Two elementary student teachers understanding of mathematical power and related pedagogy
by Susan J Adams Phillips

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University

Abstract:
This study highlighted the reform goal of “mathematical power for all students” as described in the NCTM Standards documents (NCTM 1989; NCTM 1991; NCTM 1995). The study recounted the experiences of two student teachers placed with cooperating teachers who were implementing the NCTM Curriculum Standards, for the purpose of determining how such a placement can strengthen a student teacher's understanding of mathematical power and help the student teacher develop empowering instructional strategies. The NCTM Professional Teaching Standards were used to guide and interpret observations related to the tasks, discourse, environment, and analysis of teaching and learning in each classroom. Multiple data collection methods, including observation and field notes, journals, and interviews, provided data regarding participants’ understanding of mathematical power and related pedagogy. Although both cooperating teachers were effectively implementing the Standards, and both student teachers verbally indicated an understanding of the meaning of mathematical power, one student teacher, Miss Aragon, was able to demonstrate a deeper understanding of the instructional practices which help students develop mathematical power. The instructional variable which seemed to have the strongest potential for making the processes of mathematical power visible and concrete to a student teacher, was the routine presence of empowering student discourse about challenging mathematical problems which occured daily in Classroom A. Another important feature of Miss Aragon’s student teaching experience was a schedule of responsibility which allowed her to observe empowering pedagogy being modeled by her cooperating teacher throughout the term, rather than just at the beginning. Finally, Mrs. Birch’s mentoring style, which encouraged Miss Barnaby, the less effective student teacher, to actively reflect on her teaching, was also noted to be an important influence on Miss Barnaby’s growing understanding of mathematical power.

TWO ELEMENTARY STUDENT TEACHERS' UNDERSTANDING OF MATHEMATICAL POWER AND RELATED PEDAGOGY

by

Susan J. Adams Phillips

A thesis submitted in partial fulfillment
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Montana State University
1995

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APPROVAL

of a thesis submitted by

Susan J. Adams Phillips

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

November 30, 1995 — William D. Hall
Date — Chairperson, Graduate Committee

Approved for the Major Department

11/30/95
Date — Head, Major Department

Approved for the College of Graduate Studies

12/31/95 — R. A. Brown
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Signature Susan J. Adams Phillips

Date November 30, 1995

This thesis is dedicated to my children, Jessica and Jacob,
because they thought it would be cool --
and because I really, really like them.

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ABSTRACT

This study highlighted the reform goal of "mathematical power for all students" as described in the NCTM Standards documents (NCTM 1989; NCTM 1991; NCTM 1995). The study recounted the experiences of two student teachers placed with cooperating teachers who were implementing the NCTM Curriculum Standards, for the purpose of determining how such a placement can strengthen a student teacher's understanding of mathematical power and help the student teacher develop empowering instructional strategies. The NCTM Professional Teaching Standards were used to guide and interpret observations related to the tasks, discourse, environment, and analysis of teaching and learning in each classroom. Multiple data collection methods, including observation and field notes, journals, and interviews, provided data regarding participants' understanding of mathematical power and related pedagogy. Although both cooperating teachers were effectively implementing the Standards, and both student teachers verbally indicated an understanding of the meaning of mathematical power, one student teacher, Miss Aragon, was able to demonstrate a deeper understanding of the instructional practices which help students develop mathematical power. The instructional variable which seemed to have the strongest potential for making the processes of mathematical power visible and concrete to a student teacher, was the routine presence of empowering student discourse about challenging mathematical problems which occured daily in Classroom A. Another important feature of Miss Aragon's student teaching experience was a schedule of responsibility which allowed her to observe empowering pedagogy being modeled by her cooperating teacher throughout the term, rather than just at the beginning. Finally, Mrs. Birch's mentoring style, which encouraged Miss Barnaby, the less effective student teacher, to actively reflect on her teaching, was also noted to be an important influence on Miss Barnaby's growing understanding of mathematical power.

CHAPTER 1

STATEMENT OF THE PROBLEM AND REVIEW OF THE LITERATURE

Introduction

Since the publication of the National Council of Teachers of Mathematics (NCTM)'s Curriculum and Evaluation Standards in 1989, members of the mathematics education community across the nation have seized with enthusiasm the spirit of reform, and are making efforts to "implement the Standards." These Standards call for the wide scale adoption of a new vision of mathematics teaching and learning in which the development of mathematical power is the central goal (NCTM 1989, 5; NCTM 1991, 1; NCTM 1995, 83). The traditional overemphasis on computational accuracy and the memorization of formulas will not adequately prepare students for today's and tomorrow's society. In contrast, mathematically powerful students can explore, conjecture, and reason logically; solve "messy," unfamiliar problems; connect and communicate mathematical ideas; and are flexible, persistent, curious, and confident in doing mathematics (NCTM 1989, NCTM 1991, NCTM 1995). The Standards do not provide a prescription, only a vision, for the kind of teaching that will nurture mathematical power. It is presumed the Standards can be implemented in many and various ways. But it is not possible to truly implement the spirit of the Standards if students are not becoming empowered mathematically. As we prepare new teachers for our mathematics classrooms, and provide ongoing staff development and support to help practicing teachers implement change, an understanding of mathematical power must be the central goal. This research study examined the cooperating teacher / student teacher dyad as a locus for change. It describes the classrooms of two elementary teachers who are trying to implement the Curriculum Standards, and how the student teachers assigned to these classrooms grew in their understanding of mathematical power.

Purpose of the Study

Research on student teaching indicates that the quality of the student teaching experience depends greatly on specific classroom sites (Guyton and McIntyre 1990). The purpose of this study was to examine how placement in a Standards-based classroom can strengthen a student teacher's understanding of the concept of mathematical power and the instructional decisions and strategies which support the development of mathematical power. The study took place in the classrooms of two elementary teachers from the local school district who had been actively involved in the mathematics curriculum revision process, who were knowledgeable about and committed to the vision of mathematics reform espoused in the Standards, and who had taken on the additional responsibility of supervising a student teacher. It was presumed that all four participants' beliefs, attitudes, and prior mathematics experiences would affect their understanding of mathematical power; and that the manner in which the Curriculum Standards were being implemented would affect the developing mathematical beliefs and pedagogy of the student teachers.

Research Questions

1. What prior mathematical experiences, and what attitudes and beliefs about mathematics teaching and learning, do the student teachers and cooperating teachers bring to the classroom setting?

2. What knowledge, beliefs, and criteria do the cooperating teachers use as they enact a mathematics curriculum based on the NCTM Standards, and how are these conveyed to the student teachers?

   a.) What is the nature of the tasks in which the students are engaged each day, and how are these selected?

   b.) What is the nature of the discourse among students and teachers that occurs during the mathematics lesson?

   c.) What are the features of the classroom environment associated with the mathematical empowerment of students?

   d.) How do the cooperating teacher and student teacher reflect on (analyze) the effectiveness of the program components -- the tasks, the discourse, the environment, the pedagogy -- in the empowerment of students?

3. How does placement in a Standards-based classroom influence these student teachers' understanding of mathematical power and related pedagogy?

The NCTM Standards Documents

The theoretical framework for this research study is derived from the two Standards documents: the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) and the Professional Standards for Teaching Mathematics (NCTM 1991). The first document presents a vision of a mathematics curriculum grounded in a cognitive / constructivist theory of learning which focuses on the development of mathematical power. The second document presents a guide to teachers (and other educators and policymakers) as they are considering ways to enact this vision of empowering students mathematically. The first is more concerned with the content, the second with instruction, although there is overlap. The goals of both documents include promoting the development of mathematical power in all students. The recently published Assessment Standards for School Mathematics (NCTM 1995) also calls for a shift from mastering isolated skills, to aligning assessment practices with the goal of mathematical power.

The Curriculum and Evaluation Standards

The NCTM Curriculum and Evaluation Standards (hereafter called the Curriculum Standards in this paper) were created in response to the many calls for mathematics education reform during the 1980s (e.g., A Nation at Risk 1983; The Underachieving Curriculum 1987). These publications noted that many mathematics classrooms today still reflect the mathematics that was needed for an industrial age. As a result, a broad-based task force was commissioned to arrive at a consensus concerning the kind of mathematical skills and abilities needed for the 21st century. According to the task force, new societal goals must include the development of: mathematically literate workers, opportunities for all people to learn substantive mathematics, an informed electorate, and citizens with the problem solving skills needed to be lifelong learners (NCTM 1989, 3-4). School mathematics, then, must reflect and address the importance of mathematical literacy. To this end, the Curriculum Standards describe a coherent vision of mathematical teaching and learning that will empower students in the following ways. Students will: learn to value the role of mathematics in the "real world"; develop the confidence and flexible thinking needed to solve problems they have not seen before; and learn to reason and communicate mathematically (NCTM 1989, 5). The traditional view of mathematics -- as a collection of procedures that can be transmitted to students to enable them to arrive at the correct answers to a limited variety of (usually) computational problems -- must give way to an understanding of mathematics as a sense-making process. Mathematics must be seen as a collection of attitudes, problem solving habits, and exercises in active thinking and reasoning, rather than a collection of procedures and algorithms (NCTM 1989, 7).

At the elementary level, the new emphasis is on a curriculum that will:

> Be conceptually oriented, emphasizing the development of understanding.
>
> Actively involve children in doing mathematics, offering classrooms equipped with a wide variety of physical materials and supplies.
>
> Emphasize the development of children's mathematical thinking and reasoning abilities, by building appropriate problem solving experiences into the ongoing curriculum.
>
> Include a broad range of content, including measurement, geometry, statistics, probability, and pre-algebra, which have significant and growing applications in many disciplines and occupations, and provide contexts for the use of computation skills.
>
> Make appropriate and ongoing use of calculators and computers. Although calculators do not replace the need to learn basic facts, compute mentally, or do reasonable pencil/paper computation, they unquestionably free both teachers and students to focus a more appropriate share of instructional time on real problem solving.
>
> Provide instruction that honors the unique developmental characteristics of its learners, and incorporates strategies such as cooperative learning and writing which have been shown to enhance learning across all curriculum areas.
>
> Include multiple techniques for assessing students which are integral to the instructional process (NCTM 1989, 16-18).

Thus, the Curriculum Standards provide a vision for the kind of classroom mathematics program that will enable K-12 students to develop mathematical power. These standards define a broad range of content, including mathematical applications which utilize technology, that should be included in such a curriculum; and specify the expected student outcomes associated with each standard, along with some examples of the kinds of problems and tasks which encourage powerful mathematical thinking. The first four standards (Problem Solving; Communication; Reasoning; and Mathematical Connections) are considered process standards, and are intended to provide a foundation for, or be present in, activities from the remaining content standards. (A list of the K-4 Standards areas is provided in Appendix A). It is these Curriculum Standards from which the cooperating teachers in this study are creating Standards-based instruction.

The Professional Standards for Teaching Mathematics

The enactment of the Curriculum Standards requires the creation of a curriculum and an environment for teaching and learning that are much different from predominant past and current practice, which was and often still is characterized by an overemphasis on didactic instruction, rote learning, and pencil-paper drill and practice. In 1991, the Professional Standards for Teaching Mathematics was published "to provide guidance and direction to those involved in changing mathematics teaching . . . on how to teach mathematics to enhance the development of mathematical power." This document (hereafter called the Professional Standards in this paper) rests on the assumptions "that teachers are the key figures in change and that such changes require long-term support and adequate resources." (NCTM 1991, 2).

> Five major shifts in the environment of mathematics classrooms are needed to move from current practice to mathematics teaching for the empowerment of students. We need to shift --
>
> toward classrooms as mathematical communities -- away from classrooms as simply a collection of individuals
>
> toward logic and mathematical evidence as verification -- away from the teacher as the sole authority for right answers
>
> toward mathematical reasoning -- away from merely memorizing procedures
>
> toward conjecturing, inventing, and problem solving -- away from an emphasis on mechanistic answer-finding
>
> toward connecting mathematics, its ideas, and its applications -- away from treating mathematics as a body of isolated concepts and procedures (NCTM 1991, 3).

To facilitate change in these directions, the Professional Standards offer guidelines to teachers, evaluators of teachers, university educators, and policymakers.

Section One of the Professional Standards, called the "Professional Teaching Standards," provided the constructs which enabled the researcher to relate instructional practice in the research classrooms to the idea of mathematical power. These Standards develop:

> a vision of what a teacher . . . must know and be able to do to teach mathematics as envisioned by the NCTM Curriculum and Evaluation Standards for School Mathematics and the Professional Standards for Teaching Mathematics. They are organized around a framework

emphasizing the important decisions that a teacher makes in teaching:

> Setting goals and selecting or creating mathematical ***tasks*** to help students achieve these goals;
>
> Stimulating and managing classroom ***discourse*** so that both the students and the teacher are clearer about what is being learned;
>
> Creating a classroom ***environment*** to support teaching and learning mathematics;
>
> ***Analyzing*** student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions. (NCTM 1991, 5).

These Standards provided the framework for Research Question #2 on page 3. Research related to these four areas: task, discourse, environment, and analysis, and research related to teacher beliefs, supports the notion that these are indeed key areas for the development of mathematical power in students.

Review of the Literature

Research Relating to Beliefs

The first research question explored the beliefs and attitudes about mathematics teaching and learning which participants brought to the research setting, their own prior experiences in learning mathematics, and their entry interpretations of mathematical power. It was important to establish this frame of reference, because research has shown that teachers' beliefs, attitudes, and experiences influence how they teach, (e.g., Thompson 1984), and what their students learn, in turn, will be integrally connected to how they learn it (Bruner

1966). It stands to reason, then, that a teacher's beliefs, attitudes, and experiences will influence her interpretation and understanding of mathematical power.

Beliefs about Mathematical Power. Two prominant belief systems about teaching and learning have influenced the development of mathematics education.

The first is the behaviorist orientation, whose proponents conceive of mathematical knowledge as a body of information and skills to be transmitted to the learner, and learning as a series of behavioral objectives to mastered (Post 1988, 2-4). Therefore, a student who was mathematically powerful would presumably be armed with as many skills, procedures, and formulas as possible.

The second belief system is the cognitive orientation. As defined by von Glasersfeld (1989), cognitive theorists believe that meaningful learning cannot be transmitted from one person to another; rather, it must develop from within, as the learner takes new ideas and new experiences and relates them to his own previous experiences and prior knowledge, integrating and adjusting and processing them until they make sense to him (Cooney 1994). No matter how well a teacher "explains" a concept or problem in a way that makes sense to the teacher, no matter how many tricks or devises the teacher offers the student to help him "get the answer," meaningful learning does not occur until the student does the personal and social work that enables him to make sense of the situation within his own framework. According to this belief system, mathematical power would consist of the processes and the attitudes that enable a learner to actively construct meanings and knowledge: the abilities to reason, communicate, make connections and form relationships, persevere, and solve problems from scratch by figuring them out rather than remembering a formula, all of which build confidence in one's own ability to do mathematics. This is the belief system, and the interpretation of mathematical power, upon which the NCTM Standards documents are based (NCTM 1991, 1-2).

A survey by Post (1977) determined that 96% of mathematics educators described their own primary philosophy as cognitive. Since traditional textbook-dominated classrooms are still the norm in mathematics education, such a statistic could indicate that a number of educators' beliefs and practices are not completely aligned, or perhaps that teachers use practices from both schools of thought. The Standards documents endeavor to help teachers align their practice with cognitive beliefs.

Background Experience with Mathematical Power. Another source of influence on teaching practice is prior mathematics experience. A teacher's own experience shapes her conception of mathematics as well as her conception of self as a doer of mathematics. It is commonly accepted that teachers tend to teach as they've been taught. Therefore, teachers whose own prior educational experience contributed to feelings of mathematical power and confidence would be more likely to teach for mathematical power in their classrooms. However, work by Thompson (1992) would indicate that it is possible for teachers from traditional educational backgrounds to change from a behaviorist perspective of mathematical power to a cognitive perspective of mathematical power, when exposed to new ideas and experiences. Thompson's review of the research linking conceptions to practice, concluded that teachers' "belief systems are dynamic, permeable mental structures, susceptible to change in light of experience." (Thompson 92, 140).

Research Relating to Task

In the behaviorist tradition of presenting mathematics as a set of skills, lecture and worksheets predominate as the tasks of choice. This pattern was confirmed by Porter et al. (1988) in a five year study of content determinants in elementary school mathematics. This research indicated that individual teachers have enormous power relating to content: they determine how much time is allotted to mathematics; what topics will be taught and to which students; when and in what order the topics are taught; and to what standards of achievement a topic is taught. Together these decisions influence a student's opportunity to learn mathematics, which in turn is a major influence on student achievement. Conventional wisdom would suggest that fourth grade teachers, for instance, simply teach a standard fourth grade curriculum, and that there would be little variation in the decisions cited above. However, using a three-dimensional taxonomy to determine the content of four 4th grade mathematics texts and five standardized achievement tests, minimal commonality of topics was found -- only six topics out of 385 total were common to all nine resources -- dispelling the idea of a hidden national curriculum. Computation was found to dominate the content of the classrooms studied, comprising an average of 75% of the allotted time, yet the range varied from 55% to 80% with individual teachers. The allotted time itself varied by a factor of 1.5 -- 9,000 minutes vs. 6,000 minutes across a full school year. During the remaining 25% of the time not devoted to computation, teachers covered a huge number of topics very thinly, and the choice of topics varied widely. This lack of balance among the teaching of concepts, skills, and applications is reflected in the textbook emphasis on computation, but not in the content of standardized tests, which usually contain a more balanced presentation of computation, concepts, and applications. This situation concerned the researchers, who point out that "applications are both more important and more difficult to learn than are skills, and conceptual understanding is . . . of more lasting value than either skills or applications." (Porter et al., 106). They also expressed concern that students were always given problems to solve, and rarely asked to formulate problems.

Teachers were also found to vary greatly in their knowledge of mathematics, their interest and enjoyment of mathematics, their beliefs about its importance, and their expectations for their students. What then influenced the teachers' choice of content and tasks? The teachers' own convictions and repertoires were found to be the largest determinant. "They will teach what they have taught before, what they feel comfortable with, and what they deem appropriate for their students." (Porter et al., 98). School policies such as mandated textbooks, district objectives or curriculum guide, and tests were also found to have some effect on teachers' decisions. Advice from external sources (higher grade teachers, parents, university staff) was a minor influence.

The authors point out that "only if instruction centers on important content does it have potential for being worthwhile" (Porter et al., 96) and conclude by suggesting a need for research that "starts with a judgment as to appropriate content and then seeks to design an environment that will encourage and support the teaching of that content." (Porter et al.,107).

The Professional Standards call on teachers to offer worthwhile mathematical tasks that are based on:

> sound and significant mathematics . . . and that engage students' intellect; develop students' mathematical understandings and skills; stimulate students to make connections; call for problem formulation, problem solving, and mathematical reasoning; promote communication about mathematics; represent mathematics as an ongoing human activity; display sensitivity to, and draw on, students' diverse background experiences and dispositions; and promote the development of all students' dispositions to do mathematics. (NCTM 1991, 25).

Although textbooks can be a useful resource, teachers who implement the Standards are expected to adapt or depart from texts to find tasks that will encourage the development of mathematical power.

Problem Solving Develops Mathematical Power. Problem solving is the preferred mode of mathematics learning in a Standards-based classroom. In the Agenda for Action (1980), the National Council of Teachers of Mathematics targeted problem solving as the focus for the 1980s. Schroeder and Lester (1989) describe the evolution of problem solving since that time. At first, there was a rush to get on the bandwagon and teach "about" problem solving. This involved presenting heuristics, such as Polya's "steps" and various strategies for solving nonroutine problems. Another approach was to teach "for" problem solving; that is, a concept such as division would be introduced, perhaps with manipulatives, practiced, and then students would be ready to apply the skill or concept in a problem solving setting. The current emphasis has shifted from making problem solving the focus of instruction, to making understanding the focus of instruction (Schroeder and Lester, 39). Understanding involves the ability to create relationships among mathematical ideas. Problem solving becomes the vehicle for, not just the goal of, learning. Students are given a problem at the outset, and they apply prior knowledge to construct new knowledge by solving, or working on, the problem. Teachers are encouraged to move from teaching about or for problem solving, to teaching "via" problem solving (Schroeder and Lester 1989).

Tasks Providing Real World Connections Develop Mathematical Power. Bebout and Carpenter (1989) stress the importance of providing context, or realistic settings, for problems. It is important for students to build on previous understanding in order to connect students' informal experiences with mathematics to the more formal mathematics of the classroom. Kamii (1990) reinforces these ideas. She points out that while story problems are usually presented after computational exercises, "the sequence should be reversed because children construct logico-mathematical knowledge out of daily living. Computation with numbers, which do not involve contexts, should come after a great deal of problem solving with real-life contexts." (Kamii 1990, 29).

Lampert also argues for teachers to better align school math with real math (Lampert 1991). She is referring not only to real world applications and contexts, but also to the way real mathematicians go about doing mathematics. Doing math in schools too often means learning the rules, and truth is determined by the teacher's determination of whether the answer is right or wrong. Doing math in the discipline of mathematics means testing assertions in a reasoned argument, and truth is determined by the reasoning process and the agreements that the community comes to share through these arguments (Lampert 1991, 124). An important role of the teacher, then, is to select "good problems," ones that provide learning environments that are both safe and productive. By safe, Lampert means the problem is presented in a familiar context which allows the student to call upon, reveal, and use their current knowledge about mathematical structures. By productive, she means the same problem has the potential to lead students into unfamiliar mathematical territory, allowing them to create new knowledge. An example of such a problem would be: Find a way to make $1.00 with 19 coins (Lampert 1991, 128). Cobb and Merkel also advocate problems "that can be solved in a variety of ways that make sense to pupils at different conceptual levels." By selecting such tasks, "the issue of individual differences is addressed as students use methods which make sense to them, and at the same time become more curious about math and interested in others' approaches." (Cobb and Merkel 1989, 72).

The Role of Computation in Developing Mathematical Power. Mathematical understanding is essential to the development of competence (Hiebert 1990). With the availability and incorporation of calculators and computers into current life, rote mastery of procedures is no longer essential to the development of competence. Should routine procedures play any kind of role in the selection of tasks? According to Hiebert, two distinct kinds of cognitive processes can be used in executing routine procedure: automatization and reflection. Automatization can free up mental effort, which can then be used in pursuit of understanding. Understanding comes from constructing or recognizing relationships. Reflecting on procedures means learning them, or learning about them (e.g., the two digit by two digit multiplication algorithm) in such a way that conscious consideration is given to the patterns and relationships that emerge. "Recognizing patterns does much more than help students make sense of the algorithm; it helps students make sense of the system." (Hiebert 1990, 38). He goes on to make a recommendation regarding which routines to automatize:

> If calculators are readily available, very few calculation skills would need to be automatized. A plausible suggestion is that for the sake of convenience, the whole-number arithmetic facts along with the base-ten notation rules for combining larger and smaller numbers (e.g., 200 x 40 = 8000; 0.2 x 0.4 = 0.08) should be automatized. These facts and procedures provide benchmarks that permit access to numerous sums and products and enable useful estimation skills to develop. (Hiebert, 35).

Coburn also discusses the extent to which computation should be considered a worthwhile task (Coburn 1989). According to Coburn, the thrust of current curricular reform is not to reduce the importance of computation, but to reduce the overemphasis of written computation, and to broaden the definition of computation. Computation skill continues to be useful for learning other topics, in daily life, and in most occupations. But computation modes far more useful than written algorithms, are mental math and estimation and the use of calculators to solve problems. These are the computation roads to mathematical power.

The Use of Manipulatives in Developing Mathematical Power. A final area of interest regarding task selection has to do with the use of manipulatives, long considered to be a hallmark of nontraditional classrooms. Wheatley (1992) notes that manipulatives are often used to help make abstract concepts more "apparent" or comprehensible to the student. For example, the regrouping procedure is often modeled with base ten blocks. However, the teacher is still beginning with the abstraction. Although the materials are concrete, in the sense that they are seeable and touchable objects, this does not necessarily make the concept meaningful for many students. From some constructivists' point of view, this use of manipulatives does not foster mathematical power. The manipulatives foster the teacher's intention to help students make sense, but if 2 flats, 4 longs, and 6 ones is the teacher's representation of 246 which she is trying to convey, then it is not necessarily the student's construction. Students can and often do learn to manipulate the materials just like the teacher, and still not have a personally meaningful understanding of place value (Wheatley 1992; Cobb 1991).

Kamii's approach is to have students invent their own algorithms and procedures. In the classrooms where she does research, an algorithm is never directly taught by the teacher; rather, student created procedures and strategies are shared with one another and checked and considered through lively discourse. Kamii notes that many students, allowed to do their own thinking, will adopt a left to right procedure for addition regrouping. When students receive a steady diet of externally imposed procedures, they gradually lose their inherent desire to make sense; and their purpose becomes to follow the teacher's directions and try to remember procedures (Kamii 1989).

Research Relating to Classroom Discourse

The Professional Standards highlight classroom discourse as a primary vehicle for helping students become mathematically powerful. What do we mean by mathematical discourse? The Professional Standards talk about discourse as all the ways of representing, thinking, talking, agreeing, and disagreeing, for the purpose of making sense of mathematics. Worthwhile discourse is taking place when students are explaining their thinking; listening and responding to one another; using a variety of tools (computers, calculators, diagrams, graphs, tables, metaphors and stories, concrete models, terms and symbols) to solve problems and communicate their solutions; making conjectures and exploring examples and counterexamples; and relying on mathematical evidence to determine validity and truth rather than the teacher or the answer key. Good mathematical discourse requires engaging tasks and problems, and a learning environment in which each student's ideas and thinking is respected, and all contributions are valued.

It is the teacher's responsibility to select tasks that promote thinking and reasoning and to provide a supportive environment for discourse. The teacher "orchestrates" discourse by doing less talking, less modeling, and less explaining herself, and encouraging and expecting all students to do more; by making decisions regarding which ideas to explore further so as to keep the discussion reasonably focused; by asking provocative or clarifying questions; and by deciding when to let students struggle with an idea and when to provide directed input. The student's ability to reason and communicate mathematically within a community of people collaborating to make sense of ideas becomes the central focus of the daily math lesson (NCTM 1991, 34-54).

Whitin describes an investigation from a 6th grade classroom which, although abbreviated here, illustrates the flavor of empowering discourse (Whitin 1989). Whitin had recently read that if you took a 4 digit number, reversed the digits, and subtracted, the difference would always be 6174. He tried it a few times and it worked, so he decided to share this idea with the students as a motivating way to practice subtraction. As students worked on their own problems, they quickly found some nonexamples, and a spontaneous investigation followed. They decided to keep track of their findings on the board (Numbers that Work and Numbers that Don't Work), and students added to the lists as they tested other numbers. Some began working with partners or small groups, and Mr. Whitin facilitated communication as students began raising their own questions and making their own conjectures. They explored the ideas that the sum of the digits must be an even number; that the digits must be all even or all odd; that the thousands digit must be larger than the ones digit. Good questions were being asked: Can you repeat a digit? (Yes, they agreed). Has anyone found a number that works that contains a zero? (No one had but someone did). Each conjecture uncovered more information. Persistence was the mode. One student reasoned that 6 was the only number that could be paired with zero to work (6 _ _ 0), because other combinations would produce numbers too large or too small. She displayed this finding in a chart, and other students formulated a list of 6 _ _ 0 numbers that worked. They then discovered that the difference between the middle numbers was always two; and eventually conjectured that the difference between the first and last numbers should be six. From there they eventually generated an organized list of 32 four digit numbers that "worked." This investigation thoroughly engaged the class for two straight hours.

Whitin noted several "lessons" suggested by this lesson (Whitin, 189-190): 1) Value the process of mathematizing. One student said, "We started by bungling along; then we were getting somewhere." Whitin thinks schools should promote more bungling. 2) Allow students to solve problems in their own way. He had no preconceived notion about a strategy, but supported the process through open ended questions, recognition of efforts, and trust in the students to devise meaningful solutions. 3) Recognize the power of learning as a social event. Students learn from each other by sharing, asking, challenging, and building on one another's ideas. 4) Teachers are learners too. Modeling the process of investigation and discovery can be as enlightening to students as guiding the process.

These ideas about empowering students through discourse stem from constructivist theories of learning which maintain that "learning is a process in which students actively construct mathematical knowledge as they strive to make sense of their worlds" (Cobb, Yackel, and Wood 1992, 6).

Constructivist theory has its modern roots in Piagetian developmental psychology (Steffe and Kieren 1994). Piaget postulated that children construct their own understandings of the world and how it works as they manipulate objects and interact with their environment. The mind develops in stages of increasing ability to abstract, and moves through these levels as the child is confronted with situations that no longer make sense to him from his old framework. From this state of disequilibration he begins to form new mental structures which can better accommodate and relate new information and experiences.

The Representational Point of View. Many mathematics educators took Piaget's ideas to mean that students need "hands on" learning activities and manipulatives to facilitate their development and understanding of new concepts. The mathematics was seen to be located in the materials; for example, place value concepts could be represented by Deines blocks; fraction concepts by pattern blocks. The instructional issue was simply a matter of how explicit to be. Does a teacher just put out the materials and hope the student happens to discover or construct the mathematical idea the teacher has in mind; or should the teacher clearly demonstrate (map) the relationship between the materials and the concept? (Cobb, Yackel, and Wood 1992).

The Constructivist Point of View. Cobb contrasts the current constructivist perspective with the "representational" point of view described above. He and other constructivists maintain that there is no mathematics "out there"; rather, the mathematics is in the child. The instructional dilemma described above is resolved when mathematics is no longer seen as something to transmit, show, or be discovered (Cobb, Yackel, and Wood 1992, 27-28). Instead, the mathematics will be constructed by each learner, as students actively relate existing knowledge and ways of thinking to new ideas, and the channel through which this process takes place is discourse about the task, whether or not that task is manipulative in nature. (Yackel et al. 1991).

In the constructivist classroom, the teacher plays an important role in helping students to construct their own understandings through discourse:

> . . . the teachers' role in initiating and guiding mathematical negotiations is a highly complex activity that includes highlighting conflicts between alternative interpretations or solutions, helping students develop productive small-group collaborative relationships, facilitating mathematical dialogue between students, implicitly legitimizing selected aspects of contributions, redescribing students' explanations in more sophisticated terms that are none the less comprehensible to students, and guiding the development of taken-to-be-shared interpretations when particular representational systems are established. (Cobb et al. 1991, 7).

Vygotsky's Social Constructivism. These ideas reflect "a subtle movement taking place in many quarters of the mathematics education research community from a Piagetian cognitive perspective to a Vygotskian social interactionist perspective -- a theory that ascribes greater weight to the role of social processes in the construction of knowledge." (Kieran 1994, 602).

Although Vygotsky's theories are similar to Piaget's regarding the intrinsic and developmental nature of learning, a main point of difference is Vygotsky's principle that learners reach higher levels of thinking and understanding through social interaction, and then refine and internalize their new concepts and ways of thinking individually. Jones and Thornton (1993) describe the application of Vygotsky's ideas in the classroom setting. A student has an "actual" developmental level, which is his level of performance in a problem solving situation without help, and a "potential" developmental level, at which the student can function when interacting with a teacher or more capable peer. The "zone of proximal development" is the distance between these two points, and the target area for instruction. Teachers need to provide rich, interactive settings, in which modeling of higher level thought processes helps students to bridge their zone of proximal development. This modeling is not the same as direct instruction or passive learning; rather the teacher has a strategy for the problem which may not fall within the child's zone, and the child has a strategy which obviously will fall within the zone. If the teacher's strategy is imposed, no meaningful learning will take place. Therefore, the two negotiate a meaning for the task somewhere in between -- which requires a great deal of flexible thinking and on-the-spot assessment from the teacher, and active engagement and hard work on the part of the student. Intersubjectivity is established when the two parties are in tune with each other, when "both parties are able to recognize, examine, negotiate or mutually adopt each other's perspectives." (Jones and Thornton 1993, 21). The teacher and the peer group facilitate children's learning through modeling, appropriate dialogue, feedback, and other forms of "scaffolding." (Jones and Thornton 1993).

A number of social constructivist classrooms have been described in the literature (e.g., Wheatley 1992; Kamii 1989; Yackel et al. 1990; Bauersfeld 1992; Pirie and Kieren 1992). They have in common a structure or routine in which a problem is presented, students work toward a solution strategy alone or with a small group, and then solution strategies are shared and discussed as a lively discourse among the whole class unfolds. Social norms for discussion have been established which stress sense making as the primary purpose, and respectful consideration of all points of view as the posture.

This current thinking on how meaningful learning takes place, reflected strongly in the Standards, depicts the role of the teacher not as the dispenser of knowledge and the giver of tests; nor as one who simply arranges, facilitates, and observes as learning unfolds; but as one who makes professional decisions before and during instruction about the content of the lesson, and is an active agent in guiding his or her students to construct their individual and shared mathematical knowledge through empowering discourse. (E.g., Etchberger and Shaw 1992; Knapp and Peterson 1993).

Research Relating to Environment

The Professional Standards states that the environment:

> is foundational to what students learn. More than just a physical setting with desks, bulletin boards, and posters, the classroom environment forms a hidden curriculum with messages about what counts in learning and doing mathematics: Neatness? Speed? Accuracy? Listening well? Being able to justify a solution? Working independently? If we want students to learn to make conjectures, experiment with alternative approaches to solving problems, and construct and respond to others' mathematical arguments, then creating an environment that fosters these kinds of activities is essential. (NCTM 1991, 56)

Higher Level Thinking. Peterson (1988) reports on research relating to the identification of environmental factors that seem to support and encourage the development of higher-order thinking in mathematics. Higher-order thinking, or problem solving, is one integral component of mathematical power. She points to three dimensions that are influential in facilitating higher-order thinking. These are:

1) Rote Learning versus Meaning and Understanding. Peterson found that while direct instruction improved performance in lower-order thinking skills (defined as the National Assessment of Educational Progress categories of knowledge and skill), it was insufficient to promote improvement in higher-order thinking (defined as the NAEP categories of understanding and application), and that approaches supporting children's active construction of knowledge were more effective.

2) Teaching Higher-Level Executive Processes and Strategies for Mathematics Learning. Peterson found a positive relationship between the use of specific cognitive processes during higher-order thinking tasks (such as checking answers, applying information, reworking problems, rereading directions, relating new information to prior knowledge, asking for help, using aides, and using memory strategies) and achievement. Successful students often reported specific steps or strategies, described sequences in their thinking, and verbalized insights about the nature of the task. Explicit instruction and modeling of these cognitive processes was recommended.

3) Teacher Control and Direction versus Student Autonomy and Independence. Studies undertaken at the University of Wisconsin during the Cognitively Guided Instruction Project (Carpenter and Fennema 1988) document the importance of persistence and motivation to successful higher-order thinking. The author recommends small group cooperative learning experiences to support the development of student autonomy (Peterson 1988).

Cooperative Learning. Classroom environments that nurture the development of mathematical power are collaborative communities. Cooperative learning is a powerful tool for increasing students' self-confidence (Davidson 1990). One meta-analysis of 80 studies comparing students' mathematical achievement in cooperative groups versus traditional instructional settings, found that the students in small group approaches significantly outscored control students in over 40% of the studies. In studies where the experimental teacher was not as familiar with cooperative learning methods, there was no significant difference. In only two studies did control students significantly outscore cooperative learning students (Davidson 1985). Furthermore, attitude surveys from both teachers and students consistently yield these responses regarding the advantages of cooperative learning:

> (Students) learn to cooperate . . . improve social skills, . . . communicate mathematically . . . The classroom atmosphere tends to be relaxed and informal, help is readily available, questions are freely asked and answered, and misconceptions become quickly apparent and are readily resolved. . . . Students become friends with their group members . . . usual disciplinary problems of talking and moving around are eliminated . . . students maintain a high level of interest . . . many like math more . . . students have an opportunity to pursue the more challenging and creative aspects of mathematics and to become more confident problem solvers while acquiring at least as much information and skill as when they are taught with more traditional approaches. (Davidson 1990, 60)

Johnson and Johnson have conducted extensive research in the area of cooperative learning, and advocate its use in mathematics classes for these reasons (Johnson and Johnson 1989):

> First, mathematical concepts and skills are best learned as a dynamic process with the active engagement of students. . . . Active learning

> requires intellectual challenge and curiosity, which are best aroused in discussions with other students.
>
> Second, mathematical problem solving is an interpersonal enterprise. . . . Students have more chances to explain their reasoning . . . in small groups.
>
> Third, mathematics learning groups have to be structured cooperatively to communicate effectively. Within competitive and individualistic structures, students will not engage in the intellectual interchange required for learning mathematics.
>
> Fourth, cooperation promotes higher achievement in mathematics than competitive and individualistic efforts. (Cites results of meta-analyses).
>
> Fifth, by working cooperatively, students gain confidence in their individual mathematical abilities.
>
> Sixth, choices of which mathematics courses to take and what careers to consider are heavily influenced by peers. (Johnson and Johnson, 236-237)

However, simply having students work in groups does not promote better understanding or improved communication and reasoning unless teachers ensure that key conditions are in place: the teacher clearly promotes positive interdependence in each group; students engage in promotive (assisting, supporting, encouraging) interaction during assignments; students are individually accountable; students learn and use small group skills; teachers help groups engage in regular group processing (Johnson and Johnson, 238).

Risk Taking. An environment which supports risk taking during whole group discussions is also important. In a constructivist classroom, the teacher's attitude is crucial to the development of a positive problem solving environment.

Every time a child contributes, the teacher assumes that the contribution is meaningful to that child. "By allowing a child to proceed with an explanation even when the answer is wrong, the teacher fosters a belief that the teacher is not the sole authority in the classroom to whom children have to appeal to find out if their answers are right or wrong." (Yackel et al. 1990, 18). Children learn to think for themselves, and listen to their own and others' thinking. The teacher's role is to assist, if and when appropriate, and to establish these expectations for communication: students cooperate, reach a consensus, explain their thinking, try to understand the other's thinking, and persist in trying to figure things out. Helping one's peers must be a central concern rather than a marginal activity (Yackel et al. 1990).

Research Relating to Analysis

The Professional Standards charge teachers to regularly consider what effects the tasks, discourse, and environment are having on their students' mathematical power.

> What do students seem to understand well, what only partially? What connections do they seem to be making? What mathematical dispositions do they seem to be developing? How does the group work together as a learning community making sense of mathematics? (NCTM 1991, 62)

As teachers make changes in their content, their methods, and their purposes, a dilemma arises regarding assessment (NCTM 1995, 3). Traditional assessment practices are not necessarily consistent with the new goals of mathematics. Teachers must find ways to assess their students' conceptual and procedural understandings, their ability to reason mathematically, and their dispositions. Attending to students during whole group discussions, observing them in small group discussions, conducting interviews, reading math journals, observing performances, and using traditional methods are possible means of assessing students' growth in mathematical power. Teachers also need to ask themselves questions about their selection of tasks, conduct of discourse, and features of the environment in order to better "understand the links between these and what is happening with their students" and make adaptations as needed. (NCTM 1991, 64). Thus the Standard on Analysis refers both to student assessment and to reflection about instruction.

Analysis of Students' Mathematical Power. Regarding student assessment, Wheatley (1992) reports on the assessment practices of teachers in the Math Learning Project, which are aligned with the suggestions cited above:

> Teachers assess pupils using informed professional judgment. Grades are not given for daily work and tests are not administered except for the required state and national assessments. In fact, at no time does the teacher communicate a judgment of the students' mathematics. She does not collect worksheets, mark the ones that are wrong, or have students correct their 'mistakes.' Instead, the teacher keeps notes of students' activities in which she considers their persistence, confidence, cooperation, communication, and the quality of their mathematical constructions. (Wheatley, 531)

On the state and national assessments, after two years in this Standards-based, constructivist program, students scored at grade level in computation, and well above grade level on concepts and problem solving. Prior to the program, scores had been below grade level on all subsections. "Although some importance is given to this data, it is doubtful that a standardized test is an accurate indicator of the scope of mathematical power developing in many students." (Wheatley, 532).

In the Problem Centered Instruction classrooms involved in the Purdue research project (where mathematical argumentation was emphasized and procedural instruction de-emphasized), second grade project students' conceptual understanding and problem solving abilities were superior to non-project peers on state-mandated standardized tests, and this remained true even after the next year when students were back in traditional classrooms. The Purdue studies also measured and compared beliefs and motivations, with project students placing significantly greater value on effort and understanding as reasons for success (Cobb et al. 1992).

Analysis as Reflective Practice. Is classroom instruction helping students develop mathematical power? The ideas of self-reflection and self-reliance, of teaching without an external prescription or script, permeate the Curriculum Standards and the Professional Standards and are firmly grounded in literature about the "reflective practitioner," a concept that has its roots in the work of Dewey. According to Dewey, reflective action is the "active, persistent, and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it and the consequences to which it leads," while routine action "is guided primarily by tradition, external authority, and circumstance."

(Dewey 1933).

Zeichner and Liston describe the elementary student-teaching program at the University of Wisconsin:

> (The program) emphasizes the preparation of teachers who are both willing and able to reflect on the origins, purposes, and consequences of their actions, as well as on the material and ideological constraints and encouragements embedded in the classroom, school, and societal contexts in which they work. The goals of the program are directed toward enabling student teachers to develop the pedagogical habits and skills necessary for self-directed growth and toward preparing them, individually and collectively, to participate as full partners in the making of educational policies. (Zeichner and Liston 1987, 23)

Smyth offers a four step process to help teachers "see" their own ideologies, and think about the structural conditions that are operational in their own classrooms, schools, or districts from a perspective of possible change:

1. Describe -- What do I do? (Or what is being done?)
2. Inform -- What does it mean?
3. Confront -- How did I (or things) come to be like this?
4. Reconstruct -- How might I do things differently? (Or how could things be done differently?) (Smyth, 1989)

Armaline and Hoover reinforce the importance of reflectivity, arguing that our knowledge structures are the lenses through which we make our assumptions and our interpretations of truth. Rigid knowledge structures ensure the preservation of the status quo, whether it is "working" or not. They claim that "the degree to which field experiences foster dynamic, intellectual, critical perspectives that empower teachers to empower their own students is the measure of success of teacher education." (Armaline and Hoover 1989, 42).

An Historical Perspective on Student Teaching

There are over a half million preservice teachers enrolled in teacher education programs at over 1,200 institutions each year (Doyle 1990) and each and every one of them will presumably culminate his or her experience with that ubiquitous grande finale called student teaching. However widely teacher education programs may vary in their conceptual underpinnings and goals, student teaching is common to all, and the student teaching experience is the one aspect of their teacher preparation programs that teachers consistently rate the most influential (Tom 1991; Colburn 1993; Guyton and McIntyre 1990). Yet the quality of these experiences is very diverse.

Doyle (1990) has described five possible purposes or goals to which teacher education programs might subscribe.

1) Some programs train the "good employee" by socializing the student into prevailing practices, and emphasizing the technical and experiential aspects of teaching. An effective teacher is considered one who can do it like it always has been done. School administrators and experienced teachers support this model of teacher preparation.

2) Other institutions strive to develop the "junior professor" by providing a strong academic emphasis. A successful candidate would have a strong knowledge base in the academic disciplines which they can then impart to their future students. Academics and legislators find this model appealing.

3) A third paradigm, whose proponents are often from psychology, counseling, and elementary education, values the "fully functioning person," and provides a program for students whose goal is personal understanding, clarification of values, a sense of satisfaction and purpose from teaching, and knowledge of human development and ways to promote growth. The desirable outcome of this program is psychological maturity, and an understanding of the nature of student learning and growth.

4) The fourth paradigm focuses on the "innovator," and hopes to develop new teachers who will be sources of renewal and change in schools. The emphasis of this program is training in the latest theories and practices of education; therefore, field experiences in conventional classrooms might be undesirable because students would be at risk for indoctrination into traditional practice -- lab settings would be ideal. Behavioral and social scientists, researchers, and teacher education professionals are often among the supporters of this model.

5) The final conceptual framework for teacher education is that of the "reflective professional." This model is one of the current hot topics in research, and its proponents hope to develop teachers who think critically about their work and workplace and are committed to ongoing change and improvement. The program consists of training in observation, analysis, interpretation, and decision-making. An effective teacher from this paradigm would be perpetually questioning, fine tuning, and adjusting his or her personal and professional

practice to become ever more effective (Doyle 1990, 5-6).

Do programs of teacher preparation really have such well-defined identities and mission? According to Howey (1989) 65% of faculty and students from a large number of teacher education programs surveyed identified a discernable dominant conception of teaching at their institution. Howey's categories for orientation were: skill or competency-based; clinical or problem solving; humanistic and person-oriented; inquiring and reflective; and liberal education. However, respondents from the same institution often responded differently, and it would seem that most teacher preparation programs would be likely to embrace elements of several paradigms.

The "Professional Development Standards" (Section 3 of the NCTM Professional Standards 1991) state that it is imperative for preservice and practicing teachers to understand the notion of mathematical power. This document advocates a broad based reform effort in the preparation of teachers which speaks to goals from each of the five frameworks cited above: pedagogical knowledge, content knowledge, knowledge of students as learners, socialization into renewal and reform, and the ability to reflect on and analyze one's own beliefs and practice (NCTM 1991, 123-173).

Whether or not a teacher education program has a clearly conceived and communicated sense of mission and direction, and whatever the nature of that direction, it is certainly the case that within most university / public school pairings for field service, the arena of control regarding the nature of the student teaching experience shifts at that point to the public school, or more specifically to the cooperating teacher.

This "medieval apprenticeship training model" has been the modus operandi since 1700 when the first normal school was established in France (Guyton and McIntyre, 514-515). A departure from this induction format took place in the early part of this century, when Dewey's educational theories led to the development of campus lab schools to serve as research and field experience sites. These lab schools were short-lived, enjoying a brief resurgence again in the 1960s as sites for experimenting with discovery learning, but never seriously challenging the public schools' apprenticeship approach to teacher induction. In this decade there have again been resounding calls for reform and restructuring of teacher education, with the concept of Professional Development Schools emerging as the reform model of greatest current interest.

First posited by the Holmes Group (1986), this and related reform ideas were developed at great length by Goodlad, whose extensive research convinced him of the "need to provide exemplary practice settings. . . . Some of the most unacceptable shortcomings in the settings we studied were found in this part of the program, although both students and faculty members rated student teaching highest among program components for impact." (Goodlad 1990, 280). Among Goodlad's many recommendations are 1) the placement of elementary student teachers in schools, not classrooms, so that they are exposed to a wider variety of practices and develop a broader perspective, and 2) their active involvement in the experience of renewal and reform rather than the mastery of craft.

The Research Focus

This study describes the experiences of two elementary student teachers who were placed with cooperating teachers actively involved in renewal and reform of their mathematics programs, in a school district also supportive of mathematical reform. The research project was specifically designed to investigate these student teachers' existing and developing understanding of mathematical power, and how to teach for mathematical power.

CHAPTER 2

METHODS AND PROCEDURES

A Brief Historical Perspective on Research Methods in Mathematics Education

Schoenfeld (1994) provides a concise overview of the gradual shift in methodological approaches to mathematics research that has taken place over the last two decades. By the 1960s, education research, including mathematics education research, had aligned itself firmly with statistical methodology and empirical design in an effort to lend "scientific" credibility to the field. One research model, for instance, utilized a factor analysis design, where attempts were made to find correlations between certain discrete abilities (e.g. spatial perception, computation, verbal) and performance on mathematical tasks, such as problem solving. In another typical model, researchers set up experimental designs with control groups to test the effects of various treatments or interventions, usually created by the researcher or some other source external to the actual educational environment. This treatment or intervention was usually derived from a theory about which instructional components were likely to result in more effective outcomes (e.g., higher test scores). Problematic to such designs was the fact that real classroom settings did not lend themselves well to isolating factors and controlling variables. Fragmented behaviors and isolated contexts have very little to do with real teaching and learning, which are necessarily complex and holistic and dynamic and varied -- not easily "controllable" in the scientific sense. "In short, many factors other than the ones in the statistics model -- the variables of record -- could and often did account for important aspects of the situation being modeled." (Schoenfeld 1994, 701). During this time, the behaviorist school of psychology undergirded many of the assumptions about what constituted rigorous and acceptable research: behavior and effects must be overtly observable; introspective data about what was occurring, whether from the subjects or from the researcher, was off limits, and statistical significance was what counted (so to speak).

By the late 1970s, the mathematics education community, and educational research generally, was realizing the limitations of process/product research methodology, and looking to other disciplines for new approaches. The influences of both Piaget's work and information-processing models of learning caused the clinical interview and observational methodology to gain a foothold, as the internal processes of thinking and learning once again became legitimate areas of inquiry. Examples of such research would be the constructivist teaching experiment, in which a researcher would pose a problem to a student, and then interact with the learner as he or she struggled with the problem, observing closely the thinking processes that unfolded (e.g. Cobb and Steffe 1983). This type of research has produced much valuable knowledge about learning, which was utilized by writers of the NCTM Standards.

Kilpatrick discusses the purposes of mathematics education research and their appropriate methodologies. When the goal is to explain, predict, or control -- then the empirical/analytical tradition from the natural sciences is useful. When the goal is to help students and teachers gain greater freedom and autonomy in their work, or to improve practice and involve the participants in that improvement -- then action research, introduced in sociology, can be applied. But when the goal is to understand the meanings that learning and teaching mathematics have for those engaged in the activity -- then the interpretive framework from anthropology works well. Kilpatrick reiterates that during the last decade there has been movement away from the empirical/analytical designs and toward fieldwork and interpretation, but he points out that even when empirical methods were dominant, the predominant motive for such research was more often understanding than prediction and control, which indicates that historically there may have been a frequent "mismatch" between the goals of mathematics education research and the methodology utilized (Kilpatrick 1992).

The Research Design

A qualitative research design was selected as the most appropriate method for gaining insight into the research questions posed in this study. All participants were involved in the processes of learning, of change, and of reform. According to Patton, "Qualitative inquiry is highly appropriate in studying process because depicting process requires detailed description; the experience of process typically varies for different people; process is fluid and dynamic; and participants' perceptions are a key process consideration." Further, process studies "look not only at formal activities and anticipated outcomes but they also investigate informal patterns and unanticipated interactions." (Patton 1990, 95).

Documentation of a Need for This Type of Study

A prevalent theme in current reviews of the literature is the need for naturalistic, qualitative inquiry in both student teaching and mathematics education. In the section "An Emerging Paradigm for Research on Field Experiences," from their chapter on student teaching in the Handbook of Research on Teacher Education, Guyton and McIntyre state:

> The phenomenon of the atheoretical nature of field experiences that has existed for many years can be attributed partly to the imposition of a scientific research paradigm on situations that are not compatible with the methods, purposes, philosophy, epistemology, or assumptions of the paradigm. This paradigm adopted the scientific methods of the natural sciences. A major breakthrough in the 1980s was the emergence of a new paradigm for research on field experiences, a naturalistic approach. This new paradigm has generated more meaningful results than the old and, if continued, could lead to substantive and procedural changes in the way field experiences are conceptualized and structured. . . .

> Naturalistic inquiry regards field experience as a process rather than as a variable. This systemic approach acknowledges the complexity of field experiences. Also, more naturalistic inquiry has included subjects' frames of reference. Traditionally, studies have ignored the meanings actors bring to the experience. . . . Research has moved from the more restrictive pretest-posttest design studying predetermined variables and from descriptive survey data to a discovery mode in which concepts and categories emerge from the data. Field experiences are abstruse processes, making it difficult to identify a priori the important variables, particularly given the limited knowledge base (Guyton and McIntyre 1990, 529).

Likewise Brown and Borko echo the research implications and recommendations from many mathematics education articles and reviews: "It is important that researchers focus more on teachers' classroom actions. . . . If a goal of research on becoming a mathematics teacher is to help teachers become better able to teach mathematics as envisioned in NCTM documents, then it is important to study teacher actions as well as cognitions, as well as to identify conditions under which changes in teacher cognition are likely to be accompanied by compatible changes in classroom actions." (Brown and Borko 1992, 236).

Sequence of Events Before and During the Study

The study took place in a northwestern university community of 30,000. The local school district has seven elementary (K-5) schools, and district efforts toward mathematics curriculum revision and reform had been ongoing since 1986. During 1986, the elementary teachers on the mathematics curriculum revision committee, including the researcher, took a course on current Elementary Mathematics Methods and reviewed the latest documents and publications pertaining to mathematics education, including A Nation at Risk (1983) and the NCTM publication An Agenda for Action (1980). The committee then affirmed problem solving as the cornerstone for a "new" elementary mathematics curriculum, and adopted a textbook series that the committee felt best helped teachers facilitate this goal.

In 1992, the curriculum revision cycle began again, and a new committee was formed. The cooperating teachers from this study as well as the researcher were active members of this committee, planning and participating in all curriculum processes and activities. A summary of these activities provides background information for the context of the current study.

The committee once again spent the first year immersed in the study of the current thinking in mathematics education, including the newly published Curriculum and Evaluation Standards for School Mathematics, and Professional Standards for Teaching Mathematics (NCTM 1989, NCTM 1991). During that year the committee wrote a district philosophy which reflected a rationale for mathematics education in keeping with the Standards and decided:

that no textbook series currently on the market could adequately "teach to the Standards";

that we would reaffirm our existing textbook series as an adequate resource and encourage the purchase and use of a wide variety of additional materials and resources; and

that we wanted our district's teachers, not textbooks or commercial programs, to be in charge of the management of their mathematics curriculum.

In 1993, the Math Committee goals were:

1) to provide all K-5 teachers with inservice about the <u>Curriculum and Evaluation Standards</u>

2) to develop a Scope and Sequence aligned with the Standards

3) to develop and implement a needs assessment and materials ordering process, and

4) to maintain communication regarding the Standards and the curriculum revision process.

To this end, all 100 K-5 district teachers were provided an overview of the content and spirit of the Curriculum Standards during two half-day release days in the fall of 1993, and attended a number of grade level meetings throughout the 1993-1994 school year. A needs assessment instrument was prepared to determine what materials and further inservice teachers felt they would need to begin (or continue) to implement a Standards-based curriculum in their classrooms. Meanwhile the committee began work on the collection of ten alternative assessment tasks to be used at each grade level. The thinking was that since "assessment drives curriculum," teachers who had a repertoire of rich tasks to utilize in assessing their students' mathematical understandings would want to "teach to these new tests" by planning similarly rich tasks for their students on a regular basis. Simultaneously, the committee created a curriculum guide in which each standard was analyzed in terms of learning outcomes for each K-5 grade level. In the spring of 1994, each teacher was given a budget roughly equivalent to that of a textbook adoption with which to order resource books and manipulatives. These were to be used along with, or instead of, their existing textbooks and manual to implement Standards-based instruction in their classrooms. This ordering process followed guidelines and instructions established by the District Math Committee. Presentations were made to the School Board to familiarize them with our goals, process, and progress; and meetings were held with the principals, who also attended the teachers' inservice sessions.

In the fall of 1994, "implementation" began, or in many cases continued, and it was during the 1994/95 school year that this study took place. Math Committee goals for the 94/95 school year included:

1) to help teachers begin/continue implementing a Standards-based curriculum in their classrooms, using the new Curriculum Guide (grade level learner results), their new (and old) resource books and materials, and their Alternative Assessment Packets;

2) to provide appropriate inservice opportunities and supervision to support curricular change; and

3) to create a plan for Program Assessment (i.e., evaluation of the new K-5 math curriculum at the district level).

To achieve these goals, all K-5 teachers participated in a half-day workshop in September, 1994 to become familiar with the new Curriculum Guide