Behavior of an MHD generator operating around the critical point
by Conwell James Dickey

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Electrical Engineering
Montana State University

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A summary of the current theory of steady, state MHD flow and its consequences with regard to choking was then given. A study of transient, choked MHD flow was then presented, using the previously developed numerical model, and a comparison of steady and transient flow was given. Finally, a possible means of inferring the internal state of an MHD generator, based on terminal characteristics, was introduced.

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Chairman, Examining Committee

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LIST OF SYMBOLS

Symbol		Page First Encountered
a	speed of sound, meters/second	2
A	channel cross-sectional area, square meters	14
$\underline{B}$	magnetic flux density, webers/square meter	7
B, B_z	z-component of magnetic flux density, wb/sq m	10
C	Duct circumference, meters	14
c	characteristic speed, meter/second	21
C_f	friction factor	14
C_p	specific heat at constant pressure	15
C_v	specific heat at constant volume	16
$\underline{E}$	Electric field, volts/meter	7
e	specific internal energy, N/m^2s	8
$\underline{E}'$	rest frame electric field, volts/meter	8
E_s	stagnation internal energy, N·m/kg·s	13
E_x	x-component of electric field, volts/ meter	68
E_y	y-component of electric field, volts/ meter	68
E_z	z-component of electric field, volts/ meter	68
F	friction force per unit volume, N/sq m	12
$\underline{F}(\underline{U})$	three element vector	18
$\underline{G}(\underline{U})$	three element vector	18
$\underline{G}^*(\underline{U})$	three element vector	23
$\underline{H}(\underline{U})$	three element vector	23

Symbol		Page First Encountered
h	electrode walls separation, meters	11
I	current, amperes	54
$\underline{i}$	unit vector in x-direction	7
$\underline{J}$	current density, amperes/square meter	7
$\underline{J}'$	rest frame current density, amperes/ square meter	8
$\underline{j}$	unit vector in y-direction	7
J_x	x-component of current density, amperes/ square meter	68
J_y	y-component of current density, amperes/ square meter	68
J_z	z-component of current density, amperes/ square meter	68
JP	a ratio	59
$(\underline{J}\times\underline{B})_x$	x-component of $\underline{J}\times\underline{B}$, newtons/ cubic meter	13
K	loading factor or parameter	14
$\underline{k}$	unit vector in z-direction	7
L	channel length, meters	11
ℓ	insulator walls separation, meters	14
M	Mach number	2
m	momentum density, kg/ square meters·seconds	13
N_{st}	Stanton number	15
P	pressure, newtons/ square meter	7
P_e	exit pressure, newtons/ square meter	26
P_i	inlet pressure, newtons/ square meter	57
P_o	stagnation pressure, newtons/ square meter	26

Symbol		Page First Encountered
P^*	critical pressure, newtons/ square meter	27
Q	heat loss per unit volume	12
q_w	heat transfer to the walls, Joules/ meter·second	71
R	ideal gas constant, N·m/mole·^oK	8
Re	Reynolds number	14
R_i	internal resistance, Ohms	72
R_L	load resistance, Ohms	14
r_{cp}	critical pressure ratio	28
r_o	operating pressure ratio	28
T	temperature, ^{o}Kelvin	8
T_e	exit temperature, ^{o}Kelvin	27
T_o	stagnation temperature, ^{o}Kelvin	27
T_w	wall temperature, ^{o}Kelvin	15
t	time, seconds	6
$\underline{U}$	three element vector	18
u	x-component of velocity, meter/ second	2
u_e	exit velocity, meters/ second	27
V	voltage, volts	54
$\underline{V}$	vector velocity, meters/ second	6
v	y-component of velocity, meters/ second	7
V_{oc}	open-circuit voltage, volts	73
w	z-component of velocity, meters/ second	7

Symbol		Page First Encountered
α	percent of ionization	8
β	Hall parameter	68
γ	specific heat ratio	16
Δt	differential time step	19
Δx	differential x step	14
η	viscosity, poise	14
κ_T	thermal conductivity, Joules$\cdot{}^{o}$K/second$\cdot$meter	8
μ	mobility	68
$\bar{\mu}_o$	mean molecular weight, kg/mole	8
ρ	mass density, kg/cubic meter	6
ρ_e	charge density, Coulombs/cubic meter	7
σ	conductivity, mhos/meter	9
$\overleftrightarrow{\tau}'$	shear stress	7
τ_W	shear stress at wall	14
Φ	mechanical dissipation function	8
Ψ	gravitational potential	7

ABSTRACT

The defining equations for MHD flow were presented. A numerical means of approximate solution of these equations was developed. A summary of the current theory of steady state MHD flow and its consequences with regard to choking was then given. A study of transient, choked MHD flow was then presented, using the previously developed numerical model, and a comparison of steady and transient flow was given. Finally, a possible means of inferring the internal state of an MHD generator, based on terminal characteristics, was introduced.

CHAPTER I

1.1 Introduction

Magnetohydrodynamics (MHD), as a method of energy conversion, has recently been receiving much attention due to several attractive features. MHD offers a complete lack of moving parts in the generator as well as direct thermal to electrical energy conversion and, if the MHD generator is coupled with a steam bottoming plant, efficiencies approaching 60 percent are predicted for first generation systems. These features, as well as others, are sufficient reason to continue research leading toward the eventual development of MHD as a usable means of energy conversion.

The theory behind MHD has been available since the time of Faraday, when he stated his well-known principle of magnetic induction. This principle, stated simply, says that if a conductor is moved through a magnetic field, then a current will be induced in the conductor, such that its direction of flow is perpendicular to both the direction of movement of the conductor and the direction of the magnetic field. In the case of MHD, the conductor is a fluid which is heated to such a degree that it becomes a conductor through ionization. The fluid is then forced down a duct such that the direction of flow is perpendicular to an applied magnetic field. Then, by appropriate placement of pairs of electrodes on the duct walls, electrical energy can be extracted. This description of the MHD energy conversion process is an oversimplification but will suffice until the problem is more rigorously formulated in Chapter II. For the reader who is interested in the auxilary components necessary to operate an MHD facility, Rosa (1968) is an excellent source for introductory study.

When the governing equations for an MHD generator are developed in Chapter II, it will be seen that the equations exhibit an interesting characteristic. When the fluid velocity exceeds the local speed of sound (i.e., the flow is supersonic), the equations are hyperbolic, while if the fluid velocity is less than the local speed of sound (i.e., the flow is subsonic), the equations are elliptic (Hughes, 1966). This change of form would seem to indicate that any flow which is transonic might exhibit special behavior at the sonic point. This is indeed the case, and will be more rigorously defended later. To simplify the discussion, it is usual to define a Mach number, M, as

$$M = u/a \tag{1.1}$$

where u is the fluid velocity and a is the local speed of sound. Then for subsonic flow, M is less than one, and for supersonic flow, M is greater than one.

To understand the special behavior of the MHD flow at M = 1, it is necessary to understand the significance of the speed of sound. Sound propagates as pressure disturbances and the sonic speed is actually the speed of propagation of these pressure disturbances.

For a fluid with some given velocity, the velocity with which a pressure disturbance will propagate upstream is given by

$$a - u \tag{1.2}$$

If $u < a$, $(M < 1)$, then (1.2) assumes a positive value, and pressure disturbances are able to affect the flow upstream of their occurrence. However, if $u \geq a$, $(M \geq 1)$, (1.2) assumes a nonpositive value and pressure disturbances are unable to affect the upstream flow conditions. With this in mind, it is obvious why flow for $M = 1$ exhibits such special behavior. In fact, the behavior is so special that the $M = 1$ state is usually called the critical state, and in the absences of special conditions (usually the absence of a throat at the critical point), the flow is said to be choked when it reaches its critical point. The critical point has yet another significant property, however, in that it is the axis of mirror symmetry for the flow properties. That is to say, for a given MHD channel configuration, the flow of $M < 1$ will have a mirror symmetry with the flow for $M > 1$. For instance, a subsonic diffuser will act as a nozzle for supersonic flow. This is discussed in much more depth by Shapiro (1953). Deeper study into this symmetry will show, in fact, that a generator designed to be operated with subsonic (supersonic) flow will not operate properly with supersonic (subsonic) flow. Because of this, and the mirror symmetry, it is imperative that the critical state be avoided at all points in the channel if at all possible. Based on the preceeding discussion, we are now able to define the problem which this thesis will attempt to examine.

Since it is desirable to avoid choking in the generator, this thesis will attempt to relate the terminal characteristics of an MHD generator to the internal state of the generator, such that choking can either be avoided or, predicted to allow for compensation. To accomplish this, the work will be done in the following stages.

In Chapter II, a model which reasonably predicts the steady-state and transient response of an MHD generator will be developed. This model will then be used to develop an understanding of the generator under varied operating conditions. In Chapter III, an understanding of choking based on the principles of steady, one-dimensional compressible flow will be developed. This will also include a study of the effects of the electromagnetic interaction on the flow in the channel. The end of Chapter III will contain a discussion of the effects of choking on generator operation.

In Chapter IV, the model developed in Chapter II and the theory presented in Chapter III will be used to develop an understanding of the internal transient response of the generator to changes in load and the effects of choking on terminal characteristics. A possible means of preventing choking while still allowing the desired load changes will then be presented.

Finally, Chapter V will present a summary of the results and conclusions, as well as an outline of possible areas for future research which have been suggested by this work.

CHAPTER II

2.1 Introduction

As discussed in the previous chapter, it is first necessary to develop a model of a constant area MHD generator which reasonably determines the terminal characteristics based on inlet and outlet conditions and physical constraints. Further constraints in the development of the required model are introduced because of the complexity of the defining equations for the system. The derivations will begin with the generalized system of equations, and will then proceed to reduce them to a more numerically tractable form. All quantities, unless otherwise noted, represent quantities measured in the lab frame of the system. (The lab frame is the frame in which the fluid is in motion and the generator is stationary as opposed to the rest frame in which the fluid is at rest).

2.2 Fluid Continuity Equation

Given in (2.1) is the well-known

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \underline{V}) \qquad 2.1$$

flow continuity equation, where ρ is the mass density, and $\underline{V}$ is the vector velocity, It should be noted that (2.1) is identical in form and usage to the electric current continuity equation.

2.3 Equation of Motion

The equation of motion is derived by an application of Newton's

Second Law, i.e., the sum of the forces exerted on a body is equal to the rate of change of the momentum of the body. In this case, the body is the fluid of interest and the forces will be body forces, i.e., forces per volume. The complete equation of motion

$$\rho \frac{D\underline{V}}{Dt} = - \nabla P - \rho \nabla \Psi + \nabla \cdot \overleftrightarrow{\tau}' + \underline{J} \times \underline{B} + \rho_e \underline{E} \qquad 2.2$$

is given by (2.2) (Hughes, 1966) where P is pressure, Ψ is gravitational potential, $\overleftrightarrow{\tau}'$ is the shear part of the mechanical stress tensor, $\underline{J}$ is vector current density, ρ_e is the charge density, $\underline{E}$ is electric field, $\underline{B}$ is magnetic flux density, $\frac{D}{Dt}$ is the substantial derivative and is given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u\partial}{\partial x} + \frac{v\partial}{\partial y} + \frac{w\partial}{\partial z} \qquad 2.3$$

and

$$\underline{V} = u\underline{i} + v\underline{j} + w\underline{k} \qquad 2.4$$

The left side of (2.2) represents the rate of change of the body's momentum, while the right side represents all forces acting on the body. The first term on the right side represents the pressure gradient acting on the fluid; the second, the gravitational forces; the third, the viscous forces; and the fourth and fifth, the Lorentz force.

2.4 Energy Equation

For the energy equation, a form given by (2.5) (Hughes, 1966) will be used.

$$\rho \frac{De}{Dt} = \Phi - P\nabla\cdot\underline{V} + \nabla\cdot(\kappa_T\nabla T) + \underline{J}'\cdot\underline{E}' \qquad 2.5$$

In (2.5), e represents the specific internal energy, Φ is the mechanical dissipation function which represents the effect of viscosity on internal energy, κ_T is the thermal conductivity, T is temperature, $\underline{J}'$ is current density measured in the rest frame, and $\underline{E}'$ is the rest frame electric field. Kinetic energy effects are not included in (2.5) and will be incorporated into the discussion later.

2.5 Equation of State

The fourth equation is the equation of state modified to account for the presence of two gases rather than one, and is given by

$$P = \frac{1 + \alpha}{\bar{\mu}_o} \rho RT \qquad 2.6$$

where α is the free electron concentration, $\bar{\mu}_o$ is the mean molecular weight, and R is the ideal gas constant (Sutton, 1965).

2.6 Ohm's Law

The fifth and final equation of general interest is Ohm's Law

$$\underline{J} = \sigma(\underline{E} + \underline{V}x\underline{B}) - \mu(\underline{J}x\underline{B}) \qquad 2.7$$

where σ is the fluid conductivity and μ is the electron mobility (Sutton, 1965).

2.7 System Configuration and Equations

The MHD duct is configured as Figure 2.1. L is the length of the duct, h is the electrode separation, ℓ is the insulator separation, and B_z represents the applied magnetic field in the z-direction. For this model, both electrode and insulator separation are constant, though not necessarily equal. Variable, finite segmentation of electrodes, connected in the Faraday mode, (Fig. 2.2), is assumed. The flow equations are considered in their one-dimensional form for the solution of the channel flow. Boundary layer effects. which should be treated as three-dimensional flow, are instead approximated by a method to be discussed later.

This one-dimensional approximation of channel flow allows for variation of flow variables in the x-direction only, while assuming that the flow variables, across any cross-section, assume their average value. This averaging will tend to increase the friction and heat transfer effects at the walls, and it is therefore necessary to approximate these effects.

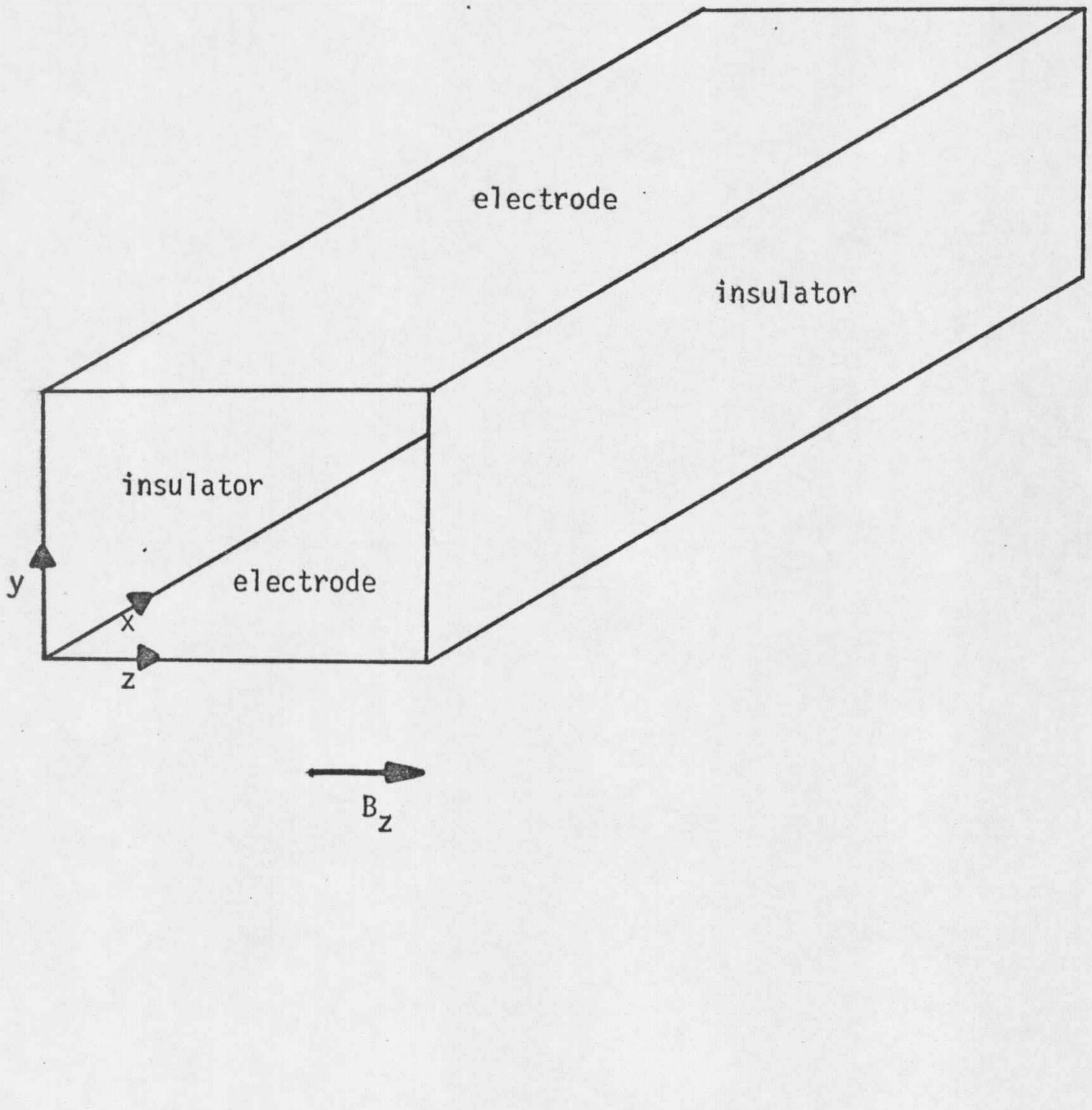

Figure 2.1 MHD Duct Configuration

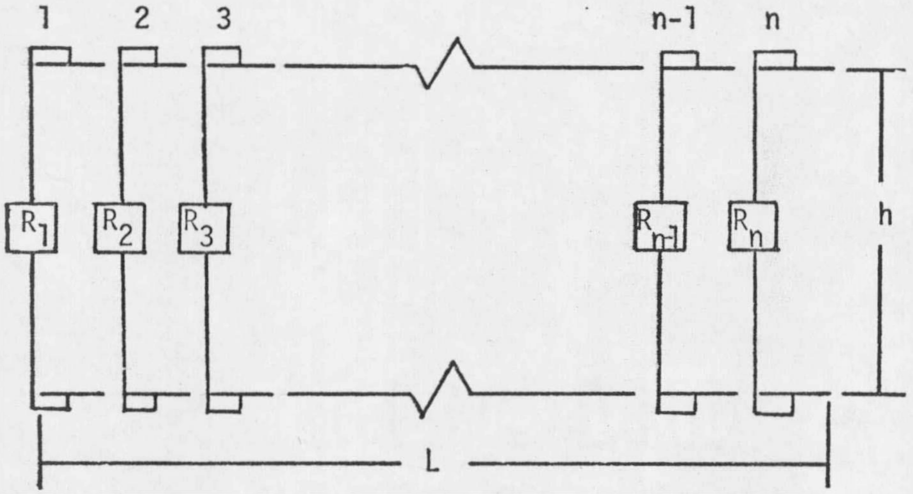

Figure 2.2 Faraday Connected Electrodes