Computer education of influence lines for continuous beams

by Richard Andrew Ehlert

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University

Abstract:
The education of fundamental engineering principles through increased use of general purpose analysis computer programs is a topic of concern among educators. The student must maximize learning efficiency if computer literacy and fundamental concepts are to be learned simultaneously.

A user-friendly, interactive, color-graphics computer program has been developed for teaching the fundamental concept of influence lines for continuous beams. The theorem of three moments and moment-area theorems are the fundamental principles presented, developed, and applied throughout the program.

Results of two types of problems using the computer program are presented. The two problems indicate the numerous capabilities for use of the program in teaching influence line concepts to students.

COMPUTER EDUCATION OF INFLUENCE LINES FOR CONTINUOUS BEAMS

by

Richard Andrew Ehlert

A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Civil Engineering

MONTANA STATE UNIVERSITY
Bozeman, Montana

November 1985

APPROVAL

of a thesis submitted by

Richard Andrew Ehlert

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citation, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

Nov. 25, 1985
Date

Chairperson, Graduate Committee

Approved for the Major Department

25 Nov '85
Date

Head, Major Department

Approved for the College of Graduate Studies

11-27-85
Date

Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a master's degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Dean of Libraries when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my permission.

Signature Richard A. Ehlert

Date November 25, 1985

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. F. F. Videon for his guidance, assistance, and encouragement during the preparation of this thesis.

A special appreciation is extended to Mr. Dimitri Nesterenko, retired Chief Structural Engineer for Stanley Consultants Inc. of Muscatine, Iowa. His concern for the education and understanding of fundamental engineering principles is reflected in this thesis.

The author also wishes to thank the following individuals and companies for their assistance:

Burt Barker, Integrated Software Systems Corporation, San Diego, California

Carol Bittinger, MSU Computing Services, Bozeman, Montana

Dal Burkhalter, MSU Computing Services, Bozeman, Montana

TABLE OF CONTENTS

TABLE OF CONTENTS—Continued

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

A	point at which influence line is developed
A_n, A_{n+1}	elastic weight diagram area, spans n and n+1 respectively
$_{n+1}A_o$	shear at left end of span n+1 when unit load occupies span n+1
$_nB_o$	shear at right end of span n when unit load occupies span n
E	modulus of elasticity
I_n, I_{n+1}	moment of inertia, spans n and n+1 respectively
M	internal beam moment
M_ℓ	support moment at left end of span n
M_{n-1}, M_n, M_{n+1}	support moment, supports n-1, n, and n+1 respectively
M_o	single-span moment due to unit load, measured at point where influence line is developed
M_r	support moment at right end of span n
M_x	moment at point of unit load application
P	concentrated point load
R	sum of elastic weight diagram reactions on either side of a given support
$R_{\ell,n}$, $R_{\ell,n+1}$	left reaction of elastic weight diagram, spans n and n+1 respectively
R_n	beam reaction at support n
$R_{r,n}$, $R_{r,n+1}$	right reaction of elastic weight diagram, spans n and n+1 respectively
V	internal beam shear
V_ℓ	shear at left end of span n

$V_{\ell,n}, V_{\ell,n+1}$	shear left of supports n and n+1 respectively
V_o	simple-span shear due to unit load, measured at point where influence line is developed
V_r	shear at right end of span n
$V_{r,n-1}, V_{r,n}$	shear right of supports n-1 and n respectively
V_x	shear at point of unit load application
a_n, a_{n+1}	distance to unit load from left end of span, spans n and n+1 respectively
b_n, b_{n+1}	distance to unit load from right end of span, spans n and n+1 respectively
c_n, c_{n+1}	distance to elastic weight diagram centroid from left end of span, spans n and n+1 respectively
d_n, d_{n+1}	distance to elastic weight diagram centroid from right end of span, spans n and n+1 respectively
ℓ_n, ℓ_{n+1}	span length, spans n and n+1 respectively
n-1, n, n+1	span or support numbers
x	distance from left end of span to point where influence line is developed in that span
$\Delta_{n-1}, \Delta_n, \Delta_{n+1}$	settlement at supports n-1, n, and n+1 respectively
Δ_o	simple-span deflection due to unit load, measured at point where influence line is developed
Δ_x	deflection at point of unit load application
θ_{n-1}	beam rotation to right of support n-1
θ_n	beam rotation to left of support n
θ'_n	beam rotation to right of support n
θ_{n+1}	beam rotation to left of support n+1

ABSTRACT

The education of fundamental engineering principles through increased use of general purpose analysis computer programs is a topic of concern among educators. The student must maximize learning efficiency if computer literacy and fundamental concepts are to be learned simultaneously.

A user-friendly, interactive, color-graphics computer program has been developed for teaching the fundamental concept of influence lines for continuous beams. The theorem of three moments and moment-area theorems are the fundamental principles presented, developed, and applied throughout the program.

Results of two types of problems using the computer program are presented. The two problems indicate the numerous capabilities for use of the program in teaching influence line concepts to students.

CHAPTER I

INTRODUCTION

The integration of computer usage into engineering curriculums presents educators and practicing engineers with an important question: How well do graduating engineers understand fundamental engineering concepts? Current engineering programs at universities throughout the country incorporate general purpose analysis computer codes as aids in teaching engineering principles [5,7,9,19]. This has led to what Yener and Ting refer to as the "black box approach" to educating engineers [21].

This "black box approach" has resulted in two diversions from quality engineering education. First, time normally spent on understanding fundamental engineering concepts is being spent on learning and using the software and hardware capabilities [12,13,15]. Knowing the capabilities allows the students to solve complex problems in minimal amounts of time. Secondly, students often accept computer solutions of complex problems without an ability to interpret and verify their validity. The combination of these two educational deficiencies can lead to a student misconception that knowledge of fundamental principles is unnecessary for engineering applications.

Influence Line Computer Program

The computer program listed in Table 8 (Appendix C) is an example of how the "black box" education of engineers can be reversed. The program was developed to serve as a teaching aid in the instruction of influence lines for continuous beams. Included within the program are three concepts fundamental to the education of influence lines:

(1) Presentation and application of theory.

(2) Interactive student computation of influence line ordinates.

(3) Application of influence lines.

History of Influence Lines

The analysis of continuous beams is believed to have been first published by Navier in his paper *Lecons* in 1826. Actual application of Navier's analysis came in 1850 with the design and construction of the Britannia Bridge over the Menai Straits by Robert Stephenson [6].

In 1857, Clapeyron reviewed continuous bridge development, citing Stephenson's Britannia Bridge as an example, in his work *Comptes Rendus*. Clapeyron is credited with being the first person to recognize that if the bending moments of the supports of a continuous beam were known, then all internal forces and deflections could be known. Even though Clapeyron first presented the theorem of three moments in 1848, it wasn't until 1855 that Bertot achieved priority for publishing the theorem [6].

Although work by Bresse in 1865 and Winkler in 1862 approached the concept of influence lines for continuous beams, it was not until 1906 that Mohr published the concept. Mohr also went on to develop theorems relating beam slope and deflection to elastic weight moment diagrams, known today as the moment-area theorems [6].

The theorem of three moments by Clapeyron and the moment-area theorems by Mohr provide the theoretical basis used in the computer program.

CHAPTER II

EQUATION DEVELOPMENT

Sign Convention

The internal force sign convention used for the development of all equations is shown in Figure 1. All externally applied loads and beam deflections are positive when acting downward. Beam reactions are positive when acting upward.

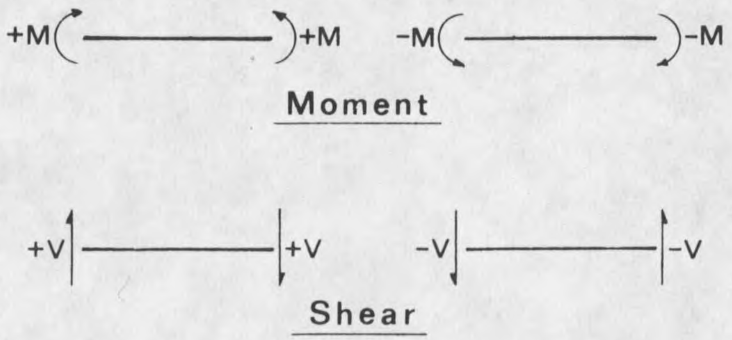


Figure 1. Sign convention.

Three-Moment Equation

The continuous beam shown in Figure 2 is a statically indeterminate structure. One method of reducing the beam in Figure 2 to a statically determinate structure is to solve for the moments at the two interior supports. Application of the general 3-moment equation (Eq. 12, Appendix A) yields:

$$2M_2\left(\frac{\ell_1}{I_1} + \frac{\ell_2}{I_2}\right) + M_3\left(\frac{\ell_2}{I_2}\right) = -6R_1 \qquad (1)$$

$$M_2 \left(\frac{\ell_2}{I_2}\right) + 2M_3 \left(\frac{\ell_2}{I_2} + \frac{\ell_3}{I_3}\right) = -6R_2 \qquad (2)$$

where R_1 and R_2 are dependent on the magnitude and location of concentrated load P. Equations 1 and 2 are solved simultaneously for unknown support moments M_2 and M_3, and the beam in Figure 2 is statically determinate. Each span of the beam is also statically determinate, and the internal forces and deflections can be computed at any point on the beam.

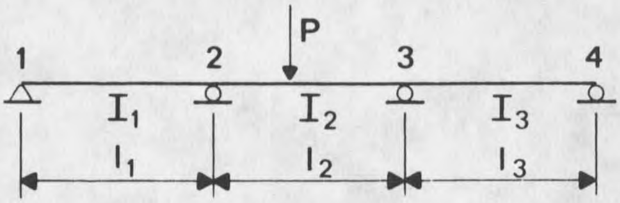


Figure 2. Three-span continuous beam.

Influence Line Equations

Influence lines can be developed for the beam in Figure 2 by setting the concentrated load P equal to unity and moving the load across the beam. Each new position of the unit load generates new values for internal support moments. Thus it is possible to develop expressions for the internal forces and deflections in a given span in terms of the support moments at each end of the span.

Every point on a continuous beam has a unique influence line for the moment, shear, and deflection at that point. Point A in Figure 3 defines the point, in an arbitrary span n, at which an influence line is to be developed. Equations have been developed in Appendix B for computing ordinates of influence lines at point A and are given in Equations 3, 4, and 5. These equations compute the influence line ordinates at the unit load position, assuming point A is located in span n.

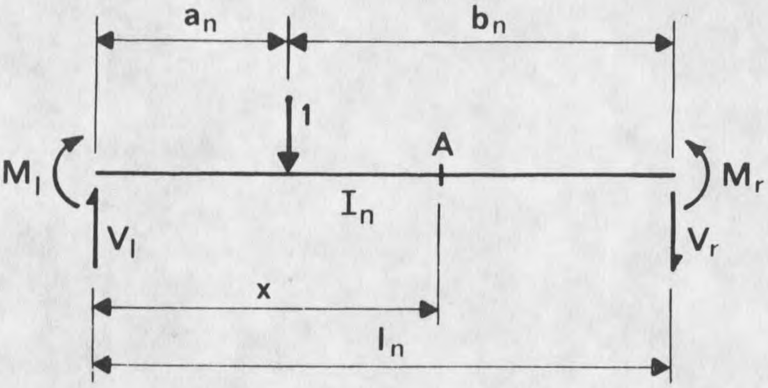


Figure 3. Free-body diagram of span n.

$$M_x = M_o + M_\ell \left(1 - \frac{x}{\ell_n}\right) + M_r\left(\frac{x}{\ell_n}\right) \tag{3}$$

$$V_x = V_o + \frac{M_r - M_\ell}{\ell_n} \tag{4}$$

$$\Delta_x = \Delta_o + \frac{x(\ell_n - x)}{6 E I_n \ell_n} \left[M_\ell (2\ell_n - x) + M_r(\ell_n + x)\right] \tag{5}$$

A general equation for reaction influence lines is also developed in Appendix B, and given in Equation 6. R_n denotes that Equation 6 is used to compute influence line ordinates for a reaction influence line at support n.

$$R_n = {}_nB_o + {}_{n+1}A_o + M_{n-1}\left(\frac{1}{\ell_n}\right) - M_n\left(\frac{1}{\ell_n} + \frac{1}{\ell_{n+1}}\right) + M_{n+1}\left(\frac{1}{\ell_{n+1}}\right) \tag{6}$$

CHAPTER III

COMPUTER PROGRAM

Hardware and Software

The computer program is written in VAX FORTRAN programming language and uses the VAX/VMS operating system, version V4.1 [8]. The program currently operates on a DEC VAX 11/780 minicomputer, and uses a TEKTRONIX 4027 color graphics terminal. A software package called DISSPLA [11] was also implemented to take advantage of the TEKTRONIX 4027 graphics capabilities.

General Characteristics

The computer program in Appendix C was developed with two basic philosophies in mind: (1) to be user-friendly and (2) to be easy to modify. Thus, the program is menu-driven and the user can proceed throughout the program by responding with appropriate alphanumeric or numeric input. All user input is checked for correctness to eliminate problems arising from non-appropriate responses. The computer code contains several comment statements for increasing the readability of the program.

Upon actual running of the program, a main menu as shown in Figure 4 is displayed. Each topic in the main menu is independent of the others and has its own execution menu. Each main menu topic has a unique educational purpose, but all topics should be explored for a complete understanding of influence lines. The content of each main menu topic will now be presented.

MAIN MENU
==== ====

(A) INTRODUCTION
(B) DEVELOPMENT OF INFLUENCE LINES
(C) APPLICATION OF INFLUENCE LINES
(D) END OF PROGRAM

PLEASE SELECT ONE OF THE ABOVE: _

Figure 4. Computer program main menu.

Introduction

The introduction segment of the program is intended to acquaint the user with all information necessary for program execution. This segment also serves as an introduction to concepts used later in the development and application segments. As shown in Figure 5, the user can choose from a variety of sub-topics. Each sub-topic contributes to a total understanding of program execution and continuous beam influence lines.

INTRODUCTION MENU
============ ====

(A) Continuous presentation of all introductory material
(B) Program purpose
(C) Graphical definition of a continuous beam
(D) Graphic examples of influence lines
(E) Presentation of theory
(F) Limitations
(G) Nomenclature
(H) Return to Main Menu

PLEASE SELECT ONE OF THE ABOVE: _

Figure 5. Computer program introduction menu.

Development of Influence Lines

The purpose of this segment is two-fold in nature: (1) present to the user a development of influence line equations from fundamental theorems and (2) allow the user to apply the influence line equations and develop any particular influence line. Sub-topics

(A) and (B) in Figure 6 present the development of influence line equations for the user. Sub-topic (C) in Figure 6 is interactive, and allows the user to specify data for a 2, 3, or 4-span continuous beam. Sub-topic (D) in Figure 6 is also interactive and the user must correctly answer questions dealing with variables in the influence line equations. Any incorrect user input results in a repeat of the previous question. An example problem using sub-topics (C) and (D) is presented in Chapter IV.

```
DEVELOPMENT MENU
=========== ====

(A) Concept of influence lines
(B) Influence line equations
(C) Beam physical data input
(D) Computation of influence line ordinates
(E) Return to Main Menu

PLEASE SELECT ONE OF THE ABOVE: _
```

Figure 6. Computer program development menu.

Application of Influence Lines

The application segment of the program allows the user to define a particular continuous beam problem and use his knowledge of influence lines to determine maximum internal forces and deflections. Through an educated trial and error procedure, the user can determine which dead load and live load combination produces worst cases.

Figure 7 shows sub-topics available to the user in the application segment. Sub-topic (A) in Figure 7 allows user definition of a specific continuous beam. Sub-topic (B) in Figure 7 allows viewing of any influence line for the beam specified in sub-topic (A). Sub-topic (C) in Figure 7 allows the user to specify a variety of dead and live loads for the beam specified in sub-topic (A). Sub-topic (D) in Figure 7 allows the user to define or re-define the beam loads, and view moment, shear, or deflection curves for the beam. An example problem using the application segment is presented in Chapter IV.

APPLICATION MENU
================

(A) Beam physical data input
(B) Display of influence lines
(C) Beam load data input
(D) Application of DL & LL to influence lines
(E) Return to Main Menu

PLEASE SELECT ONE OF THE ABOVE: _

Figure 7. Computer program application menu.

Limitations

Graphical aesthetics and scope of work have resulted in imposing several limitations on types of continuous beams and loads. These limitations are provided for the user in the introduction segment of the computer program.

Computation of all influence lines and their respective ordinates is restricted to span tenth-points. Figure 8 shows the tenth-point numbering system used throughout the computer program. In addition, all moment envelope, shear envelope, and deflection curve ordinates are computed at span tenth-points.

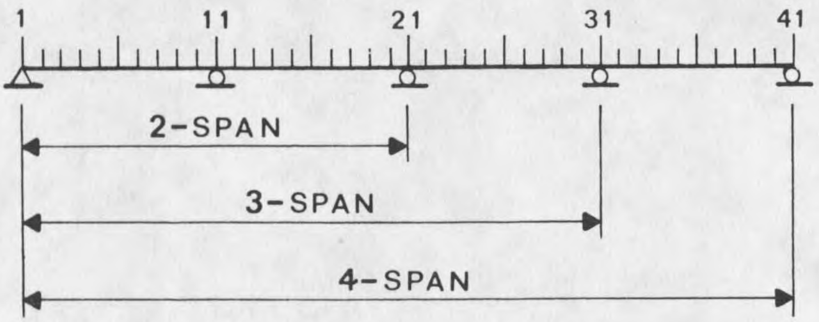


Figure 8. Span tenth-points.

CHAPTER IV

COMPUTER PROGRAM EXAMPLES

The following example problems exhibit the computer program capabilities within the development and application segments. All questions, prompts for user input, and graphical displays are as they actually appear during program execution. Unfortunately, the examples do not duplicate the color graphics and exact screen displays of the program. All user input in the examples has been double-underlined. The continuous beam shown in Figure 9 will serve as the model for both examples.

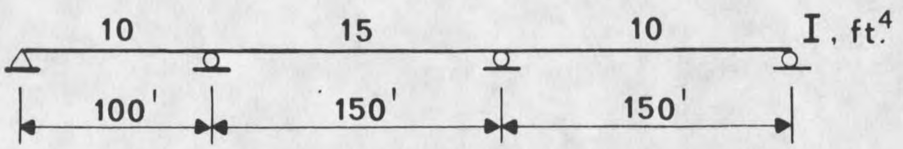


Figure 9. Application examples beam.

Development Example

The information provided in Figures 10, 11, and 12 indicates the general procedure for using the computer program to compute influence line ordinates. The shear equation in Figure 11 was developed from Equation 4. Development of Equation 4 is given in sub-topic (B) from Figure 6, and the user should view the shear equation derivation before using it to compute influence line ordinates. A hard copy of influence line ordinates requested in Figure 12 is given in Table 1.

MAIN MENU
==== ====

(A) INTRODUCTION
(B) DEVELOPMENT OF INFLUENCE LINES
(C) APPLICATION OF INFLUENCE LINES
(D) END OF PROGRAM

PLEASE SELECT ONE OF THE ABOVE: B

DEVELOPMENT MENU
=========== ====

(A) Concept of influence lines
(B) Influence line equations
(C) Beam physical data input
(D) Computation of influence line ordinates
(E) Return to Main Menu

PLEASE SELECT ONE OF THE ABOVE: C

BEAM PHYSICAL DATA INPUT

Specify number of spans (2, 3, or 4): 3

SUPPORT COORDINATE INPUT
^^^^^^^ ^^^^^^^^^^ ^^^^^

Support No. 1 coordinates are (0.0, 0.0)

Specify x, y for Support No. 2 (ft): 100.0, 0.0
Specify x, y for Support No. 3 (ft): 250.0, 0.0
Specify x, y for Support No. 4 (ft): 400.0, 0.0

MOMENT OF INERTIA INPUT
^^^^^^ ^^ ^^^^^^^ ^^^^^

Moment of Inertia for Span No. 1 (ft.^4): 10.0
Moment of Inertia for Span No. 2 (ft.^4): 15.0
Moment of Inertia for Span No. 3 (ft.^4): 10.0

Modulus of Elasticity, E (ksi): 29000.

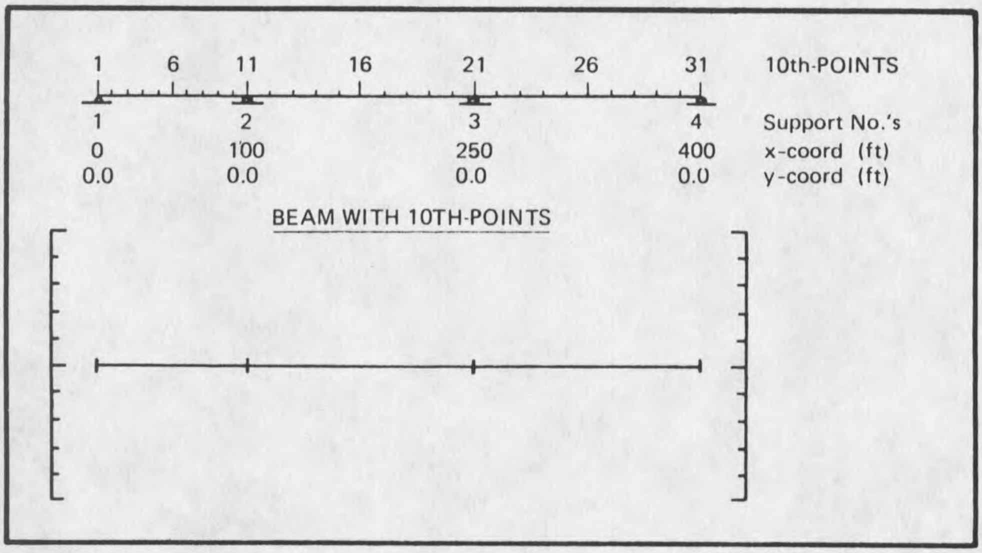
BEAM PHYSICAL DATA O.K. (Y/N): Y

DEVELOPMENT MENU
=========== ====

(A) Concept of influence lines
(B) Influence line equations
(C) Beam physical data input
(D) Computation of influence line ordinates
(E) Return to Main Menu

PLEASE SELECT ONE OF THE ABOVE: D

Figure 10. Development of influence lines, Phase I.



COMPUTATION MENU

(A) Support Moment I.L.
(B) Span Moment I.L.
(C) Shear I.L.
(D) Reaction I.L.
(E) Deflection I.L.
(F) Return to Development Menu

PLEASE SELECT ONE OF THE ABOVE: C

SHEAR I.L. ORDINATES

Please specify 10th-point for which the influence line will be developed: 18

Would you like to review the equations used for computing the ordinates (Y/N)? Y

SHEAR EQUATION

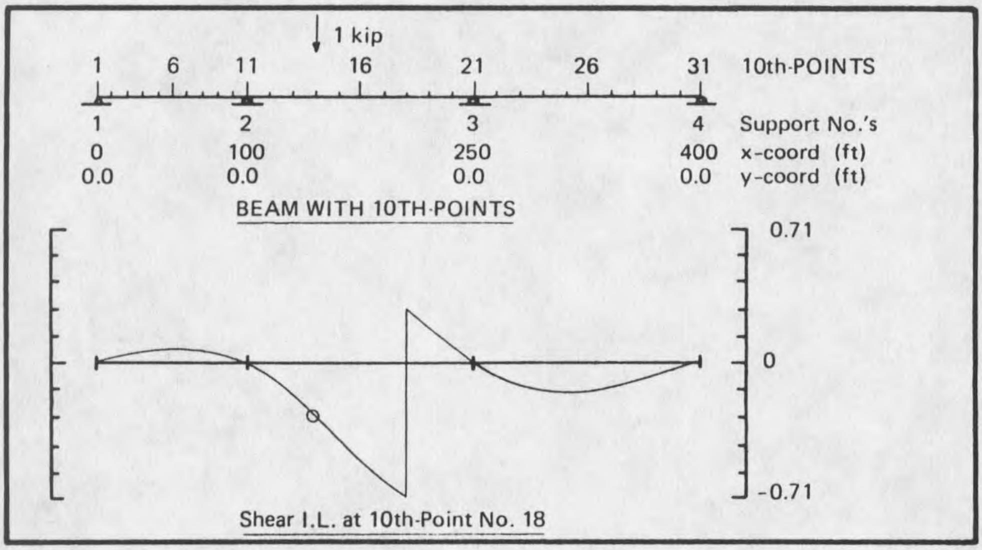
SPAN No. 2 $$V_x = V_o + \frac{M_3 - M_2}{\ell_2}$$

The "ℓ" term is independent of the unit load position and need be input only once.

Length of Span No. 2, ℓ (ft): 150.0

Specify 10th-point position of unit load: 14

Figure 11. Development of influence lines, Phase II.



With a unit load at 10th-point 14, the moments at the interior supports are:

Moment at Support No. 2 = -11.94 ft-kips
Moment at Support No. 3 = -5.80 ft-kips

Using a hand calculator and recalling equations given earlier for V_o, the student must now compute and enter the value for V_o.

$$V_o = \underline{\underline{-0.30}}$$

All terms in the equation(s) have now been defined, and solution for the I.L. ordinate is:

V_x = -0.26 kips at 10th-Point No. 14

Note that the influence line ordinate has been plotted on the beam above.

Do you want to compute another ordinate (Y/N)? N

Plot the total influence line (Y/N)? Y

Hard copy of influence line ordinates (Y/N)? Y

Computation Menu

(A) Support Moment I.L.
(B) Span Moment I.L.
(C) Shear I.L.
(D) Reaction I.L.
(E) Deflection I.L.
(F) Return to Development Menu

PLEASE SELECT ONE OF THE ABOVE: F

Figure 12. Development of influence lines, Phase III.

Table 1. Shear Influence Line Ordinates.

Type: Shear

Location: 10th-Point No. 18

SPAN	10th-POINT	I.L. ORDINATE
1		
	1	0.000
	2	0.021
	3	0.040
	4	0.057
	5	0.071
	6	0.079
	7	0.081
	8	0.075
	9	0.061
	10	0.036
	11	0.000
2		
	11	0.000
	12	-0.072
	13	-0.160
	14	-0.259
	15	-0.367
	16	-0.480
	17	-0.595
	18	-0.708
	18	0.292
	19	0.185
	20	0.086
	21	0.000
3		
	21	0.000
	22	-0.068
	23	-0.114
	24	-0.141
	25	-0.152
	26	-0.148
	27	-0.133
	28	-0.108
	29	-0.076
	30	-0.039
	31	0.000

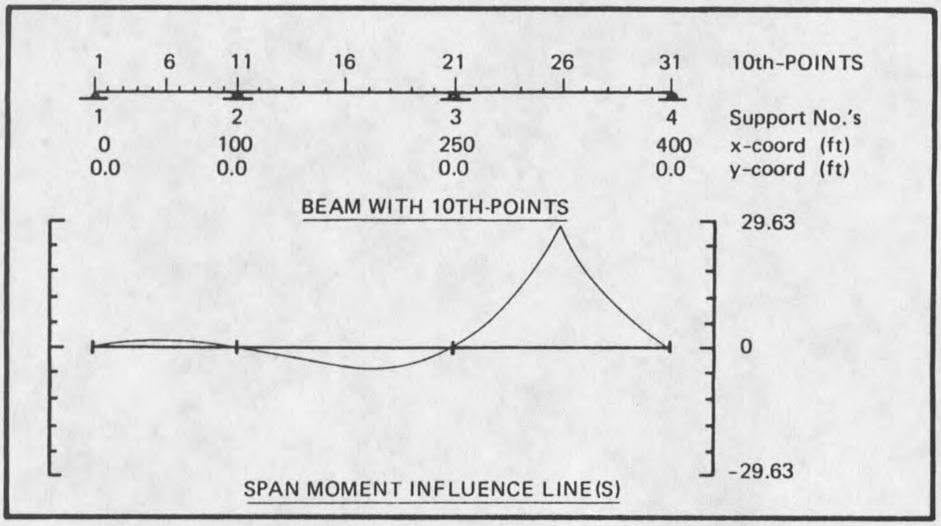
Application Example

Figure 13 provides the general procedure for viewing any type of influence line. Because the development and application segments of the computer program are independent of each other, the user must first specify beam physical data, as in Figure 10, before displaying any influence lines. A hard copy of influence line ordinates requested in Figure 13 is given in Table 2.

By viewing a variety of influence lines, the user can determine which dead load and live load combination will produce maximum moments, shears, or deflections. An example of specifying and revising beam DL and LL is provided in Figures 14, 15, and 16. Hard copies of data requested in Figures 15 and 16 is given in Tables 3, 4, 5, and 6.

Computer Analysis

Ordinates for moments, shears, reactions, and deflections were computed by directly applying dead and live loads to computed influence lines. As a result, the span tenth-point values for internal forces, reactions, or deflections may deviate slightly from exact values. Although an independent check was used to verify computer results, no guarantee is given that the method of analysis will work for all combinations of beam geometry and applied loadings.



MAIN MENU
==== ====

(A) INTRODUCTION
(B) DEVELOPMENT
(C) APPLICATION OF INFLUENCE LINES
(D) END OF PROGRAM

PLEASE SELECT ONE OF THE ABOVE: C

APPLICATION MENU
=========== ====

(A) Beam physical data input
(B) Display of influence lines
(C) Beam load data input
(D) Application of DL & LL to influence lines
(E) Return to Main Menu

PLEASE SELECT ONE OF THE ABOVE: B

INFLUENCE LINE MENU

(A) Support Moment I.L.
(B) Span Moment I.L.
(C) Shear I.L.
(D) Reaction I.L.
(E) Deflection I.L.
(F) Return to Application Menu

PLEASE SELECT ONE OF THE ABOVE: B

Specify 10th-point from above: 26

Hard copy of influence line ordinates (Y/N)? Y

Press "D" to display another influence line or "R" to return to Influence Line Menu . . . R

INFLUENCE LINE MENU

(A) Support Moment I.L.
(B) Span Moment I.L.
(C) Shear I.L.
(D) Reaction I.L.
(E) Deflection I.L.
(F) Return to Application Menu

PLEASE SELECT ONE OF THE ABOVE: F

Figure 13. Application of influence lines, Phase I.

Table 2. Span Moment Influence Line Ordinates.

Type: Span Moment

Location: 10th-Point No. 26

SPAN	10th-POINT	I.L. ORDINATE
1		
	1	0.000
	2	0.261
	3	0.505
	4	0.718
	5	0.884
	6	0.987
	7	1.011
	8	0.939
	9	0.758
	10	0.450
	11	0.000
2		
	11	0.000
	12	-0.888
	13	-1.895
	14	-2.901
	15	-3.789
	16	-4.441
	17	-4.737
	18	-4.559
	19	-3.789
	20	-2.309
	21	0.000
3		
	21	0.000
	22	3.450
	23	8.179
	24	14.045
	25	20.905
	26	28.618
	27	22.042
	28	16.034
	29	10.453
	30	5.155
	31	0.000

APPLICATION MENU
=========== ====

(A) Beam physical data input
(B) Display of influence lines
(C) Beam load data input
(D) Application of DL & LL to influence lines
(E) Return to Main Menu

PLEASE SELECT ONE OF THE ABOVE: C

BEAM LOAD DATA INPUT

The student can now apply dead loads and live loads to the beam specified in part (A). For purposes of visualizing the effects of loads on a continuous beam, the application of one dead load, one live load, and a combination of DL & LL is sufficient.

DEAD LOAD INPUT

Dead loads will be assumed as being uniformly distributed and constant for all spans.

Specify uniform DL (k/ft): 1.00

LIVE LOAD INPUT

Although a continuous beam may experience several types of live loads, it is possible to illustrate live load effects on continuous beams by using either uniformly distributed loads or AASHTO truck loads.

Live load will be: (A) zero
(B) uniformly distributed
(C) AASHTO truck load

PLEASE SELECT ONE OF THE ABOVE: B

Specify uniform LL (k/ft): 0.50

APPLICATION MENU
=========== ====

(A) Beam physical data input
(B) Display of influence lines
(C) Beam load data input
(D) Application of DL & LL to influence lines
(E) Return to Main Menu

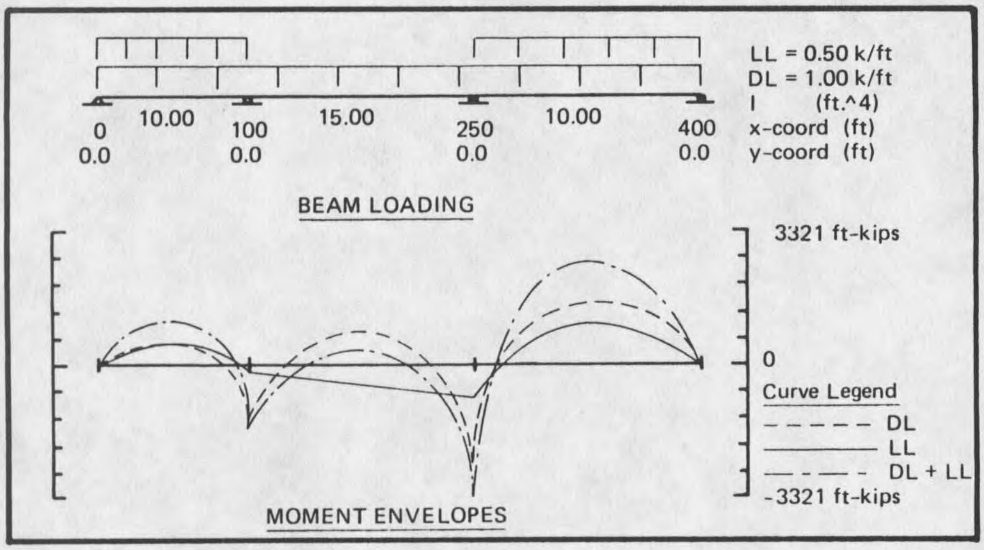
PLEASE SELECT ONE OF THE ABOVE: D

APPLICATIONS OF DL & LL TO INFLUENCE LINES

You have specified a uniformly distributed LL of 0.50 (k/ft) to be applied to various spans of the beam. Please specify which spans the LL will occupy:

LL will occupy Span No. 1 (Y/N)? Y
LL will occupy Span No. 2 (Y/N)? N
LL will occupy Span No. 3 (Y/N)? Y

Figure 14. Application of influence lines, Phase II



APPLICATION OPTIONS

(A) Display moment envelopes, shear envelopes, or deflections
(B) Revise DL or LL
(C) Return to Application Menu

PLEASE SELECT ONE OF THE ABOVE: A

DISPLAY OPTIONS

(A) Display moment envelopes (C) Display deflection curves
(B) Display shear envelopes (D) Return to Application Options

PLEASE SELECT ONE OF THE ABOVE: A

Press "C" to continue . . . C

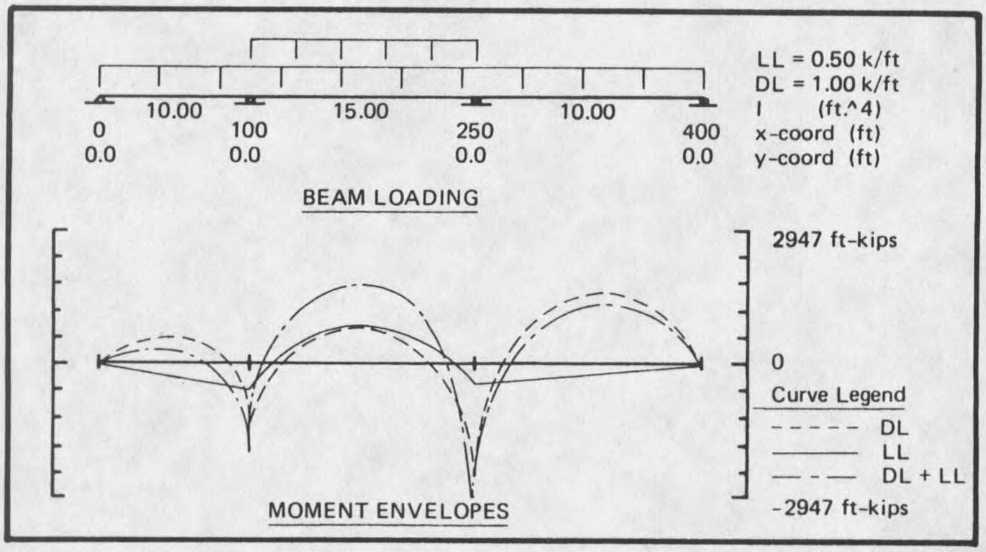
DISPLAY OPTIONS

(A) Display moment envelopes (C) Display deflection curves
(B) Display shear envelopes (D) Return to Application Options

PLEASE SELECT ONE OF THE ABOVE: D

Would you like a hard copy of all envelope ordinates and reactions for this beam (Y/N)? Y

Figure 15. Application of influence lines, Phase III.



APPLICATION OPTIONS

(A) Display moment envelopes, shear envelopes, or deflections
(B) Revise DL or LL
(C) Return to Application Menu

PLEASE SELECT ONE OF THE ABOVE: B

DL O.K. (Y/N)? Y

LL O.K. (Y/N)? N

Revise uniform LL to AASHTO truck LL (Y/N)? N

Revise uniform LL magnitude (Y/N)? N

Revise spans occupied by uniform LL (Y/N)? Y

LL will occupy Span No. 1 (Y/N)? N
LL will occupy Span No. 2 (Y/N)? Y
LL will occupy Span No. 3 (Y/N)? N

APPLICATION OPTIONS

(A) Display moment envelopes, shear envelopes, or deflections
(B) Revise DL or LL
(C) Return to Application Menu

PLEASE SELECT ONE OF THE ABOVE: A

DISPLAY OPTIONS

(A) Display moment envelopes (C) Display deflection curves
(B) Display shear envelopes (D) Return to Application Options

PLEASE SELECT ONE OF THE ABOVE: A

Figure 16. Application of influence lines, Phase IV.

Table 3. Data for Beam Loading in Figure 15.

>>>>> BEAM LOAD DATA <<<<<

SPAN	DL	LL	
		UNIFORM	AASHTO TRUCK
	(k/ft)	(k/ft)	(TYPE : DOT : DLS : RAS)**
1	1.00	0.50	---
2	1.00	0.00	---
3	1.00	0.50	---

** TYPE: H10-44, H15-44, H20-44, HS15-44, OR HS20-44

DOT : direction of truck travel

DLS : distance to front wheels from Support No. 1, feet

RAS : rear axle spacing, feet (HS15-44 & HS20-44 only)

>>>>> SUPPORT REACTIONS (kips) <<<<<

SUPPORT	DL	LL	DL+LL
1	36.16	23.94	60.10
2	131.35	21.34	152.69
3	174.21	47.65	221.86
4	58.28	32.07	90.36

>>>>> DL ENVELOPE ORDINATES <<<<<

SPAN	10th-POINT	MOMENT (ft-kip)	SHEAR (kip)	DEFLECTION++ (ft)
1				
	1	0.00	36.16	0.000000
	2	311.60	26.16	0.004230
	3	523.19	16.16	0.007753
	4	634.79	6.16	0.010064
	5	646.38	-3.84	0.010894
	6	557.98	-13.84	0.010216
	7	369.57	-23.84	0.008243
	8	81.17	-33.84	0.005424
	9	-307.24	-43.84	0.002450
	10	-795.64	-53.84	0.000253
	11	-1384.05	-63.84	0.000000
2				
	11	-1384.05	67.51	0.000000
	12	-483.90	52.51	0.003701
	13	191.25	37.51	0.009276
	14	641.40	22.51	0.014298
	15	866.55	7.51	0.017151
	16	866.69	-7.49	0.017026
	17	641.84	-22.49	0.013922
	18	191.99	-37.49	0.008648
	19	-482.86	-52.49	0.002819
	20	-1382.71	-67.49	-0.001141
	21	-2507.57	-82.49	0.000000
3				
	21	-2507.57	91.72	0.000000
	22	-1244.31	76.72	0.010592
	23	-206.05	61.72	0.028091
	24	607.20	46.72	0.046902
	25	1195.46	31.72	0.062643
	26	1558.72	16.72	0.072146
	27	1696.97	1.72	0.073452
	28	1610.23	-13.28	0.065817
	29	1298.49	-28.28	0.049708
	30	761.74	-43.28	0.026805
	31	0.00	-58.28	0.000000

++ Deflection is (+) measured downward.

Table 4. Data for Beam Loading in Figure 15.

>>>>> LL ENVELOPE ORDINATES <<<<<

SPAN	10th-POINT	MOMENT (ft-kip)	SHEAR (kip)	DEFLECTION++ (ft)
1				
	1	0.00	23.94	0.000000
	2	214.42	18.94	0.004431
	3	378.83	13.94	0.008368
	4	493.25	8.94	0.011419
	5	557.66	3.94	0.013308
	6	572.08	-1.06	0.013881
	7	536.50	-6.06	0.013105
	8	450.91	-11.06	0.011064
	9	315.33	-16.06	0.007963
	10	129.75	-21.06	0.004127
	11	-105.84	-26.06	0.000000
2				
	11	-105.84	-4.72	0.000000
	12	-176.67	-4.72	-0.005909
	13	-247.50	-4.72	-0.011183
	14	-318.33	-4.72	-0.015568
	15	-389.16	-4.72	-0.018810
	16	-459.99	-4.72	-0.020653
	17	-530.82	-4.72	-0.020845
	18	-601.65	-4.72	-0.019130
	19	-672.48	-4.72	-0.015254
	20	-743.31	-4.72	-0.008962
	21	-814.14	-4.72	0.000000
3				
	21	-814.14	42.93	0.000000
	22	-226.48	35.43	0.012047
	23	248.68	27.93	0.025415
	24	611.35	20.43	0.037545
	25	861.51	12.93	0.046482
	26	999.18	5.43	0.050877
	27	1024.34	-2.07	0.049991
	28	937.01	-9.57	0.043686
	29	737.17	-17.07	0.032434
	30	424.84	-24.57	0.017311
	31	0.00	-32.07	0.000000

++ Deflection is (+) measured downward.

>>>>> DL+LL ENVELOPE ORDINATES <<<<<

SPAN	10th-POINT	MOMENT (ft-kip)	SHEAR (kip)	DEFLECTION++ (ft)
1				
	1	0.00	60.10	0.000000
	2	526.01	45.10	0.008661
	3	902.02	30.10	0.016122
	4	1128.03	15.10	0.021482
	5	1204.05	0.10	0.024202
	6	1130.06	-14.90	0.024098
	7	906.07	-29.90	0.021348
	8	532.08	-44.90	0.016488
	9	8.09	-59.90	0.010413
	10	-665.90	-74.90	0.004379
	11	-1489.88	-89.90	0.000000
2				
	11	-1489.88	62.79	0.000000
	12	-660.57	47.79	-0.002207
	13	-56.25	32.79	-0.001907
	14	323.07	17.79	-0.001270
	15	477.38	2.79	-0.001659
	16	406.70	-12.21	-0.003628
	17	111.02	-27.21	-0.006923
	18	-409.66	-42.21	-0.010482
	19	-1155.35	-57.21	-0.012435
	20	-2126.03	-72.21	-0.010103
	21	-3321.71	-87.21	0.000000
3				
	21	-3321.71	134.64	0.000000
	22	-1470.79	112.14	0.022639
	23	42.63	89.64	0.053506
	24	1218.55	67.14	0.084447
	25	2056.97	44.64	0.109125
	26	2557.89	22.14	0.123023
	27	2721.32	-0.36	0.123442
	28	2547.24	-22.86	0.109503
	29	2035.66	-45.36	0.082142
	30	1186.58	-67.86	0.044116
	31	0.00	-90.36	0.000000

++ Deflection is (+) measured downward.

Table 5. Data for Beam Loading in Figure 16.

>>>>> BEAM LOAD DATA <<<<<

SPAN	DL (k/ft)	LL UNIFORM (k/ft)	LL AASHTO TRUCK (TYPE : DOT : DLS : RAS)**
1	1.00	0.00	---
2	1.00	0.50	---
3	1.00	0.00	---

** TYPE: H10-44, H15-44, H20-44, HS15-44, OR HS20-44

DOT : direction of truck travel

DLS : distance to front wheels from Support No. 1, feet

RAS : rear axle spacing, feet (HS15-44 & HS20-44 only)

>>>>> SUPPORT REACTIONS (kips) <<<<<

SUPPORT	DL	LL	DL+LL
1	36.16	-5.86	30.30
2	131.35	44.34	175.69
3	174.21	39.45	213.66
4	58.28	-2.93	55.35

>>>>> DL ENVELOPE ORDINATES <<<<<

SPAN	10th-POINT	MOMENT (ft-kip)	SHEAR (kip)	DEFLECTION++ (ft)
1				
	1	0.00	36.16	0.000000
	2	311.60	26.16	0.004230
	3	523.19	16.16	0.007753
	4	634.79	6.16	0.010064
	5	646.38	-3.84	0.010894
	6	557.98	-13.84	0.010216
	7	369.57	-23.84	0.008243
	8	81.17	-33.84	0.005424
	9	-307.24	-43.84	0.002450
	10	-795.64	-53.84	0.000253
	11	-1384.05	-63.84	0.000000
2				
	11	-1384.05	67.51	0.000000
	12	-483.90	52.51	0.003701
	13	191.25	37.51	0.009276
	14	641.40	22.51	0.014298
	15	866.55	7.51	0.017151
	16	866.69	-7.49	0.017026
	17	641.84	-22.49	0.013922
	18	191.99	-37.49	0.008648
	19	-482.86	-52.49	0.002819
	20	-1382.71	-67.49	-0.001141
	21	-2507.57	-82.49	0.000000
3				
	21	-2507.57	91.72	0.000000
	22	-1244.31	76.72	0.010592
	23	-206.05	61.72	0.028091
	24	607.20	46.72	0.046902
	25	1195.46	31.72	0.062643
	26	1558.72	16.72	0.072146
	27	1696.97	1.72	0.073452
	28	1610.23	-13.28	0.065817
	29	1298.49	-28.28	0.049708
	30	761.74	-43.28	0.026805
	31	0.00	-58.28	0.000000

++ Deflection is (+) measured downward.

Table 6. Data for Beam Loading in Figure 16.

>>>>> LL ENVELOPE ORDINATES <<<<<

SPAN	10th-POINT	MOMENT (ft-kip)	SHEAR (kip)	DEFLECTION++ (ft)
1				
	1	0.00	-5.86	0.000000
	2	-58.62	-5.86	-0.002316
	3	-117.24	-5.86	-0.004492
	4	-175.86	-5.86	-0.006387
	5	-234.47	-5.86	-0.007861
	6	-293.09	-5.86	-0.008773
	7	-351.71	-5.86	-0.008984
	8	-410.33	-5.86	-0.008352
	9	-468.95	-5.86	-0.006738
	10	-527.57	-5.86	-0.004001
	11	-586.18	-5.86	0.000000
2				
	11	-586.18	38.48	0.000000
	12	-65.28	30.98	0.007759
	13	343.13	23.48	0.015821
	14	639.03	15.98	0.022717
	15	822.43	8.48	0.027385
	16	893.34	0.98	0.029166
	17	851.74	-6.52	0.027806
	18	697.65	-14.02	0.023454
	19	431.05	-21.52	0.016663
	20	51.96	-29.02	0.008391
	21	-439.64	-36.52	0.000000
3				
	21	-439.64	2.93	0.000000
	22	-395.67	2.93	-0.006751
	23	-351.71	2.93	-0.011370
	24	-307.75	2.93	-0.014094
	25	-263.78	2.93	-0.015160
	26	-219.82	2.93	-0.014805
	27	-175.86	2.93	-0.013265
	28	-131.89	2.93	-0.010778
	29	-87.93	2.93	-0.007580
	30	-43.96	2.93	-0.003908
	31	0.00	2.93	0.000000

++ Deflection is (+) measured downward.

>>>>> DL+LL ENVELOPE ORDINATES <<<<<

SPAN	10th-POINT	MOMENT (ft-kip)	SHEAR (kip)	DEFLECTION++ (ft)
1				
	1	0.00	30.30	0.000000
	2	252.98	20.30	0.001914
	3	405.95	10.30	0.003261
	4	458.93	0.30	0.003677
	5	411.91	-9.70	0.003033
	6	264.88	-19.70	0.001443
	7	17.86	-29.70	-0.000741
	8	-329.16	-39.70	-0.002928
	9	-776.18	-49.70	-0.004287
	10	-1323.21	-59.70	-0.003748
	11	-1970.23	-69.70	0.000000
2				
	11	-1970.23	105.99	0.000000
	12	-549.18	83.49	0.011461
	13	534.38	60.99	0.025096
	14	1280.43	38.49	0.037015
	15	1688.98	15.99	0.044536
	16	1760.03	-6.51	0.046192
	17	1493.59	-29.01	0.041728
	18	889.64	-51.51	0.032102
	19	-51.81	-74.01	0.019482
	20	-1330.76	-96.51	0.007250
	21	-2947.20	-119.01	0.000000
3				
	21	-2947.20	94.65	0.000000
	22	-1639.98	79.65	0.003841
	23	-557.76	64.65	0.016721
	24	299.46	49.65	0.032808
	25	931.68	34.65	0.047483
	26	1338.90	19.65	0.057341
	27	1521.12	4.65	0.060187
	28	1478.34	-10.35	0.05503
	29	1210.56	-25.35	0.042128
	30	717.78	-40.35	0.022897
	31	0.00	-55.35	0.000000

++ Deflection is (+) measured downward.

CHAPTER V

SUMMARY

The trend toward using general purpose analysis computer programs for educating civil engineers can create a false sense of student understanding of fundamental engineering principles. Education of fundamental principles cannot be neglected if the integrity of the engineering profession is to be maintained.

A computer program has been developed for use as an aid in teaching the fundamental concept of influence lines for continuous beams. The speed of the computer and the user-friendly nature of the color graphics computer program follows Albert Einstein's view of teaching: ". . . Teaching should be such that what is offered is perceived as a valuable gift, and not as a hard duty" [20].

It is not possible for a student to understand the total influence line concept from use of this computer program alone. However, it does provide a good supplement to material presented in the classroom. The capability of presenting and understanding fundamental principles for all types of continuous beams is beyond the scope of the computer program and this paper. Benefits derived from use of this program would serve as an interesting topic for future research.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. American Association of State Highway and Transportation Officials, *Standard Specifications for Highway Bridges,* 13th-Ed., Washington, D.C., 1983.

2. American Institute of Steel Construction, Inc., *Manual of Steel Construction,* 8th-Ed., Chicago, Illinois, 1980.

3. Anger, Dr. -Ing. Georg, *Ten-Division Influence Lines for Continuous Beams,* 8th-Ed., New York: Frederick Ungar Publishing Co., 1956.

4. Anger, Dr. -Ing. Georg, and Tramm, Karl, *Deflection Ordinates for Single-Span and Continuous Beams,* New York: Frederick Ungar Publishing Co., 1965.

5. Chalabi, A. Fattah, "An Interactive Software System for Competency and Project Oriented Civil Engineering Education," *Proceedings of the Second Conference on Computing in Civil Engineering,* New York: American Society of Civil Engineers, 1980, pp. 406-417.

6. Charlton, T. M., "Beam Systems," In *A History of Theory of Structures in the Nineteenth Century,* Chapter 2, Cambridge, Great Britain: Cambridge University Press, 1982, pp. 14-34.

7. Craig, Robert John, "Computers in Civil Engineering Undergraduate Education," *Proceedings of the Second Conference on Computing in Civil Engineering,* New York: American Society of Civil Engineers, 1980, pp. 419-423.

8. Digital Equipment Corporation, *Programming in VAX FORTRAN,* Version 4.0, Northboro, Massachusetts, September, 1984.

9. Dill, John C., "Computer Graphics and Computer-Aided Design at Cornell's College of Engineering," *Proceedings of the National Conference on University Programs in Computer-Aided Engineering, Design, and Manufacturing,* Provo, Utah: BYU University Press, 1983, pp. 18-24.

10. Gerald, Curtis F., *Applied Numerical Analysis,* Reading, Massachusetts: Addison-Wesley Publishing Co., 1970, pp. 162-166.

11. Integrated Software Systems Corporation, *DISSPLA Users Manual,* Version 10.0, San Diego, California, 1985.

12. Kamel, Hussein A., "Interactive Graphics as a Tool in Teaching Structural Analysis," *Journal of the Structural Division,* Proceedings of the American Society of Civil Engineers, Vol. 104, No. ST8, August, 1978, pp. 1299-1314.

13. Meyer, Christian, "User-Oriented Programming," *Proceedings of the Second Conference on Computing in Civil Engineering,* New York: American Society of Civil Engineers, 1980, pp. 58-66.

14. Rogers, David F., Chalmers, David, and Richardson, J. D., "Interactive Graphics and the Uniform Beam in Engineering Education," *Proceedings of the Second Conference on Computing in Civil Engineering,* New York: American Society of Civil Engineers, 1980, pp. 436-457.

15. Scardina, J. J., Nopratrarakorn, V., and Buchanan, G. R., "Computer Graphics as a Teaching Aid," *Proceedings of the Second Conference on Computing in Civil Engineering,* New York: American Society of Civil Engineers, 1980, pp. 430-435.

16. Tektronix, Inc., *4027 Color Graphics Terminal-Operators Manual,* Beaverton, Oregon, March, 1979.

17. Tektronix, Inc., *4027 Preliminary Programmers Reference Guide,* Beaverton, Oregon, June, 1978.

18. Timoshenko, S. P., and Young, D. H., "BEAMS AND FRAMES," In *Theory of Structures,* Chapter VIII, New York: McGraw-Hill, 1945, pp. 332-355.

19. Uicker, John J., and Bollinger, John G., "Computer-Aided Engineering at the University of Wisconsin-Madison," *Proceedings of the National Conference on University Programs in Computer-Aided Engineering, Design, and Manufacturing,* Provo, Utah: BYU University Press, 1983, pp. 49-52.

20. Vild, Kathlene A., "The Civil Engineering Degree: Education or Training?", *Journal of Professional Issues in Engineering,* Vol. 110, No. 1, January, 1984, pp. 25-30.

21. Yener, Muzaffer, and Ting, Edward C., "Integrating Fundamentals and Computer Usage," *Journal of Professional Issues in Engineering,* Vol. 110, No. 1, January, 1984, pp. 31-36.

APPENDICES

APPENDIX A

THREE-MOMENT EQUATION

Two adjacent spans, n and n+1, of a transversely loaded continuous beam are shown in Figure 17. Free-body diagrams of spans n and n+1 are shown in Figure 18.

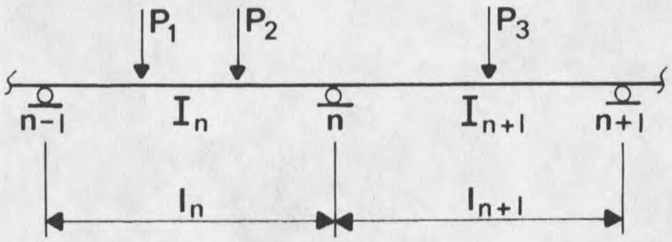


Figure 17. Two adjacent spans of a continuous beam.

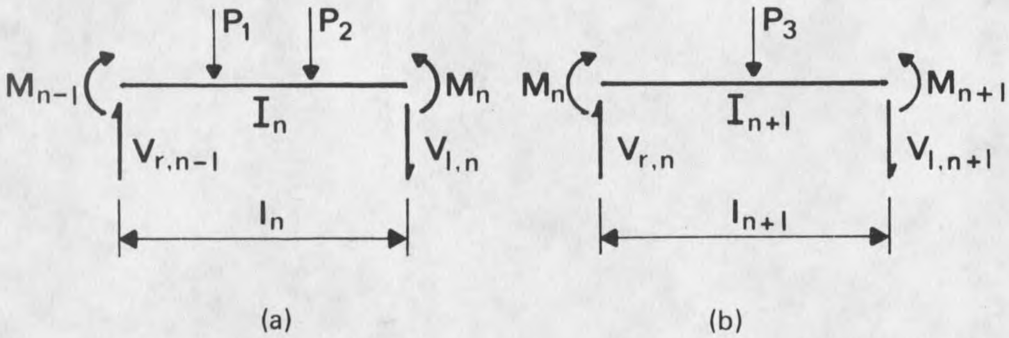


Figure 18. Free-body diagrams, spans n and n+1.

Slope-deflection relationships are applied to the elastic weight diagrams of each span to yield expressions for beam slope at supports n-1, n, and n+1. For span n, the slopes θ_{n-1} and θ_n at supports n-1 and n, respectively, are:

$$\theta_{n-1} = \frac{M_{n-1}\,\ell_n}{3\,E\,I_n} + \frac{A_n\,d_n}{E\,I_n\,\ell_n} + \frac{M_n\,\ell_n}{6\,E\,I_n} \qquad (7)$$

$$\theta_n = \frac{M_{n-1}\,\ell_n}{6\,E\,I_n} + \frac{A_n\,c_n}{E\,I_n\,\ell_n} + \frac{M_n\,\ell_n}{3\,E\,I_n} \qquad (8)$$

For span n+1, the slopes θ'_n and θ_{n+1} at supports n and n+1, respectively, are:

$$\theta'_n = \frac{M_n\,\ell_{n+1}}{3\,E\,I_{n+1}} + \frac{A_{n+1}\,d_{n+1}}{E\,I_{n+1}\,\ell_{n+1}} + \frac{M_{n+1}\,\ell_{n+1}}{6\,E\,I_{n+1}} \qquad (9)$$

$$\theta_{n+1} = \frac{M_n \ell_{n+1}}{6 E I_{n+1}} + \frac{A_{n+1} c_{n+1}}{E I_{n+1} \ell_{n+1}} + \frac{M_{n+1} \ell_{n+1}}{3 E I_{n+1}} \qquad (10)$$

Slope compatability at support n requires:

$$\theta_n = -\theta'_n \qquad (11)$$

and the resulting 3-moment equation for a straight continuous beam is:

$$M_{n-1} \left(\frac{\ell_n}{I_n}\right) + 2M_n \left(\frac{\ell_n}{I_n} + \frac{\ell_{n+1}}{I_{n+1}}\right) + M_{n+1} \left(\frac{\ell_{n+1}}{I_{n+1}}\right) = -6R \qquad (12)$$

where

$$R = \frac{A_n c_n}{I_n \ell_n} + \frac{A_{n+1} d_{n+1}}{I_{n+1} \ell_{n+1}} \qquad (13)$$

and $\frac{A_n c_n}{I_n \ell_n}$ = right reaction of transverse loading elastic weight diagram in span n,

and $\frac{A_{n+1} d_{n+1}}{I_{n+1} \ell_{n+1}}$ = left reaction of transverse loading elastic weight diagram in span n+1.

Equation 12 assumes that the modulus of elasticity, E, is constant for the entire beam and therefore does not appear.

If supports n-1, n, and n+1 in Figure 17 undergo vertical displacements, Δ_{n-1}, Δ_n, and Δ_{n+1}, then the rotations at support n become:

$$\theta_n = \frac{M_{n-1} \ell_n}{6 E I_n} + \frac{A_n c_n}{E I_n \ell_n} + \frac{M_n \ell_n}{3 E I_n} - \frac{\Delta_n - \Delta_{n-1}}{\ell_n} \qquad (14)$$

and

$$\theta'_n = \frac{M_n \ell_{n+1}}{3 E I_{n+1}} + \frac{A_{n+1} d_{n+1}}{E I_{n+1} \ell_{n+1}} + \frac{M_{n+1} \ell_{n+1}}{6 E I_{n+1}} + \frac{\Delta_{n+1} - \Delta_n}{\ell_{n+1}} \qquad (15)$$

Satisfying slope compatability at support n yields the 3-moment equation for a continuous beam with uneven supports:

$$M_{n-1}\left(\frac{\ell_n}{I_n}\right) + 2M_n\left(\frac{\ell_n}{I_n} + \frac{\ell_{n+1}}{I_{n+1}}\right) + M_{n+1}\left(\frac{\ell_{n+1}}{I_{n+1}}\right)$$

$$= -6R + 6E\left(\frac{\Delta_n - \Delta_{n-1}}{\ell_n} - \frac{\Delta_{n+1} - \Delta_n}{\ell_{n+1}}\right) \tag{16}$$

APPENDIX B

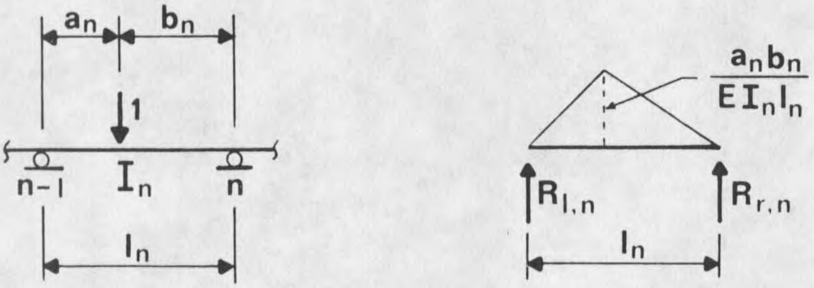
INFLUENCE LINE EQUATIONS

The principal philosophy behind developing influence lines involves positioning a unit load on a structure, and determining internal forces and deflections at all points within the structure. For each unit load position on a continuous beam, moments are generated at the supports. If the support moments are known, each span is statically determinate, and equations can be developed for computing internal forces and deflections at any point within a given span. These equations are then used to compute influence line ordinates for various types of influence lines.

Computation of Support Moments

The 3-moment equation given in Equation 12 is used to determine support moments for a beam subjected to a unit load. A continuous beam having n spans will generate n–1 equations when the 3-moment equation is applied. Moment coefficient terms on the left side of Equation 12 are functions of beam geometry and constant for any given beam.

The R term on the right side of Equation 12 is a function of unit load position and can be defined in terms of unit load position from Figure 19. The left and right reactions



(a) unit load span n (b) elastic weight diagram

Figure 19. Unit load elastic weight diagram.

of the unit load elastic weight diagram are:

$$R_{\ell,n} = \frac{b_n\,(\ell_n^2 - b_n^2)}{6\,E\,I_n\,\ell_n} \qquad R_{r,n} = \frac{a_n\,(\ell_n^2 - a_n^2)}{6\,E\,I_n\,\ell_n}$$

A unit load positioned in span n+1 yields:

$$R_{\ell,n+1} = \frac{b_{n+1}\,(\ell^2_{n+1} - b^2_{n+1})}{6\,E\,I_{n+1}\,\ell_{n+1}} \qquad R_{r,n+1} = \frac{a_{n+1}\,(\ell^2_{n+1} - a^2_{n+1})}{6\,E\,I_{n+1}\,\ell_{n+1}}$$

Therefore, the sum of elastic weight diagram reactions left and right of support n in a continuous beam is:

$$R = R_{r,n} + R_{\ell,n+1} \tag{17}$$

Equation 17 can be substituted directly for R in Equation 12 if E is eliminated from the $R_{r,n}$ and $R_{\ell,n+1}$ expressions.

Thus, for each unit load position on a continuous beam having n spans, a series of n-1 equations are solved simultaneously to yield the interior support moments. Equations can now be developed for internal forces and deflections within each span in terms of the support moments.

Moment Influence Line Equation

Cutting the beam in Figure 3 at the unit load introduces the internal forces and deflection at distance a_n as shown in Figure 20. End-moments M_ℓ and M_r in Figure 3 are the moments at supports n-1 and n resulting from a unit load positioned anywhere on a continuous beam. V_ℓ and V_r in Figure 3 are shears at the ends of span n if the unit load is in span n. The distance x denotes the location of the point at which the influence line is developed, and is measured from the left end of span n.

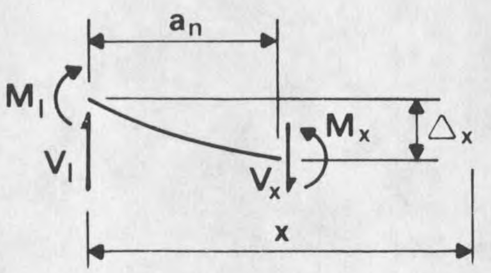


Figure 20. Partial free-body diagram of span n.

Applying static equilibrium to the partial beam in Figure 20 yields the general equation for the moment at a distance a_n:

$$M_x = M_o + M_\ell \left(1 - \frac{x}{\ell_n}\right) + M_r \left(\frac{x}{\ell_n}\right) \qquad (18)$$

where $M_o = 0$ if unit load is not in span n

$$M_o = a_n - x + \frac{b_n x}{\ell_n} \qquad \text{if } a_n \leqslant x$$

$$M_o = \frac{b_n x}{\ell_n} \qquad \text{if } a_n > x$$

Shear Influence Line Equation

Figure 20 is also used to develop a general equation for the shear at a distance a_n. Applying static equilibrium to span n in Figure 20 yields:

$$V_x = V_o + \frac{M_r - M_\ell}{\ell_n} \qquad (19)$$

where $V_o = 0$ if unit load is not in span n

$$V_o = \frac{-a_n}{\ell_n} \qquad \text{if } a_n \leqslant x$$

$$V_o = 1 - \frac{a_n}{\ell_n} \qquad \text{if } a_n > x$$

Reaction Influence Line Equation

Figure 21 shows a free-body diagram of support n in a continuous beam. Application of static equilibrium at support n yields:

$$R_n = V_{r,n} - V_{\ell,n} \qquad (20)$$

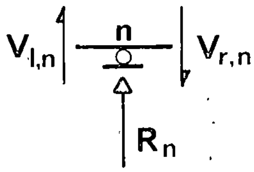


Figure 21. Free-body diagram of support n.

Assuming a unit load is positioned in span n left of support n:

$$V_{\ell,n} = -\frac{a_n}{\ell_n} + \frac{M_n - M_{n-1}}{\ell_n} \tag{21}$$

and assuming a unit load is positioned in span n+1 right of support n:

$$V_{r,n} = 1 - \frac{a_{n+1}}{\ell_{n+1}} + \frac{M_{n+1} - M_n}{\ell_{n+1}} \tag{22}$$

Substitution of Equations 21 and 22 into Equation 20 yields the general equation for the reaction of support n:

$$R_n = {}_nB_o + {}_{n+1}A_o + M_{n-1}\left(\frac{1}{\ell_n}\right) - M_n\left(\frac{1}{\ell_n} + \frac{1}{\ell_{n+1}}\right) + M_{n+1}\left(\frac{1}{\ell_{n+1}}\right) \tag{23}$$

where ${}_nB_o = 0$ if unit load is not in span n

${}_nB_o = \frac{a_n}{\ell_n}$ if unit load is in span n

${}_{n+1}A_o = 0$ if unit load is not in span n+1

${}_{n+1}A_o = 1 - \frac{a_{n+1}}{\ell_{n+1}}$ if unit load is in span n+1

Deflection Influence Line Equation

The free-body diagram in Figure 20 is again used to develop a general equation for the deflection at a distance a_n. Application of equilibrium and slope-deflection relationships to span n in Figure 20 yields:

$$\Delta_x = \Delta_o + \frac{x\,(\ell_n - x)}{6\,E\,I_n\,\ell_n}\,[M_\ell\,(2\,\ell_n - x) + M_r\,(\ell_n + x)] \tag{24}$$

where $\Delta_o = 0$ if unit load is not in span n

$$\Delta_o = \frac{a_n\,(\ell_n - x)}{6\,E\,I_n\,\ell_n}\,[b_n^2 + 2a_n b_n - (\ell_n - x)^2] \qquad \text{if } a_n \leqslant x$$

$$\Delta_o = \frac{b_n\,x}{6\,E\,I_n\,\ell_n}\,[a_n^2 + 2a_n b_n - x^2] \qquad \text{if } a_n > x$$

APPENDIX C

COMPUTER PROGRAM LISTINGS

Table 7. Tektronix 4027 Initialization Program.

```
C
C PROGRAM TO INITIALIZE TK4027 MONITOR
C
      TYPE 100
100   FORMAT(' !WOR 0 H')
      TYPE 101
101   FORMAT(' !MAR')
      TYPE 102
102   FORMAT(' !DUP')
      TYPE 103
103   FORMAT(' !ECH R')
      TYPE 104
104   FORMAT(' !BUF N')
      TYPE 105
105   FORMAT(' !MAP C0    0,100,100')
      TYPE 106
106   FORMAT(' !MAP C1 110, 50,100')
      TYPE 107
107   FORMAT(' !MAP C2 240, 50,100')
      TYPE 108
108   FORMAT(' !MAP C3  30, 60,100')
      TYPE 109
109   FORMAT(' !MAP C4 180, 50,100')
      TYPE 110
110   FORMAT(' !MAP C5 320, 50,100')
      TYPE 111
111   FORMAT(' !MAP C6  70, 50,100')
      TYPE 112
112   FORMAT(' !MAP C7   0,  0,100')
      TYPE 113
113   FORMAT(' !BEL')
      TYPE 114
114   FORMAT(' !BAU 9600')
      TYPE 115
115   FORMAT(' !MON 34 H K')
      END
```

Table 8. Influence Line Program.

```
C-----------------------------------------------------------------------C
C                                                                       C
C   PROGRAM INFORMATION                                                 C
C   ===================                                                 C
C                                                                       C
C     THE FOLLOWING PROGRAM IS AN INTERACTIVE GRAPHICS PROGRAM WHOSE    C
C     PRIMARY PURPOSE IS TO SERVE AS AN AID IN TEACHING THE THEORIES    C
C     OF INFLUENCE LINES AS THEY APPLY TO CONTINUOUS BEAMS. THE FUN-    C
C     DAMENTAL STRUCTURAL THEOREMS ARE PRESENTED, AND INFLUENCE LINE    C
C     EQUATIONS ARE DEVELOPED FOR MOMENTS, SHEARS, REACTIONS, AND       C
C     DEFLECTIONS.                                                      C
C                                                                       C
C     THE INFLUENCE LINE EQUATIONS ARE USED TO COMPUTE THE INFLUENCE    C
C     LINE ORDINATES AT THE 10TH-POINTS IN EACH SPAN FOR 2, 3, AND 4    C
C     SPAN CONTINUOUS BEAMS. DEAD AND LIVE LOADS MAY THEN BE APPLIED    C
C     TO A USER-DEFINED CONTINUOUS BEAM AND THE USER MAY VIEW MOMENT    C
C     ENVELOPES, SHEAR ENVELOPES, OR BEAM DEFLECTIONS.                  C
C                                                                       C
C                                                                       C
C   TECHNICAL INFORMATION                                               C
C   =====================                                               C
C                                                                       C
C     THE PROGRAM IS WRITTEN IN VAX FORTRAN AND OPERATES ON A VAX/VMS   C
C     SYSTEM, VERSION V4.1. HARDWARE REQUIREMENTS INCLUDE A VAX MINI-   C
C     COMPUTER, MODEL 11/780, AND A TEKTRONIX 4027 COLOR GRAPHICS       C
C     TERMINAL. THE PROGRAM IMPLEMENTS A LIBRARY OF SUBROUTINES CALL-   C
C     ED "DISSPLA", A SOFTWARE PRODUCT FROM THE 'INTEGRATED SOFTWARE    C
C     SYSTEMS CORPORATION', SAN DIEGO, CALIFORNIA.                      C
C                                                                       C
C-----------------------------------------------------------------------C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(5),B(5),D(5),D1(5),BEAM(5,3),SUPPM(41,3),SPANM(41,41),
     *SHEAR(42,44),REACT(41,5),DELTA(41,41),SC(5,2),ABC(41,3),NORD(3,2),
     *AREAM(41,4),AREAV(44,4),AREAD(41,4),VORD(40),AREAR(5,4),UNIDL(4),
     *UNILL(4),ENVEM(41,3),ENVEV(44,3),ENVED(41,3),ENVER(5,3),DX(5),
     *CONLL(3,2),AASHTO(3),YL1(30),YL2(30),YL3(30),YL4(30),YL5(31),
     *YL6(31),YL7(31),YL8(30),YL9(30),YL10(30),YL11(30),XL1(30),XL2(30),
     *NORDV(3,2),TRLLV(3,2),UDLV(3,2),TRWV(3,2),WORD(3,2),DELSM(3),
     *DELM(41),DELV(44),DELD(41),DELX(41),EDT(41,3)
      CHARACTER MM,IM,DM,EM,CMI,AM,DI,ZZ,TLL,ALL,AASHTL*7,RM,AO,DO,CIL,
     *VSP,VSUL,DOT*5,TYPE*14,LOCA*15,TITL*12,ZZDL,ZZLL,ZZLLO
C
C GRAPHICAL DATA POINTS FOR VARIOUS INFLUENCE LINES IN THE
C INTRODUCTION AND DEVELOPMENT SEGMENTS OF THE PROGRAM
C
      DATA XL1 /1.30,1.60,1.90,2.20,2.50,2.80,3.10,3.40,3.70,4.00,
     *          4.45,4.90,5.35,5.80,6.25,6.70,7.15,7.60,8.05,8.50,
     *          9.10,9.70,10.3,10.9,11.5,12.1,12.7,13.3,13.9,14.5/
      DATA XL2 /1.40,1.80,2.20,2.60,3.00,3.40,3.80,4.20,4.60,5.00,
     *          5.52,6.04,6.56,7.08,7.60,8.12,8.64,9.16,9.68,10.2,
     *          10.6,11.0,11.4,11.8,12.2,12.6,13.0,13.4,13.8,14.2/
      DATA YL1 /2.21,2.13,2.05,1.99,1.96,1.95,1.97,2.04,2.14,2.30,
     *          1.99,1.79,1.69,1.66,1.70,1.78,1.90,2.03,2.17,2.30,
     *          2.43,2.53,2.58,2.60,2.59,2.56,2.51,2.45,2.38,2.30/
      DATA YL2 /2.32,2.34,2.35,2.37,2.37,2.38,2.37,2.36,2.33,2.30,
     *          2.23,2.15,2.06,1.98,1.92,1.89,1.90,1.96,2.09,2.30,
     *          1.85,1.55,1.37,1.30,1.32,1.42,1.59,1.80,2.04,2.30/
      DATA YL3 /1.88,1.78,1.68,1.61,1.56,1.55,1.59,1.67,1.80,2.00,
     *          1.72,1.54,1.45,1.43,1.47,1.55,1.66,1.78,1.90,2.00,
     *          2.06,2.09,2.12,2.13,2.12,2.11,2.09,2.06,2.03,2.00/
      DATA YL4 /1.94,1.89,1.85,1.81,1.79,1.78,1.80,1.84,1.90,2.00,
     *          2.18,2.40,2.69,3.00,2.75,2.53,2.34,2.20,2.08,2.00,
     *          1.95,1.92,1.90,1.90,1.90,1.91,1.93,1.95,1.97,2.00/
      DATA YL5 /1.88,1.75,1.64,1.52,1.41,2.41,2.31,2.22,2.13,2.06,2.00,
     *          1.94,1.91,1.89,1.88,1.82,1.91,1.93,1.96,1.98,2.00,
     *          2.01,2.02,2.02,2.03,2.03,2.02,2.02,2.01,2.01,2.00/
      DATA YL6 /2.02,2.04,2.06,2.08,2.09,2.09,2.08,2.07,2.04,2.00,
     *          1.93,1.84,1.73,1.62,1.50,1.38,1.27,1.16,1.07,1.00,2.00,
     *          1.96,1.93,1.92,1.91,1.91,1.92,1.94,1.96,1.98,2.00/
      DATA YL7 /1.99,1.99,1.98,1.98,1.97,1.97,1.98,1.98,1.99,2.00,
     *          2.02,2.04,2.07,2.09,2.11,2.12,2.11,2.09,2.06,2.00,
     *          3.00,2.94,2.87,2.78,2.69,2.59,2.48,2.36,2.25,2.12,2.00/
      DATA YL8 /2.88,2.75,2.64,2.52,2.41,2.31,2.22,2.13,2.06,2.00,
     *          1.94,1.91,1.89,1.88,1.89,1.91,1.93,1.96,1.98,2.00,
     *          2.01,2.02,2.02,2.03,2.03,2.02,2.02,2.01,2.01,2.00/
      DATA YL9 /1.97,1.94,1.92,1.90,1.89,1.88,1.89,1.91,1.95,2.00,
     *          2.09,2.21,2.34,2.47,2.61,2.73,2.84,2.93,2.99,3.00,
     *          2.98,2.94,2.87,2.78,2.68,2.56,2.43,2.29,2.15,2.00/
      DATA YL10/2.08,2.15,2.21,2.26,2.29,2.30,2.28,2.22,2.13,2.00,
     *          1.77,1.50,1.26,1.07,1.00,1.07,1.26,1.50,1.77,2.00,
     *          2.13,2.22,2.28,2.30,2.29,2.26,2.21,2.15,2.08,2.00/
      DATA YL11/1.98,1.97,1.95,1.94,1.93,1.93,1.94,1.95,1.97,2.00,
     *          2.05,2.11,2.17,2.22,2.26,2.28,2.27,2.23,2.14,2.00,
     *          1.85,1.68,1.52,1.40,1.35,1.37,1.47,1.61,1.80,2.00/
C
C CF1 = MODULUS OF ELASTICITY CONVERSION CONSTANT, ksi-->ksf
C CF2 = MOMENT OF INERTIA CONVERSION CONSTANT, ksf-->ksf
C CF3 = LOWER ERROR BOUND CONSTANT
```

```
C CF4 = UPPER ERROR BOUND CONSTANT
C N9 = NUMBER OF CRDINATE DIVISIONS PER SPAN
C
      CF1=144.
      CF2=1.
      CF3=0.98
      CF4=1.02
      N1=0
      N9=10
      RN9=N9*1.
C
C COMMONLY USED FORMAT STATEMENTS
C
48    FORMAT(A1)
754   FORMAT(A12)
822   FORMAT(A21)
104   FORMAT(T15,A54)
646   FORMAT(F3.C)
830   FORMAT(F4.C)
842   FORMAT(F4.1)
463   FORMAT(F5.1)
943   FORMAT(F5.2)
666   FORMAT(F6.0)
889   FORMAT(F6.1)
576   FORMAT(F6.2)
552   FORMAT(F6.3)
836   FORMAT(F7.C)
855   FORMAT(F7.1)
794   FORMAT(F7.2)
386   FORMAT(F7.1,F5.2)
663   FORMAT(F8.1)
798   FORMAT(F13.7)
790   FORMAT(I1)
849   FORMAT(I2)
643   FORMAT(I4)
227   FORMAT(I5)
760   FORMAT(I6)
823   FORMAT(/)
529   FORMAT(//)
583   FORMAT(///)
C
C COLOR CODES:  C0 = WHITE     C1 = RED        C2 = GREEN     C3 = BLUE
C               C4 = YELLOW    C5 = CYAN       C6 = MAGENTA   C7 = BLACK
C
748   FORMAT(' !COL C0')
722   FORMAT(' !COL C1')
740   FORMAT(' !COL C2')
744   FORMAT(' !COL C3')
708   FORMAT(' !COL C4')
729   FORMAT(' !COL C5')
703   FORMAT(' !COL C6')
700   FORMAT(' !COL C7')
723   FORMAT(' !LIN 1')
701   FORMAT(' !MON 8 H K')
326   FORMAT(' !MON 34 H K')
151   FORMAT(' !MON')
155   FORMAT(' !WOR')
785   FORMAT(' !ERA M')
634   FORMAT(' !JUM 12,63')
595   FORMAT(' !JUM 18,63')
695   FORMAT(' !JUM 24,63')
C
C INITIALIZE "DISSPLA" COMMANDS TO TEKTRONIX 4027 TERMINAL
C
      CALL TK4027
      CALL SETDEV(0,0)
      CALL PAGE(11.,8.5)
      CALL NOBRDR
      CALL LINESP(1.5)
      CALL BASALF('L/CSTD')
      CALL MIXALF('STAND')
      CALL MX3ALF('INSTR','>')
      CALL MX4ALF('MATHE',':')
      CALL MX5ALF('GREEK','*')
C
C MAIN MENU
C
      TYPE 326
      TYPE 723
      TYPE 104,'  **  **  **  **  **  **  **  **  **  **  **  **  **  '
      TYPE 104,' **                                                ** '
      TYPE 104,'**   ANALYSIS AND BEHAVIOR OF THE CONTINUOUS BEAM   **'
      TYPE 104,' **        THROUGH THE USE OF INFLUENCE LINES      ** '
      TYPE 104,'  **                                             **   '
      TYPE 104,'   **  **  **  **           BY           **  **  **  **  '
      TYPE 104,'                **   RICK  EHLERT   **                 '
      TYPE 104,'                 **               **                   '
      TYPE 104,'                  **  **  **  **  **                   '
```

```
          TYPE 529
          TYPE 14
   14     FORMAT(T6,'The analysis of transversely loaded beams which are con
         *tinuous over three')
          TYPE 16
   16     FORMAT(T6,'or more supports is rather tedious.  A complete analysi
         *s requires the de-')
          TYPE 18
   18     FORMAT(T6,'signer to understand basic theories of continuous, inde
         *terminate struc-')
          TYPE 20
   20     FORMAT(T6,'tures.  One means of studying continuous beam behavior
         *is through the ap-')
          TYPE 22
   22     FORMAT(T6,'plication of influence lines.  The following interactiv
         *e program will let')
          TYPE 24
   24     FORMAT(T6,'the student study continuous beam behavior by developin
         *g and then apply-')
          TYPE 26
   26     FORMAT(T6,'ing various types of influence lines.'/)
          TYPE 28
   28     FORMAT(T6,'The program is divided into 3 main sections.  The main
         *menu is given be-')
          TYPE 30
   30     FORMAT(T6,'low and all first time users are urged to view the INTR
         *ODUCTION before')
          TYPE 32
   32     FORMAT(T6,'proceeding into later sections.'/)
   33     TYPE 34
   34     FORMAT(T6,'MAIN MENU')
          TYPE 36
   36     FORMAT(T6,'========='/)
          TYPE 38
   38     FORMAT(T11,'(A) INTRODUCTION')
          TYPE 40
   40     FORMAT(T11,'(B) DEVELOPMENT OF INFLUENCE LINES')
          TYPE 42
   42     FORMAT(T11,'(C) APPLICATION OF INFLUENCE LINES')
          TYPE 44
   44     FORMAT(T11,'(D) END OF PROGRAM'//)
   45     TYPE 46
   46     FORMAT(T5,' PLEASE SELECT ONE OF THE ABOVE: ',$)
          ACCEPT 48, MM
          IF(MM.LT.'A' .OR. MM.GT.'D') GO TO 45
          IF(MM.EQ.'A') GO TO 300
          IF(MM.EQ.'B') GO TO 400
          IF(MM.EQ.'C') GO TO 500
          GO TO 9999
C
C INTRODUCTION SEGMENT FROM MAIN MENU
C
  300     TYPE 326
          TYPE 785
          TYPE 302
  302     FORMAT(' INTRODUCTION')
          TYPE 303
  303     FORMAT(' ------------'/)
          TYPE 304
  304     FORMAT(T6,'The inroductory phase of this program is intended to ac
         *quaint the user')
          TYPE 305
  305     FORMAT(T6,'with all information necessary for program operation.
         *The information is')
          TYPE 306
  306     FORMAT(T6,'quite substantial and the user is encouraged to make no
         *tes for future re-')
          TYPE 307
  307     FORMAT(T6,'ference throughout the remainder of the program.'/)
  282     TYPE 319
  319     FORMAT(T6,'INTRODUCTION MENU')
          TYPE 320
  320     FORMAT(T6,'================='/)
          TYPE 308
  308     FORMAT(T11,'(A) Continuous presentation of all introductory materi
         *al')
          TYPE 309
  309     FORMAT(T11,'(B) Program purpose')
          TYPE 310
  310     FORMAT(T11,'(C) Graphical definition of a continuous beam')
          TYPE 311
  311     FORMAT(T11,'(D) Graphic examples of influence lines')
          TYPE 312
  312     FORMAT(T11,'(E) Presentation of theory')
          TYPE 313
  313     FORMAT(T11,'(F) Limitations')
          TYPE 314
  314     FORMAT(T11,'(G) Nomenclature')
```

```
          TYPE 315
    315   FORMAT(T11,'(H) Return to Main Menu'//)
    316   TYPE 46
          ACCEPT 48, IM
          IF(IM.LT.'A' .OR. IM.GT.'H') GO TO 316
          IF(IM.EQ.'H') THEN
            TYPE 785
            GO TO 33
          END IF
          IF(IM.EQ.'A' .OR. IM.EQ.'B') GO TO 321
          IF(IM.EQ.'C') GO TO 330
          IF(IM.EQ.'D') GO TO 340
          IF(IM.EQ.'E') GO TO 350
          IF(IM.EQ.'F') GO TO 360
          IF(IM.EQ.'G') GO TO 370
    321   CALL TABLET('CENTER','LONG')
          CALL LTLINE('(PROGRAM PURPOSE)$')
          CALL LTLINE('(----------------)$')
          CALL CTLINE('$')
          CALL LTLINE('(U)NDOUBTEDLY, THE MOST POWERFUL TOOL AVAILABLE FOR U
         *SE BY THE ENGINEER IS$')
          CALL LTLINE('THE COMPUTER.  (T)HE ACTUAL ANALYSIS OF THE MOST COMP
         *LEX STRUCTURES HAS$')
          CALL LTLINE('BEEN REDUCED TO NOTHING MORE THAN A FEW PAGES OF COMP
         *UTER OUTPUT.  (U)NFOR-$')
          CALL LTLINE('TUNATELY, THE STRUCTURAL ENGINEER IS STILL LEFT WITH
         *THE TASK OF INTER-$')
          CALL LTLINE('PRETTING RESULTS OF COMPUTER ANALYSIS, AND APPLYING T
         *HOSE RESULTS TOWARD$')
          CALL LTLINE('THE FINAL DESIGN.  (H)EREIN ARISES AN IMPORTANT QUEST
         *ION--(H)AS THE JUDGEMENT$')
          CALL LTLINE('MAKING ABILITY OF THE STRUCTURAL ENGINEER BEEN ADEQUA
         *TELY DEVELOPED?$')
          CALL CTLINE('$')
          CALL LTLINE('(T)HIS PROGRAM WILL DEAL WITH 2 IMPORTANT COMPONENTS
         *OF DEVELOPING SOUND$')
          CALL LTLINE('JUDGEMENT-MAKING ABILITIES :0)1:1) KNOWLEDGE OF FUNDA
         *MENTAL STRUCTURAL THEOR-$')
          CALL LTLINE('EMS AND :0)2:1) "FEEL" OF A STRUCTURE.  (T)HE PROGRAM
         * WILL USE THE CONTINUOUS$')
          CALL LTLINE('BEAM AND ASSOCIATED THEOREMS TO ILLUSTRATE THESE 2 CO
         *MPONENTS.  (T)HUS, IT$')
          CALL LTLINE('IS THE PURPOSE OF THIS PROGRAM TO PROVIDE FOR THE STU
         *DENT A QUICK MEANS OF$')
          CALL LTLINE('STUDYING THE FUNDAMENTAL THEOREMS ASSOCIATED WITH CON
         *TINUOUS BEAM ANALYSIS,$')
          CALL LTLINE('AND APPLYING THE THEOREMS SO AS TO DEVELOP A "FEEL" F
         *OR THE STRUCTURE UN-$')
          CALL LTLINE('DER VARIOUS LOADING CONDITIONS.$')
          CALL CTLINE('$')
          CALL CTLINE('$')
          CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
          CALL ENDTAB(0)
          CALL ENDPL(0)
          CALL TABLET('CENTER','LONG')
          CALL LTLINE('(T)HE (DEVELOPMENT) SEGMENT OF THIS PROGRAM IS TWO-FO
         *LD IN NATURE.  (F)IRST, THE$')
          CALL LTLINE('STUDENT WILL HAVE THE OPPORTUNITY TO EXAMINE THE DEVE
         *LOPMENT OF INFLUENCE$')
          CALL LTLINE('LINE EQUATIONS FROM FUNDAMENTAL STRUCTURAL THEOREMS.
         * (I)T IS THESE EQUA-$')
          CALL LTLINE('TIONS WHICH ARE USED TO COMPUTE INFLUENCE LINE ORDINA
         *TES.  (S)ECONDLY, SO AS$')
          CALL LTLINE('TO ASSURE THE STUDENT UNDERSTANDS THE INFLUENCE LINE
         *EQUATIONS, HE OR SHE$')
          CALL LTLINE('WILL BE ABLE TO COMPUTE ORDINATES FOR A SPECIFIC INFL
         *UENCE LINE.  (T)HIS WILL$')
          CALL LTLINE('REQUIRE STUDENT SPECIFICATION OF VARIOUS TERMS IN THE
         * INFLUENCE LINE EQUA-$')
          CALL LTLINE('TIONS.$')
          CALL CTLINE('$')
          CALL LTLINE('(T)HE (APPLICATION) SEGMENT OF THIS PROGRAM PROVIDES
         *AN IMPORTANT APPLICATION$')
          CALL LTLINE('OF INFLUENCE LINES.  (D)EAD AND LIVE LOADS CAN BE APP
         *LIED TO A SPECIFIED CON-$')
          CALL LTLINE('TINUOUS BEAM AND THE STUDENT MAY VIEW MOMENT ENVELOPE
         *S, SHEAR ENVELOPES, OR$')
          CALL LTLINE('BEAM DEFLECTION CURVES.  (O)F PARTICULAR IMPORTANCE,
         *THE STUDENT WILL BE AL-$')
          CALL LTLINE('LOWED TO RE-DEFINE OR RE-POSITION THE LIVE LOAD AND T
         *HUS DETERMINE WHICH LIVE$')
          CALL LTLINE('LOADING PRODUCES MAXIMUM MOMENTS, SHEARS, AND DEFLECT
         *IONS.$')
          CALL CTLINE('$')
          CALL CTLINE('$')
          IF(IM.EQ.'E') THEN
            CALL LTLINE('(P)RESS (RETURN) TO RETURN TO (I)NTRODUCTION (M)ENU
         *...$')
```

```
          ELSE
            CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
          END IF
          CALL ENDTAB(0)
          CALL ENDPL(0)
          IF(IM.EQ.'B') GO TO 385
330       CALL TABLET('CENTER','LONG')
          CALL LTLINE('(GRAPHICAL DEFINITION OF A CONTINUOUS BEAM)$')
          CALL LTLINE('(------------------------------------------)$')
          CALL CTLINE('$')
          CALL LTLINE('(I)T IS NOW POSSIBLE WITH THE AID OF TODAY"S COMPUTER
         * PROGRAMS TO CREATE, ANA-$')
          CALL LTLINE('LYZE, AND DESIGN THE MOST COMPLEX CONTINUOUS BEAM.  (
         *C)OMPLEX CONTINUOUS BEAMS$')
          CALL LTLINE('HOWEVER, MAY BE VIEWED AS A COMBINATION OF SIMPLE CON
         *TINUOUS BEAMS AND VAR-$')
          CALL LTLINE('IOUS PARAMETERS FUNDAMENTALLY KNOWN TO THE STRUCTURAL
         * ENGINEER.  (T)HUS, IT IS$')
          CALL LTLINE('POSSIBLE FOR THE ENGINEER TO UNDERSTAND SIMPLE CONTIN
         *UOUS BEAM BEHAVIOR AND$')
          CALL LTLINE('ANALYZE MORE COMPLEX BEAM CONFIGURATIONS.  (T)HIS PRO
         *GRAM WILL BE LIMITED TO$')
          CALL LTLINE('DISCUSSION OF RELATIVELY SIMPLE CONTINUOUS BEAMS SUCH
         * AS THE 3-SPAN BEAM$')
          CALL LTLINE('SHOWN BELOW...$')
          CALL CTSET(1)
          CALL LTLINE('(I)N GENERAL, ALL CONTINUOUS BEAMS IN THIS PROGRAM WI
         *LL HAVE THE FOLLOWING$')
          CALL LTLINE('PARAMETERS...$')
          CALL CTLINE('$')
          CALL LTLINE('                    :8)1:9) (O)NLY 2, 3, OR 4-SPAN BEAMS WILL
         *BE USED$')
          CALL CTLINE('$')
          CALL CTLINE('$')
          CALL LTLINE('                    :8)2:9) (M)OMENTS OF INERTIA MAY VARY FROM
         * SPAN TO SPAN,$')
          CALL LTLINE('                            BUT SHALL BE CONSTANT THROUGHOUT A GIV
         *EN SPAN$')
          CALL CTLINE('$')
          CALL CTLINE('$')
          CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
          CALL ENDTAB(0)
          CALL INSERT(1)
          CALL AREA2D(16,2.5)
          CALL HEIGHT(.25)
          CALL BEAM1
          CALL MESSAG('(I>L.2H.8)1',11,2.8,1.6)
          CALL MESSAG('(I>L.2H.8)2',11,7.4,1.6)
          CALL MESSAG('(I>L.2H.8)3',11,12.,1.6)
          CALL SETCLR('YELLOW')
          CALL VECTOR(1.,.5,1.,1.5,0)
          CALL VECTOR(5.,.5,5.,1.5,0)
          CALL VECTOR(10.2,.5,10.2,1.5,0)
          CALL VECTOR(14.2,.5,14.2,1.5,0)
          CALL VECTOR(1.,.7,5.,.7,1402)
          CALL VECTOR(5.,.7,10.2,.7,1402)
          CALL VECTOR(10.2,.7,14.2,.7,1402)
          CALL MESSAG('(SPAN 1)',8,2.3,.8)
          CALL MESSAG('(SPAN 2)',8,6.9,.8)
          CALL MESSAG('(SPAN 3)',8,11.5,.8)
          CALL RESET('HEIGHT')
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL TABLET('CENTER','LONG')
          CALL LTLINE('                    :8)3:9) (N)O INTERNAL HINGES WILL BE USED$
         *')
          CALL CTLINE('$')
          CALL CTLINE('$')
          CALL LTLINE('                    :8)4:9) (S)UPPORTS SHALL BE EITHER PINNED
         *OR ROLLER TYPE$')
          CALL CTLINE('$')
          CALL CTLINE('$')
          CALL LTLINE('                    :8)5:9) (S)UPPORTS MAY UNDERGO VERTICAL DI
         *SPLACEMENTS$')
          CALL CTLINE('$')
          CALL CTLINE('$')
          CALL CTLINE('$')
          CALL LTLINE('(SIGN CONVENTION)$')
          CALL LTLINE('(---------------)$')
          CALL CTLINE('$')
          CALL LTLINE('(S)O AS TO AID THE STUDENT IN UNDERSTANDING ANY OUTPU
         *T OR DISPLAYS THROUGHOUT$')
          CALL LTLINE('THE PROGRAM, THE FOLLOWING SIGN CONVENTION HAS BEEN A
         *DOPTED...$')
          CALL CTSET(2)
          IF(IM.EQ.'C') THEN
            CALL LTLINE('(P)RESS (RETURN) TO RETURN TO (I)NTRODUCTION (M)ENU
         *...$')
```

```
      ELSE
         CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
      END IF
      CALL  ENDTAB(0)
      CALL  INSERT(2)
      CALL  AREA2D(16.,4.)
      CALL  HEIGHT(.25)
      CALL  SETCLR('WHITE')
      CALL  MESSAG('>PE.4)(MOMENT)>EXU)',19,2.0,4.0)
      CALL  MESSAG('>PE.4)(SHEAR)>EXU)',18,2.0,2.5)
      CALL  MESSAG('>PE.4)(DEFLECTION)>EXU)',23,2.0,1.0)
      CALL  MESSAG('*D) IS :8)+:9) MEASURED DOWNWARD',32,5.0,1.0)
      CALL  MESSAG('*D) IS :8)-:9) MEASURED UPWARD',30,5.0,.1)
      CALL  SETCLR('CYAN')
      CALL  VECTOR(5.,4.,8.,4.,0)
      CALL  VECTOR(11.,4.,14.,4.,0)
      CALL  VECTOR(5.,2.5,8.,2.5,0)
      CALL  VECTOR(11.,2.5,14.,2.5,0)
      CALL  SETCLR('RED')
      X=5.
      Y=4.
      CALL  ARC1(X,Y)
      X=8.
      CALL  ARC2(X,Y)
      X=11.
      CALL  ARC3(X,Y)
      X=14.
      CALL  ARC4(X,Y)
      CALL  VECTOR(4.8,2.,4.8,3.,0)
      CALL  VECTOR(4.8,3.,4.7,2.8,0)
      CALL  VECTOR(8.2,3.,8.2,2.,0)
      CALL  VECTOR(8.2,2.,8.3,2.2,0)
      CALL  VECTOR(10.8,3.,10.8,2.,0)
      CALL  VECTOR(10.8,2.,10.7,2.2,0)
      CALL  VECTOR(14.2,2.,14.2,3.,0)
      CALL  VECTOR(14.2,3.,14.3,2.8,0)
      CALL  SETCLR('WHITE')
      CALL  MESSAG('(+M)',4,3.9,3.8)
      CALL  MESSAG('(+M)',4,8.6,3.8)
      CALL  MESSAG('(-M)',4,9.9,3.8)
      CALL  MESSAG('(-M)',4,14.6,3.8)
      CALL  MESSAG('TENSION BOTTOM',14,5.0,3.4)
      CALL  MESSAG('TENSION TOP',11,11.4,3.4)
      CALL  MESSAG('(+V)',4,4.2,2.3)
      CALL  MESSAG('(+V)',4,8.4,2.3)
      CALL  MESSAG('(-V)',4,10.2,2.3)
      CALL  MESSAG('(-V)',4,14.4,2.3)
      CALL  RESET('HEIGHT')
      CALL  ENDGR(0)
      CALL  ENDPL(0)
      IF(IM.EQ.'C') GO TO 385
340   CALL  TABLET('CENTER','LONG')
      CALL  LTLINE('(GRAPHIC EXAMPLES OF INFLUENCE LINES)$')
      CALL  LTLINE('(-------------------------------------)$')
      CALL  CTLINE('$')
      CALL  LTLINE('(F)OR ANY GIVEN CONTINUOUS BEAM, IT IS POSSIBLE TO CO
     *MPUTE INFLUENCE LINE$')
      CALL  LTLINE('ORDINATES AT ANY FINITE POINT ON THE BEAM.  (B)UT TO
     *COMPUTE ORDINATES AT$')
      CALL  LTLINE('EVERY FINITE POINT FOR PURPOSES OF SKETCHING THE INFL
     *UENCE LINE IS TIME$')
      CALL  LTLINE('CONSUMING, NOT TO MENTION, RIDICULOUS.  (A) COMMONLY
     *ACCEPTED MEANS OF COM-$')
      CALL  LTLINE('PUTING INFLUENCE LINE ORDINATES IS TO COMPUTE THE ORD
     *INATES AT THE 10TH-$')
      CALL  LTLINE('POINTS OF EACH SPAN.  (T)HIS PROVIDES AN ADEQUATE NUM
     *BER OF POINTS FROM$')
      CALL  LTLINE('WHICH THE INFLUENCE LINE CAN THEN BE DRAWN.  (T)HE 10
     *TH-POINT NUMBERS AND$')
      CALL  LTLINE('SUPPORT NUMBERS WHICH THE STUDENT WILL USE THROUGHOUT
     * THE PROGRAM ARE$')
      CALL  LTLINE('SHOWN BELOW...$')
      CALL  CTSET(3)
      CALL  CTLINE('$')
      CALL  LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
      CALL  ENDTAB(0)
      CALL  INSERT(3)
      CALL  AREA2D(16.,4.5)
      CALL  HEIGHT(.25)
      CALL  SETCLR('CYAN')
      CALL  VECTOR(1.,4.,13.,4.,0)
      CALL  VECTOR(.8,3.8,1.2,3.8,0)
      CALL  VECTOR(.9,3.8,1.,4.,0)
      CALL  VECTOR(1.,4.,1.1,3.8,0)
      CALL  VECTOR(3.8,3.8,4.2,3.8,0)
      CALL  VECTOR(6.8,3.8,7.2,3.8,0)
      CALL  VECTOR(9.8,3.8,10.2,3.8,0)
      CALL  VECTOR(12.8,3.8,13.2,3.8,0)
```

```
      CALL BLCIR(4.,3.9,.1,.01)
      CALL BLCIR(7.,3.9,.1,.01)
      CALL BLCIR(10.,3.9,.1,.01)
      CALL BLCIR(13.,3.9,.1,.01)
      DO 260 X=1.,13.,.3
260   CALL VECTOR(X,4.1,X,4.,0)
      DO 261 X=1.,13.,1.5
261   CALL VECTOR(X,4.2,X,4.,0)
      CALL MESSAG('1',1,.9,4.4)
      CALL MESSAG('11',2,3.8,4.4)
      CALL MESSAG('21',2,6.8,4.4)
      CALL MESSAG('31',2,9.8,4.4)
      CALL MESSAG('41',2,12.8,4.4)
      CALL MESSAG('(10TH-POINTS)',13,13.5,4.4)
      CALL MESSAG('1',1,0.9,3.3)
      CALL MESSAG('2',1,3.9,3.3)
      CALL MESSAG('3',1,6.9,3.3)
      CALL MESSAG('4',1,9.9,3.3)
      CALL MESSAG('5',1,12.9,3.3)
      CALL MESSAG('(SUPPORT NUMBERS)',17,13.5,3.3)
      CALL SETCLR('YELLOW')
      CALL VECTOR(1.,3.,1.,.3,0)
      CALL VECTOR(7.,3.,7.,1.9,0)
      CALL VECTOR(10.,3.,10.,1.1,0)
      CALL VECTOR(13.,3.,13.,.3,0)
      CALL VECTOR(1.,2.1,7.,2.1,1402)
      CALL VECTOR(1.,1.3,10.,1.3,1402)
      CALL VECTOR(1.,.5,13.,.5,1402)
      CALL MESSAG('(2-SPAN BEAM)',13,3.0,2.2)
      CALL MESSAG('(3-SPAN BEAM)',13,4.5,1.4)
      CALL MESSAG('(4-SPAN BEAM)',13,6.0,0.6)
      CALL RESET('HEIGHT')
      CALL ENDGR(0)
      CALL ENDPL(0)
      CALL TABLET('CENTER','LONG')
      CALL LTLINE('(T)HERE ARE 4 BASIC TYPES OF INFLUENCE LINES WHICH AR
     *E OF PARTICULAR IN-$')
      CALL LTLINE('TEREST TO THE STRUCTURAL ENGINEER.  (G)RAPHIC EXAMPLE
     *S OF THESE 4 TYPES$')
      CALL LTLINE('ARE SHOWN BELOW ON A 3-SPAN BEAM.$')
      CALL CTLINE('$')
      CALL LTLINE('(1. MOMENT INFLUENCE LINES)$')
      CALL LTLINE('(---------------------------)$')
      CALL CTSET(4)
      CALL NEWSET
      CALL LTLINE('(2. SHEAR INFLUENCE LINES)$')
      CALL LTLINE('(--------------------------)$')
      CALL CTSET(5)
      CALL CTLINE('$')
      CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
      CALL ENDTAB(0)
      CALL INSERT(4)
      CALL AREA2D(16.,3.)
      CALL HEIGHT(.25)
      CALL BEAM1
      CALL SETCLR('GREEN')
      CALL STRTPT(1.,2.)
      DO 251 I=1,30
251   CALL CONNPT(XL2(I),YL3(I))
      CALL VECTOR(4.,.6,6.,.6,0)
      CALL MESSAG('-(M I.L.) AT (S)UPPORT (N)O. 2',30,6.5,.5)
      CALL SETCLR('YELLOW')
      CALL STRTPT(1.,2.)
      DO 252 I=1,30
252   CALL CONNPT(XL2(I),YL4(I))
      CALL VECTOR(4.,.1,6.,.1,0)
      CALL MESSAG('+(M I.L.) AT 10TH-POINT (N)O. 15',32,6.5,0.0)
      CALL RESET('HEIGHT')
      CALL ENDGR(0)
      CALL INSERT(5)
      CALL AREA2D(16.,3.)
      CALL HEIGHT(.25)
      CALL BEAM1
      CALL SETCLR('GREEN')
      CALL STRTPT(1.,2.)
      DO 253 I=1,31
      IF(I.LT.6) THEN
        CALL CONNPT(XL2(I),YL5(I))
      ELSE
        CALL CONNPT(XL2(I-1),YL5(I))
      END IF
253   CONTINUE
      CALL VECTOR(3.,.6,5.,.6,0)
      CALL MESSAG('(V I.L.) AT 10TH-POINT (N)O. 6',30,5.5,.5)
      CALL SETCLR('YELLOW')
      CALL STRTPT(1.,2.)
      DO 254 I=1,31
      IF(I.LT.21) THEN
```

```
            CALL CONNPT(XL2(I),YL6(I))
          ELSE
            CALL CONNPT(XL2(I-1),YL6(I))
          END IF
  254     CONTINUE
          CALL VECTOR(3.,.1,5.,.1,0)
          CALL MESSAG('(V I.L.) LEFT OF 10TH-POINT (N)O. 21',36,5.5,0.0)
          CALL SETCLR('MAGENTA')
          CALL STRTPT(1.,2.)
          DO 255 I=1,31
          IF(I.LT.21) THEN
            CALL CONNPT(XL2(I),YL7(I))
          ELSE
            CALL CONNPT(XL2(I-1),YL7(I))
          END IF
  255     CONTINUE
          CALL VECTOR(3.,-.4,5.,-.4,0)
          CALL MESSAG('(V I.L.) RIGHT OF 10TH-POINT (N)O. 21',37,5.5,-.5)
          CALL RESET('HEIGHT')
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL TABLET('CENTER','LONG')
          CALL LTLINE('(3. REACTION INFLUENCE LINES)$')
          CALL LTLINE('(----------------------------)$')
          CALL CTSET(6)
          CALL NEWSET
          CALL LTLINE('(4. DEFLECTION INFLUENCE LINES)$')
          CALL LTLINE('(------------------------------)$')
          CALL CTSET(7)
          CALL LTLINE('(T)HE STUDENT WILL HAVE AN OPPORTUNITY TO COMPUTE ORD
         *INATES FOR EACH OF$')
          CALL LTLINE('THESE INFLUENCE LINE TYPES IN THE (DEVELOPMENT) SEGME
         *NT OF THIS PROGRAM.$')
          CALL LTLINE('(I)T WILL PROVE USEFUL TO THE STUDENT THROUGHOUT THE
         *PROGRAM TO KEEP IN$')
          CALL LTLINE('MIND THE GENERAL SHAPE OF EACH INLFLUENCE LINE TYPE.$
         *')
          CALL CTLINE('$')
          IF(IM.EQ.'C') THEN
            CALL LTLINE('(P)RESS (RETURN) TO RETURN TO (I)NTRODUCTION (M)ENU
         *...$')
          ELSE
            CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
          END IF
          CALL ENDTAB(0)
          CALL INSERT(6)
          CALL AREA2D(16.,3.)
          CALL HEIGHT(.25)
          CALL BEAM1
          CALL SETCLR('GREEN')
          CALL VECTOR(1.,2.,1.,3.,0)
          CALL STRTPT(1.,3.)
          DO 256 I=1,30
  256     CALL CONNPT(XL2(I),YL8(I))
          CALL VECTOR(4.,.6,6.,.6,0)
          CALL MESSAG('(R I.L.) AT (S)UPPORT (N)O. 1',29,6.5,.5)
          CALL SETCLR('YELLOW')
          CALL STRTPT(1.,2.)
          DO 257 I=1,30
  257     CALL CONNPT(XL2(I),YL9(I))
          CALL VECTOR(4.,.1,6.,.1,0)
          CALL MESSAG('(R I.L.) AT (S)UPPORT (N)O. 3',29,6.5,0.0)
          CALL RESET('HEIGHT')
          CALL ENDGR(0)
          CALL INSERT(7)
          CALL AREA2D(16.,2.5)
          CALL HEIGHT(.25)
          CALL BEAM1
          CALL SETCLR('GREEN')
          CALL STRTPT(1.,2.)
          DO 258 I=1,30
  258     CALL CONNPT(XL2(I),YL10(I))
          CALL VECTOR(3.,.6,5.,.6,0)
          CALL MESSAG('*D) (I.L.) AT 10TH-POINT (N)O. 16',33,5.5,.5)
          CALL SETCLR('YELLOW')
          CALL STRTPT(1.,2.)
          DO 259 I=1,30
  259     CALL CONNPT(XL2(I),YL11(I))
          CALL VECTOR(3.,.1,5.,.1,0)
          CALL MESSAG('*D) (I.L.) AT 10TH-POINT (N)O. 26',33,5.5,0.0)
          CALL RESET('HEIGHT')
          CALL ENDGR(0)
          CALL ENDPL(0)
          IF(IM.EQ.'D') GO TO 385
  350     CALL TABLET('CENTER','LONG')
          CALL LTLINE('(PRESENTATION OF THEORY)$')
          CALL LTLINE('(-----------------------)$')
          CALL CTLINE('$')
```

```
      CALL LTLINE('(T)HE FUNDAMENTAL THEOREM, FROM WHICH THE CONCEPT OF
     *INFLUENCE LINES IS DE-$')
      CALL LTLINE('VELOPED, IS THE PRINCIPLE OF VIRTUAL WORK.  (I)N THE
     *SIMPLEST FCRM, THE$')
      CALL LTLINE('PRINCIPLE STATES...$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL LTLINE('              (F)OR A STRUCTURE WHICH HAS DEFORMED INTO A
     *SHAPE HAVING EX-$')
      CALL LTLINE('              TERNAL AND INTERNAL DISPLACEMENTS, THE MAGNI
     *TUDE OF WORK$')
      CALL LTLINE('              DONE BY THE EXTERNAL FORCES ACTING THROUGH T
     *HEIR RESPEC-$')
      CALL LTLINE('              TIVE EXTERNAL DISPLACEMENTS IS EQUAL TO THE
     *MAGNITUDE OF$')
      CALL LTLINE('              WORK DONE BY THE INTERNAL FORCES ACTING THRO
     *UGH THEIR RE-$')
      CALL LTLINE('              SPECTIVE DISPLACEMENTS.$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL LTLINE('(F)ROM THE PRINCIPLE OF VIRTUAL WORK FOLLOW TWO ADDIT
     *IONAL FUNDAMENTAL THE-$')
      CALL LTLINE('OREMS--THE (M)OMENT-(A)REA THEOREMS, AND THEY STATE..
     *.$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL LTLINE('(M)OMENT-(A)REA (T)HEOREM (N)O. 1$')
      CALL LTLINE('(----------------------------)$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL LTLINE('              (T)HE CHANGE IN SLOPE BETWEEN ANY 2 POINTS O
     *N THE ELASTIC$')
      CALL LTLINE('              CURVE OF A BEAM, IS EQUAL TO THE AREA OF THE
     * (M/EI) DIA-$')
      CALL LTLINE('              GRAM BETWEEN THOSE 2 POINTS.$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
      CALL ENDTAB(0)
      CALL ENDPL(0)
      CALL TABLET('CENTER','LONG')
      CALL LTLINE('(M)OMENT-(A)REA (T)HEOREM (N)O. 2$')
      CALL LTLINE('(----------------------------)$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL LTLINE('              (T)HE TANGENTIAL DEVIATION OF ANY POINT (P)
     *ON THE ELASTIC$')
      CALL LTLINE('              CURVE OF A BEAM, FROM A TANGENT DRAWN AT ANY
     * OTHER POINT$')
      CALL LTLINE('              ON THE ELASTIC CURVE, IS EQUAL TO THE 1ST MO
     *MENT OF THE$')
      CALL LTLINE('              AREA OF THE (M/EI) DIAGRAM BETWEEN THOSE 2 P
     *OINTS TAKEN$')
      CALL LTLINE('              ABOUT POINT (P).$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      CALL LTLINE('(T)HESE (M)OMENT-(A)REA THEOREMS ARE THEN APPLIED TO
     *2 ADJACENT SPANS OF A CON-$')
      CALL LTLINE('TINUOUS BEAM TO YIELD THE GENERAL FORM OF THE 3-MOMEN
     *T EQUATION AS GIVEN$')
      CALL LTLINE('BELOW...$')
      CALL REQ1
      CALL CTLINE('$')
      CALL LTLINE('(F)ROM THIS EQUATION, THE MOMENTS AT THE 3 CONSECUTIV
     *E SUPPORTS CAN BE COM-$')
      CALL LTLINE('PUTED, THUS REDUCING A STATICALLY INDETERMINATE STRUC
     *TURE TO A STATICALLY$')
      CALL LTLINE('DETERMINATE STRUCTURE.  (K)NOWING THE SUPPORT MOMENTS
     * ALLOWS FOR DEVELOPMENT$')
      CALL LTLINE('OF EQUATIONS WHICH DEFINE MOMENTS, SHEARS, AND DEFLEC
     *TIONS AT ANY POINT ON$')
      CALL LTLINE('A CONTINUOUS BEAM.  (T)HESE, IN EFFECT, ARE INFLUENCE
     * LINE EQUATIONS, AND$')
      CALL LTLINE('ARE DEVELOPED IN THE (DEVELOPMENT) SEGMENT OF THIS PR
     *OGRAM.$')
      CALL CTLINE('$')
      CALL CTLINE('$')
      IF(IM.EQ.'E') THEN
        CALL LTLINE('(P)RESS (RETURN) TO RETURN TO (I)NTRODUCTION (M)ENU
     *...$')
      ELSE
        CALL LTLINE('(P)RESS (RETURN) TO CONTINUE...$')
      END IF
      CALL ENDTAB(0)
      CALL ENDPL(0)
      IF(IM.EQ.'E') GO TO 385
  360 CALL TABLET('CENTER','LONG')
```