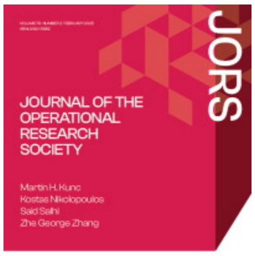

Journal of the Operational Research Society

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/tjor20

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To cite this article: Sean Harris, Ronald G. McGarvey, Andreas Thorsen & Maggie Thorsen (25 Sep 2024): Inferred attractiveness gravity-based models for estimating realized access at rural hospitals, Journal of the Operational Research Society, DOI: 10.1080/01605682.2024.2406236

To link to this article: https://doi.org/10.1080/01605682.2024.2406236

JOURNAL OF THE OPERATIONAL RESEARCH SOCIETY
https://doi.org/10.1080/01605682.2024.2406236

ORIGINAL ARTICLE

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Inferred attractiveness gravity-based models for estimating realized access at rural hospitals

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ABSTRACT

Operating obstetric units in rural America is financially challenging in part due to low birth volume. Birth volume at a hospital decreases when birthers bypass it to go to a farther hospital. Beyond financial considerations, it is important from a healthcare equity perspective for hospitals to know whether certain subgroups of birthers avoid utilizing the hospital's services. This can better inform resource allocation decisions targeting those subgroups. In this paper, we use a nonlinear programming optimization model, inferred attractiveness gravity-based model (GBM), to estimate realized access to obstetric care at hospitals in Montana. We compare three variations of GBM and benchmark our results to a regression-based conditional logit model. Results indicate that hospital attractiveness varies across the level of obstetric care provided and depends on the subgroup of birthers considered. While all GBMs produced smaller errors for hospitals with higher birth volumes, our novel variant was more accurate for low-volume hospitals. Bootstrapping analyses and resolving the models for population subgroups indicated large variations in hospital attractiveness. Research findings contribute to new knowledge about equity in access to obstetric care, the importance of considering population heterogeneity in GBMs, and the benefit of using hospital demand-based thresholds for GBMs in rural settings.

ARTICLE HISTORY
Received 19 May 2023
Accepted 16 September 2024

KEYWORDS
OR in health services; gravity-based model; obstetric bypassing; hospital choice

1. Introduction

Rural hospital-based obstetric services are declining in the US due to financial and staffing problems, leading to reduced access to services for rural birthers, particularly among disadvantaged populations (Hung et al., 2017). The low birth volume further exacerbates the financial challenges of operating obstetric units in rural areas (Zhao, 2007). This changing landscape presents challenges for birthers, including longer travel times and potentially unfavourable outcomes (Hung et al., 2018). Disparities in access to obstetrical care are greater for minority and low-income populations, who are more likely to experience long travel distances and rural OB unit closures (Hung et al., 2017; Thorsen et al., 2022).

A common perinatal strategy is to employ a decentralized regionalized system, where birthers may need to travel to nonlocal facilities to access childbirth services with the appropriate level of care for their clinical condition, thus *bypassing* their local hospital. Birthers choose whether they will bypass their local hospital and deliver at a further hospital using criteria related to their individual resources (e.g., access to transportation) and hospital system factors (e.g., access to insurance coverage) (Kozhimannil et al., 2015; Roh & Moon, 2005). Whether rural hospitals are bypassed or receive bypassing patients may help determine whether the obstetric unit will be able to remain financially viable. From a healthcare equity perspective, it is also important for hospitals to know whether certain subgroups of birthers avoid utilizing the hospital's services. This can better inform resource allocation decisions that aim to improve outcomes for those subgroups. Thus, it is important to understand how hospital and birther characteristics are associated with obstetric bypassing.

Potential and realized access are closely related to concepts of patient choice (more generally) and bypassing (specifically). Potential access, from the facility/provider perspective, assumes that improving the availability of services improves access to services, whereas realized access reflects the patient's point of view and their actual utilization of care (Hwang et al., 2017; Jang et al., 2017). Models estimating realized access can help decision makers better understand patient healthcare utilization behaviour and inform resource allocation decisions aimed at expanding access. This study focuses on the U.S. state of Montana to estimate realized access

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in hospital-based obstetric units. With the 4th largest area and 7th smallest population in the United States, Montana is an ideal location to study rural obstetric care utilization. Montana is home to twelve federally recognized tribes and seven American Indian reservations *(Montana Governor's Office of Indian Affairs)*, located in remote rural areas of the state with limited access to obstetric services. Research shows that in the state of Montana, birthers travel an average of 42 min to give birth at a hospital, 20% of birthers travel over one hour, and 5% of birthers travel over two hours (Thorsen et al., 2022). Travel time is worse for reservation-dwelling and American Indian/Alaska Native (AI/AN) birthers, who also have elevated rates of poor maternal and infant health outcomes (Thorsen et al., 2022).

Research indicates that some subgroups in Montana, including rural and AI/AN people, are more likely to bypass their local hospital-based obstetric unit to deliver elsewhere (Thorsen et al., 2023). Furthermore, the likelihood of bypassing depends on the birther's health risk, rurality, and payer type. The US healthcare system includes a complex mix of self-payers, private insurance, public insurance (e.g., Medicaid or military insurance), or Indian Health Services (IHS). IHS is a US government-funded health service delivery system for tribes, fulfilling treaty obligations in exchange for land cessions, and is an important source of health care for AI/AN birthers. IHS is not insurance *per se* but distributes funds to tribal health units and enables patients with IHS coverage to access care at non-IHS facilities vis-a-vis Purchased/Referred Care (PRC) programs (USDPHHS, 2021). Previous research has found that birthers are more likely to bypass low-volume hospitals (Bronstein & Morrisey, 1991), yet limited research has focused on the implications of obstetric bypassing for hospitals *via* lower-than-expected birth volumes. The current study considers both potential and realized access to hospitals, which is critical for understanding rural hospital financial sustainability and access to services for rural and racial minority populations.

This research study is outlined as follows:

1. We developed three inferred attractiveness GBMs to estimate realized access to obstetric care at hospitals in Montana. The inferred attractiveness GBM is an optimization model that solves for (infers) the relative attractiveness of hospitals by minimizing the difference between the actual birth volume and the model's estimated birth volume for each hospital—zip code combination.
2. We compared our GBM results to a benchmark Conditional Logit Model.
3. We examined hospital attractiveness values and model error across hospital levels to illustrate the effectiveness of our novel GBM variant.
4. To examine potential disparities in birther subgroups, in line with a healthcare equity framework:
   a. We used a bootstrapping analysis to determine whether birther subgroups produce similar model error results as the entire population.
   b. We resolved the GBMs for birther subgroups to determine whether the distance decay parameter varied and if the models produced similar hospital attractiveness values and errors.

Our general framework is used to better understand how hospital-based childbirth services are utilized across Montana for different subgroups. The research contributions of this study are:

- We apply inferred attractiveness GBMs to estimate realized access to obstetric care at hospitals. While previous studies have used GBMs to model general hospital demand, we are specifically focusing on obstetric services (giving birth), which has not been done heretofore. Furthermore, we use actual birth records instead of estimating market share via secondary data (Drezner & Drezner, 2002) or customer surveys (Drezner, 2006).
- We compare the characteristics of the solutions to GBM models with three types of distance decay functions: 1) No threshold (i.e., hospitals are assumed to have an unlimited catchment area); 2) A constant threshold across hospitals; 3) A novel data-driven threshold that varies across hospitals. We compare our results to a conditional logit model as a benchmark.
- Previous studies have not examined whether differences in facility attractiveness exist between population subgroups. Using the framework of a health equity perspective, this paper presents a bootstrapping analysis conducted to examine the extent of differences in the models' errors for certain subgroups (e.g., higher error for AI/AN birthers). We also resolved the GBMs for birther subgroups to examine differences between model results and errors. From a modelling perspective, results suggest that the birther population does not behave in a homogeneous manner, and this should be considered by researchers using GBMs in a healthcare setting. From a policy perspective, the existence of large hospital attractiveness differences between subgroups may be an important consideration for policy makers looking to improve perinatal healthcare equity.

1.1. Literature review

Gravity-based models (GBMs) have been used to model spatial choice behaviour and demand at facilities across a variety of industries including retail shopping (Drezner, 1994; D. L. Huff, 1963), hospitals (McLafferty, 1988; Teow et al., 2018), and petroleum distribution (Smith & Moses, 1996). GBMs have also been incorporated into various theoretical modelling frameworks allowing for extensions such as facility location optimization (Drezner & Drezner, 2007) or elasticity of demand (McGarvey & Cavalier, 2005).

GBMs are based on the laws of Newtonian physics, in which the attraction between two entities is directly proportional to their size and inversely proportional to their distance (Young, 1975). Reilly (1931) first used this concept to study retail competition between cities using breaking points. Huff improved this model *via* a probabilistic approach to study shopping centres, addressing shortcomings of earlier spatial preference measures (D. L. Huff, 1963, 1964). Probabilistic GBMs use relative attractiveness and distance to calculate the probability of an entity visiting a facility considering a distance decay function.

Huff's original probabilistic GBM is as follows, where the numerator in (1) may be referred to as the *utility* of facility j to customers at location i.:

$$P(C_{ij}) = \frac{\frac{S_j}{T_{ij}^{\lambda}}}{\sum_{j=1}^{n} \frac{S_j}{T_{ij}^{\lambda}}} \tag{1}$$

where

$P(C_{ij})$ represents the probability a consumer from location i visits shopping centre j.

S_j represents the attractiveness of shopping centre j.

T_{ij} is the travel time from location i to shopping centre j.

λ is a parameter used in the distance decay function $\frac{1}{T_{ij}^{\lambda}}$.

If the total number of customers in each location is known or can be estimated, then that value is multiplied by $P(C_{ij})$ to calculate the expected number of customers (market share) to visit each shopping centre. This can then be compared to the actual number of customers that visited each centre, assuming that data is available or can be estimated from secondary sources.

More comprehensive discussions of the history, evolution, and applications of probabilistic GBMs are provided by Joseph and Kuby (2011) and (Drezner, 2019). As Drezner (2019) notes, the commonality between all of these models is four variables: demand, distance, facility attractiveness, and market share. Demand is usually represented by the total number of customers (D. L. Huff, 1963, 1964) or a similar metric such as buying power (Drezner & Drezner, 2002). Traditionally, demand, distance, and facility attractiveness have been considered independent variables, and market share is the dependent variable. Drezner and Drezner (2002) proposed the inferred attractiveness GBM to estimate facility attractiveness in relation to market share. By minimizing the sum of squared error between estimated and calculated market share, the optimization model determined facility attractiveness using data on mall sales and community buying power (Drezner & Drezner, 2002).

Common distance measures include Euclidian, Manhattan (rectilinear), travel distance, travel time, travel cost, or other factors (Joseph & Kuby, 2011), along with potential transformations, such as the distance correction approach of Drezner and Drezner (1997). Several values of λ have been used for the distance decay function, ranging from 1.27 to 3.191 (Drezner, 1994, 2006; Drezner & Drezner, 2002; Huff, 1962; Huff, 1966). Alternative distance decay functions such as $e^{1.705d^{0.409}}$ have also been used (Bell et al., 1998; Drezner & Drezner, 2002). Most recently, Drezner et al. (2020) proposed replacing the facility attractiveness component with a facility-dependent distance decay function, based on the concept that more attractive facilities have a slower decay than less attractive ones.

GBMs have been applied in health services research. Spatial access GBMs having a maximum distance threshold beyond which no utility exists have been used to examine access to physicians and primary care (Luo & Wang, 2003; Schuurman et al., 2010) and medical facilities (Cheng et al., 2021). GBMs are also useful in predicting and modelling patient visitation patterns (Bucklin, 1971; McLafferty, 1988) and estimating hospital catchment areas (Jones et al., 2011) and bed demand (Teow et al., 2018).

Researchers examining birthers' decisions on birth location have utilized regression-based methods. Many of these studies have modelled a binary decision between two options (e.g., home versus hospital) using logistic regression models (Dickson et al., 2016; Sahoo et al., 2015; Semary et al., 2021). Our Montana-based scenario considers birther choice across a set of 28 different health facilities. The paper most closely related to this scenario is Hwang et al. (2017). Using a conditional logit model to estimate realized obstetric access in Korea, four components were considered (travel time, hospital level, urbanization of hospital, and number of obstetric specialists) to estimate the probability that a birther in a geographical area gives birth at a particular health facility (Hwang et al., 2017). Model results were then compared to actual patient volumes. In (Jang et al., 2019), members of this author group incorporated the conditional logit model into a facility location optimization model that aimed to improve perinatal access in under-served areas.

2. Materials and methods

2.1. Data

Montana birth records from 2014 to 2018 were used for this study. The original data set had 60,461 records. Non-residents, records with missing zip codes, home births, and births at birthing centres or hospitals that did not have an obstetric unit were excluded from the analysis. A total of 56,117 births from 374 zip codes occurring at 28 facilities were in the final data set. The hospital level of obstetric care was derived from the 2018 annual survey of the American Hospital Association. Of the 28 facilities, 19 were considered level 1 (basic care, provide services for uncomplicated cases), 6 facilities were level 2 (specialty care, provide services for most complicated cases), and 3 facilities were level 3 (sub-specialty care, provide services for all complicated cases). Distances were measured as the road miles from the zip code centroid to the hospital and calculated using a Python script to access the Google Distance Matrix API.

The birth records contained data on the birther's race, education, location of residence (including whether the birther resided on an American Indian Reservation), and payment source. The Kotelchuck Prenatal Care Index (Kotelchuck, 1994) was used to measure the adequacy of the birther's prenatal care. Data also specified whether the birth was by Caesarean, was preterm, had low birth weight, and if the birther had one or more pregnancy risk factors (e.g., gestational diabetes). Appendix A, which can be accessed *via* our public GitHub site https://github.com/rgmcgarvey/JORS235424610, summarizes descriptive information about obstetric bypassing, showing the proportion of all births and bypassing births that occurred at each (aggregated) facility obstetric level. A birth was categorized as a bypass if the birther drove at least 15 miles past their nearest facility to give birth (Thorsen et al., 2023). Bolded values in each category indicate the highest proportion for each subgroup across the OB Level, to help illustrate trends in the distribution of births across the facility obstetric level.

Figure 1 contains a series of maps that show, for each zip code, the distance to the nearest facility for different levels of obstetric care (i.e., potential access) and the average number of miles driven give birth (i.e., realized access). As Figure 1 illustrates, there are large variations in distances across the state. The hospitals that can provide a higher level of care (levels 2 and 3) are located primarily in the central and western regions. The more rural areas in the eastern and northern regions are often hundreds of miles away from these facilities. The realized access map most closely resembles the distance to the nearest level 1 facility, although it appears that many birthers in the northeastern corner choose to bypass (and travel substantial additional distance to their utilized facilities).

2.2. GBM formulations

In this section, we develop the three GBMs that are considered in this paper. Appendices C-K, which can be accessed *via* our public GitHub site https://github.

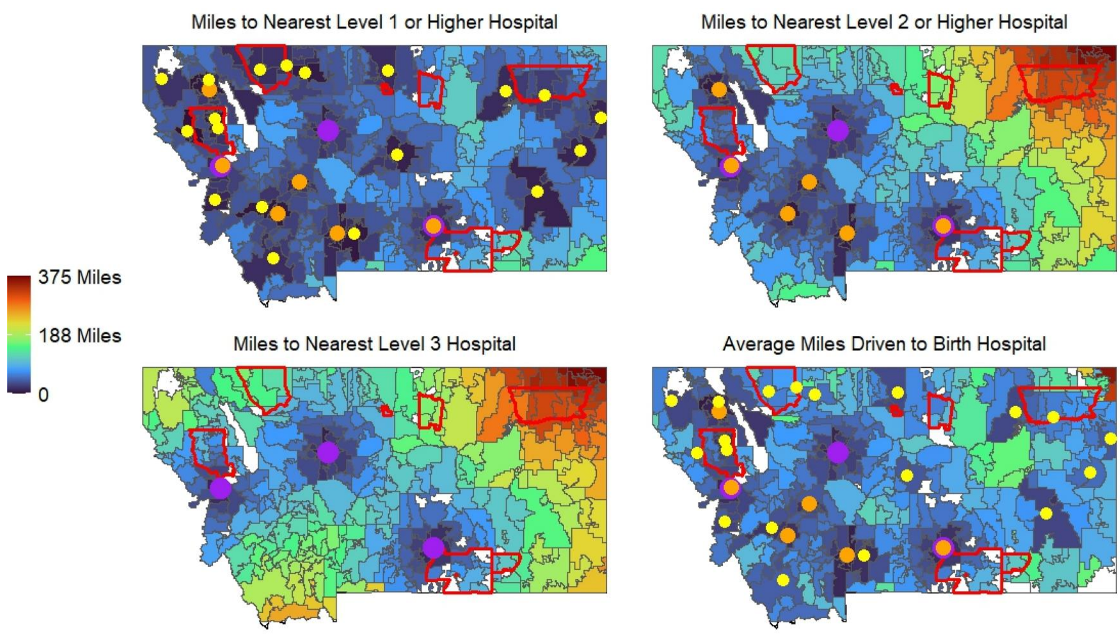

Figure 1. Zip code distances to nearest facility and average miles driven to give birth.

com/rgmcgarvey/JORS235424610, provide additional details needed to reproduce our computational results.

2.2.1. NO_MAX model

The first GBM, referred to as NO_MAX, follows a similar approach (Drezner & Drezner, 2002). The model does not contain a maximum distance parameter beyond which no utility exists. The NO_MAX formulation is as follows:

Index sets:

I : Set of zip codes i

J : Set of birth hospitals j

Decision variables:

w_j = attractiveness of birth hospital j

E_{ij} : estimated number of births from zip code i to occur at birth hospital j

$u_{i,j}$: utility of birth hospital j for birthers at zip code i

Parameters:

d_{ij} : distance (road miles) between zip code i and birth hospital j

A_{ij} : number of births from zip code i that occurred at birth hospital j

λ = distance decay parameter

Δ = distance threshold parameter beyond which no utility exists

$$\delta_{ij} = \begin{cases} d_{ij} & \text{if } d_{ij} \le \Delta \\ \infty & \text{otherwise} \end{cases}$$

Objective Function:

$$\text{Minimise } \sum_{i,j} \left| A_{ij} - E_{ij} \right| \tag{2}$$

Subject to:

$$u_{ij} = \frac{w_j}{\delta_{ij}^{\lambda}} \qquad \forall i,j \tag{3}$$

$$E_{ij} = \left(\sum_k A_{ik} \right) \frac{u_{ij}}{\sum_k u_{ik}} \qquad \forall i,j \tag{4}$$

$$w_j \ge 0 \qquad \forall j \tag{5}$$

The objective function (2) minimizes the sum of the absolute value of the difference between the number of actual births and estimated births for each zip code—hospital combination. Constraint (3) defines the utility parameter as the ratio of hospital attractiveness to the distance decay function, provided the distance is less than or equal to the distance threshold beyond which no utility exists. Recall that for the NO_MAX model, $\Delta = \infty$. Constraint (4) calculates the estimated number of births by taking the product of the total number of actual births in a zip code and the model's estimated percentage of births for the zip code—hospital pair. We define model percentage error as the total model absolute percentage error, which is the objective value divided by the total number of births:

$$\frac{\sum_{i,j} \left| A_{ij} - E_{ij} \right|}{\sum_{i,j} A_{ij}} \tag{6}$$

2.2.2. UNIV_MAX model

The second GBM, referred to as UNIV_MAX, contains a universal maximum distance parameter beyond which no utility exists. This is based on the maximum reservation distance concept from the central place models of Losch (1954) and (Christaller & Baskin, 1966). Ghosh and Craig (1991) note there is considerable empirical evidence suggesting a maximum distance that consumers are willing to travel often exists, and one might envision that a similar dynamic occurs for birthers in a large rural state where facility distances range up to several hundred miles. The other health service GBM study in a rural location, (Schuurman et al., 2010), used a similar concept that produced good results. The UNIV_MAX formulation is identical to NO_MAX, except that Δ is no longer set to ∞. We discuss our method for selecting Δ in section 3.1.

2.2.3. FAC_DEMAND model

The third GBM, referred to as FAC_DEMAND, contains a maximum distance parameter for each hospital, Δ_j, that is based upon a given percentage of births that occurred. For example, assuming a 95% threshold, if a hospital had 100 total births, then Δ_j would equal the minimum distance such that at least 95 births occurred to birthers residing within Δ_j distance of the hospital. In our data, higher-level higher-volume facilities tended to draw from a much larger area than lower-level lower-volume facilities (this approach is similar in some respects to the approach of Drezner et al. (2020) discussed above). FAC_DEMAND is an attempt to capture this dynamic in a more nuanced manner than UNIV_MAX. In this model, the parameter δ_{ij} in the NO_MAX and UNIV_MAX models is changed to reflect a facility-specific Δ_j value:

$$\delta_{ij} = \begin{cases} d_{ij} \text{ if } d_{ij} \le \Delta_j \\ \infty \text{ otherwise} \end{cases} \tag{7}$$

We discuss our method for selecting Δ_j in Section 3.1.

Figure 2 illustrates a simple example of the three GBM variants with a single zip code and two birth facilities. The ratio of actual births to estimated births for a hospital is also calculated, where a ratio greater than 1 means that a hospital had more births occur than was estimated, while a ratio less than 1 means that a hospital has fewer births occur than was estimated. There is no splitting of the data

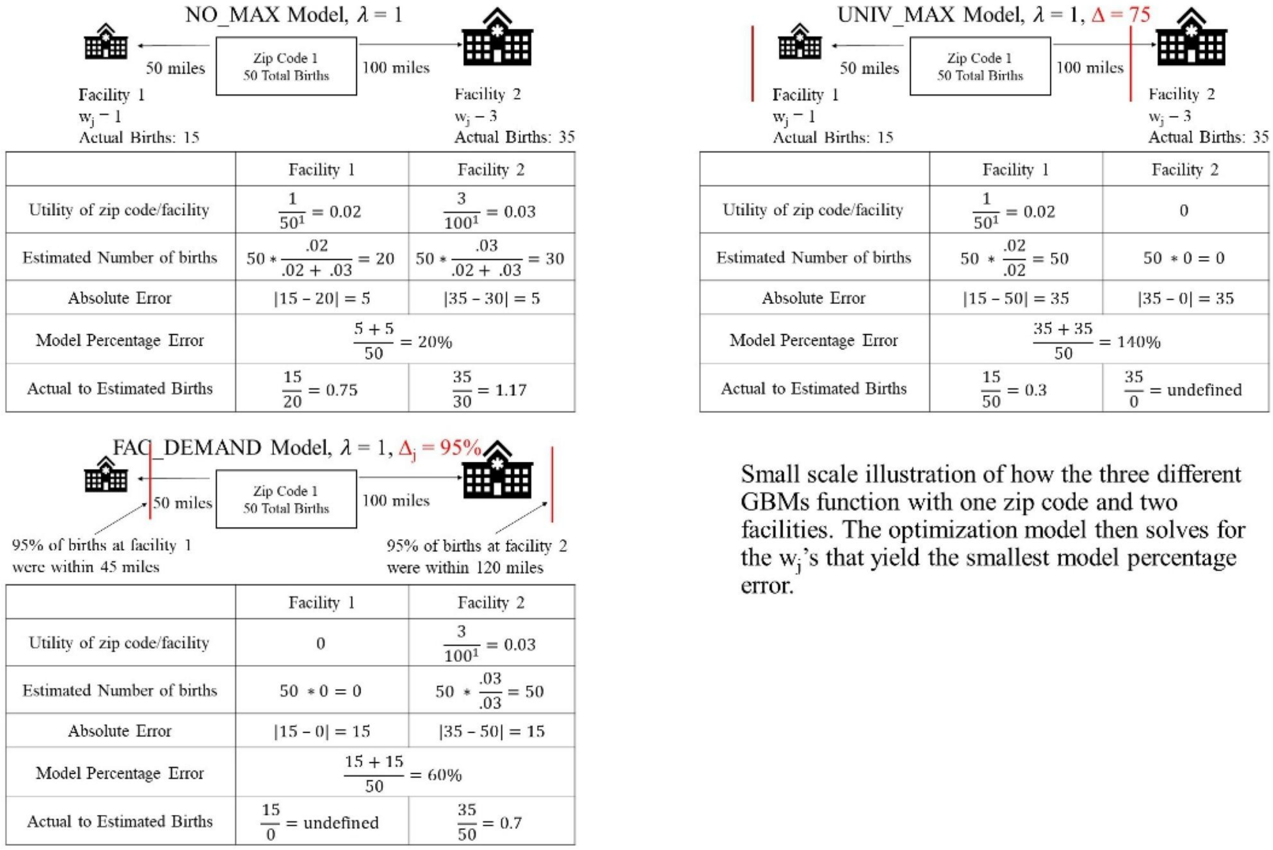

Figure 2. Small scale illustration of GBM models.

between a training and test set, similar to the inferred attractiveness models in Drezner and Drezner (2002) and (Drezner, 2006).

2.3. Conditional logit model (benchmark)

Next, we compare our GBM approach to an established benchmark: a discrete choice model, specifically the conditional logit model (CLM), first introduced by McFadden (McFadden, 1973). In a CLM, a decision maker (here, a birther) faces a choice between several alternatives (here, hospitals) and the choice is a function of the characteristics of the alternatives and the decision maker. The CLM produces a closed-form expression for the probability that a decision maker i chooses alternative j out of J as

$$P_{ij} = \frac{e^{V_{ij}}}{\sum_{j' \in J} e^{V_{ij'}}} \tag{8}$$

Hwang et al. (2017) previously validated a CLM in the same context as our problem (estimating patients' choice of delivery hospital) using travel time, urbanization of hospital location, and number of obstetricians as factors for specifying V_{ij}. Observe that the structure of the P_{ij} term in equation (8) is very similar to the GBM's E_{ij} term in equation (4).

In line with (Hwang et al., 2017), we chose similar factors for our CLM that were available in our data. First, we include Distance measured in road miles from birther i to hospital j. Second, we include the binary variable MetroHospital (equal to 1 for hospitals located in metro counties; zero otherwise). Third, recall that the hospitals in our dataset are level 1 (basic care), level 2 (specialty care), or level 3 (subspecialty care). The model includes a dummy variable OBLEV2 for level 2 care and a dummy variable OBLEV3 for level 3 care (level 1 care is the reference group). In summary, the CLM is specified as:

$$V_{ij} = \beta_1 Distance_{ij} + \beta_2 MetroHospital_j + \beta_3 OBLEV2_j + \beta_4 OBLEV3_j \tag{9}$$

3. Results

3.1. Parameter selection, model runs, and percentage error

The final parameters for each GBM were selected using a simple search procedure that minimized model percentage error. For NO_MAX, λ was varied from 1—5 in increments of 0.05. For UNIV_MAX, Δ ranged from 50—295 miles in increments of 5 miles; then, at each Δ, λ was then varied from 1—5 in increments of 0.05. For FAC_DEMAND, the percentage Δ_j threshold was set from 90—99% in increments of 1%; then, for each Δ_j, λ was varied from

1—5 in increments of 0.05. All models were run *via* the AMPL NEOS server (neos-server.org). Initial experimentation with several different nonlinear solvers showed that the Artelys Knitro solver (Byrd et al.) consistently yielded the best results.

For the benchmark approach, the CLM coefficients were computed using the CLOGIT command in Stata 16. We also evaluated measures of fit for the model and found it to be reasonable. The p-values for all coefficients are statistically significant at the $p < 0.05$ level and McFadden's R2 is 0.770. Additional details of the CLM can be found in Appendix B, which can be accessed *via* our public GitHub site https://github.com/rgmcgarvey/JORS235424610.

Table 1 summarizes the GBMs and CLM model final parameters and percentage error. Observe that each of the GBMs outperforms the CLM benchmark. The FAC_DEMAND model achieved the lowest model percentage error, almost 50% less than the CLM benchmark. The GBMs outperforming the CLM benchmark may be due to the importance of distance decay in predicting birth location, which is not among the CLM parameters.

3.2. Contrasting attractiveness of hospitals across GBMs

The inferred attractiveness GBMs have homogeneity, meaning the objective function will not change if the w_j values are multiplied by a common non-zero number (Drezner & Drezner, 2002). Thus, it is typical to set one of the w_j values equal to 1 as a reference point (Drezner & Drezner, 2002; Teow et al., 2018). However, to facilitate communication of results, we instead elected to scale the models' outputs according to the following formula:

$$\tilde{w}_j = \frac{w_j}{\max_j w_j} \tag{10}$$

For example, if the maximum hospital w_j for a model run was 120, then this hospital's $\tilde{w}_j$ value was set to 1. For this same model run, a different hospital with a w_j of 60 would have its $\tilde{w}_j$ value reported as 0.5, indicating this facility is 50% as attractive as the hospital with the highest attractiveness.

Figure 3 displays a column chart of the $\tilde{w}_j$ values for each of the 28 hospitals for each of the three GBMs. Facilities are ordered on the x-axis from highest to lowest number of actual births within each OB level. Observe that the three highest $\tilde{w}_j$ values for NO_MAX occur at level 3 hospitals, and that all $\tilde{w}_j$ values for level 1 hospitals are below 0.2 for the NO_MAX and UNIV_MAX models. Also observe that the maximum w_j value (i.e., the value used for the denominator in Equation (10)) occurs at a level 3 hospital for NO_MAX and UNIV_MAX. It appears that both the NO_MAX and UNIV_MAX models estimate greater attractiveness for level 3 hospitals. The inclusion of varied catchment areas (FAC_DEMAND) results in level 1 hospitals appearing more attractive, including one level 1 hospital achieving the second-highest attractiveness values overall. This result suggests that several level 1 hospitals successfully pull in birthers from their local market, although the catchment area is smaller than that of the higher-level hospitals. Observe that the $\tilde{w}_j$ values for level 3 facilities in FAC_DEMAND are smaller than in the other models, suggesting that while these hospitals have greater potential reach to outlying birthers (with the largest catchment areas), the most attractive hospital overall is a level 2 hospital, indicating a large market share in its surrounding area. Figure 3 demonstrates the importance of considering demand-based catchment area sizes when estimating facility attractiveness.

3.3. Examining the ratio of actual to expected births across GBMs

Figure 4 displays scatterplots of the ratio of actual births to estimated births across birth volume for the hospitals. Hospitals with ratios greater than 1 had more births than expected, while hospitals with ratios below 1 had fewer births than expected (perhaps due to birthers bypassing these facilities). High-volume hospitals with 4,000 or more births had accurate estimations, as minimizing the sum of total absolute errors favours solutions with low error for these hospitals. The ratios for level 1 hospitals in the FAC_DEMAND model were closer to 1.0, indicating better estimates for low-volume hospitals, while the NO_MAX and UNIV_MAX models had greater variability in their level 1 hospital ratios, implying greater error in estimation. The FAC_DEMAND model's ability to provide better estimates for low-volume hospitals is likely influenced by its consideration of a demand-based catchment area size when estimating facility attractiveness.

Table 1. GBMs and CLM Model Final Parameters and Percentage Error.

Model	Δ or Δ_j %	λ	V_{ij}	Model percentage error
NO_MAX	N/A	3.85	N/A	20.37%
UNIV_MAX	Δ =150 miles	3.0	N/A	18.82%
FAC_DEMAND	Δ_j = 96%*	1.4	N/A	15.73%
CLM (Benchmark)	N/A	N/A	$-0.043\text{Distance}_{ij} + 1.565\text{MetroHospital}_j + 0.055\text{OBLEV2}_j + 0.806\text{OBLEV3}_j$	30.73%

*The 96% Δ_j values ranged from 32.6 miles to 153.5 miles.

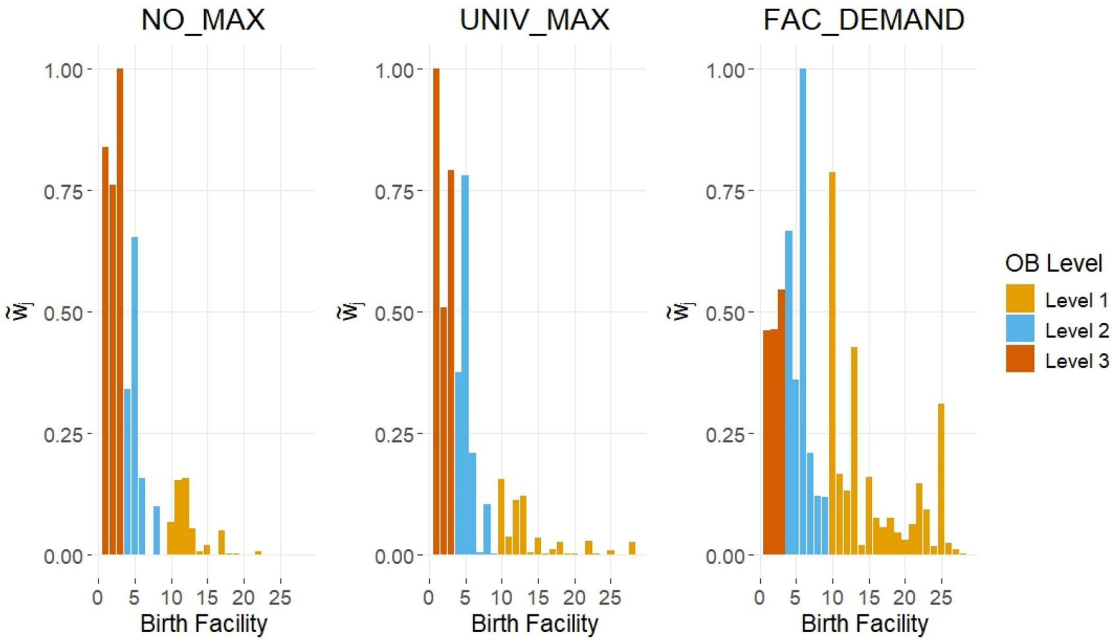

Figure 3. Scaled attractiveness values for hospitals by GBM.

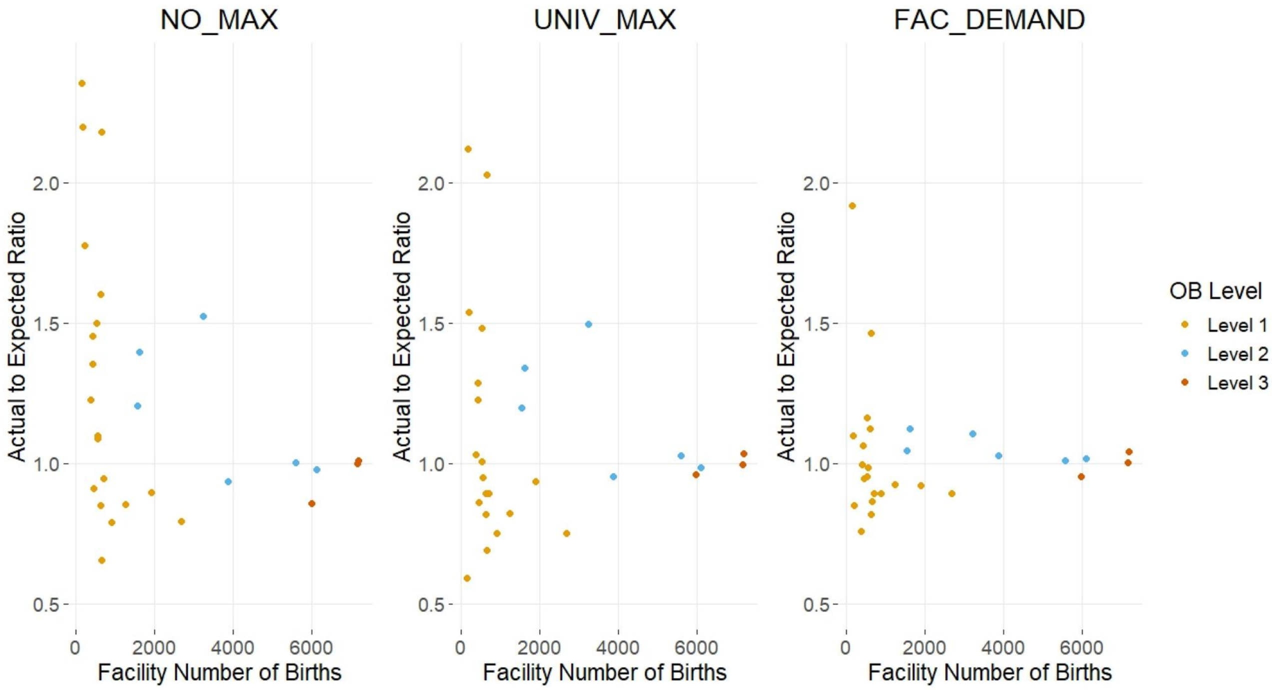

Figure 4. Scatterplots of actual births to estimated births by birth facility volume.

3.4. Examining variation across birther subgroups

We conducted further testing to determine whether the w_j values produced by the final parameters (see Table 1) worked well to estimate realized access for birther subgroups, in line with a health equity framework that emphasizes the role of fair access to resources, including health services (Peterson et al., 2021). In this approach, we used the distance decay parameters (Table 1) along with the resulting w_j values obtained in Section 3.1, changed A_{ij} from the entire population of birthers to a specific subgroup of birthers and then computed the new model percentage error. Table 2 shows the errors for each model for various subgroups of birthers. For example, the error of the NO_MAX model with $\lambda = 3.85$ increased from 20.37% for All Births ($n = 56{,}117$) to 43.21% when considering only the subgroup of Reservation-dwelling birthers ($n = 5{,}048$). The percentage error is then compared to a 99% confidence interval obtained from 1000 bootstrapped samples of the same size as the subgroup to illustrate significant differences. A (-) in Table 2 denotes the percentage error was

Table 2. Birther subgroup model percentage error compared to 99% confidence interval of 1000 bootstrapped random samples.

Population subgroup	n	%Error NO_MAX	% Error UNIV_MAX	% Error FAC_DEMAND
All births	56117	20.37%	18.82%	15.73%
Reservation dwelling status				
Non-reservation	50159	$^{(-)}$18.79%	$^{(-)}$17.23%	$^{(-)}$14.19%
Reservation	5048	$^{(+)}$43.21%	$^{(+)}$41.57%	$^{(+)}$40.09%
Race				
White	46327	$^{(-)}$20.22%	$^{(-)}$18.59%	$^{(-)}$15.15%
Black	389	32.71%	32.30%	32.18%
AI/AN	6094	$^{(+)}$32.65%	$^{(+)}$30.82%	$^{(+)}$31.27%
Asian	730	$^{(-)}$26.42%	$^{(-)}$25.27%	$^{(-)}$23.71%
Hispanic	2577	$^{(-)}$21.85%	$^{(-)}$20.42%	$^{(-)}$17.64%
Mother's education				
Less than high school	5,862	$^{(+)}$31.35%	$^{(+)}$29.08%	$^{(+)}$27.82%
High school	16,027	22.90%	21.10%	18.46%
Some college	17,487	$^{(+)}$24.32%	$^{(+)}$22.89%	$^{(+)}$20.27%
College plus	16,544	22.48%	21.04%	18.17%
Health risk factor(s)				
None	40471	20.91%	18.85%	16.23%
1 or More	15646	$^{(+)}$26.71%	$^{(+)}$25.47%	$^{(+)}$21.98%
Birth was preterm	4,764	$^{(+)}$39.52%	$^{(+)}$38.48%	$^{(+)}$36.51%
Birth was low birth weight	4,007	$^{(+)}$38.89%	$^{(+)}$37.70%	$^{(+)}$35.42%
Caesarean section	17,072	$^{(+)}$24.59%	$^{(+)}$23.28%	$^{(+)}$19.96%
Kotelchuck prenatal care index				
No prenatal care	505	$^{(+)}$49.83%	$^{(+)}$49.25%	$^{(+)}$50.21%
Inadequate care	8,601	$^{(+)}$31.73%	$^{(+)}$29.10%	$^{(+)}$27.19%
Intermediate care	4,303	$^{(+)}$37.43%	$^{(+)}$34.54%	$^{(+)}$32.67%
Adequate care	22,794	$^{(+)}$23.88%	$^{(+)}$21.82%	$^{(+)}$19.48%
Adequate plus care	19,531	$^{(+)}$26.70%	$^{(+)}$25.66%	$^{(+)}$22.28%
Payer type				
Medicaid	22902	$^{(+)}$26.69%	$^{(+)}$25.13%	$^{(+)}$22.49%
Private	28910	21.60%	20.06%	16.98%
Self-pay	1759	$^{(+)}$38.75%	$^{(+)}$36.31%	$^{(+)}$34.07%
IHS	911	$^{(+)}$77.14%	$^{(+)}$74.60%	$^{(+)}$73.95%
Military	1183	$^{(-)}$11.87%	$^{(-)}$12.61%	$^{(-)}$11.75%

($^{-}$ falls below the model error, $^{+}$ falls above the model error).

below the confidence interval and a (+) indicates that the error was above the confidence interval. Vastly different errors for subgroups (e.g., the subgroup using IHS payer) indicate that the attractiveness levels for hospitals in the models (which were trained on the entire sample) are different for birthers from certain subgroups. The higher error observed when estimating hospital utilization among some subgroups (e.g., lives on a reservation, AI/AN birthers, no prenatal care, IHS payer) may reflect that these subgroups are more likely to give birth at Level 1 facilities with low birth volumes (Thorsen et al., 2023).

Next, we attempted to reduce error for each subgroup by re-solving the GBMs. For each subgroup, the distance decay parameter λ was varied from 1-5 incrementing every 0.05 and the GBM was re-solved. Table 3 shows the error-minimizing distance decay parameter λ for each subgroup and associated model percentage error. Compared to the error percentages reported in Table 2 that used the parameter values trained on the entire sample, the models solved for each subgroup achieve substantially lower error percentages for many subgroups. Intuitively, models for subgroups with a greater likelihood to travel farther to give birth would have lower λ values while models for subgroups less likely to travel farther would have higher λ values. For example, the subgroup that had the highest bypassing rate, reservation-dwelling birthers, had the lowest λ value across all three models in Table 3. Similarly, the race subgroup that had the highest bypassing rate, AI/AN, consistently had the lowest λ across all three models. Other subgroups with health risk (one or more risk factors, preterm birth, low birth weight) also have lower λ across all three models. Observe that the distance decay parameter is smallest for the FAC_DEMAND model for each subgroup.

Figure 5 shows a scatterplot for each GBM of the distance decay λ values from Table 3 versus the subgroups' bypassing percentage rate. The FAC_DEMAND model has the strongest (negative) correlation of the three models (-0.57, versus -0.02 and -0.29 for the NO_MAX and UNIV_MAX models, respectively). It appears that a demand-based distance threshold GBM allows the search procedure to identify a distance decay function that more closely aligns with the theoretical interpretation of the λ parameter. This could be another reason that FAC_DEMAND better estimates realized access.

Figures 6–8 display the variation in $\tilde{w}_j$ values that are produced by re-solving the GBMs for each subgroup in Table 3, with the $\tilde{w}_j$ from Figure 3 (entire population) overlayed on the boxplot as an "x" for reference. Observe that the inferred attractiveness values for some hospitals are vastly different across subgroups (those with wide boxplots), while some hospitals have more consistent inferred attractiveness

Table 3. GBM λ and model percentage error after resolving for different birther sub-groups.

		NO_MAX		UNIV_MAX		FAC_DEMAND	
Population subgroup	n	λ	% Error	λ	% Error	λ	% Error
All births	56117	3.85	20.37%	3	18.82%	1.4	15.73%
Reservation dwelling status							
Non-reservation	50159	3.1	18.60%	2.3	16.97%	1.4	13.45%
Reservation	5048	2.45	33.51%	1.05	21.94%	1.05	28.31%
Race							
White	46327	3.15	19.94%	2.9	17.99%	1.4	14.55%
Black	389	4.8	20.61%	3.5	20.36%	2.95	20.30%
AI/AN	6094	2.7	26.74%	1.85	21.04%	1.15	23.60%
Asian	730	4.05	18.00%	2.35	26.20%	2.1	15.66%
Hispanic	2577	4.4	17.58%	3.45	15.85%	1.65	13.42%
Mother's education							
Less than high school	5,862	5	24.65%	3.4	23.07%	1.8	21.05%
High school	16,027	4.1	21.56%	3.3	19.61%	1.4	16.76%
Some college	17,487	3.95	23.17%	2.85	21.53%	1.35	18.53%
College plus	16,544	3.6	19.71%	2.6	17.55%	1.35	14.19%
Health risk factor(s)							
None	40,471	4.05	19.90%	3	18.04%	1.5	15.10%
1 or more	15,646	3.1	24.90%	2.6	23.01%	1.2	19.52%
Birth was preterm	4,764	4.2	28.04%	2.3	26.99%	1.15	26.37%
Birth was low birth weight	4,007	4.65	31.59%	2.55	27.95%	1.05	26.52%
Caesarean section	17,072	4.85	23.70%	3.1	21.81%	1.3	18.53%
Kotelchuck prenatal care index							
No prenatal care	505	3.9	30.92%	3.85	29.52%	2.9	27.99%
Inadequate care	8,601	3.55	29.53%	3.25	25.34%	1.25	23.59%
Intermediate care	4,303	4.35	24.82%	2.9	22.86%	1.55	21.30%
Adequate care	22,794	4.05	19.43%	2.75	17.36%	1.5	14.90%
Adequate plus care	19,531	3.2	20.64%	2.9	19.14%	1.35	15.20%
Insurance status							
Medicaid	22902	4.05	25.19%	2.45	21.71%	1.4	18.51%
Private	28910	3.15	19.42%	3	17.39%	1.45	14.34%
Self-pay	1759	5	31.09%	4.8	27.69%	1.6	26.11%
IHS	911	3.75	21.14%	1.3	16.87%	1.0	18.99%
Military	1183	3.95	8.86%	4.9	8.64%	3.6	7.94%

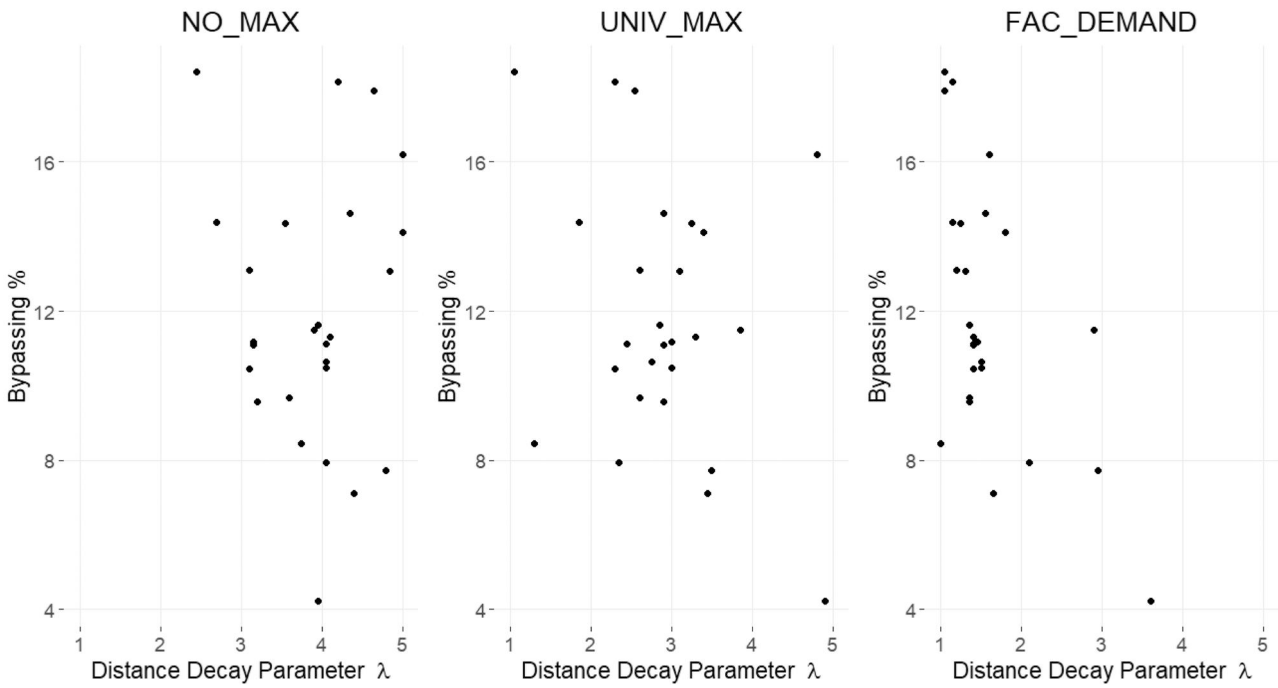

Figure 5. Scatterplots of subgroups' resolved λ versus bypassing percentage rate.

values (evidenced by narrower boxplots). Like the bootstrapping analysis, these results suggest that there is a wide variation in facility attractiveness values. Moreover, these results strengthen the argument in support of this claim, since the distance decay function parameter λ was adjusted for each sub-group, which was not the case for bootstrapping analyses.

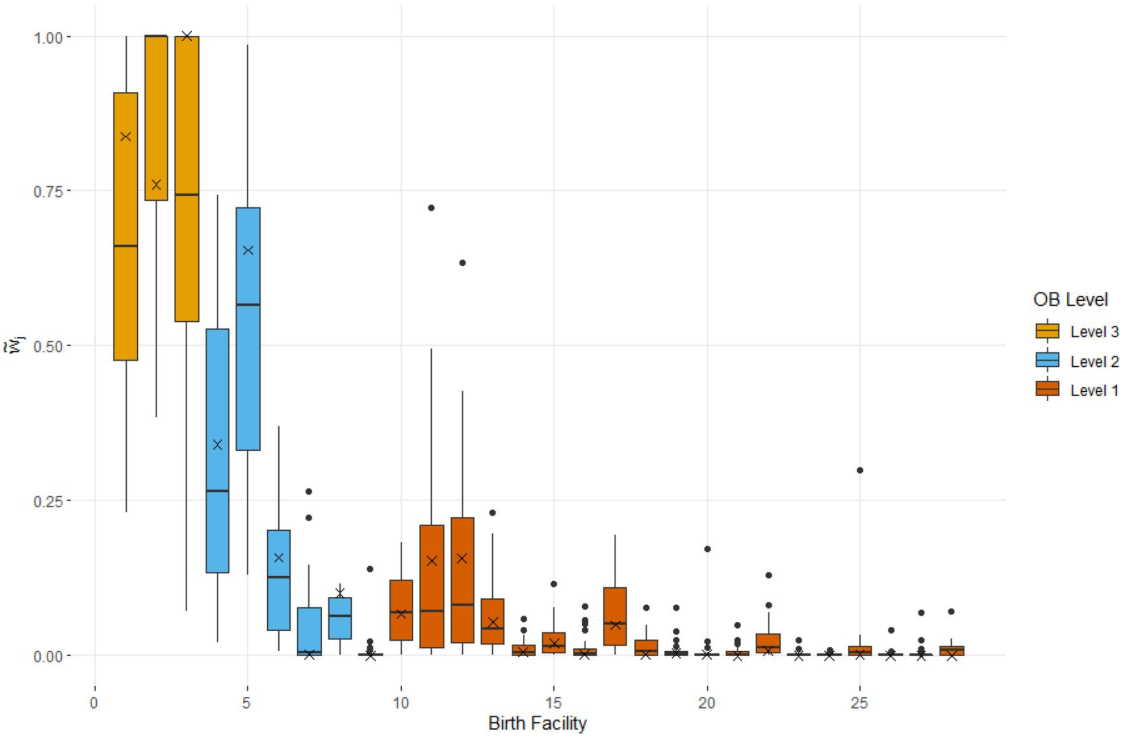

Figure 6. Box and Whisker Plot of $\tilde{w}_j$'s from resolving NO_MAX for Subgroups ("x" denotes $\tilde{w}_j$ from Figure 3).

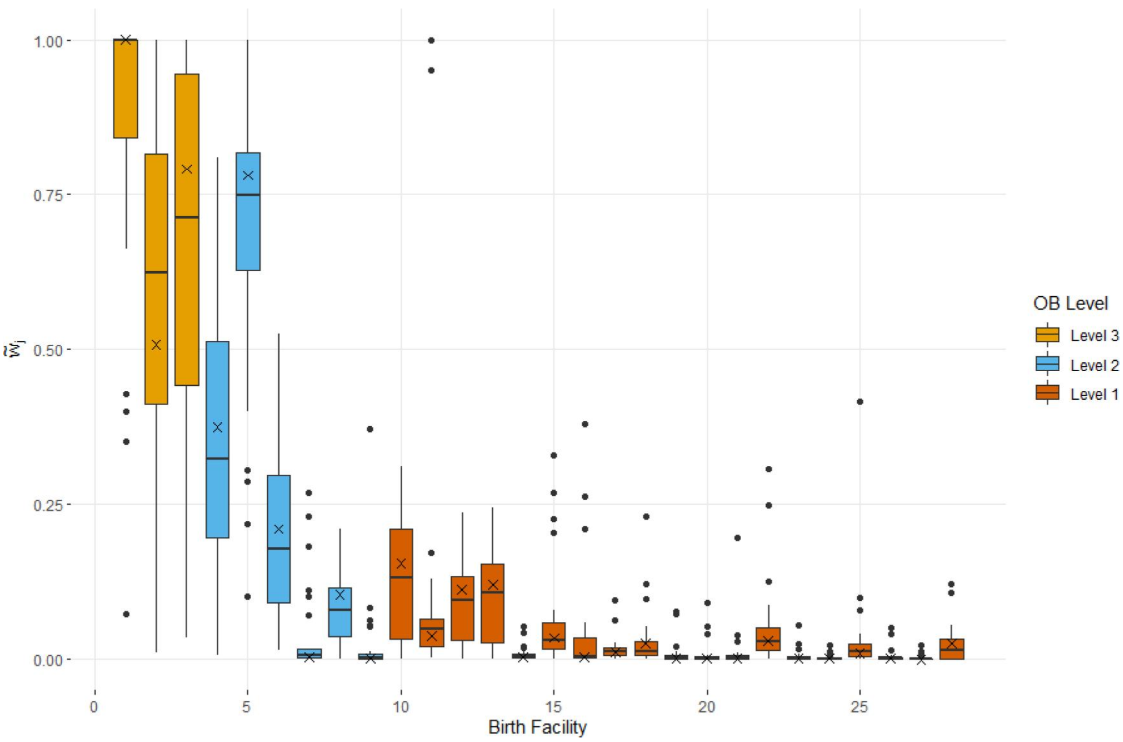

Figure 7. Box and Whisker Plot of $\tilde{w}_j$'s from resolving UNIV_MAX for Subgroups ("x" denotes $\tilde{w}_j$ from Figure 3).

4. Discussion

4.1. Advantages of FAC_DEMAND model for settings with High variation in facility concentration and travel distance

While the GBMs developed in this paper all produce smaller errors for hospitals with higher birth volume (Figure 4), the model using demand percentile-based maximum distance thresholds (FAC_DEMAND) reduced the estimates' errors for hospitals with low birth volume. Further, the FAC_DEMAND model provides a wider range of attractiveness values for level 1 hospitals compared to the other models, allowing for greater distinctions to be made between the attractiveness of these smaller hospitals. This model's assumption of facility-specific maximum

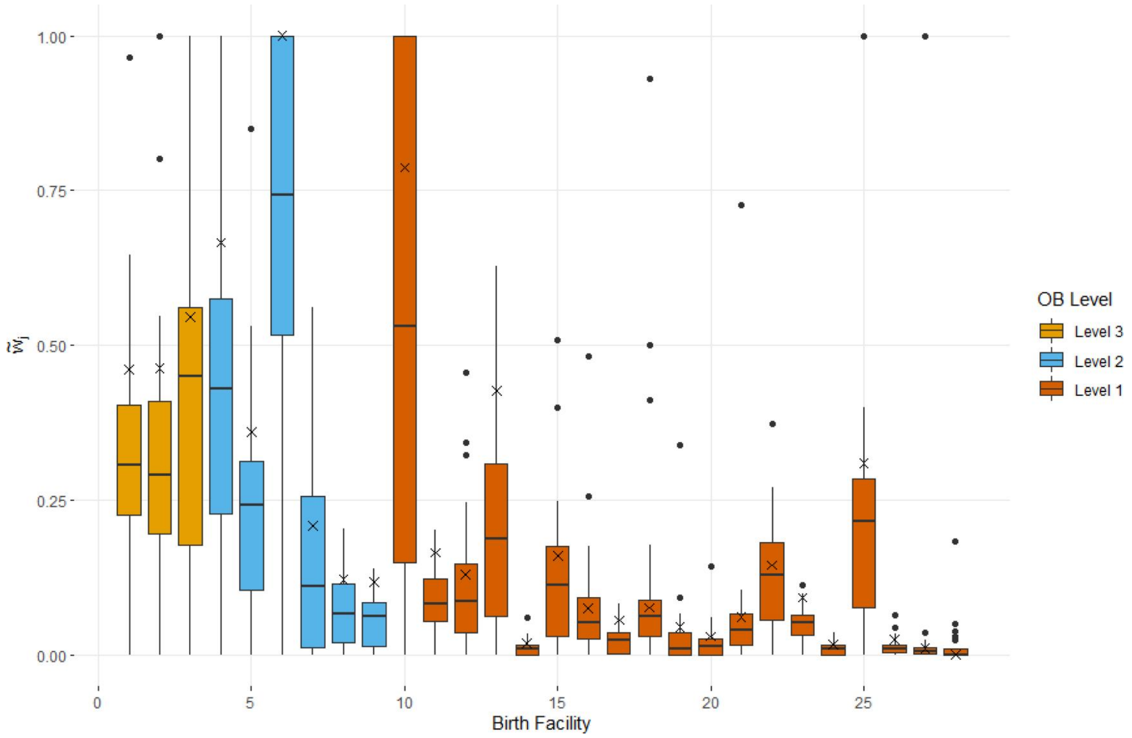

Figure 8. Box and Whisker Plot of $\tilde{w}_j's$ from resolving FAC_DEMAND for Subgroups ("x" denotes $\tilde{w}_j$ from Figure 3).

Table 4. Model Percentage Error, Hospital—Zip vs. Hospital Level Comparison.

Model	Hospital—zip percentage error (Table 1, Equation (6))	Hospital percentage error (Equation (11))
NO_MAX	20.37%	11.88%
UNIV_MAX	18.82%	10.56%
FAC_DEMAND	15.73%	5.47%
CLM (Benchmark)	30.73%	19.58%

radius is consistent with the observed data and leads to a better fit in a state like Montana, where there are significant geographic concentrations of service providers in a small number of urban areas and vast differences in distances to different patients' nearest facilities (Figure 1).

Applying the FAC_DEMAND modelling approach could help low-volume hospitals better understand the possible impacts of obstetric bypassing for their birth volume, to help with resource planning and decision-making. Regionalized hospital systems might benefit from applying this modelling approach to estimate how births are distributed within their hospital system, to plan for service utilization/demand and identify hospitals in need of additional resources (e.g., to support the additional influx of deliveries, or to understand why birthers bypass away from facilities, for example, due to insurance/payor policies or different needs of their birthing population).

From the perspective of hospital management, the FAC_DEMAND model is quite effective. Recall the GBMs minimize the error between potential and realized access for every hospital—zip code combination, and model percentage error is calculated as the total error at the hospital—zip code level (Equation (6)). However, for resource planning and decision-making purposes, hospital management would likely be more interested in estimating the total number of births at the hospital level, not the hospital—zip code level. In other words, instead of using Equation (6) to measure model percentage error, the following equation could be used:

$$\frac{\sum_j |A_j - E_j|}{\sum_j A_j} \tag{11}$$

where A_j is the actual number of births at hospital j and E_j is the estimated number of births at hospital j. Table 4 summarizes the model percentage error at the hospital level compared to the model percentage error at the hospital—zip code level (Table 1). Given the structure of the absolute value terms in Equations (6) and (11), observe that the hospital—zip code error estimate cannot be less than the hospital level estimate (moreover, these estimates could only be equal in the event that each hospital observed either all overestimates at the zip code level or all underestimates at the zip code level). Observe in Table 4 that the GBMs still outperform the CLM benchmark model, with a FAC_DEMAND hospital-level error of only 5.47%.

The intuitive negative correlation between a subgroup's distance decay λ and its bypassing rate for

FAC_DEMAND (Figure 5) suggests that the facility-specific catchment area allows for a more effective distance decay function. GBMs that have either no maximum distance threshold or a universal one appear to less accurately model subgroups that drive farther to access care. A demand-based maximum distance threshold GBM more accurately models these populations that travel farther, whether it is due to living in a rural area, having risk factors that necessitate traveling to a facility that can provide risk-appropriate care, or the birth facility accepting a specific payer type. Results suggest that researchers and policy makers should consider using a demand-based maximum distance threshold GBM such as FAC_DEMAND for modelling obstetric facility utilization patterns in rural areas. This type of GBM allows for increased discernment between the attractiveness levels of hospitals with vastly different patient volumes across large areas with significant travel distances.

4.2. Healthcare equity and subgroup variation

Bootstrapping analyses show significantly larger estimation errors for certain subgroups (e.g., AI/AN birthers), indicating that the attractiveness levels for hospitals vary across subgroups. The most pronounced differences in Table 2, i.e., the subgroups that have much higher model percentage errors (AI/AN, reservation dwelling, IHS insurance, no prenatal care, pre-term, low birthweight), also tend to be the subgroups that have more difficulty accessing obstetric services and have poorer health outcomes (Thorsen et al., 2022). Results of bootstrapping analyses underscore the importance of considering population heterogeneity when estimating healthcare utilization patterns (McGarvey et al., 2019). To advance health equity and improve healthcare access for all people, policymakers and scholars should continue to adopt modelling approaches that account for differential access and utilization patterns across population subgroups, to identify and remedy disparities in our healthcare system.

Resolving the GBMs for specific subgroups further emphasizes that attractiveness levels vary widely. Previous research suggests that factors such as cost of care, availability of complex medical services, and insurance coverage are associated with the likelihood that care is sought at a local facility or patient bypass (Adams & Wright, 1991; Radcliff et al., 2003; Roh & Moon, 2005). This is reflected in the varying attractiveness of hospitals. For certain subgroups, the attractiveness of hospital facilities may be further impacted by factors such as the availability of culturally competent care (Brooks-Cleator et al., 2018) or mistrust in providers or facilities due to perceptions of racial discrimination (Call et al., 2006), which may lead a facility to be more or less attractive for certain subgroups compared to others. Individuals who rely on IHS funding to obtain services through PRC program (i.e., contracting with non-IHS providers for health care services that cannot be obtained at an IHS facility) may be limited in the facilities where they may seek care, thereby rendering particular hospitals less attractive (Marley, 2019). Our bootstrapping analyses can help identify hospitals that are attractive or unattractive for specific subgroups and adopting this modelling approach could assist healthcare policymakers in identifying hospitals exhibiting best practices and those that may need to modify practices to improve their attractiveness for subgroups. Improving practices can help advance healthcare equity by increasing access to nearby services for birthers from health disparity groups.

The GBM models provide insight into the distance travelled by certain birthing populations, with smaller distance decay functions indicating a farther distance travelled. Reservation-dwelling and AI/AN birthers have smaller distance decay functions relative to non-reservation-dwelling and non-AI/AN birthers, likely reflecting travel burdens and lack of access to services (Thorsen et al., 2022; Thorsen et al., 2023). High-risk births also had smaller distance decay functions, indicating a greater likelihood of traveling farther to seek more risk-appropriate care. These results contribute to growing literature on the barriers to seeking care for rural birthers, particularly for AI/AN populations and individuals facing pregnancy health risks. There could be different trade-offs that people make related to travel distance and level of care, particularly for at-risk populations seeking risk-appropriate care (Lorch et al., 2021). Given the health benefits of connecting at-risk people to risk-appropriate care (Lorch et al., 2021), as well as the higher health risk profile of rural individuals who bypass (Handley et al., 2022), healthcare providers, policymakers and researchers should continue to examine the individual drivers of bypassing behaviour as well as the impacts of bypassing for the regionalized healthcare system (Handley & Lorch, 2022).

GBMs have been applied effectively in many industries, including healthcare, but they have not been used previously for birthing services. This study's results suggest that estimating utilization at low-birth-volume hospitals is challenging, particularly for certain subgroups. If resource-allocation decisions are made partly based on forecasted demand, certain targeted subgroups requiring additional resources may utilize a very different set of facilities from those forecasted, resulting in a misallocation of resources. Thus, population heterogeneity should be an important consideration in these types of forecasting models and resource allocation decisions.

4.3. Limitations and future research

Data limitations include potential transcription errors, along with the use of zip code centroids only providing approximations of driving distances. While individual health insurance information was available, information about which providers and facilities accept which particular forms of insurance was not considered.

The inferred attractiveness values in this paper are an indirect measure of multiple underlying variables, such as payer acceptance, capacity, and quality of care. The models presented cannot differentiate the effects of these factors, but identifying hospitals with high or low inferred attractiveness for certain subgroups is a crucial step towards better understanding travel burdens for birthers and potential negative outcomes for both birthers and babies (Minion et al., 2022). Future research should investigate underlying factors that influence hospital attractiveness to develop more comprehensive strategies for promoting equitable access to obstetric services.

Another potential area of future research is examining model effectiveness in an urban setting. The GBMs were applied to a rural setting with large variations in distances between facilities and birthers. The nuance in which the distance threshold (Δ or Δ_j %) for the UNIV_MAX and FAC_DEMAND models were constructed and appeared to handle this variation quite effectively. It is possible that these models would generalize to an urban setting and future research could explore if this is indeed the case.

4.4. Conclusion

In this study, we used a series of inferred attractiveness GBMs to estimate realized access to obstetric care across a large, rural, U.S. state. Our novel FAC_DEMAND GBM, in which a hospital's catchment area varies according to the distance within which a certain percentage of births occur, yielded the most accurate results, particularly for lower-volume facilities. The results suggest that varied catchment area GBMs are well-suited for large rural settings where facilities are not equally distributed geographically and there are large differences in the volume and services that facilities provide.

Birther subgroups exhibit significant variation in error and distance decay function parameters, indicating that healthcare research and policymaking should consider the heterogeneity of the birthing population. Understanding utilization patterns requires accounting for how subgroups access care. Hospital attractiveness varies across levels of obstetric care and birther subgroups. These findings emphasize the need for targeted resource allocation and consideration of population heterogeneity in GBMs for forecasting and decision-making.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

Research reported in this manuscript was supported by the National Institute of General Medical Sciences of the National Institutes of Health under Award Number P20GM104417. The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health.

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