Synthesis techniques useful in the generation of precise microwave frequencies
by Kenneth Eugene Marcotte

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in 4 Electrical Engineering
Montana State University

Abstract:
The general principles of direct and indirect methods of frequency synthesis are described. Signal operations of frequency translation, multiplication and division are analyzed and degradation of the input signal stability is discussed.

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2. The analysis of a broadband frequency divider using a Step Recovery diode is presented. The divider executes subharmonic oscillations of order 1/2 and has an operational bandwidth greater than an octave in frequency. Graphs showing performance characteristics are presented.

3. A ring modulator is described using broadband transformer coupling and matched Hot Carrier diodes to perform frequency translation through 1 GHz.

Experimental data presented for the VHF and lower UHF frequency range appears to confirm the analyses of the various signal operations under investigation.

SYNTHESIS TECHNIQUES USEFUL IN THE GENERATION OF
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Approved:

Paul E. Uhlrich
Head, Major Department

Donald K Weaver Jr
Chairman, Examining Committee

K Goering
Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

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ABSTRACT

The general principles of direct and indirect methods of frequency synthesis are described. Signal operations of frequency translation, multiplication and division are analyzed and degradation of the input signal stability is discussed.

Synthesis techniques which have application in the VHF and UHF frequency ranges under investigation are presented as follows:

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1.0 STATEMENT OF THE PROBLEM.

Frequency synthesis is the generation of sinusoidal rf signals of a desired frequency from a frequency standard in such a manner that the accuracy and stability of the derived frequency is not seriously degraded from that of the standard. Modern communication systems and precision laboratory measurement techniques in many different fields require signal sources which possess this frequency precision and stability. Electronic circuits and system techniques for use in solving the specific frequency synthesis requirement are fairly well known for the frequency range of a few cycles per second to several hundred megacycles per second [1-7]. This is not the case for the lower UHF and microwave frequencies. This study treats several synthesis techniques which have direct application in the VHF, UHF, and microwave frequency ranges.

This thesis is divided into six chapters. The statement of the problem and a discussion of techniques to be investigated for application in frequency synthesis systems are presented in Chapter 1.

The system concept of a frequency synthesizer is presented in Chapter 2. The significant differences in the overall system operation are discussed with the aid of several block diagrams of typical direct and indirect synthesis methods.

In Chapter 3 signal operations, which are used in either the direct or indirect synthesis methods, are analyzed. These operations are frequency translation, multiplication and division. The purpose of this chapter is to investigate the input signal stability degradation in the output signals with reference to input as the signal operations are performed.

The basic operating principles and analysis of techniques of performing a desired signal operation are presented in Chapter 4. Many authors have described the various performance characteristics of several of the techniques under investigation, but no one of these has treated the data presentation in a manner with sufficient depth for the general design situation. In these cases theoretical computations have been carried out and the results presented in a manner which is felt presents a much broader scope. These presentations allow performance prediction for wide ranges for variation of circuit parameters, of operating characteristics and of input or output operating frequencies. The analysis presented formulates the basic operational limitations regarding the signal operation being considered, and gives the basis by which experimental results of implemented operation may be compared.

In Chapter 5 the design, implementation and testing of the techniques analyzed in Chapter 4 are carried out to determine the performance capabilities of the practical realization.

In Chapter 6 the summary and conclusions are presented. Several specific application of the investigated signal operation techniques are discussed. Suggestions for further research investigations are also presented.

2.0 FREQUENCY SYNTHESIS

When crowding of the frequency spectrum was not a problem and frequency inaccuracies could be tolerated, the tunable L-C oscillator was a practical means of providing the required channel frequencies for various multi-channel equipment. Crowding of the spectrum required closer channel spacing and increased frequency accuracy. The crystal oscillator was used to satisfy this required increased frequency accuracy. The multiple-crystal synthesizer was developed to satisfy the requirements of multi-channel equipment. The single crystal frequency synthesizer was proposed to fulfill the accuracy and stability requirement in equipment covering a very large number of communication channels.

The single crystal frequency synthesizer retains the properties of the multiple crystal synthesizer and at the same time eliminates the problem of maintaining a large number of crystals to the same accuracy. The accuracy and stability of the output signal are essentially equal to that of the reference oscillator.

Present day frequency synthesizers, regardless of the synthesis scheme used, have the common objective of referencing all generated frequencies to a single standard frequency. This single standard frequency is generated by an extremely accurate and stable oscillator. The required output frequencies are obtained in some synthesis procedure by direct multiplication and division of the standard frequency, which are combined by the operation of either addition or subtraction to give the derived output frequency. In other schemes, the final output frequency is compared directly or divided down to a frequency which can be compared with the standard frequency. The comparator output furnishes the control signal for locking the output signal to the standard.

Synthesizers can be divided into two main categories. These are termed direct and indirect. Mixing of a set of frequencies derived directly from the standard to obtain the final specific output frequency is defined as direct synthesis. Methods which ultimately control the output of a phase-locked oscillator are defined as indirect synthesis. The common characteristic regardless of the synthesis method employed is to generate a desired frequency from a frequency standard in such a manner that the stability and accuracy of the derived frequency is not seriously degraded from that of the standard. The multiplicity of output signals available are all harmonically related to a specific subharmonic of the standard oscillator.

2.1 DIRECT FREQUENCY SYNTHESIS.

For frequency generating systems that are classed as direct synthesis the general principles of operation can be illustrated as in Fig. 2-1. From this block diagram it is clear that the output frequency is related to a subharmonic of the reference oscillator.

The selection of positive integers a and b will determine the sub-harmonic spacing between available output frequencies.

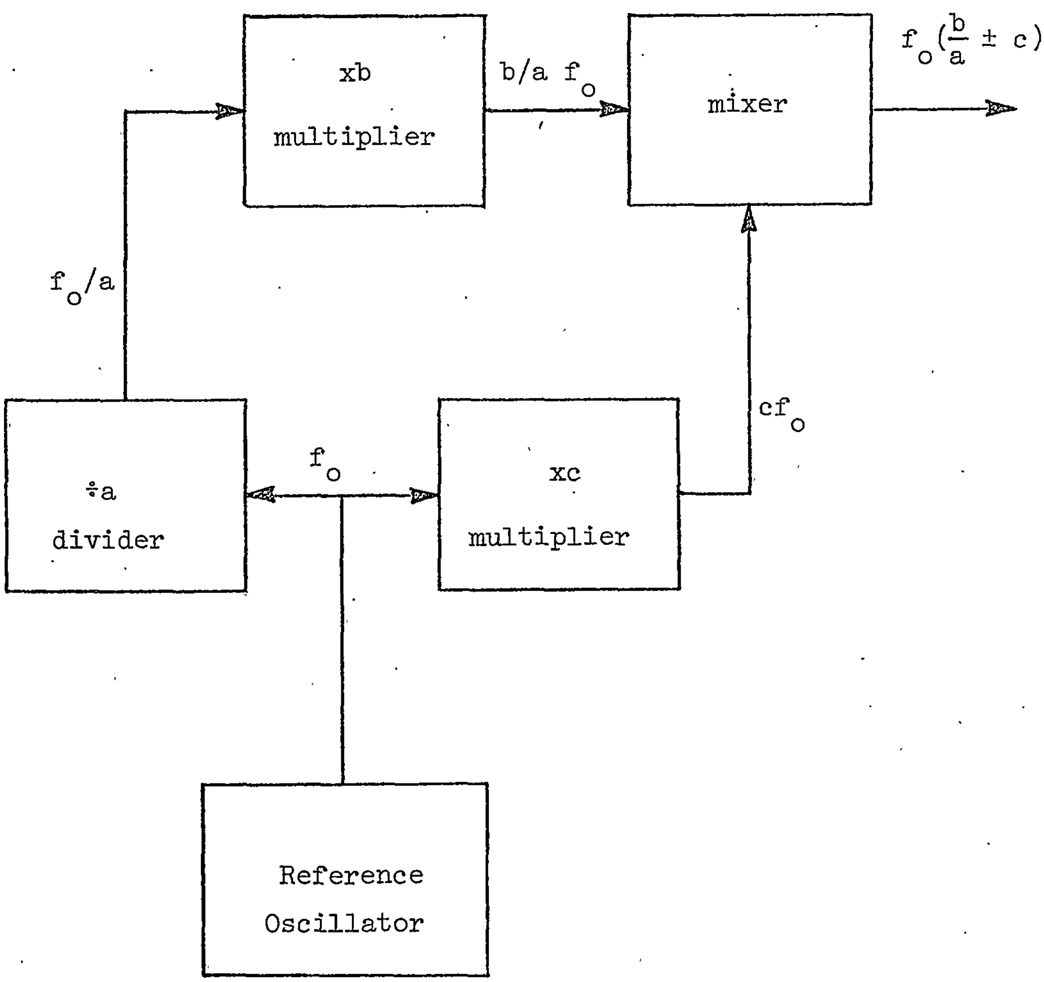


Fig. 2-1. BLOCK DIAGRAM OF DIRECT FREQUENCY SYNTHESIS.

In the actual system realization of a synthesizer using this method to obtain many possible output frequencies, a highly stable reference oscillator with multiples and sub-multiples are mixed in elaborate ways to produce the desired outputs. A more general block diagram, which indicates the complexity that may be involved in generating a set of output frequencies for a given application, is illustrated in Fig. 2-2.

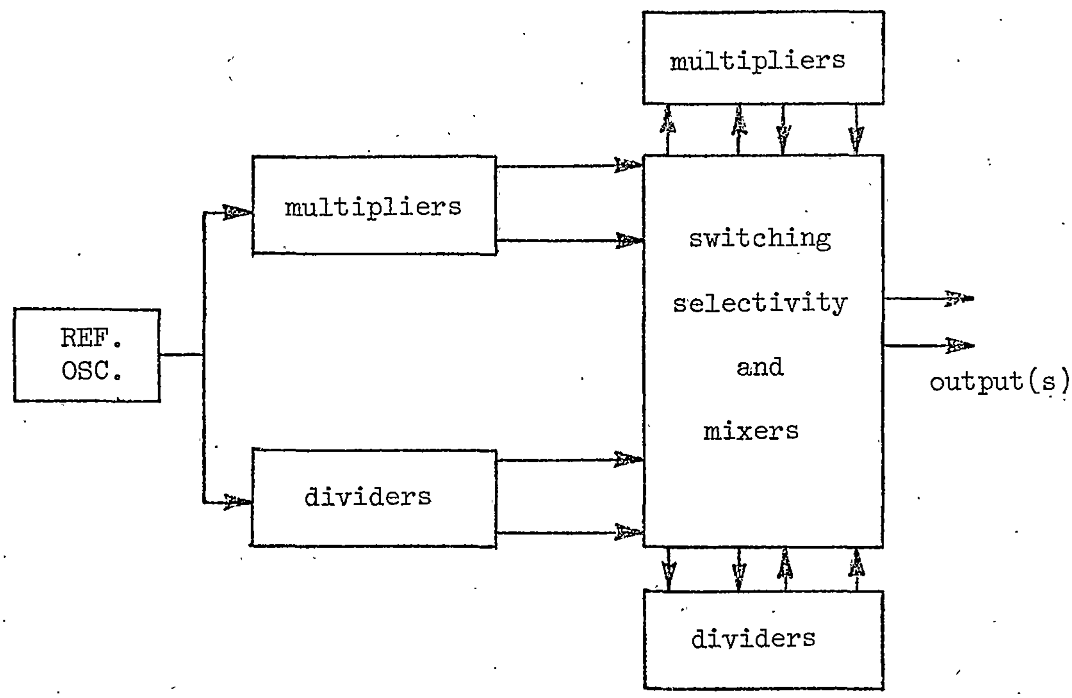


Fig. 2-2. GENERAL BLOCK DIAGRAM FOR DIRECT FREQUENCY--SINGLE SOURCE SYNTHESIZERS.

The synthesizer of least system complexity would consist of multipliers and dividers directly connected to the reference oscillator and their outputs used to generate the desired output frequencies. The next level of system complexity would be several parallel multiplier-divider combinations to generate the desired outputs. The complexity can be continued on to the multipliers and dividers that are not directly connected to the reference oscillator.

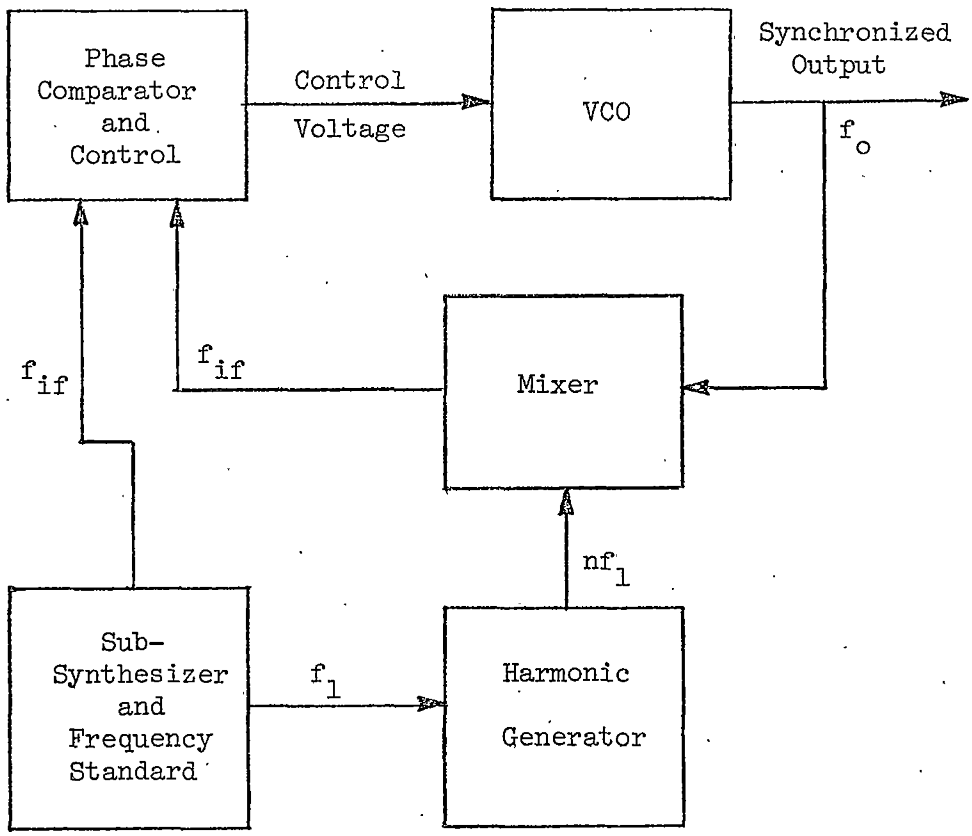
The majority of synthesizer equipments that have been built fall under this category. These systems typically provide 1 to 50 million frequencies that are manually selectable.

2.2 INDIRECT FREQUENCY SYNTHESIS.

For frequency generating systems that are classed as indirect synthesis, a controlled oscillator feeds the output terminals of the synthesizer directly. The frequency of the output oscillator is continuously monitored and compared to a reference frequency, and automatic adjustment of the oscillator frequency is made to maintain its output at precisely the desired frequency.

As in the direct synthesis class, this discussion cannot include all the many types of synthesizers that have been developed in the indirect synthesis category. Two examples have been selected that include a wide cross-section of the systems in this class.

The automatic phase control (APC) method of frequency synthesis is presented in Fig. 2-3. This system is an electronic servo that does not permit a steady-state frequency error to be developed between the reference frequency and the voltage controlled oscillator (VCO).



$f_{if} = |nf_1 \pm f_o|$

Fig. 2-3. AUTOMATIC PHASE CONTROL SYSTEM BLOCK DIAGRAM.

The VCO output is mixed with a harmonic of the sub-synthesizer output f_1. This produces an intermediate frequency signal f_{if} which is compared in the phase comparator with f_{if} from the sub-synthesizer. The comparator output is a voltage proportional to the phase error of the converted f_{if} signal to the synthesizer f_{if} signal. The comparator output is passed to the control circuitry of the VCO.

The APC system holds the VCO on frequency by varying the control voltage so that the two f_{if} signal frequencies are identical but, in general, differing in phase. This overall operation provides a frequency lock for $f_o = |nf_1 \pm f_{if}|$.

Figure 2-4 illustrates an example of digital method of indirect frequency synthesis. The output of the VCO is divided by n and fed to the phase comparator.

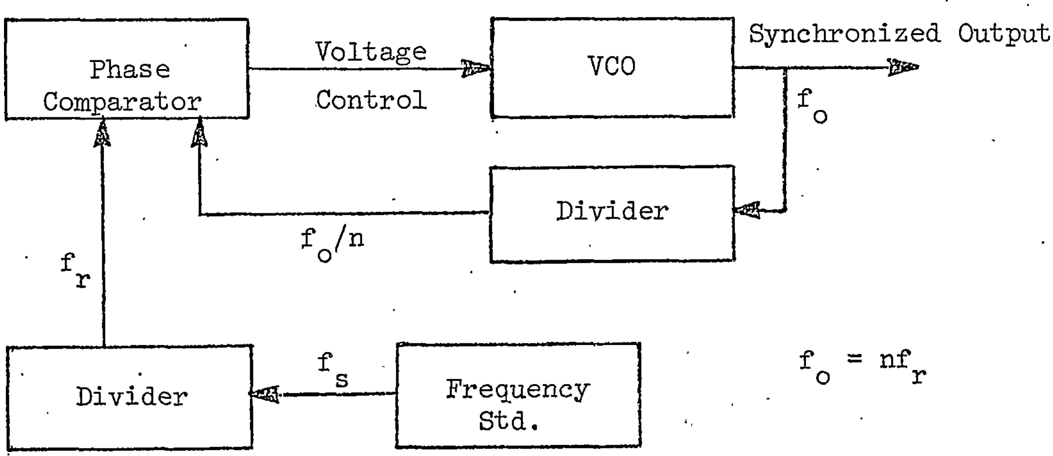


Fig. 2-4. DIGITAL FREQUENCY SYNTHESIS BLOCK DIAGRAM.

The comparator has the divided reference frequency, f_r, as the other input. The output of the phase comparator provides the VCO correction voltage if the two inputs are not identical in frequency. Therefore, the frequency of the VCO is automatically phase-locked to the reference standard.

2.3 SYNTHESIZER PERFORMANCE SPECIFICATIONS.

In general, the system application determines the performance and configuration of the synthesizer. The specific application will dictate several or all of the following performance specifications:

1) frequency coverage and resolution
2) method of frequency selection
3) speed of switching to desired output frequency
4) accuracy and stability
5) spectral purity.

The rf tuning range for a specified application (or equipment) in which the synthesizer is to be incorporated will in general set the frequency coverage needed. The system channel capability will determine the number of incremental frequencies required for communications applications.

Spectral purity is one of the more critical performance factors of a synthesizer. Complete spectral purity implies the absence of noise and spurious frequencies. In general the reference frequency standard used to drive a synthesizer will be of resonable quality and possess an output signal power spectrum as shown in Fig. 2-5.

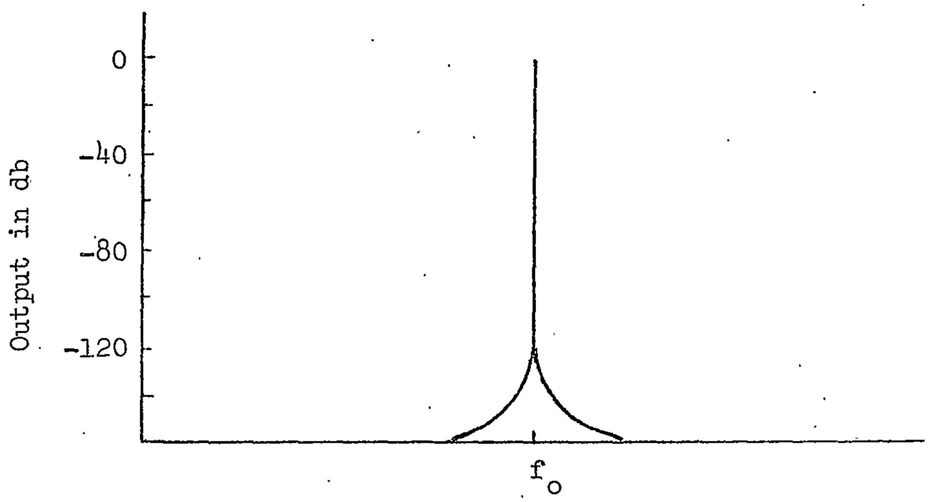


Fig. 2-5. OUTPUT SPECTRUM OF A FREQUENCY STANDARD.

The output signal of the frequency standard is specified by the harmonic distortion present, the non-harmonic related outputs, and the signal-to-noise ratio for a specific output signal bandwidth. Each of the quantities are specified in db. The db (decibel) measure is defined as

$$\text{number of db} = -10 \log \frac{P_o}{P_i}, \tag{2-1}$$

where, for the specification of a frequency standard, P_o is the power at the output frequency f_o. For the harmonic distortion specification, P_1 is the power at each of the harmonics of f_o. For the non-harmonic components, P_1 is the power at each of the spurious frequencies. For the signal-to-noise ratio P_1 is the total power associated with the specified output signal bandwidth.

A high order of spectral purity existing in the synthesizer output requires that noise and spurious frequency generation be kept to a minimum throughout the entire synthesis process. The spurious frequencies appearing in the synthesizer output are the result of either the filters used within the synthesizer presenting a finite attenuation (rather than infinite) to undesired frequency components or improper shielding of the various subsystems. In some cases the spurious outputs may be a combination of both the filtering and shielding problems. Spurious frequency generation coupled with a find incremental frequency resolution capability requires much greater system complexity.

Direct frequency synthesizers have been constructed which maintain the spurious component level at 90-100 db below the desired output signal levels. Indirect synthesizers, using filter time-constants up to 1 second in the control loop, have been able to obtain spurious levels greater than 100 db below the output level.

The stability of the synthesized outputs are dependent on both spurious levels and the frequency resolution.

In applications requiring fast switching between selected output frequencies, the design dictated is usually the direct method.

The required control loop filters in the indirect method limit their speed of response due to the bandwidth of the filter used.

2.4 SUMMARY

An attempt has been made to show that frequency synthesizer designs evolve from a few basic signal operations. The signal operations are frequency translation, frequency multiplication and frequency division. The grouping of these signal operations may be performed in a wide variety of combinations depending upon the specific system requirement. By skillful choice of operations, systems may be designed for a given application with an accuracy limited only by the primary reference oscillator.

3.0 SIGNAL REPRESENTATIONS AND OPERATIONS APPLICABLE TO FREQUENCY SYNTHESIS.

The basic signal operations that must be combined to describe a specific frequency synthesizer of the two broad classes discussed in Chapter II are frequency translation, frequency multiplication and frequency division. In all these operations the prime concern is minimum degradation of the initial standard short-term stability while performing the desired operation. These signal operations are defined and their theoretical performance criteria set down. This then will form the basis to which actual circuit performance can be compared.

3.1 SIGNAL REPRESENTATION

The signal and noise which the basic standard delivers at the output terminals is the input signal for the various signal operations considered. This signal is assumed to have a large signal-to-noise ratio along with a very narrow output signal bandwidth. It is assumed that the standard has been in operation for a sufficient length of time such that the start-up transients of the standard can be neglected. Also, the long-term effects of frequency drift in the standard output due to component drifts from aging or temperature fluctuations will not be considered in this analysis. These long-term effects can be eliminated by appropriate control of the standard. As in any system there exist possible transient responses along with the normal steady-state response for the signal operations being analyzed. The analysis that follows is concerned only with the steady-state response.

For the stated assumptions the signal being generated by the standard can be expressed as

$$s(t) = A \cos \omega_o t. \tag{3-1}$$

and the noise being considered expressed as

$$v_n(t) = v_c(t) \cos \omega_o t + v_s(t) \sin \omega_o t. \tag{3-2}$$

where

A = fixed amplitude

ω_o = standard output frequency

$v_c(t)$ and $v_s(t)$ are both narrow-band random processes which are slowly varying functions compared to the individual cycles of $s(t)$. A detailed discussion leading to equation (3-2) may be found in reference [8].

The result of adding signal and noise to obtain the output signal delivered by the standard is

$$v_1(t) = s(t) + v_n(t) \tag{3-3}$$

Substituting (3-1) and (3-2) into (3-3) yields

$$v_1(t) = (A + v_c(t)) \cos \omega_o t + v_s(t) \sin \omega_o t$$

$$= B(t) \cos (\omega_o t + \phi (t)). \tag{3-4}$$

where

$$B(t) = \sqrt{(A + v_c(t))^2 + v_s(t)^2} \qquad (3\text{-}5)$$

and

$$\tan \phi(t) = \frac{v_s(t)}{A+v_c(t)} . \qquad (3\text{-}6)$$

For large signal-to-noise ratios

$$\phi(t) \cong A^{-1} v_s(t) \qquad (3\text{-}7)$$

Equation (3-4) to (3-7) specify the input signal to be used in the analysis of the signal operations presented in this Chapter. A complete model of the standard, setting down the mechanisms by which it generates this output signal, has not been described. The signal and noise that it delivers to the next element of the system is the oscillator feature of interest.

3.2 FREQUENCY TRANSLATION.

Frequency translation has been used for many years, but the general problem of spurious frequency generation has often been treated lightly or ignored. In this section some of the general properties of frequency translation including the generation and suppression of spurious frequencies is discussed.

3.2.1. GENERAL TRANSLATION

Mixers with filtering perform the function of frequency translation in a signal chain. Mixing is essentially a modulation in which one signal is modulated by another signal producing the lower and upper side frequency components. Mixing action is achieved by means of a non-linear device and thus many frequency components may be present in the output of the mixer circuit. The operating parameters of the mixer stage will determine the ratio of the desired output(s) power level to the undesired outputs.

Frequency translation is in general accomplished by multiplying the signal to be translated by a sinusoidal signal of appropriate frequency to provide the desired translation. For the ideal case, consider $v_2(t)$ in Fig. 3-1 to be given by

$$v_2(t) = \cos \omega_2 t. \tag{3-8}$$

Then the output $v_o(t)$ is expressed as

$$v_o(t) = v_1(t) \cos \omega_2 t. \tag{3-9}$$

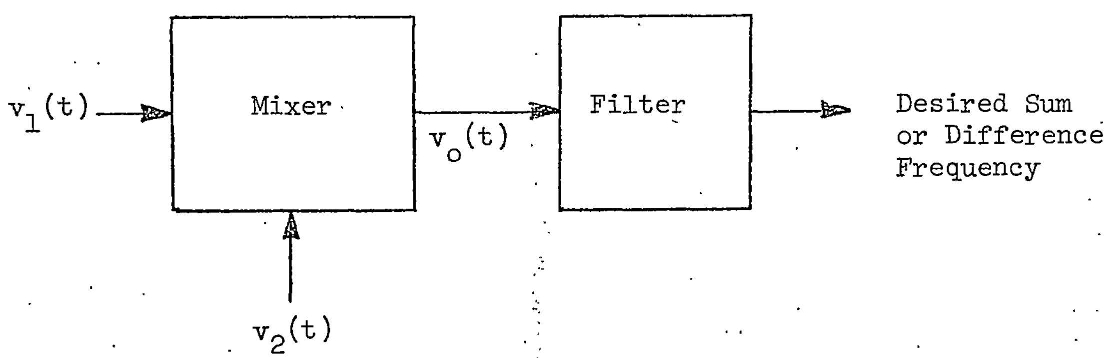


Fig. 3-1. FREQUENCY TRANSLATION.

Fourier analysis shows that this process translates the input frequency ω_1 or an input frequency spectrum by an amount $\pm \omega_2$. For a single input frequency, ω_1, the frequencies existing in the output are given by

$$\omega_o = \omega_1 \pm \omega_2. \tag{3-10}$$

The desired sum or difference frequency is selected by filtering.

3.2.2 GENERAL MIXER

As previously stated, mixing action is fundamental to the process of frequency translation and is accomplished by use of a nonlinear device; as a result, modulation products in the output will exist and many frequencies other than $\omega_1 \pm \omega_2$ along with possible additional spurious responses at ω_o. Considering a general nonlinear mixer, the output $v_o(t)$ may be expressed as

$$v_o(t) = \sum_{m=o}^{\infty} \sum_{n=o}^{\infty} a_{mn} \cos (m\omega_1 t \pm n\omega_2 t + \phi_{mn}) \tag{3-11}$$

where

a_{mn} = constant

ϕ_{mn} = constant phase shift

Spurious responses are generally produced with signals existing at undesired frequencies which satisfy the following condition

$$f_o = |mf_1 \pm nf_2| \tag{3-12}$$

where

f_1 = signal frequency

f_2 = translation frequency

f_o = desired output frequency

m and n are integers ≥ 1.

The choice of frequencies which are to be used in the translation scheme depend on several factors. This dependence in all cases relies on how well the extraneous signal can be rejected in the output of the mixer. Several means are available to aid in accomplishing the required rejection such as narrow-band filters, frequency spacing, and an effective cancellations of many components by balancing techniques. In the desired translation case the frequency of translation and the sets of harmonic sum and difference components do not fall within the output bandwidth of the mixer. In these cases the required extraneous signal rejection is provided by the filter response in the mixer output. In the cases where the translation signal frequency falls in the output signal bandwidth, balancing is required to eliminate these components. The next two sections describe unbalanced and balanced mixers.

3.2.3. SINGLE DIODE MIXER

The single diode mixer will be presented in terms of solid state diode mixers in that the conversion conductance analysis applies directly. The analysis can be extended to the transistor mixer by considering a diode mixer followed by a transistor amplifier. Then the conversion gain is the sum of the diode conversion loss at the linear signal frequency and the transistor gain at the desired output frequency.

The high conductance of a diode in the forward direction and very low conductance in the reverse direction provides the necessary non-linearity for mixing. The dc voltage-current characteristics of the diode is expressed as

$$i(t) = I_o \left[e^{\alpha(v(t)+V_o)} - 1\right]$$

$$= \sum_{k=o}^{\infty} a_k v(t)^k \qquad (3\text{-}13)$$

where V_o is the dc bias voltage, a_k is related to the diode conductance, and $v(t)$ is the total instantaneous ac voltage across the junction. Therefore, no diode mixer is truly piecewise linear nor truly square law in its operation. The commonly used diode is an exponential law mixer.

The exponential expression expanded as a Taylor series yields

$$i(t) = a_o + a_1 v(t) + a_2 v^2(t) + a_3 v^3(t) + \ldots \qquad (3\text{-}14)$$

The output voltage for a conventional mixer, Fig. 3-2, is equal to the diode current times the load resistance. The voltage, $v(t)$, is the sum of the translating and signal voltages,

$v_1(t)$ and $v_2(t)$.

Applying $v_1(t) = A_1 \sin \omega_1 t$ and $v_2(t) = A_2 \sin \omega_2 t$ to the mixer, the current expression for $v(t) = v_1(t) + v_2(t)$ becomes

$$
\begin{aligned}
i(t) = a_o &+ a_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) \\
&+ a_2 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 \\
&+ a_3 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3 \\
&+ \ldots \\
= a_o &+ a_1 A_1 \cos \omega_1 t + a_1 A_2 \cos \omega_2 t \\
&+ \frac{a_2 A_1{}^2}{2} + \frac{a_2 A_2{}^2}{2} + \frac{a_2 A_1{}^2}{2} \cos 2\omega_1 t \\
&+ \frac{a_2 A_2{}^2}{2} \cos 2\omega_2 t \\
&+ a_2 A_1 A_2 \cos (\omega_1 + \omega_2) t \\
&+ a_2 A_1 A_2 \cos (\omega_1 - \omega_2) t \\
&+ \ldots \qquad (3\text{-}15)
\end{aligned}
$$

If equation (3-14) contains high order terms then the diode current expression (3-15) will have components associated with the harmonic sum and difference frequencies.

In the output of the mixer for the case $a_n \neq o$ for $n > 2$, the frequency components present are ω_1, ω_2, $\omega_1 \pm \omega_2$, $m\omega_1$, $n\omega_2$, $m\omega_1 \pm n\omega_2$ (m and n are integers).

The signal-to-noise ratio of this mixer with an ideal translating signal and coefficients higher than a_2 being zero will be

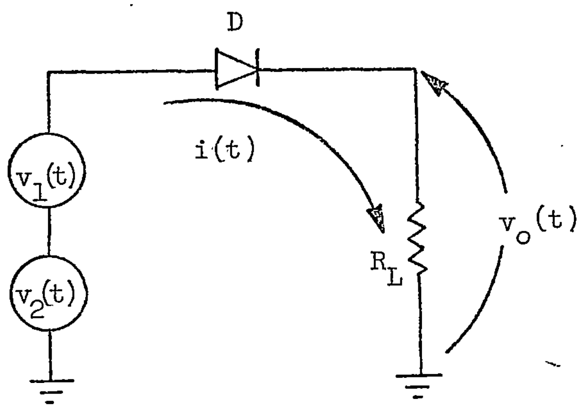


Fig. 3-2. CONVENTIONAL MIXER.

that of the signal input. However, if even ordered coefficients greater than a_2 are not zero then the output signal-to-noise ratio will be degraded since these products fall in the desired output bandwidth and therefore cannot be removed by filtering.

3.2.4 BALANCED MIXERS

The single diode mixer discussed generates a large number of spurious products. Suppression of the signal and translation frequencies along with some of the harmonic differences generated is desired. Two types of balanced mixers, generally called full lattice and half lattice, are commonly employed to meet these requirements. The half lattice modulator allows suppression of only one of the injected signals. The full lattice or ring modulator allows suppression of both injected signals and their harmonics. However, some harmonic sum and difference components still remain.

The half lattice modulator can be represented as shown in Fig. 3-3.

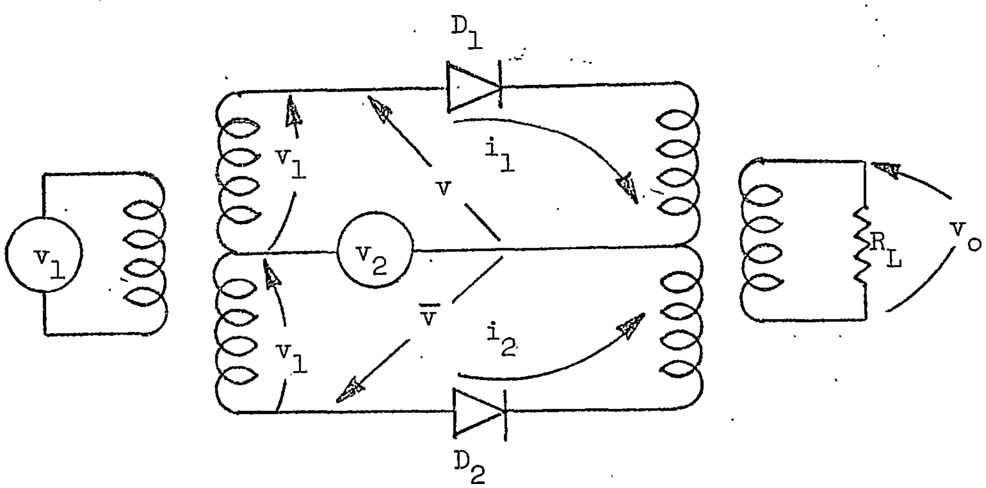


Fig. 3-3. HALF LATTICE BALANCED MODULATOR.

Assuming balanced diodes and transformers the output currents $i_1(t)$ and $i_2(t)$ are expressed as

$$i_1 = a_o + a_1\, v + a_2\, v^2 + a_3\, v^3 + \ldots$$

$$i_2 = a_o + a_1\, \bar{v} + a_2\, \bar{v}^2 + a_3\, \bar{v}^3 + \ldots \qquad (3\text{-}16)$$

where

$$v = v_1 + v_2$$

$$\bar{v} = -v_1 + v_2.$$

These voltages and currents are all functions of time as was the case in the single diode analysis. The notation has been changed merely for simplicity. The modulator output across R_L is given by

$$
\begin{aligned}
v_o &= (i_1 - i_2) R_L \\
&= R_L (2a_1 v_1 + 4a_2 v_1 v_2 + 2a_3 v_1^3 \\
&\quad + 6a_3 v_1 v_2^2 + 8a_4 v_1^3 v_2 \\
&\quad + 8a_4 v_1 v_2^3 + \ldots)
\end{aligned}
\tag{3-17}
$$

As before, if we consider only coefficients a_o, a_1, and a_2, we obtain the desired translation, $\omega_1 + \omega_2$, with no degradation in the signal-to-noise ratio at the output. The even harmonics of ω_1 and all harmonics of ω_2 have been cancelled out. For the general case, $a_n \neq o$ for $n > 2$, the frequency components present are ω_1, $\omega_1 \pm \omega_2$, $n\omega_1$, $m\omega_2 \pm n\omega_1$ (m an integer and n an odd integer). As in the single diode case, the signal-to-noise ratio will be degraded if the even ordered coefficients greater than a_2 are not zero.

The double balanced ring or full lattice modulator is shown in Fig. 3-4. Using the same analysis as used for the half lattice modulator and balanced transformers and diodes the load currents are those of the previous modulator plus two additional currents associated with the reversed polarity diodes D_3 and D_4. The load currents due to the addition of these diodes are