The effects of taxes and inflation on the composition of inputs to agriculture
by Douglas Roger Hart

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Applied Economics
Montana State University

Abstract:
The substitution effect between horsepower (farm machinery) and labor is analyzed when wage rates, tax rates, discount rates, depreciation methods, investment credits and inflation rates are varied. A simulated wheat farm is developed and the effects of the above mentioned variables are analyzed on the horsepower (machinery)/ labor ratio. When either the price of labor or horsepower (machinery) is altered directly or indirectly, there is a change in the ratio of horsepower to labor. This thesis explores the causes and extent of these changes.

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Date September 1, 1981

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ABSTRACT

The substitution effect between horsepower (farm machinery) and labor is analyzed when wage rates, tax rates, discount rates, depreciation methods, investment credits and inflation rates are varied. A simulated wheat farm is developed and the effects of the above mentioned variables are analyzed on the horsepower (machinery)/labor ratio. When either the price of labor or horsepower (machinery) is altered directly or indirectly, there is a change in the ratio of horsepower to labor. This thesis explores the causes and extent of these changes.

Chapter 1

INTRODUCTION

The composition of the agricultural community in the United States has changed dramatically over the past fifty years. It has changed from small labor intensive units to large, highly specialized, capital intensive units. This result has been the product of a number of social-political events; an agricultural depression[1/] lasting two decades, two world wars and the rapid growth of United States industry and technology during the past two decades. But these events have all subsided in the early 1970's, industrial productivity has actually been declining, yet the trend continues. The purpose of this thesis is to explore some of the underlying, and perhaps, unsuspected reasons for the changing structure of U.S. agriculture.

Fifty years ago taxes played a minor role in the operating and investment practices of most farming enterprises. But in the last couple of decades the importance of tax laws has had increasing significance on farmers' financial planning. The main objective of this thesis is to explore the effects of income tax rates, and depreciation methods on the use of labor and machinery in the

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[1/] The agricultural depression lasted from 1919 to 1939, and is documented in a number of history books such as American Epoch, by Arthur Link and William Catton, Vol. II. 4th Ed.

structure of agriculture. That is, when the composition of inputs change the structure changes. For example, when machinery is substituted for labor for any reason, the structure of agriculture becomes more machinery intensive. This study will also explore the effects of inflation on this composition, and how inflation enhances or diminishes the effects of the tax rates and depreciation methods.

The maintained hypothesis of this thesis is that with changes in income tax rates, wage rates, social security tax rates and depreciation methods, the relative prices of the inputs change. When these prices are changed relative to each other, there is a reallocation of inputs.[2/]

It may be useful to analyze this effect by showing the substitution effect between two inputs. The substitution effect is the rate at which the producer substitutes one input for another when the price of an input changes and he moves along a given isoquant.[3/] Henderson and Quandt[4/] prove that this effect is always negative; i.e., when the price of an input decreases, the quantity used of that input

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[2/] This is proved mathematically in Chapter 3.

[3/] Microeconomic Theory; a Mathematical Approach, Henderson and Quandt; 3rd Ed., p. 27; the words inputs and isoquant were substituted by the author for the words, commodity and indifference curve, respectively.

[4/] Ibid, p. 47.

increases.

For purposes of illustration, the effects of an input price change will be observed utilizing an isoquant graph. Let the isoquant represent a measure of the number of acres worked by the farmer and define this isoquant to be a function of horsepower and hours, where hours represent the amount of labor time required to work a given number of acres.

The equation of the isoquant is:

$$\text{Acres} = K \cdot HP \cdot HRS$$

where K = a constant

HP = the amount of horsepower used

HRS = the amount of labor hours used.[5/]

If acres are held constant, the equation of the isoquant will be a rectangular hyperbola, as illustrated in Figure 1. Units of horsepower are represented on the y axis and units of labor hours are measured on the x axis.

With a change in the relative prices of the inputs (Price of horsepower decreases), the budget line shifts from BB to B'B'. The least cost combination of inputs shift from point A to point C. The amount of labor used decreases (HR° -- HR') and the amount of

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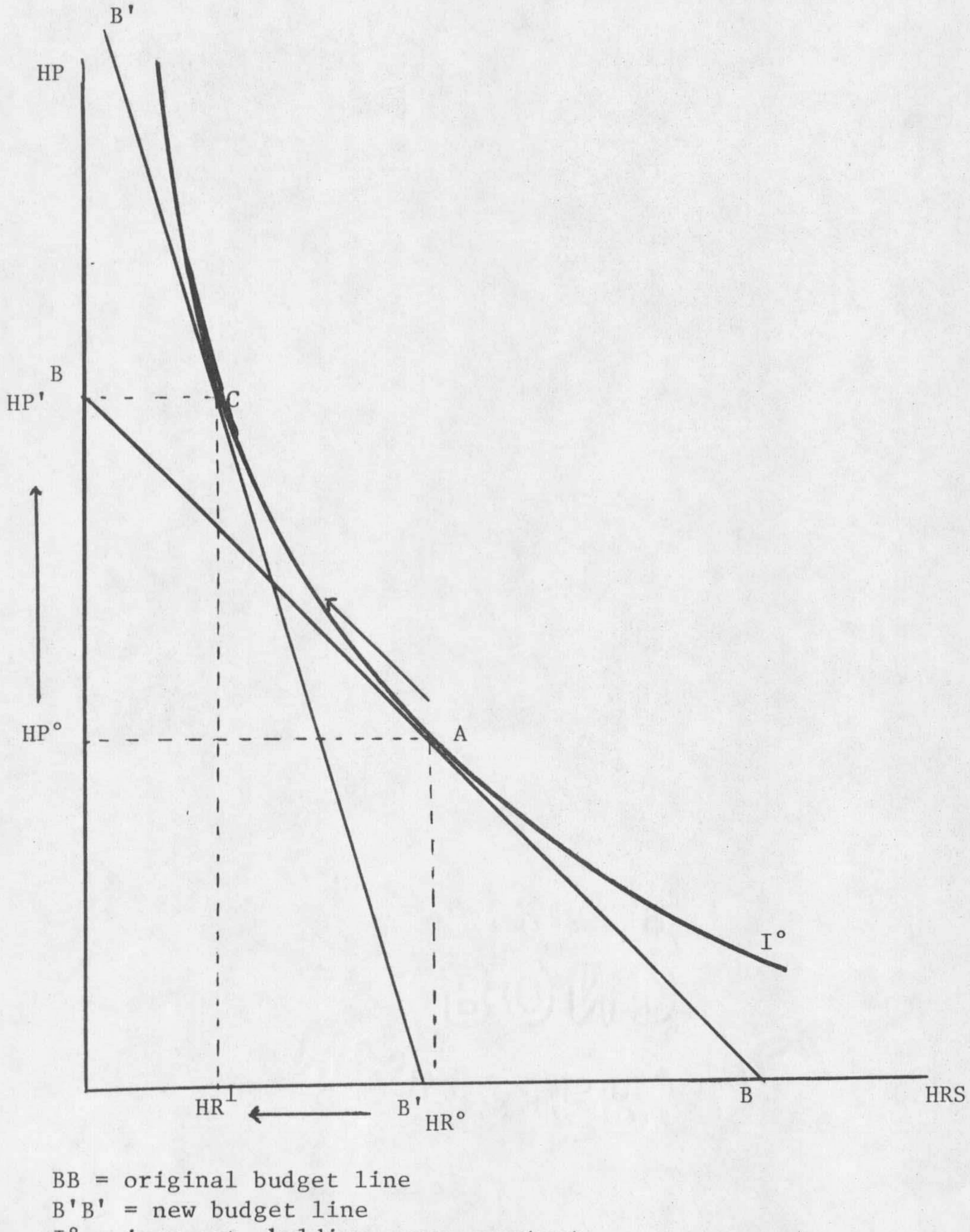
[5/] The actual derivation of this function is found in Chapter 4. K = a constant and its value can vary with given variables, this relationship is also found in Chapter 4.



BB = original budget line
B'B' = new budget line
I° = isoquant, holding acres constant

Figure 1. Demonstration of the Substitution Effect Between Two Inputs

capital (horsepower) used increases (HP° -- HP'). So, as is illustrated with a change in the relative price of an input there is a change in the least cost combinations of inputs to produce a specified level of output.

This is not the total effect. There is also a scale effect where the budget line shifts to a higher (or lower) isoquant. This effect is not explored in this thesis since the amount of acres farmed is assumed to be held constant, i.e., the isoquant is not allowed to shift.

Need for the Project

When tax reforms are initiated there should be specific objectives to be achieved. These objectives may be diverse and differ with the different environments for reform that were prevalent at the time. However, if all of the consequences of the new laws are not taken into consideration, there may be perverse effects on the system, totally opposite of the objectives that initiated the process. For example, when the amount of investment credit allowed is increased, the hoped for result would probably not include a reallocation of resources used in agriculture. But that is exactly what happens because the price of machinery, an input to agriculture, has effectively been decreased. One of the objectives of this thesis is to illustrate how this occurs and estimate the extent of the influence.

The other objectives of this thesis are specifically: 1) to measure the substitution effect between labor and machinery, for a simulated dryland wheat farm, when tax rates and depreciation methods change the relative prices of these two inputs; 2) to develop optimal replacement strategies for the farm machinery given discount rates, wage rates, and tax rates; and 3) to measure the impact of inflation on the first two objectives.

Chapter 2 is a review of the literature concerning optimal replacement strategies. A number of different methods are presented and the pertinent parts of each are discussed. Chapter 3 develops the theory and maintained hypothesis of this thesis. Chapter 4 presents the simulation model used and its assumptions. Chapter 5 contains the results of the model and the conclusions drawn from them.

Chapter 2

LITERATURE REVIEW

The thesis deals primarily with the effects of taxes, depreciation schemes and inflation on the optimal size and replacement ages of finite life, depreciable inputs in agriculture. This essentially constitutes a replacement problem. Therefore, the literature review deals with the development and theory of optimal replacement strategies.

The first section of the chapter deals with some of the pioneering work in the area of replacement theory, then traces its development to the present state of the science. The second section of the chapter deals with the application of replacement theory to agriculture and also discusses works which support some of the basic assumptions of this thesis.

A number of different replacement models will be presented in the literature review, so that the reader will have a basis by which to compare the model and replacement strategy presented in this study.

Replacement Theory

Martin Faustmann [translated 1968], a German forester, is credited with first applying the principal of discounted cash flow to a replacement problem. His article appeared in a forestry journal in

1849, and was written in response to an article which appeared just two months prior, dealing with the same problem.

Faustman addressed himself to the optimal cutting age of a stand of trees on a tract of land. The maturing age of the trees was twenty years so the tract was to be divided into twenty equal sections in order to afford an annual income.

To solve this problem, Faustmann developed and used the idea of net discounted cash flows. His symbols and equations, although not explicitly stated as such, can be reduced to:

$$\text{Net Discounted Revenue (NDR)} = \sum_{n=0}^{t} \frac{Sn - Cn}{(1+r)^n}$$

where:

Sn = sales in year n,

Cn = costs in year n,

r = the discount factor,

0-t = the planning period,

n = the year.

Thus, he maximized NDR and solved the problem of an optimal cutting age.

Faustmann's examination of problems in forestry is not as specialized and narrowly applicable as first thought. When he analyzed these problems, he really tackled the broader problem of how long capital assets should be kept before being replaced, i.e., the question of finding an optimal rate of turnover for capital stock.

As noted, this basic formulation was developed for use in forestry in 1849. It was not until years later that a similar theory was adapted for use in economics. Dr. Harold Hotelling [1925] was an early pioneer in this field, in a paper presented in 1925, he presented a model which formed the basis of many modern theories of replacement. This model was:

$$1) \quad \beta = \int_0^T [w(Q(t)) - E(t)]e^{-\int_0^T i(t)dt} dt + S(t)e^{-\int_0^T i(t)dt}$$

where:

β = original cost of a single machine

T = an unknown date at which it ought to be discarded.

w = unknown unit cost (plus interest) of the product

Q(t) = rate of production.

E(t) = combined rate of all expenses, except depreciation and interest

i(t) = rate of interest

S(t) = selling price or scrap value.

By differentiating with respect to T, the unit cost can be written:

$$2) \quad w = \frac{E(T) + i(T)S(T) - S'(T)}{Q(T)} .$$

This equation states that the cost of a unit of product is found by adding the operating cost [E(T)] of the machine, at the time when it is least efficient and about to be scrapped, to interest [$i(T)S(T)$] on scrap value and the rate of depreciation [$-S'(T)$] of the scrap value and divide this sum by the machine's rate of production. The result will be a minimum when T is determined by subtracting equation 2 from equation 1 and solving. This value of T will be the optimal time period for holding the machine.

In a 1940 publication, Gabriel A. D. Preinreich [1940] modified the Hotelling formula by defining the optimal replacement for a single machine as:

$$1)\quad V = \int_0^T [Z(Q(t)) - E(t)]e^{-it}dt + Se^{-it}$$

where:

V = capitalized value of the machine

Z = unit market price

$Q(t)$ = rate of production

$E(t)$ = combined rate of all expenses, except depreciation and interest

$i(t)$ = interest rate

$S(t)$ = selling price (scrap value)

This formulation is the rule in which a machine will not be replaced.

Preinreich then took the derivative of the valuation formula with respect to the time period T:

2) $\frac{dV}{dT} = ZQ(t) - E(t) - iS$

He then solved set (2) equal to zero and solved for the most lucrative life span (optimal replacement age) of the machine (T). T was then plugged back into equation 1) to find the capitalized value of the machine. Thus it was discovered that the economic life of the machine was independent of the price at which it was bought and sold.

Preinreich then looked at this replacement strategy for several different situations. These situations may be classified under three different headings.

A. Scope

1. A single machine;
2. A finite chain of replacements;
3. An infinite chain;
4. A number of parallel chains, whose replacement dates are evenly staggered;
5. A large plant continuously renewed in accordance with natural variations in the behavior of similar machines;

B. Limitations

1. Scarcity of new machines available for replacement;
2. Scarcity of various operating facilities or ingredients of production;

3. scarcity of demand for product;

4. scarcity of capital;

5. regulation of profit by law;

C. Economic Conditions

1. the static case where only variations due to the age of the machine are considered;

2. variations due to the number of co-operating machines;

3. change in ownership and outlook;

4. change in the type of machine used (obsolescence);

5. the general dynamic case, embracing extraneous influences as well.

After analyzing a variety of replacement problems, Preinreich concluded:

> "The general rule of replacement, which is simply the theory of maxima and minima, has a separate solution for every kind of rigid scarcity and for every volume of the supply so limited. When the volume required by a single machine becomes insignificant in comparison to the total, the problem is simplified into making the excess profit (goodwill) per unit of that ingredient a maximum. In the case of demand, that means making the cost per unit of demand (output) a minimum. In all other instances, the limitation operates at the other end of the productive process and therefore the first description applies. The excess profit per new machine, per square foot of space, per hour of labor, per ton of fuel, etc., must be made a maximum, depending on where the shortage is felt."

He observed that the reason many plants are in a rundown condition is because the resultant rise in the rate of profit hides the more significant decline in its amount. To correctly calculate replacement lives for these plants one should substitute the unknown rate of profit for the rate of interest, the original cost of a machine will always be equal to the net rental and the scrap value, discounted at the rate of profit.

Although Preinreich observed a number of variations in Hotelling's formula (properly called the Hotelling-Taylor formula), his general conclusions were that Hotelling's idea of minimizing unit costs was the most valuable single rule of thumb which can be laid down for the general guidance of entrepreneurs, at least when the number of machines is very large and no radical change in type is imminent.

Dr. Paul A. Samuelson [1937] in an article published in 1937 took a slightly different view of replacement theory. He describes N(t) as the income stream in time (t) where $0 \leq t \leq b$, 0 and b are the boundaries of the time period. The value of this stream is then defined as:

$$1) \quad V = V(t'r) = \int_t^b N(x)e^{r(t-x)}dx$$

where:

r = rate of interest

x = variable of integration.

He then defined the rate of depreciation (or appreciation) as:

$$2) \quad \frac{\partial V}{\partial t} = r[V(t)] - N(t)$$

This is to say, "The rate of depreciation at any instant of time is equal to the difference between net income and the returns on value of the investment account at that instant of time. This is equivalent to saying the net income includes the return on given value of investment plus the rate of depreciation."

If the value remains constant, and net income is considered perpetual, by performing the integration in equation 1 (if $N(X) = N$, or returns are constant):

$$3) \quad r \equiv \frac{N}{V}$$

That is, the rate of depreciation becomes equal to zero and the rate of interest may be expressed as the ratio between perpetual net income and value.

This is the same conclusion Faustmann would have reached a century earlier, if his equation had been simplified; although both theories were developed along different lines.

Samuelson observed that the value of an investment account will necessarily be given by the integration of the income stream, discounted at the market rate of interest. This follows because if the market price of the account is greater than its capitalized value, it would pay the owner to sell it and lend out the resulting sum of money at the current market rate of interest. But no one would be willing to pay any price for it above its capitalized value since they could always do better with their money elsewhere. Thus, the market price of an asset can never exceed its capitalized value.

He also proved that under a varying rate of interest, the value of an investment account is equal to:

$$V'(t) = r(t)V(t) - N(t).$$

This is the same relationship that held under a constant rate of interest. Even with a varying rate of interest, net income in his definition of the term contains a return on the value of an investment and also an amount equal to the rate of depreciation. "Thus the time shape of interest being given and income being known, the capital invested up to any time is always equal to the value of the account at that time, the value being a capitalization of subsequent income."

Samuelson's theory of replacement has been the basis of much of the work done in the area to date. It is the main theory upon which this thesis is based.

Samuelson [1962] also developed a "corallary" to his theory in which taxes and depreciation play a role. It is known as "The Fundamental Theorem of Tax Invariance" and states, "If, and only if, time loss of economic value is permitted as a tax deductible depreciation expense will the present discounted value of a cash-receipt stream be independent of the rate of tax."

This thesis will explore the effects of accelerated depreciation gimmicks on the costs of finite life inputs in agriculture. From this corollary it can be concluded that these gimmicks will affect the costs of these inputs in a negative direction. If the costs of these inputs are altered then a reallocation of these inputs relative to annual life inputs will occur. The causes and support for this hypothesis is presented in detail in the next chapter.

Replacement Theory in Agricultural Economics

There has been little actual work done on the effects taxes have on replacement theory but its occurrence has been mentioned. Kay and Rister [1976] briefly mention these effects in their article by stating,

> "The effects of these tax regulations is to lower the present value (cost) of any replacement policy. This result has, probably, in past, encouraged the trend towards larger equipment and the substitution of machinery for labor. The net result is a larger overall investment in farm machinery than would have existed without these incentives."

There has been a substantial amount of work done in the area of replacement theory in agricultural economics, although the effects of taxes and tax gimmicks have not been included in much of this work.

Dr. Mason M. Gaffney [1957] developed a procedure whereby he specified a machine should be kept another period if the marginal costs of retaining it another period were less than the "average" periodic costs of a replacement machine.

R. K. Perrin [1972] developed a model which specifies that an asset should be held to the age in which marginal revenues equal marginal opportunity costs with the latter being interpreted as the flows of earnings which would be realized from some given-year replacement policy. He also suggested calculating present values for each replacement year may be a better procedure than evaluating the marginal criteria.

Perrin also evaluated the effects of discount rates on optimal replacement ages. From this he obtained the conclusion that, "Some assets may be replaced earlier with rising discount rates while others may be replaced later; and in fact, a given asset may be replaced later up to a given rate but earlier thereafter." This depends upon the shape of the flows.

Perrin also addressed himself to the question of what the appropriate discount rate should be. The cost of capital and the return on alternative investment possibilities on the time of personal consumption may both be factors in determining what the appropriate discount rate should be. Neither of these choices are universally acceptable. The cost of capital may be appropriate if the entrepreneur faces a perfect capital market. But a particularly destitute entrepreneur may value present earnings high relative to future earnings so a high personal discount rate may be appropriate. If there were no capital markets, the internal rate of return may be the appropriate discount rate. The internal rate of return is determined by the market prices of the inputs; thus, if the internal rate is above the market rate for ventures of similar riskiness, the market price of the inputs will be driven up. Therefore, market rates of return for ventures of similar riskiness can be viewed as the appropriate discount rate if equilibrium prices of all inputs are expected to prevail by the first replacement date. The choice of appropriate discount rates for use in this thesis will be discussed in chapter four.

Anthony H. Chisholm [1974] developed a present value cost model incorporating income tax rates and investment credits. He estimated the effects of different investment credit rates and different tax rates, given certain discount rates, on the optimal replacement age of machinery. He found that the investment credit significantly influenced the optimal replacement age for machinery, and that different depreciation methods biased the optimal replacement time. He states:

> "There is no simple general rule for predicting the direction of bias on replacement age of a particular method of depreciation. Perhaps more important is the fact that in no instance was the magnitude of the bias stemming from a particular method of depreciation of sufficient size to change the optimal replacement age."

Kay and Rister [1976] also did some work on the effect of tax depreciation and investment credit on replacement ages and concluded that:

a) the after-tax discount rate had the greatest effect,

b) the tax rate causes only slight differences in replacement policy,

c) depreciation methods have little effect (they analyzed straight-line and double-declining balance).

They observed that additional first year depreciation and investment credits had the greatest affect on the optimal replacement ages. They also found that tax regulations which permit using double declining balance depreciation, additional first year depreciation and investment credit, affect the present value (cost of obtaining the constant annual stream of tractor services. This led to their comment about the effects of taxes on the allocation of capital and labor quoted at the beginning of this section.

Bates, Rayner and Custance [1979] viewed a farm tractor replacement model in a continuous-time framework. They used this model to observe the effects of inflation on replacement ages. The model they represented was:

$$PVn = \frac{1}{1-e^{-rn}} \{(Co - Cne^{-rn}) + (1-T)Rk - T(A(n)e^{-(r+f)} -$$

$$T(\int_{K=1}^{n} DKe^{-(r+f)K} dK)^{n} - Ine^{-r(r+f)} \}$$

where:

PVn = present value of the total cost in year n,

r = after-tax real discount rate,

Co = initial cost,

Cn = resale price at the end of year n,

T = marginal income tax rate,

Rk = repair cost in year K,

An = additional first year depreciation policy; with a policy in n years,

dK = regular depreciation allowance in year K,

In = investment credit

e^f = rate of inflation.

Their conclusions were that inflation affects the cost incurred by farmers in three ways:

1) Taxes are based typically on historic costs. The allowances that can be claimed for "depreciation" of equipment are a significant element in tax allowances. If inflation is significant, the model must properly allow for the loss in the real value of these allowances.

2) Receipts and benefits from tax allowances are lagged, typically by about one year. With inflation these receipts are in depreciated money.

3) When a farmer sells his equipment, the difference between the resale price and the unexpired depreciation allowance is subject to tax. In inflationary times, resale prices for any given age of equipment are likely to be increasing and may well exceed the unexpired depreciation allowances which are based on historic costs.

Watts and Helmers' [1980] research into this area explored the actual central theme of this thesis which is the substitution of capital for labor with the imposition of taxes. They incorporated a model in which an after-tax profit function was constrained by a strictly concave and continuous production function:

$Y = F(F,S)$,

where:

Y = quantity of output,

L = quantity of annual inputs,

S = quantity (size) of depreciable inputs.

The constrained present value profit function was defined as:

$$\pi = \int_0^n QYe^{-\beta ri}di - \int_0^n wLe^{-\beta ri}di - S[V(0) - V(n)e^{-\beta rn}] -$$

$$\int_0^n ST[QY - wL + V'(i)]e^{-\beta ri}di - \lambda[Y-F(L,S)].$$

where:

π = present value of after-tax profit,

Q = price of output assumed to be constant over time,

w = price of the annual input, assumed to be constant over time,

V(i) = price of a unit of depreciable input as price per unit of size of a depreciable input (which implicitly assumes price is a linear function of size at age i.

V'(i) = change in price of depreciable input as the input age (this value is negative under most circumstances),

T = the tax rate in decimal form which is assumed to be between zero and one and constant over the relative income range,

β = 1-T,

i = time or age,

n = length of planning period which is assumed to be consistent with the ownership life of the machine,

r = before-tax discount rate which is assumed to be positive.

The first order conditions for profit maximizing levels of Y, L, and S are:

$$\frac{\partial\pi}{\partial Y} = \beta Q\int_0^n e^{-\beta ri} di - \lambda = 0$$

$$\frac{\partial\pi}{\partial L} = - \beta w\int_0^n e^{-\beta ri} di + \lambda F_L = 0$$

$$\frac{\partial\pi}{\partial S} = - [V(0) - V(n) + T\int_0^n V'(i)e^{-\beta ri} di] + \lambda F_S = 0$$

$$\frac{\partial\pi}{\partial Y} = Y - F(L,S) \overset{set}{=} 0.$$

The first order conditions reduce to:

$$F_L = \frac{\beta w\int_0^n e^{-\beta ri} di}{\beta QP\int_0^n e^{-\beta ri} di} = \frac{w}{Q}$$

$$\frac{F_S}{F_L} = \frac{V(0) - V(n)e^{-\beta rn} + T\int_0^n V'(i)e^{-\beta ri} di}{\beta w\int_0^n e^{-\beta ri} di}$$

$$Y = F(L,S).$$

Since V'(i) is not constant the integral cannot be factored out, therefore the tax rate is implicit in the maximizing levels of S and L. They arbitrarily set $\frac{F_S}{F_L} = R$ and explored the derivative $\frac{\partial R}{\partial T}$ to investigate the effects of the tax rate on the optimal (maximizing) amounts of inputs to use. By exploring this derivative they reasoned that $\frac{\partial R}{\partial T}$ was likely to be less than zero which meant that as T increases, S increases and Y increases, so as long as S and L are economic substitutes, L decreases. So as the tax rate increases the amount of machinery will increase and the amount of labor will decrease. This theme will be explored in some detail in the theory part of this thesis.

In another paper, Watts and Helmers [1980] developed budgeting techniques and concepts which will be used in this thesis. For instance, assume an after-tax basis, then the original outlay for a depreciable asset is considered to be placed on an after-tax basis by the inclusion of depreciation credits. Depreciation credits are found by multiplying depreciation by the marginal tax rate. Depreciation and investment credit are on a nominal or real basis and depreciation and investment credit recapture are estimated in nominal terms. They state that:

> "The net present after-tax cost is achieved by discounting either nominal or real after-tax flows by the appropriate nominal or real after-tax discount rate. This net present cost is then placed on an annual after-tax basis by amortizing the net present cost by the real after-tax discount

> rate. The resultant real annual after tax cost can then (if desired) be converted to a before-tax basis by dividing the amortized tax cost by the complement of the marginal tax rate."

They also present a proof showing the relation between the traditional and capital budgeting fixed cost estimates of opportunity cost and depreciation. The proof is as follows:[1/]

Discrete Time Cast

Prove that: $$V(0) - \frac{V(n)}{(1+r)^n} = \sum_{i=1}^{n} \frac{D(i) + OC(i)r}{(1+r)^i}$$

where:

$D(i)$ = depreciation in year t,

$= V(i) - V(i-1)$ = market depreciation,

$OC(i)$ = opportunity cost in year i - $V(i-1)r$

$V(n)$ = value of the machine in time n.

$$\sum_{i=1}^{n} \frac{D(i) + V(i-1)r}{(1+r)^i} = \frac{V(0) - V(1) + V(0)r}{1+r} + \ldots +$$

$$V(n-1) - \frac{V(n) + V(n-1)r}{(1+r)^n}$$

$$= V(0) - \frac{V(1)}{1+r} + \frac{V(1)}{1+r} - \frac{V(2)}{1+r} + \ldots + \frac{V(n-1)}{(1+r)^{n-1}} - \frac{V(n)}{(1+r)^n}$$

1/ Myles J. Watts and Glenn Helmers, "Machinery Costs and Inflation," unpublished research, Appendix A.

$$= V(0) - \frac{V(n)}{(1+r)^n}$$

For the continuous time case show that:

$$V(0) - \frac{V(n)}{e^{rn}} = \int_0^n \frac{D(i) + OC(i)r}{e^{ri}}\, di$$

note that $d\frac{V(i)}{e^{ri}}/di = \frac{V'(i)}{e^{ri}} - \frac{V(i)r}{e^{ri}}$

$$= \frac{V'(t) - V(t)r}{e^{ri}}$$

Furthermore, since $-V'(i) = D(i)$ and $V(i)r = OC(i)$ then:

$$\int_0^n - \frac{D(i) - OC(i)}{e^{ri}}\, di = - [\frac{V(n)}{e^{rn}} - \frac{V(0)}{e^{r0}}]$$

$$= V(0) - \frac{V(n)}{e^{rn}}$$

The relevance of the proofs by Watts and Helmers and the models used by the others will become clear as the next chapter is read. Assumptions and implications of most of the models discussed in this section of the literature review are incorporated into the model used in this thesis.

Chapter 3

THEORETICAL DEVELOPMENT AND MAINTAINED HYPOTHESIS

The goal of any profit maximizing firm is to minimize costs for a given level of output and thus maximize profits for that level of output. Income tax rates, depreciation schedules and investment credit allowances are all implicit variables in the cost functions facing agricultural entrepreneurs. The effects these variables have on the cost functions and thus on the profit maximizing combination of inputs are the main subject of this thesis. This chapter will explore how these variables affect the cost functions and the profit maximizing conditions facing the farming firm. Different effects will be illustrated for different types of inputs. Annual inputs, infinite life inputs and finite life inputs, which depreciate in value over their productive lives, will all be analyzed.

Annual Inputs

Assume the farmer faces the production function $Y = F(x, w)$, where: Y = the quantity of output or product and x and w are the annual inputs. Also assume x and w are totally diminished during the production period and the total amount of Y, output, is sold at the end of the production period. The planning period is from 0 to n production periods, where n can vary.

The profit function can now be defined as the Lagrangean:

$$1) \quad \pi = \int_0^n PiYe^{-ri}di - \int_0^n V_w we^{-ri}di - \int_0^n V_x xe^{-ri}di - \lambda\,[Y-F(w,x)].$$

Which can be reduced to:

$$2) \quad \pi = Y\cdot \int_0^n Pie^{-ri}di - w\cdot \int_0^n V_w e^{-ri}di - x\cdot \int_0^n V_x e^{-ri}di - \lambda\,[Y-F(w,x)].$$

The variables are defined as:

π = Present value of profit,

r = discount rate,

0-n = planning period,

P_i = price of one unit of Y at time i,

V_w = price of one unit of w at time i,

V_x = price of one unit of x at time i.

The first-order conditions for profit maximization are:

$$3) \quad \frac{\partial\pi}{\partial Y} = \int_0^n Pie^{-ri}di - \lambda = 0$$

$$4) \quad \frac{\partial\pi}{\partial w} = - \int_0^n V_w e^{-ri}di + \lambda\,\left[\frac{\partial F(x,w)}{\partial w}\right] = 0$$

$$5)\quad \frac{\partial \pi}{\partial x} = - \int_0^n V_x e^{-ri} di + \lambda \left[\frac{\partial F(x,w)}{\partial x}\right] = 0$$

Note: $\frac{\partial F(x,w)}{\partial w}$ = marginal physical productivity of w = MPPw and $\frac{\partial F(x,w)}{\partial x}$ = marginal physical productivity of x = MPPx.

If equations 4 and 5 are solved for λ and set equal to each other, the following expression is obtained:

$$\frac{- \int_0^n V_w e^{-ri} di}{MPPw} = \frac{- \int_0^n V_x e^{-ri} di}{MPPx} ,$$

which can be reduced to:

$$\frac{- \int_0^n V_w e^{-ri} di}{- \int_0^n V_x e^{-ri} di} = \frac{MPPw}{MPPx}$$

If the input prices are constant, then

$$7)\quad \int_0^n V_w e^{-ri} di = V_w \cdot \int_0^n e^{-ri} di$$

$$8)\quad \int_0^n V_x e^{-ri} di = V_x \cdot \int_0^n e^{-ri} di.$$

Substitute these identities into equation 6 yields

$$\frac{- V_w \int_0^n e^{-ri} di}{- V_x \int_0^n e^{-ri} di} = \frac{MPPw}{MPPx} ,$$

which reduces to:

$$\frac{V_w}{V_x} = \frac{MPPw}{MPPx}$$

Thus, the ratio of input prices must equal the ratio of the marginal physical productivities of the inputs for profit to be maximized.

Similarly, if Pi = P, i.e., the price of Y is held constant, then:

$$9) \quad \int_0^n Pie^{-ri}di = \int_0^n Pe^{-ri}di = P\int_0^n e^{-ri}di.$$

If equations 3 and 4 are solved for λ and set equal to each other:

$$\int_0^n Pie^{-ri}di = \frac{\int_0^n V_w e^{-ri}di}{MPP_w} .$$

Substitute identities 7 and 9 into this equation:

$$P\int_0^n e^{-ri}di = \frac{V_w \int_0^n e^{-ri}di}{MPPw} .$$

Which reduces to:

$$\frac{P\int_0^n e^{-ri}di}{V_w \int_0^n e^{-ri}di} = \frac{1}{MPPw}$$

which further reduces to:

$$\frac{P}{V_w} = \frac{1}{MPPw},$$

and similarly $\frac{P}{V_x} = \frac{1}{MPPx}$ is obtained for input x.

This implies that the ratio of product price to input price must equal one over the marginal physical productivity of the input in order for profit to be maximized.

The time element and discount rate fall out of the profit maximizing conditions. The only determinants of the amount of inputs to be used are the prices of the inputs and the marginal physical productivities of the inputs.

If an ad-valorem tax is assessed on profit and the costs of x and w are tax deductible, assuming the prices are all still held constant, the profit function becomes:

$$\pi = \beta\ [P \cdot Y \cdot \int_0^n e^{-r\beta i} di - V_w \cdot w \cdot \int_0^n e^{-r\beta i} di - V_x\ x \int_0^n e^{-r\beta i} di] -$$

$$\lambda[Y-F(x,w)],$$

where β= the complement of the tax rate = (1-tax), and rβ is the after tax discount rate.[1]

1/ The use of r(1-t) as an appropriate discount rate is discussed in Chapter 4 of this thesis.

The first order conditions for profit maximization are:

$$\frac{\partial \pi}{\partial Y} = \beta P \int_0^n e^{-ri} di - \lambda = 0$$

$$\frac{\partial \pi}{\partial w} = - \beta V_w \int_0^n e^{-ri} di + \lambda \, MPPw = 0$$

$$\frac{\partial \pi}{\partial x} = - \beta V_x \int_0^n e^{-ri} di + \lambda \, MPPx = 0$$

If these three equations are solved in the same manner as the previous first order conditions, the following identities are obtained:

$$\frac{- \beta V_w \int_0^n e^{-ri} di}{MPPw} = \frac{- \beta V_x \int_0^n e^{-ri} di}{MPPx}$$

which reduces to

$$\frac{V(w)}{V(x)} = \frac{MPPw}{MPPx} .$$

When P is solved for, the following identities are obtained:

$$\frac{P}{V_w} = \frac{1}{MPPw} \text{ and } \frac{P}{V_x} = \frac{1}{MPPx} .$$

As can be seen, β drops out of the first order conditions, implying the amount of inputs to be used does not depend on the tax rate. In this case the inputs have a useful life of one production period. The effect of taxes on input use, where the input life is longer than one production period, will now be investigated.

Infinite Life Input

Consider an input which has an infinite life in the production function, such as land. Let:

w = quantity of the input with an infinite life,

V_w = per unit cost of w,

The profit function then becomes:

$$\pi = [\int_o^n PYe^{-r\ \beta i}di - \int_o^n (V_x)xe^{-r\ \beta i}di] - (V_w)(w) + (V_w)(w)e^{-r\ \beta n}$$
$$- \lambda\ [Y - F\ (x,w)]$$

Assuming prices are held constant as in the first section, the first order conditions reduce to:

$$\frac{rV_w}{V_x} = \frac{MPPw}{MPPx}$$

$$\frac{rV_w}{P} = MPPw$$

$$\frac{V_x}{P} = MPPx$$

If taxes are now assumed and interest, and therefore opportunity cost is tax deductible, the profit function becomes:

$$\pi = \beta\ [\int_o^n PY_e^{-r\ \beta i}di - \int_o^n (V_x)xe^{-r\ \beta i}di - (V_w)w \int_o^n re^{-\ \beta ri}di] -$$
$$[Y - F(x,w)].$$

The first order conditions yield:

$$\frac{rV_w}{V_x} = \frac{MPPw}{MPPx}$$

$$\frac{rV_w}{P} = MPPw$$

$$\frac{V_x}{P} = MPPx.$$

The tax rate is not included in the first order conditions for profit maximization, implying that the quantity produced and input use is not affected by the tax rate.

Finite Life Inputs

Consider an input with a finite life of more than one production period, the value of which diminishes (depreciates) over its productive life span. First consider the cost function of that depreciable input. Let:

V(0) = the initial purchase price of the input,

V(n) = the price of that input in time n,

then,

$$V(0) - \frac{V(n)}{e^{rn}} = \int_0^n \frac{D(i) + OC(i)r}{e^{ri}} \, di$$ [2/]

where:

2/ Machinery Costs and Inflation, by Myles J. Watts and Glenn a Helmers, unpublished research.

D(i) = depreciation in year 1

OC(i) = opportunity cost in year 1, and

not that $V(0) - V(n)e^{-rn}$ is a difference equation, $-V'(i) = D(i)$ or depreciation and $rV(i) = OC(i)$ or opportunity cost.

If w = the quantity of an input with a finite life of more than one production period and x = the quantity of annual input, then:

The profit function now becomes:

$$\pi = \int_0^n PYe^{-ri}di - \int_0^n V_x xe^{-ri}di - \int_0^n [rV(i) - V'(i)]we^{-ri}di - \lambda[Y-F(x,w)]$$

The ratio forms of the first order conditions are:

$$\frac{V_x}{P} = MPPx$$

$$\frac{\int_0^n [rV(i) - V'(i)]e^{-ri}di}{\int_0^n V_x\, e^{-ri}di} = \frac{MPPw}{MPPx}$$

$rV(i) - V'(i)$ is not constant and so cannot be factored out of the integral. Therefore, time, the planning period, becomes a factor in the first order conditions of the profit functions.

If taxes are now considered, the profit function becomes:

$$\pi = \int_o^n PYe^{-\beta ri}di - \int_o^n (V_x)(X)e^{-\beta ri}di - V(o) + V(n)e^{-\beta rn} -$$

$$T[\int_o^n PYe^{-\beta ri}di - \int_o^n (V_x)(X)e^{-\beta ri}di + \int_o^n \frac{\partial V(i)}{\partial i} e^{-\beta ri}di] -$$

$$\lambda[Y-F(x,w)].$$

where $-\frac{\partial V(i)}{\partial i}$ = tax deductible depreciation.

Therefore:

$$\pi = \int_o^n PYe^{-\beta ri}di - \int_o^n (V_x)(X)e^{-\beta ri}di - \int_o^n \left\{\frac{\partial V(i)}{\partial (i)} - \beta rV(i)\right\} e^{-\beta ri}di -$$

$$T[\int_o^n PYe^{-\beta ri}ei - \int_o^n (V_x)(X)e^{-\beta ri}di + \int_o^n \frac{\partial V(i)}{\partial i} e^{-\beta ri}di] -$$

$$\lambda[Y-F(x,w)].$$

Since $\beta = (1-T)$,

$$\pi = \beta\int_o^n PYe^{-\beta ri}di - \beta\int_o^n (V_x)(X)e^{-\beta ri}di + \beta \frac{\partial V(i)}{\partial (i)} e^{-\beta ri}di -$$

$$\int_o^n \beta\ r\ V(i)e^{-\beta ri}di - \lambda[Y-F(x,w)].$$

which reduces to:

$$\pi = \beta[\int_o^n PYe^{-\beta ri}di - \int_o^n (Vx)(x)e^{-\beta ri}di + \int_o^n \frac{\partial V(i)}{\partial (i)} e^{-\beta ri}di -$$

$$\int_o^n [rV(i)e^{-\beta ri}di]] - [Y-F(x,w)].$$

The ratio forms of the first order conditions reduce to:

$$\frac{V(x)}{P} = MPPx$$

$$\frac{\int_0^n [r\ V(i) - V'(i)]e^{-r\beta i}di}{\int_0^n Pe^{-r\beta i}di} = MPPw$$

$$\frac{\int_0^n [r\beta V(i) - V'(i)]e^{-r\beta i}di}{\int_0^n Pe^{-r\beta i}di} = MPPw$$

$$\frac{\int_0^n [r\ V(i) - V'(i)]e^{-r\beta i}di}{\int_0^n V(x)e^{-r\beta i}di} = \frac{MPPw}{MPPx}$$

As can be seen, the tax rate, as well as the discount rate, is an element inherent in the profit maximizing conditions. Therefore, the profit maximizing combinations of inputs is dependent on the tax rate.

Maintained Hypothesis

The conclusion of the previous section was that the tax rate and the planning period become elements of the profit maximizing conditions only when depreciable inputs are considered. Therefore, any policies which change the cost of a depreciable input over time will affect the first order conditions and cause a reallocation of the inputs used in that production process. Also any change in the tax rate will also influence the profit maximizing conditions and cause a reallocation of resources.

It is the maintained hypothesis of this thesis that tax gimmicks which allow a rapid rate of depreciation cause the price of finite life inputs to decrease relative to annual inputs, because the first order conditions for profit must hold for a profit maximizing firm, a reallocation of resources occur. Also, when the discount rate is altered by such things as the tax rate and inflation a reallocation of resources also occurs. This is because the discount rate is an implicit element of the profit maximizing conditions.

The effects of accelerated depreciation methods, investment credit allowances, tax rates and social security taxes will be empirically investigated in the remainder of the thesis.

Chapter 4

DEVELOPMENT OF THE SIMULATION MODEL

The purpose of developing the simulation model was to provide a means to find the input set that would minimize the total annual labor and machinery cost for operating a hypothetical farm. The total cost function is:

Total Cost = TC + PC + DC + LC

where: TC = annual tractor cost

PC = annual plow cost

DC = annual drill cost

LC = annual labor cost.

The total annual tractor cost, TC, is defined as:

TC = (NPT + (1-T)Repair T - UPT - [T(DepT)] + [T(UPT - BVT)] - INVCT + INVCTR)AFT,

where:

NPT = new price of the tractor

Repair T = total discounted repair and maintenance cost for the tractor

UPT = discounted used price of the tractor

DepT = discounted depreciation taken on the tractor

INVCT = investment credit taken on the tractor

INVCTR = investment credit recapture on the tractor

$$AFT = \frac{r(1-T) - f}{1-e^{-[r(1-T)]\partial n}}$$ = annuity discount factor for the tractor, real after tax

BVT = book value of the tractor at the replacement age.

Accumulated repair and maintenance costs are discounted by the real after-tax discount rate. In continuous time period format this discount rate = [r(1-T)-f]. Used prices are also discounted using this discount rate. Depreciation is discounted by r(1-T) since only nominal depreciation is tax deductible.[1/] The repair costs are mtuliplied by the complement of the marginal tax rate, (1-t), because repair costs (operating expenses) are direct deductions from income tax liabilities, therefore, the true cost is only (1-T) times the actual cost. Depreciation (DepT) and depreciation recapture (UPT-BVT) are multiplied by the marginal tax rate. Depreciation is multiplied by T because it is a benefit, that is, it reduces your tax liability. Depreciation recapture increases taxable income resulting in increased tax liability which is computed by multiplying the depreciation recapture by T.

Plow costs and drill costs are treated in a similar manner.

New Price Functions

The prices of inputs used in the model are important variables. The price functions for new tractors, chisel plows and drain drills were estimated. The price of tractors is estimated as a function of horsepower. The prices of chisel plows and grain drills are

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[1/] Use of [r(1-T)-f] as the appropriate after-tax discount rate is explained later in this chapter. For a more thorough explanation see Watts and Helmers [1980].

estimated as functions of width (in feet).

New price data was gathered from Gallatin Valley area implement dealers.[2/] Tractor prices were received for three brands of tractors, A, B, and C. The prices quoted were for two-wheel drive tractors, ranging from 70 to 180 horsepower with standard equipment from the factory, excluding dual tires.

Horsepower was regressed on price to determine the cost per unit of horsepower (the slope of the regression line). Various functional forms were estimated. The multiple regression using a dummy slope variable for Brand C tractors and excluding the 70 to 80 horsepower tractors for all brands yielded the best results. Omitting the 70-80 horsepower tractors was not detrimental since tractors of this size were not utilized in the model. The results of this regression are summarized in Table 4.1.

The price of the chisel plow is defined as a function of width and was determined using the same methods as when the price of the tractor was determined. Table 4.2 presents the results of this regression.

There was no new price function developed for the grain drill. Its width was set at 12 ft. and its price at $8000. This was assumed to be a reasonable estimate from the price data gathered.

[2/] To avoid any conflict, the names of the dealerships contracted have been purposely left anonymous.

Table 4.1. Multiple Regression Using a Dummy Slope Variable for Brand C Tractors and Deleting the Lower Horsepower Values.

Coefficients of*		
Horsepower	Dummy Slope Variable for Brand C	Multiple Correlation Coefficient
260.13 (13.17)	-32.57 (5.716)	0.986

*Numbers in parenthesis indicate standard errors.

The new price function for the tractor is:

NPT = 10482.80 + 260.13 HP - 32.57D.
Sample size = 18.

Table 4.2. Simple Regression on the Price of Chisel Plows as a Function of Width.

Coefficients of*	
Width	Correlation Coefficient
298.8 (114.14)	0.795

*Numbers in parenthesis indicate standard errors.

The new price function of the chisel plow is:

NPP = 1342.64 + 298.8 (width).
Sample size = 6.

To simplify the search to determine the least cost combination of machinery labor, plow prices were specified as a function of horsepower. The relationship between tractor horsepower and plow width was developed from functions featured in the Agricultural Engineers Yearbook. These functions are:

1) Field Capacity of $\frac{\text{acres}}{\text{hour}} = \frac{(\text{speed})(\text{width})(\text{efficiency})}{8.25}$

and

2) Horsepower required $= \frac{(\text{width})(\text{draft})(\text{speed})}{375}$

Solve equation 1 for acres;

3) Acres $= \frac{(\text{speed})(\text{width})(\text{efficiency})(\text{hrs})}{8.25}$

Solve equation 2 for speed;

4) Speed $= \frac{(375)(\text{horsepower})}{(\text{width})(\text{draft})}$

Substitute equation 4 into equation 3, yields

$$\text{acres} = \frac{(375)(\text{horsepower})(\text{efficiency})(\text{hours})}{(\text{draft})(8.25)}$$

$$\text{acres} = \frac{(375)(\text{hp})(\text{e})(\text{hrs})}{(\text{d})(8.25)}$$ [3/]

so, $\text{hrs} = \frac{(\text{acres})(8.25)(\text{d})}{(375)(\text{e})(\text{hp})}$

[3/] Using Farm Tractors and Machinery Efficiently, by Dr. Lee Erickson, Cooperative Extension Service, Montana State University, Bozeman, Bulletin 1229, Aug. 1980.

if equation 2 is solved for width;

$$\text{width} = \frac{(hp)(375)}{(d)(s)}$$

Hours of machine use and implement width can now be expressed as functions of horsepower. The last step is to define two new constants in order to simplify the equations.

Let:

$$K_1 = \frac{(375)(e)}{(8.25)(d)}$$

then,

$$5)\ \text{hrs} = \frac{\text{acres}}{(K_1)(hp)}$$

and let

$$K_2 = \frac{375}{(d)(s)}$$

then

$$6)\ \text{width} = (hp)(K_2)$$

Values for speed, efficiency, and draft were then exogenously assumed.[4/] Efficiency was assumed to be 80 percent, speed 5 mph, and draft, which was measured in lbs./ft. with speed in mph was assumed to be 600. These values are believed to be representative of conditions on a farm of this type.

[4/] Ibid.

Used Price and Repair Cost Functions

Used price and repair cost functions were obtained from the Agricultural Engineers Yearbook.

The repair functions were in the form:[5]

$$\frac{\text{Accumulated Repair and Maintenance Costs (ARM)}}{\text{New Price (NP)}} = C_1\left[\frac{Q}{1000}\right]^{C_2}$$

where:

C_1 and C_2 = constants

Q = hours of accumulated use.

If Q is now defined as hours of annual use and Y is defined as years of use, the equation becomes:

$$\frac{ARM}{NP} = C_1\left[\frac{Q \cdot Y}{1000}\right]^{C_2}$$

which reduces to:

$$ARM - NP \cdot C_1 \cdot \left[\frac{Q}{1000}\right]^{C_2} Y^{C_2}$$

Annual repair and maintenance cost (RM) is equal to the derivative of the accumulated repair and maintenance cost function with respect to Y (years of use):

$$\frac{\partial ARM}{\partial Y} = RM = NP \cdot C_1 \cdot \left[\frac{Q}{1000}\right]^{C_2} C_2 Y^{C_2 - 1}$$

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[5] Agricultural Engineers Yearbook, 1980-1981; 26th Ed. American Society of Agricultural Engineers, p. 253.

The actual functions obtained from the Agricultural Engineers Yearbook, for the tractor, chisel plow and grain drill are respectively:

$$\frac{ARM}{NP} = 0.0120 \left[\frac{Q}{1000}\right] 2.033$$

$$\frac{ARM}{NP} = 0.0103 \left[\frac{Q}{1000}\right] 1.40$$

$$\frac{ARM}{NP} = 0.0359 \left[\frac{Q}{1000}\right] 2.626$$

By simplifying as previously illustrated, and differentiating, the annual repair and maintenance cost functions for the tractor, chisel plow and grain drill become respectively:

$$RM = NP \cdot 0.024 \cdot \left[\frac{Q}{1000}\right] 2.033 \; {}_{Y} \; 1.033$$

$$RM = NP \cdot 0.0144 \left[\frac{Q}{1000}\right] 1.40 \; {}_{Y} \; 0.40$$

$$RM = NP \cdot 0.039 \left[\frac{Q}{1000}\right] 2.622 \; {}_{Y} \; 1.626$$

The used price functions given in the Agricultural Engineers Yearbook were in the form:[6/]

$$\frac{UP}{NP} = C_1(C_2)^n$$

where:

UP = used price of the machine

NP = new price of the machine

6/ Ibid.

C_1 and C_2 = constants

n = the number of years the machine was owned.

This equation simplifies to:

$$UP = NP(C_1)(C_2)^n.$$

The used price functions for the tractor, chisel plow and grain drill were given respectively as:

$$UPT = (NPT)(.68)(92^n)$$

$$UPP = (NPP)(.6)(885^n)$$

$$UPD = (NPD)(.6)(885^n)$$

Depreciation Functions

Depreciation is tax deductible, therefore, three methods of computing depreciation for tax purposes are considered: market, straight line and double declining balance.

Market Depreciation

Market depreciation equals the negative of the derivative of the used price function with respect to time (age). For example, the used price function of the tractor is:

1) $UPT = NPT(.68)(.92^n)$.

The partial derivative of this function with respect to time is:

$$\frac{\partial UPT}{\partial i} = .68(.92)^n(\ln.92)(NPT).$$

which is the rate of change in the value of the machine or the negative of the market depreciation. The total discounted depreciation (depT) during any given time period is:

$$depT = NPT\ [.32 - \int_o^n [.68\ (.92)^i (\ell n\ .92) e^{-r(1-T)}]\ di]$$

where:

T = tax rate

r(1-T) - after tax discount rate.

This integral can be solved in closed form since:

$$\int_o^n .68.(92)^i \ell n\ .92 e^{-r(1-T)i} di = .68\ \ell n\ .92 \int_o^n .92^i e^{-r(1-T)i} di$$

Let $(.92)^i = e^{\alpha}$, then:

$$= .68.\ell n92 \int_o^n e^{\alpha i} e^{-r(1-T)i} ei$$

$$= .68.\ell n92 [\frac{e^{[\alpha-r(1-T](n)}-1}{\alpha=r(1-T)}].$$

Therefore;

$$depT = NPT\{.32-(.68)(\ell n.92)[\frac{e^{[\alpha-4(1-T)](n)}-1}{\alpha-r(1-T)}]\}.$$

To calculate the tax benefits from using market depreciation, the marginal tax rate is multiplied by the depreciation taken:

Tax Benefits = (T)(dept) =

$$(T)(NPT)\{.32-(.68)(\ell n.92)\quad [\frac{e^{[\alpha-r(1-T)](n)}-1}{\alpha-r(1-T)}]\}$$

The equation for the tax benefits from using market depreciation, for both the plow and drill, is equal to:

Tax benefits = (T)(discounted depreciation) =

$$(T)(\text{New Price}) \quad \{.4 - (.6)(\ell n.885) \left[\frac{e^{[\alpha - r(1-T)(n)} - 1}{\alpha - r(1-T)}\right]\}$$

Depreciation recapture equals:

$$\text{dep. recapture} = T[UP-(NP - Dep.)]$$

where:

NP - Dep = book value

Dep = undiscounted total accumulated depreciation.

If market depreciation is taken, then the new price minus the used price (NP - UP), is equal to the book value or:

$$\text{dep. recapture} = T\ [UP-(NP - [NP - UP])]$$

$$= 0$$

Straight-Line Depreciation Method

Annual straight-line depreciation is:

$$= \frac{NP - .1(NP)}{IND}$$

where:

.1 (NP)= assumed salvage value

IND = the depreciable life of the asset. = the assumed value of 10.

The total discounted depreciation is equal to:

$$\text{depT} = D \cdot \left[\frac{1-e^{-r(1-T)IN}}{r(1-T)}\right]$$

where:

IN = the length of the ownership period or the depreciable life, whichever is shorter.

$\frac{1-e^{-r(1-T)IN}}{r(1-T)}$ = after-tax annuity discount factor.

The tax benefits from using straight-line depreciation are equal to (T)(dept) and the tax consequences of depreciation recapture are equal to (T) (UPT - BVT), where BVT is the book value of the machine

Double Declining Balance Depreciation Method

The book value of a machine in any given year using double declining balance depreciation is equal to:

$$BV(i) = BV(i-1) - BV(i-1)R$$

where:

R = rate of depreciation = $\frac{2}{IND}$

IND = depreciable life of the asset.

i = the year (age).

This a first-order difference equation which solves to:

$$BV(i) = (1-R)\,BV(i-1)$$

Observe the following equations:

$$BV(1) = (1-R)BV(0)$$

$$BV(2) = (1-R)BV(i) = (1-R)^2 BV(0)$$

.

.

.

$$BV(i) = (1-R)BV(i-1) = (1-R)^i BV(0)$$

The rate of depreciation will equal:

$$dep(i) = -\frac{\partial BV(i)}{\partial i} = -(1-R)^i\ [BV(0)]\ [\ell n(1-R)]$$

Let $\lambda = (1-R)$.

The rate of depreciation will then equal:

$$-\lambda^i BV(0)\ell n(\lambda).$$

The total discounted depreciation over a time period is equal to:

$$depT = \int_0^{IN} - \lambda^i [BV(0)][\ell n(\lambda)]e^{-r(1-T)i}di$$

To solve this integral in closed form, let $\lambda = e^\alpha$ and substitute:

$$depT = \int_0^{IN} (-e^{\alpha i})(BV(0)(\ell n(\lambda)(e^{-r(1-Ti})di$$

$$= (BV(0))(-\ell n(\lambda)\int_0^{IN}(e^{\alpha i})(e^{-r(1-T)i})di$$

$$=(BV(0))(-\ell n(\lambda)\int_0^{IN}(e^{[\alpha-r(1-T)]i})di$$

$$= (BV(0))(-\ell n(\lambda)[\frac{e^{[\alpha-r(1-T)]N}-1}{\alpha-r(1-T)}]$$

where:

BV(0) = NPT

The tax benefits from this type of depreciation are (T)(depT). The book value of an asset under this method of depreciation is (NPT-total discounted depreciation) and the tax consequences of depreciation recapture are the same as in the previous section.

Investment Credit

Investment credit may be taken on machinery and applied as a direct reduction of tax liabilities. A credit of 10 percent of the purchase price of a qualified investment (farm machinery used in the problem qualify) is allowed. For 1981 the regular tax credit applies against the first $25,000 of tax liabilities, plus 80 percent of the tax liability exceeding $25,000, on a joint return.

The amount of qualified investment is the sum of the basis of new "Sec. 38 property"[8/] and up to $100,000 of the cost of used "Sec. 38 property." The cost or basis that qualifies is limited if the property has a useful life of less than seven years. Only 2/3 of the cost is taken into account if the useful life is at least five and less than seven years. Only 1/3 is taken into account where the useful life is at least three and less than five years. No credit

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[8/] See 1981 U.S. Masters Tax Guide, for a description of Section 38 property.

is allowed if the useful life is less than three years.

If property with a useful life of at least seven years is disposed of before seven years then the investment credit allowed will be the same as the above equipment with the additional investment credit recaptured. For example, if a tractor was sold after four years, and the entire amount of investment credit was taken when it was purchased, 2/3 of the investment credit taken is recaptured. This recaptured investment credit is added to the tax liability.

The Discount Rate

Only real discount rates are considered in this model.[8/] Van Horne [1974] defines the real discount rate to be (r-f) under continuous times; where r = the nominal interest rate and f = the inflation rate. Since nominal interest is taxed, the real after-tax discount rate is r (1-T)-f, where r, f and T are the nominal interest rate, inflation rate and marginal tax rate, respectively. If only real interest rates were taxed the real after tax discount rate is (r-f) (1-T).

The data in Tables 4.3 and 4.4 was used to estimate an appropriate discount rate. Different methods of determining appropriate discount

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[8/] Nominal rates are used in specific instances, for example, to determine depreciation recapture; these instances are fully explained when they occur.

Table 4.3. Measurement of the Rate of Inflation.

	GNP Implicit Price Deflator (1)	Consumer Price Index (2)	Prices Paid by Farmers (3)	Prices Paid by Farmers for Tractors and Self-Propelled Machines (4)
1972	4.2	3.29	5.9	4.9
1973	5.7	6.22	15.2	7.03
1974	8.7	10.97	13.88	17.5
1975	9.3	9.14	9.75	21.11
1976	5.2	5.77	6.66	11.28
1977	5.8	12.32	5.2	9.67
1978	7.3	7.66	8.41	8.8
1979	8.5	11.26	14.16	11.58
1980	9.0	12.62*	12.00	11.76

*1980 figured from November 1979 to November 1980.

Table 4.4. Measurement of Nominal Interest Rates.

	Prime Rate Charged by Banks (5)	PCA Avg. Cost of Loans (6)
1972	5.25	7.02
1973	8.03	8.09
1974	10.81	9.43
1975	7.86	8.91
1976	6.84	8.24
1977	6.83	7.88
1978	9.06	8.83
1979	12.67	10.71
1980	15.27	(Not available)

Table 4.5. Real Interest Rates

	Col.5-Col.1	Col.5-Col.2	Col.5-Col.3	Col.5-Col.4	Col.6-Col.1	Col.6-Col.2	Col.6-Col.3	Col.6-Col.4
1972	1.05	1.96	-.65	.35	2.82	3.73	1.12	2.12
1973	2.33	1.81	-7.17	1.00	2.39	1.87	-7.11	1.06
1974	2.11	-.16	-3.07	-6.7	.73	-1.54	-4.45	-8.07
1975	-1.44	-1.28	-1.89	-13.25	-.39	-.23	-.84	-12.2
1976	1.64	1.07	.18	-4.44	3.04	2.47	1.58	-3.04
1977	1.03	-5.49	1.63	-2.84	2.08	-4.44	2.68	-1.79
1978	1.76	1.4	.65	.26	1.53	1.17	.42	.03
1979	4.17	1.41	-1.51	1.09	2.21	-.55	-3.47	-.87
1980	6.27	2.65	3.27	3.51	—	—	—	—
Avg.	2.10	.37	-.95	-2.33	1.80	.31	-1.26	-2.84

Footnotes for Tables 4.3, 4.4, and 4.5.

[1] From Table B-5, %Δ from preceding period (GNP) Implicit Price Deflator, page 239, Economic Report of the President, Jan. 1981.

[2] From Table B-50, first column, page 289, Economic Report of the President, Jan. 1981.

[3] From Table B-95, column #4, page 340, Economic Report of the President, Jan. 1980.

[4] From Table B-95, column #6, Economic Report of the President, Jan. 1980.

[5] From Table B-65, column #9, Economic Report of the President, Jan. 1980.

[6] From Table 678, column #3, Ag. Statistics, 1980, USDA.

rates were then analyzed and summarized in Table 4.5. The prime rate charged by banks and the average yearly Production Credit Union cost of loans were used as measures of the nominal interest rate. The GNP Implicit Price Deflator, Consumer Price Index, Prices Paid by Farmers, and Prices Paid by Farmers for Tractors and Self-propelled Machines were used as measures of the inflation rate. The real rate was then determined by subtracting the various mean inflation measures from the mean nominal interest rates.

The values of tables 4.3 and 4.4 are the percentage changes from the preceding period. For example, the inflation rate for 1973, column 2, table 4.1 is equal to $\frac{\text{CPI for 1973 - CPI for 1972}}{\text{CPI for 1972}}$. Using the numerical values, the rate of inflation equals 6.22% $= \frac{133.1 - 125.3}{125.3}$.

From the preceding analysis, the nine year average of real discount rates ranged from -2.84% to 2.10% depending on the method used to calculate them. As can be determined from this analysis, a real discount of 2% should approximate the value of what the real rate actually is. Real rates of 5% and 8% as well as 2%, are analyzed in this model for those who believe the real rate should actually be higher. When inflation is considered, nominal interest rates of 12% and 18% and an inflation rate of 10% are used.

The Simulation Model

The simulation model as described in the beginning of this section simulates a hypothetical dryland wheat farm located in South-central Montana. The farm consists of 2000 tillable acres of which 1000 are cropped every year and 1000 fallowed on an alternating basis. It was assumed that fallowing three times before seeding would be sufficient. The model can be altered for more or less fallowing.

The equipment used on this farm consists of a tractor, chisel plow, and grain drill (equipment was kept at a minimum in order to keep the model as simple as possible). The new price of the tractor and plow are determined endogenously in the model as a function of horsepower. These price functions were developed earlier in this chapter.

Fuel inputs are not considered because it is assumed that fuel consumption per acre is constant regardless of machine size.

Labor hours are annual inputs and are included in the analysis so that the substitution effect between annual inputs and inputs with a finite life (machinery) of more than one production period can be measured. These effects will be measured when tax rates and depreciation methods are changed. and when inflation is considered. Labor hours and horsepower are assumed to be both economic and technical substituted.

The effects on the labor to horsepower ratio is analyzed when different wage rates, interest rates, inflation rates, tax rates and depreciation methods are incorporated. The results of this analysis are summarized in the following chapter.

Chapter 5

SUMMARY AND CONCLUSIONS

The least cost combination of machinery and labor for the simulated farm are presented in this chapter. The influence of different wage rates, tax rates, discount rates and depreciation methods are featured as well as inflation, investment credits and social security tax influence.

Table 5-1 presents the least cost combination of machinery and labor when taxes, inflation, depreciation and investment credits were not taken into consideration. Table 5-1 also shows the optimal size tractor and optimal holding periods for the machinery given discount rates and wage rates. When the price of labor (wage rate) is increased, less labor and more horsepower is used. This is due to the substitution effect described in Chapter 1. When the price of horsepower is decreased, more horsepower and less labor time is used. Increasing the discount rate, in effect, increases the cost of horsepower which causes less horsepower and more labor to be used in the least cost combination.

Tables 5-2 through 5-6 show the results of the search routine when tax rates and depreciation methods are taken into consideration. Depreciation method 1 denotes actual market depreciation, method 2 denotes straight-line depreciation and method 3 denotes double declining

Table 5.1. Least Cost Combinations of Horsepower and Labor when Wage Rates and Discount Rates are Changed.

Row	PL	Rint	HP	N	M	IQ	TC	HRS
1	4	.02	100	6	15	5	10475	889
2	4	.05	93	6	18	3	11402	956
3	4	.08	86	6	18	3	12307	1034
4	8	.02	125	8	15	3	11197	711
5	8	.05	116	8	18	3	12223	767
6	8	.08	101	7	22	5	13237	880

* The following footnotes apply to Tables 5.1 through 5.6.

1) Some of the entries contain ranges of horsepower and total costs, because the total cost functions was very flat relative to horsepower.

2) The column headings stand for:

   PL = wage rate (price of labor) (in $/hour)
   Tax Rate = tax rate
   Disc. Rate = discount rate, nominal discount rate for Tables 5.3 and 5.6.
   Dep. Meth. = depreciation method.
   HP = horsepower
   N = holding period for the tractor
   M = holding period for the plow
   IQ = holding period for the drill
   TC = total annual cost of the four variables.
   HRS = amount of labor hours needed for field work.

3) The holding period for the drill never exceeded 3 years, this was caused by the large exponential constant on the repair function, described in Chapter 4, which caused the repair costs to increase rapidly.

balance depreciation.[1/] The equipment was depreciated over a ten-year period for methods 2 and 3 and the rate of depreciation for method 3 (20%) was twice the rate of method 2.

Method 2 (straight-line) is the slowest depreciation method while method 3 (double declining balance) is the fastest. One would expect the prices of horsepower to be cheaper for faster depreciation methods since the faster the depreciation method, the greater the present value of the tax depreciation benefits. This is shown by comparing the values for horsepower for the different depreciation methods while holding everything else constant. The highest horsepower to labor ratios occur when the fastest depreciation method (3) is taken, and the lowest horsepower to labor ratios are found when the slowest depreciation method (2) is taken.

Another factor indirectly affecting the price of horsepower is the tax rate. As shown in tables 5.2 through 5.9, when the tax rate increases, everything else held constant, the amount of horsepower used increases, and the amount of labor used decreases. This is caused by the lowering of an individual's after-tax discount rate when the tax rate is increased. The formula for the after-tax discount rate is: $r(1-T)$; where r = discount rate and T = the tax rate. Thus, when T increases the after-tax discount rate decreases so the

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1/ The derivation and development of these depreciation methods are fully explained in Chapter 4.

Table 5.2. The Effect of Tax Rates and Depreciation Methods on the Least Cost Combinations of Horsepower and Labor.

Runs	PL	Tax Rate	Disc Rate	Dep Meth	HP	N	M	IQ	TC	HRS
1	4	.25	.02	1	112-27	7-9	14	3	8016-19	700-94
2	4	.25	.02	2	100	6	17	3	8297	889
3	4	.25	.02	3	112-128	7-9	14	3	7977	700-94
4	4	.25	.05	1	104-11	7-8	17	3	8707	801-55
5	4	.25	.05	2	94-101	67	20	3	8880-900	880-945
6	4	.25	.05	3	105-12	7-8	17	3	8609-15	700-847
7	4	.25	.08	1	97-103	7-8	20	3	9374-84	863-916
8	4	.25	.08	2	89	6	23	3	9455	999
9	4	.25	.08	3	98	7	20	3	9227	907
10	4	.50	.02	1	135-149	9-11	14	3	5552-4	597-659
11	4	.50	.02	2	93	6	25	3	6801	956
12	4	.50	.02	3	136-157	9-12	14	3	5499-501	566-654
13	4	.50	.05	1	119-32	8-10	16	3	6017-20	673-746
14	4	.50	.05	2	83	5	27	3	7077	1071
15	4	.50	.05	3	120-46	8-12	16	3	5893-99	608-741
16	4	.50	.08	1	111-23	8-10	19	3	6462-66	723-801
17	4	.50	.08	2	81	5	31	3	7359	1098
18	4	.50	.08	3	107-37	7-12	19	3	6263-80	654-831
19	8	.25	.02	1	154-67	11-13	14	3	8653-57	533-577
20	8	.25	.02	2	127	8	17	3	9052	700
21	8	.25	.02	3	155-68	11-13	14	3	8609-10	529-573
22	8	.25	.05	1	135-47	10-12	17	3	9431	605-658
23	8	.25	.05	2	112-56	7-15	20	3	9714-35	570-794
24	8	.25	.05	3	142-53	11-13	17	3	9330-2	580-626
25	8	.25	.08	1	120-30	9-11	20	3	10177-82	684-741
26	8	.25	.08	2	106	7	23	3	10381	839
27	8	.25	.08	3	121-36	9-12	20	3	10036-43	654-736
28	8	.50	.02	1	193-99	15-16	14	3	6078-9	447-461
29	8	.50	.02	2	220	27	25	3	7173	404
30	8	.50	.02	3	200-01	16	14	3	6006	445
31	8	.50	.05	1	177-87	15-17	16	3	6600-1	476-502
32	8	.50	.05	2	196-200	27	27	3	7634	445-453
33	8	.50	.05	3	186-96	16-18	16	3	6431	453-477
34	8	.50	.08	1	159-68	16-19	19	3	7104-5	529-559
35	8	.50	.08	2	174-77	25	31	3	8095	502-511
36	8	.50	.08	3	173-181	18	19	3	6854	491-514

Table 5.3. The Effect of Tax Rates on the Least Cost Horsepower to Labor Ratio.*

	Discount Rate	Tax Rate	
		.25	.50
Wage Rate = $4	.02	$\frac{120}{747}$	$\frac{147}{610}$
	.05	$\frac{109}{774}$	$\frac{133}{675}$
	.08	$\frac{98}{907}$	$\frac{122}{743}$
Wage Rate = $8	.02	$\frac{162}{551}$	$\frac{206}{445}$
	.05	$\frac{148}{603}$	$\frac{191}{465}$
	.08	$\frac{129}{695}$	$\frac{179}{503}$

*The values in the table represent HP/HRS ratios, depreciation method 3 analyzed.

Table 5.4. The Effect of Inflation on the Least Cost Combinations of Horsepower and Labor.

Runs	PL	Tax Rate	Nominal Disc Rate	Dep Meth	HP	N	M	IQ	TC	HRS
37	4	.25	.12	1	121-39	7-9	13	3	7368-76	640-736
38	4	.25	.12	2	106	6	15	3	7783	839
39	4	.25	.12	3	119-52	7-12	13	3	7576-82	529-745
40	4	.25	.18	1	104-10	7-8	18	3	8764-75	806-855
41	4	.25	.18	2	95	6	20	3	8949	946
42	4	.25	.18	3	103-10	7-8	18	3	8826-401	808-63
43	4	.50	.12	1	162-73	9	9	3	4262-4	491-514
44*	4	.50	.12	2	212-46	20-3	19	3	5546-75	361-419
45	4	.50	.12	3	193-203	13-14	12	3	9837-42	438-61
46	4	.50	.18	1	126-34	7-8	13	3	5266-8	644-706
47	4	.50	.18	2	96	6	22	3	6522	926
48	4	.50	.18	3	187	16	16	3	5596	476
49	8	.25	.12	1	161-76	10-12	12	3	7973-4	505-52
50	8	.25	.12	2	202	17	16	3	8363	440
51	8	.25	.12	3	181-96	13-15	13	3	8113-5	454-91
52	8	.25	.18	1	128-40	9-11	17	3	9523-4	635-95
53	8	.25	.18	2	111-60	7-16	20	3	9826-30	556-801
54	8	.25	.18	3	128-56	9-14	18	3	9551-92	570-695
55	8	.50	.12	1	207	10	9	3	4731	430
56*	8	.50	.12	2	285	27	16	3	5860	312
57*	8	.50	.12	3	243	17	13	3	5216	366
58	8	.50	.18	1	176-83	11-12	12	3	5846-7	486-505
59*	8	.50	.18	2	250	29	23	3	6705	355
60	8	.50	.18	3	233	20	16	3	6017	382

Table 5.5. The Effect of Tax Rates on the Least Cost Horsepower to Labor Ratio*

	Nominal Discount Rate	Tax Rate	
		.25	.50
Wage Rate = $4	.12	$\frac{136}{637}$	$\frac{198}{450}$
	.18	$\frac{107}{836}$	$\frac{187}{476}$
Wage Rate = $8	.12	$\frac{189}{473}$	$\frac{243}{366}$
	.18	$\frac{142}{633}$	$\frac{233}{382}$

*The values in the table represent HP/HRS ratios, depreciation method 3 analyzed with an inflation rate of 10%.

Table 5.6 The Effect of Inflation on the Least Cost Horsepower to Labor Ratio Continued.*

	Nominal Discount Rate (r-f)	Inflation Rate	
		0	.10
Wage Rate = $4	.12	$\frac{147}{610}$	$\frac{198}{450}$
	.18	$\frac{122}{743}$	$\frac{187}{476}$
Wage Rate = $8	.12	$\frac{200}{445}$	$\frac{243}{366}$
	.18	$\frac{179}{503}$	$\frac{233}{382}$

*The values in the table represent HP/HRS ratios, under depreciation method 3 a 50% tax rate.

price of horsepower (machinery) is decreased which causes an increase in the horsepower to labor ratio. This effect is better shown in Tables 5.3 and 5.3.

The effects of inflation are analyzed in Table 5.4 and 5.8. An inflation rate of 10 percent was assumed, and nominal interest rates of 12 and 18 percent were analyzed. With these assumptions, the real discount rates are 2 and 8 percent respectively, since:

r–f = real discount rate,

where:

r = nominal interest rate,

f = inflation rate.

Table 5.6 illustrates the effects of inflation. By comparing the values in the table horizontally (holding everything but inflation constant), one can observe the effects of inflation on the least cost combination of horsepower and labor. When the inflation rate is increased from 0 to 10 percent, the amount of horsepower used increases dramatically (147 to 198, 122 to 187, 200 to 243, and 179 to 233). This is caused by the effect inflation has on the discount rate. As discussed in Chapter 4, an individual's real after-tax discount rate is: r(1-T)–f, where; T = the tax rate. When f increases, the individual's discount rate decreases. This causes the same effect on the horsepower to labor ratio as described in the beginning of this chapter.

Inflation increases the cost of machinery. This occurs because inflation increases depreciation recapture and erodes the value of tax deductible depreciation. Depreciation erosion apparently outweighs inflation caused reduction in the real discount rate since the amount of horsepower employed increases when the rate of inflation increases.

Comparison of the entries in Table 5.7 with Table 5.2 and 5.8 with Table 5.4 illustrate the effect of investment credit in optimal machinery holding periods and machinery size-labor relationships. When the holding period of the tractor was less than seven years it was increased to seven years and the amount of horsepower used was increased. For the larger tractors (greater horsepower) with holding periods of greater than seven years, the size of the tractors was sometimes, but not always, decreased and the holding period shortened. The same effects happened to the plow. In all cases, total annual cost was decreased. This was caused by the incidence of investment credit recapture as explained in Chapter 4. Investment credit had a more pronounced effect on the holding periods of the machines than on the horsepower to labor ratios.

Table 5.9 shows the effects of employers' social security tax. This caused the wage rate paid by the farmer to increase 6.65 percent. Thus, this tax causes a direct increase in the price of labor and because of the substitution effect, horsepower (machinery).

Table 5.7. The Effect of Investment Credit on the Least Cost Combination of Horsepower and Labor.

Runs	PL	Tax Rate	Disc Rate	Dep Meth	HP	N	M	IQ	TC	HRS
1	4	.25	.02	1	121	7	11	3	7348	735
2	4	.25	.02	2	115	7	14	3	7626	773
3	4	.25	.02	3	122	7	11	3	7223	729
4	4	.25	.05	1	112	7	13	3	7442	794
5	4	.25	.05	2	108	7	17	3	8183	823
6	4	.25	.05	3	113	7	14	3	7883	787
7	4	.25	.08	1	104	7	16	3	8590	855
8	4	.25	.08	2	101	7	20	3	8737	881
9	4	.25	.08	3	105	7	16	3	8437	847
10	4	.50	.02	1	135	7	9	3	4498	659
11	4	.50	.02	2	106	7	22	3	6183	839
12	4	.50	.02	3	136	7	9	3	4814	654
13	4	.50	.05	1	125	7	11	3	5256	711
14	4	.50	.05	2	102	7	24	3	6476	872
15	4	.50	.05	3	127	7	12	3	5157	700
16	4	.50	.08	1	116	7	13	3	5719	763
17	4	.50	.08	2	98	7	27	3	6771	907
18	4	.50	.08	3	120	7	14	3	5470	741
19	8	.25	.02	1	151	9	11	3	7958	589
20	8	.25	.02	2	168	13	14	3	8394	529
21	8	.25	.02	3	151	9	11	3	7915	589
22	8	.25	.05	1	132	8	13	3	8694	674
23	8	.25	.05	2	154	13	17	3	9090	577
24	8	.25	.05	3	133	8	14	3	8591	668
25	8	.25	.08	1	122	8	16	3	9425	729
26	8	.25	.08	2	113	7	20	3	9571	787
27	8	.25	.08	3	118	7	16	3	9235	753
28	8	.50	.02	1	172	8	9	3	5402	516
29	8	.50	.02	2	215	24	22	3	6744	414
30	8	.50	.02	3	187	11	9	3	5344	475
31	8	.50	.05	1	144-57	7-9	11	3	5916-70	566-617
32	8	.50	.05	2	198	24	24	3	7164	449
33	8	.50	.05	3	179	12	12	3	5781	497
34	8	.50	.08	1	146	9	13	3	6403	609
35	8	.50	.08	2	179	23	27	3	7586	497
36	8	.50	.08	3	172	13	14	3	6404	516

Table 5.8. The Effect of Investment Credit and Inflation on the Least Cost Combination of Horsepower and Labor.*

Runs	PL	Tax Rate	Nominal Disc Rate	Dep Meth	HP	N	M	IQ	TC	HRS
37	4	.25	.12	1	132	7	9	3	6636	674
38	4	.25	.12	2	123	7	13	3	7087	723
39	4	.25	.12	3	128	7	11	3	6975	695
40	4	.25	.18	1	111	7	13	3	8048	801
41	4	.25	.18	2	107	7	18	3	8277	831
42	4	.25	.18	3	110	7	15	3	8111	808
43	4	.50	.12	1	184	7	7	3	3442	483
44	4	.50	.12	2	290	21	17	3	5220	371
45	4	.50	.12	3	154	7	7	3	4216	577
46	4	.50	.18	1	144	7	7	3	4489	618
47	4	.50	.18	2	110	7	20	3	5936	608
48	4	.50	.18	3	171	12	12	3	5059	519
49	8	.25	.12	1	148-56	7-8	9	3	7272	569-601
50	8	.25	.12	2	197	15	13	3	7852	451
51	8	.25	.12	3	185	12	11	3	7527	480
52	8	.25	.18	1	124	7	13	3	8808	717
53	8	.25	.18	2	157	14	18	3	9232	566
54	8	.25	.18	3	148	11	15	3	8894	601
55	8	.50	.12	1	215	7	7	3	3887	414
56	8	.50	.12	2	300	26	17	3	5541	297
57	8	.50	.12	3	246	14	7	3	4704	362
58	8	.50	.18	1	167	7	7	3	5062	533
59	8	.50	.18	2	255	27	20	3	6335	349
60	8	.50	.18	3	214	15	12	3	5523	416

*Inflation Rate = .10

Table 5.9, The Effect of Adding Employers Social Security Tax to Base Wage Rate on the Least Cost Combination of Horsepower and Labor.

Runs	PL	Tax Rate	Disc Rate	Dep Meth	HP	N	M	IQ	TC	HRS
61	4.27	.25	.02	1	113-128	7-9	14	3	8063-72	700-787
62	4.27	.25	.02	2	101	6	17	3	8357	880
63	4.27	.25	.02	3	113-129	7-9	14	3	8024-31	689-787
64	4.27	.25	.05	3	106-13	7-8	17	3	8666-9	787-839
65	4.27	.25	.08	3	99	7	20	3	9287	900
66	4.27	.50	.02	3	137-58	9-12	14	3	5539	563-654
67	4.27	.50	.05	3	122-147	8-12	16	3	5934-8	605-729
68	4.27	.50	.08	3	109-138	7-12	19	3	6319-23	644-816
69	8.53	.25	.02	3	156-176	11-14	14	3	8684-6	505-570
70	8.53	.25	.05	3	149-159	12-13	17	3	9408-9	559-597
71	8.53	.25	.08	3	122-42	13-20	20	3	10125-39	626-729
72	8.53	.50	.02	3	203	14-16	14	3	6064	438
73	8.53	.50	.05	3	195-198	16-18	16	3	6491	449-461
74	8.53	.50	.08	3	179-88	20	19	3	6919	473-496

Table 5.9 shows the effects of employers' social security tax. This caused the wage rate paid by the farmer to increase 6.65 percent. Thus, this tax causes a direct increase in the price of labor and because of the substitution effect, horsepower (machinery) is substituted for labor, causing the horsepower to labor ratio to increase.

Summary

This study explored the substitution effect induced by taxes between machinery and labor under a variety of circumstances. A scale effect would also be induced by taxes. The scale effect was not investigated.

From Table 5.1 with a wage rate of $4.00 and a discount rate of .02, the horsepower to labor ratio is $\frac{100}{889}$. Under a 50 percent tax rate, nominal interest rate of 12 percent, inflation rate of 10 percent, and using market depreciation, the ratio changes to $\frac{184}{483}$ (from Table 5.8). So it may be concluded that the effects of the factors explored by this thesis are substantial.

Direct changes in the discount rate affected the least cost combination of inputs to the greatest extent. This implies that factors which affected the discount rate had the next greatest effect. These factors were the tax rate and inflation rate. Social security taxes increased the horsepower to labor ratio because it directly increased the wage rate. It did not cause a large change in the ratios because it only changed the wage rate by a small amount. The addition of investment credit to the analysis had a significant effect on the holding periods of the machines by biasing them towards seven years. This was due to the effects of investment credit recapture (discussed at length in Chapter 4). It altered the least cost combination of machinery and labor by a small amount.

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APPENDIX

SEARCH ROUTINE USED ON THE PROJECT

```
C *** RUN WITH DEPREC. * AND * I.C. ***
C *** MODIFIED BINARY SEARCH ***
      CALLUERSET(0,LEVOLD)
C     COMMON K1,K2,ALPHA,ACRES,BETA,RINT,HP,N,M,IQ,NPT,NPP,NPD
C * INPUT CONSTANTS **
C -------------- NOTE: SET METHIC = 1 ------------------------
      METHIC=1
      REAL K1,K2,N,M,IQ,NPD,NPP,NPT
      REAL T,DEP,DEPT,UPT
      REAL IN,BVT,BVP,BVD,IM,IQ1
      REAL K3,INVCT,INVCTR,INVCP,INVCPR,INVCD,INVCDR
      REAL HOURS,F
      COMMON K1,K2,ALPHA,ACRES,BETA,RINT,HP,N,M,IQ,NPT,NPP,NPD,F
      COMMON T,DEP,DEPT,UPT,DEP1,DEPP,DEP2,DEPD
      COMMON HOURS
      COMMON IN,BVT,BVP,BVD,IM,IQ1
      COMMON K3,INVCT,INVCTR,INVCP,INVCPR,INVCD,INVCDR
      UPT=UPP=UPD=0
      IN=BVT=BVP=BVD=IM=IQ1=0
      DEP=DEPT=UPT=DEP1=DEPP=DEP2=DEPD=0
      K3=INVCT=INVCTR=INVCP=INVCPR=INVCD=INVCDR=0
      EXTERNAL F1,F2,F3
      HP=100;ACRES=1000.;ALPHA=4;BETA=3.;F=.10
      K1=.045;K2=.125
      NPD=8000.
      N=M=IQ=3.5
      INTEGER PREVN,PREVM,PRIORN,PRIORM
      IBINARY=LOOP30=0
C *** CHANGE TO READ FROM A FILE FOR PRODUCTION RUN ***
      READ(105,100,END=99)PL,T,RINT,METHD
 100  FORMAT(3F10.0,I2)
      NAGGN=NAGGHP=NAGGM=0
      DIMENSION TCOST(3)
C     REAL N,M,IQ,HP
C - INITIAL STARTING POINT -
      ITER=NAGG=HPCON=MCON=MAGG=0
      NCON=1
      HP=HPORIG=210.
      N=8;M=18;IQ=6
      IQORIG=IQ;IQLL=IQLLO=1;IQUL=IQULO=15
      NORIG=N;MORIG=M
      NUL=NULO=30;NLL=NLLO=1;HPUL=HPULO=400.;HPLL=HPLLO=50.;
     *MLL=MLLO=1;MUL=MULO=40
```

```
C      OUTPUT ' ENTER: THE NUMBER OF DESIRED DEPRECIATION METHOD'
C      OUTPUT '        1 - MARKET'
C      OUTPUT '        2 - STRAIGHT LINE'
C      OUTPUT '        3 - DOUBLE DECLINING BALANCE'
C      OUTPUT '        4 - NONE'
C      INPUT METHD
 10    ITER=ITER+1
C *********** I.C. SEARCH ROUTINE LOOPERS ***
       IF(HPCON.EQ.1)IBINARY=1
       IF(HPCON.EQ.0)IBINARY=0
       IF(NCON.EQ.1.OR.MCON.EQ.1.OR.DCON.EQ.1)LOOP30=LOOP30+1
       IF(NCON.EQ.1)N=LOOP30
       IF(MCON.EQ.1)M=LOOP30
       IF(DCON.EQ.1)IQ=LOOP30
C ************** EON ***
       IF(ITER.EQ.1)NAGG=NAGG+1;MAGG=MAGG+1
C      IF(ITER.EQ.2.AND.NCON.EQ.1)N=NORIG+1
C      IF(ITER.EQ.3.AND.NCON.EQ.1)N=NORIG-1

       IF(ITER.EQ.2.AND.HPCON.EQ.1)HP=HPORIG+1
       IF(ITER.EQ.3.AND.HPCON.EQ.1)HP=HPORIG-1.
C      IF(ITER.EQ.2.AND.MCON.EQ.1)M=MORIG+1.
C      IF(ITER.EQ.3.AND.MCON.EQ.1)M=MORIG-1.
C      IF(ITER.EQ.2.AND.DCON.EQ.1)IQ=IQORIG+1
C      IF(ITER.EQ.3.AND.DCON.EQ.1)IQ=IQORIG-1
C ----- COMPUTE MACHINERY COSTS ----
C      OUTPUT ' ENTER: 1 - TO TAKE INVESTMENT CREDIT'
C      OUTPUT '        2 - TO IGNORE INVESTMENT CREDIT'
C      INPUT METHIC
C10    OUTPUT ' ENTER: HP, TRACTOR YRS, PLOW YRS, DRILL YRS'
C      INPUT HP,N,M,IQ
C -- TRACTOR SECTION
       NPT=10483. + 260*HP
       IN=N
       IF(IN.GT.10)IN=10
       IM=M
       IF(IM.GT.10)IM=10
       IQ1=IQ
       IF(IQ1.GT.10)IQ1=10
C ******* CALL IMSL ROUTINE ****
       REPAIRT=DCADRE(F1,0.0,N,0.0,.0001,ERROR,IER)
       AFP=(RINT*(1-T)-F)/(1.0-EXP(-(RINT*(1-T)-F)*M))
       AFT=(RINT*(1-T)-F)/(1.0-EXP(-(RINT*(1-T)-F)*N))
```

```
      AFD=(RINT*(1-T)-F)/(1.0-EXP(-(RINT*(1-T)-F)*IQ))
      UPT=(NPT*.68*.92**N)*EXP(-(RINT*(1-T)-F)*N)
      IF(METHD.EQ.4)GOTO 501
      IF(METHD.EQ.2)GOTO 504
      IF(METHD.EQ.3)GOTO 505
      IF(METHD.NE.1)OUTPUT ' ERROR IN DEPRECIATION METHOD CODE';GOTO 99
C******MARKET DEPRICIATION SECTION
      DEP=NPT*.68*.08338*((EXP((-.08338-(RINT*(1-T)))
     &*N)-1)/(-.08338-(RINT*(1-T))))
      DEPT=DEP+.32*NPT
      BVT=UPT
      GOTO 501
C******STRAIGHT-LINE DEPRECIATION
 504  DEP=(NPT-(.1*NPT))/10
      DEPT=DEP*((1-EXP(-RINT*(1-T)*IN))/RINT*(1-T))
      BVT=NPT-(IN*DEP)
      GOTO 501
C******DOUBLE DECLINING BALANCE
 505  DEPT=NPT*.22314*((EXP((-.22314-(RINT*(1-T)))
     &*IN)-1)/(-.22314-(RINT*(1-T))))
      BVT=NPT*(.8)**IN
 501  IF(METHIC.NE.1)GOTO 502
C******INVESTMENTCCREDIT COMPUTATION
      IF(N.GE.0.AND.N.LT.3)K3=1
      IF(N.GE.3.AND.N.LT.5)K3=.67
      IF(N.GE.5.AND.N.LT.7)K3=.34
      IF(N.GE.7)K3=0
      INVCT=.1*NPT
      INVCTR=K3*INVCT
C     TC=(NPT+REPAIRT-UPT-(T*DEPT))*AFT
 502  TC=(NPT+((1-T)*REPAIRT)-UPT-(T*DEPT)+(T*(UPT-BVT))
     &-INVCT+INVCTR)*AFT
C     OUTPUT UPT,DEPT,BVT,INVCT,INVCTR
C --  PLOW SECTION ---
      NPP=1343.+299*HP*K2
C ********** CALL IMSL ROUTINE ******
      REPAIRP=DCADRE(F2,0.0,M,0.0,.0001,ERROR,IER)
      UPP=(NPP*.6*.885**M)*EXP(-(RINT*(1-T)-F)*M)
      IF(METHD.EQ.4)GOTO 401
      IF(METHD.EQ.2)GOTO 404
      IF(METHD.EQ.3)GOTO 405
      IF(METHD.NE.1)OUTPUT ' ERROR IN DEPRECIATION';GOTO 99
```