Effect of parameter ranges upon choice between centralized and decentralized facilities

by Alfredo Roque Valderrama

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Industrial and Management Engineering
Montana State University

Abstract:
The purpose of this study was to compare the Linear Programming Technique with the Dynamic Programming Technique in solving the Transportation Problem when the constraints and the objective function are non-linear functions.

More emphasis was placed in solving a practical problem than a theoretical one. Therefore, several small businesses -in Montana were analyzed and some of them requested cooperation. At the end of this feasibility analysis, the recapping business was chosen. There are two principal reasons for solving the recapping industry problem. First, the recapping industry could be centralized or decentralized, and second, there is remarkable data and collaboration available from the small shops- in Montana.

It was concluded that the recapping industry having the State of Montana as a market area should be centralized. The Transportation cost and the unit production cost, as a function of plant size, played a great role in the solution of the problem. It also was concluded that when an optimization problem has more than one state parameter and/or more than one control variable at each stage the Dynamic Programming Approach is not practical. Therefore, only the Linear Programming Approach was worked.

The actual problem is a non-linear problem and a method was developed to adapt it to the linear case.

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ABSTRACT

The purpose of this study was to compare the Linear Programming Technique with the Dynamic Programming Technique in solving the Transportation Problem when the constraints and the objective function are non-linear functions.

More emphasis was placed in solving a practical problem than a theoretical one. Therefore, several small businesses in Montana were analyzed and some of them requested cooperation. At the end of this feasibility analysis, the recapping business was chosen. There are two principal reasons for solving the recapping industry problem. First, the recapping industry could be centralized or decentralized, and second, there is remarkable data and collaboration available from the small shops in Montana.

It was concluded that the recapping industry having the State of Montana as a market area should be centralized. The Transportation cost and the unit production cost, as a function of plant size, played a great role in the solution of the problem. It also was concluded that when an optimization problem has more than one state parameter and/or more than one control variable at each stage the Dynamic Programming Approach is not practical. Therefore, only the Linear Programming Approach was worked.

The actual problem is a non-linear problem and a method was developed to adapt it to the linear case.

CHAPTER I - INTRODUCTION

BACKGROUND AND DEVELOPMENT OF THE PROBLEM

The principal reason for the existence of the recapping industry is that the final product, i.e., the recapped (retreaded) tire, is more economical from the overall aspect of initial investment and cost per mile than a new tire (5). This fact has played a major role in the tire industry.

Until 1941 "retread tires" were a relatively insignificant part of the total tire market. Prior to this time, retread tires never achieved more than eight per cent (5) of the total tire market -- even during the sharp recession of the thirties. However, passenger retreads showed a steady growth rate averaging about ten per cent between 1930 and 1940. During World War II the proportion between new and retread tires has undergone a major but temporary change. New tire shipments dropped from 55 million units to 3 million units and retreads increased more than three times their prewar sales volume to about 10 million units per year. By 1944, however, the volume of retread units was only about 30 million per year and new tire volume was 18.5 million units per year.

After 1945, passenger tire retreading began its contemporary decline to what soon (1947) became a level almost in perfect agreement with projections based on a prewar growth trends. After World War II the output of passenger retreads grew steadily at an average rate of about 11 per cent per year until 1959. After 1959, however, the retreading industry began to change. The new tire manufacturers developed and produced the cheapest possible types of tires. These tires were shipped in car load lots to discount houses, department stores, and chain stores. Early efforts to combat this competition consisted largely of retread price reduction. As profit margins narrowed, however, it was clear to many retreaders and equipment manufacturers that the basic cost of producing a retread also had to be reduced. One way in which to do this was to increase production volume and spread production overhead costs over a larger number of units. The profit reduction in the recapping industry forced the marginal producers out of business and the better "capitalized" retreaders augmented their share of the tire market. Edwards (10) estimated that 1,200 retread shops closed between June 1961 and December 1962 throughout the United States.

The second way to reduce cost was to cheapen the product. This process has been taking place since 1961 through such means as second line retreads, lower grades of tread rubber, and reduced carcass specifications. In addition to this, production methods and equipment have been improved.

As a consequence of these processes the daily production volume per shop is increasing while the production cost per unit is decreasing. At the same time, however, the transportation cost per unit is increasing because the actual retreader is usually unable to retail his entire output through his own store.* In a sense the retreader is assuming the same marketing functions as his major new tire producer competitor, and, as is known, similar marketing functions tend to generate similar expenses.

This paper is not going to deal with the second way of reducing

*See Appendix A

costs, but will attempt to establish a technique by which the optimum combination of plant location, size of the plant, and number of plants for a given market area can be determined to yield a minimum production and distribution cost.

Essentially two methods will be presented and compared in this study. One approach employs the Linear Programming Technique. The other uses the Dynamic Programming Technique. These techniques will be described briefly in Chapters III and IV respectively.

STATEMENT OF THE PROBLEM

In recent years, due to inherent characteristics of the industry, there has been observed a tendency toward establishment of large recapping plants and an accompanying failure of plants. If this trend continues, there will be fewer plants and the existing ones will be larger with a greater volume output. This, of course, can be expected to increase transportation costs but, at the same time, decrease the unit production costs.

The State of Montana is the market area involved in this study. To facilitate the solution of the problem of cost minimization it can be assumed that the retail prices are essentially the same throughout this market area. The variables will be production and transportation costs, and if the retail selling prices are fixed then the profit will depend only on the production cost and transportation cost: Therefore, if cost is minimized the profits will maximize at the same time. This is the usual economic objective of a private enterprise.

Thus, the object of this study is to optimize the number of plants in Montana as well as the size and location of each plant in a given market area.

To reduce the problem to manageable proportions, several assumptions had to be made, as in any other case where the industrial engineer has to represent the real world by some model. The mathematical model had to be structured so that it would represent reality with a fair degree of approximation, and at the same time be as simple as possible in order to make the analytical approach practicable and economical.

To start with, it is assumed that production costs, fixed and variable, are dependent upon the size of the plant and not upon the relative location. It is also assumed that the transportation cost depends upon distance and not on the relative location of the plant in the state.

Attention is called to the fact that a contemporary retreading plant, in order to have a competitive production cost, must have a minimum production of about 80 tires daily. Montana has an average daily demand of 808 recapped tires.* Since 1955 (8) the number of retreading shops in the state has been about 20. This makes a daily

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* From Table 10, the annual demand for the state of Montana is 209,000 tires per day, and if we estimate the number of working days per year as 260, there are 209,000/260 = 808 tires recapped per day.

average output per shop of 40.0 tires. Most of these plants are not only in the retreading business but also sell batteries and automobile accessories and maintain automobile repair shops. The typical plant is too small to permit the hiring of technical personnel to determine where are the origins of its costs or the source of profit, if any. Therefore, it appears that in the very near future the marginal recapping shops will be forced to become only retailers or distribution centers.

Throughout this study an attempt is made to estimate the future situation of the recapping industry in Montana, given of course that the intrinsic conditions remain the same.

SUMMARY OF PAST WORK

Considerable study was given to the recapping industry in the past. The studies encountered during the research for this paper were concerned with the individual problems mentioned in previous sections and were dealt with as independent inputs. In other words the problems of transportation cost, production cost, marketing costs, etc. have been studied individually without provision for interaction one with another in a mathematical way.

Many studies have been made by companies who are in the tire business. Some of these deal with the production processes (23), some with production costs, (17), and others with the distribution and marketing processes (5). But actually none of these problems are isolated and independent problems of the business. The production costs depend upon the production process; the total cost depends upon the production and transportation costs.

Facts reveal the necessity of a study that shows the interaction between the production costs and the distribution and transportation costs.

METHOD OF ATTACK

As previously proposed in the second section of this chapter, the object of this study is to attempt to solve production and transportation cost problems, and not to optimize separately (or minimize in this case) first the production cost and then the transportation cost as isolated systems, but to optimize both problems simultaneously by having them interact as sub-systems of the total production - distribution system.

An attempt to minimize the total cost of the production - distribution system by two completely different mathematical techniques was also made.

The first technique used was the Linear Programming approach (22). This method is very useful in optimizing a linear function in "n" variables subjected to "m" linear constraints. Although the actual case is not a linear programming problem, it has been transformed into a linear problem (Chapter III) by making several assumptions in order to facilitate solution. It is clear that any simplification of a mathematical model will be reflected in a loss of accuracy. However, this difficulty can be overcome by the method of solution employed in Chapter III, essentially by dividing the unit production costs-volume curve into straight line segments (see Fig. 5).

The second technique employed the Dynamic Programming approach. This technique was chosen because the transportation problem can usually be stated as a multi-stage problem (see Chapter 4). Another reason for the attempt to employ the Dynamic Programming technique is due to the fact that this method does not require that the constraints and the objective function be linear and, because in many problems of this sort, Dynamic Programming has shown a high computational efficiency. This study also made it possible to arrive at tentative criteria for the efficiency of Dynamic Programming in solving the transportation problem as a non-linear programming problem.

CHAPTER II - DEVELOPMENT OF A REALISTIC FORMULATION OF THE TOTAL COSTS OF PRODUCTION AND PLANT LOCATION

PARAMETERS OF THE PRODUCTION-TRANSPORTATION PROBLEM

In the production-transportation cost problem there are two principal independent variables. One of them is the unit production cost and is a function of the plant size or daily production volume. The other variable is the transportation cost and is a function of the distance. The total production-transportation cost is given by the following mathematical expression:

$$C_{T_{ij}} = C_{P_i} + C_{t_{ij}} \qquad (2\text{-}1)$$

where, $C_{T_{ij}}$ = total cost of allocating one unit from production center i to distribution center j

C_{P_i} = unit production cost for production center i

$C_{t_{ij}}$ = unit transportation cost from production center i to distribution center j

Usually the production centers will have an upper limit of desired production volume that will be denoted by the letter b_j, and at the same time the distribution centers will have a limited volume which can be made available to retailers and will be denoted by the letter a_i.

In the actual case, production-transportation of retreads, the distance between any production center to any distribution center will be fixed by the system of transportation used i.e. railroad, truck etc.

As can be seen, the parameters of the problem will be given by the size or daily production of plant i, a_i, and by the daily demand of distribution center j, b_j. The variables will be the quantities x_{ij} to be allocated from plant i to distribution center j. The unit transportation cost from plant i to distribution center j will be constant. The restrictions will be given by the range of plant sizes that cannot produce less than 80 tires/day or more than 810 tires/day. Finally the objective function will be given by the following mathematical expression:

$$Z = \min \Sigma (C_{P_i} + C_{t_{ij}})x_{ij} \qquad (2\text{-}2)$$

COST ANALYSIS

Actually there are around thirty kinds of rubber treads for passenger and non-passenger tires. Therefore, it is not practical nor economical for a small business like a retreading shop to keep records of each one of the thirty kinds of retreaded tires produced.

From personal interviews with the manager and foreman of a medium-sized retreading shop* in Bozeman, Montana, along with available business records, the following percentages of units produced in the shop during the first half of the year, 1967, were collected. All different kinds of retreaded tires produced in the shop were summarized into five main general types as given by Table 1. The average list prices, averages selling prices, average production costs, and the average gross profit are given in the same table.**

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* Long's Tire Service, Bozeman, Montana
** Appendix B gives the dealer's data

Estimation of the Average Selling Price per Tire

From the dealer's records the following percentages of dollar sales by type of retreaded tire were obtained:

Highway Passenger Tire = 27.0%, Passenger Snow Tire = 40.0%, Truck Tire = 33.0%.

The truck tires were further divided into three main types:

Highway Truck Tire = 45.0%, Super Cross Rio Hi-Miler = 35.0%, Hi-Miler Xtra Grip = 20.0%.

TABLE 1

Average List Price, Selling Price, Production, Cost, Gross Profit and

% of Production Volume per Type of Recap Tire*

COLUMN NUMBER / TYPE OF TIRE	(1) (LIST PRICE) . (%)/100	(2) ** ACTUAL AVRG. SELLING PRICE (20% DISCOUNT)	(3) AVRG. PROD. COST (50% OF THE LIST PRICE)	(4) AVRG. GROSS PROFIT PER TYPE OF TIRE	% OF PRODUCTION VOLUME
HIGHWAY PASSENGER TIRE	14.79	11.82	7.40	4.42	20
PASSENGER SNOWTIRE	16.94	13.55	8.47	5.08	30
HIGHWAY TRUCK TIRE	38.08	26.70	19.04	7.66	22.5
TRUCK TIRE	60.69	42.49	30.35	12.14	17.5
TRUCK TIRE	28.55	20.00	14.27	5.73	10
TOTAL					100%

This table is based on cost analysis approach as illustrated on the next page.

* Source: "TIRE RETREADING AND REPAIRING PRICE LIST." THE GOOD YEAR TIRE COMPANY and also from personal interview with the manager of Long's Tire Service, Bozeman, Montana.

** Truck tires have a 30% discount.

Average Selling Prices then are:

$$\text{Per Passenger Tire} = \frac{11.82 \times 27.0 + 13.55 \times 40.0}{27.0 + 40.0} = \$12.85 \qquad (2\text{-}3)$$

$$\text{Per Non-Passenger Tire} = \frac{26.78 \times 45 + 42.49 \times 35 + 20.00 \times 20}{45 + 35 + 20} = \$30.64 \qquad (2\text{-}4)$$

$$\text{Per Average Tire} = \frac{12.85 \times 140{,}000 + 30.64 \times 69{,}000}{140{,}000 + 69{,}000} = \$18.71 \qquad (2\text{-}5)$$

Estimation of the Average Production Cost per Tire.

$$\text{Per Passenger Tire} = 12.85 \times \frac{50\%}{80\%} = \$8.03 \qquad (2\text{-}6)$$

$$\text{Per Truck Tire} = 30.64 \times \frac{50\%}{70\%} = \$22.02 \qquad (2\text{-}7)$$

$$\text{Per Average Tire} = \frac{8.03 \times 140{,}000 + 22.02 \times 69{,}000}{140{,}000 + 69{,}000} = \$12.90 \qquad (2\text{-}8)$$

The weights of 140,000 and 69,000 for passenger and non-passenger tires were used in the above calculations on the assumption that the characteristics of the demand for passenger and non-passenger tires are the same for any location of Montana. The policy of the dealers is to make a 20% discount on passenger tires and 30% discount on non-passenger tires. The average profit per unit sold is computed as follows:

Avrg. Profit/Pass. Tire = (Avrg. Selling Price/Pass. Tire) - (Avrg. Prod. Cost/ Pass. Tire = 12.85 - 8.03 = $4.82 (2.9)

Avrg. Profit/Truck Tire = (Avrg. Selling Price/Truck Tire) - (Avrg. Prod. Cost/Truck Tire) = 30.64 - 22.02 = $10.62 (2-10)

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* The weights, 140,000 and 69,000, given in eq. (2-5) correspond to the magnitude of demand for passenger and truck tires respectively (see Table 8).

Avrg. Profit/Unit Sold = (Avrg. Selling Price/Tire) - (Avrg. Prod. Cost/Tire) = 18.71 - 12.90 = \$5.81 (2-11)

Breakdown of the Production Cost for an 80 Tire/Day Plant Size

From a study made by Goerge R. Edwards (23), the percentage per item of the production cost was taken for an 80 tire/day production plant. Edwards has made this production cost breakdown by item and by type of tire. In order to obtain the average cost per item, the weighted factors concept has been applied.

$$\text{Item Avrg. Cost} = \frac{(C_1 \times \%_1 \times fr_1) + (C_2 \times \%_2 \times fr_2) + (C_3 \times \%_3 \times fr_3)}{fr_1 + fr_2 + fr_3} \quad (2\text{-}12)$$

where:

C_1 = average production cost per highway passenger tire

C_2 = average production cost per passenger snowtire

C_3 = average production cost per truck tire

$\%_1$ = percentage of the production cost per item for highway passenger tire

$\%_2$ = percentage of the production cost per item for passenger snowtire

$\%_3$ = percentage of the production cost per item for truck tire

fr_1 = frequency of sales for highway passenger tire

fr_2 = frequency of sales for passenger snowtire

fr_3 = frequency of sales for truck tire

Then, the "average cost per tire for each item of production" was computed as follows: