Optimal use of ground and surface water in the Gallatin Valley, Montana
by Kenneth Boyd Young

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY in Agricultural Economics
Montana State University
© Copyright by Kenneth Boyd Young (1971)

Abstract:
Intraseasonal interaction of the ground and surface water system figures prominently in this irrigation-planning study of a river basin. A model for optimizing use of variable streamflows and supplemental groundwater reserves is developed that simultaneously evaluates economic interaction occurring within the ground and surface water system during four discrete periods of the irrigation season.

Development of the model encompasses three stages. First, the hydrologic system is analyzed and groundwater discharge is functionally related to water table height in eight hydrologic subareas over four time periods. Second, a nonstochastic model is developed to allocate intraseasonal surface supply and utilize supplemental groundwater subject to interspatial-intertemporal groundwater interactions determined by water table balances in each period. This model is used to compute well investment levels associated with assumed fixed surface supplies and different distribution efficiencies.

Third, the probability distribution of annual streamflow is evaluated and the optimal level of well investment (within the class of well developments computed above) is determined for each field efficiency condition with stochastic surface input.

Post-optimal analysis, including sensitivity analysis of variable pumping costs, well investment costs, tolerable water table heights, and crop allotment restrictions, is also applied.

Empirical results of the model indicated that with optimal surface water use, development of groundwater reserves for irrigation supply in the Gallatin Valley would increase average net annual irrigated income by $8 per acre or 18.2 percent on the 124,416 acres in the study area. If field efficiency can be raised from the present level to the recommended level, a comparable increase in irrigated income may be attained without groundwater development. Conjunctive use of ground and surface water together with increased field efficiency in irrigation would result in a potential annual income gain of $13 per acre of 30 percent overall.

OPTIMAL USE OF GROUND AND SURFACE WATER IN THE
GALLATIN VALLEY, MONTANA

by

KENNETH BOYD YOUNG

A thesis submitted to the Graduate Faculty in partial
fulfillment of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Agricultural Economics

Approved:

Head, Major Department

Chairman, Examining Committee

Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana
June, 1971

ACKNOWLEDGMENTS

I wish to express my sincere gratitude and indebtedness to my major professor, Dr. Oscar R. Burt. His guidance and suggestions have been invaluable during the course of this study.

Gratitude is also extended to Dr. R. J. McConnen, Dr. T. L. Hanson, and Professor T. T. Williams for their careful reading and suggestions on the rough draft of this thesis.

I also wish to thank Mrs. Peggy Humphrey for her time and patience in typing this thesis

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF TABLES

LIST OF TABLES (continued)

LIST OF TABLES (continued)

LIST OF APPENDIX TABLES

LIST OF FIGURES

ABSTRACT

Intraseasonal interaction of the ground and surface water system figures prominently in this irrigation-planning study of a river basin. A model for optimizing use of variable streamflows and supplemental groundwater reserves is developed that simultaneously evaluates economic interaction occurring within the ground and surface water system during four discrete periods of the irrigation season.

Development of the model encompasses three stages. First, the hydrologic system is analyzed and groundwater discharge is functionally related to water table height in eight hydrologic subareas over four time periods. Second, a nonstochastic model is developed to allocate intraseasonal surface supply and utilize supplemental groundwater subject to interspatial-intertemporal groundwater interactions determined by water table balances in each period. This model is used to compute well investment levels associated with assumed fixed surface supplies and different distribution efficiencies. Third, the probability distribution of annual streamflow is evaluated and the optimal level of well investment (within the class of well developments computed above) is determined for each field efficiency condition with stochastic surface input.

Post-optimal analysis, including sensitivity analysis of variable pumping costs, well investment costs, tolerable water table heights, and crop allotment restrictions, is also applied.

Empirical results of the model indicated that with optimal surface water use, development of groundwater reserves for irrigation supply in the Gallatin Valley would increase average net annual irrigated income by $8 per acre or 18.2 percent on the 124,416 acres in the study area. If field efficiency can be raised from the present level to the recommended level, a comparable increase in irrigated income may be attained without groundwater development. Conjunctive use of ground and surface water together with increased field efficiency in irrigation would result in a potential annual income gain of $13 per acre of 30 percent overall.

CHAPTER I

INTRODUCTION

In many irrigation projects, the most limiting factor affecting productivity is a high water table resulting from a basic disequilibrium in the hydrologic system. The damage caused by excessive water tables has become a serious problem for irrigated agriculture in many parts of the world. In West Pakistan, for example, some 26 million acres of irrigated cropland has been damaged by waterlogging and salting, and the spread of salinity is continuing to force many more thousands of acres out of production every year. [1]

The full significance of this problem is often underestimated at the outset of irrigation development as the damage may not be evident until several years later. Accumulation of damage in the case of gradual soil salinization or loss of permeability has, generally, a very long time horizon. Damage is more easily assessed when farm income is directly affected by crop bogging or interference with harvesting.

In addition to artificial drainage, the most common method for controlling water tables, there are often several other strategies available for control of this problem. For example, the distribution system can be designed and water supply allocated so that percolation losses are minimized in the areas of the project that are most

[1] D.E. Naylor, "Control of Waterlogging and Salinity in West Pakistan," *International Journal of Agrarian Affairs*, Vol. IV, No. 1, October 1963, pp. 1-12.

susceptible to bogging. Also, where groundwater can be used to
supplement surface water, the water table may be controlled by
concentrating pumping on lower lands of the project.

In most cases, however, the economic potential of employing
these alternative strategies is not appreciated until after all irriga-
tion structures are completed and some experience is obtained in opera-
tion of the irrigation system. At this stage, artificial drainage
may be the only practical choice left for relieving high water tables
although less costly control could have been attained formerly with
another strategy, e.g., by changing the layout of the distribution
system.

Nature of the Investigation

The economic significance of high water tables to irrigated
agriculture and some alternative strategies for controlling this
problem have been discussed in general. An important implication
to be derived from this discussion is that a more systematic approach
in planning irrigation systems is needed where high water table condi-
tions are likely to be encountered. In view of the drainage problems
experienced in some completed projects, e.g., in West Pakistan, 2/
it would appear that more comprehensive planning models would be
advantageous even if based on relatively sparse hydrologic information.

2/ Ibid.

A general equilibrium model including all hydrologic relation-
ships offers several advantages over partial equilibrium methods
for analyzing water-table problems. For example, the evaluation of
alternative strategies for water-table control may be directly
incorporated within a composite criterion function for overall
optimal system organization. This is not possible in the partial
equilibrium approach. Allocation of water flows and the location
of irrigation wells within a system can be important strategies in
water-table control, particularly for localized problem areas. The
use of a composite criterion function as postulated above greatly
facilitates analysis of these more complex strategies for controlling
water tables.

The problem of surface water-groundwater interaction in a
hydrologic system may be simply explained by Darcy's Law governing
linear flow of fluids. According to Darcy's Law, groundwater
velocity for laminar flow is proportional to the hydraulic gradient
along the path of flow. 3/

The following are two examples of interaction which may be
explained by Darcy's Law:

3/ S.N. Davis and R. J. DeWiest, Hydrogeology, John Wiley & Sons,
    Inc., New York, 1966, p. 156.

1) If groundwater is pumped adjacent to a stream, water loss from the stream will tend to increase in proportion to the change in the hydraulic gradient between the stream level and the adjacent aquifer's declining water table level; and

2) If the water table in the adjacent aquifer is higher than the stream level, e.g., from heavy irrigation buildup, the surface water supply provided by the stream may be expanded by increased groundwater outflow from the adjacent aquifer.

The two examples cited are very simple cases of possible interaction between surface and groundwater. The analysis becomes more complicated when a water table change in one part of an aquifer region triggers a chain reaction of interactions throughout the rest of the system. A simulation model for evaluating this latter problem is presented in Chapter III.

In addition to the above physical interactions, some important economic interactions also take place between the ground and surface water system. A major source of interaction is the economic alternative of using the aquifer as a distribution system for irrigation supply instead of surface canals. This method of distribution is feasible in the present study as most of the aquifer area is located under agricultural land. Advantages in distribution efficiency coupled with low pumping lift associated with high water table levels make groundwater a relatively cheap alternative source of

water supply in the Gallatin Valley. Reliability of groundwater

supply is also assured because of the large volume currently in

storage and the high annual rate of recharge to the aquifer in the

Gallatin Valley.

It should be evident from the above discussion that high

water-table levels are a primary cause of surface and groundwater

interactions within a hydrologic system as well as a major threat

to productivity of an irrigation project. This further justifies

the use of a general equilibrium model for irrigation project

planning which includes all hydrologic relationships of concern

in optimal water allocation.

Purpose of the Study

This thesis involves the formulation, solution and evaluation

of a general equilibrium model for conjunctive intraseasonal use of

groundwater and surface water in the Gallatin Valley, Montana. The

purpose of the model is to optimize the combined use of available

surface and groundwater resources in the Gallatin Valley subject

to specified intertemporal and interspatial hydrologic relation-

ships that restrict the use of these resources. Since the major

restriction on water use is the level of the water table, evaluation of

water tables during the irrigation season enters prominently in

construction of the model. The identification and quantification of hydrologic relationships affecting water table balance are important features of the methodology followed in this study. The analysis in Chapter III of this study is entirely devoted to specification and quantification of hydrologic relationships.

Specific empirical objectives of the present study are to determine the economic benefits of groundwater development in the Gallatin Valley, the value of improved irrigation efficiency resulting from better farm practices, ditch consolidation and/or lining, and the relative importance of current surface water supply in the study region. Empirical objectives also include an evaluation of optimal cropping patterns in different subareas of the Valley under different water supply conditions and the optimal level of well development in each subarea. A further objective is to identify what institutional changes will be required to provide flexibility in water exchange and encourage cooperation among water users so that optimal irrigation development may be obtained in the Gallatin Valley.

Justification for the Study

The primary reason for undertaking a comprehensive water study of the Gallatin Valley is that this region provides excellent opportunity for a systems analysis of interrelated drainage and water scarcity problems within the region. The higher lands in the

Valley generally run short of surface water early in July each

season whereas the lower lands with the early water rights tend

to have an abundance of surface water supply during all of the

irrigation season. In the lower part of the valley, once fertile

lands are now bogged due to misuse of water resources in this area. 4/

It would appear that major increases in economic efficiency should

result from reallocation of the valley's water resources.

In addition to the need for a quantitative economic study of

the Gallatin Valley, the study was made feasible by the relative

abundance of empirical data available on water use and crop produc-

tion in this region. A detailed investigation of the hydrology in

the Gallatin Valley had been completed by USGS, 5/ and several

feasibility studies of proposed water storage projects had been

completed by the U.S. Bureau of Reclamation. 6/ The availability

of this type of data plus considerable local crop production data

at the Montana State University Experiment Station near Bozeman

was an important consideration in undertaking a study of this nature.

4/ H.E. Murdock, Irrigation and Drainage Problems in the Gallatin
   Valley, Mont. Agr. Expt. Sta. Bulletin 195, Bozeman, November
   1926, p. 1.

5/ Hackett, et. al., Geology and Groundwater Resources of the
   Valley, Gallatin County, Montana, Geological Survey Water-
   Supply Paper 1482, U.S. Government Printing Office, Washington,
   D.C., 1960.

6/ United States Bureau of Reclamation, Report on Three Forks
   Division, Upper Missouri Project Office, Great Falls, Montana,
   1958.

Statement of Hypotheses and Assumptions

The major hypothesis guiding this study is that a reallocation of surface water use in the Gallatin Valley, and in particular, development of groundwater pumping facilities for supplementing surface supply, would result in significant income gains and sizable improvements in efficient use of the Valley's natural resources. It is further hypothesized that the basic irrigation problem in the Gallatin Valley is resource management--making the right quantity of water available at the right place at the right time--as opposed to aggregate water shortages.

Important assumptions in the study which will not be tested are:

1) Evaluation of intertemporal and interspatial hydrologic relationships between surface and groundwater use will result in substantially more optimal use of these two resources in the Gallatin Valley than an optimization method which abstracts from this interaction between surface and groundwater use.

2) Groundwater depletion in the Gallatin Valley will not present a problem in this study as the probable annual withdrawal by pumping will not exceed annual recharge, on the average. Thus, the only effective storage function performed by the aquifer is an intraseasonal one.

3) Irrigation use of surface water in the Gallatin Valley will have no perceptible effect on the volume or quality of water supply for downstream users located on the Missouri River.

4) Dewatering of streams adjacent to irrigated lands in the Gallatin Valley will have no significant economic effect on the fishery. 7/

Scope of the Study

Following a review of the literature in the subsequent chapter, considerable attention is devoted to the development of a simple hydrologic model for approximating groundwater discharge from eight subareas of the Gallatin Valley as a linear function of water table heights in these subareas. All discharge relationships regarding groundwater flows across subarea aquifer boundaries and inter- actions with stream flow are determined from historical hydrologic data on the Gallatin Valley utilizing fundamental principles of hydrology. The linear model for predicting groundwater discharge in the system is tested for empirical validity by comparing estimated groundwater discharge from subareas adjacent to the stream with

---

7/ This assumption could not be tested in this study as the value of the fishery is not known. The shadow prices associated with stream flow constraints in the model do provide a cost estimate of maintaining minimum stream flow levels for the fishery.

measured stream flow gains attributed to groundwater discharge.
All subsequent groundwater discharge in the model is assumed to
have the same linear relationship to other water table heights as
for the particular class of water tables used in testing this model.

A theoretical general equilibrium model for irrigation optimiza-
tion is formulated in Chapter IV with a nonlinear objective function
and linear constraints largely determined from the hydrologic analysis
in Chapter III. Evaluation of the relative economic importance of
intertemporal and interspatial hydrologic relationships between
ground and surface water use will constitute an important considera-
tion in this theoretical model. Kuhn–Tucker theory is utilized to
derive economic implications of the probable effects of surface-
groundwater interactions which would result under different water
supply conditions in the model. These theoretical interpretations
provide a valuable guide for subsequent interpretation of the
empirical results and sensitivity analysis of the empirical model.

Attention in Chapter V is mainly devoted to assembly of
empirical data on water supply, crop water response and production
cost for use in a linear programming model. A further methodological
consideration is to utilize the soil reservoir for water storage
during different time periods of the irrigation season for trans-
ferring water from high streamflow periods to periods later in the

season when surface water is in relatively short supply. This
particular strategy is of value in the Gallatin Valley as the volume
of stream flow normally tends to recede as the irrigation season
progresses. Also, storage capacity of the soil reservoir is relatively
large compared with average monthly crop water requirements.
Different soil water carryover levels are considered in defining
various linear programming activities presented in Chapter V.

Formulation of the empirical linear programming model presented
in Chapter VI is similar to the former theoretical model but
simplified for ease in computation and accommodation of data.
Measurement of net welfare in the applied criterion function is
accomplished by delineating various segments of the concave
theoretical objective function and defining separate activities of
a linear program to be associated with these segments. Essentially,
the same hydrologic constraints are retained as they are already
linear in form. Additional constraints are also required for cropland
acreage, wheat allotment restrictions, and crop rotation limitations
in the eight subareas. The completed linear programming model
contains 209 constraints, 458 activities, and 2,014 matrix entries.

The linear programming model is first analyzed as a determin-
istic model using the algorithm of parametric programming to
evaluate 16 optimal levels of well development associated with
eight different surface water supplies and two different distribution

efficiencies for water supply. Probability of these surface water supplies is determined by fitting a gamma function to historical inter-seasonal streamflows. Intraseasonal flow in each of four irrigation time periods during each season is assumed to be a constant proportion of total annual flow. Expected value and standard deviation of net income are computed for each of several well development levels. These two statistics for net income in the Gallatin Valley are evaluated under various hypothesized situations and are the main quantitative measures from the economic analysis.

The two distribution efficiencies represent estimated present low field efficiency in the study area and a recommended higher field efficiency level that may potentially be adopted under a changed insti-tutional system allowing more flexibility in water exchange. Parametric changes in distribution efficiency provide estimates of the value of ditch consolidation in the study region as well as an estimate of benefits emanating from improvements in on-farm irrigation efficiency.

CHAPTER II

IRRIGATION DEVELOPMENT IN THE GALLATIN VALLEY

A Literature Review

This chapter is divided into three major parts. Part I is a brief review of the historical development of irrigation in this region. Part II is a more exhaustive review of previous studies by government agencies and researchers at Montana State University on proposed irrigation development of the Gallatin Valley. Part III is a survey of some of the methodology employed and results obtained in other area studies involving related problems.

Part I. Historical Development

Irrigation ditches were first excavated during 1864 in the Gallatin Valley. 1/ As the area became more populated, new settlers continued to add on more ditches and appropriate water rights in the local streams with little regard to the overall benefit of the irrigation system. The outcome of this ad hoc piece-meal development policy is a wasteful distribution system and an over-appropriated surface water supply. Many of these ditches still in use parallel each other and have other construction defects. Severe water shortages were reported as early as 1919. 2/

1/ Water Resources Survey, Gallatin County, State Engineer's Office, Helena, Montana, January 1953, p. 6.

2/ Murdock, H.E., Irrigation and Drainage Problems in the Gallatin Valley, Agr. Expt. Sta., Montana State University, Bulletin 195, Bozeman, November 1926.

14

Irrigation management has also been inefficient in the Gallatin
Valley. This is in part associated with early use of the western
appropriation doctrine in the acquisition of water rights. 3/ Manage-
ment was considered to be one of the key problems in 1953 in the
Gallatin Valley by the State Engineer: 4/

> Poor irrigation practices on the part of some users is
> causing a waste of water and depriving other land of its use.
> The waste of water on higher areas, and seepage from the too
> numerous ditches is causing some land in the valley bottoms to
> become water-logged. This once productive land is now of no
> use except as pasture, some of it growing nothing more than
> swamp grass.

Under the institutionally-fixed ownership of water rights, little
incentive exists for users with early surface rights to be efficient in
their use of this resource or to take proper care of their ditches.
Some, in fact, deliberately use excess water during heavy run-off
periods to build up their water tables for sub-irrigation in other
periods. This aggravates the bogging problem on lower lands.

This cursive review on historical irrigation development in the
Gallatin Valley is intended to highlight current problems in the area
and to explain their evolvement. What are some of the past proposals
for solving these problems?

13/ Gopalakrishnan, C., The Economics of Water Transfer: An Institu-
    tional Appraisal, Ph.D. Thesis, Montana State University, Bozeman,
    1967.

14/ Water Resources Survey, op. cit., p. 14.

Part II: Proposals for Development

A severe drought in 1919 provided impetus for an early extensive investigation of irrigation and drainage problems in the Gallatin Valley. 5/ Murdock's recommendations for relieving the water shortage problem were: (1) build storage reservoirs in the mountains; (2) line the irrigation canals; (3) change the irrigation system; and (4) drain the seeped lands and use this water for irrigation. ·These proposals were investigated in more depth in later studies and will be evaluated in turn along with other possible alternatives.

Surface Storage

A series of studies were begun in 1938 by the Bureau of Reclamation: (1) on the feasibility of diverting water from Hebgen Reservoir on the Upper Madison River to the Gallatin Valley; 6/ and (2) constructing a dam on the West Gallatin River below Spanish Creek for storing irrigation water. 7/ The latter alternative was later found to be more practical and the Hebgen diversion project was abandoned. Four alternate storage schemes at the Spanish Creek site were compared; the most optimal plan having a benefit-cost ratio of 1.52. In the course of this investigation, the Bureau also evaluated

5/ Murdock, op. cit.

6/ Senate Document No. 191, Missouri River Basin, 1944.

7/ U.S. Department of the Interior, Report on Three Forks Division, Bureau of Reclamation, Upper Missouri Project Office, Great Falls, Montana, Appendix M.

a groundwater pumping plan. This had a benefit-cost ratio of 2.41.
Consequently, the Bureau recommended that the groundwater plan be
adopted in place of a surface water storage project.

A storage reservoir with 8,000 acre-feet capacity was constructed
in 1948 on Middle Creek, a Gallatin River tributary, by the Montana
State Water Board.

Ditch Consolidation and Lining

Murdock's recommendations for ditch lining and consolidation
have not been pursued further or promoted any related construction
to the writer's knowledge. This may be a promising area for further
research.

Drainage

Subsurface drainage of bogged lands in Central Park and Belgrade
subareas has been investigated by the United States Soil Conservation
Service. 8/ SCS recommended that deep-interception drains be
constructed at one-mile intervals in these subareas and that provision
be made for one-half mile spacing should shorter spacing be necessary.

Artificial drainage of this scale is an expensive undertaking.
Since the bogging problem has been attributed to apparent gross

---

8/ U.S. Soil Conservation Service, Preliminary Examination Report
on Water Supply and Distribution Investigation, Gallatin Valley
Area, Gallatin County, 1948 and Survey Report on Central Park
Drainage Project, Three Rivers Soil Conservation District in
Gallatin County, Montana, 1950.

inefficiencies in surface water distribution and management,

correction of these inefficiencies may alleviate much of the prob-

lem. At least this matter could be investigated before drainage

commitments are made in the future. The present study will be

concerned with effects of groundwater pumping and variation in

surface water distribution in the valley on bogged areas.

Use of Groundwater

The Bureau of Reclamation plan to pump groundwater in the

Gallatin Valley has been investigated further by McConnen and

Mennon, 9/ Sammons, 10/ and Boyd. 11/

The Bureau proposed to pump about 92,300 acre-feet annually

from 193 wells overlying the aquifer region into established ditches

now diverting from West and East Gallatin Rivers. The pumped water

would be used to replace surface water reallocated to other water-

deficit lands outside the aquifer region.

9/ McConnen, R.J. and G.M. Mennon, Planning the Integrated Use of
    Ground and Surface Water: A Linear Programming Study of the
    Gallatin Valley, Montana, Mont. Agr. Expt. Sta, Bul. 616,
    Bozeman, 1967.

10/ Sammons, R.W., Irrigation Development: Institutional Blocks to
    Ground-Surface Water Integration in the Gallatin Valley, Montana,
    Ph.D. Thesis, Montana State University, Bozeman, 1964.

11/ Boyd, D.W., Simulation Via Time-Partitioned Linear Programming:
    A Ground and Surface Water Allocation Model for the Gallatin
    Valley of Montana, Report No. 10, Montana Water Resources Center,
    Bozeman, June 1968.

McConnen and Mennon used a linear programming model to evaluate net benefits of transferring surface water from surplus to deficit areas and replacing this water with pumped water in ditches as proposed by the Bureau. They estimated that net annual farm income under assumed Cooperative District organization in the Gallatin Valley would increase by $210,738 with the use of 190 wells.

Sammons did a case study of Highline Canal in the Gallatin System and investigated the cost of supplementing canal flows by pumping during the period July 15 to September 1 when surface water supplies are generally short. He estimated that 25 wells would be needed to meet the supplementary requirements of 7,875 acre-feet at a cost per season of $32,878. or $4.17 per acre-foot. He also analyzed institutional blocks to groundwater integration in the area. A recommendation for institutional change was: 12/

> A model of an institutional organization that would allow farmers to exchange surface water for groundwater without endangering their present water rights is needed. This institutional structure would delineate legal responsibilities for both parties during its operation and dissolution.

Boyd used linear programming to allocate water among competing uses in the Gallatin Valley: agricultural, municipal, industrial and recreational; and between surface and sub-surface storage. Adjustments were made in parameters of the model to simulate stream flow over 30 years of record during the testing phase. Pumping was

---

12/ Sammons, op. cit., p. 103.

employed to supplement surface water at a cost of $2 per acre-foot.
Boyd did not analyze effects of pumping on the water table in
different subareas or the optimal allocation of water among the
different subareas of the valley.

This perusal of previous studies on proposed irrigation develop-
ment suggests that further investigation is needed on the following
issues before an overall optimal development plan can be implemented
in the Gallatin Valley:

1) What is the optimal allocation of ground and surface water
during the irrigation season among various subareas of the valley
having different access to these resources?

2) How is the optimal allocation of ground and surface water use
affected by the presence of high water table conditions in different
subareas? Can bogging be controlled by utilizing wells for irrigation
of low lands and restricting the use of surface water?

3) What value would ditch consolidation and lining have for
conserving use of water in the system and alleviating the bogging
problem?

4) What institutional changes are needed to permit comprehen-
sive reallocation of water supplies, both ground and surface, within
the valley to conform with results of an optimization model?
Answers are required to these important questions before optimal
use can be made of water resources in the Gallatin Valley.

The following literature review is of selected studies where related questions have been investigated in other subareas. Methodology employed and results obtained in these studies will furnish valuable guidance for the present study.

Part III: A Survey of Selected Studies

Water Resource Allocation

A sizable literature has evolved on the use of allocation models and alternate methods of systems analysis in water resource studies during the past 15 years.

The Harvard Water Resources Group discusses a number of systems analysis techniques applied to allocation problems. 13/ Recent favored allocation models are inventory models and dynamic programming first used in water resource studies by Masse' and Little. 14/ An example of an inventory model applied in optimal use of a reservoir with stochastic input is a 1963 Israeli study. 15/

13/ Maass, A.M., R. Hufschmidt, H.A. Dorfman, Thomas S. Marglin, Jr. and G. Fair, Design of Water Resource Systems (Cambridge, Mass.: Harvard University Press, 1962).

14/ The idea of using dynamic programming to determine temporal allocation of water from a reservoir is attributed to Pierre Masse', Rept. to the Societe" de Statistique de Paris, Berger-Levrault, Paris, June 21, 1944. The first known application of the model in water research is reported in J.D.C. Little, "The Use of Storage Water in a Hydroelectric System," J. Operations Research Soc. of America, Vol. 3, May 1955.

15/ Avi-Itzak, B. and S. Ben-Tuvia, "A Problem of Optimizing a Collecting Reservoir System," Operations Research, Vol. II, No. 1, 1963, p. 122.

Allocation studies on conjunctive use of ground and surface water have been done by Burt, 16/17/ Leonard, 18/ Dracup, 19/ Aron, 20/ and Buras. 21/ Buras used dynamic programming in an engineering study to derive operating rules which determine the amounts of water to be allocated from a surface reservoir and a groundwater aquifer to several irrigation uses and to recharge. Burt also used this tool to derive decision rules for surface and groundwater use in an economic study. In another study, Burt considers optimal use of

---

16/ Burt, O.R., Economics of Conjunctive Use of Ground and Surface Water, Ph.D. Thesis, University of California, Berkeley, 1962.

17/ Burt, O.R., "Economics of Conjunctive Use of Ground and Surface Water," Hilgardia, Journal of Agricultural Science, University of California, Vol. 36, No. 2, December 1964.

18/ Leonard, R.L., Integrated Management of Ground and Surface Water in Relation to Water Importation: The Experience of Los Angeles County, Ph.D. Thesis, University of California, Berkeley, 1963 (unpublished).

19/ Dracup, J.A., The Optimum Use of Groundwater and Surface Water System: A Parametric Linear Programming Approach, Water Resources Center, University of California, Berkeley, Report 6-24, July 1, 1966.

20/ Aron, G., Optimization of Conjunctively Managed Surface and Groundwater Resources by Dynamic Programming, Water Resources Center, University of California, Davis, Project No. W132, June 1969.

21/ Buras, N., "Conjunctive Operation of Dams and Aquifers," Journal of the Hydraulics Division, American Society of Civil Engineers, Vol. 89, No. HY6, November 1963, pp. 111-131. There are some important distinctions between the study by Buras and the above studies by Burt concerning the evaluation of groundwater storage and the use of recharge under declining aquifer conditions. Discussion of these differences is omitted as the problem of groundwater depletion will not be an issue in the present study.

a single resource, groundwater, which may be in fixed supply o

partially renewable. By using the recursion relationship which

results from application of Bellman's "Principle of Optimality," Burt

derives approximate decision rules for determining groundwater use as

a function of current supply. 22/

In general, these allocation studies tend to have a long-run plan-

ning horizon and treat surface and groundwater as independent physical

resources related only through their joint contribution to economic

output. Under high water table conditions, as found in the Gallatin

Valley, surface supplies are not independent of groundwater use during

irrigation periods. Evaluation of these dependencies in an intra-

seasonal optimization model should serve to distinguish the present

study from most former allocation studies.

A recent Harvard study by Rogers and Smith 23/ includes an evalua-

tion of water table balance for a single irrigated area in East

Pakistan. It is assumed in this study that part of the diverted surface

water is lost to groundwater recharge, nonbeneficial evapotranspira-

tion, and surface runoff in each decision period. The remainder is

22/ Burt, O.R., "Optimal Resource Use Over Time with an Application
to Groundwater," Management Science, Vol. 11, No. 1, September
1964, pp. 80-93.

23/ Rogers, Peter, and D.V. Smith, "The Integrated Use of Ground and
Surface Water in Irrigation Project Planning," American Journal
of Agricultural Economics, Vol. 52, No. 1, February 1970, pp.
13-24.

employed for crop use. It is noted that tube wells are utilized for both supplemental irrigation supply and removing excess subsurface water. The model applied is assumed to be in a steady state thus simplifying the computation of fixed investment in canal facilities and wells.

A primary distinction between the above Harvard study and the current study is that stochastic variation in interseasonal surface water supply is an important consideration in determining the optimal level of well investment. Allowances are also made for the effects of intraseasonal water shortages upon crop response. All water table levels in eight subareas of the present model are evaluated explicitly in each decision period and all groundwater movement related to water table conditions is simulated.

An intraseasonal irrigation planning study involving allocation of limited surface water among competing irrigated crops in different time periods of the irrigation season was recently completed by Anderson, 24/ also reported by Anderson and Maass. 25/ Anderson

---

24/ Anderson, R.L., "A Simulation Program to Establish Optimum Crop Patterns on Irrigated Farms Based on Preseason Estimates of Water Supply," American Journal of Agricultural Economics, Vol. 50, No. 5, December 1968, pp. 1586-1590.

25/ Anderson, R.L., and A. Maass, "A Simulation Technique to Estimate Crop Production of Irrigation Projects, Based on Crop Response to Varying Schedules of Irrigation Water," International Commission on Irrigation and Drainage, R34, Question 23, pp. 547-558.

used a computer simulation program to allocate water supply initially among farms and then among crops on each farm during two-week intervals of the crop-growing season. Plant response to water supply in each period was estimated from evapotranspiration studies. Irrigation timing is also an important consideration in the current study.

Water Relationships

The literature on intraseasonal surface and groundwater relationships is relatively sparse. Some engineering studies have appeared on groundwater basin behavior, e.g., Tyson and Weber. 26/

Tyson and Weber simulated groundwater flows in an aquifer region divided into "polygonal zones" using an electric analog model. Equations for continuity and Darcy's Law were used in estimating flow relationships. The aquifer was not known to interact with surface water streams as is the case in the Gallatin Valley.

Ditch Consolidation Studies

A recent feasibility study on ditch consolidation was completed by Huszar 27/ in Colorado which has application to the present study.

26/ Tyson, H.N., Jr., and E.M. Weber, "Groundwater Management for
     the Nation's Future--Computer Simulation of Groundwater Basins,"
     Journal of The Hydraulics Division, Proceedings ASCE, Vol. 90,
     No. HY4, July 1964, pp. 59-77.

27/ Huszar, P.C., Economics of Irrigation System Consolidation,
     M.S. Thesis, Colorado State University, Fort Collins, March 1969.

Huszar evaluated the expected benefits of a local irrigation system (upper system) and the expected loss to downstream users (lower system) resulting from proposed consolidation in the upper system. Consolidation was found to be infeasible because of the reduction in important return flows to the lower system.

Institutional Studies

Institutional problems of ground-surface water transfers have been investigated by Hartman, 28/ Hartman and Seastone, 29/30/31/ Bittinger, 32/ Smith, 33/ and Snyder. 34/

28/ Hartman, L.M., "Economics and Groundwater Development," Ground-water, Vol. 3, No. 2, April 1965.

29/ Hartman, L.M. and D.A. Seastone, "Welfare Goals and Organization of Decision-Making for the Allocation of Water Resources," Land Economics, Vol. XLI, No. 1, February 1965, pp. 21-30.

30/ Seastone, D.A. and L.M. Hartman, "Resource Transfers and Economic Externalities in the Public Sector," Proceedings of the Fifty-Eighth National Tax Conference, New Orleans, Louisiana, November 8-12, 1965.

31/ Seastone, D.A. and L.M. Hartman, "Alternative Institutions for Water Transfers: The Experience in Colorado and New Mexico," Land Economics, Vol. XXXIX, No. 1, February 1963, pp. 32-43.

32/ Bittinger, M.W., "The Problem of Integrating Groundwater and Surface Water Use," Groundwater, Vol. 2, No. 3, 1964.

33/ Smith, S.C., "Problems in the Use of the Public District for Groundwater Management," Land Economics, August 1956.

34/ Snyder, J.H., Economic Implications and Appraisal of the Court Reference Procedure for Allocating Groundwater, Committee on the Economics of Water Resource Development, Report No. 5, 1957.

26

Hartman justifies public intervention when private interest
decisions regarding resource development and use have external
effects or repercussions upon other parties. 35/ He states that
intervention is necessary to ensure that welfare gains are balanced
against losses. This welfare criterion is applicable to both the
bogging and water shortage problem in the Gallatin Valley as they
both result from the waste of surface water by certain users.

Seastone and Hartman 36/ suggest that flexibility in water
transfer may be accomplished by: (1) organization systems such as
conservancy districts and ditch companies; (2) public policy devices
such as eminent domain, public ownership and condemnation; (3)
private purchase and sale transactions; and (4) through some combina-
tion of these institutional arrangements. These alternate methods
will be evaluated in respect to their applicability to the Gallatin
Valley after an optimal irrigation policy is estimated.

35/ Hartman, op. cit., p. 2.

36/ Seastone and Hartman, "Alternative Institutions for Water
     Transfer," p. 31.

CHAPTER III

HYDROLOGIC RELATIONSHIPS OF THE MODEL

The present chapter is basically concerned with the estimation of hydrologic relationships among subareas in the study region. These estimates are needed for simulating the operation of existing irrigation ditches in the subareas, and for allocating diverted irrigation water to different consumptive uses, to underground percolation and to surface water reuse in the system. Groundwater movement is an important factor in this analysis since irrigation water lost by underground percolation may have beneficial reuse in the system if it reappears as surface water, or it may cause crop damage through excessive watering of low-lying and poorly drained land in certain subareas.

This chapter is divided into four stages of analysis:

Part I is a general physical description of the Gallatin Valley and of polygonal subareas into which the study area in the Gallatin Valley was partitioned.

Part II involves the development of a hydrologic model for relating groundwater flow to water-table height conditions in different subareas. These estimated flow relationships and computed coefficients are specified in constraints of the system optimization model to follow in a later chapter of this study.

Part III evaluates the efficiency of the present irrigation system and the allocation of diverted surface water to different purposes.

Part IV is concerned with the validity of estimates obtained with the hydrologic model.

Part I: Description of Study Area

The Gallatin Valley

The Gallatin Valley floor is about 25 miles long and 20 miles wide; encompassing an approximate area of 540 square miles. Altitude ranges from 5,400 feet on the south end to 4,100 feet on the north with an average fall of 40 feet per mile along the valley floor. 1/ In pre-historic time, the area was largely covered by a lake. There still exists a subterranean dam or barrier near the head of the Missouri River which holds back underground water, keeping the water table high.2/

Crops Grown.--A number of different irrigated crops are grown in the Gallatin Valley, comprising most of the production for all of Gallatin County. In 1967 there were 2,900 acres of oats; 300 acres of corn; 754 acres of potatoes; 2,800 acres of spring wheat (other than durum); 7,300 acres of winter wheat; 11,100 acres of barley; and 45,300

1/ Hackett, et. al., Geology and Groundwater Resources of the Gallatin Valley, Gallatin County, Montana, Geological Survey Water-Supply Paper 1482, U.S. Government Printing Office, Washington, D.C., 1960, pp. 12-13.

2/ Water Resources Survey, p. 13.

acres of hay (all types) recorded for Gallatin County. 3/ Wheat,

barley, and hay are, currently, the most important irrigated crops

in the valley.

Soils.--Soils in the Gallatin Basin are derived chiefly from

Tertiary Age sediments which have been modified over time by wind and

stream action. 4/ While soils are quite varied over the area, they are

considered to be equally productive under irrigation. 5/ However,

their efficiency in water use is affected by structural differences.

This efficiency factor will be considered in Part III of this chapter

after subareas have been delineated.

Water Resources.--Surface water drainage streams in Gallatin

County are shown in Figure 3-1. The study area is also delineated

on this map.

Records of West Gallatin River runoff measured at Gallatin Gateway,

which date back to 1926, indicate a low flow in the 1934 water year of

295,700 acre-feet, a high in the 1952 water year of 714,600 acre-feet

and a 1926-1952 average of 520,000 acre-feet annual flow.

3/ Montana Agricultural Statistics, Montana Department of Agriculture
   and U.S. Department of Agriculture, Helena, Montana, Vol. XII,
   December 1968. Approximately 1,000 acres of potatoes were grown in
   1970 near Amsterdam mainly for seed. Gross income from this crop
   in 1970 was nearly $500,000. This crop is not analyzed in the
   present study because of the limited acreage involved.

4/ Water Resources Survey, p. 11.

5/ Ibid.

Figure 3-1. Map of Study Area

The Bureau of Reclamation 6/ has estimated that there is generally sufficient water for full irrigation of 67,434 acres of irrigated land above the head of White Ditch, three miles west and one mile south of Belgrade as shown in Figure 3-1, until July 1-5 on the average.

White Ditch diversion point is a minimum-flow point on the West Gallatin River; the river normally gains in flow downstream, and lower ditches serving some 3,849 acres are rarely short of supply.

Above White Ditch, however, the Bureau estimated that more than 30,000 acres would be short by July 31, and over 45,000 acres would be cut off by August 31. Their plan, discussed previously, was to meet these shortages using groundwater.

Groundwater flow in the Gallatin Valley was estimated at 240,000 acre-feet per year on the average during 1934-1953. 7/ The range was 120,000 to 320,000. This annual flow is in addition to confined groundwater in storage. However, only certain subareas have good access to this groundwater.

Selection of Subareas in the Model

It may be concluded from the Bureau study on surface water shortages and the USGS report on groundwater resources that the problem area

---

6/ Report on Three Forks Division, Appendix B, pp. 148-157.

7/ Ibid.

in Gallatin Valley is basically confined to regions bordered by the East and West Gallatin Rivers above their junction, and the west side of the West Gallatin River as shown in Figure 3-1. The upper end tends to have a water shortage problem in most years and the lower, end a bogging problem.

The study area, delineated in Figure 3-1, consists of eight separate subareas, i.e., (1) Camp Creek Hills Subarea, (2) West Gateway Subarea, (3) East Gateway Subarea, (4) Bozeman Fan Subarea, (5) East Belgrade Subarea, (6) West Belgrade Subarea, (7) Central Park Subarea, and (8) Manhattan Subarea. These eight subareas are shown in Figure 3-2. Also shown in Figure 3-2 are groundwater table contours estimated by USGS [8]/in 1953. These contours are utilized in estimating both the direction and velocity of groundwater flow through these eight subareas.

Part II: Hydrologic Model

General Design

The prime objective of the proposed simulation model is to follow the movement of groundwater through adjacent hydrologic subareas with respect to quantity of flow, destination and origin of flow. Major water inputs, outputs and losses are shown below.

---

[8]/ Hackett, op. cit., plate 5.

Figure 3-2. Map of the Study Area showing Water-Table Contour Lines during 1953.

A. Water Inputs to Area.   1. Diverted Surface Water
                            2. Runoff Surface Water
                            3. Groundwater Inflow
                            4. Precipitation

B. Water Losses in Area.   1. Evaporation Loss
                            2. Transpiration Use

C. Water Outputs from Area. 1. Surface Water Outflow
                            2. Groundwater Outflow

In constructing a complete simulation model, it is necessary to consider time and space dimensions (source and destination) of all basic flow relationships in the hydrologic system. The time dimension is important for water supply since the value of water, particularly for irrigation use, is directly related to the time it is available. Evaluation of the sources and destination of water movement is also necessary in planning optimal use of this resource.

Specific Design

The specific design of the simulation model should link related groundwater areas and streams together in time and space. An approximate method for estimating flow characteristics and the rate of groundwater movement is the use of transmissibility coefficients in conjunction with prepared groundwater contour maps of the study area. As defined by USGS, the coefficient of transmissibility is "the number of gallons of water per day, at the prevailing water temperature, that is transmitted through each mile strip extending the full saturated

thickness of the aquifer under a hydraulic gradient of one foot per mile." 9/

It is possible to estimate both the quantity of flow and the velocity of flow through an aquifer region since all of these units are specified in the coefficient of transmissibility. Other basic hydrologic properties utilized in the model are: (1) groundwater flow is perpendicular to the equipotential surface of the acquifer, and (2) the velocity of groundwater flow is proportional to the hydraulic gradient in an unconfined aquifer. These properties are discussed in most standard texts on hydrology. 10/

The groundwater contour lines in the study area shown in Figure 3-2 indicate that perpendicular groundwater outflow from each subarea may have several destinations. Furthermore, the contour lines do not have equal spacing, indicating that the slope of the hydraulic gradient also varies within each subarea. In the hydrologic model both the gradient slope and transmissibility coefficients are averaged in each subarea.

The destination of groundwater flow from each subarea is estimated as shown in Figure 3-3 by constructing directional lines or vectors perpendicular to groundwater contour lines in each subarea such that

9/ Ibid.

10/ For example see S.S. Butler, Engineering Hydrology, Prentice-Hall, Inc., New Jersey, 1957.

Figure 3-3. Illustration of Method Followed in Allocating
Groundwater Discharge.

all sections of land in each subarea are intercepted. Vectors are
rounded off to the nearest five degrees on the compass scale and
groundwater discharge from each aquifer subarea is allocated to each
vector in proportion to the approximate number of sections of land in
the path of flow. This approach is illustrated in Figure 3-3 for a
hypothetical square aquifer area. Outflow for each vector is then
assigned to the adjacent subarea or stream in the path of flow.

Computing Groundwater Discharge

Computations for groundwater outflow from each subarea source
(i) for time (t) and destinations of this flow (j) are based on the
following formulae:

$$GD_{it-1} = \frac{(CT_{it-1})(G_{it-1})(W_i)}{325,872.36} \tag{3.1}$$

$$GD_{ijt-1} = \frac{(GD_{it-1})(A_{ij})}{(A_i)} \tag{3.2}$$

$$GD_j(H_{it}) = \frac{(H_{it})(GD_{ijt-1})}{(H_{it-1})} \tag{3.3}$$

where: GD_{it-1} = the average rate of total groundwater discharge from subarea i during time (t-1) (A.F./day);

CT_{it-1} = average transmissibility coefficient in subarea i for time (t-1) (gpd/ft.);

G_{it-1} = average gradient slope of the water table in subarea i for time (t-1) (ft./mi.);

W_i = the average aquifer width of subarea i (miles);

GD_{ijt-1} = the estimated groundwater discharge from subarea i to destination (j) for time (t-1) (A.F./day);

$GD_j(H_{it})$ = a linear function for computing subsequent ground-water discharge as a function of GD_{ijt-1} ground-water discharge and subsequent water table height in subarea i for time (t) (A.F./day);

H_{it} = the average height of the water table in subarea i for time (t) (feet);

A_{ij} = the area of the aquifer in subarea i discharging groundwater to destination (j) (sq. miles);

A_i = the area of subarea i (sq. miles) for which total discharge is computed;

t = the time of the estimate; time (t-1) refers to the period in 1953 when the coefficients of transmissi-bility were computed.

The divisor, 325,872.36 in (3.1) is the number of gallons per acre foot.

Computed daily groundwater discharge data for each subarea on April 1, 1953 are shown in Table 3-1.

Formula 3.3 will be applied in a general equilibrium model pre-sented in Chapter IV for defining the relationship of subarea ground-water discharge to water table height in subsequent time periods.

Part III: Evaluation of the Gallatin Valley Irrigation System

Water Losses in Canal System

Measurements of actual daily flow in all irrigation ditches and streams in the Gallatin Valley were obtained by the Bureau of Reclamation for estimating efficiency of the system in 1952 and

TABLE 3-1. Groundwater Flow Data for Eight Subareas in 1953*

SA(i)	CT_{it-1}^a	G_{it-1}^b	W_i^c	GD_{it-1}^d	$D(j)^e$	A_i^f	A_{ij}^g	GD_{ijt-1}^h	H_{it-1}^i	H_A^j
1	11,000	80	24	64.64	SA3	37	5	8.73	12	20
1	11,000	80	24	64.64	SA6	37	24	41.93	12	20
1	11,000	80	24	64.64	SA8	37	8	13.98	12	20
2	275,000	40	1	33.67	WGR	16	16	33.67	43	55
3	275,000	40	1	33.67	WGR	8	8	33.67	43	55
4	48,000	80	7	92.28	EGR	61	21	28.33	140	150
4	48,000	80	7	82.28	SA5	61	40	53.95	140	150
5	206,000	30	7	132.42	SA7	53	40	99.94	160	200
5	206,000	30	7	132.42	EGR	53	7	17.49	160	200
5	206,000	30	7	132.42	WGR	53	6	14.97	160	200
6	206,000	30	2	37.83	SA8	14	14	37.83	160	200
7	86,000	30	8	63.18	EGR	40	40	63.18	20	25
8	130,000	30	3	35.81	EGR	8	8	35.81	25	40

[a]Average coefficient of transmissibility on April 1, 1953 (gpd transmitted through each mile strip of aquifer).

[b]Average gradient slope (feet/mile) on April 1, 1953.

[c]Average width of aquifer area (miles).

[d]Total estimated groundwater discharge from subarea i during time (t-1) (A.F./day).

[e]This is the destination of groundwater discharge from subarea i (SA or stream).

[f]Total area of subarea i (sq. miles).

[g]This is the estimated area of the aquifer in subarea i which discharges groundwater to destination j (sq. miles).

[h]Estimated groundwater discharge from subarea i to destination j for time (t-1) (A.F./day).

[i]Average water table height on April 1, 1953 (number of feet above aquifer floor level).

[j]Average alluvial thickness of aquifer (feet).

*Computed from data in Hackett, et. al.; pp. 104-157. As this discharge data is used in the present study for estimating subsequent discharge, the time period when this data was obtained will be subsequently referred to as t-1.

1953. 11/ Gross irrigation requirements for lands in the Gallatin

Valley were estimated by the Bureau as follows: 12/

	Acre-Feet Per Acre	Percent Diverted Surface Water
Effective Precipitation (80% of 5 dry years)	.72	--
Consumptive Use	1.31	32.8
Field Losses	1.21	30.2
Canal Losses	1.48	37.0
Gross Irrigation Requirement	4.72	

In the present study, irrigation will supplement average effective

precipitation during four discrete time periods of each irrigation

season. Consumptive use will vary with different crops selected as

activities in the model. Field losses will vary according to soil

conditions and field slopes in the subareas of the study. The

irrigation ditches are assumed to be about equally efficient and 37

percent of the diverted surface water will be assigned to canal

seepage in all subareas as estimated by the Bureau above.

Water Losses in Field

Irrigation losses in the field include: surface runoff, evapora-

tion from canals, and deep percolation. Field losses largely depend

11/ Gallatin Valley Hydrologic Survey, U.S. Bureau of Reclamation, Great Falls, Montana, 1954.

12/ Report on Three Forks Division, U.S. Bureau of Reclamation, Appendix M, September 1958, p. 41.

on the water storage capacity of the soil in the root zone, the slope

of the field, and the care exercised by the irrigator. These factors

will be evaluated in turn for the eight subareas of Gallatin Valley.

1. Soil Storage.--Water losses may be correlated with the

frequency of irrigation as some surface evaporation loss will occur

in each irrigation application before the root zone is penetrated. The

amount of soil storage capacity is therefore of interest in evaluating

irrigation efficiency. To estimate potential storage of available

moisture, it is necessary to know the depth of soil in the root zone

for different crops and the average water-holding capacity of the soil.

Descriptions of the soil profile in the upper two strata and major

soil types in the eight subareas are shown in Table A-1, Appendix A.

This data is used to compute available moisture storage capacity and

net irrigation requirements shown in Table 3-2 for the eight subareas.

In making these computations, the writer also utilized columns 2-6 of

a technical irrigation guide. 13/

2. Slope of the Field.--The slope of the field being irrigated,

together with the soil-profile characteristics, affects soil intake

of water, and the efficiency of water spreading. Average slopes of

irrigated fields in the eight subareas are estimated from a surface-

---

13/ Irrigation Guide for Intermountain and Foothill Area East of
     Continental Divide, USDA, Soil Conservation Service, Bozeman,
     Montana, May 1957, pp. 10-28.

TABLE 3-2. Estimated Available Moisture Storage Capacity of Soils and Net Irrigation Requirements in Eight Subareas of the Gallatin Valley.*

Subarea Name	A.M. Storage Capacity for crops		Extraction Depth for crops		Net Irrigation Requirement/Treatment[b]	
	Alfalfa	Other	Alfalfa	Other[a]	Alfalfa	Other
	-----Inches-----		-----Feet-----		----Inches----	
Camp Creek Hills	10.0	6.0	5.0	3.0	5.0	3.0
West Gateway	10.0	6.0	5.0	3.0	5.0	3.0
East Gateway	10.0	6.0	5.0	3.0	5.0	3.0
Bozeman Fan	10.0	6.0	5.0	3.0	5.0	3.0
East Belgrade	5.0	4.0	4.0	3.0	2.5	2.0
West Belgrade	5.0	4.0	4.0	3.0	2.5	2.0
Central Park	7.5	6.0	4.0	3.0	3.8	3.0
Manhattan	3.5	3.5	3.0	3.0	1.6	1.6

[a]Other crops are small grains, pasture, and hay.

[b]It is assumed that the crop is irrigated when available moisture in storage drops to 50 percent.

*Source: Table.A-1, Appendix A.

contour map of the Gallatin Valley. 14/ Estimated gross irrigation requirements in the subareas are shown in Table 3-3. The border method of irrigation is assumed for slopes up to 4 percent, and contour ditches for slopes over 4 percent.

It is apparent in Table 3-3 that from 30-50 percent of the irrigation water is lost to return flow, deep-percolation and evaporation. Since most Gallatin Valley soils are well-drained, evaporation losses should be relatively minor. Whether remaining losses would occur as surface return flow or deep-percolation losses depends on water-table conditions below the irrigated field and how long water is allowed to run on the field.

3. Irrigation Management.--It was stated in 1900 that farmers in the Gallatin Valley often permit irrigation water to run "wild" because of insufficient help. 15/ This matter was discussed with local SCS officials who were familiar with present irrigation practices in the Gallatin Valley. It was suggested that the farmers were probably only obtaining about half of the recommended efficiency level at present because of the shortage of labor and because the present system of water rights in the Gallatin Valley provided little incentive for

---

14/ Surface Contour Maps, USGS, Manhattan Quadrangle 1949 and Bozeman Quadrangle 1953.

15/ Fortier, S., "Duty of Water in the Gallatin Valley," Office of Experiment Station Bulletins, II, 1900, pp. 81-90.

TABLE 3-3. Estimated Gross Irrigation Requirements in Eight Subareas of the Gallatin Valley.*

Subarea Name	Ave. Slope of Fields 1/ (Percent)	Net Irrigation Requirements 2/ (Inches)		Basic Intake Rate 3/ All Crops (In. 1 Hr.)	Estimated Field Efficiency (Percent)		Water Loss (Inches)		Gross Irrigation Requirement (Inches)	
		Alf.	Other		Alf.	Other	Alf.	Other	Alf.	Other
Camp Creek Hills	4.0-8.0	5.0	3.0	1.8	50	50	5.0	.3.0	10.0	6.0
West Gateway	1.0-2.0	5.0	3.0	1.8	70	70	2.0	1.5	7.0	4.5
East Gateway	1.0-2.0	5.0	3.0	1.8	70	70	2.0	1.5	7.0	4.5
Bozeman Fan	1.0-2.0	5.0	3.0	1.8	70	70	2.0	1.5	7.0	4.5
East Belgrade	1.0-2.0	2.5	2.0	1.8	70	70	1.0	1.0	3.5	3.0
West Belgrade	1.0-2.0	2.5	2.0	1.8	70	70	1.0	1.0	3.5	3.0
Central Park	0.5-1.0	3.8	3.0	1.8	70	70	1.6	1.3	5.4	4.3
Manhattan	0.5-1.0	1.6	1.6	3.0	65	65	0.9	0.9	2.5	2.5

1/ Surface contour maps, USGS, Manhattan Quadrangle 1949 and Bozeman Quadrangle 1953.

2/ Taken from Table 3-2.

3/ Choriki, R.T., The Influence of Different Soil Types, Treatments and Soil Properties on the Efficiency of Water Storage, M.S. Thesis, Montana State University, Bozeman, August 1959, p. 53.

*Source: Irrigation Guide, loc. cit., cols. 10, 14 and 15.

users to be efficient in irrigation. The recommended higher efficiency level would also entail some increase in cost for labor and ditch management which may not be justified at the present time.

In recognition of the current management problem, the writer reasoned that the suggested low field efficiency level was a rational irrigation strategy on the part of farmers given the present system of fixed water rights, but that under a more flexible system allowing exchange of water rights, the farmers would likely increase their efficiency in water use to the level recommended in the irrigation guide cited previously. 16/ The difference in efficiency levels may then be interpreted as a cost imposed on society because of the present institutional system of water rights along with other costs resulting from the improper allocation of resources. It was therefore concluded that the investigation of water use in the Gallatin Valley should include an evaluation of both levels of efficiency to determine the effect on regional income.

The problem of surface return flow is also related to management as the irrigator could prevent any surface runoff with reasonable care in the regulation of water supply to the field. Since the subareas analyzed in this study are relatively large, there should not be much

---

16/ Irrigation Guide, op. cit., pp. 1-9.

net surface return flow from each subarea even though some individual

fields have surface runoff. It will therefore be assumed for each

subarea that there should be very little loss resulting from surface

return flow; the bulk of surface outflow is assumed to be caused by

water-table drainage which is predicted with the hydrologic model

formulated in this chapter. 17/ Consequently, all water losses in

field irrigation will be assigned to deep-percolation in the analysis

of water use in the Gallatin Valley.

Part IV: Validation of the Hydrologic Model

The hydrologic model formulated in this chapter could only be

tested for the year 1953 as estimates of groundwater discharge were

not available for other years. A comparison of predicted groundwater

discharge in 1953 from each subarea and USGS estimates of this discharge

is shown in Table 3-4. The USGS estimates of discharge are based on

measurements of gain in streamflow taken during 1953. Since the Camp

Creek Hills Subarea and the Bozeman Fan Subarea are not adjacent to

a major stream, USGS did not provide estimates for these subareas.

Except for Central Park, predicted groundwater discharge from the

subareas in Table 3-4 compares closely with USGS estimates of this

discharge during 1953. The large discrepancy for Central Park is

---

17/ USGA has also estimated that very little diverted water returns
     to the river as waste surface return flow in this region, at
     least during July and August. Hackett, op. cit., pp. 116-117.

TABLE 3-4. Model Predictions and USGS Estimates of Annual Groundwater
Discharge For 1953.*

Subarea (i)	Average Water Table Height (Feet)	Average Gradient Slope (Ft/Mi)	Predicted Discharge (Acre-Feet)	Estimated Discharge (Acre-Feet)
Camp Creek Hills (1)	12	80	23,594	na[a]
West Gateway (2)	43	40	12,290	25,000[b]
East Gateway (3)	43	40	12,290	
Bozeman Fan (4)	140	80	30,032	na[a]
East Belgrade (5)	170	80	85,412	135,000[b]
West Belgrade (6)	170	80	24,401	
Central Park (7)	20	30	23,061	300,000[c]
Manhattan (8)	25	30	12,471	14,000

[a]Not available.

[b]Estimate is aggregated over this and the following subarea.

[c]Estimate includes a considerable amount of surface water discharge since groundwater discharge is restricted by a fault.

*Source: Hackett, et. al., pp. 136-151.

explained by the geologic fault in this subarea and by large inflow of groundwater from outside the study region. Most of the groundwater leaving Central Park appears as surface water due to the severe restriction of the fault.

Most of the agricultural land in Central Park is confined to the interior part of the subarea just below East Belgrade Subarea. This interior area, encompassing about 19 square miles, is not affected by inflow from outside the study region and receives all of its ground-water recharge from the East Belgrade Subarea. During the remainder of the present study, Central Park Subarea will be defined as this interior area only.

Predicted discharge rates comparable to those shown in Table 3-4 will be utilized to predict the change in groundwater discharge associated with reallocation of water resources in the study area. As stated previously, the volume of groundwater discharge has important implications for optimal allocation of water resources in the Gallatin Valley and therefore should be evaluated in conjunction with any irrigation system optimization.

Evaluation of Recharge to Subareas

The hydrologic model also explains recharge to subareas where groundwater discharge from adjacent subareas and irrigation percolation losses are the major sources of recharge. However, there are other important sources of recharge for certain subareas which are not explained by the model.

It is stated in the Hackett report that the two Gateway subareas
and the Bozeman Fan subarea are partially recharged by runoff of small
streams entering these subareas from adjacent highlands. This outside
recharge may be accounted for in the hydrologic model by treating pre-
season water tables as a parameter. It is assumed that these small
streams are largely spring runoff, and irrigation percolation losses
are the major source of recharge to these three upper subareas during
most of the season.

West Gallatin seepage loss is a major source of recharge in the
two Belgrade subareas. This stream is influent for about 4 miles
adjacent to these subareas throughout the season. Stream loss in this
reach was estimated by USGS to be about 10 percent of flow, comprising
a total loss of 63,000 acre-feet in 1952 and 37,000 acre-feet in
1953. 18/ The 10 percent estimate will be used in the hydrologic model
for computing recharge to Belgrade subareas.

Both the West Gallatin and East Gallatin Rivers are effluent below
the Belgrade subareas, and are not a source of recharge to the
Manhattan and Central Park subareas.

General Appraisal of Hydrologic Model

The hydrologic model for predicting groundwater movement is not
claimed to be a particularly precise model for this purpose. However,

18/ Hackett, op. cit., p. 143.

it does consider the key variables affecting groundwater flow and provides useful approximations of flow relationships within the system. It is assumed that the approximations are adequate to yield significant improvements in the integrated ground-surface water model over a model that ignores the hydrologic interdependencies. In a sense, a model that assumes away hydrologic interdependencies also tacitly uses quantitative measures of flow among subareas--all flows are taken as zero.

CHAPTER IV

A GENERAL EQUILIBRIUM OPTIMIZATION MODEL

A relatively simple hydrologic model for approximating ground-
water flow as a linear function of water table height was presented
in Chapter III. The derived linear functions for eight interrelated
hydrologic subareas of the Gallatin Valley provide the necessary
link between the surface and groundwater system for general equilibrium
analysis of the overall optimal allocation of water in the study area.

The nature of water response relationships in the criterion func-
tion of the applied model will be governed by selection of crop
activities which are adaptable to the Gallatin Valley study area. For
the present, however, we need only be concerned with some hypothetical
water-response relationship for each of the eight subareas. This
response function is assumed to imply the most profitable combination
of crop activities at each level of water supply. The function could
be computed in practice by a priori specification of profit levels
by crops at various levels of water supply in a parametric program-
ming model, and then changing the water supply by increments. A
separate analysis would be required for each subarea.

In each subarea, water response depends on the timing of
irrigation deliveries as well as the total quantity of water

delivered during the irrigation season. For simplification, the
time horizon for water deliveries in the present study is divided
into four discrete time periods of the irrigation season: (1)
June 1 to July 1; (2) July 1 to August 1; (3) August 1 to September 1;
and (4) September 1 to October 1. In each of the four time periods
(subsequently identified by $t=1,...,4$), water response will be jointly
determined by the volume of water delivered in that period and in all
other periods.

Water supply to each subarea may come from various sources
depending upon access of the respective subarea to alternate streams
and groundwater reserves. Of course, water needs can be at least
partially met by natural precipitation and soil moisture in storage
at the beginning of the irrigation season. Since the latter sources
are not generally subject to control by the decision-maker, they
are not of immediate concern in formulation of the model. Delinea-
tion of water supply by ground and surface water sources is necessary
in formulation of the model for the following reasons: (1) these
two sources of supply usually have quite different procurement
costs and efficiencies in delivery; (2) their respective use is of
paramount concern in analysis of the hydrologic system, particularly
for control of water tables; and (3) some of the subareas are entirely
dependent upon surface water supply for their irrigation requirements.
The source of water supply, the subarea served and the time period
of delivery are identified in the model criterion function by

G_{it} in the case of groundwater, and by S_{it} in the case of surface water. The first subscript, i, denotes the subarea served; i=1,...,8; and the second subscript, t, denotes the time period of delivery, t=1,...,4. All surface water supply variables are prefixed by the symbol, E_i, in the criterion function to indicate possible differences in conveyance efficiency among the eight subareas. Delivery efficiency and also field efficiency of irrigation water is subject to control in project planning; for simplification, however, these efficiency levels are specified to be parameters in the theoretical model formulation.

The volume of groundwater supply available for use in each subarea is restricted by the level of development of irrigation wells. The wells represent long-term capital investments, and their optimal level of development is dependent upon both intraseasonal and interseasonal expectations of surface water supply. For simplification in presentation of the theoretical model, it is assumed that all water supply in each time period of analysis is known with certainty. The empirical model, however, will consider the problem of stochastic variation in the interseasonal surface water supply.

Net welfare is assumed to be additive over all subareas in the model since the output of each crop in the Gallatin Valley is an insignificant part of supply in the national market. Net welfare for a representative i^{th} subarea may be defined in terms of the following criterion function:

$$V_i(E_i S_{it}, G_{it}, K_i, H_{it})$$

where S_{it} denotes the vector of variables with components determined by both stream source and timing of stream diversions for irrigation, E_i is a parameter scalar representing delivery efficiency, G_{it} denotes the vector for variable groundwater supply with components determined only by time, K_i is a scalar denoting the level of irrigation well development governing effective groundwater supply, and H_{it} denotes a vector of variable water table levels. It is assumed that the water table level is not limiting groundwater supply for irrigation in the present example. In the applied version of this model, the empirical problem resulting from the water table being too high for normal production response to irrigation may be avoided by assuming this critical water table level to be an irrational production region since it may be controlled by groundwater pumping. As the groundwater table height would affect production only if above some threshold level, this influence may be evaluated by imposing an inequality constraint on the maximum level that will be tolerated. However, a more general applied model would measure the rapid decline in production resulting from progressive saturation of remaining soil space above the critical water table level. A detailed statement of all variables affecting V_i in the applied model for the Gallatin Valley area is given in Chapter VL.

Regardless of whether production is constrained by the water table level or not, the criterion function is theoretically

subject to diminishing returns in spite of the many linearities

assumed in the model regarding profits, costs, and other coefficients

for each activity. With increasing employment of certain activities

the rate of increase in net product would tend to decline as a result

of using less productive land and incurring higher distribution costs

for water supply. Crop returns are also restricted by acreage limita-

tions imposed by the government allotment program and crop rotation

requirements. The criterion function is defined to be a concave

surface due to the effect of diminishing returns.

Interdependencies that exist among different subareas regarding

the use of water resources and related hydrologic interactions are

specified in side conditions of the model. In view of the rather

complex nature of these interrelationships, the set of definitions

and notation required for statement of the mathematical constraints

is quite detailed.

Specifications of Terms in the Model

Functions: Defined for $i=1,\ldots,8$;

V_i = the net welfare or benefit obtained in the ith subarea assuming
that an optimal production program is followed. Implicit in
V_i are linear cost functions for such items as amortized costs
of well development, irrigation labor costs, crop enterprise
costs, variable pumping costs, and ditch operation and maintenance
costs.

Parameters: Defined for $i=1,\ldots,8$; $t=1,\ldots,4$;

E_i = a coefficient for determining the net amount of diverted surface water delivered at the farm headgate (proportion);

H_i^O = the initial water table height in subarea i prior to the growing season, i.e., for t=0. H_i^O is assumed to be a parameter in the present study as average annual recharge far exceeds the expected withdrawal of groundwater for irrigation use each season.

S_t^{WG} = West Gallatin surface supply available for use in time t (acre-feet);

S_t^{MC} = Middle Creek surface supply available for use in time t (acre-feet);

S_t^{SC} = Sourdough Creek surface supply available for use in time t (acre-feet);

S_t^{EG} = East Gallatin surface supply available for use in time t less appropriated water outside the study area and less S_t^{SC} tributary inflow (acre-feet);

S_t^{CC} = Camp Creek surface supply available for use in time t (acre-feet);

H_t^{WG} = the average surface water height of the West Gallatin River during time t (feet); 1/

a_i = a coefficient for estimating total groundwater outflow of subarea i in relation to water table height in subarea i (acre-feet);

a_{ji} = a coefficient for estimating total groundwater outflow of subarea j entering subarea i from water table height in subarea j (acre-feet);

b_i = a percolation-loss coefficient for field irrigation (proportion);

W_i = the average width of aquifer i (miles);

L_i = the average length of aquifer i (miles);

0.15 = the average coefficient of water storage for all subareas; and

C = average monthly pumping capacity of each well (acre-feet).

1/ This parameter is needed for estimating groundwater discharge as surface water from adjacent subareas. The stream level is compared with subarea water table height in determining the volume of groundwater discharge as surface water.

Variables: Defined for i=1,...,8; t=1,...,4

S_{it}^{WG} = the number of acre-feet of surface water diverted from the West Gallatin to subarea i during time t;

H_{it} = average water table height in the ith subarea during time t (feet above aquifer floor);

S_{it}^{MC} = the number of acre-feet of surface water diverted from Middle Creek to subarea i during time t;

S_{it}^{SC} = the number of acre-feet of surface water diverted from Sourdough Creek to subarea i during time t;

S_{it}^{EG} = the number of acre-feet of surface water diverted from the East Gallatin to subarea i during time t;

S_{it}^{CC} = the number of acre-feet of surface water diverted from Camp Creek to subarea i during time t;

G_{it} = the number of acre-feet of groundwater pumped in subarea i during time t; and

K_i = the number of 2 cfs wells developed in subarea i.

Transformations: Defined for i=1,...,8; t=1,...,4 2/

$H'_{it} = H_{it} - H_t^{WG}$, ($H'_{it}$ is the differential water table height affecting groundwater discharge to the West Gallatin--feet);

$\beta_i = 1 - E_i + b_{pi}E_i$, ($\beta_i$ is the fraction of surface water diverted which is lost in conveyance and field percolation-- proportion);

$\delta_i = 1 - b_{pi}$, (δ_i is the field consumptive use coefficient--proportion);

2/ These transformations are intended to simplify use of notation in the model. They represent combinations of parameters and variables that have been defined earlier.

$A_i = (0.15W_iL_i)$ (640 acres/mi.^{2}) (1 ft.), (A_i is the aquifer storage
volume per unit water table height
in subarea i--acre-feet);

$C_{1t} = S_t^{WG}$, (C_{1t} is total surface water supply available for use
in subareas 1, 2 and 3--acre-feet);

$C_{2t} = S_t^{WG} + S_t^{MC} + S_t^{SC}$, ($C_{2t}$ is total surface water surface available
for use in subarea 4--acre-feet);

$C_{3t} = S_t^{WG} + S_t^{MC} + S_t^{EG}$, ($C_{3t}$ is total surface water supply available
for use in subarea 5--acre-feet);

$C_{4t} = 0.90 \; S_t^{WG} + S_t^{CC}$, ($C_{4t}$ is total surface water supply available
for use in subarea 6--acre-feet);

$C_{5t} = 0.90 \; S_t^{WG} + S_t^{EG}$, ($C_{5t}$ is total surface water supply available
for use in subarea 7--acre-feet); and

$C_{6t} = 0.90 \; S_t^{WG} + S_t^{CC}$, ($C_{6t}$ is total surface water supply available
for use in subarea 8--acre-feet).

Explanation of Terms in Model

The relevance of these specified parameters, variables and trans-
formations in the model is illustrated by the following flow diagrams.
All surface flows in the system are shown in Figure 4-1.

It is evident in Figure 4-1 that there are five streams supplying
eight subareas in the study region. The West Gallatin River, however,
is the only stream accessible to all subareas. Stream flow constraints
are respectively: E. Gallatin River, S_t^{EG}, W. Gallatin River, S_t^{WG},
Sourdough Creek, S_t^{SC}, Middle Creek, S_t^{MC}, and Camp Creek, S_t^{CC}. Combined
stream constraints for each subarea were denoted earlier by C_i in the
list of transformations.

These stream constraints are at least partially relaxed by ground-
water discharge flows illustrated in Figure 4-1. The discharge flows

59

Figure 4-I. Flow diagram for study area, showing direction of stream flows, sources of diversion to subareas, and surface-water return flows.

may be rediverted for irrigation use in adjacent subarea i or in other downstream subareas.

Recall from Chapter III that groundwater outflows are functionally related to groundwater heights in subareas adjacent to streams. As defined in the hydrologic model of Chapter III, groundwater outflows arise from groundwater movement and they occur where the groundwater table intercepts a stream bed or other low elevation. The general direction of groundwater movement in the system is shown in Figure 4-2. This movement is based on groundwater flows during 1952-53 shown in Table 3-1, Chapter III. The only subareas obtaining much groundwater recharge from the West Gallatin River are East and West Belgrade as shown in Figure 4-2. West Gallatin stream loss to these subareas is estimated to be about 10 percent of flow near Cameron Bridge. 3/ The other streams in the study area are assumed to have no effect on groundwater recharge or discharge to subareas.

It may be recalled from Chapter III that the exterior part of the Central Park Subarea receives about 190,000 acre-feet of recharge annually from outside the study area. However, this part of Central Park has been omitted from the present study as very little agricultural land is involved in the region affected.

With these exceptions, all groundwater movement is determined endogenously in relation to water table heights of adjacent subareas in the study area.

3/ Hackett, op. cit., p. 143.

61

East Gallatin River

West Gallatin River

Camp Creek Hills (1)

East Gateway (3)

West Gateway (2)

Sourdough
Creek

Middle
Creek

Camp
Creek

Bozeman Fan (4)

East Belgrade (5)

West Belgrade (6)

Central Park (7)

Manhattan (8)

Missouri
River

LEGEND

Groundwater Flow _ _ _ _ _ _ _
Stream Flow

Figure 4-2. Flow model showing directions of ground water movement in study area.

The hypothesized water budget for a representative subarea i of

the Gallatin Valley for time t is shown in Figure 4-3. Three water

inputs entering subarea i during time t are precipitation, P_{it}, diverted

surface water, S_{it}, and groundwater inflow from an adjacent subarea. j,

$a_{ji}H_{jt}$. Only the volume of surface water diverted is under direct

control of the decision maker. Groundwater inflow may be indirectly

controlled to the extent that H_{jt} is regulated in subarea j adjacent

to subarea i. It is not possible for reverse flows to occur if

$H_{it} > H_{jt}$ as the aquifer area in the Gallatin Valley is steeply

inclined towards the lower northern end of the valley. The coefficient,

a_{ji}, was estimated formerly in Chapter III for all subareas to deter-

mine groundwater outflow as a linear function of water table height in

each subarea. 4/

All water inputs to subarea i are assigned to evapotranspiration

use and to groundwater storage as shown in Figure 4-3. As explained

in Chapter III, Gallatin Valley soils are generally well-drained and

no appreciable surface runoff should occur during the irrigation season.

All subarea discharge to the stream is assumed to result from high

water table drainage. It may be noted in Figure 4-3 that all precipi-

tation inflow is assigned to evapotranspiration use. Irrigation require-

ments are determined from the relationship, "evapotranspiration demand

less precipitation." However, there are conveyance losses incurred in

4/ See Table 3-1, Chapter III.

Figure 4-3. Flow chart showing computation of a hypothetical water budget for a representative subarea "i" during time "t".

63

delivering the diverted surface water to the farm headgate, and there

are further field percolation losses involved in the irrigation of

crops. The total surface water loss assigned to groundwater storage

is $\beta_i S_{it}$ as shown in Figure 4-3. Only $(1-\beta_i)S_{it}$ is left over for

evapotranspiration use. All groundwater inflow to subarea i is

alloted to groundwater storage. Groundwater may be used to supplement

surface water inflow if the latter is in short supply or if it is

desirable to lower the water table in subarea i, The benefits obtained

by irrigation pumping to keep the water table below the critical

bogging level may greatly exceed the cost of pumping even though cheap

surface water is available. Pumping for control of water tables is

an important strategy in the Gallatin Valley study.

All groundwater outflow from subarea i in Figure 4-3 is a linear

function of the average height of the water table. Part of the ground-

water outflow may be converted to surface flow if the stream level is

lower than the water table in subarea i. The coefficient, a_i, estimates

total groundwater outflow from subarea i in conjunction with H_{it}. The

difference between H_{it} and the stream level is expressed as H'_{it}. There-

fore, $a_i H'_{it}$ represents the amount of surface water outflow and the

remainder, $a_i H_{it} - a_i H'_{it}$, is the estimated amount of groundwater out-

flow discharged from subarea i.

The methodology followed in assigning water flows for subarea i

as illustrated in Figure 4-3 is the same for all subareas in the model.

However, not all subareas are adjacent to streams and some do not have access to groundwater for irrigation use. A detailed explanation of the effects of water table height upon groundwater flows and of the method followed in estimating direction of all flows within the study area was presented in Chapter III.

Water table height is viewed as a key variable in the theoretical model as it acts as a binding constraint on crop production in bogged areas, and it also determines the amount of groundwater return flow occurring in the system available for diversion again. The basic balance equation for defining water table height in any time period is:

$$H_{it} = H_{it-1} + \text{Recharge} - \text{Discharge}.$$

Using notation provided earlier in this chapter, the balance equation is explicitly defined as:

$$H_{it} = H_{it-1} + \frac{\beta_i S_{it} + \sum_j a_{ji} H_{jt}}{A_i} - \frac{a_i H_{it} + \delta_i G_{it}}{A_i} \quad (i \neq j) \qquad (4.1)$$

The first term, H_{it-1}, denotes initial water-table height. Recharge from diverted surface water is $\beta_i S_{it}$. Recharge from groundwater inflow is $\sum_j a_{ji} H_{jt}$. Discharge of groundwater is $a_i H_{it}$ and discharge of pumped water to evapotranspiration use is $\delta_i G_{it}$. All flow variables are divided by A_i to define their effect on the aquifer

in terms of change in H_{it}. As stated previously, these flows are irreversible because of the high degree of slope throughout the whole aquifer region. 5/

Complete Model Specification

The criterion function in the model is defined as:

$$\text{Max} \sum_{i=1}^{8} V_i [E_i S_{i1}^{WG}, \ldots, E_i S_{i4}^{WG}; \ E_i S_{i1}^{MC}, \ldots, E_i S_{i4}^{MC}; \ E_i S_{i1}^{SC}, \ldots, E_i S_{i4}^{SC};$$

$$E_i S_{i1}^{EG}, \ldots, E_i S_{i4}^{EG}; \ E_i S_{i1}^{CC}, \ldots, E_i S_{i4}^{CC}; \ G_{i1}, \ldots, G_{i4};$$

$$H_{i1}, \ldots, H_{i4}; \ K_i] \ \underline{6/}$$

subject to the following 27 subsets of constraints: (defined for $t=1, \ldots, 4$)

$$(1) \quad S_{1t}^{WG} \leq C_{1t}; \tag{4.2}$$

$$(2) \quad S_{1t}^{WG} + S_{2t}^{WG} - a_2 H_{2t}' \leq C_{1t}; \tag{4.3}$$

$$(3) \quad S_{1t}^{WG} + S_{2t}^{WG} + S_{3t}^{WG} - a_2 H_{2t}' - a_3 H_{3t}' \leq C_{1t}; \tag{4.4}$$

$$(4) \quad \sum_{i=1}^{4} S_{it}^{WG} + S_{4t}^{MC} + S_{4t}^{SC} - a_2 H_{2t}' - a_3 H_{3t}' \leq C_{2t} \tag{4.5}$$

$$(5) \quad \sum_{i=1}^{4} S_{it}^{WG} + S_{4t}^{MC} + S_{5t}^{MC} + S_{5t}^{EG} - \sum_{i=2}^{3} a_i H_{it}' - a_5 H_{5t}' + 0.1 S_t^{WG} \leq C_{3t}; \tag{4.6}$$

$$(6) \quad \sum_{i=1}^{6} S_{it}^{WG} + S_{6t}^{CC} - \sum_{i=2}^{3} a_i H_{it}' - \sum_{i=5}^{7} a_i H_{it}' + 0.1 S_t^{WG} \leq C_{4t}; \tag{4.7}$$

5/ Chapter III, p. 27.

6/ The vector, H_{it}, is not shown in the criterion function of the empirical model as maximum water table levels will be controlled so that they do not interfere with production response.

$$(7) \quad \sum_{i=1}^{7} S_{it}^{WG} + S_{5t}^{EG} + S_{7t}^{EG} - \sum_{i=2}^{3} a_i H_{it}' - \sum_{i=5}^{7} a_i H_{it}' + 0.1 \ S_t^{WG} \leq C_{5t}; \quad (4.8)$$

$$(8) \quad \sum_{i=1}^{8} S_{it}^{WG} + S_{5t}^{CC} + S_{7t}^{CC} - \sum_{i=2}^{3} a_i H_{it}' - \sum_{i=5}^{8} a_i H_{it}' + 0.1 \ S_t^{WG} \leq C_{6t}; \quad (4.9)$$

$$(9) \quad (A_1 + a_1)H_{1t} - A_1 H_{1t-1} - \beta_1 S_{1t}^{WG} + \delta_1 G_{1t} = 0; \quad (4.10)$$

$$(10) \quad (A_2 + a_2)H_{2t} - A_2 H_{2t-1} - a_{12} H_{1t} - \beta_2 S_{2t}^{WG} + \delta_2 G_{2t} = 0; \quad (4.11)$$

$$(11) \quad (A_3 + a_3)H_{3t} - A_3 H_{3t-1} - \beta_3 S_{3t}^{WG} + \delta_3 G_{3t} = 0; \quad (4.12)$$

$$(12) \quad (A_4 + a_4)H_{4t} - A_4 H_{4t-1} - \beta_4 (S_{4t}^{WG} + S_{4t}^{MC} + S_{4t}^{SC}) + \delta_4 G_{4t} = 0; \quad (4.13)$$

$$(13) \quad (A_5 + a_5)H_{5t} - A_5 H_{5t-1} - a_{15} H_{1t} - \beta_5 (S_{5t}^{WG} + S_{5t}^{EG} + S_{5t}^{MC}) + \delta_5 G_{5t} = 0; \quad (4.14)$$

$$(14) \quad (A_6 + a_6)H_{6t} - A_6 H_{6t-1} - a_{46} H_{4t} - \beta_6 (S_{6t}^{WC} + S_{6t}^{CC}) + \delta_6 G_{6t} = 0; \quad (4.15)$$

$$(15) \quad (A_7 + a_7)H_{7t} - A_7 H_{7t-1} - a_{57} H_{5t} - \beta_7 (S_{7t}^{WG} + S_{7t}^{EG}) + \delta_7 G_{7t} = 0; \quad (4.16)$$

$$(16) \quad (A_8 + a_8)H_{8t} - A_8 H_{8t-1} - a_{18} H_{1t} - a_{68} H_{6t} - \beta_8 (S_{8t}^{WG} + S_{8t}^{CC}) + \delta_8 G_{8t} = 0; \quad (4.17)$$

$$(17) \quad G_{it} - CK_i \leq 0 \qquad (i=1,\ldots,8); \quad (4.18)$$

$$(18) \quad G_{it} \leq H_{it} G_i^o \qquad (i=1,\ldots,8); \quad (4.19)$$

$$(19) \quad E_i S_{it}^{WG} \geq 0 \qquad (i=1,\ldots,8); \quad (4.20)$$

$$(20) \quad E_i S_{it}^{MC} \geq 0 \qquad (i=4,5); \quad (4.21)$$

$$(21) \quad E_i S_{it}^{MC} = 0 \qquad (i=1,2,3,6,7,8); \quad (4.22)$$

$$(22) \quad E_i S_{it}^{SC} \geq 0 \qquad (i=4); \quad (4.23)$$

(23) $E_i S_{it}^{SC} = 0$ $(i=1,2,3,5,6,7,8)$;   (4.24)

(24) $E_i S_{it}^{EG} \geq 0$ $(i=5,7)$;   (4.25)

(25) $E_i S_{it}^{EG} = 0$ $(i=1,2,3,4,6,8)$;   (4.26)

(26) $E_i S_{it}^{CC} \geq 0$ $(i=6,8)$;   (4.27)

(27) $E_i S_{it}^{CC} = 0$ $(i=1,2,3,4,5,7)$;   (4.28)

Explanation of Model Specification

Net welfare accruing to the system is defined in the criterion function as the sum of net welfare values obtained in all subareas. Note that net welfare is not summed over different time periods of the irrigation season as it is not possible to evaluate the marginal product of water in one time period independent of water supply in other periods of the season.

Both surface and groundwater variables are defined in the criterion function as water resources available on the farm premises for irrigation use. Actual on-farm deliveries of surface water are somewhat less than the volume of stream diversions because of canal losses incurred in distribution to farms. Distribution efficiencies in each subarea are denoted by E_i. Groundwater is assumed to be pumped on the farm premises and is therefore not subject to canal distribution loss. It may be noted in the criterion function that

all surface water variables are identified as to stream source since

the question of which streams should be diverted where, is important

in optimal allocation.

Water table height in each time period is treated as a separate

variable in the optimization model but is, of course, subject to

stringent equality constraints. This variable can have three effects

on optimization policy: (1) water table height is a primary regulator

of groundwater return flows and groundwater supply in the system, (2)

water table height may provide subirrigation benefit for different crops

if maintained at a certain level, and (3) water table height may

restrict crop production if allowed to reach a critical bogging level. 7/

The water table effects with respect to return flows are explicitly

defined in the set of model constraints.

The level of well development denoted by K_i is included among the

variables in the criterion function as the model is assumed to be in a

steady state. The efficiency of ditch development denoted by E_i could

also be treated as a variable if one wanted to analyze the benefits

obtained from ditch consolidation and lining in the system. For simpli-

fication, the present model does not consider this alternative in irri-

gation development; hence, E_i is defined as a parameter in the

criterion function.

7/ It has been assumed that this represents an irrational production
    region and maximum water table levels are restricted by side condi-
    tions in the empirical model. Subirrigation benefits could be
    analyzed with this model by simply putting upper and lower bounds
    on H_{it}. However, information was not available on the hydrologic
    requirements for subirrigation of crops and their production rela-
    tionships.

The logic involved in defining the first eight constraint subsets of the model follows directly from Figure 4-1. Care has been taken to ensure that subarea diversions do not exceed accessible stream supplies in the system. Return flows to the West Gallatin River denoted by $a_i H'_{it}$ correspond to the direction of groundwater movement shown in Figure 4-2. These return flows only apply to subareas adjacent to the West Gallatin.

Constraint subsets (9), (10), (11), (12), (13), (14), (15) and (16) are essentially water table height balance equations for the eight subareas in the model. Groundwater inflows to these subareas are defined so as to correspond to the direction of groundwater movement shown in Figure 4-2. General specification of these balance equations was explained in detail earlier in this chapter.

Constraint subset (17) defines pumping capacity of wells in relation to development in each subarea for each time period of the irrigation season. Capacity is specified as the product of average pumping capacity per well, C, and the number of wells developed in each subarea, K_i. As all wells are designed to produce an output of 2 cfs, there is no different in C among the eight subareas. The quantity of groundwater pumped, G_{it}, is limited by CK_i in each time period of the irrigation season.

Constraint subset (18) specifies that groundwater pumped in each subarea for each time period cannot exceed groundwater storage,

$H_{it}G_i^o$. The parameter G_i^o is defined as the volume of water stored

per foot of water table height in subarea i. Water table height,

H_{it}, was previously defined as a variable in the model.

The remaining nine constraint subsets specify that all model

variables cannot be negative. Some variables are set equal to zero

to simplify writing of the objective function, i.e., they are included

in the general notation of the objective function but do not actually

enter for some subscript combinations.

Economic Interpretation of the General Equilibrium Model

A general equilibrium model for optimal ground and surface water

use in the Gallatin Valley study area has been formulated and justified

primarily from a physical standpoint. Only the obvious economic

implications of the model formulation have been discussed so far in

the analysis. It was apparent at this stage of the present study

that further economic analysis of the model would be useful both for

explaining the operation of the model and for providing a better

perspective of the water allocation problem in the Gallatin Valley.

To illustrate how these economic implications may be derived,

assume a nonlinear objective function, Max $f(x_1,\ldots,x_n)$, subject to a

matrix of linear constraints, $\underline{A}\ \underline{x} \leq \underline{b}$.

The above problem is comparable to the classical maximization

problem in production economics where the production surface is

subject to diminishing returns, and the resource inputs are constrained

at some fixed level. A heuristic explanation of the application of the Kuhn-Tucker Theorem to this classical problem is given in subsequent discussion to acquaint the reader with the general use of this theorem before it is applied to the particular water-allocation problem under investigation.

In deriving economic implications of important parameter changes in the above example of a nonlinear model, it is first assumed that the optimal solutions for this maximization problem are known before the respective changes in parameter levels. According to an important theorem developed by Kuhn and Tucker, 8/ if a certain program x_1^o, x_2^o, ..., x_n^o maximizes a nonlinear objective function $f(x_1, ..., x_n)$ subject to m linear restrictions, then there must exist some nonnegative numbers P_1, P_2, ..., P_m such that for all values of i, i=1, 2, ..., n, either

(1) $x_i^o = 0$ (activity i is not used) and

$$\frac{\partial f}{\partial x_i} - \sum_j^m a_{ij} P_j \leq 0 \tag{4.29}$$

or

(2) $x_i^o > 0$ (activity i is used) and

$$\frac{\partial f}{\partial x_i} - \sum_j^m a_{ij} P_j = 0 \tag{4.30}$$

8/ Kuhn, H.W. and A.W. Tucker, "Nonlinear Programming," Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, University of California, Berkeley, 1951, pp. 481-492.

where: $\dfrac{\partial f}{\partial x_i}$ = marginal revenue product of x_i in the objective function;

a_{ij} = the coefficient value for x_i in the jth constraint;

x_i^o = the level of x_i in the optimal solution;

P_j = the imputed price of the resource constrained in the jth row 9/; and

$\displaystyle\sum_{j}^{m} a_{ij} P_j$ = the imputed value of resources employed in production of one unit of x_i.

It may be noted that there is an imputed price associated with each constraint row denoted by P_j. The imputed value, P_j, exceeds zero for constrained resources where rows are $\leq$ a particular value, and $P_j = 0$ if resource j is not constrained. In the case of an exact equality constraint, e.g., where it is desirable to fully employ a certain resource, the associated imputed price of the resource may also be negative if the constrained resource restricts returns in the objective function due to it being forced into the solution at a high level.

Equations (4.29) and (4.30) are Kuhn-Tucker conditions for a local optimum solution of the maximization problem hypothesized above. It is stated in equation (4.30) that if activity x_i is used, the unit imputed cost of x_i, estimated by $\displaystyle\sum_{j}^{m} a_{ij} P_j$, exactly equals the marginal-revenue product of x_i in the objective function denoted by $\dfrac{\partial f}{\partial x_i}$. If activity x_i is not used in the optimal solution, the unit imputed cost of x_i determined by $\displaystyle\sum_{j}^{m} a_{ij} P_j$ may be either equal to or greater than the marginal-revenue product of x_i as shown in condition (4.29).

9/ This imputed price has the same interpretation as the "shadow price" in linear programming.

Kuhn-Tucker theory may also be utilized to derive economic inter-
pretations in the general equilibrium model formulated in this chapter
provided first that activity relationships in the model are delineated
in comparable form to Kuhn-Tucker conditions set out in equations
(4.29) and (4.30). For simplification, it is assumed the model will
only include the first two subareas which jointly depend upon surface
water supply from the West Gallatin River. 10/ Subarea 1 lies above
subarea 2 and discharges groundwater to the latter subarea. Subarea 2
discharges groundwater to the stream as it is located adjacent to the
stream bank. The economic interpretations of relationships in these
two subareas are directly applicable to the remaining six subareas in
the model.

Since the model is now only composed of the first two subareas,
the criterion function may be simply defined as:

$$\text{Max} \sum_{i=1}^{2} V_i [E_i S_{i1}, \dots, E_i S_{i4}; \ G_{i1}, \dots, G_{i4}; \ H_{i1}, \dots, H_{i4}; \ K_i]$$

subject to the following six constraint subsets: (defined for t=1,...,4)

(1) $\quad S_{1t} \leq C_{1t}$ \hfill (4.31)

(2) $\quad S_{1t} + S_{2t} - a_2(H_{2t} - H_t^{WG}) \leq C_{1t}$ \hfill (4.32)

(3) $\quad (A_1 + a_1)H_{1t} - A_1 H_{1t-1} - \beta_1 S_{1t} + \delta_1 G_{1t} = 0$ \hfill (4.33)

10/ Since only one source of surface water supply is available for
   use in the two subareas, the superscript for S_{it}^{WG} is omitted to
   simplify the notation.

(4) $(A_2 + a_2)H_{2t} - A_2H_{2t-1} - a_{12}H_{1t} - \beta_2 S_{2t} + \delta_2 G_{2t} = 0$ (4.34)

(5) $G_{it} - CK_i \leq 0$ $(i=1,2)$ (4.35)

(6) $G_{it} \leq G^O_i H_{it}$ $(i=1,2)$ (4.36)

Constraint subset (1) specifies that stream diversions in each time period t cannot exceed surface water supply. The same logic applies in subset (2) for the two joint subarea users; however, the stream supply is supplemented by groundwater discharge from subarea 2. This groundwater discharge is determined by the average height of the water table in subarea 2 during each time period and the average stream level related to surface water supply. Groundwater discharge is divided into two components: discharge as surface water available for rediversion in proportion to the difference, $H_{1t} - H_t^{WG}$, and discharge as groundwater not available for rediversion which is equal to the remaining flow.

Constraint subsets (3) and (4) regarding water table balance were previously defined in equation (4.1) prior to specification of the general model in this chapter. The explanation given for equation (4.1) also applies to subareas 1 and 2.

The last two constraint subsets define limitations on groundwater use in each time period. In subset (5), groundwater use is

restricted by the level of well development, K_i, and the pumping
capacity of each well during time period t denoted by C. Subset
(6) specifies that groundwater pumping in each period cannot exceed
groundwater supply determined by the water table height. The parameter
G_i^O defines the volume of groundwater in storage for each unit of
water table height.

Necessary conditions for optimality in the model given by the
Kuhn-Tucker Theorem may be determined in a systematic way by forming
the Lagrangian Function and taking partial derivatives with respect
to each variable affecting maximization of the criterion function.
In the present application of the theorem, the variables considered
are S_{it}, G_{it}, H_{it} and K_i, t=1,...,4. The Lagrangian Function specifies
that optimization of the criterion function is constrained by the
side conditions as follows:

$$L = \sum_{i=1}^{2} V_i [E_i S_{it},...,E_i S_{i4}; G_{i1},...,G_{i4}; H_{i1},...,H_{i4}; K_i]$$

$$- \sum_{t=1}^{4} [P_{1t}(S_{1t} - C_{1t})]$$

$$- \sum_{t=1}^{4} [P_{2t}(S_{1t} + S_{2t} - a_2(H_{2t} - H_{2t}^{WG}) - C_{1t})]$$

$$+ \sum_{t=1}^{4} [P_{3t}((A_1 + a_1)H_{1t} - A_1 H_{1t-1} - \beta_1 S_{1t} + \delta_1 G_{1t})]$$

$$+ \sum_{t=1}^{4} [P_{4t}((A_2 + a_2)H_{2t} - A_2 H_{2t-1} - a_{12}H_{1t} - \beta_2 S_{2t} + \delta_2 G_{2t})]$$

$$- \sum_{t=1}^{4} [P_{5t}(G_{1t} - CK_1)]$$

$$- \sum_{t=1}^{4} [P_{6t}(G_{2t} - CK_2)]$$

$$- \sum_{t=1}^{4} [P_{7t}(G_{1t} - G_1^O H_{1t})]$$

$$- \sum_{t=1}^{4} [P_{8t}(G_{2t} - G_2^O H_{2t})]$$

Partial derivative subsets of the Lagrangian expression are: 11/

(1) $\dfrac{\partial L}{\partial S_{1t}} = \dfrac{\partial V_1}{\partial S_{1t}} - P_{1t} - P_{2t} - \beta_1 P_{3t} \leq 0$ (4.37)

(2) $\dfrac{\partial L}{\partial S_{2t}} = \dfrac{\partial V_2}{\partial S_{2t}} - P_{2t} - \beta_2 P_{4t} \leq 0$ (4.38)

(3) $\dfrac{\partial L}{\partial G_{1t}} = \dfrac{\partial V_1}{\partial G_{1t}} + \delta_1 P_{3t} - P_{5t} - P_{7t} \leq 0$ (4.39)

(4) $\dfrac{\partial L}{\partial G_{2t}} = \dfrac{\partial V_2}{\partial G_{2t}} + \delta_2 P_{4t} - P_{6t} - P_{8t} \leq 0$ (4.40)

(5) $\dfrac{\partial L}{\partial H_{1t}} = \dfrac{\partial V_1}{\partial H_{1t}} + (A_1 + a_1)P_{3t} \overset{12/}{-} A_1 P_{3t+1} - a_{12}P_{4t} - G_1^O P_{7t} \leq 0$ (4.41)

(6) $\dfrac{\partial L}{\partial H_{2t}} = \dfrac{\partial V_2}{\partial H_{2t}} - a_2 P_{2t} + (A_2 + a_2)P_{4t} \overset{12/}{-} A_2 P_{4t+1} - G_2^O P_{8t} \leq 0$ (4.42)

11/ Defined for t=1,...,4.

12/ These terms are not present when t=4.

(7) $\quad \dfrac{\partial L}{\partial K_1} = \dfrac{\partial V_1}{\partial K_1} - CP_{5t} \leq 0$ $\hspace{4cm}$ (4.43)

(8) $\quad \dfrac{\partial L}{\partial K_2} = \dfrac{\partial V_2}{\partial K_2} - CP_{6t} \leq 0$ $\hspace{4cm}$ (4.44)

It may be noted that the above Lagrangian derivative subsets now encompass the stronger form of Kuhn-Tucker conditions (4.2) and (4.3) stated previously. That is, if a given variable equals zero, the inequality holds. But if a given variable is positive, the inequality becomes a strict equality.

Assuming that all variables are positive in the above derivative subsets, the Lagrangian Multipliers, P_{1t} to P_{8t}, $t=1,\ldots,4$, are equal to the imputed values for resource restrictions which are binding upon the optimization model. Since all constraints are linear, marginal imputed cost also equals average imputed cost of resource restrictions in the model. The imputed cost of the stream constraint is measured by P_{1t} and P_{2t} where P_{1t} reflects the importance of constrained surface water supply in subarea 1 alone, and P_{2t} is an imputed cost for constrained surface water supply jointly shared by the two subareas. These prices are always positive if an increase in this resource would permit the objective function to reach a higher level. Prices, P_{5t}, P_{6t}, P_{7t}, and P_{8t}, are analogous to the imputed prices for the stream constraint as they also represent imputed costs of binding resources in the model. Ground-water use is restricted by well development in the case of P_{5t} and

to the imputed prices for the stream constraint as they also represent imputed costs of binding resources in the model. Groundwater use is restricted by well development in the case of P_{5t} and P_{6t}, and groundwater pumping is limited by water table height in the case of the latter two prices.

Prices associated with water table balance equations reflect the importance of water table height in the optimal solution. Since the constraints are equations instead of inequalities, these prices may have either sign or be equal to zero depending on general water supply conditions. If water tables are near the bogging level, a further increase in water table height may reduce net returns in the objective function or would, alternatively, indirectly impose a cost if additional groundwater pumping is required for drainage control. Under these conditions, the contribution of water table constraints in the model is obviously negative, particulary if surface water is in adequate supply and groundwater is not needed for irrigation. On the other hand, if surface water is in short supply, an increase in the water table may be of benefit in providing larger storage of groundwater for irrigation use and, in the case of subarea 2, a higher water table would also relax the constraint on stream flow.

The Lagrangian Muliplier is negative when the water table is imposing a cost on optimization as a result of bogging conditions, and the multiplier is positive when a further increase in the

water table would benefit optimization. If the water table is
not affecting optimization, the multiplier equals zero.

In addition to the interpretation of multipliers, economic
interpretations may be directly derived from the above eight sub-
sets of Kuhn-Tucker conditions for optimality.

It may be recalled in analyzing the first two subsets that
if some activity x_i is employed, the marginal net output of
x_i in the objective function is equal to the imputed value of all
resources used in the production of one unit of x_i. Hence if S_{1t}
is employed in derivative subset (4.37) then $\partial V_1 / \partial S_{1t} = P_{1t} + P_{2t} + \beta_1 P_{3t}$, i.e., the marginal net output of constrained surface
water supply in the objective function is equal to the sum of all
imputed prices for resource restrictions limiting surface water use
in subarea 1. Price, P_{1t}, is the imputed price per acre-foot for
West Gallatin stream supply which is constrained at level C_{1t}.
They have different interpretations, however, as P_{1t} is the imputed
value for water supply available to only subarea 1 whereas P_{2t} is
the imputed value for water supply jointly available to both sub-
areas.

As percolation losses result from surface water use in
irrigation, there is a direct relationship between use of S_{1t}
and water table height, H_{1t}, determined by the percolation-loss
coefficient β_1. The third price in subset (4.37) denoted by P_{3t} is

an imputed cost for increasing the water table in subarea 1. Price, P_{3t}, can be of either sign or equal to zero depending on the contribution of H_{1t} in the objective function. If H_{1t} is not restricting or enhancing V_1, the cost of surface water percolation, $\beta_1 S_{1t}$, in subarea 1 is zero. But if H_{1t} is approaching the critical level for crop production, a further increase in the water table may cause a reduction in V_1. Hence, the value of P_{3t} would be negative, since it represents the cost of raising the water table further. Another possibility is that H_{1t} is low enough that crop production is not inhibited, but an incremental rise in H_{1t} would benefit economic production through reduced groundwater pumping costs or relaxation of the aquifer storage constraint on groundwater use.

In subarea 2, the price, P_{4t}, associated with H_{2t} may be negative, zero or positive. In addition to the relationships affecting P_{3t}, it is also possible that an increase in H_{2t} may be of positive economic value to subarea 2 through relaxation of the constraint on surface water supply. The water table discharge relationship, $a_2(H_{2t} - H_{2t}^{WG})$, states that groundwater discharge to the West Gallatin River will increase in proportion to the difference between H_{2t} and the West Gallatin stream level. This additional water supply would have comparable value to an increase

in C_{1t} in constraint set (4.38). It should also be obvious from the criterion function that an increase in water distribution efficiency, E_i, would have the same effect as an increase in C_{1t}.

In derivative subsets (4.39) and (4.40), the relationship of groundwater pumping is explicitly defined for the model. The effects are exactly the same in the two subareas. Marginal net output of G_{it} is equal to the sum of the imputed resource costs related to the supply of G_{it}. The coefficient δ_i is positive for use of G_{it}, since groundwater pumping has the effect of lowering the water table in each subarea. The interpretation of prices P_{3t} and P_{4t} was discussed above. Prices, P_{5t} and P_{7t}, and their counterparts in subarea 2, P_{6t} and P_{8t}, are imputed values for increasing groundwater supply in the two subareas. Groundwater supply is restricted by the number of wells developed, K_i, and the daily output volume of the wells, C. It may be recalled earlier in this chapter that C = 2cfs. Groundwater supply may also be restricted by the height of the water table. The coefficient G_i^o defines the number of acre-feet of groundwater in storage per foot height of the water table. The above four prices may equal zero or have positive values depending on whether the wells or water tables are restricting the supply of groundwater in subareas 1 and 2.

Derivative subsets (4.41) and (4.42) define the interaction effects of H_{1t} and H_{2t} in the model. In subset (4.41) the marginal net output of H_{1t} may be either equal to zero or positive if crop

production is not affected by an increase in H_{1t} or it may be negative if H_{1t} is adversely affecting the water-response relationship in V_1. Price, P_{3t}, may be of either sign or equal to zero as stated previously since this is an imputed cost associated with the level of H_{1t}. The coefficient A_1 represents the number of acre-feet of groundwater per unit H_{1t} in subarea 1. The product of A_1 and H_{1t} estimates the number of acre-feet of groundwater in storage in time t. The coefficient, a_1, estimates groundwater discharge from subarea 1 as a linear function of H_{1t}. Subsequent water-table height, H_{1t+1}, is affected by water table height, H_{1t}, therefore P_{3t+1} in subset (4.41) is also partially determined by H_{1t}. Groundwater discharge from subarea 1 to subarea 2 is determined by the coefficient a_{12}, hence H_{1t} also affects the level of H_{2t} and the associated price, P_{4t}, of the latter water table level. Price, P_{7t}, represents the imputed value of H_{1t} for groundwater supply as explained earlier. The prices in subset (4.42) have comparable interpretations to the ones discussed in subset (4.41).

Derivative subsets (4.43) and (4.44) are concerned with the effects of well development in the model. An increase in the level of well development, K_i, reduces the level of profit obtained in V_i because of the additional amortized costs incurred in new well development. Hence, the net effect, $\dfrac{\partial V_i}{\partial K_i}$, is negative. However, the increase in well development will relax the pumping constraint imposed on the rate

of groundwater extraction in subarea i by the amount $C\Delta K_i$ in each

time period, t, of the irrigation season. In subareas 1 and 2,

the prices, P_{5t} and P_{6t}, are imputed positive values of increased

well development.

The above economic interpretations of hydrologic relationships

in subareas 1 and 2 could be similarly derived for the other six

subareas in the model as stated earlier. The height of the water

table is indirectly responsible for a considerable amount of surface

water-groundwater interaction in the system as was pointed out earlier.

In the case of short surface water supply, an increase in H_{it} may

enhance net welfare by supplementing surface water supply with a

related increase in groundwater discharge. On the other hand, an

increase in H_{it} may reduce net welfare because of crop bogging. It

is also evident that an increase in H_{it} in a given subarea in a

particular time period may trigger a related chain reaction in adja-

cent subareas and across subsequent time periods. These intertemporal

and interspatial hydrologic relationships are all specified in the

general equilibrium model.

The absence of symmetry and the several sources of surface water

preclude an analysis of the entire study area comparable to that

just given for two subareas and a single stream.

Preview of the Empirical Model

The applied version of the general equilibrium model formulated

in this chapter will be structured in Chapter VI as a linear programming

85

model to simplify the computational task. The criterion function of

the empirical model will consist of several constrained activities

which will approximate linear segments of the hypothesized concave

response surface of the theoretical general equilibrium model.

Employment of each activity will imply that certain water allocations

are made in each subarea during each period of the irrigation

season. Which activities should be employed and which sources of

water supply should be used will represent important decision

variables in the applied model. Control of water tables and the

manipulation of both interspatial and intertemporal hydrologic inter-

actions in the Gallatin Valley study area will also figure prominently

in the optimization procedure. The imputed prices of all resource

restrictions in the model discussed in the preceding section will be

computed directly as shadow prices in the linear program.

Use of a linear programming algorithm will also facilitate the

analysis of parametric changes in water supply, in distribution

efficiency, and other important coefficients in the model. Evalua-

tion of changes in water supply will proceed as follows: 12/

Assume that the linear programming model will select a column

vector $\underline{X}$ having n elements such that the linear objective function

$(\underline{C}'\underline{X})$ is maximized subject to inequality constraints $(\underline{AX} \leq \underline{b})$, and

12/ Bellman, R.E. and S. Dreyfus, Applied Dynamic Programming, Princeton University Press, N.J., 1962. Appendix II, pp. 340-343.

$(\underline{X} \geq 0)$. $\underline{C}$ is a column vector with n elements and $\underline{B}$ is a column vector with m elements. $\underline{A}$ is a coefficient matrix of dimension m by n. The maximization of $\underline{C}'\underline{X}$ subject to the above stated side conditions will provide an optimal solution to the primal problem.

Now let $f(\underline{b})$ be the maximum value of the above objective function, i.e., the maximum value is now considered as a function of the right-hand side of the constraint set $(\underline{A}\underline{X} \leq \underline{b})$. The optimal solution is now given by $f(\underline{b}) = \underline{C}'\underline{X}^O$ where $\underline{X}^O$ is the vector of activities in the optimal solution.

In analyzing the behavior of the function $f(\underline{b})$, consider a change in employment of the element X_j^O of the optimal solution. Assume that the change in X_j^O is ε and that constraint rows involving the use of X_j must be increased by the amount $\varepsilon\underline{A}^{(j)}$ to maintain feasibility in the constraint set. $\underline{A}^j$ defines the column vector $(a_{1j}, a_{2j}, \ldots, a_{mj})$ of resource input coefficients for activity X_j. Due to the above increase in employment of X_j, it may be expected that the value of the objective function should increase to $\underline{C}'\underline{Y}$ where $\underline{C}'\underline{Y} > \underline{C}'\underline{X}^O$, the former optimal solution. However, the increase in value may be attributed to the change in resource restrictions $\varepsilon\underline{A}^{(j)}$; hence, $\underline{C}'\underline{Y} = f(\underline{b} + \varepsilon\underline{A}^{(j)})$ subject to restrictions, $\underline{A}\underline{Y} \leq \underline{b} + \varepsilon\underline{A}^{(j)}$ and $(\underline{Y} \geq 0)$.

The empirical model presented in Chapter VI will be structured as a parametric programming model to allow for variation in ε on

the right-hand side vector. The optimal solution obtained will then be associated with a particular level of $\underline{b} + \epsilon\underline{A}^{(j)}$ where ϵ will represent a change in surface water supply. Several solutions will be obtained for various levels of ϵ and the optimal level of well development will be evaluated for each solution. Each well development level will then be fixed in the model and optimal solutions will be computed again for certain water supplies, $\underline{b} + \epsilon\underline{A}^{(j)}$, however, the value of ϵ will be subject to probability. The expected value of the criterion function will then be computed for each fixed well development level as follows:

$$E(\underline{C}'\underline{X}) = Pr(\epsilon_1)f(\underline{b} + \epsilon_1\underline{A}^{(j)}) + \ldots + Pr(\epsilon_n)f(\underline{b} + \epsilon_n\underline{A}^{(j)})$$

That level of well development which maximizes the expected value of the criterion function, $\underline{C}'\underline{X}$, will be considered optimal within the class of well developments computed formerly for various levels of ϵ.

It is noted that the above stochastic analysis is limited to interseasonal surface water supply; intraseasonal water supply among the four discrete time periods of the irrigation season is assumed proportional to seasonal supply.

CHAPTER V

ASSEMBLY OF DATA FOR THE EMPIRICAL MODEL

The existence of a relatively large amount of empirical data and

descriptive information on the Gallatin Valley region was an important

consideration in using a systems analysis model for the present study.

Although it was not considered imperative that the systems model

produce precise values of the potential income to be gained from

optimal resource allocation in the study area, the results should be

sufficiently accurate to at least provide useful empirical guide-

lines for development of the resource base in the Gallatin Valley.

The present chapter attempts to provide a lucid explanation of all

data sources and of the methods followed in assembling empirical

data for use in the general equilibrium model presented in Chapter

IV.

Groundwater Data

The procedure followed in mathematically determining ground-

water flows among the hydrologic subareas was initially outlined

in Chapter III and was further expanded in Chapter IV. In

addition, all empirical data related to groundwater movement have

been presented except for beginning season water-table heights

in the eight subareas. Since intraseasonal groundwater pumping

is not allowed to exceed average annual recharge, it is expected

that the beginning season water-table heights will be at approximately
the same level each year. During the 1952-53 period, the water-
tables in all subareas fluctuated less than 5 feet on the average
between April 1 and August 1. The average water-table heights
shown in Table 3-1 will be used as beginning season parameters in
the model.

Stream Flow Data

The period of record for five streams evaluated in the study is
shown in Table 5-1. Computed means and variances of stream flows
during the period of record are shown in Table 5-2.

Mean monthly stream flows shown in Table 5-2 will act as
parameter constraints in initial model optimization. Following an
analysis of the sensitivity of the results obtained, the effect of
variability in stream flows on irrigation strategies will also be
considered.

Precipitation Data

Precipitation is unevenly distributed over the study area as
shown in Figure 5-1, for the 1952 period. Bozeman has the only
weather station in the area with records dating back before 1900.
Records at the Belgrade weather station extend back to 1941.
Precipitation data in other subareas are limited to 1952.

TABLE 5-1. Drainage Area and Period of Record for Five Streams in
            Study.

Station	Drainage Area (Sq. Mi.)	Years of Record (Years)
Gallatin River (Gallatin Gateway)	828.0	1889-1894, 1930-1968
East Gallatin River (Bozeman)	149.0	1939-1960
Middle Creek (Hyalite Ranger Sta.)	48.0	1895-1904, 1934-1968
Sourdough Creek (Bozeman)	28.0	May 1950-Sept. 1953
Camp Creek (West Belgrade)	35.0	May 1950-Sept. 1953

TABLE 5-2. Estimated Monthly Stream Flows Available for Diversion in
            the Study Area.*

Name of Stream	Estimated Mean Flow				Estimated Standard Deviation of Flow			
	June	July	Aug.	Sept.	June	July	Aug.	Sept.
	(000 Acre-Feet)							
West Gallatin	152.58	67.48	33.22	25.97	71.97	34.83	12.63	9.57
East Gallatin	16.06	5.58	3.34	3.39	6.30	1.90	1.30	1.07
Middle Creek	11.28	6.83	3.91	2.26	4.62	3.10	1.98	0.93
Sourdough Creek	4.20	1.87	1.30	1.13	1.91	0.57	0.20	0.23
Camp Creek	28.85	5.10	3.60	5.55	7.28	0.28	0.14	0.78

*Source: U.S. Geological Survey, Water-Supply Papers No. 1727-29.

Figure 5-1. Map of the Gallatin Valley Showing Distribution of Precipitation in 1952.

Total precipitation in 1952 at Bozeman was about 7 percent higher

than the 50-year mean. It is assumed in this study that monthly pre-

cipitation during the irrigation season in 1952 was nearly average

for all subareas. Monthly precipitation estimates for each subarea are

presented in Table 5-3.

Crop Yield Data

Crop enterprises that are considered in the model include: (1)

dryland spring wheat; (2) dryland winter wheat; (3) dryland oats;

(4) dryland barley; (5) non-irrigated pasture; (6) non-irrigated hay;

and (7) each of the preceding crops irrigated. All irrigated crop

activities are further delineated by duration of irrigation provided

during the season. That is, crops irrigated through June, June-July,

June-August, and June-September are viewed as separate crop activities.

Irrigation before June is usually not necessary in the Gallatin Valley.

During the four time periods of the irrigation season, available

soil moisture will be maintained at or above the 50 percent level. 1/

Although crop production may be obtained under a lower moisture regime,

empirical measures of the production response for higher levels of

available moisture depletion were not available.

Estimated yields of dryland and irrigated crop activities deline-

ated by duration of irrigation, are shown in Appendix Table B-1. Hay

and pasture yields are taken from three-year experimental trials at

---

1/ The 50 percent average minimum level is recommended by local SCS
officials as a practical operating goal in the Gallatin Valley.

TABLE 5-3. Average Monthly Precipitation in Eight Subareas of
the Gallatin Valley.*

Subarea (i)	Inches of Precipitation in Month of:[a]					Annual[b] Precipitation
	May	June	July	August	September	
Camp Cr. Hills (1)	1.73	1.87	1.11	0.82	1.24	13.31
West Gateway (2)	2.26	2.45	1.45	1.08	1.63	17.56
East Gateway (3)	2.26	2.45	1.45	1.08	1.63	17.56
Bozeman Fan (4)	2.54	2.75	1.63	1.21	1.83	19.57
East Belgrade (5)	1.65	1.79	1.06	0.79	1.19	12.76
West Belgrade (6)	1.65	1.79	1.06	0.79	1.19	12.76
Central Park (7)	1.52	1.65	0.98	0.73	1.10	11.71
Manhattan (8)	1.52	1.65	0.98	0.73	1.10	11.72

[a]Monthly precipitation data for all subareas were only available for 1952 when USGS made a detailed study of water resources in the Gallatin Valley. The 50-year annual average at Bozeman, the only long-term weather station in the area, is 18.26 inches which is 7 percent less than in 1952. It is assumed, however, for all subareas that 1952 is an average water year for crop production.

[b]This data is for 1952 as explained in footnote a.

*Source: U.S. Weather Bureau records.

Bozeman. Remaining yield data are taken from 10-year experimental trials at Bozeman. According to the State Engineer's Report discussed in Chapter III, there are only minor differences in soil productivity and associated yield data among the valley subareas. Differences in irrigation efficiency among subareas are accounted for in the model.

The irrigation response data shown in Appendix Table B-1 indicate that irrigation in May does not usually benefit yield. Also, the grain crops did not benefit from irrigation after July.

Production Cost Data

Alfalfa, pasture and small grains production cost data are shown in Appendix Tables B-2, B-3, and B-4. Cost data will also vary according to the number of irrigation treatments applied. The latter computation involves a detailed accounting of projected crop water demand, expected precipitation, and available moisture storage during the season.

Although Appendix Tables B-2, B-3, and B-4 include all estimated overhead costs of crops, only the direct costs of crop production are considered in the present model. Crop returns are thus defined as net returns to land and water resources used.

There are differences in irrigation efficiencies affecting production cost among the eight subareas which were explained in Chapter III. Table 5-4 is a recapitulation of estimated efficiency

TABLE 5-4. Estimated Irrigation Efficiencies of Diverted Surface and Pumped Groundwater in Different Subareas*

Subarea i	Canal Seepage of Diverted Surface Water $(1-E_i)$	Field Seepage Loss		Consumptive Use	
		Diverted Surface Water $E_i b_{Si}$	Pumped Groundwater b_{Gi}	Diverted Surface Water $E_i(1-b_{Si})$	Pumped Groundwater $(1-b_{Gi})$
	Percent	----------------------Percent-------------------			
Camp Cr. Hills (1)	37	31	50	32	50
W. Gateway (2)	37	19	30	44	70
E. Gateway (3)	37	19	30	44	70
Bozeman Fan (4)	37	19	30	44	70
E. Belgrade (5)	37	19	30	44	70
W. Belgrade (6)	37	19	30	44	70
Central Park (7)	37	19	30	44	70
Manhattan (8)	37	22	35	41	65

*The percentages differ for ground and surface water use as only surface water is subject to canal loss.

in irrigation for the eight subareas. This table will be utilized to estimate diverted surface water and groundwater requirements for crop activities after net crop water demands are determined.

Crop water demands may be satisfied by available soil moisture in storage, by precipitation, and by irrigation. Estimated crop water requirements from all sources for irrigated crop activities considered in the optimization model are shown in Appendix Table B-5. The consumptive use data are computed via the Blaney-Criddle method applied to irrigated crops in the Missouri River Headwaters Region of Montana. 2/ Consumptive use, U, equals a constant, K, times a growing season factor, F. K will vary for different crops. A full explanation of this procedure is given in Bulletin 494. 3/

Net crop and farm delivery irrigation requirements for all sub-area crop activities are shown in Appendix Table B-6. These require-ments are estimated by subtracting monthly precipitation in Table 5-3 from consumptive use shown in Appendix Table B-5. Farm delivery requirements are estimated from subarea field efficiencies shown in Table 5-4 and net crop requirements in each subarea. Farm delivery requirements are comparable for both ground and surface water as the latter are subject to the same field losses in irrigation. The esti-mated farm delivery requirements for water supply in each time period

---

2/ Estimated Water Requirements of Crops in Irrigated Areas of
    Montana, Mont. Agr. Expt. Sta. Bull. 494, December 1953, Bozeman.

3/ Ibid.

are applicable only if no soil water storage takes place. If soil
water storage is involved between time periods, then actual farm
deliveries in each period may be changed according to the following
strategies adopted by the irrigators.

Irrigation Delivery Strategies

To determine irrigation requirements of different crop activities
average precipitation was subtracted from estimated crop consumptive
use during the irrigation season in all subareas. Available moisture
storage capacity of soils in each subarea can be utilized as a buffer,
however, to stabilize the fluctuations in irrigation demand during
the season. If available soil moisture storage capacity is large
relative to seasonal irrigation demand, there are many strategies
regarding water delivery that may be adopted during the season to
satisfy crop requirements. For example, farmers may irrigate very
heavily in June to effect a large carryover of available soil moisture
fro crop requirements in July or later months of the season when
surface water supplies are normally deficient. Over-all regional
use of these strategies could have a profound impact on conserving
water supplies during certain periods of the season.

The inclusion of these irrigation delivery strategies in the
empirical model involves some additional specification of side condi-
tions. Notation used in defining these suggested new side conditions
is presented below.

M_{ikt} = available moisture carryover for crop activity k in subarea i at the end of time period t (acre-feet);

P_{ikt} = average precipitation available to crop activity k in subarea i during time period t (acre-feet);

W_{ikt} = net irrigation delivery to crop activity k in subarea i from all sources of supply during time period t (acre-feet);

Z_{ikt} = the number of acres of crop activity k in subarea i during time period t;

C_{ikt} = the consumptive use of water per acre of crop activity k in subarea i during time period t (acre-feet); and

M_{ikt}^{o} = the available moisture storage capacity of soil for crop activity k in subarea i (acre-feet).

The side conditions may be defined in general as follows:

(1) $M_{ikt-1} + P_{ikt} + W_{ikt} \geq Z_{ikt} C_{ikt}$; (5.1)

(2) $M_{ikt} = P_{ikt} + M_{ikt-1} + W_{ikt} - Z_{ikt} C_{ikt}$; (5.2)

(3) $M_{ikt} \leq M_{ikt}^{0}$ (i=1,...,8; k=1,...,K; t=1,...,4). (5.3)

As defined, these conditions imply that the volume of irrigation water delivered in each period cannot exceed water demands for crop consumptive use in that period less precipitation and the amount of unused soil moisture storage capacity for each crop activity in each subarea. This assumes that farmers would not want to incur the expense of additional irrigation to purposely raise the water-table level, e.g., for subirrigation.

Irrigation frequency during the season can be altered by the irrigation delivery strategies adopted since the amount of available moisture stored from each irrigation treatment is limited by available capacity of the soil reservoir. Consequently, labor costs in irrigation may also vary slightly for the crop activity affected if the cost per irrigation treatment is relatively constant. It is assumed, however, in subsequent analysis that the labor cost of irrigation treatments is proportional to the amount of water applied in each treatment. Estimated average cost per treatment for each crop considered in this study is shown in Appendix Tables B-2, B-3 and B-4.

The addition of new side conditions regarding irrigation strategies as specified above would, however, require addition of other new constraints to equate alternate sources of water supply with water demand, W_{ikt}, of all irrigation strategies for each subarea. For example, in subarea 2, the required new balance equation would be:

$$\sum_{k=1}^{K} W_{2kt} Z_{2k} = (1-b_{p2}) \, E_2 S_{2t}^{WG} + G_{2t}) \qquad t=1,\ldots,4; \qquad (5.4)$$

where: W_{2kt} = variable water demand for the kth crop during time t in subarea 2; and

Z_{2k} = variable number of acres of the kth crop produced in subarea 2.

This constraint would be nonlinear as it involves the product of two variables on the left-hand side.

The above problem of nonlinear constraints may be resolved in the model by fixing the amount of end period available moisture storage at different levels. Since precipitation, P_{ikt}, is assumed to be a parameter, the level of W_{ikt} will also be fixed in relation to the level of moisture storage, M_{ikt}, that is determined a priori. Hence, all of the above constraint sets for M_{ikt} and W_{ikt} may be eliminated in the empirical model.

In the empirical model presented in Chapter VI the available soil moisture storage, M_{ikt}, will be fixed at 50 and 75 percent of maximum soil storage capacity. This will allow at least some exchange of water supply among different time periods without causing any significant change in crop production relationships. Computed water requirements of crop activities in each time period for these two levels of soil storage are shown in Appendix Tables C1 to C7.

The number of crop activities, K, is expanded to indentify the irrigation strategy followed in the production of each crop in each. subarea as shown in Appendix C, Tables C2 to C7. Equations will be constructed in the empirical model comparable to (5.4) shown for subarea 2 above to equate fixed net water requirements of all activities in each subarea with various sources of water supply available to each subarea.

Value of Production

Estimated yields of spring wheat, winter wheat, barley, oats, non-irrigated hay and irrigated hay (alfalfa) in the Gallatin Valley are valued at average state prices. Ten-year average prices of these crops, except for pasture, are shown in Appendix Table B-7. No published price is available to value the pasture enterprise.

A common method of valuing pasture production is to determine the gain in value of livestock grazing on the pasture over time. However, the feed value of pasture clippings, defined in terms of total digestible nutrients, is comparable with hay. Therefore, it may be argued that the imputed price of pasture production should also be comparable with hay. There is no simple solution to the problem of valuing pasture production. 4/ It is assumed in this study that pasture production may be valued in terms of the reported rental charge for quality pasture in the Gallatin Valley as explained in Table 5-5.

Net returns of all crop activities to land and water resources are shown in Tables 5-5 and 5-6. Average production costs and gross returns are estimated in Appendix Tables B-1, B-2, B-3, B-4 and B-7.

4/ For a discussion of the various problems of and suggested approaches to valuing pasture production, see G.L. Johnson and L.S. Hardin, Economics of Forage Evaluation, Purdue University, Agr. Expt. Sta. Bul. 623, Lafayette, Indiana, April 1955.

TABLE 5-5. Estimated Net Returns Per Acre of Crop Enterprises
Excluding Cost of Irrigation Treatments.

Crop Enterprise	Production Cost[a]	Harvesting Cost[a]	Gross Returns[b]	Net Return
		(Dollars)		
Spr. wht. (non-irr.)	23.51	3.90	43.16	15.75
(June)	26.64	6.00	66.40	33.76
(June-July)	26.64	8.10	89.64	54.90
Wtr. wht. (non-irr.)	23.51	4.95	49.83	21.37
(June)	26.64	9.00	90.60	54.96
Barley (non-irr.)	23.51	7.95	41.34	9.88
(June)	26.64	10.20	53.04	16.20
(June-July)	26.64	12.45	64.74	25.65
Oats (non-irr.)	23.51	8.85	34.22	1.86
(June)	26.64	13.65	52.78	12.49
(June-July)	26.64	18.30	70.76	25.82
Pasture (non-irr.)	9.92		41.72	32.80
(June)	10.49		46.15	35.66
(June-July)	10.49		50.58	40.09
(June-Aug.)	10.49		57.11	46.62
(June-Sept.)	10.49		59.44	48.95
Alfalfa (non-irr.)	24.04	16.81	48.84	7.99
(June)	24.38	32.85	95.46	38.23
(June-July)	24.38	40.49	117.66	52.79
(June-Aug.)	24.38	48.13	139.86	67.35

[a]These cost data are taken from Appendix Tables B-2, B-3 and B-4.

[b]Gross returns for all crops except pasture are estimated from yield
data shown in Appendix Table B-1, and price data shown in Appendix
Table B-7. Pasture production is valued at $7 per animal unit month
in the Gallatin Valley. It is assumed that one animal-unit
consumes 20 pounds of forage per day.

TABLE 5-6. Estimated Net Returns Per Acre of Crop Activities in Each Subarea Including
Irrigation Treatment Costs.*

Crop Enterprise	Camp Creek Hills	Gateway Subareas	Bozeman Fan	Belgrade Subareas	Central Park	Manhattan
			(dollars)			
Spr. wht. (non-irr.)	15.75	15.75	15.75	15.75	15.75	15.75
(June)	32.76	32.93	33.03	32.19	32.67	31.71
(June-July)	52.34	52.51	52.58	50.99	52.22	49.88
Wtr. wht. (non-irr.)	21.37	21.37	21.37	21.37	21.37	21.37
(June)	53.96	54.13	54.23	53.39	53.87	52.91
Barley (non-irr.)	9.88	9.88	9.88	9.88	9.88	9.88
(June)	15.20	15.37	15.47	14.63	15.11	14.15
(June-July)	23.09	23.26	23.33	21.74	22.97	20.63
Oats (non-irr.)	1.86	1.86	1.86	1.86	1.86	1.86
(June)	11.49	11.66	11.76	10.92	11.40	10.44
(June-July)	23.26	23.43	23.50	21.91	23.14	20.80
Pasture (non-irr.)	32.80	32.80	32.80	32.80	32.80	32.80
(June)	34.66	34.83	34.93	34.09	34.57	33.61
(June-July)	37.69	37.84	37.91	36.41	37.57	35.37
(June-August)	42.86	43.24	43.43	41.94	42.74	39.35
(June-September)	44.39	44.88	45.13	42.01	44.21	40.44
Alfalfa (non-irr.)	7.99	7.99	7.99	7.99	7.99	7.99
(June)	37.11	37.30	37.40	36.83	37.03	36.23
(June-July)	51.12	51.30	51.39	49.39	50.49	47.35
(June-August)	64.76	65.00	65.12	62.11	63.84	59.62

*The number of treatments required are estimated from Table 3-2 of Chapter III and Appendix
Table B-6. Cost per treatment for different crops is shown in Appendix Tables B-2, B-3 and
B-4. Costs of water procurement are not included as they depend on the source of water supply.

103

Irrigation treatment costs are estimated for each crop activity by

dividing total irrigation requirements for the season, estimated

in Appendix Table B-6, by 50 percent of available moisture storage

capacity, and multiplying the computed number of treatments by

average treatment cost. Water procurement costs cannot be considered

at this stage of the analysis as they depend on the source of water

used. All procurement costs are evaluated independently in the model

for different water sources.

According to Table 5-6, the three most profitable crops are

irrigated spring wheat, irrigated winter wheat and irrigated alfalfa.

However, these crops are subject to some restriction in produc-

tion. Wheats are subject to the wheat allotment program limitation,

and alfalfa in practice, also has an acreage restriction since this

crop is usually grown in rotation with other grain crops. Estimated

total acreage of agricultural land and acreage under allotment in

each subarea are shown in Table 5-7.

Alfalfa acreage is restricted in the study to five-sevenths of

total cropland acreage in each subarea to allow for some rotation

of this crop.

As there are some restrictions on all wheat and irrigated alfalfa

production, production of other crop activities may be necessary even

though they are less profitable. The next most profitable crop in

TABLE 5-7. Estimated Total Cropland Acreage and Acreage Under Allotment in Eight Subareas of the Gallatin Valley.

Acreage	Camp Creek Hills	West Gateway	East Gateway	Bozeman Fan	East Belgrade	West Belgrade	Central Park	Manhattan
Cropland[a]	21,312	4,608	9,216	35,136	30,528	8,064	10,944	4,608
Allotment[b]	3,152	372	1,343	1,681	2,094	604	1,248	175

[a]Ten percent has been deducted from total acreage of each subarea for farmsteads, river wash and other wasteland.

[b]The allotment acreage for each subarea was obtained directly from the ASCS office in Bozeman.

Table 5-6 is pasture. The remaining crops would not likely enter

the optimal solution regardless of water supply conditions unless

pasture production was also restricted.

Canal Distribution and Pumping Costs

Canal overhead and maintenance costs of some major ditch

companies in the Gallatin Valley were investigated by Sammons in

1963.[5] His cost data are utilized in the current study and are

shown in Table 5-8. There have not been any further major ditch

improvements which would affect assessment costs since 1963. The

average assessment cost per acre-foot of diverted surface water for

each subarea in 1963 is shown in Table 5-9.

The procedure adopted for estimating draw-down, horsepower

requirements, and pumping costs in each subarea is similar to that

followed by McConnen and Mennon:[6]

Drawdown is defined as: $\Delta h_i = \dfrac{264Q_i}{T_i}$,

where T_i is the average coefficient for transmissibility in subarea i,

Q_i is well discharge in acre-feet per season in subarea i, and Δh_i

is the distance of drawdown in subarea i in feet per log cycle of time.

---

[5] Sammons, op. cit., p. 71.

[6] McConnen and Mennon, op. cit., pp. 28-32.

TABLE 5-8. Average Ditch Company Assessment Costs for Surface
Water Distribution in 1963.*

Ditch Company	Subarea Served	Average Cost of Water Reported Dollars per Acre-Foot[a]
Farmers Canal	E Gateway, Bozeman Fan	0.25
Highline Canal	Camp Creek Hills	0.50
Kaghen Ditch	W Belgrade	0.12
Kleinschmidt Canal	Bozeman Fan	0.50
Lewis Ditch	Manhattan, W Belgrade	0.10
Lowline Canal	Camp Creek Hills	0.75
Lower Middle Creek Canal	E Belgrade	0.25
Spain-Ferris Ditch	E Belgrade, Manhattan	0.12
Ave. Cost of Companies		0.31

[a] This cost is based on an average diversion of four acre-feet per acre of cropland per season.

*Source: Sammons, op. cit.

TABLE 5-9. Average Ditch Assessment Costs for Subareas in 1963*

	Camp Creek Hills	E Gateway	W Gateway	Bozeman Fan	E Belgrade	W Belgrade	Central Park	Manhattan
				------Dollars------				
Assessment Cost/ acre foot	0.38	0.25	0.25[a]	0.38	0.22	0.12	0.19	0.12

[a]This cost is assumed to be the same as for West Gateway.
*Source: Table 5-8.

It is assumed as in McConnen and Mennon's study that draw-down in

all wells is constant and equal to the height resulting after 70

days of continuous pumping at the rate of 2 cfs. Only wells yielding

2 cfs are considered in this study as this volume of flow is easily

manageable for the average irrigator using surface methods of spreading

water on the field. All water is assumed to be pumped directly on

the field; not into major irrigation canals as in the above study.

Calculated values of Δh_i for 2 cfs wells, average coefficients

of transmissibility, the average depth to groundwater, average draw-

down after 70 days, required well depth, and average head after 70

days in the eight subareas are shown in Table 5-10. The water-bearing

media is not less than estimated required well depths in the subareas

except for Camp Creek Hills. However, the Bozeman Fan and Camp Creek

subareas will be excluded as they are not considered to be suitable

areas for groundwater pumping. 7/ In the remaining subareas, it is

assumed that well draw-down will be zero up to .0001 days of pumping

in applying Jacob's Method of estimating draw-down over time.

Linear functions denoting the relationship between Δh_i and estimated

draw-down by Jacob's Method are shown in Figure 5-2. Each trans-

---

7/ Bert VanDyken of VanDyken Well Drilling advised the writer that
   water yields are not reliable for irrigation in these subareas.

Figure 5-2. Estimated relationship between draw-down and time elapsed in pumping using Jacob's method.

TABLE 5-10. Calculated Average Draw-Down, Average Head, and Required Well Depth to Maintain an Output of 2 cfs for 70 days in Eight Subareas*

Subarea (i)	Ave. Coefficient of Transmissibility[a]	Δh_i[b] (2 cfs)	Draw-Down (after 70 days)	Depth to[c] Groundwater	Total Head[d]	Required Well Depth[e]
			----------Feet----------			
Camp Cr. Hills (1)	11,000	21.60	126.0	8	134.0	144.0[f]
W. Gateway (2)	275,000	.86	5.1	12	17.1	27.1
E. Gateway (3)	275,000	.86	5.1	12	17.1	27.1
Bozeman Fan (4)	48,000	4.95	28.9	10	38.9	48.9
E. Belgrade (5)	206,000	1.16	6.8	40	46.8	56.8
W. Belgrade (6)	206,000	1.16	6.8	40	46.8	56.8
Central Park (7)	86,000	2.78	16.2	5	21.2	31.2
Manhattan (8)	130,000	1.83	10.81	15	25.81	35.8

[a] Taken from Table 1-3.

[b] Δh_i is the draw-down difference in feet per log cycle of time.

[c] Taken from Table 1-3.

[d] Total head is the sum of draw-down and depth to groundwater.

[e] Well depth is determined to be: (draw-down) + (depth to groundwater) + (a 10-foot safety margin for dry years).

[f] This required well depth would exceed the average depth of water-bearing strata in this subarea.

*Source: The procedure followed in making these calculations is the Jacob Method as used by McConnen and Mennon. The Jacob Method is outlined in D. K. Todd, Groundwater Hydrology, John Wiley & Sons, Inc., New York, 1959, p. 94.

missibility zone has a different linear function defining the rate of
draw-down in linear relation to the natural log of the number of days
spent at continuous pumping. All wells are pumped at the rate of 2 cfs.
The Jacob Method of estimating draw-down of wells is discussed in
detail by Todd. 8/

Draw-down difference between 10 and 70 days of continuous pumping
is estimated to be less than 5 feet for all transmissibility zones in
the study as shown in Figure 5-2. Consequently, there is negligible
difference in the cost of well development for pumping only 10 days
or 70 days. Cost data for the 70-day period are used in the present
study to allow for possible drouth years. Horsepower and kilowatt-
hour requirements for electric motors are estimated from tables shown
in Israelsen and Hansen. 9/ These electric motors to be used in
pumping are available from manufacturers in multipliers of 5 horse-
power and they have a 15 percent overload factor. The annual amortized
costs of well development in subareas suitable for pumping were
obtained from local irrigation supply firms and well drillers. 10/
All estimated well and pumping costs are shown in Table 5-11.

8/ Todd, op. cit., p. 94.

9/ Israelsen, O.W. and V.E. Hanson, Irrigation Principles and
    Practices, John Wiley & Sons, Inc., New York, 1962, p. 73.

10/ VanDyken Drilling Company and Mountain Supply Co., Bozeman.

TABLE 5-11. Horsepower Requirements, Kilowatt Hours Used, and Average Fixed and Variable Pumping Costs for Subarea Wells in the Study

Subarea	Well Depth[a]	Horsepower Requirements Per Well[b]	Kilowatt Hours/ Acre Foot[c]	Amortized Cost of Well Development[d]	Variable Cost of Pumping/ Acre Foot[e]
	Feet	Number	Hrs/Well	$/season/well	$/Acre Foot
W. Gateway	27.1	15	96	214.68	0.96
E. Gateway	27.1	15	96	214.68	0.96
E. Belgrade	56.8	25	174	313.24	1.74
W. Belgrade	56.8	25	174	313.24	1.74
Central Park	31.2	15	96	223.88	0.96
Manhattan	35.8	15	96	234.28	0.96

[a]Estimated in Table 5-10.

[b]Israelsen and Hansen, op. cit., Table 4.1, p. 53.

[c]Ibid.

[d]All material and well digging costs were obtained from VanDyken Drilling Company and Mountain Supply Company in Bozeman. An 8 percent rate of interest is charged on average investment. Depreciation on pumps, motors, panels, and casings is computed via the straight-line method for an assumed life of 25 years. Annual service cost for motors is assumed to be $10.

[e]The electricity rate charged for irrigation is 1 cent per kilowatt-hour.

Evaluation of Water Table Levels

It was stated earlier in Chapter III that a relatively large
land area in the lower end of the Gallatin Valley, principally the
Central Park Subarea, had been subject to bogging over a period of
many years. However, this continued bogging was not known to have
resulted in any permanent destruction of the soil in this part of the
valley, the only damage being the loss of valuable crop production.
Since the value of the lost crop production (opportunity cost) could
be easily quantified, the evaluation of high water table damage was
a relatively simple problem in the present study.

Local data could not be obtained, however, on the critical water
table height for crop production in this region. It is assumed in
this study on the basis of evidence presented by Wesseling of
Holland 11/ that the average critical level for crop production in
the Gallatin Valley is two feet from the surface.

11/ Wesseling, J., Hydrology, Soil Properties, Crop Growth and Land
Drainage, Institute for Land and Water Management Research,
Technical Bulletin 57, Waneningen, The Netherlands, 1968, pp. 16-
21. Wesseling recommended that the water table be kept below
50 cm in semi-arid regions.

CHAPTER VI

RESULTS OF THE EMPIRICAL MODEL

Introduction

This chapter is concerned with empirical application of the theoretical optimization model for irrigation·development presented in Chapter IV. A linear programming model is utilized in evaluating optimal irrigation development, having ·comparable structure to the former nonlinear model but with more activities and a linear objective function.

The theoretical model assumed a known concave net benefit function for each subarea with sets of variables for surface water, groundwater, number of wells and water table height. Except for well development, each set of variables includes a separate variable for each of the four time periods analyzed during the irrigation season. In addition, surface water is partitioned into separate variables according to the stream source from which surface water is diverted.

As an empirical approximation to these functions, different crop activities with a constant profit level in each activity, are stratified according to the total number of time periods that each crop is irrigated during the season and by subarea. The nonlinear objective function associated with a given subarea could be derived

by treating the sets of variables which are arguments of that

function as fixed resources in a linear programming model of that

subarea, and tracing the objective function of the linear program

over the region associated with changing levels of the fixed

resources through the use of parametric linear programming. But

rather than go through this two-stage approach of first deriving the

nonlinear functions and then applying concave programming to the non-

linear model presented in Chapter IV, comparable results may be obtain-

ed using a large linear programming model that encompassed both

stages simultaneously.

The large linear program merely specifies many separate linear

activities (the same ones that would be used in the first stage of

the above two-stage procedure) for each subarea and introduces

additional resource constraints such as available cropland in each

subarea and certain crop restrictions.

Optimization of the linear programming model is constrained by

fixed surface water supply from five different stream sources,

limitations on groundwater pumping and water table restrictions in

each subarea of the model. For simplification, it is assumed in

the empirical model that it is more economic to restrict water table

height below the critical level for crop production than to incur

crop losses resulting from bogging conditions. This control may

be attained by utilizing groundwater in irrigation and by limiting

surface water use in order to reduce percolation losses to the groundwater table. Since maximum water table levels are restricted in each subarea, the set of water table variables is omitted from the objective function of the empirical model as the water table does not interfere with crop production at these constrained lower levels. Under the new set of constraints, water tables may be viewed as by-products of water resource use in the optimization model which have zero value in the criterion function. They are of economic significance in the model because of the cost of control involved in restricting their maximum heights and because they also regulate the volume of groundwater available for irrigation use. Surface water supply is also partially regulated by the water table as the height of the water table affects groundwater discharge to streams.

Groundwater pumping for irrigation is not feasible in subarea 1, Camp Creek Hills, and subarea 4, Bozeman Fan. Since the water table cannot be controlled by pumping in these subareas, control must be accomplished indirectly through selection of irrigated crops and the intensity of irrigation. As crop income is reduced when surface water use is restricted under high water table conditions these particular subareas would benefit considerably from an increase in irrigation efficiency.

Groundwater discharge from a given subarea may be either to an adjacent subarea or stream. Subareas located on lower lands in the

Gallatin Valley are generally more susceptible to bogging problems as they tend to have a relatively large inflow of groundwater in addition to their own surface water percolation losses. Central Park is particularly vulnerable as groundwater discharge is restricted by a geologic fault in this subarea. Care has been taken to ensure that all major hydrologic relationships among the subareas and adjacent streams are specified in the empirical model.

In view of the important hydrologic relationships among the eight subareas and the joint use of surface water supply, it was assumed that overall irrigation optimization should result from analysis of all subareas together rather than individually. The necessary coordination involved in maximizing regional welfare would require the use of some central agency to allocate surface water supply and regulate groundwater development. Groundwater depletion is not a problem in the study area as the projected seasonal withdrawal by pumping is less than average annual recharge. As the groundwater supply is not being depleted, preirrigation season water table levels are specified to be parameters in the linear programming model.

Statement of Linear Programming Model

In the theoretical model presented in Chapter IV the objective function was hypothesized to be a concave net revenue surface.

Examination of available empirical data for evaluating the water-response surface has revealed, however, that only certain subarea-crop enterprise combinations will be profitable in the Gallatin Valley. Furthermore, the perusal of data has revealed that it may be profitable to allow for exchange of water supply among the four time periods of the irrigation season by varying the level of soil moisture storage carryover and related crop water requirements among time periods.

The empirical objective function is restructured in linear form having 24 separate activities for each subarea. The activities represent different crop enterprise-irrigation strategy combinations that may be employed in each subarea.

For maximizing net revenue obtained with the linear objective function, each hydrologic subarea can be viewed as a separate sub-division of a firm whose overall goal is to maximize total firm net revenue product.

The revised linear objective function is defined as:

$$\text{Max} \sum_{i=1}^{8} [\sum_{k=1}^{K} R_{ik} Z_{ik} - C_i^d \sum_{t=1}^{4} (S_{it}^{WG} + S_{it}^{MC} + S_{it}^{SC} + S_{it}^{EG} + S_{it}^{CC}) - C_i^W W_i - \sum_{t=1}^{4} C_i^G G_{it}]$$

where: R_{ik} = the net revenue obtained per acre from the kth activity in the ith subarea exclusive of water procurement costs;

Z_{ik} = the number of acres of the kth activity produced in the ith subarea;

C_i^d = the average ditch assessment cost per acre-foot for distributing diverted surface water in the ith subarea;

C_i^W = the average amortized cost for well development in the ith subarea per well each season;

W_i = the number of 2 cfs wells developed in the ith subarea;

C_i^G = the average variable cost per acre-foot for pumping in the ith subarea; and

G_{it} = the number of acre-feet of groundwater pumped in the ith subarea.

Most of the notation used in defining the linear constraints on the above objective function was explained earlier in Chapter IV. Some new terms which will be used are:

N_i = the total available cropland acreage in the ith subarea;

N_i^A = the present wheat acreage allotment in the ith subarea;

W_{ikt}^o = the water requirement per acre in the ith subarea for the kth activity in time period t;

H_i^k = the critical water table height for production of all crop activities in the ith subarea; and

a_{ji} = a coefficient for determining groundwater discharge from the jth subarea to the ith subarea.

It may be noted that alfalfa acreage (k=18-24) is restricted to five-sevenths of total cropland acreage to allow for normal rotation of other crops with alfalfa.

Linear constraints are:

$$\sum_{k=1}^{24} Z_{ik} \leq N_i \quad i=1,\ldots,8; \quad \text{Cropland Acreage Constraint} \qquad (6.1)$$

$$\sum_{k=18}^{24} Z_{ik} \leq 5/7 N_i \quad i=1,\ldots,8; \quad \text{Alfalfa Acreage Constraint} \qquad (6.2)$$

$$Z_{i1} \leq N_i^A \quad (i=1,\ldots,8) \qquad \text{Wheat Acreage Constraint} \qquad (6.3)$$

Water Demand Constraints

$$\sum_{k=1}^{24} W_{1kt}^O Z_{1k} = (1 - b_{p1})(E_1 S_{it}^{WG}) \tag{6.4}$$

$$\sum_{k=1}^{24} W_{2kt}^O Z_{2k} = (1 - b_{p2})(E_2 S_{2t}^{WG} + G_{2t}) \tag{6.5}$$

$$\sum_{k=1}^{24} W_{3kt}^O Z_{3k} = (1 - b_{p3})(E_3 S_{3t}^{WG} + G_{3t}) \tag{6.6}$$

$$\sum_{k=1}^{24} W_{4kt}^O Z_{4k} = (1 - b_{p4})(E_4 S_{4t}^{MC} + E_4 S_{4t}^{SC} + E_4 S_{4t}^{WG}) \tag{6.7}$$

$$\sum_{k=1}^{24} W_{5kt}^O Z_{5k} = (1 - b_{p5})(E_5 S_{5t}^{MC} + E_5 S_{5t}^{EG} + E_5 S_{5t}^{WG} + G_{5t}) \tag{6.8}$$

$$\sum_{k=1}^{24} W_{6kt}^O Z_{6k} = (1 - b_{p6})(E_6 S_{6t}^{CC} + E_6 S_{6t}^{WG} + G_{6t}) \tag{6.9}$$

$$\sum_{k=1}^{24} W_{7kt}^O Z_{7k} = (1 - b_{p7})(E_7 S_{7t}^{EG} + E_7 S_{7t}^{WG} + G_{7t}) \tag{6.10}$$

$$\sum_{k=1}^{24} W_{8kt}^O Z_{8k} = (1 - b_{p8})(E_8 S_{8t}^{CC} + E_8 S_{8t}^{WG} + G_{8t}) \tag{6.11}$$

Surface Water Constraints

$$S_{1t}^{WG} \leq C_1 \tag{6.12}$$

$$S_{1t} + S_{2t} - a_2 H_{2t} \leq C_1 \tag{6.13}$$

$$S_{1t}^{WG} + S_{2t}^{WG} + S_{3t}^{WG} - a_2 H_{2t} - a_3 H_{3t} \leq C_1 \tag{6.14}$$

$$\sum_{i=1}^{4} S_{it}^{WG} + S_{4t}^{MC} + S_{4t}^{SC} - a_2 H_{2t} - a_3 H_{3t} \leq C_2 \tag{6.15}$$

$$\sum_{i=1}^{5} S_{it}^{WG} + S_{4t}^{MC} + S_{5t}^{MC} + S_{5t}^{EG} - \sum_{i=2}^{3} a_i H_{it} - a_5 H_{5t} \leq C_3 \tag{6.16}$$

$$\sum_{i=1}^{6} S_{it}^{WG} + S_{6t}^{CC} - \sum_{i=2}^{3} a_i H_{it} - \sum_{i=5}^{6} a_i H_{it} \leq C_4 \tag{6.17}$$

$$\sum_{i=1}^{7} S_{it}^{WG} + S_{5t}^{EG} + S_{7t} - \sum_{i=1}^{7} a_i H_{it} - \sum_{i=5}^{7} a_i H_{it} \leq C_5 \tag{6.18}$$

$$\sum_{i=1}^{8} S_{it}^{WG} + S_{6t}^{CC} + S_{8t}^{CC} - \sum_{i=1}^{3} a_i H_{it} - \sum_{i=5}^{8} a_i H_{it} \leq C_6 \tag{6.19}$$

$$H_{it} \leq H_i^k \qquad \begin{array}{l} i=1,\ldots,8; \\ k=1,2,\ldots \end{array} \qquad \text{Upper Bound on Water Table} \tag{6.20}$$

Water Table Balance Constraints

$$(A_1 + a_1)H_{1t} - A_1 H_{1t-1} - b_{p1} E_1 S_{1t}^{WG} - (1 - E_1) S_{1t}^{WG} = 0 \tag{6.21}$$

$$(A_2 + a_2)H_{2t} - A_2 H_{2t-1} - a_{12} H_{1t} - b_{p2} E_2 S_{2t}^{WG} + (1 - E_2) S_{2t}^{WG} +$$

$$(1 - b_{p2}) G_{2t} = 0 \tag{6.22}$$

$$(A_3 + a_3)H_{3t} - A_3 H_{3t-1} - b_{p3} E_3 S_{3t}^{WG} - (1 - E_3) S_{3t}^{WG} + (1 - b_{p3}) G_{3t} = 0 \tag{6.23}$$

$$(A_4 + a_4)H_{4t} - A_4 H_{4t-1} - b_{p4} E_4 (S_{4t}^{WG} + S_{4t}^{MC} + S_{4t}^{SC}) - (1 - E_4)(S_{4t}^{WG} +$$

$$S_{4t}^{MC} + S_{4t}^{SC}) = 0 \tag{6.24}$$

$$(A_5 + a_5)H_{5t} - A_5H_{5t-1} - a_{15}H_{1t} - b_{p5}E_5(S_{5t}^{WG} + S_{5t}^{SC} + S_{5t}^{MC}) -$$

$$0.10S_t^{WG} - (1 - E_5)(S_{5t}^{WG} + S_{5t}^{SC} + S_{5t}^{MC} + (1 - b_{p5})G_{5t} = 0$$

$$(6.25)$$

$$(A_6 + a_6)H_{6t} - A_6H_{6t-1} - a_{46}H_{4t} - b_{p6}E_6(S_{6t}^{WG} + S_{6t}^{CC}) - (1 - E_6)$$

$$(S_{6t}^{WG} + S_{6t}^{CC}) + (1 - b_{p6})G_{6t} = 0 \qquad (6.26)$$

$$(A_7 + a_7)H_{7t} - A_7H_{7t-1} - a_{57}H_{5t} - b_{pt}E_7(S_{7t}^{WG} + S_{7t}^{EG}) - (1 - E_7)$$

$$(S_{7t}^{WG} + S_{7t}^{EG}) + (1 - b_{p7})G_{7t} = 0 \qquad (6.27)$$

$$(A_8 + a_8)H_{8t} - A_8H_{8t-1} - a_{18}H_{1t} - a_{68}H_{6t} - b_{p8}E_8(S_{8t}^{WG} + S_{8t}^{CC}) -$$

$$(1 - E_8)(S_{8t}^{WG} + S_{8t}^{CC}) + (1 - b_{p8})G_{8t} = 0 \qquad (6.28)$$

$$G_{it} - CK_i \leq 0 \quad (i=1,\ldots,8) \qquad \text{Pumping Constraint vis-a-vis Wells}$$
$$(6.29)$$

$$G_{it} \leq H_{it}A_i \quad (i=1,\ldots,8) \qquad \text{Groundwater Supply Constraint}$$
$$(6.30)$$

All variables are nonnegative except for the following:

$$S_{11}^{MC} \ldots S_{34}^{MC} = \; ; S_{61}^{MC} \ldots S_{84}^{MC} = 0; S_{11}^{EG} \ldots S_{44}^{EG} = 0; S_{61}^{EG} \ldots S_{64}^{EG} = 0;$$

$$S_{81}^{EG} \ldots S_{84}^{EG} = 0; S_{11}^{CC} \ldots S_{54}^{CC} = 0; S_{71}^{CC} \ldots S_{74}^{CC} = 0; S_{11}^{SC} \ldots S_{34}^{SC} = 0;$$

$$S_{51}^{SC} \ldots S_{84}^{SC} = 0; G_{11} \ldots G_{14} = 0; G_{41} \ldots G_{44} = 0.$$

The logic involved in construction of these constraints was discussed earlier in Chapter IV. As no reverse flows of groundwater are expected to occur in the system, the constraint terms designating this flow are defined as functions of H_{it} rather than H'_{it} as shown in the theoretical model.

124

The linear programming model adopted for the Gallatin Valley irrigation area contains 209 constraints, 458 activities, and 2,014 matrix entries. A simplified format of the problem is shown in Table 6-1. In the first column shown in Table 6-1 there are 192 different water-response activities representing combinations of crops, crop response in different time periods, crop response in different sub-areas, and different strategies in water delivery during the intra-seasonal time periods analyzed.

Except for the latter item, the combinations of activities listed above are relatively commonplace in programming models for optimal water allocation. The quantity of water delivered during different periods of the irrigation season is usually determined as crop consumptive use less precipitation in each time period. In the Gallatin Valley, the capacity of the soil reservoir is relatively large in most subareas compared with crop consumptive use in each time period. It is therefore possible to store water in a given time period for use in subsequent time periods without causing undue plant stress. Since the source of surface water is limited to stream flow in the Gallatin Valley, and this supply generally diminishes as the irrigation season progresses, the exchange of water supply among the different time periods is considered to be an important part of the water allocation problem in this study.

TABLE 6-1. Linear Programming Format Activities and Constraints

Activity Sets (n) / Constraint Sets (m)	Subarea crop-time period response (192)	Subarea S. water diversions by time periods (60)	Subarea G. water pumping by time periods (24)	Subarea well development for irrigation season (6)	Subarea water-table levels by time periods (32)
Subarea cropland (8)	Crop requirements				
Subarea crop rotation (8)	Alfalfa requirements				
Subarea wheat allotment acres (8)	Wheat requirements				
Subarea irrigation requirements (32)	Irrigated crop requirements each period	Crop use, field loss canal loss each period	Crop use, field loss, each period		
Subarea water-table balance (32)		Canal loss, field loss each period	Net withdrawal each period		Determined each period in balance
Subarea S. water supply (60)		Diversion requirements each period			Water-table discharge each period
Subarea G. water well development supply (24)			Total withdrawal each period	Pumping capacity each period	

(table continued)

TABLE 6-1. Continued

Constraint Sets (m)	Activity Sets (n) Subarea crop-time period response (192)	Subarea S. water diversions by time periods (60)	Subarea G. water pumping by time periods (24)	Subarea well development for irrigation season (6)	Subarea water-table levels by time periods (32)
Subarea G. water aquifer supply (24)			Net withdrawal each period		Storage level each period
Subarea water-table limits for crops (32)					Check on water-table level each period

The hierarchy adopted in classifying this related set of activities in column 1 is as follows:

Subarea specified
|
Crop selected in subarea
|
Irrigation period(s) analyzed for each crop
|
Amount of water delivered each period for crop use

Activity sets in the second and third columns of Table 6-1 are largely self-explanatory. All groundwater is assumed to be pumped near each irrigated field; therefore, no canal losses are specified for groundwater use. Groundwater use is constrained by the level of well development and the amount of aquifer storage. Both surface and groundwater use affect water balance in each time period as is shown in Table 6-1.

Well development in each subarea is not variable in an intra-seasonal irrigation model unless the model is in a steady state. As stated earlier in Chapter IV, the initial solutions for this model will be computed for several fixed levels of surface water supply. The model is assumed to be in a steady state in analyzing the initial solutions and well development is included among the decision variables.

In the fifth column of Table 6-1, subarea water-table levels in each time period are shown as activities in the model due to their interaction with other important variables. A description of these interaction effects was given in Chapter IV.

Field efficiency in irrigation is comparable to well development as this activity is also invariant within an irrigation season except under steady-state conditions. The effect of changing field efficiency is the same as changing distribution efficiency in the present model since all water losses are assigned to deep percolation. There appeared to be a considerable gap between the present estimated field efficiency level in the Gallatin Valley and the recommended level for this region. An explanation of this difference was presented in Chapter III. In view of the economic importance of the field efficiency level for water supply and drainage, an evaluation of both efficiency levels is included in the present study. The recommended higher efficiency level is considered to be a probable operating level in the future but it could also entail some additional cost which cannot be evaluated at the present time. Nevertheless, it is of interest to this study to project what effect the higher level would have on the regional income and the allocation of resources in the Gallatin Valley.

The logic involved in the construction of constraint sets shown in Table 6-1 has been discussed at some length in previous chapters. Some new constraints have been added to limit alfalfa and wheat production. Wheat production is constrained by the recorded 1970 allotment acreage for the eight subareas in the Gallatin Valley area. As there is some question about the continuance of the present wheat allotment program in the future, solutions are also obtained with this program omitted.

Results of Deterministic Models

Optimal solutions were obtained for both the low and high field efficiency level with parametric programming of the right-hand side vector for surface water supply in each of the four time periods. Streamflow levels tested were 40, 50, 60, 70, 80, 90, 100, and 110 percent of the mean. Running time on the Sigma 7 computer at Montana State University for the 16 solutions was approximately 30 minutes.

Values of the criterion function for the above solutions are shown in Figure 6-1.. The minimum water supply level considered is 40 percent of the mean as the estimated probability of a smaller supply is only 0.021330. Aggregate net crop income is much higher for the recommended high field efficiency level than for the present lower level. It is also evident that water response diminishes much more rapidly for the high level as compared with the low level.

Table 6-2 reports the effect of parametric changes in surface water supply upon crop income, cropping patterns and pumping. It may be noted in the first half of the table that there was a 6,000 acre increase in dryland crop acreage associated with a change in surface water supply above 40 percent of the mean. The dryland crops were necessary because of high water table conditions in two subareas in which pumping was not feasible for drainage control. The only means of keeping water tables below the critical bogging level under low field efficiency in water use was to limit irrigation in these two

Figure 6-1. Income Response for Eight Surface Water Supply Levels and Two Field Efficiency Levels in the Gallatin Valley Area.

130

TABLE 6-2. The Effect of Parametric Programming of. Surface-Water Supply. upon Crop Production, Farm Income, and Pumping in the Gallatin Valley Area. [a]

	Surface Water Supply Level Proportion of Mean							
	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10
Low Field Efficiency:								
Net Farm Income	6,454	6,521	6,545	6,562	6,578	6,595	6,611	6,623
Number of Wells	311	285	255	227	198	170	140	110
G. Water Pumped(a.f.)	99,183	84,961	73,207	66,350	59,488	52,487	45,502	39,965
Dryland crops (acres)	5,108	11,115	11,115	11,115	11,115	11,115	11,115	11,115
June Irr. Crops (acres)	21,950	18,946	21,132	21,128	21,131	21,164	21,150	21,159
June-July Irr. Crops (ac.)	24,473	14,262	9,705	9,709	9,705	9,710	9,715	9,708
June-Aug. Irr. Crops (ac.)	67,633	74,836	77,207	77,208	77,207	77,169	77,119	77,177
June-Sept. Irr. Crops (ac)	5,254	5,254	5,254	5,254	5,254	5,254	5,254	5,254
High Field Efficiency:								
Net Farm Income	7,015	7,133	7,218	7,238	7,248	7,250	7,250	7,250
Number of Wells	96	77	54	24	23	23	23	23
G. Water Pumped (a.f.)	26,200	21,944	16,089	9,134	8,825	8,825	8,822	8,822
Dryland Crops (acres)	--	--	--	--	--	--	--	--
June Irr. Crops (acres)	19,381	13,116	10,669	10,669	10,669	10,669	10,669	10,669
June-July Irr. Crops (ac.)	--	--	--	--	--	--	--	--
June-Aug. Irr. Crops (ac.)	95,677	95,678	95,678	95,678	95,678	95,678	95,678	95,678
June-Sept. Irr. Crops (ac)	9,357	15,619	18,069	18,069	18,069	18,069	18,069	18,069

[a] Income is in units of $1,000.

subareas. This is a vivid example of hydrologic interactions affecting the optimal water allocation for irrigation use.

June-irrigated winter wheat appeared in all of the solutions shown in Table 6-2 as this is a relatively profitable crop. It was exactly constrained by the wheat allotment acreage limitation in all cases. Irrigation beyond June does not generally increase winter-wheat yield.

For irrigation beyond June, alfalfa is the most profitable crop. However, alfalfa yield does not usually increase for irrigation beyond August as it is normally not feasible to take a third cutting of this crop in the Gallatin Valley. All alfalfa production is constrained by a crop rotation limitation in the empirical model. Under low field-efficiency conditions, there was some shift towards irrigating alfalfa up to September rather than only to August as surface water supply increased. Under high field efficiency conditions it was always profitable to irrigate alfalfa to the end of August.

Irrigated pasture has increasing response for irrigation up to the end of September. Under low field-efficiency conditions it was evidently not profitable to irrigate much pasture land beyond June particularly where pumping was not feasible for drainage control. Under the high field efficiency level, there was a shift towards irrigating pasture to the end of September rather than to the end of June as the availability of surface water supply increased.

Aside from some slight change in cropping patterns, the major effect of increased surface water supply in Table 6-2 was a significant reduction in well and pumping costs, particularly in the case of low field efficiency. Since the level of pumping appeared to be steadily dropping for low field efficiency up to 110 percent of mean surface-water supply, higher levels of water supply were also evaluated for the model. However, the optimal amount of groundwater pumping was still nearly 40,000 acre feet for the season up to 197 percent of mean surface water supply. It was therefore estimated that approximately this amount of groundwater pumping was needed for drainage control irrespective of the level of surface water supply. This leveling off of groundwater pumping is clearly evident in the case of high field efficiency as shown in Table 6-2. Even though surface water is in slack, it still pays to pump water for irrigation use since the value of drainage control more than offsets the cost of pumping in the Gallatin Valley.

The change in shadow prices for water-table levels provides important evidence of the extent that hydrologic conditions are affecting optimal water allocation. Shadow prices for water-tables in three subareas during the four intraseasonal time periods are shown in Table 6-3. The prices are computed by the model for 40 and 50 percent of mean surface-water supply under low field efficiency conditions. Subarea 1 and 4 are the ones referred to in Table 6-2

TABLE 6-3. Computed Shadow Prices of Water-Tables in Three Subareas of the Gallatin Valley Area During Four Intraseasonal Irrigation Time Periods under Low Field Efficiency Conditions for 40 and 50 Percent of Seasonal Mean Surface Water Supply.

Water Supply/Period	Subarea 1 Water Table[a]	Price[b]	Subarea 3 Water Table[a]	Price[b]	Subarea 4 Water Table[a]	Price[b]
	Ft.	Dol.	Ft.	Dol.	Ft.	Dol.
40 Percent Mean:						
Period 1	19.4	0.00	42.1	0.27	144.2	− 5.02
Period 2	21.8	0.00	39.6	0.26	146.2	− 5.03
Period 3	20.8	0.00	37.2	0.12	148.0*	− 5.05
Period 4	19.9	0.00	37.2	0.00	147.6	(1.73)
50 Percent Mean:						
Period 1	19.2	−3.74	44.6	0.08	143.4	− 9.90
Period 2	22.0*	−3.90	42.2	0.04	146.2	− 9.93
Period 3	22.0*	−3.20	39.8	0.13	148.0*	− 0.01
Period 4	21.0	0.00	39.8	(1.14)	147.6	(1.73)

[a] The asterisk indicates that the water table was at a critical level in that particular time period (two feet from the surface).

[b] A bracketed price indicates that the high water table was of value in that time period in affecting groundwater discharge to streams when surface water was in relatively short supply.

134

that were forced to increase dryland crop acreage when surface water supply increased since it was not feasible to pump in these subareas for drainage control. Subarea 3 did not have any difficulties with high water tables but it is one of the subareas in the model that contributes to surface water supply through groundwater discharge, which, in this instance, is of benefit to all subareas since surface water is in relatively short supply.

In analyzing the water-table shadow prices shown in Table 6-3, it is evident that subarea 1 started having difficulties with a high water table after surface water supply was increased from 40 to 50 percent of mean level. Since water tables were not binding for the first supply level, the shadow prices were equal to zero. After they became binding, the shadow prices became negative. It may be recalled that this is one of the subareas which was forced to increase dryland crop acreage to reduce irrigation percolation losses to the water table since pumping was not feasible in this subarea. The negative shadow prices may be interpreted as the opportunity costs of limiting crop production to less profitable crops. Subarea 4 had water table problems for both water supply levels; they became more pronounced for the second level as indicated by the decrease (algebraically) in shadow prices. The shadow price became positive in period 4 indicating that groundwater discharge from this subarea was of benefit to other subareas in this time period. This subarea was also not able to pump water for drainage control.

Subarea 3 has positive shadow prices indicating that a further increase in the water-table height would be of benefit in the model. This subarea is located adjacent to the West Gallatin River near the upper end of the Gallatin Valley and the groundwater discharge from Subarea 1, which is estimated to be linearly related to water-table height, may be diverted for irrigation use by several other subareas located downstream.

Results of Stochastic Models

As stated earlier, the optimization of well development in a deterministic intraseasonal model is not feasible except under steady-state conditions. However, intraseasonal water supply is highly variable in the Gallatin Valley Area, and the optimal well development level will tend to fluctuate for different surface water supplies as was demonstrated in Table 6-2. Further, it cannot be assumed that mean surface-water supply is the appropriate water supply level for planning well development. The cost of unused well capacity for surface-water supply above mean level may be more than offset by the loss in crop income due to inadequate capacity when surface-water supply is below mean level. Therefore, in determining the most optimal level of well development over the long run, it is necessary to consider the probability distribution of interseasonal surface-water supply in the Gallatin Valley Area.

- 137

Distribution of Surface-Water Supply

The major sources of stream flow in the Gallatin Valley are snow
melt and precipitation in the mountains to the south of the Valley. A
considerable amount of the snow melt tends to run off in early spring and
precipitation becomes the primary source of flow during the remainder
of the irrigation season.

Since the distribution of surface-water supply has, theoretically,
a zero lower bound but no statistical upper bound, it was assumed that
the gamma distribution function could be fitted reasonably well to the
interseasonal data for West Gallatin flow. This distribution function
has performed well in other applications to precipitation data. Since
stream flows have their origin in precipitation, one would expect the
gamma to fit there too. The gamma distribution is a 2-parameter fre-
quency distribution defined as: 1/

$$f(x) = \frac{1}{\beta^\gamma \Gamma(\gamma)} x^{\gamma-1} e^{-x/\beta}; \quad \begin{array}{l} \beta > 0 \\ \gamma > 0 \end{array}$$

where: x = the argument of the gamma function;

β = a scale parameter for distribution of x;

γ = a parameter determining the shape of the distribution;

Γ = the usual gamma function; and

f(x) = zero for x < 0.

1/ The procedure followed in estimating the gamma distribution for inter-
   seasonal surface water supply is taken from H.C.S. Thom, "A Note on
   the Gamma Distribution," Monthly Weather Review, Vol. 86, April 1958,
   pp. 117-122.

Assuming that a sample of n observations has been obtained to estimate the above parameters in the theoretical density functions, $f(x)$, maximum likelihood estimates of γ and β are computed by the following procedure. The first step in fitting the distribution is to compute the value of $\log \bar{x} - \frac{1}{n}\sum_{i=1}^{n} \log x_i$, where x_i is the ith observation in the sample. For simplification, this value is subsequently identified as A. The estimated value of γ may then be computed as the positive root of the quadratic expression:

$$\gamma = [1 + \ \ 1 + 4A/3]/4A \tag{6.31}$$

The value of β is given by:

$$\hat{\beta} = \bar{x}/\hat{\gamma} \tag{6.32}$$

The above two equations provide approximations to the maximum likelihood estimates for the gamma distribution. Thom 2/ has also provided a table for adjusting errors in estimating γ since his procedure involves the use of only one term of a relatively complex asymptotic expansion for estimating γ. This approximation of γ increases in accuracy as γ becomes larger. No correction is required for $\hat{\gamma} \geq 5.6$.

After the maximum likelihood estimated values for $\hat{\gamma}$ and $\hat{\beta}$ are computed, Pearson's table 3/ may be used to determine the probability of the x-variate lying within certain intervals of the gamma distribution corresponding to $\hat{\gamma}$ and $\hat{\beta}$. Use of Pearson's table requires that $\hat{\gamma}$ and $\hat{\beta}$ ____

2/ Ibid., p. 119.

3/ Pearson, K., et. al., Tables of the Incomplete Gamma Function, Cambridge University Press, Reissue 1951.

must be converted to p and μ values, respectively, since the arguments

of his table are p and μ. The conversions are as follows:

μ = x/(β√γ̄); and

p = γ-1

The period of record for stream flow in the Gallatin Valley was

shown in Table 3-1 of Chapter III. It is assumed that all stream flow

is perfectly correlated with West Gallatin stream flow as the latter

stream provides about two-thirds of the present irrigation supply for

the Gallatin Valley. Also, much more complete records of flow were

available for the West Gallatin River as compared with the other streams

in the valley.

Intraseasonal flow in each of the four irrigation time periods,

i.e., June, July, August and September, is assumed to be a constant

proportion of total annual flow. The relative proportions are averaged

for the 50-year record of West Gallatin flow during these four irriga-

tion periods.

Using Thom's method 4/, the maximum likelihood values of γ and β

are estimated to be 9.78995 and 30.99100, respectively. The estimated

p̂ value is 8.78995. Estimated μ values are computed for 11 intervals of

the distribution for annual surface water supply as shown in Table 6-4.

The x-midpoint values corresponding to those μ values are taken as dis-

crete quantities of West Gallatin surface water supply which are subject

to the probability levels listed in Table 6-4. Coefficients in the

---

4/ Thom, op. cit.

Filmed as received

without page(s)____140____.

UNIVERSITY MICROFILMS.

TABLE 6-4. Estimation of the Probability of West Gallatin Streamflow Being Within Certain Intervals of the Computed Gamma Distribution for Interseasonal Streamflow.

X-interval (1,000 a.f.)	X-midpoint (1,000 a.f.)	μ-midpoint	$\overline{X}$-midpoint/X (1,000 a.f.)	Probability (X)
0-140	120[a]	1.44379	0.4000	0.0213300
140-180	160	1.85630	0.5333	0.0607110
180-220	200	2.26881	0.6667	0.1024240
220-260	240	2.68132	0.8000	0.1732880
260-300	280	3.09383	0.9333	0.1701260
300-340	320	3.50634	1.0667	0.1519812
340-380	360	3.91885	1.2000	0.1181551
380-420	400	4.33136	1.3333	0.0824471
420-460	440	4.74387	1.4667	0.0526739
460-500	480	5.15638	1.6000	0.0312787
500-∞	520	--	1.8667	0.0668607

[a]The discrete points chosen to represent each tail of the probability distribution are selected at the midpoint of the probability interval rather than at the x-interval midpoint, e.g., for the X-interval, 0-140, the probability is 0.0213300 that X is in this interval. The midpoint of the probability interval, 0-0.0213300, is 0.0106650 which corresponds to an X value of 120. Hence, 120 is the X-midpoint of the probability interval.

141

cluded that optimal well development should result if the above water
supply levels are specified in the linear programming model for planning
well investment in the Gallatin Valley area. It is noted that the ex-
pected values given in Table 6-5 have annual amortized costs of well
development deducted, although the original linear programming objective
functions did not.

Two optimal well investments in Table 6-5 represent the "best" plan-
ning levels for maximizing expected net income under high and low field
irrigation efficiency conditions. Stability of income, however, may be
improved with well investment. As indicated in Table 6-5, the standard
deviation of expected net income is monotonically decreasing for larger
planned investment. For example, a reduction in well development from 96
to 23 wells (planned water supply of 80 percent instead of 40 percent of
mean surface supply) under high field efficiency conditions would raise
expected net income by $16,587., but standard deviation would also in-
crease by $50,908. Thus, there is a trade-off in goals between maximizing
expected net income and minimizing income fluctuations which should be
noted in planning optimal well investment.

In Table 6-5, the rational interval of choice occurs where mean in-
come and standard deviation of income are decreasing. For smaller levels
of well investment, i.e., for larger planned surface supply, expected
income declines and standard deviation increases. This latter interval
represents an irrational planning region from a theoretical standpoint.

Decision theory to deal with this trade-off problem is essentially that of portfolio selection as first conceived by Lutz 5/ and Markowitz 6/. In order to apply this theory, however, one would first need a risk-preference schedule for irrigators in the Gallatin Valley. Their preferences regarding risk are not known at the present time.

Programming Results

Programming results regarding cropping patterns, crop restrictions, and constraint shadow prices are given in Tables 6-6 and 6-7. All values shown for low field efficiency are taken from the final tableau of solution for 80 percent mean surface water supply. High field efficiency values are taken from the solution for 60 percent mean surface water supply.

Shadow prices for the wheat allotment acreage restriction (WHTAC) represent the difference in net return per acre for wheat and the next most profitable crop to wheat. In the first two subareas, it is evident that Subarea 2 has a more profitable alternative crop than Subarea 1; therefore, the shadow price for WHTAC is higher in the case of Subarea 1. WHTAC is a binding constraint for all subareas in Tables 6-6 and 6-7.

Limitations on total cropland acreage (ACRES) are valued at the net return per acre of the least profitable crop entering the solution. Subareas 1 and 4 tend to have some low-valued crops, e.g., dryland pasture, so these subareas have relatively low shadow prices for ACRES compared with other subareas.

5/ Lutz, F., and V., The Theory of Investment of the Firm, Princeton: Princeton University Press, 1951.

6/ Markowitz, H., Portfolio Selection: Efficient Diversification of Investments, New York: John Wiley and Sons, Inc., 1962.

Table 6-5. Expected Income Values for Eight Discrete Levels of Well Development under High and Low Field-Efficiency Conditions in the Gallatin Valley Area

Planned Surface Water Supply for Well Development (Pct. of Mean)	Level of Well Development				Expected Income Field Efficiency			
	High		Low		High		Low	
	No. Wells	Annual Cost (Dols.)	No. Wells	Annual Cost (Dols.)	Mean	Std. Dev.	Mean	Std. Dev.
					——————————— (Dols.)———————————			
40	96	24,172.	311	81,232.	7,220,400.	34,104.	6,554,463.	23,030.
50	77	18,310.	285	74,631.	7,221,568.	40,094.	6,559,002.	21,916.
60	54	11,804.	255	65,234.	7,226,531.*	48,134.	6,568,251.	25,297.
70	24	5,364.	227	56,374.	7,217,379.	83,783.	6,574,534.	32,196.
80	23	5,149.	198	47,290.	7,216,987.	85,012.	6,577,337.*	46,504.
90	23	5,149.	170	39,308.	7,216,987.	85,012.	6,575,137.	64,486.
100	23	5,149.	140	30,995.	7,216,987.	85,012.	6,563,769.	89,121.
110	23	5,149.	110	24,554.	7,216,987.	85,012.	6,543,314.	111,001.

*This indicates the most optimal well development level in the class of eight well development levels compared for each field efficiency.

144

Alfalfa rotation limitations (ALFRO) also affect income in most subareas as is shown in Tables 6-6 and 6-7. Under low field efficiency conditions, ALFRO is a relatively weak restriction on income for Subareas 1 and 4 since the respective shadow prices are quite low for these two subareas.

Subarea water tables are restricted in the empirical model so as not to exceed some critical level. The cost of imposing this restriction for each subarea in the four time periods of the irrigation season is reported in Table 6-8. Shadow prices associated with these water table restrictions represent the opportunity cost of limiting crop irrigation in the case of Subareas 1 and 2, and the additional total cost of pumping for drainage control in other subareas. It is evident that the opportunity costs of foregone highvalued crop production are considerably higher than pumping costs. This outcome suggests that it is usually more profitable to utlize wells for drainage control rather than limiting irrigation in an area when this is a technical possibility.

Under high field efficiency conditions only the Central Park subarea was dependent upon pumping for drainage control. The reduction in cost from $81,000. to $1,705. for the high efficiency level indicates the value of the latter for drainage control where water allocation and pumping concentration is optimal in the Gallatin Valley area. If water allocation is not optimal, the drainage control benefit of high field efficiency may be significantly larger than this estimate. The major benefit of using the higher field efficiency level, however, is that it

TABLE 6-6.   Evaluation of the Cropping Patterns, Related Restrictions and Shadow Prices of Restrictions in Optimal Solutions for the Gallatin Valley Area under Low Field Efficiency Conditions

Crops, Restrictions and Shadow Prices	Subarea (i)							
	1	2	3	4	5	6	7	8
June wheat (acres)	3,152	372	1,343	1,681	2,094	604	1,248	175
Restrictions (June wht.)	WHTAC[a]	WHTAC	WHTAC	WHTAC	WHTAC	WHTAC	WHTAC	WHTAC
Shadow prices (WHTAC) ($)	16.53	12.57	12.47	17.59	17.28	11.89	12.86	16.10
Dryland pasture (acres)	2,937	--	--	8,178	--	--	--	--
Restrictions (Dryland past.)	ACRES[b]	--	--	ACRES	--	--	--	--
Shadow prices (ACRES) ($)	32.80	--	--	32.80	--	--	--	--
June pasture (acres)	--	--	--	--	--	--	--	--
June-July pasture (acres)	--	--	--	--	--	--	--	--
June-Aug. pasture (acres)	--	--	--	--	6,628	1,700	--	--
Restrictions (June-Aug. past.)	--	--	--	--	ACRES	ACRES	--	--
Shadow prices (ACRES) ($)	--	--	--	--	35.90	41.38	--	--
June-Sept. pasture (acres)	--	944	1,290	--	--	--	1,879	1,141
Restrictions (June-Sept. past.)	--	ACRES	ACRES	--	--	--	ACRES	ACRES
Shadow prices (ACRES) ($)	--	41.47	41.45	--	--	--	40.32	35.90
June alfalfa (acres)	3,862	--	--	6,597	--	--	--	--
Restrictions (June alfalfa)	ALFRO[c]	--	--	ALFRO	--	--	--	--
Shadow prices (ALFRO) ($)	0.50	--	--	1.91	--	--	--	--
June-July alfalfa (acres)	9,715	--	--	--	--	--	--	--
Restrictions (June-July alf.)	ALFRO	--	--	--	--	--	--	--
Shadow prices (ALFRO) ($)	0.50	--	--	--	--	--	--	--
June-Aug. alfalfa (acres)	1,645	3,292	6,583	18,681	21,805	5,760	7,817	3,292
Restrictions (June-Aug. alf.)	ALFRO	ALFRO	ALFRO	ALFRO	ALFRO	ALFRO	ALFRO	ALFRO
Shadow prices (ALFRO) ($)	0.50	20.40	20.39	1.91	19.70	20.12	19.77	19.31

[a]Wheat allotment acreage limitation.   [b]Total cropland acreage limitation.

[c]Alfalfa rotation acreage limitation.

146

TABLE 6-7. Evaluation of the Cropping Patterns, Related Restrictions and Shadow Prices of Restrictions in Optimal Solutions for the Gallatin Valley Area under High Field Efficiency Conditions.

Crops, Restrictions and Shadow Prices	Subarea (i)							
	1	2	3	4	5	6	7	8
June wheat (acres)	3,152	372	1,343	1,681	2,094	604	1,248	175
Restrictions (June wht.)	WHTAC[a]	WHTAC	WHTAC	WHTAC	WHTAC	WHTAC	WHTAC	WHTAC
Shadow prices (WHTAC) ($)	19.30	10.98	10.95	13.00	14.36	11.64	10.86	12.76
Dryland pasture (acres)	--	--	--	--	--	--	--	--
June pasture (acres)	750	--	--	--	--	--	--	--
June-July pasture (acres)	--	--	--	--	--	--	--	--
June-Aug. pasture (acres)	--	--	--	--	6,628	--	--	--
June-Sept. pasture (acres)	2,187	944	1,290	8,178	--	1,700	1,879	1,141
Restrictions (June-Sept. past.)	ACRES[b]	ACRES	ACRES	ACRES	ACRES	ACRES	ACRES	ACRES
Shadow prices (ACRES) ($)	34.45	43.12	43.13	41.15	38.92	41.70	42.74	40.09
June alfalfa (acres)	--	--	--	--	--	--	--	--
June-July alfalfa (acres)	--	--	--	--	--	--	--	--
June-Aug. alfalfa (acres)	15,223	1,656	3,128	18,505	20,891	5,760	7,817	3,292
Restrictions (June-Aug. alf.)	ALFRO[c]	ALFRO	ALFRO	ALFRO	ALFRO	ALFRO	ALFRO	ALFRO
Shadow prices (ALFRO) ($)	20.81	20.27	20.25	20.11	20.03	19.63	19.63	19.22

[a]Wheat allotment acreage limitation.　　[b]Total cropland acreage limitation.

[c]Alfalfa rotation acreage limitation.

147

Table 6-8. Evaluation of Water-Table Restrictions and Shadow Prices of Restrictions in
Optimal Solutions for Two Field Efficiency Levels in the Gallatin Valley Area

	Subarea (i)							
	1	2	3	4	5	6	7	8
Low E. 80% Mean W.S.								
t=1, water table (ft.)	19.2	47.9	50.3	143.1	163.2	166.7	19.0	27.6
t=1, shadow price ($)	0	0	0	0	0	0	0	0
t=2, water table (ft.)	22.0*	45.6	47.8	146.9	168.1	174.3	20.0*	34.1
t=2, shadow price ($)	2979.58	0	0	0	0	0	77.50	0
t=3, water table (ft.)	22.0*	43.4	45.3	148.0*	180.0	182.2	19.5	37.8
t=3, shadow price ($)	14421.32	0	0	61135.77	0	0	0	0
t=4, water table (ft.)	21.0	43.5	45.2	147.6	170.0	182.9	20.0*	38.0*
t=4, shadow price ($)	0	0	0	0	0	0	1650.06	801.74
High E. 60% Mean W.S.								
t=1, water table (ft.)	15.4	44.2	46.1	141.5	149.1	162.5	18.8	27.9
t=1, shadow price ($)	0	0	0	0	0	0	0	0
t=2, water table (ft.)	19.0	41.9	43.7	143.4	159.6	166.0	19.7	32.2
t=2, shadow price ($)	0	0	0	0	0	0	0	0
t=3, water table (ft.)	20.0	39.8	41.3	144.4	159.9	170.0	19.6	35.6
t=3, shadow price ($)	0	0	0	0	0	0	0	0
t=4, water table (ft.)	19.4	39.4	40.8	144.3	158.9	169.7	20.0*	36.3
t=4, shadow price ($)	0	0	0	0	0	0	1705.07	0

*This indicates that the water table is at a critical level.

148

greatly reduces the need for pumping to supplement short surface water supply in the study region.

Since the programming model allowed considerable flexibility in pumping to relieve short surface water supply conditions, the shadow prices for constrained surface water supply are relatively low. For both high and low field efficiency conditions, the shadow prices for this constraint are $1.68 for the month of July and $1.69 for August. They are equal to zero in other time periods.

TABLE 6-9. Parametric Programming on Variable Pumping Costs and Fixed Well Development Costs With Surface-Water Supply at 60 and 80 Percent Mean Level.[a]

Fixed and Variable Cost Levels	Programming Results For:			
	High Field Efficiency		Low Field Efficiency	
	No. Wells	Objective Value (dollars)	No. Wells	Objective Value (dollars)
Constant F.C.[b]				
VC1[b]	78	7,217,852.	198	6,580,236.
VC2[c]	78	7,215,343.	198	6,573,494.
VC3[d]	78	7,220,195.	198	6,586,692.
Constant V.C.[b]				
FC1[b]	78	7,217,852.	198	6,580,236.
FC2[c]	78	7,215,910.	198	6,575,487.
FC3[d]	78	7,219,622.	198	6,584,984.

[a] Optimal solutions were obtained at 60 percent mean surface water supply for high field efficiency conditions and at 80 percent mean water supply for low field efficiency conditions.

[b] This is the previous estimate of the cost.

[c] Previous estimated cost plus 10 percent.

[d] Previous estimated cost minus 10 percent.

The sensitivity of the optimal solution for different field efficiency levels to estimation errors in pumping costs data is tested by parametric programming of these cost coefficients in the matrix. The effects of changing these costs on the optimal level of well development and aggregate net income are shown in Table 6-9. Solutions are not sensitive to the cost changes and water allocations are also not affected.

Sensitivity of the two solutions to changes in water table limits is also tested by parametric programming of the right-hand sides constraining the water-table in each subarea. Results of the parametric programming are given in Table 6-10. In the case of high field efficiency, pumping changed only slightly in inverse proportion to the upper limit set on water table height in all subareas. Cropping patterns and water allocation are not significantly affected.

Under low field efficiency conditions, pumping increased when the water table limit was relaxed and pumping decreased when water tables were made more restrictive. This apparent paradox is explained by the change in cropping patterns occurring in the programming solution. When the water table limits were raised, both the Camp Creek Hills subarea and the Bozeman Fan subarea increased irrigated alfalfa production which expanded total water demand for irrigation. Additional pumping was required to meet this increase in irrigation demand since surface water supply was not adequate. On the other hand, when the water table limits were made more restrictive, i.e., constrained at a lower level,

the above two subareas were forced to reduce irrigation demand by

shifting to pasture crops with lower consumptive use which relaxed

the demand for surface water and in turn that for groundwater. Also,

the Central Park Subarea was forced to shift nearly half of its total

cropland acreage to dryland pasture from irrigated crops to meet the

lower water table restrictions as drainage via pumping was not adequate.

Consequently, less pumping was required in the latter solutions.

TABLE 6-10. Parametric Programming on Water-Table Limits With Surface
Water Supply Fixed at 60 and 80 Percent Mean Level. [a]

Water Table Limits	Programming Results For:			
	High Field Efficiency		Low Field Efficiency	
	No. Wells	Objective Value	No. Wells	Objective Value
(Feet)		(dollars)		(dollars)
Limit 1[b]	78	7,217,852.	198	6,580,236.
Limit 2[c]	70	7,221,171.	2 00	6,742,403.
Limit 3[d]	85	7,214,050.	170	6,285,356.

[a]These surface water supplies are for high and low field efficiency
conditions, respectively.

[b]This is the previous limit for all subareas.

[c]Previous limit on height plus 2 feet.

[d]Previous limit on height minus 2 feet.

These programming solutions are sensitive to possible errors in

determining critical water table levels for crop production, particularly

in the case of low field efficiency. It is apparent from Table 6-10

that an error of 4 feet in estimating the critical water-table level

could make a difference of almost half a million dollars in projecting

annual net income.

Since the present wheat allotment program cannot be assumed to con-

tinue indefinitely with the current price support for wheat, it was

desirable to determine the relative importance of this price-supported

crop in the solution. Alternative feed grain crops to wheat have

returns and irrigation requirements comparable to irrigated pasture as

was illustrated in Chapter IV. Removal of price-supported wheat activi-

ties from the input data for high and low field efficiency models result-

ed in the following respective revised optimal solutions:

(1) $7,063,422. and (2) $6,414,448. This represented decreases of

$154,429. and $165,788. for the two field efficiencies. Since most

alfalfa activities were previously constrained by alfalfa acreage

rotation limitations, additional irrigated pasture activities are forced

into the revised solutions to replace wheat activities.

Testing of Hypotheses

In Chapter IV it was explained how all irrigated crop activities

were stratified according to the average level of available soil mois-

ture remaining in storage at the end of each irrigation period. Details

on this stratification of activities were presented in Appendix B. It

was hypothesized in adopting this methodology that changing the level

of available soil moisture carryover among different time periods of the

irrigation season could be an important strategy for increasing the utilization of variable stream flows. Only two carryover levels are analyzed at the end of each irrigation period because of the computational problem involved with different time period-carryover level combinations for each crop in each subarea of the model. The minimum carryover level considered is 50 percent of available soil moisture storage. There is danger of possible plant stress occurring at lower levels which could not be quantified for the Gallatin Valley area.

In testing the importance of this hypothesis regarding moisture-storage carryover among the four time periods, all crop activities allowing different carryover levels are deleted from the data input of the program. The program was then rerun for both field efficiency conditions with all soil moisture storage fixed at 50 percent of capacity at the end of each time period.

This elimination of all flexibility in water exchange among the four time periods caused an income reduction of $69,466. for low field efficiency conditions and a loss of $229,569. in the case of the high level. The low field efficiency model was evidently not as sensitive to this loss of flexibility in water supply since a considerable number of wells were required primarily for drainage control in this model relative to the high field efficiency model. Since the major effect of the loss in flexibility of water transfer was an increase in total water demand in latter critical periods of the irrigation season when surface water supply was greatly reduced, the existence of a relatively large

number of available wells was of considerable advantage in the first
model for supplementing surface water supply.

It was hypothesized in Chapter I that a reallocation of ground
and surface water use in the Gallatin Valley, and in particular, more
intensive development of groundwater pumping facilities, would increase
the present level of agricultural income.

The current level of income and surface water allocation could not
be precisely quantified in the present study. Present allocation of
surface water among users in different time periods of the irrigation
season is dependent upon the recorded water rights of the users and also
intraseasonal water supply conditions. Certain operating rules are
followed during each irrigation season in allocating surface water in
consideration of both water rights and water supply conditions for each
major canal in the Gallatin Valley. Identification of these operating
rules for different water supply conditions would require that an
extensive survey be made of all decision makers involved in allocating
surface water supply. This survey was considered beyond the scope of the
present study.

Given that there is almost no well development currently, it was
considered feasible to evaluate the economic importance of new well
development for expanding agricultural income under conditions of over-
all optimal water allocation. Testing of the hypothesis that groundwater
pumping would enhance income was accomplished by comparing projected

income levels with and without groundwater development. Elimination of
all pumping activities in the model resulted in an estimated income of
$5,562,751. for low field efficiency conditions and $6,169,382. for high
field efficiency conditions. This represented a difference in income of
approximately one million dollars annually for both field efficiency
levels compared with previous solutions in the model allowing pumping
activities. Cropping patterns are changed considerably with the elimina-
tion of pumping due to both the reduced total water supply and the ab-
sence of water table drainage control

Optimal allocation of water supply, with and without pumping,
among subareas of the model in different time periods is illustrated
in Tables 6-11 and 6-12. Table 6-11 shows water allocation under
high field efficiency conditions and Table 6-12 concerns low field
efficiency.

Surface water supply was in slack for periods 1 and 4 in the case
of all solutions shown in Tables 6-11 and 6-12. Generally, greater over-
all use was made of surface water with well development since pumping
could be relied upon to control water tables in later periods if bogging
conditions developed. With no well development, the subareas susceptible
to bogging, e.g., Subarea 7 (Central Park) must limit total surface
water diversions in all time periods. Also, wells could be utilized to
maintain irrigation supply when surface water supply became short later
in the season. This tended to enhance the optimal use of surface supply

Table 6-11. Optimal Intraseasonal Allocation of Ground and Surface Water Among Eight Subareas of the Gallatin Valley under High Field Efficiency Conditions, With and Without Well Development[a]

Water Allocation[b]	t[c]	Subarea (i)							
		1[d]	2	3	4[d]	5[e]	6[e]	7	8
With Wells:									
S. water	1	21.6	2.9	5.6	20.6	21.2	5.2	3.7	3.5
G. water	1	0.0	0.0	0.0	0.0	0.0	0.0	2.7	0.0
S. water	2	24.0	1.6	1.8	25.3	26.7	7.3	4.4	4.4
G. water	2	0.0	1.2	2.5	0.0	0.0	0.0	2.7	0.0
S. water	3	10.9	0.0	0.6	14.0	17.9	5.1	1.3	3.6
G. water	3	0.0	1.2	2.5	0.0	0.0	0.0	2.7	0.0
S. water	4	1.8	0.4	0.0	3.2	0.0	0.4	0.0	0.6
G. water	4	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0
Without Wells:									
S. water	1	20.5	2.9	5.4	18.5	1.0	5.2	0.0	2.9
S. water	2	20.6	3.4	6.2	17.7	0.0	6.4	0.0	5.0
S. water	3	8.3	1.8	2.8	12.3	0.0	6.0	0.0	3.4
S. water	4	0.0	0.4	0.3	0.2	0.0	0.4	0.0	0.6

[a]Surface water supply is fixed at 60 percent of mean level in the four time periods. Field efficiency is 70 percent for subareas 2 to 7, 50 percent for subarea 1, and 65 percent for subarea 8.

[b]The units of water allocation are in terms of 1,000 acre-feet.

[c]Time periods are: (1) June, (2) July, (3) August and (4) September. September irrigation is only required for irrigated pasture activities.

[d]Subarea 1 and 4 do not have irrigation wells.

[e]Subareas 5 and 6 have relatively shallow soils requiring frequent irrigation treatments. Except for alfalfa, irrigated crop production is relatively marginal because of high labor costs. Subarea 6 has lower water conveyance costs than subarea 5.

Table 6-12. Optimal Intraseasonal Allocation of Ground and Surface Water Among Eight Subareas of the Gallatin Valley under Low Field Efficiency Conditions With and Without Well Development[a]

Water Allocation[b]:	t[c]	Subarea (i)							
		1[d]	2	3	4[d]	5[e]	6[e]	7	8
With Wells:									
S. water	1	33.7	5.7	11.1	26.6	42.4	8.1	4.1	2.5
G. water	1	0.0	0.0	0.0	0.0	0.0	0.0	7.6	2.2
S. water	2	15.9	0.0	0.0	31.4	43.7	17.2	4.1	6.5
G. water	2	0.0	3.3	6.1	0.0	4.7	0.0	7.6	2.2
S. water	3	4.2	0.0	0.0	11.9	28.5	9.4	0.6	3.9
G. water	3	0.0	3.3	6.1	0.0	4.7	0.0	7.6	2.2
S. water	4	0.0	0.8	1.1	0.0	0.0	0.0	0.0	0.0
G. water	4	0.0	0.0	0.0	0.0	0.0	0.0	1.2	0.8
Without Wells:									
S. water	1	19.3	4.7	7.7	26.1	0.0	10.5	0.0	2.5
S. water	2	24.7	5.8	8.0	24.8	0.0	12.0	0.0	4.6
S. water	3	8.9	2.2	3.1	19.0	0.0	12.0	0.0	3.0
S. water	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

[a]Surface water supply is fixed at 80 percent of mean level in each of the four time periods at the previously-determined optimal planning level for well development. Field efficiency is 35 percent for subareas 2 to 7, 25 percent for subarea 1, and 32 percent for subarea 8.

[b]Water allocations are in units of 1,000 acre-feet.

[c]Time periods are: (1) June, (2) July, (3) August and (4) September. September irrigation is only required for irrigated pasture activities.

[d]It is not feasible for subareas 1 and 4 to have irrigation wells.

[e]These subareas have shallow soils requiring frequent irrigation treatments and are therefore less profitable under irrigation. Both have relatively high pumping costs. Subarea 5 has higher surface water conveyance costs than subarea 6.

early in the season for certain crop activities. Subarea 5 (West Belgrade) has relatively high surface water conveyance cost and also high irrigation treatment costs, therefore, it is not profitable to irrigate this subarea when water supply is limited by no well development. Groundwater discharge from Subarea 5 also contributes to the bogging problem in Subarea 7 so it is desirable to keep the former subarea water table low for drainage control.

Groundwater use in Table 6-11 is limited to three subareas. Subareas 1 and 2 have a relatively low pump lift so it is more economic to pump groundwater in these two subareas as compared with most of the others in the Gallatin Valley. Subarea 7 (Central Park) has considerable pumping concentration, due to low pumping costs and because of critical water table levels.

Under low field efficiency conditions, Subarea 8 (Manhattan) also develops a critical water table in Table 6-12, requiring pumping for drainage control. Wells are more concentrated under low field efficiency conditions because of the greater shortage in surface water supply and the need for more drainage control.

The projected income levels without pumping may be much larger than current income levels in the Gallatin Valley as all projected estimates involve optimal allocation of water resources subject to hydrologic restrictions. Results of the model indicated that no surface water should be used in East Belgrade and Central Park subareas if water supply is limited. At the present time most farmers in the latter two subareas

have better than average water rights and are therefore using a considerable amount of surface water. On the other hand, farmers located in the Camp Creek Hills and Bozeman Fan subareas have generally much poorer water rights and these subareas currently tend to have their surface water supply cut off early in July each season. Since the present water allocation is obviously not optimal, income gains should be substantial for reallocating surface water diversions in the Valley even though no well development occurs.

CHAPTER VII

SUMMARY, CONCLUSIONS AND IMPLICATIONS

Irrigation Optimization

The major task of the present study was to develop a general-equilibrium optimization model for allocating water resources in the Gallatin Valley Area. In conjunction with the above task, the specification of intertemporal-interspatial hydrologic interactions affecting optimization has also figured prominently in the structure of this applied model. Inclusion of the latter relationships has extended greatly the usefulness of this model as a comprehensive planning tool in water resource allocation compared with more conventional partial-equilibrium models concerned only with cropping patterns. Results of this model, including the determination of optimal cropping patterns, also differ significantly from the probable results of conventional models which do not evaluate surface-groundwater interactions and related drainage requirements as an integral part of the optimization problem.

Attention was also focused on the importance of surface water exchange among different time periods of the irrigation season. The amount of exchange made possible by effective soil reservoir capacity for different crops in eight subareas of the Gallatin Valley is explicitly valued in the applied model. Investigation

of this strategy along with all intraseasonal restrictions on water

use in each subarea has required that the value of water response

be determined for different time periods of the irrigation season.

This is accomplished in the present study by defining all crop

activities to have different returns depending on the duration of

irrigation treatments provided during four discrete time periods of

the irrigation season.

Evaluation of field efficiency in irrigation, together with

other strategic parameters affecting optimization, is given considerable

emphasis in empirical application of the model to the Gallatin

Valley Area. Of the various parameters analyzed, the programming

solutions are most sensitive to the coefficients for field efficiency

and the limits set for critical water table levels in each subarea.

Although field efficiency is specified to be a parameter in the

present study, it is recognized that this coefficient may be changed

significantly in the future, particularly if the present institutional

system is modified to allow more flexibility in water exchange

among users. Since the optimal water allocation determined in

this study is necessarily dependent upon the existence of flexibility

in water exchange, a related increase in field efficiency is also

projected. In view of the potential for raising field efficiency

from the present estimated low level to a recommended level twice

as large, these two coefficient levels are evaluated separately

in irrigation optimization.

The optimal level of well development is a long run planning decision dependent upon interseasonal variation in surface water supply. A gamma function was fitted to interseasonal surface flows in this study and the estimated probability of streamflow is considered in determining optimal well development under the two field efficiency conditions.

A summary of different optimization strategies considered and their projected effects on net agricultural income in the Gallatin Valley Area is presented below:

1) Optimal intraseasonal water allocation without well development under present low field efficiency conditions allowing soil water in storage to be exchanged among different time periods would result in $5,562,751. net income.

2) Optimal intraseasonal water allocation without well development under recommended high field efficiency conditions. allowing water exchange among different time periods would result in $6,169,382. net income.

3) Optimal intraseasonal water allocation with well development under present low field efficiency conditions not allowing water exchange among different time periods would result in $6,510,770. net income.

4)   Optimal intraseasonal water allocation with well development under recommended high field efficiency conditions not allowing water exchange among different time periods would result in $6,988,283. net income.

5)   Optimal intraseasonal water allocation with well development under present low field efficiency conditions allowing water exchange among different time periods would result in $6,577,337. net income.

6)   Optimal intraseasonal water allocation with well development under recommended high field efficiency conditions allowing water exchange among different time periods would result in $7,226,531. net income.

The preference ordering of different irrigation strategies outlined above is summarized in Table 7-1. Well development is only slightly more effective than raising irrigation efficiency in increasing agricultural income. However, all costs are considered in using well development whereas the cost of raising field efficiency is assumed to be zero. It is possible that farmers may incur some additional cost in attaining these recommended field efficiencies. As explained earlier, farmers have no incentive at the present time to be more efficient in water use because of the institutional system for water rights existing in the Gallatin

Table 7-1. Preference Ordering of Different Irrigation Optimization
Strategies for Increasing Annual Net Agricultural Income
in the Gallatin Valley Area.

	Strategy	:Preference:	Net Income Gain[a]	
			(Dollars)	(Percent)
1.	Groundwater development	1	1,014,586.	18.2
2.	Raise field efficiency[b]	2	977,378.	17.6
3.	Intraseasonal water exchange[c]	3	66,567.	1.2
4.	Combine 1 and 2	--	1,663,780.	29.9

[a]Income gains are compared with the estimated income of $5,562,751.
which could be obtained without well development or increase in
field efficiency. The third strategy is evaluated by comparing
income levels under low field efficiency conditions and optimal
well development with and without the use of this strategy.

[b]It is assumed that average field efficiency may be raised from 35
to 70 percent without an increase in production cost.

[c]Intraseasonal water exchange is accomplished by varying soil
moisture storage carryover levels at the end of each irrigation
time period as explained in Chapter V.

Valley Area. An alternative to raising field efficiency is consolidation and/or lining of irrigation canals which currently are about 63 percent efficient. This would be of particular benefit near bogged areas of the valley.

Intraseasonal water exchange is of relatively minor significance compared with changing field efficiency or groundwater development. Nevertheless, there appears to be a profitable income payoff from this strategy for a relatively small increase in labor input. The payoff is substantially higher for smaller well development under high field efficiency conditions as reported in Chapter VI. The only problem with this strategy is that farmers may tend to be over reactive during periods of high surface flow and incur unusually high field losses in replenishing the soil reservoir. Such indiscriminate action may result in extensive bogging damage which could offset all irrigation benefits of employing this strategy.

Conjunctive use of more efficient irrigation methods and groundwater development would provide maximum income gains in the Gallatin Valley Area. Optimal use of these strategies will require some change in the present institutional system. Recommendations for institutional change are presented in a subsequent section of this chapter.

Major Conclusions of the Study

Major conclusions of the study are as follows:

1.) Overall water supply is not a limiting resource in the Gallatin Valley. Shadow prices computed for restricted surface supply approximate pumping costs for groundwater which is in ample supply.

2.) The basic "problem" regarding water supply in the Gallatin Valley is management--making the right quantity of water available at the right place at the right time. Resolution of this problem will require reorganization of water use in different subareas of the Valley.

3.) Development of groundwater pumping facilities on individual farm sites will provide an economical source of supplemental irrigation supply and also furnish the most efficient method of exploiting the groundwater basin as a distribution system. The former Bureau of Reclamation plan to pump groundwater into existing irrigation canals was less efficient for distributing groundwater to the respective users.

4.) Although not tested explicitly, groundwater development would help to stabilize agricultural income in the valley as well as raise the average level of income. There was a direct relationship between income stability and the scale of well development evaluated in the present study.

Institutional Considerations

The potential increase in net farm income to be gained from optimal water allocation with groundwater development in the Gallatin Valley Area is projected to be above one million dollars per annum. This figure represents a substantial payoff provided that water use can be reorganized to conform with the allocation made in the present study.

Reorganization of water use will entail the transfer of surface water use from subareas with access to groundwater reserves to other subareas not having access to groundwater. It is noted that the transfer would have to be relatively permanent in order to fully amortize the cost of well investment. In view of the fluctuation in surface water supply available for use in the Gallatin Valley Area, an improved system of organization should also provide for considerable flexibility in water transfer to effectively utilize variable surface flows.

A pricing system providing economic incentive for water transfers among subareas and individual users would have to be established to promote optimal water allocation and groundwater development. Irrigators in the area have traditionally guarded their surface water rights very closely and some problems may be anticipated in promoting transfer of surface water among users. As many of the users located in the aquifer region are currently in possession of early surface rights, these users would demand compensation for substituting groundwater in place of their surface water. Effective financial arrangements may also have to be made to regulate water use so that bogging damage is controlled in

the Valley.

Seastone and Hartman [1] have suggested that optimal water transfers
may be arranged through: (1) public policy devices such as eminent
domain, public ownership, and condemnation; (2) private purchase and
sale transactions; (3) organization systems such as conservancy districts
and mutual ditch companies; and (4) some combination of these institu-
tional arrangements. These three types of arrangements are evaluated in
subsequent discussion.

Public Devices

Aside from possible public regulation of minimum stream flow for
the fish habitat and recreational lands bordering the streams, there
does not appear to be any advantage to be gained from direct public
ownership or control of agricultural resources in the area. Further
public intervention may also be strongly resisted by local residents as
the proposals of the Bureau of Reclamation to control water supply
in the past have met with some social objection.

Private Arrangements

The feasibility of obtaining extensive water transfer via private
purchase and sale transactions was discussed with a local water adminis-
trator. [2] It was evident that some permanent water transfers from

---

[1] Seastone and Hartman, "Alternative Institutions for Water
Transfers," p. 31.

[2] Private conversation with Judge W. W. Lessley, Bozeman, Montana.

downstream users to upstream users had taken place but the record of sales was relatively sparse. Either permanent transfer via private sales agreements or temporary transfer via private contractual arrangements was stated [3/] to be legally acceptible provided that the transfers did not adversely affect water supply of third parties and the transfers did not complicate the supervision of surface water diversions for irrigation. There was evidently no precedent established for temporary transfer of water-use privileges in the Gallatin Valley.

Since the subareas in short surface supply are largely located upstream on the West Gallatin River, and the subareas with access to ground-water reserves are located at the lower end of the valley, there is no legal barrier to large-scale transfer of surface water use as was assumed in the study model. Promotion in the use of temporary water transfer arrangements among private parties may greatly facilitate the flexibility of water transfer in the Gallatin Valley and also pave the way for more permanent transfer arrangements needed in optimal groundwater development within this region.

Organization Systems

In view of the relatively high degree of dependence in both ground and surface water use among different hydrologic subareas of the Gallatin Valley, administration by a single agency may be the most effective organization for maximizing overall returns from

---

3 / Ibid.

irrigation. The agency could allocate water use through some direct
pricing or quota system and also help to alleviate bogging problems
through some indirect pricing method. Two alternate forms of organiza-
tion that may be adopted are the mutual incorporated ditch company and
the water conservancy district.

A mutual ditch company could be formed in the Gallatin Valley
in which all present water rights are vested. Company shares may be
distributed among water-right holders in accordance with the value of
water-use privileges vested by them in the ditch company. Depending
upon the general consent of shareholders, the company could: borrow
funds using assets of the corporation as collateral; collect assess-
ments from those stockholders who refuse to consent to needed improve-
ments and services; and sell those shares of stock when assessments
become overdue. The corporation may also enter into different contracts
and would be otherwise recognized in the state as a legal entity
having control of water supply in the Gallatin Valley. 4/

Both intraseasonal and interseasonal surface water use privileges
may be leased by the company to different parties. In view of the
relatively long-term investment in well development, it would be
desirable to enter into some long-term contracts for surface water

---

4/ The above discussion is largely based on institutional arrange-
ments reported in L. M. Hartman and D. Seastone, Water Transfers:
Economic Efficiency and Alternative Institutions., R. F. F., The
John Hopkins Press, 1970.

supply to insure that well developments costs will be recovered. On

the other hand, part of surface water supply would have to be contracted

intraseasonally due to fluctuations in overall surface supply. The

location of well development may be regulated by the company to

provide drainage control in subareas that are susceptible to bogging.

It is contemplated that the company could capture a major part of the

income gains emanating from optimal water allocation and drainage

control, and these benefits would be distributed among shareholders in

the organization.

There is growing interest in organizing a water-conservancy

district in the Gallatin Valley which could administer allocation of

water use like the mutual incorporated ditch company; and, in addition,

take on much wider responsibility for financing water development

projects and obtaining public assistance in irrigation development.

There are funds available from the federal government which may be

used in irrigation projects, e.g., under the Reclamation Act. A water

conservancy district may be established in the Gallatin Valley as a

public agency through which all water users may contract to repay

project construction costs to the federal government. As projected

in the present study, groundwater development and ditch consolidation

or lining would greatly enhance the agricultural income in the Gallatin

Valley Area. In view of the relatively high investment cost required

in construction of these projects, the best source of funding may be

the federal government.

Implications of the Study

Results of this study indicate that a substantial improvement in agricultural income can be obtained in the Gallatin Valley by the following means: (1) optimal allocation of available surface flow among different subareas; (2) development of groundwater pumping facilities for supplementing surface supply and controlling water tables; and (3) adoption of more efficient distribution methods and facilities. It is noted that considerable reorganization of water use will be required to maximize agricultural income as was pointed out in the preceding section. In addition to the need for further study of these organizational requirements, there are some empirical problems in the present study that may warrant further research.

The current allocation of surface supply among individual users in the Gallatin Valley was difficult to estimate as the amount allocated to each user varies with supply conditions depending upon the water-right priority of the user. A major survey would be required to evaluate the current allocation among individual users.

Two different field efficiency levels are evaluated in each of the eight subareas analyzed in the present study. It was assumed that farmers could attain the recommended high field efficiency level without incurring significant additional costs in production. The costs and returns associated with different levels of field irrigation efficiency should be investigated further. Distribution efficiency may also be increased by a combination of ditch consolidation and/or

lining. The present study provides an estimate of the returns associated with increased distribution efficiency but does not include an estimate of the cost involved.

Further research is also needed on hydrologic conditions in the Gallatin Valley, and on the effect of bogging upon crop production. The programming solutions obtained in this study were very sensitive to the constraints set on water table levels in each subarea. This would indicate that optimal water allocation in the valley is greatly influenced by hydrologic conditions and more attention should be devoted to the control of high water tables in future studies.

174

LITERATURE CITED

Anderson, R.L., "A Simulation Program to Establish Optimum Crop Patterns on Irrigated Farms Based on Preseason Estimates of Water Supply," American Journal of Agricultural Economics, Vol. 50, No. 5, December 1968.

Anderson, R.L. and A. Maass, "A Simulation Technique to Estimate Crop Production of Irrigation Projects Based on Crop Response to Varying Schedules of Irrigation Water," International Commission on Irrigation and Drainage, R34, Question 34, pp. 547-558.

Avi-Itzak, B. and S. Ben-Tuvia, "A Problem of Optimizing a Collecting Reservoir System," Operations Research Vol. 11, No. 1, 1963.

Aron, G., Optimization of Conjunctively Managed Surface and Groundwater Resources by Dynamic Programming, Water Resources Center, Project No. W132, University of California, Davis, June 1969.

Bellman, R.E. and S. Dreyfus, Applied Dynamic Programming, Princeton University Press, New Jersey, 1962.

Bittinger, M.W., "The Problem of Integrating Groundwater and Surface Water Use," Groundwater, Vol. 2, No. 3, 1964.

Boyd, D.W., Simulation Via Time-Partitioned Linear Programming: A Ground and Surface Water Allocation Model for the Gallatin Valley of Montana, Report No. 10, Montana Water Resources Center, Bozeman, June 1968.

Burt, O.R., Economics of Conjunctive Use of Ground and Surface Water, Ph.D. Thesis, University of California, Berkeley, 1962.

_____, "Economics of Conjunctive Use of Ground and Surface Water," Hilgardia, Journal of Agricultural Science, University of California, Vol. 36, No. 2, December 1964.

_____, "Optimal Resource Use Over Time With an Application to Groundwater," Management Science, Vol. 11, No. 1, September 1964.

Choriki, R.T., The Influence of Different Soil Types, Treatments and Soil Properties on the Efficiency of Water Storage, M.S. Thesis, Montana State University, Bozeman, August 1959.

Davis, S.N. and R.J. DeWiest, Hydrogeology, John Wiley and Sons, Inc., New York, 1966.

Dracup, J.A., The Optimum Use of a Groundwater and Surface Water System: A Parametric Linear Programming Approach, Water Resources Center, Report 6-24, University of California, Davis, June 1969.

Estimated Water Requirements of Crops in Irrigated Areas of Montana, Mont. Agr. Expt. Sta. Bulletin 494, Bozeman, December 1953.

Fortier, S., "Duty of Water in the Gallatin Valley," Office of Experiment Station Bulletins, II, 1900.

Gallatin Valley Hydrologic Survey, U.S. Bureau of Reclamation, Great Falls, Montana, 1954.

Gopalakrishnan, C., The Economics of Water Transfer: An Institutional Appraisal (With Special Reference to Montana), Ph.D. Thesis, Montana State University, Bozeman, 1967.

Guise, J.W.B. and J.C. Flinn, "The Allocation and Pricing of Water in a River Basin," American Journal of Agricultural Economics, Vol. 52, No. 3, August 1970.

Hackett, et. al., Geology and Groundwater Resources of the Gallatin Valley, Gallatin County, Montana, Geological Survey Water Supply Paper 1428, U.S. Government Printing Office, Washington D.C., 1960.

Hall, W.A. and N. Buras, "The Dynamic Programming Approach to Water Resources Development," Journal of Geophysical Research, Vol. 66, No. 2, February 1961.

Hartman, L.M. and D.A. Seastone, "Welfare Goals and Organization of Decision-Making for the Allocation of Water Resources," Land Economics, Vol. XLI, No. 1, February 1965.

_____, "Economics and Groundwater Development," Groundwater, Vol. 3, No. 2, April 1965.

Huszar, P.C., Economics of Irrigation System Consolidation, M.S. Thesis, Colorado State University, Fort Collins, March 1969.

Irrigation Guide for Intermountain and Foothill Area East of Continental Divide, United States Department of Agriculture, Soil Conservation Service, Bozeman, Montana, May 1957.

Israelsen, O.W. and V.E. Hansen, Irrigation Principles and Practices, John Wiley and Sons, Inc., New York, 1959.

Johnson, G.L. and L.S. Hardin, Economics of Forage Evaluation, Agr. Expt. Sta. Bulletin 623, Purdue University, Lafayette, Indiana, April 1955.

Leonard, R.L., Integrated Management of Ground and Surface Water in Relation to Water Importation: The Experience of Los Angeles County, Ph.D. Thesis, University of California, Berkeley, 1963 (unpublished).

Lutz, F., and V., The Theory of Investment of the Firm, Princeton: Princeton University Press, 1951.

Maass, et. al., Design of Water Resource System, Cambridge, Massachusetts: Harvard University Press, 1962.

Markowitz, H., Portfolio Selection: Efficient Diversification of Investments, New York: John Wiley and Sons, Inc., 1959.

Montana Agricultural Statistics, Vol. XII, Montana Department of Agriculture and United States Department of Agriculture, Helena, Montana, December 1968.

Murdock, H.E., Irrigation and Drainage Problems in the Gallatin Valley, Mont. Agr. Expt. Sta. Bulletin 195, Bozeman, November 1926.

McConnen, R.J. and G.M. Mennon, Planning the Integrated Use of Ground and Surface Water: A Linear Programming Study of the Gallatin Valley, Montana, Mont. Agr. Expt. Sta. Bulletin 616, Bozeman, 1964.

Naylor, D.E., "Control of Waterlogging and Salinity in West Pakistan," International Journal of Agrarian Affairs, Vol. Iv, No. 1, October 1963.

Pearson, K. et. al., Tables of the Incomplete Gamma Function, Cambridge University Press, Reissue 1951.

Rogers, P. and D.V. Smith, "The Integrated Use of Ground and Surface Water in Irrigation Project Planning," American Journal of Agricultural Economics, Vol. 52, No. 3, August 1970.

Sammons, R.W., Irrigation Development: Institutional Blocks to Ground-Surface Water Integration in the Gallatin Valley, Montana, Ph.D. Thesis, Montana State University, Bozeman, 1964.

Seastone, D.A. and L.M. Hartman, "Alternative Institutions for Water Transfers: The Experience in Colorado and New Mexico," Land Economics, Vol. XXXIX, No. 1, February 1963.

_____, "Resource Transfers and Economic Externalities in Public Sector," Proceedings of the Fifty-eighth National Tax Conference, New Orleans, Louisiana, November 8-12, 1965.

Senate Document No. 191, Missouri River Basin, 1944.

Smith, S.C., "Problems in the Use of the Public District for Groundwater Management," Land Economics, August 1956.

Snyder, J.H., Economic Implications and Appraisal of the Court Reference Procedure for Allocating Groundwater, Committee on the Economics of Water Resource Development, Report No. 5, 1957.

Surface Contour Maps, USGS, Manhattan Quadrangle 1949 and Bozeman Quadrangle 1953.

Thom, H.C.S., "A Note on the Gamma Distribution," Monthly Weather Review, Vol. 86, April 1958, pp. 117-122.

Todd, D.K., Groundwater Hydrology, John Wiley and Sons, Inc., New York, 1959.

Tyson, H.N. and E.M. Weber, "Groundwater Management for the Nation's Future--Computer Simulation of Groundwater Basins," Journal of the Hydraulics Division, Proceedings ASCE, Vol. 90, No. HY4, July 1964.

United States Bureau of Reclamation, Report on Three Forks Division, Upper Missouri Project Office, Great Falls, Montana, 1958.

United States Geological Survey, Water Supply Papers No. 1727-29.

United States Conservation Service, Preliminary Examination Report on Water Supply and Distribution Investigation, Gallatin Valley Area, Gallatin County, 1948 and Survey Report on Central Park Drainage Project, Three Rivers Soil Conservation District in Gallatin County, Montana, 1950.

Water Resources Survey, Gallatin County, State Engineer's Office, Helena, Montana, January 1953.

Wesseling, J., Hydrology, Soil Properties, Crop Growth and Land Drainage, Institute for Land and Water Management Research, Technical Bulletin 57, Waneningen, The Netherlands, 1968.

APPENDICES

TABLE 1. PROFILE DESCRIPTIONS AND MAJOR TYPES OF SOIL IN EIGHT SUBAREAS OF THE GALLATIN VALLEY

Subarea 1/ Tested	Well Profile Data 2/				Major Soil Type 3/
	Upper Stratum		Second Stratum		
	depth	description	depth	description	
	feet		feet		
West Gateway	5	sandy gravel with silt	49	sand and gravel	Gallatin Silty
	18	clayey silt	12	sandy silt	Clay Loam
average	11	silty-clay loam	30	sand	
Camp Creek Hills	15	dirt	78	gravel	Amsterdam
	23	sandy silt	9	gravel, sandy	Silt Loam
	18	clayey silt	12	sandy silt	
	24	topsoil	3	gravel	
	17	topsoil and clay	147	gravel and silt	
	5	topsoil	18	gravel	
average	17	silt loam	44	gravel	
East Gateway	5	sandy gravel with silt	49	sand and gravel	Bozeman Silt Loam
	18	clayey silt	12	sandy silt	
average	11	silt loam	30	sand and gravel	
Bozeman Fan	9	silt buff	30	silty, sandy gravel	Puffine Silt Loam
	8	topsoil	47	sand and gravel	
	4	loam	6	sandy gravel	
	3	silty loam	22	sandy gravel	
average	6	silt loam	26	sand and gravel	
East Belgrade	6	soil	4	silt,clay,and gravel	Beaverton
and West Belgrade	2	soil	26	loose gravel	Loam and
	5	silt,clay,and sand	10	silt and fine sand	Gravelly Loam
	12	gravel,sand,and silt	22	sand and gravel	

(continued)

TABLE 1. (continued) PROFILE DESCRIPTIONS AND MAJOR TYPES OF SOIL IN EIGHT SUBAREAS OF THE GALLATIN VALLEY

Subarea [1] Tested	Well Profile Data [2]				Major Soil Type [3]
	Upper Stratum		Second Stratum		
	depth	description	depth	description	
	feet		feet		
	5	soil	10	gravel	
	3	soil	13	gravel and limestone cobbles	
	5	clay	18	gravel	
	10	gravel, silt, and sand	5	gravel	
	5	sandy gravel	49	sand and gravel	
average	6	gravelly loam	17	gravel	
Central Park	5	silty loam	10	gravel	Gallatin Silt
	1	soil	14	gravel	Loam, Swampy
	4	silty soil	11	gravel and coarse sand	Phase
average	3	silt loam	12	gravel	
Manhattan	3	soil (alluvium)	4	gravel	Manhattan Fine
	1	silty soil	22	gravel	Sand Loam
average	2	fine sand loam	13	gravel	

1/ There is over-lapping on the well profile data between East and West Gateway Subareas.

2/ From Hackett, et al, loc. cit., pp 184-203.

3/ Soil Survey of The Gallatin Valley Area, Montana, 1936.

180

TABLE B-1.   ESTIMATED IRRIGATED AND DRYLAND CROP YIELDS IN THE
             GALLATIN VALLEY AREA

| Crop Enterprise | Estimated Yield for Irrigation through | | | | | Dryland Yield |
	May	June	July	August	September	
Spring wheat[a]	26 bu	40 bu	54 bu	54 bu	54 bu	26 bu
Winter wheat[a]	33 bu	60 bu	60 bu	60 bu	60 bu	33 bu
Barley[a]	53 bu	68 bu	83 bu	83 bu	83 bu	53 bu
Oats[a]	59 bu	91 bu	122 bu	122 bu	122 bu	59 bu
Pasture[b]	1.79 T	1.98 T	2.17 T	2.45 T	2.55 T	1.79 T
Hay[c]	2.2 T	4.3 T	5.3 T	6.3 T	6.3 T	2.2 T

[a]All small grain yield data are taken from Management Guides, Coopera-
tive Extension Service, Montana State University, Bozeman, January 1969.

[b]All pasture yield data are taken from "Performances of Six Grass
Species under Different Irrigation and Nitrogen Treatments," Agronomy
Journal, Vol. 54, 1962, pp. 283-288. The test data were collected at
Bozeman during 1957-1959. Some extrapolation of this data was
required to estimate yields for continued irrigation through these
calendar months.

[c]All hay yield data with the exception of May irrigation and no irriga-
tion treatments are taken from Forage Research Committee Report, 1967,
Ag. Expt. Sta., Montana State University, Bozeman, pp. 34-35. Twenty-
five alfalfa varieties were tested over a three-year period. The
remaining hay yield data are taken from C. S. Cooper, "A Comparison of
Birdsfoot Trefoil and Ladino Clover under Varying Irrigation and
Fertility Levels," Agronomy Journal, Vol. 53, 1961, pp. 180-183.

TABLE B-2.   PRODUCTION COSTS OF ALFALFA HAY*

Annual Overhead[a]	Fertilizing & Spraying	Processing Cost[b]	Irrigation Costs[c]	
			Ditch Cleaning	Irrigation Labor
----Per Acre----		Per Ton	Per Acre	Per Treatment-Acre
$30.91	$9.39	$7.64	$0.34	$1.00

[a]Annual overhead cost includes opportunity costs of land, taxes and cost of alfalfa stand. Amortized cost of the stand is $14.65.

[b]Hay processing costs include: swathing ($2.00 per ton), baling ($3.64 per ton), and stacking ($2.00 per ton).

[c]Water procurement costs are not included.

*Source: Enterprise Cost Study: Broadwater County, Bulletin 1058, Cooperative Extension Service, Bozeman, July 1968.

TABLE B-3.   PRODUCTION COSTS OF PASTURE ENTERPRISES*

Annual Overhead[a]	Harrowing, Fertilizing, and Spot Spraying	Irrigation Costs[b]	
		Ditch Cleaning	Irrigation Labor
----Per Acre----		Per Acre	Per Treatment Acre
$11.41	$9.92	$0.57	$0.94

[a]Annual overhead includes opportunity costs of land and taxes.

[b]Water procurement costs are not included

*Source: Enterprise Cost of Irrigated Pasture, Fairfield, Montana, Cooperative Extension Service, Bulletin 1057, Montana State University, Bozeman, May 1968.

TABLE B-4. PRODUCTION COSTS OF SMALL GRAIN*

Annual Overhead[a]	Fertilizing and Spraying	Preparing Land and Seeding[b]	Harvesting Costs[c]	Irr. Costs[d] Ditch Cl.	Irr. Labor
---Per Acre---			Per Bu.	Per Acre	/Trmt Acre
$16.26	$9.19	$14.32	$0.15	$3.13	$1.00

[a]Annual overhead costs include opportunity costs of land and taxes.

[b]Land preparation costs include: plowing ($4.74 per acre), disking ($1.50 per acre), and leveling ($3.14 per acre).

[c]Harvesting costs include: combining ($0.07 per bushel), trucking ($0.05 per bushel), and swathing ($0.03 per bushel).

[d]Irrigation costs do not include cost of water procurement.

TABLE B-5. ESTIMATED WATER REQUIREMENTS FOR IRRIGATED CROP ACTIVITIES IN THE GALLATIN VALLEY*

Crop Enterprise Irrigated	Estimated Consumptive Use of Water for Continued Irrigation through Months of:				
	May	June	July	August	September
	---Inches---				
Spring Wheat	4.25	4.93	5.74	5.08	3.84
Winter Wheat	4.25	4.93	5.74	5.08	3.84
Barley	4.25	4.93	5.74	5.08	3.84
Oats	4.25	4.93	5.74	5.08	3.84
Pasture	4.25	4.93	5.74	5.08	3.84
Hay	4.51	5.24	6.09	5.40	4.08

*Source: Bulletin 494, Montana State University, Agr. Expt. Sta., December 1953.

TABLE B-6. ESTIMATED CROP AND FARM DELIVERY REQUIREMENTS FOR IRRIGA-
TION IN EIGHT SUBAREAS DURING THE IRRIGATION SEASON

| | Net Crop and Farm Delivery Requirements for Irrigation in Months of:[a] | | | | | | | |
| | June | | July | | August | | September | |
Subarea/Activity	Crop	Farm	Crop	Farm	Crop	Farm	Crop	Farm
	----------------------------Inches-----------------------							
Camp Cr. Hills								
S. Grains & Past.	3.06	6.12	4.63	9.26	4.26	8.52	2.60	5.20
Hay	3.37	6.74	4.98	9.96	4.58	9.16	2.84	5.68
Gateway Subareas								
S. Grains & Past.	2.48	3.54	4.29	6.13	4.00	5.71	2.21	3.16
Hay	2.79	3.99	4.64	6.63	4.32	6.17	2.45	3.70
Bozeman Fan								
S. Grains & Past.	2.18	3.11	4.11	5.87	3.87	5.53	2.01	2.87
Hay	2.49	3.56	4.46	6.37	4.19	5.99	2.25	3.21
Belgrade Subareas								
S. Grains & Past.	3.14	4.49	4.68	6.69	4.28	6.11	2.65	3.79
Hay	3.45	4.93	5.03	7.19	4.61	6.59	2.89	4.13
Central Park								
S. Grains & Past.	3.28	4.69	4.76	6.80	4.35	6.21	2.74	3.91
Hay	3.59	5.13	5.11	7.30	4.67	6.67	2.98	4.26
Manhattan								
S. Grains & Past.	3.28	5.05	4.76	7.32	4.35	6.69	2.74	4.21
Hay	3.59	5.52	5.11	7.86	4.67	7.18	2.98	4.60

[a]It is, of course, not always profitable to irrigate during the whole season for all crops.

TABLE B-7. AVERAGE PRICES OF SIX COMMODITIES RECEIVED BY FARMERS IN MONTANA, 1959-1968*

Year	Spring Wheat[a]	Winter Wheat	Barley	Oats	Nonirrigated Hay[b]	Irrigated Hay[c]
	------------------Per Bu.-----------------				----------Per Ton-------	
1959	1.75	1.56	0.73	0.56	19.62	19.98
1960	1.74	1.60	0.66	0.59	24.69	24.96
1961	1.86	1.69	0.75	0.60	23.19	23.79
1962	2.06	1.87	0.83	0.65	24.40	21.88
1963	1.96	1.87	0.71	0.53	18.22	18.08
1964	1.55	1.48	0.76	0.54	18.96	19.04
1965	1.33	1.17	0.83	0.56	23.36	23.54
1966	1.48	1.36	0.89	0.56	23.96	24.54
1967	1.50	1.35	0.86	0.61	24.34	24.54
1968	1.36	1.16	0.82	0.63	21.52	21.62
10-Yr. Ave.	1.66	1.51	0.78	0.58	22.23	22.20

[a]Durum wheat is not included.

[b]This price is the average of all baled hay.

[c]This price is for baled alfalfa hay.

*Source: Prices Received and Prices Paid by Montana Farmers and Ranchers, Mont.
Agr. Expt. Sta. Bulletin 636, Bozeman, Marh 1970, pp. 9-11.

APPENDIX C

TABLE C-1. WATER CARRYOVER STRATEGIES FOR DIFFERENT CROP ENTERPRISES DURING THE IRRIGATION SEASON

Crop Activity	k	Percent Available Soil Moisture Carryover for Month Ending				
		May[a]	June	July	August	Sept.
Wtr. wheat (June)	1	M_{i110}^{o}	50			
Pasture (non-irr)	2	M_{i120}^{o}	--			
(June)	3	M_{i130}^{o}	50			
(July)	4	M_{i140}^{o}	50	50		
	5	M_{i150}^{o}	75	50		
(Aug.)	6	M_{i160}^{o}	50	50	50	
	7	M_{i170}^{o}	50	75	50	
	8	M_{i180}^{o}	75	50	50	
	9	M_{i190}^{o}	75	75	50	
(Sept.)	10	M_{i1100}^{o}	50	50	50	50
	11	M_{i1110}^{o}	50	75	50	50
	12	M_{i1120}^{o}	50	50	75	50
	13	M_{i1130}^{o}	50	75	75	50
	14	M_{i1140}^{o}	75	50	50	50
	15	M_{i1150}^{o}	75	75	50	50
	16	M_{i1160}^{o}	75	50	75	50
	17	M_{i1170}^{o}	75	75	75	50

(table continued)

a/ Moisture in storage at the end of May is assumed to be a constant.

TABLE C-1. (continued)

Crop Activity	k	Percent Available Soil Moisture Carryover for Month Ending:				
		May^a	June	July	August	Sept.
Alfalfa (June)	18	M^o_{i180}	50			
(July)	19	M^o_{i190}	50	50		
	20	M^o_{i200}	75	50		
(Aug.)	21	M^o_{i210}	50	50	50	
	22	M^o_{i220}	50	75	50	
	23	M^o_{i230}	75	50	50	
	24	M^o_{i240}	75	75	50	

a/ Moisture in storage at the end of May is assumed to be a constant.

APPENDIX D

Table D-1. RESULTS OF PARAMETRIC PROGRAMMING OF WELL INVESTMENT LEVELS
PLANNED FOR EIGHT FIXED SURFACE SUPPLIES AND SUBSEQUENTLY
EVALUATED FOR ELEVEN PROBABLE SURFACE SUPPLIES UNDER LOW
FIELD EFFICIENCY CONDITIONS*

Fixed Surface Supply Used For Planning Well Investment	Programming Results for Fixed Well Investment Levels Tested at 40, 107, and 187 Percent of Mean Surface Supply			
	40 percent	107 percent	187 percent	Standard Deviation
(percent mean)	(dollars)	(dollars)	(dollars)	(dollars)
40	6,454,199.0	6,565,249.0	6,581,518.0	23,030.
50	6,448,059.0	6,570,491.0	6,577,031.0	21,916.
60	6,441,673.0	6,579,888.0	6,586,429.0	25,297.
70	6,405,770.0	6,587,156.0	6,596,522.0	32,196.
80	6,345,950.0	6,598.730.0	6,602,217.0	46,504.
90	6,286,189.0	6,608,427.0	6,613,507.0	64,486.
100	6,226,759.0	6,614,891.0	6,621,659.0	89,121.
110	6,152,799.0	6,612,977.0	6,625,218.0	111,001.

* Programming results were obtained for eleven surface supplies ranging
from 40 to 187 percent of mean as explained in Chapter 6. For simplica-
tion, results of only three surface supplies used for testing well devel
opment are shown in Table D-2.

189

Table D-2. RESULTS OF PARAMETRIC PROGRAMMING OF WELL INVESTMENT LEVELS
PLANNED FOR EIGHT FIXED SURFACE SUPPLIES AND SUBSEQUENTLY
EVALUATED FOR ELEVEN PROBABLE SURFACE SUPPLIES UNDER HIGH
FIELD EFFICIENCY CONDITIONS*

Fixed Surface Supply Used For Planning Well Investment	Programming Results for Fixed Well Investment Levels Tested at 40, 107, and 187 Percent of Mean Surface Supply			
	40 percent	107 percent	187 percent	Standard Deviation
(percent mean)	(dollars)	(dollars)	(dollars)	(dollars)
40	7,014,619.0	7,229,654.0	7,229,654.0	34,104.
50	6,996,707.0	7,230,389.0	7,230,389.0	40,094.
60	6,962,356.0	7,241,953.0	7,241,953.0	48,134.
70	6,806,492.0	7,248,459.0	7,248,459.0	83,783.
80	6,800,981.0	7,248,733.0	7,248,733.0	85,012.
90	6,800,981.0	7,248,733.0	7,248,733.0	85,012.
100	6,800,981.0	7,248,733.0	7,248,733.0	85,012.
110	6,800,981.0	7,248,733.0	7,248,733.0	85,012.

* Programming results were obtained for eleven surface supplies ranging
from 40 to 187 percent of mean as explained in Chapter 6. For simplica-
tion, results of only three surface supplies used for testing well devel-
opment are shown in Table D-2.

D378
Y 85
cop 2