Testing of unit gradient soil drainage models using undisturbed cores collected by a new technique

by Mark David Tomer

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Soils
Montana State University

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Hydraulic conductivities of core samples were highly variable; therefore none of the models were shown superior in their ability to predict K(0). However, the average of 69 Ks values collected from measurements on core samples was 0.35 cm/min. This was within on order of magnitude of most Km values predicted by field models. Magnitudes of K(0) estimates given by field models generally followed the order CGA > 0 methods > W methods.

A highly satisfactory method of collecting undisturbed core samples from any depth was developed, which should prove most useful for obtaining samples from deep in the profile, where in situ studies are not possible.

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A thesis submitted in partial fulfillment
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of

Master of Science

in

Soils

MONTANA STATE UNIVERSITY
Bozeman, Montana

February, 1986

APPROVAL

of a thesis submitted by

Mark David Tomer

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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March 5, 1986 Date — Dwane G Miller, Head, Major Department

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ACKNOWLEDGMENTS

I wish to thank my advisor, Dr. Hayden Ferguson, for his help and insight which made this project possible. Thanks are also due to the other members of my graduate committee, Dr.'s Jim Bauder, Ralph Olsen, and Doug Dollhopf, for their constructive reviews of this thesis.

A project like this may only be completed with the love and support of one's family. I therefore thank my parents, Albert and Margaret Tomer, for the example and inspiration they provide me. Most of all, I thank my wife, Sandra Schwarzin, for the time she invested in preparing manuscripts, and for her love, patience and companionship.

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Symbol	Meaning
Θ	Soil water content (cm3/cm3)
Θs	Saturated water content (cm3/cm3)
Θm	Maximum field water content (cm3/cm3)
Θ'	Average water content (cm3/cm3)
Θg	Gravimetric water content (g/g)
V	Soil water flux, or flow velocity (cm/min)
H	Total hydraulic head (cm)
h	Matric soil water potential (cm)
he	Air entry potential (cm)
W	Profile water storage (cm)
K	Hydraulic conductivity (cm/min)
Km	Maximum field hydraulic conductivity (cm/min)
Ks	Saturated hydraulic conductivity (cm/min)
K(Θ/h)	Unsaturated hydraulic conductivity (cm/min)
K(Θ)	Unsaturated hydraulic conductivity as a function of soil water content (cm/min)
K(h)	Unsaturated hydraulic conductivity as a function of soil water potential (cm/min)
z	Vertical distance, or a given depth (cm)
D(Θ)	Hydraulic diffusivity as a function of soil water content (cm2/min)
x	Distance (cm)

LIST OF SYMBOLS-Continued

Symbol	Meaning
t	Time
ρ	Fluid density (g/cm3)
η	Fluid viscosity (g/cm min)
k	Intrinsic permeability (cm2)
q	Quantity of flow (cm3)
g	Acceleration of gravity (cm/sec2)
r	Pore radius (cm)
d	Pore diameter (cm)
w	Pore width (cm)
To	Surface tension (dyn/cm)
Y	Wetting angle (degree)
fi	Incremental fraction of pore area (cm2/cm2)
F	Porosity, or pore area (cm2/cm2)
F(r)	Pore size distribution
N	Empirical constant (Eq. [12])
R	Largest water filled pore (Eq. [12]) (cm)
b	Exponent in empirical relationship between 0 and h (Eq. [13])
c,d	Empirical constants in relationship between W and t (Eq. [20])
c',d'	Empirical constants in relationship between V and t (Eq. [22])
c",d"	Empirical constants in relationship between W, z, and t (Eq. [23])
I	Quantity of infiltrated water (Eq. [24]) (cm)

LIST OF SYMBOLS-Continued

Symbol	Meaning
E	Quantity of infiltrated water which evaporates (Eq. [24]) (cm)
U	Quantity of infiltrated water which drains (Eq. [24]) (cm)
ΔM	Quantity of infiltrated water which is stored (Eq. [24]) (cm)
Rat	Neutron count ratio
BD	Bulk density (g/cm3)
β	Exponent in K(0) relationship given in Eq. [32]
α	Exponential parameter in K(0) relationship given in Eq. [32]
a,j	Empirical constant in relationship between 0' and t (Eq. [44]), from "CGA" method
a'	Slope of line relating 0 and 0'
ha	Hectare
m	Meter
cm	Centimeter

ABSTRACT

Several recently proposed mathematical models use a unit gradient assumption and water content measurements collected during drainage of a flood plot to estimate K(θ). The objective of this study was to test the accuracy of five of these models for estimating hydraulic conductivity by comparison with laboratory data collected from measurements on undisturbed cores.

Models tested were the CGA (Chong et al., 1981), and four Lax algorithm models (Sisson et al., 1980), obtained from applying a θ (single measurement) and W (profile storage) approach to two possible K(θ) relationships.

A new procedure for encasing undisturbed cores was developed, which involves coating cores with a liquid rubber latex molding compound. Cores were collected around two adjacent field flood plots located on a silt loam soil. Two laboratory methods were used; direct measurement of hydraulic conductivities, and a method of predicting K(θ) from desorption and Ks data (Campbell, 1974).

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INTRODUCTION

Drainage of water from soil is an important aspect of many environmental problems. Saline seep, groundwater pollution by septic drainfields, aquifer recharge on mine disturbed lands, and irrigation system design are important examples of such problems. Characterization of soil hydrologic behavior would be helpful in finding problem solutions. Data on infiltration rates, desorption curves, and hydraulic conductivities ($K(\theta)$) which can be applied to large land areas are particularly needed. However, little of these types of data are available to land managers and engineers. Reasons for this include time and expense involved with field studies, and difficulty with extrapolating measurements from small plots to large land areas. Determination of $K(\theta)$ has proven particularly difficult for these reasons. Field methods are also inadequate for obtaining data from deep in the profile (<1.5m).

Procedures for field determination of $K(\theta)$ require constant monitoring of flood plot sites for up to one week. Reliability of tensiometers for measuring potential gradients has also been questioned (Baker et al., 1974; Luxmoore et al., 1981). Installation of tensiometer stacks is difficult, and elaborate steps are required to prevent occurence of temperature gradients which adversely affect accuracy (Schuh et al., 1984). Tensiometers therefore contribute to the expense of field studies, while at the same time providing data which may be inaccurate. Assumption of a unit hydraulic gradient, however, allows calculation of K(θ) without tensiometric data. Black et al. (1969) found the unit hydraulic gradient closely approximated actual gradient conditions in a uniform profile after thorough wetting. Since then, several mathematical models have been proposed which provide for calculation of K(θ) using only moisture content measurements at known times during drainage (Libardi et al., 1980; Sisson et al., 1980; Chong et al., 1981). However, different models give different estimates of K(θ) from the same drainage data. The first objective of this study was to determine which model, if any, is most accurate for a specific set of drainage data. It was decided that laboratory measurements would provide the best hydraulic conductivity data to test the models, and that undisturbed soil cores from the field study site would provide the best samples for measurement.

While no soil core sample is truly undisturbed, due to release of overburden pressure, and sample shearing, La Rochelle et al. (1981) have reviewed criteria for minimal soil sample disturbance. For a soil core to be considered undisturbed, it must be collected with minimal effects on structure, density, and moisture content. Also, if K(θ) is to be determined on cores in the laboratory, samples must be encased so that flow does not occur at the boundary between the soil and encasing material. Although attempts at obtaining undisturbed cores for laboratory water flow analysis have been made (Noel, 1982), no encasement method has been satisfactory. Therefore, if the unit gradient models are to be adequately tested by lab data, a satisfactory method of encasing, transporting, and performing laboratory water flow analysis on undisturbed soil cores must be developed. This was the second objective of this work. Invention of such a sampling method would also provide a way to get data from deep in the profile, where flood plot techniques are unreliable.

LITERATURE REVIEW

In recent years proper management of water resources has become more important as demands for quality water have increased. In the arid and semi arid western U.S., where water supplies are limited, improving water use efficiency and maintaining the quality of water supplies are important management objectives. Because of the important role of soils in water supply and use, knowledge of soil water behavior is crucial to achieving these objectives.

Applications of soil hydrologic characterization data include design of irrigation systems, design of septic drainfields, and prediction of aquifer recharge rates. General information often provided in soil surveys, such as drainage classes, is not specific enough to be helpful in engineering problems such as these, where flow rate predictions are required (Bouma, 1973). Knowledge of hydraulic conductivity as a function of water content is needed for these predictions.

Hydraulic Conductivity

Hydraulic conductivity (K) is the constant of proportionality in Darcy's Law (Hillel, 1982) which equates soil water flux (V) and hydraulic gradient (dH/dz):

$$V = K * dH/dz \qquad [1]$$

where K is constant for any given soil which is saturated, and dz specifies vertical flow. Under unsaturated conditions, however, the value of K (cm/min) varies greatly with small changes in soil water content or potential. Therefore Darcy's Law may be written

$$V = K(\theta/h) * d(h+z)/dz \qquad [2]$$

for unsaturated steady state conditions, where $K(\theta/h)$ implies the dependence of K on soil water content and potential conditions, and (h+z) indicates matric and gravitational components of the hydraulic gradient (Hillel, 1982).

Hysteresis is a phenomenon which may complicate determination of $K(\theta/h)$. Water contents are greater during desorption than sorption at a given potential. This has led to questions of hysteresis effects on unsaturated hydraulic conductivity. Although K(h) may be highly hysteretic, $K(\theta)$ is much less so, and effects of hysteresis on $K(\theta)$ are generally ignored. Inference of K(h) from $K(\theta)$ may be difficult, however, due to hysteresis (Topp, 1971).

Hydraulic Diffusivity

Equation [2] may be expressed as

$$V = K(\theta/h) * dH/d\theta * d\theta/dz \qquad [3]$$

$dH/d\theta$ is the reciprocal of soil specific water capacity, which is the slope of the moisture characteristic curve at a specific point. Equation [4]

$$K(\theta/h)\ dH/d\theta = D(\theta) \qquad [4]$$

is substituted into Eq. [3]; where $D(\theta)$ is hydraulic diffusivity, with dimensions x2 t-1. The result is

$$V = D(\theta)\ d\theta/dz. \qquad [5]$$

Calculation of $D(\theta)$ does not require knowledge of soil water potential, but moisture characteristic curves are required to convert $D(\theta)$ to $K(\theta)$. The specific water capacity and moisture characteristic curve is assumed to be uniform in Eq. [5]. Therefore, this equation cannot be applied to heterogeneous soil, and hysteresis effects must be negligible. The term "diffusivity" may be misleading here, as a mass flow rather than a diffusion process is being described (Hillel, 1982).

Water Flow Within Single Pores

The magnitude of K depends on the size districbution, continuity, and geometry of conducting pores, as well as the nature of the fluid. Therefore K may be written as

$$K = g\rho k/\eta, \qquad [6]$$

where ρ/η (fluid density over viscosity) accounts for the temperature dependent effect of the fluid on flow rate. Intrinsic permeability (k) indicates effects of conducting pores. Flow rates within individual pores are governed by Poiseuilles Law, given laminar flow. Equations governing flow through cylindrical and planar voids are

$$q/t = \rho\ g\ \pi\ (r**4)/8\eta * dH/dx \qquad [7]$$

and

$$q/t = \rho\ g\ w\ (d**3)/12\eta * dH/dx \qquad [8]$$

respectively, where q/t is flow rate, g is acceleration due to gravity, and r, d, and w are the pore radius, diameter, and width, respectively (Hillel, 1971). Flow rate dependence on fourth power of the radius means that small changes in pore size can result in large changes in flow rate. Flow velocities within pores also vary, according to distance from the pore boundary wall, as described by:

$$V(x) = \rho\ g\ ((r**2) - (x**2))/4\eta * dH/dx \qquad [9]$$

where x is the distance from the pore's center (Hillel, 1982). Variability of flow velocities within and between pores has important implications for water flow in natural porous systems. Piston type (Darcian) flow can be used to describe quantities and rates of water flow in such systems, but it cannot be used to describe or predict rates of solute movement, which will vary within such systems as do flow velocities.

Pore Size Distribution and Water Flow In Soils

Hydraulic conductivity will vary greatly with small changes in water content (θ). As soil water potential decreases, large pores are drained first, according to the equation

$$h = -2\ To\ (\cos Y)/(\rho g r) \qquad [10]$$

where To is the surface tension of water, Y is the wetting angle, and h is the potential in cm, expressed on a weight basis (Hillel, 1982). When a soil's largest pores are drained, water content may be only slightly reduced, since large pores generally occupy a small fraction of total porosity (except in coarse textured soils). Yet the hydraulic conductivity is greatly reduced due to dependence of flow rate on pore radius raised to the fourth power. Large soil pores (macropores) occur between structural units (peds) in natural soils, and are caused by soil animals, plant roots, and shrinkage due to dessication. Continuous macropores (>.60 microns) may allow water to be channeled rapidly through the solum (Thomas and Phillips, 1979), but there is not complete agreement on the conditions under which this will occur. Arguments based on Eq. [6] state that water tends to enter small pores (micropores) first due to greater capillary (matric) forces, and that larger pores will be filled only after micropores are saturated. Depending on the rate of water addition, however, flow through macropores may occur ahead of a wetting front during infiltration (Bevin and Germann, 1982).

Water content, tillage, and structural properties such as clay orientation and soil fabric, which determine pore size distribution and continuity within peds, will influence extent of macropore flow during infiltration (Thomas and Phillips, 1979). Water movement into peds is affected by intrapedal conductivities and hydraulic gradients, plus the spatial distribution of vertical macropores (Anderson and Bouma, 1977). Bouma (1973) found that presence of intrapedal mottles does not necessarily indicate poor drainage if interpedal pores are available for flow. The importance of macropore flow in infiltration and drainage will therefore vary with soil texture and fabric.

Measurement of Unsaturated Hydraulic Conductivity

Many steady and non-steady state approaches for estimating $K(\theta)$ in the field and lab are available. It is beyond the scope of this review to present them all. Klute (1972) and Black (1965) reviewed laboratory methods of determining $K(\theta/h)$. Herein, discussion of lab methods is limited to pore size distribution models, particularly the method of Campbell (1974). Field methods are then discussed in detail.

One point is to be made relative to lab studies utilizing undisturbed soil cores. Reliability of data from undisturbed cores has been questioned, since accuracy may be affected by changes in sample properties caused by removal from the field. Also, large sample sizes and numbers may be required to obtain representative data (Hillel, 1982). Very few data exist on spatial variability of hydraulic conductivities in the field. The major work has been done by Nielsen et al. (1973), who studied spatial variability of field hydraulic conductivities; using 20 flood plots on a 150 ha field. Maximum hydraulic conductiviy (Km) was log-normally distributed over the 20 plots, with a range nearly two orders of magnitude. There was one order of magnitude difference between mean and mode Km. Variability increased as percent saturation decreased.

Laboratory Determination of K(θ)

Pore Size Distribution Models. A soil moisture characteristic curve (the relationship between θ and h) can be used with Eq. [10] to determine pore size distribution. This may be combined with Poiseuilles Law, Eq. [7], to give an estimate of K(θ/h). Methods of determining K(θ/h) in this way are called pore size distribution models. Childs and Collis-George (1950) were the first to present such an approach. Marshall (1958), Millington and Quirk (1961), and Green and Corey (1971) have presented modifications. One example of such an equation is

$$K = F(\Sigma f_i r_i^2) \rho g/8\eta \quad [11]$$

where F is the porosity, and f_i is the fraction of the pore area (pore volume) occupied by pores of radius r_i. Eight to ten pores size classes are used in the summation. Matching factors are generally used to obtain agreement with experimental values and account for physical difficulties such as swelling of clays, air entrapment, and pore interactions (Kunze et al., 1968). Green and Corey (1971), Bruce (1972), Brust et al. (1968), and Kunze et al. (1968) were all able to obtain reasonable agreement between experimentally measured values and values calculated by various pore size distribution models with matching factors. Hysteresis will complicate direct conversion of a moisture characteristic curve to pore size distribution, particularly due to pore "bottlenecks".

Campbell (1974) proposed a method to estimate K(θ) using desorption data and a pore size distribution model given by Childs (1969) as

$$K = N \int [\int R^{**}2 \; F(r)dr] \; F(r)dr \qquad [12]$$

where R is the radius of the largest filled pore, N is a constant, and F(r) is the pore size distribution function. An empirical expression relating water content to water potential (Hillel, 1971)

$$h = he \; (\theta/\theta s)^{**}\text{-}b \qquad [13]$$

where he is air entry potential, is substituted into Eq. [10] to give

$$r = -(2To/he) \; (\theta/\theta s)^{**}b. \qquad [14]$$

This equation, and

$$F(r)dr = d\theta \qquad [15]$$

are substituted into Eq. [12]. The result is

$$K = N' \; \theta^{**}(2b+2) \qquad [16]$$

where N' includes N and constants from integration. If Ks is determined,

$$N' = Ks / \theta s^{(2b+2)} \qquad [17]$$

and

$$K(\theta) = Ks\ (\theta/\theta s)^{(2b+2)} \qquad [18]$$

A pore interaction term θp with p=1 is included (Jackson, 1972), so

$$K(\theta) = Ks\ (\theta/\theta s)^{(2b+3)} \qquad [19]$$

Campbell (1974) tested this method and found it reliable for most purposes.

<u>Field Determination of K(θ)</u>

Field methods generally fall under two categories; infiltration through an impeding layer (steady state), and drainage from a profile (non-steady state) thoroughly wetted by flooding.

<u>Field methods using flood plots</u>. Richards et al. (1956) were the first to publish results of a flood plot investigation. In their study, a tensiometer stack was installed, and there was no attempt to prevent evaporation. Water contents during drainage were determined gravimetrically from daily collected core samples. Total depth of water stored in the profile, W, above depth z was related to t by

$$W = c\ t^{-d} \qquad [20]$$

where the c and d parameters were determined through regression analysis. This equation was differentiated with respect to time to give drainage rate, dW/dt:

$$dW/dt = c(-d)\ t^{(-d-1)} \qquad [21]$$

Differences in dW/dt between adjacent depths gave flow velocities, which divided by the hydraulic gradient gave K. Flow velocities at 40cm were regressed against time to fit

$$V = c' \, t^{-d'} \qquad [22]$$

which, integrated over time, gave total drainage.

Tensiometric data were used to determine direction (upward or downward) of the hydraulic gradient, from which depth of a "static zone" was determined at different times. This zone was a boundary depth, separating water loss by evaporation (above) and drainage (below). The drainage component was subtracted from total change in W to estimate evaporation losses. Arya et al. (1975) used the static zone approach in the presence of a growing crop.

Ogata and Richards (1957) performed a drainage study in which evaporation was prevented by a plastic cover and straw mulch. Flow velocities were determined as above, except W was fitted to

$$W = c'' \, Z \, t^{-d''} \qquad [23]$$

with flow velocity determined by differentiation. Hydraulic gradients determined from tensiometer data were used to calculate K(h), which increased with depth. Sisson (1972) pointed out that d'' in Eq. [23], given unit gradient conditions, is the ratio between hydraulic conductivity and the depth of water stored in the profile above depth z at the start of drainage.

Nielsen et al. (1964) did a flood plot study on the Panoche clay loam. Their methods were expanded over previous work, in that a neutron probe was used to collect moisture data, and undisturbed cores were collected for determination of moisture characteristic curves.

Neutron probe data were used little, however, since poor depth resolution and inherent scatter of neutron probe data make it difficult to determine water content changes with the accuracy required for calculation of K by Darcy's Law. Changes in θ were therefore inferred from changes in h using desorption curves from core samples. This technique has been used frequently since (Arya et al., 1975; Davidson et al., 1969; LaRue et al., 1968; and Nielsen et al., 1973). Ahuju et al., (1980) derived a method of determining K(h) without desorption data by substituting expressions of K(h) and θ(h) into the flow equation (Eq. [2]).

Rose and associates (Rose et al., 1965; Rose and Stern, 1965) used a water budget approach for determining hydraulic conductivities. Conservation of mass dictates that the quantity of water which penetrates a soil surface (I) will evaporate (E), drain from the soil (U), or be stored in the soil (ΔM):

$$1 = E + \Delta M + U \qquad [24]$$

Recognizing that

$$\Delta M = \int \int (d\theta/dt)\ dz\ dt \qquad [25]$$

and

$$U = \int Vz\ dt \qquad [26]$$

rearrangement of Eq. [24] and substitution of Eq. [25] and Eq. [26] into the result gives

$$\int (I - E - Vz)\ dt = \int \int (d\theta/dt)\ dz\ dt \qquad [27]$$

Substituting

$$Vz = Kz + Kz\ dh/dz \qquad [28]$$

into Eq. [27] and solving for Kz gives

$$Kz = \int (I - E - (\int d\theta/dt\ dz))dt\ /\ t(dh/dz + 1) \qquad [29]$$

This approach was applied using four plots established on a fine sandy loam field soil, where moisture profiles were determined wih a neutron probe, soil water potentials were inferred from characteristic curves, and initial evaporation rate was estimated from Penman's equation (Hillel 1982). Linear regression of ln K on θ gave estimates of K(θ) at four depths. Values of K(θ) decreased with increasing depth, and scatter of results were attributed to plot differences, neglect of hysteresis effects below 100 cm depth, and measurement variability.

A commonly used method of determining K(θ) is the instantaneous profile method (IPM) (Watson 1966), designed to include possible dynamic effects of non-steady state flow in calculation of K, and first tested in the lab using soil columns. Assuming one dimensional vertical flow and no evaporation,

$$(d\theta/dt)z = (-dV/dz)t \qquad [30]$$

If water content and soil water potential profiles are monitored during drainage, velocity profiles for any time may be obtained by integrating dθ/dt over depth. Potential profiles (combining matric and gravitational components) are plotted, and the gradient at any depth is equal to slope of the plot at that depth. Hydraulic conductivity is obtained by dividing velocity at a given time and depth by corresponding potential gradient, which may then be plotted against the corresponding water content to determine the K(θ) curve.

Hillel et al. (1972) provide a summary of the data handling procedure for use of the IPM in the field, where no water table is present.

Cassel (1974) developed a computer program for calculation of average θ, flow rate, hydraulic gradient, and conductivity for all depths and times, given desorption and tensiometer data collected during drainage of a covered flood plot.

VanBavel et al. (1968) used the instantaneous flux method in the field with a static zone approach, but found that it breaks down when water movement lags behind diurnal variation in evaporative demand.

The Unit Hydraulic Gradient

Under steady state conditions, which are approached during drainage of a thoroughly wetted profile, the matric component of the potential gradient becomes negligible, and the gravitational component (equal to one) dominates. A unit hydraulic gradient results, and Eq. [2] becomes

$$V = -K \qquad [31]$$

Black et al. (1969) found that $K(\theta)$ curves determined in the lab under unit gradient conditions agreed with drainage rates found in the field, indicating unit gradient conditions in the field. Van Bavel et al. (1968) found near unit gradient conditions during drainage of a deeply wetted fallow soil. Where a unit gradient can be applied, tensiometric data are not necessary to calculate potential gradients. This greatly simplifies field determination of $K(\theta)$.

Several mathematical models have been proposed recently which assume a unit gradient, and provide for calculation of K(θ) using only water content measurements at known times during drainage. These are the Lax algorithm (Sisson et al., 1980), the "CGA" (Chong et al., 1981), and the Theta and Flux models (Libardi et al., 1980).

Lax Algorithm. Sisson et al. (1980) summarized use of the Lax algorithm. Two possible K-θ relationships are given by Watson (1967) and Davidson et al. (1969), respectively, as

$$K(\theta) = Km\ (\theta/\theta m)^{**}1/\beta \qquad [32]$$

$$K(\theta) = Km\ \exp\ \alpha(\theta - \theta m) \qquad [33]$$

where θm is maximum water content, Km is maximum hydraulic conductivity, and α and β are exponential constants to be determined. Differentiation of K with respect to θ gives

$$dK/d\theta = A\ (\theta/\theta m)^{**}(1-\beta/\beta) \qquad [34]$$

and

$$dK/d\theta = A'\ \exp\ \alpha(\theta - \theta m) \qquad [35]$$

where A = Km/βθm and A' = Kmα. Replacement of dK/dθ with z/t and rearrangement of Eq. [34] and Eq. [35] gives θ as a function of z and t:

$$\theta = \theta m\ (z/At)^{**}\beta/1-\beta \qquad [36]$$

$$\theta = \theta m + 1/\alpha\ \ln\ (z/A't) \qquad [37]$$

These can be rearranged to show a linear relationship between ln θ/θm and ln t for the Watson (1967) equation, and (θm-θ) and ln t for the Davidson et al. (1969) equation:

$$\ln \theta/\theta m = -\beta/1-\beta\ \ln A/z - \beta/1-\beta\ \ln t \qquad [38]$$

$$\theta m - \theta = 1/\alpha\ \ln A'/z + 1/\alpha\ \ln t \qquad [39]$$

Integration of Eq. [36] and Eq. [37] over depth gives

$$W = \int \theta \, dz = z \, (1-\beta) \, \theta m \, (z/At) ** \beta/1-\beta \qquad [40]$$

$$W = \int \theta \, dz = \theta m \, z + z/\alpha \, [\ln \, (z/A't) \, -1] \qquad [41]$$

where W, the total water stored above depth z, is expressed as a function of time. Rearranged, Eq. [40] and Eq. [41] show linear relationships between ln W/z and ln t (Eq. [42]) and between W/z and ln t (Eq. [43]):

$$\ln W/z = \ln \, ((1-\beta) \, \theta m \, (A/z)**-\beta/1-\beta) - \beta/1-\beta \ln t \qquad [42]$$

$$W/z = \theta m - 1/\alpha \, (1+ \ln A'/z) - 1/\alpha \ln t \qquad [43]$$

Slope and intercept results from regressions of ln W/z and W/z (dependent) versus ln t (independent) are used to calculate Km and exponential constants for the W method. Regressions for the θ method (Eq. [38] and Eq. [39]) are ln θ/θm and θm-θ (dependent) versus ln t.

CGA Method. The CGA method (Chong et al., 1981), assumes that under flood plot drainage conditions the average water content (θ') above any depth is related to time by

$$\theta' = a \, t**j \qquad [44]$$

where e is less than zero. Differentiating with respect to time yields

$$d\theta'/dt = aj \, t ** j-1 \qquad [45]$$

If Eq. [45] is multiplied by z, under the unit gradient theory

$$z \, d\theta'/dt = K \qquad [46]$$

so

$$K = zaj \, t ** j-1 \qquad [47]$$

Equation [44] is then solved for t, and the result is substituted for t in Eq. [47], leaving K(θ') as a function of (θ')

$$K(\theta) = z\ j\ a^{1/j}\ \theta'^{((j-1)/j)} \qquad [48]$$

The unknowns a and j are determined by expressing Eq. [44] in its logarithmic form:

$$\ln \theta' = \ln a + j \ln t \qquad [49]$$

and by linear regression of ln θ' on ln t using the drainage data. Note that W/z = θ', so the regression variables for the Watson W and CGA methods are identical.

Theta and Flux Methods. Libardi et al. (1980) present two methods of determining K(θ), using water content data collected during flood plot drainage and a unit gradient assumption. Combining Eq. [33], Eq. [46], and a linear relationship between θ and θ' results in

$$a'\ z\ d\theta/dt = Km \exp\ [\alpha(\theta - \theta m)] \qquad [50]$$

where a' is the slope of the θ vs θ' relation. Integrating with respect to time gives (at large times)

$$\theta m - \theta = 1/\alpha \ln t + 1/\alpha \ln \alpha Km/a'z \qquad [51]$$

which is similar to the result of the Lax algorithm where the same K(θ) expression is used. This is the Theta method. The logarithmic form of Eq. [50] is called the Flux method.

Previous Studies Utilizing Unit Gradient Models. Dane (1980) found excellent agreement between the Theta method and the IPM on a sandy soil. Luxmoore et al. (1981) found good agreement between the IPM and the flux method, although the Theta and CGA methods gave high estimates of K(θ). Jones and Wagenet (1984) found that the CGA method gives higher estimates of K(θ) than the Theta and Flux method, and that the W method (Lax algorithm) gives lower estimates.

Schuh et al. (1984) compared the IPM with the method of Ahuju et al. (1980), and Libardi's Flux and Theta methods on 5 soils. Ahuja's method and the IPM agreed well on all soils. Libardi's methods gave good estimates of drainage, but poor estimates of K(θ), particularly on layered soils. This was attributed to problems with indirect calculation of Km.

Methods Using Infiltration Through an Impeding Layer. Hillel and Gardner (1969) first proposed this method, and tested it in the lab. Impeding crusts were placed on soaked soil columns, which had two tensiometers installed at 2 and 12 cm. A minimal ponding depth was maintained over the crust until steady state flow was attained. Equation [31] and tensiometer data give one point on the K(h) curve. Increasing crust resistance led to increased tension and lower flow rates. The method has been successfully used in the field. Bouma et al. (1971) were the first to publish field results. In the field, crust resistances are decreased, resulting in K(h) data for wetting, rather than drying.

Baker et al. (1974) report that conductivities for potentials greater than -20 cm cannot be accurately determined from the IPM, and that the crust technique gives better results in this range, which is important to evaluation of septic drainfield suitability.

Gardner (1970) presented a method for calculation of D(θ) in the field, using the crust technique. Under unit gradient conditions,

$$z(d\theta/dt) = K(z) \qquad [52]$$

is expressed as

$$z \, d\theta/dh \, dh/dt = K \qquad [53]$$

or

$$z\ dh/dt = D, \qquad [54]$$

so the average diffusivity above any depth z may be calculated with a single set of tensiometer data.

PROBLEM STATEMENT

Information obtained from the K(θ) relationship includes K values at specific water contents and rate of change in K with changes in water content. Since water movement is most rapid when soil water content is high, (>30%), K values for high water contents are most important in determining the quantity of water which will rapidly drain from a soil. This project's basic objective was to test the Theta and W Lax algorithm (Sisson et al., 1980) and "CGA" (Chong et al., 1981) unit gradient models for accurate estimation of Km and K(θ) with laboratory data collected from measurements on undisturbed cores. To ensure accuracy of the lab data, the second objective was to devise a reliable procedure for encasing undisturbed cores which would minimize disturbance, and allow for safe transport and reliable water flow analysis. The models were applied to two sets of data from one soil, and undisturbed soil cores were collected around the flood plots where drainage data was obtained.

Given the extreme variabilility of hydraulic conductivities in the field (Nielsen et al., 1973), and time and expense involved in obtaining a large sample size, statistical comparison of lab measurements and field estimates of K values was not attempted. Field unit gradient models which give K(θ) estimates within the range of lab determined values were considered valid. Data were collected on the

Bozeman silt loam. Results may apply only to soils of similar texture and morphology.

MATERIALS AND METHODS

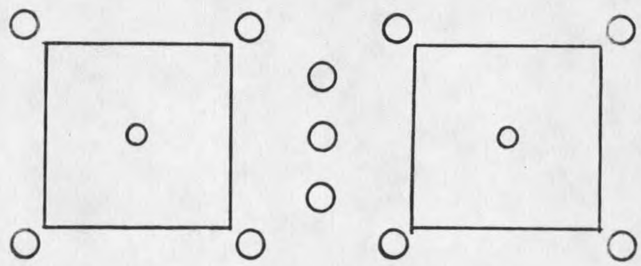
Bozeman silt loam (Pachic Argiboroll) was chosen as the field soil for this study. Two 3.65 by 3.65 m plots 3.05 m apart were established at the A.H. Post Experimental Farm, 9.7 km west of Bozeman, Montana. After the soil surface was leveled, an aluminum neutron probe access tube was installed to a depth of 2.4 m at the center of one plot (Plot A) and to a depth of 1.8 m at the center of the other (Plot B). Soil cores were extracted with a Giddings soil sampler. Cores collected around plot perimeters were 5cm in diameter, while cores extracted for access tube installation were 3.5 cm in diameter.

Flood Plot Procedure

On August 1, 1984, after each plot had been diked with 3.65 m 30.5 cm x 2.5 cm boards, they were flooded with the Post Farm's irrigation water. Neutron probe readings were taken at 15 cm increments (the first depth was at the 15 to 30 cm midpoint, 22.5 cm) before flooding began, and at 15 minute intervals for two hours thereafter. Intervals were then increased to one half hour until the wetting front reached a depth of about 1.5 m (4-5 hours after flooding began). Dikes were then broken, and infiltration ceased shortly thereafter when free water was no longer standing on the soil surface. At that time the plots were covered with a plastic tarp to prevent evaporation.

To characterize drainage, neutron probe readings were taken every twenty minutes to half hour for the first four or five hours of drainage. Reading intervals were then increased to one hour for the next 12 hours, then to two hours for the next 12 hours. Readings were then taken 3 or 4 times daily for 4 days, twice daily for 5 days, and once daily for the next ten days. Readings were then taken once every two or three days, until the last reading was taken about 29 days after drainage began.

Figure 1. Field study design showing two 3.65 m by 3.65 m plots. Small circles show neutron probe access tubes, large circles show sample collection points.



Field Study Layout

While drainage was being monitored, undisturbed cores were collected from around the plots as shown in Figure 1, and encased as described below. Cores removed for neutron probe access tube installation were similarly sectioned and encased. Eleven cores were collected outside the plots from each 15 cm depth increment, so 13 cores from each depth were collected. The coring depth was 1.8 m at all points, except that the four holes around plot A were sampled to 2.4 m. Care was taken to ensure that cores were sectioned so that the middle of each core coincided with a depth at which neutron probe readings were being taken.

Field Encasement of Core Samples

This procedure was developed to minimize handling of core samples. In the operation of the Giddings sampler, the core barrel was driven into the soil slowly (about 1/2 cm/sec), in order to minimize frictional resistance and therefore sample disturbance. The core barrel was 1.3 m in length, so the sampler was driven into the soil twice to recover soil from 1.8 or 2.4 m. There was generally full sample recovery.

After extraction, the soil core was sectioned into 15 cm lengths while still in the core barrel. The core section was then slid to the upper, open end of the core barrel, and trimmed flush. An eight inch length of PVC plastic pipe (2" ID for 3.5 cm diameter samples, 2 5/8" ID for 5 cm samples) with a fiberglass screen glued to one end was then placed over the core barrel. A small plastic bag was attached around the PVC pipe with a rubber band. A cardboard spacer was used to center the soil core inside the PVC pipe. After checking to see that the trimmed end of the core was against the screen, the tube and pipe were rotated 90 degrees to a vertical position, taking care not to disturb the core remaining at the core barrel's lower end. The PVC pipe was then lowered away from the Giddings core barrel, leaving the soil core standing balanced on the screen, inside the PVC pipe. The pipe was then clamped to a rack to facilitate sample encasement in the field.

To encase the core sample, a liquid rubber latex molding compound[1] was poured over the soil core, taking care to coat it completely. Excess latex was collected in the attached plastic bag for reuse. This rubber latex dries to form a mold around the sample, yet is viscous enough in its liquid state to prevent it from penetrating the soil itself. After the latex dried (1-2 days), wax was melted and poured over the core to prevent evaporation. The PVC pipe was then filled with a light weight concrete (75% Zonolite, ground to pass a #10 seive, 25% cement). After the cement had dried, the samples were ready for transport to the laboratory.

Neutron Probe Calibration

The neutron probe was calibrated for Bozeman silt loam at the Post Farm in September, 1984. Sixty five core samples 15 cm in length were collected for determination of gravimetric water content (θg) and bulk density (BD), from which volumetric water content (θ) was calculated by

$$\theta g \times BD = \theta. \quad [55]$$

Neutron counts were taken in the bore holes from which the cores were extracted immediately after collection. Care was taken to place the fast neutron source at the center of the 15 cm depth increment from which the core samples were taken. Moist and dry soil conditions were

[1]Product #4701, manufactured by Adhesive Products, 1660 Boone Ave., Bronx, NY 10460.

included in the sampling. Linear regression was performed, using volumetric % moisture as the dependent variable, and neutron count ratio as the independent variable. The result of the calibration was

$$\theta = 45.474*(Rat) - 5.703 \quad [56]$$

where Rat denotes neutron count ratio. The r2 of the regression was 0.95.

Treatment of Drainage Data

The Lax algorithm was applied to two forms of the K - θ relationship (Eq. [32] and [33], referred to herein as the Watson and Davidson equations, respectively) with a θ and W based approach for each, for a total of four Lax algorithm models. The CGA Method was also used, so 5 models were applied to the drainage data. Therefore five K - θ relationships were estimated at each plot and depth from the drainage data. Table 1 shows regression variables and constants determined by regression for each model. Moisture content data collected during drainage were converted to the dependent variables using a Fortran program on the CP6 computer. Regressions were run with the MREGRESS program on MSUSTAT (Lund, 1979), and the constants defining the K(θ) equations were calculated from slope and intercept results (see Appendix I).

Laboratory Procedures

Two methods of determining K(θ) were attempted in the lab using the undisturbed cores; direct measurement, and the method of Campbell (1974).

Table 1. Regression variables used in determination of K(θ) by each model tested, with constants determined by regression results for each model.

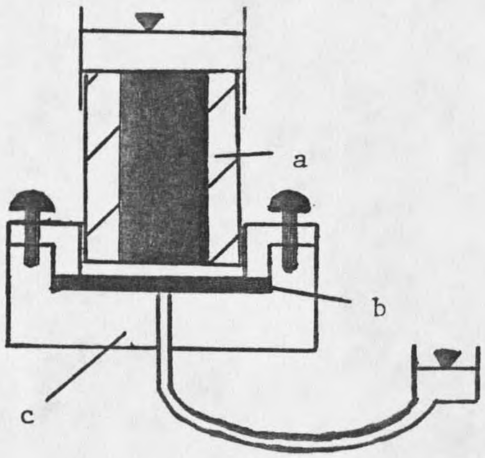
Model	Regression Variables: Dependent	Regression Variables: Independent	Constants Determined
Lax:			
θ, Watson	ln (θ/θn)	ln t	Km, β
θ, Davidson	θn-θ	ln t	Km, α
W, Watson	ln (w/z)	ln t	Km, β
W, Davidson	w/z	ln t	Km, α
CGA:	ln θ'	ln t	a,j

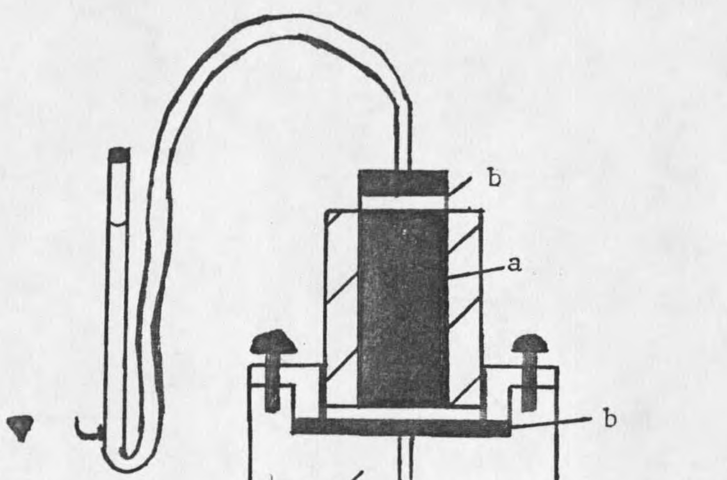
Direct measurement of Ks and K(θ) are discussed by Klute in Black (1965). Various porous plates were examined to determine their ability to conduct water rapidly (for Ks determinations), and still hold water under tensions for K(θ) determinations. Two sintered bronze powder plates were selected which could hold tensions up to 30 cm of water.[2] Two permeameter bases were built from plexiglass. Figure 2 shows the permeameter design and set-up for determining Ks and K(θ) directly.

The procedure for cutting the core samples open, saturating and performing the flow analysis was as follows. A carborundum impregnated wire gauze blade was used to cut the samples on a table saw. The PVC pipe was first cut, then the cement and rubber, so the core sample unit was rotated during cutting. The soil core itself was then broken

[2]Sintering was done by Alcan Ignot and Powders, Elizabeth, NJ.

Figure 2. Permeameter design for determination of Ks (top) and K(θ) (bottom) showing water levels (▼), soil column (a), porous pads and plates (b), and plexiglass base (c).





carefully by hand. After both ends of the core were cut and broken, the core length was measured, and outside edges of the PVC pipe were beveled with a wood rasp to allow easier installation of the core sample into the permeameter. The permeameter brace and PVC connector (for ponding water for Ks determination) were then placed around the bottom and top of the core sample, respectively. Exposed cement was then greased to ensure water flow did not occur through the cement. Any cavities at the bottom of the core were filled with fine glass beads, and an O ring was placed at the bottom of the core unit. A cellulose filter pad was saturated, placed on the bottom of the core, and sealed to the O ring with high vacuum grease. The core unit was then braced to the saturated porous plate with screws, and the soil was saturated overnight from below.

For determination of Ks, a bubbler was used to maintain a constant ponding depth over the top of the core. The outflow level was kept constant, and rate of outflow was monitored for calculation of Ks. To measure unsaturated conductivity, the water column conducting outflow was clamped, and the water ponded on the top of the core was removed. Cavities on the top of the core were filled with fine glass beads, a saturated filter pad was applied, and a weighted porous plate, attached to a burette by a water column, was placed on the pad. The burette bubbler height was then adjusted to the desired tension, and a unit gradient was established through the core. Inflow rates were measured for calculation of $K(\theta)$. Flow rates and unsaturated hydraulic conductivity were determined at 10 and 20 cm of tension. Subsamples of the cores were collected at the saturation and the tensions applied, for