



Thermal properties of extreme black hole spacetimes  
by Daniel J Lorz

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy  
Physics in  
Montana State University  
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Abstract:

Constraints are placed on the thermal properties of extreme black hole spacetimes by examining the expectation value of the stress-energy tensor for a free conformally invariant scalar field in such spacetimes.

Working in a general two dimensional static black hole spacetime, the stress-energy of the quantized field in thermal equilibrium with a black hole is found to diverge strongly on the event horizon of the black hole unless the field is in a thermal state with a temperature defined by the surface gravity of the event horizon. This divergence occurs for both extreme and nonextreme black holes in precisely the same manner. Thus, extreme black hole spacetimes cannot be assigned an arbitrary temperature without serious consequences.

Studying the stress-energy for a quantized scalar field in the reduced two dimensional charged dilatonic black hole spacetime of Garfinkle, Horowitz, and Strominger further confirms that extreme black hole spacetimes have well defined thermal properties. Despite the lack of horizons in the string description of the extreme dilatonic black hole, a unique temperature is obtained for the extreme black hole by extrapolating the temperature of the nonextreme black hole to the extreme state. Any other temperature is shown to result in a divergent stress-energy in the conformally related physical description of the extreme dilatonic black hole.

Finally, the expectation value of the stress-energy tensor for a massless conformally invariant scalar field is numerically calculated for the full four dimensional extreme dilatonic black hole of Garfinkle, Horowitz, and Strominger. Working in the string metric, the components of this stress-energy are found to be finite everywhere in the extreme dilatonic black hole spacetime. These results are the first values calculated for the stress-energy of a quantized field in a superstring black hole spacetime.

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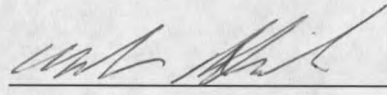
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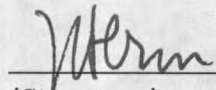
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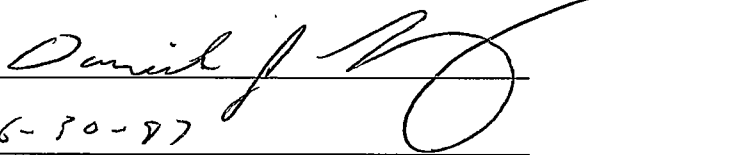
  
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*The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful.*

-Henri Poincaré

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## CONVENTIONS

Throughout this dissertation I use the conventions and notation of Misner, Thorne, and Wheeler [6]. In addition I use natural units ( $c = G = \hbar = k_B = 1$ ).

## ABSTRACT

Constraints are placed on the thermal properties of extreme black hole spacetimes by examining the expectation value of the stress-energy tensor for a free conformally invariant scalar field in such spacetimes.

Working in a general two dimensional static black hole spacetime, the stress-energy of the quantized field in thermal equilibrium with a black hole is found to diverge strongly on the event horizon of the black hole unless the field is in a thermal state with a temperature defined by the surface gravity of the event horizon. This divergence occurs for both extreme and nonextreme black holes in precisely the same manner. Thus, extreme black hole spacetimes cannot be assigned an arbitrary temperature without serious consequences.

Studying the stress-energy for a quantized scalar field in the reduced two dimensional charged dilatonic black hole spacetime of Garfinkle, Horowitz, and Strominger further confirms that extreme black hole spacetimes have well defined thermal properties. Despite the lack of horizons in the string description of the extreme dilatonic black hole, a unique temperature is obtained for the extreme black hole by extrapolating the temperature of the nonextreme black hole to the extreme state. Any other temperature is shown to result in a divergent stress-energy in the conformally related physical description of the extreme dilatonic black hole.

Finally, the expectation value of the stress-energy tensor for a massless conformally invariant scalar field is numerically calculated for the full four dimensional extreme dilatonic black hole of Garfinkle, Horowitz, and Strominger. Working in the string metric, the components of this stress-energy are found to be finite everywhere in the extreme dilatonic black hole spacetime. These results are the first values calculated for the stress-energy of a quantized field in a superstring black hole spacetime.

## CHAPTER 1

### Introduction

This dissertation examines the temperature of extreme black holes and the expectation values of the stress-energy for quantized fields in general static black hole spacetimes. Black hole radiance and stable extreme black hole states play an important role in many contemporary studies exploring the quantum nature of gravity. That black holes have temperatures and can radiate may seem an oxymoron to the reader not familiar with quantum gravity. Even the purely classical notion of an extreme black hole may be new for the lay person. This chapter introduces necessary definitions and sets the context for the work that follows.

### Black Hole Structure

Most people have a basic understanding that a black hole is an object with a strong gravitational attraction from which nothing, not even light, can escape. One does not need general relativity to imagine such objects. In the late 1700's Michell [1] and Laplace [2] independently theorized such invisible stars using Newtonian gravity

and the corpuscular theory of light. Their calculations showed that a star with Earth-like density and a radius two hundred and fifty times larger than the sun would trap its own radiation and allow no light to escape.

This intuitive idea of a black hole as a region of "no escape" is formalized by examining the causal structure of a black hole spacetime. Because of the maximum velocity defined by special relativity, not all points in a spacetime are causally connected to a given event. The boundaries of a causally connected region are defined by the trajectories followed by massless particles, such as photons. These trajectories, called null rays, mark the past and future light cones of an event. Points lying within the light cone are causally connected to the event, while points outside the light cone are not. Normally, all points in a spacetime lie in the causal past (i.e. within the past light cone) of future infinity and in the causal future (future light cone) of past infinity. However, a black hole spacetime has a region that is not causally connected to future infinity but is causally connected to past infinity. This region of points represents the black hole, and the boundary between this region and the region that is causally connected to future infinity is called the event horizon (see figures (1) and (2)). Thus, a black hole is formally defined as a region of spacetime that is not causally connected to future infinity.

The simplest black hole described by general relativity is given by the Schwarzschild metric

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (1)$$

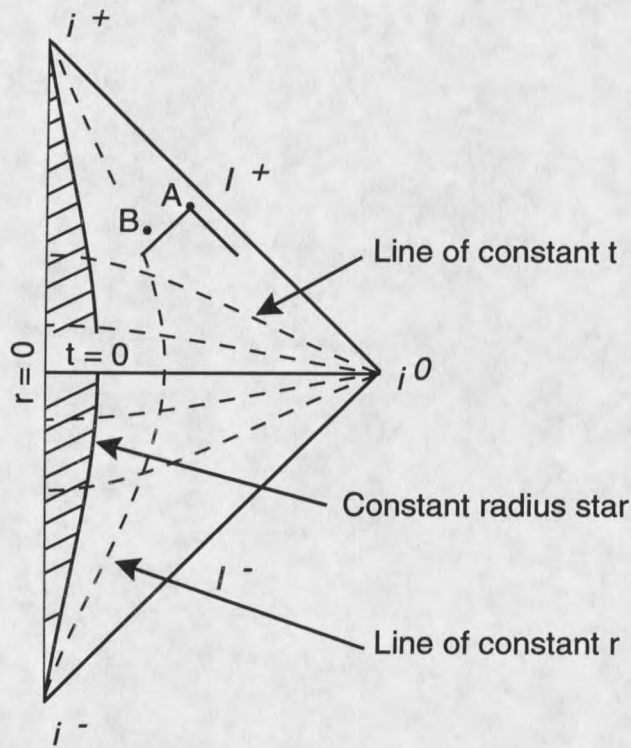


Figure 1: This figure show a conformal diagram of a constant radius star. As for all conformal diagrams, the null rays in this figure travel along lines tilted at  $45^\circ$  and infinity has been mapped to a finite boundary located at  $i$  and  $I$ . Note that events A and B are causally disconnected since event B lies outside A's light cone. In contrast, all events lie in the causal past of future infinity,  $i^+$ , since all events lie within the past light cone of  $i^+$  defined by the surface  $I^+$ . For reference, a few lines of constant  $r$  and constant  $t$  are shown. These will be suppressed in succeeding diagrams.

where  $M$  is the mass of the black hole,  $d\Omega^2$  is the metric of the two sphere,  $d\theta^2 + \sin^2\theta d\phi^2$ , and the metric components  $g_{\alpha\beta}$  are read from the interval  $ds$  by  $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$  in the usual manner. The Schwarzschild metric describes a static, spherically symmetric, vacuum spacetime with an event horizon at  $r = 2M$  and a curvature singularity at  $r = 0$  (see figure (3)). One might suspect this curvature singularity results from the spherical symmetry of the Schwarzschild geometry. However a set of proofs by

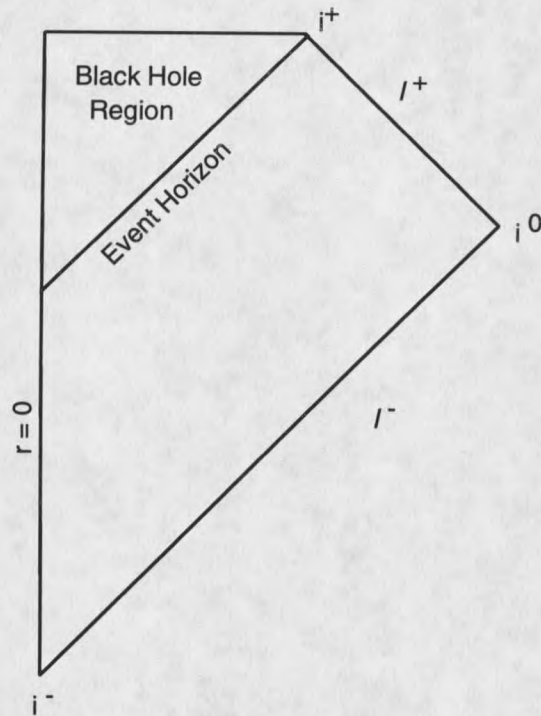


Figure 2: In this diagram, not all events lie in the causal past of future infinity,  $i^+$ . Those points of the spacetime causally disconnected from  $i^+$  represent the black hole. Note that timelike paths exist which enter the black hole and become trapped.

Hawking and Penrose show that singularities are a generic feature of black holes [3, 4]. These proofs require only that general relativity correctly describes gravity, that local energy density be positive, and that an event horizon forms during the gravitational collapse. Thus, the general relativistic picture of a black hole includes a singularity enclosed by an event horizon.

The Schwarzschild metric is the unique time independent, spherically symmetric, vacuum solution to the Einstein equations [5]. General relativity permits only two other stationary electrovacuum black hole solutions [6]. One solution generalizes

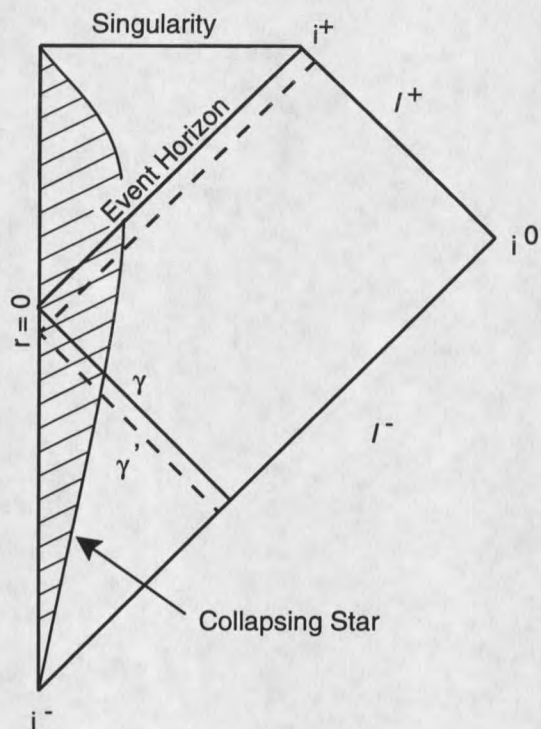


Figure 3: This conformal diagram depicts a spherically symmetric star collapsing to form a Schwarzschild black hole. The null ray  $\gamma'$  just manages to escape to infinity after reflecting from  $r = 0$ . The very next null ray,  $\gamma$ , marks the event horizon of the black hole. Any null ray reflecting from  $r = 0$  after  $\gamma$  enters the black hole and becomes trapped. Any observer traveling along a timelike geodesic that crosses the event horizon also becomes trapped within the black hole.

Schwarzschild by including an electromagnetic field and is given by the Reissner-Nordström metric

$$ds^2 = - \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad (2)$$

where  $Q$  is the charge on the black hole. The other solution generalizes Reissner-Nordström by including angular momentum and is given by the Kerr-Newman metric

$$ds^2 = - \frac{\Delta}{\rho^2} [dt - a \sin^2 \theta d\phi]^2 + \frac{\sin^2 \theta}{\rho^2} [(r^2 + a^2) d\phi - a dt]^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2, \quad (3)$$

where  $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$ ,  $\rho^2 \equiv r^2 + a^2 \cos^2 \theta$ , and  $a$  is the angular momentum of the black hole per unit mass,  $a = J/M$ . For  $Q = 0$ , the metric of Eq. (3) reduces to the Kerr metric. A general analysis of gravitational collapse remains unsolved. However, the gravitational collapse of a star with more than twice the mass of the sun is expected to form a black hole which will settle down to a final state described by the Kerr-Newman metric [3, 6, 7]. Thus, general relativity suggests a picture of complete gravitational collapse that ends in a black hole parameterized only by the black hole's mass, charge, and angular momentum. This result is often stated colloquially by saying, "black holes have no hair."

The simple picture of a black hole as a singularity enclosed by an event horizon still holds true for these more general black holes. Still, charge and angular momentum will change the structure of a black hole in some interesting ways. For the work

that follows, the most important change is the creation of an inner horizon. Unlike the Schwarzschild case, not all timelike geodesics that cross the event horizon of a Reissner-Nordström or Kerr-Newman black hole terminate on the singularity within. In fact, an observer who crosses the event horizon of a Reissner-Nordström or Kerr-Newman black hole, while forever trapped inside, is free to travel to regions within the black hole that are far from the singularity and have small curvature. However, during these maneuvers the observer eventually enters a region causally connected to the singularity. The boundary of such a region is a Cauchy horizon, and for the Reissner-Nordström black hole, this boundary marks the inner horizon (see figure (4)).

Generally, determining the horizon structure of a particular black hole geometry requires global knowledge of the entire spacetime. However, for the special case of a static spacetime, the horizons are located where  $g_{tt} = 0$ . For a Schwarzschild black hole  $g_{tt} = -(1 - 2M/r)$ , and one thus finds a single horizon at  $r_0 = 2M$ . For a Reissner-Nordström black hole, one finds inner and outer horizons ( $r_-$  and  $r_+$  respectively) at  $r_{\pm} = M \pm \sqrt{M^2 - Q^2}$ . Note that as the charge,  $Q$ , goes to zero, the inner horizon vanishes and the outer horizon moves outward to  $r_+ = 2M$ , the Schwarzschild value. On the other hand, when  $Q \rightarrow M$ , the horizons become degenerate ( $r_- \rightarrow r_+ \rightarrow M$ ), and the event horizon becomes a Cauchy horizon. In fact, if  $Q$  is greater than  $M$ , the metric component  $g_{tt}$  has no roots, no horizons exist in the spacetime, and instead of describing a black hole, Eq. (2) describes a "naked" singularity. This continuum from Schwarzschild-like black hole to naked singularity

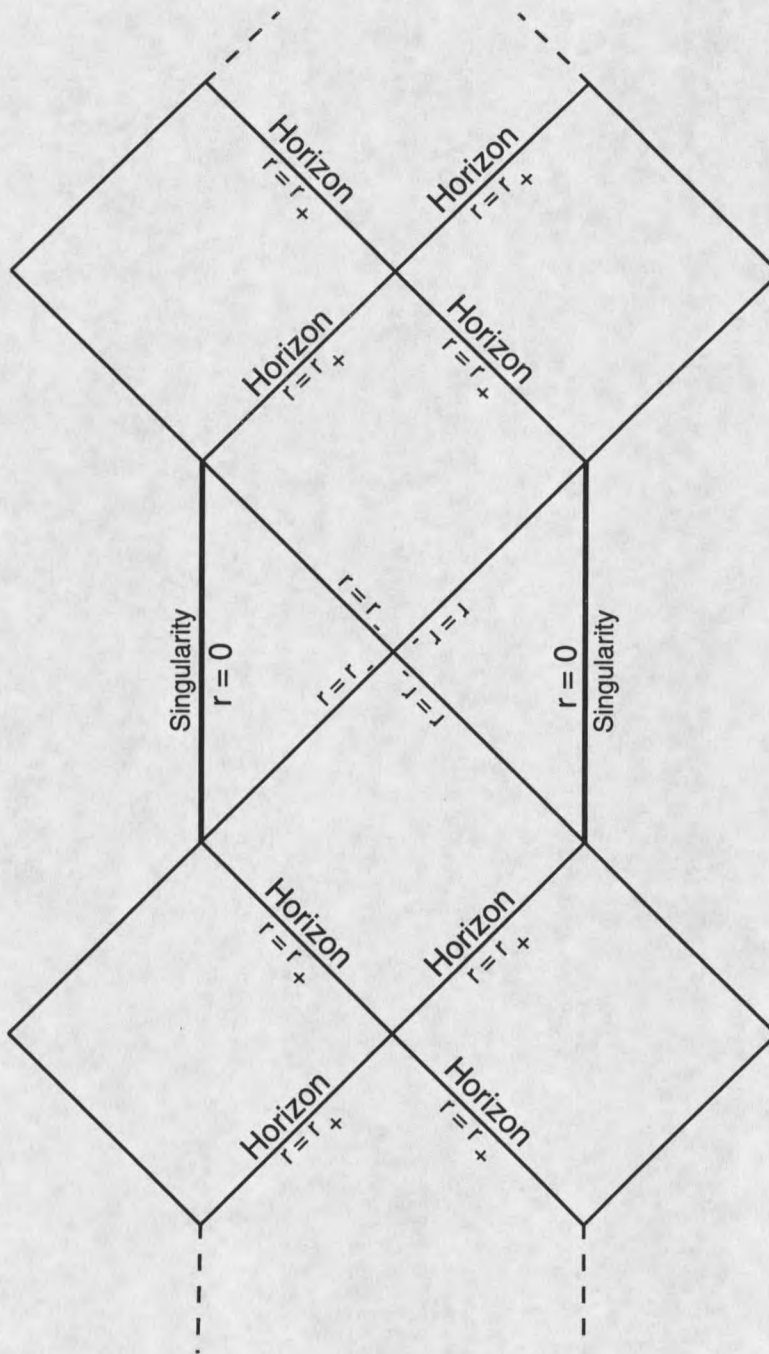


Figure 4: The Reissner-Nordström black hole has multiple horizons. An observer who crosses the inner horizon,  $r_-$ , enters a region causally connected to a singularity. The symmetry of the Reissner-Nordström solution implies this geometry continues, resulting in an infinite set of black holes within black holes.

is a general description of the possible configurations of black holes with multiple horizons. A black hole with degenerate horizons is called an extreme black hole.

A property of black holes important to this dissertation is the surface gravity,  $\kappa$ . As in Newtonian theory, surface gravity measures the force required to keep a unit test mass at rest just above the surface of a gravitating body. The general metric for a static, spherically symmetric black hole is

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2d\Omega^2. \quad (4)$$

For Eq. (4) one finds the surface gravity to be

$$\kappa = \frac{1}{2} \frac{f'}{\sqrt{fh}} \Big|_{r \rightarrow r_0}, \quad (5)$$

where  $r_0$  is the location of the event horizon, and a prime denotes differentiation with respect to  $r$ . Using Eq. (5) one finds the surface gravity for a Schwarzschild black hole to be  $\kappa = 1/(4M)$ . For a Reissner-Nordström black hole the surface gravity is  $\kappa = \sqrt{M^2 - Q^2}/r_+^2$ , where  $r_+$ , recall, is the radius of the outer horizon. The surface gravity of a Reissner-Nordström black hole thus depends on the charge as well as the mass of the black hole. As the charge increases from zero to  $M$ , the surface gravity decreases from  $1/(4M)$ , the Schwarzschild value, to zero. Hence, an extreme Reissner-Nordström black hole has zero surface gravity.

This section defined both the black hole in general and the extreme black hole con-

figuration. It also highlighted some of the physical details of black holes as presented by general relativity. As interesting as this picture may be, the general relativistic description of black holes is incomplete. One must still consider the interaction of matter fields with the black hole spacetime. The quantum characteristic of matter is well established. Accounting for this quantum aspect, one finds that black holes radiate a thermal spectrum of particles.

### Black Hole Radiation

The singularity lurking at the heart of a black hole is a disturbing presence. Singularities are locations where the description of spacetime as a manifold with pseudo-Riemannian geometry breaks down. Since all known physical laws depend on this description of spacetime, all of physics breaks down at the singularity. In the words of Stephen Hawking, "This is a great crisis for physics because it means that one cannot predict the future: One does not know what will come out of the singularity." [8] In this sense, general relativity predicts its own failure.

One might argue that the event horizon enclosing the singularity makes the breakdown of predictability a nonissue. Observers outside a black hole are causally disconnected from the singularity. They cannot see phenomena occurring within the event horizon. However, can one assume that a newly formed singularity is always safely enclosed by an event horizon? The assumption that singularities are always enclosed by an event horizon is known as the weak cosmic censorship hypothesis, and while

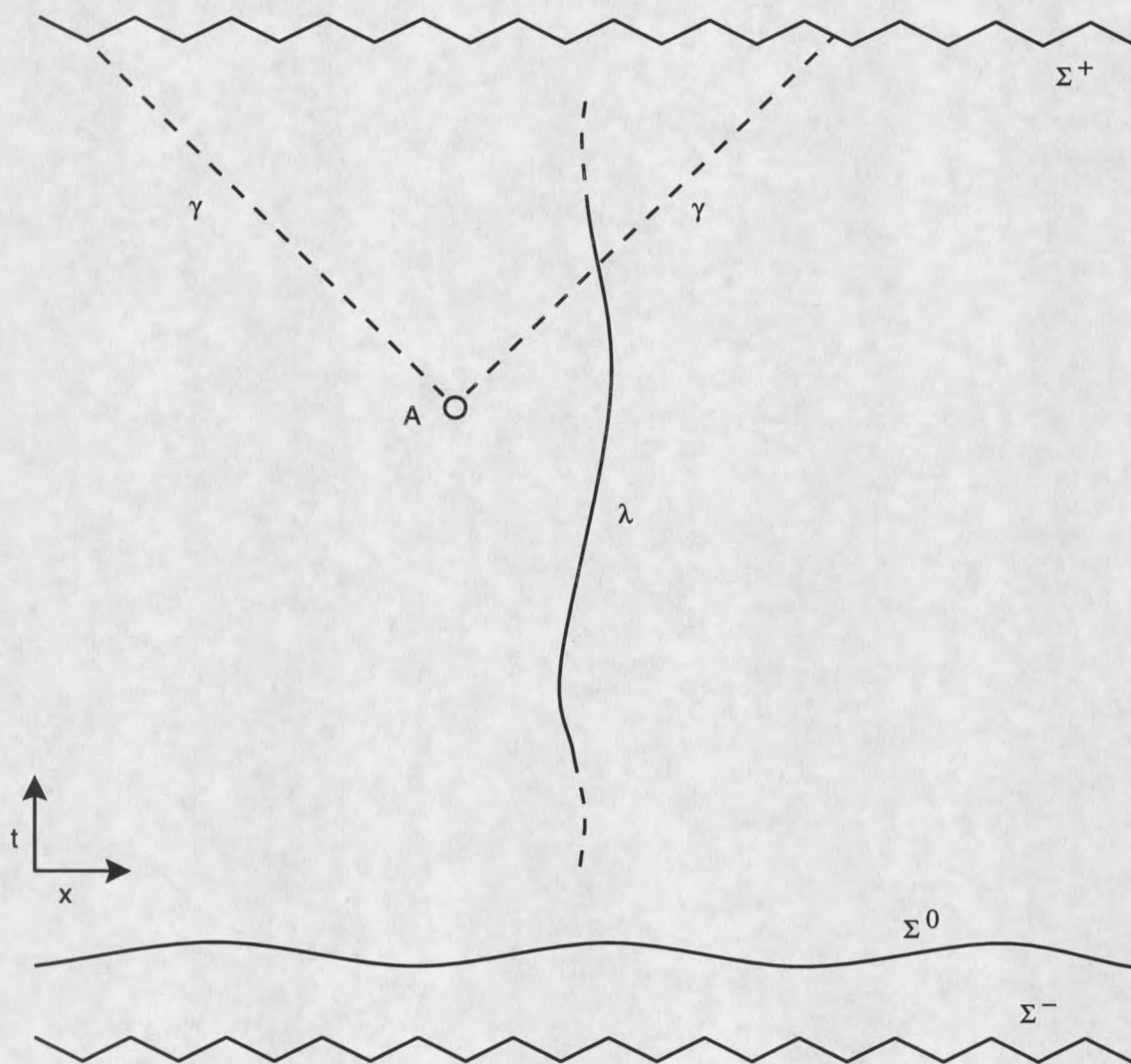


Figure 5: Three naked singularities confront an observer in this spacetime diagram. The first, located at  $\Sigma^-$ , marks the beginning of the spacetime and does not affect the time evolution of physical laws after its occurrence. Thus, initial conditions may be given along the spacelike hypersurface  $\Sigma^0$ . The singularity located at  $\Sigma^+$ , marks the end of the spacetime and goes unobserved until the observer's geodesic terminates. Only the singularity at point  $A$  causes a problem for the observer's ability to make predictions. The initial conditions given along  $\Sigma^0$  are insufficient to time evolve one's equations into the region bounded by the Cauchy horizon,  $\gamma$ .

probably true for general relativity, this hypothesis remains unproven [6].

Regardless of cosmic censorship's validity, an event horizon does not necessarily mediate the difficulties caused by singularities. For example, consider an observer who falls into a Reissner-Nordström black hole and continues across the inner horizon. Causally connected with the singularity, this observer can no longer make reliable physical predictions. This predictive failure occurs even though the observer's local spacetime can be quite normal, i.e., have negligible curvature.

Another problem arises from Hawking's discovery that black holes radiate thermal energy [8]. A clue that black holes have temperature and thus radiate, comes from entropy arguments. Imagine a collection of particles with entropy,  $S_{particles}$ , falling into a black hole. By the no hair theorems, no externally measured properties of the black hole can depend on  $S_{particles}$ . Thus, unless the black hole has its own intrinsic entropy, the entropy of the universe decreases by  $S_{particles}$  once the particles cross the event horizon, and the second law of thermodynamics is violated. Therefore, to preserve the second law of thermodynamics, one must postulate an intrinsic black hole entropy,  $S_{BH}$ , such that  $\Delta S_{BH} \geq S_{particles}$ .

An intrinsic black hole entropy is also suggested by the mathematical similarities between the second law of thermodynamics and Hawking's area theorem, which states that the total area of all event horizons in a spacetime does not decrease into the future [9, 3]. Bekenstein thus proposed that black hole entropy is proportional to the surface area,  $A$ , of the event horizon [10]. Since a black hole's surface area is, in turn, a function of the mass, charge, and angular momentum ( $M$ ,  $Q$ , and  $J$ ) of the

black hole, one has  $S_{BH} = f[A(M, Q, J)]$ . The energy,  $E$ , associated with a black hole spacetime is also a function of  $M$ ,  $Q$ , and  $J$ . Thus, one can write

$$T = \frac{dE}{dS} = F(M, Q, J). \quad (6)$$

Following the procedure above for the specific case of a Schwarzschild black hole, one finds  $T_{Sch} = \alpha/M$ , where  $\alpha$  is a constant independent of  $M$ .

This link between black holes and thermodynamics appears contradictory, unless physical meaning can be ascribed to the idea of black hole temperature. In the classical limit, black holes emit nothing and thus, must always have temperature identically equal to zero. To understand how a black hole can have nonzero temperature, one must account for the quantum aspects of the matter fields interacting with the black hole spacetime.

Consider the spacetime of an isolated object collapsing to form a black hole. (The reader can refer back to figure (3) for an example). The task is to follow the evolution of a quantized field as it interacts with the geometry of this spacetime. As a first approximation, one ignores the gravitational effects of the quantized field.

Initially, the spacetime is asymptotically flat far from the collapsing object, and one can easily choose a basis in which to write the quantum state of the field. For simplicity, let the matter field begin in a vacuum state. In the usual manner, then, all modes yield an eigenvalue of zero when operated on by the annihilation operator. The spacetime has no particles at past infinity. However, as the matter fields evolve

forward in time, the curvature of the black hole spacetime causes the positive and negative frequency modes to mix. As a result, an observer at future infinity sees a thermal spectrum of particles streaming from the black hole. This spectral flux is proportional to  $1/[\exp(\frac{2\pi\omega}{\kappa}) - 1]$ , where the frequency  $\omega$  specifies the eigenmode and  $\kappa$  is the surface gravity of the black hole [11]. (For a thorough discussion of this effect, known as Hawking radiation, see [12] and [13].) Thus, the black hole appears to radiate like a black body with temperature

$$T = \frac{\kappa}{2\pi}. \quad (7)$$

The temperature suggested by thermodynamic arguments is a result of the quantum emission of particles by the black hole.

For a Schwarzschild black hole,  $\kappa = 1/(4M)$ , and so the temperature associated with the Hawking radiation is inversely proportional to the mass of the black hole. In conventional units,  $M_{\odot}$ ,  $T = 6 \times 10^{-8} \left(\frac{M_{\odot}}{M}\right)$  Kelvin. Hence, black holes formed during stellar collapse ( $M \geq 2M_{\odot}$ ) have temperatures much smaller than the temperature associated with the cosmic background radiation. These black holes absorb energy from their surroundings, gaining mass and cooling further. (As the universe continues to expand and cool, this situation will eventually reverse itself.) However, primordial black holes, which may have formed during the early stages of the universe, can have masses much less than a solar mass. With temperatures exceeding the present 3K cosmic background value, these black holes would radiate energy. This

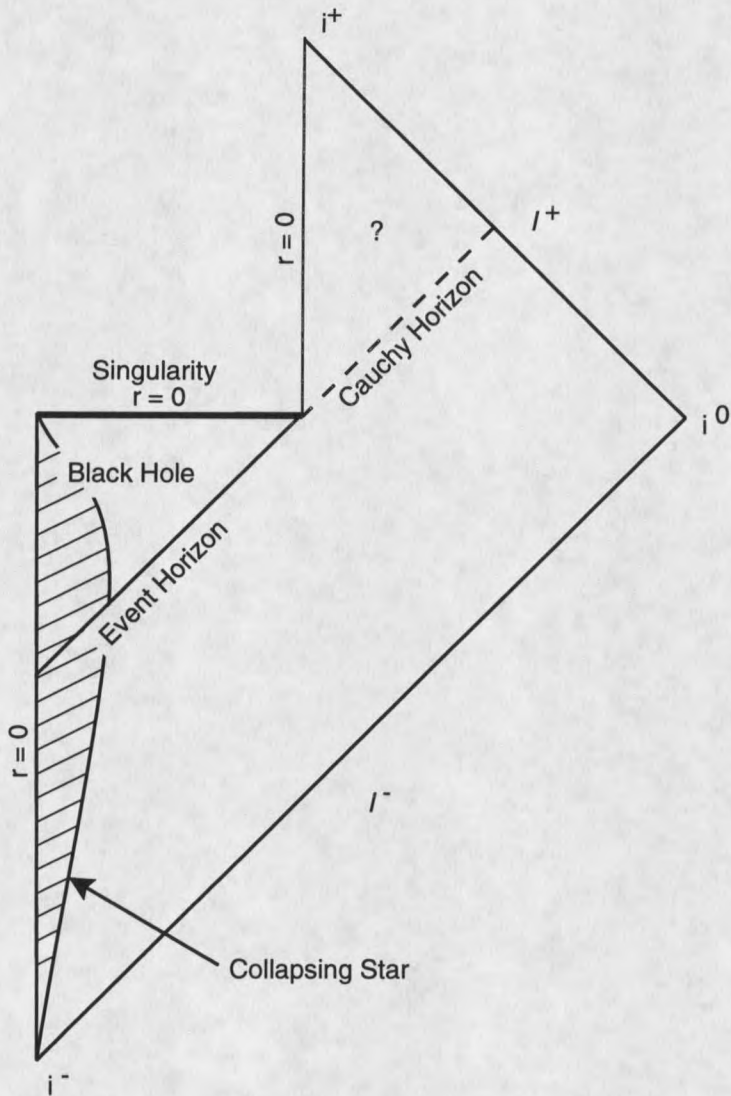


Figure 6: This conformal diagram shows an object collapsing to form a black hole which then evaporates. As always, the black hole does not lie in the causal past of future infinity. During evaporation, the event horizon approaches the singularity. The final state of black hole evaporation remains an unsolved problem.

radiated energy comes from the gravitational field, and so the black hole loses mass. However, since the temperature of the emitted flux is inversely related to a black hole's mass, the black hole becomes hotter as it radiates. Thus, a radiating black hole "evaporates", losing all of its mass in a runaway process. While the exact final state of an evaporating black hole remains unclear, at the classical level some form of naked singularity seems the probable outcome [14, 15], and one is again confronted by the breakdown of predictability caused by singularities.

At first glance, the singularity exposed at the end of black hole evaporation seems an unavoidable counterexample to cosmic censorship. However, the scales involved near the endpoint of black hole evaporation imply that a consistent treatment of the final state requires a quantum description of gravity. Unfortunately, despite decades of effort, a quantum theory of gravity remains undeveloped. The next section discusses some of the issues underlying quantum gravity and highlights research using black hole radiation and extreme black holes in attempts to solve this puzzle.

### Exploiting black hole thermal properties

Since the discovery of black hole radiance, the study of the quantum aspects of black hole spacetimes has been viewed as one of the most likely areas in which to gain further insight into the nature of quantum gravitational processes. Lacking a quantum theory for gravity, one may use semiclassical approximations to advance such studies. In the semiclassical approach, classical spacetime curvature is coupled

to quantized matter fields, modifying the Einstein equations to

$$G_{\mu\nu} = 8\pi \langle \psi | T_{\mu\nu} | \psi \rangle, \quad (8)$$

where the Einstein tensor,  $G_{\mu\nu}$ , describes the curvature of the spacetime and  $\langle \psi | T_{\mu\nu} | \psi \rangle$ , which I shall abbreviate as  $\langle T_{\mu\nu} \rangle$ , is the expectation value of the stress energy tensor for the quantized fields which are in the state  $|\psi\rangle$ . The stress-energy tensor is a frame independent description of the matter distribution in the spacetime.

The effects of quantum gravitational processes become significant at the Planck scale, defined by the length  $l_P = (\hbar G/c^3)^{1/2} \approx 1.6 \times 10^{-33}$  centimeters. (Table 1 lists various units in terms of the Planck length.) For phenomena occurring at this scale

Table 1: Quantum gravitational effects become important at the Planck scale which is defined by the length  $l_P = (\hbar G/c^3)^{1/2}$ . Using this length one finds the Planck scale for other fundamental units. A few are listed below.

$$\begin{aligned} l_P &= (\hbar G/c^3)^{1/2} \approx 1.6 \times 10^{-33} \text{ cm} \\ t_P &= l_P/c \approx 5.4 \times 10^{-44} \text{ seconds} \\ m_P &= l_P c^2/G \approx 2.2 \times 10^{-5} \text{ g} \\ E_P &= l_P c^4/G \approx 1.3 \times 10^{19} \text{ GeV} \\ \rho_P &= c^2/(l_P^2 G) \approx 5.2 \times 10^{93} \text{ g/cm}^3 \\ T_P &= E_P/k \approx 1.4 \times 10^{32} \text{ K} \end{aligned}$$

the semiclassical approximation fails and a quantum theory of gravity is needed.

Currently, researchers know of only two such phenomena that require a quantum theory of gravity, the initial singularity of the big bang, and the end point of black hole evaporation.

A third phenomenon which may require a quantum theory of gravity is the extreme black hole configuration. Recall that the temperature associated with a black hole depends on the surface gravity,  $\kappa$ , which is zero for an extreme black hole. Hence, extreme black holes apparently have zero temperature and are stable against evaporation. Because the exterior of an extreme black hole can have small curvature compared to the Planck scale, this stability against evaporation implies that one can safely apply semiclassical approximations in describing extreme black hole spacetimes. However, Trivedi has shown that the stress-energy of a quantized conformally coupled massless field diverges on the event horizon of a two dimensional, extreme black hole if the field is in a zero temperature vacuum state [16]. Thus, back reaction effects become large, and the semiclassical Einstein equations, Eq. (8), may fail.

This divergence does not occur for the four dimensional, extreme Reissner-Nordström black hole [17]. However, as will be shown in chapter two, if the quantized field is in a thermal state at any nonzero temperature, a more severe divergence in the stress-energy occurs on the event horizon. This implies that the extreme Reissner-Nordström black hole has a well defined temperature, namely  $T = 0$ .

It has been suggested that extreme black holes can be in equilibrium with quantized fields in arbitrary thermal states. In a separate line of research, Hawking, Horowitz, and Ross have pointed out that in the Euclidean sector, the distance to the event horizon of an extreme black hole is infinite in all directions (as opposed to merely in spacelike directions in the Lorentzian metric). Because the distance to the event horizon is infinite, the Euclidean geometry may be identified with an arbi-

trary period without introducing a conical singularity at the horizon, which suggests that extreme black hole spacetimes can have arbitrary temperature, regardless of the surface gravity value [18].

This issue of temperature also arises when considering extreme dilaton black holes [20, 19]. These black holes may play an important role in superstring theories as representations of massive single string states [21, 22]. They are extreme in the sense that any increase in the charge of the black hole would result in a naked singularity. However, unlike the Reissner-Nordström and Kerr-Newman extreme black hole spacetimes of general relativity, extreme dilaton black holes can have zero, finite, or even infinite surface gravities depending on the value of the dilaton coupling parameter. As a result, extreme dilaton black holes can apparently have infinite temperature unless one can assign a temperature other than that defined by the surface gravity.

This chapter began by noting that black hole radiance and extreme black holes play an important role in contemporary studies aimed at understanding quantum gravitational processes. As the discussion of this section makes clear, work in several distinct areas leads to a common central concern: Is the stress-energy of a quantized field finite on the event horizon of an extreme black hole, or does it diverge, calling into question the very existence of a meaningful semiclassical extreme black hole solution?

In chapter 2, the expectation value for the stress energy of a conformally invariant quantized field is calculated in a generic two dimensional static spacetime. By con-

sidering the boundary conditions on the quantized field, I show that in all cases the stress-energy diverges strongly on an event horizon unless the fields are in a thermal state with a temperature equal to the natural temperature defined by the surface gravity.

In chapter 3, attention is restricted to a two dimensional version of the dilaton black hole metric introduced by Garfinkle, Horowitz, and Strominger. Using a conformal transformation from the "physical" version of this metric to the "string" version, I again show that the regularity of the stress-energy expectation values depends on a judicious choice for the temperature of this spacetime.

In chapter 4, I apply the numerical method of Anderson, Hiscock, and Samuel to the four dimensional dilaton black hole metric of Garfinkle, Horowitz, and Strominger. I find that the renormalized stress-energy of a quantized conformal massless scalar field in an extreme dilaton black hole spacetime described by the metric of Garfinkle, Horowitz, and Strominger is finite every where in the spacetime.

In chapter 5, I conclude with some suggestions for future investigations.

## CHAPTER 2

# Thermal divergences on the event horizons of two dimensional black holes

## Introduction

Hawking's discovery that black holes radiate particles with a thermal (black-body) spectrum provided a fundamental link between thermodynamics and black hole physics. This thermal emission of particles implies a characteristic black hole temperature defined by the surface gravity. For a quantized field in thermal equilibrium with a black hole, a condition known as the Hartle-Hawking state, one expects the stress-energy of the quantum field to be finite on the event horizon. As discussed in chapter one, a divergent stress-energy might signal a breakdown of the semiclassical approximation.

Numerical calculations in four dimensions confirm that the stress-energy for a quantum scalar field remains finite on the horizons of Schwarzschild and Reissner-Nordström black holes [23, 24]. However, despite the no hair theorems of general relativity, interest exists in studying the stress-energy of quantized fields in more

general black hole spacetimes. First, back reaction effects due to the nonzero stress-energy of the quantized field will perturb the black hole geometry away from the classical solutions of general relativity [25, 26, 27]. Second, a fully quantum theory of gravity which resolves the fundamental inconsistency of a semiclassical theory will likely modify the black hole geometries of general relativity and may even permit a greater variety of black hole solutions. For example, the massless scalar field required in superstring theory (the dilaton) couples to the electromagnetic field. As a result, the metric describing a static, spherically symmetric, charged black hole in superstring theory is quite different from the Reissner-Nordström metric of general relativity. In chapters three and four of this dissertation, I examine the superstring metric for a charged black hole more closely.

Unfortunately, the question of whether the stress-energy of a quantized field always remains finite on the event horizon of a black hole cannot yet be answered in general in four dimensions. Calculating the expectation values of the stress-energy components in a curved four dimensional spacetime can often be difficult. However, since nonrotating black holes are expected to settle down to a final state possessing spherical symmetry, some insight can be gained by suppressing the angular dimensions and restricting one's attention to a two dimensional sector of spacetime comprised of the radial and time coordinates. In two dimensions, the stress-energy for a conformally invariant quantized field can be computed analytically for an arbitrary spacetime. Using this analytic expression, one can examine all static two dimensional black hole geometries at once.

Reducing the dimensionality of the spacetime to two simplifies the stress-energy conservation equation, and the resulting expressions can be directly integrated to find the expectation value of the stress-energy tensor for a conformally invariant quantized field. This method is well known, and similar integrations have been previously carried out for specific black hole geometries [28, 29, 16]. The new feature of the work presented in this chapter is the interpretation of the resulting integration constants in terms of the thermal state of the quantized field. As a result of this interpretation, I find that in all cases the stress-energy of a quantized scalar field diverges strongly on the event horizon of a black hole unless the field is in a thermal state with a temperature equal to the natural temperature defined by the surface gravity of the black hole. In addition, this divergence occurs regardless of whether the black hole is extreme or not. Thus, an improper choice of temperature for the thermal state of the quantum field results in a strong divergence in its stress-energy for both extreme and nonextreme black hole spacetimes. Furthermore, the analytic form of this divergence is precisely the same for both extreme and nonextreme black hole spacetimes, which is in contrast with recent suggestions that extreme black holes represent a distinct class of thermodynamic objects [18].

This work has been published in Ref. [30].

### Calculation of $\langle T_{\mu\nu} \rangle$ in two dimensions

The most general two dimensional static spacetime metric may be written in the

form

$$ds^2 = -F(R)dt^2 + \frac{1}{H(R)}dR^2. \quad (9)$$

It is always possible to transform this metric to "Schwarzschild gauge" by defining a new spatial coordinate

$$r = \int \left(\frac{F}{H}\right)^{1/2} dR, \quad (10)$$

which yields as the general metric form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2, \quad (11)$$

where  $f(r)$  is an arbitrary function of  $r$ . This metric possesses event horizons at the locations  $r = r_n$ , where  $f(r_n) = 0$ , ( $n = 0, 1, 2, \dots$ ). In this chapter, black holes with an arbitrary number of horizons are considered, but attention is restricted to the outer event horizon at  $r = r_0$ .

Conservation of stress-energy is written as

$$\langle T_{\mu;\nu}^\nu \rangle = 0. \quad (12)$$

(For the remainder of this chapter, the expectation value brackets are suppressed to simplify the notation.) For the metric of Eq. (11) the conservation of stress-energy yields the differential equations

$$T_{t,r}^r = 0, \quad (13)$$

and

$$T_{r,r}^r + \frac{1}{2} \frac{f'}{f} T_r^r - \frac{1}{2} \frac{f'}{f} T_t^t = 0, \quad (14)$$

where a prime denotes differentiation with respect to  $r$ . Equation (14) may be simplified by rewriting  $T_t^t$  in terms of the trace of the stress-energy tensor,  $T_t^t = T_\alpha^\alpha - T_r^r$ .

The conservation equations are then easily integrated to give

$$T_t^t = C_1, \quad (15)$$

and

$$T_r^r = \frac{C_2}{f} + \frac{1}{2f} \int_{r_0}^r f' T_\alpha^\alpha dr. \quad (16)$$

For a static spacetime, Eqs. (15) and (16) are the complete solution to the conservation of stress-energy equations in two dimensions. The components of the stress-energy tensor in a given static spacetime then depend on one function,  $T_\alpha^\alpha$ , and two integration constants,  $C_1$  and  $C_2$ .

If the field is chosen to be conformally invariant, then the trace is given by the conformal anomaly  $T_\alpha^\alpha = R/(24\pi)$ , where  $R$  is the Ricci scalar. Evaluated for the metric of Eq. (11), the trace becomes

$$T_\alpha^\alpha = -\frac{f''}{24\pi}. \quad (17)$$

In the case of a conformally invariant field, all information concerning the quantum

state of the field is then encoded in the two integration constants. Substituting the trace anomaly from Eq. (17) into Eq. (16), one can explicitly perform the integration to find

$$T_r^r = \frac{C_2}{f} - \frac{f'^2}{96\pi f} + \frac{\pi}{6f} (T_{Hawking})^2, \quad (18)$$

where  $T_{Hawking}$  is defined in terms of the surface gravity,  $\kappa$ , of the horizon at  $r_0$  as

$$T_{Hawking} = \frac{\kappa}{2\pi} = \frac{f'|_{r_0}}{4\pi}. \quad (19)$$

Note that  $T_{Hawking}$  does not necessarily represent the physical temperature of the black hole; rather it is merely a familiar definition used to simplify a collection of terms involving  $f'(r_0)$ .

The integration constants are fixed by choosing a particular quantum state for the field. I will consider the Hartle-Hawking state in which the black hole is in thermal equilibrium with a surrounding heat bath. The requirement of thermal equilibrium implies that  $T_\mu^\nu$  must be invariant under time reversal, and thus that  $T_t^r = C_1 = 0$ .

The remaining integration constant,  $C_2$ , is now determined by fixing the form of  $T_r^r$  in an asymptotically flat region far from the event horizon, where  $f \rightarrow \text{constant}$ : (The apparently more general asymptotically flat form  $f' \rightarrow \text{constant}$  simply amounts to choosing asymptotically Rindler coordinates rather than Minkowski). For the Hartle-Hawking state, far from the event horizon the thermal state of the quantized field is simply the thermal state of a gas of massless scalar bosons in flat space, and the  $T_r^r$

component, for example, of the stress-energy tensor is

$$T_r^r \rightarrow \frac{\pi}{6}(kT)^2, \quad (20)$$

as the metric becomes asymptotically flat. Evaluating Eq. (18) in the asymptotically flat region and using Eq.(20), one finds

$$C_2 = \frac{\pi}{6}T^2 - \frac{\pi}{6}(T_{Hawking})^2. \quad (21)$$

### $\langle T_{\mu\nu} \rangle$ on the event horizon

Having completely integrated the conservation equation and solved for the stress-energy tensor, I turn to the issue of its regularity on the event horizon. Since the coordinate system used in the metric of Eq.(11) is singular on the event horizon at  $r_0$ , I will evaluate the stress-energy tensor components in an orthonormal frame attached to a freely falling observer. The basis vectors of the frame are chosen to be the two-velocity  $e_0^\alpha = u^\alpha$  and a unit length spacelike vector  $e_1^\alpha = n^\alpha$  orthogonal to  $u^\alpha$ , so that  $n^\alpha u_\alpha = 0$  and  $n^\alpha n_\alpha = +1$ . Using the timelike Killing vector field to define a conserved energy, the geodesic equation may then be solved to find

$$u^t = \gamma/f, \quad u^r = -\sqrt{\gamma^2 - f} \quad (22)$$

and

$$n^t = -\frac{1}{f}\sqrt{\gamma^2 - f}, \quad n^r = \gamma, \quad (23)$$

where  $\gamma$  is the energy per unit mass along the geodesic. The components of the stress-energy tensor in the freely-falling orthonormal frame are then given in terms of the coordinate components by

$$T_{00} = \frac{\gamma^2 (T_r^r - T_t^t)}{f} - T_r^r, \quad (24)$$

$$T_{11} = \frac{\gamma^2 (T_r^r - T_t^t)}{f} + T_t^t, \quad (25)$$

and

$$T_{01} = -\frac{\gamma\sqrt{\gamma^2 - f} (T_r^r - T_t^t)}{f}. \quad (26)$$

Since the value of  $\gamma$  is arbitrary, the stress-energy will be regular on the horizon only if  $T_t^t$ ,  $T_r^r$ , and the combination  $(T_r^r - T_t^t)/f$  are each separately finite at  $r_0$ . Because a possible divergence in either  $T_t^t$  or  $T_r^r$  will be made stronger by the extra  $f^{-1}$  in the combination  $(T_r^r - T_t^t)/f$ , I focus on this combination as representing the strongest possible potential divergence in  $\langle T_{\mu\nu} \rangle$ . Using Eqs.(18), (19) and the trace anomaly, this combination of terms may be written as

$$\frac{T_r^r - T_t^t}{f} = \frac{2C_2}{f^2} - \frac{1}{48\pi} \frac{f'^2 - f''^2|_{r_0} - 2ff''}{f^2}. \quad (27)$$

The second term on the right hand side of Eq. (27) goes to 0/0 on the event horizon.

Applying l'Hospital's rule to this term and rewriting  $(T_r^r - T_t^t)/f$  in the limit as  $r \rightarrow r_0$  gives

$$\lim_{r \rightarrow r_0} \frac{T_r^r - T_t^t}{f} = \left( \frac{2C_2}{f^2} + \frac{1}{48\pi} \frac{f'''}{f'} \right) \Big|_{r_0}. \quad (28)$$

For an extreme black hole,  $f'|_{r_0} = 0$ , and the second term of Eq. (28) diverges. This is the unavoidable divergence Trivedi previously discovered [16]. The only escape from this divergence would be if  $f'''$  approached zero as fast or faster than  $f'$  does in the limit  $r \rightarrow r_0$ . Of course two-dimensional extreme black hole metrics for which this occurs form a set of measure zero in the space of all extreme two-dimensional black hole metrics. However, it remains to be seen whether two-dimensional gravitational dynamics, when semiclassical backreaction is included, might cause extreme black hole solutions to evolve towards such a state.

If  $C_2 \neq 0$ , then there is a far more serious divergence of the stress-energy tensor on the event horizon. The energy density and pressure seen by an infalling observer will diverge as  $f^{-2}$ , irrespective of whether the black hole is extreme or not. Both Christensen and Fulling [28] and Trivedi [16] noted this and set  $C_2 = 0$ . Here, I give a physical interpretation of the meaning of this restriction on  $C_2$ . As demonstrated in the preceding section of this chapter, the integration constant  $C_2$  may be expressed in terms of the difference between the square of the "natural" temperature of the black hole defined by the surface gravity,  $T_{Hawking}$ , and the square of the asymptotic temperature assigned to the black hole,  $T$ , as shown in Eq.(21). Thus, unless the temperature assigned to the black hole,  $T$ , is precisely equal to the natural temper-

ature defined by the surface gravity,  $T_{Hawking}$ , the stress-energy of a quantized field will diverge strongly on the event horizon. Further, the form of the divergence is independent of whether the black hole is extreme or nonextreme. Therefore, extreme black holes have a natural temperature, in precisely the same fashion as nonextreme holes, and may not be assigned arbitrary temperature without serious consequences. The divergence of  $\langle T_{\mu\nu} \rangle$  on the event horizon of the extreme Reissner-Nordström black hole in four dimensions when the temperature is chosen to be other than zero [17] is thus seen to be simply an example of the general strong divergence which occurs for all black holes when assigned an inappropriate temperature.

In conclusion, the stress-energy tensor of a conformally coupled quantized field will diverge strongly on the event horizon of any two-dimensional black hole unless the temperature of the black hole is chosen to be equal to the natural temperature defined by the surface gravity of the horizon. If the temperature is chosen to be the natural value, then the stress-energy tensor will be regular on the horizon unless the black hole is extreme. If the black hole is extreme, then there will be a weak divergence of the stress-energy on the horizon, except in a set of metrics of measure zero. Whether two-dimensional extreme black hole metrics evolve naturally towards these non-divergent cases when semiclassical backreaction is included remains to be determined.

## CHAPTER 3

# Quantized fields and temperature in charged, dilatonic black hole spacetimes

## Introduction

Garfinkle, Horowitz, and Strominger [19] (hereafter, GHS) found static spherical charged black hole solutions in the low-energy approximation to string theory. In addition to the electromagnetic field  $F_{\mu\nu}$ , these solutions possess a nonconstant scalar field called the dilaton. For a thorough discussion of black holes in superstring theory, the interested reader is directed to the review articles [31, 21, 32]. Here I will take the GHS metric as given and consider its thermodynamic consequences.

The GHS black hole solutions differ from the analogous Reissner-Nordström black hole solutions of general relativity. For example, the GHS black hole lacks inner horizons. Nonetheless, a maximum value of the charge exists which separates the black hole solutions from solutions with naked singularities. As defined in chapter one, such a black hole is extreme. However, the extreme GHS solutions again differ from the extreme Reissner-Nordström solutions. While the extreme Reissner-Nordström

black hole has a nonsingular event horizon on which quantized fields have finite stress-energy [17], the event horizon of the extreme GHS solution is singular. However, as noted by Garfinkle, Horowitz, and Strominger, the singular nature of the event horizon in the physical metric,  $g_{\mu\nu}$ , is irrelevant. Strings do not couple to the physical metric but to the conformally related metric  $e^{2\phi}g_{\mu\nu}$ , which I will call the string metric. Here  $\phi$  is the dilaton field. As discussed in the next section, in the string GHS metric, the extreme spacetime has no event horizon and is geodesically complete.

The thermodynamics of the Reissner-Nordström and GHS spacetimes also differ. In the Reissner-Nordström spacetime the black hole temperature is given by  $T_{RN} = \sqrt{M^2 - Q^2}/(2\pi r_+^2)$ , where  $r_+ = M + \sqrt{M^2 - Q^2}$ . Hence  $T_{RN}$  decreases steadily from the Schwarzschild value to zero as the extreme limit is approached ( $Q \rightarrow M$ ). In comparison, the string GHS metric has temperature,  $T_{GHS} = 1/(8\pi M e^{\phi_0})$ , independent of the charge, until the extreme solution is reached. Then the temperature apparently shifts discontinuously to zero since the extreme spacetime has no horizon. However, zero temperature for the extreme GHS black hole results in divergent stress energy of the kind discussed in chapter two.

In this chapter, working in a two-dimensional black hole metric obtained by discarding the angular portions of the GHS metric, I calculate the expectation value of the stress energy tensor for a quantized conformally coupled massless scalar field in the Hartle-Hawking state. I find that by assigning the extreme string spacetime of GHS a nonzero temperature equal to that of the nonextreme string spacetime, it is possible to render  $\langle T_{\mu\nu} \rangle$  regular everywhere outside and on the horizon for both the

string and physical metrics.

This work has been published in Ref. [33].

### The metric of Garfinkle, Horowitz, and Strominger

The effective Lagrangian for low-energy string theory used by Garfinkle, Horowitz, and Strominger is

$$L = \int d^4x \sqrt{-g} [-R + 2(\nabla\phi)^2 + e^{-2\phi} F^2], \quad (29)$$

where  $\phi$  is the dilaton field and  $F_{\mu\nu}$  is the Maxwell field. Static, spherically symmetric black hole solutions to the field equations obtained from Eq.(29) are described by[19]:

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r \left(r - \frac{Q^2 e^{-2\phi_0}}{M}\right) d\Omega^2, \quad (30)$$

$$e^{-2\phi} = e^{-2\phi_0} \left(1 - \frac{Q^2 e^{-2\phi_0}}{Mr}\right), \quad (31)$$

and

$$F = Q \sin \theta d\theta \wedge d\phi. \quad (32)$$

Here  $d\Omega^2$  is the metric of the two-sphere,  $\phi_0$  is the asymptotic value of the dilaton field, and I have corrected the published errors in the powers of  $e^{\phi_0}$  in accordance

with the Erratum to Ref.[19]. The surface

$$r = \frac{Q^2 e^{-2\phi_0}}{M} \quad (33)$$

is singular, and the extreme limit occurs when the charge is increased to a value sufficient to bring this surface into coincidence with the horizon at  $r = 2M$ , namely when

$$Q^2 = 2M^2 e^{2\phi_0}. \quad (34)$$

Note that even if the asymptotic value of the dilaton field vanishes ( $\phi_0 = 0$ ) the charge on an extreme GHS black hole still differs from that for a Reissner-Nordström black hole by a factor of  $\sqrt{2}$ .

The strings do not couple to the physical metric,  $g_{\mu\nu}$ , of Eq.(30) but rather to the conformally related string metric,  $e^{2\phi} g_{\mu\nu}$ . Applying this conformal transformation to Eq.(30) yields

$$ds_{string}^2 = -\frac{(1 - 2Me^{\phi_0}/\rho)}{(1 - Q^2 e^{-\phi_0}/(M\rho))} d\tau^2 + \frac{d\rho^2}{(1 - 2Me^{\phi_0}/\rho)(1 - Q^2 e^{-\phi_0}/(M\rho))} + \rho^2 d\Omega^2, \quad (35)$$

where the coordinates  $(\tau, \rho)$  are defined by  $\tau = e^{\phi_0} t$ ,  $\rho = e^{\phi_0} r$ . In the extreme limit,  $Q^2 = 2M^2 e^{\phi_0}$ , and Eq.(35) reduces to

$$ds^2 = -d\tau^2 + \left(1 - \frac{2Me^{\phi_0}}{\rho}\right)^{-2} d\rho^2 + \rho^2 d\Omega^2. \quad (36)$$

This extreme geometry has no horizons, and the spatial geometry of constant  $\tau$  hypersurfaces is geometrically identical to that of  $t = \text{constant}$  surfaces in the extreme Reissner-Nordström metric. As a result, the  $\rho = 2e^{\phi_0} M$  surface is at an infinite proper distance from any point in the manifold with  $\rho > 2e^{\phi_0} M$ . In the GHS extreme spacetime, the distance is infinite in any direction, whereas in the extreme Reissner-Nordström case, the distance to  $r = M$  is infinite only in spacelike directions.

### Investigation of $\langle T_{\mu\nu} \rangle$

Since the GHS metric is the low-energy approximation to full string theory black hole solutions, it makes sense to examine the physics of ordinary quantized free fields (as opposed to quantized strings) in the GHS black hole background. While obtaining values for tensor objects such as  $\langle \psi^2 \rangle$  or  $\langle T_{\mu\nu} \rangle$  for a quantized field  $\psi$  can be quite difficult in curved four-dimensional spacetimes, calculating  $\langle T_{\mu\nu} \rangle$  for a conformally coupled field in a two-dimensional spacetime is both a straightforward and often valuable exercise[34, 35, 30]. In this chapter, attention is again restricted to two dimensions by computing  $\langle T_{\mu\nu} \rangle$  for a conformally coupled quantized scalar field in the two-dimensional metrics obtained from Eqs.(30,35) by setting the angular coordinates  $\theta$  and  $\phi$  to constant values. In the next chapter, I consider the full, four dimensional metric.

The stress-energy tensor of a quantized scalar field in a two-dimensional black hole spacetime for the Hartle-Hawking state may be easily calculated. I will use the

approach of chapter two. The spatial coordinate of the black hole metric must first be transformed in order to place the metric in the "Schwarzschild gauge",

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}. \quad (37)$$

The temperature defined by the geometry is then given by

$$T = \frac{f'|_{r_0}}{4\pi}, \quad (38)$$

where  $r_0$  is the radius of the event horizon and a prime denotes differentiation with respect to  $r$ . The trace of the stress-energy tensor is given by the conformal anomaly,

$$\langle T_\alpha^\alpha \rangle = -\frac{f''}{24\pi}. \quad (39)$$

As shown in chapter two, the components may then be completely determined by quadrature of the conservation equation, with regularity of the stress-energy tensor on the black hole horizon fixing the integration constant. Specifically,

$$\langle T_r^r \rangle = -\frac{f'^2}{96\pi f} + \frac{\pi T^2}{6f}, \quad (40)$$

$$\langle T_t^t \rangle = \langle T_\alpha^\alpha \rangle - \langle T_r^r \rangle. \quad (41)$$

The physical GHS metric of Eq.(30), when restricted to two dimensions, becomes simply the two dimensional Schwarzschild metric for all values of  $Q$ . No transforma-

tion is needed and one can quickly calculate that the spacetime has a temperature of

$$T_{Sch} = \frac{1}{8\pi M}, \quad (42)$$

while the components of the stress-energy tensor for the quantized field are found to be

$$\langle T_t^t \rangle = \frac{56M^3 - 4M^2r - 2Mr^2 - r^3}{384\pi M^2 r^3}, \quad (43)$$

and

$$\langle T_r^r \rangle = \frac{1}{384\pi M^2} \left( 1 + \frac{2M}{r} \right) \left( 1 + \frac{4M^2}{r^2} \right), \quad (44)$$

and the trace anomaly is

$$\langle T_\alpha^\alpha \rangle = \frac{M}{6\pi r^3}. \quad (45)$$

To reiterate, the physical metric of the two dimensional GHS black hole is independent of charge. Thus, for the physical metric, the temperature, stress-energy components for the quantized field, and the trace anomaly are those values given in Eqs. (42 - 45) for both the extreme and nonextreme GHS black hole.

In order to calculate the stress-energy tensor of a quantized field in the GHS string metric described by Eq.(35) restricted to two dimensions, the metric must first be transformed to Schwarzschild gauge, which is accomplished by defining a new spatial coordinate  $x$ ,

$$x = \int \left( 1 - \frac{e^{-\phi_0} Q^2}{M\rho} \right)^{-1} d\rho. \quad (46)$$

The derivatives in Eqs.(38-41) are now taken with respect to the new coordinate  $x$ . I will, however, express the resulting stress-energy tensor components in terms of the original  $(\tau, \rho)$  coordinates of Eq.(35). For all nonextreme values of  $Q$ , the string metric has temperature

$$T_{string} = \frac{1}{8\pi e^{\phi_0} M}. \quad (47)$$

To obtain the components of the stress-energy tensor one may either directly integrate the conservation equation as was done for the physical metric, or utilize the relation between stress-energy tensors of conformally invariant fields in conformally related spacetimes[12]. In two dimensions, if two geometries are related by a conformal factor  $\Omega(x)$ , such that  $\bar{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}$ , then the vacuum stress-energy tensors of a conformally invariant scalar or spinor field are related by:

$$\begin{aligned} \langle \bar{T}_\mu^\nu \rangle_{ren} = & (g/\bar{g})^{1/2} \langle T_\mu^\nu \rangle_{ren} + \left( \frac{1}{12\pi} \right) \left[ (\Omega^{-3} \Omega_{;\alpha\mu} - 2\Omega^{-4} \Omega_{,\alpha} \Omega_{,\mu}) g^{\alpha\nu} \right. \\ & \left. + \delta_\mu^\nu g^{\alpha\sigma} \left( \frac{3}{2} \Omega^{-4} \Omega_{,\alpha} \Omega_{,\sigma} - \Omega^{-3} \Omega_{;\alpha\sigma} \right) \right], \end{aligned} \quad (48)$$

where all derivatives are taken with respect to the unbarred metric.

The components of the stress-energy tensor for the quantized scalar field in the string spacetime are found to be

$$\begin{aligned} \langle T_r^r \rangle = & \left[ 384\pi e^{3\phi_0} M^3 \rho^2 (e^{\phi_0} M \rho - Q^2) \right]^{-1} \left[ 8e^{5\phi_0} M^5 - 8e^{3\phi_0} M^3 Q^2 + 2e^{\phi_0} M Q^4 \right. \\ & \left. + (4e^{4\phi_0} M^4 - 4e^{2\phi_0} M^2 Q^2 + Q^4) \rho + (2e^{3\phi_0} M^3 - 2e^{\phi_0} M Q^2) \rho^2 \right] \end{aligned}$$

$$+e^{2\phi_0} M^2 \rho^3], \quad (49)$$

and

$$\begin{aligned} \langle T_\tau^\tau \rangle = & [384\pi e^{3\phi_0} M^3 \rho^3 (e^{\phi_0} M \rho - Q^2)]^{-1} [-32e^{4\phi_0} M^4 Q^2 + 16e^{2\phi_0} M^2 Q^4 \\ & + (56e^{5\phi_0} M^5 - 24e^{3\phi_0} M^3 Q^2 - 2e^{\phi_0} M Q^4) \rho - (4e^{4\phi_0} M^4 - 4e^{2\phi_0} M^2 Q^2 + Q^4) \rho^2 \\ & - (2e^{3\phi_0} M^3 - 2e^{\phi_0} M Q^2) \rho^3 - e^{2\phi_0} M^2 \rho^4], \quad (50) \end{aligned}$$

The trace anomaly is

$$\langle T_\alpha^\alpha \rangle = \frac{(Q^2 - e^{2\phi_0} M^2)(Q^2 - 2e^{\phi_0} M \rho)}{24\pi e^{\phi_0} M \rho^3 (e^{\phi_0} M \rho - Q^2)}. \quad (51)$$

It is interesting to compare the physical behavior of the stress-energy of the quantized field in the GHS charged black hole string metric, given by Eqs.(49 - 51) with those computed for the usual Reissner-Nordström black hole (these may be obtained from the expressions for the Unruh vacuum components given in Ref.[35] by a simple transformation). For the GHS black hole, the energy density as seen by a static observer  $\epsilon = -\langle T_\tau^\tau \rangle$  is always negative in the neighborhood of the horizon; the radial stress is similarly always positive, indicating a pressure. In comparison, the energy density of a quantized field near the horizon of a two-dimensional Reissner-Nordström black hole is negative for  $Q^2 < 8M^2/9$ , but is positive for larger values of  $Q^2$ . The radial stress is also positive for  $Q^2 < 8M^2/9$ , and negative for larger values of  $Q^2$ .

For the string metric, the case of the extreme GHS black hole, with  $Q^2 = 2e^{2\phi_0} M^2$ , must be treated separately, since the spacetime possesses no horizons. The lack of a horizon means no geometrically defined temperature exists in this case. Garfinkle, Horowitz, and Strominger suggest zero temperature as the natural state for this spacetime, even though the temperature is then a discontinuous function of  $Q$ . Alternatively, one could choose the temperature to be continuous by assigning a value  $T_{\text{extreme}} = 1/(8\pi M e^{\phi_0})$  to the spacetime. In fact, any temperature can apparently be assigned to the spacetime. When the metric of Eq.(36) is reduced to two dimensional form, it is flat, and hence has no trace anomaly. As a result, if an arbitrary temperature  $T_{\text{extreme}}$  is temporarily assigned to the spacetime, then the most general time-reversal symmetric (*i.e.*, equilibrium) solution of the stress-energy conservation equation is that of a simple boson gas,

$$\langle T_{\rho}^{\rho} \rangle = -\langle T_{\tau}^{\tau} \rangle = \frac{\pi}{6} T_{\text{extreme}}^2, \quad (52)$$

which is well defined for any finite temperature.

Consider now the temperatures associated with both the physical and string metrics. The physical metric is always precisely Schwarzschild (independent of  $Q$ ) in two dimensions, and hence must always have  $T = 1/(8\pi M)$ ; assignment of any other temperature would lead to a divergent stress-energy for the quantized field on the horizon[30]. Similarly, the string metric's natural temperature is  $T = 1/(8\pi M e^{\phi_0})$  for all values  $Q^2 < e^{2\phi_0} M^2$ . Again, assignment of any other temperature would cause a

strong divergence in the stress energy on the horizon. These natural, geometrically defined temperatures and the associated stress-energy tensors for quantized fields are also related directly through the conformal transformation of Eq.(48).

In the extreme case, the physical metric is still precisely Schwarzschild, and thus the spacetime must either be assigned the usual temperature or suffer divergent stress energy on the horizon. In the extreme string metric, though, there is no horizon, and hence no geometrically defined temperature for the spacetime. However, one can pick out a unique temperature for the extreme string metric by using the conformal relation of Eq.(48). If the quantized fields are to be regular in both the physical and string metrics, one must choose  $T_{extreme} = 1/(8\pi M e^{\phi_0})$ . Any other choice in the string metric will lead to divergences on the horizon of the physical metric by Eq.(48). Thus, it appears that the natural choice for the temperature of the extreme string metric is not zero, despite the lack of horizon, but rather  $T_{extreme} = 1/(8\pi e^{\phi_0} M)$ . This choice preserves continuity of the temperature in the string metric, and is the only choice which will allow quantized fields in both the string and physical metrics to be regular outside and on all horizons.

It is interesting to compare and contrast this result to the case of the extreme Reissner-Nordström black hole [17] in four dimensions. Hawking, Horowitz, and Ross [18] noted that the Euclidean section of the extreme Reissner-Nordström metric allowed one to identify the geometry with arbitrary period in Euclidean time (and hence arbitrary temperature) without creating a conical singularity in the Euclidean spacetime. However, as was shown in chapter two, the stress-energy tensor of a quantized

field will diverge strongly on the event horizon in the Lorentzian spacetime unless the temperature is chosen to be zero, the value obtained by extrapolating the form of the temperature function  $T(M, Q)$  from the nonextreme Reissner-Nordström black hole. In the present case, I again find that there will be a divergence of the stress-energy of a quantized field (in this case, for a field which takes values on the conformally related physical metric) if the temperature of the extreme GHS string metric is taken to be a value other than that extrapolated from the nonextreme GHS black hole temperature function. On the other hand, in this case, it means the extreme GHS metric must be assigned a nonzero temperature, whereas in the extreme Reissner-Nordström case the natural temperature was zero. This assignment of nonzero temperature should not trouble the reader, since as is clear from the string metric, the extreme GHS black hole differs from the more familiar general relativistic extreme black holes by lacking both a horizon and singularity.

The two dimensional analysis of chapters two and three suggest the following.

1. An extreme black hole spacetime has well defined thermal properties and cannot be assigned an arbitrary temperature.
2. The temperature of an extreme black hole is uniquely determined by extrapolating the temperature of the corresponding nonextreme black hole to the extreme state.

## CHAPTER 4

# Four dimensional analysis of stress-energy in the GHS spacetime

## Calculating $\langle T_{\mu\nu} \rangle$ in the four dimensional GHS spacetime

While one can often gain physical insight by working with a more tractable two dimensional analogue to spacetime [34, 35], one's confidence in such insights ultimately depends upon a full analysis in four dimensions. For example, Trivedi's calculation in two dimensions suggests that the stress-energy for a quantized conformally invariant field should diverge on the event horizon of an extreme Reissner-Nordström black hole. However, numerical calculations in four dimensions show that this divergence does not occur [17].

In this chapter, I apply the technique developed by Anderson, Hiscock, and Samuel [24] to calculate the stress-energy of a quantized conformally coupled massless scalar field in the four dimensional extreme black hole spacetime described by the metric of Garfinkle, Horowitz, and Strominger (GHS). As discussed in chapter one, special

interest lies in checking the stress-energy for divergences, which could imply a breakdown of the semiclassical approximation. However, the stress-energy of a quantized field in the extreme GHS black hole spacetime is also of general interest. Although the expectation value of the stress-energy,  $\langle T_{\mu\nu} \rangle$ , acts as the source term for the semiclassical Einstein equation,  $\langle T_{\mu\nu} \rangle$  values for quantized fields in black hole spacetimes have so far been calculated only for the Schwarzschild and Reissner-Nordström metrics of general relativity [24, 36, 23]. Hence, calculating  $\langle T_{\mu\nu} \rangle$  for a different black hole spacetime increases one's knowledge of how  $\langle T_{\mu\nu} \rangle$  behaves in such spacetimes. Furthermore, the extreme black holes of superstring theory play an important role as fundamental particles in that theory [37, 38]. This role depends on the existence of a consistent semiclassical description of extreme black hole spacetimes, which in turn requires knowledge of  $\langle T_{\mu\nu} \rangle$ . Thus, of the various black hole spacetimes in which the stress-energy of quantized fields has yet to be calculated, motivation exists for selecting an extreme black hole solution of superstring theory. This chapter presents the first results for the stress-energy of a quantized field in an extreme black hole spacetime that is derived from superstring theory.

Proceeding in a manner similar to Howard and Candelas' calculation of the stress-energy for a quantized conformally coupled massless scalar field in a Schwarzschild spacetime [23], Anderson, Hiscock, and Samuel developed a method for calculating  $\langle T_{\mu\nu} \rangle$  for a quantized scalar field with arbitrary mass and arbitrary curvature coupling parameter in a general static spherically symmetric spacetime. Details of this work can be found in Ref. [24]. Here, I highlight the key points of their method,

emphasizing those details unique to calculating  $\langle T_{\mu\nu} \rangle$  in an extreme GHS black hole spacetime.

First, one analytically continues the metric of interest to Euclidean space. For a static spherically symmetric metric, this analytic continuation is achieved by defining a Euclidean time,  $\tau_E$ , as  $\tau_E = it$ . The method of point splitting is then used to find[24]

$$\begin{aligned} \langle T_{\mu\nu} \rangle_{unren} = & \lim_{x' \rightarrow x} \left[ \left( \frac{1}{2} - \xi \right) (g_{\mu}^{\alpha'} G_{E;\alpha'\nu} + g_{\nu}^{\alpha'} G_{E;\mu\alpha'}) + \left( 2\xi - \frac{1}{2} \right) g_{\mu\nu} g^{\sigma\alpha'} G_{E;\sigma\alpha'} \right. \\ & - \xi (G_{E;\mu\nu} + g_{\mu}^{\alpha'} g_{\nu}^{\beta'} G_{E;\alpha'\beta'}) + 2\xi g_{\mu\nu} (m^2 + \xi R) G_E + \\ & \left. \xi (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) G_E - \frac{1}{2} m^2 g_{\mu\nu} G_E \right] \quad (53) \end{aligned}$$

Here  $m$  is the mass of the scalar field and  $\xi$  is its coupling to  $R$ , the scalar curvature. The metric is given by  $g_{\mu\nu}$ . The function  $G_E(x, x')$ , which I abbreviated as  $G_E$  in Eq. (53), is the point split Euclidean space Green's function. Before the limit  $x' \rightarrow x$  is taken, tensor quantities constructed from the Euclidean space Green's function at  $x'$  (e.g.,  $G_E(x, x')_{;\alpha'\beta'}$ ) must be parallel transported to  $x$ . This parallel transport is accomplished with the bivector,  $g_{\mu}^{\alpha'}$ .

The Euclidean space Green's function is a solution to the scalar field equation

$$(\square - m^2 - \xi R) G_E(x, x') = -g^{1/2} \delta^4(x, x'). \quad (54)$$

Specifically [39],

$$G_E(x, x') = \int d\omega \exp[i\omega(\tau_E - \tau'_E)] \sum_{l=0}^{\infty} (2l+1) P_l(\cos \gamma) C_{\omega l} p_{\omega l}(r_<) q_{\omega l}(r_>), \quad (55)$$

where  $P_l$  is a Legendre polynomial,  $\cos \gamma \equiv \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi')$ ,  $C_{\omega l}$  is a normalization constant, and  $r_>$  ( $r_<$ ) is the greater (lesser) of  $r$  and  $r'$ . The modes  $p_{\omega l}(r)$  and  $q_{\omega l}(r)$  are the homogeneous solutions to the radial equation obtained from Eq. (54). The solution  $p_{\omega l}$  is well behaved at the event horizon of the black hole and divergent at  $r = \infty$ , while the solution  $q_{\omega l}$  is well behaved at  $\infty$  and divergent at the event horizon. The normalization constant and modes also satisfy the Wronskian condition

$$C_{\omega l} \left[ p_{\omega l} \frac{dq_{\omega l}}{dr} - q_{\omega l} \frac{dp_{\omega l}}{dr} \right] = -\frac{1}{r^2} \left( \frac{g_{rr}(r)}{g_{tt}(r)} \right)^{1/2}. \quad (56)$$

If the scalar field is in a thermal state with temperature,  $T$ , not equal to zero, then the integral over the mode frequencies  $\omega$  becomes a discrete sum over  $n$ , with  $\omega = 2\pi nT$ . As argued in chapter three, the equilibrium state of a quantized scalar field in an extreme two dimensional GHS black hole spacetime either has  $T = 1/(8\pi M e^{\phi_0})$  or else the expectation value of the stress-energy tensor diverges. Attempting to find a consistent semiclassical description for the extreme GHS black hole, I will suppress this divergence by assuming the scalar field for the four dimensional GHS black hole is, in fact, in the thermal state defined by  $T = 1/(8\pi M e^{\phi_0})$ .

The Euclidean space Green's function is divergent as  $x' \rightarrow x$ , and Eq. (53) must be renormalized by subtracting off the point splitting counterterms Christensen obtained

by using the DeWitt-Schwinger expansion [40]. Schematically, one has

$$\langle T_{\mu\nu} \rangle_{ren} = \lim_{x' \rightarrow x} (\langle T_{\mu\nu} \rangle_{unren} - \langle T_{\mu\nu} \rangle_{DS}), \quad (57)$$

where  $\langle T_{\mu\nu} \rangle_{DS}$  denotes the counterterms. If one could analytically solve for the radial modes,  $p_{\omega l}$  and  $q_{\omega l}$ , Eq. (57) together with Eqs. (53) and (55) would give a general closed form solution for the expectation value of the renormalized stress-energy components. However, for most spacetimes the radial modes must be calculated numerically. Thus, the stress-energy for a quantized field in a black hole spacetime must be recalculated for each distinct spacetime of interest.

A straightforward application of the above procedure for calculating  $\langle T_{\mu\nu} \rangle_{ren}$  is extremely inefficient. As Anderson, Hiscock, and Samuel noted, the calculation is significantly improved by adding and subtracting a WKB approximation for the modes in the sums and integrals of Eq (55). This WKB approximation is in turn separated into finite terms and terms which contain the ultraviolet divergences of  $\langle T_{\mu\nu} \rangle_{unren}$ . The schematic expression of Eq. (57) for the renormalized stress-energy then becomes

$$\begin{aligned} \langle T_{\mu\nu} \rangle_{ren} &= \lim_{x' \rightarrow x} \left[ (\langle T_{\mu\nu} \rangle_{unren} - \langle T_{\mu\nu} \rangle_{WKB}) + (\langle T_{\mu\nu} \rangle_{WKB} - \langle T_{\mu\nu} \rangle_{WKBdiv}) \right. \\ &\quad \left. + (\langle T_{\mu\nu} \rangle_{WKBdiv} - \langle T_{\mu\nu} \rangle_{DS}) \right] \\ &= \langle T_{\mu\nu} \rangle_{modes} + \langle T_{\mu\nu} \rangle_{WKBfin} + \langle T_{\mu\nu} \rangle_{analytic} \end{aligned} \quad (58)$$

For large  $\omega$  and  $l$ , the difference between a particular numerically calculated mode

and its corresponding WKB approximation goes to zero, and the sums and integrals in  $\langle T_{\mu\nu} \rangle_{modes}$  can be truncated at some finite value of  $\omega$  and  $l$ . Hence, the addition and subtraction of the WKB terms amounts to using the WKB approximation for the modes at large values of  $\omega$  and  $l$ , which substantially reduces the number of modes that must be numerically calculated. The components of  $\langle T_{\mu\nu} \rangle_{analytic}$  have been calculated analytically for a quantized scalar field with arbitrary mass and arbitrary curvature coupling parameter in a general static spherically symmetric spacetime and are written in closed form in Ref. [24]. Finally, the mode sums and integrals of  $\langle T_{\mu\nu} \rangle_{WKBfin}$  cannot in general be calculated analytically. Therefore,  $\langle T_{\mu\nu} \rangle_{WKBfin}$  must be numerically calculated at each value of  $r$  where  $\langle T_{\mu\nu} \rangle_{ren}$  data is desired.

Calculating the stress-energy of a quantized scalar field with the procedure of Anderson, Hiscock, and Samuel is thus a three step process. First, one numerically calculates a finite set of mode solutions. Starting at  $\omega$  and  $l = 0$ , one calculates higher order modes until the large  $\omega$  and large  $l$  modes cancel with the WKB approximation to the limits of the numerical computation. Second, one numerically calculates the mode sums and integrals for  $\langle T_{\mu\nu} \rangle_{modes}$  and  $\langle T_{\mu\nu} \rangle_{WKBfin}$ . For  $\langle T_{\mu\nu} \rangle_{modes}$  the sums and integrals are only calculated to where the modes cancel with the WKB approximation. For  $\langle T_{\mu\nu} \rangle_{WKBfin}$  terms from  $\omega_{max} \rightarrow \infty$  and from  $l_{max} \rightarrow \infty$  must be included. These later terms are estimated by expanding the WKB approximation in inverse orders of  $\omega$  and  $l$ . Finally, these results are combined with  $\langle T_{\mu\nu} \rangle_{analytic}$  to obtain  $\langle T_{\mu\nu} \rangle_{ren}$ .

For the extreme GHS black hole spacetime, one must choose whether to calculate the expectation value of the stress-energy tensor for a quantized field using the

physical metric of Eq. (30) or the conformally related string metric of Eq. (36). In the context of this dissertation, it makes sense to choose the string metric. First, as already noted in chapter three, strings couple to the string metric, not the physical metric. Hence, the string metric is more fundamental than the physical metric in terms of superstring theory. Second, as the physical metric becomes extreme, the curvature singularity moves out to the event horizon and the two surfaces coincide. Thus, one would not be surprised to discover that the stress-energy of the quantized field diverges on the event horizon in the physical metric.

In Euclidean space, the extreme GHS black hole metric becomes

$$ds^2 = d\tau_E^2 + \left(1 - \frac{2Me^{\phi_0}}{\rho}\right)^{-2} d\rho^2 + \rho^2 d\Omega^2, \quad (59)$$

where  $\tau_E$  is the Euclidean time. For this metric, the radial wave equation for a massless scalar field is

$$\begin{aligned} \left(1 - \frac{2Me^{\phi_0}}{\rho}\right)^2 \frac{d^2 S_{\omega l}}{d\rho^2} + \frac{2}{\rho} \left(1 - \frac{2Me^{\phi_0}}{\rho}\right) \left(1 - \frac{Me^{\phi_0}}{\rho}\right) \frac{dS_{\omega l}}{d\rho} \\ - \left(\omega^2 + \frac{l(l+1)}{\rho^2} + \xi \frac{8M^2 e^{2\phi_0}}{\rho^4}\right) S_{\omega l} = 0. \end{aligned} \quad (60)$$

For numerical purposes it is useful to scale the mass of the black hole,  $M$ , and the dilaton parameter,  $\phi_0$ , out of the radial equation. This is accomplished by defining

new variables  $s$ , and  $\tilde{\omega}$  such that

$$s \equiv \frac{(\rho - 2Me^{\phi_0})}{Me^{\phi_0}}, \quad \text{and} \quad \tilde{\omega} \equiv \omega Me^{\phi_0}. \quad (61)$$

In terms of these new variables the radial equation becomes

$$\frac{d^2 S_{\omega l}}{ds^2} + 2 \frac{(s+1)}{s(s+2)} \frac{dS_{\omega l}}{ds} - \frac{1}{s^2} \left( \tilde{\omega}^2 (s+2)^2 + l(l+1) + \frac{8M}{(s+2)^2} \right) S_{\omega l} = 0. \quad (62)$$

This equation has two regular singular points at  $s = -2$  and  $s = 0$  and an irregular singular point at  $s = \infty$ . Since the radial modes are confined to the region  $s = 0$  to  $\infty$ , the singular point at  $s = -2$  can be neglected.

Initial values for the outgoing  $p_{\omega l}$  modes are determined by using a power series to solve Eq. (62) about  $s = 0$ . Specifically, near  $s = 0$  the outgoing modes are

$$p_{\omega l}(s) = \sum_{\lambda=0}^{\infty} a_{\lambda} s^{k+\lambda}, \quad (63)$$

with

$$k = \sqrt{4\tilde{\omega}^2 + l(l+1) + 2\xi} \quad (64)$$

as determined by solving the indicial equation. In this dissertation, attention is restricted to scalar fields that conformally couple ( $\xi = 1/6$ ) to the curvature. For the minimally coupled case ( $\xi = 0$ ),  $k = 0$  for the  $\omega = 0$ ,  $l = 0$  modes, and one must include a  $\log s$  factor in the series solution of Eq. (63).

The leading coefficient,  $a_0$ , is factored out of the series and absorbed into the normalization constant,  $C_{\omega l}$ . The remaining coefficients are then given by the recursion relation

$$\begin{aligned}
 a_{j+4} = & \left[ 4 \left( (k+j+4)^2 - k^2 \right) \right]^{-1} \left[ \tilde{\omega}^2 (a_j + 8a_{j+1}) \right. \\
 & - \left( (k+j+3)(k+j+2) - (24\tilde{\omega}^2 + L) \right) a_{j+2} \\
 & \left. - \left( 2(2k+2j+7)(k+j+3) - 4(8\tilde{\omega}^2 + L) \right) a_{j+3} \right] , \quad (65)
 \end{aligned}$$

where  $L \equiv l(l+1)$  and

$$a_1 = \frac{4(8\tilde{\omega}^2 + L) - 2k(2k+1)}{4((k+1)^2 - k^2)},$$

$$a_2 = \frac{(24\tilde{\omega}^2 + L) - k(k+1) - (2(2k^2 + 5k + 3) - 4(8\tilde{\omega}^2 + L)) a_1}{4((k+2)^2 - k^2)},$$

$$a_3 = \frac{8\tilde{\omega}^2 - ((k+2)(k+1) - 24\tilde{\omega}^2 - L) a_1 - (2(2k+5)(k+2) - 4(8\tilde{\omega}^2 + L)) a_2}{4((k+3)^2 - k^2)}.$$

Initial values for the incoming  $q_{\omega l}$  modes are determined by finding an asymptotic series that solves Eq. (62) at  $s = \infty$ . This series is

$$q_{\omega l}(s) = s^{-(2\tilde{\omega}+1)} e^{-\tilde{\omega}s} \sum_{\lambda=0}^{\infty} b_{\lambda} s^{-\lambda}. \quad (66)$$

As in the previous case,  $b_0$ , the leading coefficient in the series solution for the  $q_{\omega l}$  modes, is factored out of the series and absorbed (along with  $a_0$ ) into the normalization constant. The remaining coefficients are then given by the recursion relation

$$\begin{aligned}
b_{j+3} = & [2(j+3)\tilde{\omega}]^{-1} \left[ 4(L+2\xi-4\tilde{\omega}(j+1)-(j+1)^2) b_j \right. \\
& + 2(2L-4\tilde{\omega}(3j+5)-(2j+3)(j+2)) b_{j+1} \\
& \left. + (L-4\tilde{\omega}(3j+7)-(j+3)(j+2)) b_{j+2} \right] , \quad (67)
\end{aligned}$$

with

$$b_1 = \frac{L-4\tilde{\omega}}{2\tilde{\omega}}, \quad (68)$$

$$b_2 = \frac{(L-16\tilde{\omega}-2)b_1 + 4L-16\tilde{\omega}-2}{4\tilde{\omega}}, \quad (69)$$

and again  $L \equiv l(l+1)$ .

In terms of the dimensionless radial coordinate  $s$ , the  $\langle T_{\mu\nu} \rangle_{analytic}$  components for a massless scalar field in the extreme string metric are

$$\begin{aligned}
M^4 \langle T_{\tau}^{\tau} \rangle_{analytic} = & [368640\pi^2(2+s)^8]^{-1} [18688 + 24576C - 215040\xi \\
& - 245760C\xi + 737280\xi^2 + 737280C\xi^2 - 50176s \\
& + 86016Cs + 307200\xi s - 983040C\xi s + 2949120C\xi^2 s + 69888s^2 \\
& - 101376Cs^2 - 322560\xi s^2 + 1474560C\xi s^2 - 4423680C\xi^2 s^2 - 7936s^3 \\
& + 15360\xi s^3 - 3680s^4 + 1920\xi s^4 - 1344s^5 - 336s^6 - 48s^7 \\
& - 3s^8 - 24576 \ln 2 + 245760\xi \ln 2 - 737280\xi^2 \ln 2 - 86016s \ln 2
\end{aligned}$$

$$\begin{aligned}
& +983040\xi s \ln 2 - 2949120\xi^2 s \ln 2 + 101376s^2 \ln 2 - 1474560\xi s^2 \ln 2 \\
& +4423680\xi^2 s^2 \ln 2] \tag{70}
\end{aligned}$$

$$\begin{aligned}
M^4 \langle T_{\rho}^{\rho} \rangle_{analytic} &= [368640\pi^2(2+s)^8]^{-1} [5376 + 24576C \\
& +16384s + 49152Cs + 19712s^2 + 89088Cs^2 + 12032s^3 + 4000s^4 \\
& +768s^5 + 112s^6 + 16s^7 + s^8 - 30720\xi - 245760C\xi \\
& -92160s\xi - 491520Cs\xi - 107520s^2\xi - 983040Cs^2\xi - 61440s^3\xi - 17280s^4\xi \\
& -1920s^5\xi + 737280C\xi^2 + 1474560Cs\xi^2 + 2949120Cs^2\xi^2 - 24576 \ln 2 - 49152s \ln 2 \\
& -89088s^2 \ln 2 + 245760\xi \ln 2 + 491520s\xi \ln 2 + 983040s^2\xi \ln 2 - 737280\xi^2 \ln 2 \\
& -1474560s\xi^2 \ln 2 - 2949120s^2\xi^2 \ln 2] \tag{71}
\end{aligned}$$

$$\begin{aligned}
M^4 \langle T_{\theta}^{\theta} \rangle_{analytic} & [368640\pi^2(2+s)^8]^{-1} [256 - 24576C \\
& -1536s + 55296Cs - 3328s^2 - 178176Cs^2 - 2048s^3 - 160s^4 \\
& +288s^5 + 112s^6 + 16s^7 + s^8 + 245760C\xi + 15360s\xi \\
& -737280Cs\xi + 30720s^2\xi + 1966080Cs^2\xi + 23040s^3\xi + 7680s^4\xi + 960s^5\xi \\
& -737280C\xi^2 + 2211840Cs\xi^2 - 5898240Cs^2\xi^2 + 24576 \ln 2 - 55296s \ln 2 \\
& +178176s^2 \ln 2 - 245760\xi \ln 2 + 737280s\xi \ln 2 - 1966080s^2\xi \ln 2 + 737280\xi^2 \ln 2 \\
& -2211840s\xi^2 \ln 2 + 5898240s^2\xi^2 \ln 2] \tag{72}
\end{aligned}$$

Here  $C$  is Euler's constant, and I have left the curvature coupling as  $\xi$  although in this chapter I am calculating the  $\langle T_{\mu\nu} \rangle_{ren}$  for the specific case of conformal coupling ( $\xi = 1/6$ ).

## Numerical results for $\langle T_{\mu\nu} \rangle$

I now present the numerically calculated values for the renormalized stress-energy tensor of a quantized conformally coupled massless scalar field in the four dimensional extreme GHS black hole spacetime, where the quantized field is in a thermal state defined by the temperature  $T = 1/(8Me^{\phi_0})$ . Figures (7), (8), and (9) display values for the components  $\langle T_{\tau}^{\tau} \rangle$ ,  $\langle T_{\rho}^{\rho} \rangle$ , and  $\langle T_{\theta}^{\theta} \rangle$ , respectively. (Because the GHS black hole is spherically symmetric, the stress-energy components  $\langle T_{\theta}^{\theta} \rangle$  and  $\langle T_{\phi}^{\phi} \rangle$  are equal.) For each component,  $\langle T_{\mu\nu} \rangle_{ren}$  data is calculated at the radial values

$$s = .005 \rightarrow .05 \quad \text{in steps of } .005 \quad ,$$

$$s = .5 \rightarrow 5 \quad \text{in steps of } .5 \quad ,$$

$$\text{and } s = 5 \rightarrow 25 \quad \text{in steps of } 1 \quad .$$

Visually extrapolating the values for the stress-energy components to  $s = 0$  suggests that for each of the components the stress-energy of the quantized scalar field remains finite on the event horizon of the extreme GHS black hole. This statement of the stress-energy's regularity will be made more quantitative in a moment.

A few observations regarding the behavior of the renormalized stress-energy components are worth noting. First, recall that the scalar field is in a thermal state defined by the temperature  $T = 1/(8\pi Me^{\phi_0})$ . Thus, each of the renormalized stress-energy components asymptotically approaches a small nonzero value away from the

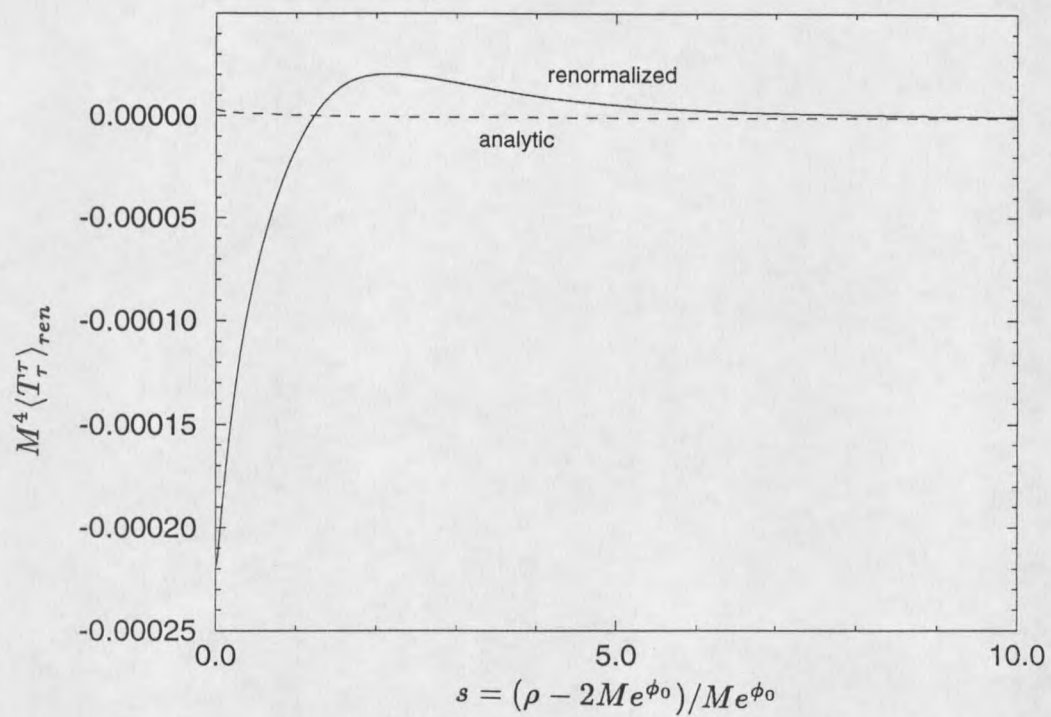


Figure 7:  $\langle T^r_r \rangle_{ren}$  as a function of the scaled radial coordinate  $s$ .

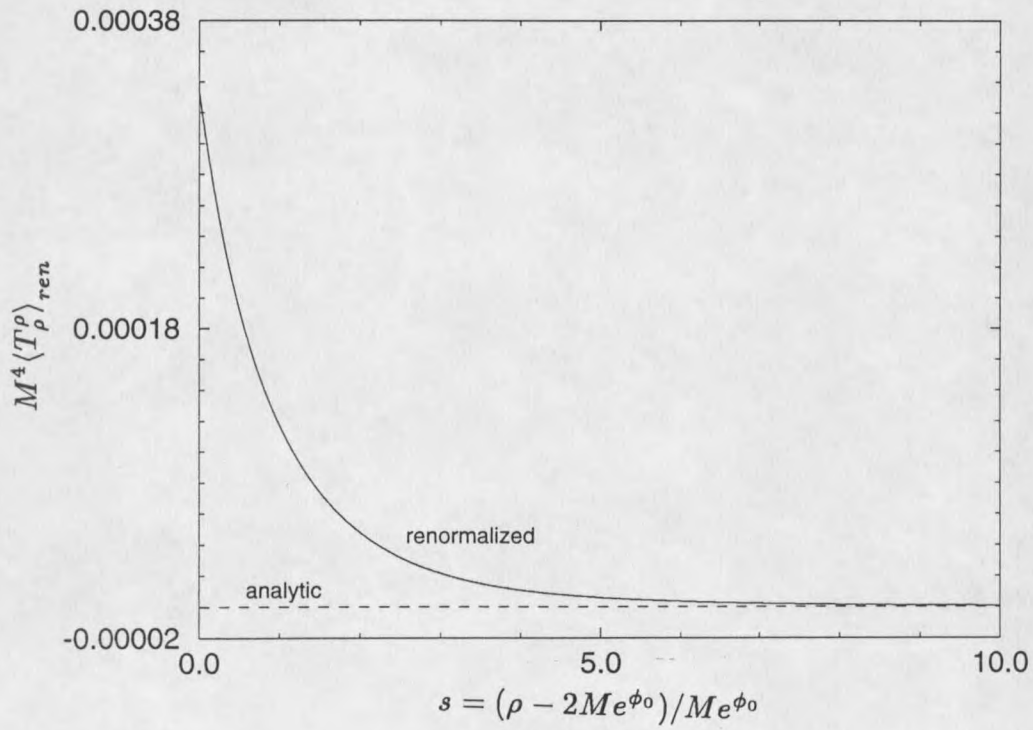


Figure 8:  $\langle T_\rho^\rho \rangle_{ren}$  as a function of the scaled radial coordinate  $s$

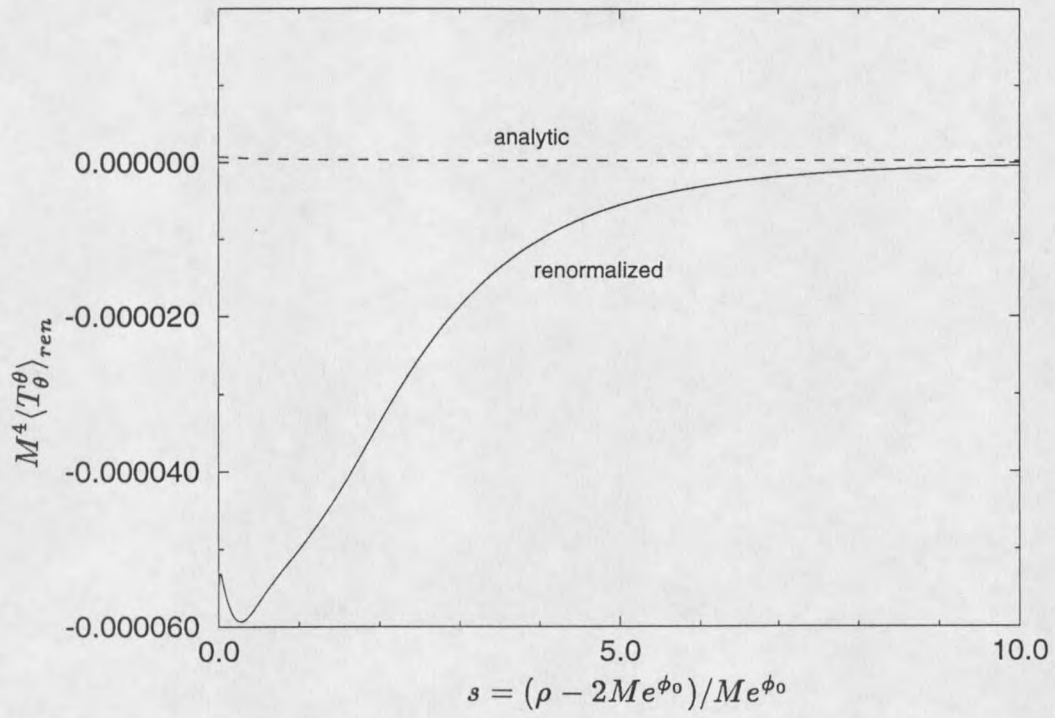


Figure 9:  $\langle T_\theta^\theta \rangle_{ren}$  as a function of the scaled radial coordinate  $s$

extreme GHS black hole. (That the asymptotic values are not zero is more easily seen in figure (10).) In contrast, a quantized scalar field in the extreme Reissner-

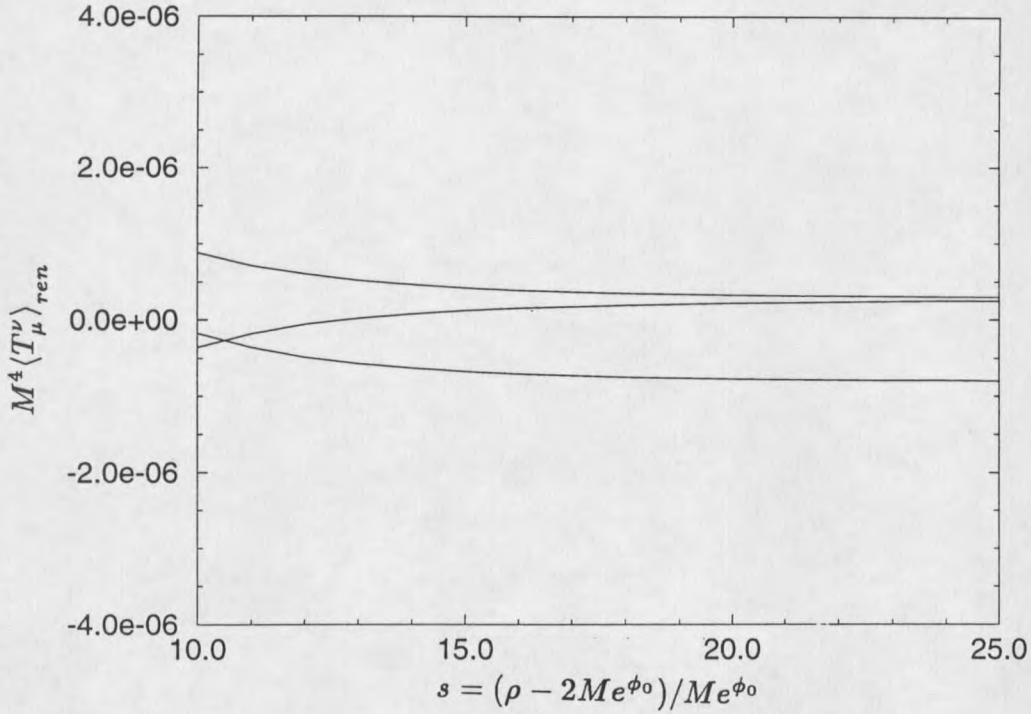


Figure 10:  $\langle T_{\mu}^{\nu} \rangle_{ren}$  as a function of  $s$ . At  $s = 15$  the curves from top to bottom are  $\langle T_{\rho}^{\rho} \rangle_{ren}$ ,  $\langle T_{\theta}^{\theta} \rangle_{ren}$  and  $\langle T_{\tau}^{\tau} \rangle_{ren}$ .

Nordström spacetime is in a thermal state defined by  $T = 0$  (or else  $\langle T_{\mu\nu} \rangle_{ren}$  of the field diverges on the event horizon), and the renormalized stress-energy components for this scalar field all approach zero near infinity. Second, the energy density of the conformal scalar field ( $\rho = -\langle T_{\tau}^{\tau} \rangle_{ren}$ ) is positive near the extreme GHS black hole, goes through a region of negative values, and then asymptotically approaches a positive value. Again for comparison, the energy density for the conformal scalar

field in the extreme Reissner-Nordström spacetime is everywhere negative. Third, the component  $\langle T_\theta^\theta \rangle_{ren}$  possesses a greater degree of structure near the event horizon (figure 11) than either  $\langle T_\tau^\tau \rangle_{ren}$  or  $\langle T_\rho^\rho \rangle_{ren}$  exhibits.

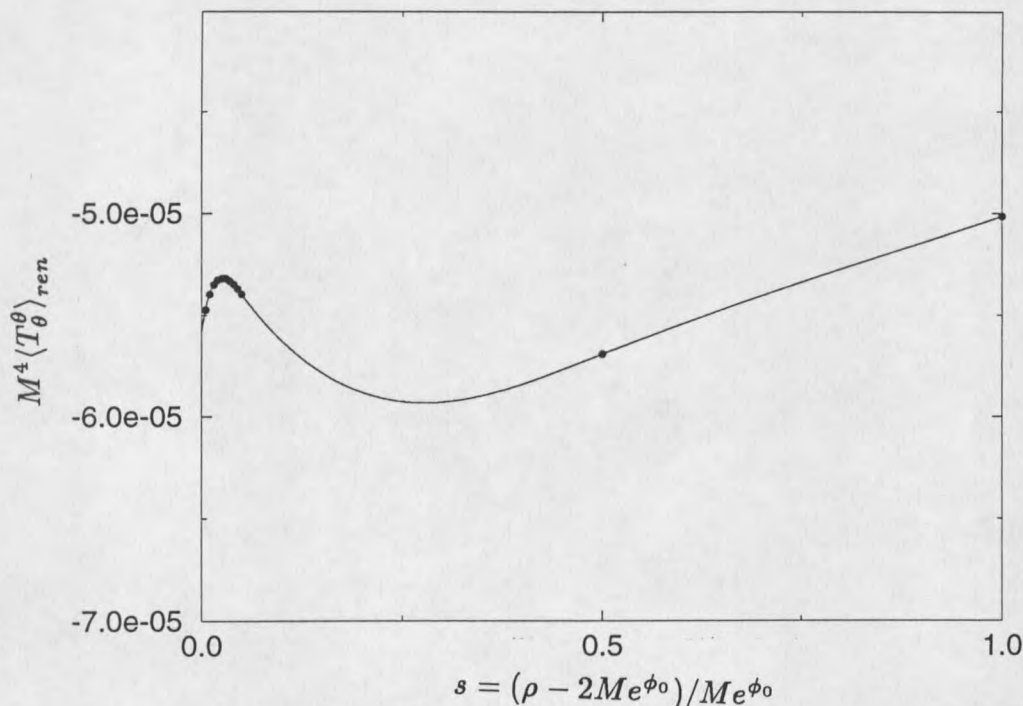


Figure 11:  $\langle T_\theta^\theta \rangle_{ren}$  near  $s = 0$ . Calculated points are explicitly shown.

Figures (7 - 9) also display the values of  $\langle T_\mu^\nu \rangle_{analytic}$  in comparison to  $\langle T_\mu^\nu \rangle_{ren}$ . (How  $\langle T_\mu^\nu \rangle_{analytic}$  behaves as a function of  $\rho$  is better displayed in figure (12).) Since  $\langle T_{\mu\nu} \rangle_{analytic}$  is conserved and, for a conformally invariant field, has a trace equal to the conformal trace anomaly, one might expect  $\langle T_{\mu\nu} \rangle_{analytic}$  to approximate  $\langle T_{\mu\nu} \rangle_{ren}$  under certain conditions. As Anderson, Hiscock, and Samuel noted, these conditions require that the stress-energy of a quantized field be slowly varying. Generally, for

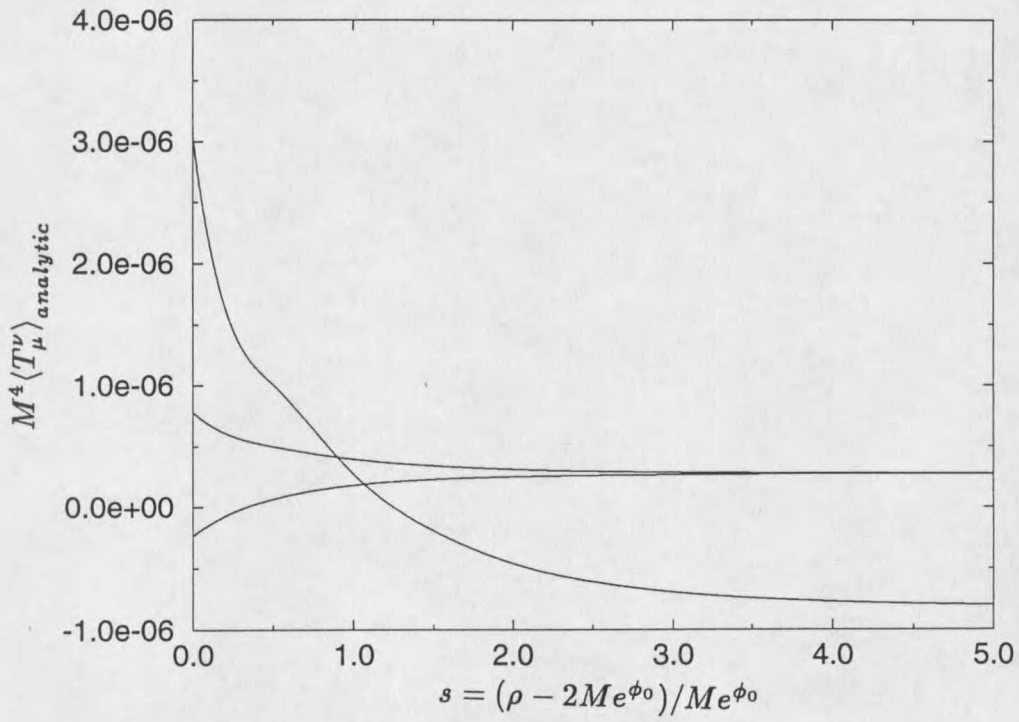


Figure 12:  $\langle T_\mu^\nu \rangle_{analytic}$  as a function of  $s$ . At  $s = 1.5$  the curves from top to bottom are  $\langle T_\theta^\theta \rangle_{analytic}$ ,  $\langle T_\rho^\rho \rangle_{analytic}$ , and  $\langle T_\tau^\tau \rangle_{analytic}$ .

a black hole spacetime, the stress-energy of a quantized field will vary slowly only in the asymptotically flat region of the spacetime. Thus, in general, one can only expect  $\langle T_{\mu\nu} \rangle_{analytic}$  to be a good approximation far from the black hole. This expectation is upheld in figures (7 - 9), where  $\langle T_{\mu}^{\nu} \rangle_{analytic}$  is observed to approach  $\langle T_{\mu}^{\nu} \rangle_{ren}$  only after the renormalized stress-energy tensor has settled down in the region far from the event horizon.

The renormalized stress-energy tensor for a conformally invariant scalar field is expected to have a trace equal to the conformal trace anomaly. Examining the fractional difference between the trace of the renormalized stress-energy tensor and the expected trace confirms this expectation. As displayed in figure (13), for a conformally invariant scalar field in the extreme GHS black hole spacetime, this fractional difference is zero to the limits of numerical noise.

Similarly, one expects that the renormalized stress-energy is conserved. Using the string metric for the extreme GHS black hole, the conservation of stress-energy gives

$$\frac{d}{d\rho} \langle T_{\rho}^{\rho} \rangle + \frac{2}{\rho} (\langle T_{\rho}^{\rho} \rangle - \langle T_{\theta}^{\theta} \rangle) = 0 \quad , \quad (73)$$

where the subscript *ren* is implied. Therefore, one can check that the renormalized stress-energy of a quantized field in the extreme GHS black hole is conserved by comparing the derivative term,  $\frac{d}{d\rho} \langle T_{\rho}^{\rho} \rangle$ , to the remaining terms  $(2/\rho)(\langle T_{\rho}^{\rho} \rangle - \langle T_{\theta}^{\theta} \rangle)$ .

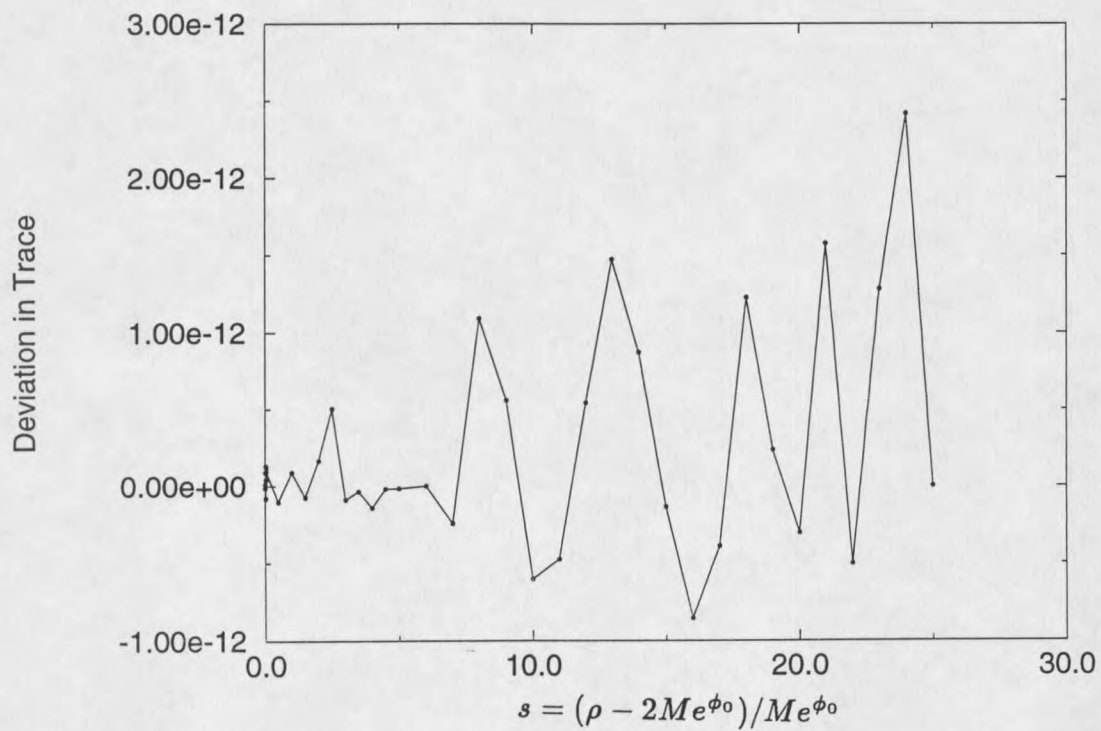


Figure 13: Fractional difference in the calculated and expected trace as function of the scaled radial coordinate  $s$ .

Specifically, I examine the fractional difference

$$\sigma_{DIV} \equiv \left| \frac{\frac{d}{d\rho} \langle T_{\rho}^{\rho} \rangle + \frac{2}{\rho} (\langle T_{\rho}^{\rho} \rangle - \langle T_{\theta}^{\theta} \rangle)}{\frac{d}{d\rho} \langle T_{\rho}^{\rho} \rangle} \right| \quad (74)$$

In order to compute the fractional difference of Eq. (74), the derivative  $\frac{d}{d\rho} \langle T_{\rho}^{\rho} \rangle$  must be calculated numerically, and unfortunately, numerically calculated derivatives are not very accurate. Although various strategies exist for improving this accuracy, such strategies assume that while the explicit form of the function to be differentiated is unknown, it is easy to compute a specific value for the function at an arbitrarily specified point. This assumption is patently false when calculating  $\langle T_{\mu\nu} \rangle_{ren}$  values. Hence, here I determine the derivative at a given radial value,  $s_i$ , by simply averaging the linear slopes of the two intervals adjacent to  $s_i$ . As an estimate of the error in this derivative, I compute  $\sigma_{slope}$ , which I define as the fractional difference between the linear slopes of these adjacent intervals. Figure (14) displays  $\sigma_{DIV}$  as a function of  $\sigma_{slope}$ . As expected, error in the derivative estimate dominates the comparison of the terms in Eq. (73). However,  $\sigma_{DIV}$  does decrease as the derivative estimate improves.

Finally, I turn to the issue of extrapolating the components of the renormalized stress-energy to the event horizon of the extreme GHS black hole. As already mentioned, visual inspection of figures (7 - 9) suggests that the renormalized stress-energy remains finite on the event horizon. This visual observation can be made more quantitative by using the method of least squares to fit the computed values of  $\langle T_{\mu\nu} \rangle_{ren}$  to a series in the dimensionless radial variable  $s = (\rho - 2Me^{\phi_0}) / (Me^{\phi_0})$ .

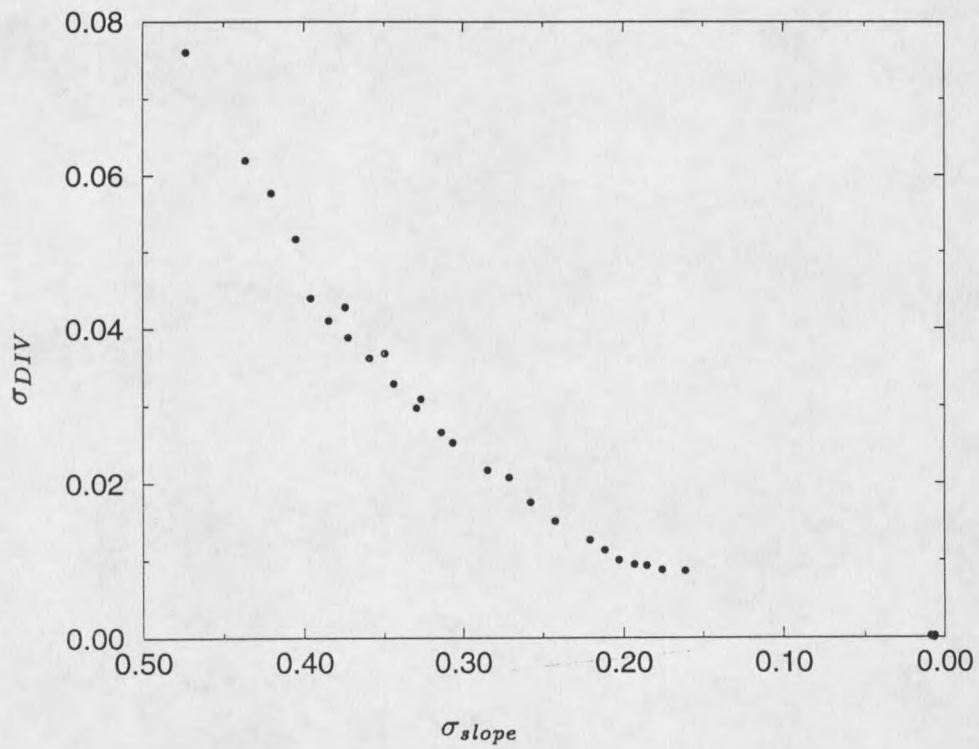


Figure 14: Estimate of the error in the divergence of the renormalized stress-energy as a function of the error in the numerical derivative. The terms  $\sigma_{DIV}$  and  $\sigma_{slope}$  are defined in the text.

Since  $\langle T_{\mu}^{\nu} \rangle_{ren}$  may actually diverge on the event horizon despite one's visual interpretation of the values computed near  $s = 0$ , the series is allowed to lead off with the terms  $s^{-1}$  and  $\ln s$ . Here I fit the values of  $\langle T_{\mu}^{\nu} \rangle_{ren}$  calculated for the set  $s = \{.005, .01, .015, .02, .025, .03, .035, .04, .045, .05\}$  to this series keeping up to six significant digits. The resulting coefficients for the terms  $s^{-1}$  and  $\ln s$  are generally of order  $10^{-4}$  and  $10^{-2}$  times the terms in the series which become finite on the event horizon. For the  $\theta\theta$  component, the stress-energy has some significant structure near the event horizon (figure 11), and the coefficient for the term  $\ln s$  in this case is of order  $10^{-1}$  times the finite terms. Specifically,

$$\begin{aligned} \langle T_{\tau}^{\tau} \rangle_{ren} M^4 &\approx -1.69384 \times 10^{-8} s^{-1} - 5.94729 \times 10^{-6} \ln s \\ &\quad - 2.49737 \times 10^{-4} + 5.43624 \times 10^{-4} s + 3.67059 \times 10^{-4} s^2 \\ \langle T_{\rho}^{\rho} \rangle_{ren} M^4 &\approx -2.58817 \times 10^{-10} s^{-1} - 7.81617 \times 10^{-8} \ln s \\ &\quad + 3.34258 \times 10^{-4} - 3.82387 \times 10^{-4} s + 2.2998 \times 10^{-4} s^2 \\ \langle T_{\theta}^{\theta} \rangle_{ren} M^4 &\approx 8.58127 \times 10^{-9} s^{-1} + 3.00794 \times 10^{-6} \ln s \\ &\quad - 4.00854 \times 10^{-5} - 8.67984 \times 10^{-5} s - 2.83075 \times 10^{-4} s^2 \end{aligned}$$

To the level of accuracy for which the data is fit to the series, these results suggest that  $\langle T_{\mu}^{\nu} \rangle_{ren}$  does not diverge on the event horizon.

The stress-energy of a quantized conformally invariant massless scalar field in

the extreme GHS spacetime appears finite everywhere in the spacetime, when the scalar field is in a thermal state defined by the temperature  $1/(8Me^{\phi_0})$ . Thus, four dimensional analysis in this extreme string metric does not contradict the results of chapters two and three. For a stronger confirmation, one would now like to examine the stress-energy of the scalar field in the physical metric. In principle, because the physical and string metrics are conformally related, the stress-energy for a quantized conformally invariant field in the physical GHS metric can be determined from the results of this chapter. Unfortunately, determining the stress-energy for a quantized conformally invariant field in one spacetime from the stress-energy of the field in a conformally related spacetime is only simple when one of the spacetimes is flat. In four dimensions, neither the physical metric nor the string metric is flat for the GHS black hole, and the calculation of  $\langle T_{\mu\nu} \rangle_{ren}$  in the physical metric is left for future research.

## CHAPTER 5

## Concluding Remarks

Although extreme black holes play an important role in contemporary studies of quantum gravity, the thermal properties of extreme black hole spacetimes has remained ambiguous. It had been suggested that an extreme black hole can couple with a quantized scalar field in an arbitrary thermal state. This dissertation reduces this ambiguity by showing that the stress-energy of a quantized scalar field in a two dimensional extreme black hole spacetime diverges strongly on the event horizon unless the quantized field is in a uniquely defined thermal state. In addition, this dissertation has also shown that this thermally related divergence in the stress-energy of a quantized scalar field occurs in precisely the same fashion for both extreme and nonextreme black hole spacetimes. Taken together, these results suggest that:

1. An extreme black hole spacetime has well defined thermal properties and cannot be assigned arbitrary temperature.
2. The temperature of an extreme black hole is uniquely determined by extrapolating the temperature of the corresponding nonextreme black hole to the extreme state.

For most black hole solutions, the surface gravity is a continuous function as the black hole approaches its extreme state. In this case, the temperature extrapolated for the extreme black hole is simply the temperature geometrically defined by the surface gravity. In the string description of the GHS black hole, no geometrically defined temperature exists for the extreme state. Yet, the temperature extrapolated from the nonextreme GHS black hole still determines the unique thermal state for a quantized field in the extreme GHS black hole spacetime. Any other thermal state for the quantized field has divergent stress-energy on the event horizon.

In the extreme Reissner-Nordström case, four dimensional analysis supports statements 1 and 2 above [17]. With the results of chapter four, one is nearly able to answer whether four dimensional analysis of the extreme GHS black hole also supports these statements. As for the two dimensional analysis of the GHS black hole found in chapter three of this dissertation, one can examine the stress-energy of a quantized field in the physical metric while the temperature associated with the spacetime described by the string metric is varied. In principle, the stress-energy tensor for a conformally invariant scalar field in the GHS black hole spacetime described by the physical metric can be determined from the results for the conformally related string metric presented in chapter four. Unfortunately, the expression relating the stress-energy tensor components from two conformally related spacetimes is only simple when one of the spacetimes is flat. For the extreme GHS black hole, neither the physical metric nor the string metric is flat, and the question of whether four dimensional analysis of the extreme GHS black hole upholds the prescriptions of statements 1 and

2 above remains to be determined.

The role played by extreme black holes in studies of quantum gravity often requires a self consistent semiclassical description for the extreme black hole spacetime. In chapter four of this dissertation I calculated the renormalized stress-energy tensor of a quantized conformal massless scalar field in an extreme GHS black hole spacetime. As one might expect for this simple spacetime, the stress-energy for such a field is finite everywhere in the spacetime. Hence, the extreme GHS black hole has a self consistent semiclassical framework.

The calculation of the renormalized stress-energy found in chapter four is also of general interest. It is the first such calculation for a superstring black hole solution. In this dissertation, I have calculated the renormalized stress-energy tensor in the string description of the extreme GHS black hole and made some initial comparisons to the stress-energy of a quantized field in the extreme Reissner-Nordström spacetime. Because the GHS solution differs from the Reissner-Nordström solution even where the curvature is small compared to Planck scale curvatures, such a comparison of the stress-energy tensors is interesting. One expects and finds significant differences for the stress-energy in the two different spacetimes. It would be interesting to compare the nonextreme spacetimes against each other.

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