Circuit analyzer for an induction motor
by Allan K Hammell

A THESIS Submitted to the Graduate faculty in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University

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Bozeman, Montana
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Allan K Hammell

ABSTRACT

The circuit analyzer for polyphase induction motors is designed for use in the Electrical Engineering laboratory at Montana State College. The circuit analyzer is used to obtain the current, voltage, power input, power output, and efficiency for various motor speeds. The long and laborious calculations of the analytical solution are unnecessary. Tests showed that the circuit analyzer results compared very favorably with those of the analytical solution.

CIRCUIT ANALYZER FOR AN INDUCTION MOTOR

INTRODUCTION

The polyphase induction motor is the most common alternating-current motor in use at the present time for industrial application. The motor is inexpensive, rugged, and has a nearly constant speed. Squirrel-cage motors have no external connections to the rotor and can be totally enclosed for use in adverse conditions where sparking or abrasive dust may be hazardous.

The wide-spread use of the polyphase induction motor makes it feasible to have a circuit analyzer for the study of induction motors.

Circuit analysis has been used in the study of transmission line short-circuit currents by means of a calculating board. The transmission line calculating board is set up with resistances connected to simulate a short circuited transmission network. Direct-current voltages are applied at points where voltage sources exist and the currents at various points in the network are measured to find the behavior of the system.

Since both inductive reactance and resistance are present in an induction motor, power factor is an important item. Consequently, an a-c system must be used in a circuit analyzer for an induction motor. The equivalent circuit of an induction motor, derived on pages 8 to 13, is used for the circuit analyzer designed for use in the Electrical Engineering laboratory at Montana State College.

INDUCTION MOTOR THEORY

Principles of Operation

The essential feature in the theory of a polyphase induction motor is a revolving field produced by the stator. The operation of the induction motor may be explained by representing the revolving field by a rotating field structure NS in Fig. 1. On the same shaft, there is a short circuited rotor R that is free to rotate in any direction.

As the field rotates, a current is induced in the windings of the rotor. With a clockwise rotation of the field, the current in the conductor under the north pole will be out of the paper. The field produced by this current will be in a counter clockwise direction and consequently, a clockwise torque will be exerted to the rotor by the interaction of the two fields. A similar torque is produced in conductors under the south pole since both the conductor current and the field flux are opposite to those of the north pole.

The rotor will not attain the same speed as the field because the existence of torque depends upon the field cutting across the conductors.

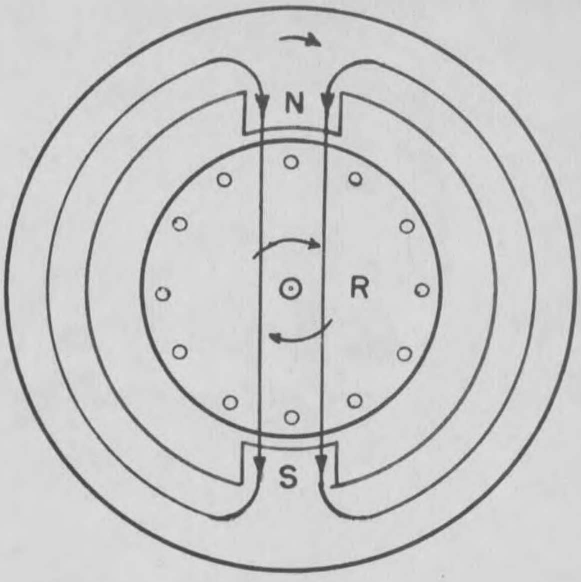

FIG. I PRINCIPLE OF INDUCTION MOTOR.

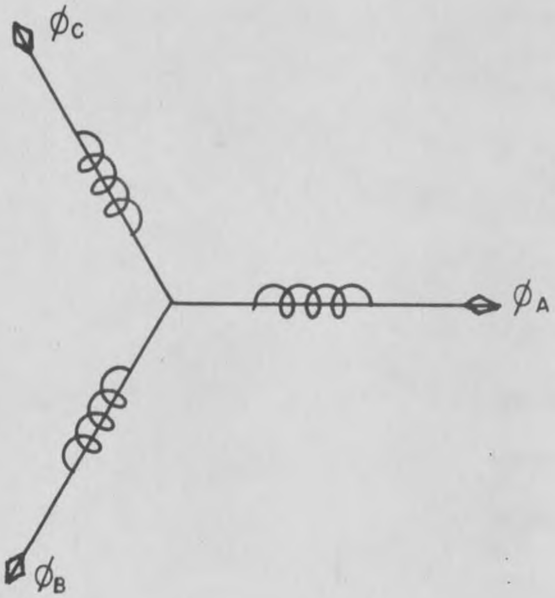

FIG. 2 STATOR FLUX DIAGRAM.

The Rotating Field

The field of an induction motor is shown to be rotating and of constant magnitude by considering the current in the stator windings and the space relationship of the windings. Although the windings are spread over the periphery of the stator, each set of windings has an axis. The axes are equally spaced in the stator with an angle of $\frac{360}{m}$ electrical degrees between them, where m is the number of phases.

The currents in a three-phase system are

$$i_a = I_M \cos \omega t \tag{1}$$

$$i_b = I_M \cos (\omega t-120°) \tag{2}$$

$$i_c = I_M \cos (\omega t+120°) \tag{3}$$

Since the flux is in phase with the inducing current, the flux equations for the three-phase stator represented in Fig. 2 may be written as

$$\phi_a = \Phi \cos \omega t = \frac{\Phi}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \tag{4}$$

$$\phi_b = \Phi \cos (\omega t-120°) = \frac{\Phi}{2}\left(e^{j(\omega t-120°)} + e^{-j(\omega t-120°)}\right) \tag{5}$$

$$\phi_c = \Phi \cos (\omega t+120°) = \frac{\Phi}{2}\left(e^{j(\omega t+120°)} + e^{-j(\omega t+120°)}\right) \tag{6}$$

The resultant flux of the stator is the vector sum of the three fluxes and is expressed by

$$\phi = \phi_a + \phi_b\, e^{j120°} + \phi_c\, e^{-j120°} \tag{7}$$

Then

$$\phi = \frac{\Phi}{2}\left[\left(e^{j\omega t} + e^{-j\omega t}\right) + e^{j120^\circ}\left(e^{j(\omega t-120^\circ)} + e^{-j(\omega t-120^\circ)}\right) + e^{-j120^\circ}\left(e^{j(\omega t+120^\circ)} + e^{-j(\omega t+120^\circ)}\right)\right] \quad (8)$$

$$\phi = \frac{\Phi}{2}\left[3e^{j\omega t} + e^{-j\omega t}\left(1 + e^{j120^\circ} + e^{j240^\circ}\right)\right] \quad (9)$$

From symmetrical components,

$$1 + e^{j120^\circ} + e^{j240^\circ} = 0 \quad (10)$$

The resultant flux

$$\phi = \frac{3}{2}\,\Phi\, e^{j\omega t} \quad (11)$$

has a constant magnitude $\frac{3}{2}\Phi$ and a rotation of line frequency. It should be noted that the rotation is in terms of electrical degrees. The frequency of rotation is converted into mechanical speed by the equation

$$n = \frac{120^\circ f_1}{p} \quad (12)$$

where n is the speed of the field in revolutions per minute;

f_1 is the frequency of the supply system in cycles per second and

p is the number of poles in the stator.

Equivalent Circuit Diagram

The polyphase induction motor has many characteristics in common with the transformer. The motor may be considered as a polyphase transformer with the stator represented by the primary and the rotor represented as a secondary that may move with respect to the primary. At standstill, there is no relative motion between the rotor and the stator, and the motor is similar to a transformer with a short-circuited secondary.

As the motor builds up speed, the frequency of the rotating field with respect to the rotor is reduced. The relationship of the rotor frequency to the synchronous frequency is called slip, which is defined as:

$$s = \frac{n_s - n}{n_s} \tag{13}$$

where s is the slip in per-unit values;

n_s is the synchronous speed in revolutions per minute; and

n is the rotor speed in revolutions per minute.

The rotor speed is $(1 - s)\, n_s$ and the frequency of the induced currents in the rotor is sf_1.

When the rotor is at standstill $(s = 1)$, the transformer analogy for one phase of the motor is shown by Fig. 3. When the rotor is revolving, the secondary voltage $\mathring{E}_{2s}$ is $s\mathring{E}_2$ and the reactance X_{2s} is sX_2.[1] Fig. 4 is the equivalent circuit with the rotor revolving. The mechanical power output of the motor may be represented by a load with a voltage $(1 - s)\,\mathring{E}_2$ applied to its terminals. Since the mechanical output is represented in electrical units, Fig. 4 may be converted to the equivalent circuit of a transformer with a ficticious load connected to the secondary.

The values of equivalent resistance and reactance representing the ficticious load may be derived from the relation

$$\mathring{E}_{2s} = s\mathring{E}_2 = \mathring{I}_{2s}\,\mathring{Z}_{2s} \tag{14}$$

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[1] See Appendix A (p. 28) for symbols.

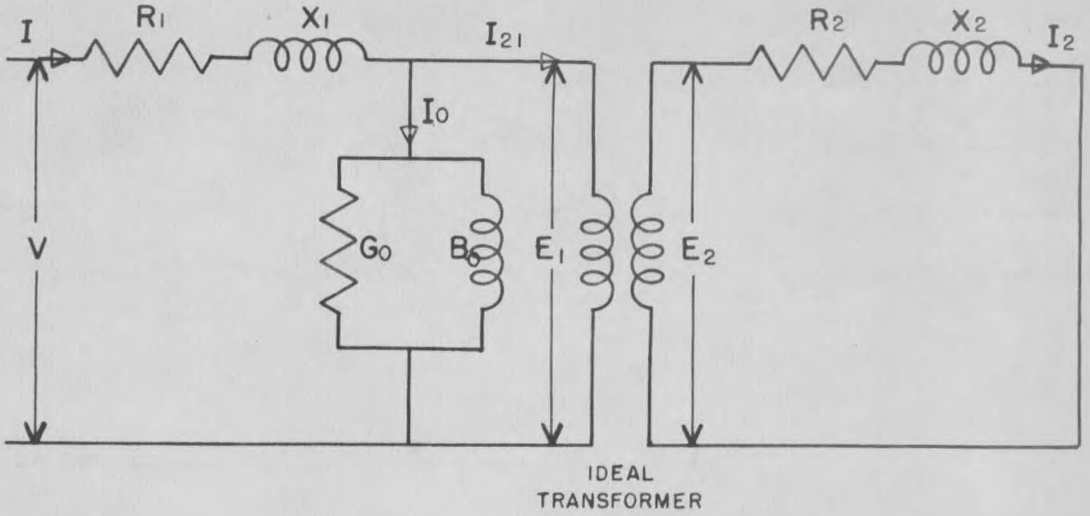

FIG. 3 EQUIVALENT CIRCUIT FOR POLYPHASE INDUCTION MOTOR AT STANDSTILL.

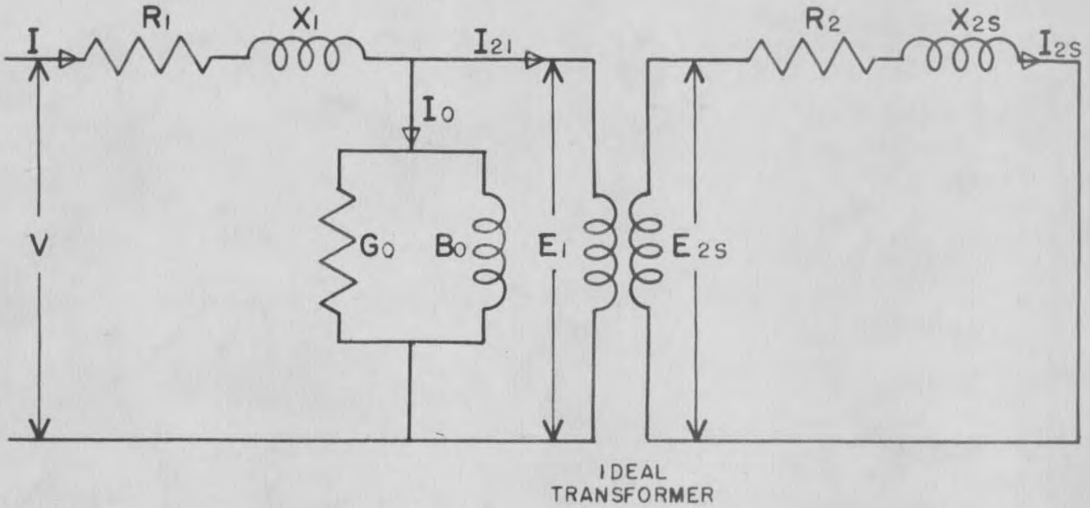

FIG. 4 EQUIVALENT CIRCUIT FOR POLYPHASE INDUCTION MOTOR AT SLIP S.

or

$$s\dot{E}_2 = \dot{I}_{2s}\,(R_2 + jX_{2s}) \qquad (15)$$

Multiplying through by $\frac{1-s}{s}$,

$$(1-s)\,\dot{E}_2 = \dot{I}_{2s} \quad \frac{1-s}{s}\,R_2 + j\,\frac{1-s}{s}\,X_{2s} \qquad (16)$$

from which the equivalent resistance and reactance are found to be $\frac{1-s}{2}R_2$ and $\frac{1-s}{s}X_{2s}$ respectively.

The equivalent circuit for the motor running at slip s is shown on Fig. 5. Since the load reactance does not represent real power, it is combined with X_{2s} as

$$\frac{1-s}{s}X_{2s} + X_{2s} = \frac{1}{s}X_{2s} \qquad (17)$$

Since

$$X_{2s} = sX_2 \qquad (18)$$

the total reactance in the secondary is X_2, which is not dependent upon the rotor frequency.

The ideal transformer may be removed by referring all secondary values to the primary in the following manner:

$$R_2{}' = a^2\,R_2 \qquad (19)$$

$$X_2{}' = a^2\,X_2 \qquad (20)$$

$$\dot{I}_2{}' = \frac{1}{a}\,\dot{I}_{2s} = \dot{I}_{21} \qquad (21)$$

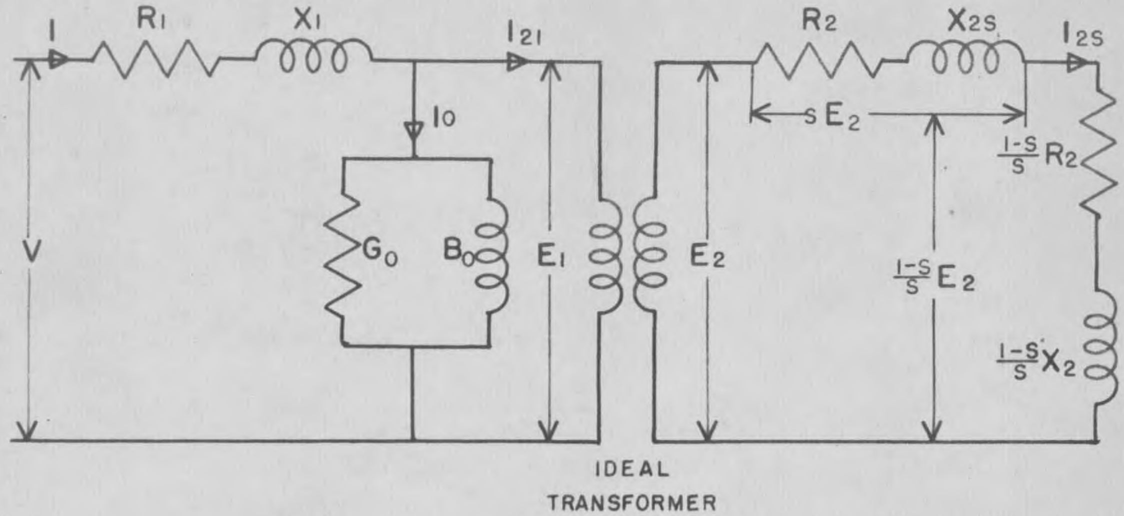

FIG. 5. EQUIVALENT CIRCUIT FOR POLYPHASE INDUCTION MOTOR.

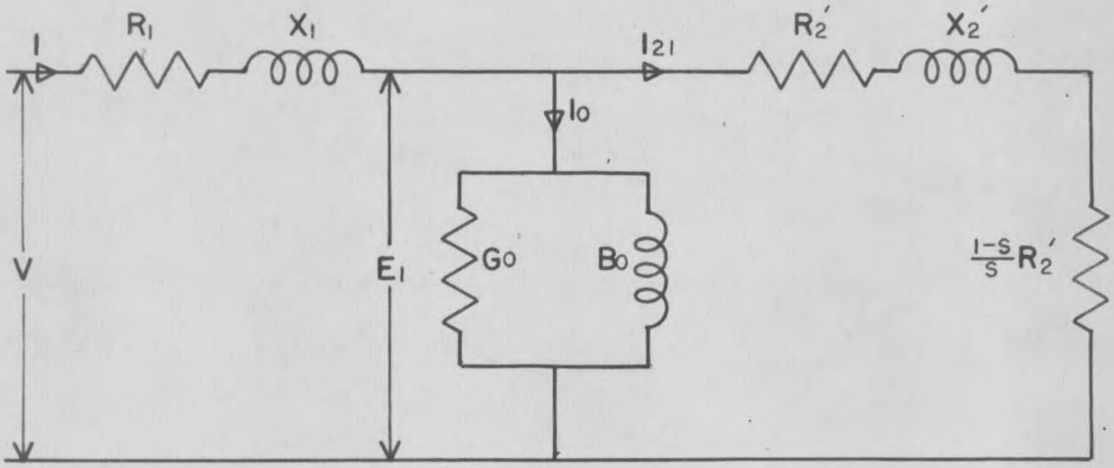

FIG. 6. EQUIVALENT CIRCUIT DIAGRAM.

$$\dot{E}_2'' = a\,\dot{E}_2 = \dot{E}_1 \tag{22}$$

Fig. 6 is the equivalent circuit diagram which is used as the circuit for the analyzer.

CIRCUIT ANALYZER DESIGN

This project was undertaken to construct a circuit analyzer for polyphase induction motors because analytical methods previously used are too long and involved. It is desired to obtain readings of current, voltage, power input, power output, efficiency, and speed directly from the board with as little calculating as possible. A per-unit base is used to reduce the power dissipated in the circuit board, and the controls and meters are arranged to simplify the operation.

Circuit Elements

The elements used in the basic circuit of the analyzer have been chosen so that any polyphase induction motor may be analyzed. The range of values for the parameters are shown in Table I. Rheostats are used in the circuit to represent the resistances of the circuit, but since a variable inductance is difficult to build, condensers are used to represent the inductance. That change will cause the power factor to lead instead of lag, but there is no other change in the behavior of the circuit. The reactance is varied by switching calibrated units of condensers in or out of the circuit. Each element, except for the ficticious load resistance, is calibrated in terms of ohms or mhos so that the board may be easily adjusted to analyze different motors.

Per-Unit Value

All resistances and reactances in the analyzer circuit are multiplied by five hundred. The voltage input to the circuit is equal to the per-phase voltage of the machine being tested. When high-voltage machines

TABLE I

ASSUMED MAXIMUM PARAMETER VARIATIONS OF POLYPHASE INDUCTION MOTORS

Parameter	Assumed Variation to be Encountered
R_1	0.01 to 2 ohms
R_2'	0.01 to 2 ohms
X_1	0.1 to 5.0 ohms
X_2'	0.1 to 5.0 ohms
B_o	0.001 to 0.01 mhos
G_o	0.01 to 0.1 mhos
s	0.0001 to 1.0 per-unit
I	2.0 to 50. amps

are to be tested, the voltage is reduced so that the per-phase voltage is about 90 to 130 volts. The use of the large per-unit value of 500 reduces the size of the condensers to a reasonable value and allows the use of radio type rheostats.

Meter Systems

Readings of current and voltage are easily obtained by placing a ammeter in series with the circuit and a voltmeter across the circuit. The power input and power output are metered by one meter in a switching circuit. For the power input, the current coil is in series with the R_1, X_1 branch and the voltage coil is across the source. The power output is obtained by switching the current coil to the load resistance branch and the voltage coil to the load resistance. A double-pole, double-throw switch is used to change from power input to power output as shown in Fig. 7. The efficiency is calculated from the input and output after the friction and windage have been subtracted from the output. Since the reduction of the friction and windage losses are compensated by the rotor core losses as the slip increases, the measured friction and windage losses are assumed to be constant for all speeds.

The speed of the machine may be calculated from the slip which is obtained from a meter circuit designed to measure the variable load resistance in terms of slip. The slip meter is a modified ohmmeter in which both the voltage and meter circuit resistances are varied in proportion to the parameter R_2'. The load resistance is dependent upon two independent variables, R_2' and s. If the voltage and resistance in an ohmmeter circuit

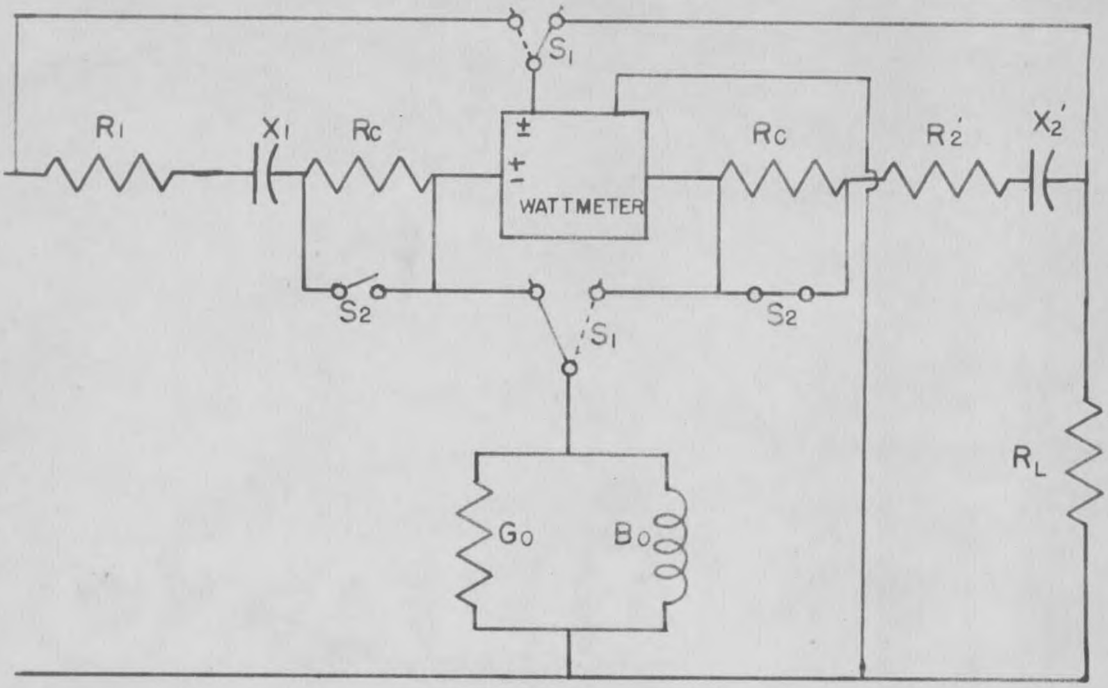

FIG. 7. COMPENSATING WATTMETER CIRCUIT.

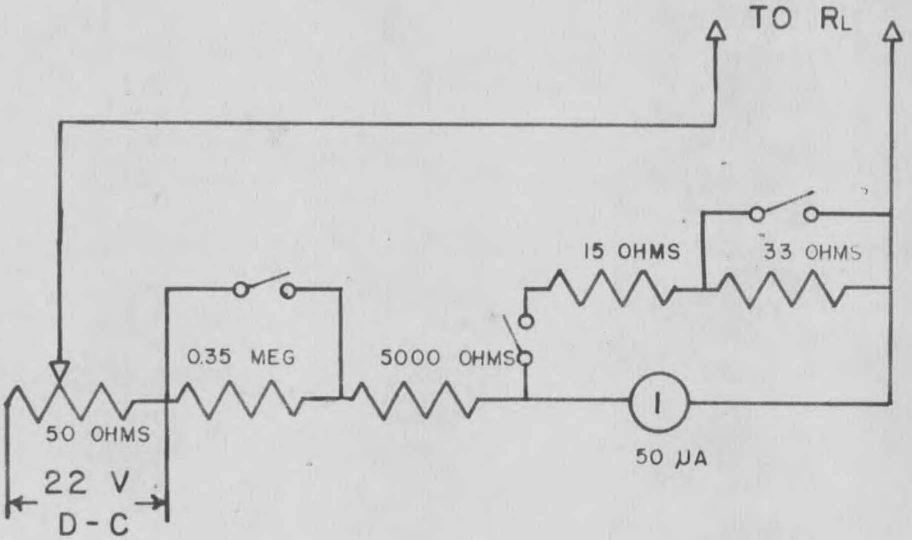

FIG. 8. SLIPMETER CIRCUIT.

are made proportional to R_2', the current will be independent of R_2' and only dependent upon s. With the ammeter calibrated, the load resistance may be adjusted in terms of slip by the circuit shown in Fig. 8.

By adding two shunt resistances across the meter and short circuiting part of the series resistance in Fig. 8, two scale ranges may be added to the meter. The voltage required by the slip meter circuit is 22 volts d-c. A 50 micro-amp d-c meter is used to measure the current.

To adjust the voltage potentiometer, the circuit is switched to a resistance that is calibrated for one per cent slip for all values of R_2'. The voltage is then increased until the slip meter reading is one per cent.

Meter Compensation

In most cases, meter resistance will have very little effect upon the circuit. When the voltage coil is placed across the load resistance, the circuit has a parallel load resistance which lowers the resistance seen by the circuit. To compensate for the lowered resistance, the load resistance is increased until the current input to the circuit is the same as that measured before the voltage coil was added.

If the resistance of the voltage coil is less than that of the load resistance, the power output cannot be metered by this method. The wattmeter used for the circuit analyzer has a voltage range of 7.5/15 volts and the voltage is stepped down with a potential transformer.

The resistance of the voltage coil is multiplied by the turns ratio squared when it is reflected to the primary. The large voltage coil resistance makes it possible to take readings for low values of slip.

For very low values of slip, where the voltage coil resistance is less than the load resistance, a different method must be used. The current in the load resistance can be measured by an ammeter in the secondary circuit, and by calculating the load resistance, the power output may be determined. Since the wattmeter will provide results for most cases, it will seldom be necessary to calculate the power output.

Compensation for the ammeter is made by subtracting the ammeter resistance from R_1. Since the ammeter resistance is in per-unit value when in the circuit, the amount of resistance subtracted from the calibrated dial for R_1 is 1/500 of the ammeter resistance. The ammeter is compensated in the calibration of R_1.

The wattmeter current coil is compensated by resistance in both the primary and secondary branches. The resistance that is in the same branch as the current coil is short circuited by a switch as shown in Fig. 7. The compensating resistances are included in the calibration of R_1 and R_2'.

The complete circuit diagram of the circuit analyzer is shown in Fig. 9. A three-pole, double-throw switch with a center off position is used to isolate the slip meter circuit from the equivalent circuit diagram while power measurements are being taken.

Fig. 10 is a photograph of the experimental model circuit analyzer and the equipment used in calibrating the circuit elements.

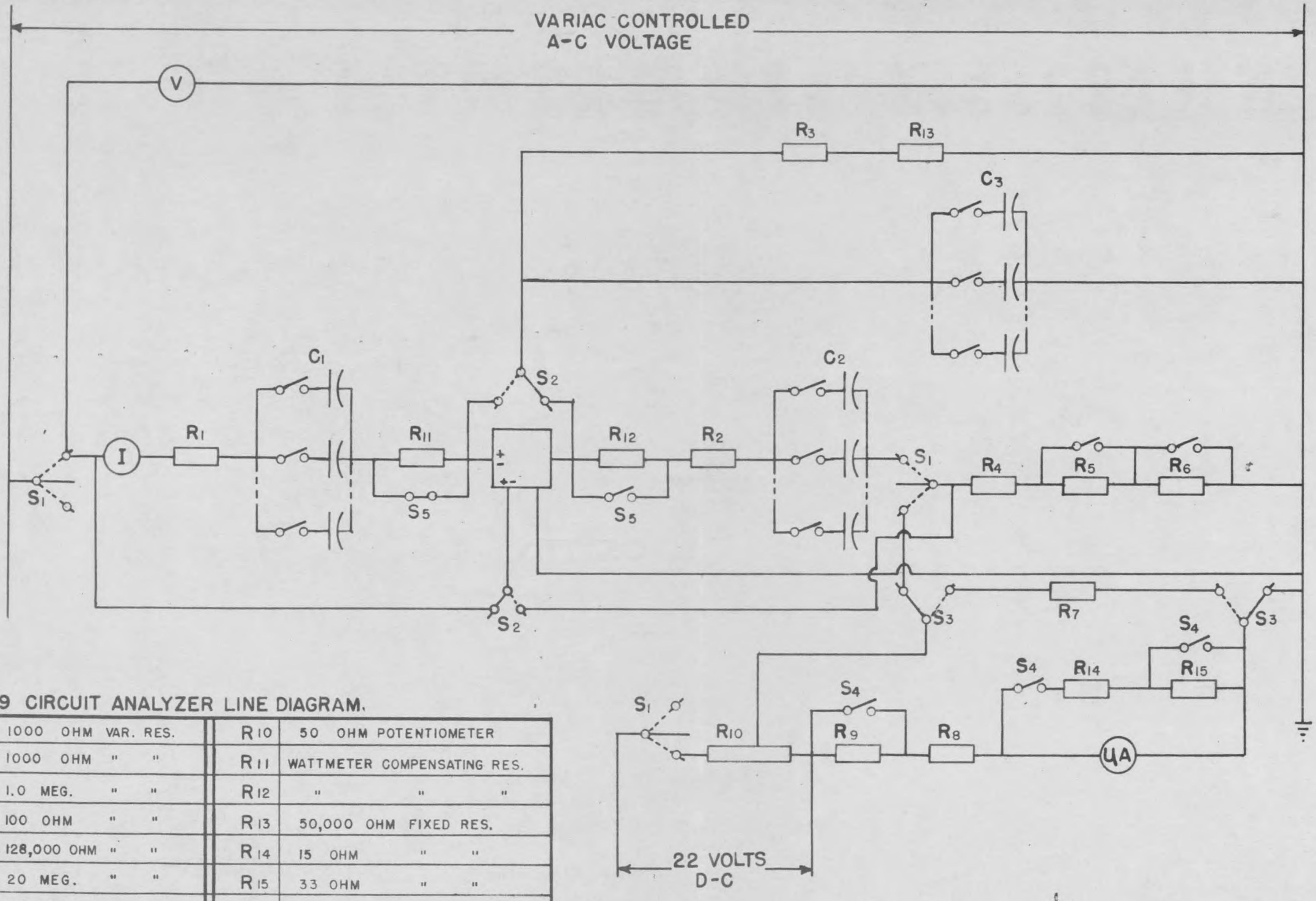

FIG. 9 CIRCUIT ANALYZER LINE DIAGRAM.

R1	1000 OHM VAR. RES.	R10	50 OHM POTENTIOMETER
R2	1000 OHM " "	R11	WATTMETER COMPENSATING RES.
R3	1.0 MEG. " "	R12	" " "
R4	100 OHM " "	R13	50,000 OHM FIXED RES.
R5	128,000 OHM " "	R14	15 OHM " "
R6	20 MEG. " "	R15	33 OHM " "
R7	0.1 MEG. " "	C1	53 μFD CONDENSER BANK
R8	5000 OHM " "	C2	53 μFD " "
R9	0.35 MEG. " "	C3	1 μFD " "

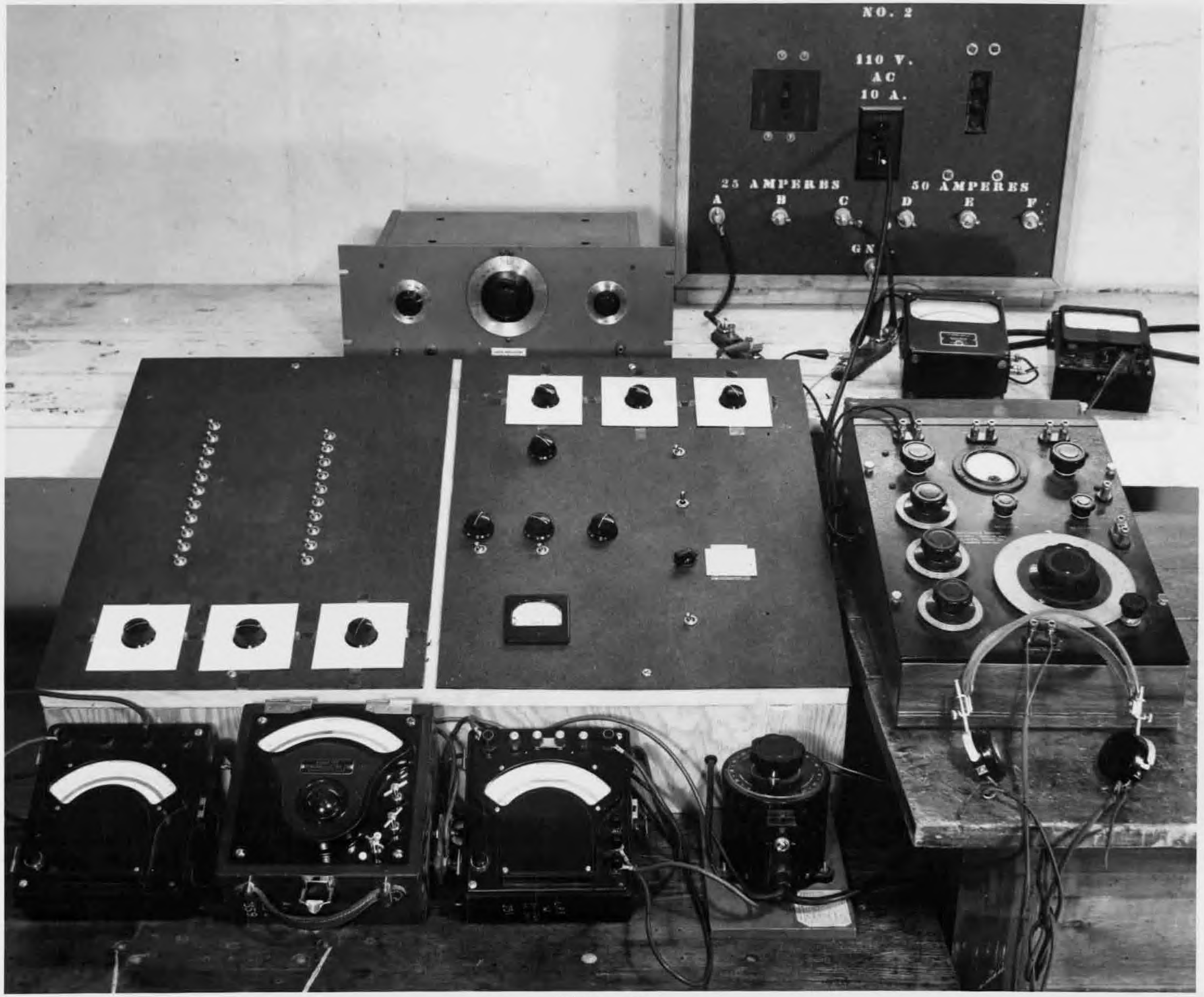

FIG. 10.

OPERATION OF THE CIRCUIT ANALYZER

A step by step process is used to operate the circuit analyzer after the induction motor parameters are determined by a running light and blocked rotor test.[2] The complete procedure for operating the circuit analyzer is described in the following paragraphs.

To set up the analyzer, a voltmeter, an ammeter, and a wattmeter are connected to the eight terminals on the front of the board. The power leads and calibrating leads are then connected to a variac controlled a-c voltage supply and a 22 volt d-c supply respectively.

The rheostats and condensers are adjusted to equal the parameter values. There are four rheostats that must be set for R_2' -- one in the equivalent circuit and three in the slip meter circuit. The slip meter is then calibrated by switching to the calibration position and increasing the slip meter circuit voltage until the meter reads one per cent slip on the low scale. The remainder of the process is repeated for each value of slip.

The load resistance is adjusted by switching the power switch to the position marked "measure" and varying the load resistance until the desired slip is indicated on the slip meter. With the wattmeter switch on the "power in" position and the power switch on the "on" position, the voltage is increased with the variac to the rated per-phase value. Readings of voltage, current, and power input may then be taken. To obtain

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[2] Methods for obtaining induction motor parameters may be found in most alternating current machinery texts.

the power output, the wattmeter switch is turned to the "power output" position, and the load resistance is increased until the current reading is the same as the reading taken while the power input was measured. The power output may then be measured since the voltage coil losses are compensated by the increase in the load resistance. The load resistance is then adjusted for another value of slip and the procedure is repeated.

If it is impossible to make the proper adjustment for current while measuring power output, the secondary current must be measured under the same conditions as the power input is measured. The power output is then calculated by the formula

$$P_{output} = \frac{1 - s}{s} R_2' (500) (I_2)^2 \qquad (23)$$

The formulas on Table II transform the data from the circuit analyzer into the actual motor values.

TABLE II

FORMULAS FOR CONVERTING CIRCUIT ANALYZER DATA INTO ACTUAL VALUE

Actual Value		Transfer Value
Line Current	(amps)	$500\ I$
Phase Voltage	(volts)	V
Power Factor	(per cent)	$\frac{P_{input}}{VI}\ (100)$
Power Input	(watts)	$3(500)\ P_{input}$
Gross Power Output	(watts)	$3(500)\ P_{output}$
Net Power Output	(watts)	$3(500)\ P_{output} - F\ \&\ W$
Efficiency	(per cent)	$\frac{3(500)\ P_{output} - F\ \&\ W}{3(500)\ P_{input}}\ (100)$
Speed	(rpm)	$(1 - s)\ n_s$
Gross Torque[3]	(lb.-ft.)	$7.045\ \frac{\text{gross power output}}{\text{speed}}$ Actual Value
Net Torque[3]	(lb.-ft.)	$7.045\ \frac{\text{net power output}}{\text{speed}}$ Actual Value

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[3] The torque formula is indeterminate when the slip is unity.

RESULTS

The results obtained from the circuit analyzer are compared to those obtained by the analytical method on Table III. Since the proper metering equipment was not available, the tests were run with a per-unit value of 100 instead of the design value of 500. The accuracy of the results was not affected by the change in per-unit value.

The results show that the circuit analyzer compares very favorably with the analytical method of solution and that involved calculations are unnecessary. With the circuit analyzer, the time required to obtain data for a complete set of curves is much less than the time required to calculate the data for one value of slip by the analytical method.

TABLE III

CIRCUIT ANALYZER TEST RESULTS COMPARED TO THOSE OBTAINED BY ANALYTICAL SOLUTION[4]

For a 5-horsepower, 3-phase, 220-volt, 6-pole, 60-cycle, wye-connected induction motor operating at a 3 per cent slip in which:

R_1 = 0.383 ohm (effective)
R_2 = 0.530 ohm (ohmic)
$X_1 = X_2$ = 1.207 ohms

g_e = 0.00260 mho
b_e = 0.0436 mho
$(F + W)_{phase}$ = 48.5 watts

	Circuit Analyzer Test	Analytical Method
Line current	8.9 amps	8.89 amps
Phase voltage	127 volts per-phase	127 volts per-phase
Power in	2548.5 watts	2532 watts
Gross power out	2250 watts	2262 watts
Net power out	2105.5 watts	2117 watts
Efficiency	82.8 per cent	83.6 per cent
Power factor	75.1 per cent	74.8 per cent
Speed	1164 rpm	1164 rpm
Net torque	12.73 lb.-ft.	12.81 lb.-ft.

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[4] Corcoran, G. S. and Reed, H. R., 1939. ELECTRICAL ENGINEERING EXPERIMENTS, p 417, John Wiley & Sons, New York

DISCUSSION AND CONCLUSIONS

The analysis of the polyphase induction motor by means of the circuit analyzer is probably the most versatile method in use.

The analyzer is designed to represent the actual induction motor and, consequently, almost any test made on the induction motor may be run with the circuit analyzer. Core loss and hysteresis tests cannot be set up on the circuit analyzer since the results depend upon the individual iron properties of the induction motor.

The effect of changing the parameters can be analyzed to obtain the design of an induction motor for a specific purpose. Since the torque can be calculated at any point except when the slip is unity, the torque-speed curves may be obtained for various rotor resistances.

The circuit analyzer results are compared to those obtained by the analytical method because similar assumptions are made in both cases. It should be noted that the accuracy of the circuit analyzer as compared to the actual motor depends upon the accuracy of the measured motor parameters.

An important use for the circuit analyzer is to check values obtained by students using an analytical solution.

In conclusion, the circuit analyzer of an induction motor provides a fast and accurate method of analyzing the polyphase induction motor without the use of long and involved calculations.

APPENDIX A

LIST OF SYMBOLS

Symbol	Meaning
R_1	primary resistance per phase to neutral
X_1	primary leakage reactance per phase to neutral
I	primary current per terminal
V	primary impressed voltage per phase to neutral
I_{21}	primary inducing current
f_1	primary frequency
E_2	secondary voltage to neutral at standstill
I_2	secondary current per phase at standstill
R_2	secondary resistance per phase
R_2'	secondary resistance per phase referred to the primary
X_2	secondary leakage reactance per phase at standstill
X_2'	secondary leakage reactance per phase as referred to the primary
s	slip in per-unit values
$1 - s$	rotor speed in per-unit values
E_{2s}	secondary voltage to neutral at slip s
I_{2s}	secondary current per phase at slip s
X_{2s}	secondary leakage reactance per phase at slip s
Z_{2s}	secondary impedance at slip s

G_o	primary exciting conductance per phase, representing core-loss
B_o	primary exciting susceptance per phase, representing the magnetizing current, the flux variation being considered linear for the range of operation
a	turns ratio between the primary and secondary
n_s	synchronous speed of motor in rpm
n	rotor speed
ϕ	instantaneous stator flux
Φ	maximum stator flux
F & W	friction and windage loss

APPENDIX B

LIST OF EQUIPMENT USED IN CONSTRUCTION OF CIRCUIT ANALYZER

1	50 ohm 25 watt potentiometer
1	100 ohm 25 watt variable resistor
2	1000 ohm 2 watt variable resistors
1	5000 ohm 2 watt variable resistor
1	128,000 ohm 50 watt variable resistor
1	0.1 megohm variable resistor
1	0.35 megohm variable resistor
1	1.0 megohm variable resistor
1	20 megohm variable resistor
2	wattmeter compensating resistors (fixed)
1	50,000 ohm fixed resistor
1	15 ohm fixed resistor
1	33 ohm fixed resistor
1	triple-pole, double-throw, center off switch
3	double-pole, double-throw switches
1	3 pole selector switch
11	double-pole, single-throw switches
11	single-pole, single-throw switches
2	parallel condenser units, 53 μfd total in each
1	parallel condenser unit, 1 μfd total
1	50 μa d-c ammeter

APPENDIX C

EXTERNAL EQUIPMENT REQUIRED TO OPERATE CIRCUIT ANALYZER

1	ammeter 100 ma a-c
1	voltmeter 150 volt a-c
1	wattmeter .8 - 1.6 watt
1	variac
1	potential transformer
1	supply 115 volt a-c
1	supply 22 volt d-c

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