



The transmission line and live foliage measurements
by Leo Setian

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY in Electrical Engineering
Montana State University
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Abstract:

A non-destructive method of determining moisture in living foliage is discussed. The use of an unbalanced transmission line set in foliage is proposed.

The theory of the open-wire transmission line with matched and mismatched terminations is discussed. Graphs showing power distribution along the transmission lines are presented.

Results are presented of the line-in-foliage measurements. A center conductor over a mesh ground plane through which the foliage grows is used. The input impedance and frequency are measured. Resonant and non-resonant lines are utilized. Concurrently, xylene tests are conducted on the foliage to measure moisture content. Graphs of impedance, permittivity, conductivity, attenuation, loss tangent and change in frequency versus percent moisture are presented.

A model of the foliage medium is discussed. The individual plants are considered as dielectric cylinders. Graphs of change in resonant frequency of the transmission line versus relative dielectric constant are presented. Theoretical and experimental values are compared.

It is concluded that it is possible to determine moisture percentage in living foliage by measuring input impedance and/or frequency. The method of measuring the resonant frequency of the transmission line is proposed as the best method.

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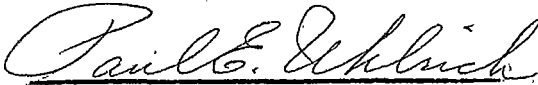
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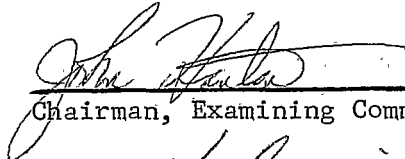
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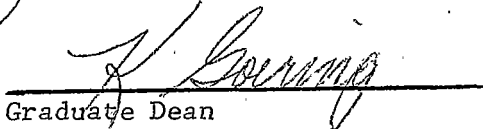
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ABSTRACT

A non-destructive method of determining moisture in living foliage is discussed. The use of an unbalanced transmission line set in foliage is proposed.

The theory of the open-wire transmission line with matched and mismatched terminations is discussed. Graphs showing power distribution along the transmission lines are presented.

Results are presented of the line-in-foliage measurements. A center conductor over a mesh ground plane through which the foliage grows is used. The input impedance and frequency are measured. Resonant and non-resonant lines are utilized. Concurrently, xylene tests are conducted on the foliage to measure moisture content. Graphs of impedance, permittivity, conductivity, attenuation, loss tangent and change in frequency versus percent moisture are presented.

A model of the foliage medium is discussed. The individual plants are considered as dielectric cylinders. Graphs of change in resonant frequency of the transmission line versus relative dielectric constant are presented. Theoretical and experimental values are compared.

It is concluded that it is possible to determine moisture percentage in living foliage by measuring input impedance and/or frequency. The method of measuring the resonant frequency of the transmission line is proposed as the best method.

GLOSSARY OF TERMS

\bar{A}	-	vector potential
\underline{a}	-	line spacing, center conductor to ground plane
\bar{B}	-	magnetic flux
C	-	capacitance, farads per meter
\underline{c}	-	velocity of light
\bar{D}	-	electric displacement
\bar{E}	-	electric field vector, volts per meter
f	-	frequency, Hertz
Δf	-	change in frequency
\bar{G}	-	conductance (shunt), mhos per meter
\bar{H}	-	magnetic field vector, amps per meter
\bar{I}	-	current, amps
\underline{i}	-	current vector, amps
\hat{i}	-	unit vector in x- direction
\bar{J}	-	current density vector, amps per square meter
\hat{j}	-	unit vector in y- direction
\hat{k}	-	unit vector in z- direction
\bar{L}	-	inductance (series), henries per meter
l	-	length of transmission line, meter
$d\ell$	-	elemental length
\bar{P}	-	Poynting's vector, watts per meter squared
R	-	resistance (series), ohms per meter
r	-	distance from conductor to field point
\bar{S}^*	-	complex Poynting vector
V	-	voltage
v	-	volume, cubic meters
Y	-	admittance, mhos per meter
Z	-	impedance, ohms
Z_0	-	characteristic impedance, ohms
α	-	attenuation, nepers per meter
β	-	phase, radians per meter
γ	-	propagation constant
δ	-	loss tangent
ϵ	-	dielectric constant
η	-	impedance of medium, ohms
λ	-	wave length, meters
μ	-	permeability
ν	-	integer
ρ	-	distance from origin to field point, meters
σ	-	conductivity, mhos per meter
u	-	velocity of wave in medium, meters per second
ω	-	radian frequency

CHAPTER I

INTRODUCTION

A. Purpose

There is interest in determining the amount of moisture in living foliage. For example, the Forest Service uses the amount of moisture to determine the Forest Fire Index. Other uses include radio propagation studies to determine attenuation and conductivity of jungles and heavily-wooded areas. The current method, boiling in xylene or baking, then measuring the amount of moisture, is a destructive method which destroys the foliage. In addition, the process is relatively lengthy in terms of hours. Thus, for at least the above two reasons, a better method of measuring percent moisture in foliage is desirable.

B. Literature Survey

Some of the electromagnetic methods tried in the past have been resonant cavities, parallel-plate capacitors, and transmission lines. The measurements made were then related to other quantities such as relative dielectric constant, conductivity, and so forth. Various geometries have been tried. Frequency has been varied.

In 1960, Kirkscether used two methods to calculate characteristic impedance.¹ The first method consisted of short- and open-circuited measurements (Z_{sc} and Z_{oc}) on earth samples that could be removed. The lengths of the samples were between 15 and 30 cm. From these, the line constants R, L, G and C were calculated; then conductivity, σ , dielectric constant, ϵ_r , and permeability, μ_r , of the earth are found from

$$\sigma = G\epsilon_0 (C')^{-1}$$

and

$$\epsilon_r = C(C')^{-1}$$

and

$$\mu_r = L(L')^{-1}$$

where the primed quantities indicate air values.

His second method used in terrain was open-circuited transmission lines introduced perpendicularly into the earth. Lines of lengths ℓ and 2ℓ were used with the relationship

$$Z_o = \sqrt{Z_{oc1} (2Z_{oc2} - Z_{oc1})}$$

instead of

$$Z_o = \sqrt{Z_{sc} Z_{oc}}$$

The samples were tested in the frequency range 0.6 to 400 MHz. Some of the parameters of interest and their variations were as follows. The earth's conductivity increased exponentially with

frequency; the dielectric constant decreased exponentially with frequency; the dissipation factor decreased exponentially with frequency; and the attenuation rose linearly with frequency first to about 10 MHz and then exponentially to 400. Kirkscester shows that by using either method of simple input impedance measurements, he could find the electrical characteristics of the earth.

More recently, Parker and Hagn did a study on the use of open-wire transmission lines, capacitors and cavities to measure electrical properties of vegetation.² A rigid open-wire line with variable length was inserted into the foliage. At 4 to 30 MHz a 4 inch diameter, 40 inch-spacing line was used and at 30 to 75 MHz a 5/8 inch diameter, 3 inch-spacing. A 4 to 1 balun matched the balanced line to the unbalanced input of the impedance meter (Boonton RX-meter). The testing was accomplished at two sites, California and Washington.

In California, values of α and ϵ_r were obtained for two living foliage samples, poison oak and fern, and for two cut samples, oak boughs and mixed boughs. Daily tests were conducted at first on the oak boughs. Later, hourly tests were made to find whether humidity effects were evident. No relationship between relative humidity and effective attenuation was evident. A similar cut-foliage test was undertaken later at 50 to 75 MHz. Both α and ϵ_r decreased with drying time as expected at both frequencies.

The Hoh rain forest in Olympic National Park was the site of the second series of tests. Here an aluminum HF line was constructed, 4 inch diameter, 40 inch spacing, in addition to the VHF line used in California. Also, the VHF line was encased in fiberglass for ease of handling. Sitka spruce, vine maple and red alder were the three live specimens tested. Comparison of the spruce results with the foliage results obtained from California indicated that the attenuation of the spruce was somewhat less, even though the spruce was considerably more dense. The authors felt that this may have been due to the varying species, different climates, or different periods of growing season. Beyond this it pointed out the great amount of cataloguing needed in studying electrical properties of foliage.

The VHF line in fiberglass was utilized in the Hoh tests. It was also used in the tests in vine maple which were divided into two sites 1.5 miles apart. Although the growth at one site was more dense and the measurements were taken during a rain, the attenuation of the vine maple was less than the second site which had experienced a recent three-day rain. Both transmission lines, HF and VHF, were used in the dense red alder and both within 10 feet of each other. The height of the VHF line was varied from 6 to 12 feet without significant change in attenuation. The values of attenuation were lower than those found in California. It was

felt that the alder may have entered its dormant season. (Measurements were taken in October.) It was also felt that this aluminum HF line was too lossy at frequencies above 30 MHz.

From the data, effective conductivity of the three specimens was plotted against frequency from 4 to 75 MHz. Some low values of conductivity were obtained for spruce but these measurements were taken before the autumn rain. In all the other cases, the earth was moist when impedances were measured. The loss tangent was also plotted against frequency. The loss tangent curve is hyperbolic, decreasing as frequency increases. The effective permittivity did not vary appreciably - the mean was 1.04. The VHF line was raised, lowered, tilted and rotated without more than a 10% change in results.

Further work on balanced wire transmission lines was done by H. Parker and Makavabhiromya when they measured electric constants in vegetation and in earth at five sites in Thailand.³ Included in the types of earth probes were: 1) brass rods spaced 2.5 cm apart for VHF, 2) 5 cm apart for HF (both 10 and 20 cm long), and 3) 1.6 cm diameter brass rods one meter long with variable spacing. The vegetation probes were: 1) 1.6 cm diameter silver-plated brass tubing with variable spacing and length, 2) 10 cm diameter aluminum pipes, 1 meter spacing with variable lengths, and 3) number 12

copper wire spaced two meters apart with variable length. With the great quantity of data taken and with the use of the computer, some interesting results were obtained.

The values of relative permittivity and permeability of the vegetation varied from 0.9 to 1.2 for ϵ_r and 0.8 to 1.1 for μ_r at frequencies between about 2 and 100 MHz. There is a question as to μ_r being not equal to 1 and ϵ_r being less than 1. Possibly the open-circuit measurements were not adequate at times. An open-circuit whose impedance is not high when compared with short-circuit is obviously not an open circuit. All the calculations in their work are dependent on the open-circuit measurements using lines of lengths l and $2l$.

The conclusions made by Parker and Makavabhiromya were that using values of unity for ϵ_r and μ_r would be sufficient for modeling forests for predicting path loss measurements. The median effective conductivity of the earth did not vary significantly between sites but showed an exponential increase with frequency. Also, since conductivity and loss tangent are directly related, as will be shown later, the same comments apply to loss tangent. At this point, a quote from the report by the above authors is in order: "The most important parameters influencing vegetation constants were stem spacing (related to stem number density) and intrinsic stem conduc-

tivity estimated to be between 0.05 and 0.5 mho/m."⁴ Stem number density is the number of stems per unit cross-sectional area.

The authors also compared median electric earth constants from eight types of soil. The results were as expected. The driest soil had the lowest dielectric constant and conductivity. Again, dielectric constant changed only slightly with frequency while conductivity showed great change.

Of major importance in the use of the transmission line as a moisture detection probe is the question of volume of foliage that is sensed by the probe when inserted in some medium. In order to find this volume, we first investigate Poynting's Theorem.

C. Physics

1. Poynting's Theorem

Poynting's Theorem states that the vector product $\bar{P} = \bar{E} \times \bar{H}$ at any point is a measure of the rate of energy flow per unit area at that point. The energy is in the direction of $\bar{E} \times \bar{H}$. If we integrate Poynting's vector over a closed surface, we obtain

$$\oint_A \bar{E} \times \bar{H} \cdot d\bar{a} = - \int_V \bar{E} \cdot \bar{J} \, dv - \frac{\partial}{\partial t} \int_V \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv$$

The first term, $\overline{\mathbf{E}} \times \overline{\mathbf{H}}$, has the units of watts per square meter and, when integrated over a closed surface, represents the rate of flow of energy outward through the surface enclosing the volume. The second term represents the power "losses" in the system. Examples of these losses are conductor dissipation, losses in the medium, and radiation. Finally the third term represents the time rate of change of the total stored energy in the volume.

2. Transmission Lines

A transmission line may be made up of any two conductors separated by a dielectric material. Conventionally, we say the current at any point along one line is equal and opposite to the current at the same point in the other line. We also note that the transmission line is considered a "low" frequency device, below 200 or 300 MHz, that supports a TEM wave. This line is represented as a distributed-constant network having a series impedance $Z = R + j\omega L$ and a shunt admittance $Y = G + j\omega C$. This is usually represented by the equivalent circuit shown in the accompanying figure, Figure 1. R, L, and C can exist in series and parallel. However, they do not appear in Figure 1 because series C would not permit current flow and parallel L would indicate an inductive medium ($\mu_r \neq 1$).

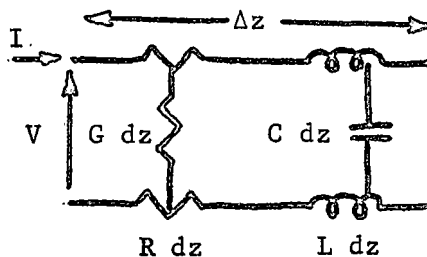


Figure 1

If R , L , G and C are the total quantities per unit length, the transmission line equations may be expressed as

$$\frac{dV}{dz} = - (R + j\omega L) I$$

and

$$\frac{dI}{dz} = - (G + j\omega C) V$$

Differentiating and combining, we get

$$\frac{d^2V}{dz^2} = \gamma^2 V$$

$$\frac{d^2I}{dz^2} = \gamma^2 I$$

where

$$\gamma^2 = (R + j\omega L) (G + j\omega C)$$

and

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

It is usual to make the location of a terminating impedance, Z_L , of a line at $z=0$ with the line to the left of $z=0$. See Figure 2.

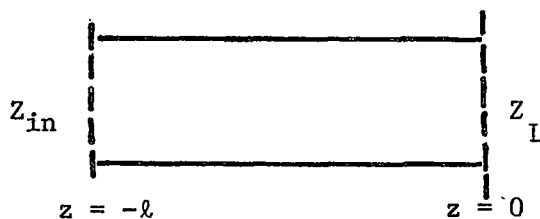


Figure 2

Solutions to the above equations may be written

$$V = A \cosh \gamma z + B \sinh \gamma z$$

$$I = C \cosh \gamma z + D \sinh \gamma z$$

where

$$V = V_L$$

and

$$I = I_L \quad \text{at } z = 0$$

Now, at the input to the line at $z = -\ell$, we write

$$\begin{aligned} Z_{in} &= \frac{V_{in}}{I_{in}} = \frac{V_L \cosh \gamma \ell + Z_0 I_L \sinh \gamma \ell}{I_L \cosh \gamma \ell + \frac{V_L}{Z_0} \sinh \gamma \ell} \\ &= Z_0 \frac{Z_L \cosh \gamma \ell + Z_0 \sinh \gamma \ell}{Z_0 \cosh \gamma \ell + Z_L \sinh \gamma \ell} \end{aligned}$$

We can also write

$$Z_0 = \sqrt{Z_{sc} Z_{oc}}$$

a. Resonance and Non-resonance

For a shorted transmission line, we have

$$\begin{aligned} Z_{sc} &= Z_0 \tanh \gamma l \\ &= Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l} \end{aligned}$$

where α is the attenuation and β is the phase and $\gamma = \alpha + j\beta$. For line lengths that are an odd multiple of a quarter wavelength, $\sin \beta l = \pm 1$ and $\cos \beta l = 0$ and the input impedance becomes

$$Z_{sc} = Z_0 \coth \alpha l$$

The shorted quarter-wave section has properties of the parallel resonant circuit. Consider a voltage induced on the line near one end. As the voltage wave travels down the line and back, it traverses one full wave in order to arrive at the "starting point" in phase with the next voltage wave. Thus the voltage builds. These voltages and currents increase until the power is equal to the I^2R losses. In general, the input impedance and resonant properties of an open- or short-circuited line are dependent on loss. Losses come mainly from conductor resistance and dielectric loss with some due to radiation, though the latter is not always considered a loss.

We summarize this by saying a short-circuited quarter wavelength line and a parallel-resonant circuit are similar in the sense that they both present high input impedance to one particular frequency;

the impedance drops rapidly if frequency varies slightly from resonance. They are inductive at frequencies below and capacitive at frequencies above resonance. The transmission line behaves so at odd multiples of its lowest resonant frequency.

For an open line

$$Z_{oc} = Z_o \coth \gamma l$$

It is similar to a series resonant circuit. It has low impedance at its resonant frequency. A lossless line is resistive at resonance, inductive at frequencies above and capacitive below resonance. Its characteristics repeat at odd multiples of the lowest resonant frequency. Thus the change in Z_{oc} is the inverse of the short-circuited line and the product of the two impedances gives the square of the characteristic impedance.

We can now discuss the behavior of the transmission line at nonresonance. Let us consider first the shorted line. From above, we noted that

$$Z = Z_o \frac{\sinh \alpha z \cos \beta z + j \cosh \alpha z \sin \beta z}{\cosh \alpha z \cos \beta z + j \sinh \alpha z \sin \beta z}$$

Rewriting and rationalizing we obtain

$$Z = Z_o \frac{\sinh \alpha z \cosh \alpha z + j \sin \beta z \cos \beta z}{\cosh^2 \alpha z \cos^2 \beta z + \sinh^2 \alpha z \sin^2 \beta z}$$

At non-resonance, the "sin βz cos βz " in the imaginary part will determine if the line is inductive or capacitive. For example, between quarter-wave ($\beta z = \pi/2$) and half-wave ($\beta z = \pi$)

$$\sin \beta z \cos \beta z < 0$$

and we see that the reactance is negative and, hence, the line is capacitive as stated above. Similarly, between half-wave ($\beta z = \pi$) and three-quarter wave ($\beta z = 3\pi/2$)

$$\sin \beta z \cos \beta z > 0$$

and the line is inductive.

For the open-circuited line

$$\begin{aligned} Z &= Z_0 \frac{\cosh \alpha z \cos \beta z + j \sinh \alpha z \sin \beta z}{\sinh \alpha z \cos \beta z + j \cosh \alpha z \sin \beta z} \\ &= Z_0 \frac{\sinh \alpha z \cosh \alpha z + j \sin \beta z \cos \beta z}{\sinh^2 \alpha z \cos^2 \beta z + \cosh^2 \alpha z \sin^2 \beta z} \end{aligned}$$

Thus where the shorted line is capacitive the open-circuited line is inductive and vice versa.

In conclusion, we note that if we can measure short-circuited and open-circuited impedance of our transmission line, or short-circuit solely, we will be able to find all the parameters of the transmission line which are related to moisture content. If some of these relationships are linear we will have a suitable handle for determining moisture

content. That is, a single-valued function will be sufficient.

Next let us examine the theoretical basis of our system.

CHAPTER II

THEORY OF EXPERIMENTAL SYSTEM

A. Balanced Wire Transmission Line With Matching Load

Consider a lossless transmission line with the lines spaced a distance $2a$ apart as shown below in Figure 3. The characteristic impedance of the line is Z_0 and it is terminated in Z_0 . We want to find the electric and magnetic fields at some point P; then find the percentage power traveling "down" the line within a circle of radius ρ .

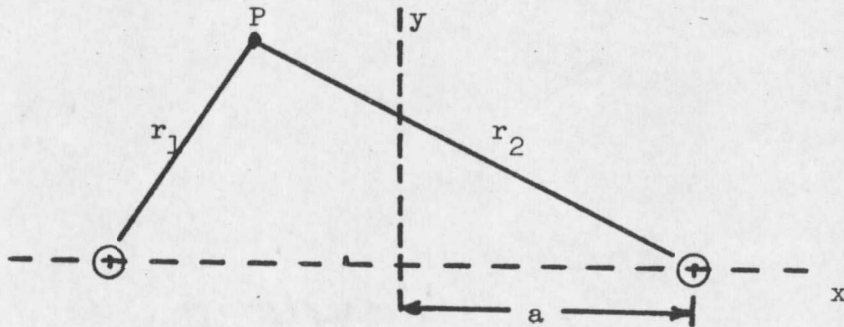


Figure 3

From Maxwell's equations, we have

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

and

$$\nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t}$$

We assume sinusoidal time variation so that

$$\nabla \times \bar{H} = \bar{J} + j\omega\epsilon\bar{E}$$

and

$$\nabla \times \bar{E} = -j\omega\bar{B}$$

By definition

$$\bar{B} = \nabla \times \bar{A}$$

From the first of Maxwell's equations above

$$\nabla \times \nabla \times \bar{A} = \mu\bar{J} + j\omega\epsilon\mu\bar{E}$$

or

$$\nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu\bar{J} + j\omega\epsilon\mu\bar{E}$$

From the second of Maxwell's equations above

$$\nabla \times (\bar{E} + j\omega\bar{A}) = 0$$

By definition

$$\bar{E} = -j\omega\bar{A} - \nabla\phi$$

so that

$$\nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu\bar{J} + \omega^2 \mu\epsilon\bar{A} - j\omega\epsilon\mu\nabla\phi$$

If we choose

$$\nabla \cdot \bar{A} = -j\omega\epsilon\mu\phi$$

then

$$\nabla (\nabla \cdot \bar{A}) = -j\omega\epsilon\mu\nabla\phi$$

and we finally obtain

