Some of the characteristics of steady and oscillatory blood flow

by Bharat Ochhavlal Shah

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Chemical Engineering
Montana State University

Abstract:
An estimate of the difference between the Poiseuillian viscosity and the apparent viscosity at a given shear rate was made for blood under steady flow conditions. Shear stress-shear rate (viscometric) data for blood under steady flow conditions were used along with some numerical methods to obtain the difference between these two viscosities. The difference is significant at low flow rates and high values of the apparent viscosity.

Experimental work was done on oscillatory flow of blood at different frequencies of oscillation in rigid circular tubes. An apparatus was built so that oscillatory pressure and flow could be measured. The apparatus was tested with Newtonian fluids, such as glycerol solution, water, saline and plasma because the theory of oscillatory flow for a Newtonian fluid is understood very well.

Different red cell suspensions such as red blood cells in plasma, red blood cells in Dextran solutions, red blood cells in albumin-saline and hardened red blood cells in 0.5% Dextran 40 solution were used for the experimental work. Flow in two rigid circular glass tubes 400 and 776 microns in diameter was investigated. The experimental suspension hematocrit was usually about 45%, although 36% hematocrit was also used.

Experimental results showed that all pressure-time curves were sinusoidal for frequencies of oscillation of 0.5 hertz through 3 hertz. (The flow-time curves were always sinusoidal because of the inherent nature of the apparatus.) The pressure-flow data are summarized in this thesis and they are explained in terms of the known properties of blood and other red cell suspensions. These properties are the rheological data of blood, the visco-elasticity of blood, the aggregation of red blood cells and the deformation of red cells.

Red blood cells in high molecular weight Dextran solutions aggregate more strongly than they do in plasma. Hence, suspensions consisting of red blood cells in such Dextran solutions need higher pressure gradients to maintain the same flow compared to pressure gradients needed by red cells in plasma. From the experimental work with red cells in saline solution and hardened red cells in saline, it was concluded that red blood cell aggregation is significant process in oscillatory blood flow when the tube diameter is 400 microns.

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Approved:

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ABSTRACT

An estimate of the difference between the Poiseuillian viscosity and the apparent viscosity at a given shear rate was made for blood under steady flow conditions. Shear stress-shear rate (viscometric) data for blood under steady flow conditions were used along with some numerical methods to obtain the difference between these two viscosities. The difference is significant at low flow rates and high values of the apparent viscosity.

Experimental work was done on oscillatory flow of blood at different frequencies of oscillation in rigid circular tubes. An apparatus was built so that oscillatory pressure and flow could be measured. The apparatus was tested with Newtonian fluids, such as glycerol solution, water, saline and plasma because the theory of oscillatory flow for a Newtonian fluid is understood very well.

Different red cell suspensions such as red blood cells in plasma, red blood cells in Dextran solutions, red blood cells in albumin-saline and hardened red blood cells in 0.5% Dextran 40 solution were used for the experimental work. Flow in two rigid circular glass tubes 400 and 776 microns in diameter was investigated. The experimental suspension hematocrit was usually about 45%, although 36% hematocrit was also used.

Experimental results showed that all pressure-time curves were sinusoidal for frequencies of oscillation of 0.5 hertz through 3 hertz. (The flow-time curves were always sinusoidal because of the inherent nature of the apparatus.) The pressure-flow data are summarized in this thesis and they are explained in terms of the known properties of blood and other red cell suspensions. These properties are the rheological data of blood, the visco-elasticity of blood, the aggregation of red blood cells and the deformation of red cells.

Red blood cells in high molecular weight Dextran solutions aggregate more strongly than they do in plasma. Hence, suspensions consisting of red blood cells in such Dextran solutions need higher pressure gradients to maintain the same flow compared to pressure gradients needed by red cells in plasma. From the experimental work with red cells in saline solution and hardened red cells in saline, it was concluded that red blood cell aggregation is significant process in oscillatory blood flow when the tube diameter is 400 microns.

INTRODUCTION

This study is an attempt to work towards the solutions of some of the problems involved in blood flow. The entire thesis is divided into two sections. Section A deals with steady flow and Section B deals with oscillatory flow. When the pressure and the flow are independent of time, the flow is termed as steady flow. When the pressure and the flow are some periodic functions of time, the flow is termed as oscillatory flow. The work shown in this thesis deals with such flows in rigid circular tubes.

General Discussion

The basic function of the human circulatory system is to provide nourishment to various parts of the body and to remove waste materials. Blood, the fluid filling the circulatory system, is a suspension of various types of non-spherical, deformable particles (red blood cells, white cells, platelets) in an aqueous solution (plasma). While the suspending medium (plasma) has Newtonian rheological properties, the suspension is a non-Newtonian fluid (5). The parameter which has the greatest influence on the suspension flow properties is the hematocrit which is the volume of red blood cells per unit volume of whole blood.

$$\text{Hematocrit} = \frac{\text{Volume of red cells}}{\text{Volume of whole blood}} \times 100$$

Normally, the hematocrit is about 42-46%, but it can vary considerably from normal values in pathological situations.

In the human circulation system, blood is pumped from the heart through arteries, arterioles, capillaries, venules and veins, which return the blood to the heart. The microcirculation consists of the arterioles, capillaries and venules. The vessels are interconnected, forming networks (5,19). The size of the vessels through which blood flows vary from about 2-10 microns to 25,000 microns (5,19). The pressure drop across the human circulation is about 100 mm of Hg but about 80% of it exists across arterioles and capillaries. In-vivo experiments have shown that there are substantial regions of flow oscillations throughout the microcirculation. Hence oscillatory flow exists in the entire circulatory system.

In-vitro, steady flow experiments have been used to determine the rheological properties of blood. It is found that under low shear rates, blood acts like a Casson fluid (which has a yield stress, and a shear-dependent viscosity) (3). At low shear rates, red blood cells tend to reversibly aggregate. The aggregates are formed only by joining together the faces of the red cells. The length of the aggregates (rouleaux) varies inversely with the shear rate (5). This kind of phenomenon can occur in the living circulatory system and also in the experimental work. This phenomenon is reversible (5). At high shear rates red blood cells are dispersed and tend to deform. Red blood cell aggregation at low shear rates and red blood cell deformation at high shear rates may exert their rheological effects through a common

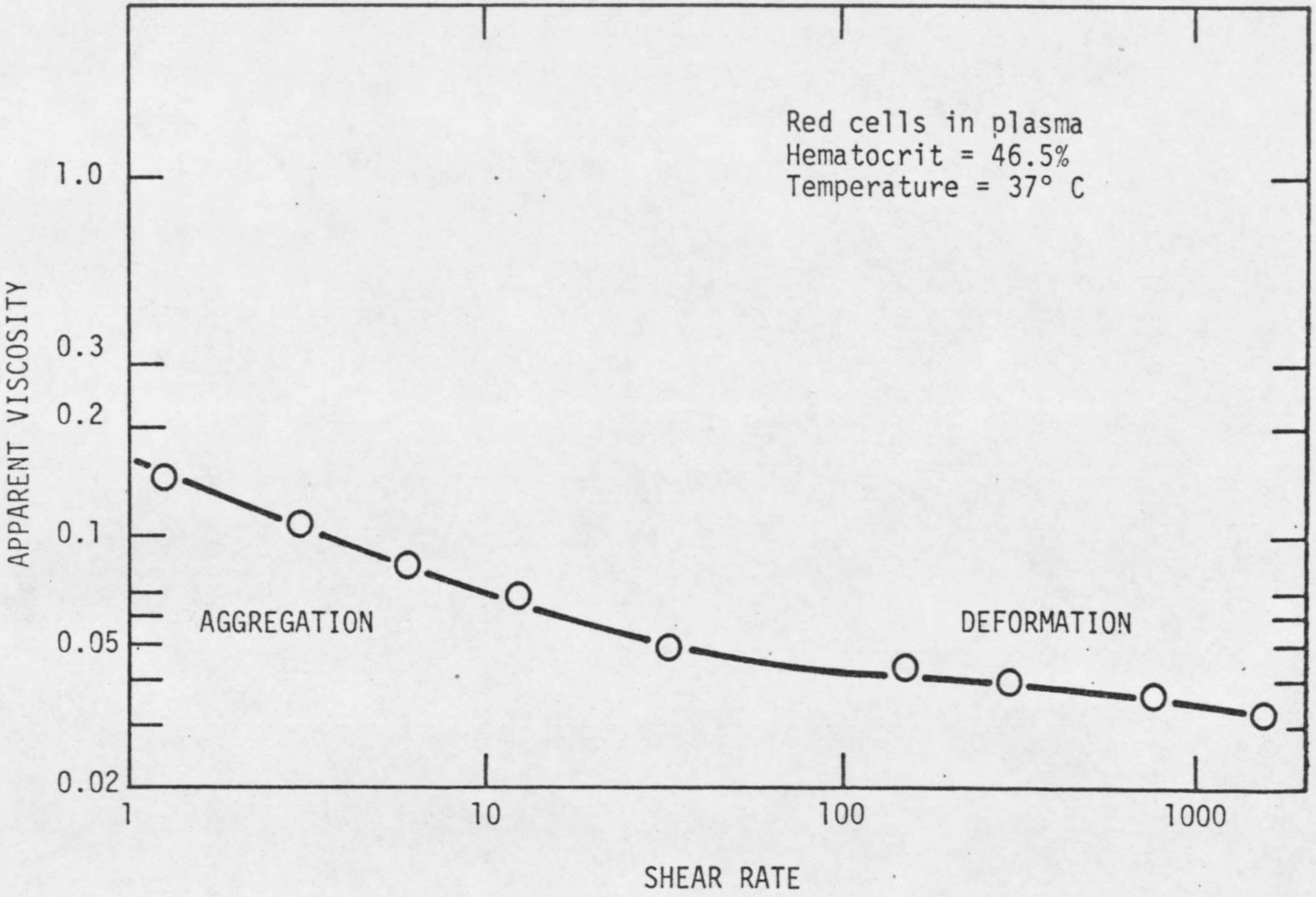


FIGURE I-1. APPARENT VISCOSITY VERSUS SHEAR RATE mechanism, shear-dependent changes in the effective cell volume. The effective cell volume is the fundamental determinant of blood viscosity (7). Apparent viscosity of blood is thus a function of shear rate and Figure I-1 shows the relation between apparent viscosity and shear rate.

Some in-vitro, oscillatory flow studies have been performed, mostly in large diameter tubes and oscillatory viscometers. The results from these studies have not been related to flow mechanisms of blood, and the interpretation of the data is often clouded by prior assumptions about the flow nature of the blood (e.g., the flow response component in phase with the flow driving function is assumed to be purely viscous while the component 90° out of phase with the driving function is assumed to be purely elastic).

Steady flow is discussed in Chapters II through VI. Oscillatory flow is discussed in Chapters VII through XII.

SECTION A

STEADY CONDITIONS

II. STATEMENT OF THE PROBLEM

This section deals with steady flow. The rheological data (the relationship between the shear stress and the shear rate) can be obtained for blood at different hematocrits using viscometers such as the concentric cylinder viscometer and the cone and plate viscometer. Such data can be used to obtain the relationship between the bulk average velocity and the pressure drop for steady flow of blood in rigid circular tubes.

In reporting the results of blood flow studies, the data are sometimes given in terms of the viscosity calculated from the Poiseuillian equation for steady, uniform laminar flow of a Newtonian fluid through a tube. The viscosity calculated by this equation is called the Poiseuillian viscosity.

$$N_p = \frac{R^2}{8U}\left(-\frac{dp}{dx}\right)$$

Where N_p = the Poiseuillian Viscosity

R = outside radius of tube

U = bulk average velocity

$\frac{dp}{dx}$ = pressure gradient per unit length of tube

This equation is valid only for Newtonian fluids and consequently its use with blood flow data is invalid, especially at lower flow rates.

This Poiseuillian Viscosity is sometimes compared to the apparent viscosity obtained from viscometric data. This comparison will not be valid, even for practical questions, if the blood flow is low enough so that the blood is very non-Newtonian. However, no measure of error due to the use of the Poiseuillian Viscosity has been reported, and the purpose of this work was to provide values of this error under various steady flow conditions. The difference between the Poiseuillian Viscosity and the apparent viscosity, for a given steady blood flow, reflects the magnitude of the non-Newtonian behavior of blood.

III. REVIEW OF PREVIOUS WORK

Poiseuille may have done the first work on blood (5). He himself found that complex fluids such as blood do not necessarily obey the Poiseuillian equation which is discussed on P. 6. Other workers also found that deviations exist between the pressure drop calculated from the Poiseuillian equation and that measured by the experimental work on blood. Fahraeus and Lindquist performed some of the classical studies of blood flow in tubes. Scott-Blair and Merrill also worked with blood using capillary viscometers. The details of such individual works are not summarized in this thesis. Fung (12) gives a review of the work done through 1969 (5).

Work published in recent years by Agarwal (1) and Lipowsky (15) shows that they used the Poiseuillian equation for calculating the apparent viscosity of blood. Lipowsky (15) indicates that the velocity of blood, the blood vessel size and the pressure drop across the blood vessel were measured and the Poiseuillian equation was used to calculate the apparent viscosity. Agarwal (1) reports the relative viscosity of blood at varying hematocrits in pulmonary circulation. He used the Poiseuillian equation in his work. Additional examples of such usage of the Poiseuillian equation can also be found in literature.

An attempt has been made here to show the magnitude of the error between viscosity calculated from the Poiseuillian equation and the apparent viscosity under the same flow conditions.

IV. THEORY AND CALCULATIONAL PROCEDURE

For a Newtonian fluid, the relationship between the pressure and the flow can be derived from the law of Conservation of Momentum for steady, laminar flow through a circular tube. The relationship obtained can be expressed by the Poiseuillian equation. This is discussed on P. 6. The viscosity derived from such equation is termed the Poiseuillian viscosity. This section describes the calculational procedure for calculating the difference between the Poiseuillian viscosity and the apparent viscosity.

The relationship between the shear stress and the shear rate (the viscometric or rheological data) at three different blood hematocrits were obtained with the help of the concentric cylinder viscometer and the cone and plate viscometer. The data are plotted and shown in Figures V-1 through V-3. The details of the viscometric apparatus and the procedures are not described here. The data for a given blood hematocrit were divided into several shear rate ranges and an analytical expression for each range was obtained. Either the linear or the non-linear regression analysis was generally used in this process. The data are represented as

$$G = f(T)$$

where G = shear rate

T = shear stress

or $$-\frac{du}{dr} = f(T) \qquad (1)$$

where u = velocity of fluid at radius r

Table V-2 summarizes the analytical expressions for different ranges at different blood hematocrits.

The relationship between the shear stress and the pressure drop per unit length of the tube for steady, uniform laminar flow of any fluid through a circular tube can be expressed as

$$T = -\frac{dp}{dx}\ \frac{r}{2} \tag{2}$$

where $\frac{dp}{dx}$ = pressure gradient per unit length of tube

r = radial coordinate

x = longitudinal coordinate

This equation is derived by simplification of the momentum equations for such flow situation.

The bulk average velocity U is given by

$$U = \frac{\text{volumetric flow rate}}{\text{cross-sectional area}}$$

or $$U = \frac{\int_0^R 2\pi ur\ dr}{\int_0^R 2\pi r\ dr}$$

or $$U = \frac{2}{R^2} \int_0^R ur\ dr$$

where R = outside radius of tube

Integrating by parts and using the boundary conditions,

$u = 0$ at $r = R$

$u = u_{max}$ at $r = 0$

$$U = \frac{1}{R^2} \int_0^R \left(-\frac{du}{dr}\right) r^2 \, dr \qquad (3)$$

Using r as an independent variable, the equations (1), (2) and (3) were solved simultaneously by numerical methods and the relationship between U and the pressure drop was obtained for different blood hematocrits. The computer program of the numerical method is shown in Appendix A.

The Poiseuillian equation was used to calculate the Poiseuillian Viscosity,

$$N_p = \frac{R^2}{8U}\left(-\frac{dp}{dx}\right)$$

where N_p = Poiseuillian Viscosity

Apparent viscosity was simultaneously calculated by the definition,

$$N_a = \frac{T}{G}$$

N_p and N_a were calculated at a given wall shear stress and the difference between the two was calculated simultaneously by computer.

It may be remarked that for a Newtonion fluid,

$$-\frac{du}{dr} = \frac{1}{\mu} T$$

where μ = viscosity, a constant.

Using equations (2) and (3), it can be shown that

$$-\frac{dp}{dx} = \frac{8\mu U}{R^2}$$

The difference between N_p and N_a is plotted against N_a and the graph is shown in Figures V-4 and V-5. $\overline{U}$ and the hematocrit are the parameters on the graph.

$$\overline{U} = \frac{U}{2R}$$

where $\overline{U}$ = reduced velocity

V. RESULTS AND DISCUSSION

Figures V-1 through V-3 show the viscometric (rheological) data. Table V-2 summarizes the fitting of the rheological data into the analytical expressions. Figures V-4 and V-5 show the results of the calculations. It is seen that the difference between the Poiseuillian Viscosity and the apparent viscosity is significant at low bulk average velocity and at high values of hematocrits. When the apparent viscosity is about 6 centipoise and hematocrit is about 42% and reduced velocity is about 1 sec^{-1}, the difference between the Poiseuillian Viscosity and the apparent viscosity is about 0.5 centipoise (which is about 8% of the actual value of the apparent viscosity).

Table V-1 shows the comparison of results obtained from literature (15) and those obtained by this work. It should be noted that results obtained by Lipowsky (15) represent in-vivo experimental data and results shown in this thesis represent in-vitro experimental data. In Table V-1, the tube hematocrit is estimated from the results of Barbee (5). These tube hematocrits are not the experimental results of Lipowsky (15). The last two columns show the corresponding results from the graphs shown in Figures V-4 and V-5. Although the error between the Poiseuillian Viscosity and the apparent viscosity is not very significant, the estimated apparent viscosity is much lower than that obtained by Lipowsky (15).

Some additional examples of the use of the Poiseuillian

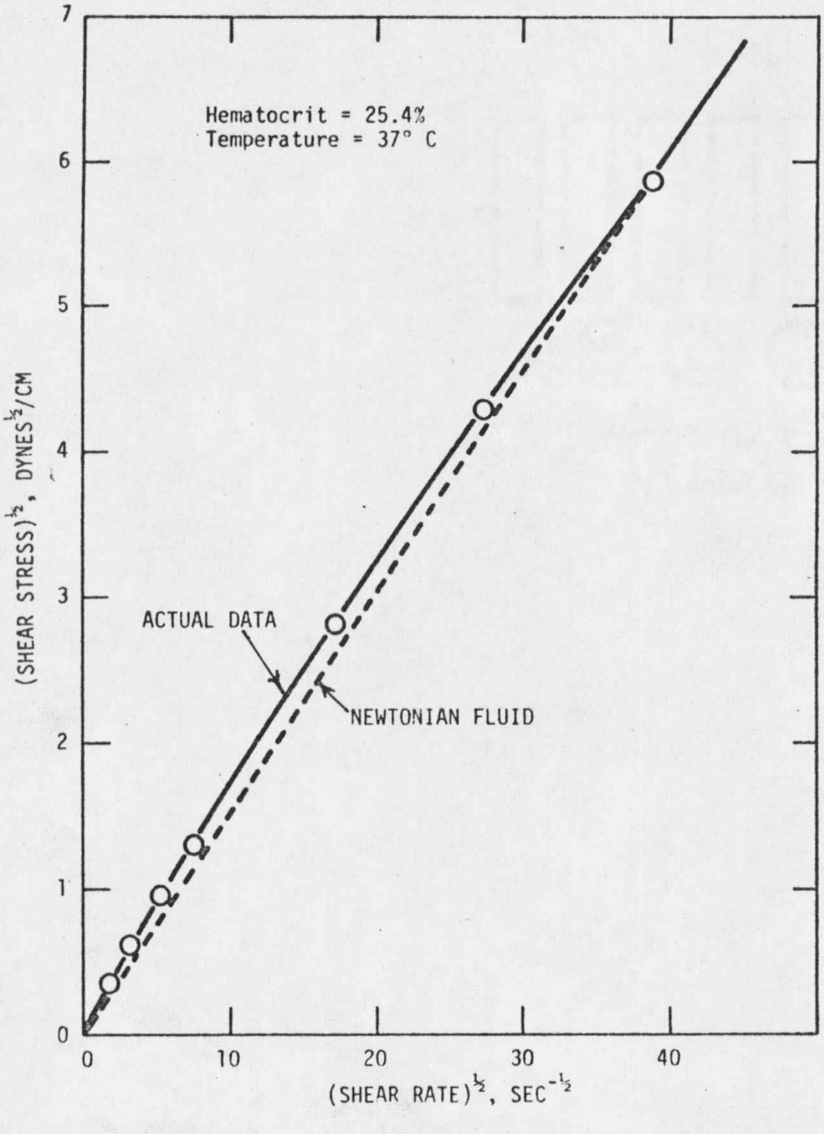


FIGURE V-1. SHEAR STRESS - SHEAR RATE DATA

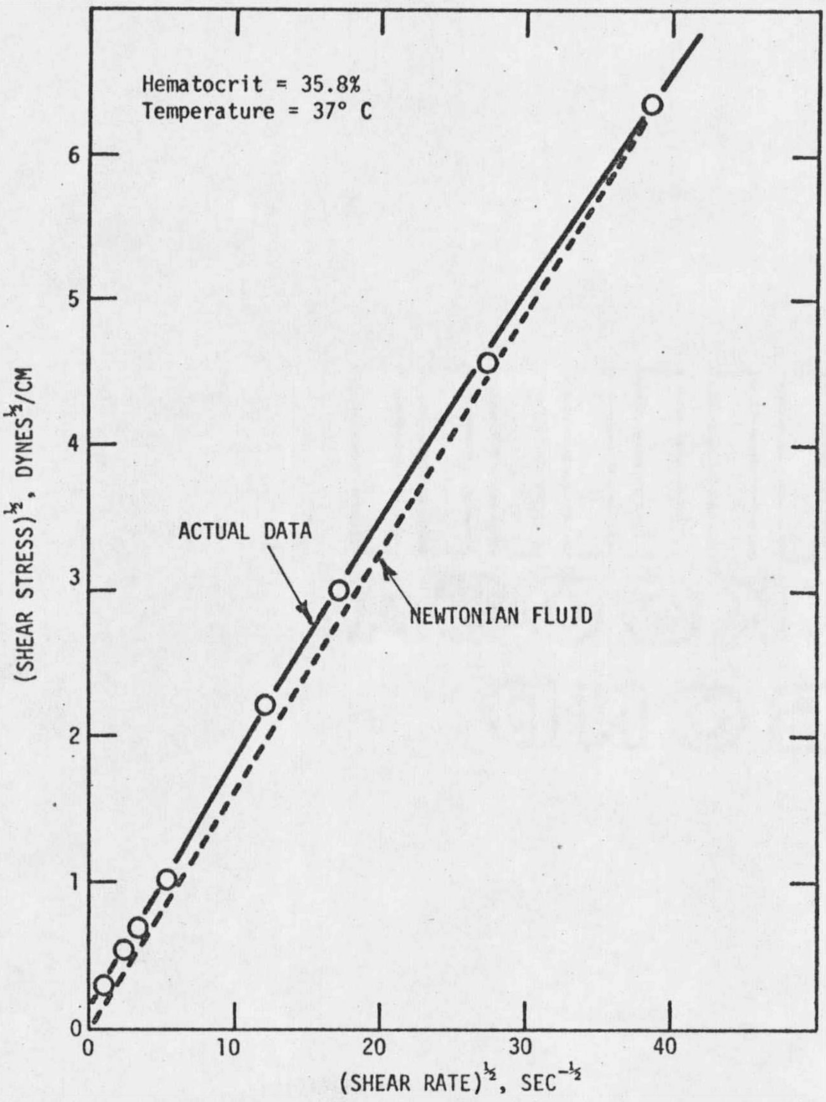


FIGURE V-2. SHEAR STRESS - SHEAR RATE DATA

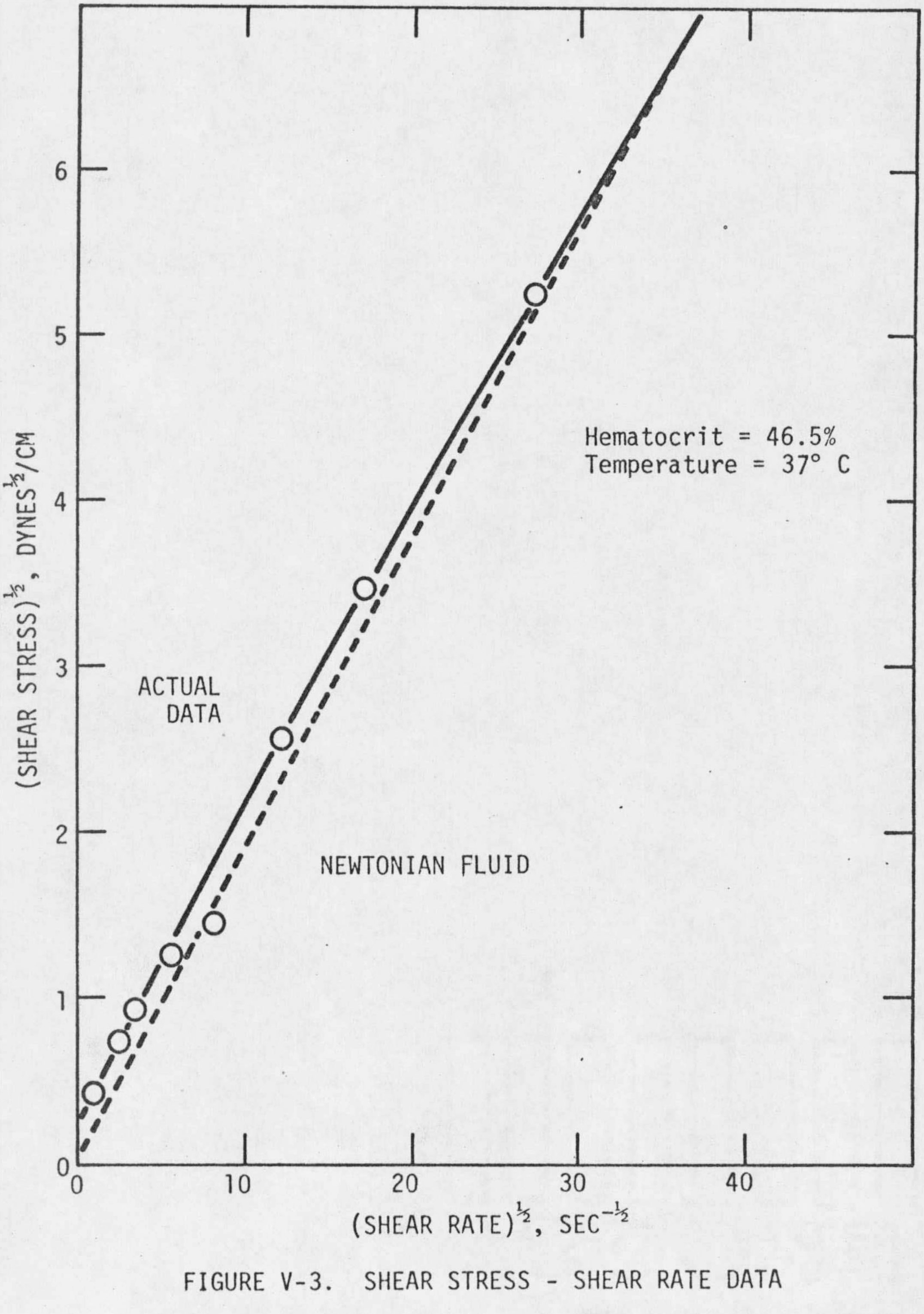


FIGURE V-3. SHEAR STRESS - SHEAR RATE DATA

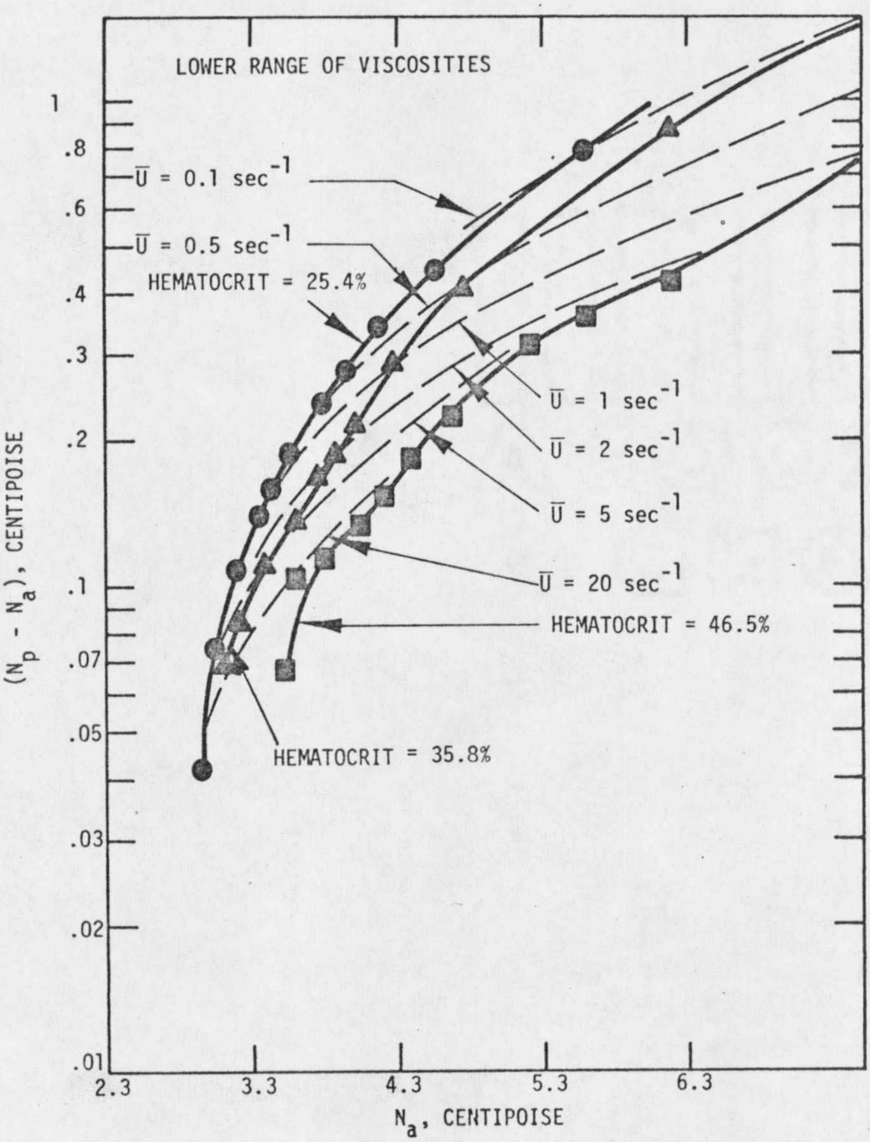


FIGURE V-4. DIFFERENCE BETWEEN N_p AND N_a

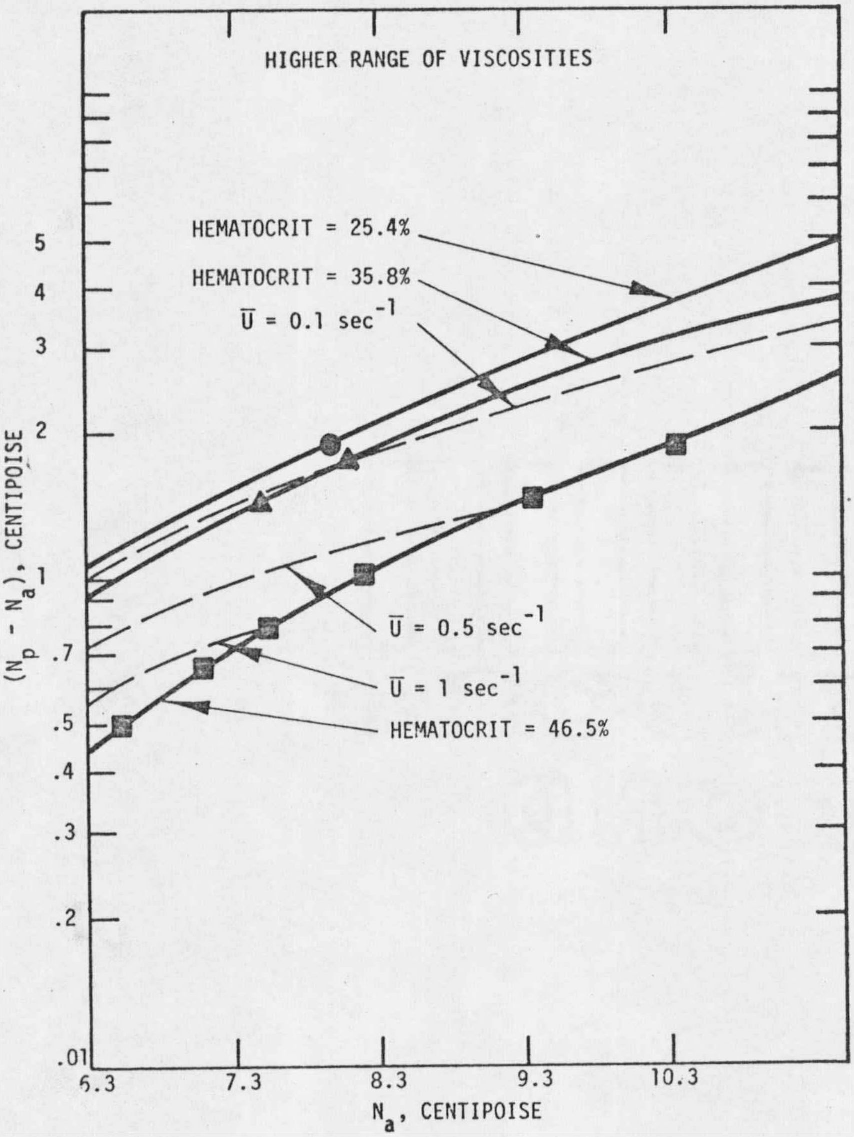


FIGURE V-5. DIFFERENCE BETWEEN N_p AND N_a

TABLE V-1

COMPARISON OF RESULTS OBTAINED FROM LITERATURE (15) AND FROM THIS WORK

Vessel Size (Avg.) (15) microns	Estimated Tube Hematocrit percentage	Reduced Velocity $\overline{U}$ (15) sec^{-1}	Apparent Viscosity (15) centipoise	Estimated Apparent Viscosity centipoise	Estimated $(N_p - N_a)$ Difference centipoise
33	28	10-20	9-4	3	0.1
38	30	5-20	20-8	3.3	0.1
43	31	2-4 9-20	25-15 30-10	3.6 3.3	0.15 0.1
48	32	6-20	20-7	3.3	0.1
58	34	15	25	3.3	0.1
39	30	8-20	25-7	3.3	0.1
50	32.5	9-20	11-7	3.3	0.1

TABLE V-2

FITTING OF RHEOLOGICAL DATA INTO ANALYTICAL EXPRESSIONS

Temperature 37° C			
	Hematocrit	Range	Analytical Expression
(1)	25.4	$0 < T \leq 0.004683$	$G = 0$
		$0.004683 < T \leq 1.0$	$G = (6.238T^{.5} - .4268)^2$
		$1.0 < T \leq 33.06$	$G = 33.005T^{1.07332}$
		$T > 33.06$	$G = 43.29T$
(2)	35.8	$0 < T \leq 0.01302$	$G = 0$
		$0.01302 < T \leq 1.04$	$G = (5.944T^{.05} - .678)^2$
		$1.04 < T \leq 37.2$	$G = (-.7027 + 5.944T^{.5} + .0463T)^2$
		$T > 37.2$	$G = 37.174T$
(3)	46.5	$0 < T \leq .04285$	$G = 0$
		$0.04285 < T \leq 1.42$	$G = (4.93105T^{.5} - 1.02074T)^2$
		$1.42 < T \leq 42.25$	$G = (-1.3727 + 5.1369T^{.5} + .065T)^2$
		$T \geq 42.25$	$G = 28.4091T$

Viscosity can be found in the literature and the errors are significant at low values of reduced velocities and hematocrits.

VI. CONCLUSIONS AND RECOMMENDATIONS

1. The difference between the Poiseuillian Viscosity and the apparent viscosity is significant at low values of reduced velocity (the ratio of bulk average velocity and tube diameter) and at low values of hematocrits.

2. In-vitro experimental results are shown in this thesis. If the actual experimental work is in-vivo, the results may differ, for reasons which are still unknown.

3. If the actual experimental work is in-vitro, the use of the Poiseuillian equation should be made only at high reduced velocities such as 30 sec^{-1} or more.

4. Additional examples of usage of the Poiseuillian Viscosity may be found from literature survey and the errors can be estimated.

SECTION B

OSCILLATORY CONDITIONS

VII. STATEMENT OF THE PROBLEMS

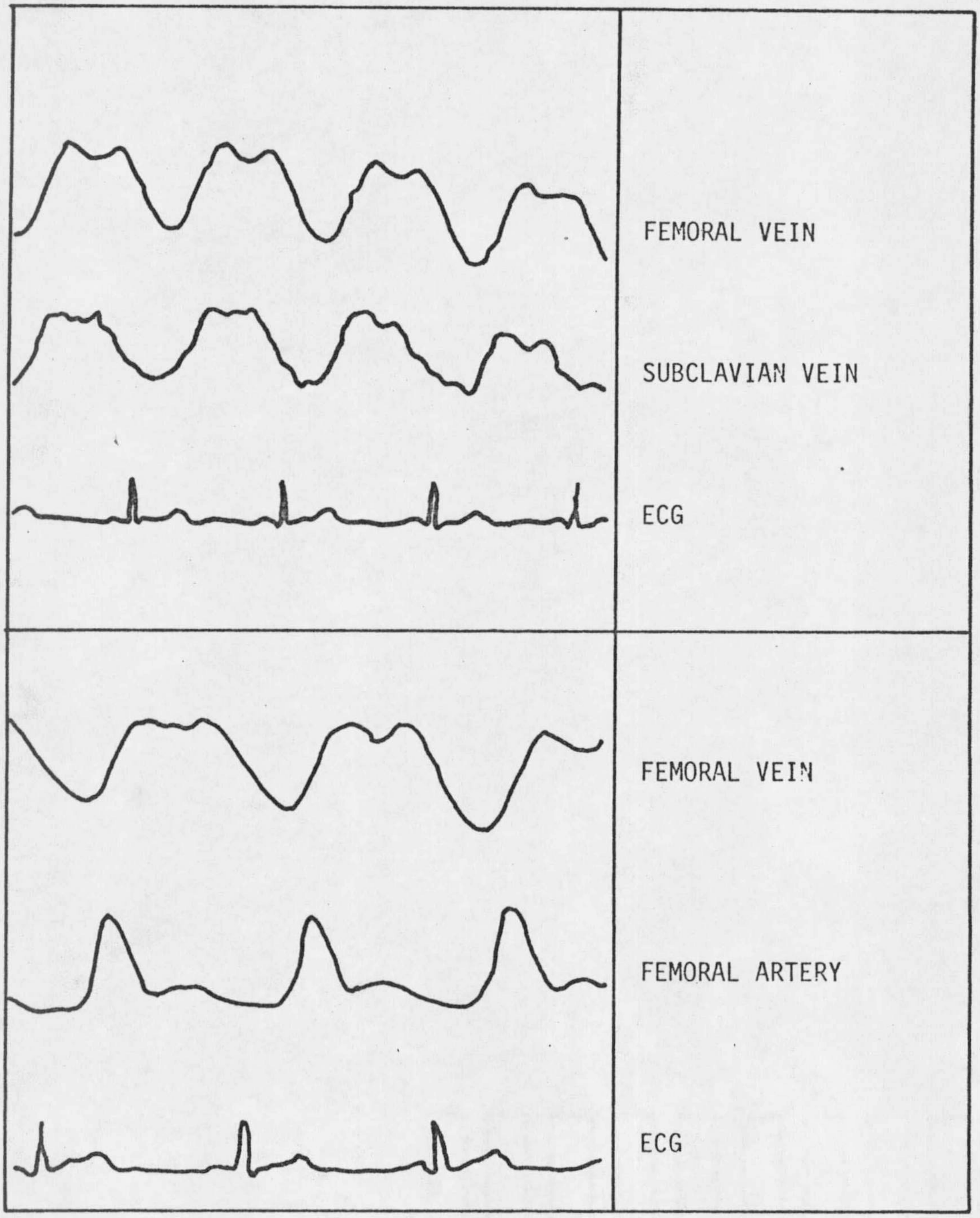
This section deals with oscillatory flow of blood. Figure VII-1 gives some idea of the nature of oscillatory flow of blood in the human circulatory system (18). The rate of heart beats usually varies from about 60-70 beats per minute at rest to about 160-170 beats per minute at peak cardiac output. Thus the useful range of frequencies of oscillation is about 1 to 3 cycles per second. Table VII-1 summarizes some typical values of mean pressure drop per unit length of the vessel at various locations (25,26).

Experimental work was done to understand some of the characteristics of oscillatory blood flow in rigid, circular glass tubes. These characteristics are the pressure gradient-flow relationship under oscillatory conditions, the values of the hematocrit of blood, the possible effects of deformation and aggregation of red blood cells, etc.

It may also be possible to calculate the pressure gradient-flow relation for blood under oscillatory conditions using the momentum equations (equations of motion) and the rheological data (obtained from steady flows in viscometers). But such data may prove to be inadequate for such calculations, because the characteristic times for red cell aggregation and red cell deformation are comparable to the times of flow oscillation. In addition, viscoelastic effects may be important.

Hence this section involves:

(i) the experimental work done on oscillatory flow.



SIMULTANEOUS RECORDINGS OF FLOW VELOCITIES

FIGURE VII-1. NATURE OF OSCILLATORY BLOOD FLOW

Table VII-1

VALUES OF MEAN PRESSURE DROP AT VARIOUS LOCATIONS (25,26)

Type of Blood Vessel	Dimensions: Diameter microns	Dimensions: Length mm	Pressure Drop Per Unit Length mm Hg/mm
Large Artery	10,000	600	-0.0183
Terminal Artery	1,600	110	-0.173
Arterioles	40	2	-15.0
Capillaries	16	1	-15.0
Venules	60	2	-4.5
Main Vein	4,000	100	-0.05
Large Vein	20,000	600	-0.00834
Descending Aorta	16,000-20,000	--	-7.01×10^{-4}
Venae Cavae	20,000	--	-3.16×10^{-4}

(ii) the discussion of the calculational procedure for pressure-flow relationship under oscillatory conditions using the steady viscometric data.

VIII. REVIEW OF PREVIOUS WORK

The literature on pulsatile blood flow has been described by several authors, such as McDonald (16), Bergel (6) and Attinger (2). Womersley (27) derived the theory of oscillatory flow for a Newtonian fluid in a circular tube. Some of the results of his theory are summarized on P. 36.

Tables VIII-1 and VIII-2 summarize some of the details of the experimental work done by Sevilla-Larrea (19), Singh (20) and Thurston (22,23). These tables summarize the tube diameters, the frequencies of oscillation, the flow amplitudes, hematocrits, the pressure amplitudes, etc., used by these workers.

Sevilla-Larrea (19) studied pulsatile flow of blood using high speed microcinematography. He measured the pressure with the help of pressure transducer but for the measurement of flow, he measured the red blood cell velocity profiles. He did not measure the flow with the help of a displacement transducer or some other flow measurement device. It also seems that the apparatus was not tested with a Newtonian fluid, i.e., the apparatus should have been checked to show that for such fluids the experimental and theoretical results agree with each other.

Singh (20) used an electromagnetic flow meter to measure oscillatory flow. His pressure measuring method was designed so that he could eliminate the entrance effects of the tube. He had two pressure taps in his tubes (1660μ, 3240μ). These taps were placed sufficiently far from the ends of the tube so that end effects were negligible. His

TABLE VIII-1

SOME OF THE DETAILS ABOUT THE PREVIOUS WORK DONE ON OSCILLATORY BLOOD FLOW

Worker	Capillary-Tube Diameter cm	Length of Tube cm	Number of Tubes in Parallel N	Hematocrit percentage	Temperature °C
Sevilla-Larrea (19)	0.004	6.5	1	5,10 20,40	25
	0.007	10.0	1	5,10 20,40	25
Singh (20)	0.0608	7.5	1	40	23
	0.166	70.0	1	40	23
	0.324	100.0	1	40	23
Thurston (22)	0.086	11.14	6	46	21.5
Thurston (23)	0.043	0.654	50	46	24
	0.0610	0.984	50	46	24
	0.0788	1.236	32	46	24
	0.094	1.401	12	46	24
	0.132	1.993	8	46	24
	0.1918	2.86	5	46	24
	0.398	5.86	2	46	24
	0.70	13.16	1	46	24

TABLE VIII-2

SOME OTHER DETAILS ABOUT THE PREVIOUS WORK DONE ON OSCILLATORY BLOOD FLOW

Worker	Frequency of Oscillation c.p.s.	Some Other Details			
		Diameter of Tube cm	Flow Amplitude cm^3/sec	Pressure Drop Amplitude $dynes/cm^3$	Nature of Flow
Sevilla-Larrea (19)	0.0 3.6 8.4	0.004	6×10^{-6} to 20×10^{-6}	8×10^{3} to 79×10^{3}	Combination of steady and oscillatory or pulsatile
		0.007	19×10^{-6} to 60×10^{-6}	4×10^{3} to 36×10^{3}	
Singh (20)	0.6 through 10	0.0608	0.08	10^{5} to 1.64×10^{5}	Purely oscillatory
		0.166	0.8	2200 to 4720	
		0.324	1.6	360 to 1800	
Thurston (22)	2	0.086	$\sim 10^{-5}$ to 10^{-3}	1 to 2×10^{3}	Steady, oscillatory and pulsatile
Thurston (23)	0.2 through 200	0.043 to 0.70	10^{-6} to 10^{-1}	10^{-1} to 100	Purely oscillatory

apparatus was tested for a Newtonian fluid. The ratio of pressure amplitude to flow amplitude is impedence. To analyze his experimental results for blood, he first plotted the impedence versus frequency for Newtonian fluids with different viscosities. The experimental impedence for blood at a given frequency was measured and then plotted on the same graph for different frequencies of oscillation. At a given frequency, the viscosity of a Newtonian fluid, which gives an impedence equal to that of blood, was considered as the blood's "equivalent viscosity." The results derived with this equivalent viscosity are only good for blood at that frequency of oscillation. Blood is non-Newtonian and the predictions about blood from a Newtonian fluid theory may not be theoretically sound. Hence, the analysis of his experimental results needs further study.

Thurston (22,23) performed both the theoretical and experimental work on oscillatory flow of blood. He considered blood as a viscoelastic fluid. A viscous material is one which flows, when stress is applied. An elastic material is one which deforms, when stress is applied. Thurston considered blood to show characteristics of both the viscous and the elastic states. He derived the theory of oscillatory flow for a linear visco-elastic fluid in a rigid circular tube (23). The tube is assumed to be filled with a fluid of complex coefficient of viscosity

$$\eta^* = \eta' - i\eta''$$

η' = real part

η'' = imaginary part

Using the Navier-Stokes equations with a complex viscosity substituted for the usual viscous viscosity, he derived equations for the complex flow. A Newtonian fluid does not show any elastic effect, while a visco-elastic fluid shows both viscous and elastic effects and they are represented by the real and imaginary terms in this theory. But such division of viscosity into two parts is possible for a purely viscous fluid also.

His experimental work on oscillatory flow involves a velocity transducer to measure the oscillatory flow. But he did not have a stirrer in his apparatus so that the red blood cells may settle and the hematocrit of blood may not remain uniform. He also had a membrane in the feed reservoir which separated the blood from water. The pressure was measured in that section of feed reservoir which contained water. His experimental results show that there is a linear relationship between the pressure and the flow at lower shear rates (i.e., at lower flow rates). This observation is in contradiction with observations of many other workers (2,3,13), because blood is found to be non-Newtonian at low shear rates and there exists a finite yield stress. Hence at lower shear rates, the relationship between the pressure and the flow should be non-linear. For these and other reasons, Thurston's experimental work needs to be repeated.

Kline (13) considered blood as a polar fluid. In polar fluid theory, blood is considered as a collection of particles and stress-tensor analysis is used to understand the oscillatory flow of blood. But his work is purely theoretical and Cowin (10) argued that polar fluid theory does not adequately model blood flow in microcirculation. Also, it is observed that the hematocrit of blood changes with radial position in a tube, but polar fluid theory does not take into consideration such a variability.

It has been mentioned on P. 24 that oscillatory flow of blood might be predictable from steady flow shear stress-shear rate (viscometric) data. But such data may prove to be inadequate because of the visco-elasticity of blood and/or because the kinetics of red cell deformation are slow compared to the oscillatory time. Hence, some of the ideas regarding visco-elasticity and the red cell aggregation and deformation are discussed in this chapter.

Lessner and co-workers (14) studied the visco-elastic properties of blood using a visco-elastomer under oscillatory conditions. They determined the relaxation time spectrum for blood over a time range between 0.3 to 0.8 second. They concluded that red cell aggregation processes have characteristic time constants of 0.8 second or less. Thus, from the relaxation time spectrum, it seems that at lower frequencies of oscillation it may be possible to predict oscillatory flow behavior of blood from the steady viscometric data.

But, the red blood cells are also deformable. Work done by Evans (11) and Waugh (24) describes the deformation of red blood cells. They considered the red blood cell as a membrane envelope containing a low viscosity Newtonian fluid (an aqueous solution of hemoglobin). For the membrane, they formulated and experimentally determined four material constants. These constants were termed the shear modulus of elasticity, the viscosity (in the visco-elastic domain), the yield "shear," and a plastic viscosity. Below a yield condition, the membrane is reversibly deformable (visco-elastic), but above the yield condition it begins to flow in a viscous, plastic manner. The red blood cell has the shape of a bi-concave disk with a mean diameter of about eight microns and thickness of about one to two microns (5). It is flexible, because it is essentially a thin membrane container incompletely filled with a fluid. It is a living cell. It is capable of responding to osmotic pressure changes in the suspending media. Waugh (24) and Evans (11) found that the time to recover the deformation occurred with the red blood cells may be of the order of 0.3 seconds. Hence, again, at lower frequency of oscillation, oscillatory flow of blood can be studied from the steady viscometric data. But at higher frequency of oscillation, it may not be possible to do so.

Work done by several workers has been described briefly in this chapter. From this discussion, the present study was made considering the following aspects.

1. A magnetic stirrer is used so that the concentration of red blood cells (hematocrit) remains uniform (i.e., cells do not settle) in the apparatus reservoirs.

2. A possible inadequacy of the use of an electromagnetic flowmeter can be avoided. Such flowmeters may record the same results for two different oscillatory flows.

3. Pressures are measured by direct contact of the blood with pressure transducers.

4. The experimental data should be explained in terms of the known properties of blood.

IX. THEORETICAL BACKGROUND

The theory of oscillatory flow for a Newtonian fluid in a circular tube was studied by Womersley (27). His results may be summarized as follows. When the pressure drop is given by

$$-\frac{dp}{dx} = P_m \cos(\omega t)$$

where P_m = amplitude of pressure drop

ω = angular frequency

$= 2\pi f$

f = linear frequency

t = instantaneous time

p = pressure

x = axial distance

the flow is given by

$$Q = \frac{\pi R^4}{\mu} P_m \frac{M_{10}^{1}}{\alpha^2} \sin(\omega t + \phi)$$

where R = radius of the tube

μ = viscosity of fluid

The quantities $\frac{M_{10}^{1}}{\alpha^2}$ and ϕ have been derived as a function of α, which is given by

$$\alpha = \left(\frac{R^2 \omega \rho}{\mu}\right)^{0.5}$$

= frequency parameter

where ρ = density of fluid

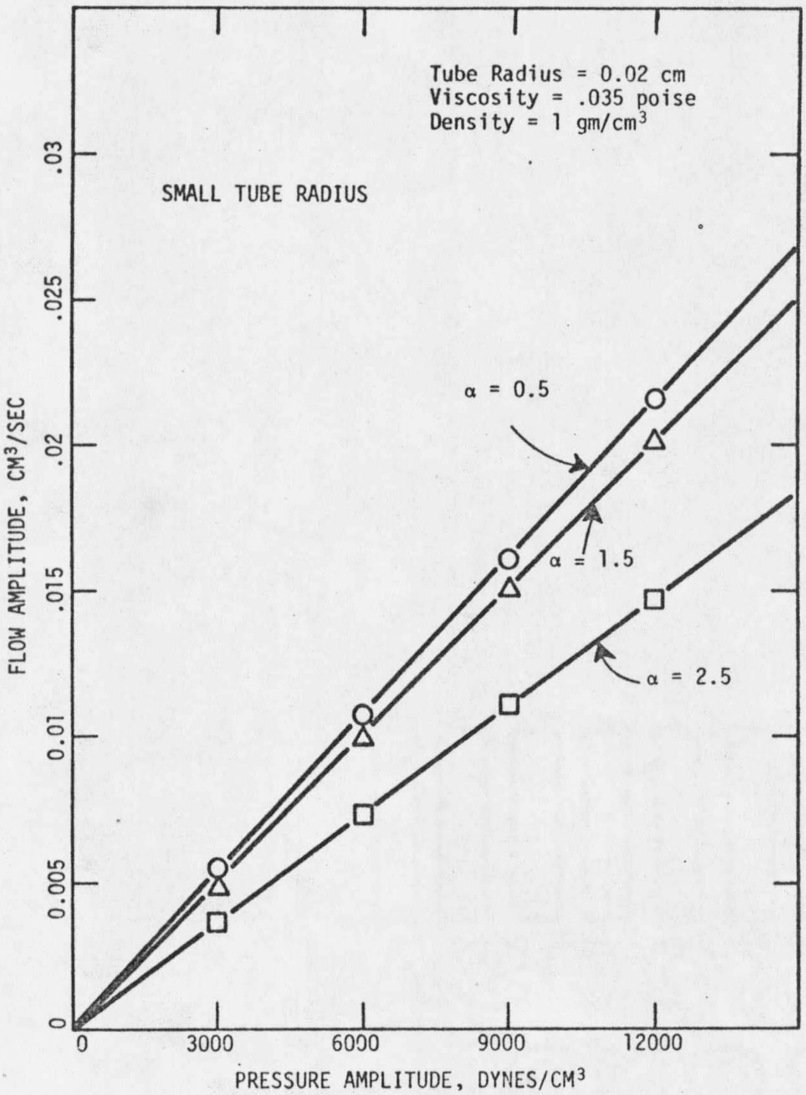


FIGURE IX-1. FLOW AMPLITUDE VERSUS PRESSURE AMPLITUDE FROM THEORY (Points shown in this figure are calculated from theory.)

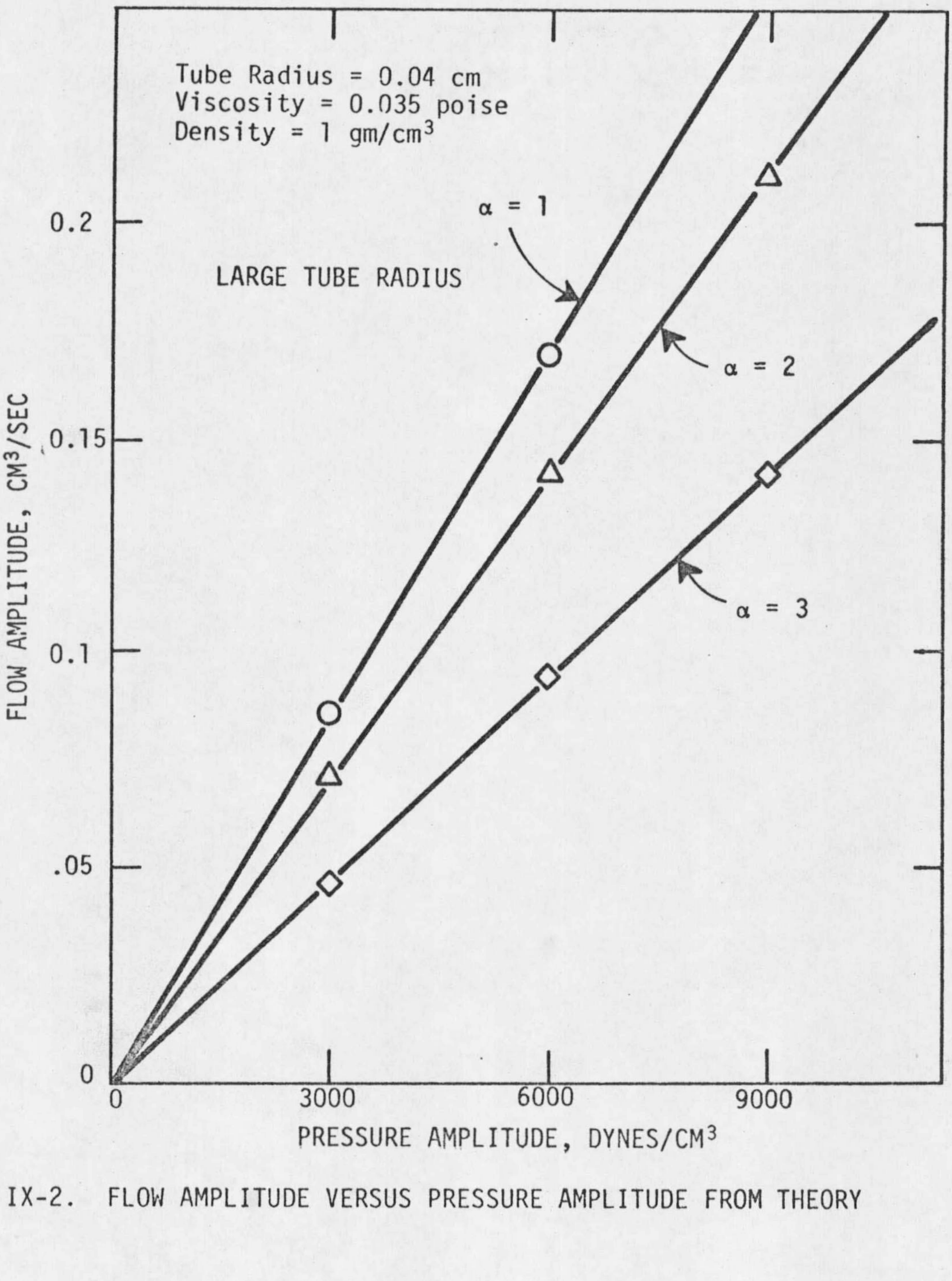


FIGURE IX-2. FLOW AMPLITUDE VERSUS PRESSURE AMPLITUDE FROM THEORY

The quantity M_{10}^{1} represents modulus of the sinusoidal oscillation of volume flow. The quantity ϕ represents the phase difference between pressure wave and flow wave. Both $\frac{M_{10}^{1}}{\alpha^2}$ and ϕ can be obtained as a function of α from a table which was constructed by Womersley (27). Figures IX-1 and IX-2 give some idea about how the flow amplitude varies with the pressure amplitude for different values of α, according to this theory.

$$\text{Flow amplitude} = \frac{\pi R^4}{\mu} P_m \frac{M_{10}^{1}}{\alpha^2}$$

$$= Q_m$$

$$\text{Pressure amplitude} = P_m$$

X. APPARATUS AND PROCEDURES

Experimental Work

A. Apparatus

The schematic diagram of the apparatus is shown in Figure X-1. The apparatus is described by considering one part after another.

Scotch-Yoke. The sinusoidal signal is generated with the help of a scotch-yoke. Figure X-2 and Figure X-3 explain the principle on which it operates. Basically it converts the circular motion of a shaft (which is driven by an electric motor) into the reciprocating (oscillating) motion of a piston, or, in other words, simple harmonic motion is generated. As shown in Figure X-2, when a particle rotates in a circulatory motion and its motion is projected on the diameter of the circulatory path, the projected motion on its diameter is called simple harmonic motion. This same analogy can be exactly applied to the scotch-yoke. As shown in Figure X-3, a pin (particle) is fitted in a groove which is drilled in part A. As part A rotates, the part B oscillates in the horizontal direction in between the guides (part C). The oscillatory motion of part B is imparted to the plunger of a syringe which, in turn, imparts the oscillatory motion to the fluid in the capillary tube. Thus, the sinusoidal signal is generated.

The scotch-yoke is driven by a variable speed electric motor, through a gear box. The motor speed is monitored by an electronic tachometer and is regulated by a feed-back circuit.

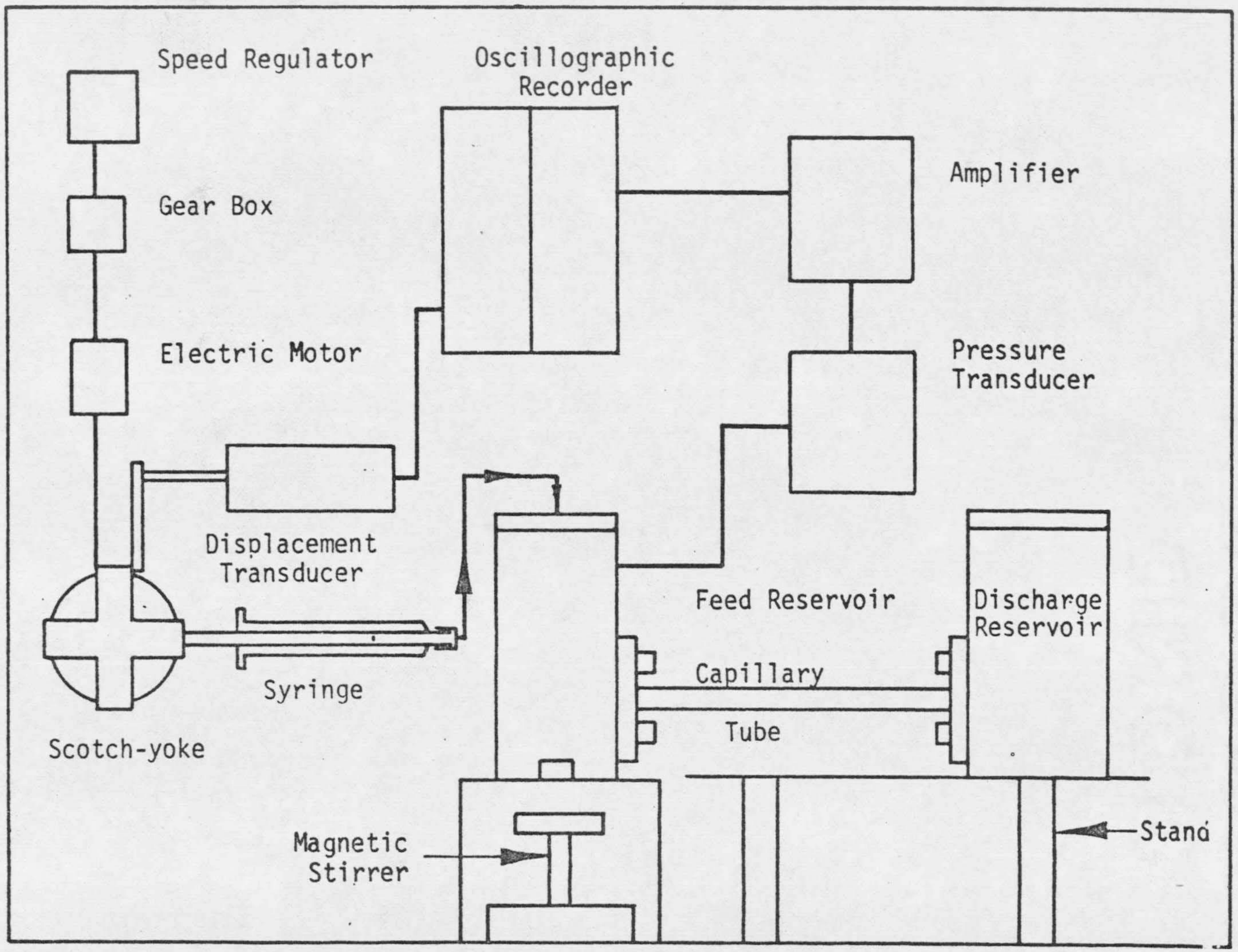


FIGURE X-1. SCHEMATIC DIAGRAM OF APPARATUS

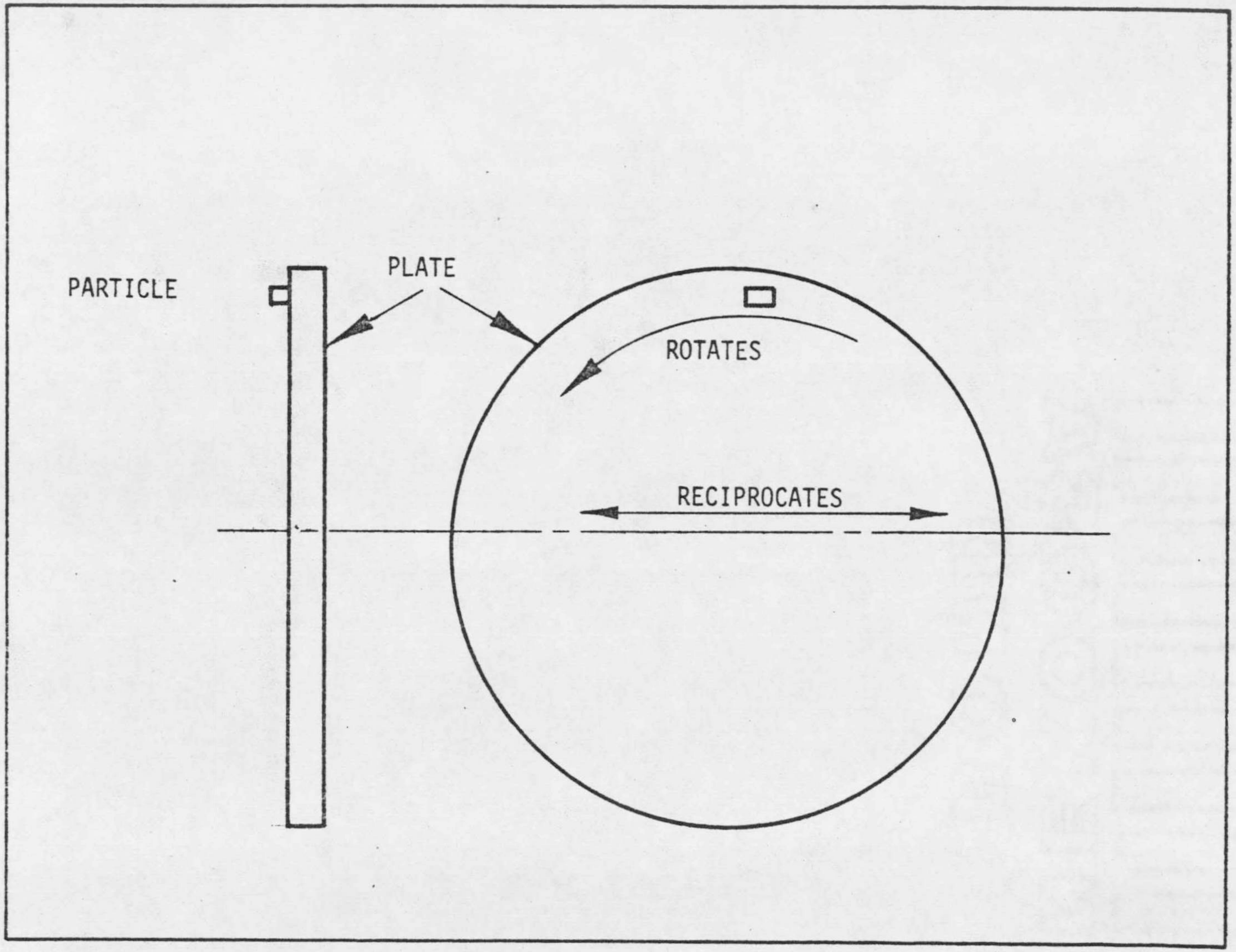


FIGURE X-2. DIAGRAM EXPLAINING PRINCIPLE OF SCOTCH YOKE

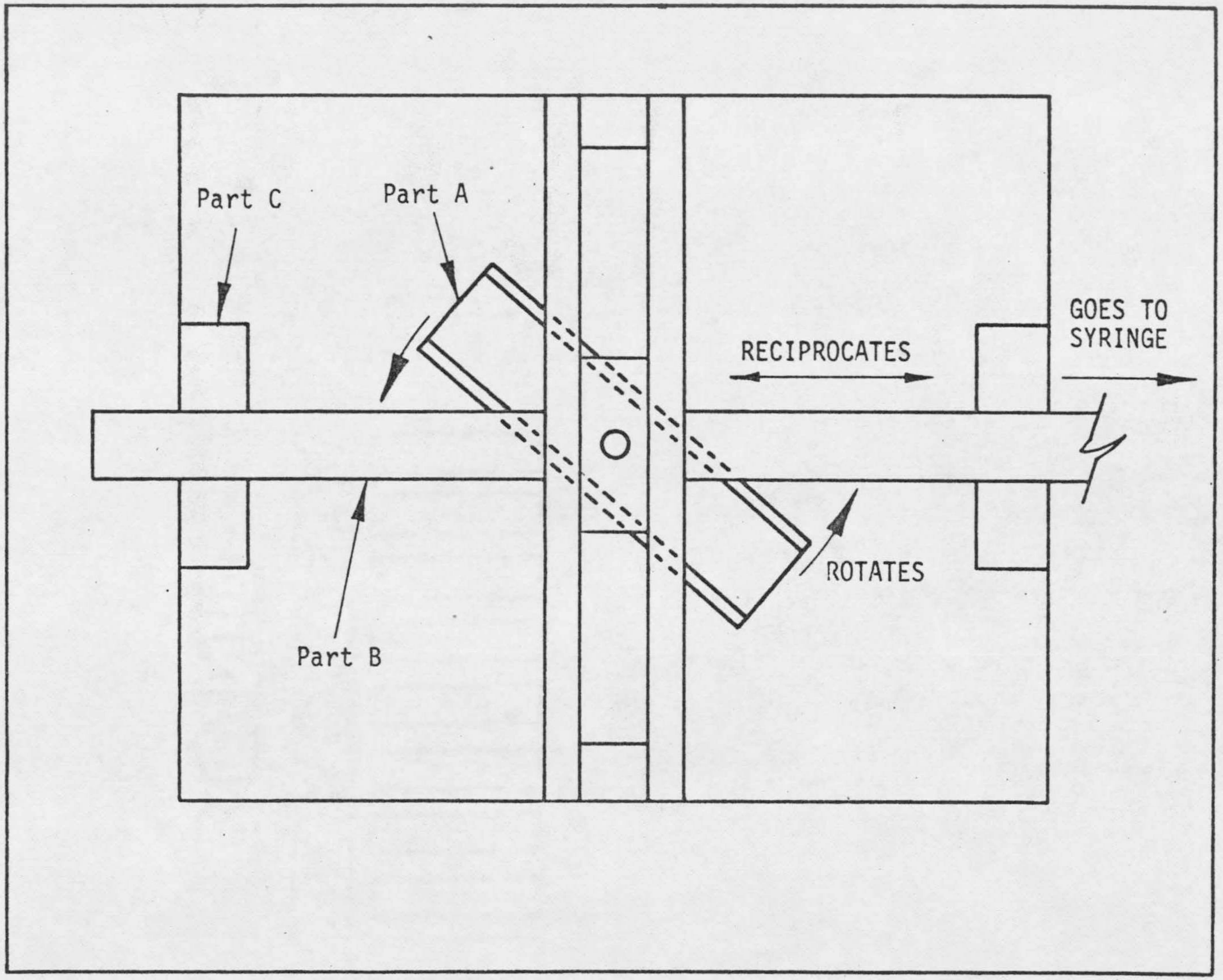


FIGURE X-3. SCOTCH-YOKE

Syringe (Appendix B). The plunger of the syringe is connected to the scotch-yoke (see Figure X-1). The syringe pumps blood into and out of the capillary tube which is fixed in between the two reservoirs.

Reservoirs and Tube (Appendix B). The reservoirs are connected to the tube with the help of square plates and screws. The plates are glued at the ends of the tube. To prevent leakage, silicone "form-a-gasket" (a rubber base adhesive sealant) is applied in between the square plates and the reservoirs.

The blood in the feed reservoir can be stirred by a magnetic stirrer (Appendix B) to maintain the hematocrit of blood uniform.

B. Pressure Measurement Device (Appendix B)

Pressure at upstream side (feed side) is measured by a Statham P23Dd pressure transducer which is connected to an amplifier. The output of the amplifier is fed to a Hewlett-Packard oscillographic recorder. The pressure in the discharge reservoir is atmospheric.

C. Flow Measurement Device (Appendix B)

The displacement of the plunger of the syringe is monitored by a displacement transducer whose output is fed to an amplifier, which is connected to the recorder. The displacement transducer measures the displacement of the plunger but from the displacement signal it is possible to calculate the flow signal.

If the displacement wave is given by,

$$S = S_m \sin \omega t$$

where S = displacement at time

t = instantaneous time

$\omega = 2\pi f$

= angular frequency

f = linear frequency

S_m = amplitude of the wave

then the plunger speed of movement is given by,

$$\frac{dS}{dt} = S_m \omega \cos \omega t$$

The fluid volumetric flow rate from (or into) the syringe is given by,

$$Q = \frac{\pi}{4} d_s^2 \frac{dS}{dt}$$

where Q = instantaneous flow

d_s = diameter of syringe plunger

Combining the last two equations,

$$Q = \frac{\pi}{4} d_s^2 S_m \omega \cos \omega t$$

This can also be written as,

$$Q = Q_m \cos \omega t$$

where $Q_m = \frac{\pi}{4} d_s^2 S_m \omega$

Thus, it is possible to calculate fluid flow amplitude from syringe plunger displacement amplitude.

Recorder (Appendix B). The Hewlett-Packard model oscillographic recorder has two channels. It can record the plunger displacement wave, and on the other channel it can simultaneously record the pressure wave.

Procedures

D. Calibration of the Pressure Transducer

The pressure transducer measures the pressure in terms of an output voltage. Hence, it is necessary to calibrate the pressure transducer. The relationship between the pressure measured in terms of volts and the pressure measured in terms of mm of Hg is termed the calibration of the pressure transducer. This calibration was obtained with the help of a precise mercury manometer (Appendix B). The relationship was found to be linear and it is shown in Figure X-4. A linear regression analysis was used to obtain an expression which relates the pressure in volts to the pressure in mm of Hg.

$$P = 114.5\ V + 1.7$$

where V = pressure in volts

P = pressure in mm of Hg.

E. Calibration of the Displacement Transducer

This transducer also measures the displacement of the plunger of the syringe in terms of an output voltage. Hence, it is necessary to obtain the relationship between the displacement measured in terms of volts and the displacement measured in terms of centimeters. This calibration was obtained using a precision micrometer. The relationship

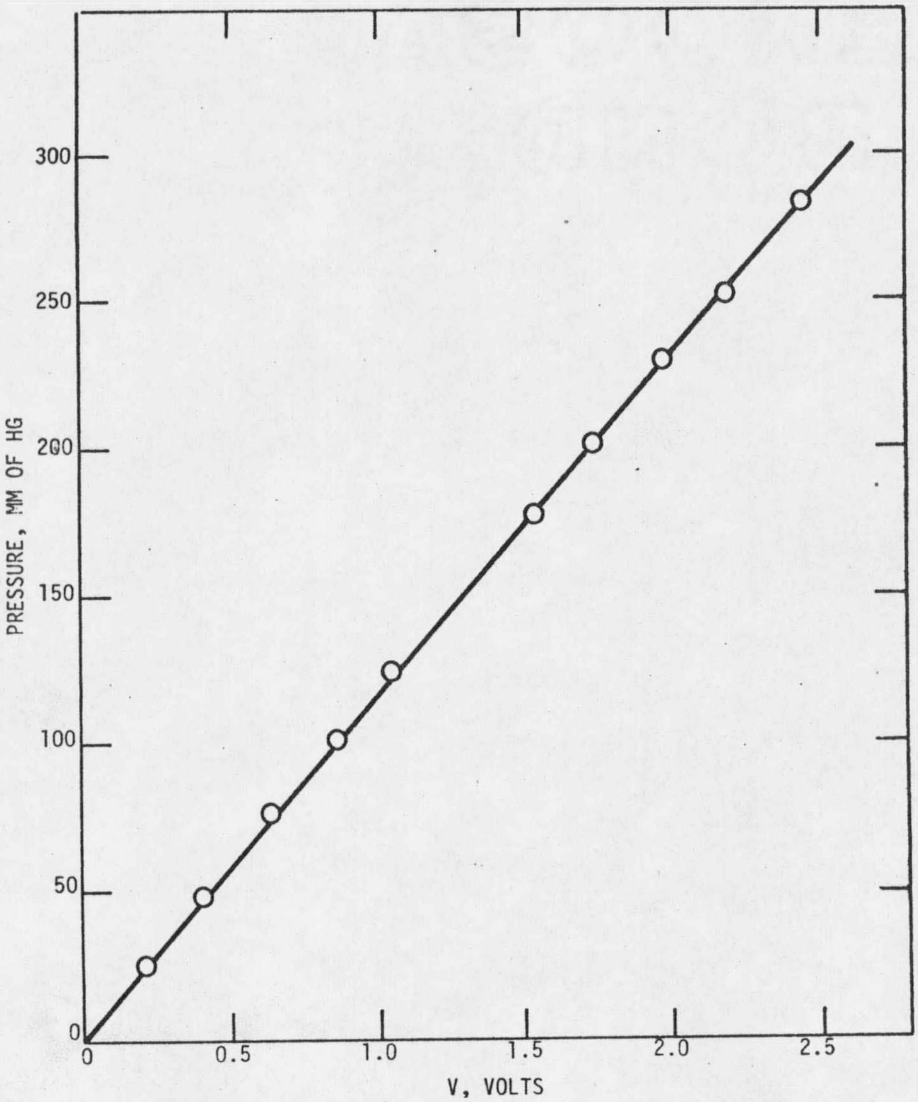


FIGURE X-4. PRESSURE TRANSDUCER CALIBRATION used is given by,

1 recorder chart division = 0.0685 cm

where 1 division corresponds to the sensitivity of 10 mv/v/full scale.

F. Measurement of the Capillary-Tube Diameter

This was done by performing an experiment in which water (a Newtonian fluid) was pumped under steady conditions through the capillary-tube with the help of a syringe pump (Appendix B). The pressure drop across the tube at various volumetric rates was measured with the help of the pressure transducer, which is fixed in the feed reservoir. The Poiseuillian equation (good for a Newtonian fluid such as water) was used to obtain the diameter of the tube.

$$R^4 \doteq \frac{Q}{\Delta p} \frac{8\mu L}{\pi}$$

where Q = volumetric flow

Δp = pressure drop across the tube

μ = viscosity of water

L = length of the tube

R = radius of the tube

Thus, the radius of the tube can be calculated. Figure X-5 shows the graph of the volumetric flow rate against the pressure drop. From the slope of the graph, the radius of the tube can be calculated.