Calculation of important design parameters for grounding systems in substations
by Arun Balakrishnan

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University

Abstract:
An important aspect of substation grounding system design is the calculation of the ground resistance offered by the grounding grid, and the values of the mesh and step voltages on the surface. The values of these parameters have to be kept within certain established Emits, keeping in view the safety of the personnel present within and without, the substation area.

Simple expressions for calculation of these parameters for square and rectangular grids are already available. Computer programs for designing substation grids, regardless of their shape, are available, but tend to be tedious in usage when the grids are not symmetrical, since a large amount of data has to be input to describe the grid configuration. Thus simplified expressions are preferred.

Another important factor in grounding grid design, is the footing resistance, which depends on the resistivity and thickness of the surface rock layer. The footing resistance, at present, can be determined by use of an infinite series, which is not very easy to evaluate. There is hence need for a simplified finite expression for calculating the footing resistance. The value of footing resistance also affects the safe values of the mesh and step voltages.

If two substation grounding grids are intertied by a bare conductor, when a fault occurs at one substation, it is important to determine, by a simple analytical procedure, the effect of this fault current on the other grounding grid, and the response of the system as a whole, to the fault current.

The objectives of this thesis may be summarized as follows: (i) To develop a new set of expressions, which may be used for calculation of the ground resistance, and the mesh and step voltages, for all practical shapes of grounding grids, once information regarding the grid configuration and soil characteristics is available.

(ii) To develop a finite expression, for calculating the footing resistance in a substation.

(iii) To design amethod to calculate the total groundresistance of a system of two substation grounding grids intertied by a bare conductor, as seen from the station where the fault occurs, and the ground potential rise at both the substations.

The development of the expressions for the calculation of the ground resistance, mesh and step voltages, footing resistance, and for evaluating the performance of intertied grids, is described in this thesis. Through a comparison with existing expressions for the calculation of these parameters, the considerable reduction in error through the use of the newly developed expressions is demonstrated. The scope for future work in this area is also discussed briefly.

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ABSTRACT

An important aspect of substation grounding system design is the calculation of the ground resistance offered by the grounding grid, and the values of the mesh and step voltages on the surface. The values of these parameters have to be kept within certain established limits, keeping in view the safety of the personnel present within and without, the substation area.

Simple expressions for calculation of these parameters for square and rectangular grids are already available. Computer programs for designing substation grids, regardless of their shape, are available, but tend to be tedious in usage when the grids are not symmetrical, since a large amount of data has to be input to describe the grid configuration. Thus simplified expressions are preferred.

Another important factor in grounding grid design, is the footing resistance, which depends on the resistivity and thickness of the surface rock layer. The footing resistance, at present, can be determined by use of an infinite series, which is not very easy to evaluate. There is hence need for a simplified finite expression for calculating the footing resistance. The value of footing resistance also affects the safe values of the mesh and step voltages.

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CHAPTER 1

INTRODUCTION

Background and Problem Definition

A grounding system in a substation has two objectives:

(1) To carry electric currents to ground under normal and fault conditions, without exceeding any operating and equipment limits, or adversely affecting continuity of service;

(2) To ensure that a person in the vicinity of the grounded facilities is not exposed to the danger of critical electric shock.

The ground return circuit is defined [1] as the circuit in which the earth or an equivalent conducting body is utilized to allow current circulation from or to its current source. The ground is a conducting connection by which an electric circuit or equipment is connected to the earth or some conducting body of relatively large extent that serves in place of the earth. A system or apparatus is provided with ground for purposes of establishing a ground return circuit and for maintaining its potential at approximately the potential of earth.

Whenever a fault occurs in a substation, the earth becomes saturated with currents flowing to ground from the grounding grid and other ground electrodes buried below the surface of the earth. The substation grounding grid is a system of horizontal, interconnected bare ground conductors, spaced a few meters apart, buried in the earth at a depth varying from 0.25 to 2.5 meters, which provides a common ground to all the equipment present in the substation.

All grounding systems offer a finite resistance (to remote earth) to the fault currents. It is very important to have a very low value of this resistance, referred to as the ground resistance of a substation grid, which is preferably about 1 ohm for large substations, and between 1 ohm and 5 ohms for distribution substations. While under normal conditions, the grounded apparatus in a substation operates at near zero ground potential, under fault conditions, the portion of the fault current that is conducted into the earth by the grounding grid causes the potential of the grid to rise with respect to remote earth. This rise in voltage of the grounding grid is referred to as the ground potential rise (GPR), and is proportional to the magnitude of the fault current, and the ground resistance of the grounding grid.

Two other important design parameters, which depend upon the value of the ground potential rise and the ground resistance of the grounding grid are the mesh voltage and the step voltage. The conductors of a grounding grid divide it into a number of meshes, and the potential on the surface of the earth, above the grounding grid, is not the same at all points. The touch voltage is defined as the potential difference between the ground potential rise, and the surface potential at the point where a person is standing, while being in contact with a grounded structure. The maximum value of the touch voltage within the substation yard is known as the mesh voltage. The step voltage is defined as the difference in the surface potential experienced by a person, with his feet one meter apart, with no part of his body being in contact with any grounded structure.

The human body, for dc, and ac power frequencies (50 and 60Hz), can be represented as a finite noninductive resistance, of value in the range 500 to 5000 ohms, and is usually approximated by a value of 1000 ohms [1]. There is hence a critical limit to the amount of shock energy that can be absorbed by a human being. Crossing this limit, which varies from person to person depending upon factors including the body weight, can prove to be dangerous. The most common physiological effects of electric current on the body are perception, muscular contraction, unconsciousness, fibrillation of the heart, respiratory nerve blockage, and burning. The fibrillation threshold, which can prove fatal, is reached when currents of magnitude 60 -

100 mA are reached. The duration for which the fault current can be tolerated by most people is given as [1]

$$(I_B)^2 t_s = S_B \tag{1.1}$$

where I_B is the rms value of current, in amperes, flowing through the body, t_s is the duration of the current flow, in seconds, and S_B is the an empirical constant related to the electric shock energy tolerated by a certain percentage of a given population.

Based on studies conducted by Dalziel [2], a current I_B that can be tolerated by 99.5% of all the population without ventricular fibrillation, has a magnitude given by

$$I_B = \sqrt{S_B/t_s} \tag{1.2}$$

Here, $\sqrt{S_B}$ has a value of 0.116 based upon the fact that the shock energy that can be survived by 99.5% of people weighing 50 kg, results in a value of S_B of 0.0135. Thus for a 50 kg body weight,

$$I_B = 0.116/\sqrt{t_s} \tag{1.3}$$

For a 70 kg body weight, the value of current I_B is obtained as

$$I_B = 0.157/\sqrt{t_s} \tag{1.4}$$

Since the above equations are based on tests for the 0.03-0.3 seconds time range, they are not valid for very short or long times. Ferris, *et al* [3], suggest a value of 100 mA as the fibrillation threshold if duration of the shock is not specified. Beigelmeier [4] has suggested values of 500 mA and 50 mA respectively, for shock durations of less, and more, than one heartbeat duration. It is thus important that the grounding system be designed in a manner to keep the shock currents below the value mentioned above. The resistance of the human body, R_B, is assumed to be about 1000 ohms for defining the limits on the step and touch voltages.

Substations usually have a layer of crushed rock spread on the soil. This usually provides a high resistivity layer below the feet of personnel in the substation. A highly simplified approach, which neglects the mutual ground resistance between the two feet of the person, and assumes a very large depth of the crushed rock layer, gives the value of the footing resistance (which is defined as the resistance of the ground below the feet), R_{foot}, as $3\rho_s$, where ρ_s is the resistivity of the crushed rock layer. This expression is derived by modelling the foot as a conducting disc of radius 8 cm. In accidental circuits for mesh and step voltages, the two feet are in parallel and series respectively; therefore, the total footing resistance is taken to be $1.5\rho_s$ and $6\rho_s$. The layer of crushed rock greatly increases the contact resistance between the feet and the substation surface, which greatly reduces the current flowing through the body of a person present in the substation.

The effect of the layer of crushed rock spread on the surface of the substation area on the value of the fault current flowing upwards into the body depends on its thickness, the relative resistivity of the crushed rock layer and the lower soil, and the resistance of the foot.

The safety of a person depends on prevention of his absorbing the critical amount of shock energy before the fault is cleared and the system re-energised. The driving voltage of any accidental circuit should not exceed the limits defined below.

The limit for the touch voltage is given as

$$E_{touch} = (R_B + 0.5R_{foot})I_B \tag{1.5}$$

With $R_B = 1000\Omega$, and $R_{foot} = 3\rho_s$, equation (1.5), for a 50kg body weight, can be written as

$$E_{touch(50)} = (1000 + 1.5 {*} C_s(h_s, K)\rho_s)0.116/\sqrt{t_s} \tag{1.6}$$

The limit for the step voltage is given as

$$E_{step} = (R_B + 2R_{foot})I_B \tag{1.7}$$

Again, with $R_B = 1000\Omega$, and $R_{foot} = 3\rho_s$, equation (1.7), again for a 50kg body weight, can be written as

$$E_{step(50)} = (1000 + 6 {*} C_s(h_s, K)\rho_s)0.116/\sqrt{t_s} \tag{1.8}$$

The factor C_s is included to account for the layer of crushed rock which is spread on top of the soil in the substation area, and is of resistivity different from that of the soil. It is a factor applied for compensating for the finite thickness of the surface crushed rock layer. This factor and its calculation form an important aspect of substation grounding grid design, and are discussed in detail later. For a uniform soil, with no upper crushed rock layer, the value of C_s equals unity. ρ_s is then the resistivity of the soil in Ω-m, and t_s is the duration of shock current in seconds. The calculated value of mesh voltage should not exceed the maximum allowable touch voltage, E_{touch}. Similarly, the calculated value of the step voltage should not exceed the maximum allowable step voltage, E_{step}. The values of $E_{step(50)}$ and $E_{touch(50)}$ are calculated assuming the body weight of the person experiencing these voltages to be 50kg.

Calculation of the mesh and step voltages, and the ground resistance, as part of a design methodology, can be made using different methods. A number of computer programs have been developed for this purpose, and give fairly accurate results. Unfortunately, programs are not always available in field sites. Also, preliminary design strategies do not require the high accuracy offered by these programs. It is hence important to have a simplified set of expressions, which may be used to compute these design parameters.

Most of the work so far has concentrated on grids which are of shapes approaching a square or a rectangle. Though simplified equations for making calculations of the ground resistance, and mesh and step voltages for such grids have been developed, and have been in use for some time, not all grids can be approximated by a square or rectangle, and for the design of such grids, the use of the existing expressions leads to erroneous results.

The following site parameters are known to have a substantial impact on the grid design: maximum fault current(I_G), fault duration(t_f), soil resistivity(ρ). The substation grid design depends on the area of the grounding system, the conductor spacing, the depth of burial of the grounding grid, and the shape of the grid.

The objectives of this thesis are to develop a new set of expressions, which can be applied to the design of substation grounding grids, of all practical shapes.

They may be summarized as:

(i) To develop a new set of expressions, which may be used for calculation of the ground resistance, and the mesh and step voltages, for all practical shapes of grounding grids.

(ii) To develop a finite expression, for calculating the footing resistance in a substation, which is easy to use, and gives fairly accurate results.

(iii)To design a method to calculate the total ground resistance of a system of two substation grounding grids intertied by a bare conductor, as seen from the station where the fault occurs, and the ground potential at both the substations.

The results of the research covered in this thesis have been presented in references [5], [6], [7], and [8].

Chapter 1 has thus far given a brief introduction to the areas of substation grounding with which the topics covered in this thesis are related. Some work regarding calculation of the ground resistance, mesh voltage, step voltage and footing resistance is available in literature, and is summarized in the next section of this chapter.

Chapter 2 covers the algorithm of the program RESIS which has been used as the primary development reference for the work involving the calculation of ground resistance and mesh and step voltages reported in this thesis. Relevant equations for developing the resistance matrix are also discussed.

In Chapter 3, the calculations involving the grounding resistance have been described, and a comparison of the new expression for ground resistance, presented in this thesis, has been made with expressions already available in literature. The accuracy of the simplified expressions is also discussed for various shapes of grounding grids.

Chapter 4 covers the development of the expressions for the calculation of mesh and step voltages for grids of irregular shapes. A comparison of the new expressions with the methodology recommended by the IEEE Std. 80 [1] is also made.

Chapter 5 describes the development of the expression for the calculation of the footing resistance. In addition, two fairly accurate analytical models to calculate the footing resistance have also been presented. A comparison of the new expression with the expressions recommended by the IEEE Std. 80-1986 is made, and the accuracy of the new expression and the two models demonstrated.

Chapter 6 provides details about the method for determining the performance of two substation grids intertied by a bare, buried conductor, when a fault occurs in one of the substations. From the suggested method, the total ground resistance of the system, as seen from the station where the fault occurs, and the ground potential rise at both substations can be calculated. A simplified method, derived using the transmission line equations, is discussed in detail.

The thesis is concluded with a brief summary of the work done, and its benefits, in Chapter 7. Possible future work is also discussed.

Literature Survey

This section gives a brief summary of the expressions already available for the calculation of the ground resistance, mesh and step voltages; and the footing resistance, and the performance of interconnected grounding systems.

A minimum value of the ground resistance offered by a grid buried in soil of uniform resistivity, can be obtained from the expression for the resistance of a circular plate, which may be applied to square grids. This expression is given as [1]:

$$R_g = \frac{\rho}{4}\sqrt{\frac{\pi}{A}} \tag{1.9}$$

where R_g is the resistance to ground of the substation grid in ohms, ρ is the resistivity of soil in ohm-meters, and A is the area of the grounding grid in square meters.

The upper limit to the ground resistance of substation grids with square shapes can be obtained by use of the formula given by Laurent [9], and Niemann [10]:

$$R_g = \frac{\rho}{4}\sqrt{\frac{\pi}{A}} + \frac{\rho}{L} \tag{1.10}$$

where L is the total length of grid conductors in meters.

The factor ρ/L accounts for the fact that the resistance of a solid plate is always lower than that of a grid of the same shape, composed of a finite number of conductors. As can be seen from the formula, the difference in resistance reduces, and approaches zero as the total length of the conductors gets infinitely large. While these expressions may be used with reasonable accuracy for square grids buried in the soil at depths less than 0.25 meters, for grids buried between a minimum of 0.25 meters and a maximum of 2.5 meters, Sverak [11], introduces a correction factor to account for the variation in the depth of burial, and develops the expression for the calculation of ground resistance as

$$R_g = \rho\left[\frac{1}{L} + \frac{1}{\sqrt{20A}}\left(1 + \frac{1}{1 + h\sqrt{20/A}}\right)\right] \tag{1.11}$$

where h is the depth of burial of the grid in meters.

The resistance of a grid, embedded in a soil of uniform resistivity, can also be calculated by use of the expression presented by Schwarz [12]. Schwarz' expression is valid for multi-grid systems, and may be simplified for a single grid, without ground rods, in a soil of uniform resistivity ρ, as

$$R_g = \frac{\rho}{\pi L}\left(\ln\frac{2L}{a^*} + K_1\frac{L}{\sqrt{A}} - K_2\right) \tag{1.12}$$

where $a^* = \sqrt{2rh}$, for conductors buried at a depth of h meters, and r is the radius of the conductors in meters.

A limitation of the expression proposed by Schwarz is that the factors K_1 and K_2 can be obtained only by use of graphs, which prevents an analytical solution using a computer. This deficiency may be overcome by use of the expressions for K_1 and K_2 derived by Kercel [13]. These expressions are given as:

$$K_1 = 1.84\sqrt{\frac{ab}{2}}\left[\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2+b^2}}{b}\right)+\frac{1}{b}\ln\left(\frac{b+\sqrt{a^2+b^2}}{a}\right)\right.$$

$$\left.+\frac{a}{3b^2}+\frac{b}{3a^2}-\frac{(a^2+b^2)^{3/2}}{3a^2b^2}\right] \tag{1.13}$$

$$K_2 = \ln\frac{4(a+b)}{b}+2K_1\frac{(a+b)}{\sqrt{ab}}-\ln\frac{(a+\sqrt{a^2+(b/2)^2})}{(b/2)}$$

$$-\frac{1}{2}\ln\frac{(b/2)+\sqrt{a^2+(b/2)^2}}{-(b/2)+\sqrt{a^2+(b/2)^2}} \tag{1.14}$$

where b is the length of the long side of the grid, and a is the length of the short side of the grid in meters. These expressions, and the graphs, can only be applied to a limited set of rectangles, with a maximum length to width ratio of 8:1.

Simplified equations have also been proposed for the calculation of the mesh and step voltages. The best expressions available so far are those developed by Sverak, and recommended by the IEEE Std. 80.

The mesh voltage, for square grids with square meshes, is calculated as

$$E_m = \frac{\rho I_G K_m K_i}{L} \tag{1.15}$$

where I_G is the maximum grid current that flows from the grid to ground in amperes.

The spacing factor for the mesh voltage, K_m is given as

$$K_m = \frac{1}{2\pi}\left[\ln\left(\frac{D^2}{16hd}+\frac{(D+2h)^2}{8Dd}-\frac{h}{4d}\right)+\frac{K_{ii}}{K_h}\ln\frac{8}{\pi(2n-1)}\right] \tag{1.16}$$

D is the spacing between parallel conductors in meters, h is the depth of burial of grid conductors in meters, and d is the diameter of grid conductors in meters, and n is the number of parallel conductors in any one dirction. K_i, K_{ii}, and K_h are correction factors, used to compensate for the fact that the actual grids are different from the model, which is based on a system of n parallel conductors in one direction. The actual grids have conductors in two directions, and these conductors are connected at the intersections.

The correction factor for grid geometry, K_i is given as

$$K_i = 0.656 + 0.172*n \tag{1.17}$$

The corrective weighting factor, K_{ii}, that adjusts the effects of inner conductors on the corner mesh is given as

$$K_{ii} = 1/(2n)^{2/n} \tag{1.18}$$

K_h, the corrective factor that emphasizes the effects of grid depth, is given as

$$K_h = \sqrt{1+h} \tag{1.19}$$

The step voltage may be calculated as

$$E_s = \frac{\rho I_G K_s K_i}{L} \tag{1.20}$$

The spacing factor for step voltage, K_s, is given as

$$K_s = \frac{1}{\pi}\left[\frac{1}{2h} + \frac{1}{D+h} + \frac{1}{D}(1-0.5^{n-2})\right] \tag{1.21}$$

For rectangular grids with square meshes, the value of n is modified as follows:

For mesh voltage calculations, n is given as

$$n = \sqrt{n_A n_B} \tag{1.22}$$

where n_A is the number of parallel conductors along one of the coordinate axis, and n_B the number along the other the axis.

For step voltage calculations, n is given as

$$n = \max(n_A, n_B) \tag{1.23}$$

These equations are recommended for use within the following limits:

$$0.25m \le h \le 2.5m \tag{1.24a}$$

$$d < 0.25h \tag{1.24b}$$

$$D > 2.5m \tag{1.24c}$$

All the expressions given above are recommended for use with grids with shapes which may be approximated by a square, or a rectangle with length to width ratio limited to less than 8:1.

Expressions are also available for the calculation of footing resistance [1]. They may be summarized as:

$$R_f = \frac{\rho_s}{4b} F(X) \tag{1.25}$$

$$F(X) = 1 + 2\sum_{n=1}^{\infty} Q \tag{1.26}$$

$$Q = K^n/[1+(2nX)^2]^{1/2} \tag{1.27}$$

$$K = (\rho - \rho_s)/(\rho + \rho_s) \tag{1.28}$$

where ρ_s is the resistivity of the surface crushed rock layer in ohm-meters, ρ is the resistivity of soil in ohm-meters, $X = h/b$, h is the thickness of the crushed rock layer in meters, b is the equivalent radius of a foot equalling 8 centimeters, and K is the reflection factor. Figure 1 shows the variation of F(X) with h, with ρ/ρ_s varying from 0.005 to 0.5.

Though the value of footing resistance may be calculated from the above expressions, due to the infinite nature of equation (1.26), they are difficult to use without the help of a computer or programmable calculator.

Seedhar, Arora and Thapar [14] have proposed a finite expression to compute the potential at any point due to a point current source located anywhere in a two layer soil. A finite expression for calculating the footing resistance may be derived from this expression.

Sverak has also proposed a simplified formula for calculating the footing resistance, which may be given as

$$R_f = \frac{\rho_s}{4b} C(X) \tag{1.29}$$

where C(X) is given as

$$C(X) = 1 - 0.106\left(\frac{1-\frac{\rho}{\rho_s}}{2h_s + 0.106}\right) \tag{1.30}$$

where h_s is the thickness of the top crushed rock layer.

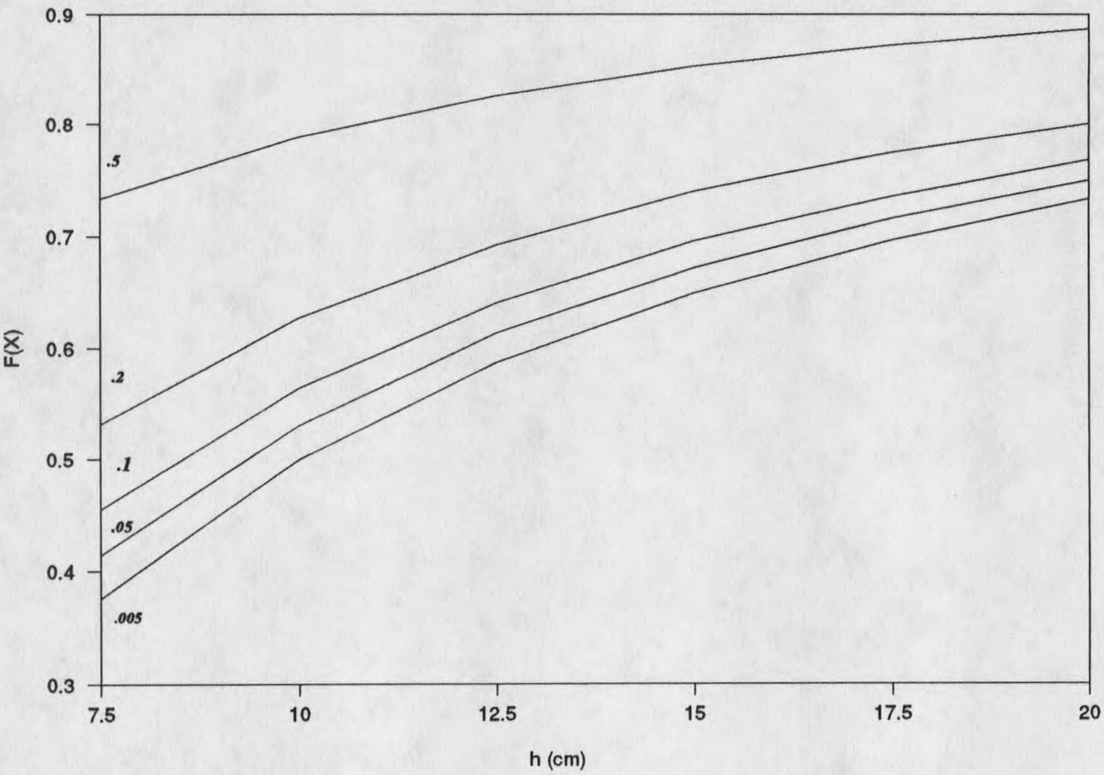

Figure 1. Variation of F(X) with h, for ρ/ρ_s = 0.005 to 0.5

Sobral, *et al* [15] have developed a method for studying interconnected grounding systems, connected by overhead ground wires. In their analysis, the mutual effect of the interconnected grids are neglected, and the system is assumed to comprise of a number of grids, and overhead ground wires, which transfer the potential of the energised grid to other connected grids.

CHAPTER 2

THE PROGRAM RESIS

Program Description

With a view to have an available reference, with which any newly developed expressions may be compared, a computer program RESIS, based on the finite element analysis, was developed. This program can handle grid configurations composed of straight linear conductors, laid in three mutually perpendicular directions. For a given grid configuration, the grid conductors are divided into small straight segments, each about 5 meters in length. The self and mutual resistances of these segments are then calculated to obtain the ground resistance of the grid.

For grids with a very large number of parallel conductors, the voltage drop may be neglected due to the lower value of current flowing per conductor. For large grids with a small number of conductors, it becomes necessary to express the voltage on each segment as a function of the applied voltage, and the voltage drops due to the leakage currents flowing through the resistance and inductance, and mutual inductances of the conductor segments. Since practical grids are rarely of this type, in the program RESIS, the conductors are assumed to be at the same potential. This is a valid assumption, as at power frequencies, the resistance and inductance of the segments are negligible when compared to the resistance between the segments and the earth. At power frequencies, the voltage drop along the conductors becomes important only if the length of the conductors becomes very large.

Whenever a fault occurs in a substation, a part of the fault current flows to the ground through the grounding grid. This fault current flowing through the grounding grid causes the potential of the grid, with respect to the remote ground, to rise. This potential rise of the conductor segments is also known as the ground potential rise (GPR) of the grid.

A part of the fault current flowing through the grid, is dissipated by each of the conductor segments. It is thus important to determine the current distribution from the segments of the grid conductors. Once the current distribution has been determined, the voltage at any point on the surface of the earth is determined by obtaining the sum of voltages at that point due to the current dissipated by each of the segments. In this fashion, the voltage profile in any direction on the surface of the earth, can be calculated.

Once the voltage profile is available, the mesh and step voltages can be computed. For a given design, based on the configuration of the grid conductors, the approximate area within which the point where the minimum potential is likely to occur, can usually be identified. Within the specified area, the potentials at a finite number of points are determined, and the minimum of these values, and its location, are identified. The difference between the GPR and this minimum voltage, gives the value of mesh voltage for the specific configuration.For grids with irregular shapes, it is likely that the point of minimum potential may not be readily identifiable, and a larger area, within which this point is likely to occur, may have to be chosen. The step voltage is calculated as the difference in voltage between the two points on the surface of the ground, one directly above a corner of the grid, and the other one meter away in the direction of the diagonal of the corner mesh extended outwards. In case of grids with irregular shapes, the step voltage at all the corner points may be determined, and the highest of these values recorded as the maximum step voltage of the grid.

The equation for the calculation of potential at a point, and subsequently the equations for the calculation of the self and mutual resistances of the various conductor segments are derived on the basis of the method of images [15]. This is done to account for the boundary between the earth and the air.

Development of the Equations

The potential at any point (x,y,z), produced by a linear conductor m parallel to the x-axis, discharging a current I in an infinite medium of resistivity ρ, is given by [16]:

$$V(x,y,z)=\frac{\rho I}{4\pi L}*\ln\frac{\sqrt{(x-x_m+L/2)^2+(y-y_m)^2+(z-z_m)^2}+x-x_m+L/2}{\sqrt{(x-x_m-L/2)^2+(y-y_m)^2+(z-z_m)^2}+x-x_m-L/2} \qquad (2.1)$$

Similar equations can be written for conductors oriented along the y-, and z-, coordinate axes.

The nomenclature used in equation (2.1) above, and in equations (2.2), (2.3), and (2.4) below are explained in Table 1.

The self ground resistance R_{mm}, of a conductor segment, is defined as the ratio of the voltage on the segment to the current flowing out of the segment, neglecting the effect of other conductor segments, and is determined by calculating the average potential on the surface of the conductor, when it discharges a unit current. R_{mm} is given as

$$R_{mm}=\frac{\rho}{4\pi L}*(2*\ln\frac{2L}{r}-2+\ln\frac{L_1+L}{L_1-L}+\frac{4h}{L}-\frac{2L_1}{L}) \qquad (2.2)$$

This equation includes the effect of the image of the conductor segment under consideration.

The mutual ground resistance, R_{mn}, between two linear conductor segments m and n, is determined by calculating the average potential on the surface of conductor segment m, when conductor segment n is discharging a unit current. To account for the effect of the image segments, the computer is programmed such that the equation is applied twice, once for the two segments, and once for one of the segments and the image of the other. The sum of these values gives us the total mutual ground resistance between the two conductor segments. The mutual resistance of segment m, as seen by the segment n is equal to the mutual resistance of segment n as seen by the segment m (ie), $R_{mn} = R_{nm}$.

TABLE 1. NOMENCLATURE FOR SELF AND MUTUAL GROUND RESISTANCE EXPRESSIONS

L	= length of the conductor segments
r	= radius of the conductors
h	= depth of burial of the grid
x_m, y_m, z_m	= x,y,z coordinates of the m[th] segment
x_n, y_n, z_n	= x,y,z coordinates of the n[th] segment

$\mid x_m - x_n \mid = a$	$\mid y_b - y_n \mid = b$	$\sqrt{L^2+4h^2} = L_1$
$\sqrt{a^2+b^2} = c_1$	$\sqrt{a^2+b^2+4h^2} = c_2$	$\sqrt{(a+L)^2+b^2} = d_1$
$\sqrt{(a+L)^2+b^2+4h^2} = d_2$	$\sqrt{(a-L)^2+b^2} = e_1$	$\sqrt{(a-L)^2+b^2+4h^2} = e_2$
$d_1+a+L = f_1$	$d_2+a+L = f_2$	$e_1+a-L = g_1$
$e_2+a-L = g_2$	$(c_1+a)^2 = j_1$	$(c_2+a)^2 = j_2$
$a+\frac{L}{2} = a_1$	$a-\frac{L}{2} = a_2$	$b+\frac{L}{2} = b_1$
$b-\frac{L}{2} = b_2$	$\sqrt{a_1^2+b_1^2} = k_1$	$\sqrt{a_1^2+b_1^2+4h^2} = k_2$
$\sqrt{a_2^2+b_1^2} = k_3$	$\sqrt{a_2^2+b_1^2+4h^2} = k_4$	$\sqrt{\sqrt{a_1^2+b_2^2}} = k_5$
$\sqrt{a_1^2+b_2+4h^2} = k_6$	$\sqrt{a_2^2+b_2^2} = k_7$	$\sqrt{a_2^2+b_2^2+4h^2} = k_8$
$\sqrt{a_1^2+4h^2} = k_9$	$\sqrt{a_2^2+4h^2} = k_{10}$	$a_1^2+4h^2 = s_1$
	$a_2^2+4h^2 = s_2$	

The effect of the image conductors may be taken into consideration by using the same expressions with one modification. Since the image conductors are not on the same plane as the source conductors, for calculation of the mutual resistances, the distance separating the i[th] conductor segment, and the image of the j[th] conductor segment, along the z-axis, is equal to twice the depth of burial of the source conductors. The distance between the i^{th} conductor segment and the image of the j^{th} conductor segment, along the other two coordinate axes, is the same as the distance between the i^{th} conductor segment and the j^{th} conductor segment. This can be easily programmed on the computer.

When the two conductor segments are parallel to the x-axis, the value of R_{mn} is given as

$$R_{mn} = \frac{\rho}{4\pi L}\left(\ln\frac{f_1 f_2}{g_1 g_2} + \frac{a}{L} * \ln\frac{f_1 f_2 g_1 g_2}{j_1 j_2} + \frac{(2c_1 + 2c_2 - d_1 - d_2 - e_1 - e_2)}{L} \right) \tag{2.3}$$

If the two segments are parallel to the y-axis, the value of R_{mn} is obtained by interchanging a and b in equation (2.3).

If the two segments are orthogonal, and parallel to the x- and y- axes respectively, the mutual ground resistance between them is given as

$$\begin{aligned} R_{mn} = \frac{\rho}{4\pi L^2}\{ & b_1 * \ln\frac{(a_1+k_1)(a_1+k_2)}{(a_2+k_3)(a_2+k_4)} + a_1 * \ln\frac{(b_1+k_1)(b_1+k_2)}{(b_2+k_5)(b_2+k_6)} \\ & - a_2 * \ln\frac{(b_1+k_3)(b_1+k_4)}{(b_2+k_7)(b_2+k_8)} - b_2 * \ln\frac{(a_1+k_5)(a_1+k_6)}{(a_2+k_7)(a_2+k_8)} \\ & + 2h(\sin^{-1}\frac{s_1+a_1k_6}{k_9(a_1+k_6)} + \sin^{-1}\frac{s_2+a_2k_4}{k_{10}(a_2+k_4)} \\ & - \sin^{-1}\frac{s_1+a_1k_2}{k_9(a_1+k_2)} - \sin^{-1}\frac{s_2+a_2k_8}{k_{10}(a_2+k_8)})\} \end{aligned} \tag{2.4}$$

A resistance matrix R, composed of the self and mutual resistances of the various conductor segments, is formed, and for a grid with j linear segments, is of order (j X j). Also, the sum of the currents being dissipated by the various conductor segments of the grid is equal to the fault current, I_G. This is the $(j+1)^{th}$ equation, and the system is of order (j+1). The system is to be solved for the current components of the fault current being dissipated by the various conductor segments, and the voltage to which each conductor segment is raised due to these current components. The voltage on a conductor segment is the sum due to the leakage current flowing through it, and the leakage currents flowing through the other segments.

The system can be represented as

$$\begin{aligned}
&R_{11}I_1 + R_{12}I_2 + \ldots + R_{1j}I_j = V \\
&R_{21}I_1 + R_{22}I_2 + \ldots + R_{2j}I_j = V \\
&\cdot \\
&\cdot \\
&\cdot \\
&R_{j1}I_1 + R_{j2}I_2 + \ldots + R_{jj}I_j = V \\
&\qquad I_1 + I_2 + \ldots + I_j = I_G
\end{aligned} \tag{2.5}$$

which can be represented in the matrix form as

$$\begin{pmatrix} R_{11} & R_{12} & R_{13} & \ldots & R_{1j} & -1 \\ R_{21} & R_{22} & R_{23} & \ldots & R_{2j} & -1 \\ R_{31} & R_{32} & R_{33} & \ldots & R_{3j} & -1 \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ R_{j1} & R_{j2} & R_{j3} & \ldots & R_{jj} & -1 \\ 1 & 1 & 1 & \ldots & 1 & 0 \end{pmatrix} * \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \cdot \\ \cdot \\ \cdot \\ I_j \\ V \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ I_G \end{pmatrix} \tag{2.6}$$

The potential at a point (x,y,0), on the surface of the earth, is the sum of the potentials at that point due to the currents dissipated by the various conductor segments, and their images. The potential at the point (x,y,0), due to each of these segments, can be determined by use of equation (2.1). This equation can also be used to calculate the mesh voltage at a point, by calculating the potential at a finite number of points on the surface, and choosing the point with the minimum potential. The difference in potential between the ground potential rise, and this value, gives the value of the mesh voltage. The step voltage may be calculated as the difference in potential between two points on the surface of the earth, which are one meter apart.

As mentioned earlier, the value of the mesh and step voltages should be less than the maximum allowable limits, calculated from equations (1.6) and (1.8).

Use of the Program

The values of ground resistance, mesh voltage and touch voltage, obtained with the program RESIS, were compared with those available in literature. Since most of the available analysis has been done for square shaped grids, only results for such grids are compared in Table 2.

It may be observed that the results from RESIS compare very well with those obtained from other sources. Since most results available in literature are for square grids, only results for such grids are compared. Once it was established that the results from RESIS were quite accurate, calculations were made for more than one hundred and fifty grids of different configurations and sizes. The shapes of the grids, and the dimensions varied are shown in Figure 2.

TABLE 2. GROUND RESISTANCE, MESH AND STEP VOLTAGES FOR SQUARE GRIDS

Depth of burial of grid = h , meters.
Diameter of grid conductors = 0.01 meters.
Number of meshes in the grid = n.
Resistivity of soil = 100 ohm-meters

SGSYS: Reference [17]
J-P Reference [18]

Grid Size	n	h	Ground Resistance			Mesh Voltage			Step Voltage		
(m X m)			(ohms)			(As a percentage of GPR)					
			RESIS	SGSYS	J-P	RESIS	SGSYS	J-P	RESIS	SGSYS	J-P
10 X 10	1	1.5	4.97	5.09	--*	45.52	46.89	--	8.19	7.28	--
20 X 20	4	1.5	2.45	2.49	--	32.69	32.82	--	7.25	6.49	--
30 X 30	9	1.5	1.61	1.62	--	28.08	27.80	--	6.61	5.91	--
40 X 40	16	1.0	1.21	1.23	1.40	26.13	26.16	25.00	8.40	7.55	7.50
50 X 50	25	1.0	0.96	0.97	1.10	24.25	24.19	24.00	7.90	7.10	6.50
60 X 60	36	1.0	0.79	0.80	0.85	22.84	22.94	22.00	7.50	6.39	6.00
70 X 70	49	0.5	0.69	0.69	0.75	23.24	23.40	25.00	11.32	9.57	12.00
80 X 80	64	0.5	0.60	0.60	--	22.27	22.41	--	10.87	9.20	--

(*) : value not available.

During these computer runs, it was observed that once the grid shape and size are fixed, the ground resistance and mesh and step voltages did not vary much with change in the radius of the conductors used, for the range of conductor sizes (2.5 millimeters - 10 millimeters) usually used in such grids. This is shown in Table 3. The radius of the conductors was thus fixed at an average value of 5 millimeters (AWG 4/0). The grid depth was varied from 0.25 meters to 2.5 meters. The meshes in all the grids were square.

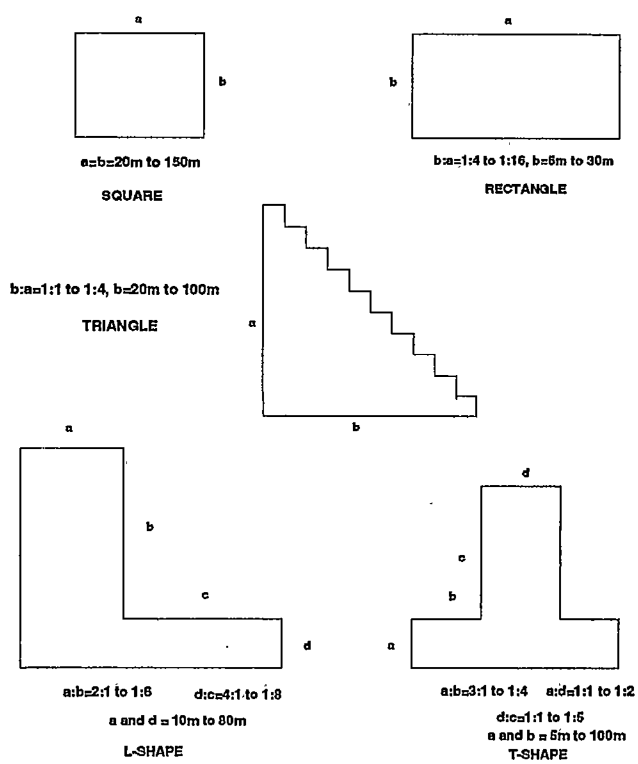

Figure 2. Shapes and Sizes of Grounding Grids.

TABLE 3. VARIATION OF DESIGN PARAMETERS WITH CONDUCTOR SIZE

Soil Resistivity	= 100 ohm-meters
Grid Size	= 30 X 30 sq.meters
Depth of burial of grid	= 0.5 meters
Number of Meshes	= 9
Conductor Radius	= r millimeters

r	Ground Resistance	Mesh Voltage	Step Voltage
(mm)	(ohms)	(As a percentage of GPR)	
2.5	1.749	32.84	13.82
5.0	1.701	30.88	14.41
7.5	1.673	29.68	14.79
10.0	1.653	28.79	15.07

CHAPTER 3

GROUND RESISTANCE CALCULATIONS

Development of the Expression

Since Sverak's formula is simple to use, and gives reasonably accurate results for square grids, it was decided to use it as the basis for any modification and improvement. The basic considerations which were kept in view when the formula was modified are:

(i) The modifications to the expression should be dimensionless.

(ii) A factor to account for the variation in shape needs to be identified. One such dimensionless factor is $\sqrt{A}/L_p$, which gives a measure of the spread of the area. It has a maximum value of 0.282 for a circular area, and decreases as the length of the spread of the area increases. The ground resistance of a conductor of given surface area decreases as the length over which the area spreads is increased. For example, the ground resistance of a circular plate is more than the ground resistance of a strip conductor of the same surface area. As the number of meshes in a grid of given dimensions is increased infinitely, by the addition of conductors, it can be approximated by a plate of the same dimensions.

(iii) A number of expressions, as a function of $\sqrt{A}/L_p$ were tried, and the expression $[2\ln(L_p\sqrt{2/A}) - 1]*(\sqrt{A}/L_p)$ was found to give the best fit with the results obtained from RESIS.

A comparison of the expression which thus evolved, with those for a strip conductor and a circular plate showed the newly developed expression to be in error for the two cases by factors of 1.437 and 1.594. A factor of 1.52 was chosen as the average of these figures, and the new expression was finally modified by a factor of 1.52, to give a new equation for the calculation of ground resistance as

$$R = 1.52*\rho*\left(\frac{1}{L_t}+\frac{1}{\sqrt{20*A}}\left\{1+\frac{1}{1+h*\sqrt{20/A}}\right\}\right)*(2*\ln[L_p\sqrt{2/A}]-1)*(\sqrt{A}/L_p) \qquad (3.1)$$

Comparison with Existing Expressions

A comparison of equation (3.1) with existing formulas for the calculation of the ground resistance of grids of various shapes, is given in Table 4. The values obtained through the use of Sverak's expressions, Schwarz' expressions, and equation (3.1), are compared with the accurate values obtained through the use of the program RESIS. The following observations can be made on the basis of the data presented in the table.

(i) Sverak's formula is quite accurate if the shape of the grid is a square. For grids with other shapes, the error is much higher, and in a few cases, is as high as 40%.

(ii) Schwarz' formula gives better results for square, and rectangular grids, but gives high errors for grids with irregular shapes.

(iii)The errors obtained from use of equation (3.1) are consistently below 10%.

Comparison with Analog Model Test Results

To compare the results obtained from use of equation (3.1), with scale model test results, analog models of grids of the shapes under consideration were made, and their resistances determined. The models are made of styrofoam of about 2cm thickness, are covered with aluminum foil and have the following physical characteristics:

1. Square.

2. Rectangle, Length : Width = 4 : 1.

3. Right angled Isosceles triangle.

4. Right angled Scalene triangle, Base : Height = 1 : 4.

5. T-shaped, b/a = 2; c/d = 2.5 (Figure 2)

6. L-shaped, b/a = c/d = 3. (Figure 2)

Three models of each shape were tested in an electrolytic tank, with water of resistivity 66.2 ohm-m used as the electrolyte. The ground resistance of each model floating on the surface of the water was determined. The area of each model in contact with the water was 314.2sq.cm. Table 5 gives the average measured values of the ground resistance.

The ground resistance in each case, as determined from equation (3.1), with $L_t = \infty$, and h = 0, is given in the second column of Table 5. It may be observed that the difference in the calculated and measured values is not more than 8% in any of the cases. Since the models tested represent plates of the same shape, it may be concluded that equation (3.1) may be even applied to grids having a very large number of parallel conductors. This is also proved analytically in the next section.

The Equation as Applied to Circular Plate and Strip Conductor

For a given area, a circular plate has the minimum peripheral length, and a strip with negligible width (as compared to its length) has the maximum peripheral length. For a given area, the circular plate gives the maximum ground resistance, and the strip, the minimum. These two shapes, can thus be assumed to represent the extremes in shapes as far as the ground resistance is considered. The ground resistance of a circular plate of radius r placed on the surface of the ground is given as

$$R = \rho/(4r) \qquad (3.2)$$

while the resistance to ground of a strip of length of length L and width W, again placed on the surface of the ground, is given as

$$R = \frac{\rho}{\pi L}\left(\ln\frac{8L}{W} - 1\right) \tag{3.3}$$

For a circular plate on the surface of ground,

$$A = \pi r^2 \qquad L_t = \infty \tag{3.4a}$$

$$h = 0 \qquad L_p = 2\pi r \tag{3.4b}$$

Substituting equations (3.4a) and (3.4b) in equation (3.1),

$$R = \rho/(4.15r) \tag{3.5}$$

A comparison of equations (3.2) and (3.5) shows that equation (3.5) gives a ground resistance value which is about 4% less as compared to equation (3.1).

For a strip conductor on the surface of the earth,

$$A = LW \qquad L_t = \infty \tag{3.6a}$$

$$h = 0 \qquad L_p = 2(L + W) = 2L, as L \gg W \tag{3.6b}$$

Substituting equations (3.6a) and (3.6b) in equation (3.1),

$$R = \frac{\rho}{0.94\pi L}\left(\ln\frac{8L}{W} - 1\right) \tag{3.7}$$

A comparison of equations (3.3) and (3.7) shows that equation (3.7) gives a ground resistance value which is about 7% more as compared to equation (3.3).

Thus, from the data in Table 5, and from the above analysis, it may be observed that the error for most shapes of grounding grids is less than 10%. Equation (3.1) may thus be used to calculate the ground resistance of grounding grids of most practical shapes.

TABLE 4. GROUND RESISTANCE OF GRIDS OF DIFFERENT SHAPES

Resistivity of soil = 100 ohm-m.
Depth of Burial of grid = 0.5 m.
Radius of conductors = 5 mm.
Number of meshes = n
Dimensions of the grid = a, b, c, d meters [Figure (1)]

(1) : Equation (3.1)
(2) : Sverak's Formula, Equation (1.11)
(3) : Schwarz' Formula, Equation (1.12)
(4) : Program RESIS

Grid Dimensions					Ground Resistance (ohms)				% Error		
n	a	b	c	d	(1)	(2)	(3)	(4)	(1)	(2)	(3)
Square Grids											
16	20	20			2.46	2.62	2.49	2.37	3.8	10.5	5.1
64	40	40			1.15	1.23	1.22	1.15	0.0	7.0	6.1
16	60	60			0.84	0.90	0.88	0.86	-2.3	4.7	2.3
100	100	100			0.46	0.49	0.49	0.47	-2.1	4.3	4.3
144	120	120			0.38	0.40	0.41	0.39	-2.6	2.6	5.1
225	150	150			0.30	0.32	0.32	0.31	-3.2	3.2	3.2
Rectangular Grids											
16	80	5			1.80	2.53	1.42	1.64	9.8	54.3	-13.4
16	80	20			1.17	1.33	1.11	1.13	3.5	17.4	- 1.8
16	160	10			0.92	1.29	0.75	0.87	5.7	48.3	-13.8
16	120	30			0.79	0.89	0.76	0.78	1.3	14.7	- 2.6
64	320	15			0.44	0.61	0.40	0.42	4.8	45.2	- 4.8
144	480	30			0.28	0.39	0.26	0.27	3.7	44.4	- 3.7
Triangular Grids											
20	40	20			1.98	2.29	2.12	1.93	2.6	18.7	9.8
40	80	20			1.27	1.57	1.36	1.21	5.0	29.8	12.4
63	90	30			1.03	1.26	1.16	1.01	2.0	24.8	14.9
60	200	50			0.51	0.64	0.57	0.52	-1.9	23.1	9.6
60	100	100			0.54	0.60	0.60	0.58	-6.9	3.4	3.4
144	320	80			0.32	0.40	0.36	0.33	-3.0	21.2	9.1
L-shaped Grids											
32	10	60	10	10	1.43	1.77	1.55	1.36	5.1	30.1	14.0
48	20	20	20	20	1.30	1.43	1.42	1.29	0.8	10.9	10.1
20	20	50	80	10	0.89	1.16	1.11	0.88	1.1	31.8	26.1
51	30	70	70	30	0.60	0.70	0.70	0.60	0.0	16.7	16.7
88	70	40	30	60	0.48	0.52	0.53	0.50	-4.0	4.0	6.0
136	80	120	120	20	0.35	0.41	0.42	0.35	0.0	17.1	20.0
T-shaped Grids											
10	5	20	5	5	2.58	3.30	2.58	2.41	7.1	36.9	7.1
32	10	20	30	10	1.43	1.77	1.72	1.42	0.7	24.6	21.1
40	10	20	50	10	1.18	1.51	1.51	1.20	-1.7	25.8	25.8
44	20	60	80	20	0.58	0.75	0.75	0.59	-1.7	27.1	27.1
72	20	80	80	40	0.45	0.58	0.57	0.46	-2.2	26.1	23.9
108	60	20	80	60	0.42	0.47	0.47	0.44	-4.5	6.8	6.8

TABLE 5. GROUND RESISTANCE OF ANALOG MODELS

	Ground Resistance (ohms)	
Shape	Measured Value (Model Test)	Calculated Value (Equation (3.1))
Square	156.0	155.0
Rectangle	138.0	146.4
Isosceles Triangle	143.0	147.9
Scalene Triangle	124.0	133.5
T-shape	133.0	137.1
L-shape	127.0	137.1

CHAPTER 4

VOLTAGE CALCULATIONS

Development of the Expressions

Equations (1.9), (1.10), (1.14), and (1.15) are based on a model comprising equally spaced infinitely long parallel conductors with no cross connections, dissipating uniform current. Correction factors K_i, K_{ii}, and K_h are used to account for the difference in the actual grids and the model. K_i, as given in equation (1.11), is based on the data obtained by Koch, from his limited experimental work [1]. This value is recommended for use for grids with the number of conductors along one direction not to exceed 25, but it was determined that a better correction factor may be obtained from a different expression for K_i. From the test cases for which RESIS was run, more than forty examples for square and rectangular grids were selected, and a new expression for K_i was derived from plotting the values of K_i obtained from the expression

$$K_i = \frac{E_m L}{\rho I_G K_m} \tag{4.1}$$

The best linear fit for K_i was obtained with the expression

$$K_i = 0.644 + 0.148 * n \tag{4.2}$$

The number of parallel conductors in one direction, n, is one of the factors used in the simplified equations. The choice of the factor n is simple for square grids, as the number of conductors is the same in both directions if the number of meshes are assumed to be the same. For rectangular grids, the IEEE Std. 80 recommends the use of equations (1.22) and (1.23). The value of n obtained from this method gives reasonably good results, provided, the length to width ratio is limited to less than 8:1. Such a strategy is not applicable to irregular shaped grids because of the following factors:

(i) The number of parallel conductors in one direction may be different from that in the other direction.

(ii)The length of all the parallel conductors in one direction may not be the same.

It is thus necessary to formulate an expression for n, which should be applicable for all shapes of grids. Such a expression would have to be a function of the shape of the grid, its dimensions, and the length of conductor used in the grid. Unlike square grids where the total length is an integer multiple of the length of one side, for irregular shaped grids, other dimensions need to be identified. These are:

L_x, the maximum length of the grid along the x-axis;

L_y, the maximum length of the grid along the y-axis;

D_m, the maximum distance between any two points on the grid periphery;

L_p, the length of the periphery of the grid;

L_t, the total length of conductor used in the grid.

Using the data generated using the program RESIS, the following expression, as a function of the lengths described above, was developed:

$$n = a*b*c*d \tag{4.3}$$

where

$$a = 2L_t/L_p \tag{4.3a}$$

$$b = (L_p/4\sqrt{A})^{1/2} \tag{4.3b}$$

$$c = (L_x L_y/A)^{0.7A/(L_x L_y)} \tag{4.3c}$$

$$d = D_m/(L_x^2 + L_y^2)^{1/2} \tag{4.3d}$$

TABLE 6. MESH AND STEP VOLTAGES IN GRIDS OF DIFFERENT SHAPES

Resistivity of soil = 100 Ω-m
Diameter of conductor = 0.01m
Depth of burial of grid = 0.5m
Number of meshes = n
Dimensions of the grid = a, b, c, d m.

(1) Modified equations {Equation 4.1 and Equation 4.2(a,b,c,d)}
(2) IEEE Std. 80, 1986.
(3) Program RESIS

Grid Dimensions					Mesh Voltage (Volts)			% Error		Step Voltage (Volts)			% Error	
n	a	b	c	d	(1)	(2)	(3)	(1)	(2)	(1)	(2)	(3)	(1)	(2)
Square Grids														
16	20	20			612	670	576	6.3	16.3	299	327	342	-12.6	- 4.4
16	40	40			376	411	349	7.7	17.8	130	143	166	-21.6	-13.8
64	40	40			225	251	224	0.5	12.1	121	135	137	-11.7	- 1.5
100	100	100			103	115	98	5.1	17.3	39	44	48	-18.8	- 8.3
144	120	120			80	90	76	5.2	18.4	31	35	35	-11.4	0.0
225	150	150			59	67	57	3.5	17.5	24	27	26	- 7.7	3.8
Rectangular Grids														
16	80	5			465	587	476	- 2.3	23.3	220	257	257	-16.8	0.0
4	80	20			553	615	543	1.8	13.2	141	203	189	-25.4	7.4
16	80	20			348	471	332	4.8	41.8	120	199	154	-22.1	29.2
16	240	15			211	270	214	- 1.4	26.2	61	175	82	-25.6	113.4
64	320	20			109	147	107	1.9	37.4	39	84	50	-22.0	68.0
144	480	30			62	87	60	3.3	45.0	23	102	29	-20.7	251.7
Triangular Grids														
20	40	20			517	595	534	- 3.2	11.4	261	372	273	- 4.4	36.3
40	80	20			299	366	291	2.8	25.8	156	315	154	1.3	104.5
63	90	30			226	277	259	-12.7	6.9	123	230	127	- 3.2	81.1
30	100	50			228	229	248	- 8.1	- 7.7	86	104	101	-14.9	2.9
72	160	80			123	162	143	-13.9	13.3	38	64	40	- 5.0	60.0
144	320	80			77	103	91	-15.4	13.2	30	73	31	- 3.2	135.5
L-shaped Grids														
32	10	60	10	10	353	432	320	10.3	35.0	182	347	187	- 2.6	85.6
48	20	20	20	20	285	282	266	7.1	6.1	152	173	162	- 6.2	6.8
52	10	60	60	10	237	367	215	10.2	70.7	126	216	126	0.0	71.4
32	20	120	20	20	219	269	194	12.9	38.7	79	150	90	-12.2	66.7
128	20	120	20	20	134	174	124	8.1	40.7	76	170	75	1.3	126.7
136	80	120	120	20	85	112	83	2.4	34.9	34	53	36	- 5.6	47.2
T-shaped Grids														
10	5	20	5	5	766	903	723	6.0	24.9	369	674	391	- 5.6	72.4
32	10	20	30	10	322	467	320	0.6	45.9	161	273	189	-14.8	44.4
40	10	20	50	10	233	294	274	-14.9	7.3	126	177	159	-20.1	11.3
44	20	60	80	20	154	249	150	2.7	66.0	55	109	71	-22.5	53.5
72	20	80	80	40	116	178	113	2.7	57.5	43	93	53	-18.9	75.5
108	60	20	80	60	97	116	91	6.6	27.5	37	51	44	-15.9	15.9

The dimensionless expression for n given in equation (4.3), reduces to the number of parallel conductors in any direction for square grids having square meshes. The factors b, c, and d reduce to unity for square grids. The factors c and d reduce to unity for square and rectangular grids. The factor d equals unity for square, rectangular and L-shaped grids.

Comparison with the Existing Methodology

A comparison of the method described in the IEEE Std. 80-1986, for the calculation of mesh and step voltages, as outlined in Chapter 1, with the same method using the new expressions, is made in Table 6. It may be observed that

(i) for mesh voltages, the new expressions give lower errors than the IEEE Std. 80 expressions, and are within 16% of the actual values obtained using RESIS. Some of the IEEE Std. 80 calculations for the mesh voltage are in error by as much as 50%.

(ii)for step voltages, the new expressions give lower errors than the IEEE Std. 80 expressions, and are within 30% of the actual values obtained using RESIS. Some the IEEE Std. 80 calculations for the step voltage are in error by more than 100%.

Since the expressions for K_i have been optimized keeping the mesh voltage calculations in view, the use of the same expressions for the step voltage results in a slightly higher error. This error in the calculation of the step voltage is acceptable, as once a design has been made safe for mesh voltage considerations, it will usually be safe from the step voltage point of view.

CHAPTER 5

CALCULATION OF FOOTING RESISTANCE

Development of the Finite Expression

The footing resistance, which is defined as the resistance of the ground below the feet of a person present in a substation, is affected by the layer of crushed rock spread on the surface of the soil. This crushed rock layer is usually 10cm - 20cm thick, and provides a high resistivity layer. As an approximation, the footing resistance is taken as $3\rho_s$, where ρ_s is the resistivity of the crushed rock layer. But this expression is arrived at based on the assumption that the layer of crushed rock is of a very large thickness, by virtue of which the effect of the boundary at the rock-soil interface is neglected. The expression is derived by considering the foot to be a conducting disc of radius 8cm, and neglecting the mutual resistance between the two feet. In the accidental circuits for step and mesh voltages, the total footing resistance is approximated to be $6\rho_s$ and $1.5\rho_s$ respectively.

The expressions for a more accurate calculation of the footing resistance recommended by the IEEE Std.80 (equations 1.25 - 1.28), take into consideration the mutual resistance between the two feet. The effect of the thin layer of crushed rock on the top of the soil is dealt with by using the two layer model of soil. While the expression derived on the basis of the above considerations is accurate, it is cumbersome to use without the help of a computer or programmable calculator. Thus, a simple set of finite expressions for calculating the footing resistance, which gives reasonably accurate values, is necessary.

Equations (1.26) and (1.27), are used in calculating the footing resistance. Equation (1.26) is given as

$$F(X) = 1 + 2\sum_{n=1}^{\infty} Q \tag{5.1}$$

$$Q = K^n/[1+(2nX)^2]^{1/2} \tag{5.2}$$

Equations (5.1) and (5.2) can be rewritten as

$$F(X) = 1 + \frac{2K}{[1+(2X)^2]^{1/2}} + 2\sum_{n=2}^{\infty} \frac{K^n}{[1+(2nX)^2]^{1/2}} \tag{5.3}$$

The infinite series expressions for potential due to a point current source in a two layer soil, when both the source S and the point P where the potential is desired, are in the upper layer, is available in literature [14]. This is given as

$$V_p = \frac{I\rho_1}{4\pi h}\{\frac{1}{\sqrt{d_h^2+(d_s-d_p)^2}} + \frac{1}{\sqrt{d_h^2+(d_s+d_p)^2}}$$

$$+\sum_{n=1}^{\infty} K^n*(\frac{1}{\sqrt{d_h^2+(2n-d_s-d_p)^2}} + \frac{1}{\sqrt{d_h^2+(2n+d_s-d_p)^2}} +$$

$$\frac{1}{\sqrt{d_h^2+(2n-d_s+d_p)^2}} + \frac{1}{\sqrt{d_h^2+(2n+d_s+d_p)^2}})\} \tag{5.4}$$

where I is the current dissipated by the point source S, ρ_1 is the resistivity of the upper layer of the soil, ρ_2 is the resistivity of the lower layer of soil, h is the depth of the upper layer, d_s = (depth of S from the ground surface)/h, d_p = (depth of P from the ground surface)/h, d_h = (horizontal distance between S and P)/h, and K is the reflection factor. All the distances have been normalized with respect to the depth of the upper layer of the soil. Equation (5.4) is the basis for the development of an empirical formula to represent the terms under the summation sign in equation (5.3). Seedhar, Arora and Thapar have suggested the following equivalence between the infinite series and an empirical expresssion:

$$\sum_{n=2}^{\infty} \frac{2K^n}{[1+(2nX)^2]^{1/2}} = \frac{1}{X}[-K-\ln(1-K)]\sqrt{\frac{13}{13+(1/X)^2}} \tag{5.5a}$$