



Natural convection heat transfer between arrays of horizontal cylinders and their enclosure  
by Robert Allen Weaver

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE  
in Mechanical Engineering  
Montana State University  
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Abstract:

The natural convection heat transfer between arrays of horizontal, heated cylinders and their isothermal, cooled enclosure was experimentally investigated. Four different cylinder arrays were used: two in-line and two staggered. Four fluids (air, water, 20 cs silicone and 96% glycerine) were used with Prandtl numbers ranging from 0.705 to 13090.0. There was no significant change in the Nusselt number between isothermal and constant heat flux conditions of the cylinder arrays. The average heat transfer coefficient was most affected by the spacing between cylinders and the total surface area of the cylinder arrays. The enclosure reduced the increase in both the average and the local heat transfer coefficients caused by changing the inner body from an in-line arrangement to a staggered arrangement of comparable spacing. An increase in fluid viscosity reduced the influence of the geometric effects.

The best empirical equation for all of the experimental data using one correlating parameter was:  $Nus = 0.214Ra^*s^{0.260}$ ;  $Ra^*s = Ras(L/Ri)$  for  $0.705 \leq Pr \leq 1.31 \times 10^4$ ;  $4.45 \times 10^4 \leq Ras \leq 1.17 \times 10^8$   $0.602 \leq L/Ri \leq 1.041$ ;  $4.63 \times 10^4 \leq Ra^*s \leq 8.15 \times 10^7$  with an average percent deviation of 12.00.

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NATURAL CONVECTION HEAT TRANSFER BETWEEN ARRAYS OF  
HORIZONTAL CYLINDERS AND THEIR ENCLOSURE

by

ROBERT ALLEN WEAVER

A thesis submitted in partial fulfillment  
of the requirements for the degree

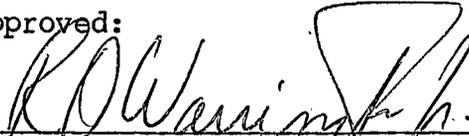
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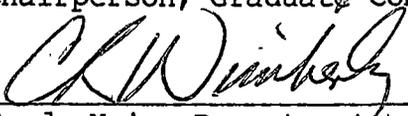
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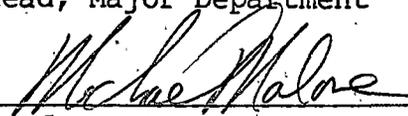
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## NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
$A_C$	Flow cross-sectional area between cylinders
$A_{CC}$	Cross-Sectional area of the copper-constantan thermocouples
$A_{htl}$	Cross-sectional area of the heat tape leads
$A_I$	Surface area of the inner body
$A_O$	Surface area of the outer body
$A_S$	Cross-sectional area of the support stems
$B$	Length of boundary layer on one cylinder, $B = \pi(d/2)$
$C_{1-4}$	Empirically determined constants
$c_p$	Specific heat at constant pressure
$d$	Diameter of a cylinder
$D_h$	Hydraulic diameter, $D_h = (4A_C Z)/A_I$
$g$	Acceleration of gravity, $9.81 \text{ m/sec}^2$ ( $32.17 \text{ ft/sec}^2$ )
$Gr_X$	Grashof number, $\beta \rho^2 g (T_I - T_O) X^3 / \mu^2$ , where X is any characteristic length
$h$	Heat transfer coefficient, $h = Q_{CONV} / A_I \Delta T$
$H$	Vertical pitch between cylinder rows
$H'$	Modified vertical pitch, $H' = H + d + [d / (\text{number of cylinder rows} - 1)]$
$k$	Thermal conductivity
$K$	Horizontal distance between cylinders
$k_{CC}$	Thermal conductivity of the copper-constantan thermocouples

<u>Symbol</u>	<u>Description</u>
$k_{htl}$	Thermal conductivity of the heat tape leads
$k_s$	Thermal conductivity of the support stems
$L$	Hypothetical gap width, $R_o - R_i$
$\Delta l$	Change in length
$Nu_X$	Nusselt number, $hX/k$ , where $X$ is any characteristic length
$Pr$	Prandtl number, $c_p \mu / k$
$Q_{COND}$	Heat transfer by conduction
$Q_{CONV}$	Heat transfer by convection
$Q_{RAD}$	Heat transfer by radiation
$Q_{TOT}$	Total amount of heat transfer, $Q_{TOT} = Q_{COND} + Q_{CONV} + Q_{RAD}$
$r$	Radius of a cylinder
$Ra_X$	Rayleigh number, $\beta \rho^2 g (T_I - T_O) X^3 c_p / \mu k$ , where $X$ is any characteristic length
$Ra^*_X$	Modified Rayleigh number, $Ra^*_X = Ra_X (L/R_i)$
$R_i$	Radius of a hypothetical sphere equal in volume to the volume of one cylinder times the number of cylinders in the cylinder array
$R_o$	Radius of a hypothetical sphere equal in volume to the outer body
$S$	Characteristic length, $S = (R_o - R_i) (A_I/A_O)$
$T_b$	Bulk temperature or average fluid temperature
$T_f$	Film temperature, $T_f = (T_b + T_I)/2$
$T_I$	Inner Body Temperature
$T_j$	Reference temperature, $T_j = T_b + 0.32(T_s - T_b)$

<u>Symbol</u>	<u>Description</u>
$T_m$	Arithmetic mean temperature, $T_m = (T_I + T_O)/2$
$T_n$	Reference temperature, $T_n = T_b + 0.20(T_s - T_b)$
$T_O$	Outer body temperature
$T_s$	Surface temperature
$\Delta T$	Temperature difference, $\Delta T = T_I - T_O$
X	Any characteristic length
Z	Height of the tube bundle
$\beta$	Thermal expansion coefficient
$\mu$	Dynamic viscosity
$\pi$	Ratio of circle circumference to diameter, 3.14159
$\rho$	Density of the fluid
$\rho_{atm}$	Density of the fluid at atmospheric pressure

## ABSTRACT

The natural convection heat transfer between arrays of horizontal, heated cylinders and their isothermal, cooled enclosure was experimentally investigated. Four different cylinder arrays were used: two in-line and two staggered. Four fluids (air, water, 20 cs silicone and 96% glycerine) were used with Prandtl numbers ranging from 0.705 to 13090.0. There was no significant change in the Nusselt number between isothermal and constant heat flux conditions of the cylinder arrays. The average heat transfer coefficient was most affected by the spacing between cylinders and the total surface area of the cylinder arrays. The enclosure reduced the increase in both the average and the local heat transfer coefficients caused by changing the inner body from an in-line arrangement to a staggered arrangement of comparable spacing. An increase in fluid viscosity reduced the influence of the geometric effects.

The best empirical equation for all of the experimental data using one correlating parameter was:

$$Nu_S = 0.214Ra^*_S{}^{0.260}; Ra^*_S = Ra_S(L/R_i)$$

for  $0.705 \leq Pr \leq 1.31 \times 10^4$ ;  $4.45 \times 10^4 \leq Ra_S \leq 1.17 \times 10^8$

$0.602 \leq L/R_i \leq 1.041$ ;  $4.63 \times 10^4 \leq Ra^*_S \leq 8.15 \times 10^7$

with an average percent deviation of 12.00.

## CHAPTER I

### INTRODUCTION

Natural convection heat transfer from a body to an infinite fluid medium has received extensive experimental and analytical study in the past. In recent years there has been a growing demand for an understanding of natural convection heat transfer within enclosures. This phenomenon has important industrial applications in areas such as nuclear reactor technology, electronic instrumentation packaging, aircraft cabin design, crude oil storage tank design, solar collector design, and energy storage systems.

This is one of the first studies of multiple body natural convection heat transfer in enclosures. Its purpose is to experimentally investigate the dissipation of heat by natural convection from arrays of heated, horizontal cylinders to a cooled, isothermal, cubical enclosure. The cylinders were subjected to both isothermal and constant heat flux conditions. This study determines the effects of cylinder geometry, and compares the results with the findings of previous studies. Four fluids and four cylinder configurations were utilized. The fluids used were air, water, 96 percent glycerine, and 20cs silicone, with Prandtl numbers ranging from 0.705 to 13090.0. The four cylinder configurations consisted of two in-line arrangements using nine and sixteen cylinders, and two

staggered arrangements using eight and fourteen cylinders.

## CHAPTER II

### LITERATURE REVIEW

Natural convection heat transfer can be classified in two ways: the external convection problem of flow about a body surrounded by an infinite fluid medium, and the internal convection problem of flow within an enclosure. The external problem has been investigated extensively in the past while relatively little attention has been given to the internal problem. This is due to the fact that internal natural convection problems are significantly more complex. For external problems the Prandtl boundary layer theory allows one to assume that the region surrounding the boundary layer is unaffected by the boundary layer. However, in the internal natural convection problem the boundary layer and the region adjacent to the boundary layer interact with each other, making it difficult to obtain analytic solutions to internal problems. The remainder of this chapter is intended to provide a useful background for this particular investigation and is not a complete survey of research in natural convection. The following discussion will be divided into the two categories mentioned above. These are (1) external natural convection to an infinite fluid medium, and (2) internal natural convection in enclosures. However, the discussion is limited to geometries which pertain to this investigation.

EXTERNAL NATURAL CONVECTION

There have been several studies of the heat transferred from single objects (e.g. plates, spheres, cylinders, etc.) to an infinite fluid medium. Morgan [1] gives a very thorough summary of the major correlations for heat transfer from smooth, horizontal, circular cylinders to an infinite fluid medium with Rayleigh numbers ranging from  $10^{-10}$  to  $10^{13}$ .

It may be shown by dimensional analysis [2] that the heat transfer from horizontal cylinders varies with the Grashof and Prandtl numbers. The resulting equation is generally of the form

$$Nu_x = A_1 + B_1 (Gr_x Pr)^{C_1}.$$

$A_1$ ,  $B_1$ , and  $C_1$  are constants and  $X$  is a characteristic length dimension where

$$Nu_x = hX/k \text{ (Nusselt number),}$$

$$Gr_x = (g \beta \rho^2 x^3 \Delta T) / \mu^2 \text{ (Grashof number),}$$

and

$$Pr = \mu c_p / k \text{ (Prandtl number).}$$

The nondimensional grouping  $(Gr_x Pr)$  is known as the Rayleigh number, which is

$$Ra_x = Gr_x Pr = (c_p \beta g \rho^2 x^3 \Delta T) / k \mu.$$

Fand, Morris, and Lum [3] presented three different correlations for natural convection heat transfer from horizontal cylinders, based on three different reference temperatures. These are

$$Nu_f = 0.474 Ra_f^{0.25} Pr_f^{0.047},$$

$$Nu_j = 0.478 Ra_j^{0.25} Pr_j^{0.050},$$

and

$$Nu_n = 0.456 Ra_f^{0.25} Pr_f^{0.057}$$

where the subscripts denote the reference temperature used in evaluating the fluid properties. The reference temperatures were

$$T_f = T_b + 0.5(T_s - T_b) \text{ (film temperature),}$$

$$T_j = T_b + 0.32(T_s - T_b),$$

and

$$T_n = T_b + 0.20(T_s - T_b).$$

Raithby and Hollands [4] have published a correlation equation for laminar and turbulent natural convection from elliptic cylinders of arbitrary eccentricity for the case of constant surface temperature. For the case of horizontal isothermal cylinders their equation takes the form

$$Nu_m = \left[ \frac{2}{\ln \left\{ (1 + \pi^{3/4}) / (f_2^{3/4} C_1 Ra^{1/4}) \right\}} \right]^m + (0.72 C_t Ra^{1/3})^m$$

where  $m = 3.337$

$$f_2 = 2.587$$

$$C_1 = (2/3) / [1 + (0.49/Pr)^{9/16}]^{4/9}$$

$$C_t = 0.14 Pr^{0.084} \text{ or } 0.15, \text{ whichever is smaller.}$$

Churchill and Chu [5] suggest that an additive constant is required in the correlation equation for natural convection heat

transfer from horizontal cylinders. They have published the following correlation equation for heat transfer by natural convection from horizontal cylinders

$$\text{Nu}_d = 0.36 + 0.518 \left[ \frac{\text{Ra}_d}{[1 + (0.559/\text{Pr})^{9/16}]^{16/9}} \right]^{0.25}$$

for all Prandtl numbers and for Rayleigh numbers ranging from  $10^{-6}$  to  $10^9$ . They recommend that for large temperature differences, such that the variation of physical properties is significant, the properties may be evaluated at the average of the bulk and surface temperatures as a first approximation.

Kim, Pontikes, and Wollersheim [6] experimentally studied the natural convection heat transfer from a horizontal cylinder with isothermal and constant heat flux surface conditions. The average free convection results were obtained by integrating the local Nusselt, Prandtl, and Grashof numbers over the test section surface area. The average Nusselt numbers for the experimental data were

$$\text{Nu}_r = 0.89 \text{ Ra}_r^{0.19}$$

for isothermal surface conditions, and

$$\text{Nu}_r = 0.57 \text{ Ra}_r^{0.20}$$

for constant heat flux surface conditions.

Although there has been extensive research performed on natural convection heat transfer from a single horizontal

cylinder, natural convection heat transfer from multiple cylinders has received little attention. Eckert and Soehngen [7], who performed one of the first studies of natural convection heat transfer from multiple cylinders, investigated the effects that one heated cylinder had on adjacent cylinders in a vertical array of horizontal, isothermal cylinders. They discovered that when one cylinder was positioned directly over another at a distance of four diameters, there was no change in the Nusselt number of the lowest cylinder as opposed to a single cylinder while the upper cylinder Nusselt number was 87 percent of the value for the lower one. A reduction in the heat transferred from the upper cylinder was said to be caused by the heated wake from the lower cylinder striking the upper cylinder. When three cylinders were arranged in a vertical array the Nusselt number of the middle cylinder was 83 percent of the Nusselt number of the bottom cylinder while the Nusselt number of the top cylinder was 65 percent of the value for the bottom cylinder. When the cylinders were staggered such that the middle cylinder was moved laterally out of line by one half of a diameter, the wake of the bottom cylinder missed the middle cylinder and the Nusselt number of the middle cylinder was 103 percent of the value for the bottom cylinder while the Nusselt number of the top cylinder was 87 percent of the value for the bottom cylinder. The increase in the heat transferred from the middle cylinder was a result of the

higher velocity of fluid movement past this cylinder which was induced by the wake from the bottom cylinder.

Liberman and Gebhart [8] investigated the interaction of natural convection wakes between a parallel array of wires. By orienting the array at different angles measured from the vertical and different spacings, they were able to determine the spacing which yielded a maximum Nusselt number for a particular angle of inclination.

Marsters [9] performed an experimental study on the natural convection heat transfer from a vertical array of heated horizontal cylinders. He concluded that the heat transfer characteristics exhibited by vertical arrays of heated horizontal cylinders are not predicted by simple superposition of single cylinder behavior. For closely spaced arrays (two diameters between cylinders), individual tube Nusselt numbers were found to be as much as 50 percent lower than for a single cylinder. For wider spacings, individual cylinder Nusselt numbers were as much as 30 percent higher than that of a single cylinder. He concluded that the overall heat transfer characteristics of an array are dependent upon array spacing as well as Rayleigh number.

Tillman [10] developed two correlation equations for natural convection heat transfer from tube bundles:

$$\text{Nu}_f = 0.057 \text{ Ra}_f^{0.5}$$

for in-line arrays and

$$\text{Nu}_f = 0.067 \text{ Ra}_f^{0.5}$$

for staggered arrays. All of the thermal properties except the coefficient of thermal expansion were evaluated at the film temperature. The coefficient of thermal expansion was evaluated at the ambient temperature. The hydraulic diameter for a compact heat exchanger was used for the characteristic length, which was defined as

$$D_h = (4A_c Z) / A_f.$$

Tsubouchi and Saito [11] conducted an experimental study of the natural convection heat transfer from arrays of uniformly heated circular cylinders in air. The cylinders were arranged in various in-line and staggered arrangements. They found that the heat transfer depended on the cylinder spacing, the number of cylinders, and the type of cylinder arrangement. They proposed the following correlation

$$\text{Nu}_M = \phi_2 (0.092) \{1 - 0.92 \exp[(d-K)/d]\} (\text{PrGr}_H)^{0.4}$$

where  $\phi_2 = 1.00$  for in-line banks and  $\phi_2 = 1.06$  for staggered banks. The number of vertical columns ranged from 3 to 5. The number of horizontal rows ranged from 3 to 7 and  $K$  was the horizontal distance between the cylinders, which had outer diameters of  $d$ . The vertical distance between the cylinders was  $H$ . The characteristic length  $H'$  was the modified vertical pitch, defined as

$$H' = H + d + d/[(\text{number of horizontal rows}) - 1].$$

They concluded by stating that the average heat transfer coefficient was affected more by a variation in the vertical pitch than by a variation in the horizontal pitch.

#### INTERNAL NATURAL CONVECTION

Internal natural convection heat transfer utilizes the same dimensionless parameters as are used for external natural convection, however an additional parameter involving a ratio of characteristic dimensions is used in internal problems.

Warrington [12] performed an in-depth experimental study of natural convection in enclosures. His work involved the heat transfer between inner bodies such as spheres, cubes, and cylinders to both spherical and cubical enclosures. Several different fluids were utilized with Prandtl numbers ranging from 0.706 to 13,800. The recommended overall correlation from his data was

$$Nu_L = 0.425 Ra_L^{0.234} (L/R_i)^{0.498}.$$

For a cylindrical inner body and a cubical enclosure the best correlation was

$$Nu_B = 0.593 Ra_B^{0.240} (L/R_i)^{0.434}.$$

Larson, Gartling, and Schimmel [13] used laser interferometry to experimentally determine the temperature field around a heated horizontal cylinder in an isothermal rectangular

box. The purpose of their study was to simulate the possible geometric configurations of a nuclear spent fuel element in a shipping cask and compare the experimental results with numerical results using finite-difference and finite-element techniques.

Dutton and Welty [14] conducted an experimental study of natural convection heat transfer in an array of uniformly heated vertical cylinders surrounded by a vertical cylindrical enclosure with mercury as the fluid medium. The cylinders were arranged in an equilateral triangular pattern. Their results indicated that the natural convection heat transfer was strongly dependent on the cylinder spacing and was less dependent on heat flux and circumferential position. In their concluding remarks they suggest that natural convection heat transfer results in the low Prandtl number range (liquid metals) are well represented by correlations involving the  $Gr_x Pr^2$  product, which is independent of viscosity.

Van De Sande and Hamer [15] studied the steady and transient natural convection heat transfer between horizontal concentric circular cylinders with constant heat flux surface conditions. Their experiment showed that a sidewise displacement of the inner cylinder did not affect the heat transfer results. However, the overall heat transfer decreased or increased depending on whether the inner cylinder was above or below the center line of the outer cylinder. An additional dimensionless group was introduced

to account for the effect of vertical eccentricity. The results were used to estimate the cooling of buried cable systems surrounded by a water layer.

Crupper [16] performed an experimental study of natural convection heat transfer between a set of four isothermal, heated cylinders and an isothermal, cooled, cubical enclosure, to determine the effect of the positioning of the cylinders within the enclosure, and to compare the results with the findings of previous studies on heat transfer from single bodies to an enclosure. Four fluids and inner body positions were utilized. The set of cylinders was oriented in both a horizontal and vertical position. The four fluids had Prandtl numbers ranging from 0.7 to  $3.1 \times 10^4$  and Rayleigh numbers, based on gap width, ranging from  $6.3 \times 10^5$  to  $6.9 \times 10^8$ . He found that the best correlations for all of the heat transfer data combined were

$$Nu_B = 0.277 Ra_B^{0.274} Pr^{0.012}$$

and

$$Nu_B = 0.286 Ra_B^{0.275}.$$

The best correlations for the cylinders in the horizontal position were

$$Nu_L = 0.498 Ra_L^{0.245} Pr^{-0.002}$$

and

$$Nu_L = 0.496 Ra_L^{0.245}.$$

All of the fluid properties were evaluated at the arithmetic mean of the inner and outer body temperatures.

Powe [17] investigated the limits of relative gap width for which available correlation equations for natural convection heat transfer in enclosures were applicable. Heat transfer rates for large relative gap widths were shown to be limited by those obtained for free convection to an infinite fluid medium, and this criteria was used to calculate a maximum relative gap width for which the enclosure equations were applicable. A minimum relative gap width for applicability of the enclosure equations was determined by the pure conduction limit.

Brown [18] experimentally studied the transfer of heat by natural convection within enclosures at reduced pressures. The best correlation which included a correction for the air density was

$$Nu_L = 0.342 Ra_L^{1/4} (\rho/\rho_{atm})^{0.129}.$$

The geometries used were cylinder-cube (inner body-outer body) and cube-cube, with the bodies mounted concentrically in both cases. The Rayleigh number ranged from  $1 \times 10^3$  to  $2 \times 10^6$  and the pressure ranged from 2670 - 86,180 Pa (20 - 646.4 mm Hg).

Powe, Warrington, and Scanlan [19] performed a detailed study of natural convection flow phenomena which occur between a body of relatively arbitrary shape and its spherical enclosure. Resulting trends in the fluid flow data were established to

facilitate better predictions of the heat transfer in problems of natural convection in enclosures.

As evidenced by this review, the amount of interest in internal natural convection has increased dramatically in the last decade. The intent of this study is to extend the work performed by Warrington and Crupper [16, 20] in the area of natural convection heat transfer between multiple bodies and an enclosure.

## Chapter III

### EXPERIMENTAL APPARATUS AND PROCEDURE

#### EXPERIMENTAL APPARATUS

The outer body used for this investigation was a cube 26.67 cm (10.5 in.) along an inner side, constructed from 1.27 cm (0.5 in.) thick, type 6061 aluminum. The assembled outer body and peripheral components are shown in Figure 3.1. A water jacket enclosure, which measured 38.1 cm (15.0 in.) on a side, surrounded the cubical test space. The water jacket consisted of six separate rectangular channels 3.175 cm (1.25 in.) in width, which gave one channel for each face of the cube. Several inlet and outlet ports on each of the channels, fed by a manifold system, ensured a uniform flow over each of the sides. The flow rate of cooling water to each of the channels was separately adjusted to maintain the cube which enclosed the test space at isothermal conditions. The cooling water was collected from the water jacket and pumped through a chiller apparatus, into an insulated storage tank, and from there back into the water jacket. A schematic of the apparatus is shown in Figure 3.2.

Access to the test chamber was accomplished through a removable rectangular cover on the water jacket and a 25.4 cm (10.0 in.) diameter circular cover on the top face of the enclosing cube. The rectangular cover was sealed with a rubber

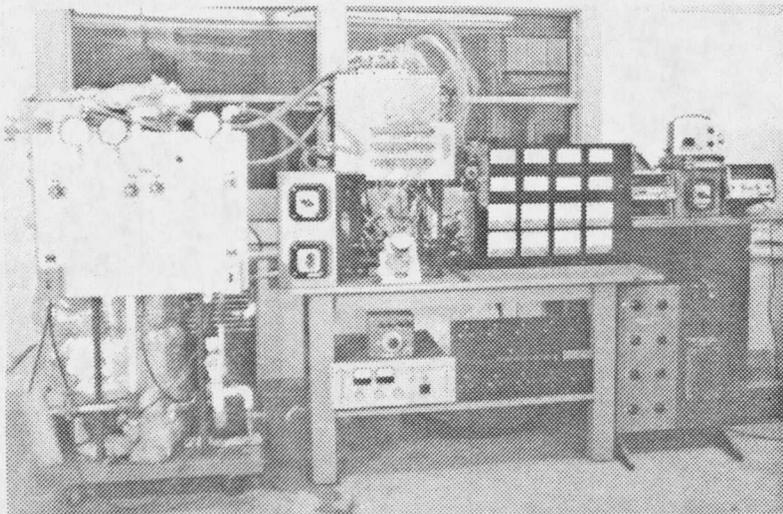


Figure 3.1 Heat Transfer Apparatus

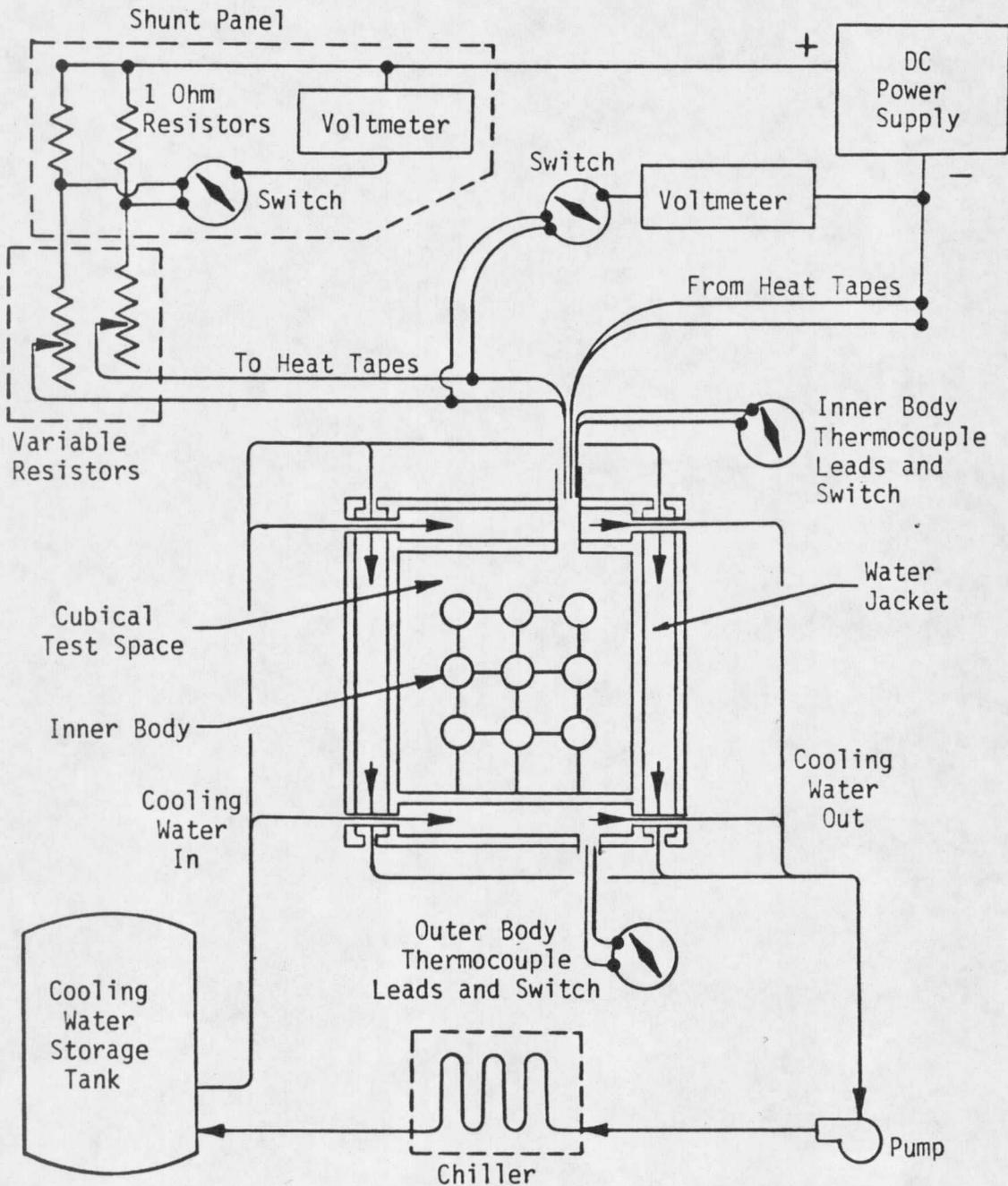


Figure 3.2 Schematic of the Heat Transfer Apparatus

gasket and the circular cover was flanged and sealed with an O-ring.

Four different arrangements of horizontal, heated cylinders were used for inner geometries in this investigation. The four geometries consisted of two in-line and two staggered arrangements. These geometries are shown in Figures 3.3 - 3.6. All of the cylinders were fabricated out of 0.36 cm (0.14 in.) thick copper pipe, 19.46 cm (7.66 in.) long, and 4.22 cm (1.66 in.) outside diameter. Copper end caps 0.25 cm (0.098 in.) thick and 4.22 cm (1.66 in.) in diameter were mounted flush to both ends of each cylinder and attached with high temperature epoxy. The length to diameter ratio of the cylinders was 4.73. Wooden dowels 0.32 cm (0.125 in.) in diameter were used to provide support and spacing for the cylinders. The ends of the wooden dowels were press fitted into .25 cm (0.1 in.) deep holes in each cylinder. Two adjacent cylinders were connected by one or two wooden dowels, depending on their location in the array of cylinders. Each wooden dowel was covered with shrink tube to preserve the wood and minimize the loss of heat. Each array of cylinders was aligned so that its axis was parallel to the sides of the cube. For each geometry, the axial distance between the ends of the cylinders and the cube was 3.35 cm (1.32 in.). The ratio of cylinder diameter to cylinder spacing (center to center) for the inner geometries was 0.55 for the eight and nine cylinder

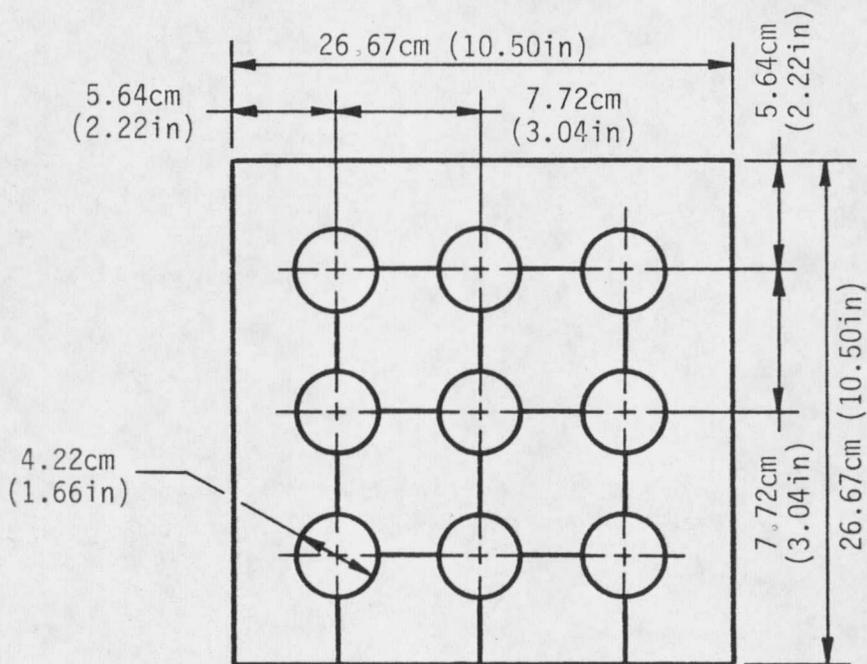
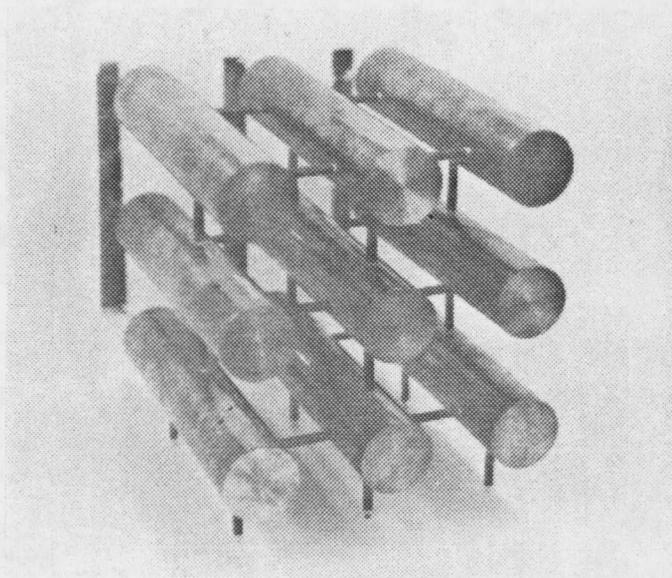


Figure 3.3 Nine Cylinder In-Line Arrangement

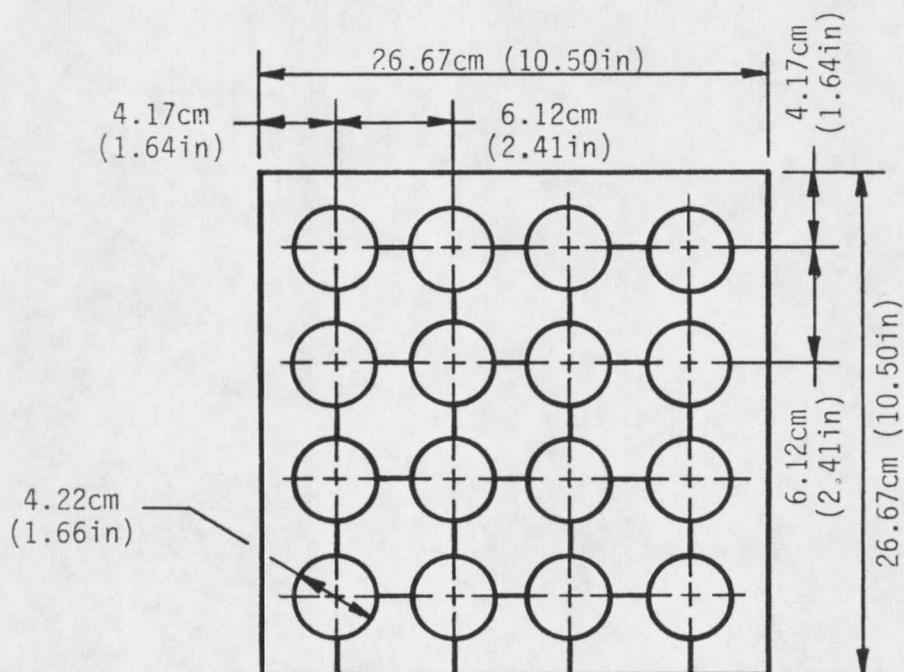
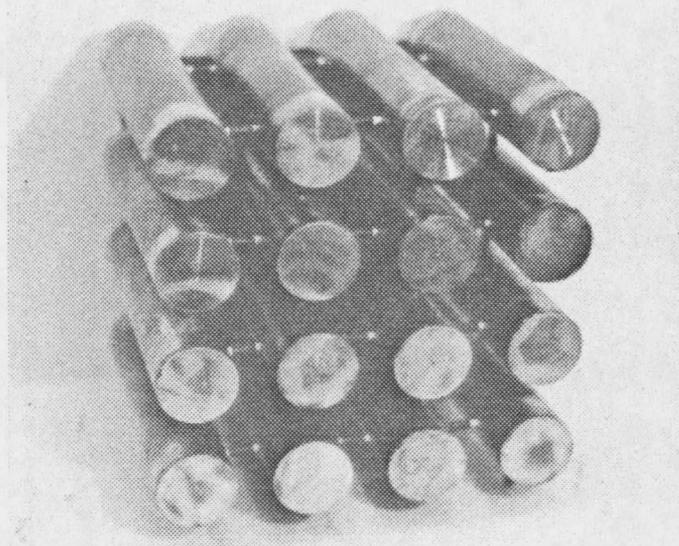


Figure 3.4 Sixteen Cylinder In-Line Arrangement

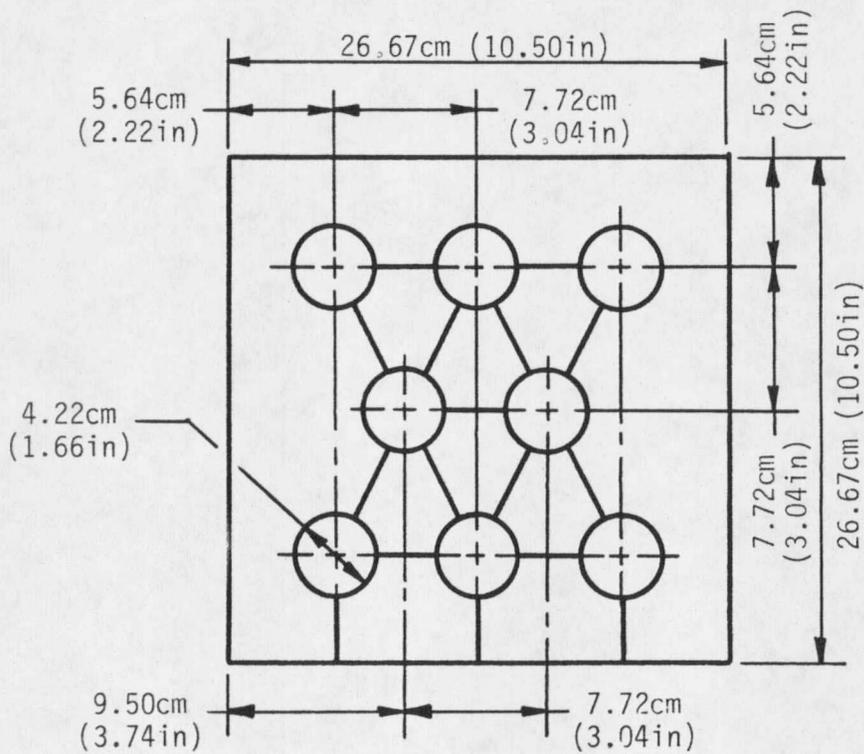
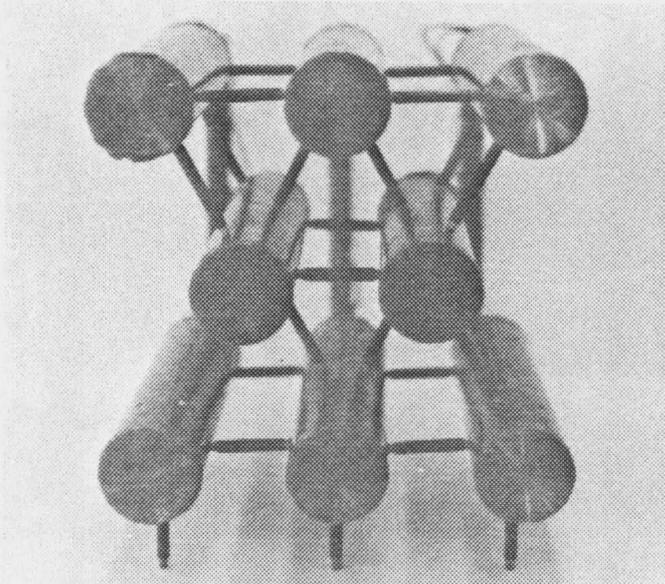


Figure 3.5 Eight Cylinder Staggered Arrangement

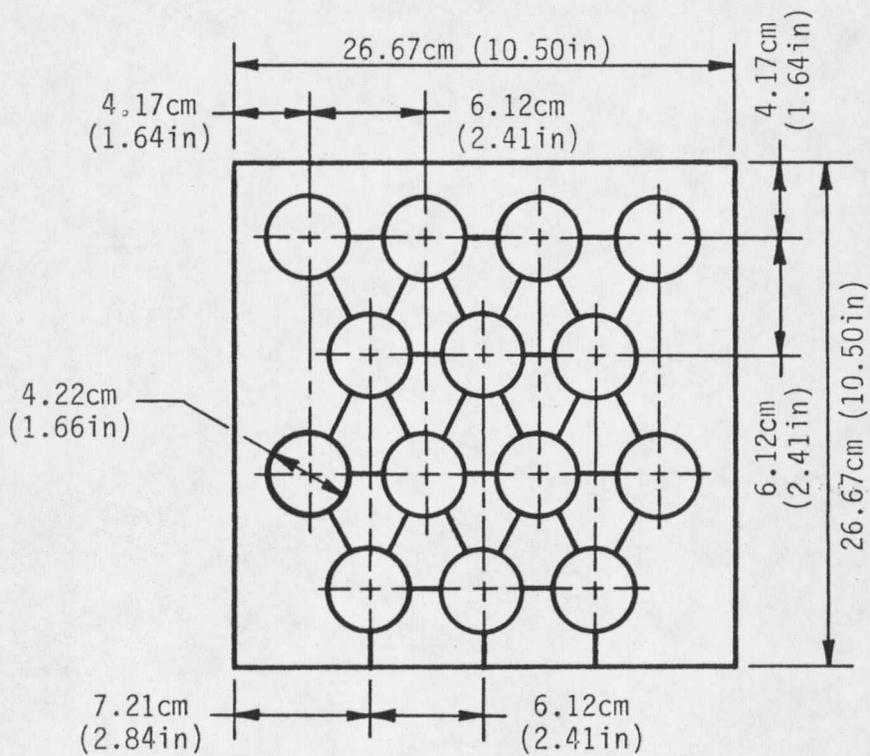
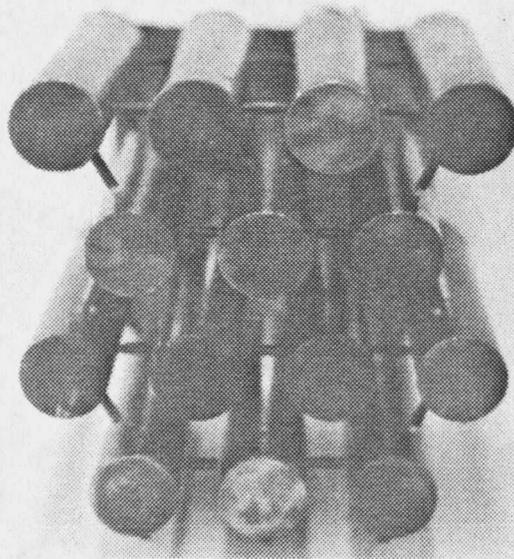


Figure 3.6 Fourteen Staggered Cylinder Arrangement

arrangements (pitch to diameter was 0.83), and 0.69 for the fourteen and sixteen cylinder arrangements (pitch to diameter was 0.45). For the staggered arrangements, the cylinder spacing was calculated as the horizontal center to center distance between cylinders.

Heat was supplied to each cylinder with electrical resistance heat tape and a direct current power source. The heat tape, 0.05 cm (0.02 in.) thick and .32 cm (.125 in.) wide, consisted of an electrical resistance wire which was rated at 28.87 ohms/m (8.8 ohms/ft.) and a maximum power of 75.46 watts/m (78.53 Btu/hr/ft). The heat tape was applied to the inner surface of each cylinder utilizing two pieces approximately 2.13 m (7 ft.) long with each piece starting at the midpoint of the cylinder and wrapped in a helix fashion toward the end. One end from each piece was connected to a power lead while the other ends were connected in series. Once the heat tapes were in place, they were coated with a high temperature sealant which kept them securely in place and provided an insulated backing. Input voltages to the tapes were controlled individually by using Ohmite variable power resistors (0-35 ohms or 0-50 ohms, 150 watts, 2.07 amperes or 1.73 amperes maximum) connected in series with the tapes. The electrical circuitry is shown in Figure 3.2. The inside of each cylinder was filled with a two-part silicone potting compound to protect the electrical connections and

minimize any convective currents which might cause an uneven temperature distribution in the cylinders.

The inside surface temperature of the enclosing cube was monitored by the use of 25 copper-constantan thermocouples epoxied 0.318 cm (0.125 in.) from the inner surface. The number of thermocouples on each face of the enclosing cube varied from 3 to 7. The thermocouples on any common side were connected in parallel to provide an average temperature for each face. It has been shown by Warrington [12] that the temperature variation on any side was less than  $.8^{\circ}\text{C}$  ( $1.5^{\circ}\text{F}$ ).

The temperature of each cylinder was monitored by two copper-constantan thermocouples epoxied from the inside of the cylinder into the wall and flush with the outer surface. The two thermocouples were placed 4.99 cm (1.97 in.) from each end of the cylinder.

The lead wires from the thermocouples and heat tape in each cylinder passed through a 0.32 cm (0.125 in.) diameter hole in one of the end caps. The lead wires were directed upward where they exited through a 1.91 cm (0.75 in.) diameter, 12.70 cm (5 in.) long PVC pipe, which was threaded into the circular cover on the top face of the enclosing cube and then passed through a hole in the rectangular cover of the water jacket. The pipe was sealed to the water jacket cover with an O-ring. At the end of the pipe, which protruded above the apparatus, the lead wires

were sealed off from the test chamber with silicone rubber cement.

#### EXPERIMENTAL PROCEDURE

The array of cylinders was placed and centered in the cubical test space. After attaching and sealing the circular cover and water jacket lid, the lead wires from the inner body were sealed off from the cubical test space with silicone rubber cement and connected as shown in Figure 3.2. With the exception of air as the working fluid, the test space was then filled with a test fluid from a reservoir located above the apparatus. The gravity fed liquid flowed through a fill stem located on the bottom of the apparatus and exited out of a hole in the top when the test space was filled.

The cooling system and power supply were activated. The cooling water flow rates were adjusted so that the desired isothermal condition was obtained for the outer body. By adjusting the current to each cylinder with the rheostats, approximately seven data points with the inner body at isothermal conditions and four data points with the inner body at constant heat flux conditions were taken for each fluid/geometry combination. When thermal equilibrium was reached (approximately two to four hours) the following data were taken: 1) The inner body and outer body thermocouple emf readings, 2) The input

voltage to each cylinder, and 3) The input amperage to each cylinder. In all, 172 data points were taken utilizing 16 different fluid/geometry combinations. A list of the partially reduced data, along with the type of test fluid, the inner body arrangement, and the inner body boundary condition is shown in Appendix II.

The percent temperature variation for either the inner or outer body was defined as

$$\text{Temperature Variation} = \frac{(T_{\text{Local, Max}} - T_{\text{Local, Min}})}{T_I - T_O} 100,$$

where  $T_{\text{Local, Max}}$  ( $T_{\text{Local, Min}}$ ) represents the maximum (minimum) inner or outer body temperature. The outer body had an average temperature variation of 7.31 percent for all of the data. The inner body had an average temperature variation of 12.92 percent for isothermal conditions and 51.47 percent for constant heat flux conditions.

The heat transferred by natural convection was obtained by subtracting the heat transfer by radiation between the inner and outer bodies and the conduction through the support system connecting the inner and outer bodies, from the total amount of heat which was transferred. This in equation form is,

$$Q_{\text{CONV}} = Q_{\text{TOT}} - Q_{\text{RAD}} - Q_{\text{COND}} \quad (3.1)$$

When air was used as a test fluid the following procedure experimentally measured the amount of heat which was transferred

by radiation and conduction. With the complete system assembled as previously explained, a vacuum pump was attached to the fill stem on the bottom of the apparatus which had been used to fill the test space. The test space between the inner and outer bodies was evacuated to a pressure below 6.67Pa (50 microns Hg), which essentially eliminated convection. Five points, over the range of inner body temperatures used in this experiment, for each inner body arrangement were collected to determine the heat loss by radiation and conduction. These data are shown in Figure 3.7. For the case of air, the average heat loss by radiation and conduction was 77.50 percent of the heat transferred by convection.

The three remaining test fluids, water, 96% glycerine and 20cs silicone were opaque to radiation while the average heat loss by conduction was 0.26 percent of the heat transferred by convection. Since the supports were insulated with shrink tube, the conduction heat loss from the inner body was calculated using a one-dimensional analysis of the conduction of heat between the bottom row of cylinders and the bottom face of the cubical test space. The conduction of heat through the lead wires from the thermocouples and heat tapes was also included in the analysis. The following equation

$$Q_{COND} = [(k_s A_s + k_{cc} A_{cc} + k_{htl} A_{htl}) / (\Delta l)] (T_I - T_O) \quad (3.2)$$

was used to calculate the conduction losses.

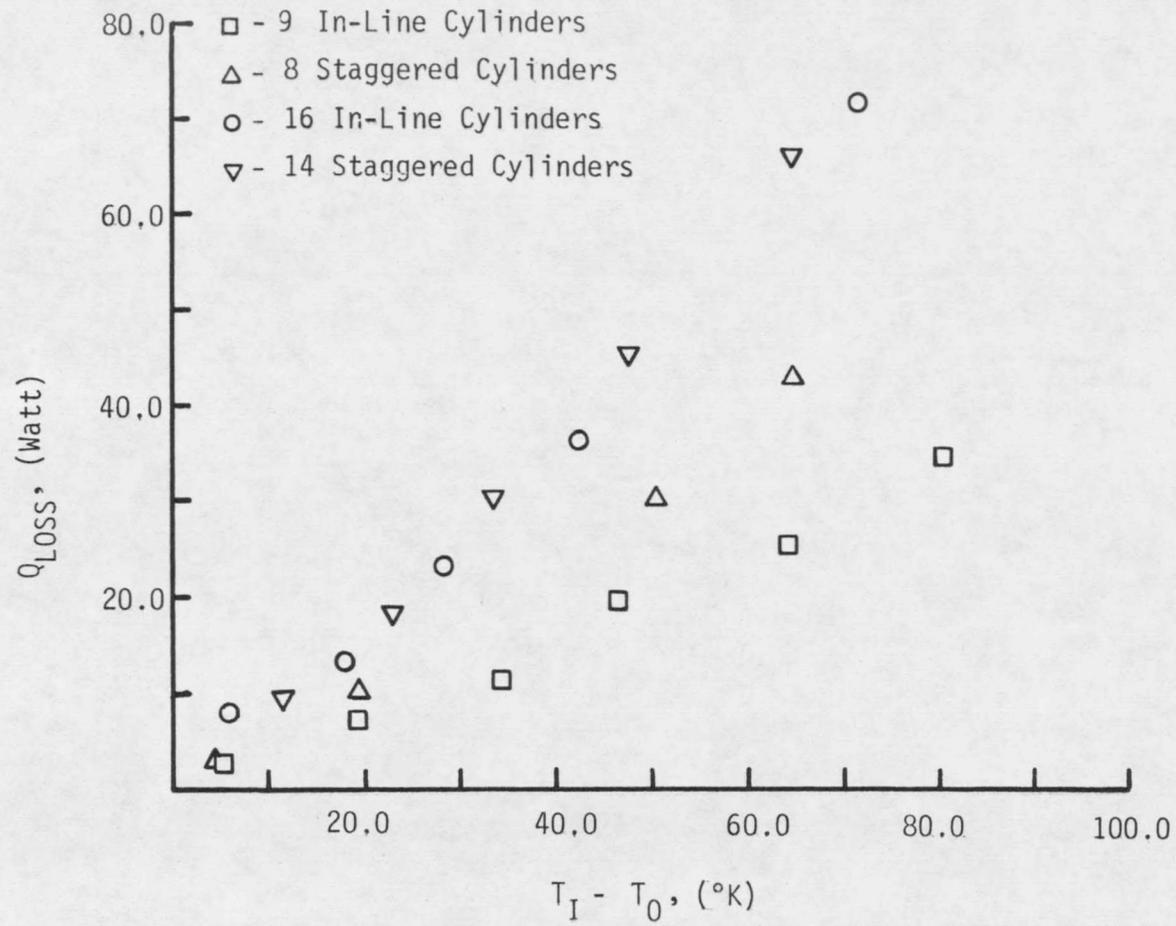


Figure 3.7 Heat Losses From Radiation and Conduction With Air as the Test Fluid

Once  $Q_{\text{CONV}}$  in equation (3.1) was determined, the average heat transfer coefficient ( $h$ ) was calculated using

$$h = (Q_{\text{CONV}}) / [(A_I)(T_I - T_O)]. \quad (3.3)$$

A program was then used to reduce and correlate the data in Appendix II; and existing subroutines [12] were used to calculate the fluid properties of the test fluids. A listing of the program and subroutines is shown in Appendix I. All property values were evaluated at the arithmetic mean fluid temperature, defined as

$$T_m = (T_O + T_I) / 2. \quad (3.4)$$

## CHAPTER IV

### RESULTS

This section will discuss the trends and correlations of the experimental data. The chapter is divided into the following sections: (1) Results of inner body boundary condition effects, (2) Results of geometric effects, (3) Results of Prandtl number effects, and (4) Results of all the experimental data combined. Nine characteristic lengths and twenty-four forms of the correlation equations were used to correlate the experimental data. The Rayleigh number, Nusselt number, Prandtl number, and a ratio of characteristic lengths were used as the independent dimensionless parameters. The following equational forms consistently provided the best results using a standard least squares method of curve fitting:

$$\text{Nu}_X = C_1 \text{Ra}_X^{C_2}, \quad (4.1)$$

$$\text{Nu}_X = C_1 \text{Ra}_X^{*C_2}, \quad (4.2)$$

$$\text{Nu}_X = C_1 \text{Ra}_X^{C_2} \text{Pr}^{C_3}, \quad (4.3)$$

$$\text{Nu}_X = C_1 \text{Ra}_X^{C_2} (L/R_i)^{C_3}, \quad (4.4)$$

and

$$\text{Nu}_X = C_1 \text{Ra}_X^{C_2} (L/R_i)^{C_3} \text{Pr}^{C_4}. \quad (4.5)$$

Three different characteristic lengths, used to calculate the Nusselt and Rayleigh numbers in the equations above, will be

used throughout this chapter. The gap width,  $L$ , is the distance between hypothetical concentric spheres of volumes equal to the actual volumes of the inner and outer bodies.  $R_i$  is the radius of a sphere which has a volume equal to the volume of one cylinder times the number of cylinders in the array.  $B$  is the distance traveled by the boundary layer on one horizontal cylinder (assuming no flow separation). This distance is defined as one-half of the outer circumference of a cylinder. Both Warrington and Crupper [12,16] found that defining  $B$  as the exact boundary layer length, determined from flow visualizations, did not improve the results.  $S$  is defined as the ratio of the inner body surface area to the outer body surface area multiplied by the gap width,  $L$ .

Several other characteristic lengths [10,11] were tried. These characteristic lengths explicitly took into account the cylinder spacing, cylinder diameter and the number of rows of cylinders of the inner body in relation to the cubical outer body. However, the three characteristic lengths,  $L$ ,  $B$  and  $S$  consistently yielded the best results, and will be used in the correlation results shown later in the text.

The ranges of the independent correlating parameters and the characteristic lengths are shown in Tables 4.1 and 4.2. Reference to parameters which require a characteristic length will be subscripted  $L$ ,  $B$  or  $S$  to denote the appropriate

TABLE 4.1  
RANGE OF DIMENSIONLESS PARAMETERS

DIMENSIONLESS PARAMETER	MINIMUM	MAXIMUM
Pr	0.705	13090.0
$Nu_L$	4.040	45.13
$Nu_B$	3.663	35.41
$Nu_S$	2.657	25.44
$Gr_L$	12.29	$8.220 \times 10^7$
$Gr_B$	9.974	$4.196 \times 10^7$
$Gr_S$	3.398	$2.143 \times 10^7$
$Ra_L$	$1.245 \times 10^5$	$3.801 \times 10^8$
$Ra_B$	$1.306 \times 10^5$	$1.964 \times 10^8$
$Ra_S$	$4.449 \times 10^4$	$1.170 \times 10^8$
$Ra_L^*$	$9.772 \times 10^4$	$3.958 \times 10^8$
$Ra_B^*$	$9.118 \times 10^4$	$1.913 \times 10^8$
$Ra_S^*$	$4.633 \times 10^4$	$8.153 \times 10^7$

TABLE 4.2

## CHARACTERISTIC DIMENSIONS OF EACH INNER BODY ARRANGEMENT

NUMBER OF INNER BODY CYLINDERS	L cm (in)	B cm (in)	S cm (in)	$L/R_i$
9 In-Line	8.115 (3.195)	6.624 (2.608)	5.004 (1.970)	0.963
8 Staggered	8.440 (3.323)	6.624 (2.608)	4.625 (1.821)	1.041
16 In-Line	6.332 (2.493)	6.624 (2.608)	6.942 (2.733)	0.620
14 Staggered	6.777 (2.668)	6.624 (2.608)	6.500 (2.559)	0.694

characteristic length.

#### RESULTS OF THE INNER BODY BOUNDARY CONDITION EFFECTS

A comparison of isothermal inner body conditions to constant heat flux inner body conditions is shown in Figure 4.1. To remove any geometric effects the graph is comprised of data from only one inner body arrangement, eight staggered cylinders. Figure 4.1 shows that for any one of the four fluid mediums, the constant heat flux data coincide very closely to the isothermal data. The three remaining inner body arrangements also exhibited a negligible difference in the Nusselt number for the different boundary conditions. Overall, the average Nusselt number of the constant heat flux data was only 0.44 percent greater than the average Nusselt number of the isothermal data.

The best correlation for all of the isothermal data was

$$Nu_B = 0.229 Ra_B^{0.257} (L/R_i)^{0.556} Pr^{0.021} \quad (4.6)$$

which had an average percent deviation of 10.48. The percent deviation at a point is defined as the quantity of the absolute difference between the data value and the equation value divided by the data value. The average percent deviation is the sum of the individual deviations divided by the number of data points.

The best correlation for all of the constant heat flux data was

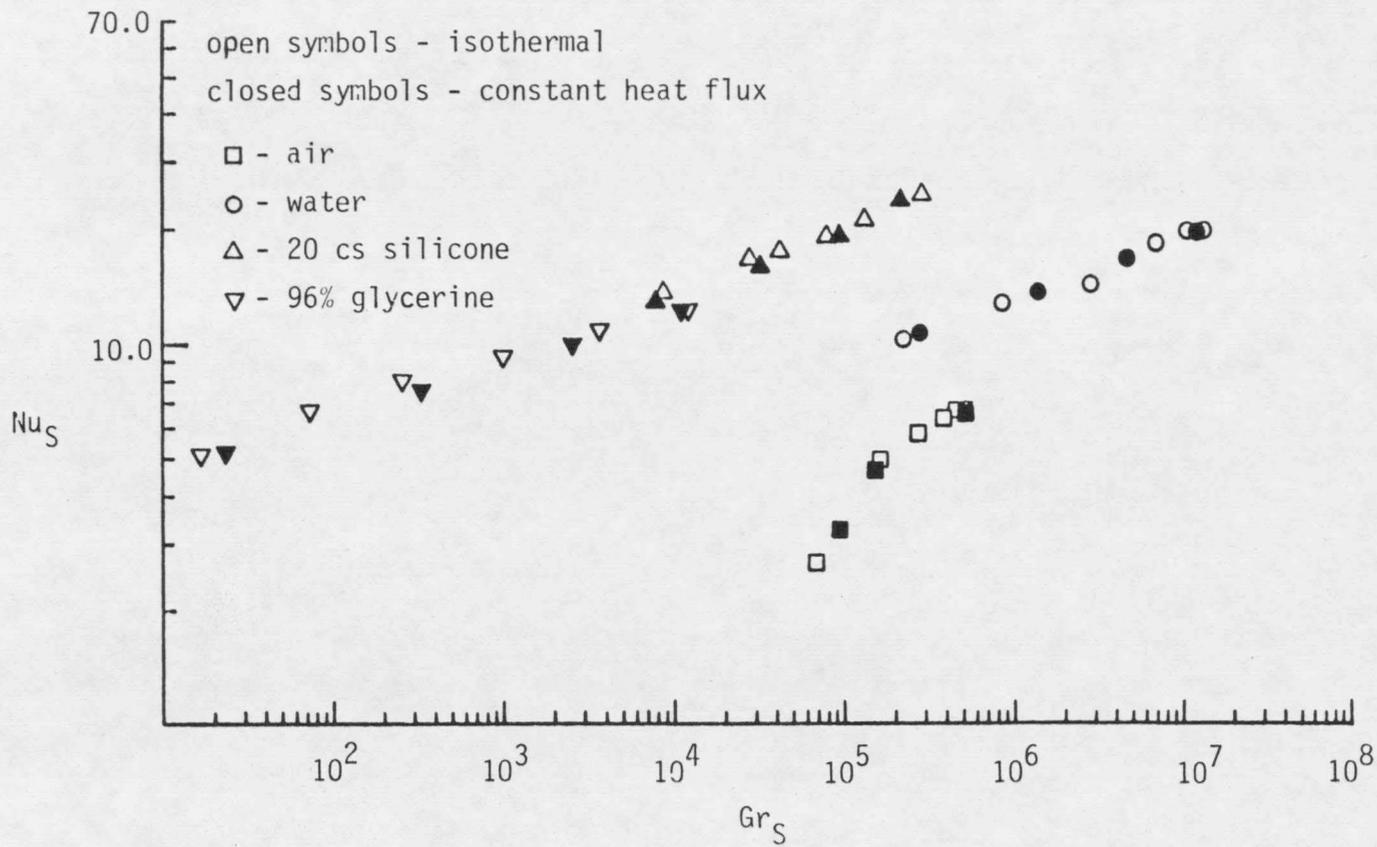


Figure 4.1 Comparison of the Heat Transfer for Isothermal and Constant Heat Flux Inner Body Conditions Using Data From the 8-Cylinder Arrangement

$$\text{Nu}_B = 0.221 \text{ Ra}_B^{0.260} (L/R_i)^{0.494} \text{ Pr}^{0.016} \quad (4.7)$$

which had an average percent deviation of 11.03.

Although there was no significant difference in the average heat transfer coefficient, Table 4.3 shows a noticeable difference in the local heat transfer coefficients when comparing isothermal and constant heat flux inner body conditions. The local heat transfer coefficient of each cylinder was calculated by using the temperature difference between the cylinder and the mean temperature of the outer body. The local heat transfer coefficient for each row of cylinders was the average of the local heat transfer coefficients of the cylinders in each row. Table 4.3 is a comparison, using all of the fluids, of the local heat transfer coefficients for each row of cylinders above the bottom row to the local heat transfer coefficient of the bottom row. The percentage comparisons were consistently higher for the constant heat flux conditions than they were for the isothermal conditions. As stated earlier, the average percent temperature variation of the inner body was 51.47 for constant heat flux conditions as opposed to 12.92 for isothermal conditions. Under constant heat flux conditions the temperatures of the upper rows of cylinders were forced to be increasingly higher than the temperature of the bottom row. Since the enclosure caused an increase in convective activity in the upper regions of the enclosure, the higher temperature of the upper rows augmented the

TABLE 4.3

COMPARISON OF THE LOCAL HEAT TRANSFER COEFFICIENT OF THE BOTTOM ROW OF CYLINDERS,  $h_{ROW1}$ , TO THE UPPER ROWS OF CYLINDERS ( $h_{ROW2}$ ,  $h_{ROW3}$ ,  $h_{ROW4}$ )

NUMBER OF INNER BODY CYLINDERS	INNER BODY BOUNDARY CONDITION	$\frac{h_{ROW2}}{h_{ROW1}} \times 100$	$\frac{h_{ROW3}}{h_{ROW1}} \times 100$	$\frac{h_{ROW4}}{h_{ROW1}} \times 100$
9 In-line	Isothermal	60.115	41.722	
8 Staggered		62.833	52.318	
16 In-line		53.910	36.819	33.925
14 Staggered		59.021	42.428	36.475
9 In-Line	Constant Heat Flux	77.515	67.752	
8 Staggered		79.118	70.178	
16 In-line		75.286	63.917	57.764
14 Staggered		74.409	64.355	57.586

driving potential for the heat transfer, resulting in higher local heat transfer coefficients for the upper rows.

The remainder of the text will not distinguish between isothermal and constant heat flux inner body conditions when discussing overall heat transfer data correlations.

#### RESULTS OF THE GEOMETRIC EFFECTS

There were two major geometric effects evident in the experimental data. First, a staggered inner body arrangement had a slightly higher heat transfer coefficient than an in-line arrangement of comparable size and spacing. Second, the heat transfer coefficient increased when the spacing of the inner body was increased and the surface area of the inner body was decreased.

These geometric effects are shown in Figures 4.2 and 4.3 which are graphs of  $Nu_L$  versus  $Ra_L$  for all of the experimental data, divided into the separate fluid mediums. These figures show that the effects of spacing and total surface area are more pronounced than the geometric effect of changing the inner body from an in-line arrangement to a staggered arrangement of comparable size and spacing.

When going from nine in-line cylinders to eight staggered cylinders or from sixteen in-line cylinders to fourteen staggered cylinders the Nusselt number increased only slightly. However,

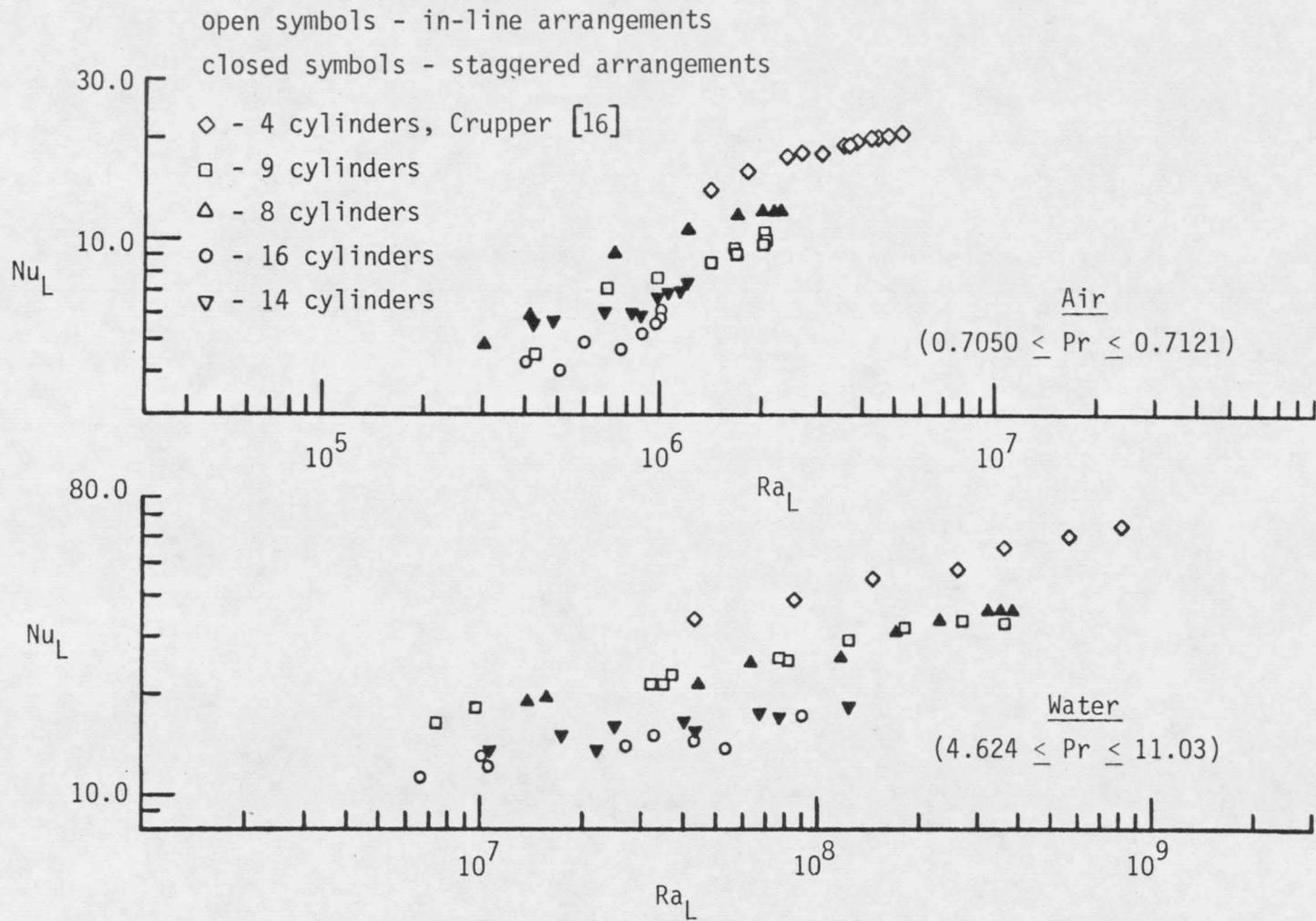


Figure 4.2 Geometric and Prandtl Number Effects for All Arrangements Using Air and Water

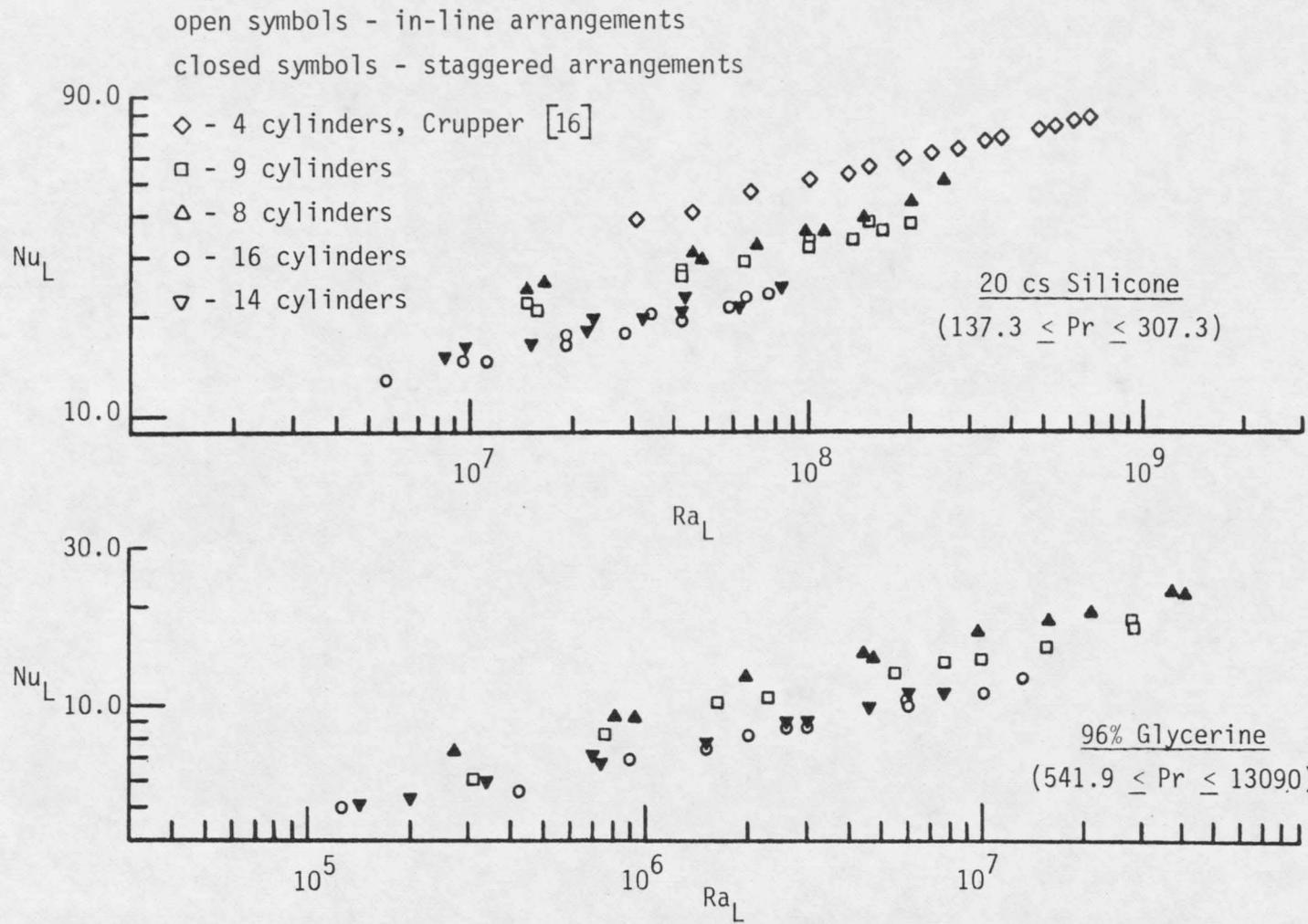


Figure 4.3 Geometric and Prandtl Number Effects for All Arrangements Using 20 cs Silicone and 96% Glycerine

there was a large increase in the Nusselt number when going from sixteen in-line cylinders and fourteen staggered cylinders (with diameter to spacing ratios of 0.69) to nine in-line cylinders and eight staggered cylinders (with diameter to spacing ratios of 0.55) respectively. Figures 4.2 and 4.3 also show that the data from Crupper [16] for four in-line cylinders (with a diameter to spacing ratio of 0.33) had a larger Nusselt number than the nine cylinder in-line arrangement. This supports the premise that an increase in the cylinder spacing and a decrease in the inner body surface area tends to increase the Nusselt number substantially. Warrington [12] also discovered that an increase in the inner body surface area reduced its capacity to transfer heat.

Crupper [16] found that the average Nusselt number from the top row of cylinders was 79.5 percent of the bottom row with a separation of three diameters between rows. Eckert and Soehngen [7], who performed a similar study using an infinite atmosphere, found that for two horizontal cylinders, placed one above the other and separated by a distance of four diameters, the Nusselt number of the top cylinder decreased to 87 percent of the Nusselt of the bottom cylinder. These same investigators performed the same investigation on three horizontal cylinders placed above each other, the resulting heat transfer from the middle cylinder was 83 percent of the bottom cylinder while the heat transfer from the top cylinder was 65 percent of the bottom cylinder.

Since the local heat transfer coefficient of an upper cylinder was decreased when the heated wake from a lower cylinder surrounded it, Eckert and Soehngen [7] investigated the effect of staggering the cylinders so that the wake from the bottom cylinder passed by the side of the middle cylinder. The Nusselt number of the middle cylinder was 103 percent of the bottom cylinder, while the Nusselt number of the top cylinder was 87 percent of the bottom cylinder.

When the results of Table 4.3 were compared to the findings of Crupper [16] it was evident that a decrease in the ratio of diameter to spacing, from 0.69 (sixteen in-line and fourteen staggered cylinders) to 0.55 (nine in-line and eight staggered cylinders) to 0.33 (four in-line cylinders [16]), led to a relative increase in the local heat transfer coefficients of the upper rows of cylinders. A similar comparison was made between the results of the eight and nine cylinder arrangements in Table 4.3 and the results of Eckert and Soehngen [7]. Their use of a smaller ratio of diameter to spacing, 0.25, contributed to their higher values for the local heat transfer coefficients of the upper rows. However, the enclosure used in this investigation, as opposed to the infinite atmosphere used by Eckert and Soehngen [7], significantly dampened the increase of the local heat transfer coefficients caused by staggering the cylinders. This was due to the recirculation of the warmer fluid.

The best correlations for the nine cylinder and the sixteen cylinder in-line arrangements were

$$\text{Nu}_S = 0.155 \text{ Ra}_S^{0.279} \text{ Pr}^{0.0065} \quad (4.8)$$

and

$$\text{Nu}_S = 0.185 \text{ Ra}_S^{0.252} \text{ Pr}^{0.029} \quad (4.9)$$

with average percent deviations of 9.00 and 11.10, respectively.

The best correlations for the eight cylinder and the fourteen cylinder staggered arrangements were

$$\text{Nu}_S = 0.227 \text{ Ra}_S^{0.255} \text{ Pr}^{0.019} \quad (4.10)$$

and

$$\text{Nu}_S = 0.245 \text{ Ra}_S^{0.239} \text{ Pr}^{0.020} \quad (4.11)$$

with average percent deviations of 11.42 and 9.94, respectively. As shown in Table 4.4, equation form 4.3, which is in terms of two correlating parameters, yielded the same average percent deviation as the more complex equation form 4.5, which is in terms of three correlating parameters. When correlating data from only one geometric arrangement, the characteristic lengths L, B, and S, each give the same average percent deviations for a particular equation form. Therefore, the correlations in Table 4.4 use only S as the characteristic length.

#### RESULTS OF THE PRANDTL NUMBER EFFECTS

As shown in Figures 4.2 and 4.3, the geometric effect of

TABLE 4.4

## CORRELATION EQUATIONS FOR EACH INNER BODY ARRANGEMENT

EQUATION FORM	X	EMPIRICAL CONSTANTS				AVERAGE % DEVIATION	MAXIMUM % DEVIATION
		C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>		
9 In-Line Cylinders							
4.1	S	.156	.280			9.275	42.029
4.2	S	.158	.280			9.275	42.029
4.3	S	.155	.279	.0065		8.995	38.868
4.4	S	.049	.280	-30.613		9.275	42.029
4.5	S	.049	.279	-30.387	.0065	8.995	37.868
8 Staggered Cylinders							
4.1	S	.234	.257			11.796	42.107
4.2	S	.232	.257			11.796	42.107
4.3	S	.227	.255	.019		11.422	33.053
4.4	S	.402	.257	-13.319		11.796	42.107
4.5	S	.401	.255	-14.103	.019	11.422	33.053
16 In-line Cylinders							
4.1	S	.176	.263			13.676	33.999
4.2	S	.200	.263			13.676	33.999
4.3	S	.185	.252	.029		11.105	31.015
4.4	S	.091	.263	-1.388		13.676	33.999
4.5	S	.101	.252	-1.265	.029	11.105	31.015
14 Staggered Cylinders							
4.1	S	.264	.240			11.555	31.542
4.2	S	.288	.240			11.555	31.542
4.3	S	.245	.239	.020		9.942	26.323
4.4	S	.140	.240	-1.732		11.555	31.542
4.5	S	.126	.239	-1.812	.020	9.942	26.323

changing the cylinder spacing and inner body surface area became less pronounced with increasing Prandtl number. However, the Prandtl number had no consistent influence when the inner body was changed from an in-line arrangement to a staggered arrangement of equal spacing. Warrington [12] found that the fluid viscosity did not influence the extent of any geometric effect, while Crupper [16], whose findings are in agreement with those in this study, found that an increase in fluid viscosity tended to damp out geometric effects. Crupper [16] postulated that the difference in the findings between himself and Warrington [12], concerning Prandtl number effects, was due to the fact that his geometric change was more radical than Warrington's [12].

The best correlation equation for the air data of all four inner body arrangements combined was

$$\text{Nu}_B = 0.0097 \text{ Ra}_B^{0.207} (L/R_i)^{0.596} \text{ Pr}^{-11.171}, \quad (4.12)$$

with an average percent deviation of 11.26. The large scatter in the data for air, as shown in Figure 4.2, was possibly caused by the large relative magnitude of the radiation and conduction losses alluded to earlier.

The best correlations in terms of three correlating parameters for water, 20cs silicone and 96% glycerine were

$$\text{Nu}_B = 1.045 \text{ Ra}_B^{0.171} (L/R_i)^{0.712} \text{ Pr}^{0.00084}, \quad (4.13)$$

$$\text{Nu}_S = 6.075 \text{ Ra}_S^{0.148} (L/R_i)^{0.131} \text{ Pr}^{-0.50}, \quad (4.14)$$

and

$$\text{Nu}_S = 0.025 \text{ Ra}_S^{0.324} (L/R_i)^{0.350} \text{ Pr}^{0.165}, \quad (4.15)$$

with average percent deviations of 5.51, 3.08, and 4.94, respectively. Table 4.5 shows the correlation results for each fluid using equation forms 4.1 through 4.5.

#### RESULTS OF ALL EXPERIMENTAL DATA COMBINED

All of the experimental data from this investigation are shown graphically in Figure 4.4, which is a graph of  $\text{Nu}_S$  versus  $\text{Ra}_S^*$ . The data for four horizontal in-line cylinders from Crupper [16] are also shown in Figure 4.4. In accordance with statements made earlier in the text, there is very little difference between the correlations for the in-line arrangements and the staggered arrangements, as shown in Figure 4.4.

The best correlation for the nine cylinder and the sixteen cylinder in-line arrangements combined was

$$\text{Nu}_S = 0.174 \text{ Ra}_S^{0.269} (L/R_i)^{0.331} \text{ Pr}^{0.017} \quad (4.16)$$

with an average percent deviation of 10.30. The best correlation for the eight cylinder and the fourteen cylinder staggered arrangements combined was

$$\text{Nu}_S = 0.247 \text{ Ra}_S^{0.248} (L/R_i)^{0.369} \text{ Pr}^{0.020} \quad (4.17)$$

TABLE 4.5

## CORRELATION EQUATIONS FOR EACH FLUID

EQUATION FORM	X	EMPIRICAL CONSTANTS				AVERAGE % DEVIATION	MAXIMUM % DEVIATION
		C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>		
Air							
4.1	S	.502	.185			13.319	51.847
4.2	L	.076	.337			13.103	33.820
4.3	B	.456	.255	2.237		16.818	46.964
4.4	B	.237	.255	.587		11.484	48.128
4.5	B	.0097	.207	.596	-11.171	11.255	50.051
Water							
4.1	S	1.182	.151			7.703	21.171
4.2	S	.933	.166			5.518	15.951
4.3	S	24.164	.040	-.596		5.631	19.755
4.4	B	1.049	.171	.712		5.506	19.097
4.5	B	1.045	.171	.712	.00084	5.507	18.098
20cs Silicone							
4.1	S	.758	.190			5.538	13.911
4.2	S	.537	.213			3.424	9.595
4.3	S	39.922	.094	-.438		3.138	10.591
4.4	S	.521	.216	.280		3.251	8.743
4.5	S	6.075	.148	.131	-.250	3.082	9.423
96% Glycerine							
4.1	S	.382	.215			5.493	37.523
4.2	S	.362	.222			4.839	40.501
4.3	S	1.452	.166	-.084		4.862	40.854
4.4	S	.364	.221	.143		4.777	39.108
4.5	S	.025	.324	.350	.165	4.938	34.958

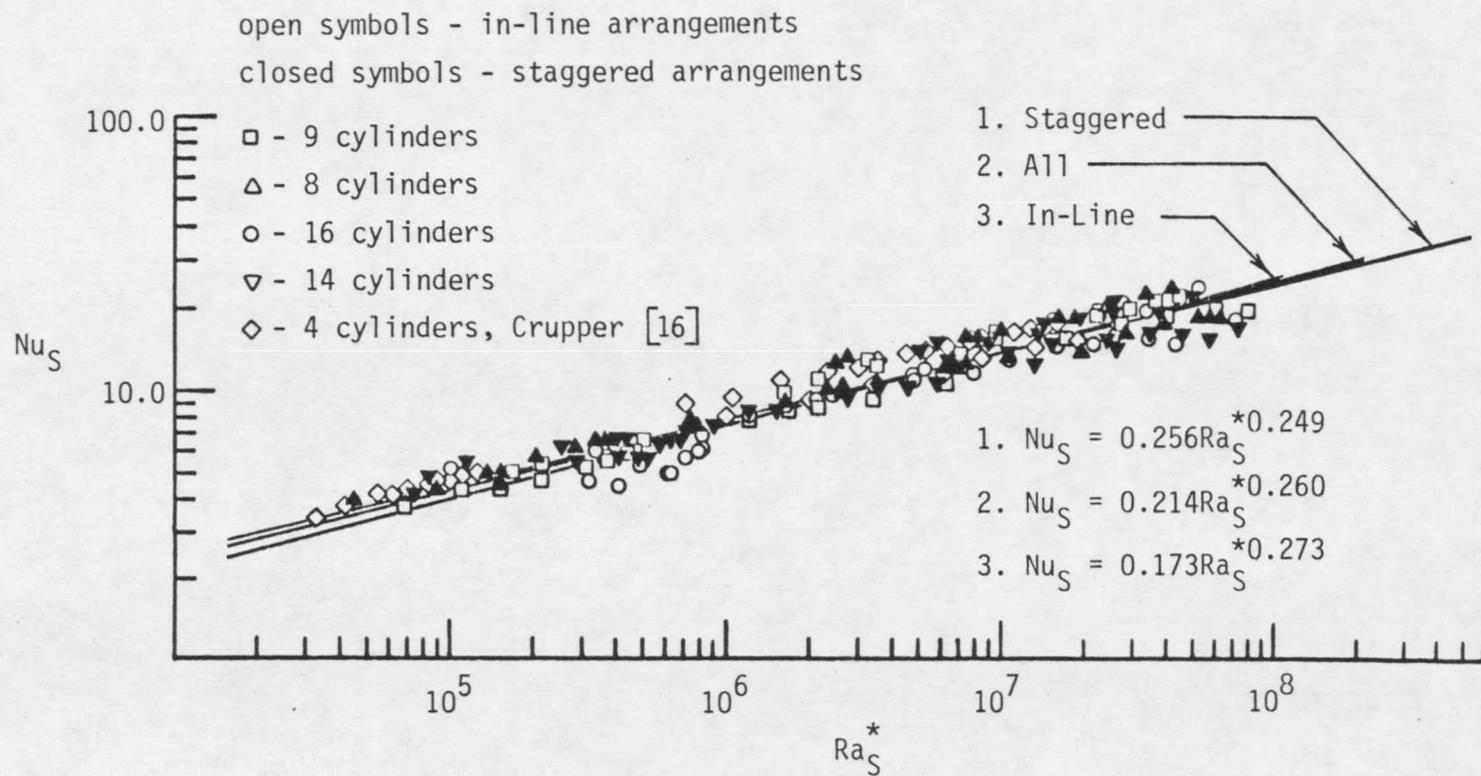


Figure 4.4 Heat Transfer Correlations for the In-Line Data, the Staggered Data, and All of the Data Combined

with an average percent deviation of 10.70. The best correlation for all of the data combined was

$$\text{Nu}_S = 0.211\text{Ra}_S^{0.258}(\text{L}/\text{R}_i)^{0.354}\text{Pr}^{0.019} \quad (4.18)$$

with an average percent deviation of 10.74. The remaining correlation results, using equation forms 4.1 through 4.5, for the in-line data, staggered data, and all of the data combined are shown in Table 4.6. In both Tables 4.5 and 4.6, the correlations employ the characteristic length which yielded the lowest average percent deviation.

Since this investigation is a continuation of the research performed by Warrington [12] and Crupper [16], their best correlations were compared with data from this study. The results of this comparison are shown in Table 4.7. The correlations from Warrington [12], based on the natural convection heat transfer between single bodies and their enclosure, fit the data from this investigation quite well. The large error which occurred from the use of Crupper's [16] correlation equations could have been caused by two things. First, the form of Crupper's [16] best correlation equations did not include a ratio of characteristic dimensions as a correlating parameter, as was used by Warrington [12] and the present author. This could reduce the accuracy of Crupper's [16] best correlation equations when applied to investigations with a different

TABLE 4.6

CORRELATION EQUATIONS FOR COMBINED IN-LINE ARRANGEMENTS,  
COMBINED STAGGERED ARRANGEMENTS, AND ALL DATA COMBINED

EQUATION FORM	X	EMPIRICAL CONSTANTS				AVERAGE % DEVIATION	MAXIMUM % DEVIATION
		C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>		
All Data Combined							
4.1	S	.222	.254			13.319	50.580
4.2	S	.214	.260			12.002	52.583
4.3	S	.216	.251	.019		12.320	40.253
4.4	S	.216	.261	.355		11.862	54.609
4.5	B	.266	.258	.533	.019	10.714	45.619
Staggered Arrangements Combined							
4.1	S	.263	.244			12.996	43.702
4.2	S	.256	.249			11.789	43.446
4.3	S	.250	.243	.020		11.932	37.605
4.4	S	.259	.249	.366		11.543	46.567
4.5	S	.247	.248	.369	.020	10.700	36.171
In-line Arrangements Combined							
4.1	S	.179	.266			12.857	45.361
4.2	S	.173	.273			11.473	43.555
4.3	S	.179	.262	.017		12.686	39.080
4.4	S	.174	.274	.336		11.404	45.125
4.5	S	.174	.269	.331	.017	10.302	37.459

TABLE 4.7

CORRELATION RESULTS USING THE DATA FROM THIS STUDY IN THE BEST CORRELATION EQUATIONS OF WARRINGTON [12] AND CRUPPER [16]

EQUATION	AUTHOR	AVERAGE % DEVIATION	MAXIMUM % DEVIATION
$Nu_L = 0.396Ra_L^{0.234} (L/R_i)^{0.496} Pr^{0.0162}$	[12]	19.547	78.520
$Nu_L = 0.425Ra_L^{0.234} (L/R_i)^{0.498}$	[12]	21.351	92.640
$Nu_B = 0.277Ra_B^{0.274} Pr^{0.012}$	[16]	74.078	165.679
$Nu_L = 0.358Ra_L^{0.257} Pr^{0.014}$	[16]	70.169	156.780

hypothetical gap with,  $L$ . Second, only one hypothetical gap width  $L$  was used during Crupper's [16] investigation. This would also suggest that Crupper's best correlations are limited to a range of hypothetical gap widths in the neighborhood of the hypothetical gap width used in his study.

CHAPTER V  
CONCLUSIONS

This investigation has extended the amount of available data for heat transfer between multiple bodies and an enclosure. In this study isothermal inner body conditions were compared to constant heat flux inner body conditions. There was no appreciable difference in the average heat transfer coefficient between the two conditions, which greatly increases the applicability of the correlations discussed in this study. However, the local heat transfer coefficients of the upper rows of cylinders, when compared to the local heat transfer coefficient of the bottom row of cylinders, were much higher for constant heat flux inner body conditions than they were for isothermal inner body conditions. The constant heat flux condition forced the upper rows to a higher temperature and augmented the driving potential for heat transfer which resulted in higher local heat transfer coefficients for the upper rows.

The distance between cylinders and the amount of inner body surface area were the dominate factors influencing the average heat transfer coefficient. The enclosure severely dampened the increase in both the average and the local heat transfer coefficients caused by changing the inner body from an in-line arrangement to a staggered arrangement.

It was observed that an increase in Prandtl number had a

dampening effect on the geometric effects. The heat transfer data correlated with the characteristic length  $S$ , which is the hypothetical gap width  $L$  multiplied by the ratio of the inner body surface area to the outer body surface area, generally provided the best correlation results.

The following equations, in terms of one, two, or three correlating parameters, are recommended by the present author for the prediction of natural convection heat transfer between arrays of heated horizontal cylinders and their cooled enclosure. These correlation equations are:

$$Nu_S = 0.2149Ra_S^{*0.260} \quad (5.1)$$

$$Nu_S = 0.216Ra_S^{0.261}(L/R_i)^{0.355} \quad (5.2)$$

$$Nu_S = 0.211Ra_S^{0.258}(L/R_i)^{0.354}Pr^{0.0189} \quad (5.3)$$

$0.620 \leq (L/R_i) \leq 1.041$ ;  $0.705 \leq Pr \leq 1.309 \times 10^4$   
 $4.449 \times 10^4 \leq Ra_S \leq 1.170 \times 10^8$ ;  $4.633 \times 10^4 \leq Ra_S^* \leq 8.153 \times 10^7$   
 which have average percent deviations of 12.00, 11.86, and 10.74, respectively.

APPENDICES

## APPENDIX I

### HEAT TRANSFER DATA REDUCTION PROGRAM

The following is a data reduction program which computes and correlates all of the dimensionless groups. All of the variables, subroutines and function subprograms are defined within the program.

C\*\*\*\*\*HEAT TRANSFER DATA REDUCTION AND CORRELATION PROGRAM

C

DIMENSION X(5,250),C(4),S(5,6),ANP(5),XNUS(3,250),RA(3,250),  
 \$ GR(3,250),RAM(3,250),PR(250),TAVO(250),TAVI(250),  
 \$ PLL(250),PP(250),ZZ(3),XNMIN(3),XNMAX(3),RMIN(3),  
 \$ RMAX(3),GMIN(3),GMAX(3),RMIN(3),RMAX(3),SAVGAP(250),  
 \$ SAVRI(250),DD(4)

C

C\*\*\*\*\*K IS A COUNTER

C\*\*\*\*\*XCUBE IS THE LENGTH OF A SIDE OF THE OUTER CUBE (IN.)

C\*\*\*\*\*JJ IS THE FLUID IDENTIFIER:

C 1-AIR  
 C 2-H2O  
 C 3-20 CS SILICONE  
 C 5-96% GLYCERINE

C\*\*\*\*\*XXX IS THE LENGTH OF ONE CYLINDER (IN.)

C\*\*\*\*\*SD IS THE DIAMETER OF ONE CYLINDER (IN.)

C\*\*\*\*\*IDB IS THE INNER BODY IDENTIFIER:

C 1-9 CYLINDER IN-LINE ARRANGEMENT  
 C 2-8 CYLINDER STAGGERED ARRANGEMENT  
 C 3-16 CYLINDER IN-LINE ARRANGEMENT  
 C 4-14 CYLINDER STAGGERED ARRANGEMENT  
 C 31-4 CYLINDER IN-LINE ARRANGEMENT

C\*\*\*\*\*P IS THE AMOUNT OF HEAT TRANSFERED BY CONVECTION (BTU/HR)

C\*\*\*\*\*PL IS THE AMOUNT OF HEAT TRANSFERED BY RADIATION AND  
 C CONDUCTION (BTU/HR)

C\*\*\*\*\*TAVGI IS THE MEAN TEMPERATURE OF THE INNER BODY (F)

C\*\*\*\*\*TAVGO IS THE MEAN TEMPERATURE OF THE OUTER BODY (F)

C\*\*\*\*\*SAIB IS THE SURFACE AREA OF THE INNER BODY (SQ. IN.)

C\*\*\*\*\*SAOB IS THE SURFACE AREA OF THE OUTER BODY (SQ. IN.)

C\*\*\*\*\*RO IS THE RADIUS OF A SPHERE WITH A VOLUME EQUAL TO THE VOLUME  
 C OF THE OUTER BODY (IN.)

C\*\*\*\*\*RI IS THE RADIUS OF A SPHERE WITH A VOLUME EQUAL TO THE VOLUME  
 C OF THE INNER BODY (IN.)

C\*\*\*\*\*GAP IS THE DISTANCE BETWEEN THESE SPHERES OR "RO-RI" (IN.)

C\*\*\*\*\*BLL IS THE LENGTH OF THE BOUNDARY LAYER ON ONE CYLINDER, ASSUMING  
 C NO FLOW SEPARATION (IN.)

C\*\*\*\*\*RAOSA IS THE RATIO OF SURFACE AREAS OF THE INNER AND OUTER  
 C BODIES, MULTIPLIED BY GAP (IN.)

C\*\*\*\*\*XNUS(I,K),RA(I,K),RAM(I,K), AND GR(I,K) ARE THE NUSSELT NUMBER,  
 C THE RALEIGH NUMBER, THE MODIFIED RALEIGH NUMBER, AND THE  
 C GRASHOF NUMBER, RESPECTIVELY. THE SUBSCRIPT "I" AND THE  
 C CORRESPONDING CHARACTERISTIC LENGTH ARE:

C 1-GAP  
 C 2-BLL  
 C 3-RAOSA

C\*\*\*\*\*PR IS THE PRANDTL NUMBER

C

C\*\*\*\*\*INPUT A: 1-FOR AN OUTPUT OF THE REDUCED DATA

C 2-FOR AN OUTPUT OF THE CURVE FIT RESULTS

C 3-FOR AN OUTPUT OF THE REDUCED DATA AND THE CURVE FIT  
 C RESULTS

C NOTE: USE ONLY ONE FLUID-INNER BODY COMBINATION PER RUN  
 C FOR OPTIONS 1 AND 3!

C

INPUT LLL

C

C\*\*\*\*\*THE FIVE EQUATIONS USED TO CURVE FIT THE DIMENSIONLESS

C RESULTS ARE:

C 1-NU=C1\*(RA\*\*C2)  
 C 2-NU=C1\*(RAM\*\*C2)

C

```

C          3-NU=C1*(RA**C2)*(PR**C3)
C          4-NU=C1*(RA**C2)*((GAP/RI)**C3)
C          5-NU=C1*(RA**C2)*((GAP/RI)**C3)*(PR**C4)
C
DO 12 I=1,3
XNMIN(I)=RMIN(I)=RMMIN(I)=GMIN(I)=PRMIN=10.**10
12 XNMAX(I)=RMAX(I)=RMMAX(I)=GMAX(I)=PRMAX=0.
K=0
PI=3.1415927
C*****READ IN A NEGATIVE INTEGER OR ZERO FOR IDB AFTER THE LAST DATA SET
50 READ(105,25) IDB,JJ,XCUBE,SD,XXX,TAVGI,TAVGO,P,PL
25 FORMAT(2I5,7F10.4)
IF(IDB)100,100,30
30 K=K+1;TAVO(K)=TAVGO;TAVI(K)=TAVGI;PP(K)=P;PLL(K)=PL;IIDDBB=IDB
RO=((3.*XCUBE**3)/(4.*PI))**(1./3.)
BLL=PI*SD/2.;J123=JJ
SAOB=XCUBE*XCUBE*6.
GO TO (1,2,3,4),IDB
GO TO 31
C*****9 CYLINDERS, IN-LINE
1 RI=(27.*SD*SD*XXX/16.)*(1./3.)
GAP=RO-RI
SAIB=9.*PI*SD*XXX+18.*PI*SD*SD/4.
RAOSA=(SAIB/SAOB)*GAP
GO TO 10
C*****8 CYLINDERS, STAGGERED
2 RI=(24.*SD*SD*XXX/16.)*(1./3.)
GAP=RO-RI
SAIB=8.*PI*SD*XXX+16.*PI*SD*SD/4.
RAOSA=(SAIB/SAOB)*GAP
GO TO 10
C*****16 CYLINDERS, IN-LINE
3 RI=(48.*SD*SD*XXX/16.)*(1./3.)
GAP=RO-RI
SAIB=16.*PI*SD*XXX+32.*PI*SD*SD/4.
RAOSA=(SAIB/SAOB)*GAP
GO TO 10
C*****14 CYLINDERS, STAGGERED
4 RI=(42.*SD*SD*XXX/16.)*(1./3.)
GAP=RO-RI
SAIB=14.*PI*SD*XXX+28.*PI*SD*SD/4.
RAOSA=(SAIB/SAOB)*GAP
GO TO 10
C*****4 CYLINDERS, IN-LINE
31 RI=(12.*SD*SD*XXX/16.)*(1./3.)
GAP=RO-RI
SAIB=4.*PI*SD*XXX+8.*PI*SD*SD/4.
RAOSA=(SAIB/SAOB)*GAP
10 CONTINUE
C*****DT IS THE TEMPERATURE DIFFERENCE BETWEEN THE INNER AND OUTER
C BODIES (F)
DT=TAVGI-TAVGO
C*****H IS THE AVERAGE HEAT TRANSFER COEFFICIENT (BTU/HR-FT**2-F)
H=P*144./(DT*SAIB)
C*****CALCULATE THE MEAN TEMPERATURE (DEG. F)
TM=(TAVGO+TAVGI)/2.
C*****TAVG, MEAN TEMPERATURE (DEG. R)
TAVG=TM+459.69

```

```

C*****CALCULATE THE FLUID PROPERTIES
C*****VISCOSITY (LBM/HR-FT)
  VIS=U(TAVG,JJ)
C*****SPECIFIC HEAT (BTU/LBM-R)
  SH=CP(TAVG,JJ)
C*****THERMAL CONDUCTIVITY (BTU/HR-FT-F)
  COND=CON(TAVG,JJ)
C*****DENSITY (LBM/FT**3)
  DEN=RHO(TAVG,JJ)
C*****THERMAL EXPANSION COEFFICIENT (1/R)
  BET=BETA(TAVG,JJ)
C*****THE PRANDTL NUMBER
  PR(K)=VIS*SH/COND
  IF(PR(K).LT.PRMIN)PRMIN=PR(K)
  IF(PR(K).GT.PRMAX)PRMAX=PR(K)
C*****THE GRASHOF NUMBER
  ZZ(1)=GAP
  ZZ(2)=BLL
  ZZ(3)=RAOSA
  Z6=32.174*BET*DT*DEN*DEN*3600.*3600./(1728.*VIS*VIS)
  DO 13 I=1,3
  GR(I,K)=Z6*(ZZ(I)**3)
  IF(GR(I,K).LT.GMIN(I))GMIN(I)=GR(I,K)
  IF(GR(I,K).GT.GMAX(I))GMAX(I)=GR(I,K)
  13 CONTINUE
C*****THE NUSSELT NUMBER
  Z7=H/(COND*12.)
  DO 14 I=1,3
  XNUS(I,K)=Z7*ZZ(I)
  IF(XNUS(I,K).LT.XNMIN(I))XNMIN(I)=XNUS(I,K)
  IF(XNUS(I,K).GT.XNMAX(I))XNMAX(I)=XNUS(I,K)
  14 CONTINUE
C*****THE RALEIGH NUMBER
  DO 15 I=1,3
  RA(I,K)=GR(I,K)*PR(K)
  IF(RA(I,K).LT.RMIN(I))RMIN(I)=RA(I,K)
  IF(RA(I,K).GT.RMAX(I))RMAX(I)=RA(I,K)
  15 CONTINUE
C*****THE MODIFIED RALEIGH NUMBER
  DO 16 I=1,3
  RAM(I,K)=RA(I,K)*GAP/RI
  IF(RAM(I,K).LT.RMMIN(I))RMMIN(I)=RAM(I,K)
  IF(RAM(I,K).GT.RMMAX(I))RMMAX(I)=RAM(I,K)
  16 CONTINUE
  SAVGAP(K)=GAP;SAVRI(K)=RI
  GO TO 50
100 CONTINUE
  IF(LLL.EQ.2)GO TO 999
C*****OUTPUT THE REDUCED DATA AND THE DIMENSIONLESS RESULTS
  IDB=IIDDDB;JJ=J123
  500 FORMAT('1')
  505 FORMAT(7/)
  510 FORMAT(//)
  515 FORMAT(34X,'*** 9 CYLINDER IN-LINE ARRANGEMENT ***')
  520 FORMAT(33X,'*** 8 CYLINDER STAGGERED ARRANGEMENT ***')
  525 FORMAT(33X,'*** 16 CYLINDER IN-LINE ARRANGEMENT ***')
  530 FORMAT(32X,'*** 14 CYLINDER STAGGERED ARRANGEMENT ***')
  535 FORMAT(34X,'*** 4 CYLINDER IN-LINE ARRANGEMENT ***')

```



```

WRITE(108,620)RMMIN(I),RM MAX(I)
WRITE(108,500)
17 WRITE(108,505)
IF(LLL.EQ.1)STOP
999 CONTINUE
C*****CORRELATE THE DIMENSIONLESS RESULTS
1010 FORMAT('1',//,40X,'*** CORRELATION NO.',I2,' ***')
1015 FORMAT(46X,'NU=C1*(RA**C2)')
1020 FORMAT(46X,'NU=C1*(RAM**C2)')
1025 FORMAT(38X,'NU=C1*(RA**C2)*((GAP/RI)**C3)')
1030 FORMAT(34X,'NU=C1*(RA**C2)*((GAP/RI)**C3)*(PR**C4)')
1040 FORMAT(41X,'NU=C1*(RA**C2)*(PR**C3)')
1060 FORMAT(32X,'*** THE CHARACTERISTIC LENGTH IS "GAP" ***//')
1065 FORMAT(32X,'*** THE CHARACTERISTIC LENGTH IS "BLL" ***//')
1070 FORMAT(31X,'*** THE CHARACTERISTIC LENGTH IS "RAOSA" ***//')
1085 FORMAT(35X,'THE COEFFICIENT,"C',I1,'"=",E14.8)
DO 1800 ICN=1,5
DO 1800 MN=1,3
WRITE(108,1010)ICN
GO TO (1200,1205,1225,1210,1215),ICN
1200 WRITE(108,1015);GO TO 1250
1205 WRITE(108,1020);GO TO 1250
1210 WRITE(108,1025);GO TO 1250
1215 WRITE(108,1030);GO TO 1250
1225 WRITE(108,1040)
1250 CONTINUE
GO TO (1255,1260,1265),MN
1255 WRITE(108,1060);GO TO 1300
1260 WRITE(108,1065);GO TO 1300
1265 WRITE(108,1070)
1300 CONTINUE
GO TO (1350,1400,1600,1450,1500),ICN
C*****NU=C1*(RA**C2)
1350 NV=2;NOB=K
DO 1355 II=1,K
X(1,II)=1.
X(2,II)=ALOG(RA(MN,II))
1355 X(3,II)=ALOG(XNUS(MN,II))
CALL CURFT(NV,NOB,X,C)
DO 1360 LOX=1,2
1360 DD(LOX)=C(LOX)
DD(1)=EXP(DD(1))
DO 1365 LOW=1,2
1365 WRITE(108,1085)LOW,DD(LOW)
CALL ERROR(NV,NOB,X,C,SSQ)
GO TO 1800
C*****NU=C1*(RAM**C2)
1400 NV=2;NOB=K
DO 1405 II=1,K
X(1,II)=1.
X(2,II)=ALOG(RAM(MN,II))
1405 X(3,II)=ALOG(XNUS(MN,II))
CALL CURFT(NV,NOB,X,C)
DO 1410 LOX=1,2
1410 DD(LOX)=C(LOX)
DD(1)=EXP(DD(1))
DO 1415 LOW=1,2
1415 WRITE(108,1085)LOW,DD(LOW)

```

```

      CALL ERROR(NV,NOB,X,C,SSQ)
      GO TO 1800
C*****NU=C1*(RA**C2)*((GAP/RI)**C3)
1450 NV=3;NOB=K
      DO 1455 II=1,K
      X(1,II)=1.
      X(2,II)=ALOG(RA(MN,II))
      X(3,II)=ALOG(SAVGAP(II)/SAVRI(II))
1455 X(4,II)=ALOG(XNUS(MN,II))
      CALL CURFT(NV,NOB,X,C)
      DO 1460 LOX=1,3
1460 DD(LOX)=C(LOX)
      DD(1)=EXP(DD(1))
      DO 1465 LOW=1,3
1465 WRITE(108,1085)LOW,DD(LOW)
      CALL ERROR(NV,NOB,X,C,SSQ)
      GO TO 1800
C*****NU=C1*(RA**C2)*((GAP/RI)**C3)*(PR**C4)
1500 NV=4;NOB=K
      DO 1505 II=1,K
      X(1,II)=1.
      X(2,II)=ALOG(RA(MN,II))
      X(3,II)=ALOG(SAVGAP(II)/SAVRI(II))
      X(4,II)=ALOG(PR(II))
1505 X(5,II)=ALOG(XNUS(MN,II))
      CALL CURFT(NV,NOB,X,C)
      DO 1510 LOX=1,4
1510 DD(LOX)=C(LOX)
      DD(1)=EXP(DD(1))
      DO 1515 LOW=1,4
1515 WRITE(108,1085)LOW,DD(LOW)
      CALL ERROR(NV,NOB,X,C,SSQ)
      GO TO 1800
C*****NU=C1*(RA**C2)*(PR**C3)
1600 NV=3;NOB=K
      DO 1605 II=1,K
      X(1,II)=1.
      X(2,II)=ALOG(RA(MN,II))
      X(3,II)=ALOG(PR(II))
1605 X(4,II)=ALOG(XNUS(MN,II))
      CALL CURFT(NV,NOB,X,C)
      DO 1610 LOX=1,3
1610 DD(LOX)=C(LOX)
      DD(1)=EXP(DD(1))
      DO 1615 LOW=1,3
1615 WRITE(108,1085)LOW,DD(LOW)
      CALL ERROR(NV,NOB,X,C,SSQ)
1800 CONTINUE
1000 END
C
C
C
C*****CURVE FIT SUBROUTINE USING THE METHOD OF LEAST SQUARES
      SUBROUTINE CURFT(NV,NOB,X,C)
      DIMENSION X(5,250),C(4),S(5,6)
      DOUBLE PRECISION S,D
C*****NV IS THE NUMBER OF INDEPENDENT VARIABLES
C*****NOB IS THE NUMBER OF OBSERVATIONS
C*****X(I,K) ARE EXPRESSIONS COMPRISED OF THE DIMENSIONLESS RESULTS

```



```

$ /,9X,91'**)
13 FORMAT(9X,91'**)
DO 2 I=1,NOB
YE=X(M,I)
YC=0.00
DO 20 IJ=1,NV
20 YC=YC+C(IJ)*X(IJ,I)
YC=EXP(YC)
YE=EXP(YE)
E=YC-YE
EP=100.*E/YE
EPA=ABS(EP)
IF (EPA-EMX) 6,6,7
7 EMX=EPA
6 DO 8 J=1,5
ACD=J*5
IF (EPA-ACD) 9,9,8
9 ANP(J)=ANP(J)+1.
8 CONTINUE
TS=TS+E*E
TE=TE+ABS(EP)
2 WRITE(108,3)I,YE,YC,E,EP
3 FORMAT(9X,'*',5X,I3,3X,4(5X,E14.8),2X,'**')
WRITE(108,13)
A=NOB
SSQ=(TS/A)**.5
TE=TE/A
WRITE (108,4) SSQ,TE
4 FORMAT('1',10/,30X,49'*/,/,30X,'* THE STANDARD DEVIATION=',E14.8,9
$X,'*',/,30X,'* THE AVERAGE PER CENT DEVIATION=',E14.8,'*')
WRITE (108,10) EMX
10 FORMAT(30X,'* THE MAXIMUM PER CENT DEVIATION=',E14.8,'*',/,10X,90
$'**)
14 FORMAT(10X,90'**)
DO 11 I=1,5
XNP=ANP(I)
XNP=100.*XNP/A
ACD=I*5
11 WRITE (108,12) XNP,ACD
12 FORMAT(10X,'*',E14.8,' PER CENT OF THE DATA ARE WITHIN ',F3.0,
$' PER CENT OF THE CORRELATION EQUATION *')
WRITE(108,14)
RETURN
END

C
C
C
C
FUNCTION U(T,JJ)
GO TO (1,2,3,4,5) JJ
C*****ABSOLUTE VISCOSITY OF AIR
1 CO=134.375
C1=6.0133834
C2=1.8432299
C3=1.3347050
U=T**C2/(EXP(C1)*(T+CO)**C3)
GO TO 50
C*****ABSOLUTE VISCOSITY OF WATER

```

```

2 TP=T-593.33203
  C1=.0071695149
  C2=.011751302
  C3=.0087791942
  C4=.81654704
  VIS=C1*TP+C2*(1.+C3*(TP**2))**.5+C4
  U=1./VIS
  GO TO 50
C*****ABSOLUTE VISCOSITY OF 20CS DOW 200 SILICONE
3 V=.03875*(4.6*10**5)/(T-359.69)**1.912
  C1=52.754684
  C2=.045437533
  C3=5.1832336/(10.**5)
  RHO=C1+(C2-C3*T)*T
  U=RHO*.949*V
  GO TO 50
4 STOP
C*****VISCOSITY OF 96% GLYCERIN
5 C1=103.51199
  C2=-.54040736
  C3=.11128595E-02
  C4=-.10526064E-05
  C5=.38202064E-09
  U=C1+C2*T+C3*T*T+C4*T**3+C5*T**4
  U=10.**U
50 RETURN
  END
C
C
C
  FUNCTION CP(T,JJ)
  GO TO (1,2,3,4,5) JJ
C*****SPECIFIC HEAT OF AIR
1 C0=2.236775/(10.**5)
  C1=.22797749
  CP=C1+C0*T
  GO TO 10
C*****SPECIFIC HEAT OF WATER
2 C1=1.3757095
  C2=.0012968965
  C3=1.1110533/(10.**6)
  CP=C1-(C2-C3*T)*T
  GO TO 10
C*****SPECIFIC HEAT OF 20CS DOW 200 SILICONE
3 TP=5.*(T-491.69)/9.
  C1=.3448334
  C2=7.7499/(10.**5)
  C3=4.167/(10.**8)
  CP=C1+(C2+C3*TP)*TP
  GO TO 10
4 STOP
C***** SPECIFIC HEAT OF 100% GLYCERIN
5 IF(T.LT.599.69) GO TO 6
  C1=.22420651
  C2=.66666701E-03
  CP=C1+C2*T
  GO TO 10
6 C1=.15481514

```

```

C2=.78571402E-03
CP=C1+C2*T
10 RETURN
END

```

C  
C  
C

```

FUNCTION CON(T,JJ)
GO TO (1,2,3,4,5) JJ
C*****THERMAL CONDUCTIVITY OF AIR
1 XP=.1
CO=-8.5964965
C1=34490.89
C2=868.23837
C3=8056583.8
7 X=XP
F=CO+C1*X+C2*X*X+C3*X*X*X-T
FP=C1+2.*C2*X+3.*X*X*C3
XP=X-F/FP
IF (ABS((XP-X)/X)-0.0001) 6,6,7
6 CON=XP
GO TO 10

```

```

C*****THERMAL CONDUCTIVITY OF WATER
2 C1=.23705417
C2=.0017156797
C3=1.1563770/(10.**6)
CON=-C1+(C2-C3*T)*T
GO TO 10

```

```

C*****THERMAL CONDUCTIVITY OF 20CS DOW 200 SILICONE
3 CON=.00034/0.004134
GO TO 10
4 STOP

```

```

C*****THERMAL CONDUCTIVITY OF 96% GLYCERIN
5 C1=.14340127
C2=.47222275E-04
CON=C1+C2*T
10 RETURN
END

```

C  
C  
C

```

FUNCTION RHO(T,JJ)
GO TO (1,2,3,4,5) JJ
C*****DENSITY OF AIR AT ATMOSPHERIC PRESS. LBM/FT3
1 A=12.5
RHO=A*144./(53.36*T)
GO TO 10
C*****DENSITY OF WATER
2 C1=52.754684
C2=.045437533
C3=5.1832336/(10.**5)
RHO=C1+(C2-C3*T)*T
GO TO 10
C*****DENSITY OF 20CS DOW 200 SILICONE
3 C1=52.754684
C2=.045437533
C3=5.1832336/(10.**5)
RHO=.949*(C1+(C2-C3*T)*T)

```

```

      GO TO 10
4 STOP
C*****DENSITY OF 96% GLYCERIN
5 C1=89.789932
  C2=-.2221163E-01
  RHO=C1+C2*T
10 RETURN
  END

C
C
C
      FUNCTION BETA (T,JJ)
      GO TO (1,2,3,4,5) JJ
C*****THERMAL EXPANSION COEFF. OF AIR
1 BETA = 1./T
  GO TO 10
C*****THERMAL EXPANSION COEFF. OF WATER
2 TP=T/100.
  IF (T-549.59) 6,6,7
6 C1=603.11841
  C2=-353.03882
  C3=68.297012
  C4=-4.3611460
  BP=C1+(C2+(C3+C4*TP)*TP)*TP
  GO TO 8
7 C1=-128.44920
  C2=68.827927
  C3=-13.858489
  C4=1.2608585
  C5=-.042495236
  BP=C1+(C2+(C3+(C4+C5*TP)*TP)*TP)*TP
8 BETA = BP/(10.**4)
  GO TO 10
C*****THERMAL EXPANSION COEFF. OF 20CS DOW 200 SILICONE
3 BETA=.00107/1.8
  GO TO 10
4 STOP
C*****THERMAL EXPANSION COEFFICIENT OF 96% GLYCERIN
5 C1=89.789932
  C2=.2221163E-01
  BETA=C2/(C1-C2*T)
10 RETURN
  END

```

APPENDIX II  
PARTIALLY REDUCED DATA

The following is a listing of all the data taken in this investigation, in partially reduced form. The column headings are:

IDG is the inner body identifier

- 1 = 9 In-Line Cylinders
- 2 = 8 Staggered Cylinders
- 3 = 16 In-Line Cylinders
- 4 = 14 Staggered Cylinders

IDBC is the inner body boundary condition identifier .

- 1 = Isothermal Conditions
- 2 = Constant Heat Flux Conditions

IDF is the fluid identifier

- 1 = Air
- 2 = Distilled Water
- 3 = Dow Corning Dimethylpolysiloxane (20 centipoise at 25°C)
- 4 = 96% Aqueous Glycerine by Weight

TAVGO is the average outer body temperature in Kelvin

TAVGI is the average inner body temperature in Kelvin

QCONV is the heat transferred by natural convection in Watts

QLOSS is the heat loss due to conduction and radiation in Watts

<u>IDG</u>	<u>IDBC</u>	<u>IDF</u>	<u>TAVGO</u>	<u>TAVGI</u>	<u>QCONV</u>	<u>QLOSS</u>
1	1	1	281.312	290.597	3.380	2.724
1	1	1	279.736	295.114	8.861	5.349
1	1	1	281.038	305.262	15.408	9.160
1	1	1	282.824	323.066	29.178	16.061
1	1	1	280.227	330.035	38.638	20.182
1	1	1	281.232	350.672	59.886	28.640
1	1	1	281.940	361.679	71.197	33.077
1	2	1	281.097	292.109	6.437	3.468
1	2	1	281.029	305.105	17.698	9.096
1	2	1	280.645	328.977	38.837	19.546
1	2	1	280.810	353.280	66.892	29.945
1	1	2	277.198	282.367	156.293	.176
1	1	2	284.799	292.020	312.992	.246
1	1	2	281.367	290.176	382.805	.300
1	1	2	287.475	298.104	534.934	.361
1	1	2	285.105	297.766	624.889	.431
1	1	2	292.258	307.748	980.162	.527
1	1	2	298.586	319.462	1379.387	.710
1	2	2	280.113	284.419	147.325	.146
1	2	2	282.950	291.431	377.156	.288
1	2	2	288.282	302.268	795.726	.476
1	2	2	297.605	315.172	1176.177	.597
1	1	3	281.282	290.064	82.479	.299
1	1	3	283.722	303.427	234.215	.670
1	1	3	283.928	311.562	364.986	.940
1	1	3	285.937	323.560	543.572	1.279
1	1	3	288.070	333.519	696.462	1.545
1	1	3	290.334	341.587	837.199	1.743
1	1	3	294.032	350.329	952.991	1.914
1	2	3	279.448	287.972	84.747	.290
1	2	3	280.945	301.735	259.969	.707
1	2	3	286.879	324.798	570.588	1.289
1	2	3	293.761	350.751	985.538	1.938
1	1	4	279.410	291.091	67.498	.397
1	1	4	278.935	299.061	157.223	.684
1	1	4	282.171	311.951	304.023	1.013
1	1	4	286.208	324.251	459.548	1.294
1	1	4	289.617	333.411	587.662	1.489
1	1	4	292.704	341.199	709.315	1.649
1	1	4	295.733	353.163	966.895	1.953
1	2	4	283.265	295.760	65.160	.425
1	2	4	283.071	307.606	238.811	.834
1	2	4	289.775	329.167	516.796	1.339
1	2	4	297.605	352.466	973.225	1.865

<u>IDG</u>	<u>IDBC</u>	<u>IDF</u>	<u>TAVGO</u>	<u>TAVGI</u>	<u>QCONV</u>	<u>QLOSS</u>
2	1	1	279.495	284.991	1.825	1.957
2	1	1	277.147	290.886	8.506	7.480
2	1	1	277.066	302.149	18.576	15.080
2	1	1	277.717	317.060	32.918	24.635
2	1	1	278.431	330.108	45.987	32.899
2	1	1	279.321	340.104	54.789	38.999
2	1	1	280.041	347.624	61.183	43.556
2	2	1	276.674	283.932	2.900	3.138
2	2	1	277.670	295.278	10.313	10.072
2	2	1	278.494	320.339	27.468	26.311
2	2	1	279.503	346.549	59.573	43.195
2	1	2	279.567	284.989	164.819	.169
2	1	2	282.588	291.768	351.008	.286
2	1	2	287.158	300.426	578.711	.414
2	1	2	292.559	309.597	796.122	.532
2	1	2	298.335	316.011	1090.437	.551
2	1	2	293.403	309.528	914.615	.503
2	1	2	300.051	319.153	1185.275	.596
2	2	2	281.291	286.258	157.602	.155
2	2	2	285.715	295.116	389.211	.293
2	2	2	292.300	305.846	701.227	.423
2	2	2	300.485	318.167	1087.938	.552
2	1	3	279.177	287.667	83.764	.265
2	1	3	281.413	300.954	233.796	.610
2	1	3	283.403	307.582	304.556	.754
2	1	3	286.041	319.706	462.703	1.050
2	1	3	289.771	333.166	656.871	1.354
2	1	3	293.999	346.009	878.013	1.623
2	1	3	297.139	355.597	1040.040	1.824
2	2	3	279.190	286.914	71.524	.241
2	2	3	283.655	303.763	230.210	.627
2	2	3	288.427	324.211	497.598	1.116
2	2	3	295.692	346.279	856.343	1.578
2	1	4	276.998	288.601	68.260	.362
2	1	4	278.163	298.404	152.249	.631
2	1	4	280.801	308.706	275.903	.871
2	1	4	282.588	319.476	434.133	1.151
2	1	4	285.907	331.750	628.645	1.430
2	1	4	290.429	345.388	883.639	1.715
2	1	4	296.187	358.373	1135.827	1.940
2	2	4	281.502	299.920	135.513	.575
2	2	4	287.887	319.820	354.299	.996
2	2	4	294.353	338.682	658.876	1.383

<u>IDG</u>	<u>IDBC</u>	<u>IDF</u>	<u>TAVGO</u>	<u>TAVGI</u>	<u>QCONV</u>	<u>QLOSS</u>
2	2	4	298.656	355.718	1046.355	1.780
3	1	1	279.516	287.858	7.880	3.616
3	1	1	278.872	297.846	15.254	14.714
3	1	1	278.159	309.658	29.056	27.787
3	1	1	278.244	323.332	40.515	41.971
3	1	1	280.544	339.745	61.229	56.704
3	1	1	281.076	353.584	81.977	70.593
3	1	1	280.020	358.336	92.106	76.657
3	2	1	278.766	286.469	7.724	2.949
3	2	1	278.027	303.078	19.038	21.057
3	2	1	279.736	325.819	42.025	43.011
3	2	1	282.037	365.308	110.805	81.828
3	1	2	282.176	286.696	219.992	.303
3	1	2	286.821	293.952	412.581	.478
3	1	2	287.233	295.839	527.228	.577
3	1	2	291.463	301.054	623.250	.643
3	1	2	288.693	301.543	789.595	.862
3	2	2	281.666	288.319	348.427	.446
3	2	2	287.762	297.319	632.830	.641
3	2	2	294.959	309.241	1116.902	.958
3	1	3	280.299	287.101	91.428	.456
3	1	3	284.045	295.846	178.808	.792
3	1	3	286.058	304.166	307.621	1.215
3	1	3	288.780	313.104	448.474	1.631
3	1	3	290.698	322.878	635.853	2.158
3	1	3	293.386	333.210	878.316	2.671
3	1	3	297.494	343.908	1132.037	3.113
3	2	3	281.401	292.234	166.665	.727
3	2	3	283.693	302.517	342.034	1.263
3	2	3	287.636	316.026	591.105	1.904
3	2	3	293.900	336.968	1012.306	2.889
3	1	4	280.632	290.320	103.230	.650
3	1	4	282.660	300.898	220.113	1.223
3	1	4	284.565	309.339	369.934	1.662
3	1	4	287.995	320.063	566.140	2.151
3	1	4	289.788	325.956	682.421	2.426
3	1	4	293.855	337.131	958.382	2.903
3	1	4	298.041	346.089	1135.092	3.223
3	2	4	290.144	315.959	419.581	1.732
3	2	4	291.173	323.556	608.203	2.172
3	2	4	295.810	336.533	928.255	2.731
3	2	4	302.728	350.836	1281.974	3.227
4	1	1	279.601	289.270	5.872	5.233

<u>IDG</u>	<u>IDBC</u>	<u>IDF</u>	<u>TAVGO</u>	<u>TAVGI</u>	<u>QCONV</u>	<u>QLOSS</u>
4	1	1	281.569	298.399	17.103	12.948
4	1	1	281.502	312.059	28.844	27.739
4	1	1	279.897	321.412	38.894	39.547
4	1	1	281.889	333.768	55.824	50.713
4	1	1	282.748	352.136	80.434	69.580
4	1	1	284.309	363.680	99.634	80.336
4	2	1	281.358	288.532	6.256	2.543
4	2	1	281.147	300.846	17.663	16.039
4	2	1	281.999	321.410	37.261	37.279
4	2	1	284.133	345.670	69.625	61.120
4	1	2	284.498	288.896	212.688	.272
4	1	2	285.678	291.605	315.462	.366
4	1	2	286.971	294.165	412.796	.444
4	1	2	288.930	297.953	541.556	.557
4	1	2	290.587	301.332	682.380	.664
4	1	2	293.284	307.133	843.062	.855
4	1	2	295.648	310.801	1022.397	.936
4	2	2	284.971	288.968	160.379	.247
4	2	2	287.267	293.618	304.453	.392
4	2	2	289.983	298.993	499.159	.557
4	2	2	293.284	305.027	738.901	.725
4	1	3	279.456	287.843	107.805	.518
4	1	3	281.615	295.046	188.364	.830
4	1	3	283.374	301.716	279.468	1.133
4	1	3	284.866	308.982	401.756	1.490
4	1	3	286.712	315.942	529.155	1.805
4	1	3	289.559	327.440	690.546	2.340
4	2	3	282.113	291.096	123.722	.555
4	2	3	284.640	303.233	309.055	1.148
4	2	3	288.448	317.477	562.422	1.793
4	2	3	295.999	339.449	900.080	2.684
4	1	4	280.907	289.909	80.268	.556
4	1	4	283.126	296.962	145.109	.855
4	1	4	284.862	304.639	237.081	1.222
4	1	4	287.042	313.183	359.723	1.615
4	1	4	289.182	320.607	500.630	1.941
4	1	4	291.127	329.253	671.090	2.355
4	1	4	293.633	337.900	864.529	2.734
4	2	4	281.316	292.468	104.340	.689
4	2	4	283.143	303.937	262.373	1.284
4	2	4	285.844	317.365	500.549	1.947
4	2	4	291.779	333.840	817.442	2.598

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