Space-based gravitational wave astrophysics
by Shane L Larson

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics
Montana State University

Abstract:
In the near future, several Earth-based and space-based gravitational wave observatories will be completed. With this imminent change in the tools available to observe the Universe, there is much work to be done in describing the fundamental response of our instruments to gravitational radiation, understanding the sources of gravitational radiation, and determining what can be learned about the fundamental nature of the gravitational interaction from the detection of these waves.

The first original result reported here is the determination of the sensitivity limits of a spaceborne gravitational wave observatory. Determining if particular sources of gravitational radiation are detectable by a specific gravitational observatory requires knowledge of the sensitivity limits of the instrument, commonly depicted as a graph of the spectral density of the dimensionless strain vs. frequency of the gravitational wave. Previous discussions of the sensitivity have relied on approximations and heuristic arguments about the shape of the transfer function to construct a sensitivity curve. This thesis details the computation of the exact sensitivity curve given a simple set of parameters describing a space-based interferometer.

The second original result presented here is an exploration of how the mass of the graviton (the fundamental quanta of the gravitational field) might be determined from observations of known sources of gravitational radiation with a space-based interferometric observatory. Current experimental limits place an upper bound on the Compton wavelength of the graviton at $\lambda g > 2.8 \times 10^{12}$ km, based on analysis of Yukawa violations to Newton's Universal law of gravitation in the solar system. The advent of state-of-the-art space-based gravitational wave interferometers will allow new bounds to be placed on the graviton mass by observing interacting white dwarf binary stars, such as AM CVn (AM Canum Venaticorum). If the graviton is massless, experimental uncertainty will be the limiting factor in computing bounds on the graviton mass. For space-based interferometric detectors, the predicted uncertainties in the observations of AM CVn will place an upper bound on the inverse mass of the graviton of $\lambda g > 1.4 \times 10^{15}$ km.

SPACE-BASED GRAVITATIONAL WAVE ASTROPHYSICS

by

Shane L. Larson

A thesis submitted in partial fulfillment
of the requirements for the degree

of

Doctor of Philosophy

in

Physics

MONTANA STATE UNIVERSITY — BOZEMAN
Bozeman, Montana

April 1999

APPROVAL

of a thesis submitted by

Shane L. Larson

This thesis has been read by each member of the thesis committee, and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style and consistency, and is ready for submission to the College of Graduate Studies.

William A. Hiscock, Ph. D. ________ April 16, 1999
(Signature) Date

Approved for the Department of Physics

John C. Hermanson, Ph. D. ________ Apr. 16, 1999
(Signature) Date

Approved for the College of Graduate Studies

Bruce R. McLeod, Ph. D. ________ 4-19-99
(Signature) Date

STATEMENT OF PERMISSION TO USE

I do not know what I appear to the world; but to myself I seem to have been only like a boy playing on a seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

- Sir Isaac Newton

ACKNOWLEDGEMENTS

Foremost among those who have contributed to the completion of this dissertation is my advisor and mentor, Dr. William Hiscock. It is only through his guidance, suggestions, and patience that this project has reached its completion. For him I wish to express my gratitude, my deepest respect, and my warmest regards. I could not have done this without him.

I would like to thank the other members of the Montana State University Relativity Group, as well as the MSU Astrophysics Group and Solar Physics Group. They have provided helpful answers and suggestions over the years, as well as many interesting and stimulating discussions. Thanks also to Dr. Ron Hellings of the Jet Propulsion Laboratory, who provided invaluable assistance during the completion of the work covered in Chapter 3.

I also owe a great debt to my undergraduate advisors, Dr. Corinne Manogue and Dr. Tevian Dray, who encouraged me to go to graduate school in relativity; thanks are also owed to Dr. Jeanne Rudzki Small, who first taught me the ins and outs of research.

My parents, Pat and Larry Larson, have been a bottomless well of support in many ways, and for them I express my deepest thanks. They perhaps did more than anyone to keep me on the path towards this end.

Lastly, I would like to thank my wife, Michelle Beauvais Larson, for her patience, her support, and her inspiration throughout this long and complicated process.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CONVENTIONS

Geometrical units are used throughout this dissertation, where $c = G = 1$, except when otherwise noted. When conventional units are required, SI (*Système Internationale*) units will be employed. Sign conventions follow those of Misner, Thorne, and Wheeler [1].

For all spacetime tensor indices, Greek indices range over temporal and spatial coordinates, taking on the values 0 to 3; Latin indices range only over the spatial coordinates, 1 to 3. The Einstein summation convention is used throughout, with repeated indices summed over.

Commas are used to denote partial derivatives

$$v^{\alpha}{}_{,\beta} = \frac{\partial v^{\alpha}}{\partial x^{\beta}}$$

and semi-colons are used to denote the metric compatible covariant derivative, *e.g.*,

$$v^{\alpha}{}_{;\beta} = \frac{\partial v^{\alpha}}{\partial x^{\beta}} + \Gamma^{\alpha}{}_{\lambda\beta} v^{\lambda} \ .$$

ABSTRACT

In the near future, several Earth-based and space-based gravitational wave observatories will be completed. With this imminent change in the tools available to observe the Universe, there is much work to be done in describing the fundamental response of our instruments to gravitational radiation, understanding the sources of gravitational radiation, and determining what can be learned about the fundamental nature of the gravitational interaction from the detection of these waves.

The first original result reported here is the determination of the sensitivity limits of a spaceborne gravitational wave observatory. Determining if particular sources of gravitational radiation are detectable by a specific gravitational observatory requires knowledge of the sensitivity limits of the instrument, commonly depicted as a graph of the spectral density of the dimensionless strain vs. frequency of the gravitational wave. Previous discussions of the sensitivity have relied on approximations and heuristic arguments about the shape of the transfer function to construct a sensitivity curve. This thesis details the computation of the *exact* sensitivity curve given a simple set of parameters describing a space-based interferometer.

The second original result presented here is an exploration of how the mass of the graviton (the fundamental quanta of the gravitational field) might be determined from observations of known sources of gravitational radiation with a space-based interferometric observatory. Current experimental limits place an upper bound on the Compton wavelength of the graviton at $\lambda_g > 2.8 \times 10^{12}$ km, based on analysis of Yukawa violations to Newton's Universal law of gravitation in the solar system. The advent of state-of-the-art space-based gravitational wave interferometers will allow new bounds to be placed on the graviton mass by observing interacting white dwarf binary stars, such as AM CVn (AM *Canum Venaticorum*). If the graviton is massless, experimental uncertainty will be the limiting factor in computing bounds on the graviton mass. For space-based interferometric detectors, the predicted uncertainties in the observations of AM CVn will place an upper bound on the inverse mass of the graviton of $\lambda_g > 1.4 \times 10^{15}$ km.

CHAPTER 1

INTRODUCTION

1.1 The Search for Gravitational Radiation

A natural question to ask in the development of any field theory is whether or not the theory has propagating, wavelike solutions. The search for wave solutions is a familiar procedure from our experience with electromagnetism, where vacuum solutions for the electromagnetic 4-potential exist that describe electromagnetic waves (light).

Similar expectations might also be held for general relativity, the classical theory of the gravitational field. Einstein himself [2] predicted the existence of gravitational waves, based on a calculation in the linear approximation to general relativity, less than one year after his publication of the field equations in 1915.

Gravitational waves interact with matter by causing small changes in the distances between particles (in extended bodies, a change in the proper length of the object would occur). These changes in distances are typically expressed as fractional changes in length, $\Delta\ell/\ell$, and are called *strains*. Typical strains created in the vicinity of Earth by astrophysical sources are expected to be quite small (fractional changes in length on the order of 10^{-21} and smaller), and to date, no direct detection of gravitational waves has occurred. Indirect, astrophysical evidence for the existence of gravitational waves exists from monitoring the famous Hulse-Taylor binary pulsar, PSR $1913+16$ [3]. General relativity predicts that the period of this binary should decrease due to gravitational radiation reaction at a rate of -2.38×10^{-12} s per s. Observations of this system show that its orbital period is changing at a rate of $(-2.40 \pm 0.09) \times 10^{-12}$ s per s [4]. This value is 1.01 ± 0.04 times the rate predicted by theory, and solidifies the belief that gravitational radiation exists and has important effects on the dynamics of astrophysical systems. The Nobel Prize in Physics was awarded to Joseph Taylor and Russell Hulse in 1993 for this discovery.

Technological innovations in laser interferometry and the advent of high performance computing are making the possibility of *direct* detection of gravitational waves within the next decade a distinct reality. Currently, efforts are underway to construct a several Earth-based gravitational wave observatories using large laser interferometer systems. There have also been several proposals to design and construct interferometric gravitational wave detectors in space. In the United States, a pair of observatories known as LIGO (Laser Interferometric Gravitational-wave Observatory) are being constructed in Hanford, Washington and Livingston, Louisiana [5]. In Europe, construction has begun on a similar observatory known as VIRGO [6] near Cascina, Italy. These instruments are slated to begin operation early in the 21^{st} century and begin a systematic search for high frequency (10 to 1000 Hz) sources of gravitational waves. Shown in Figure 1.1, a single LIGO interferometer will consist of two 4 km evacuated

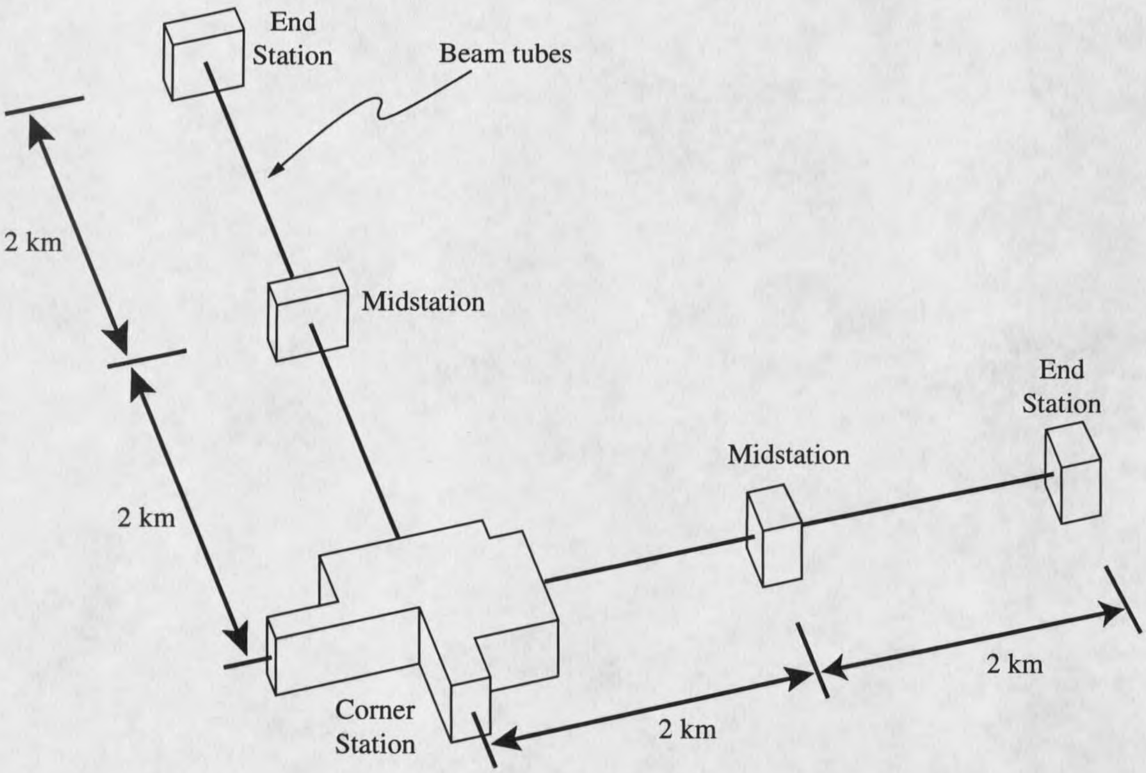

Figure 1.1: Schematic diagram of a LIGO ground based gravitational wave observatory.

arms at right angles to each other. The design of LIGO will support operation of a Michelson type interferometer, as well as several Fabry-Perot type interferometers within the 1.2 m diameter beam tubes. The midstations allow for the development, testing, and addition of equipment and secondary interferometer systems sharing the same evacuated beam lines as the primary interferometer beam. This allows the observatory to be used as a testbed for technology which will be used to increase the performance of the instrument in the coming decade. These planned improvements and modifications will greatly enhance the sensitivity and usefulness of LIGO as a gravitational observatory over the projected lifetime of the instrument.

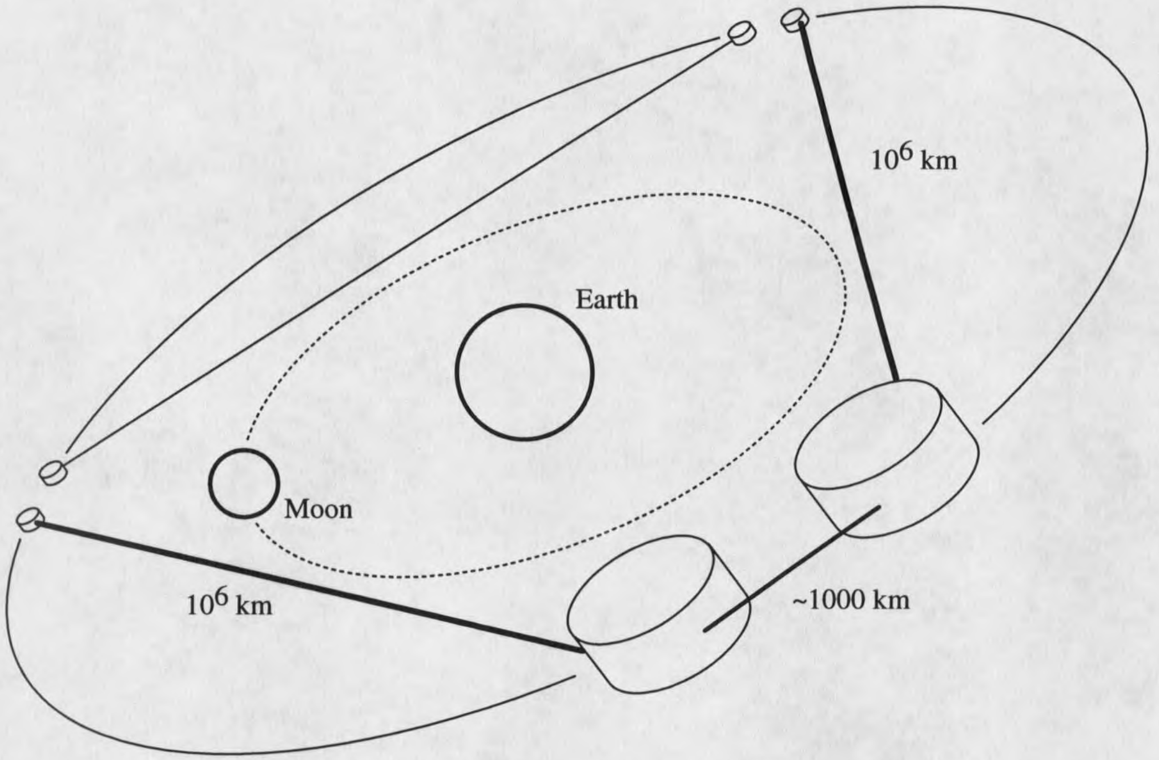

Figure 1.2: Schematic diagram of the proposed OMEGA space-based gravitational wave observatory.

Proposals have also been made to launch space-based gravitational wave observatories early in the next century. Possible designs for such an instrument include OMEGA (Orbiting Medium Explorer for Gravitational Astrophysics) [7], shown in Figure 1.2, and LISA (Laser Interferometer Space Antenna), shown in Figure 3.1 of Chapter 3 [8]. These observatories consist of a constellation of three to six spacecraft carrying lasers which allow them to be linked to form a Michelson type laser interferometer with baselines on the order of 10^9 m. These long baselines cause spaceborne interferometers to be most sensitive in the low frequency (10^{-5} to 1 Hz) gravitational waveband.

The current experimental efforts to detect gravitational waves are partly driven by the fact that gravitational radiation presents us with a fundamentally new way of observing the Universe. It is widely believed that the advent of gravitational wave astronomy will produce a revolution in our view of the Universe, similar to the revolution electromagnetic astronomy experienced when observations in the radio waveband began in the mid-1900's [9].

Whether a revolution in astronomy ensues or not, the first detections of gravitational radiation will immediately provide us with two new and interesting opportunities to study the workings of the Cosmos.

First, because gravitational waves interact very weakly with matter (a fact which makes them hard to detect), they will propagate nearly freely out of regions of space which cannot be electromagnetically imaged. In these regions, dense matter absorbs and scatters photons, continually reprocessing them so they never actually reach distant observers (*e.g.*, this occurs in the centers of galaxies, or at the center of a collapsing star). Observation of gravitational radiation could provide information about the dynamics and structure of these regions which was previously unobtainable.

Second, studies of the gravitational radiation bathing the Earth will give us the first opportunity to study general relativity in the so-called "radiative regime." It is common practice in electromagnetic field theory to study wave solutions in regions far from the source, where the waves essentially propagate freely. This region is conventionally called the *radiation zone*. This is a convenient regime to work in since the study of the physical properties of the waves are completely separated from the problems of wave generation. The generation and propagation of gravitational radiation is a considerably more complex problem to study than electromagnetic radiation because of the non-linear nature of general relativity. As was the case with electromagnetism, however, it is still convenient to define a series of zones which allow the study of gravitational waves independent of the sources [10].

The solar system lies well into the radiation zone of the strongest sources of gravitational radiation. For gravitational waves propagating through the vicinity of our detectors, the spacetime is reasonably flat, allowing study of the gravitational waves independently of the background curvature in the solar system. The detection and study of gravitational radiation under these conditions will provide the first direct experimental access to the radiative regime of general relativity. This is an important region to study gravitational radiation as it provides new tests of general relativity. Almost all metric theories of gravity predict the existence of gravitational waves [11]. Observations of gravitational radiation should make it possible to discern which theories are correct (*e.g.*, general relativity predicts two polarization states for gravitational radiation, whereas all other viable metric theories predict more than two; in the most general theory, six are predicted [12, 13]).

The remainder of this chapter develops a simple physical picture of gravitational waves. In Chapter 2, a formal description of gravitational waves is derived in the linearized approximation to general relativity. The polarization states of the waves are described, and the effects of the waves on the motion of test masses is described. Simple methods of detecting gravitational waves are also reviewed, and the formalism for describing the strength of gravitational waves from astrophysical sources is described.

Chapters 3 and 4 detail the original work which is presented in this dissertation. In Chapter 3 the *exact* response of an interferometric gravitational wave detector is used to construct the sensitivity curve for a space-based gravitational wave observatory. Previous analyses of the sensitivity of spaceborne detectors have used approximations to describe the response of the observatory to incident gravitational radiation; the results derived in this chapter are the first to evaluate the response without utilizing any approximations. Chapter 4 describes a new experiment to measure the mass of the graviton by considering the correlation between the optical and gravitational wave signals of an interacting binary white dwarf system. Assuming no net phase difference between the optical and gravitational wave signals is observed, new bounds on the graviton mass will be derived from the predicted experimental uncertainties in space-based gravitational wave measurements.

In Chapter 5 conclusions and speculations on future work in this field are suggested.

1.2 Gravity and General Relativity

The theory of general relativity is embodied in the Einstein field equations,

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \ , \tag{1.1}$$

where $G_{\mu\nu}$ is the Einstein tensor, and $T_{\mu\nu}$ is the stress-energy tensor. The Einstein tensor describes the curvature of the spacetime, and the stress-energy tensor describes the matter (and energy) content of the spacetime.

The basic premise of general relativity is that there is no "gravitational force" which acts on test particles[1]. From Einstein's viewpoint, what Newton perceived to be a gravitational force is the curvature of spacetime affecting the trajectories of particles. The deflection of a particle's path by a gravitational potential is a manifestation of mass creating curvature in the spacetime, and the particle following its straight-line[2] trajectory through the curved spacetime near the gravitating body. Particles are "freely falling" on the curved background of spacetime itself, and spacetime is curved by the presence of mass. This concept is readily embodied in the famous quote, "space tells matter how to move, matter tells space how to curve" [1].

A heuristic picture of these concepts can be imagined using a device called an embedding diagram, as shown in Figure 1.3. An embedding diagram represents the geometry of a lower dimensional curved space in a higher dimensional flat space. Let us consider the case where the curved space of interest is a two dimensional elastic sheet embedded in three dimensions. Placing a massive object on the sheet (*e.g.*, a heavy bearing, placed at point A as shown) *deforms* the shape of the sheet, producing some curvature. More massive objects (*e.g.*, a bowling ball, placed at point

[1]Extended test bodies will feel gravitational forces due to the relative acceleration between different parts of the object (tidal forces).

[2]These paths are straight in the sense that the 4–acceleration of a particle on such a trajectory is zero: $a^\mu = d^2x^\mu/d\tau^2 = 0$.

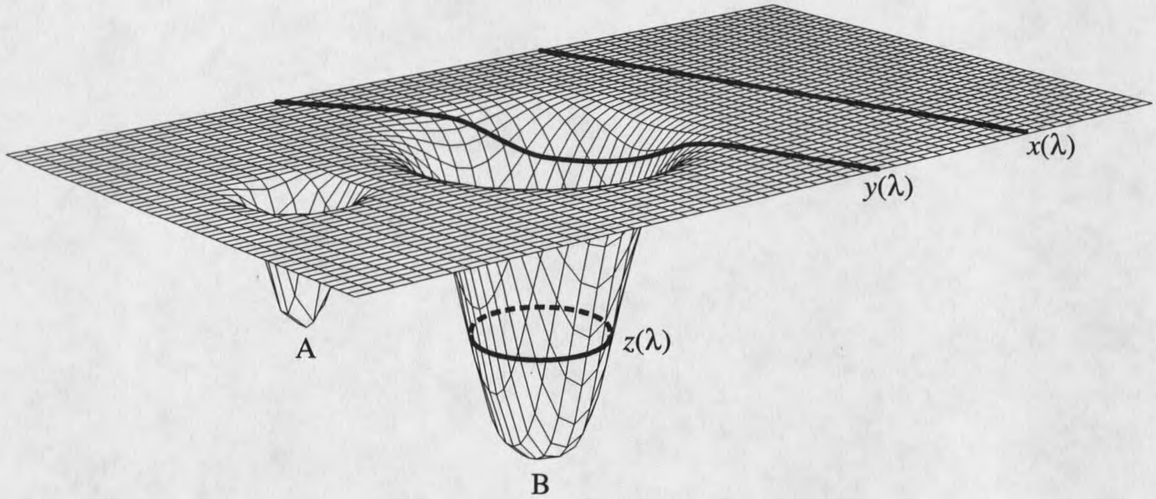

Figure 1.3: Embedding diagram showing a two dimensional surface *embedded* in a three dimensional space, making the geometry of the 2D surface readily apparent.

B as shown) produce a more serious deformation, steeply curving the sheet.

Imagine taking a ping-pong ball and rolling it across this curved landscape. The mass of the ping-pong ball will deform the sheet to some extent, but this will be much less than the deformation produced by the other objects, and one can safely ignore its contribution to the overall shape of the sheet. The ball is free to roll across the sheet, but is constrained to remain on the surface. Assume that no forces act on the ball once it is in motion (*e.g.*, there are no dissipative forces slowing the ball down)[3]. In the wide, flat regions of the sheet, far away from the masses, the ping-pong ball will roll across the sheet, tracing out a straight-line path such as $x(\lambda)$. This is the

[3]This experiment is being conducted at the surface of the Earth; gravity deforms the elastic sheet by pulling on the heavy objects, and keeps the ping-pong ball on the surface of the sheet.