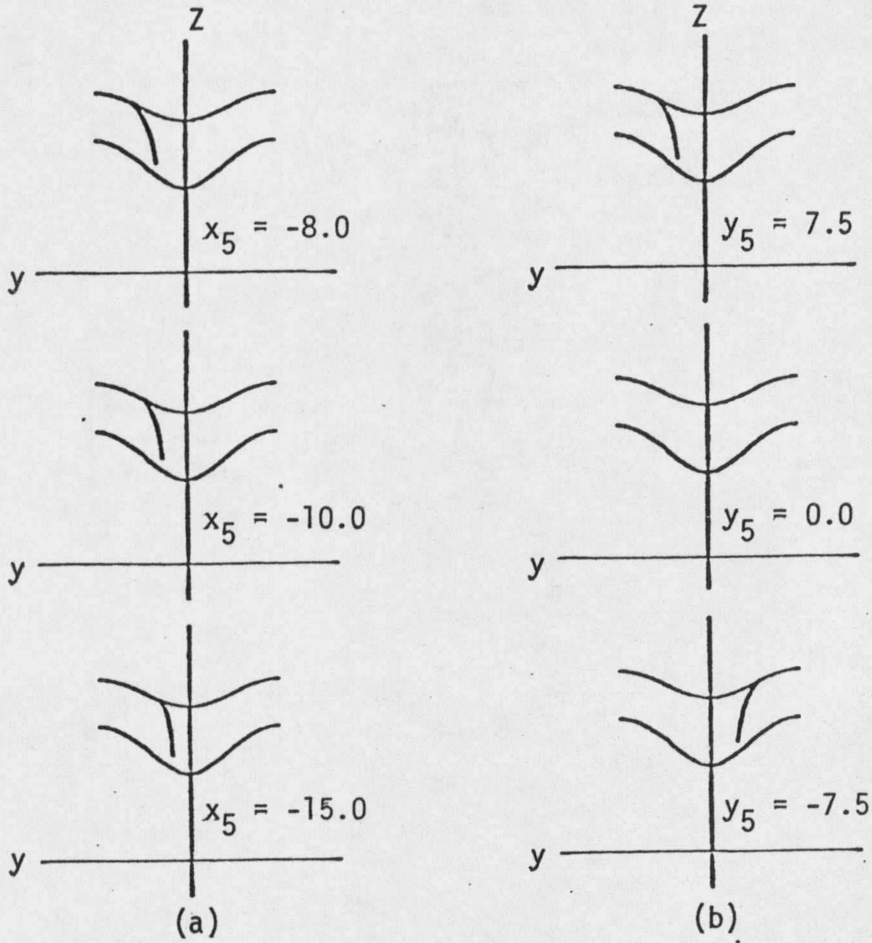
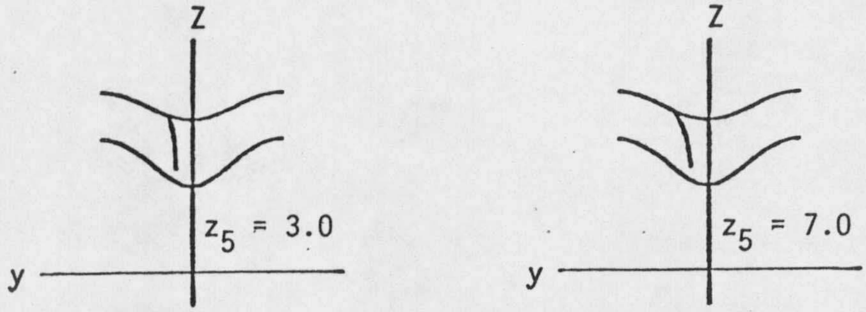


Figure 8.--Sensitivity of patellar path to effective quadriceps origin

(a) X-component (y_5=7.5;z_5=5.0)

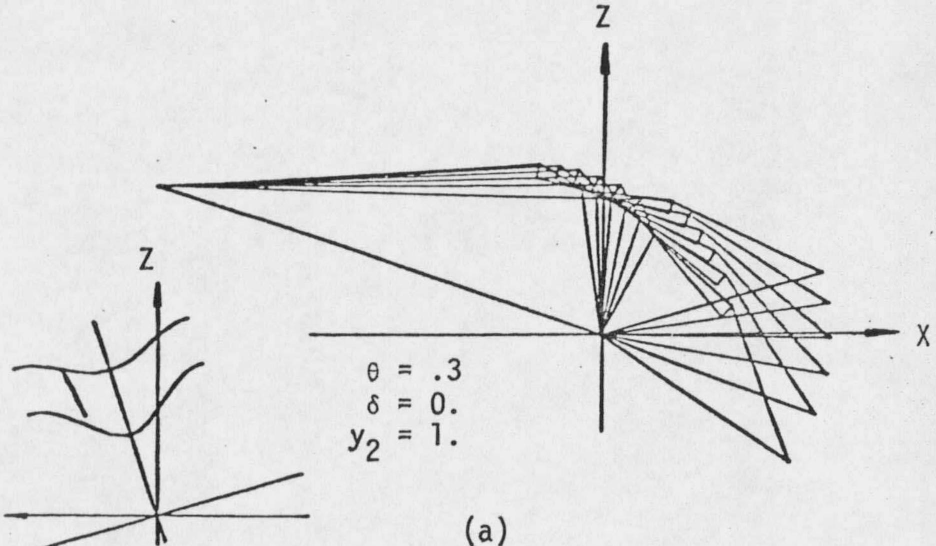
(b) Y-component (x_5=-15.;z_5=5.0)



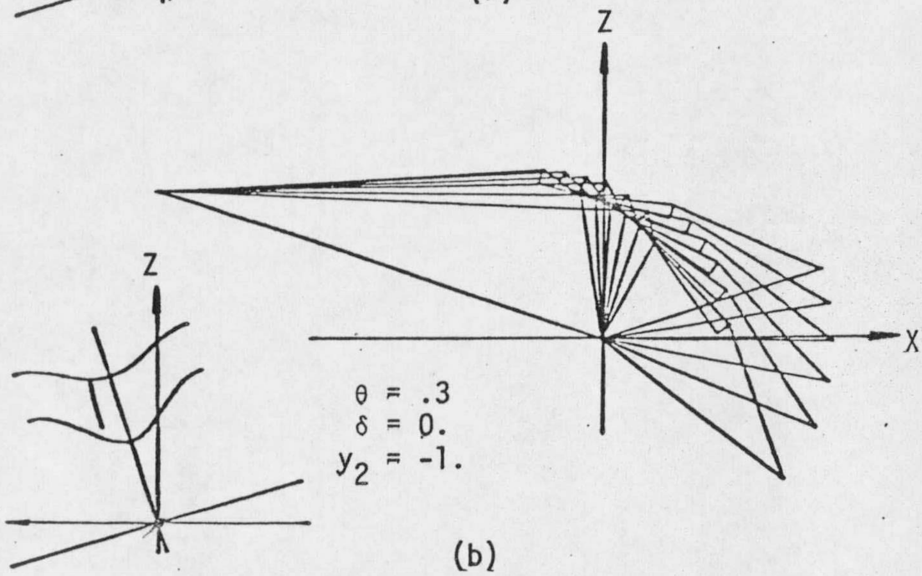
(c)

Figure 8.--Continued

(c) Z-component (x_5=-15.;y_5=7.5)

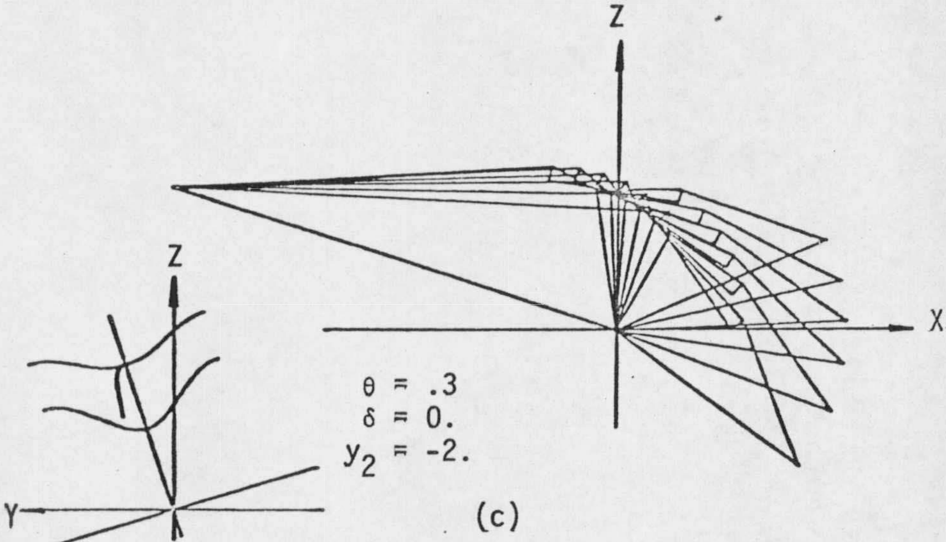


(a)

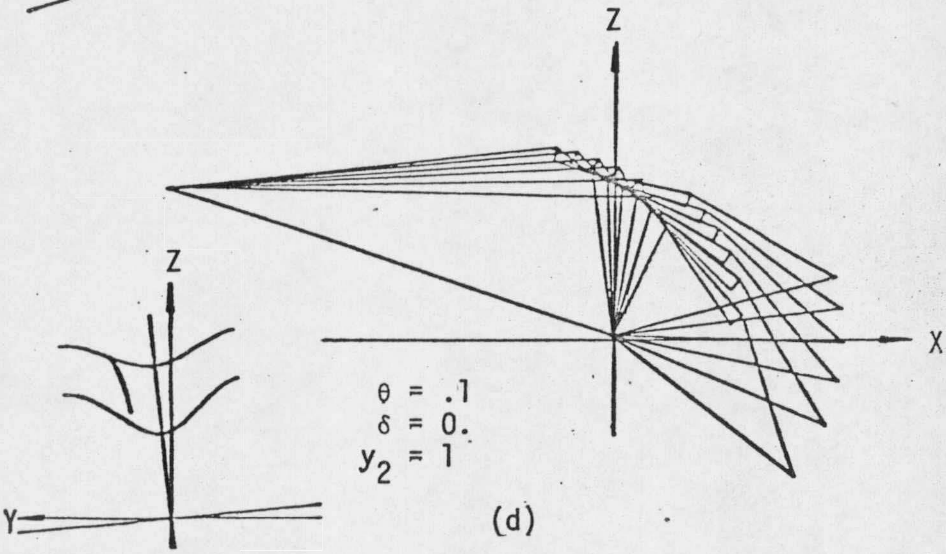


(b)

Figure 9.--Sample run of operation simulation

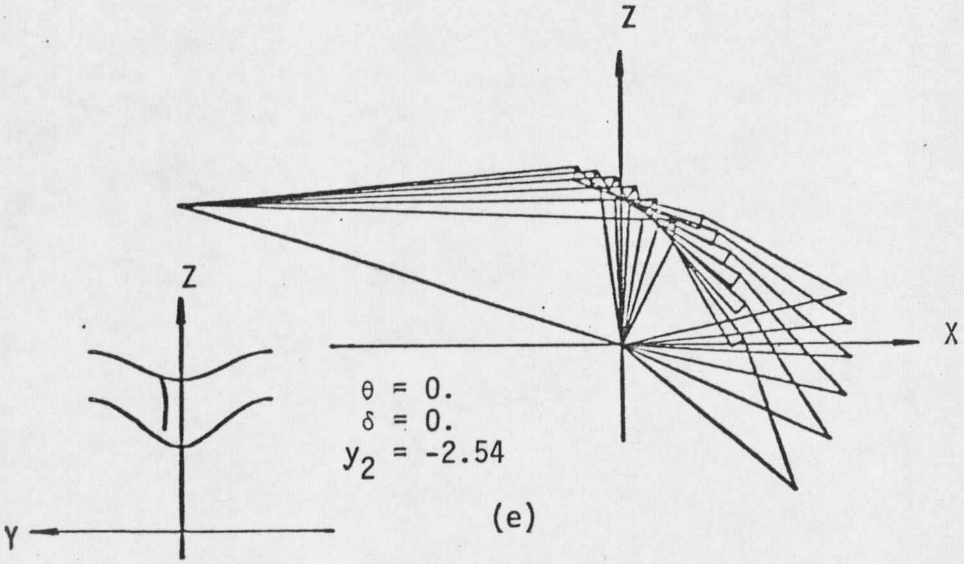


(c)

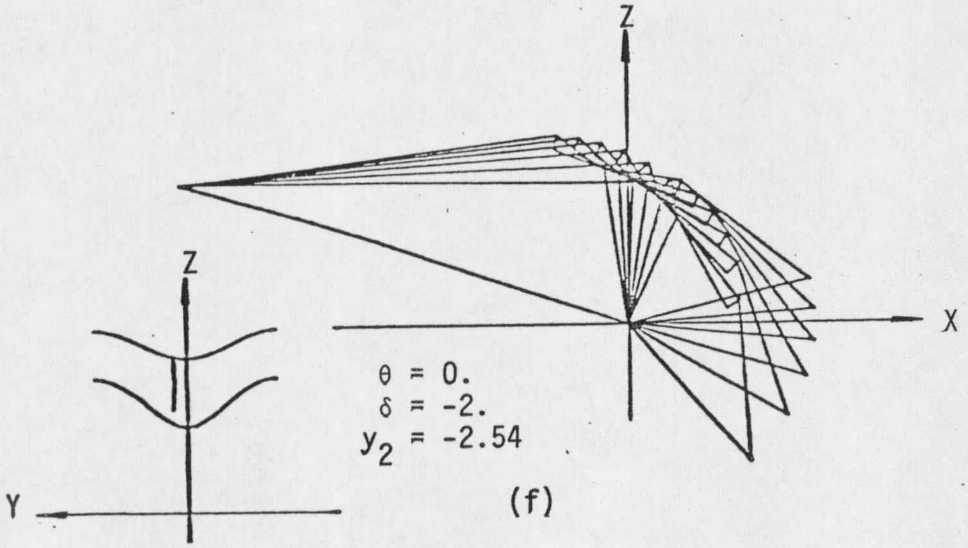


(d)

Figure 9.--Continued



(e)



(f)

Figure 9.--Continued

figures is the lateral side.

Figure 9-a represents the hypothetical patient's condition before operations are performed. Notice that the lateral side of the groove is almost flat and the path is halfway up the lateral condyle. This configuration would conceivably require only a small external lateral force to dislocate the patella, and an operation would probably be recommended.

The medial displacement of the patellar ligament is the easiest operation to perform and should therefore be the first operation simulated on the program. Figure 9-b represents the path after the ligament has been displaced two centimeters medially. This brought the path much closer to the center of the groove. Moving the ligament an additional centimeter medially brings the path even closer to the center of the groove as shown in Figure 9-c. In many instances, surgeons are tempted to stop at this point since the ligament cannot be displaced further medially, and the only alternative is to twist the femur. The rotation of the femur is a major operation and is usually avoided if at all possible. However, in this case, the lateral condyle is radically low, and a large enough lateral force could still dislocate the patella. Hence, the rotation of the femur is probably necessary in this case.

With the ligament returned to its original position, Figure 9-d demonstrates a rotation of the femur by .2 radians (approximately 11 degrees). This did not improve radically the path from Figure 9-a and it is therefore concluded that an operation involving both the displacement of the ligament and the axial rotation of the femur is necessary.

Rotating the femur .3 radians (about 17 degrees) and displacing the ligament 3.54 centimeters (1.4 inches) medially results in Figure 9-e. The lateral condyle is now even with the medial condyle, and provides a satisfactory rise on the lateral side. The path is fairly close to the center of the groove, and the patella would require a much greater force to dislocate. Figure 9-e therefore would represent the final state after the indicated operation.

Figure 9-f shows a path which is closer to the center than Figure 9-e, yet is an undesirable configuration. In this figure, the ligament attachment is moved two centimeters proximally and would therefore lessen the tension in the ligaments requiring a smaller lateral force to dislocate the patella.

It should be noted that the writer is an engineer and is therefore writing from that perspective. The above sample run may have implied conclusions which surgeons would not reach. It should also be stated that if this model indicates an operation that is radically different from what a surgeon would have done otherwise, then further investigation is recommended.

CHAPTER 4

SUMMARY

4.1 Introduction

The purpose of this research was to develop a mathematical model of the Patello-Femoral joint. The model was developed with a view to aiding surgeons in determing the optimum operation to alleviate the condition of recurrent patellar dislocation.

Vectors were used to represent the positions of the femur, tibia, patella, quadriceps group, and the patellar ligament. A set of nineteen nonlinear simultaneous equations were generated relating the nineteen unknowns. These equations were solved using Newton-Raphson's Method on the computer, resulting in an analytical model of the knee joint with the ability to simulate surgical operations.

The output is in the form of a video display indicating the path of the patella in the joint throughout flexion. Simulated operations are performed on the model until the desired patellar path is reached.

4.2 Recommendations

This model accurately defines the position of the patella in the joint with the assumption that the quadriceps group effective proximal attachment can be located accurately. Following are recommendations which would result in a more accurate model.

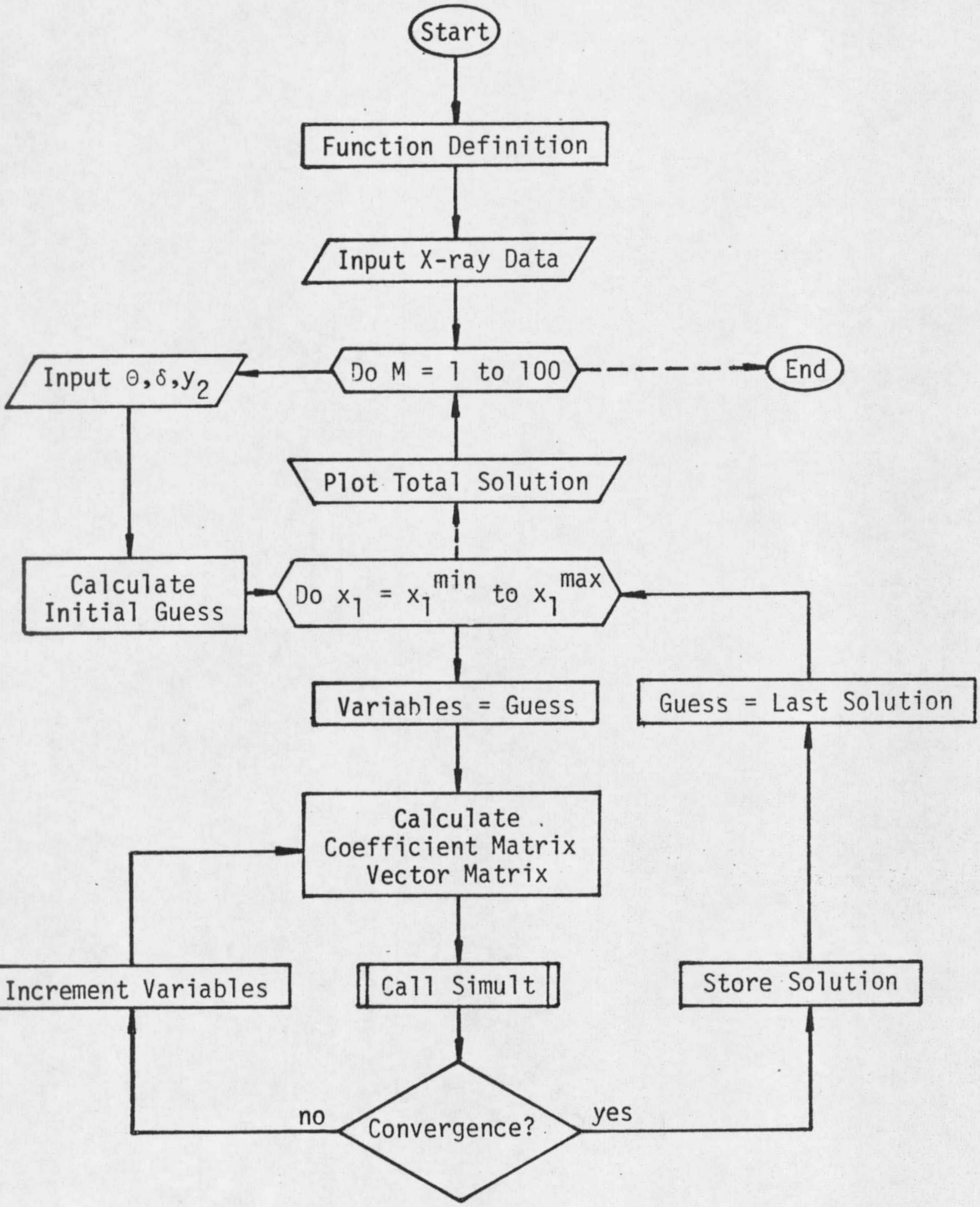
The major assumption used in the model is the approximation of the quadriceps group with a one-strand muscle. This requires the determination of an effective point of origin. In the current model, this value is input by the surgeon and remains fixed for all angles of flexion. There are two problems inherent with this assumption. One is that this effective point of origin is difficult to determine accurately, and the second is that this point changes as the flexion angle changes. In view of this, two recommendations are made.

1. An accurate relationship should be developed between the effective point of origin and the angle of flexion.
2. The second recommendation supercedes the first. Vectors should be incorporated in the model to represent all four muscles in the quadriceps group. This would add no new unknowns to the problem, enabling the same basic program to be used, but would alter the force analysis and thus Equations (17) through (19) would change. It would still be necessary to determine the effective point of origin for each of the four muscles, but this is not nearly as difficult as determining the effective origin for the total group.
3. The magnitudes of vectors $\vec{J}$ and $\vec{K}$ were assumed to be one-half the radius of the cylindrical patella for all patients. This is not necessarily true and these values could be input variables easily obtained from x-rays. This would require altering Equations (6) and (7).
4. The final recommendation is to incorporate a variable in the operation simulation to represent the movement of the patellar ligament

in the direction normal to the tibial shaft. As it now stands, the model only allows for movements in the lateral and longitudinal directions. The anterior side of the tibial shaft is V-shaped, and hence, a purely lateral displacement is not realistic.

APPENDIX I

PROGRAM KNEEPLOT FLOW CHART



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Hagelin, Jack S
An analytical model of the patello-femoral joint

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