Optimal use of ground and surface water in the Gallatin Valley, Montana
by Kenneth Boyd Young

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Agricultural Economics
Montana State University
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Abstract:
Intraseasonal interaction of the ground and surface water system figures prominently in this irrigation-planning study of a river basin. A model for optimizing use of variable streamflows and supplemental groundwater reserves is developed that simultaneously evaluates economic interaction occurring within the ground and surface water system during four discrete periods of the irrigation season.

Development of the model encompasses three stages. First, the hydrologic system is analyzed and groundwater discharge is functionally related to water table height in eight hydrologic subareas over four time periods. Second, a nonstochastic model is developed to allocate intraseasonal surface supply and utilize supplemental groundwater subject to interspatial-intertemporal groundwater interactions determined by water table balances in each period. This model is used to compute well investment levels associated with assumed fixed surface supplies and different distribution efficiencies.

Third, the probability distribution of annual streamflow is evaluated and the optimal level of well investment (within the class of well developments computed above) is determined for each field efficiency condition with stochastic surface input.

Post-optimal analysis, including sensitivity analysis of variable pumping costs, well investment costs, tolerable water table heights, and crop allotment restrictions, is also applied.

Empirical results of the model indicated that with optimal surface water use, development of groundwater reserves for irrigation supply in the Gallatin Valley would increase average net annual irrigated income by $8 per acre or 18.2 percent on the 124,416 acres in the study area. If field efficiency can be raised from the present level to the recommended level, a comparable increase in irrigated income may be attained without groundwater development. Conjunctive use of ground and surface water together with increased field efficiency in irrigation would result in a potential annual income gain of $13 per acre of 30 percent overall.

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A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

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ABSTRACT

Intraseasonal interaction of the ground and surface water system figures prominently in this irrigation-planning study of a river basin. A model for optimizing use of variable streamflows and supplemental groundwater reserves is developed that simultaneously evaluates economic interaction occurring within the ground and surface water system during four discrete periods of the irrigation season.

Development of the model encompasses three stages. First, the hydrologic system is analyzed and groundwater discharge is functionally related to water table height in eight hydrologic subareas over four time periods. Second, a nonstochastic model is developed to allocate intraseasonal surface supply and utilize supplemental groundwater subject to interspatial-intertemporal groundwater interactions determined by water table balances in each period. This model is used to compute well investment levels associated with assumed fixed surface supplies and different distribution efficiencies. Third, the probability distribution of annual streamflow is evaluated and the optimal level of well investment (within the class of well developments computed above) is determined for each field efficiency condition with stochastic surface input.

Post-optimal analysis, including sensitivity analysis of variable pumping costs, well investment costs, tolerable water table heights, and crop allotment restrictions, is also applied.

Empirical results of the model indicated that with optimal surface water use, development of groundwater reserves for irrigation supply in the Gallatin Valley would increase average net annual irrigated income by $8 per acre or 18.2 percent on the 124,416 acres in the study area. If field efficiency can be raised from the present level to the recommended level, a comparable increase in irrigated income may be attained without groundwater development. Conjunctive use of ground and surface water together with increased field efficiency in irrigation would result in a potential annual income gain of $13 per acre of 30 percent overall.

CHAPTER I

INTRODUCTION

In many irrigation projects, the most limiting factor affecting productivity is a high water table resulting from a basic disequilibrium in the hydrologic system. The damage caused by excessive water tables has become a serious problem for irrigated agriculture in many parts of the world. In West Pakistan, for example, some 26 million acres of irrigated cropland has been damaged by waterlogging and salting, and the spread of salinity is continuing to force many more thousands of acres out of production every year. 1/

The full significance of this problem is often underestimated at the outset of irrigation development as the damage may not be evident until several years later. Accumulation of damage in the case of gradual soil salinization or loss of permeability has, generally, a very long time horizon. Damage is more easily assessed when farm income is directly affected by crop bogging or interference with harvesting.

In addition to artificial drainage, the most common method for controlling water tables, there are often several other strategies available for control of this problem. For example, the distribution system can be designed and water supply allocated so that percolation losses are minimized in the areas of the project that are most

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1/ D.E. Naylor, "Control of Waterlogging and Salinity in West Pakistan," International Journal of Agrarian Affairs, Vol. IV, No. 1, October 1963, pp. 1-12.

susceptible to bogging. Also, where groundwater can be used to supplement surface water, the water table may be controlled by concentrating pumping on lower lands of the project.

In most cases, however, the economic potential of employing these alternative strategies is not appreciated until after all irrigation structures are completed and some experience is obtained in operation of the irrigation system. At this stage, artificial drainage may be the only practical choice left for relieving high water tables although less costly control could have been attained formerly with another strategy, e.g., by changing the layout of the distribution system.

Nature of the Investigation

The economic significance of high water tables to irrigated agriculture and some alternative strategies for controlling this problem have been discussed in general. An important implication to be derived from this discussion is that a more systematic approach in planning irrigation systems is needed where high water table conditions are likely to be encountered. In view of the drainage problems experienced in some completed projects, e.g., in West Pakistan, 2/ it would appear that more comprehensive planning models would be advantageous even if based on relatively sparse hydrologic information.

2/ Ibid.

A general equilibrium model including all hydrologic relationships offers several advantages over partial equilibrium methods for analyzing water-table problems. For example, the evaluation of alternative strategies for water-table control may be directly incorporated within a composite criterion function for overall optimal system organization. This is not possible in the partial equilibrium approach. Allocation of water flows and the location of irrigation wells within a system can be important strategies in water-table control, particularly for localized problem areas. The use of a composite criterion function as postulated above greatly facilitates analysis of these more complex strategies for controlling water tables.

The problem of surface water-groundwater interaction in a hydrologic system may be simply explained by Darcy's Law governing linear flow of fluids. According to Darcy's Law, groundwater velocity for laminar flow is proportional to the hydraulic gradient along the path of flow. 3/

The following are two examples of interaction which may be explained by Darcy's Law:

3/ S.N. Davis and R. J. DeWiest, Hydrogeology, John Wiley & Sons, Inc., New York, 1966, p. 156.

1) If groundwater is pumped adjacent to a stream, water loss from the stream will tend to increase in proportion to the change in the hydraulic gradient between the stream level and the adjacent aquifer's declining water table level; and

2) If the water table in the adjacent aquifer is higher than the stream level, e.g., from heavy irrigation buildup, the surface water supply provided by the stream may be expanded by increased groundwater outflow from the adjacent aquifer.

The two examples cited are very simple cases of possible interaction between surface and groundwater. The analysis becomes more complicated when a water table change in one part of an aquifer region triggers a chain reaction of interactions throughout the rest of the system. A simulation model for evaluating this latter problem is presented in Chapter III.

In addition to the above physical interactions, some important economic interactions also take place between the ground and surface water system. A major source of interaction is the economic alternative of using the aquifer as a distribution system for irrigation supply instead of surface canals. This method of distribution is feasible in the present study as most of the aquifer area is located under agricultural land. Advantages in distribution efficiency coupled with low pumping lift associated with high water table levels make groundwater a relatively cheap alternative source of

water supply in the Gallatin Valley. Reliability of groundwater supply is also assured because of the large volume currently in storage and the high annual rate of recharge to the aquifer in the Gallatin Valley.

It should be evident from the above discussion that high water-table levels are a primary cause of surface and groundwater interactions within a hydrologic system as well as a major threat to productivity of an irrigation project. This further justifies the use of a general equilibrium model for irrigation project planning which includes all hydrologic relationships of concern in optimal water allocation.

Purpose of the Study

This thesis involves the formulation, solution and evaluation of a general equilibrium model for conjunctive intraseasonal use of groundwater and surface water in the Gallatin Valley, Montana. The purpose of the model is to optimize the combined use of available surface and groundwater resources in the Gallatin Valley subject to specified intertemporal and interspatial hydrologic relationships that restrict the use of these resources. Since the major restriction on water use is the level of the water table, evaluation of water tables during the irrigation season enters prominently in

construction of the model. The identification and quantification of hydrologic relationships affecting water table balance are important features of the methodology followed in this study. The analysis in Chapter III of this study is entirely devoted to specification and quantification of hydrologic relationships.

Specific empirical objectives of the present study are to determine the economic benefits of groundwater development in the Gallatin Valley, the value of improved irrigation efficiency resulting from better farm practices, ditch consolidation and/or lining, and the relative importance of current surface water supply in the study region. Empirical objectives also include an evaluation of optimal cropping patterns in different subareas of the Valley under different water supply conditions and the optimal level of well development in each subarea. A further objective is to identify what institutional changes will be required to provide flexibility in water exchange and encourage cooperation among water users so that optimal irrigation development may be obtained in the Gallatin Valley.

Justification for the Study

The primary reason for undertaking a comprehensive water study of the Gallatin Valley is that this region provides excellent opportunity for a systems analysis of interrelated drainage and water scarcity problems within the region. The higher lands in the

Valley generally run short of surface water early in July each season whereas the lower lands with the early water rights tend to have an abundance of surface water supply during all of the irrigation season. In the lower part of the valley, once fertile lands are now bogged due to misuse of water resources in this area. 4/ It would appear that major increases in economic efficiency should result from reallocation of the valley's water resources.

In addition to the need for a quantitative economic study of the Gallatin Valley, the study was made feasible by the relative abundance of empirical data available on water use and crop production in this region. A detailed investigation of the hydrology in the Gallatin Valley had been completed by USGS, 5/ and several feasibility studies of proposed water storage projects had been completed by the U.S. Bureau of Reclamation. 6/ The availability of this type of data plus considerable local crop production data at the Montana State University Experiment Station near Bozeman was an important consideration in undertaking a study of this nature.

4/ H.E. Murdock, Irrigation and Drainage Problems in the Gallatin Valley, Mont. Agr. Expt. Sta. Bulletin 195, Bozeman, November 1926, p. 1.

5/ Hackett, et. al., Geology and Groundwater Resources of the Valley, Gallatin County, Montana, Geological Survey Water-Supply Paper 1482, U.S. Government Printing Office, Washington, D.C., 1960.

6/ United States Bureau of Reclamation, Report on Three Forks Division, Upper Missouri Project Office, Great Falls, Montana, 1958.

Statement of Hypotheses and Assumptions

The major hypothesis guiding this study is that a reallocation of surface water use in the Gallatin Valley, and in particular, development of groundwater pumping facilities for supplementing surface supply, would result in significant income gains and sizable improvements in efficient use of the Valley's natural resources. It is further hypothesized that the basic irrigation problem in the Gallatin Valley is resource management--making the right quantity of water available at the right place at the right time--as opposed to aggregate water shortages.

Important assumptions in the study which will not be tested are:

1) Evaluation of intertemporal and interspatial hydrologic relationships between surface and groundwater use will result in substantially more optimal use of these two resources in the Gallatin Valley than an optimization method which abstracts from this interaction between surface and groundwater use.

2) Groundwater depletion in the Gallatin Valley will not present a problem in this study as the probable annual withdrawal by pumping will not exceed annual recharge, on the average. Thus, the only effective storage function performed by the aquifer is an intra-seasonal one.

3) Irrigation use of surface water in the Gallatin Valley will have no perceptible effect on the volume or quality of water supply for downstream users located on the Missouri River.

4) Dewatering of streams adjacent to irrigated lands in the Gallatin Valley will have no significant economic effect on the fishery. 7/

Scope of the Study

Following a review of the literature in the subsequent chapter, considerable attention is devoted to the development of a simple hydrologic model for approximating groundwater discharge from eight subareas of the Gallatin Valley as a linear function of water table heights in these subareas. All discharge relationships regarding groundwater flows across subarea aquifer boundaries and interactions with stream flow are determined from historical hydrologic data on the Gallatin Valley utilizing fundamental principles of hydrology. The linear model for predicting groundwater discharge in the system is tested for empirical validity by comparing estimated groundwater discharge from subareas adjacent to the stream with

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7/ This assumption could not be tested in this study as the value of the fishery is not known. The shadow prices associated with stream flow constraints in the model do provide a cost estimate of maintaining minimum stream flow levels for the fishery.

measured stream flow gains attributed to groundwater discharge. All subsequent groundwater discharge in the model is assumed to have the same linear relationship to other water table heights as for the particular class of water tables used in testing this model.

A theoretical general equilibrium model for irrigation optimization is formulated in Chapter IV with a nonlinear objective function and linear constraints largely determined from the hydrologic analysis in Chapter III. Evaluation of the relative economic importance of intertemporal and interspatial hydrologic relationships between ground and surface water use will constitute an important consideration in this theoretical model. Kuhn-Tucker theory is utilized to derive economic implications of the probable effects of surface-groundwater interactions which would result under different water supply conditions in the model. These theoretical interpretations provide a valuable guide for subsequent interpretation of the empirical results and sensitivity analysis of the empirical model.

Attention in Chapter V is mainly devoted to assembly of empirical data on water supply, crop water response and production cost for use in a linear programming model. A further methodological consideration is to utilize the soil reservoir for water storage during different time periods of the irrigation season for transferring water from high streamflow periods to periods later in the

season when surface water is in relatively short supply. This particular strategy is of value in the Gallatin Valley as the volume of stream flow normally tends to recede as the irrigation season progresses. Also, storage capacity of the soil reservoir is relatively large compared with average monthly crop water requirements. Different soil water carryover levels are considered in defining various linear programming activities presented in Chapter V.

Formulation of the empirical linear programming model presented in Chapter VI is similar to the former theoretical model but simplified for ease in computation and accommodation of data. Measurement of net welfare in the applied criterion function is accomplished by delineating various segments of the concave theoretical objective function and defining separate activities of a linear program to be associated with these segments. Essentially, the same hydrologic constraints are retained as they are already linear in form. Additional constraints are also required for cropland acreage, wheat allotment restrictions, and crop rotation limitations in the eight subareas. The completed linear programming model contains 209 constraints, 458 activities, and 2,014 matrix entries.

The linear programming model is first analyzed as a deterministic model using the algorithm of parametric programming to evaluate 16 optimal levels of well development associated with eight different surface water supplies and two different distribution

efficiencies for water supply. Probability of these surface water supplies is determined by fitting a gamma function to historical interseasonal streamflows. Intraseasonal flow in each of four irrigation time periods during each season is assumed to be a constant proportion of total annual flow. Expected value and standard deviation of net income are computed for each of several well development levels. These two statistics for net income in the Gallatin Valley are evaluated under various hypothesized situations and are the main quantitative measures from the economic analysis.

The two distribution efficiencies represent estimated present low field efficiency in the study area and a recommended higher field efficiency level that may potentially be adopted under a changed institutional system allowing more flexibility in water exchange. Parametric changes in distribution efficiency provide estimates of the value of ditch consolidation in the study region as well as an estimate of benefits emanating from improvements in on-farm irrigation efficiency.

CHAPTER II

IRRIGATION DEVELOPMENT IN THE GALLATIN VALLEY

A Literature Review

This chapter is divided into three major parts. Part I is a brief review of the historical development of irrigation in this region. Part II is a more exhaustive review of previous studies by government agencies and researchers at Montana State University on proposed irrigation development of the Gallatin Valley. Part III is a survey of some of the methodology employed and results obtained in other area studies involving related problems.

Part I. Historical Development

Irrigation ditches were first excavated during 1864 in the Gallatin Valley. 1/ As the area became more populated, new settlers continued to add on more ditches and appropriate water rights in the local streams with little regard to the overall benefit of the irrigation system. The outcome of this ad hoc piece-meal development policy is a wasteful distribution system and an over-appropriated surface water supply. Many of these ditches still in use parallel each other and have other construction defects. Severe water shortages were reported as early as 1919. 2/

1/ Water Resources Survey, Gallatin County, State Engineer's Office, Helena, Montana, January 1953, p. 6.

2/ Murdock, H.E., Irrigation and Drainage Problems in the Gallatin Valley, Agr. Expt. Sta., Montana State University, Bulletin 195, Bozeman, November 1926.

Irrigation management has also been inefficient in the Gallatin Valley. This is in part associated with early use of the western appropriation doctrine in the acquisition of water rights. 3/ Management was considered to be one of the key problems in 1953 in the Gallatin Valley by the State Engineer: 4/

> Poor irrigation practices on the part of some users is causing a waste of water and depriving other land of its use. The waste of water on higher areas, and seepage from the too numerous ditches is causing some land in the valley bottoms to become water-logged. This once productive land is now of no use except as pasture, some of it growing nothing more than swamp grass.

Under the institutionally-fixed ownership of water rights, little incentive exists for users with early surface rights to be efficient in their use of this resource or to take proper care of their ditches. Some, in fact, deliberately use excess water during heavy run-off periods to build up their water tables for sub-irrigation in other periods. This aggravates the bogging problem on lower lands.

This cursive review on historical irrigation development in the Gallatin Valley is intended to highlight current problems in the area and to explain their evolvement. What are some of the past proposals for solving these problems?

13/ Gopalakrishnan, C., The Economics of Water Transfer: An Institutional Appraisal, Ph.D. Thesis, Montana State University, Bozeman, 1967.

14/ Water Resources Survey, op. cit., p. 14.

Part II: Proposals for Development

A severe drought in 1919 provided impetus for an early extensive investigation of irrigation and drainage problems in the Gallatin Valley. 5/ Murdock's recommendations for relieving the water shortage problem were: (1) build storage reservoirs in the mountains; (2) line the irrigation canals; (3) change the irrigation system; and (4) drain the seeped lands and use this water for irrigation. These proposals were investigated in more depth in later studies and will be evaluated in turn along with other possible alternatives.

Surface Storage

A series of studies were begun in 1938 by the Bureau of Reclamation: (1) on the feasibility of diverting water from Hebgen Reservoir on the Upper Madison River to the Gallatin Valley; 6/ and (2) constructing a dam on the West Gallatin River below Spanish Creek for storing irrigation water. 7/ The latter alternative was later found to be more practical and the Hebgen diversion project was abandoned. Four alternate storage schemes at the Spanish Creek site were compared; the most optimal plan having a benefit-cost ratio of 1.52. In the course of this investigation, the Bureau also evaluated

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5/ Murdock, op. cit.

6/ Senate Document No. 191, Missouri River Basin, 1944.

7/ U.S. Department of the Interior, Report on Three Forks Division, Bureau of Reclamation, Upper Missouri Project Office, Great Falls, Montana, Appendix M.

a groundwater pumping plan. This had a benefit-cost ratio of 2.41. Consequently, the Bureau recommended that the groundwater plan be adopted in place of a surface water storage project.

A storage reservoir with 8,000 acre-feet capacity was constructed in 1948 on Middle Creek, a Gallatin River tributary, by the Montana State Water Board.

Ditch Consolidation and Lining

Murdock's recommendations for ditch lining and consolidation have not been pursued further or promoted any related construction to the writer's knowledge. This may be a promising area for further research.

Drainage

Subsurface drainage of bogged lands in Central Park and Belgrade subareas has been investigated by the United States Soil Conservation Service. 8/ SCS recommended that deep-interception drains be constructed at one-mile intervals in these subareas and that provision be made for one-half mile spacing should shorter spacing be necessary.

Artificial drainage of this scale is an expensive undertaking. Since the bogging problem has been attributed to apparent gross

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8/ U.S. Soil Conservation Service, Preliminary Examination Report on Water Supply and Distribution Investigation, Gallatin Valley Area, Gallatin County, 1948 and Survey Report on Central Park Drainage Project, Three Rivers Soil Conservation District in Gallatin County, Montana, 1950.

inefficiencies in surface water distribution and management, correction of these inefficiencies may alleviate much of the problem. At least this matter could be investigated before drainage commitments are made in the future. The present study will be concerned with effects of groundwater pumping and variation in surface water distribution in the valley on bogged areas.

Use of Groundwater

The Bureau of Reclamation plan to pump groundwater in the Gallatin Valley has been investigated further by McConnen and Mennon, 9/ Sammons, 10/ and Boyd. 11/

The Bureau proposed to pump about 92,300 acre-feet annually from 193 wells overlying the aquifer region into established ditches now diverting from West and East Gallatin Rivers. The pumped water would be used to replace surface water reallocated to other water-deficit lands outside the aquifer region.

9/ McConnen, R.J. and G.M. Mennon, Planning the Integrated Use of Ground and Surface Water: A Linear Programming Study of the Gallatin Valley, Montana, Mont. Agr. Expt. Sta. Bul. 616, Bozeman, 1967.

10/ Sammons, R.W., Irrigation Development: Institutional Blocks to Ground-Surface Water Integration in the Gallatin Valley, Montana, Ph.D. Thesis, Montana State University, Bozeman, 1964.

11/ Boyd, D.W., Simulation Via Time-Partitioned Linear Programming: A Ground and Surface Water Allocation Model for the Gallatin Valley of Montana, Report No. 10, Montana Water Resources Center, Bozeman, June 1968.

McConnen and Mennon used a linear programming model to evaluate net benefits of transferring surface water from surplus to deficit areas and replacing this water with pumped water in ditches as proposed by the Bureau. They estimated that net annual farm income under assumed Cooperative District organization in the Gallatin Valley would increase by $210,738 with the use of 190 wells.

Sammons did a case study of Highline Canal in the Gallatin System and investigated the cost of supplementing canal flows by pumping during the period July 15 to September 1 when surface water supplies are generally short. He estimated that 25 wells would be needed to meet the supplementary requirements of 7,875 acre-feet at a cost per season of $32,878, or $4.17 per acre-foot. He also analyzed institutional blocks to groundwater integration in the area. A recommendation for institutional change was; 12/

> A model of an institutional organization that would allow farmers to exchange surface water for groundwater without endangering their present water rights is needed. This institutional structure would delineate legal responsibilities for both parties during its operation and dissolution.

Boyd used linear programming to allocate water among competing uses in the Gallatin Valley: agricultural, municipal, industrial and recreational; and between surface and sub-surface storage. Adjustments were made in parameters of the model to simulate stream flow over 30 years of record during the testing phase. Pumping was

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12/ Sammons, op. cit., p. 103.

employed to supplement surface water at a cost of $2 per acre-foot. Boyd did not analyze effects of pumping on the water table in different subareas or the optimal allocation of water among the different subareas of the valley.

This perusal of previous studies on proposed irrigation development suggests that further investigation is needed on the following issues before an overall optimal development plan can be implemented in the Gallatin Valley:

1) What is the optimal allocation of ground and surface water during the irrigation season among various subareas of the valley having different access to these resources?

2) How is the optimal allocation of ground and surface water use affected by the presence of high water table conditions in different subareas? Can bogging be controlled by utilizing wells for irrigation of low lands and restricting the use of surface water?

3) What value would ditch consolidation and lining have for conserving use of water in the system and alleviating the bogging problem?

4) What institutional changes are needed to permit comprehensive reallocation of water supplies, both ground and surface, within the valley to conform with results of an optimization model?

Answers are required to these important questions before optimal use can be made of water resources in the Gallatin Valley.

The following literature review is of selected studies where related questions have been investigated in other subareas. Methodology employed and results obtained in these studies will furnish valuable guidance for the present study.

Part III: A Survey of Selected Studies

Water Resource Allocation

A sizable literature has evolved on the use of allocation models and alternate methods of systems analysis in water resource studies during the past 15 years.

The Harvard Water Resources Group discusses a number of systems analysis techniques applied to allocation problems. 13/ Recent favored allocation models are inventory models and dynamic programming first used in water resource studies by Masse' and Little. 14/ An example of an inventory model applied in optimal use of a reservoir with stochastic input is a 1963 Israeli study. 15/

13/ Maass, A.M., R. Hufschmidt, H.A. Dorfman, Thomas S. Marglin, Jr. and G. Fair, Design of Water Resource Systems (Cambridge, Mass.: Harvard University Press, 1962).

14/ The idea of using dynamic programming to determine temporal allocation of water from a reservoir is attributed to Pierre Masse', Rept. to the Societe" de Statistique de Paris, Berger-Levrault, Paris, June 21, 1944. The first known application of the model in water research is reported in J.D.C. Little, "The Use of Storage Water in a Hydroelectric System," J. Operations Research Soc. of America, Vol. 3, May 1955.

15/ Avi-Itzak, B., and S. Ben-Tuvia, "A Problem of Optimizing a Collecting Reservoir System," Operations Research, Vol. II, No. 1, 1963, p. 122.

Allocation studies on conjunctive use of ground and surface water have been done by Burt, 16/17/ Leonard, 18/ Dracup, 19/ Aron, 20/ and Buras. 21/ Buras used dynamic programming in an engineering study to derive operating rules which determine the amounts of water to be allocated from a surface reservoir and a groundwater aquifer to several irrigation uses and to recharge. Burt also used this tool to derive decision rules for surface and groundwater use in an economic study. In another study, Burt considers optimal use of

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16/ Burt, O.R., Economics of Conjunctive Use of Ground and Surface Water, Ph.D. Thesis, University of California, Berkeley, 1962.

17/ Burt, O.R., "Economics of Conjunctive Use of Ground and Surface Water," Hilgardia, Journal of Agricultural Science, University of California, Vol. 36, No. 2, December 1964.

18/ Leonard, R.L., Integrated Management of Ground and Surface Water in Relation to Water Importation: The Experience of Los Angeles County, Ph.D. Thesis, University of California, Berkeley, 1963 (unpublished).

19/ Dracup, J.A., The Optimum Use of Groundwater and Surface Water System: A Parametric Linear Programming Approach, Water Resources Center, University of California, Berkeley, Report 6-24, July 1, 1966.

20/ Aron, G., Optimization of Conjunctively Managed Surface and Groundwater Resources by Dynamic Programming, Water Resources Center, University of California, Davis, Project No. W132, June 1969.

21/ Buras, N., "Conjunctive Operation of Dams and Aquifers," Journal of the Hydraulics Division, American Society of Civil Engineers, Vol. 89, No. HY6, November 1963, pp. 111-131. There are some important distinctions between the study by Buras and the above studies by Burt concerning the evaluation of groundwater storage and the use of recharge under declining aquifer conditions. Discussion of these differences is omitted as the problem of groundwater depletion will not be an issue in the present study.

a single resource, groundwater, which may be in fixed supply o partially renewable. By using the recursion relationship which results from application of Bellman's "Principle of Optimality," Burt derives approximate decision rules for determining groundwater use as a function of current supply. 22/

In general, these allocation studies tend to have a long-run planning horizon and treat surface and groundwater as independent physical resources related only through their joint contribution to economic output. Under high water table conditions, as found in the Gallatin Valley, surface supplies are not independent of groundwater use during irrigation periods. Evaluation of these dependencies in an intra-seasonal optimization model should serve to distinguish the present study from most former allocation studies.

A recent Harvard study by Rogers and Smith 23/ includes an evaluation of water table balance for a single irrigated area in East Pakistan. It is assumed in this study that part of the diverted surface water is lost to groundwater recharge, nonbeneficial evapotranspiration, and surface runoff in each decision period. The remainder is

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22/ Burt, O.R., "Optimal Resource Use Over Time with an Application to Groundwater," Management Science, Vol. 11, No. 1, September 1964, pp. 80-93.

23/ Rogers, Peter, and D.V. Smith, "The Integrated Use of Ground and Surface Water in Irrigation Project Planning," American Journal of Agricultural Economics, Vol. 52, No. 1, February 1970, pp. 13-24.

employed for crop use. It is noted that tube wells are utilized for both supplemental irrigation supply and removing excess subsurface water. The model applied is assumed to be in a steady state thus simplifying the computation of fixed investment in canal facilities and wells.

A primary distinction between the above Harvard study and the current study is that stochastic variation in interseasonal surface water supply is an important consideration in determining the optimal level of well investment. Allowances are also made for the effects of intraseasonal water shortages upon crop response. All water table levels in eight subareas of the present model are evaluated explicitly in each decision period and all groundwater movement related to water table conditions is simulated.

An intraseasonal irrigation planning study involving allocation of limited surface water among competing irrigated crops in different time periods of the irrigation season was recently completed by Anderson, 24/ also reported by Anderson and Maass. 25/ Anderson

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24/ Anderson, R.L., "A Simulation Program to Establish Optimum Crop Patterns on Irrigated Farms Based on Preseason Estimates of Water Supply," American Journal of Agricultural Economics, Vol. 50, No. 5, December 1968, pp. 1586-1590.

25/ Anderson, R.L., and A. Maass, "A Simulation Technique to Estimate Crop Production of Irrigation Projects, Based on Crop Response to Varying Schedules of Irrigation Water," International Commission on Irrigation and Drainage, R34, Question 23, pp. 547-558.

used a computer simulation program to allocate water supply initially among farms and then among crops on each farm during two-week intervals of the crop-growing season. Plant response to water supply in each period was estimated from evapotranspiration studies. Irrigation timing is also an important consideration in the current study.

Water Relationships

The literature on intraseasonal surface and groundwater relationships is relatively sparse. Some engineering studies have appeared on groundwater basin behavior, e.g., Tyson and Weber. 26/

Tyson and Weber simulated groundwater flows in an aquifer region divided into "polygonal zones" using an electric analog model. Equations for continuity and Darcy's Law were used in estimating flow relationships. The aquifer was not known to interact with surface water streams as is the case in the Gallatin Valley.

Ditch Consolidation Studies

A recent feasibility study on ditch consolidation was completed by Huszar 27/ in Colorado which has application to the present study.

26/ Tyson, H.N., Jr., and E.M. Weber, "Groundwater Management for the Nation's Future--Computer Simulation of Groundwater Basins," Journal of The Hydraulics Division, Proceedings ASCE, Vol. 90, No. HY4, July 1964, pp. 59-77.

27/ Huszar, P.C., Economics of Irrigation System Consolidation, M.S. Thesis, Colorado State University, Fort Collins, March 1969.

Huszar evaluated the expected benefits of a local irrigation system (upper system) and the expected loss to downstream users (lower system) resulting from proposed consolidation in the upper system. Consolidation was found to be infeasible because of the reduction in important return flows to the lower system.

Institutional Studies

Institutional problems of ground-surface water transfers have been investigated by Hartman, 28/ Hartman and Seastone, 29/30/31/ Bittinger, 32/ Smith, 33/ and Snyder. 34/

28/ Hartman, L.M., "Economics and Groundwater Development," Groundwater, Vol. 3, No. 2, April 1965.

29/ Hartman, L.M. and D.A. Seastone, "Welfare Goals and Organization of Decision-Making for the Allocation of Water Resources," Land Economics, Vol. XLI, No. 1, February 1965, pp. 21-30.

30/ Seastone, D.A. and L.M. Hartman, "Resource Transfers and Economic Externalities in the Public Sector," Proceedings of the Fifty-Eighth National Tax Conference, New Orleans, Louisiana, November 8-12, 1965.

31/ Seastone, D.A. and L.M. Hartman, "Alternative Institutions for Water Transfers: The Experience in Colorado and New Mexico," Land Economics, Vol. XXXIX, No. 1, February 1963, pp. 32-43.

32/ Bittinger, M.W., "The Problem of Integrating Groundwater and Surface Water Use," Groundwater, Vol. 2, No. 3, 1964.

33/ Smith, S.C., "Problems in the Use of the Public District for Groundwater Management," Land Economics, August 1956.

34/ Snyder, J.H., Economic Implications and Appraisal of the Court Reference Procedure for Allocating Groundwater, Committee on the Economics of Water Resource Development, Report No. 5, 1957.

Hartman justifies public intervention when private interest decisions regarding resource development and use have external effects or repercussions upon other parties. 35/ He states that intervention is necessary to ensure that welfare gains are balanced against losses. This welfare criterion is applicable to both the bogging and water shortage problem in the Gallatin Valley as they both result from the waste of surface water by certain users.

Seastone and Hartman 36/ suggest that flexibility in water transfer may be accomplished by: (1) organization systems such as conservancy districts and ditch companies; (2) public policy devices such as eminent domain, public ownership and condemnation; (3) private purchase and sale transactions; and (4) through some combination of these institutional arrangements. These alternate methods will be evaluated in respect to their applicability to the Gallatin Valley after an optimal irrigation policy is estimated.

35/ Hartman, op. cit., p. 2.

36/ Seastone and Hartman, "Alternative Institutions for Water Transfer," p. 31.

CHAPTER III

HYDROLOGIC RELATIONSHIPS OF THE MODEL

The present chapter is basically concerned with the estimation of hydrologic relationships among subareas in the study region. These estimates are needed for simulating the operation of existing irrigation ditches in the subareas, and for allocating diverted irrigation water to different consumptive uses, to underground percolation and to surface water reuse in the system. Groundwater movement is an important factor in this analysis since irrigation water lost by underground percolation may have beneficial reuse in the system if it reappears as surface water, or it may cause crop damage through excessive watering of low-lying and poorly drained land in certain subareas.

This chapter is divided into four stages of analysis:

Part I is a general physical description of the Gallatin Valley and of polygonal subareas into which the study area in the Gallatin Valley was partitioned.

Part II involves the development of a hydrologic model for relating groundwater flow to water-table height conditions in different subareas. These estimated flow relationships and computed coefficients are specified in constraints of the system optimization model to follow in a later chapter of this study.

Part III evaluates the efficiency of the present irrigation system and the allocation of diverted surface water to different purposes.

Part IV is concerned with the validity of estimates obtained with the hydrologic model.

Part I: Description of Study Area

The Gallatin Valley

The Gallatin Valley floor is about 25 miles long and 20 miles wide; encompassing an approximate area of 540 square miles. Altitude ranges from 5,400 feet on the south end to 4,100 feet on the north with an average fall of 40 feet per mile along the valley floor. 1/ In prehistoric time, the area was largely covered by a lake. There still exists a subterranean dam or barrier near the head of the Missouri River which holds back underground water, keeping the water table high.2/

Crops Grown.--A number of different irrigated crops are grown in the Gallatin Valley, comprising most of the production for all of Gallatin County. In 1967 there were 2,900 acres of oats; 300 acres of corn; 754 acres of potatoes; 2,800 acres of spring wheat (other than durum); 7,300 acres of winter wheat; 11,100 acres of barley; and 45,300

1/ Hackett, et. al., Geology and Groundwater Resources of the Gallatin Valley, Gallatin County, Montana, Geological Survey Water-Supply Paper 1482, U.S. Government Printing Office, Washington, D.C., 1960, pp. 12-13.

2/ Water Resources Survey, p. 13.

acres of hay (all types) recorded for Gallatin County. 3/ Wheat, barley, and hay are, currently, the most important irrigated crops in the valley.

Soils.--Soils in the Gallatin Basin are derived chiefly from Tertiary Age sediments which have been modified over time by wind and stream action. 4/ While soils are quite varied over the area, they are considered to be equally productive under irrigation. 5/ However, their efficiency in water use is affected by structural differences. This efficiency factor will be considered in Part III of this chapter after subareas have been delineated.

Water Resources.--Surface water drainage streams in Gallatin County are shown in Figure 3-1. The study area is also delineated on this map.

Records of West Gallatin River runoff measured at Gallatin Gateway, which date back to 1926, indicate a low flow in the 1934 water year of 295,700 acre-feet, a high in the 1952 water year of 714,600 acre-feet and a 1926-1952 average of 520,000 acre-feet annual flow.

3/ Montana Agricultural Statistics, Montana Department of Agriculture and U.S. Department of Agriculture, Helena, Montana, Vol. XII, December 1968. Approximately 1,000 acres of potatoes were grown in 1970 near Amsterdam mainly for seed. Gross income from this crop in 1970 was nearly $500,000. This crop is not analyzed in the present study because of the limited acreage involved.

4/ Water Resources Survey, p. 11.

5/ Ibid.

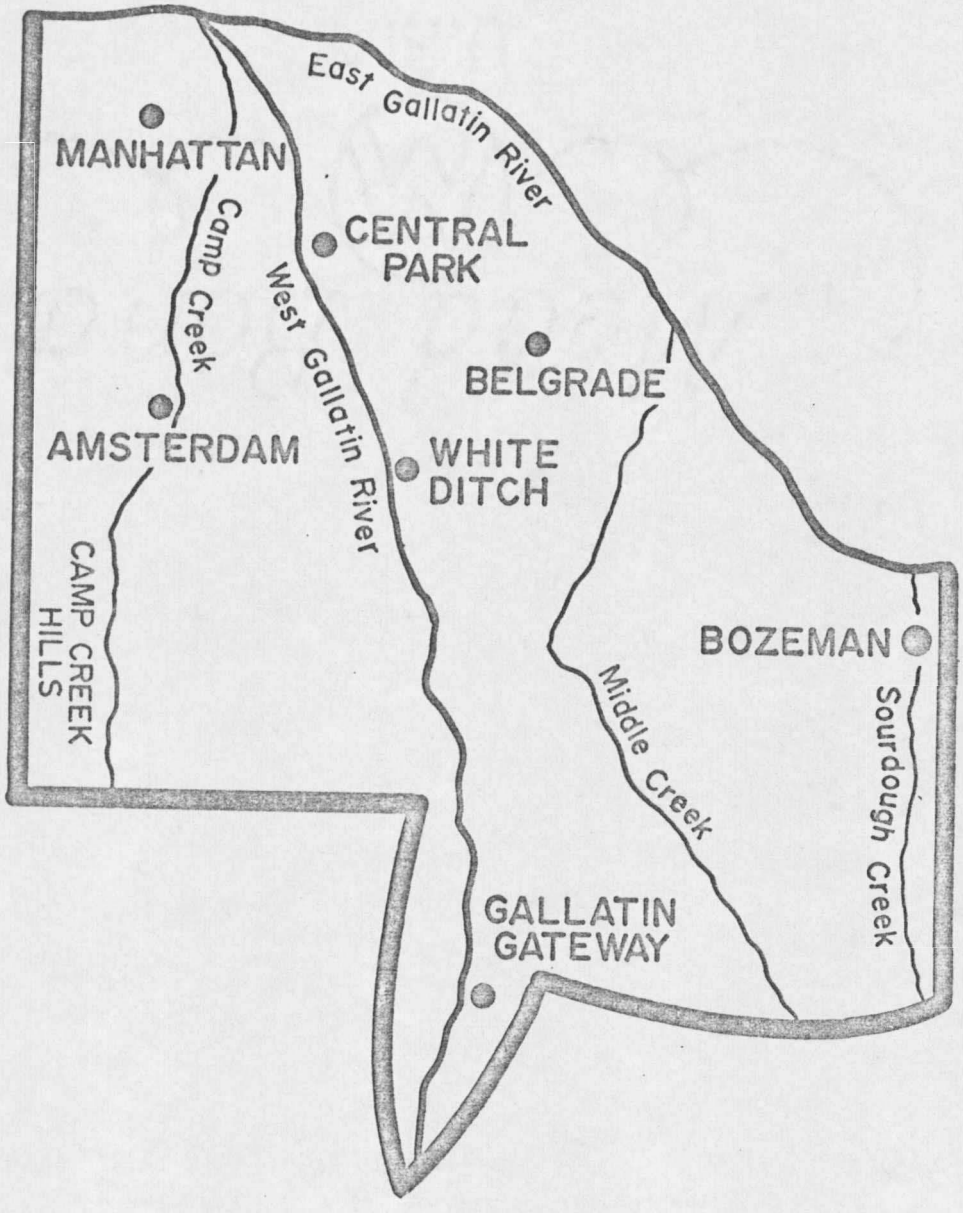

Figure 3-1. Map of Study Area

The Bureau of Reclamation 6/ has estimated that there is generally sufficient water for full irrigation of 67,434 acres of irrigated land above the head of White Ditch, three miles west and one mile south of Belgrade as shown in Figure 3-1, until July 1-5 on the average.

White Ditch diversion point is a minimum-flow point on the West Gallatin River; the river normally gains in flow downstream, and lower ditches serving some 3,849 acres are rarely short of supply.

Above White Ditch, however, the Bureau estimated that more than 30,000 acres would be short by July 31, and over 45,000 acres would be cut off by August 31. Their plan, discussed previously, was to meet these shortages using groundwater.

Groundwater flow in the Gallatin Valley was estimated at 240,000 acre-feet per year on the average during 1934-1953. 7/ The range was 120,000 to 320,000. This annual flow is in addition to confined groundwater in storage. However, only certain subareas have good access to this groundwater.

Selection of Subareas in the Model

It may be concluded from the Bureau study on surface water shortages and the USGS report on groundwater resources that the problem area

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6/ Report on Three Forks Division, Appendix B, pp. 148-157.

7/ Ibid.

in Gallatin Valley is basically confined to regions bordered by the East and West Gallatin Rivers above their junction, and the west side of the West Gallatin River as shown in Figure 3-1. The upper end tends to have a water shortage problem in most years and the lower, end a bogging problem.

The study area, delineated in Figure 3-1, consists of eight separate subareas, i.e., (1) Camp Creek Hills Subarea, (2) West Gateway Subarea, (3) East Gateway Subarea, (4) Bozeman Fan Subarea, (5) East Belgrade Subarea, (6) West Belgrade Subarea, (7) Central Park Subarea, and (8) Manhattan Subarea. These eight subareas are shown in Figure 3-2. Also shown in Figure 3-2 are groundwater table contours estimated by USGS 8/in 1953. These contours are utilized in estimating both the direction and velocity of groundwater flow through these eight subareas.

Part II: Hydrologic Model

General Design

The prime objective of the proposed simulation model is to follow the movement of groundwater through adjacent hydrologic subareas with respect to quantity of flow, destination and origin of flow. Major water inputs, outputs and losses are shown below.

8/ Hackett, op. cit., plate 5.

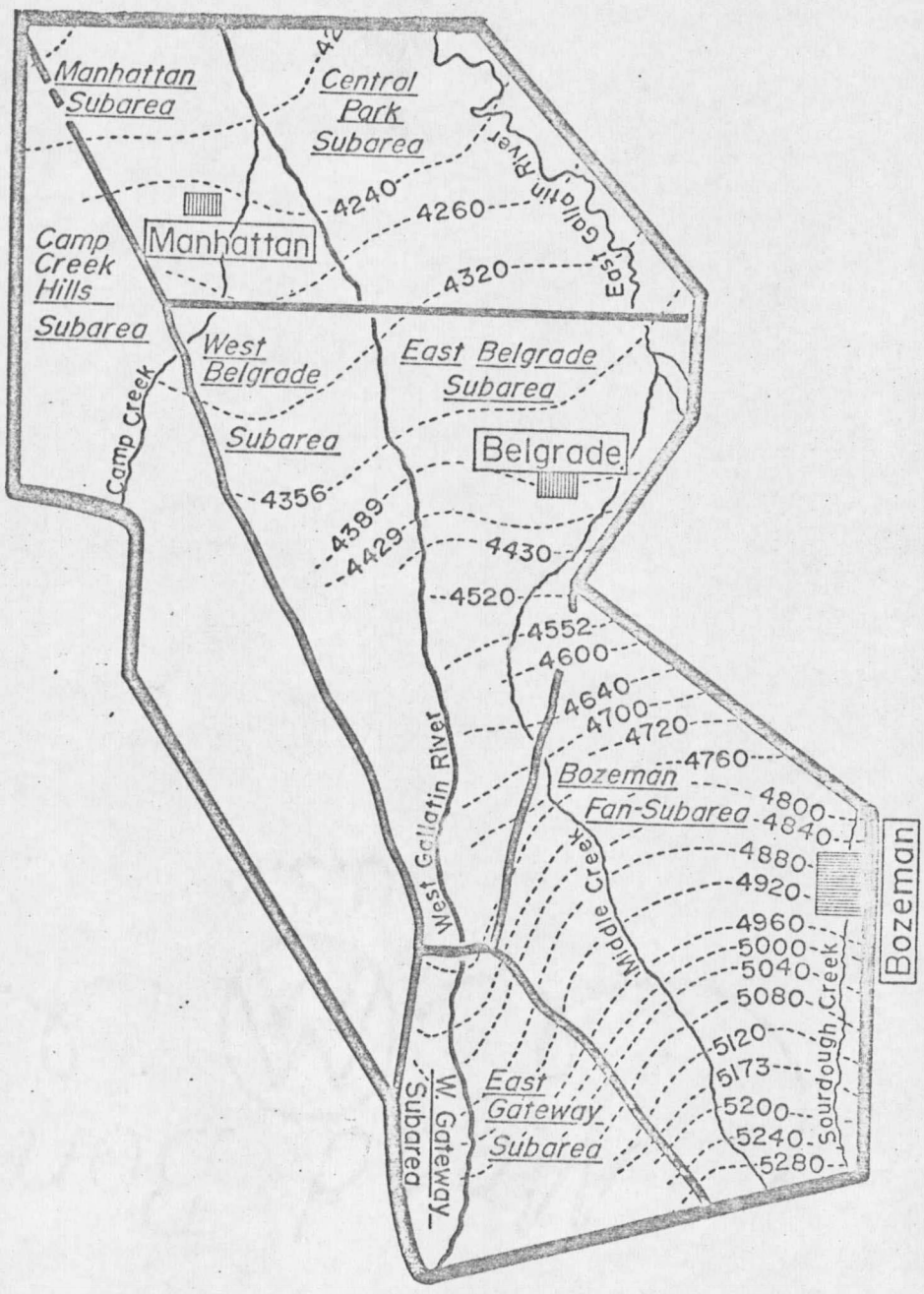

Figure 3-2. Map of the Study Area showing Water-Table Contour Lines during 1953.

A. Water Inputs to Area.	1. Diverted Surface Water 2. Runoff Surface Water 3. Groundwater Inflow 4. Precipitation
B. Water Losses in Area.	1. Evaporation Loss 2. Transpiration Use
C. Water Outputs from Area.	1. Surface Water Outflow 2. Groundwater Outflow

In constructing a complete simulation model, it is necessary to consider time and space dimensions (source and destination) of all basic flow relationships in the hydrologic system. The time dimension is important for water supply since the value of water, particularly for irrigation use, is directly related to the time it is available. Evaluation of the sources and destination of water movement is also necessary in planning optimal use of this resource.

Specific Design

The specific design of the simulation model should link related groundwater areas and streams together in time and space. An approximate method for estimating flow characteristics and the rate of groundwater movement is the use of transmissibility coefficients in conjunction with prepared groundwater contour maps of the study area. As defined by USGS, the coefficient of transmissibility is "the number of gallons of water per day, at the prevailing water temperature, that is transmitted through each mile strip extending the full saturated

thickness of the aquifer under a hydraulic gradient of one foot per mile." 9/

It is possible to estimate both the quantity of flow and the velocity of flow through an aquifer region since all of these units are specified in the coefficient of transmissibility. Other basic hydrologic properties utilized in the model are: (1) groundwater flow is perpendicular to the equipotential surface of the acquifer, and (2) the velocity of groundwater flow is proportional to the hydraulic gradient in an unconfined aquifer. These properties are discussed in most standard texts on hydrology. 10/

The groundwater contour lines in the study area shown in Figure 3-2 indicate that perpendicular groundwater outflow from each subarea may have several destinations. Furthermore, the contour lines do not have equal spacing, indicating that the slope of the hydraulic gradient also varies within each subarea. In the hydrologic model both the gradient slope and transmissibility coefficients are averaged in each subarea.

The destination of groundwater flow from each subarea is estimated as shown in Figure 3-3 by constructing directional lines or vectors perpendicular to groundwater contour lines in each subarea such that

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9/ Ibid.

10/ For example see S.S. Butler, Engineering Hydrology, Prentice-Hall, Inc., New Jersey, 1957.

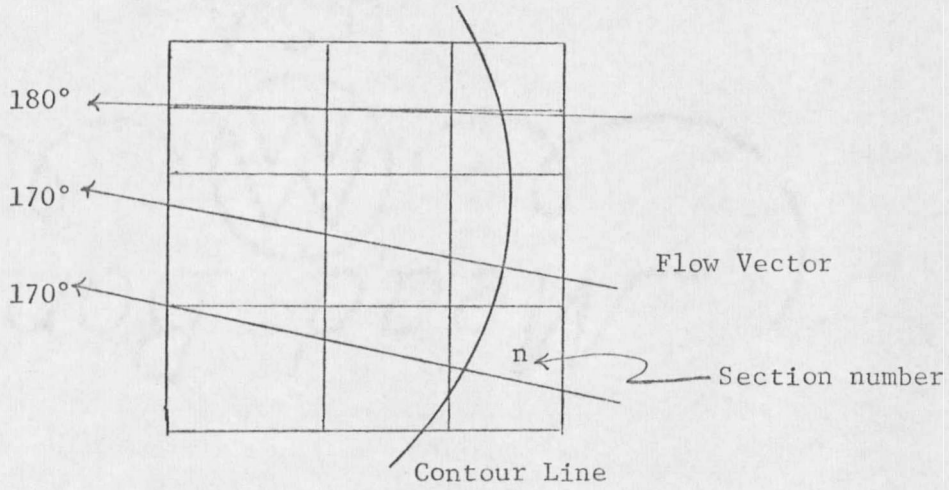

Figure 3-3. Illustration of Method Followed in Allocating Groundwater Discharge.

all sections of land in each subarea are intercepted. Vectors are rounded off to the nearest five degrees on the compass scale and groundwater discharge from each aquifer subarea is allocated to each vector in proportion to the approximate number of sections of land in the path of flow. This approach is illustrated in Figure 3-3 for a hypothetical square aquifer area. Outflow for each vector is then assigned to the adjacent subarea or stream in the path of flow.

Computing Groundwater Discharge

Computations for groundwater outflow from each subarea source (i) for time (t) and destinations of this flow (j) are based on the following formulae:

$$GD_{it-1} = \frac{(CT_{it-1})(G_{it-1})(W_i)}{325{,}872.36} \tag{3.1}$$

$$GD_{ijt-1} = \frac{(GD_{it-1})(A_{ij})}{(A_i)} \tag{3.2}$$

$$GD_j(H_{it}) = \frac{(H_{it})(GD_{ijt-1})}{(H_{it-1})} \tag{3.3}$$

where: GD_{it-1} = the average rate of total groundwater discharge from subarea i during time (t-1) (A.F./day);

CT_{it-1} = average transmissibility coefficient in subarea i for time (t-1) (gpd/ft.);

G_{it-1} = average gradient slope of the water table in subarea i for time (t-1) (ft./mi.);

W_i = the average aquifer width of subarea i (miles);

GD_{ijt-1} = the estimated groundwater discharge from subarea i to destination (j) for time (t-1) (A.F./day);

$GD_j(H_{it})$ = a linear function for computing subsequent groundwater discharge as a function of GD_{ijt-1} groundwater discharge and subsequent water table height in subarea i for time (t) (A.F./day);

H_{it} = the average height of the water table in subarea i for time (t) (feet);

A_{ij} = the area of the aquifer in subarea i discharging groundwater to destination (j) (sq. miles);

A_i = the area of subarea i (sq. miles) for which total discharge is computed;

t = the time of the estimate; time (t-1) refers to the period in 1953 when the coefficients of transmissibility were computed.

The divisor, 325,872.36 in (3.1) is the number of gallons per acre foot.

Computed daily groundwater discharge data for each subarea on April 1, 1953 are shown in Table 3-1.

Formula 3.3 will be applied in a general equilibrium model presented in Chapter IV for defining the relationship of subarea groundwater discharge to water table height in subsequent time periods.

Part III: Evaluation of the Gallatin Valley Irrigation System

Water Losses in Canal System

Measurements of actual daily flow in all irrigation ditches and streams in the Gallatin Valley were obtained by the Bureau of Reclamation for estimating efficiency of the system in 1952 and

TABLE 3-1. Groundwater Flow Data for Eight Subareas in 1953*

SA(i)	CT^{a}_{it-1}	G^{b}_{it-1}	W^{c}_{i}	GD^{d}_{it-1}	D(j)[e]	A^{f}_{i}	A^{g}_{ij}	GD^{h}_{ijt-1}	H^{i}_{it-1}	H^{j}_{A}
1	11,000	80	24	64.64	SA3	37	5	8.73	12	20
1	11,000	80	24	64.64	SA6	37	24	41.93	12	20
1	11,000	80	24	64.64	SA8	37	8	13.98	12	20
2	275,000	40	1	33.67	WGR	16	16	33.67	43	55
3	275,000	40	1	33.67	WGR	8	8	33.67	43	55
4	48,000	80	7	92.28	EGR	61	21	28.33	140	150
4	48,000	80	7	82.28	SA5	61	40	53.95	140	150
5	206,000	30	7	132.42	SA7	53	40	99.94	160	200
5	206,000	30	7	132.42	EGR	53	7	17.49	160	200
5	206,000	30	7	132.42	WGR	53	6	14.97	160	200
6	206,000	30	2	37.83	SA8	14	14	37.83	160	200
7	86,000	30	8	63.18	EGR	40	40	63.18	20	25
8	130,000	30	3	35.81	EGR	8	8	35.81	25	40

[a] Average coefficient of transmissibility on April 1, 1953 (gpd transmitted through each mile strip of aquifer).

[b] Average gradient slope (feet/mile) on April 1, 1953.

[c] Average width of aquifer area (miles).

[d] Total estimated groundwater discharge from subarea i during time (t-1) (A.F./day).

[e] This is the destination of groundwater discharge from subarea i (SA or stream).

[f] Total area of subarea i (sq. miles).

[g] This is the estimated area of the aquifer in subarea i which discharges groundwater to destination j (sq. miles).

[h] Estimated groundwater discharge from subarea i to destination j for time (t-1) (A.F./day).

[i] Average water table height on April 1, 1953 (number of feet above aquifer floor level).

[j] Average alluvial thickness of aquifer (feet).

*Computed from data in Hackett, et. al., pp. 104-157. As this discharge data is used in the present study for estimating subsequent discharge, the time period when this data was obtained will be subsequently referred to as t-1.

1953. 11/ Gross irrigation requirements for lands in the Gallatin Valley were estimated by the Bureau as follows: 12/

	Acre-Feet Per Acre	Percent Diverted Surface Water
Effective Precipitation (80% of 5 dry years)	.72	--
Consumptive Use	1.31	32.8
Field Losses	1.21	30.2
Canal Losses	1.48	37.0
Gross Irrigation Requirement	4.72	

In the present study, irrigation will supplement average effective precipitation during four discrete time periods of each irrigation season. Consumptive use will vary with different crops selected as activities in the model. Field losses will vary according to soil conditions and field slopes in the subareas of the study. The irrigation ditches are assumed to be about equally efficient and 37 percent of the diverted surface water will be assigned to canal seepage in all subareas as estimated by the Bureau above.

Water Losses in Field

Irrigation losses in the field include: surface runoff, evaporation from canals, and deep percolation. Field losses largely depend

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11/ Gallatin Valley Hydrologic Survey, U.S. Bureau of Reclamation, Great Falls, Montana, 1954.

12/ Report on Three Forks Division, U.S. Bureau of Reclamation, Appendix M, September 1958, p. 41.

on the water storage capacity of the soil in the root zone, the slope of the field, and the care exercised by the irrigator. These factors will be evaluated in turn for the eight subareas of Gallatin Valley.

1. Soil Storage.--Water losses may be correlated with the frequency of irrigation as some surface evaporation loss will occur in each irrigation application before the root zone is penetrated. The amount of soil storage capacity is therefore of interest in evaluating irrigation efficiency. To estimate potential storage of available moisture, it is necessary to know the depth of soil in the root zone for different crops and the average water-holding capacity of the soil.

Descriptions of the soil profile in the upper two strata and major soil types in the eight subareas are shown in Table A-1, Appendix A. This data is used to compute available moisture storage capacity and net irrigation requirements shown in Table 3-2 for the eight subareas. In making these computations, the writer also utilized columns 2-6 of a technical irrigation guide. 13/

2. Slope of the Field.--The slope of the field being irrigated, together with the soil-profile characteristics, affects soil intake of water, and the efficiency of water spreading. Average slopes of irrigated fields in the eight subareas are estimated from a surface-

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13/ Irrigation Guide for Intermountain and Foothill Area East of Continental Divide, USDA, Soil Conservation Service, Bozeman, Montana, May 1957, pp. 10-28.

TABLE 3-2. Estimated Available Moisture Storage Capacity of Soils and Net Irrigation Requirements in Eight Subareas of the Gallatin Valley.*

Subarea Name	A.M. Storage Capacity for crops		Extraction Depth for crops		Net Irrigation Requirement/Treatment[b]	
	Alfalfa	Other	Alfalfa	Other[a]	Alfalfa	Other
	-----Inches-----		-----Feet-----		----Inches-----	
Camp Creek Hills	10.0	6.0	5.0	3.0	5.0	3.0
West Gateway	10.0	6.0	5.0	3.0	5.0	3.0
East Gateway	10.0	6.0	5.0	3.0	5.0	3.0
Bozeman Fan	10.0	6.0	5.0	3.0	5.0	3.0
East Belgrade	5.0	4.0	4.0	3.0	2.5	2.0
West Belgrade	5.0	4.0	4.0	3.0	2.5	2.0
Central Park	7.5	6.0	4.0	3.0	3.8	3.0
Manhattan	3.5	3.5	3.0	3.0	1.6	1.6

[a]Other crops are small grains, pasture, and hay.

[b]It is assumed that the crop is irrigated when available moisture in storage drops to 50 percent.

*Source: Table A-1, Appendix A.

contour map of the Gallatin Valley. 14/ Estimated gross irrigation requirements in the subareas are shown in Table 3-3. The border method of irrigation is assumed for slopes up to 4 percent, and contour ditches for slopes over 4 percent.

It is apparent in Table 3-3 that from 30-50 percent of the irrigation water is lost to return flow, deep-percolation and evaporation. Since most Gallatin Valley soils are well-drained, evaporation losses should be relatively minor. Whether remaining losses would occur as surface return flow or deep-percolation losses depends on water-table conditions below the irrigated field and how long water is allowed to run on the field.

3. Irrigation Management.--It was stated in 1900 that farmers in the Gallatin Valley often permit irrigation water to run "wild" because of insufficient help. 15/ This matter was discussed with local SCS officials who were familiar with present irrigation practices in the Gallatin Valley. It was suggested that the farmers were probably only obtaining about half of the recommended efficiency level at present because of the shortage of labor and because the present system of water rights in the Gallatin Valley provided little incentive for

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14/ Surface Contour Maps, USGS, Manhattan Quadrangle 1949 and Bozeman Quadrangle 1953.

15/ Fortier, S., "Duty of Water in the Gallatin Valley," Office of Experiment Station Bulletins, II, 1900, pp. 81-90.

TABLE 3-3. Estimated Gross Irrigation Requirements in Eight Subareas of the Gallatin Valley.*

Subarea Name	Ave. Slope of Fields 1/	Net Irrigation Requirements 2/		Basic Intake Rate 3/	Estimated Field Efficiency		Water Loss		Gross Irrigation Requirement	
		Alf.	Other	All Crops	Alf.	Other	Alf.	Other	Alf.	Other
	(Percent)	(Inches)		(In. 1 Hr.)	(Percent)		(Inches)		(Inches)	
Camp Creek Hills	4.0-8.0	5.0	3.0	1.8	50	50	5.0	3.0	10.0	6.0
West Gateway	1.0-2.0	5.0	3.0	1.8	70	70	2.0	1.5	7.0	4.5
East Gateway	1.0-2.0	5.0	3.0	1.8	70	70	2.0	1.5	7.0	4.5
Bozeman Fan	1.0-2.0	5.0	3.0	1.8	70	70	2.0	1.5	7.0	4.5
East Belgrade	1.0-2.0	2.5	2.0	1.8	70	70	1.0	1.0	3.5	3.0
West Belgrade	1.0-2.0	2.5	2.0	1.8	70	70	1.0	1.0	3.5	3.0
Central Park	0.5-1.0	3.8	3.0	1.8	70	70	1.6	1.3	5.4	4.3
Manhattan	0.5-1.0	1.6	1.6	3.0	65	65	0.9	0.9	2.5	2.5

1/ Surface contour maps, USGS, Manhattan Quadrangle 1949 and Bozeman Quadrangle 1953.

2/ Taken from Table 3-2.

3/ Choriki, R.T., The Influence of Different Soil Types, Treatments and Soil Properties on the Efficiency of Water Storage, M.S. Thesis, Montana State University, Bozeman, August 1959, p. 53.

*Source: Irrigation Guide, loc. cit., cols. 10, 14 and 15.

users to be efficient in irrigation. The recommended higher efficiency level would also entail some increase in cost for labor and ditch management which may not be justified at the present time.

In recognition of the current management problem, the writer reasoned that the suggested low field efficiency level was a rational irrigation strategy on the part of farmers given the present system of fixed water rights, but that under a more flexible system allowing exchange of water rights, the farmers would likely increase their efficiency in water use to the level recommended in the irrigation guide cited previously. 16/ The difference in efficiency levels may then be interpreted as a cost imposed on society because of the present institutional system of water rights along with other costs resulting from the improper allocation of resources. It was therefore concluded that the investigation of water use in the Gallatin Valley should include an evaluation of both levels of efficiency to determine the effect on regional income.

The problem of surface return flow is also related to management as the irrigator could prevent any surface runoff with reasonable care in the regulation of water supply to the field. Since the subareas analyzed in this study are relatively large, there should not be much

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16/ Irrigation Guide, op. cit., pp. 1-9.

net surface return flow from each subarea even though some individual fields have surface runoff. It will therefore be assumed for each subarea that there should be very little loss resulting from surface return flow; the bulk of surface outflow is assumed to be caused by water-table drainage which is predicted with the hydrologic model formulated in this chapter. 17/ Consequently, all water losses in field irrigation will be assigned to deep-percolation in the analysis of water use in the Gallatin Valley.

Part IV: Validation of the Hydrologic Model

The hydrologic model formulated in this chapter could only be tested for the year 1953 as estimates of groundwater discharge were not available for other years. A comparison of predicted groundwater discharge in 1953 from each subarea and USGS estimates of this discharge is shown in Table 3-4. The USGS estimates of discharge are based on measurements of gain in streamflow taken during 1953. Since the Camp Creek Hills Subarea and the Bozeman Fan Subarea are not adjacent to a major stream, USGS did not provide estimates for these subareas.

Except for Central Park, predicted groundwater discharge from the subareas in Table 3-4 compares closely with USGS estimates of this discharge during 1953. The large discrepancy for Central Park is

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17/ USGA has also estimated that very little diverted water returns to the river as waste surface return flow in this region, at least during July and August. Hackett, op. cit., pp. 116-117.

TABLE 3-4. Model Predictions and USGS Estimates of Annual Groundwater Discharge For 1953.*

Subarea (i)	Average Water Table Height (Feet)	Average Gradient Slope (Ft/Mi)	Predicted Discharge (Acre-Feet)	Estimated Discharge (Acre-Feet)
Camp Creek Hills (1)	12	80	23,594	na[a]
West Gateway (2)	43	40	12,290	25,000[b]
East Gateway (3)	43	40	12,290	
Bozeman Fan (4)	140	80	30,032	na[a]
East Belgrade (5)	170	80	85,412	135,000[b]
West Belgrade (6)	170	80	24,401	
Central Park (7)	20	30	23,061	300,000[c]
Manhattan (8)	25	30	12,471	14,000

[a]Not available.

[b]Estimate is aggregated over this and the following subarea.

[c]Estimate includes a considerable amount of surface water discharge since groundwater discharge is restricted by a fault.

*Source: Hackett, et. al., pp. 136-151.

explained by the geologic fault in this subarea and by large inflow of groundwater from outside the study region. Most of the groundwater leaving Central Park appears as surface water due to the severe restriction of the fault.

Most of the agricultural land in Central Park is confined to the interior part of the subarea just below East Belgrade Subarea. This interior area, encompassing about 19 square miles, is not affected by inflow from outside the study region and receives all of its groundwater recharge from the East Belgrade Subarea. During the remainder of the present study, Central Park Subarea will be defined as this interior area only.

Predicted discharge rates comparable to those shown in Table 3-4 will be utilized to predict the change in groundwater discharge associated with reallocation of water resources in the study area. As stated previously, the volume of groundwater discharge has important implications for optimal allocation of water resources in the Gallatin Valley and therefore should be evaluated in conjunction with any irrigation system optimization.

Evaluation of Recharge to Subareas

The hydrologic model also explains recharge to subareas where groundwater discharge from adjacent subareas and irrigation percolation losses are the major sources of recharge. However, there are other important sources of recharge for certain subareas which are not explained by the model.

It is stated in the Hackett report that the two Gateway subareas and the Bozeman Fan subarea are partially recharged by runoff of small streams entering these subareas from adjacent highlands. This outside recharge may be accounted for in the hydrologic model by treating pre-season water tables as a parameter. It is assumed that these small streams are largely spring runoff, and irrigation percolation losses are the major source of recharge to these three upper subareas during most of the season.

West Gallatin seepage loss is a major source of recharge in the two Belgrade subareas. This stream is influent for about 4 miles adjacent to these subareas throughout the season. Stream loss in this reach was estimated by USGS to be about 10 percent of flow, comprising a total loss of 63,000 acre-feet in 1952 and 37,000 acre-feet in 1953. 18/ The 10 percent estimate will be used in the hydrologic model for computing recharge to Belgrade subareas.

Both the West Gallatin and East Gallatin Rivers are effluent below the Belgrade subareas, and are not a source of recharge to the Manhattan and Central Park subareas.

General Appraisal of Hydrologic Model

The hydrologic model for predicting groundwater movement is not claimed to be a particularly precise model for this purpose. However,

18/ Hackett, op. cit., p. 143.

it does consider the key variables affecting groundwater flow and provides useful approximations of flow relationships within the system. It is assumed that the approximations are adequate to yield significant improvements in the integrated ground-surface water model over a model that ignores the hydrologic interdependencies. In a sense, a model that assumes away hydrologic interdependencies also tacitly uses quantitative measures of flow among subareas--all flows are taken as zero.

CHAPTER IV

A GENERAL EQUILIBRIUM OPTIMIZATION MODEL

A relatively simple hydrologic model for approximating groundwater flow as a linear function of water table height was presented in Chapter III. The derived linear functions for eight interrelated hydrologic subareas of the Gallatin Valley provide the necessary link between the surface and groundwater system for general equilibrium analysis of the overall optimal allocation of water in the study area.

The nature of water response relationships in the criterion function of the applied model will be governed by selection of crop activities which are adaptable to the Gallatin Valley study area. For the present, however, we need only be concerned with some hypothetical water-response relationship for each of the eight subareas. This response function is assumed to imply the most profitable combination of crop activities at each level of water supply. The function could be computed in practice by a priori specification of profit levels by crops at various levels of water supply in a parametric programming model, and then changing the water supply by increments. A separate analysis would be required for each subarea.

In each subarea, water response depends on the timing of irrigation deliveries as well as the total quantity of water

delivered during the irrigation season. For simplification, the time horizon for water deliveries in the present study is divided into four discrete time periods of the irrigation season: (1) June 1 to July 1; (2) July 1 to August 1; (3) August 1 to September 1; and (4) September 1 to October 1. In each of the four time periods (subsequently identified by t=1,...,4), water response will be jointly determined by the volume of water delivered in that period and in all other periods.

Water supply to each subarea may come from various sources depending upon access of the respective subarea to alternate streams and groundwater reserves. Of course, water needs can be at least partially met by natural precipitation and soil moisture in storage at the beginning of the irrigation season. Since the latter sources are not generally subject to control by the decision-maker, they are not of immediate concern in formulation of the model. Delineation of water supply by ground and surface water sources is necessary in formulation of the model for the following reasons: (1) these two sources of supply usually have quite different procurement costs and efficiencies in delivery; (2) their respective use is of paramount concern in analysis of the hydrologic system, particularly for control of water tables; and (3) some of the subareas are entirely dependent upon surface water supply for their irrigation requirements. The source of water supply, the subarea served and the time period of delivery are identified in the model criterion function by

G_{it} in the case of groundwater, and by S_{it} in the case of surface water. The first subscript, i, denotes the subarea served; i=1,...,8; and the second subscript, t, denotes the time period of delivery, t=1,...,4. All surface water supply variables are prefixed by the symbol, E_i, in the criterion function to indicate possible differences in conveyance efficiency among the eight subareas. Delivery efficiency and also field efficiency of irrigation water is subject to control in project planning; for simplification, however, these efficiency levels are specified to be parameters in the theoretical model formulation.

The volume of groundwater supply available for use in each subarea is restricted by the level of development of irrigation wells. The wells represent long-term capital investments, and their optimal level of development is dependent upon both intraseasonal and interseasonal expectations of surface water supply. For simplification in presentation of the theoretical model, it is assumed that all water supply in each time period of analysis is known with certainty. The empirical model, however, will consider the problem of stochastic variation in the interseasonal surface water supply.

Net welfare is assumed to be additive over all subareas in the model since the output of each crop in the Gallatin Valley is an insignificant part of supply in the national market. Net welfare for a representative i^{th} subarea may be defined in terms of the following criterion function:

$$V_i(E_i S_{it}, G_{it}, K_i, H_{it})$$

where S_{it} denotes the vector of variables with components determined by both stream source and timing of stream diversions for irrigation, E_i is a parameter scalar representing delivery efficiency, G_{it} denotes the vector for variable groundwater supply with components determined only by time, K_i is a scalar denoting the level of irrigation well development governing effective groundwater supply, and H_{it} denotes a vector of variable water table levels. It is assumed that the water table level is not limiting groundwater supply for irrigation in the present example. In the applied version of this model, the empirical problem resulting from the water table being too high for normal production response to irrigation may be avoided by assuming this critical water table level to be an irrational production region since it may be controlled by groundwater pumping. As the groundwater table height would affect production only if above some threshold level, this influence may be evaluated by imposing an inequality constraint on the maximum level that will be tolerated. However, a more general applied model would measure the rapid decline in production resulting from progressive saturation of remaining soil space above the critical water table level. A detailed statement of all variables affecting V_i in the applied model for the Gallatin Valley area is given in Chapter VI.

Regardless of whether production is constrained by the water table level or not, the criterion function is theoretically

subject to diminishing returns in spite of the many linearities assumed in the model regarding profits, costs, and other coefficients for each activity. With increasing employment of certain activities the rate of increase in net product would tend to decline as a result of using less productive land and incurring higher distribution costs for water supply. Crop returns are also restricted by acreage limitations imposed by the government allotment program and crop rotation requirements. The criterion function is defined to be a concave surface due to the effect of diminishing returns.

Interdependencies that exist among different subareas regarding the use of water resources and related hydrologic interactions are specified in side conditions of the model. In view of the rather complex nature of these interrelationships, the set of definitions and notation required for statement of the mathematical constraints is quite detailed.

Specifications of Terms in the Model

Functions: Defined for i=1,...,8;

V_i = the net welfare or benefit obtained in the i^{th} subarea assuming that an optimal production program is followed. Implicit in V_i are linear cost functions for such items as amortized costs of well development, irrigation labor costs, crop enterprise costs, variable pumping costs, and ditch operation and maintenance costs.

Parameters: Defined for i=1,...,8; t=1,...,4;

E_i = a coefficient for determining the net amount of diverted surface water delivered at the farm headgate (proportion);

H_i^o = the initial water table height in subarea i prior to the growing season, i.e., for t=0. H_i^o is assumed to be a parameter in the present study as average annual recharge far exceeds the expected withdrawal of groundwater for irrigation use each season.

S_t^{WG} = West Gallatin surface supply available for use in time t (acre-feet);

S_t^{MC} = Middle Creek surface supply available for use in time t (acre-feet);

S_t^{SC} = Sourdough Creek surface supply available for use in time t (acre-feet);

S_t^{EG} = East Gallatin surface supply available for use in time t less appropriated water outside the study area and less S_t^{SC} tributary inflow (acre-feet);

S_t^{CC} = Camp Creek surface supply available for use in time t (acre-feet);

H_t^{WG} = the average surface water height of the West Gallatin River during time t (feet); 1/

a_i = a coefficient for estimating total groundwater outflow of subarea i in relation to water table height in subarea i (acre-feet);

a_{ji} = a coefficient for estimating total groundwater outflow of subarea j entering subarea i from water table height in subarea j (acre-feet);

b_i = a percolation-loss coefficient for field irrigation (proportion);

W_i = the average width of aquifer i (miles);

L_i = the average length of aquifer i (miles);

0.15 = the average coefficient of water storage for all subareas; and

C = average monthly pumping capacity of each well (acre-feet).

1/ This parameter is needed for estimating groundwater discharge as surface water from adjacent subareas. The stream level is compared with subarea water table height in determining the volume of groundwater discharge as surface water.

Variables: Defined for i=1,...,8; t=1,...,4

S_{it}^{WG} = the number of acre-feet of surface water diverted from the West Gallatin to subarea i during time t;

H_{it} = average water table height in the i^{th} subarea during time t (feet above aquifer floor);

S_{it}^{MC} = the number of acre-feet of surface water diverted from Middle Creek to subarea i during time t;

S_{it}^{SC} = the number of acre-feet of surface water diverted from Sourdough Creek to subarea i during time t;

S_{it}^{EG} = the number of acre-feet of surface water diverted from the East Gallatin to subarea i during time t;

S_{it}^{CC} = the number of acre-feet of surface water diverted from Camp Creek to subarea i during time t;

G_{it} = the number of acre-feet of groundwater pumped in subarea i during time t; and

K_i = the number of 2 cfs wells developed in subarea i.

Transformations: Defined for i=1,...,8; t=1,...,4 2/

$H'_{it} = H_{it} - H_t^{WG}$, ($H'_{it}$ is the differential water table height affecting groundwater discharge to the West Gallatin--feet);

$\beta_i = 1-E_i + b_{pi}E_i$, ($\beta_i$ is the fraction of surface water diverted which is lost in conveyance and field percolation--proportion);

$\delta_i = 1-b_{pi}$, (δ_i is the field consumptive use coefficient--proportion);

2/ These transformations are intended to simplify use of notation in the model. They represent combinations of parameters and variables that have been defined earlier.

$A_i = (0.15W_iL_i)$ (640 acres/mi.^{2}) (1 ft.), (A_i is the aquifer storage volume per unit water table height in subarea i--acre-feet);

$C_{1t} = S_t^{WG}$, (C_{1t} is total surface water supply available for use in subareas 1, 2 and 3--acre-feet);

$C_{2t} = S_t^{WG} + S_t^{MC} + S_t^{SC}$, ($C_{2t}$ is total surface water surface available for use in subarea 4--acre-feet);

$C_{3t} = S_t^{WG} + S_t^{MC} + S_t^{EG}$, ($C_{3t}$ is total surface water supply available for use in subarea 5--acre-feet);

$C_{4t} = 0.90\ S_t^{WG} + S_t^{CC}$, ($C_{4t}$ is total surface water supply available for use in subarea 6--acre-feet);

$C_{5t} = 0.90\ S_t^{WG} + S_t^{EG}$, ($C_{5t}$ is total surface water supply available for use in subarea 7--acre-feet); and

$C_{6t} = 0.90\ S_t^{WG} + S_t^{CC}$, ($C_{6t}$ is total surface water supply available for use in subarea 8--acre-feet).

Explanation of Terms in Model

The relevance of these specified parameters, variables and transformations in the model is illustrated by the following flow diagrams. All surface flows in the system are shown in Figure 4-1.

It is evident in Figure 4-1 that there are five streams supplying eight subareas in the study region. The West Gallatin River, however, is the only stream accessible to all subareas. Stream flow constraints are respectively: E. Gallatin River, S_t^{EG}, W. Gallatin River, S_t^{WG}, Sourdough Creek, S_t^{SC}, Middle Creek, S_t^{MC}, and Camp Creek, S_t^{CC}. Combined stream constraints for each subarea were denoted earlier by C_i in the list of transformations.

These stream constraints are at least partially relaxed by groundwater discharge flows illustrated in Figure 4-1. The discharge flows

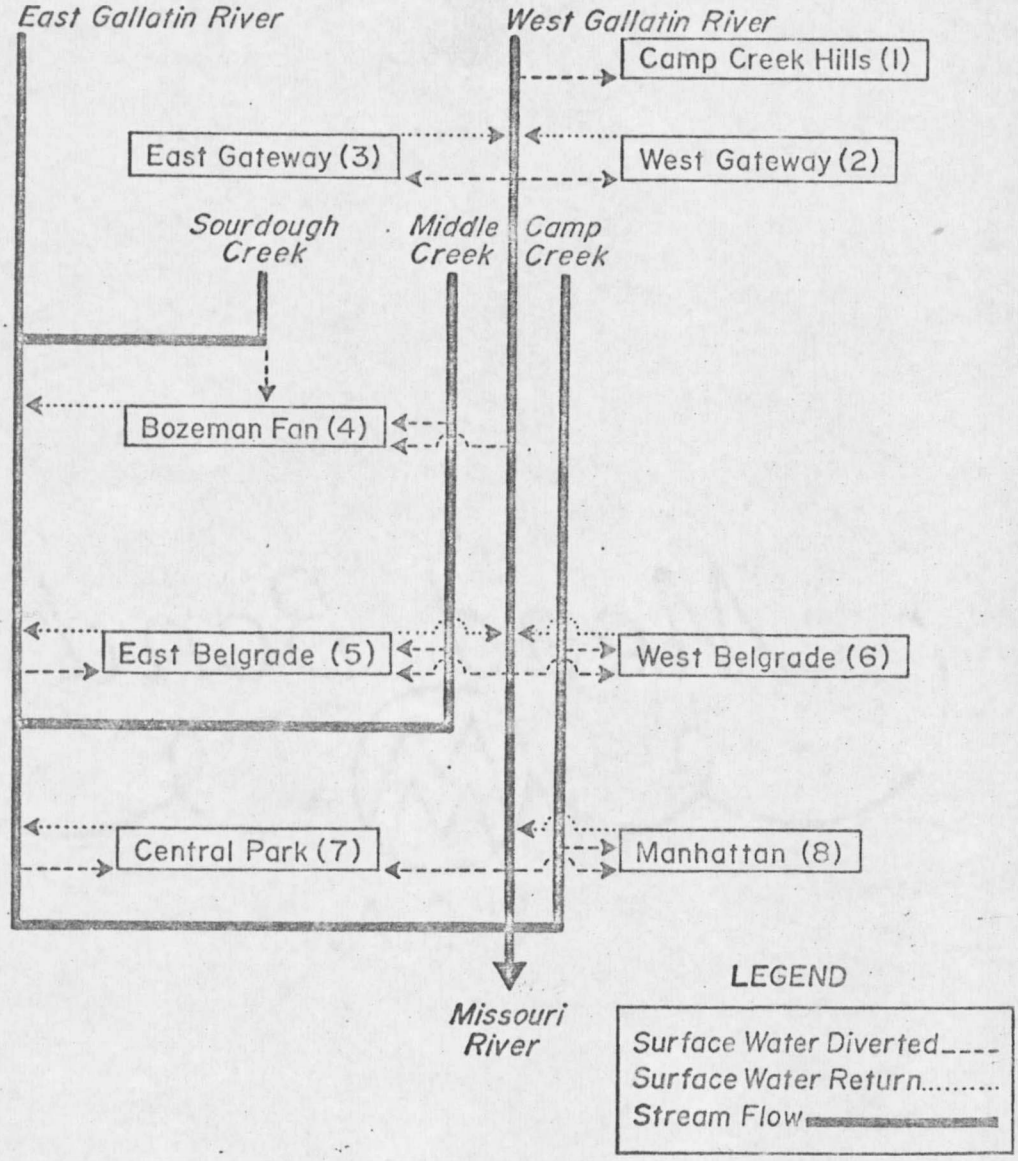

Figure 4-1. Flow diagram for study area, showing direction of stream flows, sources of diversion to subareas, and surface-water return flows.

may be rediverted for irrigation use in adjacent subarea i or in other downstream subareas.

Recall from Chapter III that groundwater outflows are functionally related to groundwater heights in subareas adjacent to streams. As defined in the hydrologic model of Chapter III, groundwater outflows arise from groundwater movement and they occur where the groundwater table intercepts a stream bed or other low elevation. The general direction of groundwater movement in the system is shown in Figure 4-2. This movement is based on groundwater flows during 1952-53 shown in Table 3-1, Chapter III. The only subareas obtaining much groundwater recharge from the West Gallatin River are East and West Belgrade as shown in Figure 4-2. West Gallatin stream loss to these subareas is estimated to be about 10 percent of flow near Cameron Bridge. 3/ The other streams in the study area are assumed to have no effect on groundwater recharge or discharge to subareas.

It may be recalled from Chapter III that the exterior part of the Central Park Subarea receives about 190,000 acre-feet of recharge annually from outside the study area. However, this part of Central Park has been omitted from the present study as very little agricultural land is involved in the region affected.

With these exceptions, all groundwater movement is determined endogenously in relation to water table heights of adjacent subareas in the study area.

3/ Hackett, op. cit., p. 143.