# Three methods of instruction in high school geometry and the effects they have on achievement, retention, and attitude

by Edward Otis Thompson

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University


Abstract:
This study investigated the effect that three methods of classroom instruction had on (a) achievement of certain geometrical concepts, (b) retention of these concepts, and (c) attitude toward geometry for students enrolled in a high school geometry course. The three methods of instruction included: (I) sma11 cooperative learning groups completing van Hiele phase-based paper and pencil activities, (II) small cooperative learning groups using the computer and accompanying software to complete similar phase-based activities, and (III) whole class instruction based on traditional textbook procedures. Independent variables were methods of instruction, school, van Hiele level, attitude toward mathematics at the start of the geometry course, attitude toward geometry prior to the treatment, pretest achievement scores, gender, age, and socio-economic background. Dependent variables were posttest achievement scores, retention test achievement scores, and attitude toward geometry at the time of the posttest and at the time of the retention test.

The eight-week study was conducted in 14 geometry classrooms at five high schools in Montana. Treatments were randomly assigned to two or three geometry classes at each school. Classroom teachers conducted all instruction and testing activities. The posttest was administered at the completion of four-week unit of instruction; the retention test was administered four weeks after the posttest.

An analysis of covariance found the mean posttest achievement scores on low cognitive level items for treatments I and II were slightly higher than those for treatment III. (No differences among the treatments were detected on mean posttest achievement scores for high cognitive level items). A similar analysis found that the mean retention score for Treatment I students was higher than the mean retention score for Treatment III students with the difference being in the performance on test items which used higher order cognitive skills. There were no differences in student attitude toward geometry among the three treatments.

Based upon the findings of this study, geometry teachers should consider instructional methods using small cooperative learning groups together with phase-based materials as viable alternatives.

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# ABSTRACT

This study investigated the effect that three methods of classroom instruction had on (a) achievement of certain geometrical concepts, (b) retention of these concepts, and (c) attitude toward geometry for students enrolled in a high school geometry course. The three methods of instruction included: (I) small cooperative learning groups completing van Hiele phase-based paper and pencil activities, (II) small cooperative learning groups using the computer and accompanying software to complete similar phase-based activities, and (III) whole class instruction based on traditional textbook procedures. Independent variables were methods of instruction, school, van Hiele level, attitude toward mathematics at the start of the geometry course, attitude toward geometry prior to the treatment, pretest achievement scores, gender, age, and socio-economic background. Dependent variables were posttest achievement scores, retention test achievement scores, and attitude toward geometry at the time of the posttest and at the time of the retention test.

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Based upon the findings of this study, geometry teachers should consider instructional methods using small cooperative learning groups together with phase-based materials as viable alternatives.

# CHAPTER 1

# INTRODUCTION

## Introduction

The role of the teacher in the early American mathematics classroom was to "state a rule, give examples, and provide problems" (NCTM, 1970, p.21); and according to more recent studies (Dossey, 1988; Sirotnik, 1983), mathematics instruction in today's classrooms is much the same. As Sirotnik describes it, "the typical classroom is still didactics, practice, and little else" (p.17). The National Council of Teachers of Mathematics (1989) advocates that a variety of instructional methods (project work, group and individual assignments, and discussion between teachers and students and among students) be used so that students can approach the learning of mathematics both independently and creatively. Venerable and modern theories on how students learn mathematics (e. g., Bruner and discovery learning, Gagne and guided learning, Piaget and constructivism, van Hiele and phase-based instruction) are continually tried and discussed by mathematics educators as they attempt to improve mathematics instruction. In addition, recent improvements in technology (e. g., computers, calculators, video cassette recorders) have given the mathematics educator new tools for instructional purposes.

At the secondary level, geometry has been one of the most controversial courses in the mathematics curriculum. The National Assessments of Educational Progress (Carpenter et. al., 1980; Carpenter, et. al., 1983; Brown, et. al., 1988) indicate that student's performance in and attitude toward geometry are low. Recent research indicates that a student's attitude toward mathematics, and geometry in particular, (Han, 1986) may be related to the method of instruction. The graphing capabilities of the modern personal computer have led to the recent development of computer software which "has the potential to change the ways teachers think about what it means to know geometry" (Lambert, 1988, p. 1). The van Hiele theory on how students learn geometry indicates that the sequencing of instructional materials may need revision (NCTM, 1988). The researcher could only locate four controlled scientific studies (Bobango, 1987; Han, 1986; Lambert, 1988; Yerushalmy, 1986) that partially investigated the effects the method of instruction, computer use in the classroom, and the van Hiele theory have on student achievement and attitude in the secondary geometry course.

Historically, student performance in geometry has been poor. This fact, coupled with the recent developments in geometry learning theory and geometry computer software, necessitates that research involving these variables be conducted to determine if student performance in geometry can improve. Mathematics educators must investigate instructional methods, learning theories, and new technology to determine which combinations of these methods, theories, and tools will have the maximum positive effect on the mathematical education of students. Only a limited number of such scientific studies have been conducted; it is the purpose of this study to add to this base of research data.

## Statement of the Problem

The primary purpose of this study was to investigate the effect that three different methods of classroom instruction had on (a) achievement of certain geometrical concepts, (b) retention of these geometrical concepts, and (c) attitude toward mathematics (and geometry in particular) for students enrolled in a high school geometry course. The three methods of instruction included: (a) small cooperative learning groups doing paper and pencil activities following the three phases (information, guided orientation, and explicitation) as described in the van Hiele theory, (b) small cooperative learning groups using the computer and accompanying software together with activities following the three phases of the van Hiele theory, and (c) whole class instruction based on traditional textbook procedures. The dependent variables were scores on the criterion-referenced tests and scores on the attitude opinionnaires administered at the end of the unit and then four weeks later. Independent variables were method of instruction, gender, age, socio-economic background, attitude toward mathematics, attitude toward geometry just prior to treatment application, pretest scores on the criterion-referenced tests, and the van Hiele level of geometrical thinking. Instruction consisted of the teaching of a unit on congruent triangles. The population was the set of high school students enrolled in traditional tenth grade geometry classes in five Montana high schools that took part in this study.

## Definitions of Terms

Terms defined for this study:

1. Achievement of certain geometrical concepts is defined as the measure (score) obtained on the criterion referenced test, Geometry Unit Test (Form A or B), constructed by the researcher and given to the students at the completion of the unit of geometry used in this research study (see Appendix A).

2. Attitude toward geometry is defined as a student's self-reported enjoyment, interest, and level of anxiety toward geometry as measured by the modified Aiken-Dreger Geometry Opinionnaire (see Appendix D).

3. Attitude toward mathematics is defined as a student's self-reported enjoyment, interest, and level of anxiety toward mathematics as measured by the Aiken-Dreger Mathematics Opinionnaire (see Appendix D).

4. Computer refers to any one of the Apple(c) II series of microcomputers.

5. Geometrical concepts refer to the topics listed in the Unit Objectives (see Appendix A).

6. Geometry course is the Montana high schools' year-long course taken by those students who have

completed one year of algebra (usually students in the tenth grade). The textbooks used by the students in this study are Rhoad, Milauskas, and Whipple's Geometry For Enjoyment and Challenge (1984) and Jurgensen, Brown, and Jurgensen's Geometry (1985).

7. IMPACT is an acronym for Integrating Mathematics Programs and Computer Technology. IMPACT was a National Science Foundation funded program to train selected mathematics teachers in Montana to provide expertise to other teachers through inservice workshops and math curriculum leadership in districts throughout the state of Montana.

8. Method of instruction refers to one of the three treatments used in this study.

a. Treatment I -- Each concept of the unit on congruent triangles is introduced to the students using worksheets which consist of activities in which the students use paper, pencil, ruler, and protractor. The activities were designed by the researcher based upon the first three phases of the van Hiele theory. The students work in small cooperative learning groups to complete these activities.

b. Treatment II -- Each concept of the unit on congruent triangles is introduced to the students using worksheets which consist of activities in which the students use a computer and the Geometric PreSupposer and Geometric Supposer: Triangles software published by Sunburst Publications. The activities were designed by the researcher based upon the first three phases of the van Hiele theory. The students work in small cooperative learning groups to complete these activities.

c. Treatment III -- Each concept of the unit on congruent triangles is introduced to the students in a class discussion format led by the teacher using only the textbook as a guide. The teacher presents the material in the same sequence as outlined in the textbook, periodically asking random members of the class various questions to check for understanding.

9. Retention of geometrical concepts is defined as the measure (score) obtained on the criterion-referenced test, Geometry Unit Test (Form A or B), constructed by the researcher and given to the students four weeks after the completion of the unit of geometry used in this research study.

10. Small cooperative learning group is defined as a group of three students (or four students when the class size is not a multiple of three) working together to complete an activity to the satisfaction of everyone in this group. As the group works through the activity, the teacher encourages the group to discuss their methods, reasoning strategies, findings and conjectures with one another. The teacher answers only procedural questions and does not give answers to the activity questions. The teacher praises the members of the group for their accomplishments.

11. Socio-economic background is defined in terms of qualification for free or reduced-cost school lunch. Poverty level means the student qualified for free school lunch; low income means the student qualified for reduced-cost school lunch; and all others means the student receive neither free nor reduced-cost school lunch.

12. The van Hiele levels of geometric thought are five levels espoused by P. M. van Hiele and his late wife, Dina van Hiele-Geldof in their model of geometry learning of students.

a. Level 0: Visual or recognition--the level at which a student recognizes geometric figures by appearance alone. The student cannot see the component parts; definitions and class inclusion have no meaning to the student at this level. For example, when asked why a given figure is a triangle, the student at this level would reply, "Because it looks like one!"

b. Level 1: Descriptive or analysis--the level at which a student analyzes the properties of geometric figures and distinguishes figures by their component parts rather than by their appearance. The student at this level neither notices nor perceives relationships between properties or figures, and definitions and class inclusion still have no meaning. For example, when asked why a given figure is an equilateral triangle, the student at this level would reply, "Because all the sides are the same length and all the angles have the same measure." The student would not perceive the relationship between these two facts.

c. Level 2: Logical order--the level at which definitions are meaningful to a student and relationships are perceived between properties and figures by a student. Class inclusion has meaning. A student at this level can give an informal argument, but the student cannot construct a deductive proof.

For example, at this level the student could understand and explain the validity of the statement: "All equilateral triangles are isosceles triangles, but not all isosceles triangles are equilateral triangles."

d. Level 3: Formal logic or deduction--the level at which a student recognizes the need for undefined terms, definitions, and axioms in a particular mathematical system. A student at this level can reason deductively and construct proofs of propositions or theorems. The student at this level cannot compare and contrast different axiomatic systems. For example, at this level the student could construct the proof (based on the Euclidean parallel postulate) that the sum of the measures of the angles of a triangle is 180 degrees, but the student could not understand how the sum of the measures of the angles of a triangle could be less than 180 degrees if the Euclidean parallel postulate were changed.

e. Level 4: Rigor--the level at which a student can compare and contrast different geometric systems. The student at this level can use symbols without referents and can manipulate the symbols according to the laws of formal logic. The student at this level understands the role and necessity of indirect proof and proof by contraposition. For example, a student at this level could understand the proof that the sum of the measures of the angles of a triangle is less than 180 degrees when in Lobachevsky geometry.

13. The van Hiele phases of the instructional process are the stages in the van Hiele model of geometry thought through which a student must pass to move from one van Hiele level of geometric thought to the next.

a. Phase 1: Information--the stage in which a student gets acquainted with the working domain through activities and general conversation with the teacher.

b. Phase 2: Guided Orientation--the stage in which a student explores the topics being considered by doing simple tasks selected by the teacher which are usually accomplished in a specified manner.

c. Phase 3: Explicitation--the stage in which a student becomes conscious of the relations resulting from phase 2 and tries to express them in words with minimal direction from the teacher. A student may use nonstandard terms, but the teacher introduces the appropriate technical language.

d. Phase 4: Free Orientation--the stage in which a student learns to resolve more complicated tasks. During this stage a student may explore his/her own ideas as well as tasks presented by the teacher.

e. Phase 5: Integration--the stage in which a student summarizes all that he/she has learned about the topics, reflects on his/her actions, and obtains an overview of the newly formed concepts and relations.

## Need for the Study

The accessibility of the computer in the classroom can have an impact on the method of mathematics instruction. Recent advances in the development of computer software are changing the way some teachers are teaching mathematics. Specifically, the Geometric Supposer, software created by the Education Development Center, Inc. and published by Sunburst Communications, Inc., appears to encourage students to conjecture about and discover geometric concepts successfully (Yerushalmy, et. al., 1987).

Curriculum materials that reflect the van Hiele levels and provide phase-based activities described in this theory are limited (NCTM, 1988).

Although cooperative learning in the mathematics classroom has been investigated by several researchers (Slavin, et. al., 1985), the availability of research data which integrates the computer, the van Hiele theory, and cooperative learning is limited. One purpose of this study is to add to this base of research data.

Five factors were crucial in establishing need for this study. They are:

1. geometry is still a major component of the high school mathematics curriculum;
2. student performance in geometry is poor;
3. even though alternatives to the "traditional" lecture-discussion type of instruction (such as small group work, individual explorations, peer instruction, use of concrete materials and manipulatives) are recommended, these methods are not being used in many mathematics classrooms;
4. instructional materials in geometry using the van Hiele theory are limited; and
5. research regarding the effects of computer assisted instruction in geometry is limited.

The following paragraphs will substantiate why these factors are crucial.

Factor 1: Geometry is still a major component of the high school mathematics curriculum.

Geometry became a standard course for high school students in the United States shortly after the Civil War. Although conferences and committees dealing with how and when geometry should be taught have affected the approaches and content of the high school geometry course, at the present it still remains a year long course in most high schools with the focus on Euclidean geometry (NCTM, 1981). The NCTM Curriculum and Evaluation Standards (1989) includes the study of geometry in two of its fourteen standards in the mathematics curriculum for grades 9-12. The Curriculum and Evaluation Standards further states (1) that "at least three years of mathematical study will be required of all secondary school students" and (2) that "four years of mathematical study will be required of all college-intending students" (NCTM, 1989. pp. 124-125). With the present configuration of high school mathematics programs, most students planning to graduate from high school will take a course in geometry.

Factor 2: Student performance in geometry is poor. Equally disturbing is the fact that nearly half of the high school students do not take geometry.

The Fourth Mathematics Assessment of the National Assessment of Educational Progress reported that about 55 percent of the eleventh-grade students in this study had completed a geometry course (Brown et. al, 1988). This assessment included forty-three geometry items that were given to eleventh-grade students. These items were placed into four categories: identification of figures, properties of figures, visualization tasks, and applications. The items focused on ideas of informal geometry, such as those taught at the elementary or middle school levels. Those students who had taken a high school geometry course did well in the identification of figures category. The students were asked to identify parallel lines, perpendicular lines, a sphere, the diameter of a circle, the radius of a circle, and the endpoints of an arc. On these items, the success rate ranged from 81 percent identifying a radius to 99 percent identifying parallel lines. However, when it came to properties of figures, only 33 percent of the students who had taken a geometry course could correctly choose the set of 3 numbers that could not be lengths of sides of a triangle and 71 percent could correctly choose the set of 3 numbers that could not be measures of angles of a triangle. In evaluating spatial-visualization skills, about two-thirds of the eleventh-grade students responded correctly; those who had taken a geometry course performed only slightly better than those who had not taken geometry. Two of the application problems involved the Pythagorean relationship; for those eleventh-grade students who took geometry, only 48 percent correctly answered the first item and only 30 percent correctly answered the second item.

Factor 3: Even though it is recommended by the National Council of Teachers of Mathematics, a variety of instructional methods is still not being used in the mathematics classroom.

The NCTM Curriculum and Evaluation Standards stresses the need for changes in the roles of both teachers and students in mathematics classes.

> A variety of instructional methods should be used in classrooms in order to cultivate students' abilities to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. In addition to traditional teacher demonstrations and teacher-led discussions, greater opportunities should be provided for small-group work, individual explorations, peer instruction, and whole-class discussions in which the teacher serves as a moderator (NCTM, 1989, p.125).

Currently, a variety of strongly recommended instructional methods is not being used in the typical mathematics classroom. From data collected in the 1986 NAEP (Dossey, 1988), it was found that about three-fourths of the eleventh graders indicated that teacher use of the chalkboard was a daily activity, but only about 26 percent indicated that they themselves frequently work problems at the board. Over half these students reported they never work mathematics problems in small groups; only 57 percent reported that frequent class discussion took place.

In "A Study of Schooling," directed by John Goodlad, similar patterns were found (Sirotnik, 1983). In the secondary mathematics classrooms, this study revealed 79.3 percent of the classroom time was instructional with the teacher talking 57.9 percent of the time and the students talking 16.4 percent of the time.

Factor 4: Instructional materials in geometry using the van Hiele theory are limited.

In 1957, Dutch educators, P. M. van Hiele and his late wife, Dina van Hiele-Geldof, presented a model of geometry learning they had developed. This model identifies five levels of thinking in geometry and phases within each level of thinking. This theory has shown promise in improving student performance when the instruction was provided in accordance with the van Hiele theory (NCTM, 1988). Except for the geometry text by Hoffer (1979), geometrical topics in the text books in the United States are not sequenced according to this theory (NCTM, 1988) and, hence, textbook-guided instruction is not utilizing this promising model.

Factor 5: Research regarding the effects of computer assisted instruction in geometry is limited.

In its An Agenda for Action, the NCTM recommended that "mathematics programs take full advantage of the power of calculators and computers at all grade levels" (NCTM, 1980, p.1). This was reiterated in the NCTM Curriculum and Evaluation Standards when the underlying assumptions included that "a computer will be available at all times in every classroom for demonstration purposes, and all students will have access to computers for individual and group work" (NCTM, 1989, p.124).

Mathematics educators (Fey, 1984; Kantowski, 1981) have pointed out that the graphics and numerical capabilities of computers allow students to manipulate and measure geometric shapes so that they can develop understanding of concepts and theorems of geometry. Several papers have been written describing the use of the computer in the geometry classroom (Lamber, 1988; Yerushalmy, 1986; Yerushalmy, et. al., 1987; Yerushalmy and Houde, 1986); however, the research on the effect of using the computer in the geometry classroom is extremely limited (Suydam, 1985; Suydam, 1987; Suydam, 1988; Suydam, 1989; Suydam and Crocker, 1990).

Questions to be Answered

The questions that this study attempted to answer fall into six categories:

1. Are there differences in achievement of geometric concepts among three groups of students receiving different methods of classroom instruction? Are there any differences in achievement between male and female students? Are there any differences in achievement among the students from different socio-economic status? Are there any differences in achievement among students performing at different van Hiele levels? Is achievement affected because of interactions between or among these independent variables?

2. Are there differences in retention of geometric concepts among three groups of students receiving different methods of classroom instruction? Are there any differences in retention between male and female students? Are there any differences in retention among the students from different socio-economic status? Are there any differences in retention among students performing at different van Hiele levels?

Is retention affected because of interactions between or among these independent variables?

3. Are there differences in attitudes toward geometry among three groups of students receiving different methods of classroom instruction? Are there any differences in attitudes toward geometry between male and female students? Are there any differences in attitudes toward geometry among the students from different socio-economic status? Are there any differences in attitudes toward geometry among students performing at different van Hiele levels? Are attitudes toward geometry affected because of interactions between or among these independent variables?

4. Is there a correlation between the attitudes toward mathematics and achievement by the students who studied geometry in each of the groups defined by the different methods of classroom instruction?

5. Is there a correlation between the attitudes toward mathematics and retention by the students who studied geometry in each of the groups defined by the different methods of classroom instruction?

6. Is there a correlation between achievement and retention by the students who studied geometry in each of the groups defined by the different methods of classroom instruction?

These questions are detailed in null hypotheses form in Chapter 3.

## Procedure of the Study

The study took place in Montana. Montana, the fourth largest state in the United States in terms of area, ranks 44th in population. Montana is classified as a rural state with 5.6 people per square mile. Only 196,000 of its 800,000 residents reside in two metropolitan areas (population over 55,000). Principal industries in the state are agriculture, mining, tourism and manufacturing (primarily wood products). Approximately 12% of the population are at the poverty level. Approximately 46,000 students are enrolled in grades 9-12 in the 163 Montana high school districts (U. S. Bureau of the Census, 1989).

This study took place at these high schools: Columbia Falls High School in Columbia Falls, Montana; Cut Bank High School in Cut Bank, Montana; Forsyth High School in Forsyth, Montana; Glasgow High School in Glasgow, Montana; and C. M. Russell High School in Great Falls, Montana. All five high schools contain grades 9-12.

Columbia Falls, population 2,942, is located in northwestern Montana. The primary industries in this community are lumber and wood products and aluminum processing. The high school district, which includes the town and several outlying community areas, has 6,142 residents with a high school enrollment of 632 students.

Cut Bank, population 3,329, is located in northern Montana. The primary industry in this community is agriculture; however, active gas and oil fields are located in the surrounding area. The high school district, which includes Cut Bank itself and the surrounding agricultural area, has 4,223 residents, and the high school population is 255.

Forsyth, population 2,178, is located in southeastern Montana. The primary industry in this community is agriculture. The high school district, with 2,811 residents, has a present enrollment of 210 students.

Glasgow, population 3,572, is located in northeastern Montana. It was once the site of a U. S. Air Force base but now the community lists agriculture as its major industry. The high school district with 4,116 residents has a present enrollment of 306 students.

Great Falls, located in northcentral Montana, is one of two cities in Montana with a population of over 55,000 (population 55,086). An active Air Force base is located adjacent to the city. The city has a diversified economy. Copper, zinc, and aluminum are processed, flour is milled from nearby wheatfields, and there is a large crude-oil refinery. The high school district contains two highs which serve a district of 67,901 residents. CMR High School (the one in this study) has 1,597 students, and GF High School has 1,743 students.

The selection of the high schools involved in this study was based upon these criteria:

1. The geometry teachers at these schools must have recently participated in the IMPACT project or an IMPACT related workshop. Through the IMPACT project or IMPACT workshops these teachers (a) received training in using the computer in the classroom and (b) had experience with cooperative learning techniques.

2. In order to maintain some uniformity in the geometry content, the geometry classes selected at these schools were using one of two identified texts (Jurgensen, et. al., 1985; Rhoad, et. al., 1984) that introduced the unit on congruent triangles (with similar objectives) at approximately the same time in the school year.

3. The administration at the schools must have been willing to cooperate with the researcher by allowing the researcher to collect confidential information about each student.

This study involved three different treatment groups. The first treatment consisted of students working in small cooperative learning groups doing paper and pencil activities designed by the researcher. These activities followed the first three phases (information, guided orientation, and explicitation) as outlined in the van Hiele theory. The last two phases (free orientation and integration) of the van Hiele theory were completed using the exercises in the student text. The second treatment consisted of students working in small cooperative learning groups using the computer and selected software (Geometric PreSupposer and Geometric Supposer: Triangles) together with activities designed by the researcher. These activities followed the first three phases of the van Hiele theory. The last two phases were completed using the exercises in the student text. In the third treatment for this study, the method of instruction was whole class instruction based on traditional textbook procedures.

At four of the five schools involved in the study, one teacher at each school taught all the geometry courses. All four of these teachers were involved in the study. At the fifth school, no mathematics teacher taught more than two sections of geometry so two teachers from this school were chosen to participate since the researcher wanted all three treatments used at a particular school whenever possible. Thus, at two schools, each teacher taught three sections of geometry and each section was randomly assigned one of the treatments. At one school (where computer facilities were limited) only treatments one and three were assigned. At the fourth school, only two sections of geometry were offered, so treatments two and three were assigned to these sections. This was done in order to balance the number of sections using each treatment. At the school where two teachers were involved, one teacher taught one class and the other teacher taught two classes in which each of the treatments was randomly assigned to one of these classes. This selection allowed limited control of the teacher variable.

Since this study was done in a natural school setting where the number of classes available in each school is limited, random assignment of students to treatment groups was impossible. Hence, the quasi-experimental pre-post design based on Campbell and Stanley's (1963) Nonequivalent Control Group Design was the appropriate design to use.

Three instruments were used in this study. (1) The Aiken-Dreger Mathematics Attitude Scale was used to measure the change in the students' attitudes toward geometry as a result of the 4-week treatments received by each of the groups in the study. (2) The CDASSG van Hiele Geometry Test was given prior to this study to determine the van Hiele level of each participant. (3) The Unit Test on Congruent Triangles, designed by the researcher, was used as pretests and posttests to measure achievement and retention of geometric concepts of the unit taught on congruent triangles.

## Limitations and Delimitations

The following limitations restrict the generalization of the findings of the study to different classes, situations, and settings:

1. The study was limited to those high school students in selected Montana high schools who were enrolled in a high school geometry course during the 1990-1991 academic year.
2. The study began after the first nine-week period had ended and the treatments were administered four continuous weeks.
3. All classes were taught by teachers having at least three years of teaching experience prior to the study. All five teachers had participated in the IMPACT project or IMPACT workshops and hence may not be representative of average geometry teachers.

4. Each class involved in the study used exactly one geometry text, but texts by two different publishers were used by the schools.

5. Individual teacher's attitude, ability, and familiarity with the materials may have influenced student attitudes and performances.

6. The use of computers was not allowed during the tests.

The following delimitations restrict the generalization of findings of this study to different situations:

1. The study was limited to an eight-week period (four weeks of instruction on a congruent triangles unit with the retention test being administered four weeks later).

2. The phase-based treatments were only for selected, rather than all, topics completed in the unit of geometry used in this research study.

## Organization of the Study

Chapter 1 presented an introduction to the problem. With the importance of the problem established, the statement, justification, and purpose of the study were discussed.

Chapter 2 begins with an historical review of geometry in the high school curriculum. It then gives a review of the related literature as it applies to (1) the van Hiele theory on how students learn geometry, (2) the use of computers in the teaching of mathematics, (3) the use of cooperative learning methods in the teaching of mathematics, and (4) the achievement and attitudes toward mathematics of high school students.

Chapter 3 describes the methodology of procedures used in the study. It includes a restatement of the problem, procedures, instruments, and treatment of the data.

Chapter 4 covers the results of the study. It comprises a restatement of the hypotheses, the results of the investigation, and an analysis of those findings.

Chapter 5 presents a summary of the study, the conclusions drawn from the findings, and recommendations based upon these conclusions.

# CHAPTER 2

# REVIEW OF LITERATURE

## Introduction

The review of the literature relative to this study is reported in this chapter. The method of organization of the reviewed literature followed these major topics: (1) an historical review of geometry in the high school curriculum, (2) the van Hiele theory, (3) the use of computers in teaching geometry, (4) cooperative learning methods in geometry classes, and (5) student achievement and attitude with respect to high school geometry.

The research studies reviewed are limited to those which pertain to this investigation and which satisfy the following criteria: (1) they must have been completed within the last 20 years, and (2) they must represent conclusions based on research and not on mere personal opinion.

## Historical Review of Geometry in the High School Curriculum

The first high school in the United States, the English High School, was founded in Boston in 1821 and by 1875 several similar high schools were well established in the northeastern part of the United States. Geometry became a high school course when the colleges made it an entrance requirement. Yale first required it in 1865 and Princeton, Michigan, and Cornell began requiring it in 1868. Harvard made it an entrance requirement in 1870 (NCTM, 1970).

Prior to 1920, geometry was essentially a course in logic and demonstration. In the NCTM's Fifth Yearbook entitled The Teaching of Geometry Swenson wrote:

> The sway which Euclid's Elements has held as a textbook for more than two thousand years is without parallel in the history of mathematics. Even the invention of Cartesian geometry in 1637 has not affected the teaching of the so-called Euclidean geometry. An almost unlimited number of textbooks have appeared in modern times but the only way in which they have differed is in the sequence of the theorems. Euclid's treatment has in the main been retained and no modern mathematical methods have been introduced. (NCTM, 1930)

There was some initial movement to change this approach at the turn of the century. John Perry, a professor of mechanics and mathematics at the Royal College of Science in London, advocated the discovery method of teaching. Perry did not approve of the strict adherence to Euclid; he wanted some experimental geometry before the study of formal geometry and he wanted a reduction in the number of theorems and proofs to be memorized. E. H. Moore of the University of Chicago, in his address before the American Mathematical Society in 1902, supported Perry's movement to emphasize the practical sides of mathematics (Moore, 1967). This influence became apparent when the geometry textbooks of the twenties began with an informal, intuitive geometry section rather than beginning with the formal logic and demonstration of the earlier texts.

A second influential component on the high school mathematics curriculum (and hence the geometry course) at this time was the College Entrance Examination Board (CEEB). The CEEB was a product of the committee originally started by Columbia University to set some uniformity in college entrance requirements. By 1903, the CEEB had begun work on syllabi that would become the guide for uniform, nationwide college entrance examinations. Before long, textbooks were designed around these syllabi, and teachers were carefully preparing their students to pass these CEEB examinations.

Then, in 1930, when critics of the mathematics curriculum questioned the rationale and content of the geometry course, the NCTM formed a committee on geometry. An early recommendation of this committee favored the combination of solid and plane geometry (NCTM, 1931). Probably the most influential derivative from this committee was the development of a text which used a metric postulate system for plane geometry. Ralph Beatley, the chairman of this committee, teamed up with George Birkhoff, editor of the NCTM's Fifth Yearbook, to write the high school text, Basic Geometry (Birkhoff, 1941).

In 1935, the Mathematical Association of America teamed with the young and aggressive National Council of Teachers of Mathematics, and together they appointed the Joint Commission consisting of members from the Mathematical Association of America and the National Council of Teachers of Mathematics. In its report, published in 1940, which was the Fifteenth Yearbook of the NCTM, the basic course in geometry was described as "a course that examines somewhat critically Euclidean geometry, and gives brief introductions to projective geometry and non-Euclidean geometry, using synthetic methods" (Joint Commission, 1940).

The next impetus for change occurred in 1955 when the CEEB again thought the time had come for it to take a leadership role in determining the school mathematics program for college-bound students. It appointed the Commission on Mathematics to carry out this function. In its report, the Commission identified three main objectives for the inclusion of geometry in the high school curriculum:

> The first objective is the acquisition of information about geometric figures in the plane and in space.... The second objective is the development of an understanding of the deductive method as a way of thinking, a reasonable skill in applying this method to mathematical situations.... The third important objective of the geometry course is the provision of opportunities for original and creative thinking by students.... (CEEB, 1959).

Many other curricular projects were initiated in the fifties, but one that had a significant impact on mathematics in the sixties and beyond was the School Mathematics Study Group (SMSG). SMSG grew directly out of two National Science Foundation (NSF) sponsored conferences, and SMSG received its first grant from NSF on 7 May 1958. Having had access to preliminary reports of the Commission on Mathematics of the CEEB, SMSG writing teams used these recommendations to design a series of textbooks that were to serve as guidelines to future mathematics textbooks. SMSG geometry used the "ruler and protractor postulates" from Birkhoff and Beatley's Basic Geometry. Suggestions from persons favoring an analytic geometry approach led SMSG to appoint a new writing team to write a text using coordinate geometry.

In the 1970's and early 1980's, the high school geometry curriculum was once again under scrutiny. Mathematicians and mathematics educators could not agree upon what the geometry curriculum should be. One only needs to look at the titles of the chapters in the Thirty-sixth Yearbook of the NCTM to see the variety of approaches that were proposed: "Conventional Approaches Using Synthetic Euclidean Geometry," "Approaches Using Coordinates," "A Transformation Approach to Euclidean Geometry," "An Affine Approach to Euclidean Geometry," "A Vector Approach to Euclidean Geometry," "Geometry in an Integrated Program," and finally "An Eclectic Program in Geometry" (NCTM, 1973).

## The van Hiele Theory

With the results from the recent National Assessments of Educational Progress showing that the geometry knowledge of students is rather minimal (Brown et. al., 1988; Carpenter et. al., 1983; Carpenter et. al., 1980), mathematics educators are beginning to look for teaching strategies that are based upon the cognitive processes of geometry students (NCTM, 1987; NCTM, 1988). One such approach to the instruction of geometry that is currently receiving considerable attention from U. S. mathematics educators is a theory espoused by Pierre and Dina van Hiele. These two Dutch educators presented this theory in 1957 and through the efforts of Izaak Wirszup (1976), American mathematics educators became aware of this theory.

The strength of the van Hiele theory is in its use of the detailed description of levels and phases which can be used in designing curriculum and instruction in geometry. The original van Hiele theory identifies five levels of thinking in geometry through which a student progresses when assisted by appropriate instructional experiences. These five levels and their characteristics are defined as follows:

Level 0: Visual or recognition--the level at which a student recognizes geometric figures by appearance alone. The student cannot see the component parts; definitions and class inclusion have no meaning to the student at this level.

Level 1: Descriptive or analysis--the level at which a student analyzes the properties of geometric figures and distinguishes figures by their component parts rather than by their appearance. The student at this level neither notices nor perceives relationships between properties or figures, and definitions and class inclusion still have no meaning.

Level 2: Logical order--the level at which definitions are meaningful to a student and relationships are perceived between properties and figures by a student. Class inclusion has meaning. A student at this level can give an informal argument, but the student cannot construct a deductive proof.

Level 3: Formal logic or deduction--the level at which a student recognizes the need for undefined terms, definitions, and axioms in a particular mathematical system. A student at this level can reason deductively and construct proofs of propositions or theorems. The student at this level cannot compare and contrast different axiomatic systems.

Level 4: Rigor--the level at which a student can compare and contrast different geometric systems. The student at this level can use symbols without referents and can manipulate the symbols according to the laws of formal logic. The student at this level understands the role and necessity of indirect proof and proof by contraposition (Mayberry,1983).

Pierre van Hiele has since revised the model into three rather than five levels of thought (Teppo, 1991). He labels these levels as level 1: visual, level 2: descriptive, and level 3: theoretical. Basically, the original van Hiele level 0 is the new level 1, the original level 1 is the new level 2 and the original levels 2, 3, and 4 have become the new level 3.

According to this theory, using either the original levels or the new levels, (van Hiele, 1986; Teppo, 1991), a student's progress through these levels has certain characteristics. They are:

(1) the levels are sequential;

(2) each level has its own language, set of symbols, and network of relations;

(3) what is implicit at one level becomes explicit at the next level;

(4) material taught to students above their level is subject to reduction of level;

(5) progress from one level to the next is more dependent on instructional experience than on age or maturation, and

(6) a student goes through various "phases" in proceeding from one level to the next.

The five phases that a student goes through as the student proceeds from one level to the next are: information, guided (or bound) orientation, explicitation, free orientation, and integration. These phases are described as follows:

Phase 1: Information--the stage in which a student gets acquainted with the working domain through activities and general conversation with the teacher.

Phase 2: Guided orientation--the stage in which a student explores the topics being considered by doing simple tasks selected by the teacher which are usually accomplished in a specified manner.

Phase 3: Explicitation--the stage in which a student becomes conscious of the relations resulting from phase 2 and tries to express them in words with minimal direction from the teacher. A student may use nonstandard terms, but the teacher introduces the appropriate technical language.

Phase 4: Free orientation--the stage in which a student learns to resolve more complicated tasks. During this stage a student may explore his/her own ideas as well as tasks presented by the teacher.

Phase 5: Integration--the stage in which a student summarizes all that he/she has learned about the topics, reflects on his/her actions, and obtains an overview of the newly formed concepts and relations.

Research projects on the van Hiele model have concentrated on determining if the five van Hiele levels can be determined for an individual student and to what extent are achievement in geometry and the van Hiele levels related. Only a limited amount of research is available which considers the implications that van Hiele phase-based instruction has on student achievement in geometry.

The purpose of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project was to "test the ability of the van Hiele theory to describe and predict the performance of students in secondary school geometry" (Usiskin, 1982). Several written tests were developed and used in this study. One of these tests, the Van Hiele Geometry Test, was used to determine the students' van Hiele levels of thought. Based upon the project's criteria for this test for determining a student's van Hiele level, it was found that about 85 percent of the nearly 1600 students in the study could be assigned a van Hiele level using this test at the beginning of the secondary geometry course. At the end of the course, this same test assigned a level to approximately 88 percent of the students. Results from this project also indicated that this test given in the fall was a good predictor of later achievement (r = 0.64).

In a study that used a subsample of the students involved in the CDASSG project, Senk (1983) tested 1520 high school geometry students and found that 30 percent of these students had no competence in proof-writing, 40 percent had some proof-writing skills, and about 30 percent achieved a mastery level of 75 percent (or above) in proof-writing. Senk found that both the student's van Hiele level and non-proof geometry achievement had a high positive correlation with proof-writing achievement.

A second project (known as the Brooklyn College Project) had four main objectives:

1. To develop and document a working model of the van Hiele levels,
2. To characterize the thinking in geometry of sixth and ninth graders in terms of the van Hiele levels,
3. To determine if teachers of grades 6 and 9 can be trained to identify van Hiele levels of geometry thinking of students and to identify van Hiele levels of geometry curriculum materials, and
4. To analyze current geometry curriculum as indicated by American text series to determine the geometry topics taught at each grade level and the

van Hiele level of these topics and to determine if the presentation of the topics was consistent with the van Hiele theory (NCTM, 1988).

Through interview procedures they developed, the researchers in this project concluded that the van Hiele model provides a reasonable structure for describing a student's geometric learning process. They also found, after analyzing three K-8 textbook series, that textbook material provides little opportunity for students to make progress to higher van Hiele levels (Fuys and Geddes, 1984).

Another project, the Oregon Project (Burger and Shaughnessy, 1986), also used clinical interviews to investigate student behaviors on geometric tasks in relation to the van Hiele theory. Specifically, the project investigated the following research questions:

1. Are the van Hiele levels useful in describing students' thinking processes on geometry tasks?
2. Can the levels be characterized operationally by student behaviors?
3. Can an interview procedure be developed to reveal predominant levels of reasoning on specific geometry tasks?

The results of this project indicated that all three questions can be answered in the affirmative. The researchers found that students who were identified as being at different van Hiele levels did in fact perform differently and use different language.

In a study which involved 24 college students, Mayberry (1983) obtained results which confirmed the theory that the van Hiele levels are sequential; i.e., students cannot function adequately at a given level without the ability to think intuitively at each preceding level. This study also found that students do not operate at the same level across all concepts.

Han (1986) conducted one of the first studies in the United States that compared the effects on student achievement and attitude of two geometry textbook programs, one consistent with the van Hiele theory, and the other program being a "traditional" program. The subjects were 478 geometry students from two high schools. One high school used the van Hiele theory based text and the other high school used the traditional text. The study concluded that there were no significant differences between the two groups in van Hiele level, within group correlation between van Hiele level and proof-writing achievement, and attitude toward geometry. The main conclusion of this researcher was that the van Hiele approach did not offer a significant advantage over the traditional one in overcoming the difficulty students have with proof.

Another study investigated the effect van Hiele phase-based instruction had on raising students' van Hiele levels and on their subsequent understanding of geometric facts, concepts, and proofs (Bobango, 1987). The study involved 72 high school students from two regular and two honors geometry classes. The computer programs, Geometric Supposer: Triangles and Geometry Supposer: Quadrilaterals (Educational Development Center, 1985), were used together with researcher-designed lessons with the phase-based treatment group. The students' achievement and van Hiele levels were compared before and after the phase-based instructional treatment. The researcher found that the phase-based instruction had a significant effect on raising regular students' van Hiele levels of thought; however, there was no difference in the achievement in standard content between the treatment and the control groups, nor was there any significant difference in proof-writing success between the two groups.

Bobango's research was the only study located which used van Hiele phase-based instruction to determine the effect on students in a high school geometry course. However, her study does not answer the questions posed and tested in this research.

## The Use of Computers in Teaching Geometry

Using the computer in the mathematics classroom with drill and practice programs has demonstrated a positive effect on the acquisition of computational skills of students (Burns and Bozeman, 1981; Hartley, 1977; Kulik, et. al., 1983). A similar positive effect for mathematical comprehension skills has been recorded when the computer and tutorial programs that test knowledge of concepts and terminology and the ability to reason have been used with students (Bridges, 1985; Burns and Bozeman, 1981; Dugdale and Kibbey, 1983; Henderson et. al., 1983; Kulik et. al., 1983). However, the use of computer simulations that could aid in the application and analysis of geometrical concepts has been limited because of lack of software. Newly developed software tools such as The Geometric Supposer: Triangles (Educational Development Center, Inc., 1985) and Geometry One: Foundations (IBM EDUCATIONAL SYSTEMS System, Inc., 1987) are now available for classroom use and the effect that these programs have on students is just beginning to be studied.

The Geometric Supposer: Triangles is one of four in a series of geometry computer programs. The user first selects a type of triangle or constructs a triangle of a particular size and shape when using this program. Then the user can construct and label points, segments, angle bisectors, medians, altitudes, circles, parallels, perpendiculars, and extensions of line segments. The user can also measure distances, angles, and areas for the figures constructed. These constructions and measurements are all completed very easily by choosing from an appropriate menu. The authors of this program identify the strengths of this program to be its ability to easily provide the user with a wealth of visual and numerical data in order to test conjectures or to find counterexamples (Yerushalmy and Houde, 1986; Schwartz and Yerushalmy, 1987).

In 1984-1985, Yerushalmy (1986) used The Geometric Supposer to investigate (1) the inductive process that takes place with high school students as they generalize conjectures from empirical data, and (2) the reasoning process that takes place in developing mathematical arguments to support these conjectures. The study compared two groups of students in the Weston, Massachusetts, high school. One group (45 students) learned geometry inductively, using The Geometric Supposer, as a learning tool, and the other group (40 students) learned traditional, deductive geometry. The results of this study showed that (1) the students in the inductive group were more willing to consider nonconventional methods of analysis, (2) the students in the traditional group showed signs of a decrease in the ability to generalize and in the use of induction whereas the opposite was true for students in the inductive group, (3) the students in the traditional group were less motivated to think about richer ideas or to change what were assumed to be key features, and (3) the students in the traditional group had a better performance in skills related to the requirements of traditional school mathematics.

The following year, a similar project was conducted in three Boston area suburbs (Yerushalmy, 1987). This study examined student learning and the issues involved in the implementation of a guided inquiry approach using <u>The Geometric Supposer</u> to teach high school geometry. The experimental group that was taught using the guided inquiry approach consisted of 39 students, and the comparison group that was taught in the traditional method had 30 students. Pretests and posttests were developed to assess students' abilities (1) to make generalizations given data or a description of a geometric situation and (2) to produce proofs. Two important and statistically significant performance differences were found between the experimental and comparison groups with regard to the ability to make generalizations. First, the experimental group produced higher level generalizations on two out of three questions on the posttest. Second, students in the experimental group produced more arguments on the posttest abstract question, even though no arguments were requested. As for the ability to produce proofs, there was no significant difference between the two groups.

Bobango (1987) used the Geometric Supposer: Triangles in her study which investigated the question of whether students who had phase-based instruction (as per the van Hiele theory) using the computer as a tool achieved significantly higher measures of van Hiele levels or higher test achievement scores when compared to a control group. She concluded that the phase-based instruction had a significant effect on raising students' van Hiele levels of thought; however, the experiment led to the conclusion that there was no difference in achievement in standard content or proof-writing success in geometry. Since there was not a treatment group using phase-based instruction without the computer, no conclusions could be made regarding the effectiveness of the computer as a tool.

Cooperative Learning in Geometry Classes

In general terms, cooperative learning involves a small group of learners who work together as a team to accomplish a common goal. Even though there are many cooperative learning structures that researchers have developed and studied, all of them have certain elements in common. Artzt and Newman (1990) identified these elements as follows:

> First, the members of a group must perceive that they are part of a team and that they all have a common goal.
> Second, group members must realize that the problem they are to solve is a group problem and the success or failure of the group will be shared by all of the members of the group.
> Third, to accomplish the group's goal, all students must talk with one another--to engage in discussion of all problems.
> Finally, it must be clear to all that each member's individual work has a direct effect on the group's success.

The results of the use of cooperative learning methods in mathematics classrooms have generally produced favorable results. Davidson (1985) reviewed over seventy studies in mathematics which compared student achievement in cooperative learning versus whole-class traditional instruction. In over 40 percent of these research studies, students in the cooperative learning groups significantly outscored the students in the whole-class traditional groups on individual mathematical achievement. In only two studies (Loomer, 1976; Johnson, et al., 1978) did the whole-class traditional students perform better, and Davidson indicated that "both of these studies had irregularities in design" (Davidson, 1990).

Only one study (Cox, et al., 1989) was located which compared students taking high school geometry using small cooperative learning groups versus whole-class traditional instruction. Two classes of high school honors geometry at a private, preparatory school in Hawaii were used in this study. One class was taught using cooperative learning methods, and the other was taught by the same teacher using traditional methods. There was no significant difference in achievement on tests between the two groups; however, the responses from the students in the cooperative learning groups indicated that this method of instruction had positive effects on their relationships with their peers and mastery of the subject and allowed a closer relationship with the teacher.

## Student Achievement and Attitude with Respect to High School Geometry

The research on student attitudes toward mathematics (in general) is quite abundant and the findings have been summarized in the writings of Aiken (1970; 1976), Kulm (1980), and Suydam (1984). These summaries of the research indicate that:

1. there is a low correlation between attitude towards mathematics and achievement scores in mathematics;

2. generally, students of all levels recognize the value of studying mathematics;

3. attitudes toward mathematics are formed primarily in grades 4-8 and have a slow but steady decline during the high school years;

4. there is evidence that male and female attitudes toward mathematics are different and that the changes in their attitudes over the grades differ in some ways; and

5. attitudes toward mathematics are probably formed and affected by many variables (In the early grades, home and classroom variables play an important role. In junior high and high school, classroom variables become less important and social interaction becomes more important. At the college level, variables relating to utility and employment opportunities play a major role.)

All of the studies located that involved student achievement and attitudes with respect to high school geometry involved comparing innovative courses with conventional ones. In general, there were no significant differences in student attitudes toward the innovative courses and the conventional courses.

Harbeck (1973) did a comparative study on students' attitudes toward geometry and proof between a group of students instructed by the flow-diagram format and a group instructed by the statement-reason format. She found no significant difference in attitudes toward geometry and proof, but the students using the flow-diagram format had significantly more favorable attitudes toward that format.

Wood (1976) compared students' attitudes for two groups of students--one group an informal geometry course and the other a formal geometry course. No difference in attitude toward geometry and no difference in attitude changes were found between the two groups.

In a study that investigated student responses to a transformational geometry course for low achieving high school students, Herot (1976) found no significant change in attitude. However, she did find that a positive attitude toward mathematics correlated with achievement for these students.

In his research project determining the effects of various levels of teacher/student verbal interaction on geometry students, Dittmer (1978) found some differences in attitude and achievement among three treatment groups. In the first treatment group, students were not permitted to talk. In the second group, the teacher/student talk ratio was 2/1. In the third group, this ratio was 1/1. The second and third groups' attitude scores toward mathematics were significantly higher than the first group's attitude scores. Also, the second and third treatment groups scored significantly higher that the first treatment group on an achievement test of abstract geometric concepts.

In eight classes of average and below average students enrolled in a "novel" informal high school geometry program, Decovsky (1978) found a slight tendency for the students to change their attitudes in the positive direction, although the change was statistically significant ($p < 0.05$) in only one class.

Cox (1979) compared students' attitudes and achievement in an informal geometry course and those in a conventional course. The results showed that the informal group expressed positive attitudes toward geometry, and the attitudes of the informal group rose slightly during the course, although the attitudes of the two groups were not significantly different. The conventional geometry group performed significantly better on a 40-item standardized test; however, the informal group's pretest/posttest achievement gains on the author's 42-item test (which excluded proof) were significantly better than the conventional group's gains.

Prince (1982) investigated the relationship among attitude, achievement, and the personality dimensions of introversion-extraversion among high school students enrolled in traditional and transformational geometry classes. The extroverts with positive attitudes were assigned to the transformational geometry class, and the introverts with positive attitudes were assigned to the traditional class. No significant differences in achievement and attitude between the two groups were found. A positive correlation between attitude and achievement was found.

In a study comparing the effects on student achievement and attitude of two textbook programs, one consistent with the van Hiele theory and the other a standard textbook, Han (1986) found no significant difference in attitude between the two groups. Significant differences between the two groups were found in favor of the traditional group in proof-writing achievement and attitude toward proof. Han also found that the combined group students' attitudes toward geometry declined in the last half of the course.

## Summary

Ever since geometry has become a standard course in the high school mathematics curriculum, it has been a course that has received special consideration. Several approaches for teaching high school geometry have been tried, but none seems to have provided any change in student success.

Fey and Good (1985) commented,

> Geometry has been a troubled strand of the curriculum for many years. Despite bold proposals for new approaches the standard experience of most students is still limited. (p.44)

Cox (1985) went on to say,

> To have any chance for success in achieving some degree of universal geometric competence, we cannot merely offer "more of the same." The population of students is diverse and many have little or no chance of achieving success in a traditional proof-oriented course on plane geometry. (p.404)

The van Hiele theory with its phase-based instruction may provide students with an instructional mode that would increase their competence in geometry, but little research with this instructional method has been published.

Computer software for geometry that makes use of the graphic capabilities of the computer is now available, but again, research data on how this software effects the students' abilities to learn geometry is somewhat limited.

According to the literature, students who were taught in a cooperative learning classroom environment had a (statistically) significantly higher achievement level than those taught in a classical classroom environment.

A review of the literature indicates a trend toward a low, but positive, relationship between attitude and achievement. When comparing attitudes of two or more treatment groups using a variety of instructional modes in high school geometry, there were no significant differences in the attitudes of the students toward geometry among the treatment groups.

# CHAPTER 3

# PROCEDURES

## Introduction

The primary purpose of this study was to determine the effect that three different methods of classroom instruction had for students enrolled in a high school geometry course on (a) achievement of certain geometrical concepts, (b) retention of these geometric concepts, and (c) attitude toward mathematics (and geometry, in particular). One method of instruction involved the students working in small cooperative groups doing paper and pencil activities which follow the first three phases (free orientation, guided orientation, and explicitation) of the van Hiele theory. The second method of instruction involved the students working in small cooperative groups using the computer and accompanying software together with activities which follow the first three phases of the van Hiele theory. The third method of instruction used the students' textbooks in a traditional whole-class format.

This chapter includes the following topics:

1. the pilot study;
2. the setting of the problem;
3. the null hypotheses of the study;
4. the instruments used in the study;
5. the nature of the treatments; and
6. the experimental design and analysis of data.

## The Pilot Study

During the 1988-89 academic year, the high school geometry teacher at Beaverhead County High School, Dillon, Montana, volunteered two geometry classes to participate in the development of the activities and the unit test for this study. When the students in the two geometry classes were studying the unit on congruent triangles, preliminary versions of the paper and pencil activity worksheets and the computer activity worksheets were completed. Any difficulties with directions or understanding were noted, and these notes were used to modify these activity worksheets so that these worksheets could be easily completed by the students with little or no teacher assistance. The researcher constructed unit test was administered before the unit began (as a pretest) and again when the unit was completed (as a posttest). Items which had no discriminating qualities were rewritten.

During the 1989-1990 academic year, teachers from three high schools in Montana (Cut Bank, Great Falls, and Billings) volunteered to pilot this project in their geometry classes. The teachers who volunteered to use these materials were familiar with the van Hiele theory, cooperative learning, and the computer software used in this study. Before the academic year began, each high school principal was contacted, and permission was granted to conduct this pilot study at the school. Permission slips were sent home with each student in these classes requesting parental permission for student participation. During the first week of school, the CDASSG van Hiele Geometry Test and the Aiken-Dreger Mathematics Attitude Scale were administered to the students. At the start of the second nine-week period and just prior to the beginning of the unit on congruent triangles the Unit Test: Congruent Triangles (Form A) and a modified version of the Aiken-Dreger Mathematics Attitude Scale were administered to the students. During the next four weeks, each teacher used the paper and pencil activity sheets with one class and the computer activity sheets with another class. These activities were used in place of the textbook to introduce the topics for the unit on congruent triangles. Each teacher kept a daily log that briefly detailed the activities' successes, the activities' compliances with the van Hiele phases, completion times, and assignments made. At the completion of the unit, the Unit Test: Congruent Triangles (Form B) and the modified version of the Aiken-Dreger Mathematics Attitude Scale were administered to the students. At this point, the teachers returned to their normal mode of teaching. After four weeks had elapsed, the Unit Test: Congruent Triangles (Form A) and the modified version of the Aiken-Dreger Mathematics Attitude Scale were administered a third time. This completed the data collection phase of the pilot study. Based upon the input from these teachers and their logs and the student responses to the activity sheets and Unit Test, final revisions were made to these items. (See Appendix B for the van Hiele phase-based activity sheets.)

Initially the researcher had concerns as to whether or not an individual teacher would be able to successfully teach three different geometry classes during the day, each using a different treatment. The teachers involved in the pilot study were able to manage successfully and effectively three such geometry classes thus removing the researcher's concerns.

## The Setting of the Problem

In June 1990, the researcher sent letters to 80 high school mathematics teachers who were to participate in a mathematics and technology workshop sponsored by Montana State University and the Montana Office of Public Instruction and invited them to participate in this study (see Appendix C for sample letter). From those who were willing to participate, six teachers at five schools were chosen. The principals at these schools were contacted for permission to conduct the study at these schools, and approval was received from all five schools. (See Appendix C for sample permission request letter).

### Schools in the Study

The study was limited to tenth grade geometry classes in five high schools in Montana. The five schools are spread across the expanse of Montana and are representative of the high schools in Montana with student enrollment in excess of 200. Three of the schools (Cut Bank, Forsyth, and Glasgow) serve communities where agriculture is the primary industry, one (Columbia Falls) serves a community in which the timber industry is predominant, and the fifth school (CMR in Great Falls) serves one of the two large metropolitan areas in Montana. The average per capita income in the various communities served by these school districts ranges from a low of $8,370 (at Columbia Falls) to a high of $10,381 at Great Falls (U. S. Bureau of Census, 1991).

For the remainder of this study, the schools will be identified simply as schools A, B, C, D, and E. A summary of the student population at these five schools is given in Table 1.

Table 1. A Summary of the Student Population in the Schools in this Study.

| High School | Total Population | Number Taking Geometry | Number of Geometry Classes | Socioeconomic Background |
|---|---|---|---|---|
| A | male 315<br>female 324 | male 64<br>female 72 | 6 | poverty level 92<br>low income 24<br>all others 523 |
| B | male 129<br>female 119 | male 34<br>female 25 | 3 | poverty level 32<br>low income 14<br>all others 202 |
| C | male 137<br>female 95 | male 16<br>female 24 | 2 | poverty level 24<br>low income 10<br>all others 198 |
| D | male 145<br>female 120 | male 31<br>female 14 | 3 | poverty level 21<br>low income 6<br>all others 238 |
| E | male 831<br>female 748 | male 131<br>female 129 | 11 | poverty level 156<br>low income 88<br>all others 1335 |

Pupils in the Study

The students in this study were high school students who had completed one year of high school algebra. Each school utilized heterogeneous grouping of the students for the geometry classes. Since each school had more than one section of geometry being offered, student placement in a geometry class was based entirely upon scheduling constraints.

At the time of the study, the total geometry population at the five schools was 540 students. Three hundred two of these were originally chosen for the study, and the total number of students completing the study was 277. Completion was defined as (a) less than four days' absence during the four-week unit, (b) completion of all activities, and (c) completion of all evaluation instruments and opinionnaires. Three geometry classes were chosen from school A, and one of the three treatments was randomly assigned to each class. Three geometry classes were chosen from school B and because of limited computer facilities, treatment 2 was not assigned to any class. Treatment 1 was randomly assigned to two classes and treatment three was assigned to the remaining class. Both geometry classes from school C were used in the study with one class receiving treatment 1 and the other class receiving treatment 2. All three geometry classes from school D were used in the study with a different treatment being randomly assigned to each of the three classes. Three of the eleven geometry classes at school E were selected, and again a different treatment was randomly assigned to each of the three classes. Sixty-three students from school A, 51 students from school B, 35 from school C, 45 from school D, and 83 from school E completed the study. One hundred forty-four males and 133 females completed the study. This data is summarized in Table 2.

Since all schools had a school lunch program, social stratification was determined as follows:

poverty level--students were given free lunch;

low income--students paid a reduced lunch cost; and

all others--students paid full lunch costs.

The number of students in socio-economic status in the sample is summarized in Table 2.

The ages of the students selected were approximately the same at each school and within each treatment. Table 3 summarizes this data.

At the beginning of the 1990-1991 academic year all 277 students in the final sample were administered the CDASSG van Hiele Geometry Test to determine whether there were any initial differences in van Hiele levels among the groups. Results are compiled in Table 4.

Table 2. Social Stratification and Gender for the Samples

| | School | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | TOTALS | |
| | N | % | N | % | N | % | N | % | N | % | N | % |
| Treatment I | | | | | | | | | | | | |
| Students | | | | | | | | | | | | |
| Males | 8 | 40.0 | 17 | 50.0 | 8 | 40.0 | 11 | 73.3 | 17 | 60.7 | 61 | 52.1 |
| Females | 12 | 60.0 | 17 | 50.0 | 12 | 60.0 | 4 | 26.7 | 11 | 39.3 | 56 | 47.9 |
| Socio-economic Background | | | | | | | | | | | | |
| Poverty Level | 1 | 5.0 | 5 | 14.7 | 0 | 0.0 | 0 | 0.0 | 2 | 7.1 | 8 | 6.8 |
| Low Income | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 1 | 3.6 | 1 | 0.9 |
| All Others | 19 | 95.0 | 29 | 85.3 | 20 | 100.0 | 15 | 100.0 | 25 | 89.3 | 108 | 92.3 |
| Treatment II | | | | | | | | | | | | |
| Students | | | | | | | | | | | | |
| Males | 9 | 47.4 | - | --- | 9 | 60.0 | 10 | 71.4 | 14 | 48.3 | 42 | 54.5 |
| Females | 10 | 52.6 | - | --- | 6 | 40.0 | 4 | 28.6 | 15 | 51.7 | 35 | 45.5 |
| Socio-economic Background | | | | | | | | | | | | |
| Poverty Level | 1 | 5.3 | - | --- | 0 | 0.0 | 1 | 7.1 | 2 | 6.9 | 4 | 5.2 |
| Low Income | 0 | 0.0 | - | --- | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 |
| All Others | 18 | 94.7 | - | --- | 15 | 100.0 | 13 | 92.9 | 27 | 93.1 | 73 | 94.8 |
| Treatment III | | | | | | | | | | | | |
| Students | | | | | | | | | | | | |
| Males | 11 | 45.8 | 11 | 64.7 | - | --- | 9 | 56.2 | 10 | 38.5 | 41 | 49.4 |
| Females | 13 | 54.2 | 6 | 35.3 | - | --- | 7 | 43.8 | 16 | 61.5 | 42 | 50.6 |
| Socio-economic Background | | | | | | | | | | | | |
| Poverty Level | 3 | 12.5 | 1 | 5.9 | - | --- | 0 | 0.0 | 0 | 0.0 | 4 | 4.8 |
| Low Income | 0 | 0.0 | 1 | 5.9 | - | --- | 1 | 6.2 | 1 | 3.8 | 3 | 3.6 |
| All Others | 21 | 87.5 | 15 | 88.2 | - | --- | 15 | 93.8 | 25 | 96.2 | 76 | 91.6 |
| All Treatments | | | | | | | | | | | | |
| Students | | | | | | | | | | | | |
| Males | 28 | 44.4 | 28 | 54.9 | 17 | 48.6 | 30 | 66.7 | 41 | 49.4 | 144 | 52.0 |
| Females | 35 | 55.6 | 23 | 45.1 | 18 | 51.4 | 15 | 33.3 | 42 | 50.6 | 133 | 48.0 |
| Socio-economic Background | | | | | | | | | | | | |
| Poverty Level | 5 | 7.9 | 6 | 11.8 | 0 | 0.0 | 1 | 2.2 | 4 | 4.8 | 16 | 5.8 |
| Low Income | 0 | 0.0 | 1 | 2.0 | 0 | 0.0 | 1 | 2.2 | 2 | 2.4 | 4 | 1.4 |
| All Others | 58 | 92.2 | 44 | 86.3 | 35 | 100.0 | 43 | 95.6 | 77 | 92.8 | 257 | 92.8 |

Table 3. A Comparison of Student Ages (Means and Standard Deviations are given in years)

| | School | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | TOTAL | |
| | Mean | S.D. | Mean | S.D. | Mean | S.D. | Mean | S.D. | Mean | S.D. | Mean | S.D. |
| Treatment I Age | 15.9 | 1.00 | 15.4 | 0.72 | 16.1 | 0.67 | 15.9 | 0.39 | 16.3 | 0.83 | 15.9 | 0.82 |
| Treatment II Age | 15.9 | 0.66 | -- | -- | 15.7 | 0.56 | 15.8 | 0.68 | 16.1 | 0.83 | 15.9 | 0.72 |
| Treatment III Age | 15.8 | 0.54 | 15.7 | 0.54 | -- | -- | 15.7 | 0.66 | 15.6 | 0.51 | 15.7 | 0.55 |
| All Treatments Age | 15.9 | 0.73 | 15.5 | 0.67 | 15.9 | 0.65 | 15.8 | 0.59 | 16.0 | 0.79 | 15.8 | 0.72 |

## Teachers in the Study

The teachers in the study were selected on the basis of the following criteria: (a) the teacher was teaching two or more sections of high school geometry, (b) the teacher was using Geometry For Enjoyment and Challenge (Rhoad, et. al., 1984) or Geometry (Jurgensen, et. al., 1985) (these two textbooks have almost identical objectives for the unit used in this study), (c) the teacher had access to an adequate number of computers (three students per computer), and (d) the teacher was familiar with using a cooperative learning method of instruction, (e) the teacher had experience using a computer in a mathematics classroom, and (f) the teacher had at least two years' experience teaching high school geometry from one of the two textbooks used in this study. A summary of the teacher experience and class size is given in Table 5.

Table 4. A Comparison of van Hiele Levels

| | School A | | School B | | School C | | School D | | School E | | Total | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | N | % | N | % | N | % | N | % | N | % | N | % |
| **Treatment I** | | | | | | | | | | | | |
| Below Level 0 | 4 | 20.0 | 7 | 20.6 | 5 | 25.0 | 0 | 0.0 | 5 | 17.9 | 21 | 17.9 |
| Level 0 | 8 | 40.0 | 18 | 52.9 | 10 | 50.0 | 5 | 33.3 | 9 | 32.1 | 50 | 42.7 |
| Level 1 | 3 | 15.0 | 6 | 17.6 | 3 | 15.0 | 5 | 33.3 | 8 | 28.6 | 25 | 21.4 |
| Level 2 | 5 | 25.0 | 1 | 2.9 | 1 | 5.0 | 3 | 20.0 | 5 | 17.9 | 15 | 12.8 |
| Level 3 | 0 | 0.0 | 2 | 5.9 | 1 | 5.0 | 2 | 13.3 | 1 | 3.6 | 6 | 5.1 |
| **Treatment II** | | | | | | | | | | | | |
| Below Level 0 | 7 | 36.8 | - | --- | 4 | 26.7 | 5 | 35.7 | 4 | 13.8 | 20 | 26.0 |
| Level 0 | 8 | 42.1 | - | --- | 9 | 60.0 | 7 | 50.0 | 10 | 34.5 | 34 | 44.2 |
| Level 1 | 3 | 15.8 | - | --- | 1 | 6.7 | 0 | 0.0 | 11 | 37.9 | 15 | 19.5 |
| Level 2 | 1 | 5.3 | - | --- | 1 | 6.7 | 1 | 7.1 | 2 | 6.9 | 5 | 6.5 |
| Level 3 | 0 | 0.0 | - | --- | 0 | 0.0 | 1 | 7.1 | 2 | 6.9 | 3 | 3.9 |
| **Treatment III** | | | | | | | | | | | | |
| Below Level 0 | 6 | 25.0 | 7 | 41.2 | - | --- | 4 | 25.0 | 5 | 19.2 | 22 | 26.5 |
| Level 0 | 6 | 25.0 | 8 | 47.1 | - | --- | 10 | 62.5 | 8 | 30.8 | 32 | 38.6 |
| Level 1 | 8 | 33.3 | 0 | 0.0 | - | --- | 1 | 6.2 | 9 | 34.6 | 18 | 21.7 |
| Level 2 | 3 | 12.5 | 2 | 11.8 | - | --- | 1 | 6.2 | 2 | 7.7 | 8 | 9.6 |
| Level 3 | 1 | 4.2 | 0 | 0.0 | - | --- | 0 | 0.0 | 2 | 7.7 | 3 | 3.6 |
| **All Treatments** | | | | | | | | | | | | |
| Below Level 0 | 17 | 27.0 | 14 | 27.4 | 9 | 25.7 | 9 | 20.0 | 14 | 16.9 | 63 | 22.7 |
| Level 0 | 22 | 34.9 | 26 | 51.0 | 19 | 54.3 | 22 | 48.9 | 27 | 32.5 | 116 | 41.9 |
| Level 1 | 14 | 22.2 | 6 | 11.8 | 4 | 11.4 | 6 | 13.3 | 28 | 33.7 | 58 | 20.9 |
| Level 2 | 9 | 14.3 | 3 | 5.9 | 2 | 5.7 | 5 | 11.1 | 9 | 10.8 | 28 | 10.1 |
| Level 3 | 1 | 1.6 | 2 | 3.9 | 1 | 2.9 | 3 | 6.7 | 5 | 6.0 | 12 | 4.3 |

A workshop detailing the purpose of this study, implementation of the treatments, use of the activity sheets, use of the computer software, and other duties to be performed by participating teachers was given by this researcher in July 1990. A complete set of instructions and activity sheets, daily log forms, assignment schedule, parental permission forms, testing instruments and answer sheets were supplied by the researcher. Upon completion, all materials (activity sheets, daily logs, permission forms, opinionnaires, and tests) were returned to the researcher for scoring and evaluation.

Table 5. Description of Teachers and Geometry Classes

| TEACHER | YEARS EXPERIENCE Total Years | At Present School | Teaching Geometry | GEOMETRY CLASSES IN THIS STUDY Period | No. of Students | Treatment |
|---|---|---|---|---|---|---|
| A | 7 | 7 | 7 | 1 | 24 | III |
| | | | | 3 | 19 | II |
| | | | | 5 | 20 | I |
| B | 18 | 6 | 17 | 3 | 18 | I |
| | | | | 5 | 16 | I |
| | | | | 7 | 17 | III |
| C | 12 | 1 | 9 | 6 | 15 | II |
| | | | | 7 | 20 | I |
| D | 19 | 19 | 15 | 3 | 14 | II |
| | | | | 5 | 15 | I |
| | | | | 7 | 16 | III |
| E-1 | 23 | 12 | 18 | 1 | 29 | II |
| | | | | 4 | 28 | I |
| E-2 | 19 | 2 | 19 | 7 | 26 | III |

## The Null Hypotheses of the Study

Questions addressed in this study are stated in the form of null hypotheses.

Hypothesis 1 Effects of Treatments on Achievement Scores

1.1 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and the corresponding pretest is controlled, in overall mean achievement among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

1.2 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and the corresponding pretest is controlled, in mean achievement on knowledge and comprehension items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

1.3 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and the corresponding pretest is controlled, in mean achievement on application and analysis items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

Hypothesis 2 Effects of Treatments on Retention Scores

2.1 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and the corresponding pretest is controlled, in overall mean retention among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

2.2 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and the corresponding pretest is controlled, in mean retention on knowledge and comprehension items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

2.3 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and the corresponding pretest is controlled, in mean retention on application and analysis items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

Hypothesis 3 Effects of Treatments on Attitude Scores

3.1 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, and attitude toward geometry before the treatments, in attitude toward geometry at the completion of the unit among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

3.2 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, and attitude toward geometry before the treatments, in attitude toward geometry four weeks after the application of the treatment among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

Hypothesis 4 Correlation between Attitude toward Geometry and Achievement

4. There is no correlation between the attitude toward geometry at the completion of the unit and the achievement of geometric concepts among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

Hypothesis 5 Correlation between Attitude toward Geometry and Retention

5. There is no correlation between the attitude toward geometry four weeks after the completion of the unit and the retention of geometric concepts among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

Hypothesis 6 Correlation between Achievement and Retention

6. There is no correlation between the achievement of geometric concepts and the retention of geometric concepts among students having treatment I (small cooperative learning groups using the paper and

pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion led by the teacher).

## The Instruments Used in the Study

The following measuring instruments were used in this study:

1. CDASSG van Hiele Geometry Test,
2. Aiken-Dreger Mathematics Opinionnaire,
3. Aiken-Dreger Geometry Opinionnaire,
4. Geometry Unit Test Form A,
5. Geometry Unit Test Form B, and
6. Participation in the "free" lunch program (poverty level, low-income level, and all others) used as the test for social stratification in each school.

Copies of instruments 1, 2, and 3 are in Appendix D and copies of instruments 4 and 5 are in Appendix A.

### CDASSG van Hiele Geometry Test

The Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project constructed this test in order to test the van Hiele theory (Usiskin, 1982). The test items were based on the van Hiele's descriptions of their five levels of geometric thought. The questions with respect to their congruence with the van Hiele descriptions of the levels were rated by the Advisory Board of the CDASSG project. After a pilot test, the CDASSG project staff consulted with P. M. van Hiele and revised some of the items (Senk, 1983). The final version of the CDASSG van Hiele Level Test was a 25-item multiple choice test with 5 foils per item and 5 items per level.

On the basis of a sample of 2699 students enrolled in a one-year geometry course, Usiskin (1982) reported Kuder-Richardson Formula 20 coefficients for the five subsets from a fall and spring administration of the instrument. For the level 0-4 subtests, the K-R 20 values were ).31, 0.44, 0.49, 0.13, and 0.10 respectively, for the fall administration and 0.39, 0.55, 0.56, 0.30, and 0.26 for the spring administration. Bobango (1987), somewhat concerned about these reliabilities, checked the test-retest reliability of this test using twenty-one advanced mathematics students who had studied formal geometry for one year. The resulting coefficient of stability was 0.60.

An interview method has been developed for identifying the van Hiele level for a student (Bobango, 1987; Burger, 1986; NCTM, 1988). Bobango (1987) correlated the van Hiele level of 16 students obtained by the interview method with the van Hiele levels determined by the CDASSG van Hiele Level Test at two different times during her study. The first set of data gave a correlation coefficient of 0.62 and the second correlation coefficient was 0.84. Both of these correlations are significant at the 0.05 level of significance. Therefore, the CDASSG van Hiele Level test was chosen to use in this study because of its acceptable coefficient of stability, its acceptable correlation with the interview method, and its convenience to administer and score.

In the CDASSG project, students were assigned van Hiele level N if they mastered levels 0,1, ... , N. Those that did not satisfy this criterion were not assigned a level. Mastery was defined as either 3 out of 5 correct (Type I error = 0.0579) or 4 out of 5 correct (Type I error = 0.0067). Usiskin (1982) reported that 85 percent of the students could be assigned a level prior to taking a formal geometry course using the 3 out of 5 criterion (with level 4 items omitted from the test) and that 92 percent of the students could be assigned a level using the 4 out of 5 criterion (under similar conditions).

In scoring this test, the researcher omitted the last five items of this test. There are two reasons for this omission. First, it is highly unlikely that any student currently enrolled in high school geometry would be at level 4. Second, as Usiskin points out, "Level [4] either does not exist or is not testable. All other levels are testable" (Usiskin, 1982, p. 79). Also, the researcher used the 4 out of 5 mastery criterion with a modification to the scoring system used in the CDASSG project. According to the CDASSG method, a student is assigned a weighted sum score in the following manner:

1 point for meeting criterion on items 1-5 (level 0);

2 points for meeting criterion on items 6-10 (level 1);

4 points for meeting criterion on items 11-15 (level 2);

8 points for meeting criterion on items 16-20 (level 3).

Assigned points were added to give the weighted sum which indicated the levels reached by each student. Thus, weighted sum scores 1, 3, 7, and 15 are equivalent to van Hiele levels 0 to 3. A score of 0 means the student has not yet mastered van Hiele level 0. Those students with weighted sum scores other than these were designated as not fitting any van Hiele level. In order to utilize the scores of those students that do not fit any level, the researcher classified the "non-fitters" to a pseudolevel as suggested by Han (1986). A non-fitter's pseudolevel is defined as the highest non-consecutive level this student has mastered (4 out of 5 correct). Thus, a weighted score of 2 is equivalent to pseudolevel 1; weighted sum scores 4, 5, or 6 are equivalent to pseudolevel 2; and weighted sum scores 8 to 14 are equivalent to pseudolevel 3.

Permission to use this test for this study was granted by Dr. Zalman Usiskin, the principal investigator of the CDASSG project. (See Appendix D for letters requesting and granting permission.)

Aiken-Dreger Mathematics Opinionnaire and Aiken-Dreger Geometry Opinionnaire

The Aiken-Dreger Mathematics Opinionnaire, a 20-item Likert scale opinionnaire developed by Aiken and Dreger (1961), was the instrument used to measure attitude toward mathematics. This opinionnaire has been used with college students, high school students, and junior high school students. This opinionnaire was selected since it would give a measure of a student's attitude toward mathematics in general before any instruction in the geometry classroom had begun.

The Aiken-Dreger Geometry Opinionnaire is simply a modification of the Aiken-Dreger Mathematics Opinionnaire made by replacing the word "mathematics" with "geometry." This enabled the researcher to measure a student's attitude toward geometry using an instrument almost identical to the instrument used to measure a student's attitude toward mathematics.

Items for the Aiken-Dreger Mathematics Opinionnaire were developed from suitable paragraphs written by 310 college students. Validity estimates were obtained from a sample of 160 female college sophomores. As a measure of this opinionnaire's content validity, Aiken and Dreger (1961) reported that scores on the attitude scale were positively correlated with numerical ability but unrelated to specified general personality variables. The authors also claim a degree of discriminating ability since a test of independence between the scores on the opinionnaire and scores on four items designed to measure attitude toward academic subjects in general indicated that attitudes specific to mathematics were being measured. The opinionnaire had a test-retest reliability of 0.94.

The responses to each item on the opinionnaire received weighted values from 4 (strongly agree) to 0 (strongly disagree). Exactly half of the items are reverse items, and hence the weights are changed accordingly. A student's score on this opinionnaire is the sum of the weighted alternatives chosen by him/her with a higher score indicating a more positive attitude toward mathematics (and geometry).

To determine if the Aiken-Dreger Geometry Opinionnaire retained the same internal consistency of the original version, the researcher used the data obtained from 136 students in the pilot study who completed both the original version and the modified version. Using the split-half technique (Popham, 1981), the correlation coefficient (obtained by correlating the two subscores derived from the odd and even items) for the original version was 0.971. The correlation coefficient for the modified version was 0.968.

The Aiken-Dreger Mathematics Opinionnaire has been placed in the public domain, so permission to use this instrument was not requested.

Geometry Unit Test Form A and B

Tests of achievement and retention of geometric concepts that were reviewed for possible use in this study were not appropriate because (1) the tests included items that did not meet the stated objectives of this unit, or (2) the tests included items in which the students had to complete detailed proofs. The students, at the time of this study, were not adept at creating detailed proofs; hence, two parallel, equivalent forms of a criterion-referenced test were developed by this researcher. The Geometry Unit Test Form A was used as the pretest, Geometry Unit Test

Form B was used as the achievement test, and Geometry Unit Test Form A was used as the retention test.

The test was designed using a table of specifications as described by Guskey (1985). The first step in this process is to determine what content is to be learned. (The content to be learned for this unit is given in Appendix S.) In the second step, student behaviors or objectives are written to reflect the content to be learned. The objectives should represent a range of categories of difficulties as described by Bloom (1956). The test developer then creates a matrix with the objectives on the vertical axis and the levels of Bloom's taxonomy on the horizontal axis. An item is written for the test and the item number is entered into the proper matrix slot. Test items can apply to more than one level of Bloom's taxonomy.

The researcher developed such a table of specifications and wrote two such tests called Unit Test Form A and Unit Test Form B. Each form contained 25 multiple choice items with 5 foils per item with the exception that two items had 3 foils and two items had 6 foils. The tests were administered to two geometry classes during the 1988-1989 academic year (as described in the Pilot Study section). Based upon the data obtained from this trial, questions were modified or rewritten. Reliability for this test was determined by the equivalence and stability method. Validity was established by the content validity method.

Popham defines stability as the "consistency of a test's measure over time" (1981, p. 128) and the stability is measured by computing the Pearson correlation coefficient for the scores of one test administered twice to the same examinees with a reasonable time interval between tests. Equivalence measures the "degree of equalness" between two forms of a test. Because criterion-referenced tests are based on very specific objectives, Popham (1981) states that it is easy to create truly equivalent forms. The equivalence coefficient of reliability of the two forms is determined by administering two forms of the test and then calculating the Pearson correlation coefficient for the examinee's scores on the two test forms. The length of the interval between the two testing occasions is, of course, a significant issue. Popham (1981) states that between-testing intervals of two or three weeks are common when establishing the reliability of achievement tests.

Thirty Algebra II students from Beaverhead County High School, Dillon, Montana, were the examinees for determining the reliability of this test. These students had completed the geometry course the previous year. Form A was administered to the entire group of thirty on 17 October 1989. On 17 November (four weeks later) fourteen of these students took Form A again and sixteen were given Form B. The stability coefficient of reliability was 0.934 and the equivalence coefficient of reliability was 0.924, above the 0.80-0.90 coefficients for which test developers strive (Popham, 1981).

In order to check for difficulty disparities, test form means on the two forms were compared. The comparison, given in Table 6, indicates that the two forms are of equal difficulty. Hence, from the data collected, the Geometry Unit Test Form A and Geometry Unit Test Form B were judged to be stable, equivalent, and of equal difficulty.

Table 6. Test Results for Unit Test Form A and Form B

| Date Administered | Form A | | Form B | |
|---|---|---|---|---|
| | Mean | S.D. | Mean | S.D. |
| 17 October 1989 | 13.97 (N=30) | 4.20 | n/a | n/a |
| 17 November 1989 | 13.43 (N=14) | 3.22 | 14.25 | 3.70 (N=16) |

Popham states that the main goal of a criterion-referenced test is to provide information on what an examinee can or cannot do and therefore "content validity is of paramount importance" (1981, p.

105). The method chosen to determine content validity is to use the table of specifications (as described earlier in the construction of this test). Both the table and test are submitted to field experts who determine whether or not the test does indeed measure the desired objectives as defined by the table of specifications (Gronlund, 1976; Popham, 1981).

The researcher submitted the test and table of specifications to three mathematics educators, one employed by the University of Montana, Missoula, Montana; the second employed by the Missoula High School District, Missoula, Montana; and the third employed by the Drummond High School District, Drummond, Montana. All three judged the test to measure the objectives (see Appendix A); hence, the test was judged to have content validity.

## Measure of Social Stratification

The instruments used to determine the social stratification of the students in this study were the federal and state guidelines used to determine whether or not a student received lunch free of charge (poverty level), qualified for reduced cost (low income level), or had to pay cost for their lunch (all others). For the academic year 1990-1991, if the total household annual income were at or below the amounts on the Gross Annual Income Chart below (Table 7), the child received free lunch (poverty level) or reduced-price lunch (low income level).

Table 7. Social Stratifications Based on Household Income.

| Socio-economic Background | Gross Annual Income | | | | |
| --- | --- | --- | --- | --- | --- |
| | Household Size | | | | |
| | 2 | 3 | 4 | 5 | over 5 |
| Poverty level | 10,946 | 13,728 | 16,510 | 19,292 | * |
| Low Income | 15,577 | 19,536 | 23,495 | 27,454 | ** |

* for each additional family member, add 2,782
** for each additional family member, add 3,959

## Nature of the Treatments

Three different experimental groups, consisting of high school geometry classes from five high schools, were established. One experimental group of classes was assigned treatment I (small cooperative learning groups using van Hiele phase-based paper and pencil activities), a second group was assigned treatment II (small cooperative learning groups using van Hiele phase-based computer activities), and a third group was assigned treatment III (traditional whole class discussion led by the teacher). The objectives for the congruent triangle unit were the same for all groups. The same problems were assigned from the Rhoad, Milauskas, and Whipple text to those classes using this text. Likewise, the same problems were assigned from the Jurgensen, Brown, and Jurgensen text to those classes using this text.

The application of the treatments began at the start of the second nine-week period of the 1990-1991 academic year and lasted for four weeks of instruction. Prior to this time, no instructional materials or methodology similar to the ones in Treatment I or Treatment II were used in any of the classes. When the four-week treatment period ended, the posttest and Geometry Opinionnaire were administered, and classroom instruction returned to the pretreatment status. Four weeks later the retention test and final Geometry Opinionnaire was administered.

In order to maximize control of the teacher variable, at four of the five schools, it was possible to select classes involved in this study that were taught by one teacher at each school. (At school E, no single teacher taught more than two geometry classes, so a second teacher was selected so that all three treatments would be present at this school.) Each geometry class was randomly assigned one of the three treatments.

A detailed description of each treatment follows.

Treatment I

Each concept of the unit on congruent triangles was introduced to the students in this treatment using a sequence of paper and pencil activities designed by the researcher based upon the first three phases of the van Hiele theory. (These activities are in Appendix B.) The students were assigned by the teacher to small cooperative learning groups and each group worked cooperatively to complete the paper and pencil activities. The teacher was always available for visitations with the groups, answering questions, encouraging discussion, and praising the groups for their efforts.

Phase 1 (information phase): Before the activity sheets were distributed, the teacher described the objectives of the activity, reviewed the prerequisite vocabulary for the activity, and checked that each group had the necessary supplies such as pencils and metric rulers. Then the activity sheets were distributed and each group proceeded to work through the activities cooperating in the completion of the assigned tasks.

Phase 2 (guided orientation phase): As the student groups proceeded to work through the activity sheets, they moved from the information phase into the guided orientation phase. The activity sheets included several tasks that required each group to draw, measure, and record the results as a means of extending the introductory concepts.

Phase 3 (explicitation phase): The activity sheets concluded with a series of questions to answer, definitions to complete, or conjectures to make that had to be completed by each group of students. Once these were completed by the groups, a spokesperson for the group shared his/her group's findings with the rest of the class. The teacher then formalized the findings so that the final conclusions matched the objectives (and technical language) of the textbook being used.

Phase 4 (free orientation phase): Problems from the text were assigned to complete phase four of the van Hiele model. The students were allowed to remain in their small cooperative groups to begin work on the homework problems as classroom time permitted.

Phase 5 (integration): Although some aspects of this phase were addressed by the students' completion of the homework assignments, there was no formal attempt to complete this phase in this study.

The students in Treatment I spent a minimum of 43% of the instructional time (school A) and a maximum of 61% of the instructional time (school C) in the mode described above. The remainder of the instructional time was devoted to teacher-led activities which included clarifying homework assignments, discussing and explaining homework problems, and administering quizzes and tests. (See Table 8 for a comprehensive analysis of how instructional time was spent at each school.)

Treatment II

Each concept of the unit on congruent triangles was introduced to the students in this treatment using a sequence of computer activities designed by the researcher based upon the first three phases of the van Hiele theory. These activities used the computer programs Geometric PreSupposer or Geometric Supposer: Triangles both from Sunburst Communications (Educational Development Center, 1986; 1985). (These activities are in Appendix B.) The students were assigned by the teacher to small cooperative learning groups, and each of the small groups worked at a computer completing the activities. The teacher was always available for visitations with the groups, answering their questions, encouraging discussion, and praising the individual groups for their efforts.

Phase 1 (information phase): Before the activity sheets were distributed, the teacher described the objectives of the activity, reviewed the prerequisite vocabulary for the activity, and made sure each group had properly "booted" the computer program. Then the activity sheets were distributed and each group of three students proceeded to work through the activities and to cooperate to complete the assigned tasks.

Phase 2 (guided orientation phase): As the student groups proceeded to work through the activity sheets, they moved from the information phase into the guided orientation phase. The activity sheets included several activities that required each group to use the computer program to draw and measure various geometric figures and to record the data obtained from the computer as a means of extending the introductory concepts.

Phase 3 (explicitation phase): The activity sheets concluded with a series of questions to answer, definitions to complete, or conjectures to make that had to be completed by each group of students. Once these were completed by the groups, a spokesperson for each group shared his/her group's findings with the rest of the class. The teacher formalized the findings so that the final conclusions matched the objectives (and technical language) of the text book being used.

Phase 4 (free orientation phase): Appropriate problems from the text were assigned to complete phase four of the van Hiele model. The students were allowed to remain in their small cooperative groups to begin work on the homework problems as classroom time permitted.

Phase 5 (integration): Although some aspects of this phase were addressed by the students' completion of the homework assignments, there was no formal attempt to complete this phase in this study.

The students in Treatment II spent a minimum of 44% of the instructional time (school D) and a maximum of 61% of the instructional time (school C) in the mode described above. The remainder of the instructional time was devoted to teacher-led activities which included clarifying homework assignments, discussing and explaining homework problems, and administering quizzes and tests. (See Table 8 for a comprehensive analysis of how instructional time was spent at each school.)

Table 8. Instructional Time (in percent) Spent in Classes Assigned Treatments I and II.

| School (Treatments) | Cooperative Learning Groups | Teacher Led Discussion | Testing | Total |
|---|---|---|---|---|
| A (I,II) | 56 | 29 | 15 | 100 |
| B (I) | 43 | 48 | 9 | 100 |
| C (II) | 61 | 29 | 10 | 100 |
| D (I, II) | 44 | 39 | 17 | 100 |
| E (I, II) | 46 | 50 | 4 | 100 |

## Treatment III

Each concept of the unit on congruent triangles was introduced in this treatment using a traditional "whole class" lecture-discussion method of instruction using only the textbook as a guide. The teacher presented the material in the same sequence as outlined in the textbook, periodically asking members of the class various questions to check for understanding. This was followed by the appropriate assignment from the textbook. The students were allowed to begin these homework problems as classroom time permitted. The students worked independently on these problems asking only the teacher for assistance if questions about a particular problem arose.

Besides presenting the new concepts to the entire class, the teacher spent part of the instructional time clarifying homework assignments, discussing and explaining homework problems, and administering quizzes and tests.

## Research Design

In this experiment, the researcher lacked full control over the ability to randomize completely the scheduling of experimental stimuli. Hence, the quasi-experimental pre-post design based on Campbell and Stanley's (1963) Nonequivalent Control Group Design was used. This design differs from the true experimental Pretest-Posttest Control Group Design in that in this quasi-experimental design the group of students comprising a particular geometry class is randomly assigned to a treatment, whereas in the true experimental design, an individual student from the common population would be assigned randomly to a treatment group.

Gay (1981) encourages the use of this model when working with intact groups. He states that using this model with intact groups minimizes the possible effects from reactive interference from variables. Campbell and Stanley (1963) add that this model controls the main effects of six contaminating variables: history, selection, testing, instrumentation, maturation, and mortality. Regression is an internal validity problem for this design; however, Stanley and Campbell indicate that this should not be a concern if treatment groups have not been selected based on extreme scores on the pretest. Since classrooms were assigned to treatments prior to any testing, the researcher assumes that this factor was minimally controlled.

Simple correlation analysis, multiple linear regression, analysis of covariance, and multiple comparisons of adjusted means were the primary methods of analyses. The analysis of covariance was chosen because it enables the researcher to identify and to adjust for sources of variation caused by the concomitant variables, thus allowing for greater control (Kerlinger, 1973).

Independent variables used in the analysis of the data are:

1. Treatment - three levels: (I) small cooperative learning groups using van Hiele phase-based paper and pencil activities, (II) small cooperative learning groups using van Hiele phase-based computer activities, and (III) traditional whole class discussion activities led by the teacher;
2. school- five levels: school A, B, C, D, and E;
3. van Hiele level - four levels: level 0, level 1, level 2, and level 3 (level 4 was not used in the study since no student scored at this level);
4. attitude toward mathematics before taking geometry;
5. attitude toward geometry prior to treatment;
6. pretest achievement total scores;
7. pretest achievement scores on knowledge and comprehension test items;
8. pretest achievement scores on application, analysis, synthesis, and evaluation test items;
9. gender - male and female;
10. age - in years as of 1 September 1990; and
11. socio-economic background - three levels: poverty, low income, and all others.

Dependent variables are:

1. posttest achievement total scores;
2. posttest achievement scores on knowledge and comprehension test items;

3. posttest achievement scores on application, analysis, synthesis, and evaluation test items;

4. retention total scores;

5. retention scores on knowledge and comprehension level test items;

6. retention scores on application and analysis level test items;

7. attitude toward geometry at the time of completion of the unit; and

8. attitude toward geometry four weeks after the completion of the unit.

The independent variables determining treatment, van Hiele level, school, gender, and socio-economic background are binary coded. All other independent and dependent variables are continuous.

In testing the hypotheses of a study, the researcher runs the risk of rejecting a null hypotheses when it is in fact true (Type I error). There is also the risk of failing to reject a null hypothesis when it is in fact false (Type II error). The primary focus of this study was to determine which instructional method tested is most effective for teaching geometry. The consequence of a Type I error could be that a geometry teacher (and the school) would decide to invest time and resources to reorganize the instructional process in the geometry class when in fact there will be no change in student behavior. The consequence of a Type II error would be to implement no change in the instructional methods and deny the students a better learning environment. Commonly accepted levels of significance (the probability of committing a Type I error, called alpha) in educational research are either 0.05 or 0.01 (Ferguson, 1981; Spatz and Johnson, 1981). This researcher believes committing a Type II error is the more serious error, and since increasing the probability of a Type I error decreases the probability of a Type II error, alpha = 0.05 was used to test all hypotheses in this study.

# CHAPTER 4

# ANALYSIS OF THE DATA

## Introduction

This chapter presents the analysis and interpretation of the data collected for this study. It is organized into four sections: (1) description of the sample, (2) methods of analysis, (3) tests of hypotheses, and (4) summary of results.

## Description of the Sample

The study was limited to high school students who had completed one year of high school algebra and were enrolled in a high school geometry course at the time of the study. The students came from one of five selected high schools in Montana. Fourteen classes were involved, three classes from each of four schools and two classes from one school. At three of the schools that had three classes participating, one teacher at each school taught the three participating classes. In the fourth school that had three classes participating, one teacher taught two of the classes and a second teacher taught the remaining class. In the fifth school where only two classes participated, both classes were taught by same teacher. The total geometry population of the five schools was 540 students at the time the study was initiated. Three hundred two of these students were initially selected to participate. Twenty-five of these students were deleted from the study because they were absent more than four days during the unit of study, did not complete all activities, or did not complete all evaluation instruments and opinionnaires. Therefore, 277 students were in the final sample. The final sample consisted of 144 males and 133 females, of which treatment I (small cooperative learning groups using the paper and pencil activities) consisted of 61 males and 56 females, treatment II (small cooperative learning groups using the computer activities) consisted of 42 males and 35 females, and treatment III (traditional whole class discussion led by the teacher) consisted of 41 males and 42 females.

## Methods of Analysis

The analysis of the data was performed using the statistical package SPSS-X Release 3.1 for VAX/VMS running on a Digital Equipment Corporation VAX 11-750.

Since "intact" classes were used as the experimental groups in this study, it was impossible to control or adjust experimentally for the effects of the independent variables age, gender, van Hiele level, attitude toward mathematics and geometry, socio-economic status, prior knowledge of geometry concepts, and school setting. Ferguson (1981) suggests using the analysis of covariance method for such a situation. Analysis of covariance uses a statistical method to control these effects. This method was used to test the first three hypotheses regarding differences among the treatments on posttest scores, retention test scores, and attitude toward geometry. In each case a step-wise linear regression analysis was used to determine which attribute-predictor variables (gender, socio-economic status, school, student age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and pretest score) could not be dropped and were needed as covariates.

If the analysis of covariance indicated there was a significant difference among mean scores for the treatment groups, the Newman-Keuls method of multiple comparisons was used to determine which pairs of treatment mean scores differed significantly. The method using the harmonic mean of the number of observations in the groups, as suggested by Ferguson (1981), was the form of Newman-Keuls method used.

To test the remaining three hypotheses regarding a correlation between attitude and posttest scores, attitude and retention test scores, and posttest and retention test scores, the Pearson product-moment correlation coefficient between the designated pair of variables was calculated for each treatment. This value was then tested to determine if it were significant. If the correlations between the two variables for two or three of the treatments were statistically significant, Fisher's $z_r$ transformation was used to test for a significant difference between these correlation coefficients.

A summary of the mean test scores by school and treatment are given in Table 9. A summary of the mean attitude scores by school and treatment measured at four different times (the start of the school year, when the treatments began at the end of the first nine weeks of school, when the treatments were completed, and four weeks later) are given in Table 10.

## Tests of Hypotheses

### Effects of Treatments on Achievement Scores of Students

One purpose of this study was to determine the effect the three treatments would have on student

Table 9. Mean Unadjusted Scores on the Pretest, Posttest, and Retention Test

| School | Treatment I | Treatment II | Treatment III |
|---|---|---|---|
| A | | | |
| pretest | 9.35 | 8.63 | 9.46 |
| posttest | 14.65 | 12.63 | 13.71 |
| retn test | 15.60 | 11.10 | 13.88 |
| B | | | |
| pretest | 9.00 | --- | 8.53 |
| posttest | 14.56 | --- | 13.00 |
| retn test | 15.62 | --- | 14.65 |
| C | | | |
| pretest | 12.75 | 11.40 | --- |
| posttest | 16.05 | 13.73 | --- |
| retn test | 18.30 | 17.13 | --- |
| D | | | |
| pretest | 10.73 | 10.93 | 10.69 |
| posttest | 17.80 | 16.86 | 15.88 |
| retn test | 17.67 | 17.93 | 16.69 |
| E | | | |
| pretest | 12.21 | 12.21 | 11.58 |
| posttest | 19.18 | 18.31 | 16.62 |
| retn test | 19.86 | 18.34 | 16.12 |
| All schools | | | |
| pretest | 10.69 | 10.93 | 10.17 |
| posttest | 16.35 | 15.75 | 14.89 |
| retn test | 17.35 | 16.24 | 15.28 |

achievement at the end of the unit of instruction as measured by the posttest. The items of the posttest were classified as low-level items (at the knowledge or comprehension level as per Bloom's taxonomy) or high-level items (at the application, analysis, synthesis, or evaluation level as per Bloom's taxonomy). Also, the effect the treatments may have

Table 10. Mean Attitude Scores as Measured at Four Different Times

| School | Trtmnt I | Trtmnt II | Trtmnt III |
|---|---|---|---|
| A | | | |
| start of school | 41.25 | 40.53 | 47.88 |
| start of trtmnt | 58.15 | 49.58 | 54.13 |
| end of treatmnt | 51.65 | 39.32 | 49.46 |
| retn test time | 46.80 | 38.95 | 48.38 |
| B | | | |
| start of school | 47.18 | --- | 35.00 |
| start of trtmnt | 38.00 | --- | 28.00 |
| end of treatmnt | 38.18 | --- | 28.77 |
| retn test time | 35.85 | --- | 32.18 |
| C | | | |
| start of school | 42.90 | 49.87 | --- |
| start of trtmnt | 41.80 | 44.73 | --- |
| end of treatmnt | --- | --- | --- |
| retn test time | 37.95 | 37.40 | --- |
| D | | | |
| start of school | 51.93 | 45.86 | 54.31 |
| start of trtmnt | 47.67 | 46.43 | 54.94 |
| end of treatmnt | --- | --- | --- |
| retn test time | 51.07 | 43.77 | 47.25 |
| E | | | |
| start of school | 41.36 | 43.41 | 49.69 |
| start of trtmnt | 34.71 | 39.00 | 42.54 |
| end of treatmnt | --- | --- | --- |
| retn test time | 37.54 | 37.28 | 43.58 |
| All schools | | | |
| start of school | 44.36 | 44.40 | 47.05 |
| start of trtmnt | 42.55 | 44.08 | 45.31 |
| end of treatmnt | 43.17 | 39.32 | 40.87 |
| retn test time | 40.43 | 38.90 | 43.34 |

on student achievement with low-level and high-level type questions was of interest.

In order to determine any possible effects, an analysis of covariance to test Null Hypotheses 1.1-1.3 was run using the researcher-designed posttest as the criterion variable, and the three treatments as the active predictor variables. A step-wise linear regression analysis was used prior to the covariance analysis to identify which attribute-predictor variables needed to be used as covariates. As a result of this linear regression analysis, the covariates used in this covariance analysis were pretest score, school, van Hiele level, age of the student, and attitude toward geometry before the treatments (see Table 11).

Table 11. Partial Correlation Coefficients between Attribute Predictor Variables and the Posttest Score.

| Variables | Partial Correlation Coefficient | F | Significance of F |
|---|---|---|---|
| Total Pretest Score | 0.402 | 52.187 | 0.000 * |
| School | 0.326 | 32.210 | 0.000 * |
| van Hiele Level | 0.239 | 16.399 | 0.000 * |
| Age of student | - 0.174 | 8.482 | 0.004 * |
| Attitude toward Geom | 0.136 | 5.137 | 0.024 * |
| Gender | 0.098 | 2.624 | 0.106 |
| Socio-economic Status | 0.077 | 1.592 | 0.208 |
| Attitude toward Math | 0.065 | 1.146 | 0.285 |

* indicates significant at 0.05 level

Null Hypothesis 1.1 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and corresponding pretest score is controlled, in overall mean achievement among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities) and treatment III (traditional whole class discussion led by the teacher).

The analysis of covariance (see Table 12) indicated that Null Hypothesis 1.1 should not be rejected at the 0.05 level of significance. Table 13 lists the observed and adjusted treatment means for the total posttest scores for each treatment.

Table 12. Analysis of Covariance Based on Total Posttest Score with Pretest Score, School, van Hiele Level, Age of Student, and Attitude toward Geometry before the Treatments as Covariates.

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 5 | 2092.65 | 418.53 | 37.83 | 0.000 |
| Treatments | 2 | 44.14 | 22.07 | 1.99 | 0.138 |
| Residual | 269 | 2976.32 | 11.06 | | |

Table 13. Observed and Adjusted Group Means for the Total Posttest Score

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 117 | 16.35 | 16.14 |
| Treatment II | 77 | 15.75 | 15.68 |
| Treatment III | 83 | 14.89 | 15.17 |

Null Hypothesis 1.2 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and corresponding pretest score is controlled, in mean achievement on knowledge and comprehension items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities) and treatment III (traditional whole class discussion led by the teacher).

The analysis of covariance (see Table 14) indicated that Null Hypothesis 1.2 should be rejected at the 0.05 level of significance. Since Null Hypothesis 1.2 was rejected, it was in order to determine which of the treatment means differed significantly from each other. Table 15 lists the observed and adjusted treatment means for the total posttest scores for each treatment. The results of the Newman-Keuls method of multiple comparisons indicated that significant differences (at the 0.05 level of significance) in mean achievement on the low-level items occurred between Treatment I and Treatment III and between Treatment II and Treatment III. There was no significant difference in mean achievement on these items between Treatment I and Treatment II.

Table 14. Analysis of Covariance Based on Knowledge and Comprehension Items Posttest Score with Pretest Score, School, van Hiele Level, Age of Student, and Attitude toward Geometry before the Treatments as Covariates.

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 5 | 218.68 | 43.74 | 15.90 | 0.000 |
| Treatments | 2 | 34.34 | 17.17 | 6.24 | 0.002 |
| Residual | 269 | 740.07 | 2.75 | | |

Null Hypothesis 1.3 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and corresponding pretest score is controlled, in mean achievement on application, analysis, synthesis and evaluation items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities) and treatment III (traditional whole class discussion led by the teacher).

Table 15. Observed and Adjusted Group Means for the Knowledge and Comprehension Items Posttest Score

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 117 | 7.62 | 7.53 |
| Treatment II | 77 | 7.44 | 7.40 |
| Treatment III | 83 | 6.59 | 6.71 |

The analysis of covariance (see Table 16) indicated that Null Hypothesis 1.3 should not be rejected at the 0.05 level of significance. Table 17 lists the observed and adjusted treatment means for the total posttest scores for each treatment.

Table 16. Analysis of Covariance Based on Application, Analysis, Synthesis, and Evaluation Items Posttest Score with Pretest Score, School, van Hiele Level, Age of Student, and Attitude toward Geometry before the Treatments as Covariates.

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 5 | 814.17 | 162.83 | 28.14 | 0.000 |
| Treatments | 2 | 3.01 | 1.51 | 0.26 | 0.771 |
| Residual | 269 | 1556.61 | 5.79 | | |

## Effects of Treatments on Retention Scores of Students

The second purpose of this study was to determine the effect the three treatments would have on student retention of geometric concepts introduced during the unit of instruction as measured by the retention test. The items of the retention test were classified as low-level items (at the knowledge and comprehension level as per Bloom's taxonomy) or high-level items (at the application, analysis, synthesis, or evaluation level as per Bloom's taxonomy). Therefore, the effect the treatments may have on student retention of low-level and high-level type questions was also of interest.

Table 17. Observed and Adjusted Group Means for the Application, Analysis, Synthesis, and Evaluation Items Posttest Score

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 117 | 8.74 | 8.58 |
| Treatment II | 77 | 8.31 | 8.35 |
| Treatment III | 83 | 8.30 | 8.41 |

In order to determine any possible effects, an analysis of covariance to test Null Hypotheses 2.1-2.3 was run using the researcher-designed retention test as the criterion variable and the three treatments as the active predictor variables. A step-wise linear regression analysis was used prior to the covariance analysis to identify what attribute-predictor variables needed to be used as covariates. As a result of this linear regression analysis, the covariates used in this covariance analysis were pretest score, school, attitude toward geometry before the treatments, and van Hiele level (see Table 18).

Table 18. Partial Correlation Coefficients between Attribute Predictor Variables and the Retention Test Score.

| Variables | Partial Correlation Coefficient | F | Significance of F |
|---|---|---|---|
| Total Pretest Score | 0.386 | 47.515 | 0.000 * |
| School | 0.325 | 32.173 | 0.000 * |
| Attitude toward Geom | 0.243 | 17.048 | 0.000 * |
| van Hiele Level | 0.205 | 11.981 | 0.001 * |
| Age of student | - 0.105 | 3.028 | 0.083 |
| Attitude toward Math | 0.058 | 0.923 | 0.338 |
| Gender | - 0.043 | 0.504 | 0.478 |
| Socio-economic Status | 0.035 | 0.340 | 0.561 |

* indicates significant at 0.05 level

Null Hypothesis 2.1 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and corresponding pretest score is controlled, in overall mean retention among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities) and treatment III (traditional whole class discussion led by the teacher).

The analysis of covariance (see Table 19) indicated that Null Hypothesis 2.1 should be rejected at the 0.05 level of significance. Since Null Hypothesis 2.1 was rejected, it was in order to determine which of the treatment means differed significantly from each other. Table 20 lists the observed and adjusted treatment means for the total posttest scores for each treatment. The results of the Newman-Keuls method of multiple comparisons indicated that significant differences (at the 0.05 level of significance) in mean achievement on all items on the retention test occurred between Treatment I and Treatment III. There was no significant difference in mean achievement on these between Treatment I and Treatment II and between Treatment II and Treatment III.

Table 19. Analysis of Covariance Based on Total Retention Test Score with Pretest Score, School, Attitude toward Geometry before the Treatments, and van Hiele Level as Covariates.

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 4 | 2215.45 | 553.99 | 44.47 | 0.000 |
| Treatments | 2 | 113.78 | 56.89 | 4.57 | 0.011 |
| Residual | 270 | 3363.62 | 12.46 | | |

Table 20. Observed and Adjusted Group Means for the Total Retention Test Score

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 117 | 17.35 | 17.13 |
| Treatment II | 77 | 16.25 | 16.06 |
| Treatment III | 83 | 15.28 | 15.67 |

Null Hypothesis 2.2 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and corresponding pretest score is controlled, in mean retention scores on knowledge and comprehension items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities) and treatment III (traditional whole class discussion led by the teacher).

The analysis of covariance (see Table 21) indicated that Null Hypothesis 2.2 should not be rejected at the 0.05 level of significance. Table 22 lists the observed and adjusted treatment means for the total posttest scores for each treatment.

Table 21. Analysis of Covariance Based on Knowledge and Comprehension Items Retention Test Score with Pretest Score, School, Attitude toward Geometry before the Treatments, and van Hiele Level as Covariates.

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 4 | 382.47 | 95.62 | 33.00 | 0.000 |
| Treatments | 2 | 7.78 | 3.89 | 1.34 | 0.263 |
| Residual | 270 | 782.37 | 2.90 | | |

Table 22. Observed and Adjusted Group Means for the Knowledge and Comprehension Items Retention Test Score

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 117 | 7.44 | 7.31 |
| Treatment II | 77 | 6.99 | 6.93 |
| Treatment III | 83 | 6.84 | 7.02 |

Null Hypothesis 2.3 There are no statistically significant differences, when variation due to gender, socio-economic background, school, age, van Hiele level, attitude toward mathematics, attitude toward geometry before the treatments, and corresponding pretest score is controlled, in mean retention scores on application, analysis, synthesis, and evaluation items among students having treatment I (small cooperative learning groups using the paper and pencil activities), treatment II (small cooperative learning groups using the computer activities) and treatment III (traditional whole class discussion led by the teacher).

The analysis of covariance (see Table 23) indicated that Null Hypothesis 2.3 should be rejected at the 0.05 level of significance. Since Null Hypothesis 2.3 was rejected, it was in order to determine which of the treatment means differed significantly from each other. Table 24 lists the observed and adjusted treatment means for the total posttest scores for each treatment. The results of the Newman-Kuels method of multiple comparisons indicated that significant differences (at the 0.05 level of significance) in mean achievement on all items on the retention test occurred between Treatment I and Treatment III. There was no significant difference in mean achievement on these between Treatment I and Treatment II and between Treatment II and Treatment III.

Table 23. Analysis of Covariance Based on Application, Analysis, Synthesis, and Evaluation Items Retention Test Score with Pretest Score, School, Attitude toward Geometry before the Treatments, and van Hiele Level as Covariates.

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 4 | 688.03 | 172.01 | 27.59 | 0.000 |
| Treatments | 2 | 70.46 | 35.23 | 5.65 | 0.004 |
| Residual | 270 | 1683.13 | 6.23 | | |

Table 24. Observed and Adjusted Group Means for the Application, Analysis, Synthesis, and Evaluation Items Retention Test Score

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 117 | 9.91 | 9.80 |
| Treatment II | 77 | 9.26 | 9.20 |
| Treatment III | 83 | 8.43 | 8.59 |

## Effects of Treatments on Attitude of Students

The third purpose of this study was to determine the effect the three treatments would have on student attitude toward geometry immediately following the treatment and then four weeks later.

In order to determine any possible effects, an analysis of covariance to test Null Hypotheses 3.1-3.2 was run using the Attitude Toward Geometry Opinionnaire as the criterion variable, and the three treatments as the active predictor variables. A step-wise linear regression analysis was used prior to the covariance analysis to identify which attribute-predictor variables needed to be used as covariates in each case. As a result of this linear regression analysis, the covariates used in each case in this covariance analysis were the same: attitude toward geometry prior to the application of the treatments, attitude toward mathematics, and gender (see Table 25 and Table 26).

Table 25. Partial Correlation Coefficients between Attribute Predictor Variables and the Attitude toward Geometry Opinionnaire Score at the End of the Treatments

| Variables | Partial Correlation Coefficient | F | Significance of F |
|---|---|---|---|
| Attitude toward Geom | 0.742 | 135.081 | 0.000 * |
| Attitude toward Math | 0.277 | 9.165 | 0.003 * |
| Gender | 0.198 | 4.488 | 0.036 * |
| Age of student | - 0.167 | 3.133 | 0.080 |
| van Hiele Level | 0.116 | 1.489 | 0.225 |
| School | 0.087 | 0.838 | 0.362 |
| Socio-economic Status | 0.031 | 0.106 | 0.746 |

* indicates significant at 0.05 level

Table 26. Partial Correlation Coefficients between Attribute Predictor Variables and the Attitude toward Geometry Opinionnaire Score Four Weeks after Treatments Ended

| Variables | Partial Correlation Coefficient | F | Significance of F |
|---|---|---|---|
| Attitude toward Geom | 0.643 | 192.913 | 0.000 * |
| Attitude toward Math | 0.194 | 10.685 | 0.001 * |
| Gender | 0.139 | 5.375 | 0.021 * |
| School | 0.104 | 2.984 | 0.085 |
| van Hiele Level | 0.074 | 1.515 | 0.219 |
| Age of student | 0.025 | 0.174 | 0.677 |
| Socio-economic Status | - 0.017 | 0.076 | 0.783 |

* indicates significant at 0.05 level

Null Hypothesis 3.1 There are no statistically significant differences, when variation due to gender, socio-economic background, age, van Hiele level, attitude toward mathematics, and attitude toward geometry before the treatments, in attitude toward geometry at the completion of the unit among the students having treatment I (small cooperative learning groups using paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion groups led by the teacher.)

The analysis of covariance (see Table 27) indicated that Null Hypothesis 3.1 should not be rejected at the 0.05 level of significance. Table 28 lists the observed and adjusted means from the Geometry Attitude Opinionnaire at completion of the unit for each treatment.

Table 27. Analysis of Covariance Based on Geometry Attitude Opinionnaire Score at Completion of the Unit with Previous Attitude toward Geometry, Attitude toward Mathematics, and Gender as Covariates

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 3 | 28367.85 | 9455.95 | 92.27 | 0.000 |
| Treatments | 2 | 582.83 | 291.41 | 2.84 | 0.063 |
| Residual | 108 | 11068.15 | 102.48 | | |

Table 28. Observed and Adjusted Group Means for the Geometry Attitude Opinionnaire Score at Completion of the Unit

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 54 | 43.17 | 43.46 |
| Treatment II | 19 | 39.32 | 37.07 |
| Treatment III | 41 | 40.88 | 42.83 |

Null Hypothesis 3.2 There are no statistically significant differences, when variation due to gender, socio-economic background, age, van Hiele level, attitude toward mathematics, and attitude toward geometry before the treatments, in attitude toward geometry four weeks after the application of the treatments among the students having treatment I (small cooperative learning groups using paper and pencil activities), treatment II (small cooperative learning groups using the computer activities), and treatment III (traditional whole class discussion groups led by the teacher.)

The analysis of covariance (see Table 29) indicated that Null Hypothesis 3.2 should not be rejected at the 0.05 level of significance. Table 30 lists the observed and adjusted means from the Geometry Attitude Opinionnaire four weeks after completion of the unit for each treatment.

Table 29. Analysis of Covariance Based on Geometry Attitude Opinionnaire Score Four Weeks after Completion of the Unit with Previous Attitude toward Geometry, Attitude toward Mathematics, and Gender as Covariates

| Source of Variation | DF | Sum of Squares | Mean Squares | F | Sig of F |
|---|---|---|---|---|---|
| Covariates | 3 | 58109.74 | 19369.91 | 139.85 | 0.000 |
| Treatments | 2 | 507.24 | 253.62 | 1.83 | 0.162 |
| Residual | 271 | 37534.75 | 138.50 | | |

Table 30. Observed and Adjusted Group Means for the Geometry Attitude Opinionnaire Score Four Weeks after Completion of the Unit

| Group | Number | Observed Mean | Adjusted Mean |
|---|---|---|---|
| Treatment I | 117 | 40.44 | 41.55 |
| Treatment II | 77 | 38.90 | 38.88 |
| Treatment III | 83 | 43.34 | 42.24 |

## Correlation between Attitude toward Geometry and Achievement

The fourth purpose of this study was to determine if student attitude toward geometry at the completion of the unit and student achievement showed significant correlation for the students for each of the three treatments.

Null Hypothesis 4 There is no correlation between the attitude toward geometry at the completion of the unit and the achievement of geometric concepts for the students having treatment I (small cooperative learning groups using paper and pencil activities), treatment II (small cooperative learning groups using computer activities) and treatment III (traditional whole class discussion led by the teacher).

The values of the Pearson product-moment correlation coefficient indicate that Null Hypothesis 4 should be rejected at 0.05 level of significance (see Table 31). Since this hypothesis was rejected, it was in order to determine if the correlations differed significantly from each other. The result of Fisher's $z_r$ transformation to test the equality of pairs of these correlation coefficients indicated that there were no significant differences at the 0.05 level of significance.

## Correlation between Attitude toward Geometry and Retention

The fifth purpose of this study was to determine if student attitude toward geometry four weeks after the completion of the unit and student retention showed significant correlation for the students for each of the three treatments.

Table 31. Pearson Product-moment Correlation Coefficients between Attitude and Posttest Scores at the Completion of the Unit

| Treatment | Sample Size | r | Sig of r |
|---|---|---|---|
| I | 54 | 0.292 | 0.032* |
| II | 19 | 0.606 | 0.006* |
| III | 41 | 0.400 | 0.010* |

* indicates significant at the 0.05 level

Null Hypothesis 5 There is no correlation between the attitude toward geometry at the completion of the unit and the achievement of geometric concepts for the students having treatment I (small cooperative learning groups using paper and pencil activities), treatment II (small cooperative learning groups using computer activities) and treatment III (traditional whole class discussion led by the teacher).

The values of the Pearson product-moment correlation coefficient indicates that Null Hypothesis 5 should be rejected at 0.05 level of significance (see Table 32). Since this hypothesis was rejected, it was in order to determine if the correlations differed significantly from each other. The result of Fisher's $z_r$ transformation to test the equality of pairs of these correlation coefficients indicated that there were no significant differences at the 0.05 level of significance.

Correlation between Achievement and Retention

The final purpose of this study was to determine if student achievement at the completion of the unit and student retention four weeks later showed significant correlation for the students for each of the three treatments.

Table 32. Pearson Product-moment Correlation Coefficients between Attitude and Retention Test Scores Four Weeks after the Completion of the Unit

| Treatment | Sample Size | r | Sig of r |
|---|---|---|---|
| I | 117 | 0.242 | 0.009* |
| II | 77 | 0.279 | 0.014* |
| III | 41 | 0.411 | 0.010* |

* indicates significant at the 0.05 level

Null Hypothesis 6 There is no correlation between the achievement of geometric concepts and the retention of geometric concepts for the students having treatment I (small cooperative learning groups using paper and pencil activities), treatment II (small cooperative learning groups using computer activities) and treatment III (traditional whole class discussion led by the teacher).

The values of the Pearson product-moment correlation coefficient indicate that Null Hypothesis 6 should be rejected at 0.05 level of significance (see Table 33). Since this hypothesis was rejected, it was in order to determine if the correlations differed significantly from each other. The result of Fisher's $z_r$ transformation to test the equality of pairs of these correlation coefficients indicated that there were no significant differences at the 0.05 level of significance.

Table 33. Pearson Product-moment Correlation Coefficients between Posttest Scores at the End of the Unit and Retention Test Scores Four Weeks after the Completion of the Unit

| Treatment | Sample Size | r | Sig of r |
|---|---|---|---|
| I | 117 | 0.724 | 0.000* |
| II | 77 | 0.654 | 0.000* |
| III | 41 | 0.697 | 0.000* |

* indicates significant at the 0.05 level

## Summary of Results

The first set of null hypotheses stated there would be no differences in achievement among the students in the three treatments as measured by the posttest at the conclusion of the unit. When considering all twenty-five test items and when considering just the fifteen high-level items (application, analysis, synthesis, and evaluation), the hypotheses were not rejected. However, when considering only the ten low-level items (knowledge and comprehension), the hypothesis was rejected. Results of the multiple comparison analysis indicated that the students in treatment I (small cooperative learning groups using paper and pencil activities) and in treatment II (small cooperative learning groups using computer activities) had the same mean score and that these mean scores were approximately one point higher (statistically significant) than the mean score for the students in treatment III (traditional class discussion lead by the teacher).

The second set of null hypotheses stated there would be no differences in retention of geometric concepts taught during the unit among the students in the three treatments. Retention was measured by a twenty-five item test (equivalent to the posttest) administered four weeks after completion of the unit. When only the ten low-level items were examined, the null hypothesis was not rejected. However, when all twenty-five items were considered and when the fifteen high-level items were considered, the null hypotheses were rejected. Using all twenty-five items, the multiple comparison analysis indicated that the mean score for treatment I (small cooperative groups using paper and pencil activities) was significantly higher (approximately two points) than the mean score for treatment III (traditional whole class discussion led by the teacher). There were no significant difference when comparing the mean scores for treatment I and treatment II (small cooperative groups using computer based activities) and again when comparing the mean scores for treatment II and treatment III. When using the fifteen high-level items, the multiple comparison analysis indicated that the mean score for treatment I was significantly higher (approximately 1.5 points) than the mean score for treatment III. Again, no other differences between mean scores were significant.

No significant differences were noted in student attitude toward geometry among the three treatments both at the time the unit ended and four weeks later when testing the third set of null hypotheses.

The fourth null hypothesis, which stated there would be no correlation between attitude toward geometry and achievement scores at the completion of the unit, was rejected. Even though the calculated Pearson product-moment correlation coefficients were 0.29 for treatment I, to 0.61 for treatment II, and 0.40 for treatment III, the differences were not statistically significant.

The fifth null hypothesis stated there would be no correlation between attitude toward geometry and scores on the retention test given four weeks after the completion of the unit. This hypothesis was also rejected. Again, even though the calculated Pearson product-moment correlation coefficients were 0.24 for treatment I, 0.27 for treatment II, and 0.41 for treatment III, the differences were not statistically significant.

The final null hypothesis stated there would be no correlation between achievement scores at the end of the unit and retention scores measured four weeks after the unit of instruction ended. This hypothesis was rejected. The Pearson product-moment correlation coefficients were 0.72 for treatment I, 0.65 for treatment II, and 0.70 for treatment III. The differences were not statistically significant.

# CHAPTER 5

# SUMMARY, CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

## Introduction

This chapter includes a summary of the findings of the study, conclusions based on analysis of the data, implications of the conclusions, and recommendations for future research.

## Summary of the Study

### Statement of the Problem

The primary purpose of this study was to investigate the effect that three different methods of classroom instruction had on (a) achievement of certain geometrical concepts, (b) retention of these geometrical concepts, and (c) attitude toward mathematics (and geometry in particular) for students enrolled in a high school geometry course. The three methods of instruction included: (a) small cooperative learning groups doing paper and pencil activities following the three phases (information, guided orientation, and explicitation) as described in the van

Hiele theory, (b) small cooperative learning groups using the computer and accompanying software together with activities following the three phases of the van Hiele theory, and (c) whole class instruction based on traditional textbook procedures.

Procedure

The nine-week study took place at five high schools in Montana. Three of the high schools with student enrollments (grades 9-12) of approximately 250 students each (Cut Bank, Forsyth, and Glasgow) serve communities where agriculture is the primary industry; one with a school enrollment (grades 9-12) of approximately 600 students (Columbia Falls) serves a community in which the timber industry is predominant; and the fifth high school with a school enrollment (grades 9-12) of approximately 1600 students (CMR in Great Falls) serves one of the two large metropolitan areas in Montana.

Five hundred forty students from a total high school population of nearly 3000 students at the five schools were enrolled in a traditional high school geometry course. Three hundred two of these high school students were originally chosen to participate and two hundred seventy-seven of these students completed the study. These students comprised 14 different geometry classes with the smallest class enrollment being 14 students and the largest class enrollment being 29 students.

In order to maximize control of the teacher variable, it was desirable to use one teacher from each of the five schools. However, six experienced teachers from these five schools were used since at one high school no teacher was assigned more than two sections of geometry, and it was desirable to have three classes from each school participate whenever possible (with each class receiving a different treatment).

Treatment I was assigned to six classes, one from each of four schools and two from the fifth school. Treatment I involved introducing the students to the concepts of a geometry unit on congruent triangles via small cooperative learning groups using paper and pencil activities (see Appendix B). These activities were designed to follow the first three phases identified in the van Hiele theory (see pp. 11-12).

Treatment II was assigned to four classes, one class from each of four of the schools. (Because of scheduling conflicts, the geometry classes at one school did not have access to the computer facilities; hence, this treatment could not be applied at this school as originally planned.) Treatment II involved introducing the students to the same concepts of the same geometry unit on congruent triangles via small cooperative learning groups using a computer and accompanying software to complete activities. (See Appendix B). These activities also followed the first three phases identified in the van Hiele theory.

Treatment III was assigned to four classes, one class from each of four schools. (Because of declining enrollments, one school dropped to only two sections of geometry being taught at the time of this study; hence, this third treatment was not applied at this school as originally planned.) Treatment III involved introducing the students to the same concepts of the same geometry unit on congruent triangles using only the textbook materials in a traditional whole class discussion led by the teacher.

In each treatment the objectives for the unit of instruction were the same, all homework assignments were from the textbooks, and the length of the instructional period was the same.

The study utilized the quasi-experimental pre-post design based on Campbell and Stanley's (1963) Nonequivalent Control Group Design. Multiple linear regression, analysis of covariance, multiple comparison of adjusted means, and simple correlation analysis were chosen as the methods of analysis. Six null hypotheses were tested using 11 predictor variables and eight criterion variables. (See the Null Hypotheses pp. 67-72 and list of variables pp. 92-93).

## Conclusions Based on Analysis of the Data

### Effects of Treatment on Achievement Scores of Students

This phase of the study was designed to determine the effect the three treatments would have on student achievement of the objectives of this geometry unit immediately following the instructional period of the unit. Achievement was measured as the number of correct items on the posttest administered at the conclusion of the unit. No significant differences in test scores occurred among the three treatments when all the test items were considered. However, when the ten items from the test classified as low level items were considered (knowledge and comprehension items as per Bloom's taxonomy), the test scores from both groups which followed the van Hiele instructional phases scored significantly higher than the group given the traditional text book based, whole class discussion treatment. This resulted in a one-question increase in adjusted mean scores from a group of eight questions. When the remaining fifteen items classified as high level items were considered (application, analysis, synthesis, and evaluation), there were no significant differences among the three treatments.

Effects of Treatment on Retention Scores of Students

This phase of the study was designed to determine the effect each of the three treatments would have on student retention of the objectives of this unit. Retention was measured as the number of correct items on a test equivalent to the posttest. The retention test was administered four weeks after the posttest. When all items on the retention test were considered, it was determined that the scores for those students in the treatment using the van Hiele phase based paper and pencil activities with small cooperative learning groups scored significantly higher than the students in the traditional textbook-based, whole class discussion treatment. No other differences among treatments were statistically significant when all items were considered.

When the retention test items were divided into the low level and high level items as was done with the posttest, the significant difference between the paper and pencil cooperative learning group treatment mean retention score and the whole class discussion treatment mean score was attributable to the difference in mean scores on the high level items. There were no significant differences among the treatments in the number of low level items that were correct.

Effect of Treatment on Attitude of Students

This phase of the study was designed to see if student attitude toward geometry was affected by the method of instruction when the objectives of the unit of instruction were the same for each method of instruction. No significant differences in attitude occurred among the three treatment groups either at the time the posttest was administered or at the time the retention test was administered.

Correlation between Attitude toward Geometry and Achievement

This phase of the study was designed to determine if the correlation between student attitude toward geometry and student achievement were significant; and if so, if the correlations were different among the treatments. It was determined that there was a positive correlation between attitude and achievement in each treatment. There was no sample evidence to indicate that any of the correlations among the treatments differed.

Correlation between Attitude toward Geometry and Retention

This phase of the study was designed to determine if the correlation between student attitude toward geometry and student retention were significant; and if so, if the correlations were different among the treatments. It was determined that there was a positive correlation between attitude and retention in each treatment. There was no sample evidence to indicate that any of these correlations among the treatments differed.

Correlation between Achievement and Retention

The final phase of the study was designed to determine if there was significant correlation between student achievement on the posttest and student retention four weeks later; and if so, if the correlations were different among the treatments. It was determined that there was significant positive correlation between achievement and retention in each treatment. There was no sample evidence to indicate that any of these correlations among the treatments differed.

## Implications of the Conclusions

The conclusions of this study indicate that the method of instruction of the unit on congruent triangles in the high school geometry course at five high schools had some effect on achievement and retention but no effect on attitude.

No significant differences in posttest scores occurred among the three treatments when all the test items were considered; however, there was a significant difference of the posttest scores (0.05 level of significance) on the low cognitive level (knowledge and comprehension) items among the treatments. Those students receiving treatments I or II outscored those receiving treatment III on these items. This difference suggests that, within a short time after receiving instruction, students who work in small cooperative learning groups using van Hiele phase-based instructional materials may have better retrieval of knowledge and comprehension level information than those receiving whole class instruction based on traditional textbook procedures. Dugdale and Kibbey (1983, 1984) also found that learning based on a student's own actions produce more accurate knowledge and recall of that knowledge than does passive, memorized learning.

The significant difference found on the retention test results was attributable to the students' performances on the high cognitive level (application, analysis, synthesis, and evaluation) items rather than on the low cognitive level questions as in the posttest results. With regard to these high cognitive level items, those students working in small cooperative learning groups using van Hiele phase-based materials (treatment I) outscored those receiving whole class instruction based on traditional textbook procedures (treatment III). (No differences in retention scores were detected between treatments I and II and between treatments II and III.) The activities completed in treatments I and II were similar, with the exception being that treatment I activities required the students to use a ruler and protractor to measure lengths and angles, whereas, in treatment II, these activities were completed by simply requesting the measurement from a menu displayed on the computer screen. These results appear to indicate that it may be more desirable to have students work with the concrete materials rather than work with the pictorial representations (computer images and indirect measurement) in order to develop higher level thinking skills with respect to certain geometrical concepts.

Since there were no statistical differences in student attitudes toward geometry among the treatments, student attitude did not identify one method of instruction as a more preferred method of instruction.

The data in Table 9 (page 100) identify the expected increase in mean test scores going from pretest to posttest for all treatments and at all schools. The data also indicate an increase in test scores going from the posttest to the retention test (this occurred in 11 of the 14 classes). Since the retention test was given four weeks after completion of the unit, one might expect test scores to decrease rather than increase. However, in the geometry course at these five high schools, the material taught following this unit made extensive use of the properties developed in this unit. Hence, the students had ample opportunity to continue to use the concepts presented in this unit and thus test scores tended to improve over time.

Even though there were no statistical differences in attitudes toward geometry among the treatments at various points in this study, there was a trend for students' attitudes toward geometry to decline over time no matter which method of instruction was used. This trend appears to agree with the slow but steady decline of student attitude toward mathematics during the high school years reported by others (Aiken, 1970, 1976; Kulm, 1980; Suydam, 1984).

It is important to note that in treatments I and II, the students spent approximately 50 percent of the classroom instructional time working with one another and not watching and listening to the teacher lecture and demonstrate. However, these students performed as well as, and perhaps better than, those students taught in the more traditional setting. The National Council of Teachers of Mathematics recommends that "a variety of instructional methods should be used in classrooms in order to cultivate students' abilities to investigate, to make sense of, and to construct meanings from new situations" and that "greater opportunity should be provided for small-group work" (NCTM, 1989, pp.125-128). Therefore, if a teacher is considering adopting an alternative instructional method for a geometry class, small cooperative learning groups using van Hiele phase-based materials should be considered knowing that the students will likely perform as well as those students taught using the whole class instruction based on traditional textbook procedures.

## Recommendations for Further Study

This study was designed and conducted to determine the effect the method of instruction in a geometry class would have on achievement, retention, and attitude toward geometry. For the unit chosen in this study, some significant differences were found. It is not certain if similar results would be obtained if a different unit in geometry had been chosen. Similar materials need to be constructed for a different geometry unit and the experiment should be repeated.

Results from this study appear to indicate students who worked with the concrete materials in small cooperative groups performed better on higher cognitive level items than those students receiving whole class instruction using typical textbook procedures; whereas, those students who worked with the semi-concrete materials (computer images and indirect measurement) in small cooperative groups did not perform any better than those students receiving whole class instruction using typical textbook procedures. Further research should be conducted to determine if instruction using concrete materials has an advantage over instruction using semi-concrete materials for developing higher level reasoning skills for high school students taking geometry.

The apparent decline in the attitude toward geometry for the students in each treatment in this study is disheartening. In a previous study, the use of cooperative learning in a geometry course produced favorable responses from students (Cox, et. al., 1989). Another study found that attitudes toward geometry in an informal geometry course rose slightly during the course (Cox, 1979). This study and the two cited above each used a different instrument to measure attitude and this may account for the apparent disagreement in results. A method to measure attitude that uses both anecdotal data and student responses to an opinionnaire should be developed and further studies should be conducted to measure changes in attitude toward geometry throughout the duration of the course.

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# APPENDICES

# APPENDIX A

# UNIT TESTS

# UNIT OBJECTIVES

## CONGRUENT TRIANGLES

At the end of this unit the student will be able to:

1. Define a triangle and identify its parts.
2. Use the words scalene, isosceles, equilateral, acute, obtuse, and right to identify triangles.
3. Identify congruent triangles and the corresponding congruent parts.
4. Use the SSS, SAS, and ASA congruence postulates to prove triangles are congruent.
5. Use congruent triangles to prove angles and segments are congruent.
6. Recognize and use overlapping congruent triangles in proofs.
7. Use the LL, LA, HA congruence theorems and the HL postulate to prove right triangles are congruent.
8. State and apply theorems about the measures of angles of isosceles and equilateral triangles.
9. Draw, identify, and describe the properties of an altitude of a triangle.
10. Draw, identify, and describe the properties of a median of a triangle.
11. Draw, identify, and describe the properties of an angle bisector of a triangle.

# BLOOM'S DESCRIPTION OF THE COGNITIVE LEVELS

| | |
|---|---|
| Knowledge: | ability to recall specifics as well as universals, methods and processes, structure, and patterns. |
| Comprehension: | ability to use materials and ideas without necessarily relating them to other materials or seeing their fullest implementation. |
| Application: | ability to use learned information in particular situations and to apply an appropriate abstraction to a new problem. |
| Analysis: | ability to break down information into its component parts thereby making clear the relative hierarchy of ideas and relationships between the ideas. |
| Synthesis: | ability to put together elements or parts to form a new whole. |
| Evaluation: | ability to make quantitative and qualitative judgments about the extent to which materials, information, method, or ideas satisfy a given set of criteria. |

## TABLE OF SPECIFICATIONS

(for Geometry Unit Test: Form A)

COGNITIVE DOMAINS

| OBJECTIVES | knowledge | comprehension | application | analysis | synthesis | evaluation |
|---|---|---|---|---|---|---|
| 1. Define a triangle and identify its parts | J: 1 3 17<br>L: 7 | S: 1 3 17<br>L: 1 3 | L: 12,17 | L: 22 | L: 9 | S: 23 |
| 2. Use scalene, isosceles, etc., to identify triangles. | S: 6 7 9 11 19 21 23 25<br>L: 7 | | L: 11 | L: 15,21,22 | L: 25 | S: 23<br>J: 23 |
| 3. Identify cong. triangles and cong. parts. | J: 4 | S: 25<br>J: 4<br>L: 4 16 | S:5 6 10 11 14 16 19 20 25<br>J: 16 25<br>L: 14 | S: 4 | L: 18 20 | |
| 4. Use SSS, SAS, and ASA to prove triangles cong. | | S:13<br>J: 5<br>L: 5 13 | S: 10 14<br>J: 5 11 | J: 10 13<br>L: 10 | J: 13 | S: 5 16<br>J:13 |
| 5. Use congruent triangles to prove angles and segments cong. | | S: 25 | S: 10<br>L: 8 | L: 11 | J: 8 | S: 14 20 |
| 6. Use overlapping triangles in proofs. | | | S: 10<br>J: 14<br>L: 10 | J: 10 | | S: 14 |
| 7. Use LL, LA, HA, and HL to prove right triangles congruent. | | | J: 6 | J: 19<br>L: 6 19 | L: 18 20 | S: 6 18 19 |
| 8. Apply theorems about angles of isosceles and equilateral tri. | S: 22 | | S: 7 11 15 21 22<br>J: 7 15 | L: 7 22 | J: 9 18 20 21<br>L: 15 | J: 23 |
| 9. Properties of an altitude of a triangle. | S: 12<br>J: 12 | S: 9<br>L: 9 | L: 12 | | | |
| 10. Properties of a median of a triangle. | S: 24<br>J: 24 | S: 21<br>J: 24 | J: 21<br>L: 9 | L: 21 24 | | |
| 11. Properties of an angle bisector of a triangle. | S: 2<br>J: 2 | J: 22<br>L: 2 | S: 8<br>L: 8 | L: 22 | | |

Identification:
S - Mrs. Sue Dolezal    J - Dr. Johnny Lott    L - Mr. Lee Brown

TABLE OF SPECIFICATIONS

(continued)

Placement of items into cognitive levels (an item was placed at the level at which at least two of the three reviewers placed it):

Unit Geometry Test: Form A

Lower cognitive (knowledge and comprehension) level items:
1, 2, 3, 4, 5, 9, 12, 13, 17, 24

Higher cognitive (application, analysis, synthesis, evaluation) level items:
6, 7, 8, 10, 11, 14, 15, 16, 18, 19, 20, 21, 22, 23, 25

Unit Geometry Test: Form B

Lower cognitive (knowledge and comprehension) level items:
1, 2, 3, 7, 8, 12, 15, 21, 23, 25

Higher cognitive (application, analysis, synthesis, evaluation) level items:
4, 5, 6, 9, 10, 11, 13, 14, 16, 17, 18, 19, 20, 22, 24

# GEOMETRY UNIT PRETEST

directions

Materials needed:

Enough test booklets (Unit Test, Congruent Triangles, Form A), answer sheets, and extra pencils for the class.

A watch or clock to time the test.

Procedure:

Have the students clear their desk and have only pencils and erasers on the desk. Each student should have at least one pencil with an eraser.

Pass out the answer sheets and have the students completely fill out the top portion (student data).

While the students are completing this, pass out the test booklets and place them face down on the students desk.

Read the following instructions to the students. (These are the same as on the Van Hiele Geometry Test.)

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on the test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.
7. If you finish the test early, check your answers, then sit quietly to allow others to work.
8. Do you have any questions?
9. You may begin.

Other details:

Write on the board "Test began: (time)."

Write on the board "Test ends: (time)."

After 25 minutes, write on the board "It is now: (time)."

After exactly 35 minutes, stop the test. Collect the answer sheets. Have the students look carefully through their test booklet and erase any pencil marks they find in it. Then collect the test booklets.

Make sure all the test booklets and answer sheets are accounted for.

If there is more time left, use it as you wish.

## GEOMETRY UNIT POSTTEST

directions

Materials needed:

Enough test booklets (Unit Test, Congruent Triangles, Form A), answer sheets, and extra pencils for the class.

A watch or clock to time the test.

Procedure:

Have the students clear their desk and have only pencils and erasers on the desk. Each student should have at least one pencil with an eraser.

Pass out the answer sheets and have the students completely fill out the top portion (student data).

While the students are completing this, pass out the test booklets and place them face down on the students desk.

Read the following instructions to the students. (These are the same as on the Van Hiele Geometry Test.)

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on the test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.
7. If you finish the test early, check your answers, then sit quietly to allow others to work.
8. Do you have any questions?
9. You may begin.

Other details:

Write on the board "Test began: (time)."

Write on the board "Test ends: (time)."

After 25 minutes, write on the board "It is now: (time)."

After exactly 35 minutes, stop the test. Collect the answer sheets. Have the students look carefully through their test booklet and erase any pencil marks they find in it. Then collect the test booklets.

Make sure all the test booklets and answer sheets are accounted for.

If there is more time left, use it as you wish.

# GEOMETRY UNIT RETENTION TEST

## directions

Materials needed:

Enough test booklets (Unit Test, Congruent Triangles, Form A), answer sheets, and extra pencils for the class.

A watch or clock to time the test.

Procedure:

Have the students clear their desk and have only pencils and erasers on the desk. Each student should have at least one pencil with an eraser.

Pass out the answer sheets and have the students completely fill out the top portion (student data).

While the students are completing this, pass out the test booklets and place them face down on the students desk.

Read the following instructions to the students. (These are the same as on the Van Hiele Geometry Test.)

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on the test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.
7. If you finish the test early, check your answers, then sit quietly to allow others to work.
8. Do you have any questions?
9. You may begin.

Other details:

Write on the board "Test began: (time)."

Write on the board "Test ends: (time)."

After 25 minutes, write on the board "It is now: (time)."

After exactly 35 minutes, stop the test. Collect the answer sheets. Have the students look carefully through their test booklet and erase any pencil marks they find in it. Then collect the test booklets.

Make sure all the test booklets and answer sheets are accounted for.

If there is more time left, use it as you wish.

# UNIT TEST
# FORM A
# CONGRUENT TRIANGLES
# ANSWER SHEET

**Please Print**

Name ______________________________________ Class period _____
Last First Middle

Math Teacher ______________________ School ______________

Grade (circle): 8 9 10 11 12 Sex (circle): M F

Birthdate ______ ______ ______
month day year

Testing date ______ ______ ______
month day year

**Cross out the correct answer** | **Space for work and drawings**

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. | a | b | c | d | e | |
| 2. | a | b | c | d | e | |
| 3. | a | b | c | | | |
| 4. | a | b | c | d | e | |
| 5. | a | b | c | d | e | |
| 6. | a | b | c | d | e | f |
| 7. | a | b | c | d | e | |
| 8. | a | b | c | d | e | |
| 9. | a | b | c | d | e | |
| 10. | a | b | c | d | e | |
| 11. | a | b | c | d | e | |
| 12. | a | b | c | d | e | |
| 13. | a | b | c | d | e | |
| 14. | a | b | c | d | e | |
| 15. | a | b | c | d | e | |
| 16. | a | b | c | d | e | |
| 17. | a | b | c | | | |
| 18. | a | b | c | d | e | |
| 19. | a | b | c | d | e | f |
| 20. | a | b | c | d | | |
| 21. | a | b | c | d | e | |
| 22. | a | b | c | d | e | |
| 23. | a | b | c | d | e | |
| 24. | a | b | c | d | e | |
| 25. | a | b | c | d | e | |

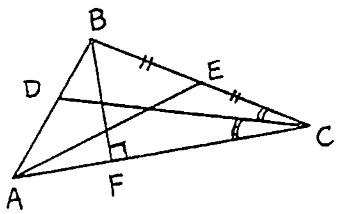
# UNIT TEST
## FORM A
## CONGRUENT TRIANGLES

**DIRECTIONS: Do not write on this test. Place all answers on the separate answer sheet.**

---

1. Which of these statements always describes a triangle?
   - a. A figure formed by three segments that join three points.
   - b. A figure formed by three segments such that each pair of segments intersect.
   - c. A figure formed by three noncollinear points.
   - d. A figure formed by three lines that pass through three noncollinear points.
   - e. A figure consisting of three noncollinear points and the line segments determined by these three points.

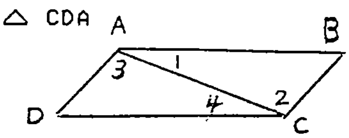
2. In △ABC at the right, which segment names an angle bisector?
   - a. $\overline{AE}$
   - b. $\overline{BF}$
   - c. $\overline{CD}$
   - d. $\overline{BE}$
   - e. $\overline{AF}$



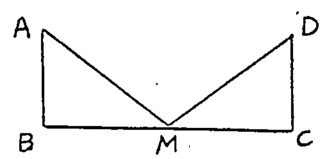

3. In △ PQR, the side opposite ∡ P is
   - a. $\overline{PQ}$
   - b. $\overline{QR}$
   - c. $\overline{PR}$

4. If △ ABC is congruent to △ XYZ, then which of the following must be true?
   - a. $\overline{AB} \cong \overline{YZ}$
   - b. $\overline{AC} \cong \overline{XY}$
   - c. $\overline{BC} \cong \overline{ZX}$
   - d. ∡CAB ≅ ∡YXZ
   - e. ∡ABC ≅ ∡XZY

5. Given ∡ 1 ≅ ∡ 4 and ∡ 2 ≅ ∡ 3, then △ ABC and △ CDA
   - a. are congruent by the SAS postulate.
   - b. are congruent by the ASA postulate.
   - c. are congruent by the SSS postulate.
   - d. cannot be congruent.
   - e. may or may not be congruent. There is not enough information given to determine this positively.

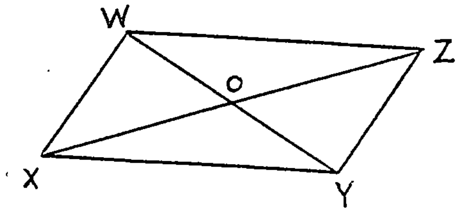



6. $\overline{AB}$ is perpendicular to $\overline{BC}$. $\overline{DC}$ is perpendicular to $\overline{CB}$. M is the midpoint of $\overline{BC}$. Then △ ABM and △ DCM
   - a. are congruent by the LL theorem.
   - b. are congruent by the LA theorem.
   - c. are congruent by the HA theorem.
   - d. are congruent by the HL postulate.
   - e. cannot be congruent.
   - f. may or may not be congruent. There is not enough information given to determine this positively.



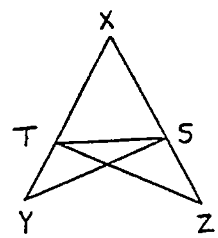
7. In an isosceles triangle, the vertex angle measures 5x degrees, one base angle measures 120 − 4x degrees and the other base angle measures 60 − x degrees. What is the value of x?
   a. 10
   b. $13\frac{1}{3}$
   c. 20
   d. 30
   e. Impossible to determine from the information given.

8. If $\triangle WXZ \cong \triangle YZX$, which congruence must be used to show $\overline{ZX}$ bisects $\measuredangle WXY$?
   a. $\measuredangle WXZ \cong \measuredangle YXZ$
   b. $\overline{WX} \cong \overline{YX}$
   c. $\overline{WZ} \cong \overline{YZ}$
   d. $\measuredangle WZX \cong \measuredangle YZX$
   e. $\measuredangle XWZ \cong \measuredangle XYZ$

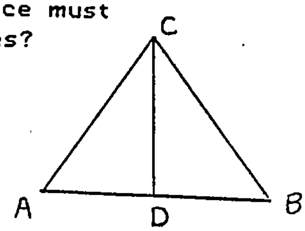



9. In $\triangle ABC$, the altitude drawn from the vertex of $\measuredangle C$ to intersect side $\overline{AB}$ bisects $\measuredangle C$. Then $\triangle ABC$ must be
   a. a scalene triangle.
   b. a right triangle.
   c. an acute triangle.
   d. an isosceles triangle.
   e. an equilateral triangle.

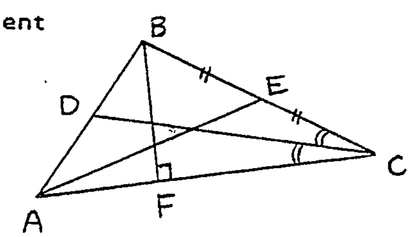
10. In the accompanying figure, $\overline{TY} \cong \overline{SZ}$ and $\overline{SY} \cong \overline{TZ}$. Therefore, $\triangle TYS$ and $\triangle SZT$
   a. are congruent by the SAS postulate.
   b. are congruent by the ASA postulate.
   c. are congruent by the SSS postulate.
   d. cannot be congruent.
   e. may or may not be congruent. There is not enough information given to determine this positively.




11. If $\triangle ADC \cong \triangle BDC$, which congruence must be used to show $\triangle ABC$ is isosceles?
   a. $\measuredangle ACD \cong \measuredangle BCD$
   b. $\overline{AC} \cong \overline{BC}$
   c. $\measuredangle ADC \cong \measuredangle BDC$
   d. $\overline{AD} \cong \overline{BD}$
   e. $\overline{CD} \cong \overline{CD}$

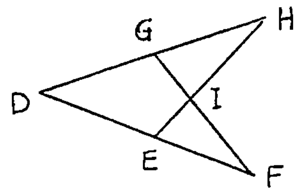



12. In $\triangle ABC$ at the right, which segment names an altitude?
   a. $\overline{AE}$
   b. $\overline{BF}$
   c. $\overline{CD}$
   d. $\overline{BE}$
   e. $\overline{AF}$

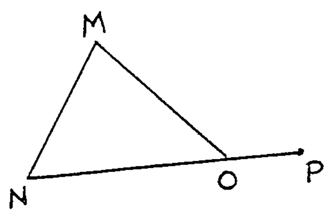


13. Which of the following statements **DOES NOT ALWAYS** describe congruent triangles?
   a. If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.
   b. If two angles and the included side of a triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.
   c. If two sides and the angle opposite one of the sides of a triangle are congruent to two sides and the corresponding opposite angle of another triangle, then the two triangles are congruent.
   d. If two angles and the side opposite one of the angles of a triangle are congruent to two angles and the corresponding opposite side of another triangle, then the two triangles are congruent.
   e. If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

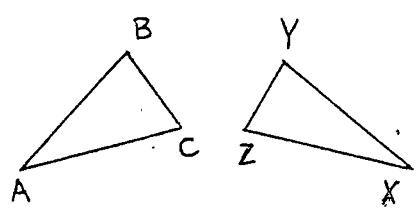
14. In the accompanying figure if $\overline{DG} \cong \overline{DE}$ and $\measuredangle$ DHE $\cong$ $\measuredangle$ DFG, then to show $\overline{EH} \cong \overline{GF}$, which congruency must you prove?
   a. $\triangle$ DEH $\cong$ $\triangle$ DFG
   b. $\triangle$ DHE $\cong$ $\triangle$ DFG
   c. $\triangle$ HED $\cong$ $\triangle$ FDG
   d. $\triangle$ GHI $\cong$ $\triangle$ EIF
   e. $\triangle$ GHI $\cong$ $\triangle$ EFI




15. In $\triangle$ MNO if $\overline{MN} \cong \overline{MO}$ and exterior $\measuredangle$ MOP measures 110°, then what is the measure of $\measuredangle$ MNO?
   a. 35°
   b. 40°
   c. 60°
   d. 70°
   e. none of these

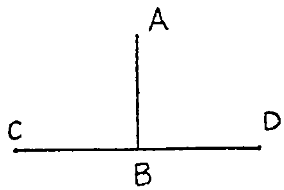



16. In the accompanying figure $\measuredangle$ A $\cong$ $\measuredangle$ X, $\measuredangle$ B $\cong$ $\measuredangle$ Y, and $\measuredangle$ C $\cong$ $\measuredangle$ Z. Then $\triangle$ ABC and $\triangle$ XYZ
   a. are congruent by the SAS postulate.
   b. are congruent by the ASA postulate.
   c. are congruent by the SSS postulate.
   d. cannot be congruent.
   e. may or may not be congruent. There is not enough information given to determine this positively.

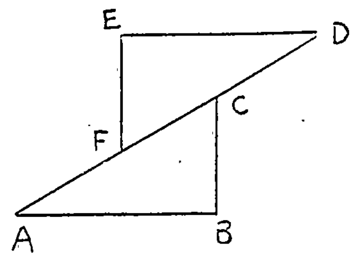



17. In $\triangle$ ABC, the angle included between $\overline{AB}$ and $\overline{CB}$ is
   a. $\measuredangle$ ACB
   b. $\measuredangle$ BAC
   c. $\measuredangle$ CBA

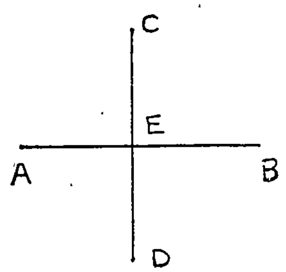
18. In the accompanying figure, $\overline{AB}$ is perpendicular to $\overline{CD}$ and B is the midpoint of $\overline{CD}$. Which statment is always true?
   a. $\overline{AB} \cong \overline{BD}$
   b. $\overline{AC} \cong \overline{AD}$
   c. $m \angle ACD = 60°$
   d. $\overline{AB} \cong \overline{AC}$
   e. $m \angle ACD = 45°$




19. In the accompanying figure $\angle ABC$ and $\angle DEF$ are right angles, $\overline{AF} \cong \overline{DC}$, and $\overline{EF} \cong \overline{BC}$. Then $\triangle ABC$ and $\triangle DEF$
   a. are congruent by the LL theorem.
   b. are congruent by the LA theorem.
   c. are congruent by the HA theorem.
   d. are congruent by the HL postulate.
   e. cannot be congruent.
   f. may or may not be congruent. There is not enough information given to determine this positively.

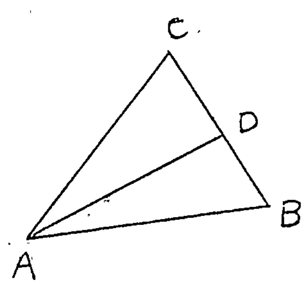



20. In the accompanying figure $\overline{CD}$ is perpendicular to $\overline{AB}$ and E is the midpoint of $\overline{AB}$. Which statement is always true?
   a. $\overline{AC} \cong \overline{AD}$
   b. $\overline{CA} \cong \overline{CB}$
   c. $\overline{CE} \cong \overline{DE}$
   d. $\overline{BE} \cong \overline{DE}$



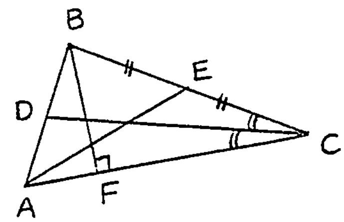

21. In $\triangle ABC$, the median drawn from the vertex of $\angle A$ to side $\overline{BC}$ is perpendicular to $\overline{BC}$. Then $\triangle ABC$ must be
   a. a scalene triangle.
   b. a right triangle.
   c. an acute triangle.
   d. an isosceles triangle.
   e. an equilateral triangle.

22. In $\triangle ABC$, $\overline{AC} \cong \overline{BC}$ and $m \angle ABC = 50°$. If $\overline{AD}$ bisects $\angle BAC$, then $m \angle BAD$ equals
   a. 15°
   b. 20°
   c. 25°
   d. 40°
   e. 50°



23. Which type of triangle cannot be drawn?
    a. An isosceles acute triangle
    b. A triangle with two acute angles
    c. A scalene right triangle
    d. A triangle with two obtuse angles
    e. An isosceles obtuse triangle

24. In △ABC at the right, which segment names a median?
    a. $\overline{AE}$
    b. $\overline{BF}$
    c. $\overline{CD}$
    d. $\overline{BE}$
    e. $\overline{AF}$




25. If △ PQR is congruent to △ DEF, then which of the following must be true?
    a. $\overline{RP} \cong \overline{DF}$
    b. $\overline{PQ} \cong \overline{EF}$
    c. $\measuredangle PQR \cong \measuredangle FDE$
    d. $\measuredangle PRQ \cong \measuredangle DEF$
    e. $\measuredangle QPR \cong \measuredangle EFD$

# UNIT TEST
# FORM B
# CONGRUENT TRIANGLES
# ANSWER SHEET

**Please Print**

Name ______________________________________________ Class period ____
 Last First Middle

Math Teacher ______________________________ School ______________

Grade (circle): 8 9 10 11 12 Sex (circle): M F

Birthdate ______ ______ ______
 month day year

Testing date ______ ______ ______
 month day year

**Cross out the correct answer** | **Space for work and drawings**

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. | a | b | c | | | |
| 2. | a | b | c | d | e | |
| 3. | a | b | c | d | e | |
| 4. | a | b | c | d | e | |
| 5. | a | b | c | d | e | |
| 6. | a | b | c | d | e | |
| 7. | a | b | c | d | e | |
| 8. | a | b | c | d | e | |
| 9. | a | b | c | d | e | f |
| 10. | a | b | c | d | e | |
| 11. | a | b | c | d | e | |
| 12. | a | b | c | d | e | |
| 13. | a | b | c | d | e | |
| 14. | a | b | c | d | e | f |
| 15. | a | b | c | d | e | |
| 16. | a | b | c | d | e | |
| 17. | a | b | c | d | e | |
| 18. | a | b | c | d | e | |
| 19. | a | b | c | d | e | |
| 20. | a | b | c | d | e | |
| 21. | a | b | c | | | |
| 22. | a | b | c | d | e | |
| 23. | a | b | c | d | e | |
| 24. | a | b | c | d | e | |
| 25. | a | b | c | d | e | |

## UNIT TEST
## FORM B
## CONGRUENT TRIANGLES

**DIRECTIONS: Do not write on this test. Place all answers on the separate answer sheet.**

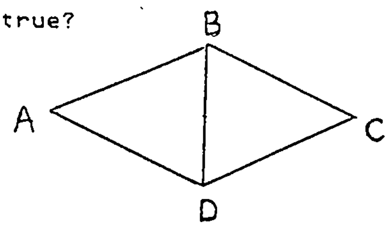
---

1. In $\triangle$ ABC, the side opposite $\measuredangle$ B is
   a. $\overline{AB}$
   b. $\overline{BC}$
   c. $\overline{AC}$

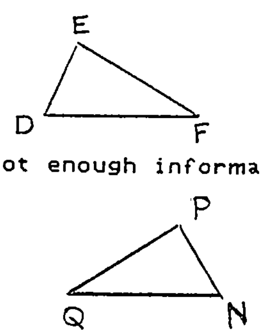
2. If $\triangle$ NPQ is congruent to $\triangle$ RST, then which of the following must be true?
   a. $\overline{PN} \cong \overline{ST}$
   b. $\overline{NQ} \cong \overline{TR}$
   c. $\overline{PQ} \cong \overline{RS}$
   d. $\measuredangle$ PQN $\cong$ $\measuredangle$ SRT
   e. $\measuredangle$QPN $\cong$ $\measuredangle$STR

3. In $\triangle$ABC, the angle bisector of $\measuredangle$A is perpendicular to $\overline{BC}$. Then $\triangle$ABC must be
   a. a scalene triangle.
   b. a right triangle.
   c. an acute triangle.
   d. an isosceles triangle.
   e. an equilateral triangle.

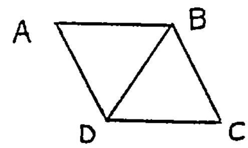
4. In the accompanying figure, $\measuredangle$ A $\cong$ $\measuredangle$ C and $\overline{BD}$ bisects $\measuredangle$ ABC. Which statement does **not** have to be true?
   a. $\overline{AB} \cong \overline{BC}$
   b. $\overline{AB} \cong \overline{BD}$
   c. $\measuredangle$ ADB $\cong$ $\measuredangle$ BDC
   d. $\overline{AD} \cong \overline{DC}$
   e. $\measuredangle$ ABD $\cong$ $\measuredangle$ CBD




5. In the accompanying figure $\measuredangle$ EFD $\cong$ $\measuredangle$ PQN, $\measuredangle$ DEF $\cong$ $\measuredangle$ NPQ, and $\measuredangle$ EDF $\cong$ $\measuredangle$ PNQ. Then $\triangle$ DEF and $\triangle$ NPQ
   a. are congruent by the SSS postulate.
   b. are congruent by the ASA postulate.
   c. are congruent by the SAS postulate.
   d. may or may not be congruent. There is not enough information given to determine this positively.
   e. cannot be congruent.

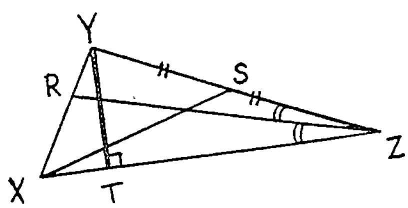


6. If $\triangle ABD \cong \triangle DCB$, then which statement does **not** have to be true?
   a. $\measuredangle ABD \cong \measuredangle BCD$
   b. $\overline{AD} \cong \overline{BD}$
   c. $\measuredangle DAB \cong \measuredangle DBA$
   d. $\measuredangle ADB \cong \measuredangle BCD$
   e. $\overline{AD} \cong \overline{BC}$

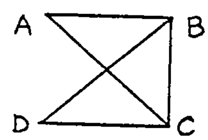



7. Which of the following statements **DOES NOT ALWAYS** describe congruent triangles?
   a. If two angles and the side opposite one of the angles of a triangle are congruent to the corresponding two angles and the opposite side of another triangle, then the two triangles are congruent.
   b. If two sides and the angle opposite one of the sides of a triangle are congruent to the corresponding two sides and the opposite angle of another triangle, then the two triangles are congruent.
   c. If two angles and the included side of a triangle are congruent to the corresponding two angles and included side of another triangle, then the two triangles are congruent.
   d. If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.
   e. If two sides and the included angle of a triangle are congruent to the corresponding two sides and included angle of another triangle, then the two triangles are congruent.

8. In $\triangle XYZ$ at the right, which segment names a median?
   a. $\overline{ZR}$
   b. $\overline{XS}$
   c. $\overline{XT}$
   d. $\overline{YT}$
   e. $\overline{XR}$

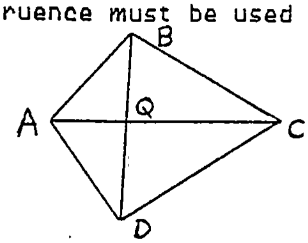



9. $\overline{AB}$ is perpendicular to $\overline{BC}$. $\overline{DC}$ is perpendicular to $\overline{CB}$. Then $\triangle ABC$ and $\triangle DCB$
   a. are congruent by the HA theorem.
   b. are congruent by the HL theorem.
   c. are congruent by the LL theorem.
   d. are congruent by the LA theorem.
   e. may or may not be congruent. There is not enough information given to determine this positively.
   f. cannot be congruent.




10. In an isosceles triangle, the vertex angle measures **210 − 9x** degrees, one base angle measures **6x − 30** degrees and the other base angle measures **3x** degrees. What is the value of **x**?
    a. 10
    b. 16
    c. 17.5
    d. 30
    e. Impossible to determine from the information given.

11. If $\triangle$ ABC $\cong$ $\triangle$ ADC, which congruence must be used to show $\triangle$ DBC is isosceles?

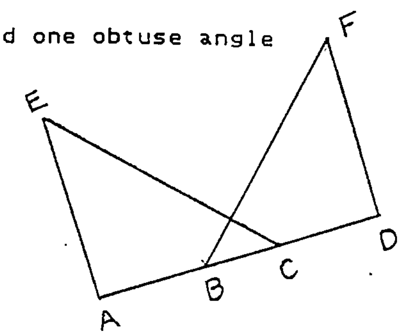



a. $\overline{AB} \cong \overline{AD}$
b. $\angle$ BCQ $\cong$ $\angle$ DCQ
c. $\overline{BC} \cong \overline{DC}$
d. $\angle$ CBA $\cong$ $\angle$ CDA
e. $\overline{BQ} \cong \overline{DQ}$

12. Which of these statements always describes a triangle?
a. A figure formed by three noncollinear points.
b. A figure formed by three segments that join three points.
c. A figure consisting of three noncollinear points and the line segments determined by these three points.
d. A figure formed by three lines that pass through three noncollinear points.
e. A figure formed by three segments such that each pair of segments intersect.

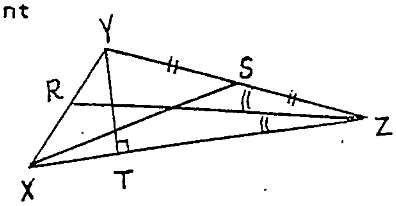
13. Which type of triangle cannot be drawn?
a. An obtuse equilateral triangle
b. A triangle with two acute angles and one obtuse angle
c. A triangle with three acute angles
d. An obtuse scalene triangle
e. An isosceles right triangle

14. In the accompanying figure $\angle$ EAB and $\angle$ FDC are right angles, $\overline{AB} \cong \overline{CD}$, and $\overline{EC} \cong \overline{FB}$. Then $\triangle$ ACE and $\triangle$ DBF



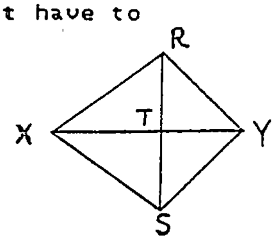

a. are congruent by the HA theorem.
b. are congruent by the HL theorem.
c. are congruent by the LL theorem.
d. are congruent by the LA theorem.
e. may or may not be congruent. There is not enough information given to determine this positively.
f. cannot be congruent.

15. In $\triangle$ XYZ at the right, which segment names an angle bisector?



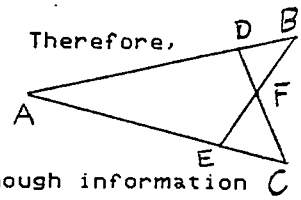

a. $\overline{ZR}$
b. $\overline{XS}$
c. $\overline{XT}$
d. $\overline{YT}$
e. $\overline{XR}$

16. In the accompanying figure $\overline{XY}$ is perpendicular to $\overline{RS}$ and T is the midpoint of $\overline{RS}$. Which statement does **not** have to be true?




a. $\overline{RT} \cong \overline{TS}$
b. $\angle$ RXT $\cong$ $\angle$ SXT
c. $\angle$ RTX $\cong$ $\angle$ RTY
d. $\overline{RX} \cong \overline{RY}$
e. $\overline{RX} \cong \overline{XS}$

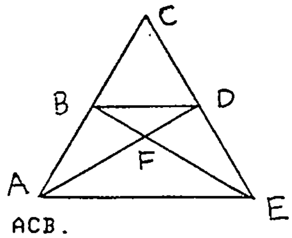
17. In the accompanying figure, $\overline{AB} \cong \overline{AC}$ and $\overline{DC} \cong \overline{EB}$. Therefore, △ ABE and △ ACD




a. are congruent by the SSS postulate.
b. are congruent by the ASA postulate.
c. are congruent by the SAS postulate.
d. may or may not be congruent. There is not enough information given to determine this positively.
e. cannot be congruent.

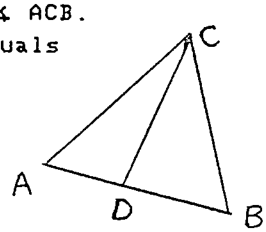
18. In the accompanying figure, $\overline{BC} \cong \overline{CD}$, ∡ BAF ≅ ∡ DEF, and ∡ C ≅ ∡ C. Using **only** these congruent parts (and no others), which congruency is established?

a. △ ABF ≅ △ EDF
b. △ ACD ≅ △ ECB
c. △ ABD ≅ △ EDB
d. △ ADE ≅ △ EBA
e. △ ACD ≅ △ ADE



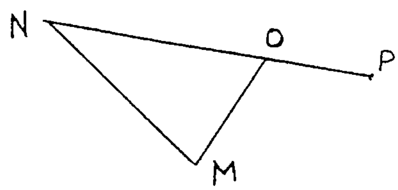

19. In △ABC, $\overline{AC} \cong \overline{AB}$ and $\overline{CD}$ bisects ∡ ACB. If m ∡ ACD = 20°, then m ∡ABC equals

a. 10°
b. 20°
c. 40°
d. 70°
e. 100°




20. In △ MNO if $\overline{MN} \cong \overline{NO}$ and exterior ∡ MOP measures 110°, then what is the measure of ∡ MNO?

a. 35°
b. 40°
c. 60°
d. 70°
e. none of these



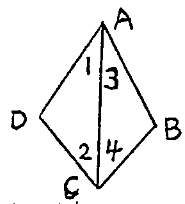

21. In △ XYZ, the angle included between $\overline{YZ}$ and $\overline{XZ}$ is

a. ∡ XYZ
b. ∡ YZX
c. ∡ ZXY

22. In △ ABC, the median drawn from the vertex of ∡ C to intersect side $\overline{AB}$ bisects ∡ C. Then △ ABC must always be

a. a scalene triangle.
b. an obtuse triangle.
c. an acute triangle.
d. an equilateral triangle.
e. an isosceles triangle.

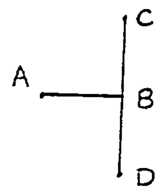
23. Given ∡ 1 ≅ ∡ 4 and ∡ 2 ≅ ∡ 3, then △ ABC and △ CDA




a. are congruent by the SSS postulate.
b. are congruent by the ASA postulate.
c. are congruent by the SAS postulate.
d. may or may not be congruent. There is not enough information given to determine this positively.
e. cannot be congruent.

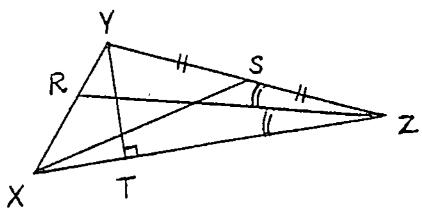
24. In the accompanying figure, $\overleftrightarrow{AB}$ is perpendicular to $\overline{CD}$ and B is the midpoint of $\overline{CD}$. Which statment does not have to be true?

a. m ∡ ABC = 90°
b. $\overline{BC} \cong \overline{BD}$
c. m ∡ ADB = 45°
d. $\overline{AC} \cong \overline{AD}$
e. m ∡ ABC = m ∡ ABD




25. In △ XYZ at the right, which segment names an altitude?

a. $\overline{ZR}$
b. $\overline{XS}$
c. $\overline{XT}$
d. $\overline{YT}$
e. $\overline{XR}$



Mathematics Department
Western Montana College
of the University of Montana
Dillon, MT 59725
<DATE>

<NAME AND ADDRESS>

Dear <NAME>:

First, let me thank you for agreeing to examine the unit test for my research project for validity.

Enclosed you will find this criterion-referenced test together with a Table of Specifications. The score that a student earns on this test will be the dependent variable in my research project. This test (and an equivalent form) will be used as the pretest and posttest for the four-week unit on congruent triangles as presented in the typical high school (tenth grade) geometry class. The form that the student is given as the pre-test will be given to this student at a later date as the post retention instrument.

To determine the validity of this instrument, I am having you, along with two other specialists in the area of mathematics, validate the content of this instrument as an appropriate measure of the subject matter as defined by the listed objectives. An abbreviated form of the objectives are on the left of the Table of Specifications and the cognitive levels are on the top of the Table. Please read the test, mark the cognitive level for each test item by writing the item number in the appropriate box. It is possible that you may decide that one item may apply to two or more objectives. If so, mark the item for all objectives to which it applies. When you have completed this task, please respond to the validation page and return all the materials to me.

I have included a page that describes the objectives in an unabbreviated form. Also, for your convenience, I have listed Bloom's description of the cognitive levels.

Again, thank you for taking time from your busy schedule to complete this for me.

Sincerely yours,

E. Otis Thompson
Assistant Professor
of Mathematics

VALIDATION OF THE CRITERION REFERENCED TEST

ON THE UNIT

CONGRUENT TRIANGLES

for

E. Otis Thompson

I have read the criterion referenced test to be used in E. Otis Thompson's research. I find the contents of this test as a measure of the subject matter as described by the objectives to be:

X Valid.

_____ Not Valid (for the reasons listed below).

Signed Lee J. Brown

Title Mathematics Department

Date 9/29/89

Reasons:

VALIDATION OF THE CRITERION REFERENCED TEST

ON THE UNIT

CONGRUENT TRIANGLES

for

E. Otis Thompson

I have read the criterion referenced test to be used in E. Otis Thompson's research. I find the contents of this test as a measure of the subject matter as described by the objectives to be:

__X__ Valid.

_____ Not Valid (for the reasons listed below).

Signed Sue Dolezal

Title Mathematics Teacher

Date 9/25/89

Reasons:

They cover each of the objectives.

The test items are at a number of levels of difficulty.

The objectives cover items (concepts) taught in standard geometry courses.

VALIDATION OF THE CRITERION REFERENCED TEST

ON THE UNIT

CONGRUENT TRIANGLES

for

E. Otis Thompson

I have read the criterion referenced test to be used in E. Otis Thompson's research. I find the contents of this test as a measure of the subject matter as described by the objectives to be:

* ✓ Valid.

_____ Not Valid (for the reasons listed below).

Signed Johnny W. Lott

Title Professor of Mathematics University of Montana

Date 9/25/89

Reasons:

* With corrections noted for symbols.

# APPENDIX B

## VAN HIELE PHASE-BASED ACTIVITY WORK SHEETS

## Geometry Activity Number One
(Paper and pencil)

Objective: To determine a minimal set of conditions needed to define a triangle.

Preparation: Review the terms **line, line segment, point, and collinear.**

Activity: Students work in groups of two or three following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

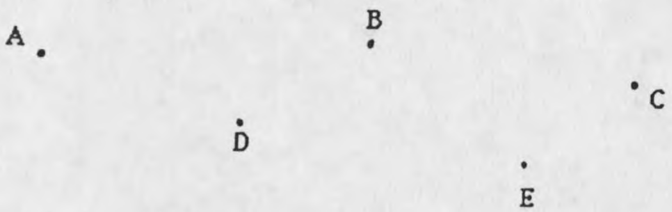
Supplies: Dittoed copies of the following pages for each group. Pencil and straightedge.

Summary: After the students have completed the activity, they report their answers to the class and discuss them. Definitions should be compared and a final (correct) version should be agreed upon by all.

## Geometry Activity Number One
**(Paper and pencil)**

**names** ______________________________

I. (a) Given the five points A, B, C, D, and E, draw all possible line segments determined by these points.




Name each line segment you drew:

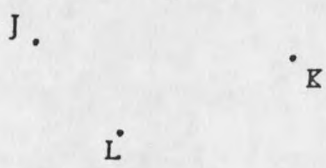
How many did you draw?

(b) Given four points F, G, H, and I, draw all possible line segments determined by these points.


Name each line segment you drew:

How many did you draw?

(c) Given the three points J, K, and L draw all possible line segments determined by these points.




Name each line segment you drew:

How many did you draw?

(d) Given the two points M and N, draw all possible line segments determined by these points.

M •

• N

Name each line segment you drew:

How many did you draw?

(e) Which of your drawings in I (a-e) above look like a triangle?

(f) With how many points must you start in order to get a triangle?

II. (a) Given the three points O, P, and Q, draw all possible line segments determined by these points.

O •

• P

• Q

Name each line segment you drew:

How many did you find?

(b) Given the points R, S, and T, draw all possible line segments determined by these points.

R • . S

• T

Name the line segments you drew:

How many did you find?

(c) Given the points U, V, and W, draw all possible line segments determined by these points.

U V W

• • •

Name the line segments you drew:

How many did you find?

(d) Which of your drawings in II (a-c) above look like a triangle?

(e) What property must the points have in order to get a triangle?

III.(a) Given the three points A, B, and C, draw all possible lines determined by these points.

A

B

C

Name the lines you drew:

How many did you find?

(b) Given the three points D, E, and F, draw all possible lines determined by these points.

E

D

F

Name the lines you drew:

How many did you find?

(c) Given the three points G, H, and I, draw all possible lines determined by these points.

G

H

I

Name the lines you drew?

How many did you find?

(d) Which of your drawings in III (a-c) above look like a triangle?

(e) Can lines be used to describe a triangle?

IV. Summary.

(a) Use the information you collected to complete the following table:

| Number of points | Number of segments determined |
|---|---|
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| n | |

(b) Form a definition of a triangle.

## Geometry Activity Number Two
### (Paper and pencil)

Objectives:
1. To create (or review) the definitions of right, acute, obtuse, scalene, isosceles, and equilateral triangles.
2. To learn to identify these types of triangles.

Preparation: Review the use of the protractor for measuring angles and the use of a metric ruler to measure a line segment to the nearest millimeter.

Activity: Students work in groups of two or three following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

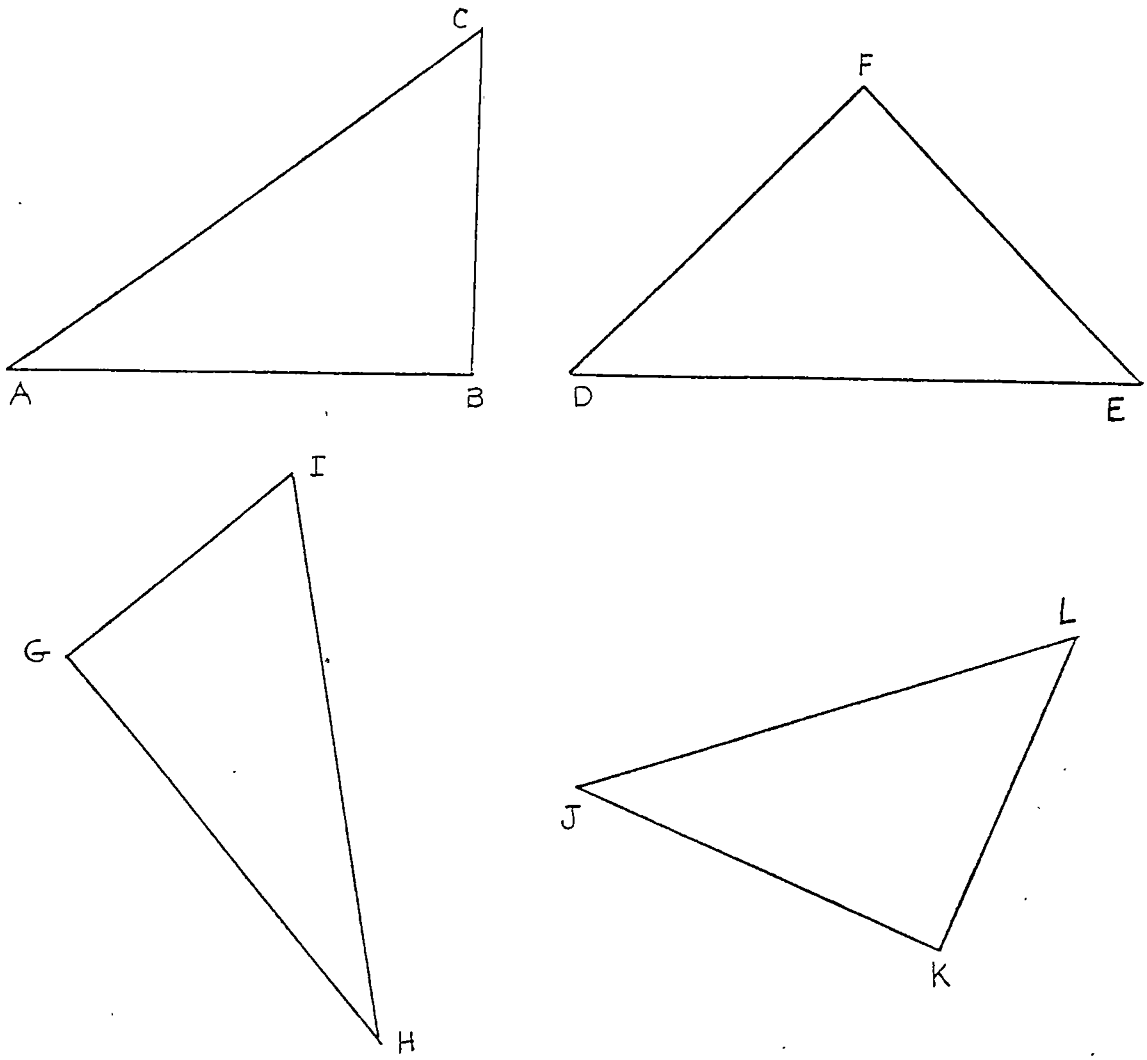
Supplies: Copies of the following pages for each group.
Pencil, protractor, and metric ruler for each group.

Summary: After the students have completed the activities, definitions should be shared and a final (correct) version for each type of triangle should be agreed upon by all. Students should be called upon to share and support their answers for the various triangles identified in the last activity.

## Geometry Activity Number Two

names________________________________________

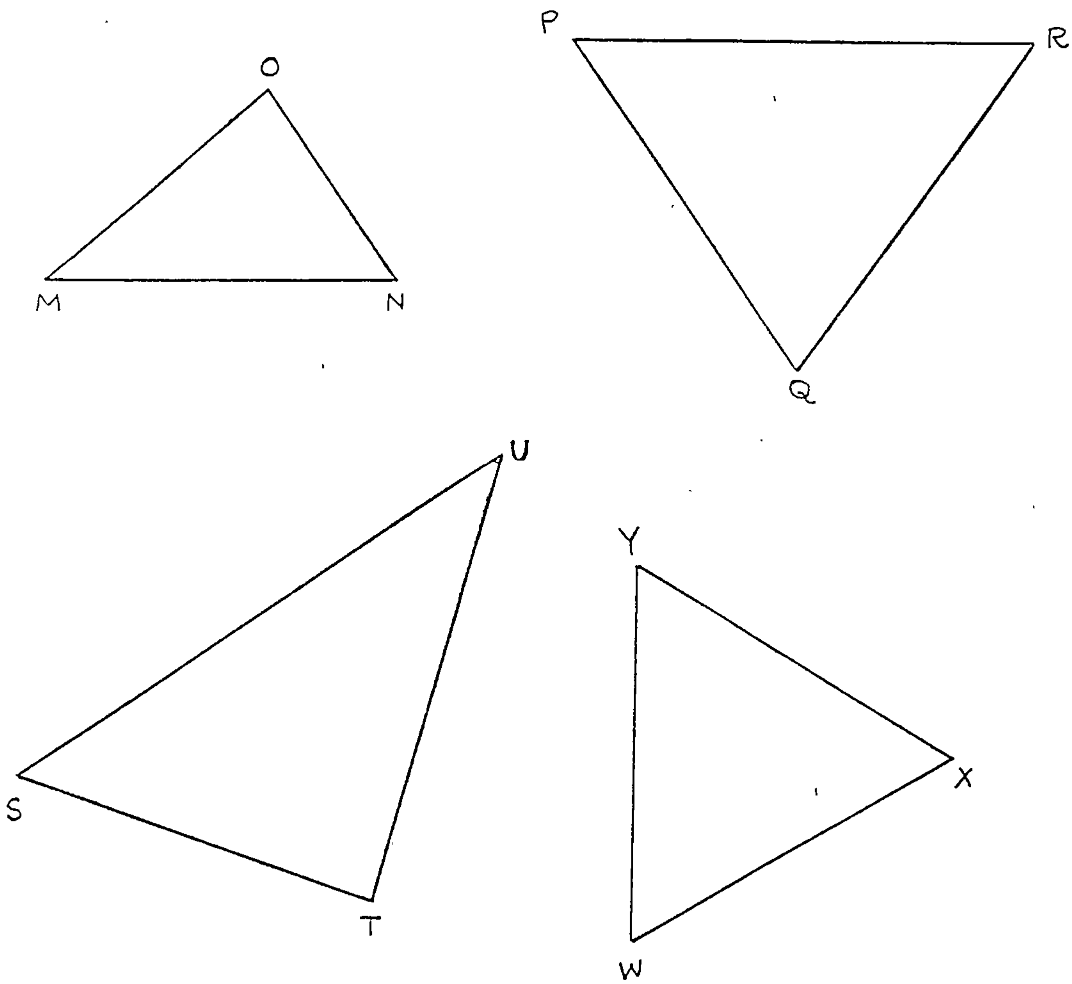
Below are four **right** triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures.




Based on your findings, complete the following definition:
A right triangle is a triangle which ____________________________________

____________________________________________________________________________

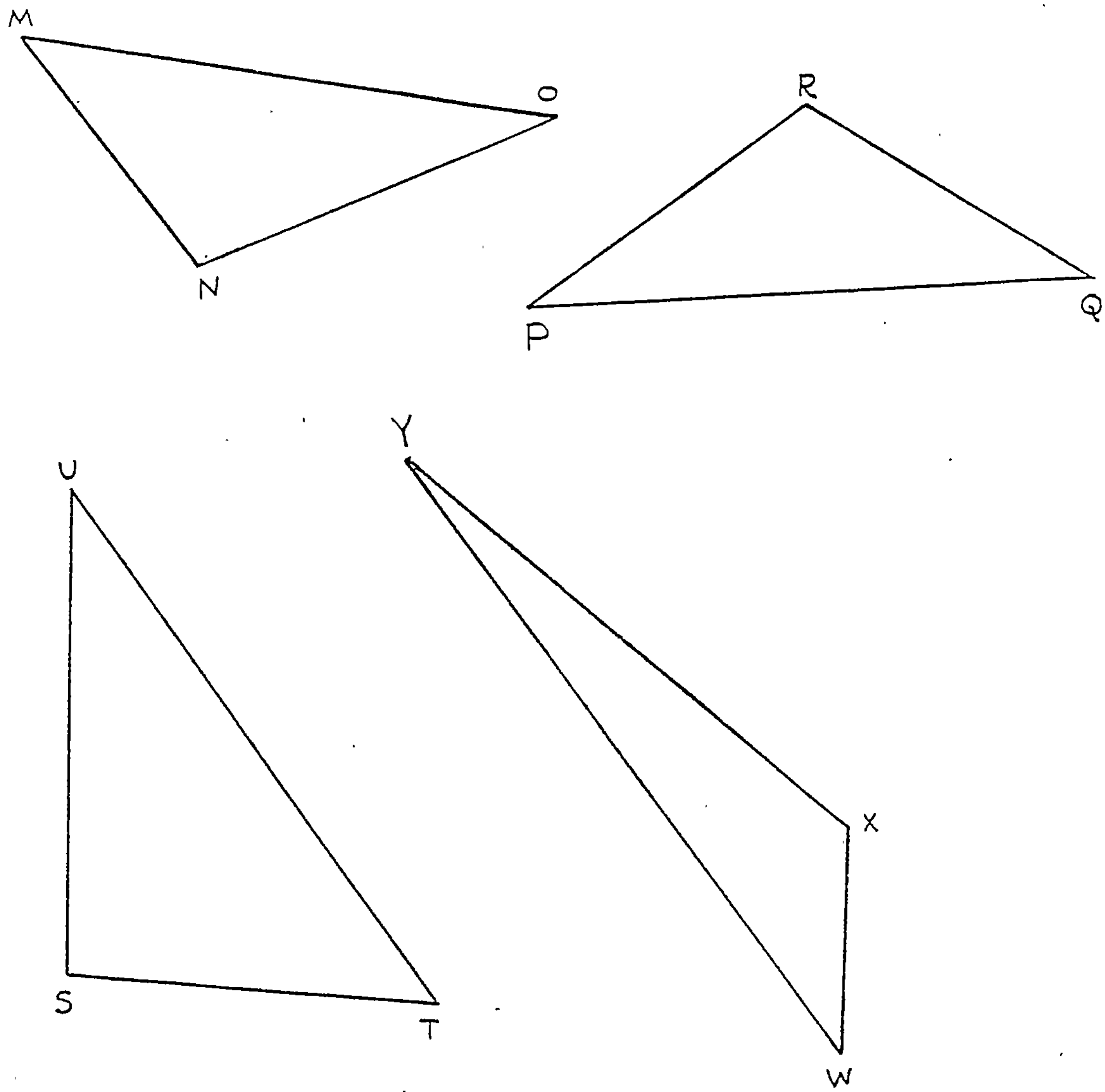
Below are four **acute** triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures.




Based on your findings, complete the following definition:
An acute triangle is a triangle which ______________________________

______________________________________________________________

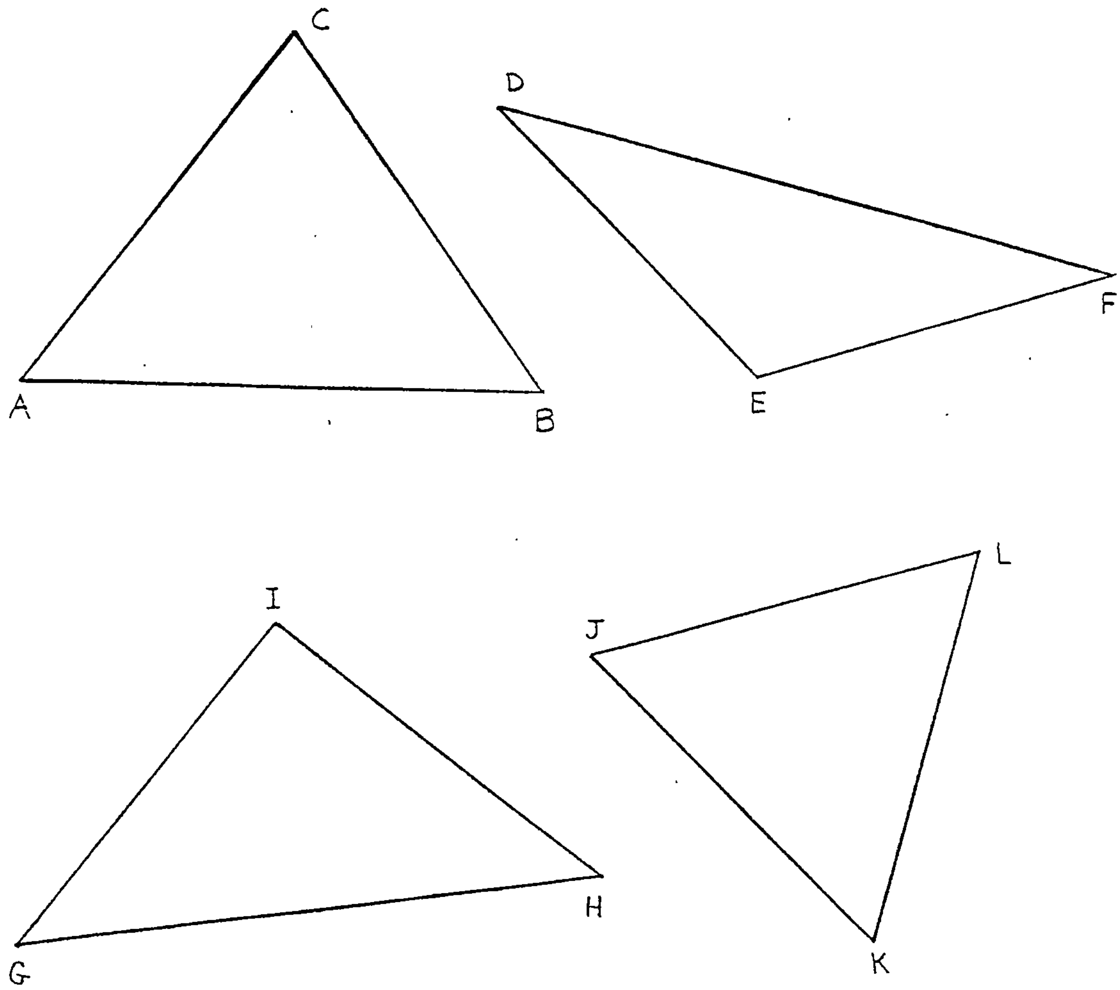
Below are four **obtuse** triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures.




Based on your findings, complete the following definition:
An obtuse triangle is a triangle which ______________________________

__________________________________________________________________

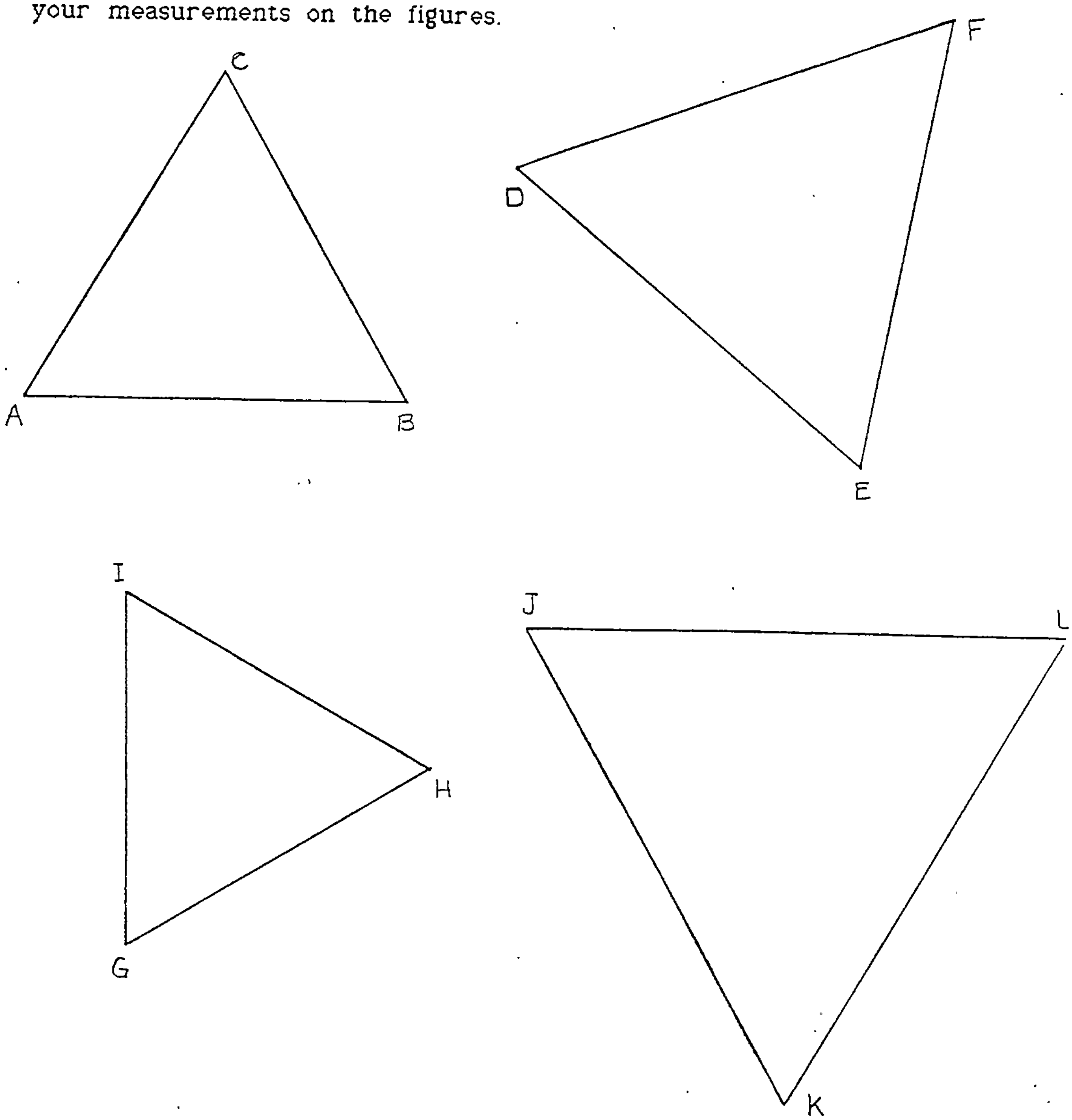
Below are four **isosceles** triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures.




Based on your findings, complete the following definition:
An isosceles triangle is a triangle which ______________________

______________________________________________________

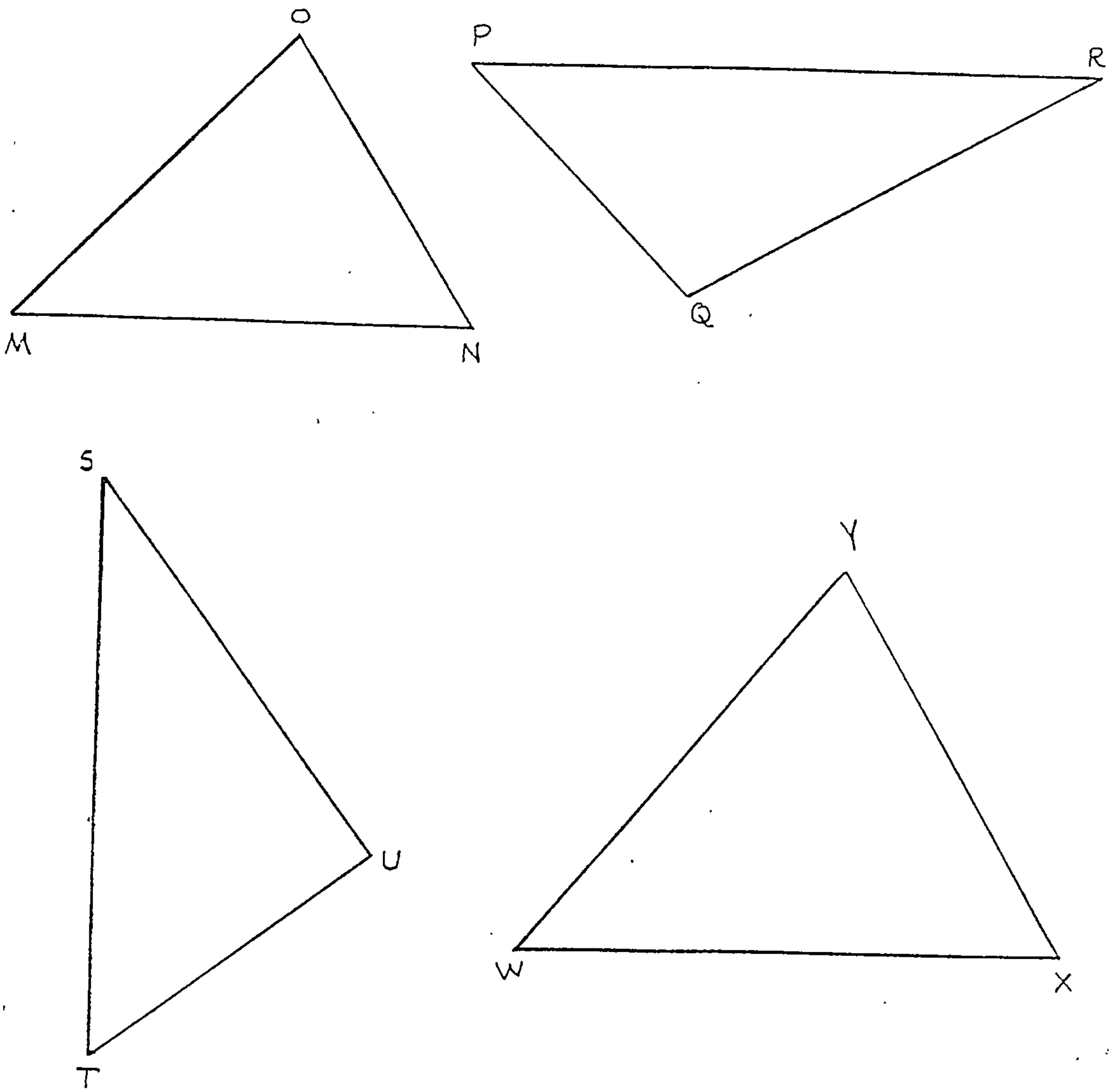
Below are four **equilateral** triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures.




Based on your findings, complete the following definition:
An equilateral triangle is a triangle which ______________________________

__________________________________________________________________________

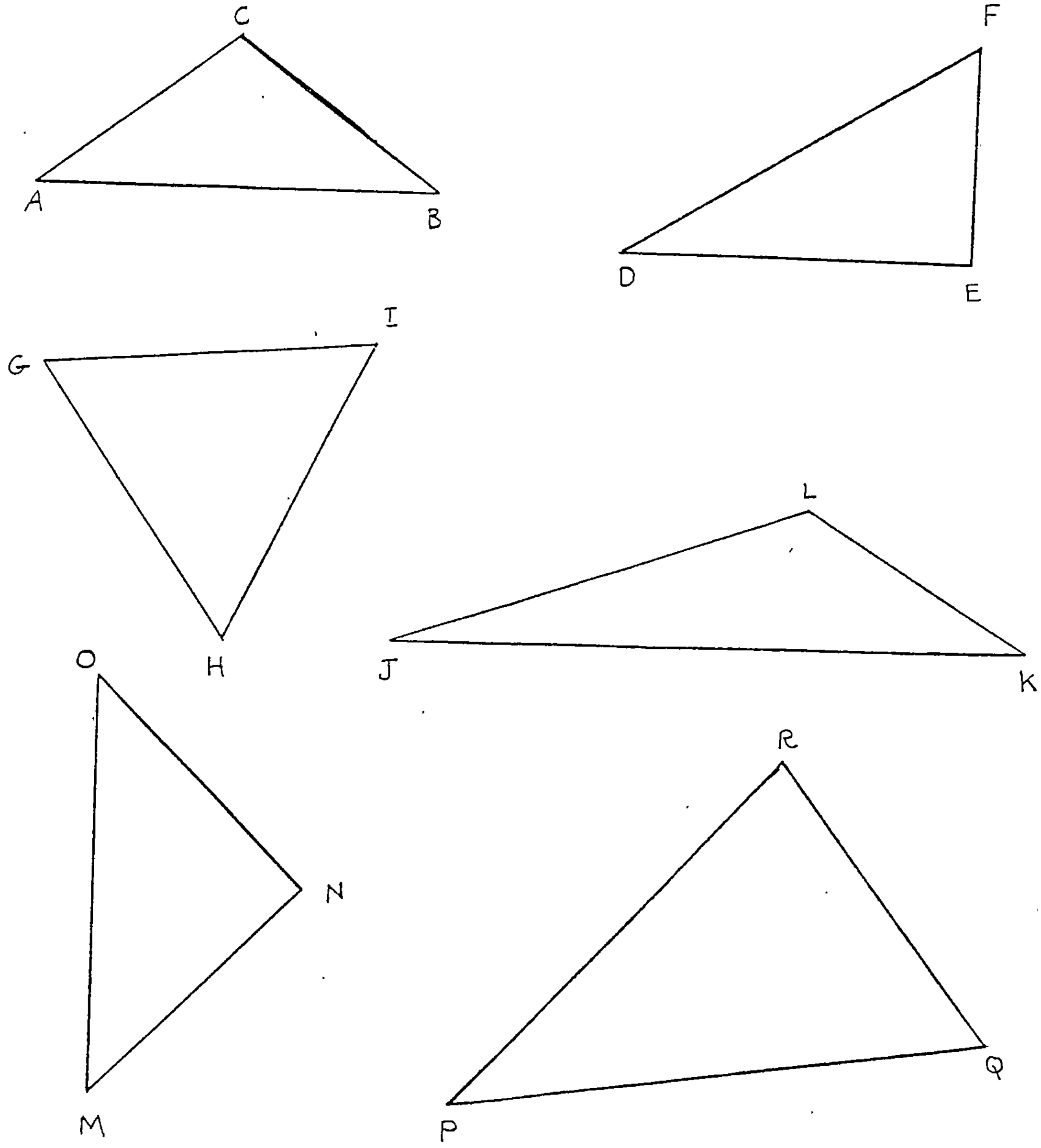
Below are four **scalene** triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures.




Based on your findings, complete the following definition:
A scalene triangle is a triangle which ______________________________

______________________________________________________________

Below are six triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures. Then describe each triangle using all the words (right, acute, obtuse, isosceles, equilateral, or scalene) that apply.



# Geometry Activity Three
## (Paper and pencil)

Objectives: To identify congruent triangles and their corresponding parts.

Preparation: Review the meaning of congruent triangles.

Activity: The students work in groups of two or three following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

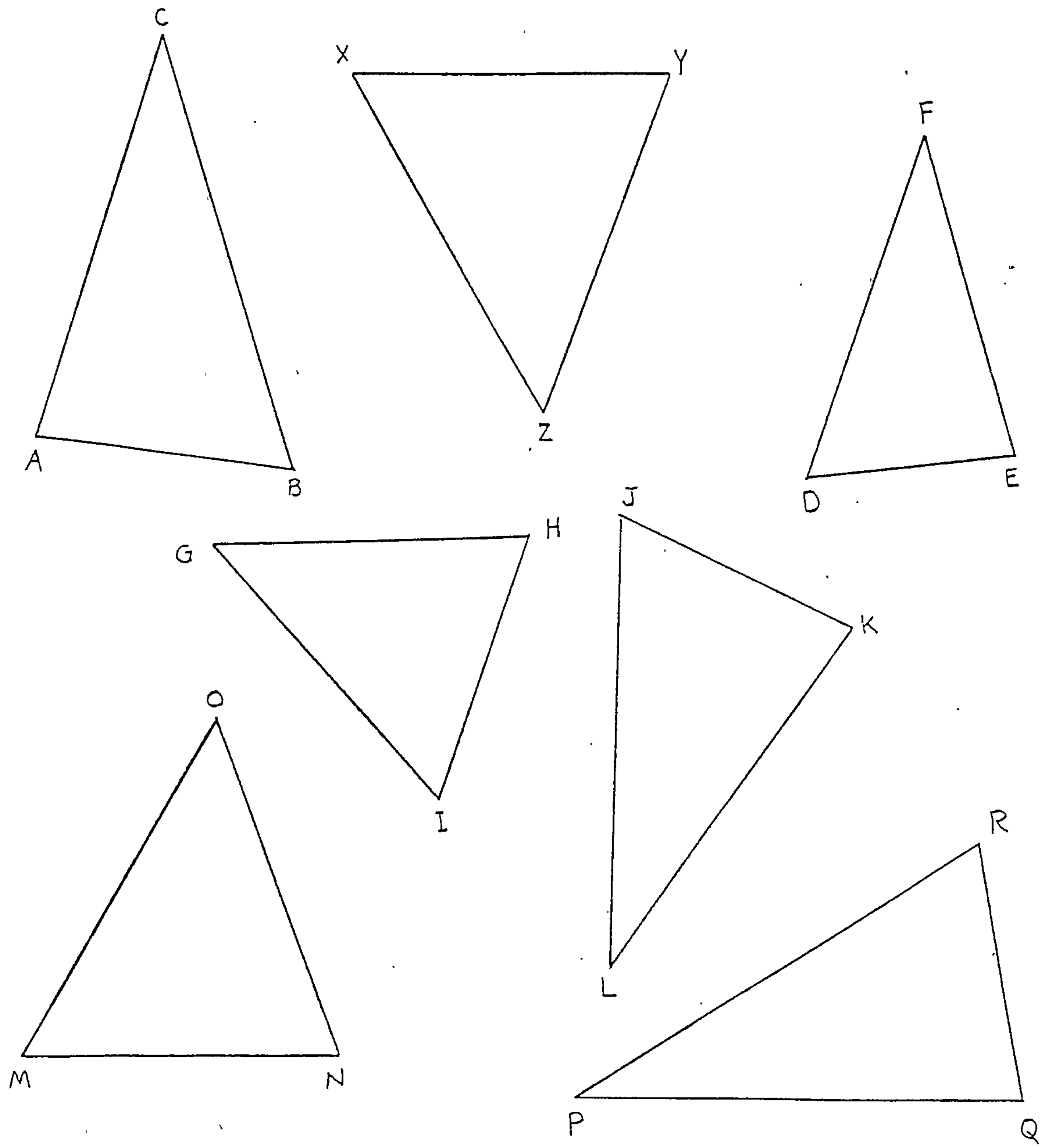
Supplies: Copies of the following two pages.
Pencils, protractors, and metric rulers.

Summary: After the students have completed the activity sheets, two or three groups should compare results and resolve any discrepancies in notation or correspondences. The teacher should make sure concensus is reached within the groups.

**Geometry Activity Number Three**

**names**____________________________

Below are several triangles. For each triangle use your protractor to measure the angles (to the nearest degree) and your metric ruler to measure the sides (to the nearest millimeter). Record your measurements on the figures.



Use the measurements you made on page 1 to identify the triangle congruent to $\triangle ABC$ and the triangle congruent to $\triangle XYZ$. Then complete the following tables:

$\triangle ABC \cong$ ____________

$\overline{AB} \cong$ ________

$\overline{AC} \cong$ ________

$\overline{BC} \cong$ ________

$\measuredangle ABC \cong$ ____________

$\measuredangle ACB \cong$ ____________

$\measuredangle BAC \cong$ ____________

$\triangle XYZ \cong$ ____________

$\overline{XY} \cong$ ________

$\overline{XZ} \cong$ ________

$\overline{YZ} \cong$ ________

$\measuredangle XYZ \cong$ ____________

$\measuredangle XZY \cong$ ____________

$\measuredangle YXZ \cong$ ____________

## Geometry Activity Four
## (Paper and pencil)

Objectives: To determine the conditions necessary for two triangles to be congruent.

Preparation: Review the definition of congruent triangles and identifying the corresponding parts.

Supplies: Copies of the following pages for each group.
Pencils, protractors, and metric rulers for each student.

Summary: After each activity page, the teacher should summarize the findings and emphasize why the particular conjecture may or may not be taken as a postulate. On page four, the teacher should make certain that at least two students have triangles that are not congruent.

**Geometry Activity Four**
**names**___________________________

Use your protractor and ruler to construct $\triangle ABC$ with $m\angle ABC = 30°$, $m\angle ACB = 90°$, and $m\angle BAC = 60°$. Measure and record the lengths of the sides of this triangle (to the nearest millimeter).

Compare your results with others in the class. Does the following conjecture appear to be true? Why or why not?

If three angles of one triangle are congruent to the corresponding angles of another triangle, then the two triangles are congruent.

Use your ruler to construct $\triangle ABC$ with AB = 10 cm, AC = 6 cm, and BC = 8 cm. (This may require some "trial and error" effort on your part.) Measure the angles of $\triangle ABC$ (to the nearest degree) and record your results. Repeat this procedure to construct $\triangle XYZ$ with XY = 15 cm, XZ = 10 cm, and YZ = 8 cm. Measure the angles of $\triangle XYZ$ (to the nearest degree) and record your results.

Compare your results with other students in your class. Does the following conjecture appear to be true? Why or why not?

If three sides of one triangle are congruent to the corresponding three sides of another triangle, then the two triangles are congruent.

Can you always make a triangle given any three numbers for the lengths of the sides. How can you be sure?

Use your protractor and ruler to construct $\triangle ABC$ using side $AB = 8$ cm, $m\angle BAC = 45°$, and side $AC = 6$ cm. Measure the remaining parts of $\triangle ABC$ and record your results. Repeat this procedure to construct $\triangle XYZ$ with $XY = 10$ cm, $m\angle YXZ = 100°$, and $XZ = 8$ cm. Measure the remaining parts of $\triangle XYZ$ and record your results.

Compare your results with other students in your class. Does the following conjecture appear to be true? Why or why not?

If two sides and the included angle of one triangle are congruent to the corresponding two sides and included angle of another triangle, then the two triangles are congruent.

Can you always make a triangle given any three numbers for the lengths of two sides and the measure of the included angle? How can you be sure?

Use your ruler and protractor to construct $\triangle ABC$ with $m\measuredangle BAC = 30°$, $AC = 10$ cm, and $BC = 6$ cm. Measure the remaining parts of $\triangle ABC$ and record your results.

Compare your results with other students in your class. Does the following conjecture appear to be true? Why or why not?

If two sides and the angle opposite one of them in one triangle are congruent to the corresponding two sides and an angle opposite in a second triangle, then the triangles are congruent.

Use your ruler and protractor to construct $\triangle ABC$ using $m\measuredangle BAC = 45°$, $AB = 6$ cm and $m\measuredangle CBA = 60°$. Measure the remaining parts of $\triangle ABC$ and record your results. Repeat this procedure to construct $\triangle XYZ$ with $m\measuredangle YXZ = 20°$, $XY = 8$ cm, and $m\measuredangle ZYX = 95°$. Measure the remaining parts of $\triangle XYZ$ and record your results.

Compare your results with other students in your class. Does the following conjecture appear to be true? Why or why not?

If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the two triangles are congruent.

Can you always make a triangle given any three numbers for the measures of the two angles and the included side? How can you be sure?

The statement "If two angles and a side opposite one of them in one triangle are congruent to the corresponding two angles and side opposite in another triangle, then the two triangles are congruent." is just a special case of the above conjecture. Why?

## Geometry Activity Number Five
(Paper and pencil)

Objective: To define the words **altitude**, **median**, **angle bisector**, and **perpendicular bisector**.

Preparation: Review the terms and notation for describing the parts of a triangle, such as vertex, side, side opposite (a vertex or angle), angle opposite a side, included angle, and included side

Activity: Students work in groups of two or three following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

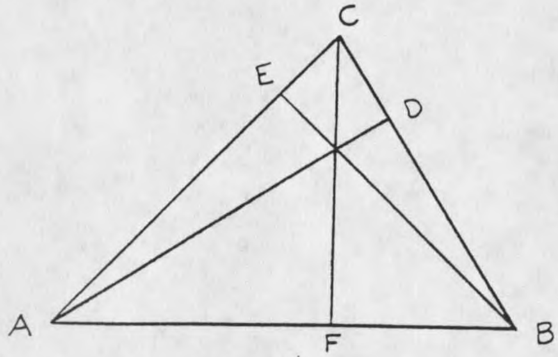
Supplies. Copies of the following pages for each group.
Pencils, protractors, and a ruler marked in millimeters.

Summary: After the students have completed the activities, they report their findings to the class and discuss them. Definition should be compared and a final (correct) version should be agreed upon by all. The teacher may want to expand on the idea that the angle bisectors, medians, altitudes, and perpendicular bisectors of the sides of any triangle are concurrent.

## Geometry Activity Number FIVE
**(Paper and pencil)**

**names** ______________________

Drawn below is △ ABC with its **altitudes** $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$. Use your protractor and metric ruler to measure the following angles and lengths. Record your results.




m ∡ ADB = ________

m ∡ BEC = ________

m ∡ CFA = ________

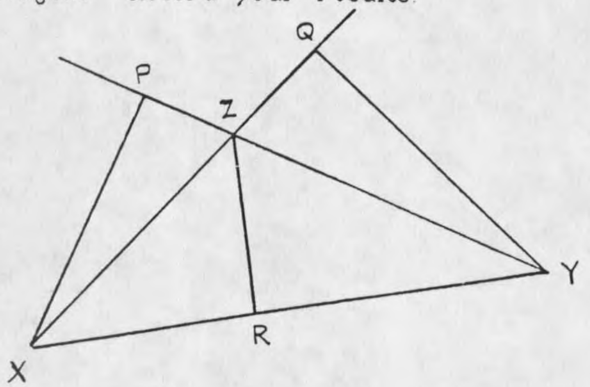
m ∡ BAC = ________

m ∡ BAD = ________

BC = ________

DC = ________

Drawn below is △ XYZ with its altitudes $\overline{XP}$, $\overline{YQ}$, and $\overline{ZR}$. Use your protractor and metric ruler to measure the following angles and lengths. Record your results.




m ∡ XPY = ________

m ∡ YQZ = ________

m ∡ ZRX = ________

m ∡ YXZ = ________

m ∡ YXP = ________

YZ = ________

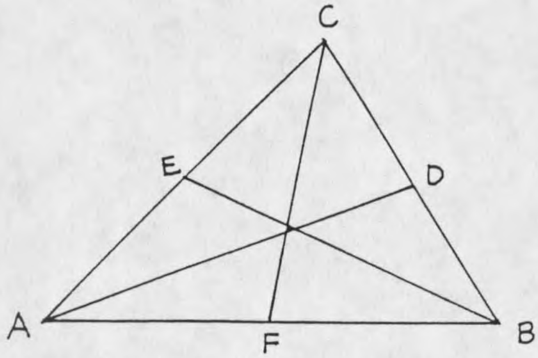
YP = ________

Complete the following definition:
An altitude of a triangle is

Use the above drawings and data to help with answering the following

1. How many altitudes does a triangle have?
2. When will an altitude of a triangle be partially outside the triangle?
3. Can the altitude of a triangle ever be a side of a triangle?

Drawn below is $\triangle$ ABC with its **medians** $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$. Use your protractor and metric ruler to measure the following angles and lengths. Record your results.




m ∡ CAD = ______

m ∡ CAB = ______

m ∡ ACF = ______

m ∡ ACB = ______

m ∡ AFC = ______

m ∡ ADB = ______

AF = ______

AB = ______

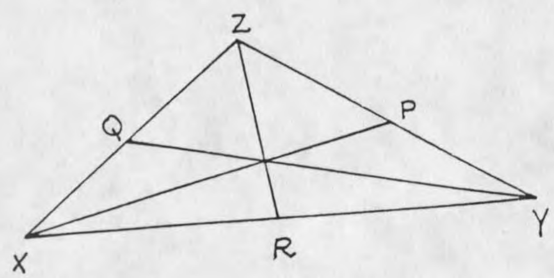
BD = ______

BC = ______

CE = ______

CA = ______

Drawn below is $\triangle$ XYZ with its **medians** $\overline{XP}$, $\overline{YQ}$, and $\overline{ZR}$. Use your protractor and metric ruler to measure the following angles and lengths. Record your results.




m ∡ ZXP = ______

m ∡ ZXY = ______

m ∡ XYQ = ______

m ∡ XYZ = ______

m ∡ XQY = ______

m ∡ XPZ = ______

YP = ______

YZ = ______

XQ = ______

XZ = ______

XR = ______

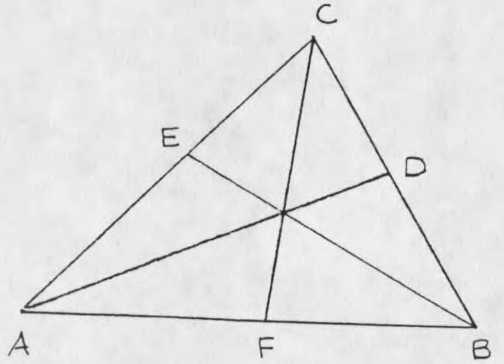
XY = ______

Complete the following definition:
A median of a triangle is

Use the above drawings and data to help with answering the following:

1. How many medians does a triangle have?
2. Does the median of a triangle, except for end points, always lie inside the triangle?
3. Does a median of a triangle ever bisect an angle of the triangle?

Drawn below is △ ABC with its **angle bisectors** $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$. Use your protractor and metric ruler to measure the following angles and lengths. Record your results.




m ∡ ABC = ______

m ∡ ABE = ______

m ∡ BCA = ______

m ∡ BCF = ______

m ∡ CAB = ______

m ∡ CAD = ______

m ∡ ADC = ______

m ∡ CFA = ______

AB = ______

AF = ______

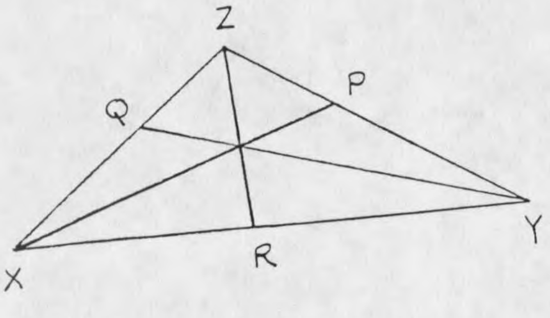
BC = ______

BD = ______

CA = ______

CE = ______

Drawn below is △ XYZ with its **angle bisectors** $\overline{XP}$, $\overline{YQ}$, and $\overline{ZR}$. Use your protractor and metric ruler to measure the following angles and lengths. Record your results.




m ∡ XYQ = ______

m ∡ XYZ = ______

m ∡ YZR = ______

m ∡ YZX = ______

m ∡ ZXP = ______

m ∡ ZXY = ______

m ∡ XQY = ______

m ∡ ZPX = ______

XQ = ______

XZ = ______

YR = ______

YX = ______

ZP = ______

ZY = ______

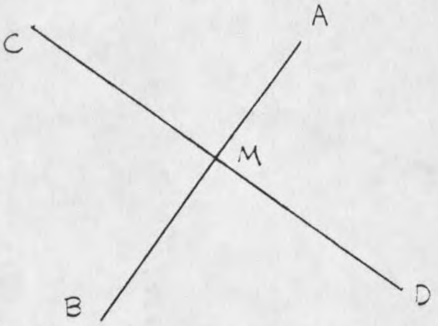
Complete the following definition:

An angle bisector of a triangle is

Use the drawings and the data to help with answering the following:

1. How many angle bisectors does a triangle have?
2. Does an angle bisector of a triangle, except for its end points, always lie inside the triangle?
3. Does an angle bisector of a triangle ever bisect a side of a triangle?

In the figure below, $\overline{AB}$ is the **perpendicular bisector** of $\overline{CD}$. Use your protractor and metric ruler to measure the angles and the segments. Record your results below.




m ∡ AMC = ________

m ∡ AMD = ________

m ∡ DMB = ________

m ∡ BMC = ________

AM = ______

BM = ______

CM = ______

DM = ______

AC = ________

BC = ________

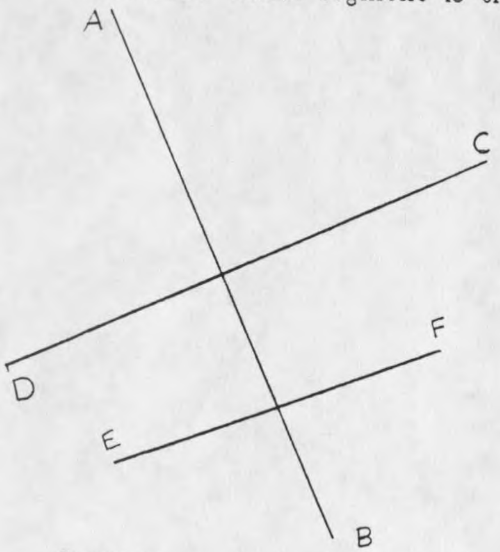
AD = ______

BD = ______

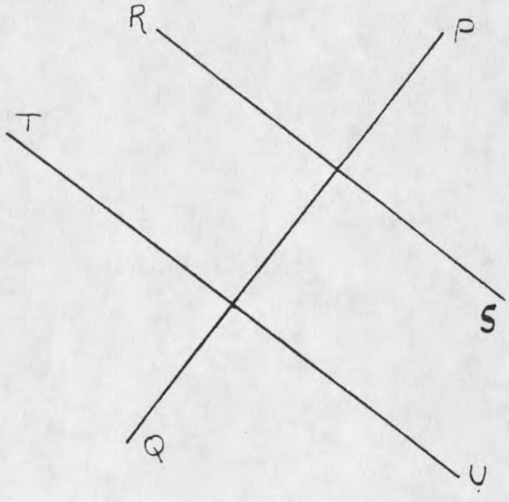
Which of the above angles are congruent?

Which of the above segments are congruent?

In each of the figures below, one of the segments is the perpendicular bisector of another segment. Measure the angles and the segments to determine which segment is the perpendicular bisector of which.




Segment ________ is the perpendicular bisector of segment ________.




Segment ________ is the perpendicular bisector of segment ________.

## Geometry Activity Number Six
## (Paper and pencil)

Objective: To test conjectures by looking for examples and counter-examples while simultaneously reinforcing the notions of angle bisectors, altitudes, medians, and perpendicular bisectors.

Preparation. Review the definitions of angle bisector, altitude, median, and perpendicular bisector. The students need to understand the difference between example and counter-example

Activity Students work in groups of two or three following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement

Supplies. Copies of the following pages for each group
Pencil, metric ruler, protractor, and scratch paper

Summary. After the students have completed the activities, they report their findings to the class and discuss them. The discussion component of these activities is very important. The students should be encouraged to justify ("prove") why their example or counter-example does what it is suppose to do The teacher may find it worthwhile to "dispose" of these exercises one by one rather than to have the students try to find the examples and counter-examples for all the exercises at one sitting.

**Geometry Activity Number SIX**
**(Paper and pencil)**
**names**____________________________

The conjectures below are always true (A), sometimes true (S), or never true (N). Read each conjecture and place the appropriate letter (A, S, or N) next to each statement. Justify your answer using one of the following procedures:

a. If you think the conjecture is always true (A), carefully draw two examples that agree with the conjecture.
b. If you think the conjecture is sometimes true (S), carefully draw two examples, one true and one false.
c. If you think the conjecture is never true (N), carefully draw two counter-examples.

CONJECTURES:

1. An altitude of a triangle bisects a side of the triangle.

2. A median of a triangle is perpendicular to the opposite side.

3. An altitude of a triangle is an angle bisector of the triangle.

4. A median of a scalene triangle is an angle bisector of this triangle.

5. A random point on a perpendicular bisector of a segment is equidistant from the end points of the segment.

6. A median of a triangle is an angle bisector of the triangle.

7. An angle bisector in a scalene triangle bisects the opposite side.

8. Portions of the altitudes of a right triangle lie outside the right triangle.

9. Except for the endpoints, the altitudes of an isosceles triangle lie inside the triangle.

10. The altitude from the vertex of a right triangle divides the original triangle into two congruent right triangles.

## Geometry Activity Number Seven
## (Paper and pencil)

Objective: To identify congruent triangles using the appropriate congruency postulates and theorems

Preparation. Review the postulates and theorems the students have encountered for establishing the congruency of triangles. Remind them that they need not check the congruency of all the corresponding parts in order to show that two triangles are congruent.

Activity: Students work in groups of two or three following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

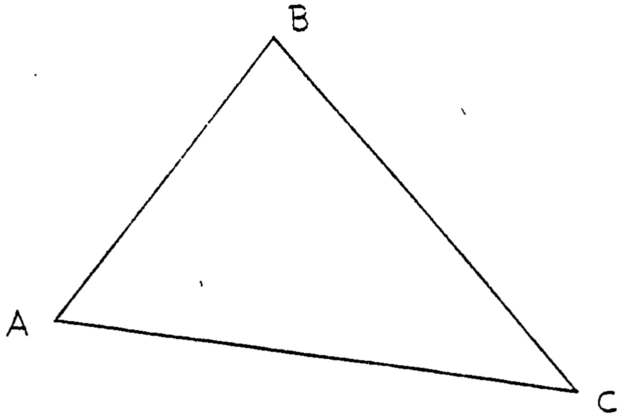
Supplies: Copies of the following pages for each group
Pencils, protractors, and metric rulers.

Summary: After the students have completed the activities, they report their findings to the class and discuss them The students should note that different postulates (or theorems) may have been used by their classmates to establish the congruencies Also, they should note that some students did not have to actually measure the angles and the sides They were able to determine some of the values from knowledge about special kinds of triangles (e.g., in an equilateral triangle, the measure of each angle is 60 degrees.) All these activity pages do not need to be completed in one period. Each activity page can "stand alone"

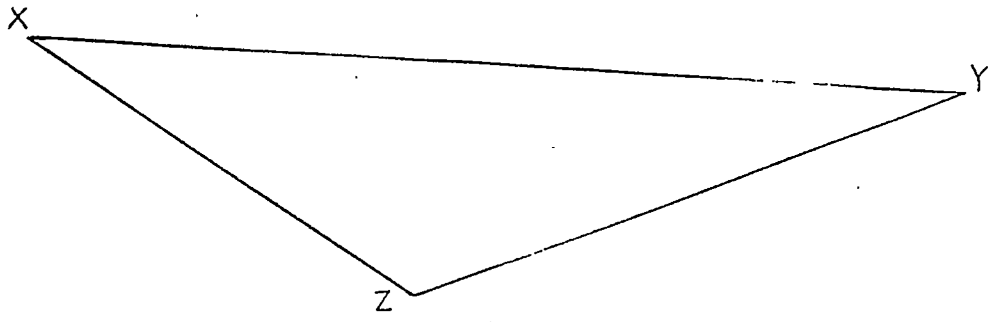
# Geometry Activity Number SEVEN
## (Paper and pencil)

**names** ______________________________

In △ABC given below, locate and label the midpoint, D, of side $\overline{AB}$. Locate and label the midpoint, E, of side $\overline{BC}$. Locate and label the midpoint, F, of side $\overline{AC}$. Draw △DEF. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all congruent triangles.

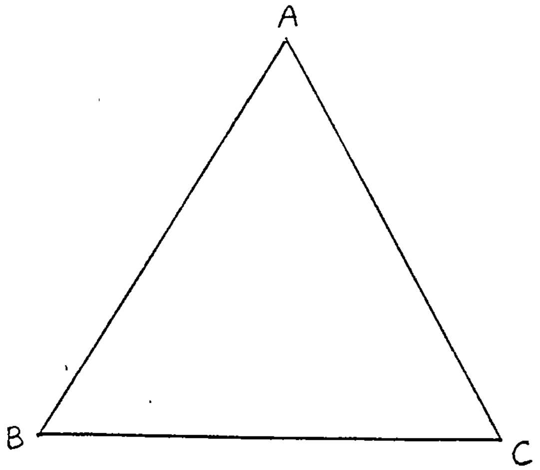



In △XYZ given below, locate and label the midpoint, R, of side $\overline{XY}$. Locate and label the midpoint, S, of side $\overline{YZ}$. Locate and label the midpoint, T, of side $\overline{XZ}$. Draw △RST. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all congruent triangles.

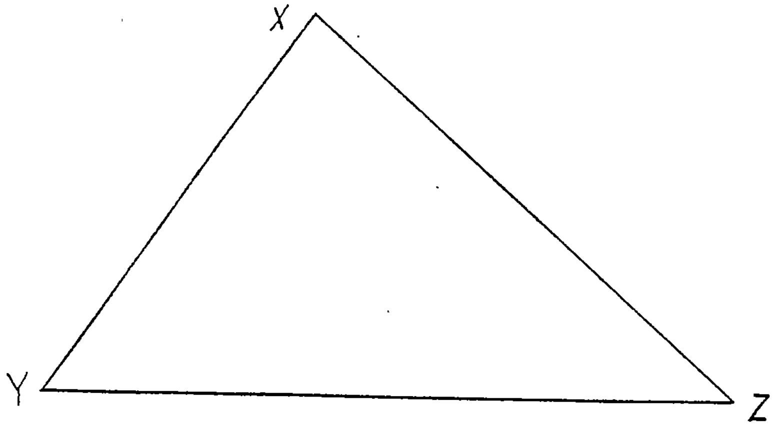



Would a similar set of congruent triangles be found if you tried this with a right triangle?

In equilateral $\triangle ABC$ given below, draw median $\overline{AD}$ from vertex A. Draw median $\overline{BE}$ from vertex B. Label the intersection of these two medians as point F. Erase segments $\overline{FE}$ and $\overline{DF}$. Draw $\overline{CF}$. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all congruent triangles.

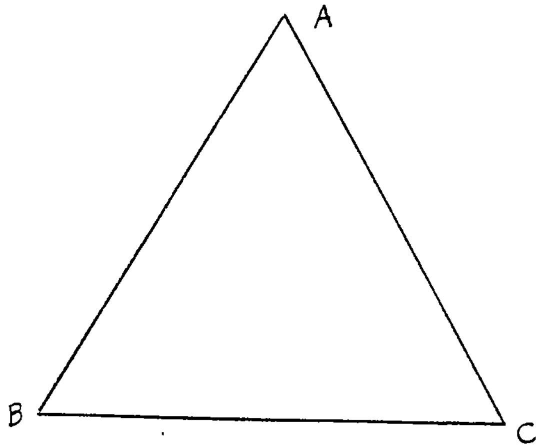



In scalene $\triangle XYZ$ given below, draw median $\overline{XR}$ from vertex X. Draw median $\overline{YS}$ from vertex Y. Label the intersection of these two medians as point T. Erase segments $\overline{ST}$ and $\overline{RT}$. Draw $\overline{ZT}$. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all congruent triangles.

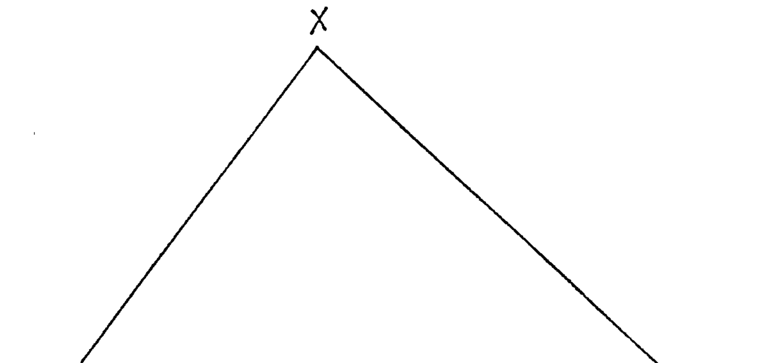



Would you find any congruent triangles if this same procedure was performed on an isosceles triangle?

In equilateral $\triangle ABC$ given below, divide segment $\overline{BC}$ into three congruent segments (label the division points D and E.) Draw $\overline{AD}$ and $\overline{AE}$. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all the congruent triangles.

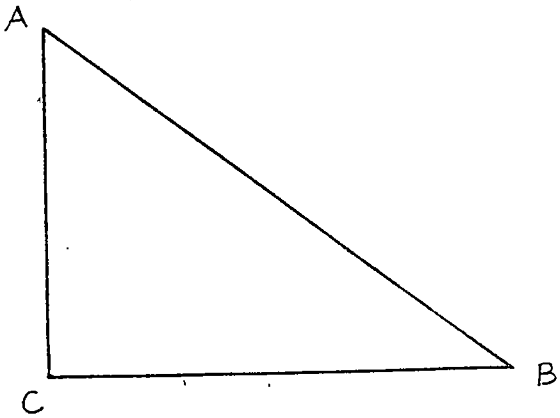



In scalene $\triangle XYZ$ given below, divide segment $\overline{YZ}$ into three congruent segments (label the division points S and T.) Draw $\overline{XT}$ and $\overline{XS}$. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all the congruent triangles.

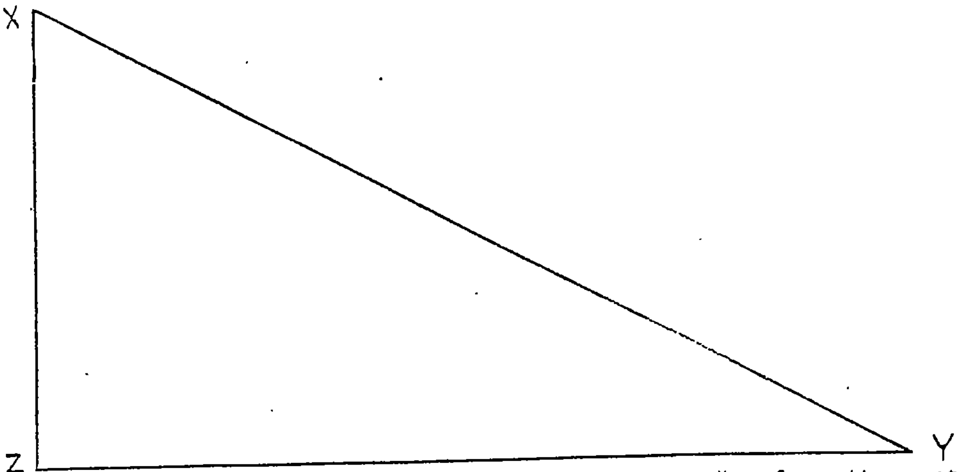



Would you find any congruent triangles if this same procedure was performed on an isosceles triangle?

In right $\triangle ABC$, locate the midpoint, D, of the hypotenuse $\overline{AB}$. Draw ray $\overrightarrow{CD}$ and locate point E such that CE = AB. Draw $\overline{EB}$. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all the congruent triangles.




In right $\triangle XYZ$, locate the midpoint, S, of the hypotenuse $\overline{XY}$. Draw ray $\overrightarrow{ZS}$ and locate point T such that ZT = XY. Draw $\overline{TY}$. Use your protractor and metric ruler to measure the necessary components that will allow you to identify all the congruent triangles in this figure. Place the numbers you need on the drawing and then list all the congruent triangles.




Did you notice that in both of these figures, the median from the vertex of the right angle is always half the length of the hypotenuse? Will this be the case in all right triangles?

## Geometry Activity Number One
## (Computer)

Objectives:
1. To learn how to operate the Geometric preSupposer computer program.
2. To determine the number of segments determined by a given number of points.
3. To determine a minimal set of conditions needed to define a triangle.

Preparation: Review the terms **line, line segment, point,** and **collinear.**

Activity: Students work in groups of two or three using the computer program Geometric preSupposer following the directions given on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

Supplies: Geometric preSupposer computer program.
Apple // computer (64K) for each group.
Dittoed copies of the following pages for each group.
Pencils.

Summary: After the students have completed the activity, they report their answers to the class and discuss them. Definitions should be compared and a final (correct) version should be agreed upon by all.

# Number of Line Segments

**(noncollinear points)**

**(using the Geometric preSupposer)**

Task 1: To determine the number of segments that can be drawn using a given number of noncollinear points.

Procedure:

- Place point A on the screen. (Press N [New shape], press 1 [Point], and then press RETURN.
- Attempt to name a segment. (Press 1 [Draw], then 1 [Segment], and then the letters to name the segment. (Note: No segment can be named, so press ESC twice to return to the first menu.) Record this information in the Drawing and Data Section on the next page.
- Place point B on the screen. (Press 2 [Label], press 4 [Moveable point], then press the arrow keys to move the point, and finally press RETURN to mark the point. Then press ESC to return to the previous menu.)
- Now draw all segments possible using these two letters. (Press 1 [Segment], then press the two letters naming the segment, then RETURN. When finished naming segments, press ESC twice to return to the main menu.) Record this data on the next page.
- Place three new points on the screen.
  (To locate point A: When you are at the main menu, press N [New shape]. Press 1 [Point]. Move the point using the arrows and press RETURN.
  To locate point B: Press 2 [Label]. Press 4 [Moveable point]. Use the arrows to move the point and then press RETURN.
  To locate point C: Press 4 [Moveable point]. Use the arrows to move the point and then press RETURN.
  To draw the segments: Press ESC and then 1 [Draw]. Press 1 [Segment] and name a segment using two letters and press RETURN. Repeat until all possible segments are named. Record this data on the next page.
- Place four new points on the screen and name all possible segments. (Follow the same procedure as when placing three points.) Record this data on the next page.
- Repeat this procedure for five and then six points.

===== *Drawings & Data* =====

Table 1.

| Points | Name of segments you can draw using the given points | Number of segments drawn |
|---|---|---|
| A | | |
| A, B | | |
| A, B, C | | |
| A, B, C, D | | |
| A, B, C, D, E | | |
| A, B, C, D, E, F | | |

===== *Conjectures* =====

Complete the following table:

Table 2.

| No. of points | No. of segments that can be drawn using these points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |

## Number of Line Segments

**(collinear points)**

**(using the Geometric preSupposer)**

**Task 2:** To determine the number of segments that can be drawn using a given number of collinear points.

**Procedure:**

- Draw three collinear points. (To do this use New Shape to place A somewhere in the drawing area. Label moveable point B. Subdivide segment AB into two sections--the subdivision point is labeled C.)
- Draw all possible segments using these three points.
- Record your picture and complete the table below.
- Draw four collinear points. (To draw four collinear points use New Shape to place A somewhere in the drawing area. Label moveable point B. Subdivide segment $\overline{AB}$ into three sections--the subdivision points are labeled C and D.)
- Draw all possible segments using these points.
- Record your picture and complete the table below.
- Repeat the above procedures using five and six points.

=====*Drawings & Data*=====================================

Table 3.

| Points | Name of segments you can draw using the given points | Number of segments drawn |
|---|---|---|
| A, B, C | | |
| A, B, C, D | | |
| A, B, C, D, E | | |
| A, B, C, D, E, F | | |

# Definition of a Triangle

**(using the preSupposer)**

**Task 2:** To develop a definition of a triangle.

**Procedure:**

- Place two points A and B on the screen.
- Draw all possible segments determined by these two points. Record your picture in the Drawings and Data section below.
- Place three points A, B, and C on the screen.
- Draw all possible segments defined by these three points. Record your picture below.
- Repeat the above procedures several times by locating the points A, B, and C in different locations each time.
- Place four points A, B, C, and D on the screen.
- Draw all possible segments defined by these three points. Record your picture below.
- Based on your findings, complete the exercises on the following pages.

=====*Drawings and Data*==========================================

====== *Summary* ======

Use the data collected in these three tasks to answer the following:

1. Based upon the data and drawings you made for Task 1, complete the following table:

| Number of noncollinear points | Number of line segments drawn | Number of lines drawn |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

2. Based upon the data and drawings you made for Task 2, complete the following table:

| Number of collinear points | Number of line segments drawn | Number of lines drawn |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

3. (a) If you used n noncollinear points, how many segments could you name?
   (b) If the n points are collinear, how many segments could you name?

4. (a) If you used n noncollinear points, how many **lines** could you name?

   (b) If you used n collinear points, how many **lines** could you name?

5. (a) When drawing all the segments named by n points, for what value(s) of n did the completed picture look like a triangle?

   (b) Will you always get a triangle in this situation?

   (c) What property must the points have in order to get a triangle?

6. Formulate a definition of a triangle.

## Geometry Activity Number Two
## (Computer)

Objectives: 1. To learn (or review) the definitions of right, acute, obtuse, scalene, isosceles, and equilateral triangles.
2. To learn to identify these types of triangles.

Preparation: Review the operation of the computer program Geometric Supposer: Triangles.

Activity: Students work in groups of two or three using the computer program Geometric Supposer: Triangles following the directions given on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

Supplies: Geometric Supposer: Triangles computer program.
Apple // computer (64K) for each group.
Copies of the following pages for each group.
Pencils.
(NO books or the students will be tempted to simply look up the definitions.)

Summary. After the students have completed the activity, definitions should be shared and a final (correct) version should be agreed upon by all. Students should be called upon to share and support their answers for the various triangles identified in the last activity.

## Right Triangle

**(using the Geometric Supposer:Triangles)**

**Task:** The object of this exercise is to develop a definition for a right triangle.

**Procedure:**

- Draw a random right triangle.
- Measure the sides and the angles of the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Repeat this process with other right triangles.
- Based on your findings, develop a definition for right triangle and write it below.

===== *Data* ==========================================================

===== *Conjecture* =====================================================

A right triangle is a triangle which ______________________________

_______________________________________________________________

_______________________________________________________________

# Acute Triangle

**(using the Geometric Supposer:Triangles)**

**Task:** The object of this exercise is to develop a definition for an acute triangle.

**Procedure:**

- Draw a random acute triangle.
- Measure the sides and the angles of the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Repeat this process with other acute triangles.
- Based on your findings, develop a definition for acute triangle and write it below.

===== *Data* =====

===== *Conjecture* =====

An acute triangle is a triangle which ______________________

______________________________________________

______________________________________________

# Obtuse Triangle

**(using the Geometric Supposer:Triangles)**

**Task:** The object of this exercise is to develop a definition for an obtuse triangle.

**Procedure:**

- Draw a random obtuse triangle.
- Measure the sides and the angles of the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Repeat this process with other obtuse triangles.
- Based on your findings, develop a definition for obtuse triangle and write it below.

===== *Data* ============================================================

===== *Conjecture* ======================================================

An obtuse triangle is a triangle which ______________________________

______________________________________________________________________

______________________________________________________________________

# Isosceles Triangle

**(using the Geometric Supposer:Triangles)**

**Task:** The object of this exercise is to develop a definition for an isosceles triangle.

**Procedure:**

- Draw a random isosceles triangle.
- Measure the sides and the angles of the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Repeat this process with other isosceles triangles.
- Based on your findings, develop a definition for isosceles triangle and write it below.

===== *Data* ==========================================================

===== *Conjecture* ====================================================

An isosceles triangle is a triangle which ______________________________

_____________________________________________________________________

_____________________________________________________________________

# Equilateral Triangle

**(using the Geometric Supposer:Triangles)**

**Task:** The object of this exercise is to develop a definition for an equilateral triangle.

**Procedure:**

- Draw a random equilateral triangle.
- Measure the sides and the angles of the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Repeat this process with other equilateral triangles.
- Based on your findings, develop a definition for equilateral triangle and write it below.

===== *Data* ==========================================================

===== *Conjecture* ====================================================

An equilateral triangle is a triangle which ________________________

____________________________________________________________

____________________________________________________________

## Scalene Triangle

**(using the Geometric Supposer:Triangles)**

**Task:** The object of this exercise is to develop a definition for a scalene triangle.

**Procedure:**

- To draw scalene triangle ABC, make your own new triangle using the SAS option with AB = 6, m∡BAC = 45°, AC = 4.
- Measure the remaining parts of the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Repeat this process for these scalene triangles:
  Use the SSS option with AB = 3, AC = 7.2, BC = 7.8
  Use the ASA option with m∡BAC = 120°, AB = 4, m∡CBA = 20°
  Use the SAS option with AB = 7, m∡BAC = 40°, AC = 6
  Use the SSS option with AB = 5, AC = 8, BC = 9
- Based on your findings, develop a definition for scalene triangle and write it below.

===== *Data* =====

===== *Conjecture* =====

A scalene triangle is a triangle which ______________________________

______________________________________________________________

______________________________________________________________

# Types of Triangles

**(using the Geometric Supposer:Triangles)**

**Task:** The object of this exercise is to determine all the names (right, acute, obtuse, isosceles, equilateral, and scalene) that apply to a given triangle.

**Procedure:**

- Draw your own triangle ABC using the SAS option with AB = 6, m∡BAC = 37°, and AC = 10.
- Measure the remaining parts of the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Based on your findings, name triangle ABC using all the words (right, obtuse, acute, isosceles, equilateral, or scalene) that apply.
- Repeat this procedure for these triangles:
  Use ASA option with m∡BAC = 44°, AB = 4.2, m∡CBA = 92°.
  Use SAS option with AB = 5.2, m∡BAC = 60°, AC = 5.2
  Use SAS option with AB = 6.2, m∡BAC = 12°, AC = 9.4
  Use ASA option with m∡BAC = 45°, AB = 8.5, m∡CBA = 45°.
  Use SSS option with AB = 4, AC = 5, BC = 6.

===== *Data* =====

===== *Conjectures* =====

The first triangle ABC is ______________________

The second triangle ABC is ______________________

The third triangle ABC is ______________________

The fourth triangle ABC is ______________________

The fifth triangle ABC is ______________________

The sixth triangle ABC is ______________________

## Geometry Activity Three
### (Computer)

Objectives: To identify congruent triangles and their corresponding parts.

Preparation: Review the meaning of congruent triangles.

Activity: The students work in groups of two or three following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

Supplies: Geometric Supposer: Triangles computer program.
Apple //e computer (64K) for each group.
Copies of the following page
Pencils.

Summary: After the students have completed the activity sheet, the teacher should check the results for correct notation and correspondences.

## Congruent Triangles I

**(using the Geometric Supposer: Triangles)**

**Task:** The object of this exercise is to identify the corresponding parts of congruent triangles.

**Procedure:**

- Draw a new obtuse $\triangle ABC$.
- Label a random point D outside $\triangle ABC$.
- Label another random point E outside $\triangle ABC$.
- Label the reflection of $\overline{AB}$ in $\overline{DE}$ as $\overline{FG}$. (You may have to scale the triangle to see $\overline{FG}$.
- Label the reflection of $\overline{AC}$ in $\overline{DE}$ as $\overline{HI}$.
- Draw $\overline{GI}$.
- Measure the six parts of $\triangle ABC$ and the six parts of $\triangle GHI$. Record your data.
- Use the repeat feature to perform these same steps on other triangles. Record your data and complete the tables below.

===== *Data* ==========================================================

| obtuse triangle | | acute triangle | | isosceles triangle | | right triangle | |
|---|---|---|---|---|---|---|---|
| $\triangle ABC$ | $\triangle GHI$ | $\triangle ABC$ | $\triangle GHI$ | $\triangle ABC$ | $\triangle GHI$ | $\triangle ABC$ | $\triangle GHI$ |
| $\overline{AB} \cong$ | ________ | $\overline{AC} \cong$ | ________ | $\measuredangle ABC \cong$ | ______ | $\measuredangle BAC \cong$ | _____ |
| $\overline{AC} \cong$ | ________ | $\overline{BC} \cong$ | ________ | $\measuredangle ACB \cong$ | ______ | $\measuredangle ACB \cong$ | _____ |
| $\overline{BC} \cong$ | ________ | $\overline{AB} \cong$ | ________ | $\measuredangle BAC \cong$ | ______ | $\measuredangle ABC \cong$ | _____ |
| $\measuredangle ABC \cong$ | _____ | $\measuredangle BAC \cong$ | _____ | $\overline{AB} \cong$ | ________ | $\overline{BC} \cong$ | ________ |
| $\measuredangle ACB \cong$ | _____ | $\measuredangle ACB \cong$ | _____ | $\overline{AC} \cong$ | ________ | $\overline{AC} \cong$ | ________ |
| $\measuredangle BAC \cong$ | _____ | $\measuredangle ABC \cong$ | _____ | $\overline{BC} \cong$ | ________ | $\overline{AB} \cong$ | ________ |

## Geometry Activity Four
## (Computer)

Objectives: To determine the conditions necessary for two triangles to be congruent.

Preparation: Review the definition of congruent triangles and identifying the corresponding parts.

Supplies: Geometric Supposer Triangles computer program.
Apple // computer (64K) for each group.
Copies of the following pages for each group.
Pencils.

Summary: After each activity page, the teacher should summarize the findings and emphasize why the particular conjecture may or may not be taken as a postulate.

## Congruent Triangles II

**(using the Geometric Supposer: Triangles)**

**Task:** The object of this exercise is to investigate what three parts of two triangles must be congruent in order that the two triangles be congruent.

**Task A:** If the angles of one triangle are congruent to the corresponding angles of a second triangle, are the triangles congruent?

**Procedure:**

- Draw your own new △ABC with AB = 8, AC = 6, and BC = 4.
- Measure the three angles of △ABC and record your data.
- Draw another △ABC of your own with AB = 4, AC = 3, and BC = 2.
- Measure the three angles of this new △ABC and record your results.
- Determine if the two triangles you drew are congruent.

===== *Data* ==========================================================

===== *Conjecture* ====================================================

Does the following conjecture appear to be true? Why or why not?

If three angles of one triangle are congruent to the corresponding three angles of another triangle, then the two triangles are congruent.

**Task B:** If the sides of one triangle are congruent to the corresponding sides of a second triangle, are the triangles congruent?

**Procedure:**

- Draw your own △ABC given the three sides AB = 9, AC = 5, and BC = 6.
- Measure the three angles of △ABC and record your data.
- Draw another △ABC of your own with sides AB = 6, AC = 9, and BC = 5.
- Measure the three angles of △ABC and record your data.
- Determine if the two triangles you drew are congruent.
- Repeat the above five steps using a different set of three numbers for the lengths of the three sides.

===== *Data* ==========================================================

===== *Conjectures* =====================================================

Does the following conjecture appear to be true? Why or why not?

If three sides of one triangle are congruent to the corresponding three sides of another triangle, then the two triangles are congruent.

Can you always make a triangle given any three numbers (that fit the screen limitations) for the lengths of the sides? How can you be sure?

**Task C:** If two sides and the included angle of one triangle are congruent to the corresponding two sides and include angle of another triangle, are the two triangles congruent?

**Procedure:**

- Draw your own △ABC using side AB = 8, m ∡BAC = 45°, and side AC = 6.
- Measure the other three parts of △ABC and record your results.
- Draw another △ABC of your own using AB = 6, m ∡BAC = 45°, and side AC = 8.
- Measure the other three parts of △ABC and record your results.
- Determine if the two triangles you drew are congruent.
- Repeat the above five steps using a different set of three numbers for the lengths of the two sides and measure of the included angle.

===== *Data* ===========================================================

===== *Conjectures* ====================================================

Does the following conjecture appear to be true? Why or why not?

If two sides and the included angle of one triangle are congruent to the corresponding two sides and included angle of another triangle, then the two triangles are congruent.

Can you always make a triangle given any three numbers (that fit the screen limitations) for the lengths of the sides and the measure of the included angle? How can you be sure?

**Task D:** If two sides and the angle opposite one them in one triangle are congruent to the corresponding two sides and angle opposite in a second triangle, are the triangles congruent?

**Procedure:**

- Draw your own △ABC using m ∡BAC = $30^0$, side AB = 6, and m ∡CBA = $30^0$.
- Measure the other three parts of △ABC and record your results.
- Draw another △ABC of your own using m ∡BAC = $30^0$, side AB = 6, and m ∡CBA = $90^0$.
- Measure the other three parts of △ABC and record your results.
- Identify that two sides and an angle opposite one of them in the first triangle are congruent to two sides and an angle opposite one of them in the second triangle.
- Determine if the two triangles you drew are congruent.

===== *Data* ==========================================================

====== *Conjectures* ==================================================

Does the following conjecture appear to be true? Why or why not?

If two sides and the angle opposite one of them in one triangle are congruent to the corresponding two sides and opposite angle in a second triangle, then the triangles are congruent.

**Task E:** If two angles and the included side of one triangle are congruent to the corresponding two angles and included side of another triangle, are the triangles congruent?

**Procedure:**

- Draw your own △ABC using m ∡BAC = $45^o$, side AB = 6, and m ∡CBA = $60^o$.
- Measure the other three parts of △ABC and record your results.
- Draw another △ABC of your own using m ∡BAC = $60^o$, side AB = 6, and m ∡CBA = $45^o$.
- Measure the other three parts of △ABC and record your results.
- Determine if the two triangles you drew are congruent.
- Repeat the above five steps using a different set of three numbers for the measures of the angles and the included side.

===== *Data* ==============================================================

===== *Conjectures* ========================================================

Does the following conjecture appear to be true? Why or why not?

If two angles and the included side of one triangle are congruent to the corresponding two angles and included side of another triangle, then the two triangles are congruent.

Can you always make a triangle given any three numbers (that fit the screen limitations) for the measures of the two angles and the included side? How can you be sure?

The statement "If two angles and a side opposite one of them in one triangle are congruent to the corresponding two angles and side opposite in another triangle, then the two triangles are congruent." is just a special case of the above conjecture. Why?

## Geometry Activity Number Five (Computer)

Objective To define the words **altitude**, **median**, **angle bisector**, and **perpendicular bisector**.

Preparation Review the terms and notation for describing the parts of a triangle, such as vertex, side, side opposite (a vertex or angle), angle opposite a side, included angle, and included side.

Activity Students work in groups of two or three using the computer program Geometric Supposer Triangles following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

Supplies. Copies of the following pages for each group.
Geometric Supposer Triangles computer program.
Apple //e computer (64K) for each group.
Pencils.

Summary After the students have completed the activities, they report their findings to the class and discuss them. Definition should be compared and a final (correct) version should be agreed upon by all. The teacher may want to expand on the idea that the angle bisectors, medians, altitudes, and perpendicular bisectors of the sides of any triangle are concurrent. Hopefully, some student will see the relationship between the ratio of the lengths of the sides and the ratio of the lengths of the segments determined by the angle bisector. This is to stress the fact that, in general, the angle bisector does not bisect the opposite side.

## Defining Altitudes

**(using the Geomtric Supposer:Triangles)**

**Task:** The object of this exercise is to formulate the definition of an altitude of a triangle.

**Procedure:**

- Draw an acute △ABC.
- Draw the altitude in △ABC from vertex B.
- Measure ∡ABD, ∡CBD, ∡BDA, and ∡BDC. Record your results.
- Measure the lengths of $\overline{AD}$ and $\overline{CD}$. Record your results.
- Repeat these last three steps for a right triangle, an obtuse triangle, an isosceles triangle, and an equilateral triangle.
- Try drawing altitudes from the other vertices of these triangles.

===== *Data* =====

===== *Conjectures* =====

Complete the following definition:

An altitude of a triangle is

Questions:

1. How many altitudes does a triangle have?

2. Do your drawings indicate that the following statement is true or false?

   The altitudes of a triangle, except for end points, are always inside the triangle.

# Defining Medians

**(using the Geomtric Supposer:Triangles)**

**Task:** The object of this exercise is to formulate the definition of a median of a triangle.

**Procedure:**

- Draw an acute △ABC.
- Draw the median in △ABC from vertex B.
- Measure ∡ABD, ∡CBD, ∡BDA, and ∡BDC. Record your results.
- Measure the lengths of $\overline{AD}$ and $\overline{CD}$. Record your results.
- Repeat these last three steps for a right triangle, an obtuse triangle, an isosceles triangle, and an equilateral triangle.
- Try drawing medians from the other vertices of these triangles.

===== *Data* ========================================================

===== *Conjectures* ====================================================

Complete the following definition:

A median of a triangle is

Questions:

1. How many medians does a triangle have?

2. Do your drawings indicate that the following statement is true or false?

   The medians of a triangle, except for end points, are always inside the triangle.

## Angle Bisectors

**(using the Geometric Supposer: Triangles)**

**Task:** The object of this exercise is to investigate some of the properties of an angle bisector of a triangle.

**Procedure:**

- Draw an acute △ABC.
- Draw the angle bisector of ∡ABC.
- Place a random point E on $\overline{BD}$.
- Measure the distance from E to $\overline{AB}$ and from E to $\overline{BC}$.
- Measure $\overline{AD}$, $\overline{DC}$, $\overline{AB}$, and $\overline{BC}$. Calculate AD/DC and AB/BC.
- Record your data and the drawing in the table below.
- Repeat these steps for each triangle listed below.
- State your conjectures.

===== *Data* =====

| Triangle / Drawing | distances from E to $\overline{AB}$ | E to $\overline{BC}$ | lengths and ratios AD / DC | AB / BC |
|---|---|---|---|---|
| Acute | | | | |
| Obtuse | | | | |
| Right | | | | |
| Isosceles | | | | |
| Equilateral | | | | |

===== *Conjectures* =====

## Defining Perpendicular Bisector

**(using the Geomtric Supposer:Triangles)**

**Task:** The object of this exercise is to formulate the definition of a perpendicular bisector of a segment.

**Procedure:**

- Draw an acute △ABC.
- Erase segment $\overline{AC}$. Erase segment $\overline{BC}$. Erase Label C.
- Draw a perpendicular bisector of AB defined by a segment eight units (u) long.
- Measure ∡EFB, ∡EFA, ∡BFD, and ∡AFD. Record your results.
- Measure the lengths of $\overline{BF}$ and $\overline{AF}$. Record your results.
- Draw another perpendicular bisector of $\overline{AB}$ defined by a segment ten units long.
- Repeat the above steps by starting with a different triangle.

===== *Data* ==========================================================

===== *Conjectures* ===================================================

Complete the following definition:

A perpendicular bisector of a segment is

Questions:

1. In a given plane, how many different lines can be a perpendicular bisector of a segment?

2. If we are not restricted to a plane, how many different lines can be a perpendicular bisector of a segment?

## Geometry Activity Number Six
## (Computer)

Objective. To test conjectures by looking for examples and counter-examples while simultaneously reinforcing the notions of angle bisectors, altitudes, medians, and perpendicular bisectors.

Preparation: Review the definitions of angle bisector, altitude, median, and perpendicular bisector. The students need to understand the difference between example and counter-example.

Activity: Students work in groups of two or three using the program Geometric Supposer: Triangles following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement.

Supplies. Copies of the following pages for each group.
Geometric Supposer: Triangles computer program.
Apple //e computer (64K) for each group
Pencils.

Summary: After the students have completed the activities, they report their findings to the class and discuss them. The discussion component of these activities is very important. The students should be encouraged to justify ("prove") why their example or counter-example does what it is suppose to do. When using this set of activities, the teacher may find it useful for the students to collect all their examples and counter-examples while at the computer one class period and then discuss the findings at the next meeting

## Altitudes, medians, angle bisectors, and perpendicular bisectors

**(using the Geometric Supposer: Triangles)**

**Task:** To collect data to support conjectures and to evaluate conditions under which a statement is true.

**Procedure:**

- The conjectures below are **always true** (A), **sometimes true** (S), or **never true** (N).
- Read each conjecture and place the appropriate letter (A, S, or N) next to each statement.
- Justify your answer using one of the following procedures:
  - if you think a conjecture is always true (A), provide two examples.
  - if you think a conjecture is sometimes true (S), provide two examples (one true and one false).
  - if you think a conjecture is never true (N), provide two examples.
- Be sure that your examples contain the appropriate measurements.

===== *Conjectures* =====================================================

1. An altitude of a triangle bisects a side of the triangle.

2. A median of a triangle is perpendicular to the opposite side.

3. An altitude of a triangle is an angle bisector of the triangle.

4. A median of a **scalene** triangle is an altitude of this triangle.

5. A random point on a perpendicular bisector of a segment is equidistant from the end points of the segment.

6. A median of a triangle is an angle bisector of the triangle.

7. An angle bisector in a **scalene** triangle bisects the opposite side.

8. Portions of the altitudes of a right triangle lie outside the right triangle.

9. Except for the endpoints, the altitudes of an isosceles triangle lie inside the triangle.

10. The altitude from the vertex of a right triangle divides the original triangle into two congruent right triangles.

## Geometry Activity Number Seven
## (Computer)

Objective. To identify congruent triangles using the appropriate congruency postulates and theorems

Preparation: Review the postulates and theorems the students have encountered for establishing the congruency of triangles. Remind them that they need not check the congruency of all the corresponding parts in order to show that two triangles are congruent.

Activity: Students work in groups of two or three using the computer program Geometric Supposer: Triangles following the directions on the activity sheets. The teacher should visit each group to offer guidance and encouragement

Supplies: Geometric Supposer: Triangles computer program.
Apple // computer (64K) for each group
Copies of the following pages for each group
Pencils.

Summary: After the students have completed the activities, they report their findings to the class and discuss them. The students should note that different postulates (or theorems) may have been used by their classmates to establish the congruencies. Also, they should note that some students did not have to actually measure the angles and the sides They were able to determine some of the values from knowledge about special kinds of triangles (e.g., in an equilateral triangle, the measure of each angle is 60 degrees) All these activity pages do not need to be completed in one period Each activity page can "stand alone."

## Finding Congruent Triangles

**(using the Geometric Supposer: Triangles)**

**Task:** The object of this exercise is to identify congruent triangles using the appropriate congruency postulates and theorems.

**Procedure:**

- Draw an acute △ABC. (Scale it to get a large triangle.)
- Label the midpoint, D, of side AB.
- Label the midpoint, E, of side BC.
- Label the midpoint, F, of side AC.
- Draw △EFD.
- Measure the necessary components, record your drawing and data, and name all the congruent triangles.
- Repeat for an obtuse triangle.
- Repeat for a right triangle.

===== *Drawings and Data* ==========================================

| Acute Triangle | Obtuse Triangle | Right Triangle |
|---|---|---|

===== *Conjecture* ==========================================

**Task:** The object of this exercise is to identify congruent triangles using the appropriate congruency postulates and theorems.

**Procedure:**

- Draw an equilateral $\triangle ABC$. (Scale it to get a large triangle.)
- Draw the median of $\triangle ABC$ from vertex A.
- Draw the median of $\triangle ABC$ from vertex B.
- Label the intersection of these two medians (point F).
- Erase segments FE and DF.
- Draw CF.
- Measure the necessary components, record your drawing and data, and identify all congruent triangles.
- Repeat for an isosceles triangle.
- Repeat for a scalene triangle with sides 4, 5, and 6.

===== *Drawings and data* ==========================================

Equilateral Triangle　　　Isosceles Triangle　　　Scalene Triangle

===== *Conjecture* ==========================================

**Task:** The object of this exercise is to identify congruent triangles using the appropriate congruency postulates and theorems.

**Procedure:**

- Draw an equilateral △ABC. (Scale it to get a large triangle.)
- Divide segment BC into three congruent segments (points D and E).
- Draw AD and AE.
- Measure the necessary components, record your drawing and data, and name all the congruent triangles.
- Repeat for an isosceles triangle.
- Repeat for a scalene triangle (you choose the lengths for the sides.)

===== *Drawings and data* ==========================================

Equilateral Triangle    Isosceles Triangle    Scalene Triangle

===== *Conjecture* ==================================================

**Task:** The object of this exercise is to identify congruent triangles using the appropriate congruency postulates and theorems.

**Procedure:**

- Draw a right △ABC. (Scale it to get a large triangle.)
- Locate the midpoint, D, of side AB.
- Locate the midpoint, E, of side BC.
- Reflect (found under "Label") BC in ED.
- Reflect AC in ED.
- Draw HF.
- Measure the necessary components, record your drawing and data, and name all the congruent triangles.
- Repeat for an isosceles triangle.
- Repeat for an equilateral triangle.

===== *Drawings and Data* ==========================================

Right Triangle | Isosceles Triangle | Equilateral Triangle

===== *Conjecture* ==================================================

# APPENDIX C

## LETTERS AND FORMS

Mathematics Department
Western Montana College
Dillon, MT 59725
<DATE>

Dear IMPACT or Mathematics Technology Workshop Participant:

Since you have indicated an eagerness to be involved in innovative mathematics education projects by your participation in IMPACT or this workshop, we are seeking your help in a study we will be conducting during the fall of 1990 with high school geometry teachers and students in the state of Montana. (This is the same study some of you piloted last fall.) I am conducting this study as partial fulfillment of the Doctorate Degree under the direction of Lyle Andersen, Bill Hall, and Maurice Burke.

The purpose of this study is to determine if different instructional methods for a unit on congruent triangles in the high school geometry course (paper and pencil group activities, computer activities using the Geometric Supposers, or the traditional approach) have any effect on student achievement or attitude.

In order to participate in this project, you must be teaching two or more sections of high school geometry during the 1990 fall term.

The responsibility of the selected teachers and schools will be as follows:

1. The teacher agree to teach the four-week unit following a detailed unit plan using one of the instructional methods in each geometry class.
2. The teacher will administer the various testing instruments at four different times during the year.
3. The teacher must complete a brief, daily log during the time the unit is presented.
4. Permission to conduct this study will have to be granted through the local school administrative personnel.

All written materials (other than the student textbook) will be provided by the project.

If you are interested in participating in this project, please contact me as soon as possible. A short training session for the participating teachers will be held during the July Math Tech Workshop at MSU.

You can contact me by writing

E. Otis Thompson
Mathematics Dept.
Western Montana College
Dillon, MT 59725

or by calling 1-800-WMC-MONT or by leaving a message for OTIS THOMPSON on Goliath.

Sincerely yours,

E. Otis Thompson
Asst. Professor of Mathematics

Mathematics Department
Western Montana College
Dillon, MT 59725
<CURRENT DATE>

<PRINCIPAL OR SUPERINTENDENT>
<ADDRESS>

Dear <PRINCIPAL OR SUPERINTENDENT>

<TEACHER NAME> has volunteered to participate in a study that I am conducting that involves using different instructional methods in teaching geometry to high school students. I am conducting this study as partial fulfillment of the Doctorate Degree I am pursuing at Montana State University and I am requesting your permission to allow <TEACHER NAME> and your school to participate.

Before outlining <TEACHER NAME>'s responsibilities if <HE/SHE> is allowed to participate, let me give you an overview of this project. The purpose of this study is to investigate the effect that three different methods of classroom instruction have on achievement of certain geometry concepts, retention of these concepts, and attitude toward geometry for students enrolled in the high school geometry course. In addition, this study will determine if gender, socio-economic background, van Hiele level of geometrical thinking, and attitude toward mathematics (in general) have any effect on these three outcomes.

The first method of instruction will introduce geometric concepts to the students by having them work in small groups doing paper and pencil activities (designed by me). These activities follow the first three phases (information, guided orientation, and explicitation) described in the van Hiele theory on how students learn geometry. The last two phases (free orientation and integration) of the van Hiele theory will be completed by using the exercises in the students' textbook. The second method of instruction will introduce these same geometric concepts to the students in another class by having them work in small groups using the computer and the Geometric Supposer software together with activities designed by me for the first three phases. Again, the last two phases will be completed using the students' textbook. In the third method of instruction, these same concepts will be introduced to the students in a third class using just the textbook materials in a "typical" whole class format.

The materials that I am designing for use in the classroom for the first two groups will be for Chapter 3 Congruent Triangles in your students' <PUBLISHING COMPANY> text. There will be ten such activities that match ten of the objectives of this unit. Each activity will take one or two class periods to complete. These are designed to replace the "whole class-teacher presentation" format usually used. If <TEACHER NAME> is allowed to participate, I would randomly assign one of <HIS/HER> geometry classes to the paper and pencil treatment group, another class to the computer treatment group, and a third class to the textbook whole class format. The teaching methods will be used during this entire unit which should last approximately five weeks.

<TEACHER NAME>'s responsibilities to me would be to administer an entry level geometry test and attitude toward mathematics survey to <HIS/HER> students at the start of the school year. Then just before <HE/SHE> begins Chapter 3 of the text, <HE/SHE> will administer a pretest and an attitude toward geometry survey to <HIS/HER> students. Then, during the course of teaching Chapter 3, <HE/SHE> will incorporate the use of the materials I have developed for this unit that replace the text material. At the completion of the unit, <TEACHER NAME> will administer a post-test and the attitude survey again. Then <TEACHER NAME> can return to <HIS/HER> own method of teaching. To determine how well the students retained this material and to see if their attitudes toward geometry has changed after a four week time period, a retention test and attitude survey will be administered one last time. Each of these assessment instruments will take the students less than 30 minutes to complete.

If you agree to allow <TEACHER NAME> to participate, parental permission to allow their children to participate will also be requested. I will also need some socio-economic background on the students that participate. Of course, the name and background information on each student will be kept strictly confidential.

I thank you in advance for considering this request and I look forward to working with <TEACHER NAME> and <HIS/HER> students in this study.

Sincerely yours,

E. Otis Thompson
Assistant Professor
of Mathematics

Mathematics Department
Western Montana College
of the University of Montana
Dillon, MT 59725
<DATE>

Dear Parent:

Mathematics educators are always searching for methods of instruction that may produce better understanding of mathematical concepts by the student. I am presently conducting a study to investigate the effect that three different methods of classroom instruction will have on achievement of certain geometry concepts, retention of these concepts, and attitude toward geometry for students enrolled in the high school geometry course.

<THE ADMINISTRATION AT THE HIGH SCHOOL> has agreed to allow the geometry students to participate and <TEACHER> has agreed to use these materials in <HIS/HER> classes. I am now requesting your permission to allow <TEACHER> to share with me some demographic and assessment data <HE/SHE> will be collecting on your child who is in <HIS/HER> geometry class. <TEACHER> will be administering an assessment instrument at four different times during the course of this study. The assessment instruments are not tests that a student might pass or fail, but simply instruments to assess the progress a student is making using certain instructional methods in class. The information collected on each student will be kept strictly confidential.

If you are willing to allow your child to participate in this study, please fill out the permission form below and return it to <TEACHER> as soon as possible. If you need further information concerning this study, you may call 683-7011 and ask for me. If I cannot be reached at that time, the operator will take your name and number and I will return your call. Thank you for your consideration.

Sincerely yours,

E. Otis Thompson
Assistant Professor of
Mathematics

*************************************************************

Geometry Study Permission Form

Student Name ______________________________

Address ____________________________________

I hereby give permission for my child to participate in the geometry study being conducted at his/her school. I also understand that the name, demographic data, and assessment data given by my child will be kept strictly confidential.

Parent's Signature ___________________________

Date _______________

Mathematics Department
Western Montana College
of the University of Montana
Dillon, MT 59725
<DATE>

Dear parent or guardian of ______________________________:

Last fall, you gave permission for your child to participate in a geometry study that I am conducting on how different methods of classroom instruction affect student achievement of certain geometry concepts. The classroom work has been completed and all the assessment instruments have been given. I now need one piece of confidential information from you to complete this study.

In educational research, social stratification can be an important factor in prescribing appropriate educational methodology for students. To determine if this variable is pertinent, I need to classify each participant in this study into a socio-economic level. I am using three classifications for participating students; namely, (1) the student receives free school lunch, (2) the student receives reduced-price school lunch, and (3) the student receives neither free nor reduced-price school lunch. Government regulations stipulate that only the parent or guardian of the student can provide me with this confidential information.

If you would be so kind as to provide me with this information, I assure you that it will be kept strictly confidential. No names are used in the study and the student is identified only by a code number in compiling the data. Simply check the appropriate box in the form below and remove it from this letter. Then have your child return the form to school and place it in the envelope provided by the child's teacher. The teacher will forward these forms to me.

I thank you for allowing your child to participate in this study and for providing this confidential information to me.

Sincerely yours,

E. Otis Thompson
Assistant Professor of Mathematics

- - - - - - - - - - - - - - - cut here - - - - - - - - - - - - - - - - - -

Student Number ________

Please check the appropriate category for this student.

This student receives free school lunch.

This student receives reduced-price school lunch.

This student receives neither free nor reduce-price school lunch.

I do not wish to release this information.

# APPENDIX D

## CDASSG VAN HIELE GEOMETRY TEST AND AIKEN-DREGER ATTITUDE OPINIONNAIRES

Western Montana College
of the University of Montana
Dillon, MT 59725
9 June 1988

Dr. Zalman Usiskin
Department of Education
University of Chicago
5835 S. Kimbark Avenue
Chicago, IL 60637

Dear Dr. Usiskin:

As partial fulfillment of the requirements for the degree Doctor of Education in the School of Education at Montana State University, I am in the process of developing a study involving high school geometry students in Montana. The purpose of this study is to determine if different instructional methods used in the geometry classroom have any effect on the achievement or attitude of these students.

I plan to use the levels of understanding in geometry developed by the van Hieles as one of the predictor variables in my study. In the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project, in which you were the principal investigator, a test was developed to determine the van Hiele level of a student. I am seeking your permission to reproduce this test to use in my study.

If permission is granted to use this instrument, it would be administered to approximately 300 geometry students in Montana high schools at the beginning of the school term this fall.

Thank you for considering this request. I anxiously await your reply.

Sincerely yours,

E. Otis Thompson
Assistant Professor
of Mathematics

UNIVERSITY OF CHICAGO
Department of Education
5835 S. Kimbark Avenue
Chicago, IL 60637

July 13, 1988

Professor E. Otis Thompson
Department of Mathematics
Western Montana College
Dillon, MT 59725

Dear Professor Thompson:

I am embarrassed to have taken so long to respond to your letter. Your letter became buried on my desk; I simply lost track of it.

I assume you have (or have access to) a copy of the 219-page report in which the various options for grading of the van Hiele test are described.

We customarily give permission for the van Hiele and other tests developed by the CDASSG project provided that a description of the study is given (this has been done in your letter, though it would be interesting to know what "different instructional methods" you are studying) and with the assurance that we will receive a copy of any written report using results of the test.

If you agree with those conditions, I am happy to give permission for the test to be duplicated. Again I apologize for the delay in my response.

Sincerely,

Zalman Usiskin
Professor of Education

# VAN HIELE GEOMETRY TEST
## ANSWER SHEET

Name________________________________________ Class Period________
last first middle

Test Date ____________ School____________________________
mo/day/yr

Cross out the correct answer

space for drawing or figuring
(You may also use the other side)

| | | | | | |
|---|---|---|---|---|---|
| 1. | A | B | C | D | E |
| 2. | A | B | C | D | E |
| 3. | A | B | C | D | E |
| 4. | A | B | C | D | E |
| 5. | A | B | C | D | E |
| 6. | A | B | C | D | E |
| 7. | A | B | C | D | E |
| 8. | A | B | C | D | E |
| 9. | A | B | C | D | E |
| 10. | A | B | C | D | E |
| 11. | A | B | C | D | E |
| 12. | A | B | C | D | E |
| 13. | A | B | C | D | E |
| 14. | A | B | C | D | E |
| 15. | A | B | C | D | E |
| 16. | A | B | C | D | E |
| 17. | A | B | C | D | E |
| 18. | A | B | C | D | E |
| 19. | A | B | C | D | E |
| 20. | A | B | C | D | E |
| 21. | A | B | C | D | E |
| 22. | A | B | C | D | E |
| 23. | A | B | C | D | E |
| 24. | A | B | C | D | E |
| 25. | A | B | C | D | E |

# VAN HIELE GEOMETRY TEST

## directions

Do not begin this test until you are told to do so.

This test contains 25 questions. It is not expected that you know everything on this test.

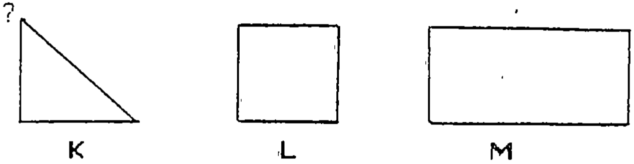
When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on the test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.

## VAN HIELE GEOMETRY TEST

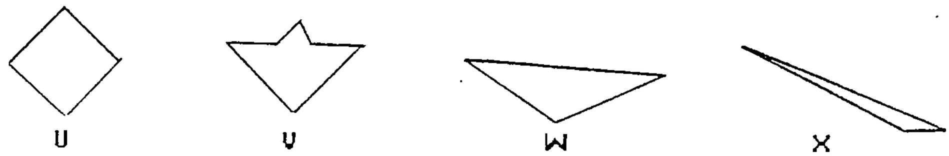
**PLEASE DO NOT WRITE ON THIS TEST!**

1. Which of these are squares?



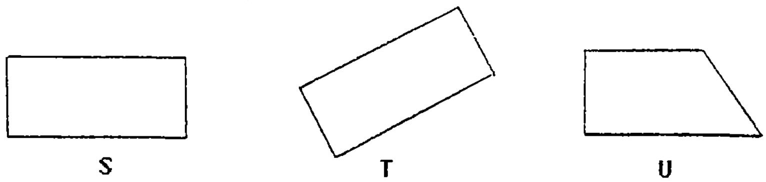

(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares

2 Which of these are triangles?



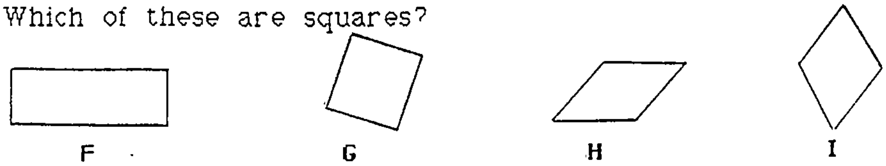

(A) None of these are triangles
(B) V only
(C) W only
(D) W and X only
(E) V and W only

3 Which of these are rectangles?



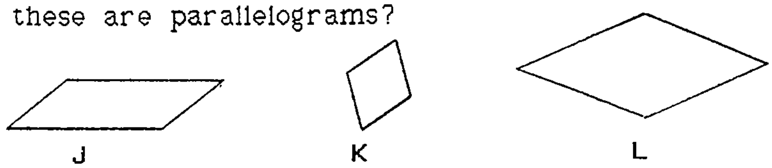

(A) S only
(B) T only
(C) S and T only
(D) S and U only
(E) All are rectangles

4. Which of these are squares?



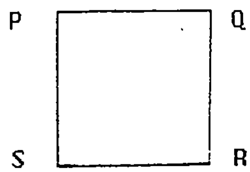

(A) None of these are squares
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares

5. Which of these are parallelograms?



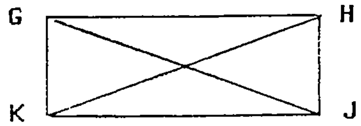

(A) J only
(B) L only
(C) J and K only
(D) None of these are parallelograms
(E) All are parallelograms

6. PQRS is a square.
Which relationship is true in all squares?



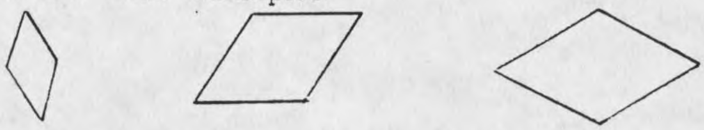

(A) $\overline{PR}$ and $\overline{RS}$ have the same length
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular
(D) $\overline{PS}$ and $\overline{QS}$ have the same length
(E) Angle Q is larger than angle R

7 In a rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are diagonals




Which of (A) through (D) is <u>not</u> true in <u>every</u> rectangle?
(A) There are four right angles
(B) There are four sides
(C) The diagonals have the same length
(D) The opposite sides have the same length
(E) All of (A) through (D) are true in every rectangle

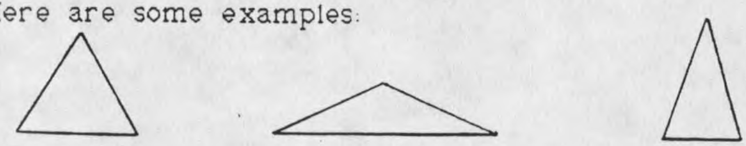
8. A <u>rhombus</u> is a 4-sided figure with all sides of the same length. Here are some examples:



Which of (A) through (D) is <u>not</u> true in every rhombus?

(A) The two diagonals have the same length
(B) Each diagonal bisects two angles of the rhombus
(C) The two diagonals are perpendicular
(D) The opposite angles have the same measure
(E) All of (A) through (D) are true in every rhombus

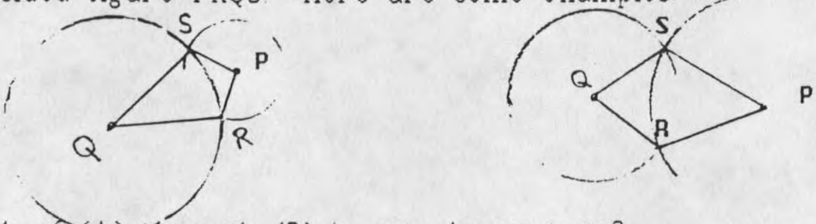
9. An isosceles triangle is a triangle with two sides of equal length. Here are some examples:



Which of (A) throught (D) is true in every isosceles triangle?

(A) The three sides must have the same length
(B) One side must have twice the length of another side
(C) There must be at least two angles with the same measure
(D) The three angles must have the same measure
(E) None of (A) through (D) is true in every isosceles triangle

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are some examples:




Which of (A) through (D) is <u>not</u> always true?

(A) PRQS will have two pairs of sides of equal length
(B) PRQS will have at least two angles of equal measure
(C) The lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ will be perpendicular
(D) Angles P and Q will have the same measure
(E) All of (A) through (D) are true

11. Here are two statements.

    Statement 1: Figure F is a rectangle
    Statement 2: Figure F is a triangle

    Which is correct?

    (A) If 1 is true, then 2 is true.
    (B) If 1 is false, then 2 is true.
    (C) 1 and 2 cannot both be true.
    (D) 1 and 2 cannot both be false.
    (E) None of (A) through (D) is correct.

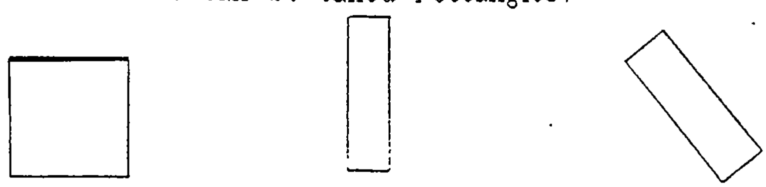
12. Here are two statements.

    Statement S: △ABC has three sides of the same length.
    Statement T: In △ ABC, ∠ B and ∠ C have the same measure.

    Which is correct?

    (A) Statement S and T cannot both be true.
    (B) If S is true, then T is true.
    (C) If T is true, then S is true.
    (D) If S is false, then T is true.
    (E) None f (A) through (D) is correct.

13. Which of these can be called rectangles?



P Q R

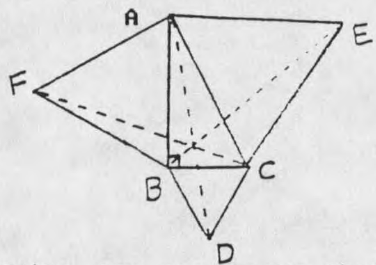
    (A) All can
    (B) Q only
    (C) R only
    (D) P and Q only
    (E) Q and R only

14. Which is true?

    (A) All properties of rectangles are properties of squares.
    (B) All properties of squares are properties of rectangles.
    (C) All properties of rectangles are properties of parallelograms.
    (D) All properties of squares are properties of parallelograms.
    (E) None of (A) through (D) is true.

15. What do all rectangles have that some parallelograms do not have?
(A) opposite sides equal
(B) diagonals equal
(C) opposite sides parallel
(D) opposite angles equal
(E) none of (A) through (D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.




From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?
(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
(B) In some, but not all, right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
(C) In any right triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
(D) In any triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.
Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?
(A) D implies S which implies R.
(B) D implies R which implies S.
(C) S implies R which implies D.
(D) R implies D which implies S.
(E) R implies S which implies D.

18. Here are two statements:

    I. If a figure is a rectangle, then its diagonals bisect each other.

    II. If the diagonals of a figure bisect each other, then the figure is a rectangle

    Which is correct?

    (A) To prove I is true, it is enough to prove that II is true.
    (B) To prove II is true, it is enough to prove that I is true.
    (C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.
    (D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
    (E) None of (A) through (D) is correct.

19. In geometry:

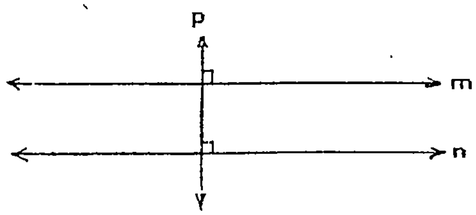
    (A) every term can be defined and every true statement can be proved true.
    (B) every term can be defined but it is necessary to assume that certain statements are true.
    (C) Some terms must be left undefined but every true statement can be proved true.
    (D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.
    (E) None of (A) through (D) is correct

20 Examine these three sentences.

    (1) Two lines perpendicular to the same line are parallel.
    (2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.
    (3) If two lines are equidistant, then they are parallel.

    In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

    (A) (1) only
    (B) (2) only
    (C) (3) only
    (D) Either (1) or (2)
    (E) Either (2) or (3)



21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are {P, Q}, {P, R}, {P, S}, {Q, R}, {Q, S}, and {R, S}.

. P

Q .

R . . S

Here are how the words "intersect" and "parallel" are used in F-geometry. The line {P, Q} and {P, R} intersect at P because {P, Q} and {P, R} have P in common. The lines {P, Q} and {R, S} are parallel because they have no points in common.
From this information, which is correct?
(A) {P, R} and {Q, S} intersect.
(B) {P, R} and {Q, S} are parallel.
(C) {Q, R} and {R, S} are parallel.
(D) {P, S} and {Q, R} intersect.
(E) None of (A) through (D) is correct.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?
(A) In general, it is impossible to bisect angles using only a compass and unmarked ruler.
(B) In general, it is impossible to trisect angles using only a compass and marked ruler.
(C) In general, it is impossible to trisect angles using any drawing instruments.
(D) It is still possible that in the future someone may find a general way to trisect angles using only a compass and unmarked ruler.
(E) No one will ever be able to find a general method for trisecting angles using only a compass and unmarked ruler

23. There is a geometry invented by a mathematician J in which the following is true:

> The sum of the measures of the angles of a triangle is less than 180 degrees.

Which is correct?

(A) J made a mistake in measuring the angles of the triangle.
(B) J made a mistake in logical reasoning.
(C) J has a wrong idea of what is meant by "true."
(D) J started with different assumptions than those in the visual geometry.
(E) None of (A) through (D) is correct.

24. Two geometry books define the word rectangle in different ways. Which is true?

(A) One of the books has an error.
(B) One of the definitions is wrong. There cannot be two different definitions for rectangle.
(C) The rectangle in one of the books must have different properties from those in the other book.
(D) The rectangle in one of the books must have the same properties as those in the other book.
(E) The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I. If p, then q.
II. If s, then not q.

Which statement follows from statements I and II?

(A) If p, then s.
(B) If not p, then not q.
(C) If p or q, then s.
(D) If s, then not p.
(E) If not s, then p.

## MATHEMATICS OPINIONNAIRE

name______________________ School______________ Period____ Date______

Directions: Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to express on a five-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), and Strongly Agree (SA). You are to circle the letter which best indicates how closely you agree or disagree with the feeling expressed to each statement as it concerns you.

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|---|---|---|
| 1. | I do not like mathematics. I am always under a terrible strain in the mathematics class. | SD D U A SA |
| 2. | I do not like mathematics, and it scares me to have to take it. | SD D U A SA |
| 3. | Mathematics is very interesting to me. I enjoy this mathematics course. | SD D U A SA |
| 4. | Mathematics is fascinating and fun. | SD D U A SA |
| 5. | Mathematics makes me feel secure, and at the same time it is stimulating. | SD D U A SA |
| 6. | I do not like mathematics. My mind goes blank and I am unable to think clearly when doing mathematics. | SD D U A SA |
| 7. | I feel a sense of insecurity when attempting mathematics. | SD D U A SA |
| 8. | Mathematics makes me feel uncomfortable, restless, irritable, and impatient. | SD D U A SA |
| 9. | The feeling that I have toward mathematics is a good feeling. | SD D U A SA |
| 10. | Mathematics makes me feel as though I'm lost in a jungle of numbers and I can't find my way out. | SD D U A SA |
| 11. | Mathematics is something which I enjoy a great deal. | SD D U A SA |
| 12. | When I hear the word mathematics, I have a feeling of dislike. | SD D U A SA |
| 13. | I approach mathematics with a feeling of hesitation--hesitation resulting from a fear of not being to do mathematics. | SD D U A SA |
| 14. | I really like mathematics. | SD D U A SA |
| 15. | Mathematics is a course in school which I now like and enjoy studying. | SD D U A SA |
| 16. | I don't like mathematics. It makes me nervous to even think about having to do a mathematics problem. | SD D U A SA |
| 17. | I do not like mathematics, and it is my most dreaded subject. | SD D U A SA |
| 18. | I love mathematics. I am happier in the mathematics class than in any other class. | SD D U A SA |
| 19. | I feel at ease in mathematics, and I like it very much. | SD D U A SA |
| 20. | I feel a definite positive reaction to mathematics; it is enjoyable. | SD D U A SA |