Optimal decision rules for marketing and storage of wheat and corn
by James William Mjelde

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Applied Economics
Montana State University

Abstract:
Expected profit maximizing decision rules for marketing and storage of winter wheat and corn were determined in this study. Decision alternatives considered were: 1) sell the crop at time of harvest, 2) go into the Farmer Owned Reserve, 3) take out a regular CCC loan on the crop and 4) store the grain without any federal loans taken out on the crop. The optimal decision rules, which were obtained from a stochastic dynamic programming model, depend on grain prices and the type of program under which the grain is held. Validity of the decision rules depend on future validity of the time series model for grain prices that were used to estimate Markovian transition probabilities.

Results indicate that the Farmer Owned Reserve can increase the value of a bushel of grain to an individual producer. When market prices are low for both winter wheat and corn a producer should enter the Farmer Owned Reserve. If market prices are high at either harvest or after harvest a producer should sell.

Unconditional and conditional probabilities of being in the various marketing alternatives while following the optimal policy were calculated. These probabilities show that if producers follow the optimal policy, a large amount of the U.S. grain crop will be held in the FOR program for the duration of the three year FOR contract.

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ABSTRACT

Expected profit maximizing decision rules for marketing and storage of winter wheat and corn were determined in this study. Decision alternatives considered were: 1) sell the crop at time of harvest, 2) go into the Farmer-Owned Reserve, 3) take out a regular CCC loan on the crop and 4) store the grain without any federal loans taken out on the crop. The optimal decision rules, which were obtained from a stochastic dynamic programming model, depend on grain prices and the type of program under which the grain is held. Validity of the decision rules depend on future validity of the time series model for grain prices that were used to estimate Markovian transition probabilities.

Results indicate that the Farmer-Owned Reserve can increase the value of a bushel of grain to an individual producer. When market prices are low for both winter wheat and corn a producer should enter the Farmer-Owned Reserve. If market prices are high at either harvest or after harvest a producer should sell.

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Chapter 1

INTRODUCTION

Grain producers are now faced with several grain marketing alternatives; they can sell their crop at the time of harvest, store their grain, go into the Farmer Owned Reserve (FOR), or take out a regular CCC loan on the crop. Each of these alternatives has different expected benefits and cost. For example, if a producer sells at harvest he will incur no storage cost or interest on loans taken out on the crop, but the market price the producer will receive is usually at its low point in its yearly cycle, due to the increased supply.

The primary objective of this study was to determine the optimal grain storage and marketing strategy[1] for an expected profit maximizing producer faced with the above alternatives. Although this problem has been submitted to cursory economic analysis by several researchers (see, for example, Johnson, Rizzi, Short, Fulton (1982); Meyers and Ryan (1981); Steele (1981); and McConnen, Stauber, Baldridge and Taylor (1982)), it has not been analyzed with the rigorous multiperiod optimization techniques.[2]

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[1]For brevity, "marketing strategy" will henceforth be used to refer to the joint grain-storage, grain-marketing strategy.

[2]Several studies (Gustafson (1958); Burt, Koo and Dudley (1980); Cochrane and Danin (1976); Alaouze et al. (1978, 1979); Kennedy (1979); Taylor and Talpaz (1979)) have employed rigorous optimization techniques to address grain storage and grain release at the national level. However, results from these studies are not applicable to the individual producer's problem for several reasons.

The study was comprised of two parts. One part was to develop time-series models to describe price movements for winter wheat and corn. These time-series models were used to calculate price transition probabilities for a stochastic dynamic programming (DP) model, which provided the foundation for the second part of the study. The DP model was used to find the decision rule that maximized the expected present value of profit over a three-year period, given the decision alternatives of: (a) sell at harvest; (b) privately store the grain; (c) participate in the 1982 CCC loan program; and (d) participate in the 1982 FOR program. At harvest time, the decision is based on current and past market prices. Starting the first month after harvest and each month thereafter, the decision for that month is based on the type of program under which the grain is held as well as current and past market prices. It should be cautioned that the validity of the decision rule depends on the future validity of the transition probabilities.

Dynamic programming was selected as the optimization technique because it is a powerful analytical and computational method for handling Markovian multistage decision processes (Burt (1982), and

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First, these studies incorporated price response, whereas the price received by an individual does not depend on the quantity he stores or sells. Second, storage decisions were assumed to be made yearly, while much shorter decision periods are appropriate for individuals. Third, the aggregate studies focused on determining an optimal national program, while this study focuses on optimal marketing strategies given the national programs for the 1982 crop.

Burt and Allison (1963)).[3] Furthermore, stochastic DP is by far the most efficient method for solving stochastic multistage optimization problems.

This study is divided into five chapters. The first chapter is an introduction. The second chapter gives a detailed discussion of the alternatives facing a producer. In this chapter the provisions of the regular CCC loan and FOR programs relevant to this study are discussed. The third chapter deals with the methodology used in developing the time-series models for winter wheat and corn, and the DP model. Assumptions used in the study are summarized in Chapter Four. Then the results of various DP solutions are discussed. Chapter five discusses limitations of the study and recommendation for further study.

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[3]The Markovian requirements states that an optimal policy in a given state depends only on the state of the process in that stage and not on the state at proceeding stages. This requirement allows us to separate past decisions from current and future decisions when formulating and solving the DP model.

Chapter 2

PROVISIONS OF FOR AND REGULAR CCC LOAN PROGRAMS

The four most viable grain marketing and storage options currently available to a producer at the time of harvest are: (a) sell; (b) privately store the grain; (c) participate in the 1982 Farmer Owned Reserve program; or (d) participate in the 1982 CCC loan program. The option to sell at the time of harvest is available to all producers and under certain conditions may be the optimal decision. Selling at harvest saves the producer storage costs, but putting the grain into private storage gives the producer more flexibility. He is able to sell at a later date when the market price is usually higher or, if conditions dictate, the producer can go into either the FOR program or the regular CCC loan program. A producer pays for this increased flexibility by incurring storage costs for the grain.

The FOR program and the regular CCC loan program were continued by the Agriculture and Food Act of 1981. Operating regulations for the program are under the control of the Secretary of Agriculture. The FOR was instituted for wheat and rice in April 1977, and was later extended to other crops including corn. This policy, an outgrowth of debates and studies of the volatile behavior of the grain markets in the 1970s, is an attempt to stabilize grain markets. The provisions of these two federal programs for the 1982 crop will be presented as they pertain to the model developed in this study.

A major assumption of this study is that the producer has complied with the regulations of the Agriculture and Food Act of 1981, so the producer is eligible for these two federal programs. An outline of the essential compliance (eligibility) regulations follows.

1. To participate in the program for wheat, wheat acreage for 1982 crops must be reduced at least 15 percent from the established base.

2. To participate in the program for corn, corn acreage for 1982 crops must be reduced at least 10 percent from the established base.

3. The farm program acreage base for a particular crop will generally be the higher of (a) 1981 planted acreage; or (b) the average of 1980-81 planted acreages. For farms that have been following a definite crop rotation pattern, the established bases will reflect these variations.

4. Cross compliance is not required in 1982. A grain grower may participate in the program for one crop and not participate in the program for another crop.

5. When acreage is reduced in order to participate in the program, it is required that an equal amount of acreage be placed in a conserving use.

Nine month regular CCC loans and the three year FOR loans are available to eligible producers at a specified loan rate for the crop under contract. One similarity between these two programs is that producers retain ownership of the grain that is used as collateral for either loan. A second similarity is that any time between harvest and March 31 of the following year the producer can take out a regular loan or go into the FOR program.

An appealing feature of the regular CCC loan program is that the contract can be redeemed (loan + interest) without penalty at any time during the term of the loan. At maturity a producer may forfeit his loan, whereby CCC forgives both the principal and interest owed on the regular loan in exchange for the grain which was held as collateral, or the producer may retain ownership of the grain by paying principal plus interest. In the past a producer has been able to extend these nine month loans for twelve month periods, but no decision about extensions has been made for the 1982 crop. In the model developed for this study these regular loans were assumed to be extendable up to three years from harvest.

Another feature of the regular CCC loan program is that once the producer has taken out a nine month loan, he has the option of transferring the loan contract to a three year FOR contract if market price is below the set release price (Table 1). This transfer can only take place if the loan has not reached maturity or if the producer has received an extension on his loan. By transferring to the FOR program the producer will receive a premium loan (Table 1) plus storage payments. These two benefits make this an attractive alternative under certain conditions.

Table 1. Provisions of the 1982 regular CCC and FOR loan program for winter wheat and corn.

	Corn (National Average)	Winter Wheat (National Average)
	- - - - - $ per bushel	- - - - -
FOR loan price	2.90	3.92
FOR release price	3.25	4.65
Target price	2.70	4.05
FOR premium loan	.35	.45
FOR storage payments	.265	.265
Price Support (CCC loan price)	2.55	3.47

The interest rate charged for both the regular CCC loan program and the FOR program is the prevailing CCC interest rate, which is the rate at which it costs the Treasury to borrow money. This rate changes the first of every month. The interest rate charged on a particular loan contract (regular or FOR) is the rate that was prevailing at the time of the transaction. This rate remains in effect from the time of the transaction to December 31 of that same year. An annual rate change occurs on January 1. This new rate is applied to the remainder of the term of the loan contract. To simplify this study it was assumed that the interest rate remained constant throughout the three years of the study period. Since March 1978, interest on FOR loans has been waived after the first year of the three year contract. Another simplifying assumption used in this study is that interest is paid throughout the three year contract; that is, no interest payments are waived.

The FOR program has some added benefits that the regular loan program does not have. One such benefit is that the loan rate (support price plus premium loan) for FOR (Table 1) is greater than the loan rate (support price) for the regular loan program (Table 1). A second benefit is the storage payments received under the FOR program, which are slightly less than the cost of commercial storage. Another benefit as stated earlier is the waiver of interest payments after the first year of the contract.

For these added benefits the producer faces a major constraint. This constraint regulates the redemption of the reserve grain contract.

Voluntary redemption for a FOR contract before maturity is permitted without penalty only after the market price has reached or exceeded a specified release price (Table 1). If a producer redeems his FOR contract before maturity and the market price is at or above the release price the producer pays loan principal plus interest (if applicable). On the other hand, a penalty is assessed if the producer redeems the FOR contract when market price is below the release price. The penalty (liquidation damages) for redemption of the contract is given by the formula:

(contract + liquidation damages) = (principal of loan) + (interest rate) x (principal of loan) + (storage payments received) + (interest rate) x (storage payments) + (50% of interest rate) + (principal of loan).

Another aspect of the FOR program is that if the market price is above the release price, the producer does not have to redeem the contract. However, after two consecutive months of market price being above the release price, storage payments will stop. The storage payments will begin again once market price goes below the release price. The release price also regulates whether a producer can enter the FOR program. If market price is greater than or equal to the release price, a producer cannot take out a FOR loan.

Upon maturity of the FOR contract, redemption is much like that of the regular loan program. A producer may redeem the contract and pay principal plus interest (if applicable). The second option is to forfeit the contract whereby CCC forgives both the principal and interest owed in exchange for the grain.

To summarize, the producer foregoes the options of taking out a FOR contract or a regular CCC loan contract if he sells at harvest, but at the same time he does not pay any interest. Going into private storage at harvest gives a producer more alternatives (sell at a later date, FOR, or regular CCC loan program) on how and when he will market his grain. The regular loan and FOR contracts give interim financing to the participant so that the grain does not have to be sold at harvest time when market prices are usually lower. Regular CCC loan contracts have no constraints on redemption. The FOR program has redemption constraints, but the participant receives storage payments and a premium loan. Both the regular CCC loan and the FOR programs eliminate down side risk. The regular loan program does not sacrifice on the upside due to its redemption failures, except for interest payments. However, the FOR program sacrifices some upside potential when market price is between the loan price and release price. A participant can not redeem the contract in this market price range without substantial penalty.

Chapter 3

METHODOLOGY

Formulation of the DP Model

Successful application of the DP technique to a multiperiod optimization problem depends on appropriate specification of stages, state variables, decision alternatives and the Markovian relationship between state variables for each decision alternative at different stages (Nemhauser (1966); Bellman (1957)). With DP, as with other optimization techniques, one must balance the realism of model specification against cost of solving the model. A discussion of the compromises made for the grain storage-marketing problem follows.

Stages and States

In order to be cast in a DP framework a multiperiod process must be divided into time intervals or stages, with a decision being made at each stage. Stages for the stochastic DP model developed in this study are the day of harvest and each month thereafter for three years. Using months as stages gives a marketing interval that allows for seasonal fluctuations in prices, but is not so short that it gives an excessive number of stages and an unwieldy decision rule.

At each stage of the process, state variables are used to describe the state of the process. State variables used to describe the optimization problem at hand are of two types. The first type of state variable measures the market price for grain. Price is used as a state variable because the market price that a producer receives for his grain is a major determinant of the profit from marketing grain. Market price is treated stochastically in the model. Future cash market prices are not known with certainty; thus treating price stochastically makes the model more realistic. The second type of state variable indicates the type of program under which the grain is currently held. Stock states were defined as:

1) grain held in the FOR program,

2) grain held in a regular CCC loan program,

3) grain held in private storage without a loan, and

4) no grain in storage; the grain has already been sold.

Other variables, such as the date grain went into the FOR, were not specified as state variables because they would signfiicantly increase computational cost. Moreover, such variables would not likely have an appreciable impact on the decision rule.

Decision Alternatives

The decision made at a given stage deterministically or probabilistically controls the state in which the process will be found in the next stage. Decision alternatives considered in the model are:

0) Keep grain in the current stock state;

1) sell the grain;

2) With grain in the regular CCC loan program go into FOR program;

3) with grain in storage without a loan, obtain a regular CCC loan; and

4) with grain in storage without a loan, go into the FOR program.

Stock state transitions are deterministic; that is, once a decision is made at a particular stage the stock state at the next stage is known with certainty given a decision. However, price state transitions are stochastic, but the decision made at a particular stage has no effect on the price state at the next stage. The price state transition is a function of the values of current and past prices and the month of the year.

Recursive Equation

The objective function for the dynamic programming model is maximization of expected net present value of returns from one bushel of grain, given the various alternatives available at harvest. Consider now the DP recursive equation which provides the foundation for solving a multiperiod optimization problem with the DP technique.

A recursive equation must possess the following three properties. First, at any stage t a decision is to be made. Second, a decision together with the state of the process at stage t stochastically determines the state of the process at the next stage. Third, for any stage t, the state and the decision determine expected returns for that stage. Fourth, the process must be fully described at any stage t by the state at stage t. State variables in any other stage do not help describe the state at stage t.

The DP principle of optimality is the fundamental concept which provides the basis for the formulation of a recursive equation and for the solution technique. This principle states that an optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the initial decision. An optimal policy is defined as the sequence of decisions that optimizes the objective function.

Application of the principle of optimality to the grain marketing problem gives the following recursive relationship for both winter wheat and corn:

$$V_t\ (S_t, PS_t) = \max_{D_t}\ [\pi_t(S_t,\ PI_t,\ D_t) + \beta EV_{t+1}(S_{t+1},\ PI_{t+1})]$$

where

t = time (months since harvest);

$V_t\ (S_t,\ PI_t)$ = return from following an optimal policy from the current time period t through the final time period t_j;

PI_t = price information vector at time t;

Max = maximization operator;

D_t = decision made at time t;

π_t = expected immediate returns which depends on the stock state, price state, and the decision;

E = expectation operator; and

β = discount factor.

Using this recursive equation, the solution procedure moves backwards stage by stage, finding an optimal policy for each state at every stage. Due to the provisions of the Agriculture and Food Act of 1981, which make different decision alternatives applicable at some stages than at other stages, the stages of the DP model were divided into four subsets based on time intervals. The first time interval is the last stage ($V_T(S_T,PI_T)$), at which the regular CCC or FOR contracts are assumed to come due. In this stage, the highest value of the two different ways in which the regular CCC loans or FOR loans can be redeemed is calculated. The two alternative ways of redeeming contracts are: (a) sell the grain and pay back the loan; or (b) forfeit the contract and transfer ownership of the grain to the CCC. If the grain is in private storage without a loan, the terminal value is market price minus storage cost for that month.

In the second time interval, the recursive equation is solved for the next-to-last stage, (T-1), through time period 7 where decision alternative 3 (in private storage -- obtain a regular CCC loan) and alternative 4 (in private storage -- go into FOR program) are not available to the producer. The provisions of the regular loan program do not allow a producer to take out a regular CCC loan on his crop after March 31 (assumed to be six months after harvest). The producer cannot go into the FOR program after March 31, unless prior to March 31 the producer took out a regular CCC loan on the crop.

In the third and fourth time intervals of the DP model, all of the decision alternatives are viable options for the producer. The recursive equation for time periods six through one are solved in the third time interval, and returns for the day of harvest are calculated in the fourth interval, thus completing the solution process.

If the decision D_t is to sell at time t, the value of $V_t(S_t,PI_t)$

is $\pi_t(S_t, PI_t, D_t)$ and the producer has no future returns $V_{t+1}(S_{t+1}, PI_{t+1}) = 0$. Any decision, except sell at time t, gives a positive value of $V_{t+1}(S_{t+1}, PI_{t+1})$. This value is dependent on the stock state and the price information vector. The decision D_t and stock state S_t at time t determine the stock state S_{t+1} at time t+1. PI_{t+1} is probabilistically dependent on PI_t and the month of the year. The price information vector PI_t is independent of the decision alternatives.

The major difference between the winter wheat and the corn recursive equation is in the market price information vector (PI_t). For winter wheat this vector contains only current price. Current market price is used to find the value of a bushel of winter wheat if it is sold. This price is also used to generate first order Markovian transition probabilities, which are probabilities of prices next month given the current price.

The price information vector for corn contains two values, current market price and price lagged one month. The current price gives the value of a bushel of corn if sold. Both current and lagged prices are used to generate second order Markovian transition probabilities for price the next period.

Estimation of the Price Forecasting Equations

Two time-series models, one for corn and one for winter wheat, were estimated for the purpose of probabilistically predicting future values of market prices. The price forecasting equations were used to generate Markovian transition probabilities (see Appendix A), which give the probabilities of price next month given previous prices.

Price forecasting equations must be kept simple in terms of the number of variables in the equation in order to keep the DP model computationally manageable. Any variable in the equation except time must become a state variable in the DP model. Two reasons for this needed simplicity are the curse of dimensionaltiy and any variable in the equation other than lagged price, such as stockpile of U.S. grain, must have its own transition probabilities computed. In theory a DP model is able to incorporate any number of state variables, but in practical applications this is not possible from a computational standpoint.[4]

If a variable other than lagged prices is added to the equation, the problem of computing transition probabilities for this nonprice variable is encountered. Calculation of these nonprice variable transition probabilities would complicate the DP model, yet it was deemed that they would not significantly improve the accuracy of the decision rule.

Using the above information, it was assumed that market price could be forecast by a time series model using lagged prices and

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[4]The curse of dimensionality affects both the number of computations and computer storage requirements. As the total number of states increase, the computational and storage requirements increase rapidly since a value for every possible state combination must be calculated and stored. The increase in storage requirements and cost, due to the increase in the number of states, is the curse of dimensionality; thus the number of states must be kept reasonably small.

monthly variations

$$P_t = f(P_{t-1}, P_{t-2}, \ldots, P_{t-s}, \text{monthly variation})e^{\mu t}$$

where,

P_t = market price at time t; and

μ_t = error term.

As with any time-series model, this equation does not give a structural explanation for changes in market price. However, this model does replicate past market price behavior in such a way that it will help forecast future behavior of price, unless the structure of the market changes.

One would expect grain prices to have relatively higher variability when the market price is high. A multiplicative error term better accounts for this higher variability at high prices than an additive error term. When estimating this equation it is desirable to smooth the monthly variations. For corn this smoothing was accomplished by polynomial constraints on twelve monthly dummies. Winter wheat monthly variations were postulated to follow a sine function, which symmetrically smooths the variations. The monthly dummies for corn and the monthly variation (sine function) for winter wheat tell us nothing about what is causing the variations in market prices. However, they do give a quantitative measure of monthly variations that influence transition probabilities.

Although economic and statistical theory tells us that a time-series model can be constructed using past values of a variable, it does not tell us the functional form of the equation. The specification of an equation can take two forms: linear or nonlinear in parameters. Statistical results for both specifications are presented in Table 2. To compare adjusted R^2 and standard error of the estimates between two equations, the dependent variables must be in the same form. Since the dependent variables of the linear and nonlinear equation used here are not in the same form, a measure of relative dispersion is needed to compare the forms. When an equation is estimated in natural units, the coefficient of variation gives a measure of relative dispersion. For an equation that is estimated in log form, the standard error of the estimate is a measure of relative dispersion. By comparing the appropriate values in Table 2, it can be seen that the nonlinear forms have smaller relative dispersion. This method of comparing relative dispersion is a practical method, but in no way is it fool proof.[5]

The nonlinear form was used to develop the transition probabilities. Two reasons for this decision are 1) the smaller relative dispersion, and 2) the nonlinear form made it easier to handle the multiplicative error term. As stated earlier this type of error structure helps in overcoming the differing variability in prices when market price is either high or low. During the Russian wheat deal period, heteroscedasticity in the error term was prevalent when using an additive error term; a multiplicative error term helped overcome this heteroscedasticity problem.

[5]Personal communication with Dr. Oscar R. Burt.