



The determination of bubble velocity in fluidized beds  
by Jeffrey Edward Surma

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in  
Chemical Engineering  
Montana State University  
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**Abstract:**

Bubble velocity in a fluidized bed was determined with the use of an assembly of optical probes. Two probe assemblies were developed, one employed the use of three optical probes, the second used four. The four probe assembly proved most adequate in use for local bubble velocity determination. The volumes of bubbles detected by the four-probe assembly were also determinable from the probe outputs.

The four probe assembly was used to determine bubble velocities in a fluidized bed of sand. The sand was irregular in shape and had an average particle diameter of  $515 \pm 68 \mu\text{m}$ . Experimental results were compared to the Davidson and Harrison equation and a modification of this equation. The modified equation,  $U_b = C^*0(U_0 - U_{mf}) + U_{br}$ , incorporated a distribution coefficient  $C^*0$  which compared local bubble flow to the cross-sectional average.

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MONTANA STATE UNIVERSITY  
Bozeman, Montana

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Jeffrey Edward Surma

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## NOMENCLATURE

$A_T$	Cross-sectional area of the bed
$C_o$	Distribution coefficient defined by Equation 5
$C_o^*$	Distribution coefficient defined in Table 3
$D$	Diameter of the bed
$d_b$	Equivalent spherical diameter of bubble
$f_w$	Bubble wake fraction
$g$	Gravitational constant
$j$	Volumetric flux density of mixture
$K_b$	Bubble coefficient used in Equation 1
$L_c$	Defined by Equation 8
$r$	Radial distance
$r_{max}$	Radius of bed
$R_n$	Frontal radius of bubble
$T$	Total time
$t_{bi}$	Time of bubble contact with probe
$t_d$	Time of bubble contact with probe A
$t_p$	Time for bubble to rise from probe A to probe D
$U_b$	Absolute velocity of bubble
$U_{bo}$	Superficial velocity of bubble phase
$U_{br}$	Bubble natural rise velocity
$U_{mf}$	Minimum fluidization velocity
$U_o$	Superficial gas velocity

$X$	Horizontal dimension of bubble used in Equation 8
$X_p$	Distance from bed center of probes B and C
$Y$	Vertical dimension of bubble used in Equation 8
$Y_p$	Vertical separation of probes A and D
$\delta_b$	Volume fraction of bubbles
$\delta_{bi}$	Local volume fraction of bubbles cross-sectionally
$\epsilon_f$	Void fraction of bubbling bed as a whole
$\epsilon_{mf}$	Void fraction at minimum fluidization
$\rho_b$	Density of bubble phase
$\rho_d$	Density of dense phase
$\langle \rangle$	Averaged over bed cross-sectional area
—	Arithmetic mean value or a weighted mean value

## ABSTRACT.

Bubble velocity in a fluidized bed was determined with the use of an assembly of optical probes. Two probe assemblies were developed, one employed the use of three optical probes, the second used four. The four probe assembly proved most adequate in use for local bubble velocity determination. The volumes of bubbles detected by the four-probe assembly were also determinable from the probe outputs.

The four probe assembly was used to determine bubble velocities in a fluidized bed of sand. The sand was irregular in shape and had an average particle diameter of  $515 \pm 68 \mu\text{m}$ . Experimental results were compared to the Davidson and Harrison equation and a modification of this equation. The modified equation,

$$U_b = C_o^* (U_o - U_{mf}) + U_{br} ,$$

incorporated a distribution coefficient  $C_o^*$ , which compared local bubble flow to the cross-sectional average.

## INTRODUCTION

A fluidized bed is a column in which solid particles are suspended by an upward fluid flow. The phenomenon of fluidization occurs when there exists a balance between the upward drag force exerted on the particles by the fluidizing fluid and the downward gravitational force on the particles.

Fluidization was first successfully employed for industrial purposes in 1922 when Fritz Winkler demonstrated the use of fluidized beds for the gasification of coal. Since Winkler's gasification process, fluidization technology has had many successful breakthroughs. Although there are many successful fluidized bed operations in a wide variety of applications in industry, there have been numerous costly failures in the development of new fluidization processes. These failures can be attributed to the lack of predictable knowledge of what is happening within the fluidized bed. For this reason much of the recent research and development efforts in fluidization technology have been in the area of fluidization fundamentals such as fluid-particle interactions, and the hydrodynamic properties of fluidized beds.

### Regimes of Fluidization

Gas-solid systems comprise to a large extent the majority of fluidization processes. In gas-solid systems the state of fluidization varies widely depending on gas velocity and particle properties. Except for a limited range of conditions under which individual particles can be said to be uniformly dispersed, particles in gas-solid systems aggregate, giving rise to several distinct flow regimes. There are at least four distinct regimes that have been observed [1]. They are:

1. Particulate regime
2. Bubbling/slugging regime
3. Fast fluidization
4. Dilute-phase flow

The particulate regime is that in which the gas velocity is above the minimum velocity required to incipiently fluidize the solid particles but below that at which bubbles are formed. The particulate regime is characterized by a smooth expansion of the bed with the particles evenly spaced and the fluid smoothly passing through the interstices without the formation of bubbles. In most gas-solid systems particulate fluidization is only observed for a narrow range of gas velocities.

As the velocity of the gas is increased beyond the range of particulate fluidization, the particles aggregate and voids or bubbles are formed. The phenomenon of bubbling in a fluidized bed is thought to be the result of instabilities in the lower regions of the bed [2]. Bubbles grow in size as the velocity of gas is further increased beyond that at which bubbles first form. If the bed is of sufficient height, the bubbles will coalesce and the formation of a single slug results. The diameter of the slug approaches the radial dimension of the bed with further increases in gas velocity.

The regime termed fast fluidization refers to that state at which bubble or slug stability diminishes. The bed becomes turbulent and considerable entrainment of solids results. The bed is described as a "dense entrained suspension characterized by an aggregative state in which much of the solid is, at any given moment, segregated in relatively large densely packed strands and clusters" [1].

Dilute-phase flow results when the concentration of solids in the bed is low and the gas velocity is well above the terminal velocity of the solid particles. All solids are entrained and carried out of the bed in the exiting gas.

### Motivation for Fluidization Research

Many of the industrial applications of fluidized beds and those of practical interest require that the operation of the bed be in the bubbling/slugging regime. Bubbling in gas fluidized beds is important to mass and heat transfer as well as mixing in the bed. It has been determined one of the most important factors governing the extent of chemical conversion in a fluidized bed reactor operating in the bubbling regime is the diameter and velocity of bubbles in the bed. Generally, the smaller the diameter bubble, the greater is the extent of chemical reaction. Much interest therefore lies in the determination of the characteristics of bubbles within fluidized beds.

### Research Objectives

It is the objective of this research to develop an optical probe assembly for use in the determination of local bubble properties in a freely bubbling gas-solid fluidized bed.



## PREVIOUS RELATED RESEARCH AND BACKGROUND

Interest in the bubble phase in fluidized beds has led to the development of various techniques with which to investigate bubble characteristics and properties. Considerable stores of information pertaining to bubbles in fluidized media are present in the literature, yet to date there exists no adequately accurate direct measurements available on the size and corresponding velocity of bubbles in three-dimensional beds. Methods used to determine bubble velocities indirectly or by analogy are given in the following.

### Two Dimensional Fluidized Beds

The use of two-dimensional fluidized beds and cine photography proved to be a viable means of qualitatively measuring and observing bubble properties and characteristics. The two-dimensional bed consists of two transparent parallel plates in close proximity allowing the observation of bubbles as they contact the transparent surfaces. Much insight into the mechanisms of bubble growth and coalescence, as well as information on the spatial distribution and shape of bubbles within fluidized beds, has been gained using two-dimensional beds. However, many differences between two and three-dimensional beds exist, and no accurate analogy can always be made between the two types of beds. Geldart [3] has proposed such an analogy, but with insufficient experimental data there is doubt in its validity.

### X-ray Techniques

Other investigators used X-ray techniques to observe bubbles within three-dimensional beds [4,5,6]. Although the X-ray method of bubble observation has had wide application, the method is of limited value, due to the difficulty of identifying a particular bubble

when high concentrations of bubbles are present. The bed dimension is also limited in the direction of ray transmission to about 30 centimeters.

### Probe Techniques

The technique most commonly employed for the detection of bubbles within fluidized beds is the use of intrusive probes. Under normal operating conditions bubbles are well distributed over the cross section in the lower regions of the fluidized bed. As bubbles rise, they coalesce with adjacent bubbles and grow. At sufficient heights above the gas distributor a single train of bubbles rises along the bed center line. Techniques limited to the detection of undisturbed, single rising bubbles such as X-ray, and the early probes which were large in size, were of limited use in the lower regions of the bed. With the development of smaller probes, local bubble measurements within groups of bubbles in the lower regions of the bed have been made possible. Probes have been used extensively in the measurements of fluidization hydrodynamics. Various types of probes have been developed; the resistivity probe, the inductance probe, the capacitance probe, the pneumatic probe, and the optical probe are the typical probes used. Of these, the capacitance, pneumatic, and the optical probes have had the highest degree of success in bubble detection.

### Capacitance Probes

The early capacitance probes consisted of two opposing plates. Plate-type capacitance probes never gained acceptance as a means of bubble detection, due largely to the shape of the probes which caused destruction of the rising bubbles rather than the determination of the local state of bubble hydrodynamics. However, Werther and Molerus [7] developed an inobtrusive needle capacitance probe which proved adequate in its utility as a means of bubble detection. Though capacitance probes have had extensive use in the study of bubble phenomena, they suffer three major shortcomings: (1) the signal-to-noise ratio becomes

unacceptably low when probes and cables are long, (2) the dependence of probe calibration on bed material and bed operating conditions, and (3) the need for sophisticated and expensive equipment for signal analysis.

### Pneumatic Probes

Pneumatic techniques have had success in the measurement of bubble properties. One major advantage of this technique is the capability of bubble detection at high temperatures and pressures. The determination of gas velocity in the free board (i.e., region of bed directly above fluidized media) and within bubbles is possible when the probe is used as an anemometer. A probe of small dimension developed by Flemmer [8] has proven adequate as a bubble measurement device. The pneumatic probes have the drawback of slow response time due to the dead volume of the system (pressure transducer and piping), limiting the detection of bubbles with frequencies ranging from 0-4 Hz.

### Optical Probes

The design of the optical probe has forgone various changes since the technique was first used by Yasui and Johnson [9]. Opto-electronic components have been reduced in size to where the implementation of these small electronic devices are well suited to the optical probe. Such a probe was developed by Wen and Dutta [10]. The optical probes suffer none of the shortcomings that exist for the capacitance probe, and unlike pneumatic probes are capable of detecting bubbles with frequencies much greater than 4 Hz. Optical probes are simple to implement in a bubble detection scheme and require no expensive equipment for their use. The small size leads to the use of multi-probe configurations from which many bubble characteristics are determinable.

Table 1 lists some of the probes that have been implemented in the study of fluidization characteristics in gas-solid systems along with the individual investigators.

Table 1. Probes Used in Bubble Investigation.

Probe Type	Investigators
Capacitance	Toei et al. [4] Geldart and Kelsey [11] Watkins and Greasy [12] Harrison and Leung [13] Davidson et al. [14] Werther and Molerus [7]
Pneumatic	Svoboda et al. [15] Flemmer [8] Ran [16] Viswanathan [17]
Optical	Yasui and Johnson [9] Winter [18] Wen and Dutta [10] Whitehead et al. [19]
Resistivity	Burgess [20]

#### Measurement of Bubble Velocity

At normal operating conditions there are groups or clouds of bubbles rising in the bed. With the use of external devices such as X-ray, capacitance plates, etc., the capability of bubble discrimination is greatly diminished. Thus, bubble velocity measurements in three-dimensional beds are most successfully accomplished with the use of internal probes.

A bubble can be detected by two probes of known vertical separation. The delay in detection is utilized in the determination of the bubble velocity. Bubble size can also be approximated from the probe signal duration. Most bubble velocity data reported in the literature were determined via this method.

The method requires assumptions about the path of the bubble center in relation to the probe end. It is generally assumed that the bubble moves vertically along a line. The probe may in fact see an edge of the bubble, and the probe signal registered results in the length of a chord rather than the diameter.

An effort to determine bubble orientation with respect to the probe requires a matrix of probes which may in fact destroy the integrity of bubble shape. If a minimum number of miniature probes are used, it may be possible to develop a procedure to accurately determine bubble velocity, and bubble orientation with respect to the probes.

One such multi-probe system was developed by Burgess [20]. It consisted of five resistivity probes situated in such a manner that velocity, and bubble shape, as well as bubble orientation with respect to the probes could be determined. No other efforts in this area have been reported in the literature.

#### General Characteristics of Bubbles

Bubbles in fluidized media are in many respects similar to gas bubbles in a liquid. The bubbles have a precise edge or boundary and are essentially free of particles, except for a trailing wake. The bubbles are approximately spherical in shape but the lower wake region is indented as depicted in Figure 1. Perturbations about the stable or idealized shape are common due to the effects of neighboring bubbles and the vessel wall.

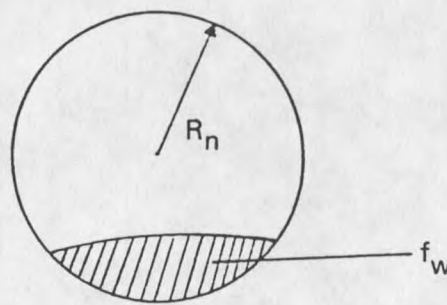


Figure 1. Idealized bubble shape.

The effect of bringing bubbles in close proximity to each other (i.e., increasing the concentration of bubbles) is to promote coalescence during which considerable distortion

of the bubble shape occurs. When the bed is operated at reasonably high superficial velocities the stable bubble shape may never exist. The bubble shape appears to be a characteristic of the fluidized medium. The discernible feature that characterizes a bubble in one material from that in another is the wake fraction,  $f_w$ , of the bubble. There exists no information for the reliable prediction of the wake fraction from particle properties, but the bed voidage,  $\epsilon_f$ , has been noted to affect the wake fraction. The greater the voidage at minimum fluidization,  $\epsilon_{mf}$ , the smaller the bubble wake fraction.

#### Rise Velocity of Bubbles

Bubbles in fluidized beds rise with a velocity that is dependent on the bubble size, bubble concentration (i.e., bubble volume fraction,  $\delta_b$ ), and the properties of the fluidized material.

The principal parameter affecting bubble velocity is that of size. Larger bubbles rise with a greater velocity than do smaller bubbles. Bubbles whose diameter is much smaller than the diameter of the column ( $d_b/D < 0.1$ ) rise with a velocity that is independent of vessel diameter. When this situation exists the bed is said to be a bubbling bed. When the relative size of a bubble is increased its rise velocity becomes increasingly influenced by the vessel walls. As the bubble diameter is yet further increased in size and approaches that of the column, its velocity becomes independent of its volume and is solely a function of column diameter. When this situation exists the bed is referred to as a slugging bed, refer to Figure 2. There exists a transition region between the bubbling and slugging beds where the velocity of a bubble is affected both by the bubble and column diameters.

The concentration of bubbles can have a marked effect on the velocity of bubbles. As the concentration is increased, the bubble velocity increases. This is due to an upward flow of solids which augments the bubble rise velocity.

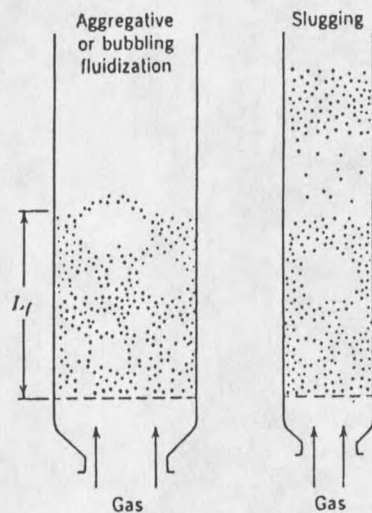


Figure 2. Bubbling and slugging beds.

Bubble velocity can also be affected by the presence of other bubbles at a higher axial position in the bed. As a bubble rises, it pulls with it a wake of solids with an upward velocity approximately equal to that of the bubble. When another bubble is drawn into this wake it will quickly catch and coalesce with the preceding bubble. In almost all cases the mechanism by which two bubbles coalesce is by the lower bubble being absorbed through the base of a preceding bubble. In general the preceding bubble suffers little distortion and rises at its normal velocity.

Properties of the solid have been shown to have only slight effects on bubble velocity [21]. The only observed effect is the slowing of bubbles with increasing wake fraction. This probably is the effect of energy dissipation in the wake of the bubble.

Isolated bubbles in a quiescent bed with a superficial gas velocity just beyond that of minimum fluidization have been observed to rise with a relatively constant velocity. This constant velocity is termed the natural rising velocity of the bubble,  $U_{br}$ . The form of the equation which predicts this velocity is,

$$U_{br} = K_b (gR_n)^{1/2} \quad (1)$$

which results from application of the steady-state mechanical energy balance to describe the rate of rise of a single spherical bubble in a low viscosity medium.  $R_n$  is the frontal radius of curvature as indicated in Figure 1. Davies and Taylor [22] developed the equation for the rise of gas bubbles in a liquid, but it was also shown to be applicable to gas-solid fluidized beds.

Application of theory to fluidized systems necessitated the postulation of the two-phase theory of fluidization. A model of aggregative fluidization may be set up by considering a fluidized bed as a two-phase system consisting of (1) a particulate phase in which the fluid flow-rate is equal to the flow-rate required to incipiently fluidize the bed, and (2) a bubble phase which carries the additional flow of fluidizing fluid.

There is evidence both supporting and negating the validity of the two-phase theory, but improved experimental determination of bubble properties are needed to test the theory.

Starting with the two-phase model, Davidson's [23] analysis of bubble motion in a fluidized bed led to the same form equation as the Davies and Taylor equation

$$U_{br} = 0.711(gd_b)^{1/2} \quad (2)$$

where  $d_b$  is the diameter of a sphere having the same volume as the spherical cap bubble. The constant  $K_b$  was determined experimentally to be 0.71 by many investigators, but values have been reported in the literature ranging from 0.57 to 0.85 for a bubbling bed. A value of 0.35 for  $K_b$  has been reported by most investigators of slugging beds where  $d_b$  is the bed diameter.

Davidson's model successfully accounted for the movement of both gas and solid as well as the pressure distribution about rising bubbles. Models of fluidization have also been developed by Jackson [24] and Murray [25], but will not be elaborated here.



The next consideration was that of the rise velocity of a crowd of bubbles. Nicklin [26] developed the first successful approximation for gas-liquid systems, which was later applied to fluidized beds by Davidson and Harrison [21]. It was assumed the relative velocity between bubble and emulsion is unaffected, but that the emulsion has an upward velocity of  $U_o - U_{mf}$ , from the two-phase theory. With this approximation the absolute rise velocity of bubbles in a freely bubbling bed is given by

$$U_b = U_o - U_{mf} + U_{br} \quad (3)$$

where

$$U_{br} = 0.711(gd_b)^{1/2} \quad (2)$$

Equation 3 was derived based on the concept of a single bubble rising in an infinite medium and from analogy with slug flow. Hence, the equation cannot account for interference of other bubbles, or the effects of nonuniform bubble flow. An equation developed by Weimer and Clough [27] has attempted to account for those nonidealities. The equation was derived from concepts quite different from those of previous investigators, yet the result is similar to that of Equation 3. The development of the equation was obtained assuming churn-turbulent flow throughout the fluidized bed and by considering a multiple-particle drag coefficient. The improved bubble velocity equation has the form

$$\bar{u}_b = C_o \langle u_{bo} \rangle + 0.71 \left[ \frac{g \bar{d}_b (\rho_d - \rho_b) (1 - \langle \delta_b \rangle)}{\rho_d} \right]^{1/2} \quad (4)$$

where  $C_o$  is a distribution coefficient. This coefficient represents the effect of non-uniform bubble flow and volume fraction. A value of  $C_o$  equal to 1 is indicative of uniform bubble flow cross-sectionally. Values greater than 1 represent bubble flow tending toward the bed's axial center, and values less than 1 represent bubble flow near the wall. The coefficient is determined from









































































































































