An Economic analysis of nitrogen fertilization of winter wheat grown in south central Montana
by Christopher Watson Wessells

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Applied Economics
Montana State University

Abstract:
The objective of this research was to estimate profit maximizing rates of nitrogen fertilizer applied to hard red winter wheat. The relationship between protein and yield response of winter wheat to nitrogen fertilizer and various soil and climatic factors were determined by using data from 30 fertilizer experiments conducted in south central Montana during the years of 1976 through 1980. The yield and protein response functions were estimated by a generalized nonlinear least squares regression.

The variables important in explaining yield and protein response were applied nitrogen, applied phosphorus, soil phosphorus, precipitation (composed of logged summer and winter precipitation), various interaction terms, and locational and periodic binary variables.

A wheat protein-premium function was estimated with simple regression to determine the relationship of premium paid for various levels of grain protein. The function was specified with a hyperbolic tangent form.

Profit functions were formulated from the response functions and protein-premium function. Profit was specified in an unconditional context, and it was specified conditionally on given levels of winter precipitation. Analysis of profit in these two contexts facilitated a measure of the additional expected profit gained from knowledge concerning winter precipitation. Furthermore, because fertilization is a prospect involving risk, based on random events such as precipitation, profit was analyzed with a safety first risk constraint. This constraint required the farmer to forego expected profit until meeting the probability constraint that expected marginal profit is negative no more than 25 percent of the time.

Analysis determined that approximately 88 pounds per acre of applied nitrogen was optimal for unconditional profit without the risk constraint; and about 64 pounds per acre with the risk constraint. Optimal profit was also obtained for various conditional profit functions. In comparison, the expected conditional and unconditional profit suggested that only a slight additional amount of profit could be expected from gauging nitrogen fertilization according to levels of winter precipitation. Finally, it was evident from comparing constrained and unconstrained profit, that nitrogen fertilization of winter wheat is a relatively riskless prospect.

AN ECONOMIC ANALYSIS OF NITROGEN FERTILIZATION OF WINTER WHEAT GROWN IN SOUTH CENTRAL MONTANA

by

Christopher Watson Wessells

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APPROVAL

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citation, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

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CHAPTER I

INTRODUCTION

Background

North America's remarkable agricultural productive capacity and export position have made substantial contributions to global economic development and to the world hunger problem. For example, American wheat is a significant food staple for many countries, and comprises the largest proportion of all agricultural commodities traded throughout the world. Within the United States, Montana farmers are among the leading producers of wheat. In fact, Montana's total annual output of winter wheat, approximately 90 million bushels per year, ranks sixth within the United States. The environment is well suited for dryland winter wheat production, which requires a cool climate at seeding, a freezing period when the wheat remains dormant, thus promoting vernalization, a moderate amount of precipitation as the wheat matures, and a warm climate at harvest [52]. These conditions, along with soil structure and management practices, result in a grain with a relatively high protein content. Growing periods in Montana typically begin with planting in late September and early October and end with harvest in August through early September. Finally, from 1960 through 1980, the area of hard red winter wheat harvested in Montana has consistently remained near 2.5 million acres. Although in 1983, Montana wheat acreage decreased considerably because of the government Payment In Kind (PIK) program. The benefits for the 1984 PIK program appear to be less lucrative for producers, and consequently, one would expect an increase in acreage harvested in hard red winter wheat over the 1983 level.

Problem Statement

Agricultural production is characterized by a decision making unit which selects combinations of scarce inputs, or factors of production, to yield some quantity and quality of product. More specifically, with intentions of increasing or at least maintaining yields, grain quality, and profit levels, virtually all agricultural producers include fertilization techniques in their farm management programs. This holds for Montana producers who have increased consumption of nitrogen, phosphorus, and potassium fertilizers approximately 60, 11, and 89 percent respectively between 1976 and 1981.[1] In spite of this trend, one should not infer that continual increases in applied fertilizers will yield optimal economic outcomes.

Frequently people presume that greater application of fertilizer is responsible for higher yields, and so, eventually, results in larger profits. This presumption is false for a number of reasons: The wheat producer must realize that exogeneous factors such as soil moisture, soil type, soil nutrients, and climate vary between farms and can greatly influence the effect of nitrogen application levels on yield. Furthermore, essentially all agricultural response functions exhibit some degree of diminishing marginal productivity. In other words, as equal and successive increments of nitrogen are applied to soil, the resulting physical product (winter wheat in this case), will increase at a decreasing rate; and, beyond some point where yield is maximized, additional applications of nitrogen will have the exasperating effect of causing a decline in the total product. Finally, in order to maximize profit, wheat producers must take into account input and output prices in conjunction with the information concerning marginal productivity and marginal factor use. Thus, economists reject the notion that augmented fertilizer applications always lead to higher yield and profits.

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[1] See page 20 of [67].

Montana winter wheat producers employ a variety of decision criteria for determining fertilizer application levels. Not surprisingly, this is due to insufficient information regarding fertilization of winter wheat. Prior to 1960, fertilizers were practically absent from the dryland farming sector in Montana [39]. Following the introduction of commercial fertilizers, producers expanded their use rapidly and, correspondingly, generated a need for accurate information regarding application rates: first, they must know the response of winter wheat (in terms of yield and protein) with respect to different levels of nitrogen and phosphorus fertilizers, changing climatic conditions, and varying soil conditions. Second, the producer requires knowledge of marginal changes in yield and protein content of the wheat caused by incremental changes in factor inputs. Third, the farmer needs accurate estimates of fertilizer prices and winter wheat prices, the latter of which has historically displayed large fluctuations. Over the past three years, from 1981 through 1983, the price of nitrogen has remained stable between 24 and 28 cents per pound. From 1972 to 1982 the nominal cost of nitrogen, in the form of ammonium nitrate, varied between 8 and 28 cents per pound.[2] During the same period, nominal winter wheat prices fluctuated between 1.56 and 6.05 dollars per bushel.[3] To further perplex the situation, wheat prices contain a "protein premium price structure." That is, prices received for wheat are based on the percentage of protein content of the grain. Relatively high protein grain sells for a higher price, or, more formally, a premium is paid. Fourth, the producer must determine his risk preferences toward recovering input costs from expected profit. In other words, the farmer selects a frequency at which he is willing to tolerate a loss. If the above information is available, then the farmer can apply it to a more scientific criterion in order to select an economically optimal fertilizer program.

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[2] These fertilizer prices are yearly averages taken from [22].

[3] These are actually prices paid for hard red winter wheat upon delivery to Portland, Oregon. Note the low price is for ordinary protein and the high price is for thirteen percent protein.

Purpose of the Study

In brief, the primary goal of this study is to design a decision criterion that will assist Montana winter wheat producers in selecting optimal levels of nitrogen fertilizer. Second, the project serves to expand upon previous agricultural economic research concerning Montana wheat production, by Garnick [31] in 1977, and by Simons [88] in 1980. More precisely, the objectives of this project are as follows:

1. To estimate the yield and protein response of winter wheat with respect to nitrogen fertilization and other significant variables such as applied phosphorus, applied potassium, soil nutrients, and precipitation.

2. To determine the optimal amount of applied nitrogen by considering the properties of the response function, input prices, the protein premium price structure of wheat, the level of winter precipitation, and a risk constraint.

3. To compile and present the information and conclusions of this study in a format that will aid farmers in determining a profit-maximizing fertilization program.

CHAPTER II

LITERATURE REVIEW

The contents of this chapter summarizes previous research related to the topic of winter wheat production. The literature is reviewed in three sections. The first section presents the founding studies in plant response research. The second section provides a general overview of qualitative and quantitative studies concerning the factors influential to crop growth and management. Finally, the third section analyzes the procedures and results of a few selected articles which include fertilizer recommendations.

Scholars have conducted extensive research concerning the relationship of yield and protein response of crops to a variety of agronomic factors. Generally, Dr. Justus Liebig is credited with the first scientific attempt to explain the relationship between plant growth and nutrient levels. His model related the factors of growth linearly to yield, and restricted plant growth by any single factor diminishing to a "minimum" state, which he referred to as the "law of the minimum" [53].

Since Liebig's pioneering studies in 1840, scientists have discovered that crop responses to nutrients are significantly more complex than initially thought. Mitscherlich [65] conducted numerous greenhouse experiments between 1911 and 1921 on corn, wheat, and other crops in order to empirically test his hypothesized yield-nutrient relationship. In 1921, Mitscherlich proposed the function: $Y = A(1 - e^{-cx})$ for the single factor case, where Y is yield, a is the approximated maximum yield, c is the coefficient associated with the single input x [66].

Briggs [15] points out the central deficiencies of the Mitscherlich function. Namely, it assumes increasing returns even at infinite applications of input x, and it implies an instantaneous and irreversible response to the input. Instead, Briggs suggests either a quadratic function, $Y = a + bx - cx^2$, or a function of the nature, $Y = a(x/x + c)$, in order to better represent a "true" plant growth function, which includes diminishing returns. Also, he postulates that some crop response functions may be cubic in form.

Proceeding three years after Briggs' criticism of the Mitscherlich equation, in 1928 Balmurkand [6] reveals Mitscherlich's failure to statistically test the precision of the values obtained for his parameter estimates, leaving the reader only to speculate upon the accuracy of the Mitscherlich studies. Furthermore, the original Mitscherlich equation was constrained to the single factor case, thus disregarding any interaction between nutrients. Balkmurkand proposed a modest variation of Liebig's inverted yield equation, with the following resistance formula: $Y = (a_n/n + N) - (b_n/(n + N)^2) + 1/c$, where a,b, are parameters, c is the maximum yield, n is a measure of soil nitrate, and N is the amount of fertilizer. Additionally, to improve readers' confidence in the presentation of agronomic research, he further proposed that researchers conduct analysis of variance tests on their estimated coefficients. Incidentally, Balmurkand intended this suggestion to improve the statistical precision and quality of early plant and soil science research, which it undoubtedly succeeded in doing. Yet unfortunately, a majority of contemporary agronomy, soil science, and plant science research still employs analysis of variance as a sole measure of statistical accuracy, when other known statistical tests and models exist that would serve as more precise measures, and thus give more credibility to modern agronomy research.

The founding work of Liebig and his colleagues has fostered the development of more sophisticated physiological plant growth models. In 1976, Smith presented an analytical model which simulates nitrogen, phosphorus, and potassium utilization in the plant-soil system for a variety of crops including corn, beans, oats, rutabagas, and pine trees [94]. To paraphrase Smith, the model is a measure of general physiology of many species subjected to a wide range of nutrient availability.

Along with the advancements in biological and physiological knowledge of crop growth, a general awareness of the technical relationship between inputs and products has increased. From inspection of the contemporary yield-response literature, one observes a variety of agronomic factors affecting protein and yield levels.

Many research projects have shown that both yield and protein content of winter wheat can be greatly enhanced from applications of nitrogen fertilizer [31,41,54,61,88, 90]. In 1950, agronomists found that Tennessee wheat grain yields and quality, in other words, yields and protein, are influenced by nitrogen fertilizer, with the most potent yield response occurring from November applications of nitrogen. Later fertilization resulted in a smaller response, with practically no response resulting from applications made in the late spring, after May fourth [54]. Conclusions drawn in the preceding study coincide with results from earlier projects. In particular, research regarding the response of soft winter wheat grown in Ohio revealed that applications of nitrogen made at or near seeding tended to increase yield; and, in contrast, nitrogen applied in spring tended to inhibit yields [9]. Yet on other locations, where growing conditions differ greatly from those in Ohio or Tennessee, fall application of nitrogen is frequently less effective than spring application in increasing yield. For example, in the prairie provinces of Canada, scientists report substantial losses of fall applied nitrogen fertilizers over winter [81]. Urea, KNO_3, and $(NH_4)_2SO_4$ comprise three forms of nitrogen tested in this experiment. The loss of nitrogen, occurring in early spring, was primarily due to denitrification rather than leaching. In fact, in the extreme case, denitrification accounted for 87 percent of the fall applied KNO_3 lost. Consequently, agronomists advise Canadian farmers to use a starter fertilizer, then apply larger treatments of nitrogen in the springtime. Additionally, agronomists found that late spring applications of nitrogen improved yields and grain protein for both spring and winter wheat grown in Illinois [41]. Clearly, as Williams and Smith [111] emphasize, the timing of fertilization will vary between geographic locations, depending upon soil and climatic indigeneous to a given area.

Commonly, yield studies include at least one explanatory variable that represents a soil characteristic such as soil nitrate (NO_3-N), soil phosphorus (in soil solution, organic, and inorganic forms [26]), or soil moisture, and often these studies analyze interactions between nitrogen, phosphorus, and potassium. Extensive soil potassium tests conducted in Montana from 1972 through 1974 reveal that "nearly all major agricultural soils of Montana are on the 'high' category of K (potassium)."[4] Meaning that"... crop responses to potassium fertilizer would occur only infrequently."[5] Nevertheless, the study further states that on potassium deficient soils, on average, winter wheat yields increase by 5 bushels per acre, provided the presence of adequate nitrogen and phosphorus. Experiments by Koch and Mengel [49] showed that potassium improves the translocation of nitrogeneous compounds in wheat, which, in turn, increases the formation of amino acids and protein during grain formation.

Similar to potassium, phosphorus is a critical component in plant and soil systems [12]. Canadian researchers observed pronounced, and frequently variable, crop response to phosphate fertilizers, to plant utilization of applied fertilizers, and to interactions between nitrogen and phosphate [42]. They conclude that plant failure to mobilize phosphorus may be due to one of the following causes [42]:

1. The absorption capacity of the roots may be damaged from moisture stress.
2. Changing physical conditions of moisture in the soil may result in decreased availability of soil phosphorus in the root zone.

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[4] See page 1 of [92].

[5] Ibid.

Experiments involving spring wheat and barley production, grown on four different soils in North Dakota, suggest that phosphorus alone has no influence on yield, but its interaction with nitrogen and water significantly affects yield. A study completed in Nebraska [70], using wheat and oats, indicates that combined nitrogen and phosphorus functions synergistically as compared to the effect of the same elements individually.

Field experiments on a diversity of crops show that efficient combinations of the three paramount fertilizers, render increased yields [26,56,57]. In particular, this is true for soils which contain relatively low levels of nutrients. For example, Michaelson et al. [64] estimated multiple regression equations to predict barley grain yields on nutrient deficient subarctic soils. They found significant parameters for nitrogen and phosphorus, in addition to significant interactions between nitrogen and phosphorus, nitrogen and potassium.

Numerous investigations show that soil chemicals and nutrients essential to plant growth can limit crop yields. In many areas, the most limiting soil component is soil nitrate [78,101,107]. This is especially true in countries where relative prices of nitrogen fertilizers are extremely high or where availability of applied nitrogen is scarce or nonexistent [19]. In Nebraskan corn and winter wheat experiments [71] soil nitrate significantly influenced yields and grain protein percentages. Young et al. used multiple regression techniques to determine explanatory yield factors. They found that at seeding, soil nitrate to a depth of 61 centimeters has a significant relationship with yield response of spring cereal grains to applied nitrogen [115]. Other research, particularly relevant to winter wheat crops grown in Montana [51,89], concludes that frozen soil profiles impair the translocation of soil nutrients to plant roots. In wheat and rye experiments, Gashaw and Mugwira [32] discovered that applied ammonium (NH_4-N) increases the uptake of nitrogen, phosphorus, and iron, while soil nitrate increases the uptake of magnesium, calcium, and manganese. Traditionally, soil scientists have maintained that soil nitrate substitutes perfectly for applied nitrogen [35,50]. Although, in contrast to that idea, recent research indicates that nitrogen fertilizer is approximately three times more efficient that soil nitrate in producing winter wheat [19,36].

At this point, concerning soil nutrients, it is worthwhile to note that considerable controversy exists between soil scientists as to the accuracy of soil testing. Most soil scientists profess that procedures for measuring soil nutrients are accurate [35]. However, others believe that no precise tests exist for certain chemicals and micronutrients [43]. Thus one must maintain discretion when one undertakes research involving soil nutrients.

In addition to edaphatic influences on crop yields and grain protein, other factors such as soil moisture, precipitation, and temperature usually exhibit significant relationships to successful crop production [5,13,91]. Canadian greenhouse experiments demonstrate that increased wheat yields may be obtained from larger quantities of soil moisture (up to three-quarters of field capacity) in loam but not in loamy sand [21]. Wheat and barley field experiments show that yield responses to nitrogen fertilizer on nonfallowed soils are significantly affected by growing season precipitation and available stored soil moisture at seeding. In fact, the sum of growing season precipitation and soil moisture comprised 40.3 percent of the yield response to nitrogen [8]. In 1924, Fisher used linear regression techniques to predict wheat yields from rainfall distribution at Rothamsted, England [25]. Since then, many researchers have utilized Fisher's methods on different crops in dissimilar environments. One such study, completed in India [30], concluded that 75 percent of the yield variation was due to rainfall distribution. Furthermore, the study presents response curves depicting expected changes in yield caused by marginal increases in precipitation at any point in time. These response curves show that precipitation exceeding the average during the period of a month prior to seeding and during the period of germination is generally beneficial to the crop, while precipitation at tillage is damaging [30].

A Montana study [14] examining the long term fertilizer and climatic influences on morphology and yield of spring wheat shows that water use efficiency increases with increasing rates of applied nitrogen and phosphorus. The most efficient combination of nitrogen (45 kg/ha) and phosphorus (180 kg/ha) resulted in water utilization of 91 kg/ha/cm by the plant.

Variation in management practices, such as tillage [7,104,110]; timing and rates of seeding [74]; types and application methods of fertilizers [37,111], insecticides, and herbicides; all influence crop yields. Pendleton and Dungan [74] present evidence that yield response to the seeding rates and dates of applied nitrogen differs between four varieties of winter wheat. The Knox variety showed the strongest response to nitrogen, and maintained the highest yields as seeding was increased from three to eighteen pecks per acre. Moreover, they explain that,

> Variety selection showed the greatest effect on grain yield, heading date, test weight, plant height, straw yield, ratio of straw to grain yield, and kernel weight. Application of nitrogen exhibited the greatest effect on plant erectness, clover stands, and protein content of the grain. Seeding rate . . . (influenced) . . . the number of grain-bearing heads per plant [74]. page 312.

A Montana study [76] concludes that rates at which winter wheat is seeded possess little influence on yields. However, in a general sense, the study recommends seeding dates from September tenth through the twenty-fifth for western and central sections of the state, and from September first through the tenth for eastern Montana.

Typically, dry fertilizers are either side-dressed or broadcast, and liquid fertilizers are either injected into the soil or sprayed on the surface. Soil scientists have also observed successful results from other techniques of application. For instance, Altman et al. [1] experimented with foliar sprayed urea, applied to immature spikes of hard red winter wheat. They discovered increases in average grain protein percentage and in average yield by thirteen and six percent respectively, over a control plot which received no urea. Other studies [93] have similarly found foliar sprayed fertilizer quite effective in increasing protein and yield of wheat grain.

Research also shows that efficient use and absorption of nitrogen by plants depends highly on the amount and type of nitrogen available [32]. Peterson et al. [75] concluded that, in general, the broadcasting method of fertilization is frequently less effective than row applications. However, the effectiveness ratio of broadcast to row applications of phosphorus appears to depend solely upon the level and availability of soil phosphorus present in the plant root zone. Often for cereal crops, agronomists recommend split applications of fertilizers, with treatments in the fall and spring, in order to enhance production of grain and protein. Winter wheat experiments [41] involving a series of three spring nitrogen applications (April 2, 23, and May 9) displayed markedly greater yields and higher percent protein over plots which received a single dose of fertilizer (on April 2) equivalent to the sum of the split nitrogen treatments.

In addition to a fertilizer plan, the manager must also select a crop rotation scheme. Research concerning dryland wheat production in the southern Great Plains [4] suggests that the optimal cropping pattern, based on long term precipitation records, is an alternating wheat-fallow, and wheat-sorghum-fallow rotation. An innovative farm management study [16] demonstrates how stochastic dynamic programming can be used to select an optimal policy, either crop or fallow, for any given state (soil moisture at seeding) at each stage (a year within the farmer's planning horizon). The resulting long-run expected return per year under the optimal policy, the continuous wheat policy, and the alternating fallow and wheat policy are, respectively, $25.60, $22.56, and $20.60 per acre.

Cochran et al. [18] found that direct drilling of winter wheat into soil covered with crop residue promotes more efficient water use, and decreased soil erosion, but tends to result in stunted plants and lower grain yields. Ecofallow [110], which minimizes tillage within a crop rotation, and chemical fallow [79], which employs herbicides and insecticides, have both shown fruitful results in regard to crop water management and soil conservation. Yet the central drawback of those methods is lower mean yields compared with yields obtained after traditional fallow practices. Note that chemical fallow, nontillage, and limited tillage practices considerably reduce the farmers' diesel fuel costs. Studies have been undertaken to explore the profitability of various fallow techniques in cereal grain production [7,79].

The crop response literature contains extensive analysis of protein yields for grain crops. Most experiments [42,60,90] show that an inverse relationship exists between grain yield and protein content of grain. This relationship is intuitively appealing because, in general, one would expect that as yield or grain mass increases, protein becomes diluted in a sizably larger kernel.

In order to clarify conflicting observations on the nature of the relationship between yield and protein, McNeal et al. [62] engaged in meticulous experimentation with eight wheat varieties, grown under many conditions, in locations near Kalispell, Bozeman, and Huntley, Montana. They confirmed an inverse relationship between yield and protein for wheat grown in Montana. Furthermore they suggest that grain protein formation depends on a combination of genetic and environmental variables.

> (Specifically) . . . it appears that grain protein percentages were entirely dependent upon the amount of carbohydrate (amino acids) translocated into grain. This may have been influenced by the number of kernels present to act as carbohydrate sinks, and by the ability of plants to translocate carbohydrates into kernels. Both kernel number and kernel weight are affected by environment, but are also under genetic control [62]. page 601.

Moreover, the two paramount ingredients for protein formation in wheat grain are nitrogen and potassium [40]. In short, within the plant, nitrogen is restructured into amino acids, primarily lysine and methionine. Then elemental potassium and water carry the free amino acids to the kernels, where conversion of the amino acids builds the four types of grain protein: Albumin, globulin, prolamin, and glutelin [63].

In a study concerning irrigation experiments, Terman et al. [101] explain that the primary effect of applied nitrogen in the presence of adequate water was to increase yields, while in a situation with low water levels, nitrogen mainly increased grain protein. From the same article, when dryland wheat experiments displayed significant grain yield response to nitrogen, simultaneously, scientists observed the unusual result of increasing percent grain protein. Although, when yield response to nitrogen was negligible, an extremely potent protein response resulted from applied nitrogen. Furthermore, in a third experiment, Terman and his colleagues stated that,

> Average protein contents of about 20 hard red winter wheat varieties at different locations in Nebraska in 1966 and 1967 were . . . negatively related to grain yields. Protein contents varied more widely among locations than among varieties at each location [101]. page 755.

A project conducted in Belgrade, Montana [61] tested the protein contents of five spring wheat varieties, including short, medium, and tall genotypes. Grain yields were similar for all varieties, but grain protein percentages decreased as the grain to straw ratio increased. This implies that the quantity of top growth serves an important role in extracting soil nutrients. Also, water use by all five varieties increased with increasing rates of applied nitrogen. Oswalt and Schlehuber [72] point out three fundamental factors responsible for protein development in grain. Specifically they mention the environment or climate, composition of the soil, and wheat variety. Environmental elements are especially important because the availability of sufficient precipitation or soil moisture dictates the amount of nitrogen and other essential nutrients transferred in the soil and within the plant, then engendering protein formation in the kernels. As early as 1942 researchers [73] employed a “regressional integral” technique to show the importance of rainfall to protein content in wheat. Oswalt and Schlehuber conclude that nitrogen has greater influence on protein content than on grain yield.

A majority of yield and protein response studies lead to a fertilizer recommendation for a crop grown under specific geographic conditions. These recommendations are often based on experiments with fertilizer applied at, for example, rates of 0, 30, 60, and 90 pounds per acre; then researchers [41,64,70,75,77] mistakenly report one of the four levels of fertilization as "optimal," using highest average yield as a criterion. Furthermore, the most glaring error in the plant science, soil science, and agronomy literature is the total disregard for input and output prices [44,85]. For many years economists have known that prices are among the essential components for making appropriate profit maximizing production decisions [2,19,38]. Thus, failure to include prices in analyses leads to suboptimal recommendations. In sophisticated studies [71,74], atypical to the majority of current literature, researchers properly apply statistics and calculus to their data in order to maximize profit surfaces. Fertilizer recommenations based on this type of logic are much more accurate than those founded on maximum yield or protein, on fertilizer budget models [85], or on personal opinions.

De Janvry's [19] effort to estimate production of Argentine wheat and corn under risk exemplifies one reasonable process which leads to economic fertilizer recommendations. Five environmentally different sites, representative of growing conditions in Argentina, served as data sources for observations on wheat response to nitrogen and phosphorus fertilization. Grain yields were estimated for each location with Cobb-Douglas functional specifications. The independent variables included in the regressions were: nitrogen, phosphorus, organic matter in the soil, rainfall at seeding, rainfall at plant emergence, rainfall at tasseling, and dummy variables representing location and wheat variety. Nitrogen recommendations were formulated from a "fertilization possibilities frontier,"[6] depending on

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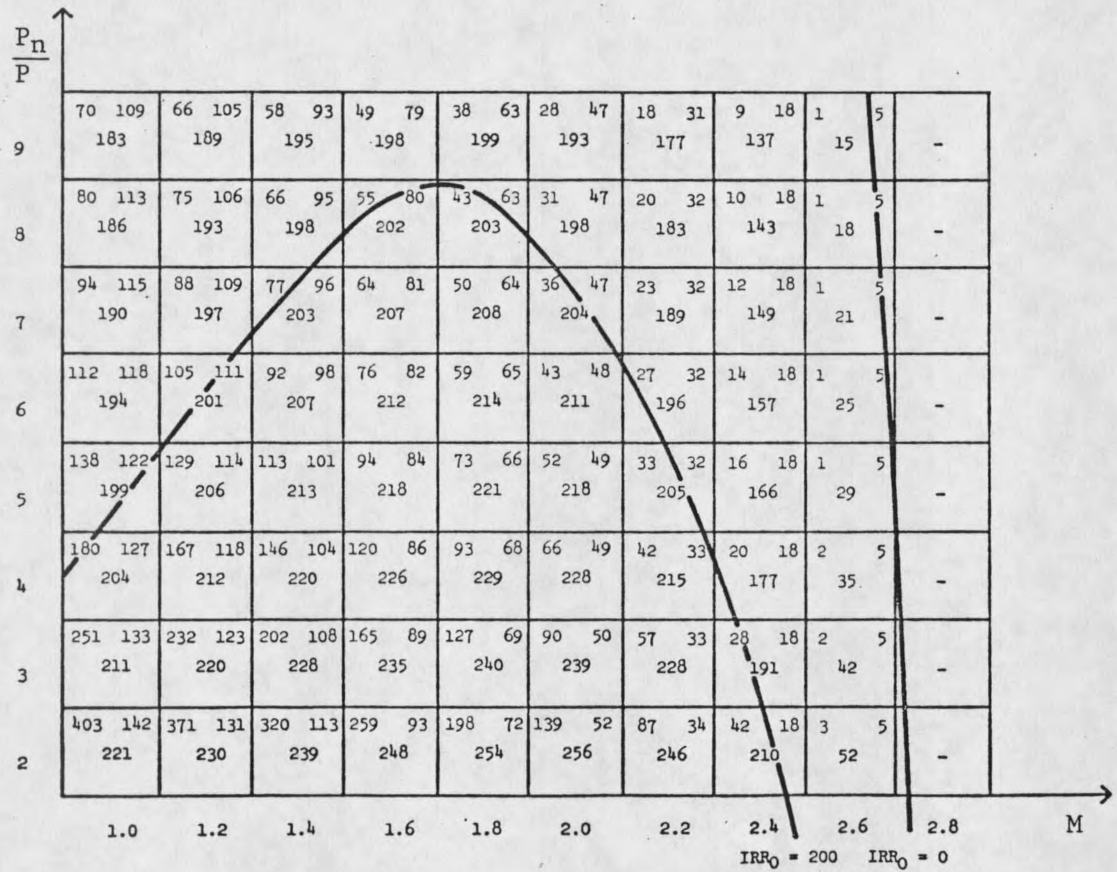
[6] See De Janvry [19], page 4, for the mathematical derivation of the "fertilization possibilities frontier."

various price ratios, organic matter, probabilities of incurring revenue less than the cost of fertilization, and other variables fixed at their means.

Figure 1, copied from De Janvry's article, displays two "fertilization possibility frontiers" as a function of soil organic matter for southern Argentina farms.



$\frac{P_n}{P}$ \ M	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8
9	70 109 183	66 105 189	58 93 195	49 79 198	38 63 199	28 47 193	18 31 177	9 18 137	1 5 15	-
8	80 113 186	75 106 193	66 95 198	55 80 202	43 63 203	31 47 198	20 32 183	10 18 143	1 5 18	-
7	94 115 190	88 109 197	77 96 203	64 81 207	50 64 208	36 47 204	23 32 189	12 18 149	1 5 21	-
6	112 118 194	105 111 201	92 98 207	76 82 212	59 65 214	43 48 211	27 32 196	14 18 157	1 5 25	-
5	138 122 199	129 114 206	113 101 213	94 84 218	73 66 221	52 49 218	33 32 205	16 18 166	1 5 29	-
4	180 127 204	167 118 212	146 104 220	120 86 226	93 68 229	66 49 228	42 33 215	20 18 177	2 5 35	-
3	251 133 211	232 123 220	202 108 228	165 89 235	127 69 240	90 50 239	57 33 228	28 18 191	2 5 42	-
2	403 142 221	371 131 230	320 113 239	259 93 248	198 72 254	139 52 256	87 34 246	42 18 210	3 5 52	-

Figure 1. Optimum level of wheat fertilization in southern Argentina. (Reproduced from De Janvry, Table 4 [19]).[7]

If the producer could obtain accurate estimates of organic matter, precipitation, and prices, in conjunction with risk preferences, then they could apply their information to a table similar to the one listed, and determine optimal levels of nitrogen and phosphorus to maximize profit.

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[7] The data that appear in each cell are the optimum nitrogen (N_O) and phosphorus (F_O) in kg/ha; the corresponding internal rate of return (IRR_O) in percent.

Recent efforts to quantify winter wheat yield and protein response under Montana conditions were made by Garnick [31] in 1977. His data set included observations on yield; percent protein; applied nitrogen, phosphorus, and potassium; soil nitrate; soil water; and summer precipitation from 43 experiments on 38 locations in Montana. Final yield and protein response functions were specified as third degree polynomials with interaction variables and multiplicative disturbance terms. Incidentally, with a multiplicative error contained in the model, the standard deviation of the disturbance is proportional to the mean of the dependent variable. Parameter estimates were generated by method of GNLR, a nonlinear regression computer program written by Dr. Oscar Burt (Montana State University). Garnick found that "only applied nitrogen, precipitation, and soil nitrate were significant in explaining yield response."[8] His final estimate exhibited appropriate theoretical behavior. That is, for each variable, the response function had a maximum and exhibited diminishing marginal returns. As with yield, protein showed significant reponse from nitrogen, precipitation, and soil nitrate. However, accompanying these variables, protein was also affected by applied phosphorus.

Eventually, Garnick combines the response functions with a crude estimate of protein premium to form a profit function. Optimal rates of nitrogen are displayed in two-way tables with varied rates of precipitation and price ratios on the two axes. His results show that accounting for the protein premium structure of wheat prices boosts optimal rates of nitrogen by approximately 15 pounds per acre, as compared to recommendations without a premium. Finally, a 25 percent marginal rate of return was specified as a constraint, which resulted in a slight reduction in the optimal levels of nitrogen.

Similar wheat response research was done by Simons [88] in 1980. His estimates of polynomial yield and protein functions revealed applied nitrogen, phosphorus, potassium,

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[8] See page 72 of [31].

soil nitrate, soil phosphorus, a water variable, and interactions among the factors as significant explanatory elements. Protein premium was not included in the optimization. Simons made recommendations for phosphorus and nitrogen based on three different price levels for wheat with various capital constraints. Depending on different price and constraint scenarios, optimal nitrogen varied from 45 to 85 lbs/ac, and optimal phosphorus rates were between 0 and 19 lbs/ac.

The preceding discussion of wheat production is by no means exhaustive. In fact, the list of works which explore factors contributing to optimal wheat production could conceivably fill volumes of texts. Nevertheless, this literature review should sufficiently point out that a number of variables and interactions influence grain response. To reiterate the literature, applied nitrogen, phosphorus, and potassium fertilizers have been found to be positively correlated with grain yield and protein response. Studies also demonstrate that soil nitrate substitutes imperfectly for nitrogen. The interactions of nitrogen and potassium, and also, nitrogen and phosphorus are essential for proper grain protein formation. The presence of micronutrients in the soil, along with soil nitrate contribute positively to yields and protein, but the evidence of exactly how these nutrients interact to promote plant growth is at best vague. This problem is primarily due to our dependence on imprecise tests for measuring soil nutrients. In general, increasing water availability, in either the form of precipitation or soil water, is postively correlated with yield and has a negative relationship to protein. Management practices, such as crop rotation, varietal selection, methods of fertilizer application, and timing of planting and fertilization, exemplify a few more important factors in determining crop response. This project is intended to carefully analyze Montana winter wheat production and improve upon the existing knowledge in the field of agricultural production economics.

CHAPTER III

DEVELOPMENT AND ESTIMATION OF YIELD AND PROTEIN RESPONSE FUNCTIONS

Methodology

The data used to fit the yield and protein response functions were generated by personnel of the Southern Agricultural Research Center, Huntley, Montana, on experimental plots located on 18 farms in south central Montana. Dr. Vincent A. Haby[9] directed a series of fertilizer experiments which provided the data set. A total of 30 treatments of various combinations of nitrogen, phosphorus, and potassium were applied to each of 30 total sites over five years (1976-1980). Note that periodically the experiment was replicated some years on 18 farms, hence accounting for 30 total experiments. Furthermore, among the 30 fertilizer treatments per site, there were seven replications. The product of 30 treatments and 30 site-years result in 900 observations. Appendix A summarizes information on experimental location and year.

Selection of experimental sites was based on a specific set of characteristics. Namely, the locations chosen were adjacent sites in a crop-fallow rotation sharing uniform soil conditions, unhindered by insects, weeds, and other detrimental factors. In further efforts to achieve consistent soil types scientists examined surface color and texture, and strata concentrations of soil nitrate, elemental phosphorus, and potassium. After scientists deemed an experimental site acceptable, then individual plots (2.44 X 6.1 m) containing

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[9] Currently Dr. Haby is an associate professor at the Agricultural Research and Extension Center of Texas A&M University.

eight rows of wheat were delineated. Sites which either incurred hail damage to crops or suffered from unusually high saline content were excluded from this analysis.

In the fall, prior to seeding, soil samples were analyzed for each plot. Soil nitrate was measured by depth of 0 through 15, 15 through 30, 30 through 60, 60 through 90, and 90 through 120 cm using the "CTA" method of Haby and Larson [35]. Tests were run to determine quantities of soil organic matter and the minerals potassium, calcium, and magnesium. Scientists also calculated the levels of elemental soil phosphorus by depths of 0 through 15, 15 through 30, and 30 through 60 cm. In the spring, before applications of nitrogen, soil nutrients were measured again to check for any appreciable change over winter.

Measurements of weather conditions were made for each site. Summer precipitation (April through July), temperature, and estimates of evapotranspiration were the primary factors subject to observation. Any errors in measurement were substituted with similar data from the nearest weather station. Researchers did not compile observations regarding winter precipitation (September through March). Consequently, that data was taken from records of the nearest weather station that was within the same average precipitation gradient of the site in question.[10]

Applications of nitrogen, phosphorus, and potassium were each calibrated precisely at five rates and followed a nested-cube statistical design developed by Lund and Linell [55]. The nitrogen fertilizer, ammonium nitrate, was top dressed in spring at rates varying from 0 to 89 kg/ha in increments of 22.25 kg. Phosphorus, in the form of triple superphosphate, was banded with seed at rates ranging from 0 to 35.7 kg/ha in increments of 8.925 kg. Potassium, as muriate of potash, was rototilled to a depth of 10 cm at rates of 0 to 96 kg/ha in increments of 24 kg.

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[10] This precipitation gradient map designates areas of equivalent average annual precipitation within the State of Montana. It was prepared by the U.S. Soil Conservation Service, Box 970, Bozeman, MT 59715.

Winter wheat (the Centurnk variety) was seeded in 30 cm row widths at a rate of 50 kg/ha. Weeds were sprayed with 0.42 kg/ha of (2,4-dichlorophenoxy) acetic acid (2,4-D) amine plus 0.125 kg/ha Banvel. Grain protein content was determined as a percentage of kernel weight, and grain yield was calculated in kg/ha after harvesting the middle four of the eight-row plots. See Appendix B for summary statistics of the data.

Model Description

The emergence of production economics originated from research conducted by both Ricardo [80] and Von Thünen [106] during the late eighteenth and early nineteenth centuries. These two individuals were the first to examine the concepts of marginal changes in total product. Their research serves as a basis for many important theoretical [17] and applied studies in production economics.

Since the era of Ricardo and Von Thünen, economists have come to realize that productions functions can take on a variety of algebraic forms. Heady and Dillon [38] present a general synopsis of the response functions representative of agricultural production. In selecting a functional form the researchers must carefully examine the physical and economic ramifications imposed by various models. Obviously, the function which best simulates the nature of the topic should be chosen. For this project a somewhat modified polynomial equation (i.e., it contained some logged variables) was selected for estimating yield response. Beattie and Taylor point out certain virtues of polynomial functions.

> The quadratic production function . . . is appealing because it gives a second-order local approximation to any function . . . A cubic specification . . . is even more appealing because it gives a third-order approximation to any function. Both the quadratic and cubic functions are relatively easy to statistically estimate . . . (10). page 117.

Generally, agricultural economists have experienced successful use of polynomial equations to fit crop-yield data [31,38,84,88], but more importantly, the properties associated with polynomial functions seem most appropriate for the data under consideration.

The general functional specification for yield takes the following form. Note that the function is nonlinear because X_n is actually composed of quadratic and interactive variables

$$Y = \beta_0 + \sum_{n=1}^{N} B_n X_n + \sum_{j=1}^{J} \tau_j D_j + \sum_{t=1}^{T} \phi_t Z_t + \epsilon_{ijt} \tag{III.1}$$

where:

Y = total wheat yield

β_0 = autonomous yield (intercept)

X_n = the nth explanatory variable (representing linear, squared and interaction terms)

β_n = unknown parameter associated with the nth explanatory variable

D_j = the jth locational binary variable

τ_j = unknown parameter associated with the jth locational binary variable

Z_t = the tth periodic binary variable

ϕ_t = unknown parameter associated with the tth periodic binary variable

ϵ_{ijt} = the random component disturbance term

Σ = the summation operator

For clarity, Table 1 contains a list of all the explanatory variables used in the final model specifications.

One can readily observe from Equation III.1 and Table 1 that the yield equation is linear with respect to parameters and contains some third order polynomial terms. Note that the explanatory variables are organized by re-defining nonlinear variables, that is, the quadratic and third degree terms, as linear variables, thus facilitating the linear relationship in the parameters.

A point mentioned in Chapter II suggested that investigation concerning grain yields demonstrate that the effect of certain nutrients on yield are significantly influenced by the presence of other nutrients. Therefore, the model includes interaction variables which

Table 1. Explanatory Variables Used in Estimating Wheat Yield Response.

X_0 = binary variable for missing data
X_1 = applied nitrogen
X_2 = applied nitrogen squared
X_3 = applied phosphorus
X_4 = applied phosphorus squared
X_5 = composite precipitation $= \beta_{15} X_{15} + (1 - \beta_{15})X_{14}$
X_6 = composite precipitation squared
X_7 = soil phosphorus
X_8 = soil phosphorus squared
X_9 = applied nitrogen × applied phosphorus
X_{10} = applied nitrogen × composite precipitation
X_{11} = applied nitrogen × composite precipitation squared
X_{12} = applied phosphorus × composite precipitation
X_{13} = applied phosphorus × soil phosphorus
X_{14} = log of winter precipitation
X_{15} = log of summer precipitation
D_1 = binary variable for the Warren corp.
D_2 = binary variable for the K. Hanson farm
D_3 = binary variable for the Eastlick farm
D_4 = binary variable for the Keller farm
D_5 = binary variable for the Logan farm
D_6 = binary variable for the Haines farm
D_7 = binary variable for the A. Hanson farm
D_8 = binary variable for the Herzog farm
D_9 = binary variable for the Holland farm
D_{10} = binary variable for the McFarland farm
D_{11} = binary variable for the Mosdal farm
D_{12} = binary variable for the Sire farm
D_{13} = binary variable for the S.A.R.C.
D_{14} = binary variable for the Lee farm
D_{15} = binary variable for the Becker farm
D_{16} = binary variable for the Schaff farm
D_{17} = binary variable for the Micheal farm
Z_1 = binary variable for the year 1977
Z_2 = binary variable for the year 1978
Z_3 = binary variable for the year 1979
Z_4 = binary variable for the year 1980

reflect the non-additive characteristics of some inputs. The binary variables listed in Table 1 represent fixed effects associated with location and year. They will be explained in further detail in the section presenting ideas underlying the disturbance term. Furthermore, an inquisitive reader may be interested in the functional specifications and regression results which lead to the final set of variables in Table 2, and their parameter estimates in Table 4 on page 38. Initial and intermediate regressions will be discussed later in this chapter in the section concerning the derivation of relevant variables.

For a well behaved (i.e., strictly concave or strictly quasiconcave) production function which exhibits diminishing returns, in theory, we would expect the parameter estimates to have the following signs:

The parameters associated with the linear and logged variables:

$\beta_0, \beta_1, \beta_2, \beta_3, \beta_5, \beta_7, \beta_{14}, \beta_{15} > 0$ for positive product

The parameters associated with squared variables:

$\beta_2, \beta_4, \beta_6, \beta_8 < 0$ for diminishing marginal productivity

When considering the coefficients for the interaction variables, it would be imprudent to make statements regarding their signs prior to estimation. That is, when estimating a complex polynomial with many different variables and interactions, it is difficult to make inferences concerning the technical interrelationship between factors. On the other hand, when examining a simple polynomial production function with one interaction, then one may feel more comfortable in stating a hypothesized sign for the interaction.

Due to the complex structure of the specification, that is, with linear, quadratic, and interaction terms, it would not be surprising to see atypical signs for some of the parameter estimates. This could be considered a problem, yet more importantly, one should focus attention on the behavior of the partial derivatives of yield with respect to specific explanatory variables. If the partial derivatives exemplify reasonable behavior, along with satisfactory second order conditions for convexity, then, in some cases, parameter estimates possessing theoretically unusual signs may be permissible.

The yield equation was specified with an additive error term because previous yield research by Garnick [31] and Simmons [88] shows that Montana wheat yield and protein functions can be satisfactorily fitted with an additive disturbance. For this project, no functional specifications included a multiplicative disturbance. Incidentally, researchers often use a multiplicative model for its appealing characteristics. Namely, the mean of the dependent variable has a proportional relationship with variation or the standard deviation of the estimate. Additionally, if the observations on the dependent variable covers a wide range, then, frequently, the multiplicative model is more appropriate than the additive model. A formal test to compare the two models is presented by Just and Hallam [46].

Another notable characteristic of the regression model is that the error term for the pooled time series and cross sectional data was decomposed into systematic components, or effects, which are typically thought to be either random or fixed, depending on the nature of the effect [45]. Mundlock [68] explains that the selection of an estimation procedure, that is, utilizing dummy variables, employing stochastic error components, or using a combination of the two methods, depends on assumptions made about the constant or intercept parameters. To exemplify this point, consider the following hypothetical model:

$$y_{jt} = \alpha_{1j} + \sum_{n=2}^{N} \beta_n x_{njt} + \epsilon_{jt} \qquad \text{(III.2)}$$

where:

J = 1, 2, . . ., J cross-sectional units (such as location)

t = 1,2, . . ., T time periods (such as years)

y_{jt} = the observed dependent variable for the jth cross-sectional unit and tth period

x_{njt} = the nth observed explanatory variable for the jth cross-sectional unit in the tth period

ϵ_{jt} = the random error, independently distributed over cross-sectional units and time

β_n = slope parameters associated with the explanatory variables

α_{1j} = the intercept parameters which are constant over time and different for cross-sectional units

If α_{1j} is characteristically random then the intercept parameter may assume the form $\alpha_{1j} = \tilde{\alpha}_1 + \psi_j$, where $\tilde{\alpha}_1$ is an unknown parameter and ψ_j are random independent and identically distributed with mean zero and constant variance. Thus, the assumption of a random effect implies the use of the stochastic error components model. Specifically, Equation III.2 may be written as:

$$y_{jt} = \tilde{\alpha}_1 + \sum_{n=2}^{N} \beta_n x_{njt} + \psi_j + \epsilon_{jt} \tag{III.3}$$

Maddala [58] shows that for this error component model the generalized least squares estimator is the best linear unbiased estimator.

On the other hand, if, by nature, α_{1j} is fixed, then the use of dummy variables is appropriate, and Equation III.2 is respecified as:

$$y_{jt} = \sum_{k=1}^{J} \alpha_{1k} D_{kt} + \sum_{n=2}^{N} \beta_n x_{njt} + \epsilon_{jt} \tag{III.4}$$

where D_{kt} are "zero-one" dummary variables which correspond to cross-sectional units, k. Moreover,

$$D_{kt} = \begin{cases} 0 & \text{for } k \neq j \\ 1 & \text{for } k = j \end{cases}$$

Usually one cannot easily determine the nature of an effect, that is, deciding if it is fixed or random. However, Mundlak indicates that frequently "... the whole approach which calls for a decision on the nature of the effect, whether it is random or fixed, is both, arbitrary and unnecessary."[11] Basically, he shows that in both cases the ψ_j can be used as a random component of the disturbances, although, when employing the dummy variable technique, the inference that the ψ_j are random is conditional on the effects ψ_j in the sample. The central difference between the component error model and the dummy variable model pertains to specific assumptions of the effect ψ_j. The error components model necessitates assumptions regarding the distribution of ψ_j; whereas the conditional inference implied by the dummy variable method requires no specific assumptions. This added flexibility of the dummy variable method permits its application to a larger variety of problems. For presentations which include applications of component error models and dummy variables to agricultural production topics, the interested reader is encouraged to consult Mundlak and Hellinghausen [69] and Antle [3]. Chapter 15 of Graybill [33] and an article by Dielman [20] provide more elaborate explanations of the statistical theory for error structures related to pooled data.

The component error term associated with the wheat yield and protein regression models may be written in general as:

$$\epsilon_{ijt} = \nu_{ijt} + \psi_{1i} + \psi_{2j} + \psi_{3t} + \omega_{jt} \qquad \text{(III.5)}$$

where:

ν_{ijt} = random component associated with the ith treatment, jth location, and tth year

ψ_{1i} = random component associated with the ith treatment

ψ_{2j} = random component associated with the jth location

ψ_{3t} = random component associated with the tth year

ω_{jt} = an interaction component for the jth location and tth year

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[11] See page 70 of [68].

The disturbance associated with treatments, ψ_{1i}, is tacitly assumed to be zero because fertilizer applications followed a random design. Also, in the course of research, dummy variables were incorporated into the yield and protein specifications to explicitly account for effects of location and year. Generally, use of dummy variables, in conjunction with pooled cross section and time series data, stems from concern about biased estimates of the slope coefficients. The logic supporting their use in a pooled cross section time series context is synonymous to that which serves as a basis for trend, seasonal, and age variables employed in a time series framework.[12] The parameters associated with the dummy variables Z_1 through Z_{17} and D_1 through D_4 account for fixed effects for location and year, respectively, which are not attributable to the causal variables, X_1 through X_{15}. In other words, the dummy variables usurp the independent effects of location and time period from the error components ψ_{2j} and ψ_{3t}. Finally, ω_{jt} is the interactive contribution of location and year which is not explained by the causal variables or dummy variables. Consequently, Equation III.5, the error components, now becomes,

$$\epsilon_{ijt} = \nu_{ijt} + \omega_{jt} \tag{III.6}$$

where the first source of random variation is represented by ν_{ijt}, which is independently and identically distributed across treatments, i, locations, j, and years, t. Similarly, the second source of variation, ω_{jt}, is independently and identically distributed across locations, j, and years, t.

The statistical assumptions associated with the error model, Equation III.6, are designed in the proceeding section.

1. $\nu_{ijt} \sim N(0, \sigma_\nu^2)$
2. $\omega_{jt} \sim N(0, \sigma_\omega^2)$

Also, the components ν_{ijt} and ω_{jt} are assumed to be independent, so that,

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[12] See Wonnacott and Wonnacott [112], Chapters 4 and 6, concerning dummy variables, and Chapter 16 concerning adjustments for season and/or trend.

3. $E(\nu_{ijt}\,\omega_{jt}) = E(\nu_{ijt}) \cdot E(\omega_{jt}) = 0,$ for all i, j, and t

The fact that any linear combination of normally distributed random variables is normal.

4. $\epsilon_{ijt} \sim N(0, \sigma_\epsilon^2)$

From assumptions 1, 2, and 3, it is relatively easy to show that

5. $\mathrm{Var}(\nu_{ijt}) = \sigma_\nu^2 = E(\nu_{ijt}^2)$

6. $\mathrm{Var}(\omega_{jt}) = \sigma_\omega^2 = E(\omega_{jt}^2)$

The homoscedastic structure of the variance of the error components and their independence implies that

$$\mathrm{Var}(\epsilon_{ijt}) = \sigma_\epsilon^2 = E(\epsilon_{ijt}^2) - [E(\epsilon_{ijt})]^2 = E(\epsilon_{ijt}^2)$$

where:

$$E(\epsilon_{ijt}^2) = E[(\nu_{ijt} + \omega_{jt})^2] = E(\nu_{ijt}^2) + E(\omega_{jt}^2) + 2[E(\nu_{ijt}) \cdot E(\omega_{jt})], \text{ i.e.,}$$
$$+ 2[E(\nu_{ijt}) \cdot E(\omega_{jt})]$$

7. $\sigma_\epsilon^2 = \sigma_\nu^2 + \sigma_\omega^2$

Serially correlated errors are assumed to be absent from the topic. In other words, the following combinations of random variables are independent.

8. $E(\nu_{ijt}\,\nu_{i'j't'}) = 0$, for any $i \neq i'$, and/or $j \neq j'$, and/or $t \neq t'$

9. $E(\omega_{jt}\,\omega_{j't'}) = 0$, for any $i \neq i'$, and/or $j \neq j'$, and/or $t \neq t'$

The covariance between ϵ_{ijt} and $\epsilon_{i'jt}$, $i \neq i'$, is

$$\mathrm{Cov}(\epsilon_{ijt}, \epsilon_{i'jt}) = E[(\epsilon_{ijt} - E(\epsilon_{ijt}))(\epsilon_{i'jt} - E(\epsilon_{i'jt}))] = E[\epsilon_{ijt}\,\epsilon_{i'jt}]$$

where:

$$E[\epsilon_{ijt}\,\epsilon_{i'jt}] = E[(\nu_{ijt} + \omega_{jt})(\nu_{i'jt}\,\omega_{jt})] = E(\nu_{ijt}\,\nu_{i'jt}) + E(\nu_{ijt}\,\omega_{jt}) + E(\nu_{i'jt}\,\omega_{jt})$$
$$+ E(\omega_{jt}^2)$$

Then, employing assumptions 3, 8, 9 we get

10. $\mathrm{Cov}(\epsilon_{ijt}, \epsilon_{i'jt}) = E(\epsilon_{ijt}\,\epsilon_{i'jt}) = E(\omega_{jt}^2) = \sigma_\omega^2$ for $i \neq i'$

The correlation between treatments for a given location and year is:

$$11.\quad \rho\,(\epsilon_{ijt}\,\epsilon_{i'jt}) = \frac{\mathrm{Cov}\,(\epsilon_{ijt}, \epsilon_{i'jt})}{[\mathrm{Var}(\epsilon_{ijt})\mathrm{Var}(\epsilon_{i'jt})]^{1/2}} = \frac{\sigma_\omega^{\,2}}{\sigma_\epsilon^{\,2}}\,, \quad \text{for } i \neq i'$$

One can also show that the covariance between different locations and the covariances between different years are both equal to zero. This is a direct result of the dummy variable method where ψ_{2j} and ψ_{3t} are replaced by fixed effects τ_j and ϕ_t for j equals 1 through 17 and t equals 1 through 4.

Estimation Technique

Parameter estimates for the response functions were obtained by means of maximum likelihood, computed with a generalized nonlinear regression program, GNLR. Chapter 9 of Malinvaud [59] contains a detailed explanation of the statistical theory underlying GNLR. The statistical assumptions associated with the component disturbance are a special to which the computer program is applicable. The interested reader may consult other sources [26,47,102] for an explanation of maximum likelihood estimation procedures.

Statistical Tests

T-ratios for the coefficients served as the primary test to determine the significance of individual parameters and their associated variables. On occasion, the likelihood ratio test was employed to evaluate the significance of several explanatory variables taken jointly. The likelihood ratio test essentially compares the plausibility of one value of the parameter vector θ against another, conditional on the sample $y_1, y_2, \ldots, y_n$.

Theil [102] provides a more technical presentation of likelihood ratio tests. Briefly, he shows a general theorem that $-2\ \ln\lambda$ asymptotically converges to a Chi-square distribution with degrees of freedom equivalent to the number of restrictions, where λ is the likelihood ratio statistic. The Chi-square test statistic is given below.

$$T \ln \left[\frac{Dr}{Dg}\right] \sim \chi_q^2. \qquad \text{(III.10)}$$

where:

T = number of experiments

D = determinant of the residual covariance matrix

r = restricted model

g = general model with fewer restrictions

q = difference in the number of independent parameters between g and r.

Determining Relevant Variables

In the course of deciphering the explanatory factors which contribute to yield, a myriad of specifications were estimated. The initial polynomial specifications were as high as fifth degree, with up to fifty variables, including numerous interaction variables.

Factors which display little or no statistical significance were omitted from the maintained hypothesis, thus transferring their effects to the error components. Observations on soil water were never specified in a model because previous research by both Garnick [31] and Simons [88] found that soil water, as it was measured in Dr. Haby's experiments, exhibited nonsensical economic behavior. In all the initial estimates, the coefficients associated with applied potassium, that is, the parameters associated with both the linear and squared variables, consistently displayed insignificant t-ratios. Additionally, one could observe high correlation between the parameters associated with linear and quadratic potassium, thus leading to the conclusion that the effect of the squared variable is insignificant and could be eliminated. These results imply that the dispersion on observations for applied potassium was too small to estimate a linear response. This suggests that the range of application for potassium (0-40 kg/ha) was sufficiently small enough to result in estimation of an approximately linear portion of the nonlinear function. In an attempt to deal with potassium's insignificance, the quadratic term was removed from the function. Unfortunately, this attempt was unsuccessful and applied potassium was removed from further consideration. Future experiments involving potassium should be designed with a wider application range, for instance 0 to 100 kg/ha. However, a possibility exists that the soils yield no response to potassium fertilizer.

Early in the estimation process, the only measure of precipitation was spring through summer (April through July) rainfall. When observing the t-ratios, frequently the estimates for precipitation and precipitation squared were insignificant at a 20 percent significance level. Additionally, the correlation between the two parameters was very high. When recording weekly measurements of precipitation, it is difficult to account for evapotranspiration from plants and evaporation from the soil and the precipitation collection device. In an attempt to find a better description of precipitation in the model, winter precipitation was included in the functional specification.[13] Initially, parameter estimates of winter and summer precipitation as both linear and squared variables suggested that increasing marginal returns occurred for all levels of precipitation. In order to alleviate this unusual description of precipitation, the natural logarithms of winter and summer precipitation were generated and combined as a weighted sum to create a composite precipitation variable. That is, composite precipitation is defined as,

$$X_5 = \beta_{15} X_{15} + (1 - \beta_{15}) X_{14} \qquad \text{(III.11)}$$

where:

X_{14} = the natural log of winter precipitation

X_{15} = the natural log of summer precipitation

The purpose in using logged variables was to impose diminishing returns on each winter

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[13] This data was collected from monthly records at climatological stations that are located within the same precipitation gradient as the experimental sites.

and summer precipitation individually. As an added benefit, the use of a composite precipitation variable as opposed to using separate seasonal precipitation variables conserves degrees of freedom in the model by greatly reducing the number of interaction variables.

The dummy variables for location and year relate indirectly to misspecification of the precipitation variable. Basically the use of location dummy variables implicitly assumes that the dependent variable is influenced by site specific characteristics such as different terrain, soil permeability, and drainage. The periodic dummy variables, representing each experimental year, extract climatic effects such as temperature and precipitationn effects which are not captured by the primary variables.

Many yield functions, involving a multitude a soil nitrate variables, were specified and estimated. The variables assumed exponents from the square root power through the fifth power, including a number of higher order interaction variables. Yet even after trying a wide variety of polynomial specifications, no logical result could be found for soil nitrate. Essentially the dilemma of the soil nitrate variables stems from the fact that virtually all the parameter estimates were highly significant, but, simultaneously, often they possessed atypical signs and unreasonable partial derivatives. That is, the preliminary results indicated increasing marginal returns for soil nitrate well beyond the mean of 36 pounds per acre. The parameters also indicated a negative incremental response for levels of soil nitrate less than 30 pounds per acre.

Some outlying observations on soil nitrate appeared to adversely affect the regression estimate. As a result, attempts were made to restrict the definition of the soil nitrate variable. One method of averaging soil nitrate across experimental plots on each farm achieved success in reducing the standard error of the estimated equation and increased the significance of the parameter estimates, but failed to yield estimates which model reasonable biological behavior. A second technique was to fix all observations below 30 pounds per acre equal to 30 pounds per acre. It was hoped that this procedure would remove the problem of decreasing yield response from increasing incremental soil nitrate between 0 and 30 pounds per acre. Again, this idea did not ameliorate the predicament with soil nitrate.

These initial results seem incomprehensible and suggest that possibly the observations on soil nitrate are incorrect. One hypothesis may be that the test for soil nitrate indirectly measures some anonymous soil components, and thus is a surrogate measure of many site specific soil attributes. This contention is supported by some agronomists who believe that "... no test for (soil) nitrogen has proven successful enough to justify a recommendation ..."[14]

On the other hand, in consideration of a different cause for inaccurate soil nitrate observations, one might speculate that the measure of the nutrient made during the fall was in fact correct for that time. However, because soil is a dynamic macrocosm with a continuously changing network of organic and inorganic elements, the amount of nitrate measured in the fall is not necessarily the same quantity which is available to the plant in spring or summer. For example, leaching of soil nutrients from winter and spring precipitation could dramatically alter levels of soil nitrate present for summer plant growth. Obviously, in such a situation, the treatment of a fall observation of soil nitrate as an exclusive observation would imprecisely represent the true soil nitrate variables.

Finally, after an exhaustive series of regression estimates, the hope to discover a meaningful functional specification, which included soil nitrate, proved futile. Consequently, all soil nitrate variables were excluded from further analysis and estimation.

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[14] See page 32 of [43].

Yield Estimation

The variables specified in the final yield estimate are listed in Table 2, along with their means, standard errors, and adjusted coefficients of variation. Table 3 lists location and periodic dummy variables which were included in the regression estimate. Furthermore, final parameter estimates with other relevant statistics are listed in Table 4.

The estimated yield response function is characterized by maximum yields and declining marginal productivity to the resources under consideration, meaning that the function is strictly concave. All the estimated coefficients for the linear and squared variables possess their hypothesized signs.

Analysis of the cross partial derivatives of the interacting factors reveals the following information concerning "factor interdependence" [10].

$$\frac{\partial^2 Y}{\partial X_1 \partial X_5} = \frac{\partial^2 (-.1786\, X_{10} + .1061\, X_{11})}{\partial X_1\, \partial X_5}$$

$$= \frac{\partial^2 (-.1786\, X_1 X_5 + .1061\, X_1 X_5{}^2)}{\partial X_1\, \partial X_5}$$

$$= -.1786 + .2122\, X_5 \overset{\text{set}}{=} 0$$

$$X_5 = .8416$$

Consequently, applied nitrogen, X_1, and composite precipitation, X_5, are technical complements for values of composite precipitation greater than .8416 (note the range of observations on X was between 1.01 and 2.38).

Applied nitrogen and applied phosphorus are technical complements because the parameter estimate, $\hat{\beta}_9$, for the interaction is positive. Similarly, the positive sign on $\hat{\beta}_{12}$ suggests that applied phosphorus and composite precipitation are technical complements. Finally, the negative sign attached to $\hat{\beta}_{13}$ implies that applied phosphorus and soil phosphorus are technically competing factors.

Table 2. Explanatory Variables for the Final Winter Wheat Yield Equation.

Variable	Variable Name	Units	High	Low	Mean	Standard Deviation	Adjusted Coefficient of Variation
Y	grain yield	bu/ac	76.16	10.76	37.21	12.33	0.4661
X_0	missing data dummy		1.0	1.0	1.0	0.0	0.0
X_1	applied nitrogen	kg/ha	89.0	0.0	44.5	30.4	0.6837
X_2	applied nitrogen squared		7921	0.0	2905	2848	0.9805
X_3	applied phosphorus	kg/ha	35.7	0.0	17.83	12.21	0.6844
X_4	applied phosphorus squared		1275	0.0	466.8	458.5	0.9822
X_5	composite precipitation		2.38	1.01	1.664	0.3189	0.4874
X_6	composite precipitation squared		5.65	1.02	2.871	1.072	0.5790
X_7	soil phosphorus	kg/ha	27.4	0.7	8.449	3.976	0.5131
X_8	soil phosphorus squared		751	0.49	87.18	83.36	0.9619
X_9	applied nit. X applied phos.		3177	0.0	793	877	1.1047
X_{10}	applied nit. X composite precipitation		212	0.0	74.07	53.47	0.7219
X_{11}	applied nit. X comp. prec. squared		503	0.0	128	105	0.8198

Table 2 (continued).

Variable	Variable Name	Units	High	Low	Mean	Standard Deviation	Adjusted Coefficient of Variation
X_{12}	applied phos. X comp. prec.		84.83	0.0	29.69	21.45	0.7225
X_{13}	applied phos. X soil phos.		871	0.0	151	137	0.9068
X_{14}	logged winter precipitation		2.442	0.802	1.773	0.4588	0.4723
X_{15}	logged summer precipitation		2.624	0.820	1.556	0.4328	0.5880

Table 3. Binary Variables Representing Location and Year Used in the Final Wheat Yield Equation.

Binary Variable	Variable Name
D_1	F. Warren Corp.
D_2	K. Hanson Farm
D_3	G. Eastlick Farm
D_4	D. Keller Farm
D_5	S. Logan Farm
D_6	G. Haines Farm
D_7	A. Hansen Farm
D_8	D. Herzog Farm
D_9	D. Holland Farm
D_{10}	C. McFarland Farm
D_{11}	L. Mosdal Farm
D_{12}	K. Sire Farm
D_{13}	Southern Ag. Resch. Center
D_{14}	D. & C. Lee Farm
D_{15}	B. Becker Farm
D_{16}	N. Schaff Farm
D_{17}	R. Michael Farm
Z_1	Year of 1977
Z_2	Year of 1978
Z_3	Year of 1979
Z_4	Year of 1980

Table 4. Final Parameter Estimates for the Winter Wheat Yield Equation.

Parameter	Parameter Estimate	t-Ratio	Variable Name
β_0	14.265	0.4947	Missing Data Dummy
β_1	0.1822	1.7622	Applied Nitrogen
β_2	-0.00102	-5.9498	Applied Nitrogen Squared
β_3	0.09193	1.2641	Applied Phosphorus
β_4	-0.02721	-2.5586	Applied Phosphorus Squared
β_5	10.327	0.32588	Composite Precipitation
β_6	-3.9299	-0.44942	Composite Precipitation Squared
β_7	0.5875	2.7484	Soil Phosphorus
β_8	-0.01494	-1.6784	Soil Phosphorus Squared
β_9	0.001122	3.3242	Applied Nitrogen × Applied Phosphorus
β_{10}	-0.1786	-1.4396	Applied Nitrogen × Composite Precipitation
β_{11}	0.1061	2.8992	Applied Nitrogen × Composite Precip. Squared
β_{12}	0.1046	2.8759	Applied Phosphorus × Composite Precipitation
β_{13}	0.0275	-4.2225	Applied Phosphorus × Soil Phosphorus
β_{15}	0.6680	12.744	Logged Winter Precipitation
τ_1	3.2103	-0.62108	F. Warren Corp.
τ_2	-5.0613	-1.3679	K. Hanson Farm
τ_3	-56.380	-1.5632	G. Eastlick Farm
τ_4	8.8377	2.2444	D. Keller Farm
τ_5	2.4693	0.04809	S. Logan Farm
τ_6	-3.8427	-0.92438	G. Haines Farm
τ_7	31.362	6.3882	A. Hansen Farm
τ_8	11.558	2.3609	D. Herzog Farm
τ_9	7.8006	1.5909	D. Holland Farm
τ_{10}	0.1698	0.03249	C. McFarland Farm
τ_{11}	-2.9035	-0.59287	L. Mosdal Farm
τ_{12}	13.255	2.6255	K. Sire Farm
τ_{13}	23.943	4.7029	S.A.R.C.
τ_{14}	15.625	2.6917	D. & C. Lee Farm
τ_{15}	4.6410	0.74779	B. Becker Farm.
τ_{16}	1.1414	0.18334	N. Schaff Farm
τ_{17}	20.138	3.3655	R. Micheal Farm
ϕ_1	4.5863	1.0425	Year of 1977
ϕ_2	11.099	2.3225	Year of 1978
ϕ_3	4.8981	1.1598	Year of 1979
ϕ_4	-9.2421	-1.9902	Year of 1980

Standard Error of the Estimate = 5.62
Multiple R^2 = 0.805.
Correlation Among Treatments (for a given site and year) = 0.4256.
Number of Observations = 900.

The critical factor, which carries more weight than the signs of the coefficients, is the analysis of the response resulting from infinitesimal changes in the level of an input (i.e., the partial derivatives of yield with respect to any selected input). The partial derivatives in Tables 5, 6, and 7 show evidence that the parameter estimates reasonably model winter wheat yield.

Table 5 illustrates the partial derivative of yield with respect to summer precipitation. This marginal response decreases as summer precipitation increases due to the logged structure of the composite precipitation variables. Recall that composite precipitation is a weighted sum of logged winter and summer precipitation. Additionally, the marginal response associated with summer precipitation increases as applied nitrogen is increased. This is due to an overall positive interaction composed of both the second and third degree interaction terms involving composite precipitation and applied nitrogen (i.e., X_{11} and X_{12}) as arguments.

Likewise, Table 6, which displays the partial derivative of yield with respect to winter precipitation, may be analyzed in the same manner as Table 5 was interpreted. That is, the marginal responsiveness of yield decreases slightly as the level of winter precipitation increases. Also, the marginal response associated with winter precipitation increases with progressively higher levels of applied nitrogen.

Table 7 categorizes the partial derivative of yield with respect to nitrogen for various levels of applied nitrogen and phosphorus. The marginal response decreases as nitrogen is increased due to the negative coefficient associated with the quadratic nitrogen variable.

Table 5. The Partial Derivative of Yield With Respect to Summer Precipitation for Various Levels of Applied Nitrogen (lbs/ac) and Summer Precipitation (inches).

Nitrogen	Summer Precipitation										
.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00
.00	.34	.00	-.15	-.23	-.28	-.30	-.31	-.32	-.32	-.31	-.32
10.00	.58	.25	.09	.00	-.06	-.09	-.12	-.13	-.14	-.15	-.15
20.00	.83	.50	.33	.23	.16	.11	.08	.05	.04	.02	.01
30.00	1.07	.75	.57	.46	.38	.32	.28	.24	.21	.19	.17
40.00	1.31	1.01	.82	.69	.60	.53	.47	.43	.39	.36	.33
50.00	1.56	1.26	1.06	.92	.82	.73	.67	.61	.57	.53	.50
60.00	1.80	1.51	1.30	1.15	1.03	.94	.87	.80	.75	.70	.66
70.00	2.04	1.76	1.54	1.38	1.25	1.15	1.06	.99	.93	.87	.82
80.00	2.28	2.01	1.79	1.61	1.47	1.36	1.26	1.18	1.10	1.04	.99
90.00	2.53	2.26	2.03	1.84	1.69	1.56	1.45	1.36	1.28	1.21	1.15
100.00	2.77	2.51	2.27	2.07	1.91	1.77	1.65	1.55	1.46	1.38	1.31

Table 6. The Partial Derivative of Yield With Respect to Winter Precipitation for Various Levels of Applied Nitrogen (lbs/ac) and Winter Precipitation (inches).

Nitrogen	Winter Precipitation										
.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
.00	-.01	-.04	-.06	-.07	-.08	-.08	-.08	-.08	-.08	-.08	-.08
10.00	.12	.07	.04	.02	.01	.00	-.01	-.01	-.02	-.02	-.02
20.00	.25	.18	.14	.11	.09	.07	.06	.05	.05	.04	.03
30.00	.37	.29	.24	.20	.17	.15	.13	.12	.11	.10	.09
40.00	.50	.40	.34	.29	.26	.23	.21	.19	.17	.16	.15
50.00	.63	.42	.44	.38	.34	.31	.28	.26	.24	.22	.21
60.00	.75	.63	.54	.47	.42	.38	.35	.32	.30	.28	.26
70.00	.88	.74	.64	.56	.51	.46	.42	.39	.36	.34	.32
80.00	1.01	.85	.74	.66	.59	.54	.50	.46	.43	.40	.38
90.00	1.13	.96	.84	.75	.67	.62	.57	.53	.49	.46	.43
100.00	1.26	1.07	.94	.84	.76	.69	.64	.59	.56	.52	.49

Table 7. The Partial Derivative of Yield With Respect to Applied Nitrogen for Various Levels of Applied Nitrogen (lbs/ac) and Applied Phosphorus (lbs/ac).

Nitrogen	Phosphorus										
.00	.00	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00
.00	.50	.51	.52	.52	.53	.53	.54	.54	.55	.55	.56
10.00	.48	.49	.50	.50	.51	.51	.52	.52	.53	.53	.54
20.00	.46	.47	.47	.48	.49	.49	.50	.50	.51	.51	.52
30.00	.44	.45	.45	.46	.47	.47	.48	.48	.49	.49	.50
40.00	.42	.43	.43	.44	.45	.45	.46	.46	.47	.47	.48
50.00	.40	.41	.41	.42	.43	.43	.44	.44	.45	.45	.46
60.00	.38	.39	.39	.40	.40	.41	.42	.42	.43	.43	.44
70.00	.36	.37	.37	.38	.38	.39	.40	.40	.41	.41	.42
80.00	.34	.35	.35	.36	.36	.37	.38	.38	.39	.39	.40
90.00	.32	.33	.33	.34	.34	.35	.36	.36	.37	.37	.38
100.00	.30	.31	.31	.32	.32	.33	.33	.34	.35	.35	.36

Similarly, the marginal response associated with nitrogen increases with augmented applications of phosphorus. This is caused by the positive interaction between nitrogen and phosphorus. Other interesting partial derivatives for yield are listed in Appendix C.

When observing the t-ratios in Table 4, one can see that estimates for linear and quadratic composite precipitation are statistically insignificant; and clearly there is a negligible interaction between nitrogen and composite precipitation. The justification for retaining these variables in the model is that other interaction variables containing composite precipitation (X_{11} and X_{12}) show strong significance. In this case, as in many previous studies, the effect of precipitation on yield is vague, and therefore difficult to model. One may be inclined to simply remove insignificant water variables from the specification. Yet, indisputable scientific evidence reveals that precipitation and/or soil water influence crop yields and protein formation. The fundamental problem arises from finding the correct mathematical description of a complex explanatory factor. After extensively analyzing precipitation, the best functional description of that factor was employed in the final model, in Table 4.

The parameter estimate for linear phosphorus is insignificant at a 90 percent confidence level. This variable remained in the final yield estimate because of the highly significant quadratic phosphorus coefficient. From an economic viewpoint, to retain the quadratic variable and discard the linear variable would yield an illogical description of applied phosphorus.

The standard error of the estimated equation appears to be reasonable, at 5.6 bushels per acre. The multiple correlation coefficient, $R^2 = .805$, is remarkably high considering the nature of the data used in this project. Usually less emphasis is placed on the correlation coefficient when using pooled cross section and time series data than would be the case with purely time series data. One would expect high unexplained variation with cross section data for numerous reasons that are not accounted for by the causal variables. For instance, soil temperature, air temperature, management practices, timing of precipitation, and other elements undoubtably differ between locations. Nonetheless, apparently the locational and periodic dummy variables satisfactorily extract a large quantity of the unexplained variation, resulting in a high R^2.

Random variation between treatments, across given locations and time, is measured by the correlation among the endogenous variables, which is 0.4256. In addition, then 0.5744 is the variation for a specific treatment for different locations and years.

With relatively little manipulation, the yield estimate can provide information concerning expected yields for incremental levels of various inputs, while treating other factors exogenously. Tables 8, 9, and 10 illustrate this type of supplemental information regarding yield response.

Table 8. Wheat Yields (bu/ac) Predicted by the Estimated Yield Equation for Various Levels of Applied Nitrogen (lbs/ac) and Applied Phosphorus (lbs/ac).

Nitrogen	Phosphorus										
.00	.00	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00
.00	24.31	25.06	25.67	26.15	26.49	26.70	26.77	26.71	26.51	26.17	25.70
10.00	26.10	26.91	27.58	28.11	28.51	28.77	28.90	28.89	28.75	28.47	28.05
20.00	27.69	28.55	29.28	29.87	30.33	30.65	30.83	30.88	30.79	30.56	30.20
30.00	29.08	30.00	30.78	31.43	31.94	32.31	32.55	32.65	32.62	32.45	32.15
40.00	30.26	31.23	32.07	32.78	33.34	33.78	34.07	34.23	34.25	34.14	33.89
50.00	31.24	32.27	33.17	33.93	34.55	35.04	35.39	35.60	35.68	35.63	35.43
60.00	32.02	33.10	34.06	34.87	35.55	36.09	36.50	36.77	36.91	36.91	36.77
70.00	32.59	33.73	34.74	35.61	36.35	36.95	37.41	37.74	37.93	37.99	37.91
80.00	32.96	34.16	35.22	36.15	36.94	37.60	38.12	38.50	38.75	38.86	38.84
90.00	33.13	34.38	35.50	36.49	37.33	38.05	38.62	39.06	39.37	39.53	39.57
100.00	33.09	34.40	35.58	36.62	37.52	38.29	38.92	39.42	39.78	40.00	40.09

Table 9. Wheat Yields (bu/ac) Predicted by the Estimated Yield Equation for Various Levels of Applied Nitrogen (lbs/ac) and Winter Precipitation (inches).

Nitrogen	Winter Precipitation										
.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
.00	26.48	26.45	26.39	26.32	26.25	26.17	26.09	26.00	25.92	25.84	25.76
10.00	28.19	28.29	28.34	28.36	28.38	28.38	28.37	28.36	28.34	28.32	28.30
20.00	29.71	29.92	30.08	30.20	30.30	30.38	30.45	30.50	30.55	30.60	30.63
30.00	31.02	31.35	31.62	31.83	32.02	32.18	32.32	32.45	32.57	32.67	32.77
40.00	32.13	32.58	32.95	33.26	33.54	33.78	34.00	34.19	34.37	34.54	34.69
50.00	33.04	33.61	34.08	34.49	34.85	35.17	35.46	35.73	35.98	36.21	36.42
60.00	33.74	34.43	35.01	35.51	35.96	36.36	36.73	37.07	37.38	37.67	37.94
70.00	34.24	35.05	35.73	36.33	36.87	37.35	37.79	38.20	38.58	38.93	39.26
80.00	34.54	35.46	36.25	36.95	37.57	38.13	39.65	39.13	39.57	39.98	40.37
90.00	34.63	35.67	36.57	37.36	38.07	38.72	39.31	39.85	40.36	40.84	41.29
100.00	34.52	35.68	36.68	37.57	38.37	39.09	39.76	40.37	40.95	41.49	41.99

Table 10. Wheat Yields (bu/ac) Predicted by the Estimated Yield Equation for Various Levels of Applied Nitrogen (lbs/ac) and Summer Precipitation (inches).

Nitrogen	Summer Precipitation									
.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00
.00	26.33	26.48	26.39	26.20	25.94	25.65	25.34	25.02	24.70	24.38	24.06
10.00	27.78	28.17	28.34	28.38	28.34	28.26	28.16	28.03	27.89	27.75	27.60
20.00	29.02	29.67	30.08	30.35	30.55	30.68	30.78	30.84	30.89	30.91	30.93
30.00	30.06	30.96	31.62	32.13	32.55	32.89	33.19	33.45	33.67	33.88	34.06
40.00	30.90	32.05	32.95	33.70	34.34	34.90	35.40	35.85	36.26	36.63	36.98
50.00	31.54	32.93	34.08	35.07	35.93	36.71	37.41	38.05	38.64	39.19	39.70
60.00	31.97	33.61	35.01	36.23	37.32	38.31	39.21	40.04	40.82	41.54	42.22
70.00	32.20	34.09	35.74	37.20	38.51	39.71	40.81	41.84	42.79	43.69	44.54
80.00	32.23	34.37	36.26	37.95	39.49	40.90	42.21	43.43	44.56	45.64	46.65
90.00	32.05	34.44	36.58	38.51	40.27	41.90	43.40	44.81	46.13	47.38	48.56
100.00	31.67	34.31	36.69	38.86	40.85	42.69	44.40	45.99	47.50	48.92	50.26

Protein Estimation

An estimate of a generalized protein response function was ascertained from the experimental wheat data. Similar to yield, protein was specified with an additive disturbance with locational and periodic dummy variables. The same underlying statistical assumptions regarding the error structure for yield also hold for protein. Parameter estimates were generated by means of GNLR with a maximum likelihood procedure.

Results of fitting the polynomial protein equation are listed in Table 11. Initially, the protein function followed the final yield specification, but during the course of estimation, some of the variables contained in yield were removed from the protein model due to highly insignificant parameter estimates.

Aside from the binary variables, the t-ratios in Table 11 indicate that all but three parameter estimates were significantly different from zero at least at a 95 percent confidence level. The linear and quadratic composite precipitation variables were retained in the final protein model because of the significance of summer precipitation, and because of the extreme importance of interacting nitrogen and composite precipitation.

Apparently, as the coefficients reveal, applied nitrogen has a linear relationship with protein response. Results from the regression also indicate that the coefficients for applied phosphorus were both significantly different form zero at the .05 level. This importance of applied phosphorus in explaining protein response is somewhat surprising because of the insignificance of soil phosphorus.

Predicted wheat protein percentages from the estimated equation are presented in Tables 12, 13, and 14 for varying levels of inputs. Each table shows a positive relationship between applied nitrogen and percent protein. Furthermore, with the exception of high

Table 11. Final Parameter Estimates for the Winter Wheat Protein Equation.

Parameter	Parameter Estimate	t-Ratio	Variable Name
β_0	17.048	3.5693	Missing Data Dummy
β_1	0.05317	8.4738	Applied Nitrogen
β_2	0.000034	0.92063	Applied Nitrogen Squared
β_3	−0.02640	−3.0912	Applied Phosphorus
β_4	0.000492	2.1507	Applied Phosphorus Squared
β_5	−6.6615	−1.2728	Composite Precipitation
β_6	1.5561	1.0682	Composite Precipitation Squared
β_{10}	−0.01418	−4.4730	Applied Nitrogen X Composite Precipitation
β_{15}	0.58261	5.9719	Logged Summer Precipitation
τ_1	1.7450	2.1722	F. Warren Corp.
τ_2	2.8918	5.2019	K. Hanson Farm
τ_3	0.50599	0.9409	G. Eastlick Farm
τ_4	1.2294	2.1008	D. Keller Farm
τ_5	−0.85561	−1.0768	S. Logan Farm
τ_6	1.5591	2.4919	G. Haines Farm
τ_7	4.8702	6.6071	A. Hansen Farm
τ_8	−1.7869	−2.3629	D. Herzog Farm
τ_9	−1.5710	−2.1522	D. Holland Farm
τ_{10}	1.6552	2.1370	C. McFarland Farm
τ_{11}	−1.3031	−1.7244	L. Mosdal Farm
τ_{12}	0.03682	0.04839	K. Sire Farm
τ_{13}	4.1043	5.3364	S.A.R.C.
τ_{14}	−1.0514	−1.1785	D. & C. Lee Farm
τ_{15}	−1.3442	−1.4366	B. Becker Farm
τ_{16}	0.89983	0.94126	N. Schaff Farm
τ_{17}	0.36509	0.34849	R. Micheal Farm
ϕ_1	−1.9125	−2.8770	Year of 1977
ϕ_2	−1.1768	−1.6315	Year of 1978
ϕ_3	−1.1821	−1.8434	Year of 1979
ϕ_4	3.1217	3.6588	Year of 1980

Standard Error of the Estimate = 1.0624.
Multiple R^2 = 0.842
Correlation Among Treatments (for a given site and year) = 0.2510.
Number of Observations = 900.

Table 12. Wheat Protein (percent) Predicted by the Estimated Protein Equation for Various Levels of Applied Nitrogen (lbs/ac) and Applied Phosphorus (lbs/ac).

Nitrogen	Phosphorus								
.00	.00	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00
.00	10.17	10.05	9.95	9.88	9.83	9.81	9.82	9.84	9.90
10.00	10.45	10.33	10.24	10.17	10.12	10.10	10.10	10.13	10.18
20.00	10.75	10.63	10.53	10.46	10.42	10.40	10.40	10.43	10.48
30.00	11.05	10.93	10.84	10.77	10.72	10.70	10.70	10.73	10.78
40.00	11.36	11.24	11.14	11.07	11.03	11.01	11.01	11.04	11.09
50.00	11.67	11.55	11.46	11.39	11.34	11.32	11.32	11.35	11.40
60.00	12.00	11.88	11.78	11.71	11.66	11.64	11.65	11.67	11.73
70.00	12.32	12.21	12.11	12.04	11.99	11.97	11.98	12.00	12.06
80.00	12.66	12.54	12.45	12.37	12.33	12.31	12.31	12.34	12.39
90.00	13.00	12.88	12.79	12.72	12.67	12.65	12.65	12.68	12.73
100.00	13.35	13.23	13.14	13.07	13.02	13.00	13.00	13.03	13.08

Table 13. Wheat Protein (percent) Predicted by the Estimated Protein Function for Various Levels of Applied Nitrogen (lbs/ac) and Summer Precipitation (inches).

Nitrogen	Summer Precipitation									
.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00
.00	10.41	10.08	9.88	9.76	9.68	9.64	9.61	9.61	9.61	9.62	9.63
10.00	10.74	10.39	10.17	10.04	9.95	9.89	9.86	9.84	9.83	9.84	9.85
20.00	11.08	10.70	10.47	10.32	10.22	10.15	10.11	10.08	10.07	10.06	10.06
30.00	11.43	11.03	10.78	10.61	10.49	10.42	10.36	10.33	10.31	10.29	10.29
40.00	11.78	11.36	11.09	10.90	10.78	10.69	10.63	10.58	10.55	10.53	10.52
50.00	12.14	11.69	11.41	11.21	11.07	10.97	10.90	10.84	10.81	10.78	10.76
60.00	12.51	12.04	11.73	11.52	11.37	11.26	11.17	11.11	11.07	11.03	11.01
70.00	12.89	12.39	12.06	11.84	11.67	11.55	11.46	11.39	11.33	11.29	11.26
80.00	13.27	12.75	12.40	12.16	11.98	11.85	11.75	11.67	11.61	11.56	11.52
90.00	13.65	13.11	12.75	12.49	12.30	12.15	12.04	11.96	11.89	11.83	11.79
100.00	14.05	13.48	13.10	12.83	12.62	12.47	12.35	12.25	12.17	12.11	12.06

Table 14. Wheat Protein (percent) Predicted by the Estimated Protein Function for Various Levels of Applied Nitrogen (lbs/ac) and Winter Precipitation (inches).

Nitrogen	Winter Precipitation										
.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
.00	10.17	10.01	9.90	9.81	9.76	9.71	9.68	9.65	9.64	9.62	9.61
10.00	10.48	10.31	10.19	10.10	10.03	9.98	9.94	9.91	9.89	9.87	9.86
20.00	10.81	10.62	10.49	10.39	10.31	10.26	10.21	10.18	10.15	10.12	10.11
30.00	11.14	10.94	10.79	10.69	10.60	10.54	10.49	10.45	10.41	10.38	10.36
40.00	11.48	11.26	11.11	10.99	10.90	10.83	10.77	10.72	10.68	10.65	10.63
50.00	11.82	11.59	11.43	11.30	11.20	11.12	11.06	11.01	10.96	10.93	10.90
60.00	12.17	11.93	11.75	11.62	11.51	11.43	11.36	11.30	11.25	11.21	11.17
70.00	12.42	12.27	12.09	11.94	11.83	11.74	11.66	11.60	11.54	11.49	11.46
80.00	12.89	12.62	12.43	12.27	12.15	12.05	11.97	11.90	11.84	11.79	11.75
90.00	13.26	12.98	12.77	12.61	12.48	12.38	12.29	12.21	12.15	12.09	12.04
100.00	13.64	13.35	13.13	12.96	12.82	12.71	12.61	12.53	12.46	12.40	12.35

levels of summer precipitation and low nitrogen levels of summer precipitation and low nitrogen levels, both winter and summer precipitation display a negative relationship with protein. Evidently, high levels of water in combination with low nitrogen levels result in depressed yields and higher levels of protein. In other words, plant growth suffered from inadequate nitrogen, while ironically, the concentration of protein was larger (on a percentage basis) due to an immature kernel size.

One may have observed that protein response to applied phosphorus is decreasing at a decreasing rate up to approximately 25 to 30 pounds per acre, where the response becomes slightly positive at higher levels of phosphorus. That is, the protein response function with respect to applied phosphorus is parabolic or convex in curvature. This is evident from the signs on the estimated coefficients associated with linear and squared phosphorus terms in Table 11; this unusual function shape is supported by the protein response across all levels of nitrogen in Table 12. It is difficult to fathom the cause of such a relationship, nevertheless, a negative then slightly positive response to phosphorus appears to exist for the experiments under consideration. To speculate, perhaps protein formation of the grain is inhibited then stimulated only after the phosphorus requirements for growth are satisfied, or more probably, the estimated protein function incorrectly describes the response from applied phosphorus for levels greater than 30 lbs/ac. That is, the increasing protein response beyond 30 lbs/ac of phosphorus is most likely due to a slight curve fitting problem.

Appendix C contains various tables of partial derivatives for the estimated protein response equation with respect to important factors of production.

CHAPTER IV

ESTIMATION OF A WINTER WHEAT PRICE FUNCTION INCLUDING PROTEIN PREMIUM

Background

In order to find an optimal level of applied nitrogen, some form of an output price, in this case the price of wheat, must be selected and incorporated in a profit function. Previous crop response studies [31] have simply chosen representative output prices or input-output price ratios for the profit function. Relatively few studies explicitly account for premiums paid for high protein wheat. Simons [88] attempted to determine profit maximizing levels of nitrogen, phosphorus, and potassium fertilizers with a system of equations which embodied a protein premium structure for wheat prices. His effort faltered because he could not find an algorithm which could solve the first order conditions of the equations.

Basically, daily market prices for wheat are determined by factors affecting supply and demand for wheat. In general, for hard red winter wheat, a base price is paid for 10 percent protein, or "ordinary," wheat; and typically a premium is paid for grain with higher concentrations of protein. This phenomena can be observed in Figure 2. Evidently, wheat prices are actually described by a discontinuous "step" function which assumes different shapes as daily price changes occur.

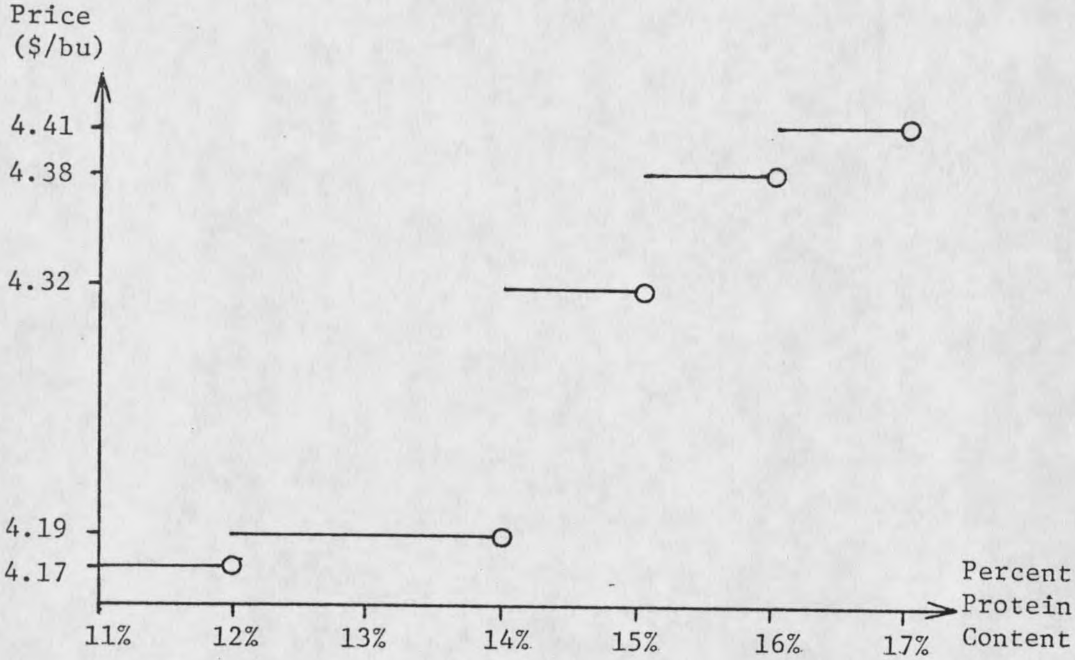

Figure 2. Minneapolis cash grain price, 11% through 17% protein content in hard red winter wheat, July 22, 1981.

Source of Price Data

Data used to estimate a protein premium wheat price function were obtained from the U.S. Wheat Association in Portland, Oregon. The data set was composed of 44 observations on eleven years of average annual prices (F.O.B.) for ordinary, 11%, 12%, and 13% grain protein for hard red winter wheat. The actual observations on the independent variable employed in the regression were for 10.5%, 11.5%, 12.5%, and 13.5% protein. In the author's opinion using the median values in each protein interval serves as more representative measure of protein than selecting a value of protein at either end of the interval. For example, it is posited that 10.5% protein is the best measure of protein in the interval 10%

through 10.99% protein. Furthermore, "Prices Received by Farmers for Food Grains,"[15] served as an index to deflate observations on average annual wheat prices (1980 was used as a base). Substantial price variation for wheat exists from year to year. In order to reduce a major portion of this variability premium paid was divided by price, that is, the dependent variable is defined as,

$$\frac{P_{n\%} - P_{10.5\%} + .01}{P_{n\%} + .01} = \frac{\text{Premium}_{n\%}}{\text{Price}_{n\%}} \qquad \text{(IV.1)}$$

where $n = 10.5, 11.5, \ldots, 13.5$

Estimation of a Winter Wheat Premium Function

The observations on price and percent protein were fitted with a hyperbolic tangent function. That type of trigonometric function is extremely flexible in fitting any sigmoid type curve, and it possesses desirable properties which are appropriate for the topic on hand. Specifically, the function used to fit the protein--premium data assumed the following form.[16]

$$P = \frac{e^{\lambda x}}{\eta + \gamma e^{\lambda x}} \qquad \text{(IV.2)}$$

The following algebraic manipulations and logarithmic transformation were utilized to make Equation IV.2 amendable for estimation with ordinary least squares.

$$P(\eta + \gamma e^{\lambda x}) = e^{\lambda x}$$

$$P\eta = e^{\lambda x}(1 - P\gamma)$$

$$\frac{P}{(1 - P\lambda)} = \frac{e^{\lambda x}}{\eta}$$

---

[15] Taken from page 414 of [105].

[16] Frequently this function is called a "logit" function, a term coined by Berkson [11], however, Thiel [102] shows that the so called "logit" function is actually a hyperbolic tangent function.

$$\ln \frac{P}{(1 - P\gamma)} = \ln \frac{1}{\eta} + \lambda x \qquad \text{(IV.3)}$$

Moreover, Equation IV.3 is rewritten simply as,

$$Z = \alpha + \lambda x \qquad \text{(IV.4)}$$

where $Z = \ln \left[\frac{P}{(1 - P\gamma)}\right]$

x = percent protein content

P = premium-price ratio, defined in Equation IV.1

$\alpha = \ln \frac{1}{\eta}$

The parameters γ, η, and λ define certain characteristics of the curve. For instance, $1/\gamma$ defines an upper asymptote; η is merely a constant which effects the upper and lower tails of the distribution; and λ relates to the horizontal and vertical shape of the curve. Note that a series of 26 initial regression estimates were computed (using SHAZAM). This meant that the dependent variable Z assumed new values for each fixed value of γ. The final regression employed a value of 7.05 for γ, which imposes an upper asymptote of .1418. This number seems reasonable since the largest value of P was .1379. Table 15 presents the final least squares estimates of α and λ contained in Equation IV.4, along with other pertinent information.

Table 15. Final Regression Estimates for the Winter Wheat Price Equation.

$\hat{\lambda}$ = 0.88168
(12.985)[17]
$\hat{\alpha}$ = -13.81
(-17.424)
γ = 7.05, a fixed number
$\bar{R}^2$ = 0.668
SSE = 1.1917

---

[17] The number in parentheses are t-ratios associated with the estimated coefficients.

With relative ease one can transform these parameter estimates for IV.4 back to the coefficients in the original Equation IV.3. That is,

$$P = \frac{e^{.88168x}}{e^{13.81} + 7.05\, e^{.88168x}} \tag{IV.5}$$

Therefore, $\eta = e^{-\alpha} = e^{13.81}$

$\lambda = .88168$

$\gamma = 7.05$

Then, Equation IV.5 may be converted to predict a premium induced from any level of percent protein, given a base price. Recall Equation IV.1 which defined P as the ratio of premium to price. Thus,

$$\text{premium} = \frac{e^{.88168x}}{e^{13.81} + 7.05\, e^{.88168x}} \cdot \text{(base price)} \tag{IV.6}$$

Given a base price of \$3.50 per bushel, the curve describing premium for various levels of protein takes on the following graphical form.

Without question, there exist some problems regarding the capability of the fitted curve to accurately predict protein premium. For instance, Figure 3 shows that premiums are paid for progressively higher levels of protein, between eight and fourteen percent. This is in fact wrong, for the reason that no premium is paid for wheat containing less than 10 percent protein. As a final word of caution, in reality wheat prices fluctuate dramatically over time; the fitted curve, Equation IV.6, simply describes the premium that might be generally expected. In fact, on occasion when substantial quantities of high protein wheat are harvested or released for sale, frequently the premium is depressed to practically nothing.

Keeping in mind the pitfalls mentioned above, the estimated premium-protein function will still be included in the derivation of optimal fertilizer recommendations for winter wheat.

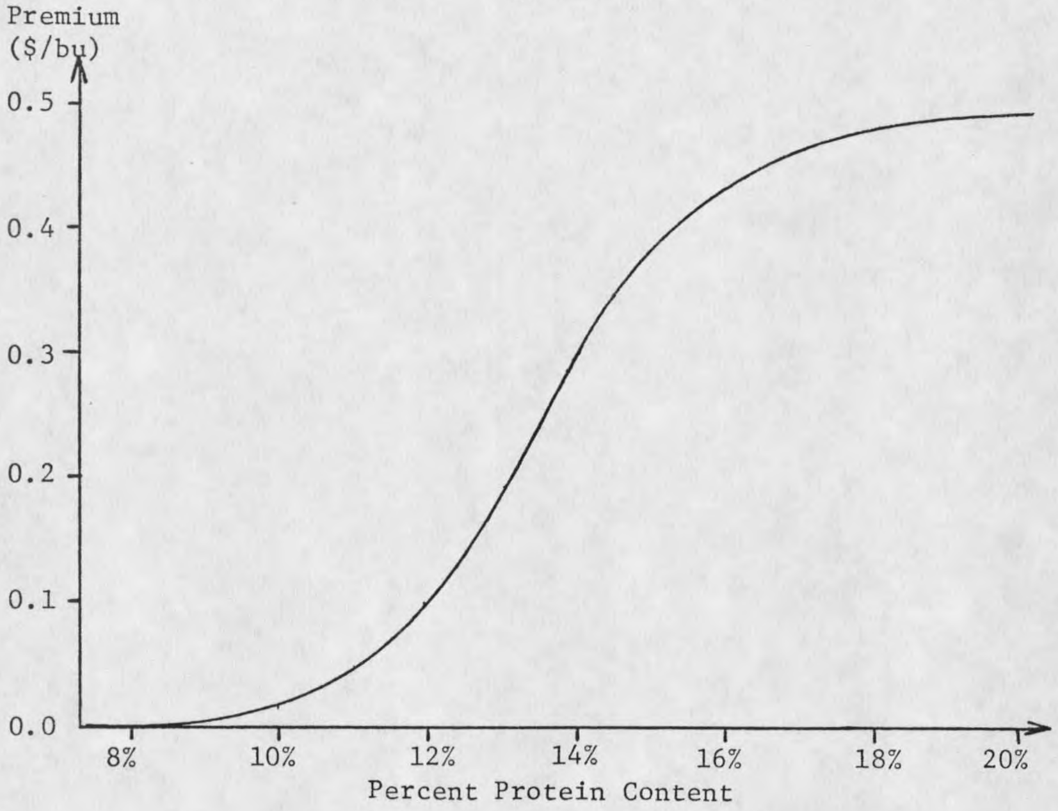

Figure 3. Graph of predicted premium from various wheat grain protein concentrations.

CHAPTER V

OPTIMAL APPLICATION LEVELS OF NITROGEN FERTILIZER

The Decision Model

One may remember from Chapter I that the primary objective of this research is to determine the optimal amount of applied nitrogen for profitable winter wheat production. In order to derive the optimal levels of nitrogen, profit maximization is used as a decision criterion. The following discussion will analyze expected profit in an unconditional context, $E(\pi)$, and when expected profit is conditioned on the level of winter precipitation, $E(\pi | WP)$. In other words, in the second case, spring fertilizer recommendations, based on maximizing profit, are calculated given the additional information concerning precipitation received during the winter. At a later point in this chapter, a comparison of the conditional and unconditional scenarios reveals the value obtained from utilizing the additional knowledge regarding winter precipitation.

One should note that unconstrained maximization of expected profit, whether in the conditional or unconditional framework, is not entirely acceptable because considerable variability in profits may exist. The physical crop response to fertilization is stochastic due to its dependence upon noncontrollable factors such as precipitation. Logically it follows that profit, or economic returns from fertilization, is intrinsically stochastic. Therefore, it is essential to quantify the level of risk which producers are willing to assume when they apply nitrogen fertilizer to their wheat crop. Because profit is stochastic, the decision criterion becomes maximizing a profit function subject to a risk constraint. The "strict safety-first principle" [87] is used to define risk as the probability that marginal profit is non-positive.

Previous studies typically define risk as a probability that either profit or the internal rate of return [19] is negative. For this project, it was discovered that expected profits were rarely negative, except in a few situations where nitrogen is applied at very high rates and precipitation is extremely low. As a consequence, the safety first risk constraint is based on expected marginal profit rather than expected profit. A graph of the decision model is presented in Figure 4.

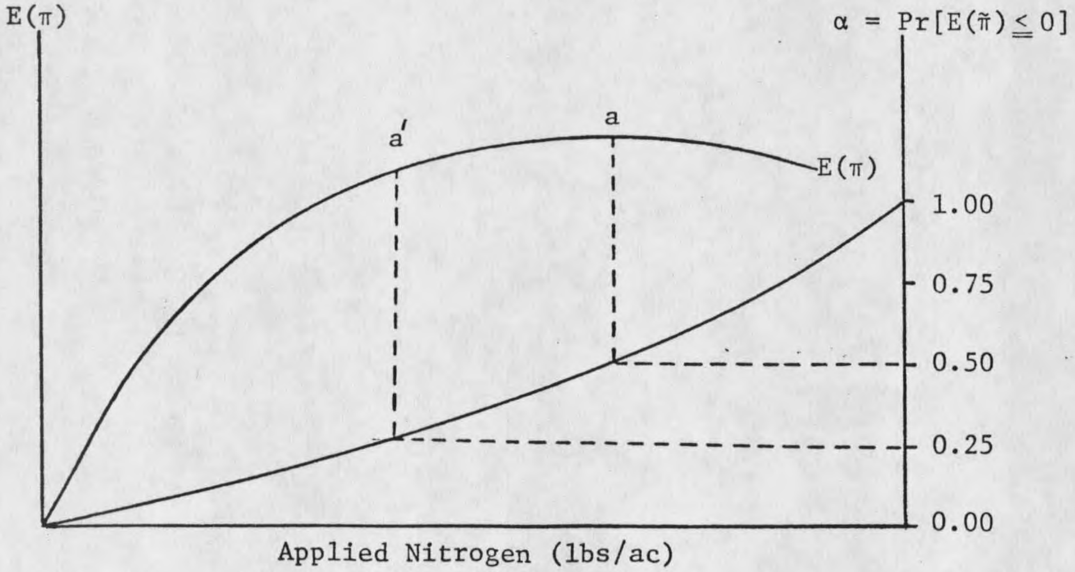

Figure 4. Illustration of the decision model which describes the risk constraint.

Essentially, Figure 4 suggests that if mean profit is normally distributed, then one would expect marginal profit, to be less than zero fifty percent of the time at maximum expected profit, point a. Selection of a probability level, α less than fifty percent implies that the producer is willing to forego expected profit in order to reduce the probability of negative marginal profit. Note that α is equivalent to $\Pr[E(\tilde{\pi}) \leqslant 0]$.

The class of model described above was originally proposed by Schackle [87] as a variation of the principle of "safety-first," which Roy [83] developed to analyze risk in holding financial assets. Many versions of the "strict safety-first principle" have been applied to agricultural topics. For example, Telser [100] used the principle to evaluate risk associated with various hedging strategies in the futures market. In terms of production under risk, Frohberg and Taylor [29], Talpaz and Taylor [96], and De Janvry [19] each applied the technique to formulate optimal fertilizer recommendations for crops grown under variable climatic conditions. Finally, Watts et al. [108] presented a method to incorporate a "safety-first" risk constraint into a mixed integer programming model for whole farm planning.

In a mathematical form, the objective function and risk constraint for unconditional profit are stated below:

$$\underset{N}{\text{Max:}}\ E[(P\cdot Y(R(SP,WP)|N)) - D\cdot p\cdot N) - (P\cdot Y(R(SP,WP)|N=0))] \qquad \text{(V.1)}$$

Or equivalently, Equation V.1 is written in a more simple form below:

$$\underset{N}{\text{Max:}}\ E[\pi(R(SP,WP)|N)] \qquad \text{(V.2)}$$

$$\text{Subject to: } \Pr[\tilde{\pi}(R(SP,WP)|N) \leqslant 0] = \alpha$$

where:

E = the expected value operator

Y = yield per acre

N = applied nitrogen

SP = a stochastic summer precipitation variable

WP = a nonstochastic winter precipitation variable

R = a stochastic precipitation variable composed of winter and summer precipitation

D = a discount rate

π = profit

$\tilde{\pi}$ = marginal profit

Pr = probability symbol

α = a prespecified subjective level of risk

p = the price of nitrogen

P = the price of wheat $= B + [f(q(R(SP,WP)|N)\cdot B)]$

q = percent grain protein

f = premium paid above base price

B = a prespecified subjective base price for wheat

Similarly, the mathematical form of the conditional criterion is written as follows:

$$\underset{N}{\text{Max}}:\ E[\pi(R(SP|WP)|N)] \qquad \text{(V.3)}$$

$$\text{Subject to: } Pr[\tilde{\pi}(R(SP|WP)|N) \leqslant 0] = \alpha$$

An Alternative Method for Incorporating Risk Into Fertilizer Decisions

Before explaining the procedures for calculation of the profit distributions, it is worth mentioning other decision methods which could have been used to evaluate fertilization under risk. Anderson, Dillon and Hardaker [2] present an exceptional overview of Decision Analysis as applied to agriculture. Another method to evaluate decisions involving risks, aside from the safety first method, is the "stochastic dominance" technique.

To familiarize the reader with the concept of stochastic dominance, first consider two probability density functions f(x) and g(x) which are defined only when x does not take values outside the interval [a,b]. Also, assume x is continuous over the range [a,b], which necessitates continuous density functions. The cumulative density functions are defined mathematically in Equations V.4 and V.5.

$$F_1(R) = \int_a^R f(x)dx \qquad R \epsilon [a,b] \qquad (V.4)$$

$$G_1(R) = \int_a^R g(x)dx \qquad R \epsilon [a,b] \qquad (V.5)$$

These cumulative density functions define the area under their respective probability density functions in a cumulative manner. F_1 (R) and G_1 (T) are the central focus of "first order stochastic dominance," which will be explained momentarily.

Using exactly the same procedure, one can determine the area under the cumulative density functions.

$$F_2(R) = \int_a^R F_1(x)dx \qquad R \epsilon [a,b] \qquad (V.6)$$

$$G_2(R) = \int_a^R G_1(x)dx \qquad R \epsilon [a,b] \qquad (V.7)$$

The density functions F_2 (R) and G_2 (R) are used in "second order stochastic dominance." Obviously, the aforementioned techniques of obtaining higher order density functions may be applied an infinite number of times. However, frequently stochastic dominance is not pursued beyond the third order because fourth order or greater adds restrictions which have no relationship to risk preference assumptions.

First order stochastic dominance assumes that an individual prefers more to less of the variable x. Specifically, the rule for first order stochastic dominance is explained below:

> F is said to dominate G in the sense of first-degree stochastic dominance (FSD) if, $F_1(R) \leqq G_1(R)$ for all possible R in the range [a,b] with at least a strict inequality (i.e., the < holds for at least one value of R) [2]. page 282.

This rule states that a cumulative density function, say F_1, dominates all other cumulative density functions provided that the other functions lie nowhere to the right of F_1. The rule is illustrated in Figure 5.

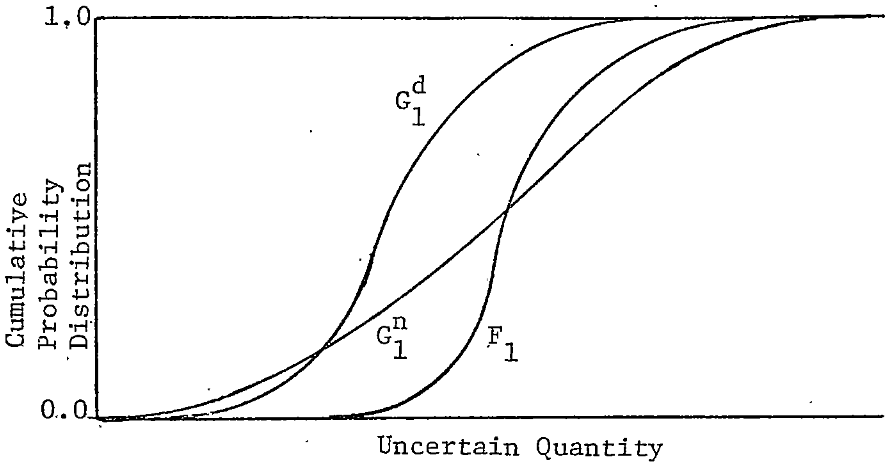

Figure 5. Illustration of first order stochastic dominance (Reproduced from Anderson et al., Figure 9.1 [2]).[19]

Second order stochastic dominance further presumes that equal and successive increments of x diminish in value to the individual. Behaviorally, if the individual possesses preferences which coincide with this presumption, then the individual is risk adverse. Analogous to the rule for first order stochastic dominance, the condition for second order stochastic dominance is stated in the following words.

> F is said to dominate G in the sense of SSD if $F_2(R) \leqq G_2(R)$ for all possible R with at least one strict inequality [2]. page 285.

This second case may be more easily visualized in graphic form. Figure 6 displays cumulative distributions where F_1 dominates G_1, based on the second order rule.

In order to use the stochastic dominance technique to evaluate risk in fertilizing wheat in south central Montana, the following steps are proposed. First, profit could be treated as a function of random variable r, spring and summer rainfall. A total of 78 observations on r are generated from the series of 78 years of monthly precipitation data from

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[19] The cumulative density function F_1 dominates G_2^d but not G_1^n.

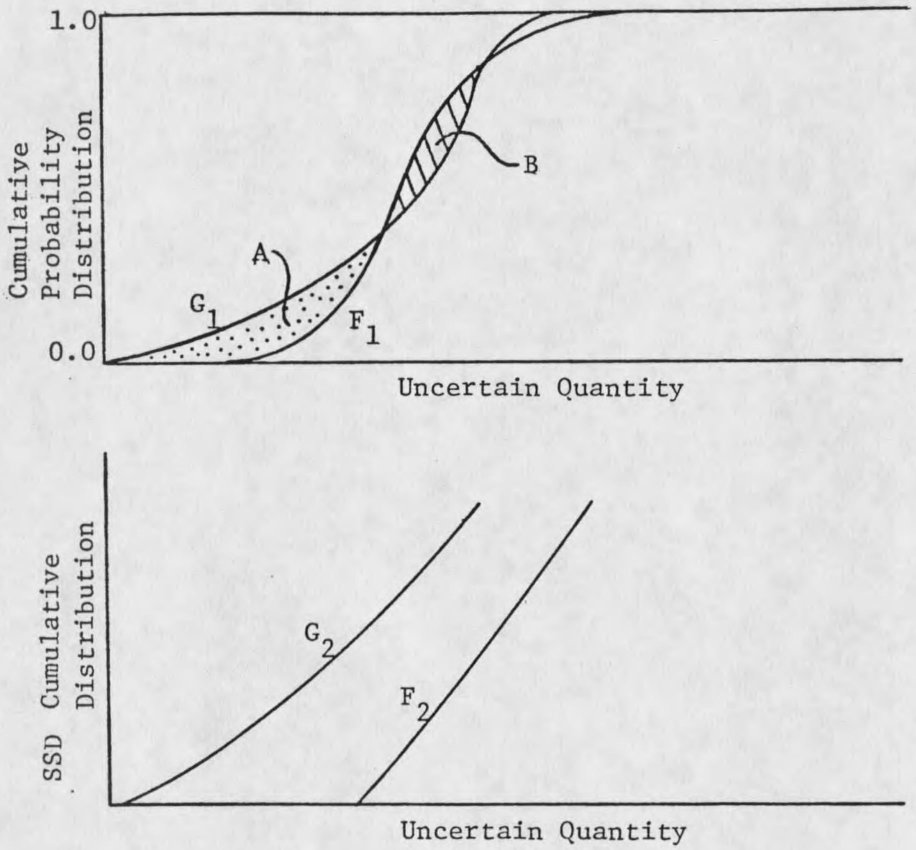

Figure 6. Illustration of second order stochastic dominance (Reproduced from Anderson et al., Figure 9.3 [2]).[20]

Billings, Montana.[21] Second, make profit conditional on nitrogen, in say five pound increments. That is, $\pi = F(r|n)$ with all other factors exogenous. Third, from the conditional profit function one can formulate 78 values of profit for each given level of nitrogen by simply inserting each value of r. Fourth, now cumulative distributions of profit may be calculated for each level of nitrogen. This might be accomplished by first ordering profit from

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[20] This is the case when the cumulative density functions cross twice, and area A exceeds area B.

[21] This data was taken from precipitation records (1905-1983) from the climatological service, courtesy of Dr. Caprio, Dept. of Plant and Soil Science, MSU, Bozeman.

lowest to highest for the 78 values of profit. Next, one might form fractiles for the values of profit, then calculate the sum of the number of observations on profit within each fractile added to the total number of observations from the preceding fractile. If this method of summing the observations was performed in a cumulative manner from the lowest through the highest, then the total number of cumulative observations on profit within each fractile constitutes one bar of a histogram. Thus, histograms of profit for each given level of nitrogen may be fit with a curve or "smoothed" to form a set of cumulative probability density functions. Schematically, this may be observed from the hypothetical curves in Figure 7. Fifth, and finally, the fitted cumulative distributions would be integrated to find higher order cumulative density functions. A dominance comparison is made between the cumulative functions by means of the stochastic dominance rules presented earlier.

The main deficiency of the stochastic dominance method lies in the results. That is, often the method produces a set of stochastically efficient solutions, thus leaving the decision maker with no definitive choice.

Stochastic dominance is a method used to evaluate risky prospects, many similar techniques exist which are summarized by Anderson et al. [2].

Development of Data For Unconditional and Conditional Profit and Marginal Profit

In order to maximize expected profit, or expected conditional profit, distributions must be organized for profit and marginal profit. A series of 78 years of monthly precipitation data[22] for Billings, Montana was used in this analysis. Each year of precipitation was divided to form winter (September through March) and summer precipitation (April through July). A FORTRAN computer program which encompassed the estimated yield,

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[22] Ibid., footnote 21.

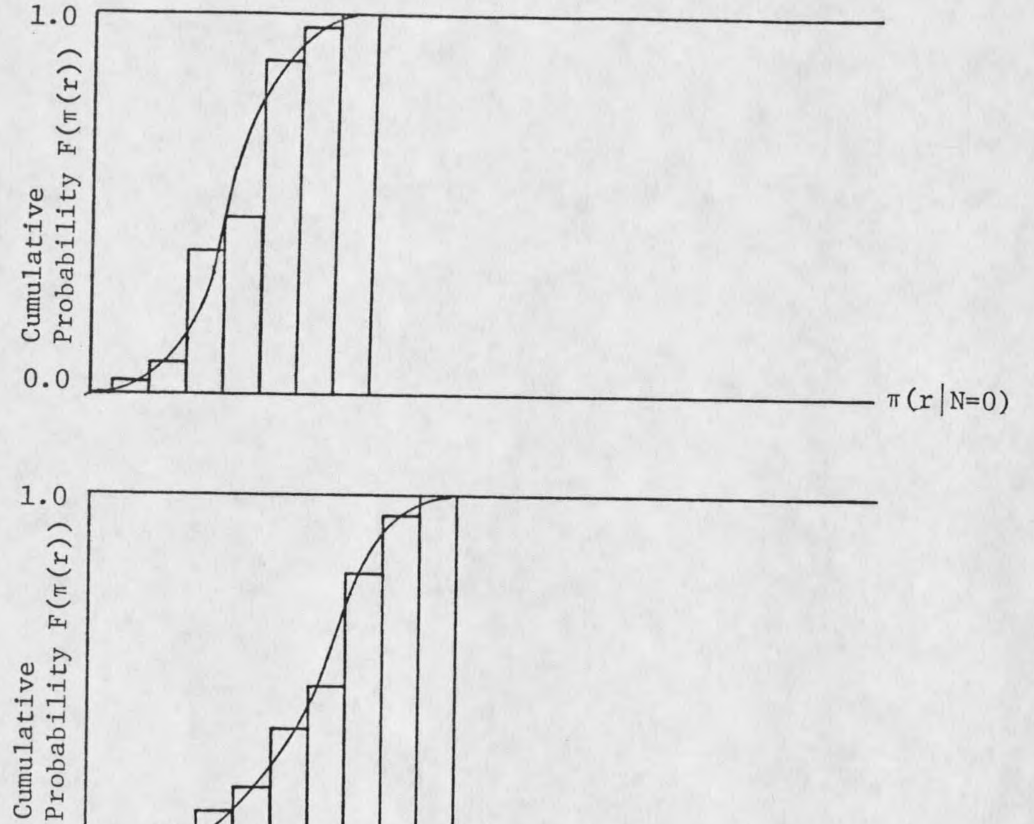

Figure 7. Hypothetical cumulative density functions for profit fitted from histograms.

protein, and price equations was used to calculate conditional profit and conditional marginal profit by specified levels of winter precipitation (2, 4, . . ., 12 inches) and specified levels of applied nitrogen (0, 5, . . ., 100 lbs/ac). Similar figures for unconditional profit and unconditional marginal profit were also computed. Therefore, profit distributions were derived for specified levels of applied nitrogen and winter precipitations. Also, an unconditional profit distribution was derived with winter precipitation at its actual observed value.

Estimation of Conditional and Unconditional Expected Profit Functions

After assembling the distributions, they were used to generate conditional and unconditional expected profit. That is, in the conditional case, within each category of winter precipitation, the mean profit was calculated for each prespecified level of nitrogen. The series of tables constituting Appendix D list the unconditional and conditional expected profits, variances, standard deviations, and coefficients of variation for incremental levels of nitrogen. The table below summarizes expected profit associated with the conditional and unconditional distributions.

To more easily observe the relationship between nitrogen and expected profit, the points describing each distribution are displayed in Figures 11 through 17. These graphs indicate that expected profit may be a quadratic function of nitrogen. In fact, a quadratic specification proved quite successful in fitting each data set. Parameter estimates for the fitted curves, along with their t-ratios (in parentheses), are listed in Equations V.8 through V.14. Later in this chapter, the fitted expected profit functions will serve an instrumental role in the search for optimal nitrogen recommendations. At this stage, it would be prudent to examine the distributions of marginal profit.

Table 16. Unconditional and Conditional Expected Profit per Acre Associated With Various Levels of Nitrogen.

	Mean Profit ($/ac)						
	Conditional						
Nitrogen (lbs/ac)	WP=2 Inches	WP=4 Inches	WP=6 Inches	WP=8 Inches	WP=10 Inches	WP=12 Inches	Unconditional
5	1.59	2.16	2.60	2.97	3.28	3.55	2.58
10	3.04	4.16	5.04	5.77	6.39	6.93	5.00
15	4.34	6.00	7.32	8.41	9.34	10.15	7.26
20	5.50	7.69	9.45	10.89	12.12	13.20	9.37
25	6.51	9.23	11.41	13.21	14.75	16.09	11.31
30	7.38	10.61	13.22	15.38	17.21	18.82	13.11
35	8.12	11.86	14.88	17.38	19.52	21.39	14.75
40	8.70	12.95	16.39	19.24	21.67	23.81	16.25
45	9.15	13.90	17.75	20.94	23.67	26.07	17.59
50	9.44	14.71	18.97	22.50	23.52	28.18	18.79
55	9.58	15.38	20.04	23.91	27.23	30.14	19.85
60	9.56	15.90	20.96	25.17	28.78	31.95	20.76
65	9.38	16.28	21.75	26.29	30.19	33.62	21.53
70	9.02	16.51	22.39	27.27	31.46	35.14	22.15
75	8.49	16.59	22.89	28.11	32.59	36.52	22.62
80	7.76	16.51	23.24	28.80	33.57	37.76	22.95
85	6.84	16.28	23.44	29.35	34.41	38.86	23.12
90	5.71	15.87	23.49	29.75	35.11	39.82	23.13
95	4.36	15.28	23.39	30.01	35.66	40.63	22.98
100	2.80	14.52	23.11	30.10	36.07	41.30	22.66

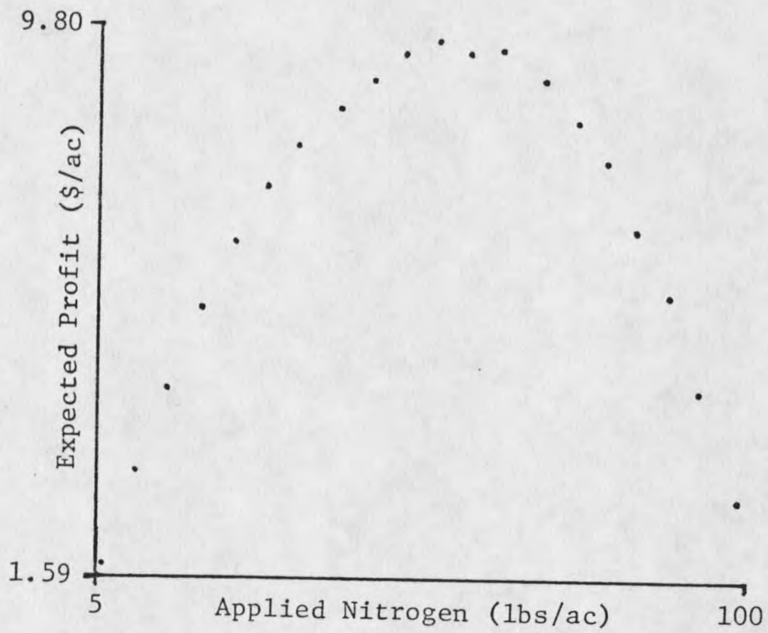

Figure 8. Points of expected profit conditioned on winter precipitation equal to 2 inches, plotted against increments of nitrogen at 5 lbs/ac (from 5 to 100 lbs/ac).

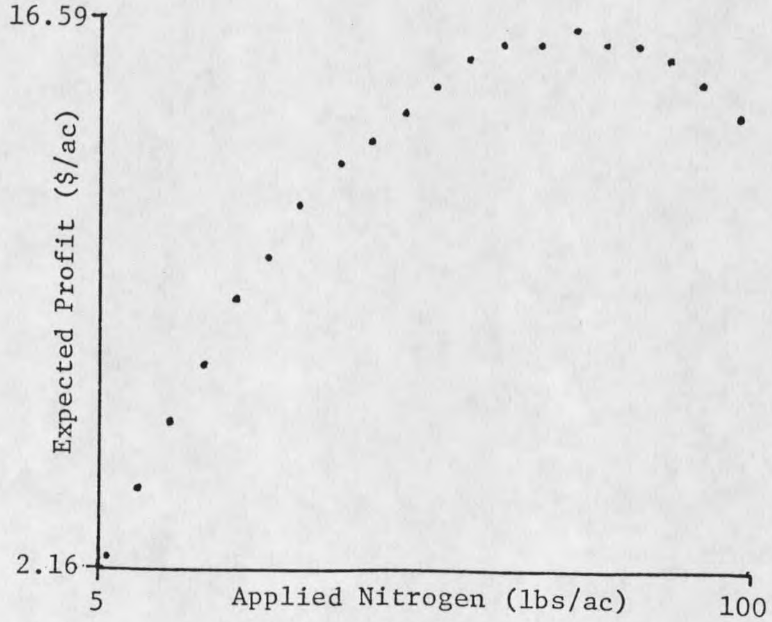

Figure 9. Points of expected profit conditioned on winter precipitation equal to 4 inches, plotted against increments of nitrogen at 5 lbs/ac (from 5 to 100 lbs/ac).

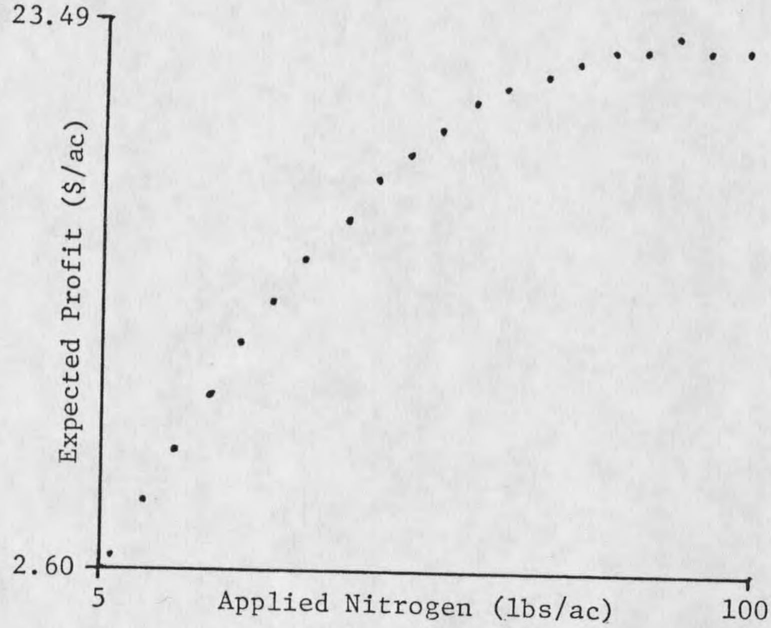

Figure 10. Points of expected profit conditioned on winter precipitation equal to 6 inches, plotted against increments of nitrogen at 5 lbs/ac (from 5 to 100 lbs/ac).

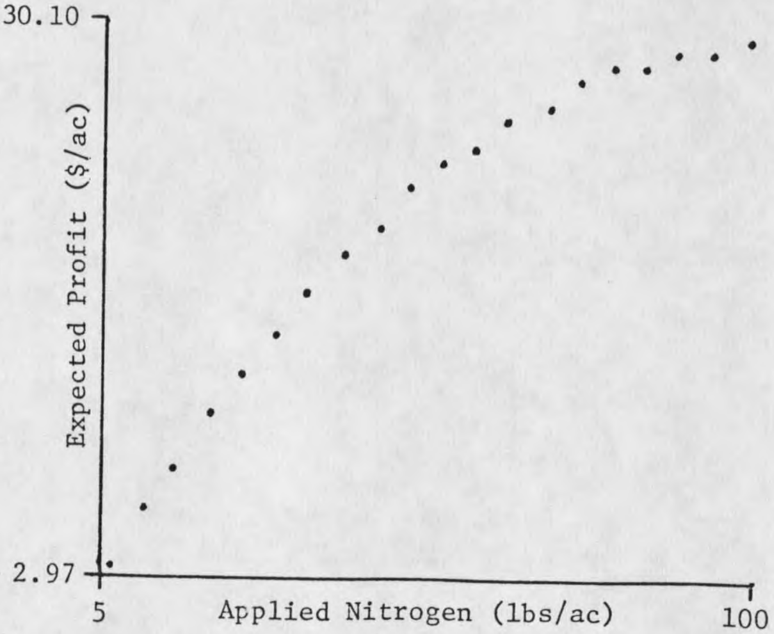

Figure 11. Points of expected profit conditioned on winter precipitation equal to 8 inches, plotted against increments of nitrogen at 5 lbs/ac (from 5 to 100 lbs/ac).

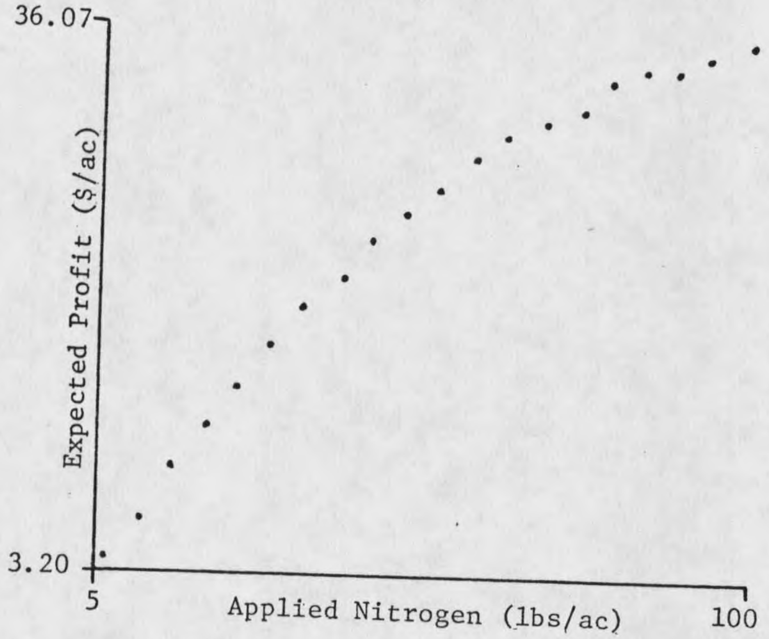

Figure 12. Points of expected profit conditioned on winter precipitation equal to 10 inches, plotted against increments of nitrogen at 5 lbs/ac (from 5 to 100 lbs/ac).

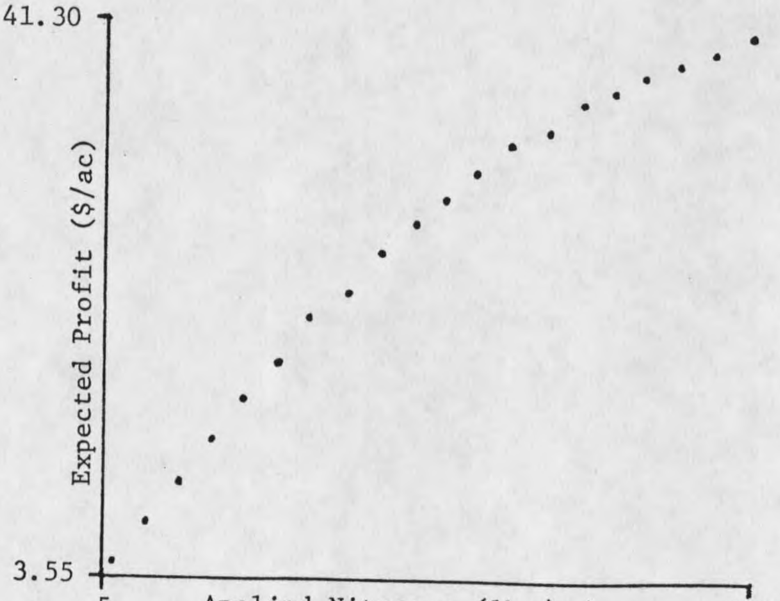

Figure 13. Points of expected profit conditioned on winter precipitation equal to 12 inches, plotted against increments of nitrogen at 5 lbs/ac (from 5 to 100 lbs/ac).

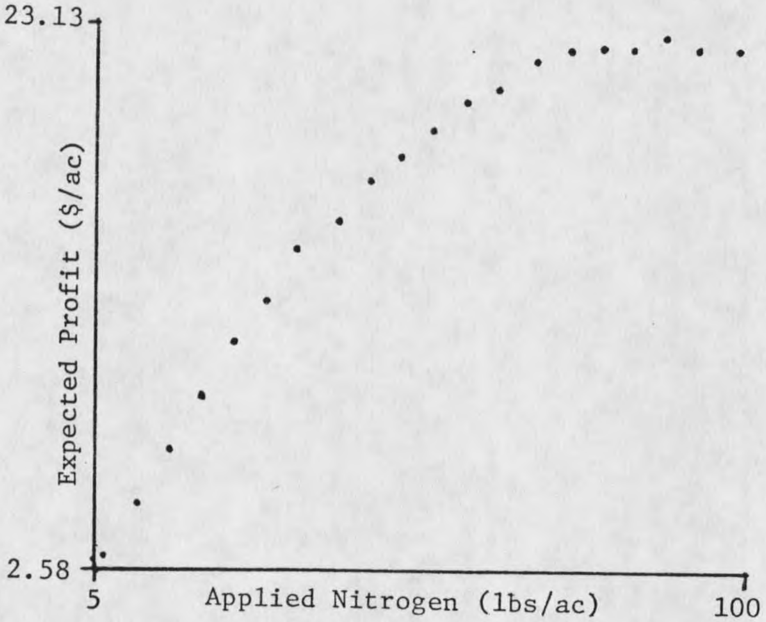

Figure 14. Points of unconditional expected profit, plotted against increments of nitrogen at 5 lbs/ac (from 5 to 100 lbs/ac).

$$E(\pi \mid WP = 2) = \underset{(138.0)}{0.34614\,N} - \underset{(99.5)}{0.00314\,N^2} \tag{V.8}$$

$$E(\pi \mid WP = 4) = \underset{(744.0)}{0.44356\,N} - \underset{(395.7)}{0.00297\,N^2} \tag{V.9}$$

$$E(\pi \mid WP = 6) = \underset{(962.6)}{0.52885\,N} - \underset{(430.2)}{0.00298\,N^2} \tag{V.10}$$

$$E(\pi \mid WP = 8) = \underset{(744.7)}{0.60063\,N} - \underset{(295.4)}{0.00300\,N^2} \tag{V.11}$$

$$E(\pi \mid WP = 10) = \underset{(699.1)}{0.66216\,N} - \underset{(253.4)}{0.00302\,N^2} \tag{V.12}$$

$$E(\pi \mid WP = 12) = \underset{(691.3)}{0.71618\,N} - \underset{(233.1)}{0.00304\,N^2} \tag{V.13}$$

$$E(\pi) = \underset{(1289.8)}{0.52607\,N} - \underset{(582.1)}{0.00299\,N^2} \tag{V.14}$$

number of observations = 20

Estimation of Marginal Profit Distributions

In this study, the central underlying component of the safety first risk constraint is marginal profit, which was previously mentioned to be stochastic. Because of the random nature of marginal profit, a probability density function can be fit to each conditional case and the unconditional case of marginal profit.

To ease the task of estimating the marginal profit distributions, the previously mentioned categories of conditional and unconditional marginal profit were re-organized from smallest to largest.[23] That is, marginal profit was ranked from the smallest level to the largest level for each given 5 pound increment of nitrogen between 5 and 100 pounds per acre; the ranking of the random variable was based on an assigned "... cumulative frequency value to each observation."[24] This procedure is similar to the earlier presentation regarding the ordering of the data to describe cumulative distributions for stochastic dominance analysis. Following Taylor's presentation, one obtains the cumulative frequency values with the following technique (the transformation will be explained momentarily):

> Given a sample of $n > 1$ observations on Y for the rth value of X ($r = 1, 2, \ldots,$ R), ranked from smallest to largest, $Y_{1r} < Y_{2r} < \ldots < Y_{ir} \ldots < Y_{nr}$. We can assign to each a cumulative frequency $F(Y_{ir}\ X_r) = i/n$. Then, all $F(Y_{ir}\ X_r)$ except $F(Y_{nr}|X_r)$ can be transformed ... to give a finite Z; the problem with $F(Y_{nr}|X_r)$ is that Z_{nr} is infinite [99]. page 3.

For the topic at hand, Y may be considered marginal profit, say $\tilde{\pi}$, and X is nitrogen, to be denoted N. Within one group of marginal profit conditioned on winter precipitation (2, 4, . . ., 12 inches) and unconditional marginal profit, these frequency values with their associated observations on marginal profit and nitrogen were formulated for each year of the 78 years of precipitation data.

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[23] Re-organization was accomplished by means of SORT [99].

[24] See Appendix A of [98].

A hyperbolic tangent transformation procedure, outlined by Taylor [99], was used to estimate the probability density functions. Actually the hyperbolic tangent provides the cumulative distribution function, and its derivative, the probability density function, is the square of the hyperbolic secant. The hyperbolic tangent may be written as follows,

$$F(\tilde{\pi}|N) = .5 + .5\tanh[P(\tilde{\pi},N)] \tag{V.15}$$

where,

$F(\tilde{\pi}|N)$ = marginal profit conditioned on nitrogen, or cumulative density function
$P(\tilde{\pi},N)$ = a polynomial in $\tilde{\pi}$ and N
$\tilde{\pi}$ = marginal profit
N = nitrogen

and, for example, $\tanh f(x) = \dfrac{e^{f(x)} - e^{-f(x)}}{e^{f(x)} + e^{-f(x)}}$

Taylor suggests the use of ordinary least squares to obtain initial estimates of β, a vector of parameters associated with $P(\tilde{\pi},N)$. Thus, to make Equation (V.15) conformable for ordinary least squares estimation, a transformation was imposed which gives,

$$Z = .5 \ln \left[\frac{F(\tilde{\pi}|N)}{1 - F(\tilde{\pi}|N)}\right] = P(\tilde{\pi},N) \tag{V.16}$$

where: Z = a constant

Additionally, Z could be thought of as the cumulative frequency value generated from the previously discussed SORT program. That is, Z was constructed from the previously described results, i/n.

The sorted data file, composed of observations on Z and the variables in the polynomial (i.e., $\tilde{\pi}$, $\tilde{\pi}^2$, $\tilde{\pi}^3$, N, N^2, N^3, $\tilde{\pi}N$, $\tilde{\pi}^2N$, $\tilde{\pi}N^2$), served to estimate Equation V.16 with ordinary least squares. One should keep in mind that these parameter estimates are just preliminary estimates, or starting values to simplify estimation of Equation V.15. Appendix E lists the initial parameter estimates of $P(\tilde{\pi},N)$ in Equation V.16, for the six marginal profit distributions conditioned on winter precipitation and for the single unconditional distribution.

Results from the initial estimates foster the implementation of SECANT,[25] a FORTRAN program which utilizes a numerical search routine to locate maximum likelihood estimates of Equation V.15. For more explicit information regarding SECANT, the reader might study pages 3, 4, and Appendix B of [98].

The maximum likelihood parameter estimates for Equation V.15, including relevant statistical information, are tabulated in Appendix F. In short, the estimates produced by SECANT characterize the various cumulative density functions for marginal profits.

Optimal Application of Nitrogen Fertilizer

The ultimate purpose of this project was to obtain profit maximizing levels of nitrogen fertilizer. Specifically, the goal is to maximize profit without violating the safety first risk constraint. Respectively, Equations V.1 and V.2 state this objective for unconditional and conditional expected profit.

The quadratic profit functions facilitate optimization of Equations V.1 and V.2, by means of a FORTRAN optimization program. Basically, the computer program uses the first derivative of profit with respect to nitrogen, set equal to zero, to scan for the global maximum, provided the risk constraint, is not violated. Recall that a subjective probability must be assigned to the constraint that marginal profit is negative or equal to zero. For instance, assume a value of .05 is selected for α. This implies that the farmer will tolerate negative marginal profits in only one of twenty seasons of production. The size of the probability constraint, α, reflects an individual's risk preference. Progressively smaller values of α indicate increasing risk adversity.

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[25] See page 3 of [98].

Thus, to make the optimization program functional, a value of .25 was specified for α, along with the quadratic profit functions and marginal profit distributions. Additionally, specification of certain constants in Equations V.1 and V.2 must occur before optimizing the functions. Namely, p, the price of nitrogen was set equal to 23 cents per pound; D, the discount factor was fixed at 1.14; and B, the base price of wheat was specified as 3.50 dollars per bushel.

Table 17 summarizes the optimal levels of nitrogen and profit for the unconditional and conditional cases. One can see in Table 17 that, in most cases, the optimal levels of nitrogen fertilizer decreases approximately 25%, with imposition of the safety first constraint. Another point to note is that as more risk averse levels of α are chosen, then the profit maximizing rate of nitrogen, with the safety first constraint satisfied, will decrease. Furthermore, from the conditional distributions one can observe that optimal rates of nitrogen increase with greater accumulation of winter precipitation. This characteristic seems plausible, for as a greater quantity of water accumulates in the soil, the plant has greater capability to extract nutrients from the root zone. In addition, the reader may recall that the estimated yield response function revealed that applied nitrogen and composite precipitation were technical complements. In the event that nutrients become more dilute with greater precipitation, or with build up of soil water, then, higher rates of applied nitrogen would be required to sustain optimal yields of grain and protein. The optimal nitrogen for unconstrained profit conditioned on winter precipitation greater than 8 inches exceeds 100 pounds per acre. These optimal levels of nitrogen exceed the highest rate of nitrogen applied in Dr. Haby's experiments, 100 pounds per acre. Consequently, when profit is conditioned on winter precipitation greater than 8 inches, the figures for optimal nitrogen in the unconstrained framework should not be treated as fertilizer recommendations since those rates exceed the upper bound on nitrogen observations in the data.

Table 17. Summary of Optimal Profit and Their Associated Optimal Nitrogen Levels for Unconditional and Conditional Profit with $\alpha = 0.25$ (Maximization of Equations V.1 and V.2).

Type of Maximized Expected Profit	No Safety First ($/ac)	Associated Level of Nitrogen (lbs/ac)	Safety First ($/ac) $\alpha = 0.25$	Associated Level of Nitrogen (lbs/ac)
Unconditional	23.12	88	21.42	64
Profit| WP=2	9.39	55	9.16	44
Profit| WP=4	16.54	75	15.53	56
Profit| WP=6	23.47	89	21.91	66
Profit| WP=8	30.03	100	28.11	75
Profit| WP=10	36.24	109	33.85	81
Profit| WP=12	42.13	118	39.81	90

As a final and paramount observation from Table 17, one can see that winter wheat fertilization in south central Montana is not a very risky prospect.

Finally, Table 18 enables the reader to see various categories of information associated with incremental levels of nitrogen. The numbers in Table 18 were calculated from the previously estimated yield, protein, and protein-premium equations. The profit and marginal profit categories are unconstrained and not conditioned on winter precipitation. Total revenus is merely the product of price and yield. The linear and squared observations on summer and winter precipitation were generated from the 78 years of precipitation data from Billings, Montana. All inputs other than nitrogen remained fixed at their mean values. More specifically, expected values of the primary variables (in Table 2) were generated from the experimental data and used to formulate the numbers listed below.

The optimal level of nitrogen in the context described above is approximately 90 pounds per acre, which roughly coincides with the optimal of 88 pounds per acre for unconditional unconstrained profit in Table 17. The predicted levels of protein in Table 18 seem reasonable. They are between about 10 and 13 percent protein. Also, the predicted premium paid, varying from 3 to 22 cents, is typical of historical wheat premiums.

Table 18. Various Information Associated with Incremental Levels of Applied Nitrogen.

Nit	Yield	Protein	Premium	Price	Total Revenue	Profit (returns to nitrogen)	Derivative of Profit
0	23.57	10.16	.03	3.53	83.20	0	
5	24.64	10.30	.03	3.53	86.98	2.57	2.57
10	25.67	10.44	.03	3.53	90.62	4.97	2.41
15	26.65	10.58	.04	3.54	94.34	7.22	2.25
20	27.58	10.72	.04	3.54	97.63	9.31	2.09
25	28.45	10.87	.05	3.55	101.00	11.24	1.93
30	29.27	11.01	.05	3.55	103.91	13.02	1.78
35	30.05	11.16	.06	3.56	106.98	14.64	1.63
40	30.77	11.31	.07	3.57	109.85	16.12	1.48
45	31.44	11.46	.07	3.57	112.24	17.45	1.33
50	32.06	11.61	.08	3.58	114.77	18.64	1.19
55	32.63	11.76	.09	3.59	117.14	19.69	1.05
60	33.15	11.92	.10	3.60	119.34	20.60	.91
65	33.62	12.08	.11	3.61	121.37	21.37	.77
70	34.04	12.24	.13	3.63	123.57	22.00	.63
75	34.41	12.40	.14	3.64	125.22	22.50	.50
80	34.72	12.56	.16	3.66	127.08	22.85	.36
85	34.99	12.72	.17	3.67	128.41	23.07	.21
90	35.20	12.89	.19	3.69	129.89	23.13	.07
95	35.36	13.06	.21	3.71	131.19	23.04	-.09
100	35.48	13.22	.22	3.72	131.99	22.79	-.25

Derivation of the Additional Profit Gained from Fertilization Given the Knowledge of Winter Precipitation

Theoretically, if a producer possesses information concerning the amount of precipitation which accumulates over winter, then he might gauge spring fertilization rates more accurately, and achieve a greater profit. Stated differently, it is hypothesized that fertilizer recommendations which utilize information on winter precipitation will yield greater profit over recommendations which do not contain knowledge of winter rainfall.

To evaluate this idea, a comparison is made between unconstrained profit in the conditional and unconditional contexts. The ordered pairs of optimal conditional profit without the risk constraint and the winter precipitation level in Table 19 were fitted with a line using ordinary least squares regression. Similarly, observations on unconstrained optimal unconditional profit were generated for incremental levels of winter precipitation, with nitrogen fixed at its optimal level of 87.89 pounds per acre. A quadratic function provided the best specification for the unconditional profit data. Regression estimates for the two functions are presented in Equations V.17 and V.18. The parenthetical values are t-ratios.

Curves derived from the estimated equations are presented in Figure 18. Figure 18 reveals that some additional profit may be captured by knowing the quantity of winter precipitation. In each regression the number of observations was six.

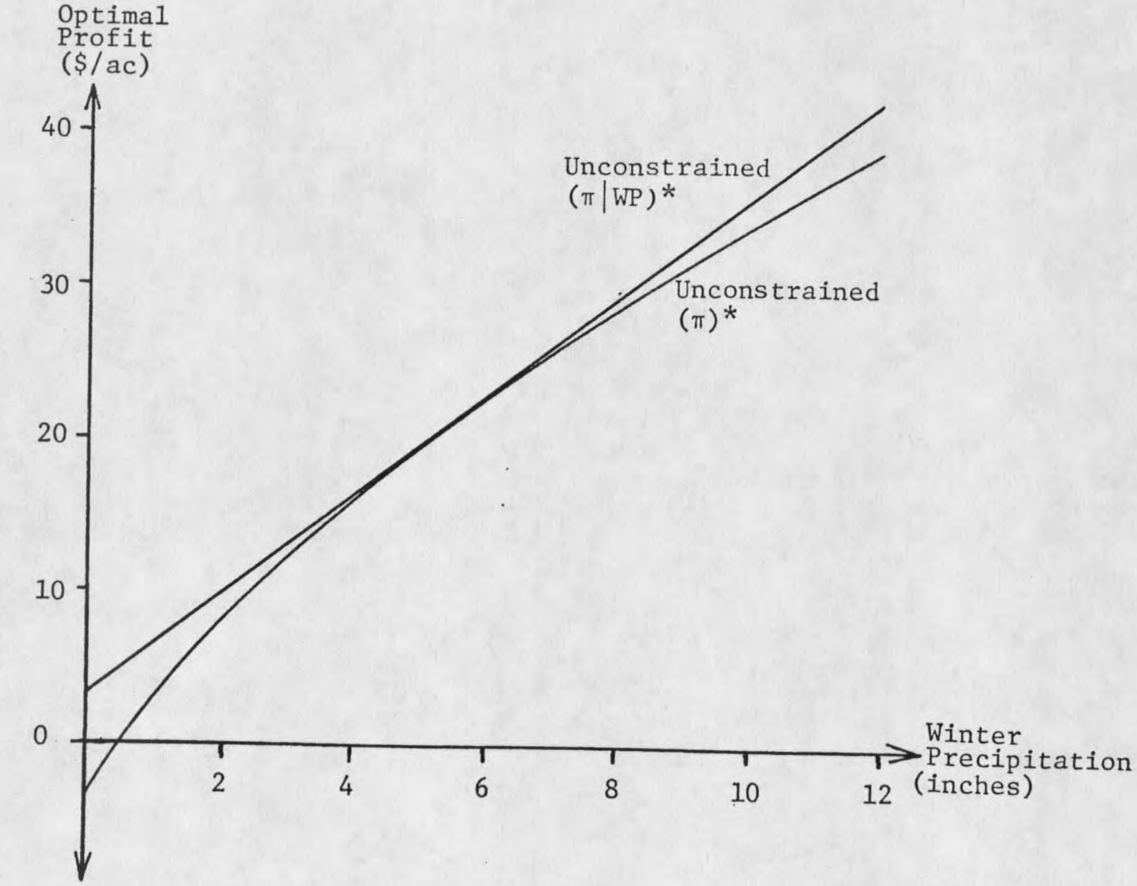

Figure 15. Curves for conditional and unconditional optimal profit, with no safety first constraint.

$$(\pi | \mathrm{WP})^* = \underset{(7.947)}{3.4547} + \underset{(58.54)}{3.2672}\,\mathrm{WP} \qquad \text{(V.17)}$$

$$(\pi)^* = \underset{(4.653)}{-3.7582} + \underset{(20.65)}{5.4561}\,\mathrm{WP} - \underset{(8.433)}{0.1565}\,\mathrm{WP}^2 \qquad \text{(V.18)}$$

To calculate the amount of profit a producer might expect in each scenario, first a distribution must be estimated to characterize winter precipitation. In other words, winter precipitation is treated as a random variable with a probability distribution. Secondly, the winter precipitation distribution will be multiplied by the estimated profit functions in Equations V.17 and V.18, hence yielding expected profit for the conditional and unconditional cases.

Thom [103] presents a convincing argument, supported by empirical evidence, which suggests that meteorological data follows a gamma distribution. A random variable is said to have a gamma distribution if the density function follows the mathematical form of Equation V.19,

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} \qquad \text{for } 0 < x < \infty \qquad \text{(V.19)}$$

where:

x = a random variable

α = a parameter to be estimated

β = a parameter to be estimated

e = Euler's constant

Γ = a gamma function

Actually, the gamma function is equivalent to the following integral,

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y}\, dy \qquad \text{(V.20)}$$

where: $y = x/\beta$ for $0 < x < \infty$

Observations on winter precipitation come from 78 years of climatological data collected at Billings, Montana. The parameters, α and β, were estimated with a FORTRAN program designed specifically for that purpose. Values of 8.8176 and 0.6862 were obtained as estimates of α and β, respectively.

These parameter estimates were incorporated into another computer program which calculates expected profit associated with a given function. Specifically, the gamma function, Equation V.20, is calculated, then the program estimates the probability density function, V.19. The density function is multiplied by either the conditional or unconditional optimal profit function, Equations V.17 and V.18 respectively, and the result is numerically integrated to arrive at the expected profit.[26]

Results from the series of computations point out that the value of additional knowledge concerning winter precipitation is relatively small. To be exact, the expected optimal conditional profit equals 23.19 dollars per acre, in contrast to 22.85 dollars per acre in the unconditional situation.

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[26] The following FORTRAN computer programs, GAMMA, OPTGAMMA, and IMSL routines, MGAMA, DCADRE were used in this sequence of computations.

CHAPTER IV

CONCLUDING REMARKS

Summary

Thinking retrospectively for the moment, this study employed data from 30 fertilizer experiments conducted over five years in south central Montana. The experiments were performed on hard red winter wheat grown under dryland conditions on 30 different sites. This data served as a base to determine profit maximizing rates of nitrogen fertilizer for winter wheat. This final objective was accomplished through a general sequence of carefully orchestrated steps.

First, both polynomial yield and protein response functions were estimated with a generalized nonlinear least squares algorithm. Causal variables which displayed statistical significance in explaining yield and protein response were applied nitrogen, applied phosphorus, soil phosphorus, composite precipitation (made up of logged winter and summer precipitation), a variety of interaction variables, and dummy variables representing fixed effects of experimental location and year. The response functions were specified with an additive disturbance term comprised of two stochastic components. Significance of the parameter estimates was analyzed by calculating t-ratios for each coefficient. Slope and curvature of the response functions were determined from the partial derivatives of yield and protein with respect to the various inputs.

The second procedure involved fitting a curve to describe the premium and received by farmers for various concentrations of grain protein. A hyperbolic tangent functional specification was used in the estimation of the protein-premium function. Ordinary least squares regression served to generate parameter estimates.

As a final step, profit functions were formulated by combining the response functions with the protein-premium function. Unconditional profit and profit conditioned on specified levels of winter precipitation were computed. Profit was also compared in a constrained and unconstrained framework. That is, a safety first risk constraint was imposed which required expected marginal profit to be negative no more than one out of four years of production. A base price of 3.50 dollars per bushel of wheat and a discount factor of 1.14 were incorporated into the profit functions. Finally, the unconditional and conditional profit functions were maximized for optimal levels of applied nitrogen in both a constrained and unconstrained framework.

The optimal rate of nitrogen, with winter precipitation at its mean value, was 88 pounds per acre with an associated unconditional profit level of 23.12 dollars per acre. However, given information regarding winter precipitation, optimal rates of nitrogen varied between 55 pounds per acre for 2 inches of precipitation and 118 pounds per acre for 12 inches of winter precipitation. Including a safety first constraint, to incorporate a measure of risk associated with fertilization, suggested a reduction of approximately 25 percent in each level of optimal nitrogen. This reduction in expected profit is traded off for reduced frequency of negative expected marginal profit.

A comparison was made between the conditional approach, where fertilization was based on a given level of winter precipitation, and the unconditional situation. The analysis revealed optimal application of nitrogen in the conditional case produced only slightly greater profit on average than when using the unconditional approach. In fact, the expected unconditional profit was 22.85 dollars per acre and expected conditional profit was 23.19 dollars per acre. Furthermore, results from comparing unconstrained and constrained expected profit indicated that farmers assume relatively inconsiderable risk when applying nitrogen fertilizer. In other words, the difference between unconstrained and constrained optimal profit is, on average, very small.

Limitations

A few comments concerning the potential limitations in the usefulness of this study should be mentioned. First, the entire area of winter wheat production in south central Montana is subject to highly variable geographic, soil, and climatic conditions. The methods utilized in explaining yield and protein response attempt to account for any possible variability, yet no model or estimation technique is absolutely flawless. Second, the unconstrained optimal fertilization rates seem rather high, nevertheless, this reflects that on average, higher rates of nitrogen produce higher protein grain, which allows the producer to capture a substantial premium. Provided one possesses faith in the data and quantitative methods which were used to arrive at the recommended fertilization rates, then one must accept these rates as being truly optimal. Third, the response functions were specified as polynomials. Perhaps, some other functional form would provide a more precise estimate of yield and protein response. Fourth, and finally, the protein-premium function is only a generalized function. From historical data one may observe highly variable wheat prices, therefore, the protein-premium estimate could reflect highly inaccurate premiums in situations when either a large supply of wheat exists, or when wheat is very scarce.

Future Research

This thesis may have been refined and expanded upon in many areas, however, three primary areas for possible future research seem most noteworthy. First, an individual might undertake the task of building a more accurate model to predict premium paid for wheat grain protein. If the model were designed to predict mean protein level of grain harvested in the Southwest and the Great Plains, then producers in the northern plain states could gauge their spring fertilizer decisions appropriately and possibly increase their profit. Undoubtedly the quality of grain from those southern areas will influence the price received for wheat grown in Montana, which is harvested at a much later date. Also, one may gather information concerning the storage and blending practices of the various grain elevator operators. Regional differences in protein content of grain should be accounted for in the model.

Second, an individual may estimate other functional specifications of yield and protein response which may include soil nitrates as a variable.

Third, and finally, one might simply use the results from this project to compare different techniques to evaluate risk. For instance, a comparison of strict safety first could be made to first and second order stochastic dominance.

As a final note, it was intended that the information rendered from this project might assist Montana wheat producers in selecting the most profitable rate to apply nitrogen. Hopefully, the analysis has succeeded in that respect, along with adding to the technical economic knowledge of wheat production in Montana.

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APPENDICES

APPENDIX A

TABLE OF EXPERIMENTAL SITES AND YEARS

Table 19. Experimental Location and Year.

Farm	Location	Year
1. Art Hansen Farm	Melville, MT	1976
2. Don Herzog Farm	Rapelje, MT	1976
3, Don Holland Farm	Rosebud, MT	1976
4. Jim Larson Farm	Forsyth, MT	1976
5. Clinton McFarland Farm	Molt, MT	1976
6. Lloyd Mosdal Farm	Rapelje, MT	1976
7. Floyd Warren Corp.	Hardin, MT	1976
8. George Eastlick Farm	Molt, MT	1977
9. Ken Hanson Farm	Melville, MT	1977
10. Jim Larsen Farm	Forsyth, MT	1977
11. Southern Ag. Resch. Center	Huntley, MT	1977
12. Ken Sire Farm	Ballantine, MT	1977
13. Floyd Warren Corp.	Hardin, MT	1977
14. George Eastlick Farm	Molt, MT	1978
15. Ken Hanson Farm	Melville, MT	1978
16. Dick Keller Farm	Custer, MT	1978
17. Jim Larsen Farm	Forsyth, MT	1978
18. Dennis and Charles Lee Farm	Billings, MT	1978
19. Stan Logan Farm	Huntley, MT	1978
20. Ben Becker Farm	Billings, MT	1979
21. George Eastlick Farm	Molt, MT	1979
22. George Haines Farm	Forsyth, MT	1979
23. Ken Hanson Farm	Melville, MT	1979
24. Dick Keller Farm	Custer, MT	1979
25. Stan Logan Farm	Huntley, MT	1979
26. Ned Schaff Farm	Lavine, MT	1979
27. Floyd Warren Corp.	Hardin, MT	1979
28. George Haines Farm	Forsyth, MT	1980
29. Jim Larsen Farm	Forsyth, MT	1980
30. Robert Micheal Farm	Huntley, MT	1980

APPENDIX B

TABLE OF SUMMARY STATISTICS FOR WINTER WHEAT DATA

Table 20. Summary Statistics for Experimental Wheat Data.

Observation	Units	Mean	Standard Deviation	Range	
				Min.	Max.
Yield	bu/ac	37.2	12.3	10.8	76.2
Protein	%	12.1	2.6	7.1	20.1
Soil Nitrate	kg/ha	40.2	23.7	13.6	139.1
Soil Phosphorus	kg/ha	8.5	4.0	0.7	27.4
Summer Precipitation	inches	4.7	1.5	2.3	13.8
Winter Precipitation	inches	5.9	1.6	2.2	11.5
Applied Nitrogen	kg/ha	44.5	30.4	0	89.0
Applied Phosphorus	kg/ha	17.8	12.2	0	35.7
Applied Potassium	kg/ha	48.0	–	0	96.0

APPENDIX C

ADDITIONAL YIELD AND PROTEIN PARTIAL DERIVATIVES

Table 21. The Partial Derivative of Yield With Respect to Winter Precipitation for Various Levels of Applied Nitrogen (lbs/ac) and Summer Precipitation (inches).

Nitrogen	Summer Precipitation										
.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00
.00	.08	.00	-.06	-.11	-.15	-.18	-.22	-.24	-.27	-.29	-.31
10.00	.13	.08	.03	.00	-.03	-.06	-.08	-.10	-.12	-.14	-.15
20.00	.19	.15	.13	.10	.09	.07	.05	.04	.03	.02	.01
30.00	.25	.23	.22	.21	.20	.20	.19	.18	.18	.17	.17
40.00	.30	.31	.31	.32	.32	.32	.32	.33	.33	.33	.33
50.00	.36	.38	.40	.42	.44	.45	.46	.47	.48	.49	.49
60.00	.41	.46	.50	.53	.55	.58	.60	.61	.63	.64	.66
70.00	.47	.54	.59	.63	.67	.70	.73	.76	.78	.80	.82
80.00	.52	.61	.68	.74	.79	.83	.87	.90	.93	.96	.98
90.00	.58	.69	.78	.85	.90	.96	1.00	1.04	1.08	1.11	1.14
100.00	.64	.77	.87	.95	1.02	1.08	1.14	1.18	1.23	1.27	1.30

Table 22. The Partial Derivative of Yield With Respect to Summer Precipitation for Various Levels of Applied Nitrogen (lbs/ac) and Winter Precipitation (inches).

Nitrogen	Winter Precipitation										
.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
.00	-.01	-.09	-.15	-.20	-.24	-.28	-.32	-.35	-.38	-.41	-.43
10.00	.19	.13	.09	.05	.02	-.01	-.04	-.06	-.08	-.10	-.12
20.00	.38	.35	.32	.30	.27	.26	.24	.23	.21	.20	.19
30.00	.58	.56	.55	.54	.53	.53	.52	.51	.51	.50	.50
40.00	.77	.78	.79	.79	.79	.80	.80	.80	.80	.81	.81
50.00	.97	1.00	1.02	1.04	1.05	1.07	1.08	1.09	1.10	1.11	1.12
60.00	1.17	1.21	1.25	1.28	1.31	1.34	1.36	1.38	1.40	1.41	.143
70.00	1.36	1.43	1.48	1.53	1.57	1.61	1.64	1.66	1.69	1.71	1.74
80.00	1.56	1.65	1.72	1.78	1.83	1.87	1.92	1.95	1.99	2.02	2.05
90.00	1.76	1.86	1.95	2.02	2.09	2.14	2.19	2.24	2.28	2.32	2.36
100.00	1.95	2.08	2.18	2.27	2.35	2.41	2.47	2.53	2.58	2.62	2.67

Table 23. The Partial Derivative of Yield With Respect to Applied Nitrogen for Various Levels of Applied Nitrogen (lbs/ac) and Summer Precipitation (inches).

Nitrogen	Summer Precipitation										
.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00
.00	.35	.43	.51	.58	.64	.70	.76	.81	.85	.90	.94
10.00	.33	.41	.49	.56	.62	.68	.73	.79	.83	.88	.92
20.00	.31	.39	.47	.54	.60	.66	.71	.77	.81	.86	.90
30.00	.29	.37	.45	.52	.58	.64	.69	.75	.79	.84	.88
40.00	.27	.35	.43	.50	.56	.62	.67	.72	.77	.82	.86
50.00	.25	.33	.41	.48	.54	.60	.65	.70	.75	.80	.84
60.00	.23	.31	.39	.46	.52	.58	.63	.68	.73	.78	.82
70.00	.21	.29	.37	.44	.50	.56	.61	.66	.71	.76	.80
80.00	.19	.27	.35	.42	.48	.54	.59	.64	.69	.74	.78
90.00	.17	.25	.33	.40	.46	.52	.57	.62	.67	.72	.76
100.00	.15	.23	.31	.38	.44	.50	.55	.60	.65	.70	.74

Table 24. The Partial Derivative of Yield With Respect to Applied Nitrogen for Various Levels of Applied Nitrogen (lbs/ac) and Winter Precipitation (inches).

Nitrogen	Winter Precipitation										
.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
.00	.44	.48	.51	.54	.56	.59	.61	.63	.65	.66	.68
10.00	.42	.46	.49	.52	.54	.57	.59	.61	.63	.64	.66
20.00	.40	.44	.47	.50	.52	.55	.57	.59	.60	.62	.64
30.00	.38	.42	.45	.48	.50	.53	.55	.57	.58	.60	.62
40.00	.36	.40	.43	.46	.48	.51	.53	.55	.56	.58	.60
50.00	.34	.38	.41	.44	.46	.48	.51	.53	.54	.56	.58
60.00	.32	.36	.39	.42	.44	.46	.49	.51	.52	.54	.56
70.00	.30	.34	.37	.40	.42	.44	.47	.49	.51	.54	.54
80.00	.28	.32	.35	.38	.40	.42	.45	.46	.48	.50	.52
90.00	.26	.29	.33	.36	.38	.40	.42	.44	.46	.48	.50
100.00	.24	.27	.31	.33	.36	.38	.40	.42	.44	.46	.48

Table 25. The Partial Derivative of Protein With Respect to Applied Nitrogen for Various Levels of Applied Nitrogen (lbs/ac) and Applied Phosphorus (lbs/ac).

Nitrogen	Applied Phosphorus										
.00	.00	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00
.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
10.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
20.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
30.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
40.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
50.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
60.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
70.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
80.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
90.00	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
100.00	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04

Table 26. The Partial Derivative of Protein With Respect to Winter Precipitation for Various Levels of Applied Nitrogen (lbs/ac) and Winter Precipitation (inches).

Nitrogen	Winter Precipitation										
.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
.00	-.20	-.13	-.09	-.07	-.05	-.04	-.03	-.02	-.02	-.01	-.01
10.00	-.21	-.14	-.10	-.08	-.06	-.04	-.03	-.03	-.02	-.02	-.01
20.00	-.22	-.16	-.11	-.09	-.07	-.05	-.04	-.03	-.03	-.02	-.02
30.00	-.24	-.17	-.12	-.09	-.07	-.06	-.05	-.04	-.03	-.02	-.02
40.00	-.25	-.18	-.13	-.10	-.10	-.08	-.05	-.04	-.04	-.03	-.02
50.00	-.27	-.19	-.14	-.11	-.09	-.07	-.06	-.05	-.04	-.03	-.03
60.00	-.28	-.20	-.15	-.12	-.10	-.08	-.06	-.05	-.05	-.04	-.03
70.00	-.30	-.21	-.16	-.13	-.10	-.08	-.07	-.06	-.05	-.04	-.04
80.00	-.31	-.23	-.17	-.14	-.11	-.09	-.08	-.06	-.05	-.05	-.04
90.00	-.33	-.24	-.18	-.14	-.12	-.10	-.08	-.07	-.06	-.05	-.05
100.00	-.34	-.25	-.19	-.15	-.12	-.10	-.09	-.07	-.06	-.06	-.05

Table 27. The Partial Derivative of Protein With Respect to Summer Precipitation for Various Levels of Applied Nitrogen (lbs/ac) and Summer Precipitation (inches).

Nitrogen	Summer Precipitation										
.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00
.00	-.43	-.25	-.15	-.10	-.06	-.03	-.02	.00	.01	.01	.02
10.00	-.46	-.27	-.17	-.11	-.07	-.04	-.03	-.01	.00	.01	.01
20.00	-.49	-.29	-.19	-.12	-.08	-.05	-.03	-.02	-.01	.00	.01
30.00	-.52	-.31	-.20	-.14	-.09	-.06	-.04	-.03	-.02	-.01	.00
40.00	-.54	-.33	-.22	-.15	-.11	-.07	-.05	-.04	-.02	-.01	-.01
50.00	-.57	-.35	-.24	-.16	-.12	-.09	-.06	-.04	-.03	-.02	-.01
60.00	-.60	-.37	-.25	-.18	-.13	-.10	-.07	-.05	-.04	-.03	-.02
70.00	-.63	-.39	-.27	-.19	-.14	-.11	-.08	-.06	-.05	-.03	-.03
80.00	-.65	-.42	-.29	-.21	-.15	-.12	-.09	-.07	-.05	-.04	-.03
90.00	-.68	-.44	-.30	-.22	-.16	-.13	-.10	-.08	-.06	-.05	-.04
100.00	-.71	-.46	-.32	-.23	-.18	-.14	-.11	-.09	-.07	-.06	-.04

APPENDIX D

UNCONDITIONAL AND CONDITIONAL MEAN PROFIT ASSOCIATED WITH VARIOUS LEVELS OF NITROGEN, INCLUDING OTHER STATISTICS

Table 28. Unconditional Mean Profit, Variance, Standard Deviation, and Coefficient of Variation for Increments of Nitrogen, 5 lbs/ac (from 5 to 100 lbs/ac).

Nitrogen	Mean Profit	Variance	Standard Deviation	Coefficient of Variation
5	2.58	1.03	1.02	.393
10	5.00	4.10	2.03	.405
15	7.26	9.18	3.03	.417
20	9.37	16.24	4.03	.430
25	11.31	25.21	5.02	.444
30	13.11	36.09	6.01	.458
35	14.75	48.80	6.99	.474
40	16.25	63.36	7.96	.490
45	17.59	79.73	8.93	.508
50	18.79	97.89	9.89	.526
55	19.85	117.90	10.86	.547
60	20.76	139.77	11.82	.569
65	21.53	163.58	12.79	.594
70	22.15	189.46	13.76	.621
75	22.62	217.58	14.75	.652
80	22.95	248.09	15.75	.686
85	23.12	281.22	16.77	.725
90	23.13	317.31	17.81	.770
95	22.98	356.57	18.88	.822
100	22.66	399.37	19.98	.882

Table 29. Conditional (Winter Precipitation = 2 inches) Mean Profit, Variance, Standard Deviation, and Coefficient of Variation for Increments of Nitrogen, 5 lbs/ac (from 5 to 100 lbs/ac).

Nitrogen	Mean Profit	Variance	Standard Deviation	Coefficient of Variation
5	1.59	.26	.51	.322
10	3.04	1.04	1.02	.335
15	4.34	2.29	1.51	.348
20	5.50	4.00	2.00	.364
25	6.51	6.16	2.48	.381
30	7.38	8.77	2.96	.401
35	8.12	11.84	3.44	.424
40	8.70	15.39	3.92	.451
45	9.15	19.48	4.41	.483
50	9.44	24.19	4.92	.521
55	9.58	29.61	5.44	.568
60	9.56	35.87	5.99	.626
65	9.38	43.10	6.57	.700
70	9.02	51.47	7.17	.795
75	8.49	61.14	7.82	.921
80	7.76	72.27	8.50	1.095
85	6.84	84.98	9.22	1.348
90	5.71	99.42	9.97	1.747
95	4.36	115.68	10.76	2.464
100	2.80	133.78	11.57	4.131

Table 30. Conditional (Winter Precipitation = 4 inches) Mean Profit, Variance, Standard Deviation, and Coefficient of Variation for Increments of Nitrogen, 5 lbs/ac (from 5 to 100 lbs/ac).

Nitrogen	Mean Profit	Variance	Standard Deviation	Coefficient of Variation
5	2.16	.60	.77	.359
10	4.16	2.38	1.54	.371
15	6.00	5.30	2.30	.384
20	7.69	9.35	3.06	.398
25	9.23	14.48	3.80	.412
30	10.61	20.67	4.55	.428
35	11.86	27.89	5.28	.445
40	12.95	36.13	6.01	.464
45	13.90	45.38	6.74	.484
50	14.71	55.67	7.46	.507
55	15.38	67.02	8.19	.532
60	15.90	79.52	8.92	.561
65	16.28	93.23	9.66	.593
70	16.51	108.31	10.41	.630
75	16.59	124.91	11.18	.674
80	16.51	143.19	11.97	.725
85	16.28	163.40	12.78	.785
90	15.87	185.74	13.63	.859
95	15.28	210.44	14.51	.949
100	14.52	237.76	15.42	1.062

Table 31. Conditional (Winter Precipitation = 6 inches) Mean Profit, Variance, Standard Deviation, and Coefficient of Variation for Increments of Nitrogen, 5 lbs/ac (from 5 to 100 lbs/ac).

Nitrogen	Mean Profit	Variance	Standard Deviation	Coefficient of Variation
5	2.60	.84	.92	.353
10	5.04	3.36	1.83	.363
15	7.32	7.52	2.74	.374
20	9.45	13.31	3.65	.386
25	11.41	20.69	4.55	.399
30	13.22	29.63	5.44	.412
35	14.88	40.10	6.33	.426
40	16.39	52.09	7.22	.440
45	17.75	65.57	8.10	.456
50	18.97	80.52	8.97	.473
55	20.04	96.96	9.85	.491
60	20.96	114.93	10.72	.511
65	21.75	134.43	11.59	.533
70	22.39	155.56	12.47	.557
75	22.89	178.46	13.36	.584
80	23.24	203.24	14.26	.613
85	23.44	230.11	15.17	.647
90	23.49	259.23	16.10	.685
95	23.39	290.95	17.06	.729
100	23.11	325.45	18.04	.781

Table 32. Conditional (Winter Precipitation = 8 inches) Mean Profit, Variance, Standard Deviation, and Coefficient of Variation for Increments of Nitrogen, 5 lbs/ac (from 5 to 100 lbs/ac).

Nitrogen	Mean Profit	Variance	Standard Deviation	Coefficient of Variation
5	2.97	1.04	1.02	.343
10	5.77	4.14	2.03	.352
15	8.41	9.28	3.05	.362
20	10.89	16.45	4.06	.372
25	13.21	25.61	5.06	.383
30	15.38	36.75	6.06	.394
35	17.38	49.83	7.06	.406
40	19.24	64.82	8.05	.418
45	20.94	81.73	9.04	.432
50	22.50	100.51	10.03	.446
55	23.91	121.17	11.01	.460
60	25.17	143.70	11.99	.476
65	26.29	168.14	12.97	.493
70	27.27	194.52	13.95	.511
75	28.11	222.90	14.93	.531
80	28.80	253.45	15.92	.553
85	29.35	286.26	16.92	.576
90	29.75	321.51	17.93	.603
95	30.01	359.47	18.96	.632
100	30.10	400.33	20.01	.665

Table 33. Conditional (Winter Precipitation = 10 inches) Mean Profit, Variance, Standard Deviation, and Coefficient of Variation for Increments of Nitrogen, 5 lbs/ac (from 5 to 100 lbs/ac).

Nitrogen	Mean Profit	Variance	Standard Deviation	Coefficient of Variation
5	3.28	1.20	1.09	.334
10	6.39	4.79	2.19	.342
15	9.34	10.74	3.28	.351
20	12.12	19.07	4.37	.360
25	14.75	29.72	5.45	.370
30	17.21	42.69	6.53	.380
35	19.52	57.96	7.61	.390
40	21.67	75.49	8.69	.401
45	23.67	95.27	9.76	.412
50	25.52	117.28	10.83	.424
55	27.23	141.52	11.90	.437
60	28.78	167.95	12.96	.450
65	30.19	196.63	14.02	.464
70	31.46	227.53	15.08	.479
75	32.59	260.77	16.15	.496
80	33.57	296.38	17.22	.513
85	34.41	334.48	18.29	.531
90	35.11	375.22	19.37	.552
95	35.66	418.82	20.47	.574
100	36.07	465.51	21.58	.598

Table 34. Conditional (Winter Precipitation = 12 inches) Mean Profit, Variance, Standard Deviation, and Coefficient of Variation for Increments of Nitrogen, 5 lbs/ac (from 5 to 100 lbs/ac).

Nitrogen	Mean Profit	Variance	Standard Deviation	Coefficient of Variation
5	3.55	1.34	1.16	.326
10	6.93	5.35	2.31	.334
15	10.15	12.02	3.47	.342
20	13.20	21.33	4.62	.350
25	16.09	33.27	5.77	.358
30	18.82	47.83	6.92	.368
35	21.39	64.98	8.06	.377
40	23.81	84.71	9.20	.387
45	26.07	106.98	10.35	.397
50	28.18	131.80	11.48	.407
55	30.14	159.14	12.62	.419
60	31.95	188.99	13.75	.430
65	33.62	221.36	14.88	.443
70	35.14	256.28	16.01	.456
75	36.52	293.80	17.14	.469
80	37.76	333.90	18.27	.484
85	38.86	376.78	19.41	.500
90	39.82	422.48	20.55	.516
95	40.63	471.21	21.71	.534
100	41.30	523.17	22.87	.554

APPENDIX E

INITIAL POLYNOMIAL PARAMETER ESTIMATES FOR SECANT, FOR THE UNCONDITIONAL AND CONDITIONAL DISTRIBUTIONS OF MARGINAL PROFIT

Table 35. Polynomial Parameter Estimates for Unconditional Marginal Profit.

Variable Name	Estimated Coefficient	t-Ratio
$\tilde{\pi}$	12.452	36.474
$\tilde{\pi}^2$	-13.447	-26.697
$\tilde{\pi}^3$	6.6562	27.739
N	0.07226	21.305
N^2	-0.000443	-8.1266
N^3	0.0000013	4.2306
$\tilde{\pi}N$	-0.16033	-19.789
$\tilde{\pi}^2N$	0.000681	13.611
$\tilde{\pi}N^2$	0.13741	25.858
intercept	-3.7057	-48.851

Adjusted R^2 = .9856.
Standard Error of the estimate = .1108.
Number of observations = 780.

Table 36. Polynomial Parameter Estimates for Conditional (Winter Precipitation = 2 inches) Marginal Profit.

Variable Name	Estimated Coefficient	t-Ratio
$\tilde{\pi}$	37.480	28.240
$\tilde{\pi}^2$	-69.711	-23.788
$\tilde{\pi}^3$	53.064	24.340
N	0.23950	23.140
N^2	-0.003070	-17.068
N^3	0.000016	15.445
$\tilde{\pi}N$	-0.98812	-22.071
$\tilde{\pi}^2N$	0.007845	21.482
$\tilde{\pi}N^2$	1.1953	24.672
intercept	-6.5046	-33.572

Adjusted R^2 = .9687.
Standard Error of the estimate = .1631.
Number of observations = 780.

Table 37. Polynomial Parameter Estimates for Conditional (Winter Precipitation = 4 inches) Marginal Profit.

Variable Name	Estimated Coefficient	t-Ratio
$\tilde{\pi}$	27.476	41.318
$\tilde{\pi}^2$	-46.437	-36.327
$\tilde{\pi}^3$	30.273	37.761
N	0.16025	30.729
N^2	-0.001608	-19.933
N^3	0.0000063	-31.447
$\tilde{\pi}N$	-0.57058	-31.447
$\tilde{\pi}^2N$	0.003437	28.628
$\tilde{\pi}N^2$	0.60921	39.991
intercept	-5.5825	-49.438

Adjusted R^2 = .9841.
Standard Error of the estimate = .1162.
Number of observations = 780.

Table 38. Polynomial Parameter Estimates for Conditional (Winter Precipitation = 6 inches) Marginal Profit.

Variable Name	Estimated Coefficient	t-Ratio
$\tilde{\pi}$	23.442	48.330
$\tilde{\pi}^2$	-35.390	-43.178
$\tilde{\pi}^3$	20.528	45.477
N	0.13675	34.078
N^2	-0.001204	-20.325
N^3	0.0000041	13.653
$\tilde{\pi}N$	-0.42032	-36.119
$\tilde{\pi}^2N$	0.002161	31.114
$\tilde{\pi}N^2$	0.39074	43.093
intercept	-5.4914	-57.917

Adjusted R^2 = .9882.
Standard Error of the estimate = .0999.
Number of observations = 780.

Table 39. Polynomial Parameter Estimates for Conditional (Winter Precipitation = 8 inches) Marginal Profit.

Variable Name	Estimated Coefficient	t-Ratio
$\tilde{\pi}$	21.309	51.598
$\tilde{\pi}^2$	-29.325	-46.636
$\tilde{\pi}^3$	15.527	49.548
N	0.12491	34.892
N^2	-0.000995	-19.420
N^3	0.0000031	11.807
$\tilde{\pi}N$	-0.34350	-37.881
$\tilde{\pi}^2N$	0.001584	31.123
$\tilde{\pi}N^2$	0.28602	45.627
intercept	-5.5711	-61.678

Adjusted R^2 = .9898.
Standard Error of the estimate = .0932.
Number of observations = 780.

Table 40. Polynomial Parameter Estimates for Conditional (Winter Precipitation = 10 inches) Marginal Profit.

Variable Name	Estimated Coefficient	t-Ratio
$\tilde{\pi}$	19.925	53.389
$\tilde{\pi}^2$	-25.295	-48.482
$\tilde{\pi}^3$	12.384	51.727
N	0.11835	35.211
N^2	-0.0008813	-18.726
N^3	0.0000025	10.621
$\tilde{\pi}N$	-0.29758	-38.729
$\tilde{\pi}^2N$	0.001278	30.820
$\tilde{\pi}N^2$	0.22571	46.739
intercept	-5.6973	-63.837

Adjusted R^2 = .9906.
Standard Error of the estimate = .0894.
Number of observations = 780.

Table 41. Polynomial Parameter Estimates for Conditional (Winter Precipitation = 12 inches) Marginal Profit.

Variable Name	Estimated Coefficient	t-Ratio
$\tilde{\pi}$	18.859	53.822
$\tilde{\pi}^2$	-22.431	-49.102
$\tilde{\pi}^3$	10.288	52.579
N	0.11211	34.484
N^2	-0.000774	-17.326
N^3	0.0000020	8.9958
$\tilde{\pi}N$	-0.26340	-38.431
$\tilde{\pi}^2N$	0.0010563	29.446
$\tilde{\pi}N^2$	0.18686	46.779
intercept	-5.7996	-64.333

Adjusted R^2 = .9910.
Standard Error of the estimate = .0875.
Number of observations = 780.

APPENDIX F

MAXIMUM LIKELIHOOD PARAMETER ESTIMATES FOR THE MARGINAL PROFIT DISTRIBUTIONS

(Calculated from SECANT)

Table 42. Maximum Likelihood Parameter Estimates of the Polynomial $P(\tilde{\pi},N)$, Which Characterize the Cumulative Density Function for Unconditional Marginal Profit.

Variable	Parameter Estimate	Variance of Estimate	Asymptotic t-Ratio
intercept	-4.0063	0.4241	-9.4475
$\tilde{\pi}$	14.1443	1.7471	8.0958
$\tilde{\pi}^2$	-16.0448	2.6939	-6.1031
$\tilde{\pi}^3$	8.1459	1.4584	5.5855
N	0.08460	0.02098	4.0323
N^2	-0.000593	0.000371	-1.5996
N^3	0.0000018	0.0000021	0.8290
$\tilde{\pi}N$	-0.1998	0.04543	-4.3989
$\tilde{\pi}^2N$	0.000898	0.000306	2.9387
$\tilde{\pi}N^2$	0.1657	0.03139	5.2776

Table 43. Maximum Likelihood Parameter Estimates of the Polynomial $P(\tilde{\pi},N)$, Which Characterize the Cumulative Density Function for Conditional Marginal Profit (Winter Precipitation = 2 inches).

Variable	Parameter Estimate	Variance of Estimate	Asymptotic t-Ratio
intercept	-6.9776	0.6367	-10.9584
$\tilde{\pi}$	39.9604	4.0734	9.8099
$\tilde{\pi}^2$	-71.0413	8.9518	-7.9359
$\tilde{\pi}^3$	48.1948	6.8177	7.0690
N	0.2664	0.03597	7.4056
N^2	-0.003548	0.000670	-5.2932
N^3	-0.000018	0.000004	4.5486
$\tilde{\pi}N$	-1.0551	0.1439	-7.3342
$\tilde{\pi}^2N$	0.008259	0.001247	6.6239
$\tilde{\pi}N^2$	1.1726	0.1456	7.5857

Table 44. Maximum Likelihood Parameter Estimates of the Polynomial $P(\tilde{\pi},N)$, Which Characterize the Cumulative Density Function for Conditional Marginal Profit (Winter Precipitation = 4 inches).

Variable	Parameter Estimate	Variance of Estimate	Asymptotic t-Ratio
intercept	-5.3895	0.5356	-10.0627
$\tilde{\pi}$	25.5209	2.8069	9.0923
$\tilde{\pi}^2$	-41.2963	5.3161	-7.7681
$\tilde{\pi}^3$	26.0294	3.5072	7.4216
N	0.1523	0.02669	5.7055
N^2	-0.001506	0.000457	-3.2977
N^3	0.0000059	0.0000026	2.2949
$\tilde{\pi}N$	-0.5120	0.08067	-6.3457
$\tilde{\pi}^2N$	0.003049	0.000577	5.2841
$\tilde{\pi}N^2$	0.5282	0.07188	7.3482

Table 45. Maximum Likelihood Parameter Estimates of the Polynomial $P(\tilde{\pi},N)$, Which Characterize the Cumulative Density Function for Conditional Marginal Profit (Winter Precipitation = 6 inches).

Variable	Parameter Estimate	Variance of Estimate	Asymptotic t-Ratio
intercept	-5.1869	0.5227	-9.9229
$\tilde{\pi}$	21.0176	2.3559	8.9209
$\tilde{\pi}^2$	-30.1948	3.8971	-7.7479
$\tilde{\pi}^3$	17.1619	2.2530	7.6174
N	0.1245	0.02419	5.1484
N^2	-0.001058	0.000403	-2.6249
N^3	0.0000036	0.0000023	1.5837
$\tilde{\pi}N$	-0.3592	0.05996	-5.9904
$\tilde{\pi}^2N$	0.001824	0.000392	4.6531
$\tilde{\pi}N^2$	0.3239	0.04557	7.1084

Table 46. Maximum Likelihood Parameter Estimates of the Polynomial $P(\tilde{\pi},N)$, Which Characterize the Cumulative Density Function for Conditional Marginal Profit (Winter Precipitation = 8 inches).

Variable	Parameter Estimate	Variance of Estimate	Asymptotic t-Ratio
intercept	-5.3275	0.5455	-9.6107
$\tilde{\pi}$	18.9386	2.2101	8.5691
$\tilde{\pi}^2$	-21.7124	3.2769	-7.5415
$\tilde{\pi}^3$	12.8305	1.6940	7.5742
N	0.1124	0.02378	4.7256
N^2	-0.000858	0.000386	-2.2219
N^3	0.0000026	0.0000022	1.2093
$\tilde{\pi}N$	-0.2902	0.05175	-5.6076
$\tilde{\pi}^2N$	0.001323	0.000319	4.1441
$\tilde{\pi}N^2$	0.2346	3.4826	6.7358

Table 47. Maximum Likelihood Parameter Estimates of the Polynomial $P(\tilde{\pi},N)$, Which Characterize the Cumulative Density Function for Conditional Marginal Profit (Winter Precipitation = 10 inches).

Variable	Parameter Estimate	Variance of Estimate	Asymptotic t-Ratio
intercept	-5.6952	0.5772	-9.8663
$\tilde{\pi}$	19.9137	2.1835	9.1201
$\tilde{\pi}^2$	-25.3045	2.9948	-8.4494
$\tilde{\pi}^3$	12.3692	1.4307	8.6457
N	0.1184	0.02377	4.9812
N^2	-0.000882	0.000376	-2.3450
N^3	0.0000025	0.0000021	1.2042
$\tilde{\pi}N$	-0.2977	0.0482	-6.1995
$\tilde{\pi}^2N$	0.001281	0.000283	4.5247
$\tilde{\pi}N^2$	0.2250	0.02986	7.5355

Table 48. Maximum Likelihood Parameter Estimates of the Polynomial P($\tilde{\pi}$,N), Which Characterize the Cumulative Density Function for Conditional Marginal Profit (Winter Precipitation = 12 inches).

Variable	Parameter Estimate	Variance of Estimate	Asymptotic t-Ratio
intercept	-5.4921	0.5936	-9.2523
$\tilde{\pi}$	16.8776	2.0495	8.2350
$\tilde{\pi}^2$	-18.9849	2.5800	-7.3585
$\tilde{\pi}^3$	8.5234	1.1338	7.5179
N	0.1014	0.02381	4.2592
N^2	-0.000671	0.000373	-1.8003
N^3	0.0000017	0.0000021	0.8382
$\tilde{\pi}N$	-0.2241	0.04325	-5.1827
$\tilde{\pi}^2N$	0.000888	0.000250	3.5472
$\tilde{\pi}N^2$	0.1541	0.02435	6.3299