The effect of high school calculus background on achievement in post-secondary calculus
by William John Swartz, Jr

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University

Abstract:
The problem of this study was to determine, for students at Montana State University in Bozeman and Montana College of Mineral Science & Technology in Butte, the effects of one-semester and two-semester high school calculus courses on achievement in Calculus 1, the first calculus course at the college/university level.

The Calculus 1 student sample from each institution was divided into several subsamples, reflecting each student's freshman or non-freshman status. Data of each subsample and both complete institutional samples were analyzed separately. Students in each group were categorized as having in high school (1) no exposure to calculus, (2) a one semester calculus course, or (3) a year course. Achievement in Calculus 1 was measured by scores of examinations given in the course.

Two main questions occur concerning the merit of calculus as a high school course: (a) What are the effects of advanced placement in college/university mathematics courses? (b) What are the effects on student achievement in Calculus 1 for students who had exposure to calculus in high school but did not receive advanced placement in mathematics at the college/university level? This study addresses the latter question.

Multiple Linear Regression was employed to correlate calculus background and other predictor variables with Calculus 1 achievement. ANOVA and the Newman-Keuls method were used to compare mean achievement among the high school background groups. The Chi Square Test of Independence was used to test for independence of background and achievement.

In several instances, students with a year high school course outperformed either those with no high school background or both other background groups. In some instances, there was no difference in achievement among the three background groups. In no case did the semester group achieve better than the no-calculus group.

There appears to be a positive link between a year of high school calculus background and achievement in Calculus 1. A semester course, however, offers no apparent benefit.

THE EFFECT OF HIGH SCHOOL CALCULUS BACKGROUND ON ACHIEVEMENT IN POST-SECONDARY CALCULUS

by

William John Swartz Jr.

A thesis submitted in partial fulfillment
of the requirements for the degree

of

Doctor of Education

MONTANA STATE UNIVERSITY
Bozeman, Montana

September 1992

APPROVAL

of a thesis submitted by

William John Swartz Jr.

This thesis has been read by each member of the graduate committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

Sept. 21, 1992 — Donald Robson
Date — Co-Chairperson, Graduate Committee

Oct. 28, 1992 — Lyle Andersen
Date — Co-Chairperson, Graduate Committee

Approved for the Major Department

Oct 30, 1992 — Duane Melling
Date — Head, Major Department

Approved for the College of Graduate Studies

10/30/92 — R L Brown
Date — Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a doctoral degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library. I further agree that copying of this thesis is allowable only for scholarly purposes, consistent with "fair use" as prescribed in the U.S. Copyright Law. Requests for extensive copying or reproduction of this thesis should be referred to University Microfilms International, 300 North Zeeb Road, Ann Arbor, Michigan 48106, to whom I have granted "the exclusive right to reproduce and distribute copies of the dissertation in and from microfilm and the right to reproduce and distribute by abstract in any format."

Signature William John Swartz Jr

Date Sept. 15, 1992

ACKNOWLEDGEMENTS

I am grateful to my graduate committee co-chairmen, Dr. Lyle Andersen and Dr. Donald Robson, for their considerable investment of time, expertise, and encouragement. Without their dedication to the task, this study could not have been done.

I also wish to thank the other members of the graduate committee for their assistance (Dr. Maurice Burke, Dr. Gary Conti, Dr. Eric Strohmeyer, and Dr. Gregory Young). The helpful cooperation of the registrars of Montana State University and Montana Tech (Mr. Joseph Frazier and Mr. Charles Nelson), in providing the volumes of data essential for this study, is appreciated.

My grandmother, Mrs. Eli Swartz, generously provided my housing while this work was in progress. My greatest debt of gratitude is to my parents, Dr. William and Doris Swartz, who have strongly supported and influenced all my educational endeavors.

TABLE OF CONTENTS

TABLE OF CONTENTS--Continued

LIST OF TABLES

LIST OF TABLES--(Continued)

LIST OF TABLES--(Continued)

LIST OF TABLES--(Continued)

ABSTRACT

The problem of this study was to determine, for students at Montana State University in Bozeman and Montana College of Mineral Science & Technology in Butte, the effects of one-semester and two-semester high school calculus courses on achievement in Calculus 1, the first calculus course at the college/university level.

The Calculus 1 student sample from each institution was divided into several subsamples, reflecting each student's freshman or non-freshman status. Data of each subsample and both complete institutional samples were analyzed separately. Students in each group were categorized as having in high school (1) no exposure to calculus, (2) a one semester calculus course, or (3) a year course. Achievement in Calculus 1 was measured by scores of examinations given in the course.

Two main questions occur concerning the merit of calculus as a high school course: (a) What are the effects of advanced placement in college/university mathematics courses? (b) What are the effects on student achievement in Calculus 1 for students who had exposure to calculus in high school but did not receive advanced placement in mathematics at the college/university level? This study addresses the latter question.

Multiple Linear Regression was employed to correlate calculus background and other predictor variables with Calculus 1 achievement. ANOVA and the Newman-Keuls method were used to compare mean achievement among the high school background groups. The Chi Square Test of Independence was used to test for independence of background and achievement.

In several instances, students with a year high school course outperformed either those with no high school background or both other background groups. In some instances, there was no difference in achievement among the three background groups. In no case did the semester group achieve better than the no-calculus group.

There appears to be a positive link between a year of high school calculus background and achievement in Calculus 1. A semester course, however, offers no apparent benefit.

CHAPTER 1

PROBLEM, NEED AND DEFINITIONS OF TERMS

Introduction

Mathematics programs in elementary and secondary schools in the United States underwent considerable revision in the late 1950s and early 1960s. This revamping was one notable manifestation of Sputnik and its ripple effects in American society. The changes that were made in elementary and secondary mathematics curricula are classified by Rash (1977) into two general categories:

1. The use of unifying concepts.
2. Accelerating the content.

The former usually took the form of the so-called "new math" curricula (namely the teaching of number bases in the middle grades and emphasis on set concepts), and the latter led to the introduction of calculus as an advanced course in the high school (Rash). While much discussion and research has centered around the merits and demerits of the "new math," less attention has been devoted to the question of effectiveness of the high school calculus programs and the ramifications for placement in college or university mathematics courses. A recounting of research that has been done in this area will follow in chapter 2.

Statement of the Problem

The problem of this study was to investigate the effects of one-semester and two-semester high school calculus courses on achievement in the traditional first course in calculus for mathematics, science, and engineering majors at the college/university level.

Need of the Study

In the period from 1973 to 1982 in the United States, the number of students enrolled in calculus courses (that is, mathematics courses treating such topics as limits, differentiation, and often integration) in high school grew at a rate of more than 10% annually. In 1982, 234,000 students passed a high school calculus course, with 148,600 of them receiving a grade of B- or better (Carroll, 1984). By 1987 about 300,000 high school students were enrolled annually in some type of calculus course (Steen, 1987).

Concurrently, about 600,000 students enroll annually in first-year Calculus 1 in American colleges and universities (Anderson & Loftsgarden, 1987). Many of these are the same students who had exposure to calculus in high school. However, Calculus 1, the classic introductory calculus course at the college/university level suffers a 35% national failure rate (Douglas, 1988). Even more disturbing is the fact that only 46% of these students completed the first year of calculus (two semesters or three quarters) with a grade average of D or better (Anderson & Loftsgarden). These circumstances clearly warrant concern and raise important questions. As Steen (1986, p. 157) notes,

> Each year about 500,000 students in the United States study [post-secondary] calculus . . . Calculus is big business with far reaching consequences for students, for colleges and universities, for the mathematical community, and for our nation.

While some research efforts have addressed questions relating to the connection between high school calculus and achievement in college/university mathematics, there is a clear need for further investigation in this important area. Studies by Austin (1975), Bergeson (1967), Burton (1989), Dickey (1985), Fry (1973), McKillip (1965), Paul (1970), Pocock (1974), Robinson (1970), Shimizu (1969), Sorge and Wheatley (1977), and Tillotson (1962), each dealing with some aspect of this complex issue, will be discussed in the Review of Literature in chapter 2.

In assessing the worth or benefits of calculus as a high school course, two main issues must be examined:

1. The effects of advanced placement in college/university mathematics courses.

2. The effect on student achievement in Calculus 1 for students who have had exposure to calculus in high school but did not receive advanced placement in mathematics at the college or university.

Studies discussed in chapter 2 provide evidence that advanced placement students who have had calculus as a high school course and have passed either the AB or BC mathematics Advanced Placement Examination generally perform better in the traditional second college/university-level calculus course, and also in subsequent mathematics coursework, than their regular placement counterparts.

Other investigations noted below dealt with the question of whether students receiving two semesters of calculus in high school demonstrated greater achievement in Calculus 1 at the college or university than students without that high school background. However, the studies cited do not satisfactorily resolve this question. Given the considerable and varied changes in secondary and college/university curricula and teaching methods in the last two decades, studies by Paul (1970), Robinson (1970), and Sorge and Wheatley (1977) may be considered outdated. Furthermore, Paul's data were derived only from students at The Ohio State University. Robinson's data came exclusively from the University of Utah, and Sorge and Wheatley's sample was limited to Purdue University. The locations of these studies indicate a need for further investigation which has stronger generalizability to students attending a college or university in Montana or comparable institutions in the Northern Rocky Mountain region. As Robinson notes, "Further studies of this type, conducted at different universities (involving students from other geographic regions) are necessary to determine whether the results can be generalized" (p. 60). This study utilized data from both Montana State University in Bozeman, and Montana College of Mineral Science & Technology (Montana Tech) in Butte. Montana State University is a land grant institution with strong programs in agriculture and engineering, as well as degree programs in various education, fine arts, liberal arts, and science areas. By contrast, Montana Tech is noted for degree programs in engineering areas such as environmental science, geophysics, metallurgy, mining, and petroleum, as well as computer science. The multi-institutional aspect of this study distinguishes it from earlier studies done on this subject.

The manner in which information regarding the content of high school mathematics courses was obtained, directly from school personnel, is unique to the present study.

Another question concerns the achievement in Calculus 1 of students who have had one semester of calculus in high school versus those with little or no exposure to the subject in high school. The literature reviewed below, regarding this issue, is inconclusive indicating a need for further study. Of the five known studies that have been done in this area, one was completed in 1962 and another in 1965, when calculus as a high school subject was in its infancy. The likelihood of change in content, emphasis or teaching methods for such a course in the ensuing decades renders those studies of questionable use today. The three more recent studies, one completed in 1970, and two in 1977, are also not very current. These latter studies have resulted in conflicting conclusions. Paul (1970) found that students with one semester of high school calculus had greater achievement in university Calculus 1 than those students without such a background. Both Rash (1977) and Sorge and Wheatley (1977) found otherwise (no difference in achievement) in their studies. Hence there is a need for further research.

Another question compares the merits of one semester versus two semesters of high school calculus, as measured by achievement in Calculus 1. Again, the findings of previous research fail to resolve the issue. For example, Sorge and Wheatley (1977) note that their data "indicate that the amount of calculus in high school does not seriously affect a student's chances for a particular grade" in university Calculus 1 (p. 644). By contrast, McKillip's results (1965) indicated that a one-semester high school calculus course produced no significant effect on students' grades but that a two-semester high school calculus course did improve the grades in Calculus 1.

Some previous studies in this realm have distinguished between two-semester high school calculus courses offered under the auspices of the Advanced Placement Program and two-semester calculus courses which are not a part of the Advanced Placement Program. The present study did not differentiate between these two categories of courses, since there are very few high school calculus courses offered under the aegis of the Advanced Placement Program represented in the sample of the study.

Another reason for conducting this study was that the potentially interactive effects of several important predictor variables have been ignored by previous research done in this area. Therefore, this study will account for the effects on college mathematics achievement of such predictor variables as student age, student's major, class standing, high school size, and gender. There exists considerable evidence that gender often plays a significant role as a variable in mathematics achievement. Stent (1977) and Tobias (1976) concluded that due to the influence of sex role socialization, mathematics anxiety is more common and more severe among women than among men. Betz (1978) in turn found that higher levels of mathematics anxiety are related to lower mathematics achievement test scores, suggesting that gender should be included as a predictor variable in the present study.

Other included variables may also play significant predictive roles in Calculus 1 achievement. For example, age may correlate closely with mathematics background once attained but forgotten, that is, a "rustiness" factor, particularly for students over traditional age. Perhaps students choosing major areas of study that are strongly mathematics oriented (such as mathematics, engineering, and the physical sciences) achieve at a higher level in Calculus 1 than those who gravitated to majors in subjects that are less mathematical.

Gaining further insight into the effects of one or two semesters of high school calculus on achievement in Calculus 1 is useful for at least five reasons:

1. A common motivation among students for taking a one or two-semester calculus course in high school is the hope that it will enhance subsequent achievement in college/university Calculus 1 (Rash, 1977).

2. If a one or two-semester high school calculus course does not generally demonstrate a positive effect on achievement in Calculus 1, this would be a relevant consideration in deciding whether to discontinue offering calculus as a one or two-semester course at the secondary level (McKillip, 1965; Sorge & Wheatley, 1977). Under such circumstances, a one-semester course could conceivably be eliminated in favor of a two-semester course if the latter showed a significantly greater positive effect on Calculus 1 achievement. Another possibility is that a high school offering a one-semester calculus course may choose to eliminate calculus entirely from its curriculum if budgets, low enrollment, lack of a qualified instructor or other factors make a year-long calculus course unfeasible. This is a plausible scenario in view of both current fiscal problems plaguing many school districts and the plethora of different mathematics courses offered by many high schools in recent years. Whitesitt (1980) notes, for example, that Bozeman Senior High School in Montana offered only four mathematics courses during the 1957-1958 school year, but offered a total of thirteen mathematics courses in the 1977-1978 school year. Similar figures could be noted in high schools throughout the United States. If neither a one or two-semester high school calculus course is shown to enhance achievement in Calculus 1,

that knowledge would be a germane consideration in a decision to eliminate calculus from the high school curriculum.

3. According to data reported by Sorge and Wheatley (1977), students may devote time and effort prematurely studying calculus in high school at the expense of adequately mastering important prerequisite concepts in geometry, algebra and/or trigonometry; thus hampering their efforts in subsequent mathematics coursework. Therefore, the possibility exists that the study of calculus in high school may not only fail to yield justifying benefits, but as Steen (1986) believes, may in fact be counterproductive to the students who pursue further mathematical study at the college/university level.

4. The recently published National Council of Teachers of Mathematics (NCTM) standards suggest that discrete mathematics and probability and statistics be included in the high school curriculum, perhaps in lieu of calculus. The conclusions of this study thus have ramifications for the related question of alternative courses of study for the high school senior mathematics student.

5. Many high schools in predominantly rural areas of Montana and other sparsely populated western states serve relatively few students. In 1989-90 in Montana, one hundred five of the one hundred eighty-four accredited high schools had enrollments of fewer than one hundred students (grades 9 - 12). Of these, more than half had enrollments of fewer than fifty students ("High School Enrollments" 1990). The constraints of small enrollment will necessarily limit the breadth of

courses a high school may offer. Under this circumstance, is the lack of a high school calculus course a deficiency? Is the offering of such a course warranted for very small numbers of students? Among the high schools represented in the student sample of this study, most of those offering a calculus course had enrollments of 100 or more students. However, some very small high schools also provided calculus in their course offerings. For example, Froid High School (enrollment 37 in grades 9 - 12) and Turner High School (enrollment 28 in grades 9 - 12), both in Montana, offered a semester calculus course. Among other Montana schools, Clyde Park High School (enrollment 57 in grades 9 - 12) and Lustre Christian School (enrollment 34 in grades 9 - 12) offered a year-long calculus course.

Addressing the present, Ferrini-Mundy and Gaudard (1992, p. 57) assert, "The research base for understanding [high school calculus] is not well-developed." In summary, Rash (1977, p. 280) observes, "Calculus is an academic and a financial luxury high schools can afford only if it can be shown that some of the outcomes are fruitful." This study sheds needed light on one such outcome - achievement in college/university Calculus 1.

General Questions to be Answered

This study has attempted to answer the following four major questions:

1. What correlation does successful completion of a one-semester/survey calculus course in high school have with achievement in Calculus 1 at the college/university level?

2. What correlation does successful completion of a two-semester calculus course in high school have with achievement in Calculus 1 at the college/university level?

3. Do students in the three categories (1) no high school calculus, (2) one-semester/survey high school calculus, and (3) two-semester high school calculus demonstrate the same mean achievement in college/university Calculus 1?

4. As measured by achievement in college/university Calculus 1, does any one high school calculus option (no high school calculus, one-semester/survey course, or two-semester course) demonstrate superior worth over the other two options?

General Procedures

A copy of the proposed study was presented to the appropriate administrative personnel at Montana State University and Montana Tech. Upon the approval of the proposal, the study was explained to the mathematics department heads and registrars of both schools. Their support and cooperation facilitated gathering most of the needed data.

At both institutions, class rosters and examination scores were obtained from the instructors involved. The Math Anxiety Rating Scale (MARS) instrument was administered to each class section (see Appendix A). Other data such as ACT and SAT scores, gender, minority status, age and class standing were obtained from computerized student records with the assistance of the registrars. Information including high school mathematics courses taken, year of high school graduation, and high school alma mater was obtained by the researcher from direct examination of high school transcript records on file with the registrars' offices. High school enrollment figures were obtained from other sources.

Each student was categorized as a "new" freshman or other undergraduate depending on whether the student graduated from high school the spring preceding the study (1989). Each student was categorized as having taken (1) no high school calculus, (2) a one-semester/survey calculus course, or (3) a two-semester calculus course. The categorization was based on the content of the senior high school mathematics course taken, if any. In numerous instances, the appropriate categorization was not apparent from the high school mathematics course title because of its nebulous nature. Such vague rubrics included "Math IV," "Senior Math," "Advanced Math," and others. In these instances, the high school in question was contacted (usually by mail) and the nature of the course was ascertained by a topic questionnaire. The student's high school calculus background was then categorized based on this information.

The Montana State University and Montana Tech data were analyzed separately. Separate analysis was necessary since Calculus 1 at the two institutions were different courses in some important respects. The scope was different since one was a semester-duration course (Montana Tech) and the other was a quarter-duration course. The two courses also utilized different textbooks. Also, student achievement was measured differently in the two courses.

The student sample from Montana State University was further divided into three different subsamples for purposes of statistical analysis: (1) new freshmen, (2) the complement group, i.e., those other than new freshmen, and (3) the combined sample consisting of both the new freshmen and the complement. The student sample at Montana Tech was divided into three subsamples as described above, and one additional subsample. Subsample definitions are provided in the Definition of Terms later in this chapter.

The data of each subsample were subjected to several analysis techniques, including Multiple Linear Regression, One-way Analysis of Variance, and, for the Montana State University subsample only, Chi Square Test of Independence. The researcher, using these methods, investigated the correlation between high school calculus background and certain other independent variables and achievement in college/university Calculus 1. The researcher also explored the difference in mean achievement among the three high school calculus background categories.

Limitations and Delimitations

The following were limitations and delimitations of this study:

1. The study directly considered only the effects of one and two-semester high school calculus courses and not any other high school mathematics background.

2. The study samples consisted of all students enrolled in Calculus 1 (Math 181) at Montana State University during autumn quarter of the 1989 - 1990 academic year, as well as approximately half of all Calculus 1 (Math 121) students at Montana Tech during fall semester 1989, and all Math 121 students during spring semester 1990.

3. Calculus 1 students for whom no high school transcripts were available (typically transfer students from other colleges or universities) were not included in this study. Those for whom other necessary data were unavailable (such as SAT or ACT scores) were also excluded from the study.

4. The study was supported by the resources available through Montana State University's Renne Library, the registrars' student record files at Montana State University and Montana Tech, and information gathered directly from high schools represented by students in the study samples.

Definition of Terms

The following definitions are provided by the researcher or correspond to those used in other sources as indicated by citation and are to be considered as operational definitions.

Achievement: Achievement in Calculus 1 at Montana State University was measured in this study by the aggregate raw score (500 points possible) attained by the student on three midterm examinations worth 100 points each and the comprehensive final examination given at the conclusion of the course weighted 200 points (see Appendixes B, C, D and E). At Montana Tech achievement was measured by the score (150 points possible) attained by the student on a comprehensive final examination given at the conclusion of the course (see Appendix F).

Advanced Placement (AP): The Advanced Placement Program, a cooperative educational endeavor of the College Entrance Examination Board and secondary and post-secondary schools.

Advanced Placement Calculus AB and Calculus BC Courses: Mathematics courses offered by accredited participating high schools designed to prepare the student for either the Calculus AB examination or the Calculus BC examination, both of which are written, administered, and scored by the Advanced Placement Program. The following course descriptions are provided by the College Entrance Examination Board (1990, p. 5).

Calculus AB: This course is intended for students who have a thorough knowledge of college preparatory mathematics, including algebra, axiomatic geometry, trigonometry, and analytic geometry (rectangular and polar coordinates, equations and graphs, lines, and conics). Calculus AB is a course in introductory calculus with elementary functions. The list of topics for Calculus AB given [in Appendix G] is intended to indicate the scope of the course but not necessarily the order in which the topics are to be taught.

Calculus BC: This course is intended for students who have a thorough knowledge of analytic geometry and elementary functions in addition to college preparatory algebra, geometry, and trigonometry. Calculus BC is considerably more extensive than Calculus AB. All of the calculus topics in Calculus AB are included. The list of topics given [in Appendix G] is intended to indicate the scope of the course but not necessarily the order in which the topics are to be studied.

<u>Calculus 1</u>: The first course in the traditional mathematics analysis sequence, designed for engineering, mathematics and science majors, as defined by the school catalogs of Montana State University and Montana Tech. The course description at Montana State University (Montana State University, 1988, p. 179) is:

Math 181M Calculus and Analytic Geometry 1
Prerequisites: Math 165. [trigonometry]

Functions, graphs of functions, operations on functions, limits of functions, continuity of functions, one sided limits and continuity, limits at infinity, infinite limits, vertical and horizontal asymptotes, derivatives, differentiation formulas, derivatives of trigonometric functions, the chain rule, implicit differentiation, higher derivatives, extrema of functions, the mean value theorem, first and second derivative tests, rectilinear motion, related rates, L'Hopital's rule, definition and properties of the Riemann integral, the fundamental theorem of calculus, indefinite integrals.

The course description at Montana Tech (Montana College of Mineral Science & Technology, 1988, p. 135) is:

> Math 121 Analytic Geometry and Calculus
> 5 Cr (Hrs:5 lec.)
> Covers functions, limits, derivatives, integrals, techniques of integration, exponential, logarithmic and hyperbolic functions. Prerequisite: Math 105 (College Algebra) and Math 106 (College Trigonometry) or their equivalent.

One Semester High School Calculus Course: Any mathematics course of one semester duration offered by an accredited secondary school with content including, but not necessarily limited to, the following topics: functions, graphs of functions, operations on functions, limits of functions, derivatives, differentiation formulas, the chain rule, extrema of functions, first and second derivative tests. The treatment of these topics was usually method-oriented and nontheoretical. For example, epsilon-delta ideas were generally not treated in a one-semester high school calculus course. There was little or no emphasis on proofs of theorems or derivations of formulas. The study of functions, limits and derivatives infrequently extended to trigonometric, exponential or logarithmic functions. Other topics which also were occasionally part of such a course were applications, asymptotes, and an introduction to integration. A two-semester course offering a survey of calculus, as reflected by the same attenuated list of calculus topics and minimal rigor, was classified in this category for purposes of this study.

Subsample 1: Montana State University New Freshmen: Fall quarter 1989 Montana State University students enrolled in Calculus 1 who graduated from high school in spring 1989.

Subsample 2: Montana State University Complement: Fall quarter 1989 Montana State University students enrolled in Calculus 1 who graduated from high school prior to spring 1989.

Subsample 3: Montana State University Combined: All fall quarter 1989 Montana State University students enrolled in Calculus 1 for whom complete data were available.

Subsample 4: Montana Tech New Freshmen: Fall semester 1989 and spring semester 1990 Montana Tech students enrolled in Calculus 1 who had no other college mathematics coursework, and who graduated from high school in spring 1989.

Subsample 5: Montana Tech Freshmen Complement: Spring semester 1990 Montana Tech students enrolled in Calculus 1 who had taken prior college level mathematics coursework (such as trigonometry or college algebra) during fall semester 1989; and who graduated from high school in spring 1989.

Subsample 6: Montana Tech Complement: Fall semester 1989 and spring semester 1990 Montana Tech students enrolled in Calculus 1 who graduated from high school prior to spring 1989.

Subsample 7: Montana Tech Combined: All fall semester 1989 and spring semester 1990 Montana Tech students enrolled in Calculus 1 for whom complete data were available.

Two-Semester High School Calculus Course: Any mathematics course of two semesters duration offered by an accredited secondary school which included (but was not necessarily limited to) most or all of the topics covered in the Advanced Placement Calculus AB course (see Appendix G). Any two-semester high school mathematics course not meeting the content criteria specified by this definition but fulfilling the definitional criteria given for a one-semester/survey high school calculus course was, for purposes of this study, classified in the latter category.

WF Grade: At Montana State University, a grade assigned to a student who was failing at the time of withdrawal from a course. For purposes of computing quarter and cumulative grade point average, a WF grade is treated the same as an F grade (Montana State University Office of Publications and News Services, 1988).

WP Grade: At Montana State University, a grade assigned to a student who was passing at the time of withdrawal from a course. A WP grade does not affect a student's quarter or cumulative grade point average (Montana State University Office of Publications and News Services, 1988).

CHAPTER 2

REVIEW OF RELATED LITERATURE

The question being addressed by this study is "What are the effects of one-semester and two-semester high school calculus courses on achievement in the traditional first course in calculus at the college/university level?" The problem has not been conclusively answered by previous research. This study has considered the possible effects of other potentially intervening variables ignored to a considerable extent, as noted earlier, by other researchers.

The review of literature chapter will first focus on various points of view in favor of and against offering calculus in the high school curriculum. This is followed by a review of studies that investigated the effects of high school calculus on achievement in Calculus 1, Calculus 2, and more advanced post-secondary mathematics coursework. The chapter concludes with an examination of research on the effects of advanced placement in college/university mathematics courses.

High School Calculus: Pro and Con

The Advanced Placement Program in Mathematics, formally adopted by the College Entrance Examination Board (CEEB) in 1955 grew from 368 students taking the Calculus AB or BC examinations in

1956 to 10,675 taking these examinations in 1967 and 14,673 in 1971 (College Entrance Examination Board, 1990). Presently, approximately 31% of American high schools participate, serving about 17% of their college-bound students through advanced placement testing (College Entrance Examination Board, 1987). Acceleration, by moving students to more advanced courses as a means of providing for the needs of academically talented students, has enjoyed considerable support over the last three decades. The Advanced Placement Program "is based on the assumption that some twelfth-grade students can do college freshman work" (College Entrance Examination Board, 1964, p. 24). The assertion of the College Entrance Examination Board is that for colleges not to reward such work completed in high schools would result in needless duplication and, worse, retard the vertical progress of students in a subject field.

There are several reasons commonly proposed in favor of teaching calculus as a high school subject. One purpose is to accelerate the program of the more able students. Having had a college-equivalent calculus course in high school, a student may receive credit for one or more semesters or quarters of college/university calculus. Bypassing one or more college calculus courses, with or without college credit for the high school course, enables the student to take more advanced courses in college, proponents reason.

Another purpose given for teaching calculus in high school is it provides students an additional opportunity to develop problem-solving and logical reasoning skills as well as cultivating the students' ability to deal with abstract concepts.

A third purpose is to give secondary students a "head start" on college/university mathematics. The hope and intent is students with prior high school exposure to calculus will achieve greater success in comparable college/university coursework and/or in more advanced college/university mathematics courses than those without such background.

The validity of the first and third purposes have been investigated by several researchers as discussed below. Relatively little quantitative research appears to have been done regarding the intent to develop problem-solving and logical reasoning skills and cultivate the students' ability to deal with abstraction. Ferrini-Mundy and Gaudard's study (1992), summarized below, explored the question of conceptual learning (as contrasted to procedural competence). Currently, many mathematics educators question whether high school calculus is effective in developing problem-solving and logical reasoning skills and fostering the ability to deal with abstraction. Typical of such views is that of Rash (1977, p. 78), who believes that the skills alluded to earlier

> can be cultivated with good teacher guidance, regardless of the content. When a [high school level] calculus course does not have only the best students, the teacher must proceed at a slower pace and give more explanations (or omit proofs and difficult concepts). Thus, less challenging learning activities are done independently by the majority of students. Students develop a false impression of the

> amount of productive study which is necessary for a college course and an erroneous sense of security in their own knowledge and ability.

Small (1986) concurs with this view and further suggests that students receiving an attenuated calculus course in high school proceed to the college/university level overconfident and possessed with ". . . an unfounded feeling of subject mastery . . . that can lead to serious problems in college calculus courses" (p. 134).

Rash (1977) and Small (1986) go on to assert that there are often other less laudable underlying motives for offering calculus as a high school course: (1) The teachers want to teach calculus. (2) Offering calculus is a status symbol for the school. (3) Students consider calculus a prestigious course. (4) The school is yielding to political pressure from parents, school board officials, counselors and school administrators, as they demonstrate a competitive-type pride in their school's offering of calculus. (5) No one in the school has taken the initiative to consider other alternatives for advanced courses. No data exist to substantiate or disprove these claims. This is not surprising since it would seem difficult to obtain accurate data regarding the issue of subconscious or ulterior motives.

Rash (1977) believes that in lieu of high school calculus, greater emphasis should be placed on precalculus preparation in areas such as algebra, trigonometry and analytic geometry. Sorge and Wheatley (1977) concur, based on statistics from their study which suggest that in the rush to take calculus in high school, the acquisition of adequate background in the aforementioned precalculus areas is being seriously sacrificed. They note from 1962 to 1975 a thirteen year pattern of steady decline nationally in mean mathematics scores on the Scholastic Aptitude Test. In a roughly concurrent observation, Rash (1977) notes that in 1972-73 there were four times as many students studying calculus in high school as there were in 1960. As further evidence, Sorge and Wheatley's data (1977) indicated that in the period 1969-1977 there was a marked decline in the ratio of students who had received analytic geometry in high school. Concurrently, the ratio of students who took a course titled "Precalculus" or "Calculus" increased. According to Sorge and Wheatley, among students in the first course of each of three different freshman calculus sequences at Purdue University (each tailored for student groups with different abilities as measured by such factors as high school grades, high school mathematics courses completed, and SAT mathematics scores), percentages of those who had taken high school trigonometry or analytic geometry decreased, while percentages of those who had taken calculus in high school increased, from 1970 to 1975. Sorge and Wheatley's data (p. 646) are summarized in Table 1. The first courses of each of the three different freshman calculus sequences are denoted with the rubrics MA 155, MA 161 and MA 163 in Table 1. The course MA 155 is the least advanced and MA 163 is the most advanced.

Table 1. Percent of Respondents Who Indicated They Had Taken a Particular Course in High School from the Sorge and Wheatley Study.

Course	Academic Year 1969-70	1974-75	Percent Change
	Trigonometry		
MA 155	90	84	-6
MA 161	96	88	-8
MA 163	98	95	-3
	Analytic Geometry		
MA 155	52	37	-15
MA 161	63	46	-17
MA 163	87	72	-15
	Calculus		
MA 155	15	20	+5
MA 161	28	31	+3
MA 163	47	63	+16

Table 1 indicates a drop in all three groups in the percentage of students who have taken trigonometry in high school and a more pronounced drop in the percentage of those who have taken analytic geometry. The data also shows a concurrent increase in the percentage of high school students who took a calculus course in high school.

Effect on College Performance

The nature of the high school calculus course is itself a variable which must be contended with in designing studies in this area. Obviously, as noted by Burton (1989), Ferrini-Mundy and Gaudard (1992), Small (1986), Steen (1986), and others, the content and level of rigor embodied in the high school calculus courses will vary widely. Hence in considering the questions of effect of high school calculus on college/university-level mathematics performance, several distinctions must be made. Burton and also Ferrini-Mundy and Gaudard based their research on classifying students into four groups:

1. No previous calculus experience.
2. A one-semester course which is a brief, mostly nonrigorous introduction to calculus topics.
3. A full year of non-Advanced Placement high school calculus.
4. A full year of Advanced Placement high school calculus.

For investigative purposes, this study categorized exposure to calculus in high school in a similar manner, except no distinction was made between Advanced Placement and non-Advanced Placement full year high school calculus courses. Grouping separately those students who had an Advanced Placement high school calculus course would have rendered samples too small to be useful for statistical analysis in the present study. Among Subsample 3 (Montana State University Combined), only 14 of 422 students (3.3%) had received such a course. Of the 14 students who had an AP course, 8 were from high schools outside Montana. By contrast, in Ferrini-Mundy and Gaudard's Calculus 1 sample, 103 of 686 students (15%) had taken an AP calculus course. Evidently the Advanced Placement Program is more prevalent in other areas of the country than in Montana. Grouping all students together who had a year of high school calculus, rather than separately categorizing those who had an AP calculus course, is defensible in the present study for two reasons: (1) Since the Advanced Placement Program impacts very few students in the population of this study, it is largely irrelevant to that population. (2) The mean achievement level in Calculus 1 of the few students in this study who had an AP calculus course would likely not differ significantly from the mean achievement level of students who had a full year non-AP calculus course in high school. This contention is supported by the results of Ferrini-Mundy and Gaudard's research, wherein the outcome measures of the two groups were statistically the same.

In some of the research to be cited, each participating student with a high school calculus background was paired with a student of comparable ability (as measured by such yardsticks as class rank if from the same high school or SAT scores) who had no high school calculus background. In this way, it was sought to remove the effects of difference in scholastic ability and thus isolate the effects of high school calculus background on subsequent college mathematics achievement.

Another approach employed in some studies was to use Multiple Linear Regression to derive a predictor of achievement in a given college/university mathematics course without benefit of exposure to calculus in high school. Then the difference between a student's achievement (as measured by an achievement test, in some instances actual examination(s) from the course in question) and his predicted achievement was tested for significance.

Other studies utilized multiple comparison of means methods to compare achievement levels among various high school calculus background groups. In one instance, analysis of covariance, with mathematics aptitude (as measured by mathematics SAT score) as covariate, was used to explore differences in Calculus 1 achievement among several groups of students with different high school mathematics backgrounds.

Investigation of the effects of a high school calculus course on achievement in Calculus 1 and/or Calculus 2 has yielded mixed results.

What is the effect of one semester of calculus in high school on achievement in Calculus 1 at the college/university level? Several studies have addressed this question. Results of the following researchers indicate that students who have had one semester of high school calculus do not achieve significantly better in Calculus 1 at the college/university level: Ferrini-Mundy and Gaudard (1992), McKillip (1965), Rash (1977), Sorge and Wheatley (1977), and Tillotson (1962). Contrary findings were reported by Paul (1970). In his study, students with one semester of high school calculus did outperform their otherwise comparable counterparts who did not have that background.

What is the effect of two semesters (that is, a full year) of calculus in high school on achievement in Calculus 1 at the college/university level? Studies by Burton (1989), Ferrini-Mundy and Gaudard (1992), Paul (1970), Robinson (1970) and Sorge and Wheatley (1977) all rendered the same results: Students who had two semesters of calculus in high school demonstrated greater achievement in Calculus 1 at the college or university than their counterparts who did not have two semesters of calculus in high school. Other data collected by Robinson indicated that students who had two semesters of calculus in high school also outperformed their counterparts without such background in Calculus 2 at the university level.

Austin (1975), in studying the effects of high school calculus on college/university mathematics achievement did not differentiate between those having one semester of calculus and those having two semesters in high school. He tested for achievement with his own instruments in three areas: manipulative skills, problem solving skills and theoretical concepts. Overall achievement was defined to be the sum of these subscores. In his data, there were significant differences between the means of the scores of those students who had studied calculus in high school and those who had not studied the subject there, in overall achievement and in all three subareas of achievement.

Shimizu (1969) compared achievement in Calculus 1 among students who (1) had taken a high school calculus course, (2) had taken an alternate advanced twelfth grade course, namely Analytic Geometry, Survey of Mathematics, or Probability, and (3) had not taken an advanced twelfth grade mathematics course (a control group). She found that students who had taken an advanced course (including calculus) outperformed the control group. However, the calculus students did not achieve better than the students who took the other advanced courses. Shimizu noted that some of the advanced twelfth grade courses were semester courses while others were year courses. The duration of the high school calculus course was not specified.

Two studies are noteworthy by virtue of their currentness. Burton (1989) and also Ferrini-Mundy and Gaudard (1992) used data gathered at the University of New Hampshire. In both studies, students in Calculus 1 were categorized as having either (1) no previous calculus experience, (2) a brief (one semester or less) introduction to calculus, (3) a year-long high school calculus course, or (4) a full year of Advanced Placement high school calculus. From her analysis, Burton claims (p. 351) "The lack of the year of high school calculus [either an AP or non-AP course] can seriously handicap the first year university student" attempting Calculus 1. Burton's data (p. 352) comparing Calculus 1 course grade outcomes of those having a year of high school calculus with those having a semester or less, are reproduced in Table 2.

Table 2. Burton's Calculus 1 Achievement Data.

	A	B	C	D	F
Minimal (or no) previous calculus (370)	9	53	99	94	115
Full year of high school calculus (313)	47	119	95	33	19

Burton notes that overall, the results for students with minimal prior calculus experience is disappointing. Meanwhile, the grades of students with a year of high school calculus preparation concentrate in the B and C brackets, but not in the A bracket. Despite their background, these students do not appear to be overqualified for this so-called introductory course, according to Burton.

Burton (1989) also studied high school calculus background in conjunction with another predictor of Calculus 1 success, a Mathematical Association of America test in algebra and trigonometry, which was given as a calculus pretest during the first week of the Calculus 1 course. Data relating to this is presented in Table 3 (p. 352). Burton makes several observations from these data. (1) Students who pass the pretest are more likely to be successful in Calculus 1. (2) The apparent disadvantage of lack of prior calculus experience is lessened for students who can pass the calculus pretest. (3) The group at greatest risk is those students with little or no calculus background and who could not

Table 3. Burton's Calculus Pretest Results.

		A	B	C	D	F
Minimal (or no) previous calculus:	Failed pretest	0	9	37	31	80
	Passed pretest	8	43	57	57	30
Full year of high school calculus:	Failed pretest	3	20	35	18	12
	Passed pretest	42	96	59	13	6

pass the calculus pretest. (4) Students represented by the second line of the table, while lacking calculus background, nonetheless reflect the stated prerequisites of Calculus 1 by virtue of passing the algebra-trigonometry pretest. Yet, their letter-grade results are disappointingly poor. Burton concludes that students with at least some prior exposure to calculus (even a "year-long but watered-down" course (p. 351)) are at a comparative advantage in Calculus 1 over students for whom the subject is new.

The study of Ferrini-Mundy and Gaudard (1992) investigated the effects of the aforementioned levels of high school calculus on achievement in Calculus 1 in the same sample of students utilized by Burton (1989) at the University of New Hampshire. To control the parallel effects of mathematical aptitude, they employed analysis of covariance in making multiple comparison of achievement means. The covariate measuring mathematics aptitude (as contrasted to achievement) was the student's SAT mathematics score. Achievement was measured on three examinations and a final examination. For each high school background group and each examination, a mean score was calculated. From this, an adjusted mean was calculated adjusting for the effects of the covariate, the SAT mathematics score. No adjusted means were calculated for Examination 1 since the parallelism assumption necessary for analysis of covariance was not confirmed for that measure. The course grade scale corresponds an 11 to A, 10 to A-, 9 to B+, et cetera. Selected data from Ferrini-Mundy and Gaudard's study (p. 62) are given in Table 4.

Table 4. Ferrini-Mundy and Gaudard's Calculus 1 Achievement Data.

High School Calculus Level	Exam 2	Exam 3	Final Exam	Course Grade
No calculus Adjusted M	53.3	54.8	15.8	3.2
Brief introduction Adjusted M	55.6	58.0	17.1	3.8
Year Adjusted M	66.0	70.6	20.4	5.9
Year AP Adjusted M	70.7	74.0	21.8	6.7

Note: Brackets join all groups which did not differ significantly at the 0.05 level.

Examination of the data in Table 4 as well as other data reported in their study led Ferrini-Mundy and Gaudard to several conclusions: Students who had a year of high school calculus (an AP course or otherwise) achieved at a significantly higher level than students who had either no high school calculus experience or a brief introductory course. A brief (typically one semester) introductory course offered very little advantage in comparison to no high school calculus background. Students who had a full year high school calculus course performed significantly better than other groups throughout the Calculus 1 course. This advantage was manifested more strongly in procedural than in conceptual test items. Students with more high school calculus background were more likely to continue into the second semester college calculus course, Calculus 2. However, there were no significant differences in achievement among the four high school calculus groups in Calculus 2. This suggests that any benefits of high school calculus background are relatively short-lived.

What effect does a background of high school calculus have on achievement in advanced mathematics coursework, that is, coursework beyond second semester calculus? Contrary to Ferrini-Mundy and Gaudard's conclusions (1992), Pocock's research (1974) suggests that those who have had calculus in high school are more successful in second year calculus and post-calculus college/university mathematics.

Effects of Advanced Placement

What are the effects of Advanced Placement in college/university mathematics courses concomitant with having had calculus in high school?

Using a self-designed calculus achievement test, Dickey (1986) sought to determine whether students in Advanced Placement high school calculus courses are learning calculus at a level similar to that of college students. He found that the high school students scored as well or better than their college counterparts.

Concerning placement, Fry's study (1973) rendered several relevant results: Advanced Placement students omitting Calculus 1 at the college or university showed greater achievement than regular placement students in Calculus 2 and Calculus 3. Furthermore, the two groups demonstrated equal achievement in a post-calculus linear algebra course. Also, Advanced Placement students omitting both Calculus 1 and Calculus 2 showed equal achievement with their regular placement counterparts in Calculus 3 and significantly greater achievement in Calculus 4 and linear algebra than the regular placement students. These results are mostly consistent with those of Bergeson (1967). In his study, advanced placement of students who had calculus in high school did not adversely affect performance in advanced mathematics courses. The works of Dickey (1986), Fry, and Bergeson all indicate that students who have successfully completed a high school calculus course can (and perhaps should) receive advanced placement in college/university mathematics courses.

NCTM and CUPM Positions

The NCTM has adopted a neutral stance regarding high school calculus courses per se. The NCTM "does *not* advocate the *formal* study of calculus in high school for all students or even for college-intending students" (NCTM, 1989, p. 180). Nor does the NCTM categorically oppose such study. Rather, they advocate systematic inclusion of natural extensions of topics in algebra, geometry, trigonometry and coordinate geometry to motivate the basic ideas of calculus, such as the limit, area under a curve, the rate of change and slope of a tangent line, and infinite sums (series). Such instruction should be exploratory and based on numerical and geometric activities that utilize both the calculator and instructional computer software.

> Although developing a foundation for the future study of calculus remains a goal of the 9 - 12 curriculum for college-intending students, equally important is the development of prerequisite understandings for further study of statistics, probability, and discrete mathematics. [The NCTM] calls for a new balance of skills, concepts and applications in that portion of the curriculum traditionally associated with preparation for calculus. Instead of devoting large blocks of time to developing a mastery of paper-and-pencil manipulative skills, more time and effort should be spent on developing a conceptual understanding of key ideas and their applications. (NCTM, 1989, p. 182)

The Committee on the Undergraduate Program in Mathematics (CUPM) Panel on Calculus Articulation (1987) has recommended that calculus taught in a high school setting should be a year-long course incorporating the content of the Advanced Placement syllabus. In lieu of this, the CUPM suggests that a sound background in algebra, trigonometry and coordinate geometry is preferable to the common one-semester introductory calculus course.

Effects of Mathematics Anxiety and Gender

Richardson and Suinn (1972, p. 551) define mathematics anxiety as

> feelings of tension and anxiety that interfere with manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations.

Studies by Austin-Martin, Waddell, and Kincaid (1980) and Betz (1978) both indicated that mathematics anxiety has an adverse impact on mathematics achievement at the post-secondary level. Therefore, in the present study, data measuring mathematics anxiety was collected for each student to account for the variance in achievement in Calculus 1 attributable to mathematics anxiety. Aichele (1978, p. 36) believes mathematics anxiety is more widespread in women, noting that "Women in our culture have not been expected to excel in mathematics. . . . the 'mathematical mind' has been considered a male attribute." Tobias (1976, p. 57) echoes this view, asserting that mathematics anxiety in women is rooted in

> . . . a culture that makes mathematic ability a masculine attribute, that punishes women for doing well in mathematics, that soothes the slower mathematics learner by telling her she does not have a "mathematic mind."

Data collected by Betz confirmed that women, more than men, suffer from the effects of mathematics anxiety. Since gender has been shown to be a relevant factor in mathematics achievement, it was included as an attribute variable in this study.

CHAPTER 3

PROCEDURES, METHODS, AND ANALYSES OF DATA

Procedures

The purpose of this study was to investigate the effects of one-semester and two-semester high school calculus courses on achievement in the traditional first quarter course in calculus for mathematics, science, and engineering majors at Montana State University and the first semester course in calculus at Montana College of Mineral Science & Technology.

Chapter 3 begins with a description of the population and the sampling procedure used, and is followed by the questions to be answered, expressed in hypothesis form. The investigative categories are defined and measures to control contaminating variables are then discussed. Next, the methods of data collection are outlined along with measures to assure accuracy of the data. Chapter 3 concludes with a list of statistical hypotheses and an explanation of the data analysis techniques that were employed.

Population Description and Sampling Procedure

The population of the Investigative Study consisted of all students in Math 181 (Calculus 1) at Montana State University (including the honors section) during the 1989 autumn quarter, and all students in Math 121 (Calculus 1) at Montana Tech during the 1989-1990 academic year. To be included in the population, the student had to participate in the course to the extent of receiving any one of the following grades for the course: A, B, C, D, F, or WF (withdraw fail). Those receiving a grade of I (incomplete), P (pass) or WP (withdraw pass) were not included in the population. Foreign students and other students completing their high school or equivalent education outside the United States were excluded from the population. Also omitted were students completing their secondary education through the General Educational Development Program (GED). As noted earlier, some students had to be dropped from the sample because of incomplete data.

Montana State University students receiving either an F or WF grade in Math 181 who did not take the final examination were included in the population if they took all three regular term examinations. In these instances, the student's Math 181 achievement score was defined as the student's raw total score on the three regular term examinations (300 points possible) plus a "presumed" final examination score calculated in the manner illustrated: An individual with a missing final examination had a total score on the three midterm examinations that was (for example) 84% of the corresponding mean for the entire sample group. The student's presumed final examination score was then defined to be 84% of the final examination mean for the entire sample group that took the final examination. For a student in the Montana State University population who took the final examination and was missing only one midterm score, a presumed midterm examination score was calculated and used in a like manner to determine an achievement score. Only a small portion of the student sample required the insertion of a presumed score (approximately 20 out of 350). However, it was desirable to include in the sample failing students who did not take the final examination or who missed one midterm examination, since excluding them would bias the sample in a systematic way by excluding only students who were experiencing serious difficulty in Calculus 1. Since the achievement measure for Montana Tech Math 121 students is the final examination score only, F and WF students not taking the final examination were excluded from the population. The sample for this study included all students in the population as it is defined here except for two (of four) autumn Math 121 sections at Montana Tech. Due to administrative difficulties beyond the researcher's control, suitable data from the omitted sections could not be obtained.

Students in the Montana State University sample for whom there were two or three missing examination scores were not assigned an achievement score and were utilized only in the Chi Square Analysis which allows utilization of students whose achievement score data is only nominal in nature. For this purpose, students were grouped by achievement into one of three categories per whether they (1) did not complete the course, as indicated by no achievement score, (2) attained an achievement score in the lower 50th percentile for the sample group as a whole, or (3) achieved in the upper 50th percentile for the sample group as a whole.

Chi Square Analysis to include students with no achievement score was not done for the Montana Tech population, since achievement score was based wholly on the final examination and students who did not take the final examination were typically expunged from the instructor's grade records.

Montana State University, located in Bozeman, Montana, is an accredited, comprehensive land grant institution established in 1893 under the authorization of the Morrill Act of 1862. Bozeman is a small city of approximately 25,000 people. The university offers undergraduate and graduate programs in liberal arts, basic sciences, the professional areas, agriculture, architecture, business, nursing, education and engineering. The student population in 1987 was approximately 9900 (58% men, 42% women), of which 84% were Montana residents; 29% were over 25 years of age. Montana State University has approximately 500 resident faculty members, more than two-thirds of whom hold doctorates. The student faculty ratio is approximately 18:1 (Montana State University Office of Publications and News Services, 1988).

Montana State University operated on the academic quarter system when the study was conducted.

The Department of Mathematical Sciences at Montana State University consists of approximately 25 full-time instructional faculty members, about an equal number of graduate teaching assistants, and several part-time faculty members. Department records indicate that at the conclusion of autumn quarter 1989, there were approximately 407 students enrolled in 14 sections of Math 181. The average class size was therefore approximately 29 students per section. These figures include students who received a grade of WF for the course, but do not include those receiving a grade of I, P, or WP for the course.

Montana Tech, one of six units along with Montana State University which comprise the Montana University System, is located in Butte, Montana, population 35,000. This institution is an accredited engineering, science and technology-based school offering undergraduate and graduate programs with a strong orientation toward business and industry. Established in 1895 as the Montana School of Mines, the college has historically focused on mineral and energy-related professional engineering programs. In a comprehensive, wide-ranging evaluation of college and university programs in 1987, <u>U.S. News & World Report</u> gave Montana Tech the top ranking nationwide for undergraduate science programs in a category of smaller comprehensive colleges. ". . . when presidents of the smaller comprehensive colleges were asked to name top schools in the area of science and technology, Montana Tech was their first choice largely because of a superb program in minerals engineering" (p. 67).

Recently, degree programs have been added in the basic sciences supporting the engineering programs and in fields related to the administration, application and societal impact of the engineering programs. There are approximately 100 academic faculty members. Montana Tech operates on the academic semester system.

In autumn, 1989, there were nine full-time mathematics faculty members. Montana Tech offers B.S. degrees in mathematics and computer science. Student enrollment in Math 121 was approximately 180 in autumn semester 1989 and approximately 100 in spring semester 1990.

Investigative Categories

Investigating the effect of prior high school calculus on achievement in college/university Calculus 1 was the focus of this study. For this purpose, the Montana State University and Montana Tech student samples were subdivided into the seven subsamples defined earlier. These were (a) Subsample 1, Montana State University New Freshmen; (b) Subsample 2, Montana State University Complement; (c) Subsample 3, Montana State University Combined; (d) Subsample 4, Montana Tech New Freshmen; (e) Subsample 5, Montana Tech Freshmen Complement; (f) Subsample 6, Montana Tech Complement; and (g) Subsample 7, Montana Tech Combined. Each student was placed in the appropriate subsample according to the date of high school graduation noted on his high school transcript, and for Montana Tech freshmen, consideration of any college precalculus mathematics courses taken.

Each subsample was divided into three investigative categories:

<u>Category 1: No Calculus</u>: Students who have completed no high school mathematics course meeting the criteria of either Categories 2 or 3.

<u>Category 2: One-Semester Calculus</u>: Students who have completed a one-semester high school calculus course.

<u>Category 3: Two-Semester Calculus</u>: Students who have completed a two-semester high school calculus course.

Each student was placed in an investigative category per his high school calculus background. In many cases, the appropriate categorization for a student was apparent from the mathematics course titles on the student's high school transcript. For example, a transcript listing only mathematics coursework up to "Algebra II" , "Algebra III," "Trigonometry," or "Trig/Analytic Geometry" indicated a placement in Category 1, No Calculus. Transcript course titles such as "Calculus" (sem) or "Intro to Calc" (sem) dictated assignment to Category 2, One-Semester Calculus. Several course titles clearly indicated placement in Category 3, Two-Semester Calculus. These included "Calculus" (yr), "AP Calculus," and "Math 5 Calculus" (yr).

Many course titles were ambiguous with respect to course content, including such rubrics as "Math 4," "Advanced Math," "College Math," and "Senior Math." In these cases the high school in question was contacted by the researcher, in most instances by mail, for specific information regarding course content. This information was solicited in the form of a topic questionnaire (see Appendix H) prepared by the researcher. Topics listed on the questionnaire include the usual assortment of concepts generally included in a first calculus course(s). These are topics in limits, differentiation, and integration of real functions of one variable. Categorization was based on the course content and duration as indicated by the school's response to the topic questionnaire. Eighty-two Montana high schools and sixteen high schools outside Montana were contacted for this information.

Sample sizes and Calculus 1 achievement means of all subsamples and investigative categories thereof are given in Table 5.

Variable Controls

The multiple sections of Calculus 1 at Montana State University are tested under a common hour examination system. All sections of the course are given the same midterm examinations and same final examination at a common evening hour. Each problem is graded on all the examinations by a single designated instructor, with all the course instructors utilized as graders to an approximately equal extent. Under this system, uniformity in grading within and across the sections is greatly enhanced. Because of this uniformity, course examinations were utilized in this study as the measure of student achievement for the

Table 5. Sample Sizes and Achievement Means.

Group		Sample Size	Achievement Mean
Subsample 1	MSU FROSH	277	322.2996
Category 1	NOHSCALC	119	312.0672
Category 2	SEMCALC	74	326.5405
Category 3	YRHSCALC	84	366.0357
Subsample 2	MSU COMPLEMENT	87	317.6092
Category 1	NOHSCALC	54	312.5000
Category 2	SEMCALC	21	303.5238
Category 3	YRHSCALC	12	365.2500
Subsample 3	MSU COMBINED	364	328.7885
Category 1	NOHSCALC	173	312.2023
Category 2	SEMCALC	95	321.4526
Category 3	YRHSCALC	96	365.9375
Subsample 4	MT TECH FROSH	42	103.5238
Category 1	NOHSCALC	14	95.5714
Category 2	SEMCALC	17	101.1176
Category 3	YRHSCALC	11	117.3636
Subsample 5	MT TECH FROSH COMPLEMENT	77	95.7013
Category 1	NOHSCALC	39	88.1538
Category 2	SEMCALC	25	99.6800
Category 3	YRHSCALC	13	110.6923
Subsample 6	MT TECH COMPLEMENT	93	88.9140
Category 1	NOHSCALC	78	89.1410
Category 2	SEMCALC	11	90.0000
Category 3	YRHSCALC	4	81.5000
Subsample 7	MT TECH COMBINED	135	93.4593
Category 1	NOHSCALC	92	90.1196
Category 2	SEMCALC	28	96.7500
Category 3	YRHSCALC	15	107.8000

Montana State University sample. Course examination scores were deemed suitable as the achievement measure for two other reasons: (1) The examinations were designed by experienced faculty specifically for content validity in the Calculus 1 course as it was taught at Montana State University. (2) The examination scores were the main determining factor (83%) of the student's course grade. The examinations are included in Appendixes B, C, D and E. The 500 possible examination points were combined with a 100 point component based on each instructor's choice of such evaluation measures as homework, quizzes, and attendance. Each student's course grade was based on the student's point total according to the scale shown in Table 6. The instructor-designed 100 point component was not used in this study. This omission was to further control the effects of the instructor variable.

Table 6. Grade Scale of Math 181 (Calculus 1) at Montana State University.

Point Total	Grade
540 - 600	A
480 - 539	B
420 - 479	C
360 - 419	D
0 - 359	F

Since there was no provision for common hour testing at Montana Tech, only the final examination was used as the measure of student achievement. A comprehensive final examination was constructed for content validity by mathematics faculty at Montana Tech. Unknown to the students, this same final examination was used in all sections of Calculus 1 constituting the Montana Tech student sample. Like the Montana State University examinations, all final examinations at Montana Tech were group graded in a manner that assured uniform scoring across sections. The Montana Tech Final Examination is shown in Appendix F.

The Math Anxiety Rating Scale (MARS) is a widely used yardstick for measuring mathematics anxiety in post-secondary students (see Appendix A). This instrument is a 98 item self-rating scale that was administered to all subjects in the study sample. Each item on the scale describes a mathematics-oriented situation which may cause anxiety for the student. The student indicates the degree of anxiety aroused by choosing among the alternatives of "not at all," "a little," "a fair amount," "much," or "very much." Once the student has decided the level of anxiety associated with a specific test item, he indicates that choice by marking the appropriate circle on the answer sheet. Directions are included on each test blank for the student to read. Responses are to indicate a student's anxieties as they currently exist. For scoring, each "not at all" response is numerically weighted 1, each "a little" response is weighted 2, and so forth, with the greatest anxiety level, "very much," weighted 5. The raw sum of the numerical weights of a student's responses constitutes his score.

Reliability and validity of the MARS have been established by at least two studies. Richardson and Suinn (1973) tested and seven weeks later retested 35 University of Missouri students. The first testing yielded a mean MARS score of 235.08 (SD = 51.26). The mean score for the second testing was 232.97 (SD = 56.46). The Pearson product-moment correlation compares favorably with the short-term reliabilities of other measures of social and test anxieties cited by the authors. In the same study, an internal consistency reliability coefficient was found to be alpha = 0.97 (N = 397). In essence, this indicates that the average intercorrelation of the items in the test is quite high. It verifies that the test is reliable and shows that the test items are dominated by one single factor, presumably mathematics anxiety. Item-total correlations for all items were also calculated; over half the correlations were greater than 0.50.

Suinn, Edie, Nicoletti and Spinelli (1973) found a test-retest reliability coefficient of .78 after two weeks, significant at $p < 0.001$. This compares favorably to reliabilities of other anxiety scales in common use cited by the authors.

Concerning normative data, the mean score for Richardson and Suinn's (1973) Missouri students (N = 397) was 215.38 with a standard deviation of 65.29. Suinn, Edie, Nicoletti and Spinelli (1973) obtained a mean MARS score of 187.3 and standard deviation of 55.5 from a sample of 119 students. The mean and standard deviation of MARS scores of students in the present study are comparable to these normative results; the mean being 205.507 and the standard deviation being 62.347.

A placement test score was included in the data collected for each student in the sample. This information was included as an independent variable in the Multiple Regression Analysis as a measure of each student's scholastic aptitude. Data procured from the registrars' student records indicates approximately half the students had taken the ACT, half had taken the SAT and some had taken both. For purposes of statistical analysis, these scores were normalized, that is, converted to Z (Standard Normal Distribution) scores. This converts both ACT and SAT scores to a common scale, with a mean of zero and one standard deviation equal to one. Use of both the ACT and SAT are universal in the United States and the validity and reliability of these instruments are well established.

All other variables considered in the study were demographic in nature: age, gender, minority status, class (freshman, sophomore, et cetera), size of graduating high school, academic major, and whether the student took a mathematics course in her senior year of high school. This information was obtained from examination of student college records and high school transcripts.

Method of Collecting Data

The high school transcript of each sample student, obtained from Montana State University or Montana Tech student records, was examined. Students whose transcripts showed no exposure to calculus were placed in Category 1. Those who completed a one-semester calculus course were placed in Category 2, and those who had completed a year-long study of calculus in high school were placed in Category 3. For each student whose status was uncertain, school officials at the student's graduating high school were contacted. The necessary information about the content of the student's senior mathematics course was obtained via the course questionnaire. This information was the basis for placing the student in the appropriate investigative category.

The MARS was administered during regularly scheduled class meetings to all sample students within the first two weeks of the Calculus 1 course. The response sheets were electronically scored and tabulated by the Montana State University Testing Service.

Examinations used to measure student achievement were given under the usual controlled classroom conditions at regular intervals during the quarter at Montana State University. The comprehensive final examinations at both Montana State University and Montana Tech were given at the conclusion of the term. All examination scores were obtained directly from instructor records.

The information related to all other variables in the study was obtained from Montana State University and Montana Tech student records.

Method of Organizing Data

All hypotheses were tested separately for the Montana State University and Montana Tech student samples. For this purpose, the

Montana State University data and Montana Tech data were maintained in separate files. For both bodies of data, information on each sample student was coded per the variable definitions indicated in Table 7.

Table 7. Variable Definitions.

Variable	Number	Category
Predictor Variables		
X_1	NOHSCALC	*no high school calculus
X_2	SEMCALC	*one semester of high school calculus
X_3	AGE	current age of student
X_4	CLASS	freshman, sophomore, et cetera
X_5	GENDER	*gender
X_6	HSMATH	*took mathematics high school senior year
X_7	HSSIZE	enrollment size of graduating high school
X_8	MAJOR	*academic major
X_9	MARS	MARS score
X_{10}	MINORITY	*minority status
X_{11}	PTS	placement test score
Criterion Variable		
Y		Calculus 1 achievement score

*These variables are binary coded. All other variables are continuous.

High school calculus background was coded with two binary variables:

A student having no high school calculus was coded

NOHSCALC $X_1 = 2$,

SEMCALC $X_2 = 1$.

A student having one semester of high school calculus was coded

NOHSCALC $X_1 = 1$,

SEMCALC $X_2 = 2$.

Students having a year of high school calculus were coded

NOHSCALC $X_1 = 1$,

SEMCALC $X_2 = 1$.

Several variables were binary coded, as indicated in Table 7. The variable (Class) X_4 was assigned a 1 for freshman, 2 for sophomore, and so forth. Academic majors were grouped as follows: The value

(MAJOR) $X_8 = 1$

represented a mathematics, science or engineering major. The value

$X_8 = 2$

indicated a major other than mathematics, science or engineering. Placement Test Score (PTS) X_{11} is a continuous variable representing the Z score-equivalent of the student's ACT or SAT score. The other variables, (MARS) X_9, (HSSIZE) X_7, (AGE) X_3, and (ACHIEV) Y are also continuous, assuming number values of the parameters they represent.

Within both the Montana State University and Montana Tech data files, the year of high school graduation for each student was included. This information was not utilized in any statistical analysis of the data, since it presumably would correlate very strongly with the age variable. However, this data item did provide a convenient means of accessing the different subsamples in an obvious way: New freshmen were those with a 1989 graduation date, while the complement consists of all those with an earlier graduation date.

Statistical Hypotheses Tested

To investigate the effects of high school calculus on achievement in college/university Calculus 1, this study tested several null hypotheses. Hypotheses 1, 2, and 3 were tested for each Montana State University and Montana Tech subsample using the Multiple Linear Regression procedure.

1. There is no significant correlation between lack of high school calculus background and achievement in Calculus 1.
2. There is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.
3. There is no subset of the variables X_1 - X_{11} which provides a unique and significant contribution to the prediction of achievement in Calculus 1 (variable Y). Restated, there is no subset of the variables X_1 - X_{11} which significantly correlates with Y.

Hypothesis 4 was tested for each Montana State University and Montana Tech subsample using ANOVA. In the instances when Hypothesis 4 was rejected, the Newman-Keuls procedure was used to make multiple comparison of means.

4. There is no significant difference in mean achievement in Calculus 1 among students who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

Hypothesis 5 was tested for each Montana State University subsample using the Chi Square Test of Independence. For purposes of Hypothesis 5, a student's success in Calculus 1 is measured by whether the student (1) did not complete the course, as indicated by no achievement score, (2) attained an achievement score in the lower 50th percentile for the sample group as a whole, or (3) achieved in the upper 50th percentile for the sample group as a whole.

5. Success in Calculus 1 is independent of high school calculus background.

Experimental Design

This study investigated the effects of one-semester and two-semester calculus courses on achievement in the first course in calculus at the college/university level. The data were analyzed using three methods of statistical inference: (a) Multiple Linear Regression, (b) One-way Analysis of Variance, and (c) Chi Square Test of Independence.

According to Ferguson (1981), Multiple Linear Regression determines the collective and separate contribution of two or more independent variables to the variability of a dependent variable. This study sought to account for the variance in student achievement in Calculus 1 by knowledge of high school calculus background and other independent variables. The multiple correlation coefficient R^2 indicates the amount (percentage) of variability of the dependent variable that is accounted for by knowledge of the independent variables. While the focus of this study was high school calculus background, the study attempted to maximize R^2 and to thus reduce the amount of error or unexplained variability of the Calculus 1 student achievement dependent variable by adding the other independent variables, X_3 through X_{11} (see Table 7), to the regression model. For each of the seven Montana State University and Montana Tech subsamples, a linear regression model,

$$Y = B_1X_1 + B_2X_2 + \ldots + B_{11}X_{11} + \text{Constant}$$

or

$$Y_z = \beta_1Z_1 + \beta_2Z_2 + \ldots + \beta_{11}Z_{11}$$

was constructed by the stepwise method, as follows. The independent variable demonstrating the highest (simple) Pearson correlation to Y was the first variable incorporated into the model:

$$Y = B_iX_i + \text{Constant}; \quad \text{for some } i, \quad 1 \leq i \leq 11.$$

Next was entered the variable resulting in the greatest increase in R^2; that is, the variable giving maximum increase in the variability of Y that is accounted for collectively by the variables in the model. The model then assumed the form

$$Y = B_iX_i + B_jX_j + \text{Constant}; \quad \text{for } i, j; \ 1 \leq i,j \leq 11.$$

This process of including in the model, one by one, the "most important" of the remaining unused variables continued until all the independent variables were part of the model, or until none of the remaining unused variables would significantly increase (at the 0.05 level of significance) the R^2 from the previous model.

The value of R^2 represents the proportion (percentage) of variability of the dependent variable that is accounted for by knowledge of the independent variables in the regression model. The adjusted R^2 value is the expected degree of fit of the regression model to the total population represented by the sample data. Stated differently, the adjusted R^2 reflects the expected percentage of variability of the dependent variable Y explained by knowledge of the independent variables when the model is applied to the full population. Shrinkage is defined as the difference,

$$\text{shrinkage} = R^2 - \text{adjusted } R^2.$$

Shrinkage can be lessened by increasing the ratio of sample size to variables in the regression model (Ferguson, 1981).

Since the eleven independent variables, X_1 - X_{11}, are measured in different units, it is difficult to determine the relative importance of each variable on the basis of raw-score (B value) coefficients alone. Since the relative contribution of each variable was of ancillary interest, standardized regression coefficients (beta weights, β_i) were calculated. Each beta weight value indicates the incremental contribution of the corresponding independent variable in the presence of the other included variables. That is, β_i indicates the unique contribution of X_i (or Z_i) to the variance of the dependent variable Y. Therefore, performing a T Test of Significance on β_i is a means of evaluating whether X_i (or Z_i) makes a significant contribution with the influence of the other included independent variables controlled. At each step in the process, an F Test tested the significance of R^2, against the null hypothesis,

$$H_o: R^2 = 0.$$

At each step in the process, a T Test of Significance for each beta weight in the model tested the null hypothesis,

$$H_o: \beta_i = 0.$$

Rejection implied that X_i provided a significant and unique contribution to the prediction of Y after the other included variable(s) had been taken into account.

One-way Analysis of Variance (ANOVA) was performed with each subsample to test for equivalence of Calculus 1 mean achievement ($\bar{Y}$) among the three high school calculus background categories. Retention of the null hypothesis

$$H_o: \bar{Y}_1 = \bar{Y}_2 = \bar{Y}_3$$

indicates no significant difference in Calculus 1 mean achievement among the different high school calculus background categories. Rejection of H_o indicates a significant difference between at least one pair of student achievement means (Ferguson, 1981). When this outcome occurred, each of the three pairs of means was tested for equivalence using the Newman-Keuls Multiple Comparison Technique.

The achievement mean and sample size are given for each high school calculus group. An Analysis of Variance summary table for mean student achievement was generated for each test of Hypothesis 4. Included are degree of freedom values for both between-group and within-group sources of variance. Sum-of-square (SS) and mean-square (MS) values for both sources were included. The F statistic

$$F = (MS_{Between})/(MS_{Within})$$

was tested at the level of significance alpha = 0.05. This test determined whether H_0 was retained or rejected in Hypothesis 4.

When H_0 was rejected in Hypothesis 4, multiple pairwise comparison of means proceeded using the Newman-Keuls procedure as prescribed by Ferguson (1981), Christensen (1977), and Howell (1987):

Step 1: Arrange the student achievement means $\bar{Y}_J$ for the three high school calculus background groups in order from largest to smallest.

Step 2: Compute all differences of pairs of means. These differences form a triangular matrix of values $D_{J,K}$, where $D_{J,K}$ represents the difference

$$D_{J,K} = \bar{Y}_J - \bar{Y}_K\,, \qquad (J > K).$$

See Table 8.

Table 8. Pairwise Differences of Achievement Means.

	Group 1	Group 2	Group 3
Group 1			
2	$D_{2,1}$		
3	$D_{3,1}$	$D_{3,2}$	

Step 3: Using the formula for unbalanced data since the different investigative groups in no case have the same number of students, compute $S_{\bar{X}}$:

$$S_{\bar{X}} = (MS_W/2)(1/N_L + 1/N_s),$$

where MS_W is obtained from the ANOVA table and N_L and N_s are the number of students in the largest and smallest investigative groups respectively.

Step 4: Divide each difference $D_{J,K}$ in Table 8 by S_X, forming a table of studentized range statistic values (Q), as illustrated in Table 9.

Table 9. Studentized Range Statistic Values.

	Group 1	Group 2	Group 3
Group 1			
2	$Q_{2,1}$		
3	$Q_{3,1}$	$Q_{3,2}$	

Step 5: Test $D_{3,1}$ for significance by comparing $Q_{3,1}$ to the critical value of Q for alpha = 0.05 and appropriate degrees of freedom. If $D_{3,1}$ is significant, similarly test $D_{2,1}$. If $D_{3,1}$ is not significant, then $D_{2,1}$ (a smaller difference) can be assumed insignificant without testing. Similarly test $D_{3,2}$ for significance.

A table indicating significantly different pairs of achievement means will be given for each subsample.

Multiple Linear Regression Analysis and ANOVA procedures were used to test hypotheses for each subsample, using only those sample students for whom complete data were available, including the achievement score in Calculus 1. The Chi Square Test of Independence was done for each Montana State University subsample. Each student was placed in one of three categories reflecting his success in Calculus 1. One category was comprised of students who attained an achievement score in the upper 50th percentile of the subsample. Those whose achievement scores were in the lower 50th percentile of the subsample constituted another category. The third category was composed of students who enrolled in Calculus 1 but had no achievement score by virtue of dropping or withdrawing from the course, or otherwise not completing the required three examinations to tally an achievement score. Testing for independence of Calculus 1 success and high school calculus background by Chi Square Analysis allowed utilization of students in the Montana State University sample for whom only nominal achievement data were available, namely those who did not attain an achievement score. These sample students could not be utilized in the other methods of analysis, Multiple Linear Regression and ANOVA. Since information regarding dropped and withdrawn Montana Tech students was not available, Chi Square Analysis was done only with the Montana State University sample.

Analysis of Data

For each inference procedure, One-way Analysis of Variance, Chi Square Test of Independence and Multiple Linear Regression, the analysis of the data was completed in a similar manner for each subsample. The Chi Square Test of Independence utilized all students in the Montana State University sample. For the Analysis of Variance, only students for whom there were Calculus 1 achievement scores were included. Multiple Linear Regression procedures utilized only sample students for whom there was complete data, relating to all the variables enumerated in Table 7. The null hypotheses were tested at the 0.05 level of significance. When testing at the 0.05 level of significance, there are five chances in one hundred of rejecting a true null hypothesis. The consequence of rejecting a true null (Type I error) is the possibility of concluding that there is a greater link between high school calculus background and achievement in Calculus 1 than actually exists. The consequence of a Type II error (retaining a false null hypothesis) is the possibility of failing to recognize an important link between high school calculus background and Calculus 1 achievement. This Type II error could in turn discourage the offering of calculus as a high school subject when it could generally benefit students. Since the consequence of the Type II error was of more importance to this study, the level of significance was set to 0.05 rather than 0.01. This increased the likelihood of a Type I error, but reduced the chance of a Type II error (Ferguson, 1981).

Precautions Taken for Accuracy

In the few instances where manual computation was done (such as calculation of presumed achievement scores for students missing one examination score), the calculations were duplicated and results compared to ensure accuracy. Results of the MARS inventory were electronically determined by the Montana State University Testing Service. All calculations relating to statistical inference were done by the computer facilities at Montana State University. The computer served to expedite the analysis and eliminate mathematical errors.

CHAPTER 4

FINDINGS, INTERPRETATIONS AND DISCUSSION

Findings and Interpretations

The general area of concern of this study was whether a relationship exists between high school calculus background and achievement in college/university Calculus 1. This issue led to the formulation of five hypotheses to be tested by the investigation. These hypotheses were stated in chapter 3 in null hypothesis form. To test these hypotheses, three types of statistical analyses were performed: Multiple Linear Regression, One-way Analysis of Variance (and the Newman-Keuls Multiple Comparison of Means, where appropriate), and the Chi Square Test of Independence.

Each statistical analysis except the Chi Square Analysis was performed on three Montana State University subsamples and four Montana Tech subsamples. Chi Square Analysis was done only on the three Montana State University subsamples. Except for the Chi Square procedure, the Montana State University subsamples included only students for whom sufficient data were available to determine a Calculus 1 achievement score. Only students for whom complete data could be compiled were included in the Montana Tech sample.

Multiple Linear Regression

This statistical analysis relates to Hypotheses 1, 2, and 3, and was repeated for each of the seven subsamples. Null Hypotheses 1-1, 2-1, and 3-1, along with Tables 10-12, pertain to the Multiple Linear Regression Analysis of Subsample 1, Montana State University New Freshmen.

Null Hypothesis 1-1: Among Montana State University New Freshmen, there is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

Null Hypothesis 2-1: Among Montana State University New Freshmen, there is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.

Null Hypothesis 3-1: Among Montana State University New Freshmen, there is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

The independent variables included in the stepwise regression model, as shown in Table 10, are, in order of inclusion, Placement Test Score, X_{11}; High School Size, X_7; No High School Calculus, X_1; One Semester High School Calculus, X_2; and Gender, X_5. The inclusion of each of these variables significantly increased (alpha = 0.05) the value of

R^2, which is the amount (percentage) of variability of Y that is accounted for by the included independent variables.

Table 10. Subsample 1. Stepwise Construction of Linear Regression Model. Hypotheses 1-1, 2-1, and 3-1.

Step	Vars. in Model		R^2	Increase of R^2	$P(R^2)$
1.	X_{11}	PTS	.07486	.07486	.0000
2.	X_7	HSSIZE	.14498	.07012	.0000
3.	X_1	NOHSCALC	.18536	.04038	.0000
4.	X_2	SEMCALC	.21574	.03038	.0000
5.	x_5	GENDER	.24090	.02516	.0000

The variables not included in the regression model by virtue of not significantly increasing R^2 were: Age, X_3; Class, X_4; High School Mathematics Taken as Senior, X_6; Major, X_8; MARS, X_9; and Minority, X_{10}. The addition of all these remaining variables would have increased R^2 from 0.24090 to 0.26816. Thus inclusion of these six additional variables would have collectively accounted for only an additional 2.726% of the variability of Y. The five variables included in the model collectively account for 24.090% of the variability of Y, as indicated by the R^2 value in Table 11. The F Statistic value given in Table 11 indicates that R^2 is significant at an alpha level less than 0.00005. The adjusted R^2 value given in Table 11,

$$\text{adjusted } R^2 = 0.22488,$$

is the expected degree of fit of the regression model to the total population represented by the sample data used. The shrinkage of R^2 in Subsample 1 is

$$\text{shrinkage} = R^2 - \text{adjusted } R^2 = 0.01602.$$

Table 11. Subsample 1. Multiple Linear Regression. Hypotheses 1-1, 2-1, and 3-1.

Multiple R	.49081
R square	.24090
Adjusted R square	.22488
Shrinkage	.01602
Standard error	61.73280

ANALYSIS OF VARIANCE

	DF	Sum of Squares	Mean Square
Regression	5	286624.68028	57324.93606
Residual	237	903192.38968	3810.93835

F = 15.04221 P(F) = .0000

The predictive model created by Multiple Linear Regression for Subsample 1 assumes either of two forms:

$$Y = B_{11}X_{11} + B_7X_7 + B_1X_1 + B_2X_2 + B_5X_5 + C$$

or

$$Z_y = \beta_{11}Z_{11} + \beta_7Z_7 + \beta_1Z_1 + \beta_2Z_2 + \beta_5Z_5.$$

The variables Z_y and Z_j (j = 1, 2, 5, 7, 11) are the standard normal distribution equivalents of Y and X_j respectively. The raw-score coefficient values B_j and corresponding beta weights β_j are shown in Table 12.

Table 12. Subsample 1. Variables in Regression Equation. Hypotheses 1-1, 2-1, and 3-1.

Variable	B	SE B	Beta	T	sig T
X_{11} PTS	36.44569	7.14343	.29366	5.102	.0000
X_7 HSSIZE	.02218	.00627	.20918	3.537	.0005
X_1 NOHSCALC	-42.42100	9.50346	-.29834	-4.464	.0000
X_2 SEMCALC	-32.84675	10.85266	-.20676	-3.027	.0027
X_5 GENDER	24.65546	8.79649	.16023	2.803	.0055
C (Constant)	346.16732	30.53143		11.338	.0000

This model accounts for 24.090% of the variability of Y, since

$$R^2 = 0.24090.$$

Since R^2 differs significantly from zero (P(F) = .0000), Null Hypothesis 3-1 is rejected for the variables included in the model, namely X_1, X_2, X_5, X_7, and X_{11}. Presumably, since

$$\text{adjusted } R^2 = 0.22488,$$

the model would account for 22.488% of the variability of Y if applied to the total population of Montana State University students satisfying the criteria for membership in Subsample 1.

Null Hypotheses 1-1 and 2-1 are both rejected since β_1 and β_2 both differ significantly from zero (alpha < 0.00005), as indicated by the T-statistic values given for β_1 and β_2 in Table 12. Therefore, variables X_1 and X_2 both provide a significant and unique contribution to the prediction of Y after the other variables in the model have been taken into account. Since the beta values for X_1 and X_2 are both negative, these variables, No High School Calculus and One Semester High School Calculus, both correlate negatively with Calculus 1 achievement, Y. This implies that students in Subsample 1 with either no high school calculus background or one semester of high school calculus do not achieve as high as all other students, namely those who took a full year of calculus in high school.

Null Hypotheses 1-2, 2-2, and 3-2, along with Tables 13-15, pertain to the Multiple Linear Regression Analysis of Subsample 2, Montana State University Complement.

> <u>Null Hypothesis 1-2</u>: Among the Montana State University Complement, there is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

> <u>Null Hypothesis 2-2</u>: Among the Montana State University Complement, there is no significant correlation between one

semester of high school calculus background and achievement in Calculus 1.

Null Hypothesis 3-2: Among the Montana State University Complement, there is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

Table 13. Subsample 2. Stepwise Construction of Linear Regression Model. Hypotheses 1-2, 2-2, and 3-2.

Step	Vars. in Model		R^2	Increase of R^2	$P(R^2)$
1.	X_2	SEMCALC	.23527	.23527	.0163

The only independent variable included in the stepwise regression model, as shown in Table 13, was One Semester High School Calculus, X_2. The inclusion of this variable significantly increased (alpha = 0.05) the value of R^2. With only one variable in the model, R is actually the simple Pearson correlation coefficient between X_2 and Y, which is significant at the 0.05 level.

The variables not included in the regression model by virtue of not significantly increasing R^2 were No High School Calculus, X_1; Age, X_3; Class, X_4; Gender, X_5; High School Mathematics Taken as Senior, X_6; High School Size, X_7; Major, X_8; MARS, X_9; Minority, X_{10}; and Placement Test Score, X_{11}. The addition of all these remaining variables would have increased R^2 from 0.23527 to 0.58762. Thus inclusion of these ten variables would have collectively accounted for an additional 35.235% of the variability of Y. The variable in the model, X_2, accounts for 23.527% of the variability of Y, as indicated by the R^2 value in Table 14. The F statistic value given in Table 14 indicates that R^2 is significant at an alpha level 0.0163. The adjusted R^2 value given in Table 14,

$$\text{adjusted } R^2 = 0.20051,$$

is the expected degree of fit of the regression model to the total population represented by the sample data of Subsample 2. The shrinkage of R^2 in Subsample 2 is

$$\text{shrinkage} = 0.03476.$$

Table 14. Subsample 2. Multiple Linear Regression. Hypotheses 1-2, 2-2, and 3-2.

Multiple R	.48504
R square	.23527
Adjusted R square	.20051
Shrinkage	.03476
Standard error	49.26673

ANALYSIS OF VARIANCE

	DF	Sum of Squares	Mean Square
Regression	1	16428.00000	16428.00000
Residual	22	53398.62500	2427.21023

F = 6.76826 P(F) = .0163

The predictive model created by Multiple Linear Regression for Subsample 2 assumes either of two forms:

$$Y = B_2X_2 + C$$

or

$$Z_y = \beta_2 Z_2.$$

The raw score coefficient value B_2 and corresponding beta weight β_2 are shown in Table 15.

Table 15. Subsample 2. Variables in Regression Equation. Hypotheses 1-2, 2-2, and 3-2.

Variable	B	SE B	Beta	T	sig T
SEMCALC	-55.50000	21.33312	-.48505	-2.602	.0163
C (Constant)	406.87500	30.16959		13.486	.0000

This model accounts for 23.527% of the variability of Y since

$$R^2 = 0.23527.$$

Since R^2 differs significantly from zero ($P(F) = 0.0163$), Null Hypothesis 3-2 is rejected for variable X_2. Presumably, since

$$\text{adjusted } R^2 = 0.20051,$$

the model would account for 20.051% of the variability of Y if applied to the total population of Montana State University students satisfying the criteria for membership in Subsample 2.

Null Hypothesis 1-2 is retained since X_1 was not included in the regression model. Null Hypothesis 2-2 is rejected since β_2 differs significantly from zero (alpha = 0.0163), as indicated by the T statistic value given in Table 15. Therefore, variable X_2 provides a significant and unique contribution to the prediction of Y. Since the beta value for X_2 is negative, this variable, One Semester High School Calculus, correlates negatively with Calculus 1 achievement, Y. This implies that students in Subsample 2 with one semester of high school calculus do not achieve as high as all other students, namely those with no high school calculus background or a full year of high school calculus.

Null Hypotheses 1-3, 2-3, and 3-3, along with Tables 16-18 pertain to the Multiple Linear Regression Analysis of Subsample 3, Montana State University Combined.

<u>Null Hypothesis 1-3</u>: Among the Montana State University Combined sample, there is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

<u>Null Hypothesis 2-3</u>: Among the Montana State University Combined sample, there is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.

<u>Null Hypothesis 3-3</u>: Among the Montana State University Combined sample, there is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

The independent variables included in the stepwise regression model, as shown in Table 16, are, in order of inclusion, Placement Test Score, X_{11}; High School Size, X_7; No High School Calculus, X_1; One

Semester High School Calculus, X_2; and Gender, X_5. The inclusion of each of these variables significantly increased (alpha = 0.05) the value of R^2.

Table 16. Subsample 3. Stepwise Construction of Linear Regression Model. Hypotheses 1-3, 2-3, and 3-3.

Step	Vars. in Model		R^2	Increase of R^2	$P(R^2)$
1.	X_{11}	PTS	.07188	.07188	.0000
2.	X_7	HSSIZE	.14447	.07259	.0000
3.	X_1	NOHSCALC	.17839	.03392	.0000
4.	X_2	SEMCALC	.21547	.03708	.0000
5.	X_5	GENDER	.24222	.02675	.0000

The variables not included in the regression model by virtue of not significantly increasing R^2 were Age, X_3; Class, X_4; High School Mathematics Taken as Senior, X_6; Major, X_8; MARS, X_9; and Minority, X_{10}. The addition of all these remaining variables would have increased R^2 from 0.24222 to 0.26343. Thus inclusion of these six variables would have collectively accounted for an additional 2.121% of the variability of Y. The five variables included in the model collectively account for 24.222% of the variability of Y, as indicated by the R^2 value in Table 17. The F statistic value given in Table 17 indicates that R^2 is significant at an alpha level less than 0.00005. The adjusted R^2 value given in Table 17,

$$\text{adjusted } R^2 = 0.22770,$$

is the expected degree of fit of the regression model to the total population represented by the sample data of Subsample 3. The shrinkage of R^2 in Subsample 3 is

$$\text{shrinkage} = 0.01452.$$

Table 17. Subsample 3. Multiple Linear Regression. Hypotheses 1-3, 2-3, and 3-3.

Multiple R	.49216
R square	.24222
Adjusted R square	.22770
Shrinkage	.01452
Standard error	60.49661

ANALYSIS OF VARIANCE

	DF	Sum of Squares	Mean Square
Regression	5	305327.28724	61065.45745
Residual	261	955218.20340	3659.83986

F = 16.68528 P(F) = .0000

The predictive model created by Multiple Linear Regression for Subsample 3 assumes either of two forms:

$$Y = B_{11}X_{11} + B_7X_7 + B_1X_1 + B_2X_2 + B_5X_5 + C$$

or

$$Z_y = \beta_{11}Z_{11} + \beta_7Z_7 + \beta_1Z_1 + \beta_2Z_2 + \beta_5Z_5.$$

The raw-score coefficient values B_j and corresponding beta weights β_j are shown in Table 18.

Table 18. Subsample 3. Variables in Regression Equation. Hypotheses 1-3, 2-3, and 3-3.

Variable	B	SE B	Beta	T	sig T
X_{11} PTS	34.52160	6.64813	.28357	5.193	.0000
X_7 HSSIZE	.02175	.00587	.20974	3.706	.0003
X_1 NOHSCALC	-41.82402	8.93764	-.30037	-4.680	.0000
X_2 SEMCALC	-36.24291	10.23774	-.23408	-3.540	.0005
X_5 GENDER	24.68029	8.13167	.16454	3.035	.0026
C (Constant)	353.14810	28.64839		12.327	.0000

This model accounts for 24.222% of the variability of Y since

$$R^2 = 0.24222.$$

Since R^2 differs significantly from zero (P(F) = .0000), Null Hypothesis 3-3 is rejected for the variables included in the model, namely X_1, X_2, X_5, X_7, and X_{11}. Presumably , since

$$\text{adjusted } R^2 = 0.22770,$$

the model would account for 22.770% of the variability of Y if applied to the total population of Montana State University students satisfying the criteria for membership in Subsample 3.

Null Hypotheses 1-3 and 2-3 are both rejected since β_1 and β_2 both differ significantly from zero, as indicated by the T statistic values given for β_1 and β_2 in Table 18. (For β_1, alpha < 0.00005, and for β_2, alpha = .0005.) Therefore, variables X_1 and X_2 both provide a significant and unique contribution to the prediction of Y after the other variables in the model have been taken into account. Since the beta values for X_1 and X_2 are both negative, these variables, No High School Calculus and One Semester High School Calculus, both correlate negatively with Calculus 1 achievement, Y. This implies that students in Subsample 3 with either no high school calculus background or one semester of high school calculus do not achieve as high as all other students, namely those who took a full year of calculus in high school.

Null Hypotheses 1-4, 2-4, and 3-4, along with Tables 19-21, pertain to the Multiple Linear Regression Analysis of Subsample 4, Montana Tech New Freshmen.

Null Hypothesis 1-4: Among Montana Tech New Freshmen, there is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

Null Hypothesis 2-4: Among Montana Tech New Freshmen, there is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.

Null Hypothesis 3-4: Among Montana Tech New Freshmen, there is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

The only independent variable included in the stepwise regression model, as shown in Table 19, was Gender, X_5. The inclusion of this variable significantly increased (alpha = 0.05) the value of R^2. With only one variable in the model, R is actually the simple Pearson correlation coefficient between X_5 and Y, which is significant at the 0.05 level.

Table 19. Subsample 4. Stepwise Construction of Linear Regression Model. Hypotheses 1-4, 2-4, and 3-4.

Step	Vars. in Model		R^2	Increase of R^2	$P(R^2)$
1.	x_5	GENDER	.21321	.21321	.0035

The variables Class, X_4; Major, X_8; and Minority, X_{10} were not included in the regression procedure since no correlation could be computed for them. This is because they were constants. All students in Subsample 4 were (by definition of Subsample 4) freshmen. All had the same Major classification, namely

$$X_8 = 1,$$

representing a mathematics, science, or engineering major. All were nonminority students.

The variables not included in the regression model by virtue of not significantly increasing R^2 were No High School Calculus, X_1; One Semester High School Calculus, X_2; Age, X_3; High School Mathematics Taken as Senior, X_6; High School Size, X_7, MARS, X_9; and Placement

Test Score, X_{11}. The addition of all these remaining variables would have increased R^2 from 0.21321 to 0.35076. Thus inclusion of these seven variables would have collectively accounted for an additional 13.755% of the variability of Y. The variable in the model, X_5, accounts for 21.321% of the variability of Y, as indicated by the R^2 value in Table 20. The F statistic value given in Table 20 indicates that R^2 is significant at an alpha level

$$\text{alpha} = 0.0035.$$

The adjusted R^2 value given in Table 20,

$$\text{adjusted } R^2 = 0.19136,$$

is the expected degree of fit of the regression model to the total population represented by the sample data of Subsample 4. The shrinkage of R^2 in Subsample 4 is

$$\text{shrinkage} = 0.02185.$$

Table 20. Subsample 4. Multiple Linear Regression. Hypotheses 1-4, 2-4, and 3-4.

Multiple R	.46175
R square	.21321
Adjusted R square	.19136
Shrinkage	.02185
Standard error	24.20293

ANALYSIS OF VARIANCE

	DF	Sum of Squares	Mean Square
Regression	1	5714.72073	5714.72073
Residual	36	21088.14769	585.78188

F = 9.75571 P(F) = .0035

The predictive model created by Multiple Linear Regression for Subsample 4 assumes either of two forms:

$$Y = B_5X_5 + C$$

or

$$Z_y = \beta_5Z_5.$$

The raw-score coefficient value B_5 and corresponding beta weight β_5 are shown in Table 21.

Table 21. Subsample 4. Variables in Regression Equation. Hypotheses 1-4, 2-4, and 3-4.

Variable	B	SE B	Beta	T	sig T
X_5 GENDER	-25.84923	8.27595	-.46175	-3.123	.0035
C (Constant)	139.92923	11.78072		11.878	.0000

This model accounts for 21.321% of the variability of Y since

$$R^2 = 0.21321.$$

Since R^2 differs significantly from zero (P(F) = 0.0035), Null Hypothesis 3-4 is rejected for the variable included in the model, namely X_5. Presumably, since

$$\text{adjusted } R^2 = 0.19136,$$

the model would account for 19.136% of the variability of Y if applied to the total population of Montana Tech students satisfying the criteria for membership in Subsample 4.

Null Hypotheses 1-4 and 2-4 are both retained since X_1 and X_2 were not included in the regression model. Retention of these hypotheses indicates that neither variable, No High School Calculus, X_1, or One Semester High School Calculus, X_2, accounts for a significant increase in R^2 in the presence of X_5, the variable in the model.

Null Hypotheses 1-5, 2-5, and 3-5, along with Tables 22-24, pertain to the Multiple Linear Regression Analysis of Subsample 5, Montana Tech Freshmen Complement.

Null Hypothesis 1-5: Among the Montana Tech Freshmen Complement, there is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

Null Hypothesis 2-5: Among the Montana Tech Freshmen Complement, there is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.

Null Hypothesis 3-5: Among the Montana Tech Freshmen Complement, there is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

Table 22. Subsample 5. Stepwise Construction of Linear Regression Model. Hypotheses 1-5, 2-5, and 3-5.

Step	Vars. in Model		R^2	Increase of R^2	$P(R^2)$
1.	X_6	HSMATH	.16541	.16541	.0015

The only independent variable included in the stepwise regression model, as shown in Table 22, was High School Mathematics Taken as Senior, X_6. The inclusion of this variable significantly increased (alpha = 0.05) the value of R^2. With only one variable in the model, R is actually the simple Pearson correlation coefficient between X_6 and Y, which is significant at the 0.05 level.

The variables Class, X_4; Major, X_8; and Minority, X_{10} were not included in the regression procedure since no correlation could be computed for them. This is because they were constants. All students in Subsample 5 were (by definition) freshmen. All had the same Major classification,

$$X_8 = 1,$$

representing a mathematics, science, or engineering major. All were nonminority students.

The variables not included in the regression model by virtue of not significantly increasing R^2 were No High School Calculus, X_1; One Semester High School Calculus, X_2; Age, X_3; Gender, X_5; High School Size, X_7; MARS, X_9; and Placement Test Score, X_{11}. The addition of all these remaining variables would have increased R^2 from 0.16541 to 0.29616. Thus inclusion of these seven variables would have collectively accounted for an additional 13.075% of the variability of Y. The variable in the model, X_6, accounts for 16.541% of the variability of Y, as indicated by the R^2 value in Table 23. The F statistic value given in Table 23 indicates that R^2 is significant at an alpha level

$$\text{alpha} = 0.0015.$$

The adjusted R^2 value given in Table 23,

$$\text{adjusted } R^2 = 0.15051,$$

is the expected degree of fit of the regression model to the total population represented by the sample data of Subsample 5. The shrinkage of R^2 in Subsample 5 is

$$\text{shrinkage} = 0.01490.$$

Table 23. Subsample 5. Multiple Linear Regression. Hypotheses 1-5, 2-5, and 3-5.

Multiple R	.40671
R square	.16541
Adjusted R square	.15051
Shrinkage	.01490
Standard error	26.13609

ANALYSIS OF VARIANCE

	DF	Sum of Squares	Mean Square
Regression	1	7581.77810	7581.77810
Residual	56	38253.34259	683.09540

F = 11.09915 P(F) = .0015

The predictive model created by Multiple Linear Regression for Subsample 5 assumes either of two forms:

$$Y = B_6X_6 + C$$

or

$$Z_y = \beta_6X_6.$$

The raw-score coefficient value B_6 and corresponding beta weight β_6 are shown in Table 24.

Table 24. Subsample 5. Variables in Regression Equation. Hypotheses 1-5, 2-5, and 3-5.

Variable	B	SE B	Beta	T	sig T
X_6 HSMATH	-45.12037	13.54340	-.40671	-3.332	.0015
C (Constant)	148.49074	14.87863		9.980	.0000

This model accounts for 16.541% of the variability of Y since

$$R^2 = 0.16541.$$

Since R^2 differs significantly from zero (P(F) = 0.0015), Null Hypothesis 3-5 is rejected for the variable included in the model, namely X_6. Presumably, since

$$\text{adjusted } R^2 = 0.15051,$$

the model would account for 15.051% of the variability of Y if applied to the total population of Montana Tech students satisfying the criteria for membership in Subsample 5.

Null Hypotheses 1-5 and 2-5 are both retained since X_1 and X_2 were not included in the regression model. Retention of these hypotheses indicates that neither variable, No High School Calculus, X_1, or One Semester High School Calculus, X_2, accounts for a significant increase in R^2 in the presence of X_6, the variable in the model.

Null Hypotheses 1-6, 2-6, and 3-6 pertain to the Multiple Linear Regression Analysis of Subsample 6, Montana Tech Complement.

Null Hypothesis 1-6: Among the Montana Tech Complement, there is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

Null Hypothesis 2-6: Among the Montana Tech Complement, there is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.

Null Hypothesis 3-6: Among the Montana Tech Complement, there is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

In an attempt to form a linear regression model correlating the variables X_1 - X_{11} with Calculus 1 achievement, Y, no variable X_j ($1 \leq j \leq 11$) demonstrated an R^2 value (that is, a simple Pearson r value in the first step) that was significantly different from zero (alpha = 0.05). Therefore, knowledge of none of the variables X_1 - X_{11} accounts for a significant amount of variance of Y. Consequently, Null Hypotheses 1-6, 2-6, and 3-6 are all retained. These results imply that in Subsample 6, there is no relation between high school calculus background (or any of the other independent variables X_3 - X_{11}) and achievement in Calculus 1.

Null Hypotheses 1-7, 2-7, and 3-7, along with Tables 25-27, pertain to the Multiple Linear Regression Analysis of Subsample 7, Montana Tech Combined.

Null Hypothesis 1-7: Among the Montana Tech Combined sample, there is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

Null Hypothesis 2-7: Among the Montana Tech Combined sample, there is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.

Null Hypothesis 3-7: Among the Montana Tech Combined sample, there is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

Table 25. Subsample 7. Stepwise Construction of Linear Regression Model. Hypotheses 1-7, 2-7, and 3-7.

Step	Vars. in Model		R^2	Increase of R^2	$P(R^2)$
1.	X_1	NOHSCALC	.07351	.07351	.0195

The only independent variable included in the stepwise regression model, as shown in Table 25, was No High School Calculus, X_1. The inclusion of this variable significantly increased (alpha = 0.05) the value of R^2. With only one variable in the model, R is actually the simple Pearson correlation coefficient between X_1 and Y, significant at the 0.05 level.

The variables not included in the regression model by virtue of not significantly increasing R^2 were One Semester High School Calculus, X_2;

Age, X_3; Class, X_4; Gender, X_5; High School Mathematics Taken as Senior, X_6; High School Size, X_7; Major, X_8; MARS, X_9; Minority, X_{10}; and Placement Test Score, X_{11}. The addition of all these remaining variables would have increased R^2 from 0.07351 to 0.23286. Thus inclusion of these ten variables would have collectively accounted for an additional 15.935% of the variability of Y. The variable in the model, X_1, accounts for 7.351% of the variability of Y as indicated by the R^2 value in Table 26. The F statistic value given in Table 26 indicates that R^2 is significant at an alpha level

$$\text{alpha} = 0.0195.$$

The adjusted R^2 value given in Table 26,

$$\text{adjusted } R^2 = 0.06064,$$

is the expected degree of fit of the regression model to the total

Table 26. Subsample 7. Multiple Linear Regression. Hypotheses 1-7, 2-7, and 3-7.

Multiple R	.27113
R square	.07351
Adjusted R square	.06064
Shrinkage	.01287
Standard error	27.52245

ANALYSIS OF VARIANCE

	DF	Sum of Squares	Mean Square
Regression	1	4327.36287	4327.36287
Residual	72	54538.92092	757.48501

F = 5.71280 P(F) = .0195

population represented by the sample data of Subsample 7. The shrinkage of R^2 in Subsample 7 is

$$\text{shrinkage} = 0.01287.$$

The predictive model created by Multiple Linear Regression for Subsample 7 assumes either of two forms:

$$Y = B_1X_1 + C$$

or

$$Z_y = \beta_1Z_1.$$

The raw-score coefficient value B_1 and corresponding beta weight β_1 are shown in Table 27.

Table 27. Subsample 7. Variables in Regression Equation. Hypotheses 1-7, 2-7, and 3-7.

Variable	B	SE B	Beta	T	sig T
X_1 NOHSCALC	-15.38433	6.43656	-.27113	-2.390	.0195
C (Constant)	123.35403	10.50198		11.746	.0000

This model accounts for 7.351% of the variability of Y since

$$R^2 = 0.07351.$$

Since R^2 differs significantly from zero (P(F) = 0.0195), Null Hypothesis 3-7 is rejected for the variable included in the model, namely X_1. Presumably, since

$$\text{adjusted } R^2 = 0.06064,$$

the model would account for 6.064% of the variability of Y if applied to the total population of Montana Tech students satisfying the criteria for membership in Subsample 7.

Null Hypothesis 1-7 is rejected since β_1 differs significantly from zero, as indicated by the T statistic value given for β_1 in Table 27. (For β_1, alpha = 0.0195.) Therefore, variable X_1 provides a significant and unique contribution to the prediction of Y. Since the beta value for X_1 is negative, this variable, No High School Calculus, correlates negatively with Calculus 1 achievement, Y. This implies that students in Subsample 7 with no high school background do not achieve as high as all other students, namely those who took either a full year or one semester of calculus in high school.

Null Hypothesis 2-7 is retained since X_2 was not included in the regression model. Retention of this hypothesis indicates that the variable X_2 did not account for a significant increase in R^2 in the presence of X_1, the variable in the model.

One-Way Analysis of Variance

ANOVA relates to Hypothesis 4 and was repeated for each of the seven subsamples. In the instances when Null Hypothesis 4 was rejected, the Newman-Keuls procedure was used to make multiple comparison of means. The results are summarized in Tables 28-46. Tables 28-30 pertain to Null Hypothesis 4-1.

<u>Null Hypothesis 4-1</u>: Among Montana State University New Freshmen, there is no significant difference in mean achievement in Calculus 1 among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

The sample sizes and achievement means for the three high school calculus subgroups of Subsample 1 are given in Table 28.

Table 28. Subsample 1. Achievement Means. Hypothesis 4-1.

Category		Count	Mean
Category 1	NOHSCALC	119	312.0672
Category 2	SEMCALC	74	326.5405
Category 3	YRHSCALC	84	366.0357
Total		277	322.2996

Analysis of variance statistics are given in Table 29. The F statistic value,

$$F = 15.6167,$$

is significant at

$$\text{alpha} < 0.00005,$$

dictating the rejection of Null Hypothesis 4-1.

Table 29. Subsample 1. ANOVA. Hypothesis 4-1.

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between groups	2	146769.3965	73384.6983	15.6167	.0000
Within groups	274	1287554.733	4699.1049		
Total	276	1434324.130			

The results of the Newman-Keuls procedure for pair-wise comparison of means are summarized in Table 30.

Table 30. Subsample 1. Comparison of Means. Hypothesis 4-1.

Mean	Category	1	2	3
312.0672	1 NOHSCALC			
326.5405	2 SEMCALC			
366.0357	3 YRHSCALC	*	*	

* Denotes pairs of categories significantly different at the 0.05 level.

The mean achievement of Category 3 (YRHSCALC) is significantly different from (higher than) the mean achievements of both Category 1 (NOHSCALC) and Category 2 (SEMCALC). Both differences are significant at the 0.05 level. In summary, students in Subsample 1, who all had a year of high school calculus, achieved at a significantly higher level than those who had either a semester of high school calculus or no high school calculus. There was no significant difference in mean achievement between those who had no high school calculus and those who had one semester of high school calculus.

Tables 31-33 pertain to Null Hypothesis 4-2.

Null Hypothesis 4-2: Among the Montana State University Complement, there is no significant difference in mean achievement in Calculus 1 among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

The sample sizes and achievement means for the three high school calculus subgroups of Subsample 2 are given in Table 31.

Table 31. Subsample 2. Achievement Means. Hypothesis 4-2.

Category		Count	Mean
Category 1	NOHSCALC	54	312.5000
Category 2	SEMCALC	21	303.5238
Category 3	YRHSCALC	12	365.2500
Total		87	317.6092

Analysis of variance statistics are given in Table 32. The F statistic value,

$$F = 4.0667$$

is significant at

alpha = 0.0206,

dictating the rejection of Null Hypothesis 4-2.

Table 32. Subsample 2. ANOVA. Hypothesis 4-2.

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between groups	2	32811.7245	16405.8623	4.0667	.0206
Within groups	84	338874.9881	4034.2260		
Total	86	371686.7126			

The results of the Newman-Keuls procedure for pair-wise comparison of means are summarized in Table 33.

Table 33. Subsample 2. Comparison of Means. Hypothesis 4-2.

Mean	Category	1	2	3
312.5000	1 NOHSCALC			
303.5238	2 SEMCALC			
365.2500	3 YRHSCALC	*	*	

* Denotes pairs of categories significantly different at the 0.05 level.

The mean achievement of Category 3 (YRHSCALC) is significantly different from (higher than) the mean achievements of both Category 1 (NOHSCALC) and Category 2 (SEMCALC). Both differences are significant at the 0.05 level. In summary, students in Subsample 2, who all had a year of high school calculus, achieved at a significantly higher level than those who had either a semester of high school calculus or no high school calculus. There was no significant difference in mean achievement between those who had no high school calculus and those who had one semester of high school calculus.

Tables 34-36 pertain to Null Hypothesis 4-3.

Null Hypothesis 4-3: Among the Montana State University Combined sample, there is no significant difference in mean achievement in Calculus 1 among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

The sample sizes and achievement means for the three high school calculus subgroups of Subsample 3 are given in Table 34.

Table 34. Subsample 3. Achievement Means. Hypothesis 4-3.

Category		Count	Mean
Category 1	NOHSCALC	173	312.2023
Category 2	SEMCALC	95	321.4526
Category 3	YRHSCALC	96	365.9375
Total		364	328.7885

Analysis of variance statistics are given in Table 35. The F statistic value,

$$F = 20.4431,$$

is significant at

$$\text{alpha} < 0.00005,$$

dictating the rejection of Null Hypothesis 4-3.

Table 35. Subsample 3. ANOVA. Hypothesis 4-3.

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between groups	2	185189.6306	92594.8153	20.4431	.0000
Within groups	361	1635109.081	4529.3880		
Total	363	1820298.712			

The results of the Newman-Keuls procedure for pair-wise comparison of means are summarized in Table 36.

Table 36. Subsample 3. Comparison of Means. Hypothesis 4-3.

Mean	Category	1	2	3
312.2023	1 NOHSCALC			
321.4526	2 SEMCALC			
365.9375	3 YRHSCALC	*	*	

* Denotes pairs of categories significantly different at the 0.05 level.

The mean achievement of Category 3 (YRHSCALC) is significantly different from (higher than) the mean achievements of both Category 1 (NOHSCALC) and Category 2 (SEMCALC). Both differences are significant at the 0.05 level. In summary, students in Subsample 3, who all had a year of high school calculus, achieved at a significantly higher level than those who had either a semester of high school calculus or no high school calculus. There was no significant difference in mean achievement between those who had no high school calculus and those who had one semester of high school calculus.

Tables 37 and 38 pertain to Null Hypothesis 4-4.

<u>Null Hypothesis 4-4</u>: Among Montana Tech New Freshmen, there is no significant difference in mean achievement in Calculus 1 among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

The sample sizes and achievement means for the three high school calculus subgroups of Subsample 4 are given in Table 37.

Table 37. Subsample 4. Achievement Means. Hypothesis 4-4.

Category		Count	Mean
Category 1	NOHSCALC	14	95.5714
Category 2	SEMCALC	17	101.1176
Category 3	YRHSCALC	11	117.3636
Total		42	103.5238

No two group means are significantly different at the 0.05 level.

Analysis of variance statistics for Subsample 4 are given in Table 38. The F statistic value,

$$F = 2.2849,$$

is not significant at an alpha level of

$$\text{alpha} = 0.05.$$

Therefore Null Hypothesis 4-4 is retained. We conclude there is no significant difference, in Subsample 4, in mean achievement in Calculus 1 among the three high school calculus subgroups.

Table 38. Subsample 4. ANOVA. Hypothesis 4-4.

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between groups	2	3090.7375	1545.3687	2.2849	.1153
Within groups	39	26377.7387	676.3523		
Total	41	29468.4762			

Tables 39 - 41 pertain to Null Hypothesis 4-5.

<u>Null Hypothesis 4-5</u>: Among the Montana Tech Freshmen Complement, there is no significant difference in mean achievement in Calculus 1 among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

The sample sizes and achievement means for the three high school calculus subgroups of Subsample 5 are given in Table 39.

Table 39. Subsample 5. Achievement Means. Hypothesis 4-5.

Category		Count	Mean
Category 1	NOHSCALC	39	88.1538
Category 2	SEMCALC	25	99.6800
Category 3	YRHSCALC	13	110.6923
Total		77	95.7013

Analysis of variance statistics are given in Table 40. The F statistic value,

$$F = 3.5495,$$

is significant at an alpha level

$$\text{alpha} = 0.0337,$$

dictating the rejection of Null Hypothesis 4-5.

Table 40. Subsample 5. ANOVA. Hypothesis 4-5.

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between groups	2	5538.8437	2769.4219	3.5495	.0337
Within groups	74	57737.2862	780.2336		
Total	76	63276.1299			

The results of the Newman-Keuls procedure for pair-wise comparison of means are summarized in Table 41.

Table 41. Subsample 5. Comparison of Means. Hypothesis 4-5.

Mean	Category	1	2	3
88.1538	1 NOHSCALC			
99.6800	2 SEMCALC			
110.6923	3 YRHSCALC	*		

* Denotes pairs of categories significantly different at the 0.05 level.

The mean achievement of Category 3 (YRHSCALC) is significantly different from (higher than) the mean achievement of Category 1 (NOHSCALC). This difference is significant at the 0.05 level. In summary, students in Subsample 5, who all had a year of high school calculus, achieved at a significantly higher level than those who had no high school calculus. There was no significant difference in mean achievement between those who had no high school calculus and those who had one semester of high school calculus. There was no significant difference in mean achievement between those who had a semester of high school calculus and those who had a year of high school calculus.

Tables 42 and 43 pertain to Null Hypothesis 4-6.

<u>Null Hypothesis 4-6</u>: Among the Montana Tech Complement, there is no significant difference in mean achievement in Calculus 1

among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

The sample sizes and achievement means for the three high school calculus subgroups of Subsample 6 are given in Table 42.

Table 42. Subsample 6. Achievement Means. Hypothesis 4-6.

Category		Count	Mean
Category 1	NOHSCALC	78	89.1410
Category 2	SEMCALC	11	90.0000
Category 3	YRHSCALC	4	81.5000
Total		93	88.9140

No two group means are significantly different at the 0.05 level.

Analysis of variance statistics for Subsample 6 are given in Table 43. The F statistic value,

$$F = 0.1358,$$

is not significant at an alpha level of

$$\text{alpha} = 0.05.$$

Therefore Null Hypothesis 4-6 is retained. We conclude there is no significant difference, in Subsample 6, in mean achievement in Calculus 1 among the three high school calculus subgroups.

Table 43. Subsample 6. ANOVA. Hypothesis 4-6.

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between groups	2	236.8631	118.4316	.1358	.8752
Within groups	90	78474.4487	871.9383		
Total	92	78711.3118			

Tables 44 and 45 pertain to Null Hypothesis 4-7.

Null Hypothesis 4-7: Among the Montana Tech Combined sample, there is no significant difference in mean achievement in Calculus 1 among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

The sample sizes and achievement means for the three high school calculus subgroups of Subsample 7 are given in Table 44.

Table 44. Subsample 7. Achievement Means. Hypothesis 4-7.

Category		Count	Mean
Category 1	NOHSCALC	92	90.1196
Category 2	SEMCALC	28	96.7500
Category 3	YRHSCALC	15	107.8000
Total		135	93.4593

No two group means are significantly different at the 0.05 level.

Analysis of variance statistics for Subsample 7 are given in Table 45. The F statistic value,

$$F = 2.6499,$$

is not significant at an alpha level of

$$\text{alpha} = 0.05.$$

Therefore Null Hypothesis 4-7 is retained. We conclude there is no significant difference, in Subsample 7, in mean achievement in Calculus 1 among the three high school calculus subgroups.

Table 45. Subsample 7. ANOVA. Hypothesis 4-7.

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between groups	2	4414.1911	2207.0956	2.6499	.0744
Within groups	132	109941.3348	832.8889		
Total	134	114355.5259			

Chi Square Test of Independence

The Chi Square Test of Independence was performed on all three Montana State University subsamples. For purposes of this analysis, the three Calculus 1 success categories were defined as follows: Category 1 consisted of those students who had no achievement score by virtue of dropping or withdrawing, or otherwise failing to complete three of the four Calculus 1 examinations. Category 2 consisted of students whose

Calculus 1 achievement scores were in the lower 50th percentile for that subsample. Category 3 consisted of students whose achievement scores were in the upper 50th percentile. The high school calculus background categories were the same as those used in the regression and analysis of variance procedures.

The results of the Chi Square Test of Independence are given in the form of total frequency distributions. Tables 46-48 report the Chi Square Test results for the three Montana State University subsamples. In every case, the first number in each cell is the actual count for that cell, and the second number is the expected value. Table 46 pertains to Null Hypothesis 5-1.

<u>Null Hypothesis 5-1</u>: Among Montana State University New Freshmen, success in Calculus 1 is independent of high school calculus background.

Table 46. Subsample 1. Chi Square. Hypothesis 5-1.

Calculus 1 Achievement Group	High School Calculus Background Category		
	NOHSCALC	SEMCALC	YRHSCALC
1	24 16.0	7 9.1	4 9.9
2	73 63.7	41 36.1	25 39.2
3	46 63.3	33 35.8	59 38.9
Chi Square 30.37444	Significance .00000		

Since the value of Chi Square in Table 46 is significant at a level less than 0.000005, Null Hypothesis 5-1 is rejected. The conclusion follows that achievement in Calculus 1 is related to high school calculus background in Subsample 1. Examination of the statistics in Table 46 indicates that a disproportionate number of those with no high school calculus achieved poorly in Calculus 1. Vis-a-vis, a disproportionate number of those who had a year of high school calculus had good success in Calculus 1, achieving in the upper 50th percentile. Stated another way, among the students in Subsample 1, having no high school calculus background is associated with unexpectedly poor performance in Calculus 1. Having a year of high school calculus is associated with unexpectedly high performance in Calculus 1.

Table 47 pertains to Null Hypothesis 5-2.

Null Hypothesis 5-2: Among the Montana State University Complement, success in Calculus 1 is independent of high school calculus background.

Since the value of Chi Square in Table 47 is significant at a level

alpha = 0.01440,

Null Hypothesis 5-2 is rejected. The conclusion follows that achievement in Calculus 1 is related to high school calculus background in Subsample 2. Examination of the statistics in Table 47 indicates that a disproportionate number of those who had a year of high school calculus had good success in Calculus 1, achieving in the upper 50th percentile. That is, having a year of high school calculus background is associated with unexpectedly high performance in Calculus 1 among students in Subsample 2.

Table 47. Subsample 2. Chi Square. Hypothesis 5-2.

Calculus 1 Achievement Group	High School Calculus Background Category		
	NOHSCALC	SEMCALC	YRHSCALC
1	19 15.3	4 5.2	0 2.5
2	22 22.6	11 7.7	1 3.7
3	32 35.2	10 12.0	11 5.8
Chi Square 12.43375	Significance .01440		

Table 48 pertains to Null Hypothesis 5-3.

Null Hypothesis 5-3: Among the Montana State University Combined sample, success in Calculus 1 is independent of high school calculus background.

Since the value of Chi Square in Table 48 is significant at a level less than 0.000005, Null Hypothesis 5-3 is rejected. The conclusion follows that achievement in Calculus 1 is related to high school calculus background in Subsample 3. Examination of the statistics in Table 48 indicates that a disproportionate number of those with no high school calculus achieved poorly in Calculus 1; that is, did not complete

Table 48. Subsample 3. Chi Square. Hypothesis 5-3.

Calculus 1 Achievement Group	High School Calculus Background Category		
	NOHSCALC	SEMCALC	YRHSCALC
1	43 29.7	11 14.6	4 13.7
2	102 93.7	54 46.0	27 43.4
3	71 92.6	41 45.5	69 42.9
Chi Square 43.46159	Significance .00000		

Calculus 1 or achieved in the lower 50th percentile. Vis-a-vis, a disproportionate number of those who had a year of high school calculus had good success in Calculus 1, achieving in the upper 50th percentile.

Discussion of Findings

Chapter 4 contains the tests of hypotheses for the Investigative Study. Null Hypotheses 1 and 2 were tested for each of the seven subsamples included in the study. Null Hypotheses 1 and 2 are the following:

> Null Hypothesis 1: There is no significant correlation between lack of high school calculus background and achievement in Calculus 1.

Null Hypothesis 2: There is no significant correlation between one semester of high school calculus background and achievement in Calculus 1.

Null Hypothesis 1 was rejected at the 0.05 level for the following three subsamples: Subsample 1 (Montana State University New Freshmen), Subsample 3 (Montana State University Combined), and Subsample 7 (Montana Tech Combined). In these subsamples, there was statistically significant negative correlation between lack of high school calculus background and achievement in Calculus 1. Null Hypothesis 1 was retained for the other four subsamples.

Null Hypothesis 2 was rejected at the 0.05 level for the following three subsamples: Subsample 1 (Montana State University New Freshmen) Subsample 2 (Montana State University Complement), and Subsample 3 (Montana State University Combined). In these subsamples, there was statistically significant negative correlation between one semester of high school calculus and achievement in Calculus 1. Null Hypothesis 1 was retained for the four Montana Tech subsamples.

In Subsamples 1 and 3, both variables X_1 and X_2 showed significant negative correlation with Calculus 1 achievement, implying a year of high school calculus correlates positively with achievement in Calculus 1.

Null Hypothesis 3 was tested for each of the seven subsamples included in this study.

<u>Null Hypothesis 3</u>: There is no subset of the variables X_1 - X_{11} which significantly correlates with Calculus 1 achievement.

Null Hypothesis 3 was rejected for the following six subsamples and indicated independent variables of significance: The variables accounting for significant variability of Y (Calculus 1 achievement) in Subsamples 1 and 3 were X_1 (NOHSCALC), X_2 (SEMCALC), X_5 (GENDER), X_7 (HSSIZE), and X_{11} (PTS). The variable accounting for significant variability of Y in Subsample 2 was X_2 (SEMCALC). The variable accounting for significant variability of Y in Subsample 4 was X_5 (GENDER). The variable accounting for significant variability of Y in Subsample 5 was X_6 (HSMATH). The variable accounting for significant variability of Y in Subsample 7 was X_1 (NOHSCALC). Null Hypothesis 3 was retained for Subsample 6 since no variables accounted for significant variability of Y. The results of testing Null Hypotheses 1-3 on Subsamples 1-7 are summarized in Table 49. Table 50 shows the independent variables included in the Multiple Linear Regression models of Subsamples 1-7.

Table 49. Results of Testing Hypotheses 1-3 on Subsamples 1-7.

Hypothesis	1	2	3	4	5	6	7
Null Hypothesis 1	*		*				*
Null Hypothesis 2	*	*	*				
Null Hypothesis 3	*	*	*	*	*		*

* Denotes rejected null hypothesis (P = 0.05).

Table 50. Variables Included in Multiple Regression Models of Subsamples 1-7.

Subsample	Variables Included					
1	X_1	X_2	X_5		X_7	X_{11}
2		X_2				
3	X_1	X_2	X_5		X_7	X_{11}
4			X_5			
5				X_6		
6						
7	X_1					

Null Hypothesis 4 was tested for each of the seven subsamples included in the study. In the instances when Null Hypothesis 4 was rejected, the Newman-Keuls procedure was used to make multiple comparison of means.

<u>Null Hypothesis 4</u>: There is no significant difference in mean achievement in Calculus 1 among those who had no high school calculus background, those who had one semester of high school calculus, and those who had a year of high school calculus.

Null Hypothesis 4 was rejected for Subsamples 1, 2, 3, and 5. In all three Montana State University subsamples, the mean achievement of those having a year of high school calculus was significantly higher than the mean achievement for both other subgroups. In all three groups, there was no significant difference in achievement between those having no high school calculus and those having a semester of high school calculus. In Subsample 5, the only significant difference in mean achievement was between those having a year of high school calculus and those having no high school calculus. A summary of results from multiple comparison of means in Subsamples 1, 2, 3, and 5 is given in Table 51.

Table 51. Significantly Different Pairs of Achievement Means.

High School Calculus Subgroup	1	2	3
1	--		
2	(None)	--	
3	Subsamples 1, 2, 3, 5	Subsamples 1, 2, 3	—

Null Hypothesis 4 was retained for Subsamples 4, 6, and 7. This indicates there was no significant difference in mean achievement in Calculus 1 between any pair of high school calculus subgroups in these subsamples.

Null Hypothesis 5 was tested for each of the three Montana State University subgroups, using the Chi Square Test of Independence.

<u>Null Hypothesis 5</u>: Success in Calculus 1 is independent of high school calculus background.

Null Hypothesis 5 was rejected for each Montana State University subsample at a 0.05 level of significance. In all three subsamples, students with a year of high school calculus had an unexpectedly high rate of success in Calculus 1. In both Subsamples 1 and 3, those with no high school calculus had an unexpectedly large incidence of low achievement in Calculus 1.

CHAPTER 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The purpose of this investigation was to study the effects of one-semester and two-semester high school calculus courses on achievement in the traditional first course in calculus at the college/university level. The independent variables of primary importance in the study were three levels of high school mathematics background: (1) little or no exposure to calculus in high school, (2) one semester high school calculus course, and (3) full year high school calculus course including those with Advanced Placement Program designation. Other independent variables of secondary interest included age of student, class (freshman, sophomore, et cetera), gender, whether the student took mathematics as a high school senior, enrollment size of the student's graduating high school, academic major, MARS score (a measure of mathematics anxiety), minority status, and placement test score (an aptitude measure). The placement test score was based on the student's ACT or SAT score. The dependent variable was student achievement in college/university Calculus 1, as measured by examination scores attained in the course.

The general questions underlying the purpose of this investigation were:

1. What correlation does successful completion of a one-semester calculus course in high school have with achievement in Calculus 1 at the college/university level?

2. What correlation does successful completion of a two-semester calculus course in high school have with achievement in Calculus 1 at the college/university level?

3. Do students in the three categories (1) no high school calculus, (2) one semester high school calculus, and (3) two-semester high school calculus, demonstrate the same mean achievement in Calculus 1?

4. As measured by achievement in Calculus 1, does any one high school calculus option (no high school calculus, one-semester course, or two semester course) demonstrate superior worth over the other two options?

Subjects

The subjects for the Montana State University sample included all students in the fourteen sections of Calculus 1 at Montana State University in Fall quarter 1989 who graduated from an accredited United States high school and for whom the necessary data could be obtained. The Montana Tech sample consisted of all such students in two of four Fall semester 1989 sections of Calculus 1, and the students in the single Spring semester 1990 section of the course.

The Montana State University sample was divided into three subsamples for purposes of statistical analysis. The three subsamples were (1) students who graduated from high school in Spring 1989 (Montana State University New Freshmen), (2) those who graduated from high school prior to Spring 1989 (Montana State University Complement), and (3) the entire Montana State University sample (Montana State University Combined Sample).

The Montana Tech sample was divided into four subsamples: (1) students who graduated from high school in Spring 1989 and had no prior college mathematics coursework (Montana Tech New Freshmen), (2) Spring semester 1990 Montana Tech students enrolled in Calculus 1 who had taken prior college-level mathematics coursework (precalculus courses) during Fall 1989, and who graduated from high school in Spring 1989 (Montana Tech Freshmen Complement), (3) students who graduated from high school prior to Spring 1989 (Montana Tech Complement), and (4) the entire Montana Tech sample (Montana Tech Combined Sample).

Since different items of data were required for the three statistical inference procedures employed in the Investigative Study, the effective size of each subsample varied with each inference procedure used.

Instrumentation

The subjects were all administered the Mathematics Anxiety Rating Scale (MARS) at the outset of the Calculus 1 course. The MARS consists of 98 five-point Likert-type statements designed to measure a student's level of mathematics anxiety.

All sample students had taken either the ACT or SAT prior to entering Calculus 1. The students' ACT or SAT scores were converted to a common scale by calculating the standard normal distribution equivalent (Z value) of each ACT or SAT score. The Z value was defined herein as the student's Placement Test Score and was used as a measure of the student's scholastic aptitude.

Student achievement in Calculus 1 at Montana State University was measured by the aggregate score attained on three 100 point midterm examinations and the comprehensive 200 point final examination. These examinations included both conceptual and procedural test items. Student achievement in Calculus 1 at Montana Tech was measured by the student's score on the 150 point comprehensive final examination administered at the conclusion of the course.

Duration

The Investigative Study was conducted over the duration of Fall quarter 1989 at Montana State University and both Fall semester 1989 and Spring semester 1990 at Montana Tech.

At Montana State University, student achievement was measured at approximately regular intervals during the quarter by the three midterm examinations and the final examination. At Montana Tech, student achievement was measured by the (same) final examination at the conclusion of each semester. In both samples, the MARS was administered at the outset of the term to gauge the mathematics anxiety that each student brought into Calculus 1.

Research Design

Separate Multiple Linear Regression Analyses were performed for each of the seven subsamples to test Null Hypotheses 1, 2, and 3. Analysis of Variance and the Newman-Keuls Multiple Comparison of Means (where appropriate) were performed for each of the seven subsamples, to test Null Hypothesis 4. The Chi Square Test of Independence was done for the three Montana State University subsamples to test Null Hypothesis 5. The hypotheses were tested at the 0.05 level of significance. All statistical analyses were executed on Montana State University mainframe computer facilities using SPSS Version 4.0 software.

Findings

The following are findings obtained from the analyses of the data collected for this Investigative Study.

1. Null Hypothesis 1 was rejected for Subsamples 1, 3, and 7, all significant at the 0.05 level. Significant correlation, therefore, exists between lack of high school calculus background and achievement in Calculus 1 in these subsamples. In each instance, the correlation was

negative. Null Hypothesis 1 was retained for Subsamples 2, 4, 5, and 6 at the 0.05 level.

2. Null Hypothesis 2 was rejected for Subsamples 1, 2, and 3, all significant at the 0.05 level. Significant correlation, therefore, exists between one semester of high school calculus background and achievement in Calculus 1 in these subsamples. In each instance, the correlation was negative. Null Hypothesis 2 was retained for Subsamples 4, 5, 6, and 7 at a significance level of 0.05.

3. Null Hypothesis 3 was rejected for Subsamples 1, 2, 3, 4, 5, and 7. In Subsamples 1, 3, and 7, variable X_1 (NOHSCALC) showed significant correlation with Calculus 1 achievement Y. Variable X_2 (SEMCALC) was significant in Subsamples 1, 2, and 3. Variable X_5 (Gender) was significant in Subsamples 1, 3, and 4. Variable X_6 (HSMATH) was significant in Subsample 5. Variable X_7 (HSSIZE) was significant in Subsamples 1 and 3. Variable X_{11} (PTS) was significant in Subsamples 1 and 3. All correlations were significant at a level of 0.05.

4. Null Hypothesis 4 was rejected for Subsamples 1, 2, 3, and 5. Multiple comparison of means revealed significant differences in mean achievement between the NOHSCALC and YRHSCALC groups in Subsamples 1, 2, 3, and 5. Significant differences in mean achievement between the SEMCALC and YRHSCALC groups occurred in Subsamples 1, 2, and 3. In all cases, the YRHSCALC means were higher. In no instance was there any significant difference in mean achievement

between the SEMCALC and NOHSCALC groups. All differences were significant at a level of 0.05.

5. Null Hypothesis 5 was rejected for each Montana State University subsample at a 0.05 level of significance. In all three subsamples, the YRHSCALC groups had a disproportionately high rate of success in Calculus 1. In Subsamples 1 and 3, the NOHSCALC groups had a disproportionately large rate of low achievement in Calculus 1.

Conclusions and Discussion

Several conclusions are offered from the analyses of the data collected in this study.

1. Lack of high school calculus background is associated with low achievement in college/university Calculus 1.

2. Completion of a one-semester high school calculus course (only) is associated with low achievement in Calculus 1.

Statistical analyses revealed significant negative correlation between lack of high school calculus background and Calculus 1 achievement for Null Hypotheses 1-1, 1-3, 1-7, 3-1, 3-3, and 3-7. Null Hypotheses 1-2, 1-4, 1-5, and 1-6 showed no significance at the 0.05 level.

Analyses showed significant negative correlation between one semester of high school calculus background and Calculus 1 achievement for Null Hypotheses 2-1, 2-2, 2-3, 3-1, 3-2, and 3-3. Null

Hypotheses 2-4, 2-5, 2-6, and 2-7 showed no significance at the 0.05 level.

The variables X_1 (NOHSCALC) and X_2 (SEMCALC), when significant, demonstrated only negative correlations with Calculus 1 achievement. This occurred in Subsamples 1, 3, and 7 for X_1, and in Subsamples 1, 2, and 3 for X_2. In no instance did either of these variables demonstrate a significant positive correlation with achievement in Calculus 1. The multiple regression correlation coefficients (beta weights) for X_1 and X_2, taken from Tables 12, 15, 18, and 27, are the following:

1. -0.29834 for X_1 and -0.20676 for X_2 in Subsample 1.
2. -0.48505 for X_2 in Subsample 2.
3. -0.30037 for X_1 and -0.23408 for X_2 in Subsample 3.
4. -0.27113 for X_1 in Subsample 7.

In both Subsamples 1 and 3, variables X_1 and X_2 provided a significant and unique contribution to the prediction of Calculus 1 success after variables X_5, X_7, and X_{11} had been taken into account. In Subsample 2, variable X_2 was alone in making a significant and unique contribution to the prediction of Calculus 1 success. In Subsample 7, variable X_1 was the only variable of significance in predicting Calculus 1 achievement. In Subsamples 4 and 5, only predictors other than X_1 and X_2 were significant (see Table 50). For Subsample 6, Null Hypotheses 1, 2, and 3 were retained.

3. Students who take a year-long calculus course in high school achieve at a higher level in college/university Calculus 1 than students who receive little or no exposure to calculus in high school.

4. Montana State University students who take a year-long calculus course in high school achieve at a higher level in Calculus 1 than students who take a one-semester calculus course in high school.

5. Students who take a one-semester calculus course in high school achieve at the same level in Calculus 1 as students with little or no exposure to calculus in high school.

Null Hypothesis 4 was retained for Subsamples 4, 5, and 7, and was rejected for Subsamples 1, 2, 3, and 5. Subsequent multiple comparison of means in the latter four instances with the Newman-Keuls procedure indicated significantly higher levels of mean achievement in Calculus 1 for the YRHSCALC students compared to the NOHSCALC students in all four subsamples. In Subsamples 1, 2, and 3, the YRHSCALC subgroups had significantly higher mean achievement than the SEMCALC subgroups. All tests were at a significance level of 0.05. In no instance did students in the SEMCALC subgroup demonstrate significantly greater mean achievement than students in the NOHSCALC subgroup.

6. Students who complete a year-long high school calculus course demonstrate a higher success rate in college-university Calculus 1 than other students.

7. Students who take no calculus in high school typically demonstrate a lower success rate in Calculus 1 than other students.

Null Hypothesis 5 was tested only for Subsamples 1, 2, and 3; and was rejected in each case. In all three instances, students in the YRHSCALC subgroup showed disproportionately high rates of success in Calculus 1. In Subsamples 1 and 3, students in the NOHSCALC subgroup had disproportionately large rates of low achievement in Calculus 1.

In summary, the outcomes of testing Null Hypotheses 1-1, 1-3, 1-7, 3-1, 3-3, 3-7, 4-1, 4-2, 4-3, 4-5, 5-1, 5-2, and 5-3 all indicate that students who have taken a year of calculus in high school are more successful in Calculus 1 than those who had no exposure to calculus in high school. These results are consistent with those of Burton (1989), Ferrini-Mundy and Gaudard (1992), Paul (1970), Robinson (1970), and Sorge and Wheatley (1977).

The outcomes of testing Null Hypotheses 2-1, 2-2, 2-3, 3-1, 3-2, 3-3, 4-1, 4-2, and 4-3 all indicate that students who have taken a year of calculus in high school are more successful in Calculus 1 than those who took a one-semester calculus course in high school. These results are consistent with those of Ferrini-Mundy and Gaudard (1992), McKillip (1965), and Sorge and Wheatley (1977).

In no subsample or hypothesis test did students with a semester of high school calculus outperform students with no high school calculus background. These results confirm those of Ferrini-Mundy and Gaudard

(1992), McKillip (1965), Rash (1977), Sorge and Wheatley (1977), and Tillotson (1962). These results contradict those of Paul (1970).

Recommendations

Recommendations for Education

The following recommendations for education are offered based on the analyses of data collected in this study.

1. When making high school mathematics curriculum decisions, policy makers should consider calculus as one of several alternatives in choosing extensions of core topics for acceleration courses. Other options include statistics, probability, and discrete mathematics. A four-year high school course of mathematical study should culminate in an extension of core topics, with calculus not viewed as the only possible capstone course.

2. If school resources and student demand make a high school calculus course feasible, the school should preferably offer the Advanced Placement AB or BC course. In lieu of an AP course, the high school should only offer a year course with content and evaluation standards comparable to either version of the AP course. Introductory or survey-type calculus courses of one semester duration or less, which typically concentrate on computational procedures, are specifically discouraged. The results of this study and others strongly indicate that such a course does not represent a fruitful investment of pedagogical resources and students' time. This recommendation is consistent with

the position of the CUPM Panel on Calculus Articulation (1987) and other researchers cited above.

3. The researcher believes that the above recommendations, indicated by the results of the present study, should be offered concurrent with a final recommendation, which reflects current trends in secondary school mathematics education as espoused by the NCTM. High school calculus courses, while conforming to the duration, content, and standards described above, should concurrently incorporate recent NCTM (1989) topical recommendations. A sampling of topics that should receive decreased emphasis include calculations using tables, interpolations and paper and pencil calculations; excessive procedural drill, excessive reliance on formulas as a problem-solving strategy, and the expression of function equations in standardized form in order to graph them. Topics that should receive increased emphasis include fostering an awareness of, and appreciation for the historical origins and cultural significance of the calculus; numerical methods, applications to real-world phenomena, and conceptual underpinnings of calculus. An integral facet of any calculus course should be appropriate use of both graphing calculators and instructional computer technology.

Recommendations for Research

The following recommendations are offered for further research.

1. This study should be replicated in the future at post-secondary institutions of varying size and type, located in different demographic

areas of the country. Calculus is clearly an ingrained part of the secondary school curriculum that will remain extant in various forms for the foreseeable future. Its effectiveness and desirability therefore require ongoing examination in the face of rapid changes in content, and instructional strategies and techniques at both the secondary and post-secondary level.

2. Further study should be conducted to investigate comparative benefits, for the college-bound student, of calculus and other advanced high school mathematics courses such as probability, statistics, and discrete mathematics.

3. Further research should be conducted to ascertain the effects of advanced placement in college/university Calculus 2 or 3 of students who have taken a year calculus course in high school.

4. Further investigation is recommended to measure the longitudinal effects of a year of high school calculus. Such research would verify whether any benefit of high school calculus carries over to the second semester of college/university coursework and beyond.

5. Further research may explore possible interaction between the high school calculus background variable and attribute variables such as gender, minority status, age, and placement test score; as they affect achievement in Calculus 1.

REFERENCES CITED

Aichele, D. B. (Ed.). (1978). Mathematics teacher education: Critical issues and trends. Washington, D. C.: National Education Association.

Anderson, R. D., & Loftsgarden, D. (1987). A special calculus survey: Preliminary report. In L. A. Steen (Ed.), Calculus for a new century: A pump, not a filter. MAA Notes No. 8. Washington, D.C.: Mathematical Association of America.

Austin, H. W. (1975). The effects of high school calculus on achievement during the first semester in college calculus for first year students. Dissertation Abstracts International, 36, 2728A. (University Microfilms No. 75-26, 078).

Austin-Martin, G., Waddell, L., & Kincaid, M. B. (1980, October 15). Correlates of math anxiety in female college freshmen. Paper presented at Eleventh Annual Conference of Rocky Mountain Educational Research Association, Las Cruces, NM.

Bergeson, J. B. (1967). Academic performance of college students granted advanced standing as a result of participation in the advanced placement program. Journal of Educational Research, 61, 151-152.

Betz, N. (1978). Prevalence, distribution and correlates of math anxiety in college students. Journal of Counseling Psychology, 25, 441-448.

Burton, M. B. (1989). The effect of prior calculus experience on "introductory" college calculus. The American Mathematical Monthly, 96, 350-354.

Carroll, C. D. (1984, April 3). High school and beyond tabulation: Mathematics courses taken by 1980 high school sophomores who graduated in 1982. National Center for Educational Statistics. (LSB).

Christensen, H. B. (1977). Statistics step by step. Atlanta: Houghton Mifflin Comnpany.

College Entrance Examination Board. (1987). A guide to the Advanced Placement Program. New York: Author.

College Entrance Examination Board. (1964). A guide to the Advanced Placement Program, 1964-1965. New York: Author.

College Entrance Examination Board. (1990). Advanced placement course description mathematics. United States: Author.

Committee on the Undergraduate Program in Mathematics Panel on Calculus Articulation. (1987). Problems in from the transition from high school calculus to college calculus. The American Mathematical Monthly. 94. 776-785.

Dickey, E. M. (1986). A comparison of advanced placement and college students on a calculus achievement test. Journal for Research in Mathematics Education, 17 (2), 140-144.

Douglas, R. G. (1988, January 20). Today's calculus courses are too watered down and outdated to capture the interest of students. Chronicle of Higher Education, p. 34.

Ferguson, G. A. (1981). Statistical analysis in psychology and education. New York: McGraw Hill Book Company.

Ferrini-Mundy, J., & Gaudard, M. (1992). Secondary school calculus: preparation or pitfall in the study of college calculus? Journal for Research in Mathematics Education, 23 (1), 56 - 71.

Fry, D. E. (1973). A comparison of the college performance in college-level mathematics courses between regular-process students and advanced placement students. Dissertation Abstracts International, 34, 1768A. (University Microfilms No. 73-23,333).

High school enrollments. (1990, March 7). Billings Gazette, p. 1-B.

Howell, D. C. (1987). Statistical methods for psychology. Boston: Duxbury Press.

McKillip, W. D. (1966). The effects of secondary school analytic geometry and calculus on students' first semester calculus grades at the University of Virginia. Dissertation Abstracts, 26, 5920-5921.

Montana College of Mineral Science & Technology. (1988). Montana Tech 1988-1989 catalog. Butte, MT: Author.

Montana State University. (1988). Montana State University undergraduate catalog 1988-1991. Bozeman, MT: Montana State University Office of Publications and News Services.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: The Council.

Paul, H. W. (1971). The relationship of various high school mathematics programs to achievement in the first course in college calculus. Dissertation Abstracts International, 31, 3396A. (University Microfilms No. 70-26,346).

Pocock, R. C. (1974). Advanced placement calculus as a factor in the study of college mathematics. Dissertation Abstracts International, 35, 850A. (University Microfilms No. 74-18,732).

Rash, A. (1977). Is calculus an appropriate high school course?. High School Journal, 60, 277-283.

Richardson, T., & Suinn, R. M. (1973). The Mathematics Anxiety Rating Scale: Psychometric data. Journal of Counseling Psychology, 19, 551-554.

Robinson, W. B. (1970). The effects of two semesters of secondary school calculus on students' first and second quarter calculus grades at the University of Utah. Journal for Research in Mathematics Education, 1, 57-60.

Shimizu, M. T. (1969). Achievement in senior advanced mathematics and first-year college mathematics. Math Teacher, 62, 311-315.

Small, D. (1986). Calculus articulation between high school and college. In Toward a lean and lively calculus: Report of the Conference Workshop to Develop Curriculum and Teaching Methods for Calculus at the College. (pp. 129-142). Mathematics Association of America.

Sorge, D. H., & Wheatley, G. H. (1977). Calculus in high school - At what cost?. American Mathematical Monthly, 84, 644-647.

Steen, L. A. (1986). Twenty questions for calculus reformers. In Toward a Lean and Lively Calculus: Report of the Conference Workshop to Develop Curriculum and Teaching Methods for Calculus at the College. (pp. 157-165). Mathematics Association of America.

Steen, L. A. (Ed.). (1987). Calculus for a new century: A pump, not a filter. MAA Notes No. 8. Washington, D.C.: Mathematical Association of America.

Stent, A. (1977). Can math anxiety be conquered?. Change, 9, 40-43.

Suinn, R. M. (1972). Math Anxiety Rating Scale (MARS) information for users. Fort Collins, CO: RMBSI, Inc.

Suinn, R. M., Edie, C. A., Nicoletti, J., & Spinelli, P. R. (1973). The MARS, a measure of mathematics anxiety: Psychometric data. Journal of Clinical Psychology, 28, 373-375.

Tillotson, D. B. (1963). The relationship of an introductory study of calculus in high school to achievement in a university calculus course. Dissertation Abstracts, 24, 577-578.

Tobias, S. (1976, September). Math anxiety. MS Magazine, 5(1), pp. 56-59, 92.

U.S. News & World Report (1987, October 26). America's best colleges, pp. 48-90.

Whitesitt, J. D. (1980). Entrance level competencies needed for beginning mathematics courses at Montana State University. Doctoral thesis, Montana State University.

APPENDIXES

APPENDIX A

MATHEMATICS ANXIETY RATING SCALE (MARS)

Name ____________________ Total Score __________

MATHEMATICS ANXIETY RATING SCALE (MARS)

The items in the questionnaire refer to things and experiences that may cause tension or apprehension. For each item, place a check (√) in the circle under the column that describes how much you would be made anxious by it. Work quickly, but be sure to think about each item.

How anxious . . .	Not at all	A little	A fair amount	Much	Very much
1. Deciding how much change you should get back from buying several items.	O	O	O	O	O
2. Having someone watch you as you add up a column of numbers.	O	O	O	O	O
3. Having someone watch you as you divide a five digit number by a two digit number.	O	O	O	O	O
4. Being asked to add up 976 + 777 in your head.	O	O	O	O	O
5. Dividing a five digit number by a two digit number in private with pencil and paper.	O	O	O	O	O
6. Figuring out a simple percentage, like the sales tax on something you buy.	O	O	O	O	O
7. Listening to a salesman show you how you would save money by buying a higher priced product because it reduces long term expenses.	O	O	O	O	O
8. Listening to a person explain how your share of expenses on a trip was figured out (including meals, transportation, housing, etc.).	O	O	O	O	O
9. Having to figure out how much it will cost to buy a product on credit (figuring in the interest rates).	O	O	O	O	O

	How anxious . . .	Not at all	A little	A fair amount	Much	Very much
10.	Adding up a bill for a meal when you think you have been over-charged.	O	O	O	O	O
11.	Telling the cashier that you think the bill for the meal was wrong and watching the cashier add up the bill again.	O	O	O	O	O
12.	Being treasurer for a club.	O	O	O	O	O
13.	Adding up the dues received and the expenses for a club you belong to.	O	O	O	O	O
14.	Adding up 976 + 777 on paper.	O	O	O	O	O
15.	Doing a word problem in algebra.	O	O	O	O	O
16.	Solving a problem such as: If x = 11, and y = 3, then the result of x/y is equal to ______?	O	O	O	O	O
17.	Solving the problem such as: If x = 12, and y = 4, then the ratio of x to y is equal to ______?	O	O	O	O	O
18.	Figuring out your grade average for last term.	O	O	O	O	O
19.	Reading an article on the basketball team, showing what percentage of free throws each player made, the percentage of field goals made, the total number attempted, etc.	O	O	O	O	O
20.	Reading a novel with many dates in it.	O	O	O	O	O
21.	Counting the number of pages left in a novel you are reading.	O	O	O	O	O
22.	Being asked to guess how many people are at a large gathering you are attending.	O	O	O	O	O
23.	Receiving a math textbook.	O	O	O	O	O

How anxious . . .	Not at all	A little	A fair amount	Much	Very much
24. Watching someone work with math tables.	O	O	O	O	O
25. Watching a teacher work an algebra problem on the blackboard.	O	O	O	O	O
26. Signing up for a math course.	O	O	O	O	O
27. Listening to another student explain a math formula.	O	O	O	O	O
28. Walking into a math class.	O	O	O	O	O
29. Having to figure the miles per gallon of gas for a car.	O	O	O	O	O
30. Watching someone work with a calculator.	O	O	O	O	O
31. Looking through the pages of a math text.	O	O	O	O	O
32. Working on an income tax form.	O	O	O	O	O
33. Reading your W-2 form (or other statement showing your annual earnings and taxes).	O	O	O	O	O
34. Studying for a math test.	O	O	O	O	O
35. Starting to read a new chapter in a math book.	O	O	O	O	O
36. Walking to class and thinking about a math course.	O	O	O	O	O
37. Meeting your math teacher while walking in the hall.	O	O	O	O	O
38. Reading the word "Statistics".	O	O	O	O	O
39. Sitting in a math class and waiting for the teacher to begin.	O	O	O	O	O

How anxious . . .	Not at all	A little	A fair amount	Much	Very much
40. Solving a square root problem.	O	O	O	O	O
41. Signing up for a course in Geometry.	O	O	O	O	O
42. Checking over your monthly bank statement.	O	O	O	O	O
43. Taking the math section of a standardized test, like an achievement test.	O	O	O	O	O
44. Having someone explain bank interest rates while describing savings accounts.	O	O	O	O	O
45. Raising your hand in a math class to ask a question about something you do not understand.	O	O	O	O	O
46. Reading and interpreting graphs or charts.	O	O	O	O	O
47. Reading a cash register receipt after you buy something.	O	O	O	O	O
48. Figuring the sales tax for something that costs more than $1.00.	O	O	O	O	O
49. Having a person show you the best way to divide your money into a savings and a checking account.	O	O	O	O	O
50. Figuring how you would make more money: by taking a job that has a lower salary, but includes, room, meals, and travel; or a job that has a higher salary, but no other benefits.	O	O	O	O	O
51. Reading a formula in chemistry.	O	O	O	O	O
52. Hearing a lecture in a social studies class where the teacher is commenting on some figures, like the percentage of each socio-economic group who voted Republican.	O	O	O	O	O

How anxious . . .		Not at all	A little	A fair amount	Much	Very much
53.	Taking an examination (quiz) in a math course.	O	O	O	O	O
54.	Taking an examination (final) in a math course.	O	O	O	O	O
55.	Hearing two of your friends talking about the best way to figure out the actual cost of a product.	O	O	O	O	O
56.	Having someone ask you to recheck the numbers in a simple calculation, such as division or addition.	O	O	O	O	O
57.	Being asked by a friend to answer the question: "How long will it take to get to the state capital if I drive at 30 miles per hour?"	O	O	O	O	O
58.	Studying for a driver's license test and memorizing the figures involved, such as the distances it takes to stop a car going at differing speeds.	O	O	O	O	O
59.	Hearing friends quote the odds on a game as they make bets.	O	O	O	O	O
60.	Playing cards where numbers are involved, like poker or blackjack.	O	O	O	O	O
61.	Having a friend try to teach you how to do a math problem and finding that you cannot understand what is being said.	O	O	O	O	O
62.	Making a schedule for your daily routine, setting aside times for classes, study time, meals, recreation, etc.	O	O	O	O	O
63.	Juggling class times around at registration to determine the best schedule.	O	O	O	O	O

	How anxious . . .	Not at all	A little	A fair amount	Much	Very much
64.	Deciding which courses to take in order to come out with enough credit hours for promotion or graduation.	O	O	O	O	O
65.	Working a <u>concrete, everyday</u> application of mathematics that has meaning to you, e. g., figuring out how much you can spend on recreational activities after paying other bills.	O	O	O	O	O
66.	Working on a math problem which seems less important in your life, such as "If x = outstanding bills, and y = total income, calculate how much is left for recreational purposes."	O	O	O	O	O
67.	Being given a set of addition problems to solve on paper.	O	O	O	O	O
68.	Being given a set of subtraction problems to solve on paper.	O	O	O	O	O
69.	Being given a set of multiplication problems to solve on paper.	O	O	O	O	O
70.	Being given a set of division problems to solve on paper.	O	O	O	O	O
71.	Picking up your math textbook to begin working on a homework assignment.	O	O	O	O	O
72.	Being given a homework assignment of many difficult math problems, which is due the next time the class meets.	O	O	O	O	O
73.	Thinking about an upcoming math test one week before.	O	O	O	O	O
74.	Thinking about an upcoming math test one day before.	O	O	O	O	O
75.	Thinking about an upcoming math test one hour before.	O	O	O	O	O

	How anxious . . .	Not at all	A little	A fair amount	Much	Very much
76.	Thinking about an upcoming math test five minutes before.	O	O	O	O	O
77.	Checking the time and figuring out whether or not you can stop in two more stores and still meet a friend at the exact time you said you would.	O	O	O	O	O
78.	Waiting to get a math test returned on which you expected to do well.	O	O	O	O	O
79.	Waiting to get a math test returned on which you expected to do poorly.	O	O	O	O	O
80.	Walking to math class.	O	O	O	O	O
81.	Realizing that you have to take a certain number of math classes to meet the requirements for graduation.	O	O	O	O	O
82.	Picking up a math textbook to begin a difficult reading assignment.	O	O	O	O	O
83.	Being called on to answer a question in a math class on a topic you have spent some time studying.	O	O	O	O	O
84.	Not knowing the formula needed to solve a particular problem.	O	O	O	O	O
85.	Receiving your final math grade on your report card.	O	O	O	O	O
86.	Opening a math or statistics book and seeing a page full of problems.	O	O	O	O	O
87.	Being responsible for collecting the dues for a club and keeping track of the amount received.	O	O	O	O	O

	How anxious . . .	Not at all	A little	A fair amount	Much	Very much
88.	Getting ready to study for a math test.	O	O	O	O	O
89.	Listening to a lecture in a math class.	O	O	O	O	O
90.	Figuring out your monthly budget.	O	O	O	O	O
91.	Being given a "pop" quiz in a math class.	O	O	O	O	O
92.	Seeing a computer printout.	O	O	O	O	O
93.	Having to use the tables in the back of a math book.	O	O	O	O	O
94.	Being told how to interpret probability statements.	O	O	O	O	O
95.	Asking your math teacher to help you with a problem that you don't understand.	O	O	O	O	O
96.	Being asked to explain how you arrived at a particular answer for a problem.	O	O	O	O	O
97.	Adding up the results of a survey or poll.	O	O	O	O	O
98.	Acting as secretary, keeping track of the number of people signing up for an event.	O	O	O	O	O

APPENDIX B

MATH 181
EXAM 1

Math 181 - Exam 1 Name: ____________

October 17, 1989 Section/Instructor: ____________

SHOW ALL WORK FOR FULL CREDIT.

DO NOT USE CALCULATORS.

FOR SCORING ONLY

Prob	Points	Score
1	27	
2	30	
3	27	
4	16	
Tot	100	

In problems 1-4, fill in the blank with the letter of the best choice. Each problem is worth 3 points.

____ 1. The range of the function $f(x) = -5\cos 2x$ is

a) $[-2,2]$ b) $(-\infty,\infty)$

c) $[-5,5]$ d) $(-2,2)$

____ 2. The function $f(x) = x\tan x$ is

a) even b) odd

c) neither even nor odd d) both even and odd

____ 3. $x + 3 - |2x - 4| =$

a) $\begin{cases} -x + 7 & \text{if } x < 2 \\ 3x - 1 & \text{if } x \geq 2 \end{cases}$ b) $\begin{cases} -x - 1 & \text{if } x < 2 \\ 3x - 1 & \text{if } x \geq 2 \end{cases}$

c) $\begin{cases} 3x + 7 & \text{if } x < 2 \\ -x - 7 & \text{if } x \geq 2 \end{cases}$ d) $\begin{cases} 3x - 1 & \text{if } x < 2 \\ -x + 7 & \text{if } x \geq 2 \end{cases}$

____ 4. The slope of a line perpendicular to $2y + 3x = 1$ is

a) $-3/2$ b) $2/3$ c) $-2/3$ d) $3/2$

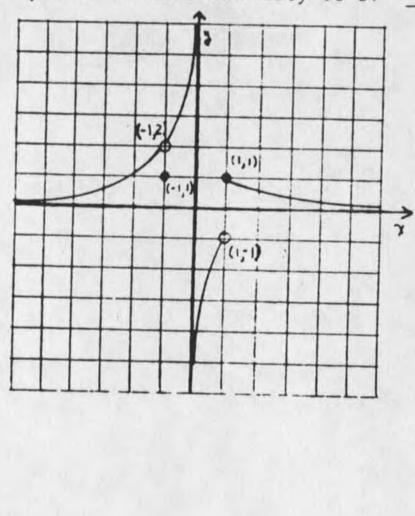
5. (15 pts) In the picture below of the graph of $y = f(x)$ the solid dots at the points (1,1) and (-1,1) signify that $f(1) = 1$ and $f(-1) = 1$. Based on the picture, give the values of the following limits if they exist or are $-\infty$ or ∞. Otherwise, write DNE (for "does not exist").

a) $\lim\limits_{x\to-\infty} f(x) =$ ____________ b) $\lim\limits_{x\to-1} f(x) =$ ____________

c) $\lim\limits_{x\to0+} f(x) =$ ____________ d) $\lim\limits_{x\to1} f(x) =$ ____________

e) $\lim\limits_{x\to1-} f(x) =$ ____________ f) $\lim\limits_{x\to\infty} f(x) =$ ____________

g) What are the points of discontinuity of f? ____________



6. (30 pts) Calculate the following limits if they exist or are $-\infty$ or ∞. Otherwise write DNE (for "does not exist").

a) $\lim_{x\to 1} \frac{x^2 - 1}{x^2 + 1} =$

b) $\lim_{t\to 5} \frac{t^2 - 4t - 5}{t - 5} =$

c) $\lim_{x\to -1^-} \frac{|x + 1|}{x + 1} =$

d) $\lim_{x\to\infty} \frac{2 - 3x + 5x^2}{1 - 2x^2} =$

e) $\lim_{x\to\pi/2+} \tan x =$

f) $\lim_{x\to -\infty} \frac{\sqrt{x^2 + 1}}{x - 1} =$

g) $\lim_{t\to 0} \frac{1 - \cos t}{t^2} =$

h) $\lim_{x\to 1} \frac{1}{(x - 1)^3} =$

i) $\lim_{x\to\infty} \frac{\cos(x^2)}{x} =$

j) $\lim_{x\to\infty} \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + x} =$

7. (9 pts) Prove, using the ε-δ definition of limit, that

$$\lim_{x \to 1} (1 - 5x) = -4.$$

8. (10 pts) Given $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x & \text{if } x > 2 \end{cases}$

a) Is f continuous at $x = 2$? Justify your answer.

b) Is f differentiable at $x = 2$? Justify your answer.

9. (8 pts) Does the equation $x^4 - 9x + 1 = 0$ have a solution in the interval $[0,2]$? Justify your answer.

10. (8 pts) Use the definition of the derivative as a limit of a difference quotient to calculate $f'(1)$ given $f(x) = 2x^2 - x$.

11. (8 pts) At what point does the tangent line to the graph of $f(x) = \sqrt{x}$ at $(4,2)$ cross the x-axis?

APPENDIX C

MATH 181
EXAM 2

MATH 181 - Exam #2 Name: ____________________

November 7, 1989 Section/Instructor: ________________

SHOW ALL WORK FOR FULL CREDIT.
DO NOT USE CALCULATORS.

FOR SCORING ONLY

Page	Points	Score
1	30	
2	36	
3	20	
4	14	
Tot	100	

In problems 1-4, fill in the blank with the letter of the best choice. Each problem is worth 4 points.

_____ 1. Given $f(x) = \sqrt[4]{x^5}$, $f'(x) =$

a) $20x^4$ b) $20x^{1/4}$

c) $5/4\ x^4$ d) $5/4\ x^{1/4}$

_____ 2. Suppose that f is twice differentiable at x. Then $f''(x) =$

a) $\lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h}$ b) $\lim_{h\to 0} \frac{(f(x+h))^2 - (f(x))^2}{h}$

c) $\lim_{h\to 0} \frac{f(x+h^2) - f(x)}{h^2}$ d) $\lim_{h\to 0} \frac{f''(x+h) - f''(x)}{h}$

_____ 3. If $f(x) = \sec x$, then $D^{-1}(f'(x)) =$

a) $\sec x \tan x + c$ b) $\sec x + c$

c) $x + c$ d) $\tan^2 x + c$

_____ 4. Suppose that $F(x) = (f \circ g)(x)$, $f'(2) = 3$, $g'(2) = 5$, $g(2) = 4$, and $f'(4) = -1$. Then $F'(2) =$

a) -4 b) 15 c) -5 d) 12

_____ 5. $D^{20}\sin x$ (the 20th derivative of $\sin x$) =

a) $\sin x$ b) $\cos x$

c) $-\sin x$ d) $-\cos x$

6. (10 pts) Find an equation for the tangent line to the graph of $y^2 + 3xy + x^2 = -1$ at the point $(-1,2)$.

7. (36 points) In the following, do not simplify your answers.

a) Given $y = (x^2 + 1)^7(x^3 + 2x)^5$, $\frac{dy}{dx} =$

b) Given $f(x) = (x + \sin x)^5$, $f'(x) =$

c) Given $s = \frac{\sqrt{2t+1}}{t^2}$, $\frac{ds}{dt} =$

d) Given $g(x) = 4\cos(x^2)$, $g'(x) =$

e) Given $h(x) = \tan 2x$, $h'(x) =$

$h''(x) =$

f) $D^{-1}(2x + 1)^9 =$

8. (5 pts) Suppose that an object moves along a straight line with velocity at time t given by $v(t) = 5 \sin t$, and suppose that the position at time $t = 0$ is 10. Find the position $s(t)$ at any time t. (Recall that $s'(t) = v(t)$.)

9. a. (7 pts) Find an equation of the tangent line to the graph of $f(x) = \sqrt{2x+1}$ at the point (4,3).

   b. (3 pts) Use the equation of the tangent line found in part (a) to approximate $f(4.1) = \sqrt{9.2}$. (Your answer may be left as a fraction.)

10. (5 pts) Let $f(x) = \frac{1}{x^2}$. Does Rolle's Theorem (or the mean value theorem) guarantee the existence of a point c such that $-1 < c < 1$ and $f'(c) = 0$? Why or why not?

11. (10 pts) Let $f(x) = x^3 - 6x^2$, $a = 0$, $b = 3$. Find a number c that satisfies: $a < c < b$ and $f'(c) = \frac{f(b) - f(a)}{b - a}$.

12. (4 pts) Calculate $\lim_{h \to 0} \frac{\tan\sqrt{\pi^2+h} - \tan\sqrt{\pi^2}}{h}$

(Hint: Consider the derivative of $f(x) = \tan\sqrt{x}$.)

APPENDIX D

MATH 181
EXAM 3

MATH 181 - Exam 3 Name: ______________

November 28, 1989 Section/Instructor: ______________

SHOW ALL WORK FOR FULL CREDIT.

DO NOT USE CALCULATORS.

FOR SCORING ONLY

Page	Points	Score
1	20	
2	30	
3	25	
4	25	
Tot	100	

In problems 1-5, fill in the blank with the letter of the best choice. Each problem is worth 4 points.

_____ 1. If f is a continuous function on the closed and bounded interval [a,b] then

a) f has a global maximum and a global minimum on [a,b].

b) f may have neither a global maximum nor a global minimum on [a,b].

c) There is at least one point c, $a \leq c \leq b$, such that $f'(c) = 0$.

d) f must have a local maximum or a local minimum at some point c, $a < c < b$.

_____ 2. If $f''(x) = 0$ for some x then

a) f has neither a local maximum nor a local minimum at x.

b) f has an inflection point at x.

c) $f'(x) = 0$

d) $\lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h} = 0$

_____ 3. Suppose that $f(x) = ax^2 + bx + c$, where a, b, and c are constants and $a > 0$. Then

a) f is concave down on $(-\infty,\infty)$.

b) f has a global maximum on $(-\infty,\infty)$.

c) f is concave up on $(-\infty,\infty)$.

d) f is increasing on $(-\infty,\infty)$.

_____ 4. The graph of $x^4 - y^3 = 1$ is symmetric about

a) the x-axis

b) the y-axis

c) the origin

d) none of the above

_____ 5. An asymptote of the graph of $f(x) = \frac{2x^3 - 2}{x^2}$ is

a) $y = 0$

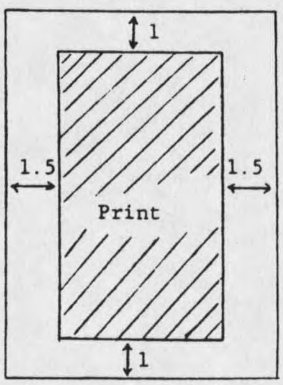
b) $x = 1$

c) $y = 2x$

d) $y = -2$

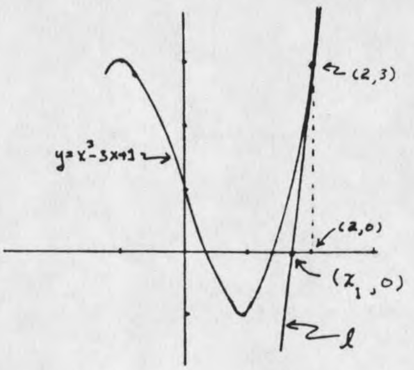
6. (10 pts) What is the global minimum of $f(x) = x^3 - 3x + 6$ on the interval $[-3, 2]$?

7. (20 pts) The rectangular area of print on the page of a book is to be 54 square inches. If the margins on each side of the page are 1.5 inches and the margins on the top and bottom of the page are 1 inch, what should the dimensions of the page be in order that the area of the page be a minimum?



8. (15 pts) An object moves along the graph of $x^2 - y^2 = 9$ in such a way that its x-coordinate is increasing at a rate of 8 cm/sec. What is the time rate of change of the y-coordinate of the object when the object passes through the point (-5,4)?

9. (10 pts) Pictured below is an application of Newton's method to approximate a root of $f(x) = x^3 - 3x + 1 = 0$ using the initial guess $x_0 = 2$.



a) What is the equation of the line ℓ?

b) What is the exact value of x_1?

10. (25 pts) Given $f(x) = \frac{4x - 4}{x^2}$, then $f'(x) = \frac{-4x + 8}{x^3}$ and

$f''(x) = \frac{8x - 24}{x^4}$.

a) On what intervals is f increasing?

b) On what intervals is f decreasing?

c) On what intervals is f concave up?

d) On what intervals is f concave down?

e) At what points does f have:

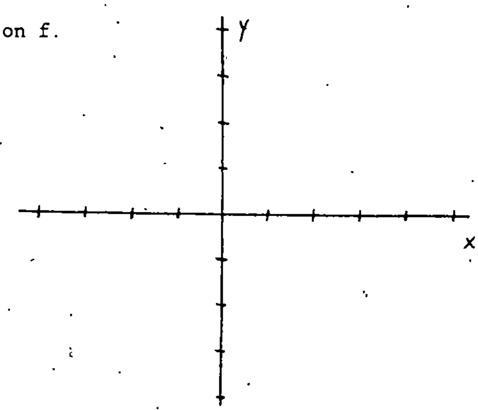
A local maximum?

A local minimum?

f) What are the points of inflection (give both x and y coordinates)?

g) What are the asymptotes of f?

h) Graph the function f.



APPENDIX E

MATH 181
FINAL EXAM

MATH 181 - FINAL EXAM   Name: ____________

December 14, 1989   Section/Instructor: ____________

SHOW ALL WORK FOR FULL CREDIT.

DO NOT USE CALCULATORS.

FOR SCORING ONLY

Page	Points	Score
1	24	
2	36	
3	30	
4	35	
5	35	
6	40	
Tot	200	

In problems 1-10, fill in the blank with the letter of the best choice. Each problem is worth 3 points.

____ 1. The function $f(x) = x^2 + x \sin x$ is

a) even b) odd

c) both even and odd d) neither odd nor even

____ 2. $\lim_{x \to 0^-} \frac{|x|}{x} =$

a) $+\infty$ b) $-\infty$ c) 1 d) -1

____ 3. $\lim_{x \to 0} \frac{\sin 2x}{x} =$

a) 1 b) 1/2 c) 2 d) The limit does not exist

____ 4. $\lim_{x \to -\infty} \frac{3 - x - 2x^3}{x^3 - x^2} =$

a) 2 b) -2 c) 3 d) The limit does not exist

____ 5. $\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} =$

a) 0 b) sin x c) -sin x d) The limit does not exist

____ 6. If $f'(c) = 0$, f'' is continuous on an open interval containing c, and $f''(c) > 0$ then

a) f has a local minimum at x = c.
b) f has a local maximum at x = c.
c) f has an inflection point at x = c.
d) perhaps none of the above

____ 7. If f is continuous (but not necessarily differentiable) on the interval [-1,1], $f(-1) = -1$, and $f(1) = 1$, then there must be a c, $-1 < c < 1$, such that

a) $f(c) = 0$ b) $f'(c) = 0$

c) $f''(c) = 0$ d) $f'(c) = 1$

____ 8. $D^{-1}(x\cos(x^2)) =$

a) $x\sin(x^2) + c$ b) $\sin 2x + c$

c) $\frac{1}{2}\sin(x^2) + c$ d) $\cos(x^2) - 2x^2\sin(x^2) + c$

____ 9. The coordinates of the center of the ellipse $x^2 + 2x + 4y^2 - 4y = 2$ are

a) (1, -1/4) b) (-1, 1/4) c) (1, 1/2)

d) (-1, 1/2)

____ 10. Which of the following lines is an asymptote for the hyperbola defined by $x^2 - 4y^2 + 8y = 5$?

a) $y = -1/2\ x$ b) $y = 1/2\ x + 1$

c) $y = -1/2(x + 1)$ d) $y = -1/2\ x - 1$

In problems 11-20, fill in the blanks with a T (true) or F (false). Each problem is worth 3 points.

____ 11. If the function f is differentiable at $x = c$ then f is continuous at $x = c$.

____ 12. If the function f is continuous on the interval $[1,\infty)$ and $\lim_{x\to\infty} f(x) = \infty$ then f has a global minimum on $[1,\infty)$.

____ 13. If $f''(c) = 0$ then the function f must have an inflection point at $x = c$.

____ 14. The product rule states that if the functions f and g are differentiable at x then $D(f(x)g(x)) = (Df(x))(Dg(x))$.

____ 15. If the function f is differentiable on the interval (a,b) and, on the interval [a,b] f has a global maximum at $x = c$ with $a < c < b$, then $f'(c) = 0$.

____ 16. $f(x) = |x + 1|$ is differentiable at $x = -1$.

____ 17. The line $x = \pi/2$ is an asymptote for the graph of $f(x) = \sec x$.

____ 18. The line $y = 0$ is an asymptote for the graph of $g(x) = \dfrac{\sin x}{x}$.

____ 19. $D^5(x^5 + x^3) = 120$

____ 20. The function $f(x) = x^3 - 3x$ is decreasing on the interval (-1,1).

21. (5 pts) $\lim_{x \to 2} \frac{x^2 - 4}{x + 2} =$

22. (5 pts) $\lim_{x \to 1^-} \frac{|x| - 1}{|x - 1|} =$

23. (5 pts) $\lim_{x \to -\infty} \frac{\sqrt{x^2 + x} - x}{x} =$

24. (5 pts) $\lim_{x \to 0} \frac{\sin x - \sin x \cos x}{x^2} =$

25. (10 pts) Use the definition of the derivative as a limit of difference quotients to calculate $f'(2)$ given $f(x) = x^2 + 1$.

26. (5 pts) Given $f(x) = (x + 1)^2(x^2 - 1)^3$, find $f'(x)$.

27. (5 pts) Given $(s + t)^2 = t$, find $\frac{ds}{dt}$.

28. (5 pts) Given $g(x) = \tan(3x + \pi)$,

a) $g'(x) =$

b) $g''(x) =$

29. (10 pts) Given that $f'(t) = -32t + 2$ and $f(2) = 0$, find $f(t)$ for all t.

30. (10 pts) Is the function $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$

continuous at $x = 1$? Justify your answer.

31. (10 pts) Given that $f(x) = \sqrt{x}$ and $g(x) = 4 + \sin 2x$, find $(f \circ g)'(0)$.

32. (10 pts) Find all critical points of $f(x) = \frac{1}{5}x^5 - \frac{1}{3}x^3 + 1$ and classify each as a local maximum, a local minimum, or neither.

33. (15 pts) A rectangular box with square base is to have a volume of 16 cubic feet. If the material used in the construction of the sides and top of the box weighs 1 pound per square foot and the material used in the construction of the base of the box weighs 3 pounds per square foot, find the dimensions of the box of minimum weight.

34. (15 pts) A spherical pig is increasing in volume at the rate of 1/2 cubic foot per day. At what rate is the surface area of the pig increasing when the radius of the pig is 2 feet?

(A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.)

35. (15 pts) A curve is parameterized by $x = t^3 - 2t$, $y = t^2$.

a) Find the slope of the tangent line to the curve at the point (-1,1) on the curve corresponding to $t = 1$.

b) Is the curve concave up or concave down at the point (-1,1)?

36. (10 pts) Given $g(x) = x\cos x + \sin x$, is there a c, $0 < c < \frac{\pi}{2}$, such that $g'(c) = \frac{2}{\pi}$? Justify your answer.

APPENDIX F

MATH 121
FINAL EXAM

MATH 121

Final Exam

NAME ______________________

1. Evaluate the following: (If no limits exist, state - no limit)

   a) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} 1-x^2 & \text{for } x<0 \\ x+1 & \text{for } x \geq 0 \end{cases}$

   b) $\lim_{x \to 2} \frac{x^2}{x-2}$

   c) $\lim_{x \to \infty} \frac{2x^2}{e^x}$

2. Differentiate the following with respect to x:

   a) $g(x) = \sqrt{1 + \cos(x)}$

   b) $f(x) = xe^{2x}$

   c) $h(x) = \ln(\sqrt{2x-1})$

3. Integrate the following:

a) $\displaystyle\int \frac{\sin(x)}{1+\cos(x)}\,dx$

b) $\displaystyle\int \sqrt{3x+4}\,dx$

c) $\displaystyle\int x\ln(x)\,dx$

d) $\displaystyle\int \frac{dx}{\sqrt{2-x^2}}$

4. Find the area enclosed by the two curves: $f(x) = 2x^2$ and $g(x) = x^3$

5. Given the function $f(x) = 2x^3+3x^2-12x-3$

   a) Find all critical points of $f(x)$

   b) Sketch the function $f(x)$

6. The area of a square is increasing at a rate of 250 $\frac{ft^2}{sec}$. How fast is the perimeter changing at the instant one side is equal to 100 feet?

7. Two non-negative numbers are such that the first plus the square of the second is 10. Find the numbers if their sum is to be as large as possible.

APPENDIX G

COMPARISON OF TOPICS IN CALCULUS AB AND CALCULUS BC

Comparison of Topics in Calculus AB and Calculus BC

Column I below lists topics that should be covered in a Calculus AB course or in preparation for it. It is assumed that each topic in Column I will have been studied as a prerequisite for or as a part of the Calculus BC course. The topics in Column II are additional topics that are to be covered in Calculus BC but are not included in Calculus AB.

Note: The examples that are given with some of the following topics are not intended to be exhaustive, but rather to illustrate the kinds of questions or functions that might appear on the test.

Column I. Topics covered in Calculus AB

A. Elementary Functions
(algebraic, trigonometric, exponential, and logarithmic)

1. Properties of functions
   a. Definition, domain, and range
   b. Sum, product, quotient, and composition
   c. Absolute value, e.g., $|f(x)|$ and $f(|x|)$
   d. Inverse
   e. Odd and even
   f. Periodicity
   g. Graphs; symmetry and asymptotes
   h. Zeros of a function
2. Properties of particular functions
   a. Fundamental identities and addition formulas for trigonometric functions
   b. Amplitude and periodicity of $A \sin (bx + c)$ and $A \cos (bx + c)$
   c. a^x ($a > 0$, $a \neq 1$) and $\log_a x$ ($a > 0$, $a \neq 1$, and $x > 0$) and their inverse relationship

Column II. Additional topics covered in Calculus BC

i. Vector functions and parametrically defined functions

j. Graphs in polar coordinates

Column I. Topics covered in Calculus AB

Column II. Additional topics covered in Calculus BC

3. Limits
   a. Statement and applications of properties, e.g., limit of a constant, sum, product, and quotient
   b. The number e such that
   $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
   c. $\lim_{x\to 0}\frac{\sin x}{x} = 1$
   d. Nonexistent limits, e.g.,
   $\lim_{x\to 0}\frac{1}{x^2}$, $\lim_{x\to 0}\sin\frac{1}{x}$, and $\lim_{x\to 0}\frac{|x|}{x}$
   are each, for different reasons, nonexistent.
   e. Continuity
   f. Statements and applications but not proofs of continuity theorems: if f is continuous on $[a, b]$, then f has a maximum and a minimum on $[a, b]$; the intermediate value theorem
   g. Epsilon-delta definition

B. Differential Calculus

1. The derivative
   a. Definitions of the derivative; e.g.,
   $$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \text{ and}$$
   $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
   b. Derivatives of elementary functions
   c. Derivatives of sum, product, quotient
   d. Derivative of a composite function (chain rule)
   e. Derivative of an implicitly defined function
   f. Derivative of the inverse of a function (including Arcsin x and Arctan x)
   l. Derivatives of vector functions and parametrically defined functions

Column I. Topics covered in Calculus AB	*Column II. Additional topics covered in Calculus BC*
g. Logarithmic differentiation	
h. Derivatives of higher order	
i. Statement (without proof) of the Mean Value Theorem; applications and graphical illustrations	
j. Relation between differentiability and continuity	
k. Use of l'Hôpital's rule (quotient indeterminate forms)	m. l'Hôpital's rule (exponential and other indeterminate forms)
2. Applications of the derivative	
a. Slope of a curve; tangent and normal lines to a curve (including linear approximations)	g. Tangent lines to parametrically defined curves
b. Curve sketching	
(1) Increasing and decreasing functions	
(2) Critical points; relative (local) and absolute maximum and minimum points	
(3) Concavity	
(4) Points of inflection	
c. Extreme value problems	
d. Velocity and acceleration of a particle moving along a line	h. Velocity and acceleration vectors for motion on a plane curve
e. Average and instantaneous rates of change	
f. Related rates of change	

C. Integral Calculus

1. Antiderivatives	
2. Applications of antiderivatives	
a. Distance and velocity from acceleration with initial conditions	
b. Solutions of $y' = ky$ and applications to growth and decay	
3. Techniques of integration	
a. Basic integration formulas	
b. Integration by substitution (use of identities, change of variable)	d. Integration by trigonometric substitution: trigonometric integrals

Column I. Topics covered in Calculus AB

c. Simple integration by parts, e.g., $\int x \cos x\, dx$ and $\int \ln x\, dx$

4. The definite integral
   a. Concept of the definite integral as an area
   b. Approximations to the definite integral by using rectangles
   c. Definition of the definite integral as the limit of a sum
   d. Properties of the definite integral
   e. Fundamental theorems: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$ and $\int_a^b f(x)dx = F(b) - F(a)$, where $F'(x) = f(x)$

5. Applications of the integral
   a. Average value of a function on an interval
   b. Area between curves
   c. Volume of a solid of revolution (disc, washer, and shell methods) about the x- and y-axes or lines parallel to the axes

Column II. Additional topics covered in Calculus BC

e. Repeated integration by parts, e.g., $\int e^x \cos x\, dx$

f. Integration by partial fractions

f. Approximations: upper and lower sums, trapezoidal rule

g. Recognition of limits of sums as definite integrals

h. Functions defined by integrals, e.g., $f(x) = \int_0^{x^2} e^{-t^2}dt$

d. Area bounded by polar curves

e. Volumes of solids with known cross sections

f. Length of a path including parametric curves

g. Improper integrals

D. Sequences and Series

1. Sequences of real numbers and of functions; convergence
2. Series of real numbers
   a. Geometric series

Column I. Topics covered in Calculus AB	*Column II. Additional topics covered in Calculus BC*

b. Tests for convergence: comparison (including limit comparison), ratio, and integral tests
c. Absolute and conditional convergence
d. Alternating series and error approximation

3. Power series
   a. Radius of convergence; interval of convergence
   b. Taylor series
   c. Taylor polynomials with remainder and error approximation
   d. Manipulation of series, i.e., addition of series, substitution, term-by-term differentiation and integration

E. Elementary Differential Equations

1. First-order, variables separable
2. First-order, linear with constant coefficients, i.e., equations of the form
   $$\frac{dy}{dx} + ky = g(x),$$
   where k is a constant
3. Applications with initial conditions

APPENDIX H

TOPIC QUESTIONNAIRE

1. School______________________________
2. Instructor______________________________
3. Course title in 1988-89______________________________
4. Duration of course ___full year
   ___one semester
   ___other (specify)______________
5. Title and author of text used______________________________
6. This course was (check one)
   ____ an Advanced Placement Program Course (AB Version)
   ____ an Advanced Placement Program Course (BC Version)
   ____ not under the auspices of the Advanced Placement Program.
7. ____No course was offered featuring any of the topics listed below.
8. ____I would like further information about the study being conducted.

Place a check beside each topic that was discussed in the calculus course offered in the **1988-1989** school year.

____ intuitive definition of limit
____ epsilon-delta definition of limit
____ one-sided limits
____ limits at infinity
____ "sandwich" or "squeezing" limit theorem
____ continuity
____ l'Hopital's Rule
____ tangent lines of curves
____ limit definition of derivative
____ sum, product, quotient rules of differentiation
____ Chain Rule
____ higher derivatives
____ implicit differentiation
____ tangent line approximations and the differential
____ increasing/decreasing functions
____ finding max/min points with derivatives
____ second derivative test
____ Rolle's Theorem and/or Mean Value Theorem
____ concavity and inflection points
____ applied max/min problems
____ related rates applications
____ limit-summation definition of integral
____ computation of integral by above method for simple functions

____ linearity properties of integrals, i.e., $\int(f + g) = \int f + \int g$ and $\int cf = c\int f$
____ Fundamental Theorem of Calculus (integrals as antiderivatives)
____ indefinite integrals
____ area computation by integration
____ volumes: the disk method
____ volumes: the shell method
____ arc length and/or surface area
____ force-work problems with integration
____ the natural logarithm as $\int_1^x dt/t$
____ integration by parts
____ integration by u-substitution
____ integrals of trigonometric functions
____ integrals by trigonometric substitution
____ integration by partial fractions
____ Trapezoidal and/or Simpson's Rule
____ improper integrals
____ inverse functions
____ continuity and/or derivatives of inverse functions
____ derivatives and/or integrals involving inverse trigonometric functions
____ differentiation of exponential functions
____ differentiation of logarithm functions
____ integration of exponential functions
____ integration of logarithm functions
____ exponential growth and decay
____ hyperbolic functions
____ graphing conic sections, focus points, asymptotes, etc.
____ rotation of axes in graphing conic sections
____ infinite sequences
____ infinite series (Briefly specify series topics discussed.)__________
____
____ introduction to differential equations

Teacher typically proved major theorems in class. (Check one.)
____almost always ____often ____occasionally ____rarely