Natural convection heat transfer between arrays of horizontal cylinders and their enclosure

by Robert Allen Weaver

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
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Abstract:
The natural convection heat transfer between arrays of horizontal, heated cylinders and their isothermal, cooled enclosure was experimentally investigated. Four different cylinder arrays were used: two in-line and two staggered. Four fluids (air, water, 20 cs silicone and 96% glycerine) were used with Prandtl numbers ranging from 0.705 to 13090.0. There was no significant change in the Nusselt number between isothermal and constant heat flux conditions of the cylinder arrays. The average heat transfer coefficient was most affected by the spacing between cylinders and the total surface area of the cylinder arrays. The enclosure reduced the increase in both the average and the local heat transfer coefficients caused by changing the inner body from an in-line arrangement to a staggered arrangement of comparable spacing. An increase in fluid viscosity reduced the influence of the geometric effects.

The best empirical equation for all of the experimental data using one correlating parameter was: Nus = 0.214Ra*s^O.260; Ra*s = Ras(L/Ri) for 0.705 ≤ Pr ≤ 1.31xl0^4; 4.45xl0^4 ≤ Ras ≤ 1.17xl0^8 0.602 ≤ L/Ri ≤ 1.041; 4.63x10^4 ≤ Ra*s ≤ 8.15x10^7 with an average percent deviation of 12.00.

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Signature Robert A. Weaver

Date 17 February, 1982

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ROBERT ALLEN WEAVER

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NOMENCLATURE

Symbol	Description
A_c	Flow cross-sectional area between cylinders
A_{cc}	Cross-Sectional area of the copper-constantan thermocouples
A_{htl}	Cross-sectional area of the heat tape leads
A_I	Surface area of the inner body
A_O	Surface area of the outer body
A_s	Cross-sectional area of the support stems
B	Length of boundary layer on one cylinder, $B = \pi(d/2)$
C_{1-4}	Empirically determined constants
c_p	Specific heat at constant pressure
d	Diameter of a cylinder
D_h	Hydraulic diameter, $D_h = (4A_cZ)/A_I$
g	Acceleration of gravity, 9.81 m/sec^2 (32.17 ft/sec^2)
Gr_X	Grashof number, $\beta\rho^2 g(T_I - T_O)X^3/\mu^2$, where X is any characteristic length
h	Heat transfer coefficient, $h = Q_{CONV}/A_I\Delta T$
H	Vertical pitch between cylinder rows
H'	Modified vertical pitch, H' = H + d + [d/(number of cylinder rows - 1)]
k	Thermal conductivity
K	Horizontal distance between cylinders
k_{cc}	Thermal conductivity of the copper-constantan thermocouples

Symbol	Description
k_{htl}	Thermal conductivity of the heat tape leads
k_s	Thermal conductivity of the support stems
L	Hypothetical gap width, $R_o - R_i$
$\Delta \ell$	Change in length
Nu_X	Nusselt number, hX/k, where X is any characteristic length
Pr	Prandtl number, $c_p \mu / k$
Q_{COND}	Heat transfer by conduction
Q_{CONV}	Heat transfer by convection
Q_{RAD}	Heat tranfser by radiation
Q_{TOT}	Total amount of heat transfer, $Q_{TOT} = Q_{COND} + Q_{CONV} + Q_{RAD}$
r	Radius of a cylinder
Ra_X	Rayleigh number, $\beta \rho^2 g (T_I - T_O) X^3 c_p / \mu k$, where X is any characteristic length
Ra^*_X	Modified Rayleigh number, $Ra^*_X = Ra_X (L/R_i)$
R_i	Radius of a hypothetical sphere equal in volume to the volume of one cylinder times the number of cylinders in the cylinder array
R_o	Radius of a hypothetical sphere equal in volume to the outer body
S	Characterictic length, $S = (R_o - R_i)(A_I/A_O)$
T_b	Bulk temperature or average fluid temperature
T_f	Film temperature, $T_f = (T_b + T_I)/2$
T_I	Inner Body Temperature
T_j	Reference temperature, $T_j = T_b + 0.32(T_s - T_b)$

Symbol	Description
T_m	Arithmetic mean temperature, $T_m = (T_I + T_O)/2$
T_n	Reference temperature, $T_n = T_b + 0.20(T_s - T_b)$
T_O	Outer body temperature
T_s	Surface temperature
ΔT	Temperature difference, $\Delta T = T_I - T_O$
X	Any characteristic length
Z	Height of the tube bundle
β	Thermal expansion coefficient
μ	Dynamic viscosity
π	Ratio of circle circumference to diameter, 3.14159
ρ	Density of the fluid
ρ_{atm}	Density of the fluid at atmospheric pressure

ABSTRACT

The natural convection heat transfer between arrays of horizontal, heated cylinders and their isothermal, cooled enclosure was experimentally investigated. Four different cylinder arrays were used: two in-line and two staggered. Four fluids (air, water, 20 cs silicone and 96% glycerine) were used with Prandtl numbers ranging from 0.705 to 13090.0. There was no significant change in the Nusselt number between isothermal and constant heat flux conditions of the cylinder arrays. The average heat transfer coefficient was most affected by the spacing between cylinders and the total surface area of the cylinder arrays. The enclosure reduced the increase in both the average and the local heat transfer coefficients caused by changing the inner body from an in-line arrangement to a staggered arrangement of comparable spacing. An increase in fluid viscosity reduced the influence of the geometric effects.

The best empirical equation for all of the experimental data using one correlating parameter was:

$$Nu_S = 0.214{Ra^*_S}^{0.260};\quad Ra^*_S = Ra_S(L/R_i)$$

for $0.705 \leq Pr \leq 1.31 \times 10^4;\ 4.45 \times 10^4 \leq Ra_S \leq 1.17 \times 10^8$

$0.602 \leq L/R_i \leq 1.041;\ 4.63 \times 10^4 \leq Ra^*_S \leq 8.15 \times 10^7$

with an average percent deviation of 12.00.

CHAPTER I

INTRODUCTION

Natural convection heat transfer from a body to an infinite fluid medium has received extensive experimental and analytical study in the past. In recent years there has been a growing demand for an understanding of natural convection heat transfer within enclosures. This phenomenon has important industrial applications in areas such as nuclear reactor technology, electronic instrumentation packaging, aircraft cabin design, crude oil storage tank design, solar collector design, and energy storage systems.

This is one of the first studies of multiple body natural convection heat transfer in enclosures. Its purpose is to experimentally investigate the dissipation of heat by natural convection from arrays of heated, horizontal cylinders to a cooled, isothermal, cubical enclosure. The cylinders were subjected to both isothermal and constant heat flux conditions. This study determines the effects of cylinder geometry, and compares the results with the findings of previous studies. Four fluids and four cylinder configurations were utilized. The fluids used were air, water, 96 percent glycerine, and 20cs silicone, with Prandtl numbers ranging from 0.705 to 13090.0. The four cylinder configurations consisted of two in-line arrangements using nine and sixteen cylinders, and two staggered arrangements using eight and fourteen cylinders.

CHAPTER II

LITERATURE REVIEW

Natural convection heat transfer can be classified in two ways: the external convection problem of flow about a body surrounded by an infinite fluid medium, and the internal convection problem of flow within an enclosure. The external problem has been investigated extensively in the past while relatively little attention has been given to the internal problem. This is due to the fact that internal natural convection problems are significantly more complex. For external problems the Prandtl boundary layer theory allows one to assume that the region surrounding the boundary layer is unaffected by the boundary layer. However, in the internal natural convection problem the boundary layer and the region adjacent to the boundary layer interact with each other, making it difficult to obtain analytic solutions to internal problems. The remainder of this chapter is intended to provide a useful background for this particular investigation and is not a complete survey of research in natural convection. The following discussion will be divided into the two categories mentioned above. These are (1) external natural convection to an infinite fluid medium, and (2) internal natural convection in enclosures. However, the discussion is limited to geometries which pertain to this investigation.

EXTERNAL NATURAL CONVECTION

There have been several studies of the heat transferred from single objects (e.g. plates, spheres, cylinders, etc.) to an infinite fluid medium. Morgan [1] gives a very thorough summary of the major correlations for heat transfer from smooth, horizontal, circular cylinders to an infinite fluid medium with Rayleigh numbers ranging from 10^{-10} to 10^{13}.

It may be shown by dimensional analysis [2] that the heat transfer from horizontal cylinders varies with the Grashof and Prandtl numbers. The resulting equation is generally of the form

$$Nu_X = A_1 + B_1 \ (Gr_X Pr)^{C_1}.$$

A_1, B_1, and C_1 are constants and X is a characteristic length dimension where

$$Nu_X = hX/k \text{ (Nusselt number)},$$

$$Gr_X = (g \beta \rho^2 X^3 \Delta T)/\mu^2 \text{ (Grashof number)},$$

and

$$Pr = \mu c_p/k \text{ (Prandtl number)}.$$

The nondimensional grouping $(Gr_X Pr)$ is known as the Rayleigh number, which is

$$Ra_X = Gr_X Pr = (c_p \beta g \rho^2 X^3 \Delta T)/k\mu.$$

Fand, Morris, and Lum [3] presented three different correlations for natural convection heat transfer from horizontal cylinders, based on three different reference temperatures. These are

$$Nu_f = 0.474\ Ra_f^{0.25}\ Pr_f^{0.047},$$

$$Nu_j = 0.478\ Ra_j^{0.25}\ Pr_j^{0.050},$$

and

$$Nu_n = 0.456\ Ra_f^{0.25}\ Pr_f^{0.057}$$

where the subscripts denote the reference temperature used in evaluating the fluid properties. The reference temperatures were

$$T_f = T_b + 0.5(T_s - T_b) \text{ (film temperature)},$$

$$T_j = T_b + 0.32(T_s - T_b),$$

and

$$T_n = T_b + 0.20(T_s - T_b).$$

Raithby and Hollands [4] have published a correlation equation for laminar and turbulent natural convection from elliptic cylinders of arbitrary eccentricity for the case of constant surface temperature. For the case of horizontal isothermal cylinders their equation takes the form

$$Nu_m = \left[\frac{2}{\ln\{(1+\pi 2^{3/4})/(f_2^{3/4} C_l\, Ra^{1/4})\}}\right]^m + (0.72 C_t\ Ra^{1/3})^m$$

where $m = 3.337$

$f_2 = 2.587$

$C_l = (2/3)/[1 + (0.49/Pr)^{9/16}]^{4/9}$

$C_t = 0.14\, Pr^{0.084}$ or 0.15, whichever is smaller.

Churchill and Chu [5] suggest that an additive constant is required in the correlation equation for natural convection heat transfer from horizontal cylinders. They have published the following correlation equation for heat transfer by natural convection from horizontal cylinders

$$Nu_d = 0.36 + 0.518 \left[\frac{Ra_d}{[1 + (0.559/Pr)^{9/16}]^{16/9}}\right]^{0.25}$$

for all Prandtl numbers and for Rayleigh numbers ranging from 10^{-6} to 10^9. They recommend that for large temperature differences, such that the variation of physical properties is signficant, the properties may be evaluated at the average of the bulk and surface temperatures as a first approximation.

Kim, Pontikes, and Wollersheim [6] experimentally studied the natural convection heat transfer from a horizontal cylinder with isothermal and constant heat flux surface conditions. The average free convection results were obtained by integrating the local Nusselt, Prandtl, and Grashof numbers over the test section surface area. The average Nusselt numbers for the experimental data were

$$Nu_r = 0.89\ Ra_r^{0.19}$$

for isothermal surface conditions, and

$$Nu_r = 0.57\ Ra_r^{0.20}$$

for constant heat flux surface conditions.

Although there has been extensive research performed on natural convection heat transfer from a single horizontal cylinder, natural convection heat transfer from multiple cylinders has received little attention. Eckert and Soehngen [7], who performed one of the first studies of natural convection heat transfer from multiple cylinders, investigated the effects that one heated cylinder had on adjacent cylinders in a vertical array of horizontal, isothermal cylinders. They discovered that when one cylinder was positioned directly over another at a distance of four diameters, there was no change in the Nusselt number of the lowest cylinder as opposed to a single cylinder while the upper cylinder Nusselt number was 87 percent of the value for the lower one. A reduction in the heat transferred from the upper cylinder was said to be caused by the heated wake from the lower cylinder striking the upper cylinder. When three cylinders were arranged in a vertical array the Nusselt number of the middle cylinder was 83 percent of the Nusselt number of the bottom cylinder while the Nusselt number of the top cylinder was 65 percent of the value for the bottom cylinder. When the cylinders were staggered such that the middle cylinder was moved laterally out of line by one half of a diameter, the wake of the bottom cylinder missed the middle cylinder and the Nusselt number of the middle cylinder was 103 percent of the value for the bottom cylinder while the Nusselt number of the top cylinder was 87 percent of the value for the bottom cylinder. The increase in the heat transferred from the middle cylinder was a result of the higher velocity of fluid movement past this cylinder which was induced by the wake from the bottom cylinder.

Liberman and Gebhart [8] investigated the interaction of natural convection wakes between a parallel array of wires. By orienting the array at different angles measured from the vertical and different spacings, they were able to determine the spacing which yielded a maximum Nusselt number for a particular angle of inclination.

Marsters [9] performed an experimental study on the natural convection heat transfer from a vertical array of heated horizontal cylinders. He concluded that the heat transfer characteristics exhibited by vertical arrays of heated horizontal cylinders are not predicted by simple superposition of single cylinder behavior. For closely spaced arrays (two diameters between cylinders), individual tube Nusselt numbers were found to be as much as 50 percent lower than for a single cylinder. For wider spacings, individual cylinder Nusselt numbers were as much as 30 percent higher than that of a single cylinder. He concluded that the overall heat transfer characteristics of an array are dependent upon array spacing as well as Rayleigh number.

Tillman [10] developed two correlation equations for natural convection heat transfer from tube bundles:

$$Nu_f = 0.057\ Ra_f^{0.5}$$

for in-line arrays and

$$Nu_f = 0.067\ Ra_f^{0.5}$$

for staggered arrays. All of the thermal properties except the coefficient of thermal expansion were evaluated at the film temperature. The coefficient of thermal expansion was evaluated at the ambient temperature. The hydraulic diameter for a compact heat exchanger was used for the characteristic length, which was defined as

$$D_h = (4A_cZ)/A_I.$$

Tsubouchi and Saito [11] conducted an experimental study of the natural convection heat transfer from arrays of uniformly heated circular cylinders in air. The cylinders were arranged in various in-line and staggered arrangements. They found that the heat transfer depended on the cylinder spacing, the number of cylinders, and the type of cylinder arrangement. They proposed the following correlation

$$Nu_M = \Phi_2\ (0.092)\ \{1 - 0.92\ \exp[(d-K)/d]\}\ (PrGr_{H'})^{0.4}$$

where Φ_2 = 1.00 for in-line banks and Φ_2 = 1.06 for staggered banks. The number of vertical columns ranged from 3 to 5. The number of horizontal rows ranged from 3 to 7 and K was the horizontal distance between the cylinders, which had outer diameters of d. The vertical distance between the cylinders was H. The charactristic length H' was the modified vertical pitch, defined as

$$H' = H + d + d/[(\text{number of horizontal rows}) - 1].$$

They concluded by stating that the average heat transfer coefficient was affected more by a variation in the vertical pitch than by a variation in the horizontal pitch.

INTERNAL NATURAL CONVECTION

Internal natural convection heat transfer utilizes the same dimensionless parameters as are used for external natural convection, however an additional parameter involving a ratio of characteristic dimensions is used in internal problems.

Warrington [12] performed an in-depth experimental study of natural convection in enclosures. His work involved the heat transfer between inner bodies such as spheres, cubes, and cylinders to both spherical and cubical enclosures. Several different fluids were utilized with Prandtl numbers ranging from 0.706 to 13,800. The recommended overall correlation from his data was

$$Nu_L = 0.425\, Ra_L^{0.234}\, (L/R_i)^{0.498}.$$

For a cylindrical inner body and a cubical enclosure the best correlation was

$$Nu_B = 0.593\, Ra_B^{0.240}\, (L/R_i)^{0.434}.$$

Larson, Gartling, and Schimmel [13] used laser interferometry to experimentally determine the temperature field around a heated horizontal cylinder in an isothermal rectangular box. The purpose of their study was to simulate the possible geometric configurations of a nuclear spent fuel element in a shipping cask and compare the experimental results with numerical results using finite-difference and finite-element techniques.

Dutton and Welty [14] conducted an experimental study of natural convection heat transfer in an array of uniformly heated vertical cylinders surrounded by a vertical cylindrical enclosure with mercury as the fluid medium. The cylinders were arranged in an equilateral triangular pattern. Their results indicated that the natural convection heat transfer was strongly dependent on the cylinder spacing and was less dependent on heat flux and circumferential position. In their concluding remarks they suggest that natural convection heat transfer results in the low Prandtl number range (liquid metals) are well represented by correlations involving the $Gr_X Pr^2$ product, which is independent of viscosity.

Van De Sande and Hamer [15] studied the steady and transient natural convection heat transfer between horizontal concentric circular cylinders with constant heat flux surface conditions. Their experiment showed that a sidewise displacement of the inner cylinder did not affect the heat transfer results. However, the overall heat transfer decreased or increased depending on whether the inner cylinder was above or below the center line of the outer cylinder. An additional dimensionless group was introduced to account for the effect of vertical eccentricity. The results were used to estimate the cooling of buried cable systems surrounded by a water layer.

Crupper [16] performed an experimental study of natural convection heat transfer between a set of four isothermal, heated cylinders and an isothermal, cooled, cubical enclosure, to determine the effect of the positioning of the cylinders within the enclosure, and to compare the results with the findings of previous studies on heat transfer from single bodies to an enclosure. Four fluids and inner body positions were utilized. The set of cylinders was oriented in both a horizontal and vertical position. The four fluids had Prandtl numbers ranging from 0.7 to 3.1 x 10^4 and Rayleigh numbers, based on gap width, ranging from 6.3 x 10^5 to 6.9 x 10^8. He found that the best correlations for all of the heat transfer data combined were

$$Nu_B = 0.277\ Ra_B^{0.274}\ Pr^{0.012}$$

and

$$Nu_B = 0.286\ Ra_B^{0.275}.$$

The best correlations for the cylinders in the horizontal position were

$$Nu_L = 0.498\ Ra_L^{0.245}\ Pr^{-0.002}$$

and

$$Nu_L = 0.496\ Ra_L^{0.245}.$$

All of the fluid properties were evaluated at the arithmetic mean of the inner and outer body temperatures.

Powe [17] investigated the limits of relative gap width for which available correlation equations for natural convection heat transfer in enclosures were applicable. Heat transfer rates for large relative gap widths were shown to be limited by those obtained for free convection to an infinite fluid medium, and this criteria was used to calculate a maximum relative gap width for which the enclosure equations were applicable. A minimum relative gap width for applicability of the enclosure equations was determined by the pure conduction limit.

Brown [18] experimentally studied the transfer of heat by natural convection within enclosures at reduced pressures. The best correlation which included a correction for the air density was

$$Nu_L = 0.342\ Ra_L^{1/4}\ (\rho/\rho_{atm})^{0.129}.$$

The geometries used were cylinder-cube (inner body-outer body) and cube-cube, with the bodies mounted concentrically in both cases. The Rayleigh number ranged from 1×10^3 to 2×10^6 and the pressure ranged from 2670 - 86,180 Pa (20 - 646.4 mm Hg).

Powe, Warrington, and Scanlan [19] performed a detailed study of natural convection flow phenomena which occur between a body of relatively arbitrary shape and its spherical enclosure. Resulting trends in the fluid flow data were established to facilitate better predictions of the heat transfer in problems of natural convection in enclosures.

As evidenced by this review, the amount of interest in internal natural convection has increased dramatically in the last decade. The intent of this study is to extend the work performed by Warrington and Crupper [16, 20] in the area of natural convection heat transfer between multiple bodies and an enclosure.

Chapter III

EXPERIMENTAL APPARATUS AND PROCEDURE

EXPERIMENTAL APPARATUS

The outer body used for this investigation was a cube 26.67 cm (10.5 in.) along an inner side, constructed from 1.27 cm (0.5 in.) thick, type 6061 aluminum. The assembled outer body and peripheral components are shown in Figure 3.1. A water jacket enclosure, which measured 38.1 cm (15.0 in.) on a side, surrounded the cubical test space. The water jacket consisted of six separate rectangular channels 3.175 cm (1.25 in.) in width, which gave one channel for each face of the cube. Several inlet and outlet ports on each of the channels, fed by a manifold system, ensured a uniform flow over each of the sides. The flow rate of cooling water to each of the channels was separately adjusted to maintain the cube which enclosed the test space at isothermal conditions. The cooling water was collected from the water jacket and pumped through a chiller apparatus, into an insulated storage tank, and from there back into the water jacket. A schematic of the appartus is shown in Figure 3.2.

Access to the test chamber was accomplished through a removable rectangular cover on the water jacket and a 25.4 cm (10.0 in.) diameter circular cover on the top face of the enclosing cube. The rectangular cover was sealed with a rubber

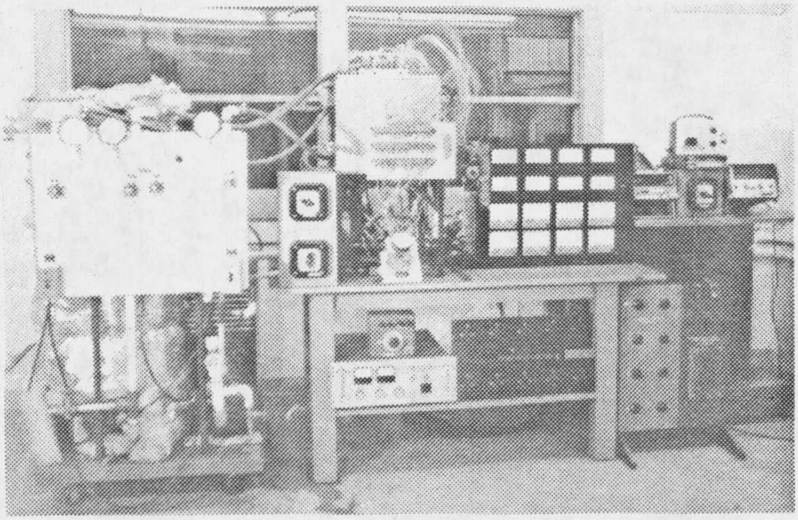

Figure 3.1 Heat Transfer Apparatus