THE SMALL SAMPLE PROPERTIES OF A NONSTANDARD ESTIMATOR IN THE CONTEXT OF FIRST ORDER AUTOCORRELATION

by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Applied Economics

MONTANA STATE UNIVERSITY
Bozeman, Montana

March 1987

APPROVAL

of a thesis submitted by

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliograhic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ACKNOWLEDGMENTS

I wish to thank Dr. Jeffrey T. LaFrance for his guidance, patience, and commitment in helping me complete this thesis. I would also like to thank Dr. Gail Cramer for his support and encouragement in my undergraduate and graduate career at Montana State University.

Special thanks goes to my parents, Bob and Sarah, and my brother, John. My parents for their love and support and my brother for his hard work and commitment to the family farm that allowed me to pursue my academic goals.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES--Continued

LIST OF FIGURES

ABSTRACT

The purpose of this study is to compare the small sample properties of a nonstandard estimator for first order autocorrelated errors in a time series equation with those of the more widely used estimators by using Monte Carlo experiments. The estimation method of interest arises either from the assumption that the presample residuals are not generated from an autoregressive process or from fixing the estimates of the presample values of the residuals at their unconditional expectations. This method has several nice properties. First, the estimator that is obtained is asymptotically equivalent to the standard methods. Second, the initial observations in the sample are retained, which overcomes problems that can arise in small samples when the independent variables are trended. Third, the data transformation that is used to estimate the unknown parameters of the model can be generalized to any order autoregressive process without any substantial increase in complexity.

The results indicate that this nonstandard estimator performs very well relative to the other estimators considered for most experimental designs. This implies that the costs of using this more convenient estimation technique in terms of accuracy of parameter estimates is low relative to the other techniques considered.

CHAPTER 1

INTRODUCTION

Autoregressive errors are phenomena in stochastic processes in which random disturbances have effects across observations as well as upon the observation in which the random disturbance originates. These autoregressive errors can occur in both cross-sectional data and time series data but are usually of most interest in time series analysis.

In time series analysis, autoregressive errors imply that a random event or disturbance in time t_0 has effect in time t_1 as well as in time t_0, and possibly in other subsequent time periods. Such errors arise when the time series is of relatively short duration so that a random disturbance does not have full impact in the current time period but flows into proceeding time periods.

Therefore, autoregressive errors are of interest to economists in applied work in time series analysis since accounting for such correlation among the residuals in regression can significantly improve the efficiency of parameter estimates.

The general form of a linear model is $Y = X\beta + \epsilon$ where Y is a $T \times 1$ vector of dependent observations, X is a $T \times K$ matrix of independent observations, β is a $K \times 1$ vector of parameters, and ϵ is a $T \times 1$ vector of normally distributed errors in which $E(\epsilon) = 0$ and $E(\epsilon\epsilon') = \sigma^2 I_T$ where I_T is a $T \times T$ identity matrix. The ordinary least squares solution for the parameter estimates is $b = (X'X)^{-1}X'Y$ and the sample variance of the parameter estimates is $\sigma^2(X'X)^{-1}$.

Under autoregressive errors the general form of a linear model is $Y = X\beta + u$ where Y, X, and β are vectors as defined previously, but u is a T x 1 vector of autoregressive errors in which $E(u) = 0$ and $E(uu') = \Psi$ is the covariance matrix. This is illustrated as follows,

$$\Psi = \sigma^2 \begin{bmatrix} 1 & \rho_1 & \rho_2 & . \; . \; . & . & \rho_{T-1} \\ \rho_1 & 1 & \rho_1 & . \; . \; . & . & . \\ \rho_2 & \rho_1 & 1 & . \; . \; . & . & . \\ . & . & . & & . & . \\ . & . & . & & . & \rho_1 \\ \rho_{T-1} & . & . & . \; . \; . & \rho_1 & 1 \end{bmatrix} \tag{1.1}$$

where ρ_j is the correlation coefficient, $\rho_j = E(\mu_t \mu_{t-j}) / E(\mu_t^2)$, $j = 0, \pm 1, \pm 2, \ldots$

If ordinary least squares is used when the errors are autoregressive, there are three main consequences (Johnston, 1972). The first is that ordinary least squares estimates are unbiased; the second is that ordinary least squares is not efficient; and the third is that the usual least squares formula for the sampling variance, $s^2(X'X)^{-1}$, with $s^2 = 1/(n - k)\Sigma\epsilon_i^2$, gives biased estimates and under the appropriate circumstances the sampling variance can be seriously understated. The result of the latter consequence is inefficient predictions.

Theory suggests that to improve the performance of the least squares estimator, the correlation among the errors should be accounted for as in the generalized least squares estimator. Generalized least squares minimizes the following sum of squared errors,

$$S(\beta,\Psi) = (Y - X\beta)'\Psi^{-1}(Y - X\beta) \tag{1.2}$$

which results in the best linear unbiased solution, $\hat{\beta} = (X'\Psi^{-1}X)X'\Psi^{-1}Y$, and a sample variance of $\sigma^2(X'\Psi^{-1}X)^{-1}$. If u has a multivariate normal distribution, the joint probability density function for $Y = X\beta + u$ is

$$f(Y) = (2\pi)^{-T/2} \, |\sigma^2\Psi|^{-1/2}\exp\{-(Y - X\beta)'\Psi^{-1}(Y - X\beta)/2\sigma^2\} \qquad (1.3)$$

which can be manipulated to the following concentrated log likelihood function, neglecting a constant,

$$L(\beta,\Psi) = -T/2 \; \ell\eta[(Y - X\beta)'\Psi^{-1}(Y - X\beta)] - 1/2\ell\eta|\Psi|. \qquad (1.4)$$

The maximum likelihood solutions are those values of β and the unknown element in the covariance matrix, Ψ, that maximize this concentrated likelihood function. These solutions are equivalent to minimizing

$$S_L(\beta,\Psi) = |\Psi|^{1/T}S(\beta,\Psi) \qquad (1.5)$$

where $S(\beta,\Psi)$ is defined in equation (1.2).

Judge et. al (1985) have shown that if the covariance matrix, Ψ, is assumed to be a known real positive definite symmetric matrix, there exists a unique transformation matrix, P, such that $P\Psi P' = I_T$ where I_T is the identity matrix. This simplifies to $\Psi^{-1} = PP'$. Substituting this definition into equation (1.2) results in the following sum of squared error

$$S(\beta,\Psi) = (PY - PX\beta)'(PY - PX\beta). \qquad (1.6)$$

This transformation causes the errors to be independent and identically distributed. Examples of such estimation techniques are the two-stage Cochrane-Orcutt estimator and the two-stage Prais-Winston estimator. In addition to these more widely used estimation techniques, Judge et al. (1985) allude to an additional nonstandard estimation technique. This nonstandard estimation technique arises either from the assumption that the first observation is not generated by an autoregressive process or from fixing the estimates of the presample residuals at their expected values. These assumptions result in a specific transformation matrix when the errors are generated by a first order autocorrelated process. A first order autocorrelated process is defined as

$$u_t = \rho u_{t-1} + \epsilon_t \qquad t = 0, \pm 1, \pm 2, \ldots \tag{1.7}$$

where the errors, ϵ_t, are assumed to be independently and identically distributed as $N(0, \sigma_\epsilon^2)$. The initially nonstationary approximation results in the following transformation matrix,

$$Q = \begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 & 0 & 0 \\ -\rho & 1 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & -\rho & 1 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -\rho & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & -\rho & 1 \end{bmatrix} \tag{1.8}$$

The estimator that utilizes this transformation matrix is called "initially nonstationary." The reason for this designation is that the second moment of the first or initial transformed error is time dependent and the estimator is therefore nonstationary. This nonstationarity can be easily demonstrated. Define e = Qu where e is a T x 1 and u is a vector defined earlier. This results in the following transformed errors,

$$\begin{aligned} e_1 &= u_1 = \epsilon_1/(1 - \rho^2) \\ e_2 &= u_2 - \rho u_1 = \epsilon_2 \\ e_3 &= u_3 - \rho u_2 = \epsilon_3 \\ &. \quad . \quad . \\ &. \quad . \quad . \\ e_T &= u_T - \rho u_{T-1} = \epsilon_T \end{aligned} \tag{1.9}$$

The $Var(e_1) = \sigma_\epsilon^2 /(1 - \rho^2)$, while $Var(e_t) = \sigma_\epsilon^2$ for $t = 2,\ldots,T$. It is clear from this that the initial error term has a second moment that is dependent upon time.

Theil (1971) suggests that setting the initial observation equal to some starting value, in this case ϵ_1, may in fact be a very reasonable assumption since for some data the process generating the disturbances may have been affected, for example, by a major war. This is opposed to the assumption that all the observations are generated from a process that reaches back infinitely into the past.

All of the estimation methods suggested are asymptotically equivalent but differ in their small sample properties. However, since the small sample properties are difficult to derive analytically, econometricians have had to rely on Monte Carlo evidence in choosing among different estimation techniques. Little is known, though, about the small sample properties of the initially nonstationary estimator. The objective of this research is to examine and rank the small sample properties of the previously suggested estimation techniques under several experimental designs using Monte Carlo experiments in the context of first order autocorrelation. The results of such an examination will be useful for applied econometricians in choosing among estimation methods that deal with autocorrelation.

Literature Review

Cochrane and Orcutt (1949) published an article on their research in the application of least squares regression when first order autocorrelation is present in the error term. This article has become a classic in applied econometrics, and the method proposed is still in widespread use today.

The method suggested to deal with autocorrelation is to transform all the variables according to the autoregressive structure of the error term and then apply least squares regression to the transformed variables. When the true autoregressive structure is known, such a transformation results in best linear unbiased regression coefficients (Cochrane and Orcutt, 1949).

When the true autoregressive structure is unknown, however, Cochrane and Orcutt suggested an iterative two-stage procedure in which ordinary least squares is applied to the raw data to obtain a series of residuals. These residuals are used to estimate the correlation coefficient, rho, of the autoregressive process. Using this estimate of rho, difference the data by using $Y_t - \rho Y_{t-1}$ and $X_t - \rho X_{t-1}$ and apply ordinary least squares to the transformed data. Continue this procedure until the change in the estimated correlation coefficient is within some prescribed tolerance. It was the opinion of Cochrane and Orcutt that since it was necessary to make the residuals only approximately random, such an iterative process is necessary only once or twice. This procedure has been labeled the two-stage Cochrane-Orcutt estimation procedure.

Cochrane and Orcutt (1949) expressed some reservation concerning this method,

> The real difficulty with this procedure is that the series of residuals will be strongly biased towards randomness and therefore the autoregressive transformation based in the above way on the residuals may not in fact go far enough in randomizing the error term.

There has also been expressed in the literature some concern as to the convergence of the Cochrane-Orcutt procedure to a local minima as opposed to a global minimum or the possibility of multiple solutions (Johnston, 1972). Sargan (1969) has shown that the Cochrane-Orcutt procedure will at least result in a local minimum.

It is pertinent to this research to note that the Cochrane-Orcutt method of estimation ignores the first error term and uses only T - 1 transformed observations. The justification for leaving the first term out is that asymptotically it will make no difference in the performance of the estimator. Further, deleting the first observation has the advantage of simpler algebraic expressions utilized in the transformation matrix. The resulting (T - 1) x T transformation matrix is

$$\begin{bmatrix} -\rho & 1 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & -\rho & 1 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -\rho & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & -\rho & 1 \end{bmatrix} \qquad (1.10)$$

Another method of estimation is the two-stage Prais-Winston estimator. In an unpublished Cowles Foundation discussion paper, Prais and Winston (1954) derived the correct transformation matrix that takes the first observation into account. The resulting T x T transformation matrix is

$$\begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & -\rho & 1 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -\rho & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & -\rho & 1 \end{bmatrix} \tag{1.11}$$

The first observation is assigned a weight of $\sqrt{1-\rho^2}$, whereas in the Cochrane-Orcutt method the first observation is ignored. The reason for the weight of $\sqrt{1-\rho^2}$ on the first term is that such a weight allows the first transformed error to have the same variance as the other transformed errors, thereby defining the transformation to be homoskedastic.

Asymptotically, including the first observation in the way described makes no difference. Both estimation techniques converge in probability to the true parameter values. In small samples, however, inclusion of the first observation may result in a significant improvement in efficiency. As Rao and Griliches (1969) reflect on the Prais-Winston article,

> The loss in efficiency entailed in the usual procedure [the Cochrane-Orcutt procedure] depends critically on how different the beginning X value is from the average, and that it could be quite high for trend like X's.

The procedure of Prais-Winston estimation is similar to the Cochrane-Orcutt procedure in that it is also a two-stage estimator. The first stage estimates parameter values again using ordinary least squares and the resulting residuals are used to estimate rho. In the second stage the T x T transformation matrix is used in differencing the data instead of the (T - 1) x T transformation matrix in the Cochrane-Orcutt procedure.

Neither of the previous estimation procedures is maximum likelihood when rho has to be estimated (Beach and MacKinnon, 1978). Therefore, Beach and MacKinnon propose and derive a full maximum likelihood procedure that utilizes the first observation, as the Prais-Winston method does, while at the same time incorporating the a priori assumption of stationarity. As they state in their article, such an estimation procedure is better on purely theoretical arguments.

The estimation technique is based upon maximizing the concentrated log likelihood function with respect to beta and rho while holding rho and beta fixed, respectively. The solution for rho is in terms of beta, and the solution for beta is in terms of rho. Therefore, Beach and MacKinnon proposed a two-step procedure similar to that of Corchrane and Orcutt in which initially rho is set equal to zero and an iterative process is continued until two successive values of rho are within some prescribed tolerance. The maximum likelihood solutions for beta and rho are as follows,

$$\tilde{\beta} = (X^{*\prime}X^{*})^{-1}X^{*\prime}Y^{*}, \tag{1.12}$$

$$\tilde{\rho} = -2\sqrt{p/3}\,\cos(\varphi/3 + \pi/3) - a/3, \tag{1.13}$$

where $X_1^* = \sqrt{1 - \rho^2}X_1$, $Y_1^* = \sqrt{1 - \rho^2}Y_1$ and $X_t^* = X_t - \tilde{\rho}X_{t-1}$ $Y_t^* = Y_t - \tilde{\rho}Y_{t-1}$, $t = 2, \ldots, T$

and

$$\varphi = \cos^{-1}\left[(q\sqrt{27})/(2p\sqrt{-p})\right], \quad 0 \leq \varphi \leq \pi, \tag{1.14}$$

$$p = b - a^2/3, \tag{1.15}$$

$$q = c + ab/3 + 2a^3/27, \tag{1.16}$$

$$a = -(T - 2)\Sigma A_t A_{t-1} / \left[(T - 1)(\Sigma A_{t-1}^2 - A_1^2)\right], \tag{1.17}$$

$$b = \left[(T - 1)A_1^2 - T\Sigma A_{t-1}^2 - \Sigma A_t^2\right] / \left[(T - 1)(\Sigma A_{t-1}^2 - A_1^2)\right], \tag{1.18}$$

$$c = T\Sigma A_t A_{t-1} / \left[(T - 1)(\Sigma A_{t-1}^2 - A_1^2)\right], \tag{1.19}$$

where

$$A_t = Y_t - \tilde{\beta} X_t \tag{1.20}$$

and the summations run from $t = 2$ to T.

Since this full maximum likelihood approach is a two-step procedure, as in the Cochrane-Orcutt procedure, there also exists the question of whether the resulting solution is a local maximum or a global maximum.

Computationally, this full maximum likelihood algorithm works very quickly. As Beach and MacKinnon (1978) point out, "This algorithm works extremely rapidly, requiring, on the average, between four and seven least squares computations to achieve better than five digit accuracy." Even though one iteration of the full maximum likelihood estimation procedure is more expensive than the conventional Cochrane-Orcutt, the full maximum likelihood estimation procedure arrives at a solution on the average in fewer iterations than the Cochrane-Orcutt procedure, and therefore, is on the average less expensive.

The fourth commonly used estimation technique is an iterative nonlinear least squares estimation method, of which the Cochrane-Orcutt method is an example. This estimation procedure seeks a value of beta and rho simultaneously that minimizes the following sum of squared errors,

$$S = \Sigma_{t=2}^{T} \epsilon_t^2 = \Sigma_{t=2}^{T} (Y_t - \rho Y_{t-1} - \beta X_t + \beta\rho X_{t-1})^2 \tag{1.21}$$

and imposes the constraint that $\widehat{\beta\rho} = \hat{\beta} \times \hat{\rho}$ (Rao and Griliches, 1969). This structural form is the result of the differencing suggested by Cochrane and Orcutt. The literature suggests several methods of minimization. These methods are Gauss-Newton (linearization), steepest descent, or Marquardt's compromise (Draper and Smith, 1966).

Since each of these techniques are asymptotically equivalent and the small sample properties are difficult to derive analytically and difficult to interpret, Monte Carlo studies have become useful as a way of analyzing the small sample properties of each estimator. Rao and Griliches (1969) published an article on their research on the small sample properties of several two-stage estimators when autocorrelation exists in the errors. These various estimation techniques are generalized least squares when rho is known, ordinary least squares, Durbin's estimator, the two-stage Cochrane-Orcutt method, the two-stage Prais-Winston method, and an iterative nonlinear least squares method which Rao and Griliches claimed to be maximum likelihood (conditional on the first observation). The Durbin method is essentially the same as the Cochrane-Orcutt method except that a different estimator for rho is used. This article published by Rao and Griliches was the first attempt to deal with the small sample properties of the estimators comprehensively.

Rao and Griliches' Monte Carlo study was based upon first order autocorrelation and a sample size of 20, with 50 such samples being drawn. The model specification was

$$Y_t = 0 + 1.0X_t + u_t , \qquad (1.22)$$

$$u_t = \rho u_{t-1} + \epsilon_t, \qquad (1.23)$$

$$X_t = \lambda X_{t-1} + v_t, \qquad (1.24)$$

where $E(v_t) = E(w_t) = E(v_t w_t) = E(w_t w_{t-1}) = E(v_t v_{t-1}) = 0$ and $E(v_t^2) = \sigma_v^2$, $E(w_t^2) = \sigma_w^2$ and rho and lambda are compared at intervals of 0.1 and 0.2, respectively.

Rao and Griliches concluded for estimates of rho that none of the estimation techniques are unbiased. Of the methods used, however, the Durbin estimates have the smallest bias for positive values of rho, and ordinary least squares and nonlinear techniques have the smallest bias for low and high values of rho, respectively. They noted that for negative values of rho, the Durbin estimator has bias very near to that of ordinary least squares and nonlinear estimators. This suggests that over the whole parameter space of rho, the Durbin estimator for rho is most acceptable.

In considering the estimates of the slope coefficient, Rao and Griliches used the mean square error of parameter estimates of each estimation procedure as a comparison method. The result of the Monte Carlo experiments in regard to estimates of beta was that no one procedure had a lower mean square error for beta for all values of rho and lambda. Ordinary least squares did perform quite well for $|\rho| \leq .20$, and in fact for these values of rho, ordinary least squares had a consistently lower mean square error. In a more detailed pairwise analysis, the importance of retaining the first observation in Prais-Winston as compared to Cochrane-Orcutt was discussed and evaluated by a ratio of the mean square error of the Cochrane-Orcutt estimator to the mean square error of the Prais-Winston estimator. This ratio indicated that Prais-Winston was more efficient than Cochrane-Orcutt except when rho was close to unity. This seems reasonable since as rho approaches one, the Prais-Winston method becomes identical to the Cochrane-Orcutt method. However, when there was a strong trend, the Prais-Winston method actually lost efficiency in comparison to the Cochrane-Orcutt method.

In regards to the iterative nonlinear approach, Rao and Griliches showed that there was little to be gained from this more complex method as compared to the other two-stage estimators. The iterative nonlinear approach does yield better estimates when rho and lambda are negative and large in magnitude. This circumstance, however, seems unlikely to occur in actual applications.

Therefore, Rao and Griliches (1969) concluded that,

> There is a significant gain in efficiency to be had from using two stage estimation procedures for moderate and high levels of serial correlation in the residuals [$|\rho| > .30$] and very little loss from using such methods even when the true ρ is small,

and later recommended that

> Where computational costs are a consideration, a compromise mixed strategy of switching to a second-stage only if the estimated $|\rho| \geq .30$ should do relatively well over the whole parameter range.

Gallant and Goebel (1976) discussed the small sample properties of nonlinear regression, which is essentially the Prais-Winston procedure, when the errors are autocorrelated. Their hypothesis, although similar to the conclusions of Rao and Griliches, is noteworthy. They suggested that when ordinary least squares residuals appear to at least reasonably satisfy the structure of an autoregressive process, a nonlinear estimation process which takes this autocorrelation into account would result in better estimates than estimators that ignore such correlation.

Their research used Monte Carlo evidence to answer two basic questions: (1) Does in fact a nonlinear regression that accounts for autocorrelation provide better estimates than circular estimators? and (2) If the answer to the first question is yes, then does a two-stage autoregressive nonlinear estimator do better than a one-stage nonlinear estimator?

The result of Gallant and Goebel's Monte Carlo experiment suggests that nonlinear estimation that takes autocorrelation into account has better small sample efficiency than estimators that ignore such correlation. However, it was not clear from the results whether a two-stage estimator did better than a one-stage estimator. They concluded that a two-stage estimator certainly did not decrease the efficiency of parameter estimates, and there was some evidence of possible gains in efficiency. Based upon this evidence the authors recommended using a two-stage estimator in applications.

The article by Rao and Griliches was the foundation for other research projects that used Monte Carlo studies to analyze the small sample properties in the presence of first order autocorrelation. Maeshiro (1976) published an article on the efficiency of the Cochrane-Orcutt estimator relative to ordinary least squares when the independent variable is trended, autocorrelation exists in the error term, and the magnitude of the autocorrelation is known. It was his belief that under a commonly observed condition, trended independent variables, that ordinary least squares would outperform one iteration of the Cochrane-

Orcutt transformation in terms of efficiency of parameter estimates, which is contrary to the findings of Gallant and Goebel and Rao and Griliches.

The design of Maeshiro's experiment was similar to that of Rao and Griliches in that this study had the following specification,

$$Y_t = \alpha + \beta X_t + u_t , \qquad (1.25)$$

$$u_t = \rho u_{t-1} + \epsilon_t , \qquad (1.26)$$

where $|\rho| < 1.0$, $E(\epsilon_t) = 0$, $Var(\epsilon_t) = \sigma^2$, and $E(\epsilon_t \epsilon_r) = 0$ for $t \neq r$. Sample sizes included are 10, 20, 50, and 100. The independent variables are constructed according to five different specifications. The various specifications are a geometric series, $X_t = \lambda X_{t-1}$; arithmetic series, $X_t = Bt$; U.S. real GNP series; randomized GNP series; and Federal Reserve Board capacity utilization series for manufacturing. Maeshiro calculated the variance of beta under each estimation technique and compared each using a ratio.

Maeshiro concluded that for all trended independent variables, the Cochrane-Orcutt transformation reduces the efficiency of parameter estimates compared to ordinary least squares for nonnegative values of rho. This loss of efficiency can be substantial for select values of rho and lambda. For instance, when $\rho = .80$ and $\lambda = .90$, a 90% loss in efficiency was recorded.

Maeshiro (1976) suggested a reason why he thought this was true,

> Unless the gain in the efficiency of estimators acquired by correcting the autocorrelation in the disturbance term outweighs the loss in efficiency due to this reduction in the variability [from differencing the data], the C-O [Cochrane-Orcutt] estimator reduces the efficiency of estimators rather that increases it.

The results of varying the sample size are what one would expect. As the sample size increases, the relative efficiency of transformed Cochrane-Orcutt to ordinary least squares increases. But, as Maeshiro points out, when the independent variable contains a trend, it would be erroneous to conclude that transformed Cochrane-Orcutt is more efficient than ordinary least squares simply because the sample size is large.

Maeshiro (1979) published another article on the effects of trended independent variables. In this article, using Monte Carlo studies, Maeshiro examined the relative efficiency of estimation procedures that retain the first observation with weight $\sqrt{1 - \rho^2}$ (generalized least squares or Prais-Winston when rho is known) as compared to the application of ordinary least squares to Cochrane-Orcutt transformed data. His expectation was that under positively autocorrelated disturbances and trended independent variables, generalized least squares that retain the first observation would be more efficient than had been demonstrated in earlier Monte Carlo work such as in the Rao and Griliches study referenced earlier.

Maeshiro reasoned that retaining the first observation is important because when the data is transformed by a positive autoregressive structure - in this case first order autocorrelation - these new variables are more similar. The weighted first observation of $\sqrt{1 - \rho^2}X_1$ tends to take on a less similar value. Therefore, the contribution of the first observation to the variability of the transformed data is significant. However, the first observation becomes less important to the efficiency of the estimator as the sample size increases, since the marginal contribution of the first observation to the variability of the data becomes less as the sample size increases.

The design of the Monte Carlo experiment was exactly that of Maeshiro's earlier article. However, instead of using the Federal Reserve Board capacity utilization series for manufacturing as a design for the independent variable, a stable first order autocorrelated process that is independent of the process generating the disturbance was used. Two sample sizes were used in this experiment, T = 20 and T = 50, instead of the four sample sizes used previously.

In the experiment Maeshiro compared two estimators, the application of least squares to autoregressively transformed data (what he called ARG) and generalized least squares. He calculated the marginal relative contribution (MRC) of the first observation according to the following specification,

$$MRC(\rho) = \left\{Var[ARG(\rho)] - Var[GLS(\rho)]\right\} / \, Var[GLS(\rho)]. \qquad (1.27)$$

The result of the analysis is that the marginal relative contribution for the case of a trended independent variable is much larger than for the case of a random independent variable. This is especially true for middle to high values of rho. Specifically, for trended independent variables, the omission of one observation under appropriate combinations of rho and lambda can increase the variance of an estimator by as much as 50%. The marginal relative contribution of the first observation, however, was negligible when the original data possessed a high degree of variability, i.e. $\lambda \geq 1.12$. This implies that if the original data has a high enough degree of variability so that after it is transformed it still has a relatively high degree of variability, the importance of including the first observation is much less.

The effect of sample size is somewhat contrary to the expected results. It appeared that as the sample size increased from 20 to 50, the importance of retaining the first observation remained quite high. In fact, for the case of real GNP as the design for the independent variable, Maeshiro set the sample size to 100. The result for this specific experiment was that it was still costly in terms of efficiency to omit the first observation. Therefore, Maeshiro concluded that even when the sample size is large the contribution of the first observation can still be significant.

In addition to the above Monte Carlo study when rho is known, Maeshiro compared the relative efficiency of the two-step Cochrane-Orcutt and the two-step Prais-Winston estimators when rho is unknown. The intent was to confirm the importance of retaining the first observation when rho in unknown. The results, however, did not bear out such a distinction. Maeshiro reported no substantial difference between the two procedures.

Spitzer (1979) duplicated the Monte Carlo studies of Rao and Griliches and arrived at contrary results. He pointed out that the nonlinear estimator that Rao and Griliches used and equated with a maximum likelihood estimator was actually only asymptotically equivalent to a maximum likelihood estimator. Therefore, Spitzer saw the opportunity to examine the small sample properties of what he called small sample maximum likelihood.

His small sample maximum likelihood estimator amounted to maximization of the following concentrated log likelihood function,

$$L(\rho) = -T/2\ell\eta(\sigma^2) + 1/2\ell\eta(1 - \rho^2). \qquad (1.28)$$

This is the same objective function as Beach and MacKinnon used in deriving their full maximum likelihood estimator, but here Spitzer uses a Gauss-Newton method of optimizing both the nonlinear and the maximum likelihood objective function.

The design of the Monte Carlo experiment is exactly that of Rao and Griliches except that twice as many samples are taken. The results are generally contrary to those of Rao and Griliches, although Spitzer agreed that in the presence of autocorrelation, ordinary least squares performs very poorly in comparison to the other estimators. His results showed that often the additional costs of the more complex maximum likelihood and nonlinear least squares methods are justified in terms of better parameter estimates. For $|\rho| > .60$ the maximum likelihood procedure is better in terms of lower mean square error than Durbin, and Prais-Winston is slightly superior to nonlinear. The nonlinear approach for very large $|\rho|$ resulted in lower mean square error than Durbin. These results are averaged over lambda. Based upon such, Spitzer ranked the various estimators over the range of rho for optimal estimator performance. The rank is as follows: for values of $|\rho| \leq .20$, ordinary least squares is optimal; for values of rho where $.20 < |\rho| \leq .50$, Prais-Winston is optimal followed by maximum likelihood for values of $|\rho| > .50$. Spitzer also noted that each of these optimal estimators use T observations rather than T - 1 observations, which is similar to the results of Maeshiro's research under trended independent variables.

In estimating rho, Rao and Griliches concluded that the Durbin estimator had the lowest bias for the range of values for rho. Spitzer's results show that both the maximum likelihood and nonlinear estimators of rho are superior to Durbin and decisively superior to ordinary least squares estimates of rho. For large absolute values for rho, maximum likelihood estimates are less biased than nonlinear estimates. In addition, Spitzer compared the mean square error of rho averaged over the range of lambda and concluded that for $|\rho| > 0.3$ both the nonlinear and maximum likelihood estimates of rho had a smaller mean square error than Durbin and ordinary least squares.

Beach and MacKinnon (1978) also conducted their own Monte Carlo experiment comparing their full maximum likelihood estimator with the conventional Cochrane-Orcutt estimation procedure. The design of the experiment was similar to all the previous experiments in that,

$$Y_t = \alpha + \beta X_t + u_t , \tag{1.29}$$

$$u_t = \rho u_{t-1} + \epsilon_t , \tag{1.30}$$

$$\epsilon_t \sim N(0, \sigma_\epsilon^2), \tag{1.31}$$

and

$$X_t = \exp(.04t) + w_t , \tag{1.32}$$

$$w_t \sim N(0, \sigma_w^2). \tag{1.33}$$

The sample sizes are set at 20 and 50; and rho is varied between the values .60, .80, and .99, with 200 replications of each experiment. The results are that in every case, gains, measured by lower root mean square error, are recorded by using full maximum likelihood over the conventional Cochrane-Orcutt procedure. This is true for estimates of alpha, beta, and rho. Beach and MacKinnon also experimented with randomly distributed independent variables. In this case, there was very little improvement in estimates of beta using the full maximum likelihood procedure over the Cochrane-Orcutt procedure, but the full maximum likelihood did result in better estimates of alpha than did the Cochrane-Orcutt procedure.

The conclusion of varying the sample size from 20 to 50 is that it may take a very large sample size in order for these two estimation procedures to be asymptotically equivalent. The advantage of using the full procedure is still high, even when $T = 50$.

It is obvious from the previous discussion that there is varying opinion on the importance of correcting for autocorrelation and on the importance of retaining the initial observation. Taylor (1981) published an article on his efforts to explain such discrepancies. These discrepancies have come primarily from the conclusions of the research of Rao and Griliches (1969) and of Maeshiro (1976). Rao and Griliches concluded that when $|\rho| > .20$, the Cochrane-Orcutt method was more efficient than ordinary least squares. Maeshiro showed that when the independent variable contained a moderate trend, ordinary least squares was more efficient than Cochrane-Orcutt and that estimation techniques that include the initial observation were usually more efficient. Maeshiro theorized that the Cochrane-Orcutt transformation reduced the variability in the data and this in turn reduced the efficiency of parameter estimates. In an effort to determine whether or not this is true, Williams (1981) derived the variances of an autoregessively transformed independent variable and the variance of an untransformed independent variable. He showed that $Var(X_t^*)/Var(X_t) =$

$1 - 2\rho\lambda + \rho^2$, $t > 1$, where $X_t^* = X_t - \rho X_{t-1}$. Therefore, he concluded that whenever $\rho > 0$ and $\rho > 2\lambda$ or $\rho < 0$ and $\rho < 2\lambda$, the Cochrane-Orcutt transformation increases the variance of the independent variable and ultimately increases the efficiency of parameter estimates.

He also pointed out that there exists a unique asymmetry in the contribution of the first observation that holds even as the sample size grows large. This unique case is when the model specification of the independent variable is nonstachastic and trended, and $\rho = \lambda$. If this is the case, then the first observation is the only relevant information for estimating beta. This is clearly a useless manipulation of the data. Therefore, Williams analyzed the importance of the first observation when the independent variable is generated according to a stochastic specification, and concluded that when this is the case, any single observation will become negligible asymptotically.

In general, comparison of ordinary least squares to Cochrane-Orcutt hinges on how the independent variable is generated. Williams has shown that when the independent variable is constructed according to a stochastic specification, as in Rao and Griliches, Cochrane-Orcutt always does better than ordinary least squares; and when the independent variable is generated by a nonstochastic process, as in Maeshiro's research, ordinary least squares performs better than Cochrane-Orcutt.

The conclusion from the research in the area of estimation when the errors are autocorrelated is that correcting for this correlation significantly increases estimator performance. In small samples, estimation techniques that retain the initial observation while correcting for autocorrelation prove also to do better, especially when the independent variable is trended. This has motivated the improvement of the efficiency of conventional techniques such as the Cochrane-Orcutt procedure. One of these improvements has been the retention of the first observation, as in Prais-Winston. Another has been the derivation of the first order conditions for maximizing the concentrated log likelihood function in small samples. Monte Carlo experiments bear out the superiority of maximum likelihood procedures in small samples as in research done by Spitzer (1979) and Beach and MacKinnon (1978). Asymptotically, all of the methods presented are equivalent, but as has been demonstrated in the literature cited, each of these estimators has its own unique small sample properties and it is these properties which the econometrician should note.

Methodology

This research examines the small sample properties of several estimators that take autocorrelation of errors into account. The methods of interest are ordinary least squares, the two-stage Cochrane-Orcutt estimator, the two-stage Prais-Winston estimator, the iterative Cochrane-Orcutt estimator, the iterative Prais-Winston estimator, the iterative full maximum likelihood estimator, and the iterative initially nonstationary estimator that utilizes a nonstandard transformation matrix and a nonlinear optimization technique. This nonstandard T x T transformation matrix is as follows,

$$\begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 & 0 & 0 \\ -\rho & 1 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & -\rho & 1 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -\rho & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & -\rho & 1 \end{bmatrix} . \qquad (1.34)$$

Since the small sample properties of these estimators are difficult to derive and even more difficult to interpret, this research relies on Monte Carlo experiments to provide an indication of the small sample properties of each of these estimators.

Monte Carlo studies describe the process of specifying the structure of a model, its parameter values, and the distribution of the error term. The independent variables are constructed according to a specific specification, i.e. trended, random, etc., and used to generate dependent variables. Once this has been completed, the model's parameter values can be estimated using the independent and dependent variables and a particular estimation technique. The parameter estimates can be compared to the true parameters by measurements of bias, variance, or the mean square error.

In the study, the model structure that will be used in the Monte Carlo experiment is that of an intercept and a single explanatory variable as follows,

$$Y_t = \alpha + \beta X_t + u_t \qquad (1.35)$$

where u_t follows a first order autocorrelation error structure expressed by the following equation,

$$u_t = \rho u_{t-1} + \epsilon_t \qquad (1.36)$$

where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$.

The construction of the independent variable will follow two specifications, a dampened trend and a growth trend. The dampened trend variables will be generated by the following stochastic structure,

$$X_1 = \theta + w_1$$

$$X_t = \lambda X_{t-1} + w_t \text{ for } t = 2, \ldots, T \tag{1.37}$$

where theta is an intercept and $w_t \sim N(0, \sigma_w^2)$. The random case occurs when $\lambda = 0.0$ in equation (1.37). The growth trend will be generated by

$$X_t = \exp(.04t) + w_t \tag{1.38}$$

where $w_t \sim N(0, \sigma_w^2)$. This growth trend is the same as the design for the independent variable in Beach and MacKinnon's Monte Carlo study [see equations (1.32) and (1.33)].

The sample sizes included in this Monte Carlo analysis are 25, 50, and 100, with 500 replications of each experiment. Parameter values for rho range from -.80 to .80 at intervals of .20 and from -.99 to -.90, and .90 to .99 at intervals of .01, and values for lambda will range from -.75 to .75 at intervals of .25.

The alternative estimation techniques will be compared and evaluated by using the estimated mean square error values for the parameter estimates of alpha, beta, and rho of the 500 experiments.

As indicated in the introduction, the goal of this research is a ranking of the various methods of estimation that account for autocorrelation. This research also should indicate how the ranking changes under the two experimental designs suggested and under various combinations of rho and lambda. This ranking, however, is not the only information sought in this research.

In addition, since the Cochrane-Orcutt method of estimation does not include the initial observation, there is the question of how the inclusion of this observation will impact the performance of the other estimators. As the literature has suggested, when there is a substantial amount of trend, the importance of retaining this initial observation is critical to estimator performance. This research should estimate and document how substantial this trend actually must be before it is critical.

It is clear that as the sample size gets larger these estimation techniques are equivalent in their efficiency of estimating. But how large should the sample size be before practical equivalence is assured, and how large a sample size is necessary before the contribution of the initial observation is negligible? This relationship between sample size and the initial observation is crucial. In small samples, the estimation methods that include the first observation in their derivation are theoretically superior. Knowing how large a sample size needs to be before the benefits of these more complex methods become negligible is of pivotal importance to the applied econometrician. Therefore, this is the additional information that this research seeks to address.

In the following chapters, Chapter 2 will explain the theoretical motivations for each of the estimators used in this study and a few of the computer program particulars, Chapter 3 will detail the actual Monte Carlo experiments and their results, and Chapter 4 will summarize the results into conclusions as these relate to the objectives of this study.

CHAPTER 2

THEORY OF ESTIMATION

This research explores the small sample properties of seven different estimators. Those seven estimators are ordinary least squares, two-step Cochrane-Orcutt, two-step Prais-Winston, iterative Cochrane-Orcutt, iterative Prais-Winston, iterative maximum likelihood, and the iterative initially nonstationary estimator. In this chapter the theoretical motivations of each of these estimators will be explained, the theoretical basis for the specific estimator of the autocorrelation coefficient, ρ, used in the two-step Cochrane-Orcutt and Prais-Winston estimators will be explained, and the method of generating normally distributed errors used in the Monte Carlo experiments will also be explained.

Ordinary least squares solutions for parameter estimates are the result of minimizing the sum of squared errors. The sum of squared errors is as follows,

$$S = (Y - X\beta)'(Y - X\beta) \tag{2.1}$$

where X is T x K, Y is T x 1, and β is K x 1. Therefore, the least squares solution is $b = (X'X)^{-1}X'Y$. This is motivated by five assumptions: (1) a normally distributed error term; (2) an expected value of the error term of zero, i.e. $E(\epsilon_t) = 0$; (3) homoskedasticity, $E(\epsilon_t^2) = \sigma_\epsilon^2$ for all t = 1,...,T; (4) nonautoregression, $E(\epsilon_t \epsilon_r) = 0$ for all s $\neq$ t; and (5) explanatory variables that are stochastically independent of ϵ.

When the error terms are autoregressive, ordinary least squares is unbiased but not efficient, as has been discussed previously. Cochrane and Orcutt (1949) derived the transformed objective function that takes first order autocorrelation into account. This same approach can be extended to higher orders of autocorrelation as well. The specification used by Cochrane and Orcutt is

$$Y_t = \alpha + \beta X_t + u_t \tag{2.2}$$

where u_t is a first order autocorrelated error term. This first order autocorrelated error term is

$$u_t = \rho u_{t-1} + \epsilon_t \tag{2.3}$$

where the ϵ_t are independent and identically distributed as

$$\epsilon_t \sim N(0, \sigma_\epsilon^2). \tag{2.4}$$

This results in the following difference equation, excluding the first observation,

$$Y_t - \rho Y_{t-1} = \alpha - \rho\alpha + \beta X_t - \rho\beta X_{t-1} + \epsilon_t \tag{2.5}$$

which results in the sum of squared error equation,

$$S = \Sigma_{t=2}^{T}\left[(Y_t - \alpha - \beta X_t) - \rho(Y_{t-1} - \alpha - \beta X_{t-1})\right]^2. \tag{2.6}$$

It is clear that the first order conditions for the above sum of squared errors cannot be solved explicitly in terms of alpha, beta, and rho. This is because of the nonlinear term $\rho\beta$. Therefore, econometricians have resorted to other methods for finding optimal solutions to (2.6).

The two most common approaches are a search procedure and an iterative procedure. A search procedure involves setting rho, a priori, equal to some reasonable value. Then the values of alpha and beta are estimated using the estimators derived from the objective function conditional on rho. These estimates of alpha and beta and the selected value of rho are then substituted into the objective function. The resulting value of the objective function is examined for optimality in relationship to other values of the objective function using different values of rho, alpha, and beta.

The iterative procedure, which is used in this research, initially sets the value of rho equal to zero and estimates alpha and beta using ordinary least squares. These estimates of alpha and beta are used in the estimation of rho which then can be substituted back into the estimator for alpha and beta. After a series of substitutions or iterations, the underlying objective function will converge to some optimal value.

The search method is usually computationally more expensive than the iterative procedure, but has the advantage of guaranteeing a global optimum, whereas the iterative procedure does not (Judge et al., 1982). The iterative procedure, however, is more widely used and has been accepted as the conventional method.

In the context of this research, several of the methods used are noniterative and stand as building blocks to the iterative methods. One example of this is the two-step Cochrane-Orcutt estimator. In this case, solutions for alpha and beta, in the two-parameter case, are derived by two applications of ordinary least squares. Initially, the ordinary least squares estimates of alpha and beta are used to obtain a set of residuals which then are used to estimate rho by some estimation procedure, one of which will be discussed later. This is the first stage of the estimation procedure. In the second stage, the data is differenced using the estimate of rho and ordinary least squares is applied to this new set of data in estimating alpha and beta.

Let $X_t^* = X_t - \hat{\rho}X_{t-1}$ and $Y_t^* = Y_t - \hat{\rho}Y_{t-1}$ and substitute these definitions into the sum of squared error objective function for the two-parameter case. This yields the following sum of squared errors,

$$S = \Sigma_{t=2}^{T}\left[Y_t^* - \alpha(1 - \hat{\rho}) - \beta X_t^*\right]^2. \qquad (2.7)$$

Solving this with respect to alpha and beta results in ordinary least squares solutions, with $\left[(1 - \hat{\rho})\mathbf{1},\ X^*\right]$ as the independent variables and Y^* as the dependent variable where $\mathbf{1}$ is a vector of ones. These solutions are

$$\hat{\beta} = \Sigma_{t=2}^{T}(X_t^* - \bar{X}^*)(Y_t^* - \bar{Y}^*)/\ \Sigma_{t=2}^{T}(X_t^* - \bar{X}^*)^2 \qquad (2.8)$$

and

$$\hat{\alpha} = \left[\bar{Y}^* - \hat{\beta}\bar{X}^*\right] / (1 - \hat{\rho}). \qquad (2.9)$$

When the sample sizes are small, it has been thought that the inclusion of the first observation is an addition of vital information. This was the motivation behind the research of Prais and Winston (1954). In their research they sought to weight the first observation so that it would be homoskedastic. In order to make u_1 homoskedastic, we need to know the variance of u_1. The following is a derivation of the variance of u_1.

For the case of first order autocorrelation, it can be shown that the autocorrelated error, u_t, by recursive substitution is

$$\mu_t = \epsilon_t + \rho\epsilon_{t-1} + \rho^2\epsilon_{t-2} + \rho^3\epsilon_{t-3} + \ldots. \qquad (2.10)$$

or rather, $u_t = \Sigma_{r=0}^{\infty} \rho^r \epsilon_{t-r}$. For any t we know that

$$\mathrm{Var}(u_t) = E(u_t^2) - [E(u_t)]^2 \tag{2.11}$$

and that

$$\begin{aligned}[E(u_t)]^2 &= [E(\Sigma_{r=0}^{\infty} \rho^r \epsilon_{t-r})]^2 \\ &= [\Sigma_{r=0}^{\infty} \rho^r E(\epsilon_{t-r})]^2\end{aligned} \tag{2.12}$$

whose expected value is zero by assumption. All that is needed, then, to find the $\mathrm{Var}(u_t)$ is to find $E(u_t^2)$.

$$\begin{aligned}E(u_t^2) &= E[(\Sigma_{r=0}^{\infty} \rho^r \epsilon_{t-r})^2] \\ &= E[\Sigma_{r=0}^{\infty} \rho^{2r} \epsilon_{t-r}^2 + 2\Sigma_{i<j} \rho^{i+j} (\epsilon_{t-i} \epsilon_{t-j})] \\ &= \Sigma_{r=0}^{\infty} \rho^{2r} E(\epsilon_{t-r}^2) + 2\Sigma_{i<j} \rho^{i+j} E(\epsilon_{t-i} \epsilon_{t-j}).\end{aligned} \tag{2.13}$$

We know that $E(\epsilon_{t-i} \epsilon_{t-j}) = 0$, since each error term is assumed to be independent for $i \neq j$. Therefore,

$$E(u_t^2) = \Sigma_{r=0}^{\infty} \rho^{2r} E(\epsilon_{t-r}^2) \tag{2.14}$$

and again by assumption, $E(\epsilon_{t-r}^2) = \sigma_\varepsilon^2$, so that

$$E(u_t^2) = \Sigma_{r=0}^{\infty} \rho^{2r} \sigma_\varepsilon^2. \tag{2.15}$$

This is a geometric series which converges to $\sigma_\varepsilon^2/(1 - \rho^2)$ for values of rho such that $|\rho| < 1$. Thus it has been shown that for any t, $\mathrm{Var}(u_t) = \sigma_\varepsilon^2/(1 - \rho^2)$. Therefore, the weight needed to make the first observation homoskedastic is $\sqrt{1 - \rho^2}$. This can be illustrated as follows,

$$\begin{aligned}\mathrm{Var}(\sqrt{1 - \rho^2} u_1) &= (1 - \rho^2)\mathrm{Var}(u_1) \\ &= (1 - \rho^2)[\sigma_\varepsilon^2/(1 - \rho^2)] \\ &= \sigma_\varepsilon^2.\end{aligned} \tag{2.16}$$

This variance is consistent with the variance of the remaining transformed errors.

$$\text{Var}(u_t - \rho u_{t-1}) = \text{Var}(\epsilon_t) = \sigma_\epsilon^2 \tag{2.17}$$

where $u_t = \rho u_{t-1} + \epsilon_t$.

For the Prais-Winston estimator, the appropriate sum of squared errors then becomes,

$$S_{PW} = (1 - \rho^2)(Y_1 - \alpha - \beta X_1)^2 + \Sigma_{t=2}^{T}\left[(Y_t - \alpha - \beta X_t) - \rho(Y_{t-1} - \alpha - \beta X_{t-1})\right]^2. \tag{2.18}$$

The optimizing procedure of the two-stage Prais-Winston estimator parallels that of the two-stage Cochrane-Orcutt estimator. In the first stage, ordinary least squares solutions for alpha and beta are used to generate an estimate of rho. In the second stage, the data is differenced using this estimate of rho. The inclusion of the first observation with weight $\sqrt{1 - \hat{\rho}^2}$, however, makes estimating alpha and beta slightly more complicated than in the Cochrane-Orcutt method. As indicated earlier, the least squares solution for parameter estimates is $b = (X'X)^{-1}X'Y$. Let b, then, be a 2 x 1 vector such that $b = [\hat{\alpha}\ \hat{\beta}]'$, and define $\tilde{X}$ and $\tilde{Y}$ as follows,

$$\tilde{X} = \begin{bmatrix} \sqrt{1-\rho^2} & \sqrt{1-\rho^2}X_1 \\ (1-\rho) & X_2 - \rho X_1 \\ (1-\rho) & X_3 - \rho X_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ (1-\rho) & X_T - \rho X_{T-1} \end{bmatrix}$$

$$\tilde{Y} = \begin{bmatrix} \sqrt{1-\rho^2}Y_1 \\ Y_2 - \rho Y_1 \\ Y_3 - \rho Y2 \\ \cdot \\ \cdot \\ \cdot \\ Y_T - \rho Y_{T-1} \end{bmatrix}. \tag{2.19}$$

Using matrix algebra it can be shown that $b = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{Y}$ yields the correct solutions for $\hat{\alpha}$ and $\hat{\beta}$. These solutions are the estimates used in the Prais-Winston and maximum likelihood procedures.

One of the iterative procedures, Beach and MacKinnon's (1978) maximum likelihood estimation procedure, has already been discussed at length in the previous chapter. However, there are a few points that should be re-emphasized and another that needs to be addressed. The concentrated log-likelihood function mentioned earlier is as follows,

$$£ = \text{const.} + 1/2\ell\eta(1 - \rho^2) \qquad (2.20)$$
$$- T/2\ell\eta\left[(1 - \rho^2)(Y_1 - X_1\beta)^2 + \Sigma_{t=2}^{T}(Y_t - X_t\beta - \rho Y_{t-1} + \rho X_{t-1}\beta)^2\right].$$

Beach and MacKinnon point out that the $(1 - \rho^2)(Y_1 - X_1\beta)^2$ term ensures that the first term will have some impact upon the estimates of beta and rho. The other term, $1/2\ell\eta(1 - \rho^2)$, constrains the estimates of rho to be stationary. As previously indicated, such theoretical properties make this estimator more attractive than most others.

The final point in relation to the maximum likelihood estimator is the theory behind the estimator of rho. The first order condition for the concentrated log-likelihood function results in a third order polynomial. Beach and MacKinnon refer to Uspensky's (1948) general solution for a unique real valued solution to such a problem. The third order polynomial is $f(\rho) = \rho^3 + a\rho^2 + b\rho + c = 0$. The definition for a, b, and c and the solution for rho are given in the previous chapter, equations (1.13) to (1.19).

The iterative Cochrane-Orcutt, iterative Prais-Winston, and iterative initially nonstationary estimators use a Gauss-Newton method of optimizing. This technique provides for a straightforward algorithm that can be easily programmed on a computer.

Let $\theta = [\alpha, \beta, \rho]$ and $S(\theta)$ be the objective function. The objective function is the sum of squared errors. Given a set of starting values, $\theta^o = [\alpha^o, \beta^o, \rho^o]$, $S(\theta)$ can be approximated near θ^o using a Taylor series expansion. This is derived as follows,

$$S(\theta) = S(\theta^o) + (\theta - \theta^o)'\left[\partial S(\theta^o)/\partial\theta\right] \tag{2.21}$$

$$+ \ 1/2(\theta - \theta^o)'\left[\partial^2 S(\theta^o)/\partial\theta\partial\theta'\right](\theta - \theta^o) + o(\|\theta - \theta^o\|^3),$$

where $\|\theta - \theta^o\| = \sqrt{(\alpha - \alpha^o)^2 + (\beta - \beta^o)^2 + (\rho - \rho^o)^2}$ and the limit as θ goes to θ^o of $\{o(\|\theta - \theta^o\|^2\} = 0$.

Minimizing $S(\theta)$ can be accomplished by iteratively minimizing the Taylor series expansion with respect to $(\theta - \theta^o)$. This results in the following first order condition,

$$\partial S(\theta^o)/\partial\theta + \left[\partial^2 S(\theta^o)/\partial\theta\partial\theta'\right](\theta - \theta^o) = 0. \tag{2.22}$$

Solving this expression for θ using matrix algebra yields,

$$\theta^1 = \theta^o - \left[\partial^2 S(\theta^o)/\partial\theta\partial\theta'\right]^{-1}\partial S(\theta^o)/\partial\theta. \tag{2.23}$$

The matrix of second order derivatives, $\partial S(\theta^o)/\partial\theta\partial\theta'$, is simply the Hessian matrix of $[\alpha, \beta, \rho]$,

$$H = \partial^2 S(\theta^o)/\partial\theta\partial\theta', \tag{2.24}$$

and the optimal solutions become

$$\theta^1 = \theta^o - H^{-1}\partial S(\theta^o)/\partial\theta. \tag{2.25}$$

In summary, using the starting values of θ^o, the Taylor series expansion solutions update the parameter estimates in an optimal direction. Such an updating, or iterating, can be continued until the parameter estimates converge, or no further improvement is made in $S(\theta)$.

The Gauss-Newton algorithm has been known to sometimes converge very slowly, if at all, requiring many iterations and may oscillate widely, reversing direction of parameter updates frequently (Draper and Smith, 1966). Therefore, in this research, utilizing the stochastic properties of the second order derivatives, the Gauss-Newton optimization technique was modified to always move in the correct direction and speed up the rate of convergence by modifying the Hessian matrix to make it positive definite so long as the matrix of transformed independent variables has full rank. This is a sufficient condition to guarantee that the direction of change of theta is toward a lower value of $S(\theta)$. This modification is to set specific second order derivatives equal to their expectations of zero. These specific second order derivatives and their expectations are,

$$E(\partial^2 S(\theta)/\partial\alpha\partial\rho) = 0, \tag{2.26}$$

$$E(\partial^2 S(\theta)/\partial\beta\partial\rho) = 0, \tag{2.27}$$

and by using Youngs' theorem, the appropriate transposes in the Hessian matrix are also zero. In general then, the modified Hessian matrix becomes

$$\tilde{H} = \begin{bmatrix} \partial^2 S(\theta)/\partial\alpha^2 & \partial^2 S(\theta)/\partial\alpha\partial\beta & 0 \\ \partial^2 S(\theta)/\partial\alpha\partial\beta & \partial^2 S(\theta)/\partial\beta^2 & 0 \\ 0 & 0 & \partial^2 S(\theta)/\partial\rho^2 \end{bmatrix}. \tag{2.28}$$

The objective function for the iterative Cochrane-Orcutt estimator is the following sum of squared errors,

$$S_{co}(\theta) = \Sigma_{t=2}^{T}\left[(Y_t - \alpha - \beta X_t) - \rho(Y_{t-1} - \alpha - \beta X_{t-1})\right]^2. \tag{2.29}$$

It can be seen that the sum of squared errors for Prais-Winston and the initially nonstationary estimator equal the sum of squared errors for the iterative Cochrane-Orcutt procedure plus the first observation with different weights. Therefore, the sum of squared errors for Prais-Winston and initially nonstationary estimators is as follows,

$$S_{PW} = (1 - \rho^2)(Y_1 - \alpha - \beta X_1)^2 + S_{CO}(\theta) \tag{2.30}$$

$$S_{IN} = (Y_1 - \alpha - \beta X_1)^2 + S_{CO}(\theta). \tag{2.31}$$

This is a useful property and will be utilized in deriving the first and second order derivatives.

The first order derivatives for Cochrane-Orcutt are,

$$\partial S_{CO}/\partial\alpha = -2(1 - \rho)\Sigma_{t=2}^{T}\left[(Y_t - \alpha - \beta X_t) - \rho(Y_{t-1} - \alpha - \beta X_{t-1})\right] \tag{2.32}$$

$$\partial S_{CO}/\partial\beta = -2\Sigma_{t=2}^{T}(X_t - \rho X_{t-1})\left[(Y_t - \alpha - \beta X_t) - \rho(Y_{t-1} - \alpha - \beta X_{t-1})\right]$$

$$\partial S_{CO}/\partial\rho = -2\Sigma_{t=2}^{T}(Y_t - \alpha - \beta X_t)\left[(Y_t - \alpha - \beta X_t) - \rho(Y_{t-1} - \alpha - \beta X_{t-1})\right].$$

The necessary second order derivatives in the revised Hessian matrix for Cochrane-Orcutt are,

$$\begin{aligned} \partial^2 S_{CO}/\partial\alpha^2 &= 2(T - 1)(1 - \rho)^2 \\ \partial^2 S_{CO}/\partial\alpha\partial\beta &= 2(1 - \rho)\Sigma_{t=2}^{T}(X_t - \rho X_{t-1}) \\ \partial^2 S_{CO}/\partial\beta^2 &= 2\Sigma_{t=2}^{T}(X_t - \rho X_{t-1})^2 \\ \partial^2 S_{CO}/\partial\rho^2 &= 2\Sigma_{t=2}^{T}(Y_{t-1} - \alpha - \beta X_{t-1})^2. \end{aligned} \tag{2.33}$$

The first order derivatives for Prais-Winston are,

$$\begin{aligned} \partial S_{PW}/\partial\alpha &= -2(1 - \rho^2)(Y_1 - \alpha - \beta X_1) + \partial S_{CO}/\partial\alpha \\ \partial S_{PW}/\partial\beta &= -2(1 - \rho^2)X_1(Y_1 - \alpha - \beta X_1) + \partial S_{CO}/\partial\beta \\ \partial S_{PW}/\partial\rho &= -2\rho(Y_1 - \alpha - \beta X_1)^2 + \partial S_{CO}/\partial\rho. \end{aligned} \tag{2.34}$$

The necessary second order derivatives for Prais-Winston are,

$$\begin{aligned} \partial^2 S_{PW}/\partial\alpha^2 &= 2(1 - \rho^2) + \partial^2 S_{CO}/\partial\alpha^2 \\ \partial^2 S_{PW}/\partial\alpha\partial\beta &= 2(1 - \rho^2)X_1 + \partial^2 S_{CO}/\partial\alpha\partial\beta \\ \partial^2 S_{PW}/\partial\beta^2 &= 2(1 - \rho^2)X_1^2 + \partial^2 S_{CO}/\partial\beta^2 \\ \partial^2 S_{PW}/\partial\rho^2 &= -2(Y_1 - \alpha - \beta X_1)^2 + \partial^2 S_{CO}/\partial\rho^2. \end{aligned} \tag{2.35}$$

The first order derivatives for the initially nonstationary estimator are,

$$\partial S_{IN}/\partial\alpha = -2(Y_1 - \alpha - \beta X_1) + \partial S_{CO}/\partial\alpha$$
$$\partial S_{IN}/\partial\beta = -2X_1(Y_1 - \alpha - \beta X_1) + \partial S_{CO}/\partial\beta \qquad (2.36)$$
$$\partial S_{IN}/\partial\rho = \partial S_{CO}/\partial\rho.$$

The necessary second order derivatives for the initially nonstationary estimator are,

$$\partial^2 S_{IN}/\partial\alpha^2 = 2 + \partial^2 S_{CO}/\partial\alpha^2$$
$$\partial^2 S_{IN}/\partial\alpha\partial\beta = 2X_1 + \partial^2 S_{CO}/\partial\partial\alpha\partial\beta \qquad (2.37)$$
$$\partial^2 S_{IN}/\partial\beta^2 = 2X_1^2 + \partial^2 S_{CO}/\partial\beta^2$$
$$\partial^2 S_{IN}/\partial\rho^2 = \partial^2 S_{CO}/\partial\rho^2 .$$

It is obvious at this point how the different weights on the first observation affect the sum of squared errors and the subsequent first and second order derivatives.

In regard to the estimation of rho, many alternative estimators have been suggested in the literature. Judge et al. (1985) suggest four different estimators. In this research, a modified sample correlation coefficient was used in the two-stage Cochrane-Orcutt and the two-stage Prais-Winston estimation methods. The attractive property of this estimator is that the estimate of rho is constrained to lie between -1 and 1. This property is appealing since for the two-step Prais-Winston estimator, values for $\hat{\rho}$ exceeding unity in absolute value cause the estimator to be undefined. Rao and Griliches (1969), in their Monte Carlo analysis, commented on this. When they encountered values of $\hat{\rho}$ exceeding unity, they set $\hat{\rho}$ equal to either -1 or 1, as the case would define. The advantage of using the sample correlation coefficient as an estimator of rho is that it will be defined for all estimation techniques and will allow for a consistent estimator across estimation techniques.

The sample correlation coefficient estimator suggested by Judge et al. (1985) is

$$r_1 = \Sigma_{t=2}^{T} \hat{e}_t \hat{e}_{t-1} / \Sigma_{t=1}^{T} \hat{e}_t^2 . \tag{2.38}$$

The estimator for rho used in Rao and Griliches' Cochrane-Orcutt estimation technique is

$$\hat{\rho} = \Sigma_{t=2}^{T} \hat{e}_t \hat{e}_{t-1} / \Sigma_{t=2}^{T} \hat{e}_t^2 . \tag{2.39}$$

The difference between the two estimators is the inclusion of the first error term in the denominator of r_1. The inclusion of this one observation guarantees that the estimate of rho in r_1 is less than unity in absolute value. The inclusion of this one error term, however, is more restrictive than necessary to ensure that $|\hat{\rho}| < 1$. Therefore, using the Cauchy-Schwarz inequality, the sample correlation coefficient was modified slightly, making a less restrictive estimator. This modified estimator is

$$\tilde{\rho} = \Sigma_{t=2}^{T} \hat{u}_t \hat{u}_{t-1} / \left[\Sigma_{t=2}^{T-1} \hat{u}_t^2 + 1/2(\hat{u}_1^2 + \hat{u}_T^2)\right] \tag{2.40}$$

where $\hat{u}_t$ are estimated autocorrelated errors from the residuals. This modification allows the sample correlation coefficient to be as least restrictive as possible. The following is a derivation of this modification.

From the Cauchy-Schwarz inequality we know that

$$0 \leq \Sigma_{t=2}^{T} (u_t + u_{t-1})^2 . \tag{2.41}$$

Expanding this yields

$$\Sigma_{t=2}^{T}(u_t + u_{t-1})^2 = \Sigma_{t=2}^{T} u_t^2 + \Sigma_{t=1}^{T-1} u_t^2 + 2\Sigma_{t=2}^{T} u_t u_{t-1} \tag{2.42}$$
$$= 2\Sigma_{t=2}^{T-1} u_t^2 + u_1^2 + u_T^2 + 2\Sigma_{t=2}^{T} u_t u_{t-1}$$

which implies that

$$-1 \leq \Sigma_{t=2}^{T} u_t u_{t-1} / \left[\Sigma_{t=2}^{T-1} u_t^2 + 1/2(u_1^2 + u_T^2)\right] = \tilde{\rho}. \tag{2.43}$$

Conversely, we also know that

$$0 \leq \Sigma_{t=2}^{T}(u_t - u_{t-1})^2. \tag{2.44}$$

Again expanding this yields

$$\Sigma_{t=2}^{T}(u_t - u_{t-1})^2 = \Sigma_{t=2}^{T} u_t^2 + \Sigma_{t=1}^{T-1} u_t^2 - 2\Sigma_{t=2}^{T} u_t u_{t-1} \tag{2.45}$$
$$= 2\Sigma_{t=2}^{T-1} u_t^2 + u_1^2 + u_T^2 - 2\Sigma_{t=2}^{T} u_t u_{t-1}$$

which implies that

$$1 \geq \Sigma_{t=2}^{T} u_t u_{t-1} / \left[\Sigma_{t=2}^{T-1} u_t^2 + 1/2(u_1^2 + u_T^2)\right] = \tilde{\rho}. \tag{2.46}$$

Therefore, it has been shown that $-1 \leq \tilde{\rho} \leq 1$, and as such it is no more restrictive in this sense than necessary. This is the estimation technique used in the two-step Cochrane-Orcutt and two-step Prais-Winston estimators.

In this study, in the construction of the independent variable X and dependent variable Y, a stochastic element was included. This stochastic element, as defined by the Monte Carlo experiment, is a normally distributed error with zero mean. Therefore, it was necessary to generate a series of errors with these properties. This was accomplished by using the Box and Muller transformation of a uniform distribution suggested by Hogg and Craig (1978).

The Box and Muller transformation is a one-to-one transformation of a random sample from the uniform distribution into a random sample from a normal distribution with zero mean and a variance of one. This requires the generation of a series of uniformly distributed numbers. The multiplicative congruent algorithm suggested by Law and Kelton (1982) was the method employed. This method is given by the following,

$$X(I+1) = (7^5)*X(I)MOD((2^{31}) - 1) \tag{2.47}$$

$$U = X(I+1)/((2^{31}) - 1)$$

where MOD is a Fortran intrinsic function which returns the remainder from subtracting the greatest integer no larger than the ratio of the first argument divided by the second. All that is needed to begin generating numbers from the uniform distribution is an initial value for X, or a seed. The Box and Muller specification is,

$$Z_1 = (-2\ell\eta U_1)^{1/2}\cos(2\pi U_2) \tag{2.48}$$

$$Z_2 = (-2\ell\eta U_1)^{1/2}\sin(2\pi U_2).$$

If U_1 and U_2 are independently identically distributed UNIFORM(0, 1), then Z_1 and Z_2 are independently identically distributed as NORMAL(0, 1).

A test was conducted on the statistical properties of this NORMAL(0, 1) pseudo-random number generator. The first four moments, i.e. the mean, variance, skewness, and kurtosis, were compared to the true values of these moments for a NORMAL(0, 1) distribution. Obviously, the mean and variance should be zero and one, respectively. The true value of the skewness should be zero and the kurtosis should be 3.0 for a standard normal distribution. In a sample size of 100,000, with a seed of 101182, the results for the first four moments are as follows,

sample mean	-0.00025
sample variance	1.00176

sample skewness	-0.00414
sample kurtosis	3.03700.

Therefore, it is clear that such a transformation yields very adequate results for a pseudo-normal random number generator.

This concludes a theoretical examination of each estimation procedure as used in the study as well as an explanation of the random number generator. Chapter 3 will detail the results of the actual Monte Carlo experiments performed.

CHAPTER 3

MONTE CARLO EXPERIMENTS

In Chapter 1, the design of the Monte Carlo experiment used in this research project was outlined. This discussion, however, left several parameters unspecified and a few experimental particulars unexplained. This chapter will detail such parameter specifications and experimental particulars as well as document the experiments conducted and their respective results.

The unspecified parameters are the values of alpha and beta in equation (1.35), the variance of ϵ in equation (1.36), and the values of theta and variance of w in equation (1.37). Choosing values for alpha, beta, and the variance of ϵ are arbitrary. Therefore, the specification of these parameters will be consistent with those used by Beach and MacKinnon (1978) in their Monte Carlo experiments. The values of alpha and beta are both set to 1.0 and the variance of ϵ is set to 0.0036 for all experiments.

Appendix A contains the basic Fortran program used in this research. Each different experimental design changed the input variables and the values and increments of lambda and rho in the main program.

One particular of the program is the convergence criterion of the iterative procedures. For the case of the iterative maximum likelihood estimator, the convergence criterion is to see if the value

$$|\hat{\rho}_i - \hat{\rho}_{i-1}| / \max(\hat{\rho}_i, 0.5) \qquad (3.1)$$

is less than a prescribed tolerance for the i^{th} iteration. For the other iterative techniques, Cochrane-Orcutt, Prais-Winston, and initially nonstationary, two tests for convergence are used. The first test is similar to that of the maximum likelihood estimator in that if the maximum of the following values is less than a prescribed tolerance, then it is no longer necessary to iterate. These values are

$$|\hat{\alpha}_i - \hat{\alpha}_{i-1}| / \max\left[|\hat{\alpha}_i - \hat{\alpha}_{i-1}|, 1.0\right], \qquad (3.2)$$

$$|\hat{\beta}_i - \hat{\beta}_{i-1}| / \max\left[|\hat{\beta}_i - \hat{\beta}_{i-1}|, 1.0\right],$$

$$|\hat{\rho}_i - \hat{\rho}_{i-1}| / \max\left[|\hat{\rho}_i - \hat{\rho}_{i-1}|, 0.5\right].$$

The prescribed tolerance is 0.00001 for all experiments.

The second convergence test is to examine the value of the sum of squared error. If the new estimates of alpha and beta do indeed decrease the value of the sum of squared error, then there is still some benefit to continue iterating; otherwise it is no longer necessary to iterate.

In the development of the program it became clear that an iterative limit was necessary. This iterative limit applies to the iterative Cochrane-Orcutt, iterative Prais-Winston, iterative maximum likelihood, and iterative initially nonstationary estimators. This iterative limit was set to limit the number of iterations at a maximum of 100 in order to avoid the computer problems associated with an infinite loop.

The program was also designed to indicate if this iteration constraint was binding and if it was, how many times in 500 replications of an experiment it was binding. In this respect, the iterative

Cochrane-Orcutt by far performed the worst. For example, when the independent variable, X, was defined by a dampened trend with an intercept value of $\theta = 20$, the iterative Cochrane-Orcutt hit the iterative limit 136 times over all the values of rho and lambda. This is compared to eleven times for the iterative initially nonstationary, seven times for the iterative maximum likelihood, and one time for the iterative Prais-Winston. In one specific experimental design, when $\rho = .97$, $\lambda = .75$, $\theta = 20$, and the sample size was 25, the iterative Cochrane-Orcutt hit the limit 15 times.

In the case of a dampened trend, the Cochrane-Orcutt estimator was observed to have its difficulty converging when $\rho \geq .90$ for all values of lambda and theta, but did show improvement as the sample size increased. The iterative initially nonstationary estimator also had difficulty in converging for values of $\rho \geq .90$, but not to the extent that the iterative Cochrane-Orcutt estimator did. This conclusion was consistent for all values of lambda and theta, but did not improve appreciably as the sample size increased, as in the iterative Cochrane-Orcutt estimator. In regards to iterative maximum likelihood, this estimation procedure generally had little difficulty in converging for all situations, but did demonstrate an increasing occurrence of not converging as theta increased from 10 to 30. Tables 7, 8, 9, and 10, found in Appendix B, contain the average iterations for the iterative Cochrane-Orcutt estimator, iterative Prais-Winston estimator, iterative maximum likelihood estimator, and the initially nonstationary estimator, respectively, for most combinations of rho and lambda.

When the independent variable, X, was defined by a growth trend, all the iterative estimators did relatively well in regards to converging. In fact, the iterative maximum likelihood and the iterative Prais-Winston estimator always converged on their own. Both the iterative Cochrane-Orcutt and iterative initially nonstationary estimators did again have difficulty converging when $\rho \geq .90$. Tables 11 and 12, found in Appendix B, contain the average iterations for each iterative estimator under two specific experimental designs. Table 11 defines the variance of w to be 1.0 and Table 12 specifies the variance of w to be 0.0009.

Another program particular is the generation of the initial autocorrelated error term for each replication of a Monte Carlo experiment. The structure of a first order autocorrelated error is as follows,

$$u_t = \rho u_{t-1} + \epsilon_t \quad \text{for } t = 2,3,\ldots$$

where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. However, when t = 1, there is no value for μ_0. Therefore, in order for the Monte Carlo experiment to be run, the first autocorrelated error term, μ_1, must be defined. This error term has been defined for the Monte Carlo experiment used in this study as follows,

$$u_1 = \epsilon_1 / \sqrt{1 - \rho^2}$$

where $\epsilon_1 \sim N(0, \sigma_\epsilon^2)$ and the value for rho is the true parameter value. The reason for this weight on the first error term is that this makes the variance of the first transformed error term, using the Prais-Winston transformation matrix as discussed in Chapter 2, consistent with the variance of the other transformed errors. This defines the autocorrelated error process to be stationary.

In order for the experiments to be executed for the case of a dampened trend, the parameters needed to be specified are theta and the variance of w. Excluding the impacts of sample size, the relationship between theta and the variance of w determines how quickly the trend line of X stabilizes to random perturbations about the expected value of X. This expectation converges to zero. When theta is large the trend line stabilizes less quickly than when theta is small. The variance of w determines the magnitude of the random perturbations about this trend line. The respective magnitudes of these parameters determine at what point the process switches from the trend dominating the randomness to the randomness dominating the trend. This relationship was modeled by letting theta range from 10 to 20 to 30 while holding the variance of w fixed at 1.0. These values are clearly subjective choices.

This particular experiment generated a large amount of information. Upon examining the results, the average estimate and mean square error, it was evident that the results were consistent across values of theta, those being 10, 20, and 30. Therefore, to reduce the quantity of output reported, only the case where $\theta = 20$ will be reported as a representation of the complete set of data. Furthermore, since this research focuses on the small sample properties, only the results of the sample size of 25 will be reported. A complete listing of the results can be obtained by either contacting Dr. Jeffrey LaFrance, Assistant Professor of Agricultural Economics, Montana State University, or the author. Tables 13, 14, and 15 in Appendix B contain the average estimate and mean square error results for alpha, beta, and rho, respectively. Only the border values of -.99, -.95, -.90, .90, .95, and .99 for rho are reported as representative of the more complete set. The notations in the tables are abbreviations for the iterative methods examined in this study. These methods and their abbreviations are iterative Cochrane-Orcutt, ITCO; iterative Prais-Winston, ITPW; iterative maximum likelihood, ITML; and iterative initially nonstationary, ITIN. These abbreviations and their definitions will be consistent throughout the tables.

The results show that the two-stage Prais-Winston, iterative Prais-Winston, iterative maximum likelihood, and the iterative initially nonstationary estimation techniques are all superior in terms of a uniformly lower mean square error in relationship to the other estimators considered in estimating alpha, beta, and rho. This is generally true for all experimental designs and sample sizes. Of these estimators, the iterative maximum likelihood generally had the lowest mean square error.

The performance of the ordinary least squares estimator is consistent with the ranking by Spitzer, as discussed in Chapter 1. Spitzer concluded that for $|\rho| \leq .20$, ordinary least squares was optimal in the estimation of beta. This study shows that ordinary least squares is very similar in terms of mean square error in estimating alpha, beta, and rho when $|\rho| \leq .40$, and generally optimal for $|\rho| \leq .20$ in comparison to the estimation techniques that take first order autocorrelation into account. This result holds for all values of lambda and theta and is consistent across different sample sizes. The level of improvement, however, of ordinary least squares over the methods that take first order autocorrelation into account is usually very small.

One of the more important results of this study is an indication that the untested iterative initially nonstationary estimation technique is very inefficient in estimating beta when $\rho < -.80$. For example, when $\theta = 20$, $\lambda = 0.0$, $\rho = -.99$, and the sample size is 25, the mean squared error of the initially nonstationary estimator for beta is about 14 times less efficient than that of the maximum likelihood estimator. This, however, is not the case when rho is large and positive. The results indicate that the initially nonstationary estimator is a very poor estimator for beta when $\rho < -.80$, regardless of the values of lambda, theta, or sample size.

Tables 1, 2, and 3 detail the relationship of the mean square errors of the initially nonstationary estimation technique to that of the maximum likelihood estimator by a ratio. Table 1 reports the relative ratios for alpha followed by Tables 2 and 3 for beta and rho, respectively. It is clear from Table 2 that the iterative initially nonstationary estimator is quite inefficient in comparison to the maximum likelihood estimator for all values of lambda when $\rho < -.80$. Otherwise, this estimator for alpha and rho and for beta when $\rho \geq -.80$ for all experimental designs is quite good. Figure 1 plots the relative ratios of the mean square error for beta of the iterative initially nonstationary estimator and the iterative maximum likelihood estimator for the worst case, when $\lambda = 0.0$, $\theta = 20$, $\rho = -.99$, and the sample size is 25. The iterative Prais-Winston is also very comparable to the

Table 1. Relative efficiency of initially nonstationary to maximum likelihood in estimating alpha.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	1.101	1.188	1.186	1.228	1.280	1.393	1.682
-.95	0.996	1.017	1.018	1.025	1.050	1.066	1.118
-.90	1.003	1.003	1.011	1.020	1.024	1.033	1.109
-.80	0.999	0.995	0.998	0.999	1.004	0.998	1.031
-.60	1.001	0.998	0.998	1.004	1.001	1.002	0.999
-.40	0.999	1.000	0.999	0.999	1.000	0.998	1.000
-.20	1.000	1.000	1.001	1.001	0.999	0.998	0.998
0.0	0.999	1.000	1.001	1.000	1.000	1.000	1.002
.20	1.011	1.011	1.006	1.008	1.000	1.001	1.003
.40	1.018	1.019	1.132	1.009	1.023	1.001	1.000
.60	1.916	1.946	1.378	1.133	1.154	1.107	1.004
.80	2.637	2.164	1.966	1.777	1.893	1.483	1.308
.90	1.569	1.862	1.804	1.974	1.827	1.490	1.398
.95	1.417	1.483	1.499	1.515	1.536	1.423	1.336
.99	1.153	1.053	1.095	1.154	1.108	1.075	1.135

* Sample size = 25, theta = 20.
[Mean square error(IN)] / [mean square error(ML)].

Table 2. Relative efficiency of initially nonstationary to maximum likelihood in estimating beta.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	7.339	9.297	10.598	13.829	11.605	11.754	6.196
-.95	2.025	2.819	2.619	3.190	2.805	2.537	2.078
-.90	1.183	1.548	1.733	1.768	1.699	1.663	1.492
-.80	1.020	1.129	1.262	1.213	1.186	1.172	1.190
-.60	1.010	0.944	0.994	1.073	1.010	1.067	1.000
-.40	1.015	0.978	0.990	0.968	0.991	0.991	1.009
-.20	1.016	0.999	1.000	0.978	0.989	1.000	0.991
0.0	1.004	1.001	0.994	0.999	0.993	0.997	1.003
.20	1.001	0.998	0.995	0.992	0.998	1.000	1.004
.40	1.004	0.996	1.009	0.977	0.989	0.999	0.997
.60	1.010	1.042	1.034	1.033	1.003	0.984	0.992
.80	1.029	1.068	1.100	1.108	1.094	1.067	1.060
.90	1.046	1.089	1.074	1.138	1.161	1.085	1.047
.95	1.032	1.095	1.090	1.123	1.101	1.106	1.096
.99	1.017	1.051	1.089	1.060	1.102	1.087	1.060

* Sample size = 25, theta = 20.
[mean squre error(IN)] / [mean square error(ML)].

Table 3. Relative efficiency of initially nonstationary to maximum likelihood in estimating rho.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	0.877	0.984	0.995	1.108	1.132	1.346	1.399
-.95	0.898	0.957	0.960	1.018	1.060	1.071	1.195
-.90	0.851	0.910	0.971	0.980	1.027	1.072	1.113
-.80	0.908	0.948	0.956	0.979	1.030	1.059	1.052
-.60	1.018	1.000	1.006	0.992	1.000	1.015	1.037
-.40	1.067	1.055	1.048	1.021	1.020	1.025	1.007
-.20	1.072	1.083	1.075	1.054	1.046	1.021	1.005
0.0	1.042	1.065	1.072	1.069	1.058	1.048	1.028
.20	1.020	1.043	1.043	1.042	1.042	1.029	1.028
.40	1.013	1.015	1.011	1.011	1.010	1.011	1.016
.60	1.079	1.080	1.034	0.985	0.978	0.968	0.967
.80	1.103	1.043	1.030	0.999	0.969	0.944	0.917
.90	0.954	0,977	0.935	0.947	0.942	0.921	0.883
.95	0.901	0.906	0.921	0.914	0.901	0.903	0.888
.99	0.817	0.843	0.861	0.856	0.873	0.883	0.880

* Sample size = 25, theta = 20.
[mean square error(IN)] / [mean square error(ML)].

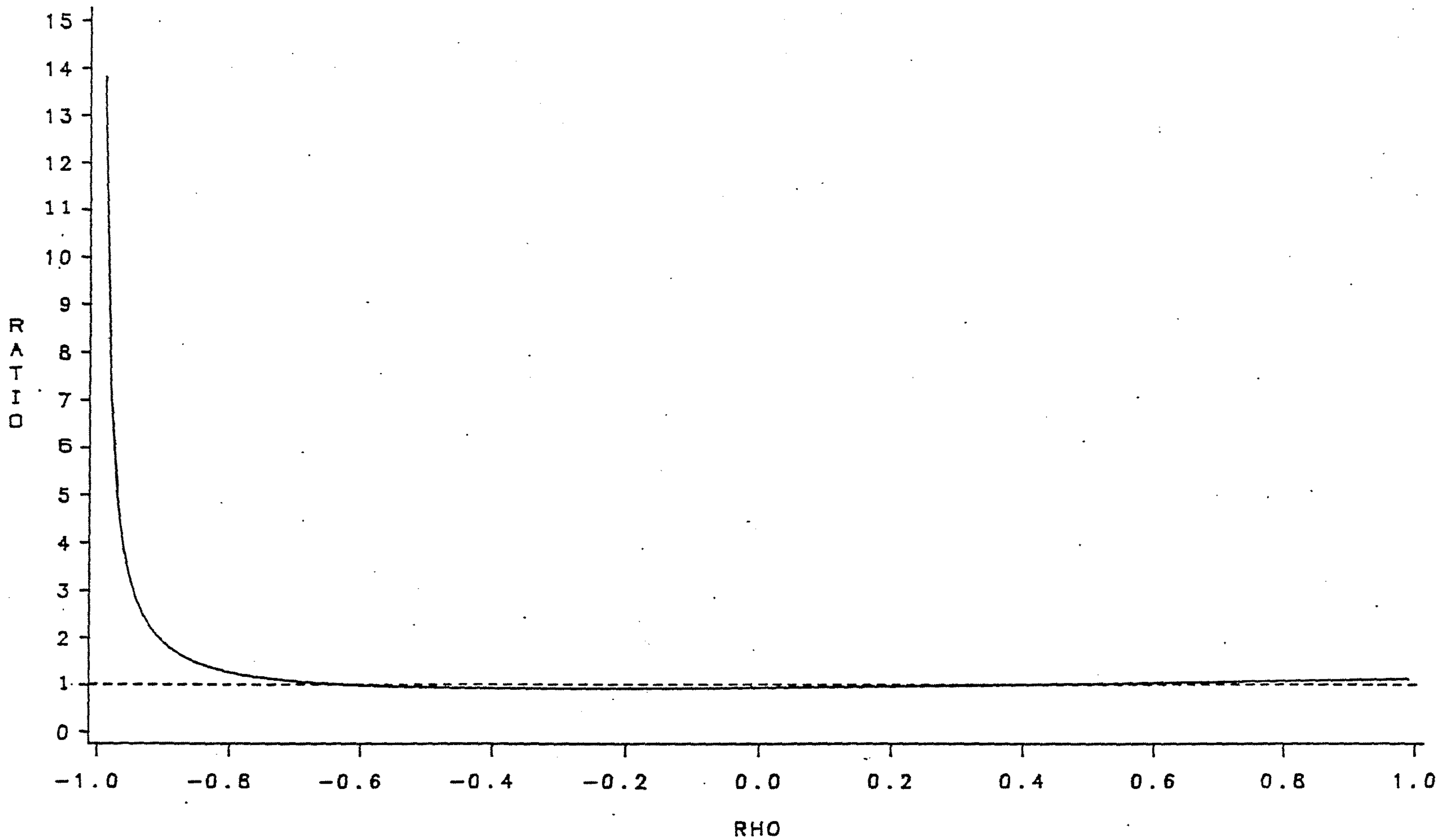

Figure 1. Relative Mean Square Error of Initially Nonstationary to Maximum Likelihood.

iterative maximum likelihood estimator. Tables 4, 5, and 6 depict the relative ratios of the iterative Prais-Winston estimator to that of the iterative maximum likelihood estimator for alpha, beta, and rho, respectively.

Another interesting result is the apparent importance of including the initial observation. The two methods that ignore the first observation are the iterative Cochrane-Orcutt and the two-stage Cochrane-Orcutt. Both of these estimators have consistently high mean square errors in estimating alpha and beta when $\rho \geq .80$ when compared to methods that include the initial observation. In the case of estimating beta, this difference is still very noticeable even when $\rho > .40$, and is slightly noticeable when $\rho > -.40$. This result is true for all values of lambda and sample size.

Considering the differences between iterative procedures and two-stage procedures, the results are interesting. The mean square error values for the estimates of alpha, beta, and rho for the two-stage Cochrane-Orcutt estimator are consistently better than the iterative Cochrane-Orcutt. The two-stage Prais-Winston estimator is, likewise, more efficient than the iterative Prais-Winston estimator, but the difference is small and usually negligible. The iterative Prais-Winston is slightly more efficient in estimating rho when $|\rho| > .80$. These results are consistent for all experimental designs and different sample sizes.

For the case of a growth trend, the only parameter that needs to be specified is the variance of w in equation (1.38). Experiments were conducted using two different values for the variance of w. These

Table 4. Relative efficiency of Prais-Winston to maximum likelihood in estimating alpha.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	1.043	0.997	0.998	0.996	0.998	0.995	1.000
-.95	1.008	1.003	1.004	1.005	0.998	1.000	1.000
-.90	1.002	1.001	1.000	0.999	0.998	0.998	0.995
-.80	1.001	1.004	1.001	1.001	1.000	1.001	1.000
-.60	1.002	1.000	1.001	0.999	1.000	1.000	1.000
-.40	0.999	1.000	1.000	1.001	1.000	1.000	1.000
-.20	1.001	1.000	1.001	1.000	1.000	1.000	1.000
0.0	1.000	1.001	1.001	1.001	1.001	1.001	1.002
.20	1.002	1.001	1.001	1.003	1.000	1.001	1.003
.40	1.005	1.002	1.006	1.004	1.004	1.002	1.000
.60	1.026	1.013	1.012	1.009	1.011	1.015	1.000
.80	1.056	1.021	1.018	1.008	1.015	1.009	1.020
.90	1.006	1.044	1.009	1.011	1.044	1.012	1.013
.95	1.970	1.165	2.200	4.499	1.037	1.018	1.028
.99	1.068	1.063	1.033	1.001	1.129	1.021	1.000

* Sample size = 25, theta = 20.
[mean square error(PW)] / [mean square error(ML)].

Table 5. Relative efficiency of Prais-Winston to maximum likelihood in estimating beta.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	2.273	1.236	1.167	1.132	1.032	1.068	0.997
-.95	1.538	1.153	1.102	1.042	1.055	1.016	1.004
-.90	1.313	1.142	1.070	1.030	1.021	1.008	1.001
-.80	1.102	1.034	1.019	1.016	1.013	1.007	1.001
-.60	1.046	1.035	1.006	1.003	1.007	1.002	1.003
-.40	1.067	1.018	1.007	1.010	1.003	1.004	1.000
-.20	1.008	1.009	1.004	1.007	1.003	1.002	1.002
0.0	1.004	1.003	1.005	1.003	1.003	1.004	1.004
.20	1.003	1.005	1.003	1.004	1.001	1.006	1.007
.40	1.000	1.001	0.999	1.006	1.003	1.001	1.001
.60	1.002	0.996	1.002	0.997	1.004	1.016	1.007
.80	0.999	0.997	0.998	0.995	1.000	1.003	1.019
.90	0.991	1.001	0.992	1.002	1.013	0.988	1.007
.95	0.992	0.988	1.057	1.128	0.973	0.956	1.025
.99	0.990	0.991	1.002	0.994	1.068	0.998	0.960

* Sample size = 25, theta = 20.
[mean square error(PW)] / [mean square error(ML)].

Table 6. Relative efficiency of Prais-Winston to maximum likelihood in estimating rho.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	1.003	1.012	1.021	1.080	0.996	1.250	0.971
-.95	0.988	0.948	0.960	0.940	0.964	0.960	0.991
-.90	0.926	0.941	0.976	0.973	0.989	1.002	0.999
-.80	0.961	1.028	0.998	1.015	1.033	1.032	1.056
-.60	1.037	1.067	1.047	1.076	1.083	1.093	1.116
-.40	1.078	1.085	1.095	1.113	1.100	1.123	1.120
-.20	1.089	1.107	1.101	1.106	1.104	1.111	1.118
0.0	1.092	1.090	1.091	1.090	1.091	1.092	1.091
.20	1.080	1.072	1.066	1.068	1.057	1.064	1.060
.40	1.032	1.027	1.020	1.028	1.025	1.019	1.009
.60	0.983	0.985	0.980	0.974	0.969	0.976	0.959
.80	0.899	0.899	0.901	0.900	0.903	0.908	0.910
.90	0.843	0.860	0.850	0.851	0.874	0.875	0.869
.95	0.825	0.836	0.847	0.853	0.844	0.849	0.871
.99	0.818	0.823	0.812	0.828	0.832	0.851	0.862

* Sample size = 25, theta = 20.
[mean square error(PW)] / [mean square error(ML)].

values are 1.0, consistent with the previous dampened trend experiments, and a value of 0.0009, consistent with the Monte Carlo experiments of Beach and MacKinnon. This was done so that the results of this study can be compared directly with those of Beach and MacKinnon. In these two experiments only sample sizes of 25 and 50 were considered.

The results of the different estimators for the case of a growth trend are similar to those for the dampened trend. The two-stage Prais-Winston, iterative Prais-Winston, iterative maximum likelihood, and the iterative initially nonstationary estimators all appear to be very good estimation techniques in terms of lower mean square errors when comparing across experimental designs. Tables 16 and 17 in Appendix B contain the average estimates and mean square error values for alpha, beta, and rho for a sample size of 25. Table 16 reports the results for the experiment when the variance of w is 1.0, and Table 17 reports the results for a variance of 0.0009.

Ordinary least squares again performs consistent with the ranking by Spitzer. Ordinary least squares appears to be similar for $|\rho| \leq .20$ and is generally optimal for $|\rho| \leq .10$ in terms of a uniformly lower mean square error in comparison to the other estimators in estimating alpha, beta, and rho. This is true for the two different variances of w considered, 1.0 and 0.0009, and the two different sample sizes, 25 and 50.

The iterative initially nonstationary estimation technique again is inefficient in estimating beta for $\rho < -.80$ in comparison to the other estimation methods that take first order autocorrelation into account. For example, when the sample size is 25, $\rho = -.99$, and the variance of w = 1.0, the mean square error value in estimating beta for the iterative initially nonstationary technique is about twice the mean square error value for the iterative maximum likelihood estimator.

The iterative initially nonstationary technique is also very inefficient in estimating alpha. For $|\rho| \geq .80$, the iterative initially nonstationary estimator has significantly higher mean square error values than the other estimators. This result is consistent across sample size.

There is little difference in terms of mean square error between the two-stage Cochrane-Orcutt estimator and the iterative Cochrane-Orcutt estimator in the estimation of alpha and beta. The two-stage Cochrane-Orcutt estimator, however, usually has a lower mean square error than that of the iterative Cochrane-Orcutt estimator in the estimation of rho. There is also very little difference between the mean square errors of the two-stage Prais-Winston and the iterative Prais-Winston estimators in the estimation of all parameters, alpha, beta, and rho.

When the independent variable, X, is defined by a growth trend, the importance of retaining the initial observation is still high in the estimation of alpha, beta, and rho. Those methods that include the initial observation usually have a lower mean square error value than the two Cochrane-Orcutt methods. The difference is, however, small. This reflects the fact that for a growth trend, where the initial observation is not that much different from the next observation, the initial observation does not contain as much information as in the case of the dampened trend. Even in the change in sample size from 25 to 50, this importance of retaining the first observation is still noticeable.

Beach and MacKinnon (1978) analyzed the relationship between their maximum likelihood estimator and the two-stage Cochrane-Orcutt estimator using just three different experimental designs and a growth trend in the independent variable. These experiments were defined by using three different values of rho, .60, .80, and .99. It is clear from the results of this research that the iterative initially nonstationary estimator would appear to be very efficient in comparison to the iterative maximum likelihood estimator if only these values of rho were considered. This is especially true in estimating beta with $\rho < -.80$.

This concludes the explanation of the Monte Carlo experiments actually done in this research, along with the general results of the experiments. The following chapter will summarize these results into conclusions and make suggestions as to possible future research.

CHAPTER 4

CONCLUSIONS

There are four main objectives for this research. The first is to rank the various estimators being considered and determine how this ranking changes with different experimental designs. The second is to document the importance of retaining the first observation, noting how the relative importance of the first observation is affected by a trend component in the independent variable. The third objective is to assess the effects of different sample sizes on the ranking of the estimators and on the importance of retaining the initial observation. The last objective, and the primary focus of this study, is ascertaining the relative costs of using the iterative initially nonstationary estimator in comparison to the other more commonly used techniques. Each of these topics was directly or indirectly discussed in Chapter 3. In this chapter, concluding statements about each of these objectives will be made, along with suggestions for further research that this study prompts.

In regards to the ranking of the various estimators, four estimators are uniformly superior in estimating alpha, beta, and rho. These estimators are the two-stage Prais-Winston estimator, iterative Prais-Winston estimator, iterative maximum likelihood estimator, and the iterative initially nonstationary estimator. It is true, however, that the relative efficiency of the iterative initially nonstationary estimator quickly decreases in estimating beta for $\rho < -.80$. The consequences of this inefficiency are relatively small because of the low probability of a large negative autocorrelation occurring in practice. Of these four estimators, the iterative Prais-Winston and the iterative maximum likelihood are generally the better estimation techniques for estimating alpha, beta, and rho in terms of a universally low mean square error. The difference between these four estimators is, however, usually very small. In general, the ordinary least squares estimator for the two designs for the independent variable is optimal for $|\rho| \leq .20$ in estimating alpha, beta, and rho.

One surprising result of this study is the importance of including the first observation. The results indicate that the magnitude of the trend has little impact on the importance of the first observation for the case of a dampened trend. The magnitude of the autocorrelation, however, affected the importance of the initial observation. When the independent variable is generated by a dampened trend, those methods that ignore the first observation have a consistently higher mean square error for $\rho \geq .80$ in estimating alpha and beta. When the independent variable is generated by a growth trend, these same estimators again have higher mean square error, but this holds for all values of rho. In this case, the difference between the mean square error of the techniques that include the initial observation and those that do not is usually small.

The effects of the sample size on both the ranking and the importance of retaining the initial observation are consistent with the results of Maeshiro's (1979) research. Changing the sample size from 25 to 50 to 100 for the case of a dampened trend does not affect the ranking. The four estimators, two-stage Prais-Winston, iterative Prais-Winston, iterative maximum likelihood, and the initially nonstationary estimator, still are uniformly the best estimators for all experimental designs. This is the same conclusion reached for the different sample sizes used in a growth trend.

As indicated earlier, the iterative initially nonstationary estimator ranks among the better estimators examined in this research in estimating alpha, beta, and rho. It does lose efficiency in estimating beta for $\rho \leq -.80$ for a dampened and growth trend. It also loses relative efficiency in estimating alpha when $|\rho| \geq .80$ for just a growth trend. This method is, however, very convenient in that its first and second order derivatives are very straightforward, as Chapter 2 demonstrates; the transformation matrix is simple, and this estimator can be easily extended to higher orders of autocorrelation.

The costs in terms of losses in efficiency of parameter estimates for using the iterative initially nonstationary estimator in estimating beta is a maximum of 26% loss for $\rho > -.80$ and a maximum of 15% for $\rho > -.60$. This is noticeable in Table 2 (Chapter 3), where the mean square error of the iterative initially nonstationary estimator for beta is compared to the mean square error for the iterative maximum likelihood estimator by a ratio. In the case of a growth trend, the iterative initially nonstationary estimator is also inefficient in estimating alpha. For $|\rho| \geq .40$ the iterative initially nonstationary estimator is at most 15% less efficient than the iterative maximum likelihood estimator in terms of mean square error. When $|\rho| \geq .60$, the level of inefficiency increases to a maximum of 50%.

Therefore, it is clear that the convenience of the iterative initially nonstationary estimator has associated with it a substantial cost in terms of losses in efficiency under certain circumstances. The iterative initially nonstationary estimator, however, is at a disadvantage to the other estimators that account for autocorrelation. The Cochrane-Orcutt, Prais-Winston, and maximum likelihood techniques are all based upon the assumption of stationarity. The iterative initially nonstationary estimator is not. The design of the Monte Carlo experiments of this research assume the process generating the autocorrelated errors is stationary. Therefore, the Cochrane-Orcutt, Prais-Winston, and maximum likelihood techniques correctly transform the autocorrelated errors.

When the performance of the iterative initially nonstationary estimator is judged in light of this severe disadvantage, its consistent perfomance in remarkable. Based upon this, its performance, and the attractiveness of its assumptions and its convenience, the iterative initially nonstationary estimator is a very powerful estimation technique.

This study on the analysis of the small sample properties of several estimators in the context of first order autocorrelation suggests further research. The interesting extensions of this research are two-fold. The first extension is to examine the small sample properties of the initially nonstationary estimator in comparison to the other estimators when in fact the process generating the autocorrelated errors in the Monte Carlo experiment allows the first autocorrelated term to be nonstationary.

In Chapter 3 it was explained why the first autocorrelated error term, u_1, was defined to be

$$u_1 = \varepsilon_1 / \sqrt{1 - \rho^2}$$

in the Monte Carlo experiment [see equation (3.3)]. In this case it would be interesting to note how the initially nonstationary estimator would perform relative to the other estimation techniques when the first autocorrelated error was defined as $u_1 = \varepsilon_1$ [see equation (1.9)]. This is equivalent to assuming that the random process generating the autocorrelated errors begins at some point in time. This is exactly the assumption behind the initially nonstationary estimator considered in this research.

The second extension of this research would be to examine the small sample properties of the same estimators considered here, but to do this in the context of second order autocorrelation. The assumption used to derive the transformation matrix for the initially nonstationary estimator is easily extended to second order autocorrelation.

In conclusion, this research has shown that the iterative initially nonstationary estimator is very similar in its estimation qualities when compared to the more commonly used techniques and as such can be used by applied econometricians with a high degree of confidence.

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APPENDICES

APPENDIX A:

FORTRAN PROGRAM

Figure 2. Monte Carlo Fortran Program.

```
C
C  VARIABLE DECLARATIONS
C
      INTEGER            BIGFL1, BIGFL2, BIGFL3, BIGFL4
      INTEGER            BIGCT1, BIGCT2, BIGCT3, BIGCT4
      INTEGER*4          ISEED
      REAL               LAMBDA
      DIMENSION          X(100), Y(100), W(100), E(100)
      DOUBLE PRECISION   ACO, BCO, RCO, APW, BPW, RPW, AML, BML, RML
      DOUBLE PRECISION   AOLS, BOLS, ROLS, AICO, BICO, RICO
      DOUBLE PRECISION   AIPW, BIPW, RIPW, ANL, BNL, RNL
      DOUBLE PRECISION   ABCO, BBCO, RBCO, ABPW, BBPW, RBPW
      DOUBLE PRECISION   ABML, BBML, RBML, ABOLS, BBOLS, RBOLS
      DOUBLE PRECISION   ABICO, BBICO, RBICO, ABIPW, BBIPW, RBIPW
      DOUBLE PRECISION   ABNL, BBNL, RBNL
      DOUBLE PRECISION   AMSECO, BMSECO, RMSECO, AMSEPW, BMSEPW, RMSEPW
      DOUBLE PRECISION   AMSEML, BMSEML, RMSEML, AMSENL, BMSENL, RMSENL
      DOUBLE PRECISION   AMSEOLS, BMSEOLS, RMSEOLS
      DOUBLE PRECISION   AMSEICO, BMSEICO, RMSEICO
      DOUBLE PRECISION   AMSEIPW, BMSEIPW, RMSEIPW
      COMMON/GEN1/       LAMBDA, THETA, WVAR, EVAR
      COMMON/GEN2/       W, E
      COMMON/ITT1/       ALPHA, BETA, RHO, EPS
      COMMON/ITT2/       X, Y
C
C  PROGRAM CODE
C
      OPEN(5, FILE = 'MONTE.CTL', STATUS = 'OLD')
      OPEN(6, FILE = 'MONTE.OUT', STATUS = 'NEW')
      TOTML = 0.0
      TOTNL = 0.0
      TOTPW = 0.0
      TOTCO = 0.0
      EPS = 0.00001
      ISEED = 793716
C
C  get parameter values from file 5, MONTE.CTL
C
      READ (5, 10) ALPHA, BETA, EVAR, WVAR, NR, IT
10    FORMAT(4F10.5,2I5)
C
C iterate for all combinations of THETA, RHO and LAMBDA
C
      EVAR = SQRT(EVAR)
      WVAR = SQRT(WVAR)
      DO 1600, THETA = 10.0,30.0,10.0
      WRITE (6, 20) ALPHA, BETA, THETA, EPS, EVAR*EVAR, WVAR*WVAR,
     & NR, IT
```

```
20     FORMAT('1',1X,'Original parameter values:',8x,'Alpha',12X,F10.5,
     & /36X,'Beta',13X,F10.5,/36X,'Theta',12X,F10.5,/36X,'Epsilon',
     & 10X,F10.5,/36X,'Variance of E',4X,F10.5,/36X,'Variance of W',
     & 4X,F10.5,/36X,'Sample size',13X,I3,/36X,'Number of replications',
     & 2X,I3,/)
C       LAMBDA = .04
       DO 1500, LAMBDA = -0.75,0.75,0.25
        DO 1400, IRUN = 1,9,1
            IF ( IRUN .EQ. 1 ) THEN
                 RHO = -0.80
            ELSE
                 RHO = RHO + 0.20
            ENDIF
C
C initializing
C
       BIGFL1 = 0
       BIGFL2 = 0
       BIGFL3 = 0
       BIGFL4 = 0
       BIGCT1 = 0
       BIGCT2 = 0
       BIGCT3 = 0
       BIGCT4 = 0
       CTCO = 1.0D0
       CTPW = 1.0D0
       CTNL = 1.0D0
       CTML = 0.0D0
       ABCO = 0.0D0
       BBCO = 0.0D0
       RBCO = 0.0D0
       AMSECO = 0.0D0
       BMSECO = 0.0D0
       RMSECO = 0.0D0
       ABPW = 0.0D0
       BBPW = 0.0D0
       RBPW = 0.0D0
       AMSEPW = 0.0D0
       BMSEPW = 0.0D0
       RMSEPW = 0.0D0
       ABML = 0.0D0
       BBML = 0.0D0
       RBML = 0.0D0
       AMSEML = 0.0D0
       BMSEML = 0.0D0
       RMSEML = 0.0D0
       ACO = 0.0D0
       BCO = 0.0D0
       RCO = 0.0D0
       APW = 0.0D0
       BPW = 0.0D0
```

```
      RPW = 0.0D0
      AML = 0.0D0
      BML = 0.0D0
      RML = 0.0D0
      ABICO = 0.0D0
      BBICO = 0.0D0
      RBICO = 0.0D0
      ABIPW = 0.0D0
      BBIPW = 0.0D0
      RBIPW = 0.0D0
      ABNL = 0.0D0
      BBNL = 0.0D0
      RBNL = 0.0D0
      AMSEICO = 0.0D0
      BMSEICO = 0.0D0
      RMSEICO = 0.0D0
      AMSEIPW = 0.0D0
      BMSEIPW = 0.0D0
      RMSEIPW = 0.0D0
      AMSENL = 0.0D0
      BMSENL = 0.0D0
      RMSENL = 0.0D0
      AICO = 0.0D0
      BICO = 0.0D0
      RICO = 0.0D0
      AIPW = 0.0D0
      BIPW = 0.0D0
      RIPW = 0.0D0
      ANL = 0.0D0
      BNL = 0.0D0
      RNL = 0.0D0
      AOLS = 0.0D0
      BOLS = 0.0D0
      ROLS = 0.0D0
      ABOLS = 0.0D0
      BBOLS = 0.0D0
      RBOLS = 0.0D0
      AMSEOLS = 0.0D0
      BMSEOLS = 0.0D0
      RMSEOLS = 0.0D0
C
C run IT batches of observations
C
      DO 300, J = 1,IT
      CALL NORMAL (NR, W, ISEED)
      CALL NORMAL (NR, E, ISEED)
C
C calculate arrays of u, x, and y with parameters
C
```

```
      CALL GENXY (A, B, AOLS, BOLS, ABOLS, AMSEOLS, BBOLS, BMSEOLS, NR)
C
C estimate parameters ALPHA, BETA, RHO and add their bias
C
      CALL CO (ABCO, BBCO, RBCO, AMSECO, BMSECO, RMSECO,
     & ACO, BCO, RCO, ROLS, RBOLS, RMSEOLS, A, B, R, NR)
      CALL PW (ABPW, BBPW, RBPW, AMSEPW, BMSEPW, RMSEPW,
     & APW, BPW, RPW, A, B, R, NR)
      CALL MLIT (ABML, BBML, RBML, AMSEML, BMSEML, RMSEML,
     & CTML, AML, BML, RML, A, B, BIGFL4, BIGCT4, NR)
      CALL COPWNL( ABICO, AMSEICO, BBICO, BMSEICO, RBICO, RMSEICO,
     & ABIPW, AMSEIPW, BBIPW, BMSEIPW, RBIPW, RMSEIPW, ABNL, AMSENL,
     & BBNL, BMSENL, RBNL, RMSENL, AICO, BICO, RICO, AIPW, BIPW, RIPW,
     & ANL, BNL, RNL, CTCO, CTPW, CTNL, A, B, R, BIGFL1, BIGFL2,
     & BIGFL3, BIGCT1, BIGCT2, BIGCT3, NR)
300   CONTINUE
C
C update statistics from each estimation for parameters
C
      ABCO = ABCO / IT
      BBCO = BBCO / IT
      RBCO = RBCO / IT
      AMSECO = AMSECO / IT
      BMSECO = BMSECO / IT
      RMSECO = RMSECO / IT
      ACO = ACO / IT
      BCO = BCO / IT
      RCO = RCO / IT
      ABPW = ABPW / IT
      BBPW = BBPW / IT
      RBPW = RBPW / IT
      AMSEPW = AMSEPW / IT
      BMSEPW = BMSEPW / IT
      RMSEPW = RMSEPW / IT
      APW = APW / IT
      BPW = BPW / IT
      RPW = RPW / IT
      ABML = ABML / IT
      BBML = BBML / IT
      RBML = RBML / IT
      AMSEML = AMSEML / IT
      BMSEML = BMSEML / IT
      RMSEML = RMSEML / IT
      AML = AML / IT
      BML = BML / IT
      RML = RML / IT
      ABICO = ABICO / IT
      BBICO = BBICO / IT
      RBICO = RBICO / IT
      ABIPW = ABIPW / IT
```

```
      BBIPW = BBIPW / IT
      RBIPW = RBIPW / IT
      ABNL = ABNL / IT
      BBNL = BBNL / IT
      RBNL = RBNL / IT
      AMSEICO = AMSEICO / IT
      BMSEICO = BMSEICO / IT
      RMSEICO = RMSEICO / IT
      AMSEIPW = AMSEIPW / IT
      BMSEIPW = BMSEIPW / IT
      RMSEIPW = RMSEIPW / IT
      AMSENL = AMSENL / IT
      BMSENL = BMSENL / IT
      RMSENL = RMSENL / IT
      AICO = AICO / IT
      BICO = BICO / IT
      RICO = RICO / IT
      AIPW = AIPW / IT
      BIPW = BIPW / IT
      RIPW = RIPW / IT
      ANL = ANL / IT
      BNL = BNL / IT
      RNL = RNL / IT
      AOLS = AOLS / IT
      BOLS = BOLS / IT
      ROLS = ROLS / IT
      ABOLS = ABOLS / IT
      BBOLS = BBOLS / IT
      RBOLS = RBOLS / IT
      AMSEOLS = AMSEOLS / IT
      BMSEOLS = BMSEOLS / IT
      RMSEOLS = RMSEOLS / IT
      CTML = CTML / IT
      CTCO = CTCO / IT
      CTPW = CTPW / IT
      CTNL = CTNL / IT
      TOTML = TOTML + CTML
      TOTNL = TOTNL + CTNL
      TOTPW = TOTPW + CTPW
      TOTCO = TOTCO + CTCO
C
C outputting the results
C
      WRITE(6,500)
      WRITE(6,600)
      WRITE(6,400) RHO, LAMBDA
      WRITE(6,401)
      WRITE(6,700) AOLS, AMSEOLS, ABOLS
      WRITE(6,800) ACO, AMSECO, ABCO
      WRITE(6,900) APW, AMSEPW, ABPW
      WRITE(6,801) AICO, AMSEICO, ABICO
```

```
      WRITE(6,901) AIPW, AMSEIPW, ABIPW
      WRITE(6,1000) AML, AMSEML, ABML
      WRITE(6,1140) ANL, AMSENL, ABNL
      WRITE(6,403)
      WRITE(6,700) BOLS, BMSEOLS, BBOLS
      WRITE(6,800) BCO, BMSECO, BBCO
      WRITE(6,900) BPW, BMSEPW, BBPW
      WRITE(6,801) BICO, BMSEICO, BBICO
      WRITE(6,901) BIPW, BMSEIPW, BBIPW
      WRITE(6,1000) BML, BMSEML, BBML
      WRITE(6,1140) BNL, BMSENL, BBNL
      WRITE(6,405)
      WRITE(6,700) ROLS, RMSEOLS, RBOLS
      WRITE(6,800) RCO, RMSECO, RBCO
      WRITE(6,900) RPW, RMSEPW, RBPW
      WRITE(6,801) RICO, RMSEICO, RBICO
      WRITE(6,901) RIPW, RMSEIPW, RBIPW
      WRITE(6,1000) RML, RMSEML, RBML
      WRITE(6,1140) RNL, RMSENL, RBNL
      WRITE(6,1300) CTML
      WRITE(6,1350) CTCO, CTPW, CTNL
      IF( BIGFL1 .EQ. 1 ) THEN
            WRITE(6,1375) BIGCT1
            ENDIF
      IF( BIGFL2 .EQ. 1 ) THEN
            WRITE(6,1380) BIGCT2
            ENDIF
      IF( BIGFL3 .EQ. 1 ) THEN
            WRITE(6,1385) BIGCT3
            ENDIF
      IF( BIGFL4 .EQ. 1 ) THEN
            WRITE(6,1390) BIGCT4
            ENDIF
1400  CONTINUE
1500  CONTINUE
      TOTML = TOTML / 77.0D0
      TOTNL = TOTNL / 77.0D0
      TOTPW = TOTPW / 77.0D0
      TOTCO = TOTCO / 77.0D0
      WRITE(6,1550) TOTML, TOTNL, TOTPW, TOTCO
1600  CONTINUE
400   FORMAT( 5X,'For rho = ',F5.2,1X,'and lambda = ',F5.2)
401   FORMAT(/,' ALPHA')
403   FORMAT(/,1X,'BETA')
405   FORMAT(/, ' RHO')
500   FORMAT('1',/,'                         estimated value',
     & '    squared error','          bias')
600   FORMAT( '_______________________________________________',
     &        '______________________________',/)
700   FORMAT( ' OLS                  ',E13.7,5X,E13.7,5X,E13.7)
800   FORMAT( ' Cochrane-Orcutt      ',E13.7,5X,E13.7,5X,E13.7)
```

```
801   FORMAT( ' Iterative C-O     ',E13.7,5X,E13.7,5X,E13.7)
900   FORMAT( ' Prais-Winston     ',E13.7,5X,E13.7,5X,E13.7)
901   FORMAT( ' Iterative P-W     ',E13.7,5X,E13.7,5X,E13.7)
1000  FORMAT( ' Iterative M-L     ',E13.7,5X,E13.7,5X,E13.7)
1140  FORMAT( ' Iterative N-L     ',E13.7,5X,E13.7,5X,E13.7)
1300  FORMAT(/,' AVERAGE ITERATIONS   Maximum Likelihood = ',F5.2)
1350  FORMAT( ' AVERAGE ITERATIONS FOR TAYLOR SERIES:',
     & ' CO = ',F7.2,' PW = ',F5.2,' NL = ',F5.2,/)
1375  FORMAT( ' CO HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
1380  FORMAT( ' PW HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
1385  FORMAT( ' NL HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
1390  FORMAT( ' ML HIT THE ITERATIVE LIMIT (100)',I3,' TIMES.')
1550  FORMAT(/,' Total average iterations: ML = ',F5.2,2X,' NL = ',
     & F5.2,2X,' PW = ',F5.2,2X,' CO = ',F5.2)
      CLOSE(5)
      CLOSE(6)
      STOP
      END
C
      SUBROUTINE GENXY(A, B, AOLS, BOLS, ABOLS, AMSEOLS, BBOLS,
     & BMSEOLS, NR)
C
C VARIABLE DECLARATIONS
C
      REAL                LAMBDA
      DIMENSION           X(100), Y(100), U(100), W(100), E(100)
      DOUBLE PRECISION    XBAR, YBAR, XVAR, XYDEV
      DOUBLE PRECISION    AOLS, BOLS, ABOLS, AMSEOLS, BBOLS, BMSEOLS
      COMMON/GEN1/        LAMBDA, THETA, WVAR, EVAR
      COMMON/GEN2/        W, E
      COMMON/ITT1/        ALPHA, BETA, RHO, EPS
      COMMON/ITT2/        X, Y
C
C SUBROUTINE CODE
C
      X(1) = EXP(LAMBDA) + W(1)*WVAR
      U(1) = (E(1) * EVAR) / SQRT(1.0 - RHO*RHO)
      Y(1) = ALPHA + BETA*X(1) + U(1)
      XBAR = X(1)
      YBAR = Y(1)
      XVAR = X(1)*X(1)
      XYDEV = X(1)*Y(1)
      DO 20 I=2,NR
          X(I) = EXP(LAMBDA*I) + W(I)*WVAR
          U(I) = RHO*U(I-1) + E(I)*EVAR
          Y(I) = ALPHA + BETA*X(I) + U(I)
          XBAR = XBAR + X(I)
          YBAR = YBAR + Y(I)
          XVAR = XVAR + X(I)*X(I)
          XYDEV = XYDEV + X(I)*Y(I)
20    CONTINUE
```

```
      XBAR = XBAR / NR
      YBAR = YBAR / NR
      XVAR =  ( XVAR / NR) - XBAR*XBAR
      XYDEV = ( XYDEV / NR) - XBAR*YBAR
C caculating alpha and beta
      B = XYDEV/ XVAR
      A = YBAR - B*XBAR
      AOLS = AOLS + A
      BOLS = BOLS + B
      ABOLS = ABOLS + (A - ALPHA)
      BBOLS = BBOLS + (B - BETA)
      AMSEOLS = AMSEOLS + (A - ALPHA)**2
      BMSEOLS = BMSEOLS + (B - BETA)**2
      RETURN
      END
C
C end of GENXY subroutine
C
      SUBROUTINE CO (ABCO, BBCO, RBCO, AMSECO, BMSECO, RMSECO,
     & ACO, BCO, RCO, ROLS, RBOLS, RMSEOLS, A, B, R, NR)
C
C  VARIABLE DECLARATIONS
C
      INTEGER            FSTPS2
      DIMENSION          X(100), Y(100), UHAT(100)
      DOUBLE PRECISION   ROLS, RBOLS, RMSEOLS
      DOUBLE PRECISION   ABCO, BBCO, RBCO, AMSECO, BMSECO, RMSECO
      DOUBLE PRECISION   ACO, BCO, RCO
      DOUBLE PRECISION   UHATSQ, UHATCO
      DOUBLE PRECISION   XBARST, YBARST, XDIFSQ, XYDEV
      COMMON/ITT1/       ALPHA, BETA, RHO, EPS
      COMMON/ITT2/       X, Y
C
C SUBROUTINE CODE
C
      ALPHAH = A
      BETAH  = B
      FSTPS2 = 0
C
C make array of uhat and
C find sum of squares and sum of consecutive products for uhat
C
20    UHATSQ = 0.0D0
      UHATCO = 0.0D0
      UHAT(1) = Y(1) - ALPHAH - BETAH*X(1)
      DO 40, I = 2,NR
         UHAT(I) = Y(I) - ALPHAH - BETAH*X(I)
         IF ( I .LT. NR ) THEN
              UHATSQ  = UHATSQ + UHAT(I)*UHAT(I)
         ENDIF
         UHATCO  = UHATCO + (UHAT(I)*UHAT(I - 1))
```

```
40    CONTINUE

      UHATSQ = UHATSQ + 0.5*(UHAT(1)*UHAT(1) + UHAT(NR)*UHAT(NR))
      RHOH = UHATCO / UHATSQ
C
C calculate alpha and beta
C
      XBARST = 0.0D0
      YBARST = 0.0D0
      XDIFSQ = 0.0D0
      XYDEV = 0.0D0
      DO 50, I = 2,NR
           XDIF = X(I) - RHOH*X(I-1)
           YDIF = Y(I) - RHOH*Y(I-1)
           XDIFSQ = XDIFSQ + XDIF*XDIF
           XYDEV = XYDEV + XDIF*YDIF
           XBARST = XBARST + XDIF
           YBARST = YBARST + YDIF
50    CONTINUE
      XBARST = XBARST / (NR - 1)
      YBARST = YBARST / (NR - 1)
      XDIFSQ = (XDIFSQ / (NR - 1)) - XBARST*XBARST
      XYDEV = (XYDEV / (NR - 1)) - XBARST*YBARST
      BETAH = XYDEV / XDIFSQ
      ALPHAH = (YBARST - BETAH*XBARST) / (1 - RHOH)
      IF ( FSTPS2 .EQ. 0 ) THEN
           R = RHOH
           ROLS = ROLS + RHOH
           RBOLS = RBOLS + (RHOH - RHO)
           RMSEOLS = RMSEOLS + (RHOH - RHO)**2
           FSTPS2 = 1
           GO TO 20
      ENDIF
C
C add to sum the bias and the variance for ALPHA, BETA, and RHO
C
      ABCO = ABCO + (ALPHAH - ALPHA)
      BBCO = BBCO + (BETAH  - BETA)
      RBCO = RBCO + (RHOH   - RHO)
      AMSECO = AMSECO + (ALPHAH - ALPHA) ** 2
      BMSECO = BMSECO + (BETAH  - BETA ) ** 2
      RMSECO = RMSECO + (RHOH   - RHO  ) ** 2
      ACO = ACO + ALPHAH
      BCO = BCO + BETAH
      RCO = RCO + RHOH
      RETURN
      END
C
C END OF CO SUBROUTINE
C
```

```
      SUBROUTINE PW (ABPW, BBPW, RBPW, AMSEPW, BMSEPW, RMSEPW,
     & APW, BPW, RPW, A, B, R, NR)
C
C VARIABLE DECLARATIONS
C
      INTEGER            FSTPS1
      DOUBLE PRECISION   APW, BPW, RPW
      DIMENSION          X(100), Y(100), UHAT(100)
      DOUBLE PRECISION   ABPW, BBPW, RBPW, AMSEPW, BMSEPW, RMSEPW
      DOUBLE PRECISION   UHATSQ, UHATCO
      DOUBLE PRECISION   A11, A12, A22, B1, B2
      COMMON/ITT1/       ALPHA, BETA, RHO, EPS
      COMMON/ITT2/       X, Y
C
C SUBROUTINE CODE
C
      ALPHAH = A
      BETAH  = B
      RHOH = R
      FSTPS1 = 0
C
C calculating alpha and beta
C
20    A12 = 0.0D0
      A22 = 0.0D0
      B1  = 0.0D0
      B2  = 0.0D0
      DO 50, I = 2,NR
           XDIF = X(I) - RHOH*X(I-1)
           YDIF = Y(I) - RHOH*Y(I-1)
           A12 = A12 + XDIF
           A22 = A22 + (XDIF*XDIF)
           B1  = B1  + YDIF
           B2  = B2  + XDIF*YDIF
50    CONTINUE
      A11 = (1 - RHOH*RHOH) + (NR - 1)*((1 - RHOH)**2)
      A12 = (1 - RHOH*RHOH)*X(1) + (1 - RHOH)*A12
      A22 = (1 - RHOH*RHOH)*X(1)*X(1) + A22
      B1  = (1 - RHOH*RHOH)*Y(1) + (1 - RHOH)*B1
      B2  = (1 - RHOH*RHOH)*X(1)*Y(1) + B2
      ALPHAH = (A22*B1 - A12*B2)/(A11*A22 - A12*A12)
      BETAH  = (-1.0D0*A12*B1 + A11*B2)/(A11*A22 - A12*A12)
      IF ( FSTPS1 .EQ. 0 ) THEN
           FSTPS1 = 1
           UHATSQ  = 0.0D0
           UHATCO  = 0.0D0
           UHAT(1) = Y(1) - ALPHAH - BETAH*X(1)
           DO 40, I = 2,NR
           UHAT(I) = Y(I) - ALPHAH - BETAH*X(I)
           IF ( I .LT. NR ) THEN
                UHATSQ  = UHATSQ + UHAT(I)*UHAT(I)
```

```
              ENDIF
              UHATCO  = UHATCO + (UHAT(I) * UHAT(I - 1))
40            CONTINUE
              UHATSQ = UHATSQ + 0.5*(UHAT(1)*UHAT(1) + UHAT(NR)*UHAT(NR))
              RHOH = UHATCO / UHATSQ
              GO TO 20
          ENDIF
C
C add to sum the bias and the variance for ALPHA, BETA, and RHO
C
          ABPW = ABPW + (ALPHAH - ALPHA)
          BBPW = BBPW + (BETAH  - BETA)
          RBPW = RBPW + (RHOH   - RHO)
          AMSEPW = AMSEPW + (ALPHAH - ALPHA)**2
          BMSEPW = BMSEPW + (BETAH  - BETA)**2
          RMSEPW = RMSEPW + (RHOH   - RHO)**2
          APW = APW + ALPHAH
          BPW = BPW + BETAH
          RPW = RPW + RHOH
          RETURN
          END
C
C END OF PW SUBROUTINE
C
          SUBROUTINE MLIT (ABML, BBML, RBML, AMSEML, BMSEML, RMSEML,
         & CTML, AML, BML, RML, A, B, BIGFL4, BIGCT4, NR)
C
C  VARIABLE DECLARATIONS
C
          INTEGER             BIGFL4, BIGCT4
          DIMENSION           X(100), Y(100)
          DOUBLE PRECISION    ABML, BBML, RBML, AMSEML, BMSEML, RMSEML
          DOUBLE PRECISION    AML, BML, RML
          DOUBLE PRECISION    A11, A12, A22, B1, B2
          DOUBLE PRECISION    ACOEF, BCOEF, CCOEF, PHI, P, Q
          DOUBLE PRECISION    UTLSUM, UTSUM, UCOSUM
          DOUBLE PRECISION    PI, RT27D2
          COMMON/ITT1/        ALPHA, BETA, RHO, EPS
          COMMON/ITT2/        X, Y
C
C SUBROUTINE CODE
C start iterative loop
C
          PI = 3.14159265358979323846
          RT27D2 = 2.5980762113533159403
          HALF = 0.5
          ALPHAH = A
          BETAH = B
          CTML = CTML + 1.0
          ML = 0
          RHOH  = 0.0
```

```
25     PREVR = RHOH
       UTSUM =  0.0D0
       UTLSUM = 0.0D0
       UCOSUM = 0.0D0
       U1SQ = (Y(1) - ALPHAH - BETAH*X(1))**2
       DO 40, I = 2,NR
          UT = Y(I) - ALPHAH - BETAH*X(I)
          UTLS1 = Y(I-1) - ALPHAH - BETAH*X(I-1)
          UTLSUM = UTLSUM + UTLS1*UTLS1
          UTSUM = UTSUM + UT*UT
          UCOSUM = UCOSUM + UT*UTLS1
40     CONTINUE
       ACOEF = -1.0D0*(NR - 2)*UCOSUM/((NR - 1)*(UTLSUM - U1SQ))
       BCOEF = ((NR - 1)*U1SQ - NR*UTLSUM - UTSUM) /
      & ((NR - 1)*(UTLSUM - U1SQ))
       CCOEF = NR*UCOSUM/((NR - 1)*(UTLSUM - U1SQ))
       P = BCOEF - (ACOEF*ACOEF/3.0D0)
       Q = CCOEF - (ACOEF*BCOEF/3.0D0) + (2.0D0*(ACOEF**3)/27.0D0)
       PHI = DACOS(RT27D2*Q/(P*DSQRT(-P)))
       RHOH = -2.0D0*(DSQRT(-P/3.0D0))*DCOS((PHI + PI)/3.0D0)
      & - (ACOEF/3.0D0)
C
C calculating alpha and beta
C
       A12 = 0.0D0
       A22 = 0.0D0
       B1  = 0.0D0
       B2  = 0.0D0
       DO 50 , I = 2,NR
            XDIF = X(I) - RHOH*X(I-1)
            YDIF = Y(I) - RHOH*Y(I-1)
            A12 = A12 + XDIF
            A22 = A22 + (XDIF*XDIF)
            B1  = B1  + YDIF
            B2  = B2  + XDIF*YDIF
50     CONTINUE
       A11 = (1 - RHOH*RHOH) + (NR - 1)*((1 - RHOH)**2)
       A12 = (1 - RHOH*RHOH)*X(1) + (1 - RHOH)*A12
       A22 = (1 - RHOH*RHOH)*X(1)*X(1) + A22
       B1  = (1- RHOH*RHOH)*Y(1) + (1 - RHOH)*B1
       B2  = (1 - RHOH*RHOH)*X(1)*Y(1) + B2
       ALPHAH = (A22*B1 - A12*B2)/(A11*A22 - A12*A12)
       BETAH  = (-1.0D0*A12*B1 + A11*B2)/(A11*A22 - A12*A12)
C
C end iterative loop
C
       TAU = ABS(RHOH - PREVR) / AMAX1( ABS(RHOH), HALF )
       IF ( TAU .GE. EPS .AND. ML .LT. 100 ) THEN
          CTML = CTML + 1.0
          ML = ML + 1
          GO TO 25
```

```
      END IF
      IF( ML .EQ. 100 ) THEN
         BIGFL4 = 1
         BIGCT4 = BIGCT4 + 1
      ENDIF
C
C add to sum the bias and the variance for ALPHA, BETA, and RHO
C
      ABML = ABML + (ALPHAH - ALPHA)
      BBML = BBML + (BETAH  - BETA)
      RBML = RBML + (RHOH   - RHO)
      AMSEML = AMSEML + (ALPHAH - ALPHA) ** 2
      BMSEML = BMSEML + (BETAH  - BETA ) ** 2
      RMSEML = RMSEML + (RHOH   - RHO  ) ** 2
      AML = AML + ALPHAH
      BML = BML + BETAH
      RML = RML + RHOH
      RETURN
      END
C
C END OF MLIT SUBROUTINE
C
       SUBROUTINE COPWNL( ABICO, AMSEICO, BBICO, BMSEICO, RBICO,
     & RMSEICO, ABIPW, AMSEIPW, BBIPW, BMSEIPW, RBIPW, RMSEIPW, ABNL,
     & AMSENL, BBNL, BMSENL, RBNL, RMSENL, AICO, BICO, RICO, AIPW,
     & BIPW, RIPW, ANL, BNL, RNL, CTCO, CTPW, CTNL, A, B, R, BIGFL1,
     & BIGFL2, BIGFL3, BIGCT1, BIGCT2, BIGCT3, NR)
C
C VARIABLE DECLARATIONS
C
       INTEGER            FLAG1, FLAG2, FLAG3
       INTEGER            BIGFL1, BIGFL2, BIGFL3
       INTEGER            BIGCT1, BIGCT2, BIGCT3, CO, PW
       DIMENSION          X(100), Y(100)
       DOUBLE PRECISION   SCO1, SCO2, SPW1, SPW2, SNL1, SNL2
       DOUBLE PRECISION   UHATSQ, UHATCO
       DOUBLE PRECISION   ACODIR, BCODIR, RCODIR
       DOUBLE PRECISION   APWDIR, BPWDIR, RPWDIR
       DOUBLE PRECISION   ANLDIR, BNLDIR, RNLDIR
       DOUBLE PRECISION   COA11, COA12, COA22, COA33
       DOUBLE PRECISION   PWA11, PWA12, PWA22, PWA33
       REAL*8             NLA11, NLA12, NLA22, NLA33
       DOUBLE PRECISION   AICO, BICO, RICO, AIPW, BIPW, RIPW
       DOUBLE PRECISION   ANL,BNL,RNL
       DOUBLE PRECISION   ABICO, AMSEICO, BBICO, BMSEICO, RBICO, RMSEICO
       DOUBLE PRECISION   ABIPW, AMSEIPW, BBIPW, BMSEIPW, RBIPW, RMSEIPW
       DOUBLE PRECISION   ABNL, AMSENL, BBNL, BMSENL, RBNL, RMSENL
       COMMON/ITT1/       ALPHA, BETA, RHO, EPS
       COMMON/ITT2/       X, Y
```

```
C
C SUBROUTINE CODE
C
      ONE = 1.0
      HALF = 0.5
C
C do loop to calculate the first sum of squared errors
C
      SCO1 = 0.0D0
      DO 60, I= 2,NR
           SCO1 = SCO1 + (((Y(I) - A - B*X(I)) -
     &     R*(Y(I-1) - A - B*X(I-1)))**2)
60    CONTINUE
      SPW1 = SCO1 + (1 - R*R)*((Y(1) - A - B*X(1))**2)
      SNL1 = SCO1 + ((Y(1) - A - B*X(1))**2)
C
C initializing
C
      CO = 0
      PW = 0
      NL = 0
      FLAG1 = 0
      FLAG2 = 0
      FLAG3 = 0
      ICONV1 = 0
      ICONV2 = 0
      ICONV3 = 0
      ACO2 = A
      BCO2 = B
      RCO2 = R
      APW2 = A
      BPW2 = B
      RPW2 = R
      ANL2 = A
      BNL2 = B
      RNL2 = R
C
C reestimating alpha, beta and rho using Cochrane-Orcutt method
C
75    IF ( ICONV1 .EQ. 0 ) THEN
           CALL DERIV( ACODIR, BCODIR, RCODIR, COA11, COA12,
     &     COA22, COA33, ACO2, BCO2, RCO2, NR )
           QCO = COA11*COA22 - (COA12*COA12)
           ACOT = (COA22 / QCO)*ACODIR - (COA12 / QCO)*BCODIR
           BCOT = ( -1*COA12 / QCO)*ACODIR + (COA11 / QCO )*BCODIR
           RCOT = RCODIR / COA33
           ACO1 = ACO2
           BCO1 = BCO2
           RCO1 = RCO2
           ACO2 = ACO1 - ACOT
           BCO2 = BCO1 - BCOT
```

```
              RCO2 = RCO2 - RCOT
              CTCO = CTCO + 1.0
              CO = CO + 1
         ENDIF
C
C calculating the sum of squares for Cochrane-Orcutt
C
         IF ( ICONV1 .EQ. 0 ) THEN
              SCO2 = 0.0D0
              DO 100, I = 2,NR
                   SCO2 = SCO2 + ((Y(I) - ACO2 - BCO2*X(I)) - RCO2*
     &             (Y(I-1) - ACO2 - BCO2*X(I-1)))**2
100           CONTINUE
              AMAXCO = ABS( ACOT ) / AMAX1( ABS(ACO1), ONE )
              BMAXCO = ABS( BCOT ) / AMAX1( ABS(BCO1), ONE )
              RMAXCO = ABS( RCOT ) / AMAX1( ABS(RCO1), HALF )
         ENDIF
C
C decision rule whether or not to iterate
C
         IF ( SCO2 .GE. SCO1 .OR. AMAX1(AMAXCO, BMAXCO, RMAXCO)
     & .LE. EPS .OR. CO .EQ. 100 ) THEN
              ICONV1 = 1
         ELSE
              SCO1 = SCO2
         ENDIF
C
C statement to set flag in order to determine which contraint holds
C
         IF ( SCO2 .GE. SCO1 .AND. ICONV1 .EQ. 1 ) THEN
              FLAG1 = 1
         END IF
         IF ( CO .EQ. 100 .AND. ICONV1 .EQ. 1 ) THEN
              BIGFL1 = 1
              BIGCT1 = BIGCT1 + 1
         ENDIF
C
C  reestimating alpha, beta, and rho using Prais-Winston
C
         IF ( ICONV2 .EQ. 0 ) THEN
              CALL DERIV( APWDIR, BPWDIR, RPWDIR, PWA11, PWA12,
     &        PWA22, PWA33, APW2, BPW2, RPW2, NR )
              APWDIR = (-2*(1 - RPW2*RPW2)*(Y(1) - APW2 - BPW2*X(1)))
     &        + APWDIR
              BPWDIR = (-2*(1 - RPW2*RPW2)*X(1)*(Y(1) - APW2 - BPW2*X(1)))
     &        + BPWDIR
              RPWDIR = (-2*RPW2*((Y(1) - APW2 - BPW2*X(1))**2)) + RPWDIR
              PWA11 = 2*(1 - RPW2*RPW2) + PWA11
              PWA12 = 2*(1 - RPW2*RPW2)*X(1) + PWA12
              PWA22 = 2*(1 - RPW2*RPW2)*X(1)*X(1) + PWA22
```

```
            PWA33 = -2*((Y(1) - APW2 - BPW2*X(1))**2) + PWA33
            QPW = PWA11*PWA22 - (PWA12*PWA12)
            APWT = (PWA22 / QPW)*APWDIR - (PWA12 / QPW)*BPWDIR
            BPWT = ( -1*PWA12 / QPW)*APWDIR + (PWA11 / QPW)*BPWDIR
            RPWT = RPWDIR / PWA33
            APW1 = APW2
            BPW1 = BPW2
            RPW1 = RPW2
            APW2 = APW1 - APWT
            BPW2 = BPW1 - BPWT
            RPW2 = RPW1 - RPWT
            CTPW = CTPW + 1.0
            PW = PW + 1
       ENDIF
C
C calculating the sum of squared errors for Prais-Winston
C
       IF ( ICONV2 .EQ. 0 ) THEN
            SPW2 = 0.0D0
            DO 200, I = 2, NR
                 SPW2 = SPW2 + ((Y(I) - APW2 - BPW2*X(I)) - RPW2*
     &           (Y(I-1) - APW2 - BPW2*X(I-1)))**2
200         CONTINUE
            SPW2 = SPW2 + (1 - RPW2*RPW2)*((Y(1) - APW2 - BPW2*
     &      X(1))**2)
            AMAXPW = ABS( APWT ) / AMAX1( ABS(APW1), ONE )
            BMAXPW = ABS( BPWT ) / AMAX1( ABS(BPW1), ONE )
            RMAXPW = ABS( RPWT ) / AMAX1( ABS(RPW1), HALF )
       ENDIF
C
C decision rule whether or not to iterate
C
       IF ( SPW2 .GE. SPW1 .OR. AMAX1(AMAXPW, BMAXPW, RMAXPW)
     & .LE. EPS .OR. PW .EQ. 100 ) THEN
            ICONV2 = 1
       ELSE
            SPW1 = SPW2
       ENDIF
C
C statement to set flag in order to determine which constraint holds
C
       IF ( SPW2 .GE. SPW1 .AND. ICONV2 .EQ. 1 ) THEN
            FLAG2 = 1
       ENDIF
       IF( PW .EQ. 100 .AND. ICONV2 .EQ. 1 ) THEN
            BIGFL2 = 1
            BIGCT2 = BIGCT2 + 1
       ENDIF
```

```
C
C reestimating alpha, beta, and rho using nonlinear technique
C
      IF ( ICONV3 .EQ. 0 ) THEN
           CALL DERIV( ANLDIR, BNLDIR, RNLDIR, NLA11, NLA12,
     &     NLA22, NLA33, ANL2, BNL2, RNL2, NR )
           ANLDIR = (-2*(Y(1) - ANL2 - BNL2*X(1))) + ANLDIR
           BNLDIR = (-2*X(1)*(Y(1) - ANL2 - BNL2*X(1))) + BNLDIR
           NLA11 = 2 + NLA11
           NLA12 = 2*X(1) + NLA12
           NLA22 = 2*X(1)*X(1) + NLA22
           QNL = NLA11*NLA22 - (NLA12*NLA12)
           ANLT = (NLA22 / QNL)*ANLDIR - (NLA12 / QNL)*BNLDIR
           BNLT = ( -1*NLA12 / QNL)*ANLDIR + (NLA11 / QNL)*BNLDIR
           RNLT = RNLDIR/ NLA33
           ANL1 = ANL2
           BNL1 = BNL2
           RNL1 = RNL2
           ANL2 = ANL1 - ANLT
           BNL2 = BNL1 - BNLT
           RNL2 = RNL1 - RNLT
           CTNL = CTNL + 1.0
           NL = NL + 1
      ENDIF
C
C calculate the sum of squared errors for nonlinear
C
      IF ( INCONV3 .EQ. 0 ) THEN
           SNL2 = 0.0D0
           DO 250, I = 2, NR
                SNL2 = SNL2 + ((Y(I) - ANL2 - BNL2*X(I)) - RNL2*
     &          (Y(I-1) - ANL2 - BNL2*X(I-1)))**2
250        CONTINUE
           SNL2 = SNL2 + ((Y(1) - ANL2 - BNL2*X(1))**2)
           AMAXNL = ABS( ANLT )/ AMAX1 ( ABS(ANL1), ONE )
           BMAXNL = ABS( BNLT )/ AMAX1 ( ABS(BNL1), ONE )
           RMAXNL = ABS( RNLT )/ AMAX1 ( ABS(RNL1), HALF )
      ENDIF
C
C decision rule whether or not to iterate
C
      IF ( SNL2 .GE. SNL1 .OR. AMAX1 (AMAXNL, BMAXNL, RMAXNL)
     & .LE. EPS .OR. NL .EQ. 100 ) THEN
           ICONV3 = 1
      ELSE
           SNL1 = SNL2
      ENDIF
C
C statement to set flag in order to determine which contraint holds
C
      IF ( SNL2 .GE. SNL1 .AND. ICONV3 .EQ. 1 ) THEN
```

```
          FLAG3 = 1
     END IF
     IF( NL .EQ. 100 .AND. ICONV3 .EQ. 1 ) THEN
          BIGFL3 = 1
          BIGCT3 = BIGCT3 + 1
     ENDIF


     IF ( ICONV1 .EQ. 0 .OR. ICONV2 .EQ. 0 .OR. ICONV3 .EQ. 0 ) THEN
          GO TO 75
     ENDIF
     IF ( FLAG1 .EQ. 0 ) THEN
          ACOH = ACO1
          BCOH = BCO1
          RCOH = RCO1
          CTCO = CTCO - 1.0
      ELSE
          ACOH = ACO2
          BCOH = BCO2
          RCOH = RCO2
      ENDIF
      IF ( FLAG2 .EQ. 0 ) THEN
          APWH = APW1
          BPWH = BPW1
          RPWH = RPW1
          CTPW = CTPW - 1.0
      ELSE
          APWH = APW2
          BPWH = BPW2
          RPWH = RPW2
      ENDIF
      IF ( FLAG3 .EQ. 0 ) THEN
          ANLH = ANL1
          BNLH = BNL1
          RNLH = RNL1
          CTNL = CTNL - 1.0
      ELSE
          ANLH = ANL2
          BNLH = BNL2
          RNLH = RNL2
      ENDIF
C
C add to sum the bias and the variance for ALPHA, BETA,  and RHO
C
      ABICO = ABICO + ( ACOH - ALPHA )
      BBICO = BBICO + ( BCOH - BETA )
      RBICO = RBICO + ( RCOH - RHO )
      AMSEICO = AMSEICO + ( ACOH - ALPHA )**2
      BMSEICO = BMSEICO + ( BCOH - BETA )**2
      RMSEICO = RMSEICO + ( RCOH - RHO )**2
      ABIPW = ABIPW + ( APWH - ALPHA )
```

```
      BBIPW = BBIPW + ( BPWH - BETA )
      RBIPW = RBIPW + ( RPWH - RHO )
      AMSEIPW = AMSEIPW + ( APWH - ALPHA )**2
      BMSEIPW = BMSEIPW + ( BPWH - BETA )**2
      RMSEIPW = RMSEIPW + ( RPWH - RHO )**2
      ABNL = ABNL + ( ANLH - ALPHA )
      BBNL = BBNL + ( BNLH - BETA )
      RBNL = RBNL + ( RNLH - RHO )
      AMSENL = AMSENL + ( ANLH - ALPHA )**2
      BMSENL = BMSENL + ( BNLH - BETA )**2
      RMSENL = RMSENL + ( RNLH - RHO )**2
      AICO = AICO + ACOH
      BICO = BICO + BCOH
      RICO = RICO + RCOH
      AIPW = AIPW + APWH
      BIPW = BIPW + BPWH
      RIPW = RIPW + RPWH
      ANL = ANL + ANLH
      BNL = BNL + BNLH
      RNL = RNL + RNLH
      RETURN
      END
C
      SUBROUTINE DERIV (ADIR, BDIR, RDIR, A11, A12, A22, A33,
     & A, B, R, NR)
C
C DECLARATIONS
C
      DOUBLE PRECISION        ADIR, BDIR, RDIR
      DOUBLE PRECISION        A11, A12, A22, A33
      DIMENSION               Y(100), X(100)
      COMMON/ITT2/            X, Y
C
C PROGRAM CODE
C intializing
C
      ADIR = 0.0D0
      BDIR = 0.0D0
      RDIR = 0.0D0
      A12 = 0.0D0
      A22 = 0.0D0
      A33 = 0.0D0
C
C do loop to calculate derivatives and respective entries in the
C Fisher information matrix
C
      DO 100, I= 2, NR
          U1 = Y(I) - A - B*X(I)
          U2 = Y(I-1) - A - B*X(I-1)
          DIF =  U1 - R*U2
          ADIR = ADIR + DIF
```

```
          BDIR = BDIR + (X(I) - R*X(I-1))*DIF
          RDIR = RDIR + (Y(I-1) - A - B*X(I-1))*DIF
          A12 = A12 + (X(I) - R*X(I-1))
          A22 = A22 + ((X(I) - R*X(I-1))**2)
          A33 = A33 + ((Y(I-1) - A - B*X(I-1))**2)
100    CONTINUE
       ADIR = -2.0D0*(1.0D0 - R)*ADIR
       BDIR = -2.0D0*BDIR
       RDIR = -2.0D0*RDIR
       A11 = 2.0D0*(NR - 1)*((1.0D0 - R)**2)
       A12 = 2.0D0*(1.0D0 - R)*A12
       A22 = 2.0D0*A22
       A33 = 2.0D0*A33
       RETURN
       END
C
      DOUBLE PRECISION FUNCTION UNIF(IX)
C
C     THIS FUNCTION GENERATES UNIFORM (0,1) RANDOM NUMBERS
C     USING THE MULTIPLICATIVE CONGRUENT ALGORITHM GIVEN BY
C          IX(I+1) = (7**5)*IX(I) MOD((2**31) - 1)
C          U = IX(I+1)/((2**31) - 1)
C
      DOUBLE PRECISION A,P,X
      INTEGER*4 IX
      A = 16807.D0
      P = 2147483647.D0
      X = IX*A
      IX = DMOD(X,P)
      UNIF = IX/P
      RETURN
      END
C
      SUBROUTINE NORMAL(N,Z,IX)
C
C     This subroutine generates Normal (0,1) random numbers using
C     the method of Box and Muller.  N is the dimension of the
C     vector of i.i.d. n(0,1) observations, Z.  IX is the seed
C     for the Uniform (0,1) random number generator.  IX must be
C     in the interval [1, (2**31) - 1].  IX is updated on each
C     call to the function UNIF(IX).
C
      DOUBLE PRECISION UNIF
      DIMENSION Z(N)
      INTEGER*4 IX
      PI = 3.14159265358979323846
      M = 1
      L = N/2
      IF(N.EQ.1) GO TO 200
      DO 100 I = 1,L
      U1 = UNIF(IX)
```

```
    U2 = UNIF(IX)
    Z(M) = ((-2.0*ALOG(U1))**0.5)*COS(2.0*PI*U2)
    Z(M+1) = ((-2.0*ALOG(U1))**0.5)*SIN(2.0*PI*U2)
    M = M + 2
100 CONTINUE
200 L2 = 2*L

    IF(N.NE.L2) THEN
       U1 = UNIF(IX)
       U2 = UNIF(IX)
       Z(N) = ((-2.0*ALOG(U1))**0.5)*COS(2.0*PI*U2)
    END IF
    RETURN
    END
```

APPENDIX B:

TABLES

Table 7. Average iterations for Cochrane-Orcutt estimator.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	8.19	7.15	6.01	5.39	4.68	4.33	3.83
-.95	7.44	6.83	5.91	5.07	4.59	4.12	3.66
-.90	6.88	7.04	6.09	5.07	4.52	3.91	3.66
-.80	6.90	6.67	6.41	5.33	4.55	4.01	3.56
-.60	7.88	7.36	7.29	6.07	4.71	4.08	3.49
-.40	6.30	7.17	7.41	7.60	5.89	4.32	3.52
-.20	4.82	7.53	7.56	7.75	6.64	4.88	3.63
0.0	4.06	6.27	8.24	7.95	7.60	6.38	4.09
.20	3.70	5.70	7.28	7.79	7.76	7.50	5.02
.40	3.80	4.71	6.75	8.02	8.41	7.78	6.72
.60	4.02	5.01	6.00	7.04	8.24	8.55	8.14
.80	5.08	5.53	6.47	7.51	9.03	8.99	9.48
.90	5.97	6.99	7.30	8.65	10.62	10.58	10.80
.95	6.77	7.72	9.04	9.73	10.68	11.80	11.94
.99	7.54	8.05	9.79	10.24	11.62	13.03	15.16

* Sample size = 25, theta = 20.

Table 8. Average iterations for Prais-Winston estimator.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	9.17	7.79	6.64	6.09	5.42	4.99	4.36
-.95	8.68	7.13	6.22	5.61	5.25	4.73	4.23
-.90	7.59	6.68	6.10	5.32	4.99	4.54	4.25
-.80	6.08	6.07	5.72	5.32	4.90	4.46	4.07
-.60	4.95	5.40	5.10	5.05	4.55	4.41	3.86
-.40	4.55	4.80	4.89	4.71	4.58	4.20	3.71
-.20	3.70	4.25	4.26	4.44	4.22	3.93	3.49
0.0	3.44	3.77	3.98	4.02	3.93	3.84	3.43
.20	3.53	4.03	4.11	4.17	4.09	4.09	3.70
.40	3.78	4.09	4.58	4.75	4.97	4.50	4.05
.60	4.24	4.79	5.15	5.29	5.40	5.45	5.05
.80	4.74	5.20	5.78	6.04	6.13	6.45	6.29
.90	5.09	5.67	6.20	6.79	6.66	6.84	7.40
.95	5.16	5.69	6.20	6.95	7.08	7.53	7.79
.99	5.09	5.64	6.26	6.73	6.91	7.55	8.50

* Sample size = 25, theta = 20.

Table 9. Average iterations for Maximum Likelihood estimator.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	5.30	4.85	4.47	4.27	4.14	3.91	3.63
-.95	5.90	5.25	4.75	4.52	4.34	4.03	3.78
-.90	5.98	5.31	5.00	4.52	4.40	4.08	3.90
-.80	5.43	5.34	5.09	4.73	4.51	4.22	3.94
-.60	4.88	5.11	4.89	4.78	4.53	4.37	3.91
-.40	4.76	4.96	4.88	4.95	4.66	4.38	3.92
-.20	4.14	4.65	4.77	4.61	4.47	4.29	3.96
0.0	4.21	4.24	4.46	4.42	4.60	4.54	4.14
.20	4.01	4.44	4.48	4.48	4.46	4.56	4.24
.40	4.06	4.29	4.68	4.80	5.10	4.76	4.55
.60	4.27	4.67	4.93	4.97	5.11	5.53	5.20
.80	4.52	4.74	5.13	5.26	5.40	5.88	5.98
.90	4.54	4.90	5.15	5.53	5.60	5.96	6.58
.95	4.62	4.97	5.17	5.58	5.85	6.30	6.64
.99	4.60	4.99	5.28	5.49	5.74	6.28	7.00

* Sample size = 25, theta = 20.

Table 10. Average iterations for Initially Nonstationary estimator.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99	4.30	3.91	3.76	3.68	3.46	3.36	3.19
-.95	4.33	3.95	3.69	3.53	3.40	3.21	3.10
-.90	4.55	3.97	3.73	3.50	3.38	3.21	3.16
-.80	4.82	4.23	3.88	3.68	3.50	3.20	3.10
-.60	4.85	4.44	4.07	3.85	3.63	3.37	3.20
-.40	4.83	4.53	4.27	3.97	3.77	3.53	3.20
-.20	3.84	4.17	4.09	4.05	3.73	3.53	3.18
0.0	3.39	3.58	3.89	3.89	3.76	3.69	3.23
.20	3.42	3.83	3.96	4.02	3.89	4.04	3.81
.40	3.70	3.97	4.32	4.46	4.82	4.38	4.40
.60	5.23	5.65	5.71	5.21	5.15	5.11	5.50
.80	8.27	8.02	8.44	7.86	7.20	6.71	6.47
.90	9.37	9.49	0.16	0.07	8.73	8.39	7.91
.95	8.99	9.60	9.59	0.46	9.88	8.62	8.53
.99	9.20	9.20	9.81	9.59	9.33	9.08	9.20

* Sample size = 25, theta = 20.

Table 11. Average iterations for growth trend.*

Rho	Iterations ITCO	ITPW	ITML	ITIN
-.99	4.24	4.43	3.60	4.12
-.95	4.11	4.26	3.75	4.04
-.90	4.24	4.32	3.91	4.11
-.80	4.35	4.39	4.05	4.21
-.60	4.60	4.77	4.44	4.56
-.40	4.81	4.91	4.75	4.76
-.20	4.79	4.78	4.82	4.68
0.0	5.02	4.94	5.04	4.81
.20	5.38	5.25	5.39	5.28
.40	5.81	5.64	5.51	6.05
.60	6.35	6.20	5.62	7.87
.80	7.45	6.36	5.56	10.56
.90	11.30	6.47	5.44	10.91
.95	12.19	6.43	5.40	11.37
.99	18.55	6.47	5.36	11.22

* Sample size = 25, variance of trend component = 1.0.

Table 12. Average iterations for growth trend.*

	Iterations			
Rho	ITCO	ITPW	ITML	ITIN
-.99	4.12	4.16	3.29	3.87
-.95	3.84	4.08	3.54	3.64
-.90	3.75	4.00	3.67	3.59
-.80	3.62	3.94	3.69	3.51
-.60	3.64	3.91	3.78	3.48
-.40	3.46	3.65	3.72	3.36
-.20	3.49	3.62	3.73	3.33
0.0	3.51	3.42	3.69	3.26
.20	3.71	3.38	3.75	3.55
.40	4.32	3.97	4.05	4.35
.60	5.33	4.62	4.53	6.21
.80	7.68	5.84	5.11	9.70
.90	9.68	6.59	5.43	10.75
.95	10.64	6.76	5.49	11.77
.99	12.46	7.29	5.69	12.49

* Sample size = 25, variance of trend component = 0.0009.

Table 13. Average estimate and mean square error in estimating alpha.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99							
ITCO	1.0000	0.9996	1.0009	0.9999	1.0000	0.9993	0.9999
	(0.3763E-04)	(0.3982E-04)	(0.3859E-04)	(0.4069E-04)	(0.4122E-04)	(0.4172E-04)	(0.4918E-04)
ITPW	1.0001	0.9996	1.0009	0.9999	1.0000	0.9993	0.9999
	(0.3812E-04)	(0.3996E-04)	(0.3835E-04)	(0.4083E-04)	(0.4126E-04)	(0.4151E-04)	(0.4915E-04)
ITML	1.0000	0.9996	1.0009	0.9999	0.9999	0.9993	0.9999
	(0.3655E-04)	(0.4006E-04)	(0.3844E-04)	(0.4098E-04)	(0.4134E-04)	(0.4172E-04)	(0.4914E-04)
ITIN	0.9999	0.9997	1.0008	1.0000	0.9999	0.9992	1.0002
	(0.4023E-04)	(0.4758E-04)	(0.4557E-04)	(0.5032E-04)	(0.5292E-04)	(0.5811E-04)	(0.8267E-04)
-.95							
ITCO	1.0005	1.0005	0.9996	1.0002	0.9999	1.0002	0.9999
	(0.4102E-04)	(0.4214E-04)	(0.4057E-04)	(0.3956E-04)	(0.4468E-04)	(0.4457E-04)	(0.5123E-04)
ITPW	1.0005	1.0004	0.9996	1.0003	0.9999	1.0002	1.0000
	(0.4013E-04)	(0.4152E-04)	(0.4044E-04)	(0.3915E-04)	(0.4470E-04)	(0.4414E-04)	(0.5102E-04)
ITML	1.0005	1.0004	0.9996	1.0003	0.9999	1.0002	1.0000
	(0.3982E-04)	(0.4141E-04)	(0.4029E-04)	(0.3896E-04)	(0.4477E-04)	(0.4413E-04)	(0.5102E-04)
ITIN	1.0004	1.0003	0.9995	1.0003	0.9998	1.0002	0.9999
	(0.3966E-04)	(0.4213E-04)	(0.4100E-04)	(0.3993E-04)	(0.4702E-04)	(0.4706E-04)	(0.5702E-04)

Table 13--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.90							
ITCO	0.9996 (0.4738E-04)	1.0006 (0.4431E-04)	1.0002 (0.4210E-04)	1.0001 (0.4658E-04)	1.0005 (0.4549E-04)	0.9996 (0.5124E-04)	1.0003 (0.5508E-04)
ITPW	0.9998 (0.4435E-04)	1.0006 (0.4331E-04)	1.0001 (0.4177E-04)	1.0002 (0.4643E-04)	1.0005 (0.4542E-04)	0.9996 (0.5100E-04)	1.0003 (0.5588E-04)
ITML	0.9998 (0.4427E-04)	1.0006 (0.4328E-04)	1.0001 (0.4179E-04)	1.0002 (0.4646E-04)	0.9999 (0.4553E-04)	0.9996 (0.5110E-04)	1.0003 (0.5615E-04)
ITIN	0.9998 (0.4440E-04)	1.0006 (0.4343E-04)	1.0002 (0.4226E-04)	1.0002 (0.4737E-04)	1.0006 (0.4660E-04)	0.9996 (0.5277E-04)	1.0004 (0.6229E-04)
-.80							
ITCO	0.9996 (0.5220E-04)	0.9999 (0.5158E-04)	1.0003 (0.4604E-04)	0.9993 (0.5236E-04)	0.9997 (0.4875E-04)	0.9999 (0.4744E-04)	1.0005 (0.6256E-04)
ITPW	0.9996 (0.4949E-04)	1.0000 (0.4849E-04)	1.0002 (0.4519E-04)	0.9993 (0.5176E-04)	0.9997 (0.4801E-04)	0.9999 (0.4698E-04)	1.0004 (0.6213E-04)
ITML	0.9996 (0.4945E-04)	1.0000 (0.4832E-04)	1.0002 (0.4513E-04)	0.9993 (0.5173E-04)	1.0010 (0.4799E-04)	0.9999 (0.4692E-04)	1.0004 (0.6215E-04)
ITIN	0.9996 (0.4939E-04)	1.0000 (0.4809E-04)	1.0002 (0.4506E-04)	0.9993 (0.5168E-04)	0.9997 (0.4820E-04)	0.9999 (0.4681E-04)	1.0004 (0.6408E-04)

Table 13--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.60							
ITCO	1.0001 (0.5493E-04)	0.9999 (0.6378E-04)	0.9999 (0.5705E-04)	0.9998 (0.6260E-04)	0.9998 (0.6636E-04)	0.9997 (0.7086E-04)	0.9993 (0.9108E-04)
ITPW	1.0002 (0.5309E-04)	1.0000 (0.6068E-04)	0.9997 (0.5506E-04)	0.9999 (0.5972E-04)	0.9999 (0.6576E-04)	0.9996 (0.7056E-04)	0.9993 (0.8962E-04)
ITML	1.0002 (0.5300E-04)	1.0000 (0.6067E-04)	0.9997 (0.5499E-04)	0.9999 (0.5976E-04)	1.0010 (0.6576E-04)	0.9996 (0.7057E-04)	0.9993 (0.8958E-04)
ITIN	1.0002 (0.5305E-04)	1.0000 (0.6058E-04)	0.9997 (0.5487E-04)	0.9999 (0.6000E-04)	0.9999 (0.6584E-04)	0.9996 (0.7072E-04)	0.9993 (0.8947E-04)
-.40							
ITCO	0.9997 (0.8779E-04)	1.0003 (0.7813E-04)	1.0002 (0.8621E-04)	1.0002 (0.9213E-04)	1.0000 (0.8713E-04)	1.0004 (0.8571E-04)	1.0002 (0.1227E-03)
ITPW	0.9999 (0.8458E-04)	1.0003 (0.7047E-04)	1.0002 (0.8292E-04)	1.0000 (0.8634E-04)	1.0000 (0.8391E-04)	1.0003 (0.8134E-04)	1.0002 (0.1196E-03)
ITML	0.9999 (0.8464E-04)	1.0003 (0.7045E-04)	1.0002 (0.8293E-04)	1.0000 (0.8625E-04)	1.0010 (0.8390E-04)	1.0003 (0.8136E-04)	1.0002 (0.1196E-03)
ITIN	0.9999 (0.8455E-04)	1.0003 (0.7044E-04)	1.0002 (0.8288E-04)	1.0000 (0.8619E-04)	1.0001 (0.8389E-04)	1.0004 (0.8121E-04)	1.0002 (0.1196E-03)

Table 13--continued

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.20							
ITCO	0.9993 (0.1024E-03)	1.0006 (0.1171E-03)	1.0003 (0.1053E-03)	1.0000 (0.1263E-03)	0.9993 (0.1092E-03)	0.9999 (0.1362E-03)	0.9992 (0.1670E-03)
ITPW	0.9993 (0.9644E-04)	1.0007 (0.1105E-03)	1.0002 (0.1020E-03)	1.0001 (0.1161E-03)	0.9993 (0.1011E-03)	1.0000 (0.1308E-03)	0.9990 (0.1576E-03)
ITML	0.9993 (0.9638E-04)	1.0007 (0.1105E-03)	1.0002 (0.1020E-03)	1.0001 (0.1161E-03)	1.0010 (0.1011E-03)	1.0000 (0.1308E-03)	0.9990 (0.1576E-03)
ITIN	0.9993 (0.9636E-04)	1.0007 (0.1105E-03)	1.0002 (0.1020E-03)	1.0001 (0.1162E-03)	0.9993 (0.1010E-03)	1.0000 (0.1306E-03)	0.9989 (0.1573E-03)
0.0							
ITCO	1.0004 (0.1486E-03)	1.0001 (0.1564E-03)	1.0006 (0.1567E-03)	1.0005 (0.1545E-03)	1.0002 (0.1762E-03)	1.0000 (0.1770E-03)	0.9990 (0.2385E-03)
ITPW	1.0006 (0.1441E-03)	1.0001 (0.1455E-03)	1.0007 (0.1454E-03)	1.0004 (0.1426E-03)	1.0001 (0.1639E-03)	1.0001 (0.1646E-03)	0.9994 (0.2307E-03)
ITML	1.0006 (0.1441E-03)	1.0001 (0.1454E-03)	1.0007 (0.1453E-03)	1.0004 (0.1425E-03)	1.0010 (0.1638E-03)	1.0001 (0.1645E-03)	0.9994 (0.2303E-03)
ITIN	1.0006 (0.1440E-03)	1.0001 (0.1455E-03)	1.0007 (0.1455E-03)	1.0004 (0.1425E-03)	1.0001 (0.1638E-03)	1.0001 (0.1644E-03)	0.9994 (0.2308E-03)

Table 13--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.20							
ITCO	0.9996 (0.2421E-03)	1.0011 (0.2513E-03)	1.0002 (0.2366E-03)	1.0000 (0.2528E-03)	1.0012 (0.2599E-03)	1.0003 (0.2657E-03)	0.9995 (0.3605E-03)
ITPW	0.9995 (0.2285E-03)	1.0010 (0.2302E-03)	1.0004 (0.2242E-03)	1.0000 (0.2500E-03)	1.0010 (0.2370E-03)	1.0004 (0.2409E-03)	0.9997 (0.3081E-03)
ITML	0.9995 (0.2282E-03)	1.0010 (0.2300E-03)	1.0004 (0.2240E-03)	1.0000 (0.2494E-03)	1.0010 (0.2371E-03)	1.0004 (0.2406E-03)	0.9997 (0.3072E-03)
ITIN	0.9995 (0.2306E-03)	1.0010 (0.2325E-03)	1.0005 (0.2255E-03)	1.0000 (0.2514E-03)	1.0010 (0.2370E-03)	1.0004 (0.2408E-03)	0.9997 (0.3081E-03)
.40							
ITCO	0.9978 (0.3977E-03)	0.9990 (0.4028E-03)	0.9994 (0.4166E-03)	1.0014 (0.4330E-03)	1.0000 (0.4625E-03)	0.9995 (0.4816E-03)	1.0007 (0.6698E-03)
ITPW	0.9982 (0.3632E-03)	0.9992 (0.3734E-03)	0.9996 (0.3757E-03)	1.0019 (0.4031E-03)	1.0002 (0.4095E-03)	0.9993 (0.4394E-03)	1.0005 (0.5773E-03)
ITML	0.9982 (0.3614E-03)	0.9992 (0.3726E-03)	0.9996 (0.3736E-03)	1.0019 (0.4014E-03)	1.0010 (0.4081E-03)	0.9993 (0.4385E-03)	1.0006 (0.5771E-03)
ITIN	0.9983 (0.3681E-03)	0.9993 (0.3796E-03)	1.0000 (0.4230E-03)	1.0020 (0.4051E-03)	1.0002 (0.4173E-03)	0.9993 (0.4391E-03)	1.0006 (0.5770E-03)

Table 13--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.60							
ITCO	0.9982	0.9998	1.0002	0.9986	0.9997	1.0008	0.9977
	(0.8716E-03)	(0.9334E-03)	(0.1066E-02)	(0.9665E-03)	(0.9399E-03)	(0.1102E-02)	(0.1258E-02)
ITPW	0.9986	1.0005	0.9992	0.9996	1.0000	1.0002	0.9977
	(0.7807E-03)	(0.8185E-03)	(0.9400E-03)	(0.8620E-03)	(0.8710E-03)	(0.8689E-03)	(0.1076E-02)
ITML	0.9986	1.0005	0.9993	0.9996	1.0010	1.0001	0.9976
	(0.7612E-03)	(0.8082E-03)	(0.9292E-03)	(0.8541E-03)	(0.8614E-03)	(0.8564E-03)	(0.1076E-02)
ITIN	0.9988	1.0004	0.9973	0.9999	0.9999	1.0003	0.9977
	(0.1458E-02)	(0.1572E-02)	(0.1280E-02)	(0.9674E-03)	(0.9944E-03)	(0.9477E-03)	(0.1080E-02)
.80							
ITCO	1.0041	0.9963	1.0001	0.9937	0.9997	1.0002	1.0007
	(0.4390E-02)	(0.3913E-02)	(0.5401E-02)	(0.6059E-01)	(0.5026E-02)	(0.4449E-02)	(0.1048E-01)
ITPW	1.0040	0.9987	0.9991	0.9996	0.9998	1.0008	0.9989
	(0.2617E-02)	(0.2853E-02)	(0.2946E-02)	(0.3180E-02)	(0.2774E-02)	(0.3020E-02)	(0.3871E-02)
ITML	1.0038	0.9988	0.9990	0.9999	1.0010	1.0007	0.9992
	(0.2477E-02)	(0.2795E-02)	(0.2893E-02)	(0.3155E-02)	(0.2733E-02)	(0.2994E-02)	(0.3795E-02)
ITIN	1.0027	1.0013	0.9981	1.0001	0.9983	1.0006	0.9978
	(0.6532E-02)	(0.6047E-02)	(0.5687E-02)	(0.5606E-02)	(0.5173E-02)	(0.4440E-02)	(0.4963E-02)

Table 13--continued

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
.90							
ITCO	0.9860 (0.5798E-01)	1.0021 (0.1179E+00)	1.0085 (0.3972E-01)	1.0021 (0.2802E-01)	0.9945 (0.2423E-01)	0.9772 (0.3693E+00)	1.0063 (0.3984E-01)
ITPW	0.9975 (0.9938E-02)	1.0072 (0.9505E-02)	1.0047 (0.8219E-02)	0.9983 (0.8297E-02)	0.9982 (0.8825E-02)	0.9993 (0.9262E-02)	1.0071 (0.1018E-01)
ITML	0.9978 (0.9877E-02)	1.0060 (0.9102E-02)	1.0049 (0.8146E-02)	0.9993 (0.8208E-02)	0.9999 (0.8452E-02)	0.9990 (0.9154E-02)	1.0071 (0.1004E-01)
ITIN	0.9942 (0.1550E-01)	0.9978 (0.1695E-01)	1.0048 (0.1469E-01)	0.9980 (0.1621E-01)	0.9992 (0.1544E-01)	0.9991 (0.1364E-01)	1.0048 (0.1404E-01)
.95							
ITCO	0.9539 (0.4279E+00)	0.9990 (0.2964E+00)	0.9923 (0.4664E+01)	1.0044 (0.2670E+00)	1.0414 (0.2808E+00)	0.9906 (0.2686E+01)	0.9711 (0.3400E+00)
ITPW	0.9923 (0.4619E-01)	1.0038 (0.2844E-01)	1.0149 (0.5643E-01)	0.9891 (0.1069E+00)	1.0117 (0.2456E-01)	0.9943 (0.2651E-01)	0.9961 (0.2766E-01)
ITML	0.9973 (0.2345E-01)	1.0021 (0.2441E-01)	1.0072 (0.2565E-01)	1.0018 (0.2377E-01)	0.9999 (0.2369E-01)	0.9950 (0.2606E-01)	0.9955 (0.2689E-01)
ITIN	0.9932 (0.3324E-01)	1.0088 (0.3620E-01)	1.0189 (0.3845E-01)	1.0119 (0.3600E-01)	1.0146 (0.3639E-01)	0.9889 (0.3709E-01)	0.9953 (0.3593E-01)

Table 13--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.99							
ITCO	1.0002 (0.4011E+00)	0.9795 (0.4534E+00)	1.2114 (0.2056E+02)	1.0043 (0.6694E+00)	0.9284 (0.1320E+01)	0.8814 (0.1076E+01)	0.9094 (0.4810E+01)
ITPW	1.0027 (0.1755E+00)	0.9774 (0.1823E+00)	1.0359 (0.1685E+00)	1.0104 (0.1520E+00)	0.9855 (0.1816E+00)	0.9719 (0.1697E+00)	1.0086 (0.1622E+00)
ITML	1.0045 (0.1643E+00)	0.9693 (0.1715E+00)	1.0336 (0.1631E+00)	1.0094 (0.1518E+00)	0.9999 (0.1609E+00)	0.9760 (0.1663E+00)	1.0106 (0.1622E+00)
ITIN	1.0098 (0.1894E+00)	0.9697 (0.1805E+00)	1.0338 (0.1787E+00)	1.0082 (0.1751E+00)	0.9987 (0.1782E+00)	0.9799 (0.1788E+00)	1.0067 (0.1841E+00)

* Mean square error values in parenthesis and reported in exponential form; sample size = 25, theta = 20, true parameter value for alpha is one.

Table 14. Average Estimate and Mean Square Error in Estimating Beta.*

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
-.99							
ITCO	1.0000	0.9999	1.0000	0.9999	0.9997	0.9999	1.0001
	(0.9502E-04)	(0.3918E-04)	(0.1852E-04)	(0.9616E-05)	(0.5964E-05)	(0.3013E-05)	(0.1716E-05)
ITPW	1.0003	1.0001	0.9999	0.9999	0.9997	0.9999	1.0001
	(0.9613E-04)	(0.3366E-04)	(0.1714E-04)	(0.9373E-05)	(0.5574E-05)	(0.3082E-05)	(0.1685E-05)
ITML	0.9999	0.9998	0.9998	0.9999	1.0002	0.9999	1.0001
	(0.4229E-04)	(0.2723E-04)	(0.1468E-04)	(0.8278E-05)	(0.5403E-05)	(0.2886E-05)	(0.1690E-05)
ITIN	0.9988	0.9987	0.9999	0.9993	1.0000	0.9998	0.9999
	(0.3103E-03)	(0.2532E-03)	(0.1556E-03)	(0.1145E-03)	(0.6270E-04)	(0.3392E-04)	(0.1047E-04)
-.95							
ITCO	0.9992	0.9994	0.9999	1.0001	0.9999	0.9999	1.0001
	(0.1304E-03)	(0.4752E-04)	(0.2227E-04)	(0.1058E-04)	(0.6439E-05)	(0.3787E-05)	(0.1869E-05)
ITPW	0.9999	0.9998	0.9999	1.0001	0.9999	0.9999	1.0001
	(0.5518E-04)	(0.2114E-04)	(0.1689E-04)	(0.8440E-05)	(0.5577E-05)	(0.3467E-05)	(0.1812E-05)
ITML	0.9998	0.9998	0.9999	1.0001	1.0002	0.9999	1.0001
	(0.3588E-04)	(0.1834E-04)	(0.1533E-04)	(0.8096E-05)	(0.5284E-05)	(0.3411E-05)	(0.1806E-05)
ITIN	0.9999	1.0001	0.9999	1.0000	1.0000	0.9999	1.0001
	(0.7265E-04)	(0.5170E-04)	(0.4016E-04)	(0.2582E-04)	(0.1482E-04)	(0.8652E-05)	(0.3752E-05)

Table 14--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.90							
ITCO	1.0010	1.0002	0.9999	0.9998	1.0002	1.0000	0.9999
	(0.1647E-03)	(0.7894E-04)	(0.2911E-04)	(0.1409E-04)	(0.7414E-05)	(0.3884E-05)	(0.2054E-05)
ITPW	1.0001	1.0001	1.0001	0.9998	1.0001	1.0000	0.9999
	(0.4099E-04)	(0.2089E-04)	(0.1331E-04)	(0.8964E-05)	(0.6111E-05)	(0.3468E-05)	(0.1983E-05)
ITML	0.9999	1.0000	1.0001	0.9998	1.0002	1.0000	0.9999
	(0.3122E-04)	(0.1829E-04)	(0.1244E-04)	(0.8699E-05)	(0.5985E-05)	(0.3441E-05)	(0.1981E-05)
ITIN	0.9997	1.0001	0.9999	0.9999	1.0000	1.0000	0.9999
	(0.3693E-04)	(0.2830E-04)	(0.2156E-04)	(0.1538E-04)	(0.1017E-04)	(0.5721E-05)	(0.2956E-05)
-.80							
ITCO	0.9999	0.9999	0.9999	0.9999	0.9998	0.9999	1.0000
	(0.1784E-03)	(0.8694E-04)	(0.4181E-04)	(0.1704E-04)	(0.9337E-05)	(0.4503E-05)	(0.2051E-05)
ITPW	1.0001	1.0000	0.9998	1.0001	0.9998	0.9999	1.0000
	(0.2574E-04)	(0.1848E-04)	(0.1133E-04)	(0.9371E-05)	(0.6297E-05)	(0.3697E-05)	(0.1959E-05)
ITML	1.0000	1.0000	0.9998	1.0001	1.0000	0.9999	1.0000
	(0.2336E-04)	(0.1787E-04)	(0.1112E-04)	(0.9223E-05)	(0.6214E-05)	(0.3671E-05)	(0.1958E-05)
ITIN	1.0000	1.0000	0.9997	1.0001	0.9999	0.9999	1.0000
	(0.2383E-04)	(0.2018E-04)	(0.1404E-04)	(0.1119E-04)	(0.7370E-05)	(0.4301E-05)	(0.2331E-05)

Table 14--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.60							
ITCO	1.0001 (0.1493E-03)	1.0007 (0.1664E-03)	1.0005 (0.8967E-04)	0.9999 (0.4041E-04)	0.9999 (0.1553E-04)	1.0001 (0.6583E-05)	1.0001 (0.3205E-05)
ITPW	0.9998 (0.1344E-04)	1.0000 (0.1212E-04)	1.0001 (0.1225E-04)	1.0002 (0.8185E-05)	0.9999 (0.6706E-05)	1.0001 (0.4456E-05)	1.0001 (0.2664E-05)
ITML	0.9998 (0.1285E-04)	1.0000 (0.1171E-04)	1.0001 (0.1217E-04)	1.0002 (0.8163E-05)	1.0000 (0.6658E-05)	1.0001 (0.4449E-05)	1.0001 (0.2657E-05)
ITIN	0.9998 (0.1298E-04)	0.9999 (0.1105E-04)	1.0001 (0.1210E-04)	1.0002 (0.8762E-05)	0.9999 (0.6725E-05)	1.0001 (0.4748E-05)	1.0001 (0.2657E-05)
-.40							
ITCO	1.0001 (0.6824E-04)	0.9999 (0.1813E-03)	1.0000 (0.1666E-03)	1.0000 (0.1049E-03)	1.0000 (0.3414E-04)	1.0000 (0.9750E-05)	1.0000 (0.4068E-05)
ITPW	0.9999 (0.8607E-05)	0.9998 (0.1171E-04)	1.0000 (0.9697E-05)	0.9999 (0.8737E-05)	0.9999 (0.7237E-05)	1.0000 (0.5039E-05)	1.0000 (0.3171E-05)
ITML	0.9999 (0.8066E-05)	0.9998 (0.1150E-04)	1.0000 (0.9625E-05)	0.9999 (0.8650E-05)	1.0000 (0.7211E-05)	1.0000 (0.5020E-05)	1.0000 (0.3172E-05)
ITIN	0.9999 (0.8188E-05)	0.9998 (0.1125E-04)	1.0000 (0.9533E-05)	0.9999 (0.8373E-05)	0.9999 (0.7150E-05)	1.0000 (0.4975E-05)	1.0000 (0.3201E-05)

Table 14--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.20							
ITCO	0.9998 (0.2079E-04)	0.9998 (0.1257E-03)	1.0007 (0.1882E-03)	1.0008 (0.1462E-03)	0.9998 (0.7674E-04)	1.0000 (0.2055E-04)	1.0000 (0.6935E-05)
ITPW	0.9998 (0.5751E-05)	0.9998 (0.8516E-05)	1.0000 (0.9813E-05)	1.0000 (0.9028E-05)	1.0001 (0.7616E-05)	0.9998 (0.6420E-05)	1.0001 (0.4444E-05)
ITML	0.9998 (0.5706E-05)	0.9998 (0.8437E-05)	1.0000 (0.9779E-05)	1.0000 (0.8961E-05)	1.0000 (0.7591E-05)	0.9998 (0.6404E-05)	1.0001 (0.4436E-05)
ITIN	0.9998 (0.5796E-05)	0.9998 (0.8425E-05)	1.0000 (0.9777E-05)	1.0000 (0.8768E-05)	1.0001 (0.7504E-05)	0.9998 (0.6402E-05)	1.0001 (0.4395E-05)
0.0							
ITCO	0.9997 (0.1004E-04)	0.9999 (0.5789E-04)	1.0002 (0.1482E-03)	1.0008 (0.1535E-03)	0.9995 (0.1203E-03)	1.0004 (0.4971E-04)	1.0002 (0.1126E-04)
ITPW	0.9999 (0.4171E-05)	0.9999 (0.6754E-05)	0.9999 (0.8800E-05)	0.9999 (0.9064E-05)	0.9999 (0.9862E-05)	0.9998 (0.7862E-05)	0.9999 (0.5535E-05)
ITML	0.9999 (0.4156E-05)	0.9999 (0.6731E-05)	0.9999 (0.8760E-05)	0.9999 (0.9041E-05)	1.0000 (0.9830E-05)	0.9998 (0.7827E-05)	0.9999 (0.5516E-05)
ITIN	0.9999 (0.4174E-05)	0.9999 (0.6740E-05)	0.9999 (0.8706E-05)	0.9999 (0.9031E-05)	0.9999 (0.9763E-05)	0.9998 (0.7800E-05)	0.9999 (0.5534E-05)

Table 14--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.20							
ITCO	1.0001 (0.4567E-05)	0.9999 (0.2406E-04)	0.9997 (0.9260E-04)	0.9995 (0.1641E-03)	1.0007 (0.1701E-03)	0.9998 (0.9327E-04)	1.0002 (0.3056E-04)
ITPW	0.9999 (0.2879E-05)	0.9999 (0.6066E-05)	0.9999 (0.7530E-05)	1.0000 (0.1017E-04)	1.0000 (0.1003E-04)	1.0000 (0.8688E-05)	0.9999 (0.6355E-05)
ITML	0.9999 (0.2870E-05)	0.9999 (0.6037E-05)	0.9999 (0.7510E-05)	1.0000 (0.1013E-04)	1.0000 (0.1002E-04)	1.0000 (0.8637E-05)	0.9999 (0.6312E-05)
ITIN	0.9999 (0.2872E-05)	0.9999 (0.6026E-05)	1.0000 (0.7472E-05)	1.0000 (0.1006E-04)	1.0000 (0.9998E-05)	1.0000 (0.8640E-05)	0.9999 (0.6338E-05)
.40							
ITCO	1.0000 (0.3164E-05)	1.0001 (0.1202E-04)	1.0002 (0.4695E-04)	1.0004 (0.1193E-03)	0.9994 (0.1806E-03)	1.0002 (0.1398E-03)	0.9999 (0.7139E-04)
ITPW	1.0000 (0.2367E-05)	1.0001 (0.4815E-05)	1.0001 (0.7364E-05)	1.0000 (0.1007E-04)	0.9999 (0.1162E-04)	1.0002 (0.1113E-04)	0.9998 (0.1034E-04)
ITML	1.0000 (0.2366E-05)	1.0001 (0.4808E-05)	1.0001 (0.7368E-05)	1.0000 (0.1001E-04)	1.0000 (0.1159E-04)	1.0002 (0.1112E-04)	0.9998 (0.1033E-04)
ITIN	1.0000 (0.2375E-05)	1.0001 (0.4790E-05)	1.0002 (0.7437E-05)	1.0000 (0.9787E-05)	0.9999 (0.1147E-04)	1.0002 (0.1111E-04)	0.9998 (0.1030E-04)

Table 14--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.60							
ITCO	1.0000 (0.2326E-05)	0.9999 (0.1006E-04)	0.9999 (0.1832E-04)	1.0001 (0.4548E-04)	0.9995 (0.1388E-03)	0.9997 (0.1912E-03)	1.0004 (0.1370E-03)
ITPW	1.0000 (0.1842E-05)	1.0001 (0.4261E-05)	0.9999 (0.6325E-05)	1.0003 (0.8654E-05)	1.0000 (0.1285E-04)	0.9998 (0.1541E-04)	1.0001 (0.1697E-04)
ITML	1.0000 (0.1839E-05)	1.0001 (0.4277E-05)	0.9999 (0.6310E-05)	1.0003 (0.8679E-05)	1.0000 (0.1280E-04)	0.9997 (0.1518E-04)	1.0001 (0.1685E-04)
ITIN	1.0000 (0.1858E-05)	1.0001 (0.4457E-05)	0.9999 (0.6522E-05)	1.0003 (0.8966E-05)	1.0000 (0.1284E-04)	0.9997 (0.1494E-04)	1.0001 (0.1671E-04)
.80							
ITCO	1.0001 (0.2016E-05)	0.9999 (0.4331E-05)	1.0001 (0.1093E-04)	1.0001 (0.2411E-04)	1.0000 (0.7078E-04)	1.0003 (0.1803E-03)	1.0002 (0.1810E-03)
ITPW	1.0001 (0.1800E-05)	1.0000 (0.3226E-05)	1.0002 (0.6219E-05)	1.0001 (0.9796E-05)	1.0000 (0.1371E-04)	0.9999 (0.2043E-04)	1.0000 (0.2921E-04)
ITML	1.0001 (0.1802E-05)	1.0000 (0.3235E-05)	1.0002 (0.6231E-05)	1.0001 (0.9843E-05)	1.0000 (0.1371E-04)	0.9999 (0.2037E-04)	1.0000 (0.2867E-04)
ITIN	1.0000 (0.1854E-05)	1.0000 (0.3455E-05)	1.0001 (0.6856E-05)	1.0001 (0.1091E-04)	0.9999 (0.1500E-04)	0.9999 (0.2174E-04)	1.0000 (0.3040E-04)

Table 14--continued

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
.90							
ITCO	1.0000 (0.1302E-05)	1.0000 (0.4485E-05)	0.9999 (0.7806E-05)	1.0000 (0.2040E-04)	0.9995 (0.6072E-04)	0.9999 (0.1279E-03)	1.0002 (0.2196E-03)
ITPW	1.0000 (0.1240E-05)	1.0000 (0.3313E-05)	1.0000 (0.5462E-05)	0.9999 (0.9571E-05)	0.9997 (0.1469E-04)	1.0001 (0.2198E-04)	0.9999 (0.3971E-04)
ITML	1.0000 (0.1251E-05)	1.0000 (0.3309E-05)	1.0000 (0.5504E-05)	0.9999 (0.9555E-05)	1.0002 (0.1449E-04)	1.0002 (0.2226E-04)	0.9998 (0.3944E-04)
ITIN	1.0000 (0.1309E-05)	1.0000 (0.3605E-05)	1.0000 (0.5910E-05)	0.9999 (0.1088E-04)	0.9997 (0.1683E-04)	1.0002 (0.2414E-04)	0.9998 (0.4130E-04)
.95							
ITCO	1.0000 (0.1332E-05)	0.9999 (0.3333E-05)	0.9999 (0.8561E-05)	1.0000 (0.1894E-04)	0.9998 (0.4452E-04)	0.9996 (0.9697E-04)	1.0009 (0.2191E-03)
ITPW	1.0000 (0.1219E-05)	0.9999 (0.2791E-05)	1.0000 (0.6116E-05)	1.0000 (0.9880E-05)	0.9999 (0.1399E-04)	1.0002 (0.2494E-04)	1.0006 (0.5835E-04)
ITML	1.0000 (0.1229E-05)	0.9999 (0.2824E-05)	1.0000 (0.5784E-05)	1.0001 (0.8755E-05)	1.0002 (0.1438E-04)	1.0001 (0.2608E-04)	1.0004 (0.5692E-04)
ITIN	1.0000 (0.1267E-05)	0.9999 (0.3092E-05)	1.0000 (0.6302E-05)	1.0001 (0.9834E-05)	0.9999 (0.1584E-04)	1.0003 (0.2886E-04)	1.0004 (0.6239E-04)

Table 14--continued

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
.99							
ITCO	1.0000 (0.1274E-05)	1.0000 (0.3503E-05)	0.9998 (0.8406E-05)	1.0002 (0.1574E-04)	0.9998 (0.4363E-04)	0.9997 (0.1039E-03)	0.9993 (0.2057E-03)
ITPW	1.0000 (0.1228E-05)	1.0001 (0.2904E-05)	0.9998 (0.5561E-05)	1.0002 (0.8449E-05)	1.0002 (0.1557E-04)	1.0000 (0.2996E-04)	0.9999 (0.5484E-04)
ITML	1.0000 (0.1240E-05)	1.0001 (0.2930E-05)	0.9998 (0.5552E-05)	1.0002 (0.8501E-05)	1.0002 (0.1457E-04)	1.0000 (0.3003E-04)	1.0000 (0.5714E-04)
ITIN	1.0000 (0.1261E-05)	1.0001 (0.3078E-05)	0.9998 (0.6048E-05)	1.0002 (0.9008E-05)	1.0002 (0.1606E-04)	1.0000 (0.3263E-04)	1.0000 (0.6059E-04)

* Mean square error values in parenthesis and reported in exponential form: sample size = 25, theta = 20, true parameter value for beta is one.

Table 15. Average Estimate and Mean Square Error in Estimating Rho.*

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.99							
ITCO	-0.9393	-0.9496	-0.9557	-0.9571	-0.9514	-0.9595	-0.9538
	(0.1503E-01)	(0.9994E-02)	(0.7488E-02)	(0.7657E-02)	(0.8727E-02)	(0.5957E-02)	(0.7491E-02)
ITPW	-0.9987	-1.0035	-1.0044	-1.0093	-1.0020	-1.0141	-1.0035
	(0.1091E-01)	(0.7960E-02)	(0.5993E-02)	(0.5820E-02)	(0.6184E-02)	(0.4740E-02)	(0.4773E-02)
ITML	-0.9441	-0.9523	-0.9556	-0.9592	-0.8911	-0.9638	-0.9571
	(0.1088E-01)	(0.7862E-02)	(0.5869E-02)	(0.5389E-02)	(0.6206E-02)	(0.3793E-02)	(0.4916E-02)
ITIN	-0.9650	-0.9685	-0.9703	-0.9698	-0.9611	-0.9664	-0.9575
	(0.9543E-02)	(0.7735E-02)	(0.5842E-02)	(0.5971E-02)	(0.7028E-02)	(0.5106E-02)	(0.6877E-02)
-.95							
ITCO	-0.8840	-0.8987	-0.8932	-0.9020	-0.8968	-0.8961	-0.9046
	(0.2168E-01)	(0.1459E-01)	(0.1512E-01)	(0.1269E-01)	(0.1406E-01)	(0.1389E-01)	(0.1180E-01)
ITPW	-0.9390	-0.9479	-0.9431	-0.9488	-0.9476	-0.9459	-0.9534
	(0.1801E-01)	(0.1114E-01)	(0.1223E-01)	(0.9703E-02)	(0.1115E-01)	(0.1134E-01)	(0.9192E-02)
ITML	-0.8859	-0.9009	-0.8964	-0.9044	-0.8911	-0.9008	-0.9086
	(0.1822E-01)	(0.1175E-01)	(0.1274E-01)	(0.1033E-01)	(0.1156E-01)	(0.1182E-01)	(0.9271E-02)
ITIN	-0.9079	-0.9174	-0.9099	-0.9144	-0.9069	-0.9029	-0.9083
	(0.1635E-01)	(0.1124E-01)	(0.1223E-01)	(0.1051E-01)	(0.1226E-01)	(0.1266E-01)	(0.1108E-01)

Table 15--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.90							
ITCO	-0.8168	-0.8266	-0.8441	-0.8499	-0.8502	-0.8573	-0.8456
	(0.2810E-01)	(0.2591E-01)	(0.1731E-01)	(0.1817E-01)	(0.1582E-01)	(0.1395E-01)	(0.1761E-01)
ITPW	-0.8605	-0.8728	-0.8922	-0.8909	-0.8943	-0.8981	-0.8935
	(0.2295E-01)	(0.2099E-01)	(0.1440E-01)	(0.1514E-01)	(0.1358E-01)	(0.1212E-01)	(0.1511E-01)
ITML	-0.8167	-0.8317	-0.8506	-0.8508	-0.8911	-0.8572	-0.8517
	(0.2479E-01)	(0.2230E-01)	(0.1475E-01)	(0.1555E-01)	(0.1373E-01)	(0.1209E-01)	(0.1513E-01)
ITIN	-0.8387	-0.8475	-0.8616	-0.8623	-0.8600	-0.8633	-0.8494
	(0.2109E-01)	(0.2029E-01)	(0.1432E-01)	(0.1525E-01)	(0.1409E-01)	(0.1296E-01)	(0.1684E-01)
-.80							
ITCO	-0.7191	-0.7583	-0.7430	-0.7536	-0.7566	-0.7577	-0.7644
	(0.3534E-01)	(0.2464E-01)	(0.2544E-01)	(0.2184E-01)	(0.2010E-01)	(0.2046E-01)	(0.2022E-01)
ITPW	-0.7530	-0.7943	-0.7810	-0.7913	-0.7949	-0.7961	-0.8014
	(0.3019E-01)	(0.2000E-01)	(0.2338E-01)	(0.2100E-01)	(0.1858E-01)	(0.1865E-01)	(0.1978E-01)
ITML	-0.7184	-0.7583	-0.7456	-0.7563	0.1079	-0.7610	-0.7659
	(0.3140E-01)	(0.1946E-01)	(0.2343E-01)	(0.2068E-01)	(0.1799E-01)	(0.1808E-01)	(0.1873E-01)
ITIN	-0.7438	-0.7778	-0.7588	-0.7661	-0.7662	-0.7644	-0.7680
	(0.2851E-01)	(0.1844E-01)	(0.2239E-01)	(0.2024E-01)	(0.1853E-01)	(0.1914E-01)	(0.1971E-01)

Table 15--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.60							
ITCO	-0.5609	-0.5736	-0.5592	-0.5686	-0.5786	-0.5809	-0.5955
	(0.3888E-01)	(0.2945E-01)	(0.3600E-01)	(0.3106E-01)	(0.3057E-01)	(0.2537E-01)	(0.2400E-01)
ITPW	-0.5660	-0.5969	-0.5826	-0.5986	-0.6054	-0.6100	-0.6226
	(0.3838E-01)	(0.2911E-01)	(0.3418E-01)	(0.3111E-01)	(0.3158E-01)	(0.2695E-01)	(0.2550E-01)
ITML	-0.5412	-0.5709	-0.5576	-0.5725	0.1079	-0.5835	-0.5960
	(0.3701E-01)	(0.2730E-01)	(0.3265E-01)	(0.2892E-01)	(0.2916E-01)	(0.2466E-01)	(0.2285E-01)
ITIN	-0.5631	-0.5897	-0.5739	-0.5831	-0.5879	-0.5871	-0.5989
	(0.3767E-01)	(0.2730E-01)	(0.3284E-01)	(0.2869E-01)	(0.2915E-01)	(0.2502E-01)	(0.2369E-01)
-.40							
ITCO	-0.3652	-0.3909	-0.3901	-0.3936	-0.3932	-0.4121	-0.4129
	(0.4606E-01)	(0.3966E-01)	(0.3933E-01)	(0.3995E-01)	(0.3985E-01)	(0.2859E-01)	(0.2831E-01)
ITPW	-0.3752	-0.3986	-0.4119	-0.4206	-0.4175	-0.4307	-0.4319
	(0.4289E-01)	(0.3684E-01)	(0.3778E-01)	(0.4040E-01)	(0.3935E-01)	(0.3248E-01)	(0.3158E-01)
ITML	-0.3584	-0.3814	-0.3943	-0.4024	0.1079	-0.4125	-0.4137
	(0.3978E-01)	(0.3395E-01)	(0.3449E-01)	(0.3630E-01)	(0.3575E-01)	(0.2892E-01)	(0.2820E-01)
ITIN	-0.3736	-0.3960	-0.4069	-0.4115	-0.4060	-0.4174	-0.4160
	(0.4243E-01)	(0.3581E-01)	(0.3615E-01)	(0.3708E-01)	(0.3646E-01)	(0.2965E-01)	(0.2840E-01)

Table 15--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
-.20							
ITCO	-0.1844 (0.4366E-01)	-0.2371 (0.4720E-01)	-0.2141 (0.3999E-01)	-0.2114 (0.4651E-01)	-0.2131 (0.4554E-01)	-0.2314 (0.3734E-01)	-0.2438 (0.3491E-01)
ITPW	-0.1894 (0.4524E-01)	-0.2335 (0.4556E-01)	-0.2229 (0.4010E-01)	-0.2317 (0.4528E-01)	-0.2312 (0.4291E-01)	-0.2449 (0.4096E-01)	-0.2556 (0.3994E-01)
ITML	-0.1813 (0.4155E-01)	-0.2235 (0.4117E-01)	-0.2134 (0.3642E-01)	-0.2218 (0.4095E-01)	0.1079 (0.3888E-01)	-0.2345 (0.3686E-01)	-0.2447 (0.3573E-01)
ITIN	-0.1886 (0.4454E-01)	-0.2330 (0.4460E-01)	-0.2216 (0.3917E-01)	-0.2281 (0.4314E-01)	-0.2263 (0.4068E-01)	-0.2371 (0.3764E-01)	-0.2456 (0.3592E-01)
0.0							
ITCO	-0.0124 (0.3888E-01)	-0.0241 (0.4504E-01)	-0.0384 (0.4425E-01)	-0.0348 (0.4718E-01)	-0.0462 (0.5051E-01)	-0.0606 (0.5086E-01)	-0.0732 (0.4866E-01)
ITPW	-0.0118 (0.4167E-01)	-0.0148 (0.4093E-01)	-0.0314 (0.4266E-01)	-0.0414 (0.4625E-01)	-0.0566 (0.4824E-01)	-0.0721 (0.4909E-01)	-0.0789 (0.5155E-01)
ITML	-0.0112 (0.3818E-01)	-0.0142 (0.3755E-01)	-0.0300 (0.3909E-01)	-0.0396 (0.4241E-01)	0.1079 (0.4420E-01)	-0.0690 (0.4494E-01)	-0.0756 (0.4725E-01)
ITIN	-0.0127 (0.3976E-01)	-0.0161 (0.3997E-01)	-0.0319 (0.4189E-01)	-0.0409 (0.4534E-01)	-0.0548 (0.4678E-01)	-0.0691 (0.4709E-01)	-0.0750 (0.4856E-01)

Table 15--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.20							
ITCO	0.1702 (0.3777E-01)	0.1507 (0.4407E-01)	0.1201 (0.5681E-01)	0.1286 (0.5214E-01)	0.1107 (0.5343E-01)	0.1274 (0.5831E-01)	0.1007 (0.5365E-01)
ITPW	0.1787 (0.4170E-01)	0.1612 (0.4385E-01)	0.1398 (0.4802E-01)	0.1424 (0.4793E-01)	0.1127 (0.5224E-01)	0.1194 (0.5718E-01)	0.1020 (0.5548E-01)
ITML	0.1711 (0.3862E-01)	0.1544 (0.4091E-01)	0.1338 (0.4506E-01)	0.1363 (0.4487E-01)	0.1079 (0.4941E-01)	0.1144 (0.5375E-01)	0.0975 (0.5236E-01)
ITIN	0.1723 (0.3939E-01)	0.1566 (0.4267E-01)	0.1371 (0.4701E-01)	0.1408 (0.4674E-01)	0.1123 (0.5149E-01)	0.1203 (0.5529E-01)	0.1026 (0.5383E-01)
.40							
ITCO	0.3186 (0.4370E-01)	0.3123 (0.4640E-01)	0.2975 (0.5030E-01)	0.2897 (0.6191E-01)	0.3004 (0.5499E-01)	0.2784 (0.6308E-01)	0.2829 (0.5587E-01)
ITPW	0.3342 (0.4483E-01)	0.3284 (0.4595E-01)	0.3233 (0.4293E-01)	0.3125 (0.5505E-01)	0.3187 (0.4935E-01)	0.2812 (0.6247E-01)	0.2840 (0.5254E-01)
ITML	0.3198 (0.4342E-01)	0.3145 (0.4473E-01)	0.3094 (0.4207E-01)	0.2991 (0.5354E-01)	0.1079 (0.4813E-01)	0.2690 (0.6129E-01)	0.2718 (0.5209E-01)
ITIN	0.3234 (0.4401E-01)	0.3203 (0.4540E-01)	0.3161 (0.4255E-01)	0.3082 (0.5412E-01)	0.3158 (0.4861E-01)	0.2802 (0.6197E-01)	0.2826 (0.5291E-01)

Table 15--continued

	Lambda						
Rho	-.75	-.50	-.25	0.0	.25	.50	.75
.60							
ITCO	0.5048 (0.4040E-01)	0.4935 (0.4630E-01)	0.4971 (0.4248E-01)	0.4820 (0.5219E-01)	0.4709 (0.5799E-01)	0.4689 (0.6079E-01)	0.4469 (0.6603E-01)
ITPW	0.5308 (0.3926E-01)	0.5237 (0.4217E-01)	0.5252 (0.4027E-01)	0.5057 (0.4719E-01)	0.5003 (0.4714E-01)	0.4835 (0.5668E-01)	0.4588 (0.5946E-01)
ITML	0.5081 (0.3992E-01)	0.5007 (0.4282E-01)	0.5022 (0.4107E-01)	0.4840 (0.4844E-01)	0.1079 (0.4865E-01)	0.4618 (0.5807E-01)	0.4383 (0.6203E-01)
ITIN	0.5207 (0.4310E-01)	0.5142 (0.4624E-01)	0.5161 (0.4249E-01)	0.4985 (0.4772E-01)	0.4955 (0.4757E-01)	0.4793 (0.5619E-01)	0.4577 (0.5999E-01)
.80							
ITCO	0.6622 (0.4579E-01)	0.6600 (0.4710E-01)	0.6495 (0.5287E-01)	0.6370 (0.5618E-01)	0.6472 (0.5442E-01)	0.6235 (0.7126E-01)	0.5960 (0.8354E-01)
ITPW	0.7029 (0.3805E-01)	0.6952 (0.4044E-01)	0.6906 (0.4329E-01)	0.6733 (0.4791E-01)	0.6834 (0.4548E-01)	0.6567 (0.5742E-01)	0.6256 (0.7009E-01)
ITML	0.6712 (0.4234E-01)	0.6645 (0.4496E-01)	0.6599 (0.4802E-01)	0.6434 (0.5322E-01)	0.1079 (0.5037E-01)	0.6267 (0.6323E-01)	0.5958 (0.7703E-01)
ITIN	0.7094 (0.4668E-01)	0.6987 (0.4690E-01)	0.6909 (0.4946E-01)	0.6745 (0.5316E-01)	0.6824 (0.4883E-01)	0.6525 (0.5967E-01)	0.6237 (0.7060E-01)

Table 15--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.90							
ITCO	0.7374 (0.5011E-01)	0.7372 (0.5162E-01)	0.7393 (0.4819E-01)	0.7281 (0.5445E-01)	0.7079 (0.6887E-01)	0.6965 (0.7691E-01)	0.6684 (0.9308E-01)
ITPW	0.7824 (0.3807E-01)	0.7869 (0.4026E-01)	0.7861 (0.3798E-01)	0.7777 (0.4173E-01)	0.7566 (0.5357E-01)	0.7367 (0.6115E-01)	0.7169 (0.6750E-01)
ITML	0.7468 (0.4516E-01)	0.7505 (0.4683E-01)	0.7501 (0.4470E-01)	0.7418 (0.4902E-01)	-0.8911 (0.6131E-01)	0.7022 (0.6989E-01)	0.6815 (0.7771E-01)
ITIN	0.7986 (0.4309E-01)	0.8035 (0.4575E-01)	0.8017 (0.4180E-01)	0.7893 (0.4641E-01)	0.7623 (0.5773E-01)	0.7388 (0.6435E-01)	0.7161 (0.6860E-01)
.95							
ITCO	0.7741 (0.5345E-01)	0.7683 (0.5622E-01)	0.7687 (0.5963E-01)	0.7572 (0.6485E-01)	0.7448 (0.6787E-01)	0.7467 (0.7373E-01)	0.7031 (0.1025E+00)
ITPW	0.8270 (0.3886E-01)	0.8288 (0.4076E-01)	0.8217 (0.4574E-01)	0.8193 (0.4761E-01)	0.7964 (0.5298E-01)	0.7924 (0.5659E-01)	0.7531 (0.7779E-01)
ITML	0.7882 (0.4713E-01)	0.7880 (0.4873E-01)	0.7825 (0.5401E-01)	0.7785 (0.5585E-01)	-0.8911 (0.6274E-01)	0.7520 (0.6662E-01)	0.7130 (0.8933E-01)
ITIN	0.8453 (0.4247E-01)	0.8486 (0.4413E-01)	0.8380 (0.4974E-01)	0.8352 (0.5102E-01)	0.8070 (0.5650E-01)	0.7957 (0.6015E-01)	0.7519 (0.7933E-01)

Table 15--continued

Rho	Lambda -.75	-.50	-.25	0.0	.25	.50	.75
.99							
ITCO	0.7943 (0.6058E-01)	0.7861 (0.6474E-01)	0.7870 (0.6369E-01)	0.7855 (0.6589E-01)	0.7688 (0.7700E-01)	0.7616 (0.8359E-01)	0.7327 (0.1101E+00)
ITPW	0.8584 (0.4263E-01)	0.8493 (0.4719E-01)	0.8545 (0.4354E-01)	0.8452 (0.4934E-01)	0.8287 (0.5534E-01)	0.8161 (0.6593E-01)	0.7818 (0.8465E-01)
ITML	0.8145 (0.5214E-01)	0.8064 (0.5733E-01)	0.8100 (0.5362E-01)	0.8026 (0.5962E-01)	-0.8911 (0.6650E-01)	0.7731 (0.7744E-01)	0.7382 (0.9824E-01)
ITIN	0.8836 (0.4257E-01)	0.8719 (0.4832E-01)	0.8710 (0.4617E-01)	0.8636 (0.5104E-01)	0.8411 (0.5803E-01)	0.8229 (0.6836E-01)	0.7865 (0.8650E-01)

* Mean square error values in parenthesis and reported in exponential form, sample size = 25, theta = 20.

Table 16. Average estimate and mean square error values for a growth trend.*

Rho	Estimates		
	Alpha	Beta	Rho
-.99			
ITCO	1.0000 (0.2160E-03)	0.9999 (0.5644E-04)	-0.9600 (0.5476E-02)
ITPW	1.0000 (0.2171E-03)	0.9999 (0.5644E-04)	-1.0111 (0.4484E-02)
ITML	1.0001 (0.2143E-03)	0.9999 (0.5609E-04)	-0.9623 (0.3895E-02)
ITIN	1.0009 (0.4470E-03)	0.9996 (0.1103E-03)	-0.9606 (0.5425E-02)
-.95			
ITCO	1.0001 (0.2398E-03)	1.0000 (0.6431E-04)	-0.9053 (0.1158E-01)
ITPW	1.0001 (0.2420E-03)	1.0000 (0.6474E-04)	-0.9514 (0.8977E-02)
ITML	1.0001 (0.2432E-03)	1.0000 (0.6521E-04)	-0.9081 (0.9455E-02)
ITIN	1.0000 (0.3242E-03)	1.0001 (0.8329E-04)	-0.9062 (0.1139E-01)

Table 16--continued

Rho	Estimates		
	Alpha	Beta	Rho
-.90			
ITCO	1.0003 (0.2400E-03)	0.9999 (0.6901E-04)	-0.8515 (0.1545E-01)
ITPW	1.0003 (0.2386E-03)	0.9999 (0.6895E-04)	-0.8943 (0.1342E-01)
ITML	1.0004 (0.2377E-03)	0.9999 (0.6867E-04)	-0.8526 (0.1369E-01)
ITIN	1.0003 (0.2519E-03)	0.9998 (0.7295E-04)	-0.8522 (0.1540E-01)
-.80			
ITCO	0.9997 (0.2628E-03)	1.0001 (0.7659E-04)	-0.7709 (0.2002E-01)
ITPW	0.9997 (0.2564E-03)	1.0001 (0.7497E-04)	-0.8109 (0.2006E-01)
ITML	0.9996 (0.2562E-03)	1.0001 (0.7488E-04)	-0.7745 (0.1828E-01)
ITIN	0.9996 (0.2589E-03)	1.0002 (0.7540E-04)	-0.7714 (0.2011E-01)
-.60			
ITCO	0.9996 (0.3748E-03)	1.0002 (0.1019E-03)	-0.5847 (0.3036E-01)
ITPW	0.9996 (0.3703E-03)	1.0001 (0.1010E-03)	-0.6141 (0.3286E-01)
ITML	0.9996 (0.3709E-03)	1.0001 (0.1012E-03)	-0.5868 (0.2963E-01)
ITIN	0.9997 (0.3749E-03)	1.0001 (0.1016E-03)	-0.5849 (0.3049E-01)

Table 16--continued

Rho	Estimates Alpha	Beta	Rho
-.40			
ITCO	1.0003 (0.4679E-03)	0.9997 (0.1277E-03)	-0.4103 (0.3556E-01)
ITPW	0.9999 (0.4406E-03)	0.9999 (0.1216E-03)	-0.4307 (0.3868E-01)
ITML	0.9999 (0.4404E-03)	0.9999 (0.1215E-03)	-0.4119 (0.3451E-01)
ITIN	0.9999 (0.4392E-03)	0.9999 (0.1211E-03)	-0.4118 (0.3480E-01)
-.20			
ITCO	1.0006 (0.5867E-03)	0.9994 (0.1412E-03)	-0.2342 (0.4313E-01)
ITPW	1.0002 (0.5266E-03)	0.9996 (0.1323E-03)	-0.2459 (0.4742E-01)
ITML	1.0002 (0.5251E-03)	0.9996 (0.1316E-03)	-0.2350 (0.4258E-01)
ITIN	1.0001 (0.5194E-03)	0.9996 (0.1310E-03)	-0.2352 (0.4281E-01)
0.0			
ITCO	0.9996 (0.6354E-03)	1.0000 (0.1446E-03)	-0.0427 (0.4897E-01)
ITPW	0.9992 (0.5980E-03)	1.0001 (0.1369E-03)	-0.0447 (0.5367E-01)
ITML	0.9992 (0.5946E-03)	1.0001 (0.1359E-03)	-0.0429 (0.4887E-01)
ITIN	0.9992 (0.5956E-03)	1.0001 (0.1359E-03)	-0.0434 (0.4909E-01)

Table 16--continued

Rho	Estimates: Alpha	Beta	Rho
.20			
ITCO	1.0000 (0.7556E-03)	1.0003 (0.1520E-03)	0.1371 (0.4664E-01)
ITPW	1.0000 (0.6743E-03)	1.0003 (0.1447E-03)	0.1430 (0.5013E-01)
ITML	1.0000 (0.6711E-03)	1.0003 (0.1436E-03)	0.1364 (0.4671E-01)
ITIN	1.0000 (0.6686E-03)	1.0003 (0.1433E-03)	0.1378 (0.4688E-01)
.40			
ITCO	0.9977 (0.1004E-02)	1.0000 (0.1532E-03)	0.2951 (0.5455E-01)
ITPW	0.9971 (0.9199E-03)	1.0002 (0.1482E-03)	0.3110 (0.5702E-01)
ITML	0.9971 (0.9146E-03)	1.0002 (0.1472E-03)	0.2970 (0.5520E-01)
ITIN	0.9965 (0.1036E-02)	1.0002 (0.1477E-03)	0.2994 (0.5592E-01)
.60			
ITCO	1.0008 (0.1554E-02)	0.9998 (0.1299E-03)	0.4849 (0.5256E-01)
ITPW	1.0014 (0.1321E-02)	0.9999 (0.1244E-03)	0.5118 (0.5096E-01)
ITML	1.0014 (0.1311E-02)	0.9999 (0.1249E-03)	0.4885 (0.5178E-01)
ITIN	1.0018 (0.1963E-02)	0.9999 (0.1255E-03)	0.5000 (0.5636E-01)

Table 16--continued

	Estimates		
Rho	Alpha	Beta	Rho
.80			
ITCO	1.0025 (0.6060E-02)	1.0003 (0.1108E-03)	0.6521 (0.5056E-01)
ITPW	0.9990 (0.3399E-02)	1.0003 (0.1088E-03)	0.6892 (0.4363E-01)
ITML	0.9991 (0.3332E-02)	1.0004 (0.1103E-03)	0.6573 (0.4877E-01)
ITIN	0.9960 (0.6657E-02)	1.0003 (0.1133E-03)	0.6925 (0.5220E-01)
.90			
ITCO	0.9848 (0.1215E+00)	0.9999 (0.9677E-04)	0.7439 (0.5077E-01)
ITPW	1.0006 (0.1052E-01)	0.9999 (0.9585E-04)	0.7941 (0.3984E-01)
ITML	1.0005 (0.8905E-02)	0.9998 (0.9777E-04)	0.7555 (0.4585E-01)
ITIN	1.0042 (0.1776E-01)	0.9998 (0.9995E-04)	0.8085 (0.4556E-01)
.95			
ITCO	1.0941 (0.6079E+01)	1.0002 (0.1044E-03)	0.7609 (0.6033E-01)
ITPW	0.9987 (0.4881E-01)	1.0002 (0.1028E-03)	0.8157 (0.4531E-01)
ITML	1.0018 (0.2524E-01)	1.0001 (0.1030E-03)	0.7762 (0.5409E-01)
ITIN	1.0141 (0.3792E-01)	1.0002 (0.1037E-03)	0.8387 (0.4814E-01)

Table 16--continued

	Estimates		
Rho	Alpha	Beta	Rho
.99			
ITCO	0.9924 (0.2024E+01)	1.0006 (0.7981E-04)	0.7970 (0.6155E-01)
ITPW	0.9906 (0.2011E+00)	1.0006 (0.7655E-04)	0.8654 (0.4126E-01)
ITML	1.0016 (0.1450E+00)	1.0006 (0.7615E-04)	0.8191 (0.5072E-01)
ITIN	0.9945 (0.1617E+00)	1.0005 (0.7735E-04)	0.8843 (0.4379E-01)

* Mean square error values in parenthesis and reported in exponential form, sample size = 25, variance on trend component = 1.00, true parameter values for alpha and beta are one.

Table 17. Average estimate and mean square error values for a growth trend.*

	Estimates		
Rho	Alpha	Beta	Rho
-.99			
ITCO	1.0001 (0.5476E-03)	1.0000 (0.1700E-03)	-0.9603 (0.5369E-02)
ITPW	1.0002 (0.5447E-03)	0.9999 (0.1692E-03)	-1.0116 (0.4412E-02)
ITML	1.0002 (0.5419E-03)	0.9999 (0.1686E-03)	-0.9628 (0.3784E-02)
ITIN	1.0020 (0.1265E-02)	0.9991 (0.3415E-03)	-0.9616 (0.5260E-02)
-.95			
ITCO	1.0009 (0.5830E-03)	0.9995 (0.1794E-03)	-0.9054 (0.1164E-01)
ITPW	1.0009 (0.5880E-03)	0.9995 (0.1808E-03)	-0.9516 (0.9099E-02)
ITML	1.0010 (0.5901E-03)	0.9995 (0.1814E-03)	-0.9081 (0.9513E-02)
ITIN	1.0012 (0.7775E-03)	0.9994 (0.2244E-03)	-0.9063 (0.1150E-01)

Table 17--continued

Rho	Estimates		
	Alpha	Beta	Rho
-.90			
ITCO	1.0007 (0.5276E-03)	0.9997 (0.1666E-03)	-0.8526 (0.1456E-01)
ITPW	1.0006 (0.5245E-03)	0.9998 (0.1660E-03)	-0.8951 (0.1264E-01)
ITML	1.0005 (0.5227E-03)	0.9998 (0.1657E-03)	-0.8536 (0.1292E-01)
ITIN	1.0000 (0.5733E-03)	1.0000 (0.1776E-03)	-0.8534 (0.1446E-01)
-.80			
ITCO	1.0017 (0.6282E-03)	0.9989 (0.1965E-03)	-0.7741 (0.1880E-01)
ITPW	1.0018 (0.6190E-03)	0.9989 (0.1940E-03)	-0.8137 (0.1890E-01)
ITML	1.0019 (0.6188E-03)	0.9988 (0.1937E-03)	-0.7774 (0.1701E-01)
ITIN	1.0022 (0.6594E-03)	0.9987 (0.2045E-03)	-0.7749 (0.1869E-01)
-.60			
ITCO	0.9992 (0.7956E-03)	1.0004 (0.2285E-03)	-0.5869 (0.2882E-01)
ITPW	0.9990 (0.7840E-03)	1.0005 (0.2263E-03)	-0.6159 (0.3107E-01)
ITML	0.9990 (0.7852E-03)	1.0005 (0.2267E-03)	-0.5889 (0.2786E-01)
ITIN	0.9990 (0.7998E-03)	1.0005 (0.2299E-03)	-0.5877 (0.2876E-01)

Table 17--continued

Rho	Estimates Alpha	Beta	Rho
-.40			
ITCO	1.0001 (0.1030E-02)	0.9998 (0.3131E-03)	-0.4254 (0.3011E-01)
ITPW	0.9996 (0.9847E-03)	1.0001 (0.2997E-03)	-0.4450 (0.3408E-01)
ITML	0.9996 (0.9840E-03)	1.0001 (0.2995E-03)	-0.4260 (0.2992E-01)
ITIN	0.9996 (0.9877E-03)	1.0000 (0.3006E-03)	-0.4263 (0.3003E-01)
-.20			
ITCO	1.0011 (0.1500E-02)	0.9992 (0.4549E-03)	-0.2567 (0.3863E-01)
ITPW	1.0004 (0.1379E-02)	0.9995 (0.4270E-03)	-0.2683 (0.4299E-01)
ITML	1.0004 (0.1378E-02)	0.9995 (0.4267E-03)	-0.2569 (0.3834E-01)
ITIN	1.0004 (0.1373E-02)	0.9996 (0.4253E-03)	-0.2573 (0.3879E-01)
0.0			
ITCO	1.0007 (0.2206E-02)	0.9994 (0.6501E-03)	-0.0784 (0.4651E-01)
ITPW	1.0001 (0.1979E-02)	0.9997 (0.6019E-03)	-0.0815 (0.5060E-01)
ITML	1.0001 (0.1977E-02)	0.9997 (0.6014E-03)	-0.0781 (0.4634E-01)
ITIN	1.0001 (0.1976E-02)	0.9997 (0.6011E-03)	-0.0785 (0.4671E-01)

Table 17--continued

Rho	Estimates		
	Alpha	Beta	Rho
.20			
ITCO	0.9993 (0.3103E-02)	1.0005 (0.8718E-03)	0.0953 (0.4721E-01)
ITPW	0.9995 (0.2460E-02)	1.0004 (0.7395E-03)	0.0988 (0.4977E-01)
ITML	0.9995 (0.2459E-02)	1.0005 (0.7391E-03)	0.0946 (0.4731E-01)
ITIN	0.9994 (0.2447E-02)	1.0005 (0.7368E-03)	0.0962 (0.4760E-01)
.40			
ITCO	0.9916 (0.1010E-01)	1.0025 (0.1916E-02)	0.2362 (0.6689E-01)
ITPW	0.9942 (0.4879E-02)	1.0018 (0.1370E-02)	0.2464 (0.6686E-01)
ITML	0.9942 (0.4860E-02)	1.0018 (0.1367E-02)	0.2358 (0.6651E-01)
ITIN	0.9945 (0.5304E-02)	1.0017 (0.1457E-02)	0.2395 (0.6723E-01)
.60			
ITCO	0.9994 (0.3871E-01)	0.9990 (0.4619E-02)	0.4114 (0.7307E-01)
ITPW	1.0047 (0.9695E-02)	0.9980 (0.2730E-02)	0.4309 (0.6800E-01)
ITML	1.0046 (0.9700E-02)	0.9980 (0.2731E-02)	0.4120 (0.7126E-01)
ITIN	1.0047 (0.1042E-01)	0.9979 (0.2853E-02)	0.4235 (0.7223E-01)

Table 17--continued

Rho	Estimates Alpha	Beta	Rho
.80			
ITCO	0.9896 (0.9689E-01)	1.0050 (0.1225E-01)	0.5416 (0.1047E+00)
ITPW	0.9944 (0.2420E-01)	1.0031 (0.6257E-02)	0.5770 (0.9117E-01)
ITML	0.9935 (0.2359E-01)	1.0036 (0.6080E-02)	0.5506 (0.9955E-01)
ITIN	0.9942 (0.2591E-01)	1.0032 (0.6831E-02)	0.5823 (0.9669E-01)
.90			
ITCO	1.0007 (0.1964E+00)	0.9978 (0.2220E-01)	0.6154 (0.1176E+00)
ITPW	1.0071 (0.5127E-01)	0.9952 (0.1384E-01)	0.6570 (0.9714E-01)
ITML	1.0071 (0.5160E-01)	0.9962 (0.1386E-01)	0.6267 (0.1091E+00)
ITIN	1.0074 (0.5533E-01)	0.9961 (0.1426E-01)	0.6658 (0.1013E+00)
.95			
ITCO	0.9984 (0.2238E+00)	0.9945 (0.2565E-01)	0.6197 (0.1485E+00)
ITPW	1.0268 (0.8314E-01)	0.9868 (0.1647E-01)	0.6602 (0.1254E+00)
ITML	1.0269 (0.8348E-01)	0.9859 (0.1686E-01)	0.6295 (0.1402E+00)
ITIN	1.0279 (0.8405E-01)	0.9895 (0.1707E-01)	0.6727 (0.1276E+00)

Table 17--continued

Rho	Estimates Alpha	Beta	Rho
.99			
ITCO	0.9844 (0.4851E+00)	1.0102 (0.3571E-01)	0.6352 (0.1644E+00)
ITPW	0.9884 (0.2338E+00)	1.0077 (0.2765E-01)	0.6852 (0.1330E+00)
ITML	0.9887 (0.2346E+00)	1.0078 (0.2846E-01)	0.6520 (0.1502E+00)
ITIN	0.9928 (0.2335E+00)	1.0049 (0.2790E-01)	0.6985 (0.r1361E+00)

* Mean square error values in parenthesis and reported in exponential form, sample size = 25, variance on trend component = 0.0009, true parameter values for alpha and beta are one.