Phase separation in the flow of suspensions through a bifurcation with applications to blood flow in the microcirculation

by Robert Darrell Olson

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in CHEMICAL ENGINEERING
Montana State University

Abstract:
Quantitative analysis of blood is complicated by several factors.

One observable phenomenon is plasma skimming. Plasma skimming is the reduction in concentration present in the more slowly flowing branch of a bifurcation.

Due to the problems associated with quantitative measurements on systems the size of vessels in microcirculation, and because of the complex nature of non-Newtonian blood flow in small vessels, plasma skimming was investigated by means of a model using a Newtonian suspension of rigid spheres.

The present research investigated situations of vertical flows through a bifurcation having equal diameter branches. Also considered were horizontal flows through 45° and 90° bifurcations having equal diameter branches and a 90° bifurcation whose side branch was two-thirds the diameter of the main branch. The Reynolds numbers for vertical and horizontal flows, based on flows through the upstream branch and using the physical properties of the suspending media, were 3 x 10^-2 and 160 respectively. Concentration changes were measured using tube and mixing cup concentration ratios.

On the basis of this research, it was concluded that the ratio of relative flow rates in the two branches downstream from the bifurcation and the ratio of the diameters of these branches were important parameters in determining the extent of plasma skimming. The angle of bifurcation and the upstream concentration had little effect for the 45° and 90° bifurcations with equal diameter branches. The 90° bifurcation having the smaller diameter side branch exhibited a dependence on upstream concentration. The degree of plasma skimming being less at the higher concentrations. Of unknown importance is the particle to tube size ratio, the shape and flexibility of the particle, and the density of differences between particles and the fluid. There is also the possibility that inertial effects may be present in the horizontal flows.

The mixing cup and tube concentration ratios are identical for bifurcations having equal diameter downstream branches. For bifurcations having different diameter downstream branches, the mixing cup and tube concentration ratios are not identical.

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Date August 7, 1973

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August, 1973

ACKNOWLEDGMENTS

The author wishes to thank the entire faculty of the Chemical Engineering Department of Montana State University for their advice and assistance. A special note of thanks do Dr. G. R. Cokelet under whose supervision and guidance this project was completed.

Financial support for this study was provided by the National Heart and Lung Institute, Grant #HL 12723.

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ABSTRACT

Quantitative analysis of blood is complicated by several factors. One observable phenomenon is plasma skimming. Plasma skimming is the reduction in concentration present in the more slowly flowing branch of a bifurcation.

Due to the problems associated with quantitative measurements on systems the size of vessels in microcirculation, and because of the complex nature of non-Newtonian blood flow in small vessels, plasma skimming was investigated by means of a model using a Newtonian suspension of rigid spheres.

The present research investigated situations of vertical flows through a 45° bifurcation having equal diameter branches. Also considered were horizontal flows through 45° and 90° bifurcations having equal diameter branches and a 90° bifurcation whose side branch was two-thirds the diameter of the main branch. The Reynolds numbers for vertical and horizontal flows, based on flows through the upstream branch and using the physical properties of the suspending media, were 3×10^{-2} and 160 respectively. Concentration changes were measured using tube and mixing cup concentration ratios.

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INTRODUCTION

The smallest vessels of the circulation system are the capillaries. The capillaries form a multiple branched network connecting the arteries and veins. It is in this highly branched network that the transfer of metabolites between the blood stream and adjacent cells occurs. This transfer occurs via passive diffusion and active transport. Because of their basic survival function, the problems of flows through capillaries provides a challenging field of study.

Capillary blood vessels were first discovered by Marcello Malpighi in 1661. One would think that during the 300 years since then, the mechanics of blood flow in the capillaries would be well understood; this is not the case however. The lack of understanding is due in part to the fact that capillaries are very small. Systemic capillaries in humans are about 6×10^{-6} meters (6 microns) in diameter and several hundred microns in length. Also, the mechanical properties of the red blood cells and the cells which compose the capillary are not well defined. It is reasonable to say that the investigation of the mechanics of blood flow in the microcirculation is still exploratory in nature (11).

In 1921, Krogh (18) found that when flow in a small artery branching from a larger vessel was slowed by partial contraction the hematocrit or volume fraction of red blood cells in the artery may be markedly reduced Occasionally to the extent that practically no cells could be seen in the side branch. This phenomenon came to be known as plasma skimming. While posing an interesting fluid dynamics problem, plasma skimming may be of some considerable importance physiologically. It should be noted that blood flow in the microcirculation is under a complex system of vascular controls. These controls operate via three basic mechanisms; 1) basal tone and local control, 2) neurogenic control, and 3) hormonal control. These controls have pronounced effects on blood flows and distributions in the microcirculation(8). These mechanisms of control are by no means completely understood but are presented to give "plasma skimming" perspective. The degree of plasma skimming is the resultant of the effects of the various control mechanisms. By understanding the phenomenon, perhaps a better understanding of its causes will follow.

Few systematic studies of plasma skimming exist. Those that do are of three basic types with special problems and difficulties inherent to each. In vivo or experiments performed on the animal, have special problems of measurements and minimization of unwanted effects. In vitro experiments, invclve exteriorizing the experiment from the animal but uses the same physical set up of animal. Modeling experiments use a convenient scale to perform the experiments. Care must be taken so that the model accurately represents the physical situation which is the basis of the model, known as the prototype.

Svanes and Zweifach (26) observed that by partially occluding capillaries there was a slowing of the flow in the capillary due to the increased resistance and a striking reduction in capillary hematocrit. In situations where the flow in the parent branch was rapid, the capillary hematocrit was reduced to nearly zero. When the flow in the parent branch was slow, the reduction in hematocrit was only slight. There was no significant change in vessel diameter except at the point of compression. While providing some useful information, these data are of purely qualitative nature and do little to quantify the parameters important to the phenomenon.

Johnson, Blaschke, et al, (15) simultaneously measured red cell spacing and red cell velocity in the mesenteric capillaries of the cat and obtained some semi-quantitative data on plasma skimming. Under control conditions, most capillaries exhibited reasonably stable hematocrits, although some showed erratic changes and a few exhibited a well defined periodicity. During reductions in gross flow, capillary hematocrit was generally unchanged though some showed a reproducible increase or decrease. They found that in some cases, capillary hematocrit was affected by vasomotion. Though providing no quantitative information about the important parameters for plasma skimming, Johnson's work did present values of the relative hematocrit change associated with a change in capillary flow. Of importance is the fact that these experiments by their very nature, may have a significant effect upon the degree of plasma skimming. The major weakness of these experiments is that they provide no quantitative values for the relative size of the flows involved.

Gelin (12) perfused blood through channels etched in blocks of polymethyl methacrylate. The two channels crossed to form a perpendicular junction. Investigation of channels of various diameters indicated that plasma skimming would be significant in vessels smaller than 110 microns. It was found that increasing the degree of aggregation of the red blood cells by the addition of high molecular weight dextran, the degree of plasma skimming increased. The usefulness of this information is limited even though it provides the first quantitative data on plasma skimming. The most important objection to the data is that one rarely sees two branches leaving the parent vessel at the same location in the microcirculation. Also, the flows in the three branches were nearly the same and provided no information for other flow situations.

The most comprehensive treatment of the subject is the work of Bugliarello and Hsiao (5). Modeling blood with a suspension of rigid, neutrally buoyant spheres, they investigated the importance of total flow rate, concentration, angle of bifurcation and relative flow rates in the branches, in determining the degree of plasma skimming. The suspension particles were chosen so that the diameter of the tube was ten times the particle diameter, a size ratio in the range where plasma skimming has been observed. Using bifurcations made from Lucite blocks and under situations of vertical flow, the concentration of spheres in the side branch was less than for the main branch for almost all flow conditions. The concentrations measured were the outflow or mixing cup concentrations. The magnitude of the total flow rate had only a minor effect on the degree of plasma skimming. For bifurcations of 45° and 90°, whose side branch is the same diameter as the main branch, the effect of the angle of branching was negligible. Using a bifurcation of 45° whose side branch was one half the diameter of the main branch, it was found that the degree of plasma skimming was less than for the corresponding situation when the branches were of equal diameters. It was found that higher concentrations lessen the effect of plasma skimming, all other factors being equal.

Of unknown importance in attempting to relate observations discussed here is the fact that the concentrations measured or observed are of different types. In the work by Johnson, the concentrations measured were those in the vessel while Gelin and Bugliarello determined the concentration of the outflow, known as the mixing cup concentration. Superficially, this seems to be a relatively minor consideration. However, because of the nature of the Fahraeus effect, significant complications to correlations between the two types of measurements may result. Fahraeus and Lindqvist (7) observed in 1931 that for blood flowing in small capillary tubes, the calculated viscosity of blood decreases as the tube diameter decreases from 500 to 40 microns. This dependence of viscosity upon tube diameter has been explained by assuming a cell free plasma layer at the tube wall and a core of the same concentration as the feed reservoir. This latter assumption is erroneous and results in a calculated plasma layer much too large. Data of Barbee and Cokelet (1) show that the average tube hematocrit is less than the hematocrit for the reservoir feeding the tube. It was also found that for tubes larger than 20-25 microns, the discharge or mixing cup hematocrit was the same as the feed reservoir hematocrit while for tubes smaller than 20-25 microns, the mixing cup hematocrit was less than that for the feed reservoir. Research now indicates that at normal physiological hematocrits, the reduced cell concentrations at the vessel walls is due primarily to physical exclusion of cells and is known as the "smooth-wall" or "Vand" effect (4, 20, 27). It is apparent that extreme caution is required when attempting to correlate data obtained by different techniques, due to the fact the concentration in the vessels is not the same as the concentration of the outflow.

The phenomenon of plasma skimming has been attributed to the nature of the fluid elements deflected into the side branch. Since the fluid at the periphery of the upstream branch moves slower, it is easier to deflect into the side branch. The composition of this fluid will determine the extent to which plasma skimming is observed. In order to understand plasma skimming, it is first necessary to understand to a certain extent, the behavior of suspensions flowing in a straight tube. It would be useful to understand the hydrodynamic aspects of flow division at a bifurcation. Perhaps these concepts can be applied to the more complicated case of suspensions flowing through a bifurcation. With these considerations, a sufficient condition for plasma skimming would be that a radial concentration gradient exist upstream. Specifically, the concentration near the wall must be less than the concentration in the center of the tube. This radial gradient has a theoretical basis which is supported by experimental data. The most substantial evidence for a radial concentration gradient is that single particles and very dilute suspensions exhibit radial migration. The force responsible for this displacement originates in the inertia of the fluid and it is postulated to be akin to the Magnus effect where a transverse force arises from the combination of rotary and translatory motion relative to the undisturbed flow of the fluid. Segre and Silverberg (23) observed that single spheres migrated radially inward from the wall and outward from the axis until a equilibrium position was reached. The effects of the radial velocity gradient in Poiseuille flow is sufficient to explain inward radial migration but cannot account the outward movement. Inward migration can be evaluated by analogy to the theory of Rubinow and Keller (27) for a rotating sphere.

$\vec{F} = \pi a^3 \rho_f (\vec{\omega} \times \vec{V})$ Where $\vec{F}$ is the force on the sphere; $\vec{V}$ the particle velocity; $\vec{\omega}$ the rotational velocity.

While this equation was not derived for Poiseuille flows, it is reasonable to assume that it yields a force of the correct order of magnitude for such a flow. Repetti and Leonard (21) were able to explain the outward migration by modifying the above equation and by considering that the sphere will influence the fluid to some distance δ past the sphere surface. This modification produces moderately good agreement with experimental data. This was presented only to show that there exists a force which would tend to produce radial concentration gradients for Poiseuille flow. It should be noted that this effect is expected to be much less pronounced at higher concentrations because of particle-particle interactions. At concentrations above 35%, the concentration decrease at the vessel wall can be attributed almost entirely to mechanical exclusion by the wall (16). These higher concentrations produce a velocity profile which is blunted considerably from the normal parabolic. This blunting is found to occur with blood (13). as well as suspensions of rigid spheres (16).

Also since inflow and mixing cup hematocrits are equal even though the tube hematocrits are lower, for blood flow in small straight tubes, there must be a radial red cell concentration gradient. Otherwise, a material balance on the tube could not be closed.

The nature of flows at a bifurcation was studied by Barnett and Cochrane (2) for Newtonian fluids. The limiting streamline, the streamline which just passes by the side branch without entering, was determined for bifurcations of 45° and 90°. It was found that the position of the critical streamline was determined by the relative flow rates, the angle of branching, the viscosity of the fluid, and the diameter of the side branch. The slower the flow in the side branch compared to the flow in the downstream branch, the further the streamline is from the center of the upstream tube. For a given value of relative flow rates, the smaller angle withdraws fluid more toward the center of the upstream branch. This effect is not extreme however. The higher viscosity fluid withdraws more fluid from the center of the upstream branch, all other factors being equal.

It has been demonstrated that the fluid deflected into a branch is the fluid closest to that branch. While the presence of particles in a suspension may have an effect, it is not expected that the basic nature of the flow division would be affected drastically.

In engineering, it is commonplace to evaluate performance characteristics of equipment and units by test procedures using a conveniently sized model. Models are useful whenever the size or expense of the prototype is such that experimentation on it is impractical. Model theory is dependent upon several criteria of similarity. Geometric similarity exists if all the corresponding dimensions of the model and prototype are in the same ratio. Kinematic similarity exists if all the velocities at corresponding positions have a constant ratio. Dynamic similarity is a point-to-point correspondence between inertial, normal, shear, and field forces for the two systems. Geometric similarity is a prerequisite to kinematic and dynamic similarity because of the requirement of corresponding positions (9). If there is complete similarity between systems, all the corresponding dimensionless groups have the same values in the model and prototype. The solutions to the differential equations of flow then will be the same for the two systems.

Dimensional analysis consists of characterizing a dimensionally homogeneous equation by a relationship among a complete set of dimensionless products. Dimensional homogeneity means that each term in an equation has the same dimensions. There are several possible methods of dimensional analysis but the Buckingham Pi Method has the advantages of relative ease of computation and is applicable even when the differential equation of flow describing the situation is unknown (3).

Analysis starts by postulating that some physical process Q is a function of the n dimensional variables Q_1, Q_2, $\cdots$ Q_n.

$$Q = f\ (Q_1, Q_2, \cdots Q_n)$$

Since the equation describing the behavoir of the systems must be dimensionally homogeneous, this equation can be reduced to

$$0 = f(\pi_1, \pi_2, \cdots \pi_i)$$

where π_1, π_2, $\cdots\pi_i$ are dimensionless groups.

The number of dimensionless groups i is given by

$$i = n - r$$

where n = number of variables

r = maximum number of these variables which will not form a dimensionless group

The dimensionless groups are expressed by

$$\pi_1 = Q_1^{a1} \quad Q_2^{b1} \cdots Q_j^{j1}$$

$$\pi_2 = Q_1^{a2} \quad Q_2^{b2} \cdots Q_j^{j2}$$

$$\pi_i = Q_1^{ai} \quad Q_2^{bi} \cdots Q_j^{ji}$$

where the exponents are such that the π's are dimensionless.

There are serious limitations to the use of dimensional analysis. Dimensional analysis gives no indication of the fundamental mechanism of the process. Also, the analysis is invalid if any significant parameter is neglected (17).

The primary objective of this research is the quantification of the important parameters of plasma skimming. The first step involved in such an undertaking is the identification of all possible parameters that could possibly have an effect on the phenomenon. From these parameters, dimensionless groups are formed; these will provide the basis of analysis. Through a careful consideration of the fluid dynamic aspects of the flow of suspensions and the relatively complex geometry of the flow situation, the parameters associated with this situation become apparent. Table I presents the parameters considered to have a possible effect on plasma skimming. Also listed are the appropriate symbols and the dimensions for each quantity.

TABLE I. IMPORTANT PARAMETERS

Parameter	Symbol	Dimensions
Upstream Concentration	ϕ	None
Total Flow Rate	V	L^3/T
Relative Flow Rates, Side to Downstream	Vs/Vds	None
Angle of Bifurcation	θ	None
Ratio of particle to upstream tube diam.	Dp/Dt	None
Ratio of the diameter of the side branch to downstream branch	Ds/Dds	None
Diameter of the branch	D	L
Density of the media	ρ	M/L^3
Density of the particle	ρ_p	M/L^3
Orientation relative to gravity		
Shape of the particle	Φ	
Yield Stress	τ_y	F/L^2 or M/LT^2
Flexibility of the particle	E	F/L^2 or M/LT^2
Viscosity of the media	μ	M/LT

F = Force; M = Mass; L = Length; T = Time

Since a number of the parameters are dimensionless, no further characterization is necessary. However, the remaining dimensional quantities must be combined to form dimensionless groups. The result of such an analysis combined with the dimensionless parameters is given in Table II.

TABLE II. DIMENSIONLESS GROUPS

ϕ	Φ	θ
Vs/Vds	$U^2\rho/E$	Orientation relative to gravity
Dp/Dt	E/τ_y	
Ds/Dds	$U\mu/DE$	

Where $U= 4V/\pi D^2$

The orientation relative to gravity may have an effect and will be considered but not included in the analysis of dimensionless groups. A word must be said about the parameters of shape, flexibility and yield stress.

A red blood cell is a biconcave disc and as such, a rather difficult shape to reproduce in quantity on a macroscopic sacle. Also there is the problem of how to characterize the shape. There have been a variety of methods used ranging from determining an equivalent spherical diameter to using ratios of the radii of curvature.

The flexibility of the red cell poses a rather complicated problem for modeling on a macroscopic scale. Values of elastic moduli for the red cell membrane of 10^4 and 10^8 dynes per centimeter squared have been reported (11, 14). These widely different values are explained as the results of measurements of two different stress mechanisms. Apparently the red cell membrane deforms in two different ways. A change in shape with constant membrane area requires only small stresses, while changes in membrane area require large stresses.

A practial definition of yield stress is that critical stress below which fluids plastically deform but do not viscously flow, given a reasonable period of observation. Blood has a yield stress whereas most suspensions do not.

Since these parameters involve quantities which would be difficult if not impossible to model on a macroscopic scale, the model will be simplified by using rigid spherical particles in a suspension having no yield stress.

Eliminating the three parameters discussed leaves the dimensionless groups listed in Table III. Also shown are representative values of these groups based on blood flow in microcirculation along with corresponding values for the model.

TABLE III VALUES FOR THE VARIOUS DIMENSIONLESS GROUPS

Dimensionless Group	Micro-circulation	Vertical Flow	Horizontal Flow
ϕ	$<40\%$	20%	20% - 10%
V_s/V_{ds}	0 - 1.0	.33 - 2.0	.03 - 3.0
θ	≤ 90	45°	45°, 90°
D_p/D_t	$>.07$	.333	.333
D_s/D_{ds}	<1.0	1.0	1.0, .667
$DU\rho/\mu$	$10^2 - 10$	3.2×10^{-2}	40 - 160
ρ_p/ρ	1.07	1.08	~ 1.0

The group identifiable as a type of Reynolds number can be defined in a variety of ways to account for the particles and their size relationship to the tube. As long as the definitions are consistent between cases, no complications result. For simplicity, it will be defined in terms of flow through the upstream branch and the physical properties are those for the suspending media.

EXPERIMENTAL APPARATUS AND PROCEDURES

This research was performed in two parts; in one case the flow was vertically downward, while in the other, the flow was horizontal. The apparatus and experimental methods pertinent for these two situations will be discussed separately.

Vertical Flow

For the vertical flow portion, the suspension consisted of one-eighth inch polystyrene spheres suspended in Dow Corning 200 Fluid. The size tolerance for the spheres is .002 inch and the specific gravity of the polystyrene is in the range of 1.04 - 1.065. The suspending media, a polymer of dimethylsiloxane, was an equal proportion mixture of fluids of two different viscosities. The two base fluids have viscosities of 10 and 100 poise while the resultant mixture has a viscosity of 26.2 poise and a density of 0.975 g/cm^3 at 22°C. Due to the density differences between the particles and the fluid, the spheres had a sedimentation rate of 1.2 centimeters per minute.

The apparatus consisted of a reservoir, a connecting section of Tygon tubing, the tubes associated with the various branches of the bifurcation and the block which forms the junction, as shown in Figure 1. The bifurcation was a Lucite block through which three-eighths inch channels have been drilled such that they formed a junction with each other having an angle of 45°. The transparent acrylic tubes which formed the various branches of the bifurcation, were precision

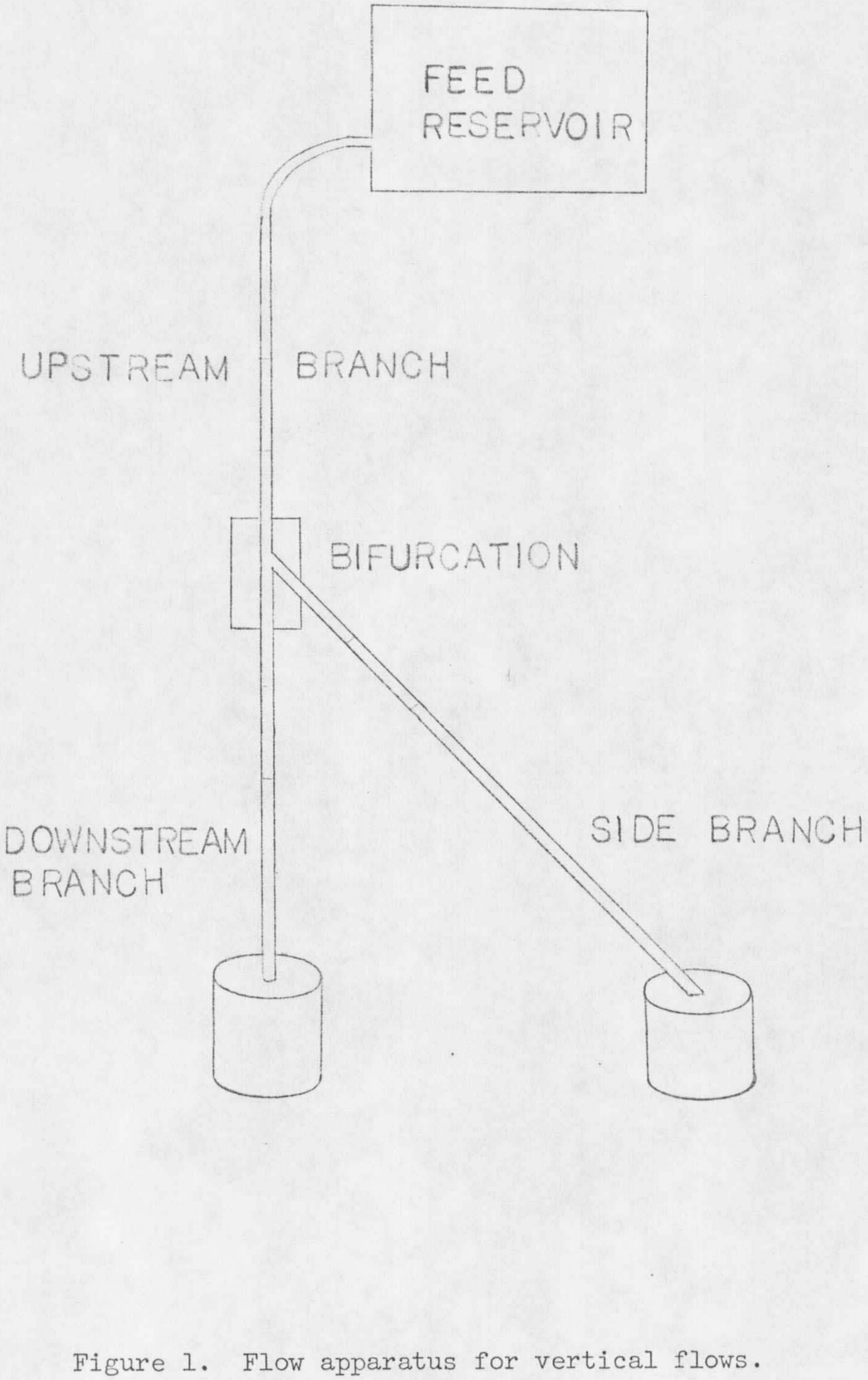

Figure 1. Flow apparatus for vertical flows.

machined. When threaded into the block, the inside of the tube formed a smooth continuous surface with the channels in the block, as shown in Figure 2. Various lengths of tubing were used to alter the flow resistance and hence the relative flow rates in the downstream branches. The suspension flows from the reservoir through the connecting tubing and into the upstream branch of the bifurcation. The flow is divided at the bifurcation and the outflow of the two downstream branches was collected in large beakers. The volume of the reservoir was so large compared to the amount of flow, that the change in pressure head was only one inch in ten feet, and can be neglected.

The procedures involved in gathering of experimental data, began with manually stirring the suspension in the reservoir to resuspend the particles that had settled to the bottom. The stirring was done manually because the high viscosity of the fluid made it impractical to use some automatic method. The system was filled initially with particle free fluid. Flow was initiated and the suspension allowed to fill the system. After the system was completely filled with suspension, the efflux from the side and downstream branches was collected in graduated cylinders and the collection period timed with a stopwatch. Flow was stopped by plugging the ends of the tubes. The number of spheres in several six-inch test sections was determined for each of the three branches. From the average number of spheres in a

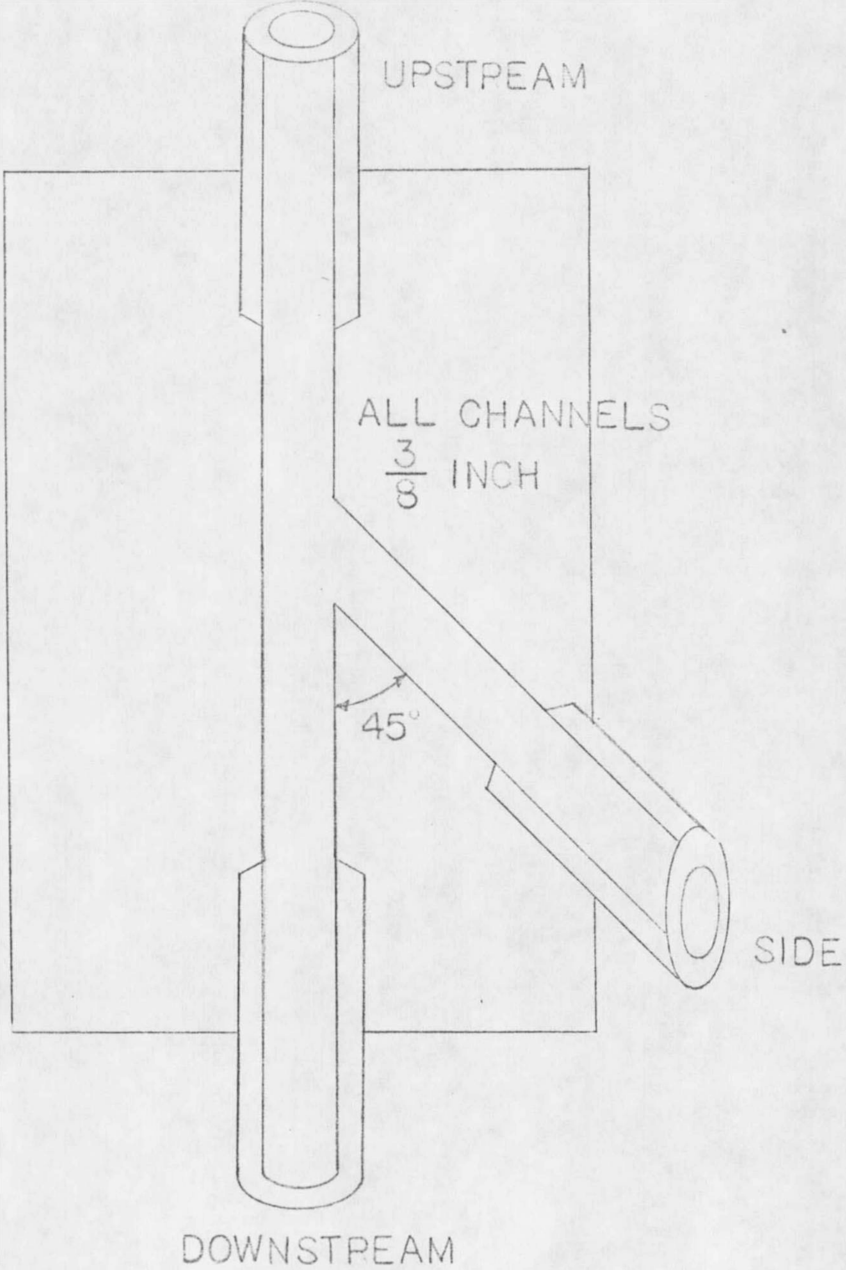

Figure 2. Sketch of a 45° bifurcation with equal diameter branches.

given section, the average tube concentration was computed. The flow rates in each branch could be determined from the values for the timed efflux for the appropriate branch.

After each such run, the system was purged of particles by resuming flow while preventing the particles from leaving the reservoir by placing a screen over the tube entrance. This procedure was repeated a number of times and the average values of the concentrations and flow rates determined. When sufficient data had been gathered, the geometry of the system was altered to change the relative size of the flows in the downstream branches. The entire procedure was repeated for this new value of relative flow rate.

Horizontal Flows

For the situations where the flows were horizontal, the suspension consisted of the same polystyrene spheres as used previously but this time the fluid was a mixture of glycerine and water. The fluid composition was approximately 21% glycerine by volume and had a viscosity of 1.94 centipoise at 24°C. It was expected that by using glycerine and water, the fluid density could be matched exactly to the particle density by adjusting the fluid's composition; thus sedimentation effects could be eliminated. However, it was found that the density variations among the particles was such that at this particular fluid composition, approximately equal proportions of spheres settled to the bottom and floated to the surface when the suspension was allowed to stand undisturbed for a while. To prevent fluid composition changes due to the loss of water by evaporation, the top of the reservoir was covered with Saran Wrap. A small amount of detergent was added as a surfactant to reduce the surface tension and disperse the spheres. Sodium azide was added as a fungicide to prevent the growth of a slime found to be using the glycerine as a food source.

The reservoir consisted of a plexiglas box into which a three-eights inch acrylic tube had been threaded so that the tube extended more than one particle diameter into the suspension as shown by Figure 3A. This tube was coupled to the upstream branch of the bifurcation by means of a Tygon tubing sleeve. The sleeve kept the two tubes aligned so that blockages would not occur at their junction. The suspension in the reservoir was kept stirred by means of an electromagnetic stirrer. The reservoir and stirrer apparatus are shown in Figure 3B.

By means of appropriately sized Tygon tubing, the outflow of the side and downstream branches flowed into a fluid filled 250 milliliter Erlenmeyer flask. A fluid and air tight seal was obtained by fitting the Tygon tubing through a hole bored in a #6 rubber stopper. A six inch 14 gauge hypodermic needle with a bevelled end is fitted through the stopper and the entire assembly sealed with Dow Corning 780 sealant. This portion of the apparatus is shown in Figure 4;

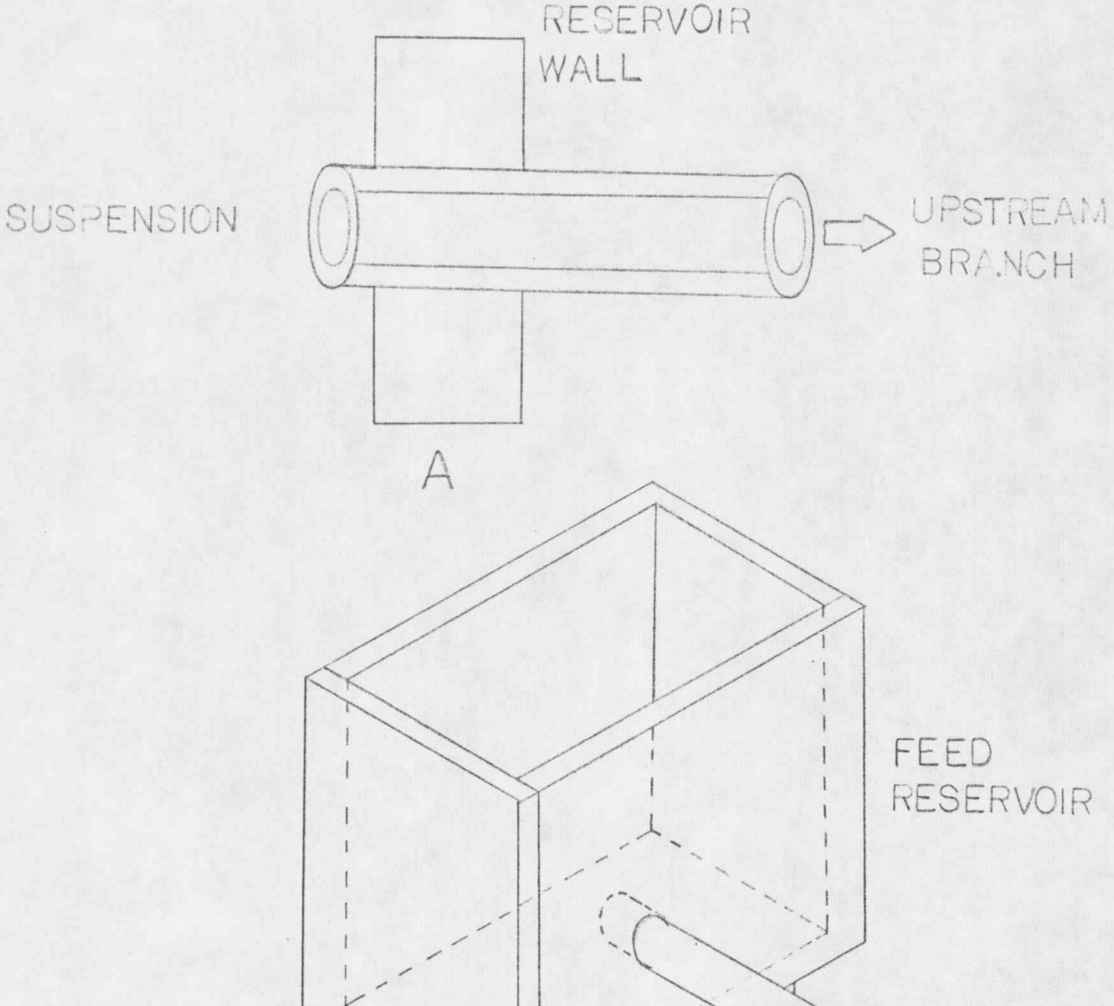

Figure 3. A. Sketch of the exit tube and reservoir wall.
B. Reservoir and magnetic stirrer assembly.

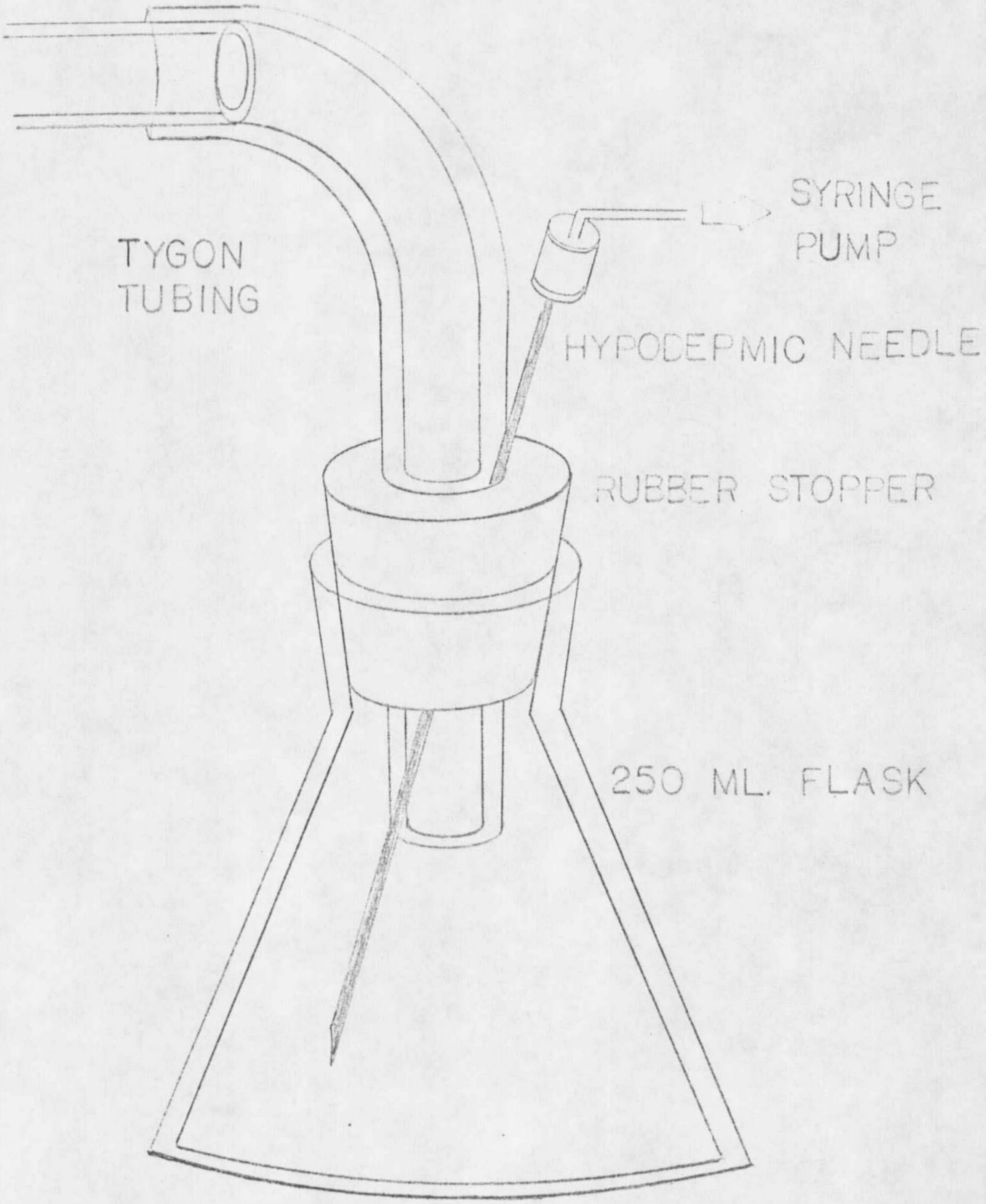

Figure 4. Flask apparatus for the separation of the spheres from the fluid.

however, the sealant has been omitted for clarity. Only fluid could flow out through the needle so the spheres accumulated in the flask. Medical Grade polyethylene tubing having an inside diameter of 0.105 inches was used to connect the hypodermic needle to a Harvard Apparatus Series 900 single action syringe pump with a three-way valve intermediate. The three way valve permitted the syringe on the pump to be emptied while the flow through the apparatus was stopped.

Three bifurcations were used in the study with horizontal flows. The method and materials of construction were similar in all cases. The blocks were Lucite and the tubing a transparent acrylic. The characteristics of the three bifurcations were:

1) The diameter of all the branches was the same and equal to three-eighths of an inch. The angle of branching was 45°.
2) All the branches had diameters of three-eighths of an inch, while the angle of branching was 90°.
3) A 90° bifurcation whose side branch was two-thirds the diameter of the upstream and downstream branches. The diameter of the side branch was one-fourth an inch and the diameter of the upstream and downstream branches was three-eighths of an inch.

Figure 2 shows the 45° bifurcation and figure 5 showns the two 90° bifurcations.

The interior of all the tubing was siliconized with Siliclad to

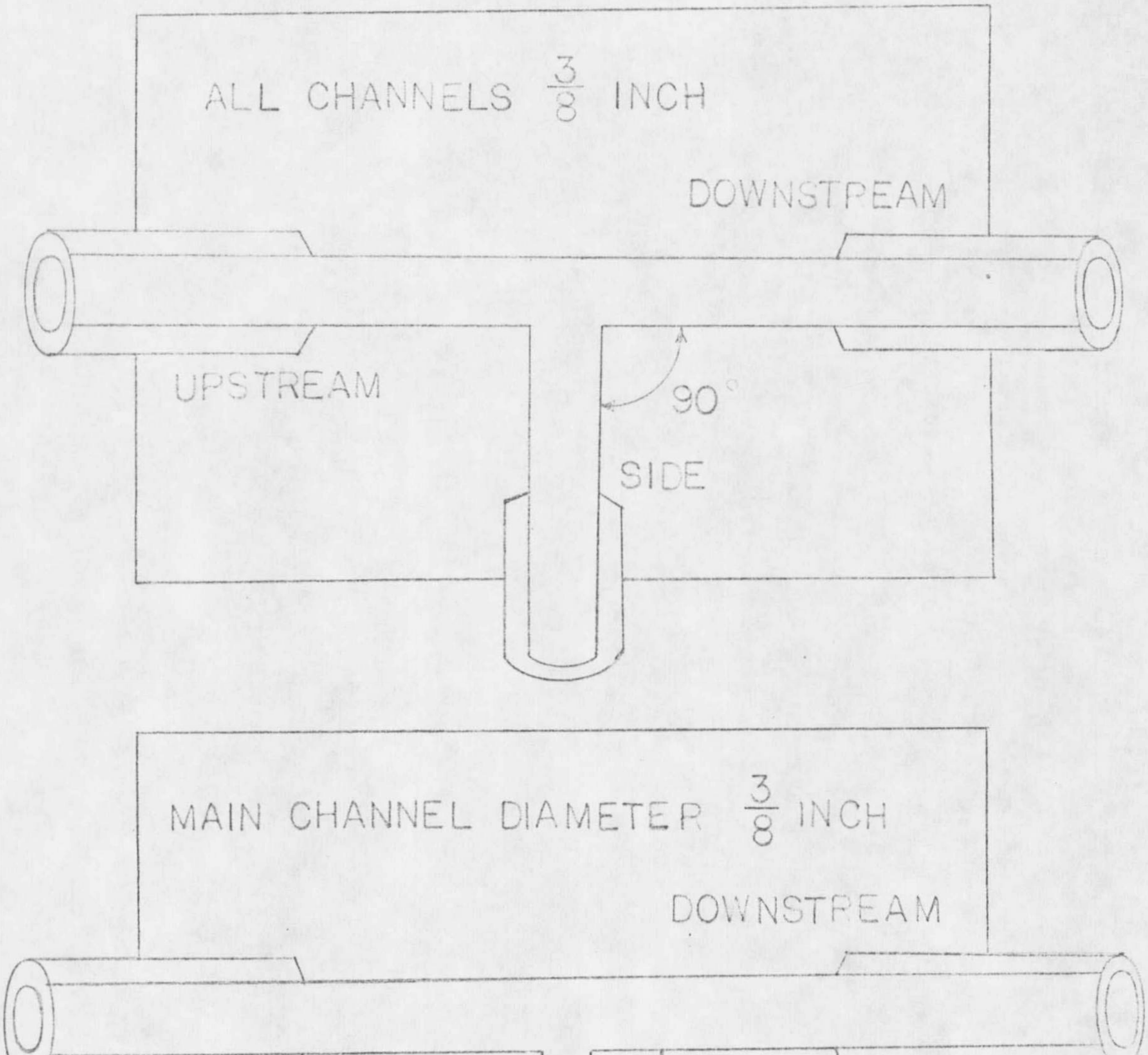

Figure 5. Sketches of the two 90° bifurcations.

facilitate flow by eliminating interaction effects of the tube wall with the particles of the suspension. Prior to siliconing, it was observed that when flow was stopped and then resumed, the particles in contact with the tube wall adhered and would not flow. Siliconing the tubes eliminated this effect.

A diagram of the assembled apparatus is presented in Figure 6.

While stirring the suspension in the reservoir continuously, flow was initiated by simultaneously turning on the two syringe pumps which withdrew fluid from the downstream branches at a constant rate. The rate of withdrawal was determined by the size of the syringe and the gear setting on the syringe pump. When one of the syringes was filled to capacity, the flow was stopped by turning off the pumps and the syringes emptied into graduated cylinders by means of the three-way valve. Flow was resumed after the syringes had been emptied. This procedure was repeated until the apparatus was filled with suspension, at which time a photograph was taken of a six-inch test section of each branch. Photographs were taken at intervals corresponding to the filling of the syringes on the pumps. In all cases, the amount of flow between photographs was such that all of the particles in the test section in one photograph were washed out of the section and did not appear in the succeeding photograph. The photographs were taken with an Agfa 35 millimeter single lens reflex camera using Kodak Tri-X Pan film. The camera was fitted with a Vemar

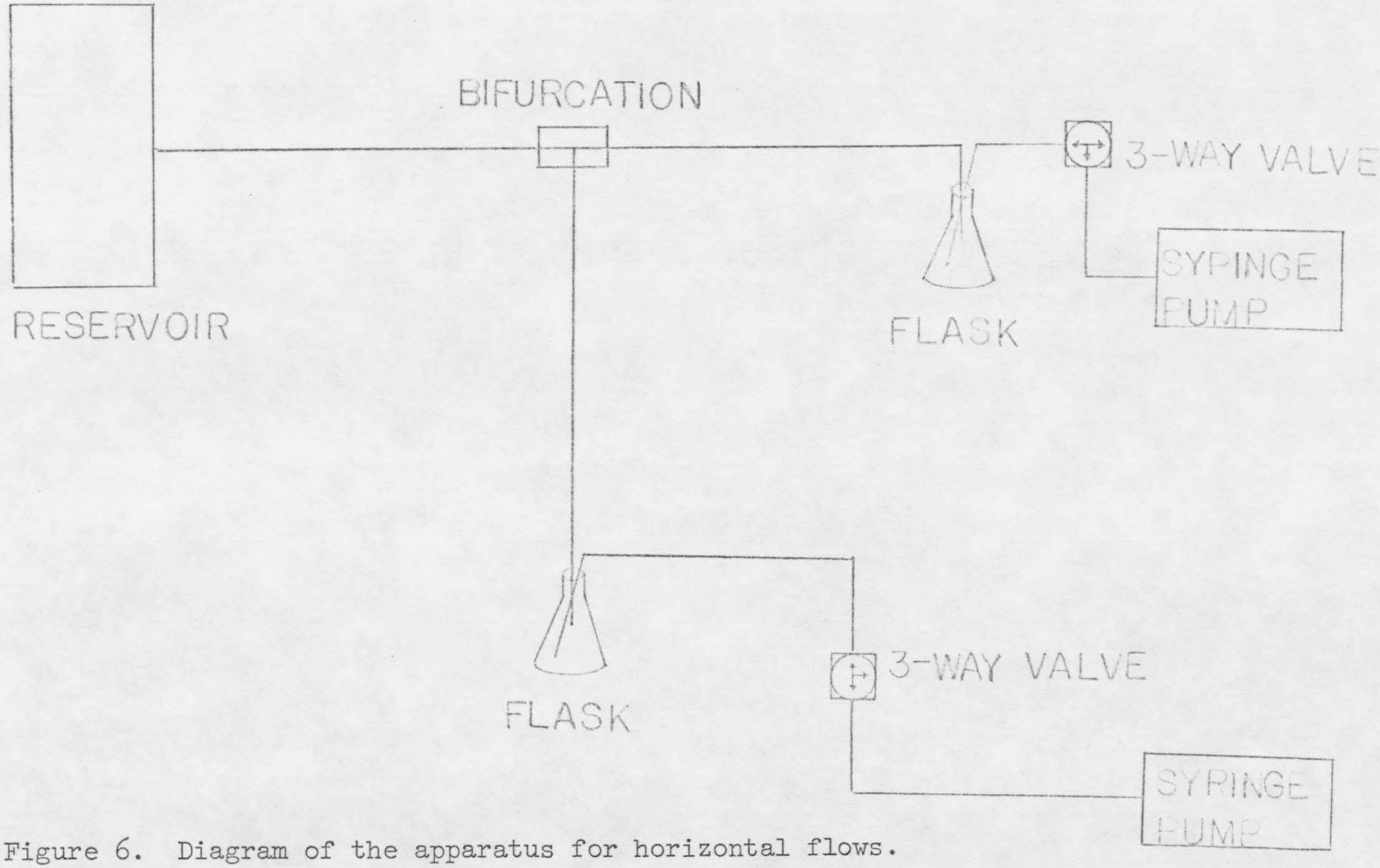

Figure 6. Diagram of the apparatus for horizontal flows.

#3 close up lens, which reduced the focal length from 2.8 to 1.0 feet. This procedure was continued until 21 photographs, seven of each branch, were taken. At that time, the entrance to the upstream branch was blocked using a stopper and the two flasks downstream detached. The accumulated spheres were removed, washed, dried, and weighed to the nearest hundredth of a gram. The upstream branch was then unstopped and a screen fitted over the entrance which prevented the spheres from entering the system. The apparatus was flushed free of particles and reassembled. From the mass of spheres in the flasks and the volume of fluid withdrawn from each branch, the average concentration of the outflow for each branch can be calculated. The film was developed commercially and the spheres present in the test section were counted and an average tube concentration for each branch determined. The details of those computations are enumerated in the appendix.

EXPERIMENTAL RESULTS AND DISCUSSION

Vertical Flow

For the case of vertical flow through a 45° bifurcation with equal diameter branches the amount of plasma skimming is found to be a strong function of the relative flow rates in the branches. Plasma skimming is measured by means of the dimensionless ratio of the concentration in the side branch to the concentration in the downstream branch. This ratio is plotted as a function of another dimensionless ratio, the volumetric flow rate in the side divided by the flow rate in the downstream branch, in Figure 7. It can be seen that the ratio $\phi s/\phi ds$ varies from 0.75 to 1.4 as the relative flow rates, Qs/Qds, vary from 0.33 to 1.75 respectively. The concentrations used in determining the concentration ratio are tube concentrations. Of unknown importance for this case is the amount of sedimentation and its effect, if any, on the side branch concentration. Since the spheres tend to settle at a rate of about 1.2 centimeters per minute, which is appreciable when one considers that the bulk average velocity in the side branch is only about 20 centimeters per minute. This settling tends to move spheres from a region of high velocity, the center, to a region of low velocity, the tube wall. This translation could result in an accumulation of spheres in the side branch. If accumulation in the side branch is occurring, the graph in Figure 7 may be translated upward at each point by the extent to which accumulation has increased the side branch concentration over the

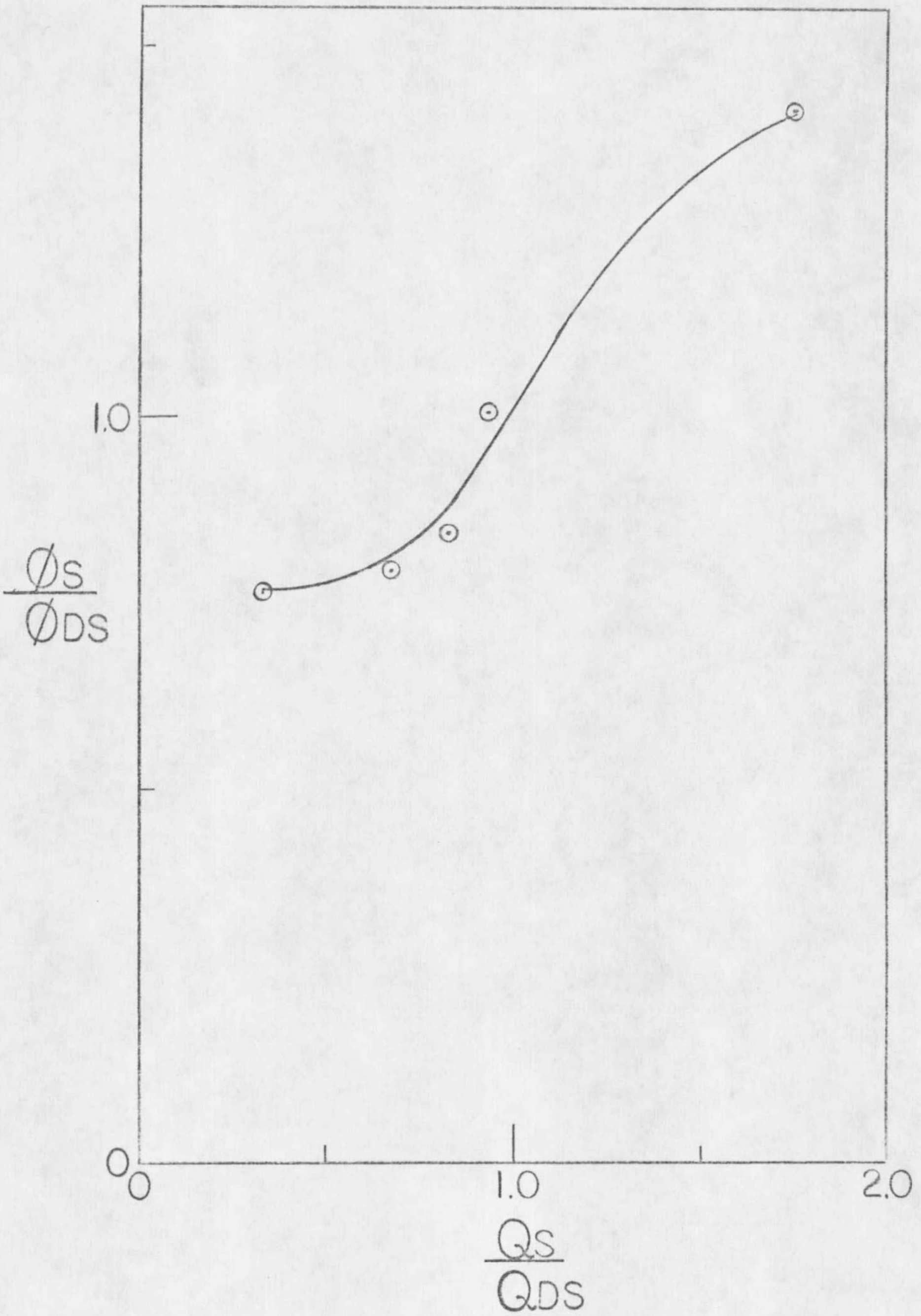

Figure 7. Tube concentration ratios for vertical flow through a 45° bifurcation.

corresponding value had the spheres been neutrally buoyant.

From the results of later investigations, this sedimentation and accumulation effect is believed to have occurred. While it is unknown as to the extent to which the curve must be translated to represent the case for neutrally buoyant particles, the values obtained are of use in determining, at least semiquantitatively, the effect of changes in relative flow rates. Considering that the slowest flows in the side branch result when the ratio of the flow rates is less than unity, one suspects that this portion of the curve is the region where sedimentation and accumulation effects would be most pronounced. It should be noted that accumulation would occur only in the side branch since the downstream branch is vertical, sedimentation would not tend to cause the particles to move to the tube walls.

Horizontal Flow

The decision to switch fluids and the attendant change from vertical to horizontal flow is based on two considerations. First, it is hoped that by changing fluids to one whose density more closely matches that of the spheres, the effects of sedimentation could be minimized. The choice of fluids is limited by the fact that most hydrocarbons attack the polystyrene and the specific gravity of the required fluid has to be greater than unity. Secondly, it is desired to investigate the effects of total flow rate, as well as a wide range of relative flow rates, on the degree of plasma skimming. This requires some method of pumping as opposed to altering the flows by means of resistances. The switch from vertical to horizontal is a convenience.

Using the apparatus detailed previously, investigation is begun into the various aspects of plasma skimming. It is also desired to investigate plasma skimming in terms of mixing cup concentrations, which was not done during measurements with vertical flows. It is well documented that for blood suspensions flowing through small tubes between two reservoirs, the concentrations in the reservoirs are identical even though the concentration in the connecting tube is less than the reservoir concentrations. When the tube diameter is smaller than a certain critical size, apparently two cell diameters for blood, a screening effect is observed. The resultant effect is that the exit reservoir concentration is less than the feed reservoir concentration. It is felt that the screening effect should be more prominent when one considers a suspension of rigid particles instead of blood because the deformability of the red blood cells should facilitate their entrance into the tube. In the model study, the possiblity exists that there may be screening of particles occurring at the entrance since the tube diameter is only three times the diameter of the sphere. Table IV presents results obtained in attempting to measure the degree of screening which occurs when the suspension flows into and through a straight tube.

TABLE IV. MEASUREMENTS ON SCREENING AS A FUNCTION OF CONCENTRATION AND FLOW RATE

Flow Rate ml/min	Reservoir Concentration	Mixing Cup Concentration	$\frac{\phi MC}{\phi R}$
190	40%	22.6%	.564
114	40%	23.5%	.588
75	40%	28.4%	.710
190	20%	10.6%	.532
114	20%	12.8%	.642
75	20%	11.3%	.565

The experiment is performed to determine whether the phenomenon is independent of flow rate. The amount of screening is measured by the quantity $\phi MC/\phi R$, where ϕMC is the outlet concentration and ϕR is the reservoir concentration. This ratio should have a value of 1.0 if no screening occurred. The degree of screening is dependent on flow rate when the reservoir concentration is 40%. Screening is found to increase with increasing flow rate. This is reasonable if one accepts the postulate that at high flow rates, the spheres are so crowded together at the entrance, that they hinder each other's entrance into the tube to the extent that at the higher flow rates proportionately more fluid is drawn into the tube than spheres. The differences shown in Table IV are statistically significant when comparing the two high flow rate values to the low flow rate value of $\phi MC/\phi R$ at a reservoir concentration of 40%. The differences are not significant for any of the differences when the reservoir concentration is 20% nor is the difference between the two highest flow rate values at a reservoir concentration of 40% significant. Values of the appropriate statistics may be found in the appendix.

From a knowledge of the Fahraeus Effect, the average concentration in the upstream branch of a bifurcation is expected to be less than the feed reservoir concentration. It has just been shown that a screening effect occurs at the tube entrance when the reservoir concentration is 40% spheres by volume. Since this screening effect is a function of flow rate, one anticipates that the upstream tube concentration should also be a function of flow rate at least to the same extent as the different degrees of screening at the entrance. Another effect may be the accumulation of spheres within the tube at the slower flow rates. Seshadri and Sutera (24) found that stirring the suspension in the reservoir has an effect on the tube concentration in their experiments. Because it is desired to keep the feed suspension homogeneous by stirring, the possible effect of stirring is considered. Figure 8 provides information concerning the effect of the two parameters, stirrer speed and flow rate, on the upstream tube concentration. It is apparent that the tube concentrations for

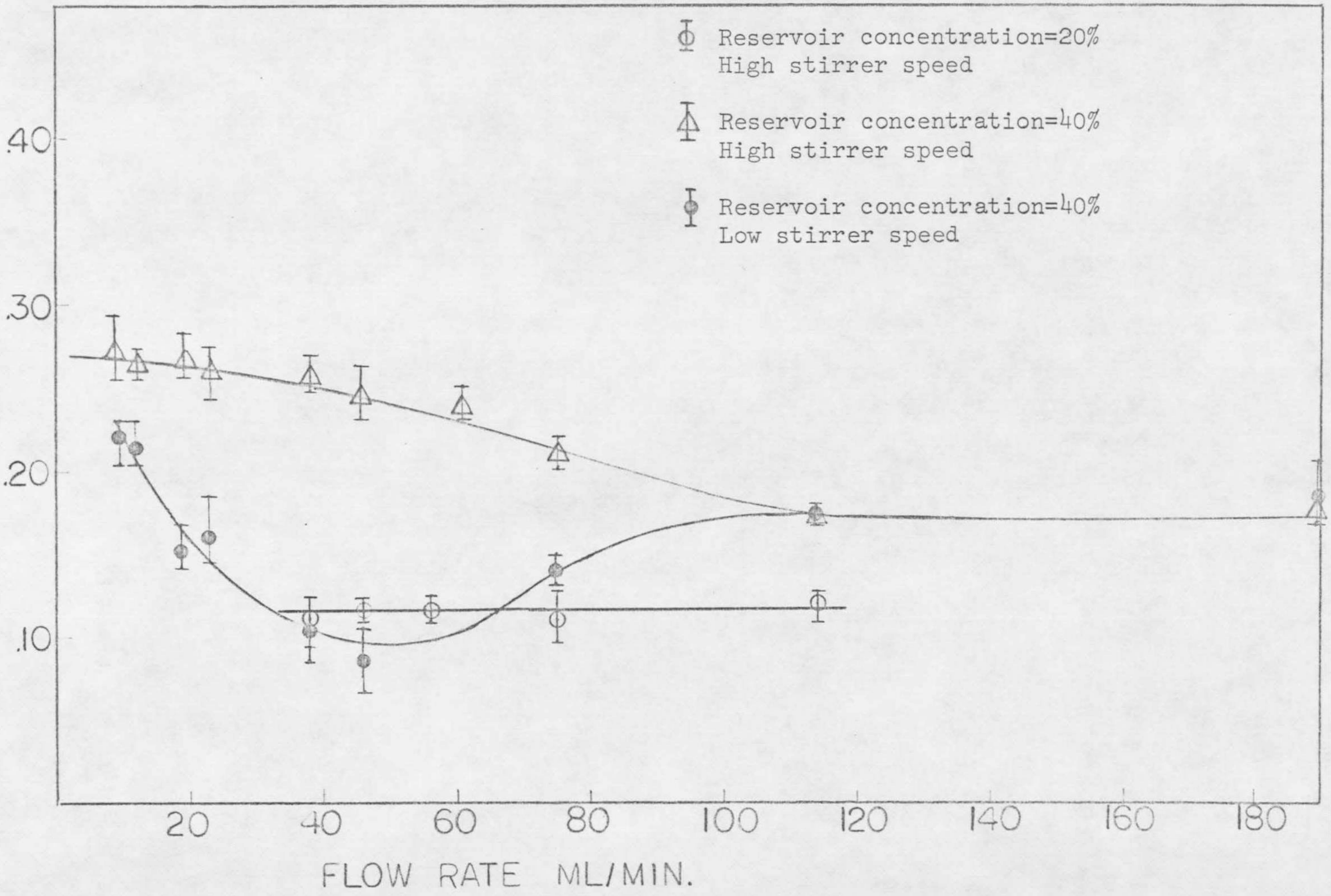

Figure 8. Upstream tube concentration as a function of flow rate, reservoir concentration and stirrer rate. These values are for horizontal flows with a glycerine and water media.

the two stirring rates approach values of 20% and 30% for high and low flow rates respectively, when the feed reservoir concentration is 40%. For intermediate values of flow rate, there is a large discrepancy between concentrations obtained at the different stirring rates. A plausible explanation of this would be that changing the stirring rate affects the flow pattern at the tube entrance, thus providing a reason for the differences observed at intermediate flow rates. The convergence of the values obtained at high and low flow rates for the two stirring rates can be explained thusly:

1) The convergence at high flow rates occurs because the flow pattern at the entrance has little effect in determining tube concentration because the flow rate is so large compared to the flows in the tank.
2) The convergence at low flow rates occurs because the same steady state concentration is reached regardless of stirrer speed, because of the accumulation of spheres within the tube.

This accumulation at low flow rates is exactly the effect that is supposedly eliminated by the use of a neutrally buoyant media of glycerine and water. However, there is a region in which the effects of accumulation are minimal. Specifically, the region of flows larger than 100 milliliters per minute. Thus by choosing a total flow rate well into this range, the flows in the branches should be such that accumulation would be of lesser importance. All data presented here-after is obtained at the higher level of stirrer speed.

Investigation of the effect of total flow rate on plasma skimming proceeded using the 45° bifurcation of Figure 2 and one value of relative flow rate, namely Qs/Qds equal to 0.60. The curves in Figure 9 represent values of concentration ratios obtained over a range of flow rates from 30 to 300 millileters per minute, with a reservoir concentration of 40%, when the bifurcation is horizontal. Values for both mixing cup and tube concentration ratios approach unity for small flow rates. For higher flows, the values of the ratios are less than unity. The curve for the tube concentration ratios consists of two straight line segments. It is shown statistically in the Appendix that the differences between the tube ratios for 50 and 120 milliliters per minute are significant, while the differences between 120 and 300 milliliters per minute flows are not significant. The non-horizontal segment of the line corresponds to the region of flows where accumulation occurs. It appears that accumulation occurs to the extent that at low flows the concentrations in the two tubes approaches the same steady state value.

There seems to be no ready explanation for the behavior of the mixing cup values at differing flow rates. The mixing cup ratios decrease with increasing flow rate from a value near unity at low flows through a minimum of about 0.7 at 120 milliliters per minute and increases to about 0.85 at the highest flow rate. The differences

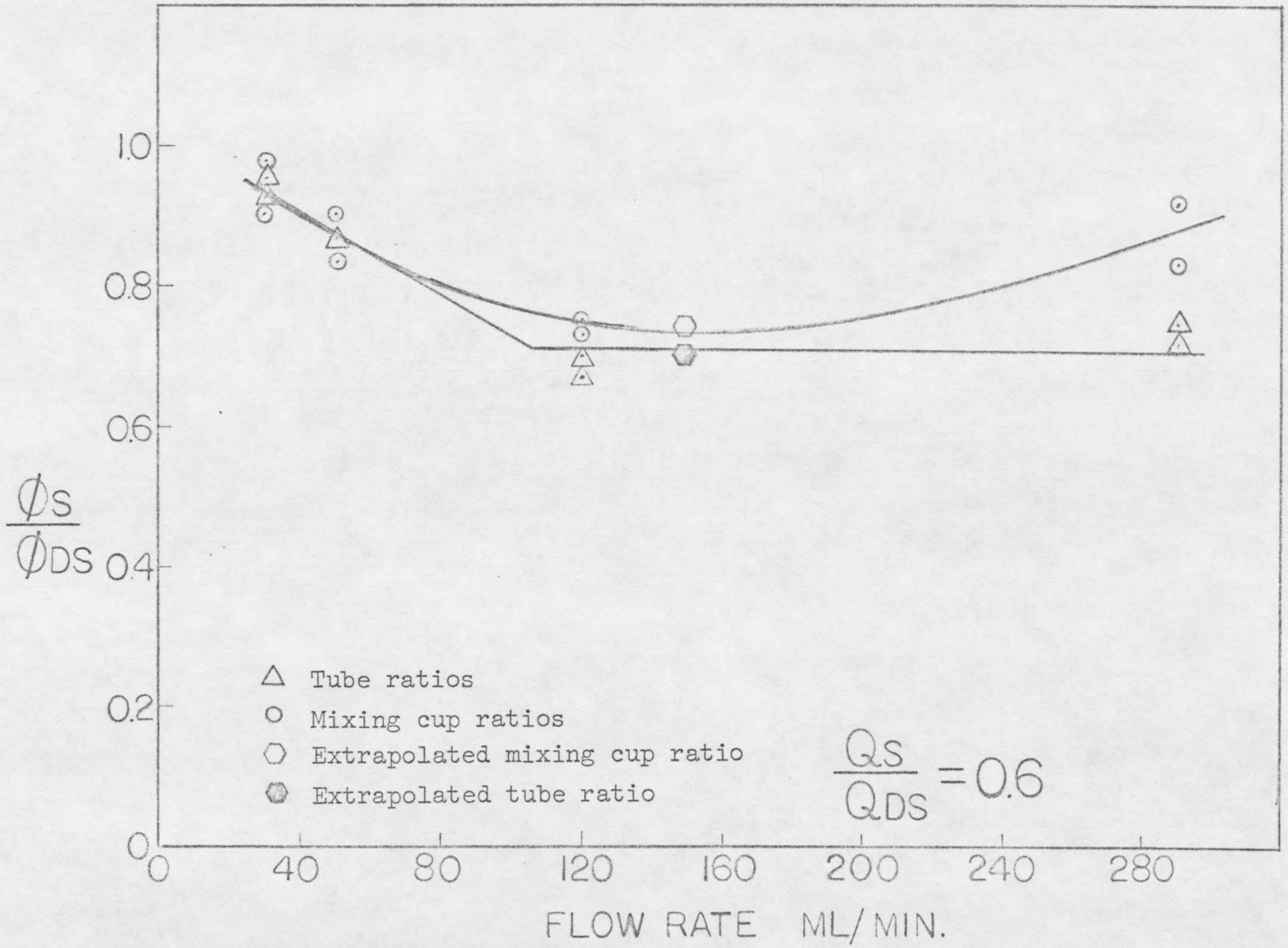

Figure 9. Plasma skimming as a function of total flow rate for a 45° bifurcation and under situations of horizontal flows.

between the ratios at 50 and 120 milliliters per minute are probably significant as are the differences between the values at 120 and 300 milliliters per minute.

Investigation of the effect of total concentration and relative flow rate upon plasma skimming begins with choosing to investigate these factors when the total flow rate is approximately 150 milliliters per minute. This total flow rate is chosen for two reasons. First, at this total flow rate, the curves in Figure 9 are horizontal or very nearly so for a range of flow rates. This is important to assure that small changes in total flow rate do not produce profound changes in plasma skimming. Second, this total flow rate provides the optimum number of possibilites for pairs of flows whose combination is approximately 150 milliliters per minute. This is necessary to assure the widest range of relative flow rates possible at a nearly constant value of total flow rate.

For a tank concentration of 40% and bifurcations of 45° and 90° having branches of equal diameters, Figure 10 presents information for values of ratios of mixing cup concentrations as a function of relative flow rates in the downstream branches. To reiterate one more time, plasma skimming is measured by the dimensionless ratio of the concentration for the side branch to the concentration of the downstream branch. This ratio is plotted as a function of the dimensionless ratio of volumetric flow rates, the flow rate in the

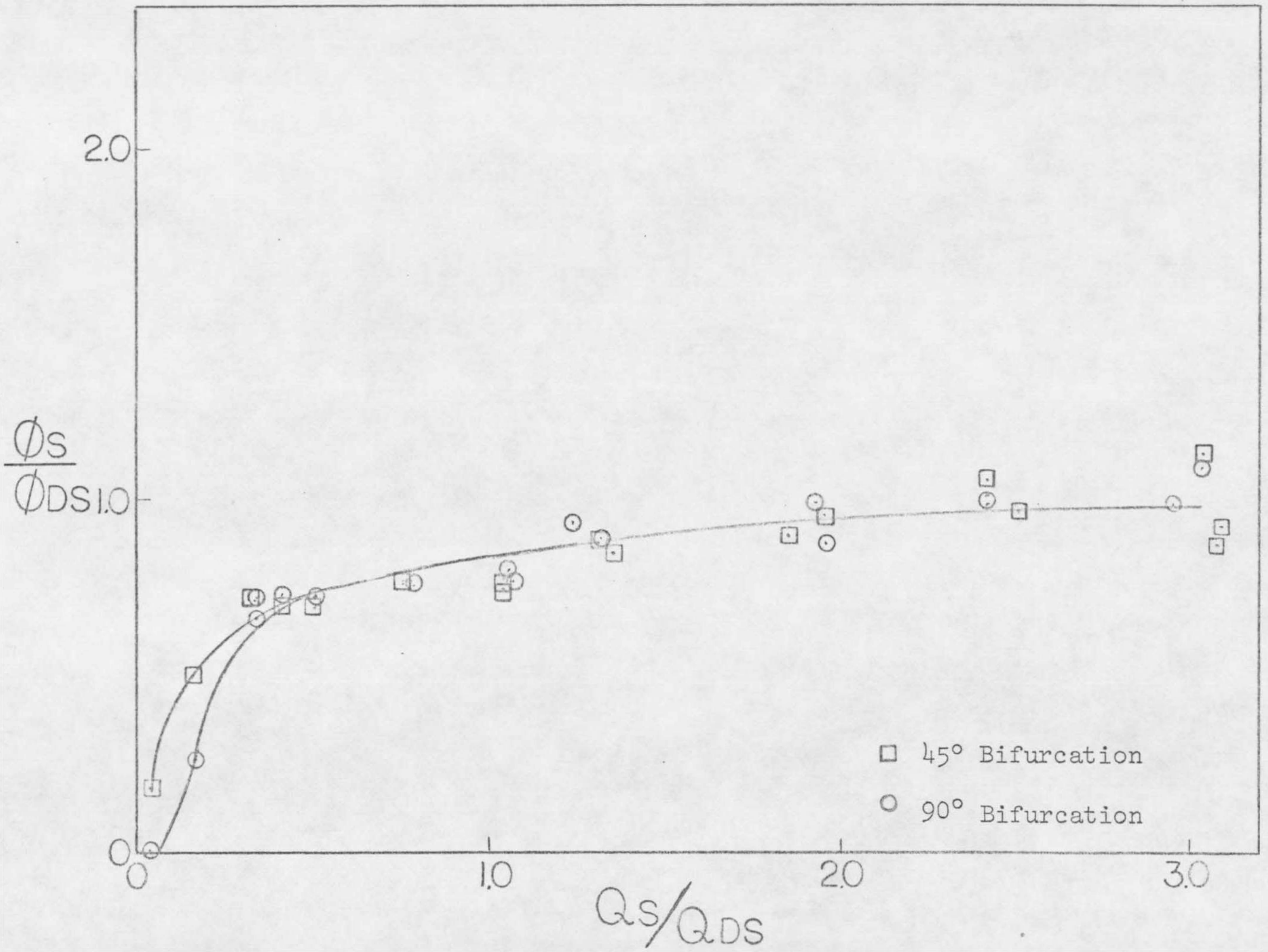

Figure 10. Mixing cup concentration ratios for the 45° and 90° bifurcations having equal diameter branches. Reservoir concentration is 40%.

in the side branch divided by the flow rate in the downstream branch. It is found for the 45° bifurcation, that the concentration ratio is relatively constant over a wide range of relative flows, increasing from 0.70 to about 1.0 as the relative flow to side branch is increased from 0.33 to 3.0 respectively. For relative flows less than 0.33, the concentration ratio decreases rapidly to about 0.2 as the relative flow is decreased to about .04. For the 90° bifurcation, the same trends are observed. In fact, statistical analysis shows that there are no significant differences between values for the 90° and 45° bifurcation except at flow ratios less than 0.33. The pertinent statistics are given in the Appendix.

Figure 11 gives values of concentration ratios for various flow rates for the two bifurcations but this time the reservoir concentration is 20%. For this tank concentration, there are no significant differences between concentration ratios obtained at the same relative flow rates for the two different bifurcations. The value of the concentration ratio increases sharply from zero to 0.6 as the ratio of flows is increased from 0.35 to 0.33 respectively. As the relative flow ratio is increased to about 3.0, the concentration ratio increases slowly to a value of approximately 1.2. Statistics for the comparison of values between the two bifurcations may be found in the Appendix. It is important to remember that the concentration ratio is evaluated from mixing cup concentrations for the above situations.

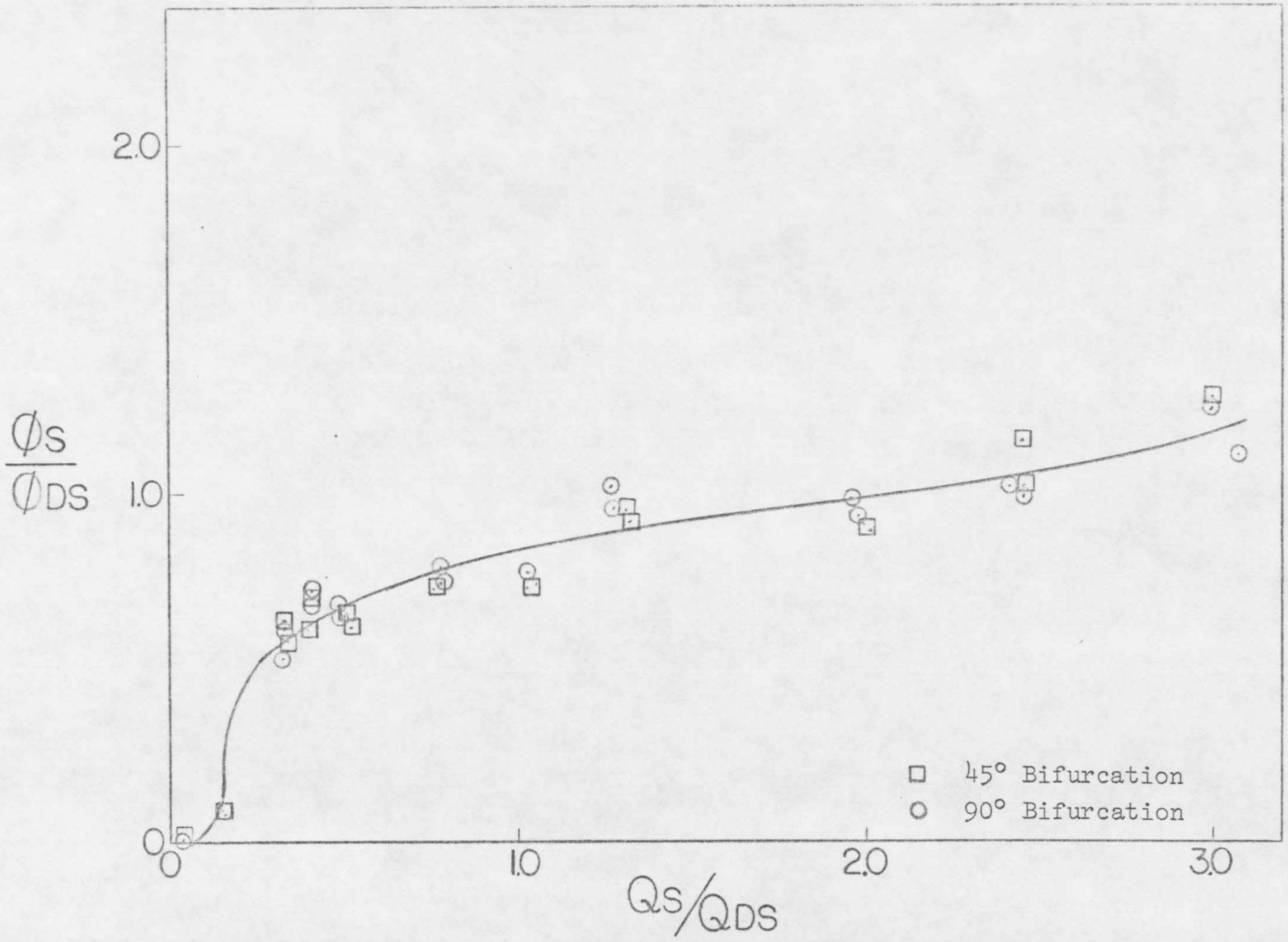

Figure 11. Mixing cup concentration ratios for the 45° and 90° bifurcations. Reservoir concentration is 20%.

When the concentration ratio is determined from tube concentrations a complication arises due to the possibility of unequal accumulation in the different downstream branches because of different flow rates. For the cases where concentration ratios are determined from tube concentrations, the actual data will be presented as well as data which have been adjusted to account for the effects of accumulation. It is found that at low reservoir concentrations, accumulation effects are not present over the range of applicable flows. The reader is referred to Figure 8 for evidence of this. However, for high concentrations, accumulation is found to be a factor. In applying the relationship for adjusting the tube ratios, it will be assumed that the increase in tube concentration at lower flow rates as shown in Figure 8, is due entirely to accumulation because of the reduced flow. While this assumption may not be completely valid, it will supply an upper limit for possible values. The actual data provides a lower limit as it assumes no accumulation occurs. Therefore, the true curve should be bracketed by these two curves.

The values one obtains for concentration ratios defined in terms of tube concentrations and the corresponding adjusted ratios for a 45° bifurcation with a reservoir concentration of 40% are given in Figure 12. The unadjusted concentration ratio is found to rise sharply from zero to 0.70 and then slowly rises to about 1.0 as the ratio of relative flows is increased from 0.035 to 0.33 and then to 3.0

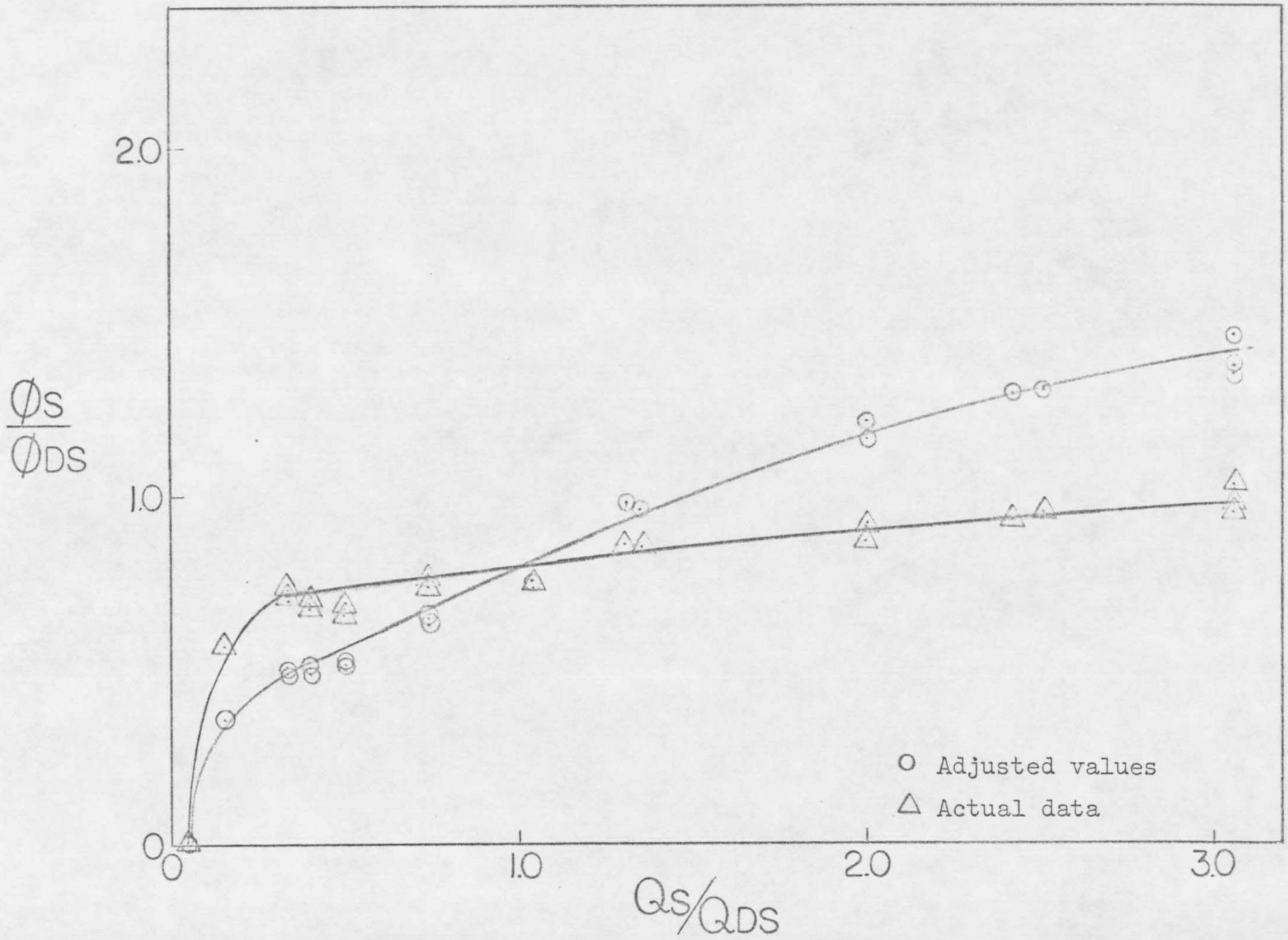

Figure 12. Tube concentration ratios for the 45° bifurcation. Reservoir concentration is 40%.

respectively. The effect of adjusting these ratios is that the portions of the curve for relative flows greater and less than unity are shifted upward and downward respectively. The magnitude of the shift is dependent upon the relative amount of accumulation in each branch as determined from Figure 8. The curve for the adjusted values is very closely approximated by a portion of a parabola. The adjusted ratio increases smoothly from zero to about 1.4 as the ratio of relative flows increases from 0.035 to 3.0. The differences between values obtained at high and low values of relative flow rates are highly significant as shown in the Appendix.

For the 90° bifurcation and the high reservoir concentration the adjusted and unadjusted ratios for various relative flows are plotted in Figure 13. The unadjusted values of the concentration ratio rise sharply from zero to 0.7 and slowly to 0.9 as the relative flow ratio increases from 0.035 to 0.5 and then to 3.0. The adjusted ratios increase smoothly from zero to 1.3 as the flow increases through the same range as stated previously. Here again the significance of values obtained at high and low flow ratios is high. For more information concerning the statistics involved, refer to the Appendix.

For low concentrations, there is no adjusting factor. Figure 14 presents data for tube concentration ratios for the 90° and 45° bifurcations at low reservoir concentrations. Detailed calculations

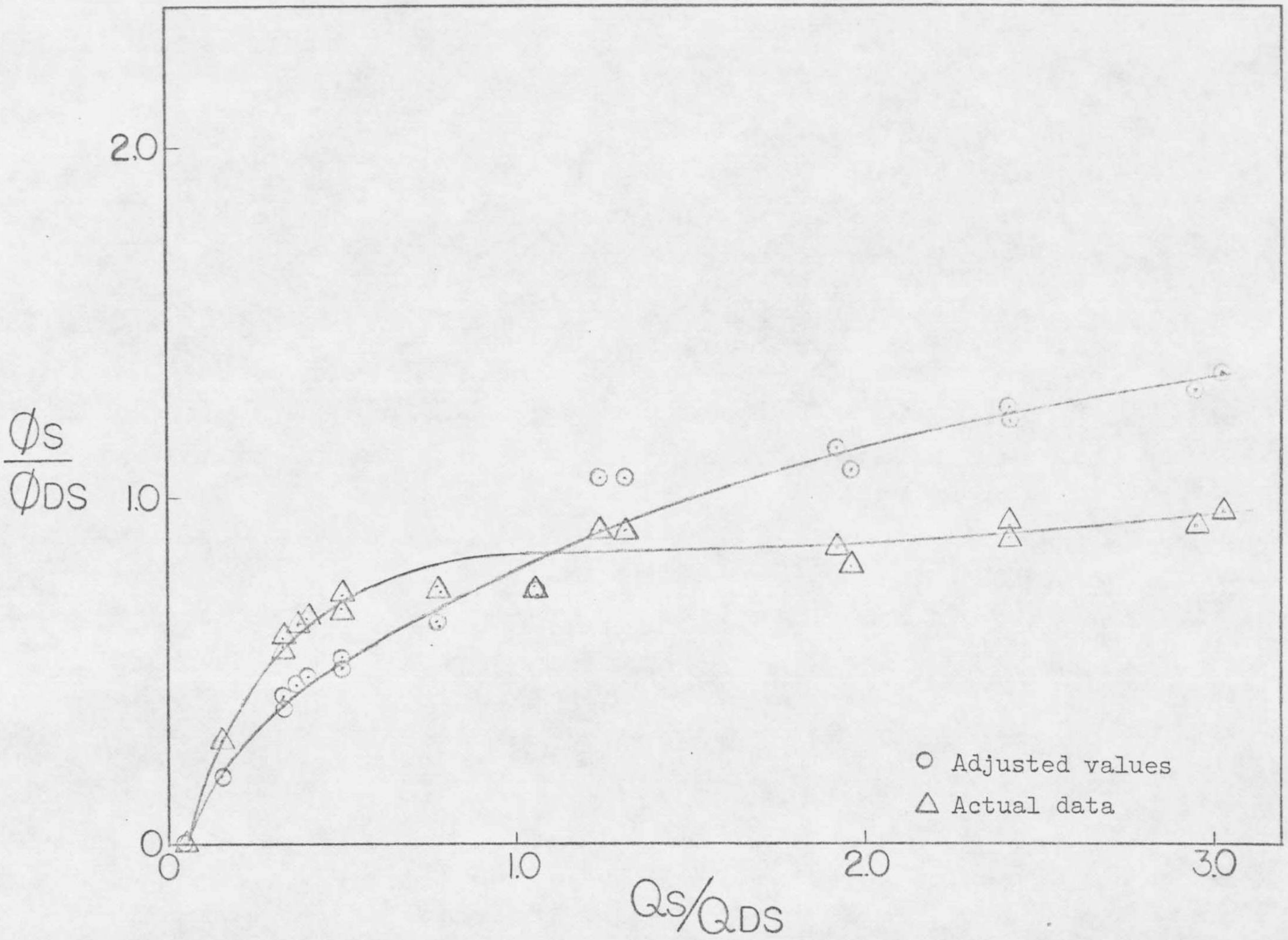

Figure 13. Tube concentration ratios for the 90° bifurcation. Reservoir concentration is 40%.

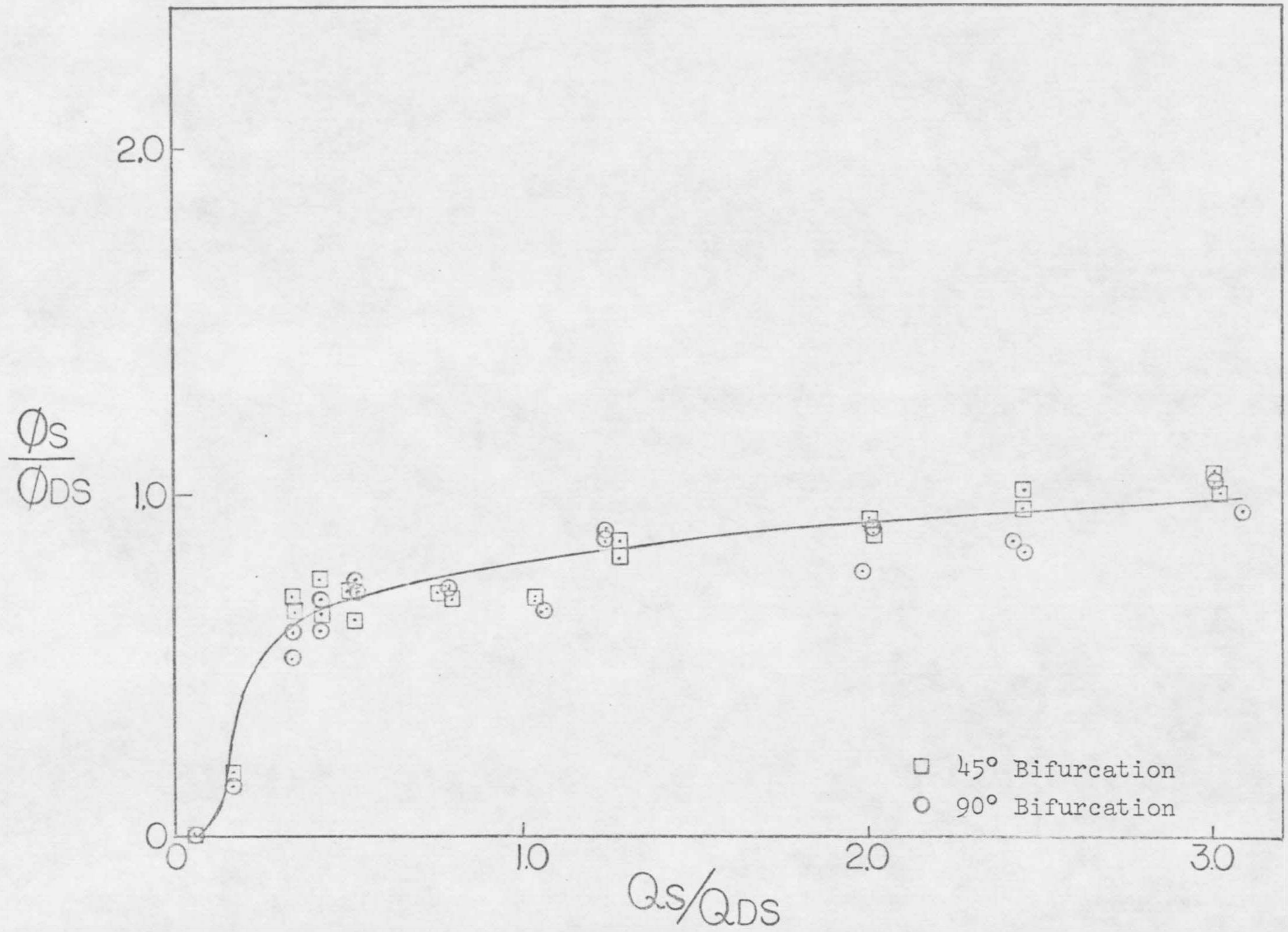

Figure 14. Tube concentration ratios for the 45° and 90° bifurcations. Reservoir concentration is 20%.

presented in the appendix show that the differences of values obtained for the two bifurcations are not significant. The concentration ratio rises sharply from zero to 0.70 and then slowly to 0.95 as the ratio of the relative flow is first increased from 0.035 to 0.33 and then to 3.0.

From an examination of Figures 11, 12, 13 and 14, an interesting point can be seen. As the ratio of the relative flows changes from 1.0 to 1.3, there is a rather significant increase in concentration ratio. In other words, as the flow in the side branch becomes larger than the flow in the downstream branch, there is a significant increase in the concentration in the side branch compared to the concentration in the downstream branch. This increase is rather drastic when compared to the changes observed for flow changes of a similar magnitude. This large change can be attributed to the effect of the angle.

The final bifurcation investigated has a 90° angle of branching and the diameter of the side branch is two-thirds the diameter of the downstream branch. The diameter of the side branch is twice the nominal average diameter of the spheres. Since at all times any sphere in the side branch must have some portion at the centerline, accumulation in the side branch will be neglected. Also of importance for this case is the size distribution of the spheres. Since a number of the spheres have diameters slightly larger than half the diameter of the side branch it is possible that two spheres the sum of whose diameters is larger than the tube diameter might attempt to enter the side branch simultaneously and cause an occlusion of the tube. This is found to occur with increased frequency with increasing relative flow through the side branch. The data presented here are for the entire range of flow rates obtainable. The dotted portion of the curve is an extrapolation based on values obtained at relative flow rates such that blockages occurred repeatedly.

Figure 15 presents data on concentration ratios determined from mixing cup concentrations. The curve for a reservoir concentration of 40% is sigmoid shaped. The concentration ratio varies from zero for a relative flow of 0.04 to about 0.85 for a relative flow ratio of 0.25. For relative flow ratios greater than 0.25, blockages occur repeatedly at the side branch entrance. For a reservoir concentration of 20%, the curve increases smoothly from 0.024 to 0.60 for flow ratio increases from 0.114 to 0.35. At flow ratios larger than 0.35, the side branch plugs repeatedly.

Concentration ratios based on tube concentrations are plotted in Figure 16 for two levels of reservoir concentration, 40% and 20%. For the lower reservoir concentration, the concentration ratio increases from zero to 0.50 as the flow ratio is increased from 0.05 to 0.34. The ratio of concentrations increases from zero to 0.50 for flow ratios from 0.05 to 0.25. Plugging of the side branch occurs repeatedly at