AN OPTIMAL REPLACEMENT POLICY FOR BEEF COWS: A STATE MODEL UNDER CONDITIONS OF UNCERTAINTY

by

Patrick M. Graham

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APPROVAL

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Patrick M. Graham

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8-17-94
Date

Steve Stanbro
Chairperson, Graduate Committee

Approved for the Major Department

8-17-94
Date

R. Clyde Green
Head, Major Department

Approved for the College of Graduate Studies

8/22/94
Date

R. L. Bruns
Graduate Dean

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ABSTRACT

Cattle ranchers are periodically faced with the decision of whether or not to cull/replace a brood cow. Assets such as beef cows are subject to stochastic elements that affect the cull/replacement decision. Stochastic elements include events such as death, infertility, and the inability to meet specific performance standards. Observations suggest that many Montana ranchers use a period-by-period cull/replacement decision-making strategy. However, varying management practices, stochastic elements, and the lengthy biological process associated with cattle production limit the validity of a single long-run replacement policy applied across all cattle ranches. This thesis examines a long-run optimal replacement policy based on economic criterion that individual cattle ranchers can adapt to their specific operation.

Burt's discrete time model for optimal replacement under stochastic conditions is used as the analytical model. Published data measuring the likelihood of stochastic events, Billings, Montana cattle prices, and the long-run average variable costs from a case study of an individual ranch in Eastern Montana are used in the empirical model. Prices and costs are varied to reflect varying cattle prices over time and the heterogeneous nature of management practices with respect to costs.

The results indicate that, although a single optimal planned replacement age is not derived, voluntary replacement should occur between cow ages of six and eleven years. These results are insensitive to varying price/cost combinations used in the study. Given the insensitivity of the results to the varying price/cost levels it is likely that the probabilities measuring the stochastic events are the main determinants in deriving an optimal replacement age. The results also show that cattle ranching is profitable except in high cost with average to below average model price conditions.

CHAPTER 1

INTRODUCTION

Cattle ranchers are faced with durable asset replacement decisions periodically, whether the asset is capital equipment or beef cows. Faced with the decision of whether or not to cull\replace a beef cow, ranchers need a decision strategy based on an economic criterion.

Ranchers may cull brood cows for a variety of reasons. These reasons, or criteria, are often applied in the following order. The first criterion is health related. Culling because of physical impairments is an objective decision. Either a cow is impaired to the degree that future production is affected, and therefore culled, or the cow is healthy and retained if she passes the next two criteria. The second criterion depends on fertility status and also is an objective decision. If a cow is tested open, she is marketed. If pregnant, she may be retained depending on the third criterion, which is a subjective test. If the cow is healthy and pregnant she is judged on a performance criterion specific to a particular management practice. If it is expected that the cow will meet performance criterion, then she will be retained in the herd.

Observations suggest that this culling\replacement decision process is practiced by many Montana ranchers. Factors such as prices, costs, and range conditions may

alter the second and third criteria somewhat, but on many Montana ranches this replacement decision process is well entrenched. The sequential application of these criteria constitutes a period-to-period replacement strategy but does not constitute a long-run optimal replacement strategy in the face of productivity uncertainty.

Purpose of Study

Given the dynamic nature of cattle prices and the heterogeneous nature of cattle ranches with respect to management practices and costs, calculation of a general optimal replacement age for beef cows is a difficult task. Cattle ranch heterogeneity, coupled with the lengthy biological process and stochastic elements associated with cattle production, limits the validity of any predetermined long-run optimal replacement age for a beef cow. This points out the need for a long-run optimal replacement decision framework that cattle ranchers can employ and adapt to their specific ranch operations. The purpose of this study is to address this need by formulating such a replacement decision framework. Using published data, which measure the likelihood of occurrence of stochastic elements, and a representative Eastern Montana cattle ranch with specified management practices, this study will define a procedure for individual cattle ranchers to use in determining a long-run optimal beef cow replacement age for their individual operations.

CHAPTER 2

REVIEW OF LITERATURE

The Principle of Discounted Cash Flow

The basic construct used in establishing criteria for determining optimum asset replacement policy has been traced to Faustmann (1849). His work investigated the optimal time to harvest a stand of trees. Faustmann's formula is alleged to be the earliest known application of the principle of discounted cash flow (CFI institute paper, 1968). Discounted cash flow is now the commonly used technique by which the present value of a stream of future earnings is determined.

Present Value: Static Models

Over the past three decades asset replacement research specific to livestock has used Faustmann's approach to formulate the principle of maximizing net present value. The most referenced research efforts using this approach were those conducted by Chisholm (1966) and Perrin (1972).

Chisholm, in determining the criteria for computing the optimum replacement pattern, criticized and expanded on earlier work by Faris (1960). Faris approached the optimum asset replacement problem in terms of maximizing net

revenue over time. The problem defines an asset, such as a beef cow in a herd, as an enterprise which returns an uneven flow of revenue over time. Thus the present value of the net revenue from period "t" of a potential replacement asset can be converted into a constant annual flow of revenue. This annualized return from a potential replacement can then be compared with the marginal net revenue from the existing asset. Faris postulated the optimum time to replace the asset is when the marginal net revenue from the existing asset is equal to the maximum annuity obtainable from the potential replacement asset.

Chisholm also assumed the aim of the replacement decision was to select the production period, which over a specific planning horizon yields the maximum net present value of future profits. In addition, Chisholm correctly identified the interest on the total revenue obtainable from the asset's sale as an overlooked component of marginal cost when computing the present value of the asset's income stream.

Perrin published a general replacement criterion based on the principle of maximizing net present value. His model incorporated the basic marginal principle that the gains from keeping an asset one more period are to be compared to the opportunity gains realized from a replacement asset during the same period of time.

Research by Melton (1980) utilized Perrin's model. Melton specified his optimum asset replacement model for application to a Florida cow herd. He derived asset replacement criteria in the presence of genetic progress, which he postulated was achieved by a producer raising his own replacements. His results under conditions of certainty indicate that without genetic progress the optimal culling age is 11 years of

a random event occurring, leading to unplanned replacement, is measured in terms of a probability. Therefore, derivation of the long-run optimal replacement age must use expected net present value as a criterion.

Burt's (1965) discrete time model was selected to address the beef cow replacement problem. Revenues are received at the end of an annual production period, while annual variable costs for cows are assumed to occur at the midpoint of the period. Replacement, whether voluntary or unplanned, takes place at the end of the period when calves are weaned and the cows are pregnancy tested, vaccinated, and sorted in preparation for winter. An infinite planning horizon is specified and constant prices and costs are used. The probabilities associated with involuntary replacement are invariant with respect to time. Burt (1965) and Perrin (1972) showed that the optimal replacement age is unaffected by the acquisition age of the replacement cow. Therefore an asset that is replaced will always be replaced with a new one. This model defines a new asset as a six-month-old weaned replacement heifer calf. A constant herd size is assumed, which is implied by involuntary replacement of cows culled, open, or dead. Consequently, the expected net present value must be viewed as associated with a particular slot in the herd rather than a specific cow whose presence is steadily diminished due to stochastic elements.

Burt's Model

Burt defines the expected present value of net returns from a particular asset over its life up to some planned replacement time T as;

$$(1) \quad EPV_T = \beta R_1 + \beta^2 P_1 R_2 + \beta^3 P_1 P_2 R_3 + \ldots + \beta^T P_1 P_2 \ldots P_{T-1}(R_T - P_T C_T)$$

where

R_t = conditional expected value of an asset's net revenue, including cost of involuntary replacement due to stochastic elements, during period t (where t = 1, ..., T);

P_t = the probability that an asset in period t will reach period t+1 with normal productivity (where t = 1, ..., T);

C_T = the cost of voluntary replacement; and

β = the discount factor, 1/(1+i), where i is the applicable annual interest rate.

R_t is conditional upon the asset not being replaced due to stochastic events prior to period t. The cost associated with voluntary replacement, C_T, occurs with certainty given that the asset has not been subject to unplanned replacement prior to period T. Given the probabilities associated with the random events that lead to involuntary replacement, the asset's life has a probability distribution as shown in Table 1.

Table 1. Probability distribution of an asset's life [L(T)].

Asset age (t)	Probability that L(T) = t
1	$(1 - P_1)$
2	$P_1(1 - P_2)$
.	.
.	.
.	.
T−2	$P_1P_2 \ldots P_{T-3}(1-P_{T-2})$
T−1	$P_1P_2 \ldots P_{T-2}(1-P_{T-1})$
T	$P_1P_2 \ldots P_{T-1}$

where

$L(T) =$ realized replacement age, which is a random variable that defines the expected life of the asset, given involuntary replacement; and

$T =$ the planned replacement age in which voluntary replacement occurs with certainty conditional upon the asset's survival to age T.

Burt defines the expected net present value of an asset over an infinite planning horizon as

$$(2) \qquad ERPV_T = \frac{EPV_T}{1 - E\beta^{L(T)}},$$

where

$EPV_T =$ the expected present value of net returns from a particular asset over its life up to some time T, and

$1 - E\beta^{L(T)} =$ one minus the expected discount factor, $1-\beta$, which is the annuity factor.

Since the asset's life has a probability distribution, then the discount factor, β, must be weighted by the probability that L(T) = t. This is expressed by (3).

(3) $$E\{\beta^{L(T)}\} = (1-P_1)\beta + P_1(1-P_2)\beta^2 + \ldots +$$
$$P_1 \ldots P_{T-2}(1-P_{T-1})\beta^{T-1} + \ldots + P_1 \ldots P_{T-1}\beta^T$$

Let w_t represent the product of the discount factor times the probability that L(T) = t. It follows that if $w_1, w_2, \ldots, w_T = 1, \beta P_1, \beta^2 P_1 P_2, \ldots, \beta^{T-1}P_1, \ldots, P_{T-1}$, respectively, then (3) simplifies to

(4) $$E\beta^{L(T)} = 1 - (1-\beta)\sum_{t=1}^{T} w_t$$

Substitution of (4) into (2) gives

(5) $$ERPV_T = \frac{EPV_T}{(1-\beta)\sum_{t=1}^{T} w_t}$$

T represents the production period t in which voluntary (planned) replacement occurs over an infinite planning horizon. $ERPV_T$ is calculated for each possible replacement cycle of length T to find the maximum $ERPV_T$, which will reveal the optimal replacement age. Equation (5) is the capitalization of a weighted average of expected net revenues into perpetuity. In order to maximize $ERPV_T$ with respect to T (planned replacement age), data describing the production periods relative to cow age and year, stochastic elements, prices, costs, and the applicable interest rate must be specified.

CHAPTER 4

AN EMPIRICAL MODEL OF OPTIMAL REPLACEMENT OF BEEF COWS

Production Periods

The stochastic elements associated with beef cows are, by their very nature, a function of cow age and are also associated with a particular time of year. Therefore there are implied relationships between production periods, cow age, and the calendar year which must be defined. The annual production period begins November 1 of year n and terminates October 31 of the following calendar year, n+1. Decisions concerning replacement are made at the end of the period, October 31. Asset (cow) age is referred to as the midpoint of the annual production period. For example, a one year old cow is .5 years old on November 1 of year n, 1 year old at the midpoint of year n+1, and 1.5 years old on October 31 of year n+1. The calendar year and intervals for cow ages from one to thirteen years (over which the data exists) are presented in Table 2.

Table 2. Calendar year/age intervals by cow age.

Cow age (t)	Age interval (year)	Calendar year interval (year = n)
1	.5 - 1.5	$Nov.1_n$ - $Oct.31_{n+1}$
2	1.5 - 2.5	$Nov.1_{n+1}$ - $Oct.31_{n+2}$
3	2.5 - 3.5	.
4	3.5 - 4.5	.
5	4.5 - 5.5	.
6	5.5 - 6.5	.
7	6.5 - 7.5	.
8	7.5 - 8.5	.
9	8.5 - 9.5	.
10	9.5 - 10.5	.
11	10.5 - 11.5	.
12	11.5 - 12.5	.
13	12.5 - 13.5	$Nov.1_{n+12}$ - $Oct.31_{n+13}$

Probabilities

The data for the stochastic elements are measured in terms of probabilities and are published in the Arizona Rancher's Management Guide (1993) for cows aged two to thirteen years. The data were collected over the period from 1983 to 1990 from a USDA herd in Arizona. The data include probabilities associated with calf survival which affect the net revenue contributed by the cow, as well as the probabilities associated with random events that affect replacement of the cow.

The probabilities that affect replacement of the cow describe the likelihood of a cow being in one of four particular states at the end of period t, given she was in the herd at the beginning of the period. A cow could either be a) open, b) culled, c) dead, or d) pregnant. These probabilities are biological functions of a cow's age.

A cow that is open (not pregnant), culled (unable to meet performance standards), or dead by the end of the period is replaced by a six-month-old weaned heifer calf, which is referred to as an involuntary or unplanned replacement. A culled cow may be either pregnant or open, but her pregnancy condition is irrelevant in the face of her inability to meet performance standards. The culled state may be due to a physical impairment or an unacceptable production performance. A cow that is classified as open is assumed to have met performance criterion and would have been retained had she tested pregnant. Therefore the probability of being pregnant is an implied joint probability of being pregnant and meeting performance standards, and shall be referred to hereafter as the probability that a cow is pregnant.

The probabilities of being open, culled, dead, and pregnant sum to unity and describe the likelihood of all possible states a cow might be in at the end of a period. Therefore the summation of the probabilities of being open, culled, and dead is a measure of the total probability that the cow is not pregnant. Consequently, the state of a cow by the end of period t (which is not the planned replacement period T) can be described by one of two conditions; 1) pregnant or 2) not pregnant. The probability of being pregnant plus the probability of not being pregnant sum to unity. Since a cow that is not pregnant is involuntarily replaced, one minus the probability

of being pregnant describes the probability of involuntary replacement due to stochastic elements.

In the planned replacement period, T, the cow is replaced with unit probability regardless of her status. This model examines the periods from one to thirteen. But Tronstad and Gum's data exist for cows of age two to thirteen; therefore it was necessary to assume probability values for the states of open, culled, dead, and pregnant for t=1. All probabilities of being open, culled, dead, pregnant, and one minus the probability of being pregnant as functions of a cow's age are shown in Table 3.

Table 3. Probabilities of a calf being born live, surviving to weaning, and sex as functions of cow age.

Cow age (t)	Probability that a cow is open (PO_t)	Probability that a cow is culled (PC_t)	Probability that a cow is dead (PD_t)	Probability that a cow is pregnant (P_t)	Probability that a cow is not pregnant ($1-P_t$)
1	.03	.01	.01	.95	.05
2	.1476	.0128	.0136	.8260	.1740
3	.1483	.0142	.0202	.8172	.1828
4	.1488	.0190	.0269	.8053	.1947
5	.1491	.0270	.0335	.7904	.2096
6	.1492	.0383	.0402	.7724	.2276
7	.1491	.0527	.0469	.7513	.2487
8	.1489	.0702	.0537	.7271	.2729
9	.1486	.0908	.0606	.6999	.3001
10	.1482	.1146	.0676	.6696	.3304
11	.1477	.1416	.0746	.6361	.3639
12	.1472	.1719	.0817	.5992	.4008
13	.1466	.2057	.0888	.5588	.4412

The probabilities pertaining to a calf's survival describe the likelihoods of a calf's survival at two separate stages. In the first stage, a probability measures the likelihood that a calf will survive pregnancy and birth given that the cow was pregnant at the beginning of period t. This probability is also a function of the cow's age. In the second stage, a probability describes the likelihood that a calf will survive until weaning given the calf survived pregnancy and birth. This probability is independent of a cow's age and is a constant equal to .98. Other probabilities relevant to calves are the probabilities that the calf is a heifer or steer calf. These probabilities are also constant for all t and are each equal to .50. These probabilities are shown in Table 4.

Table 4. Probabilities of a calf being born live, surviving to weaning, and sex as functions of cow age.

Cow Age (t)	Probability that a calf is born live (PLB_t)	Probability that a calf survives to weaning (PCSTW)	Probability that a calf is a heifer (PHFR)	Probability that a calf is a steer (PSTR)
1	0.00	0.00	0.00	0.00
2	.9858	.98	.50	.50
3	.9783	.98	.50	.50
4	.9722	.98	.50	.50
5	.9677	.98	.50	.50
6	.9647	.98	.50	.50
7	.9632	.98	.50	.50
8	.9633	.98	.50	.50
9	.9648	.98	.50	.50
10	.9678	.98	.50	.50
11	.9724	.98	.50	.50
12	.9785	.98	.50	.50
13	.9861	.98	.50	.50

Cow and Calf Weights

Revenue that is generated by a cow from marketing as an open or culled cow is a function of weight and the relevant slaughter price. Revenue that is generated from selling the calf of the cow is a function of the calf's weight and calf price. Tronstad and Gum (1994) estimated cow weight and calf weight, measured in pounds, as quadratic functions of cow age at the end of period t. Steer and heifer weights were not estimated individually, but rather averaged and referred to as calf weight. Equations 6 and 7 estimate cow weight and calf weight, respectively.

$$(6)\quad COWWT_t = 853.96 + 81.02(TAGE_t) - 5.24(TAGE_t)^2 + V_1 \qquad \bar{R}^2 = .17$$
$$\qquad\qquad (79.993)\quad (24.456)\quad (-21.620)$$

$$(7)\quad CALFWT_t = 412.34 + 28.05(TAGE_t) - 2.05(TAGE_t)^2 + V_2 \qquad \bar{R}^2 = .07$$
$$\qquad\qquad (42.675)\quad (9.382)\quad (-10.072)$$

where

$COWWT_t$ = the expected cow weight at the end of period t;

$CALFWT_t$ = the expected calf weight at the end of period t;

$TAGE_t$ = the age of the cow at the end of period t, which is t + .5 years;

$V_i (i=1,2)$ = the error term which is assumed to be distributed $N(0, \sigma^2)$; and

$\bar{R}^2$ = the adjusted coefficient of determination.

The t values are in parentheses below the estimated parameters. The t values indicate that cow age significantly influences cow weight and calf weight. Since steer and heifer weights were not defined individually, it was arbitrarily assumed that steer weights are 1.07 times greater than heifer weights. Therefore for any calf weight,

which is the average weight of the steer and heifer weights, the following relationships hold.

(8) $HFRWT_t = .9662 * CALFWT_t$

(9) $STRWT_t = 1.0338 * ALFWT_t$

where

$HFRWT_t$ = a heifer calf's weight at the end of period t; and

$STRWT_t$ = a steer calf's weight at the end of period t.

A weaned heifer calf selected as a replacement was assumed to be a larger and heavier calf than the average heifer calf. The expected weight of a replacement heifer calf was calculated as the simple average of the three largest heifer calf weights as a function of cow age. Weaned replacement heifer calf weight is equal to 490 pounds and constant for all production periods. Calf weight, heifer weight, steer weight, and weaned replacement heifer calf weight with their corresponding relationships are presented in Table 5.

Table 5. Cow and calf weights as functions of cow age.

Production period (t)	Cow age at end of t (TAGE)	Cow weight ($COWWT_t$)	Average Calf Weight ($CALFWT_t$)	Heifer Calf Weight ($HFRWT_t$)	Steer Calf Weight ($STRWT_t$)	Weaned Replacement Heifer Calf Weight (WRHWT)
	(pounds)					
1	1.5	963.7	0.0	0.0	0.0	0.0
2	2.5	1023.8	469.7	453.8	485.5	490.0
3	3.5	1073.3	485.4	469.0	501.8	490.0
4	4.5	1117.4	497.1	480.3	513.9	490.0
5	5.5	1141.1	504.6	487.5	521.7	490.0
6	6.5	1159.2	508.1	490.9	525.2	490.0
7	7.5	1166.9	507.4	490.3	524.6	490.0
8	8.5	1164.0	502.7	485.7	519.6	490.0
9	9.5	1150.7	493.8	477.1	510.5	490.0
10	10.5	1127.0	480.9	464.6	497.1	490.0
11	11.5	1092.7	463.8	448.1	479.5	490.0
12	12.5	1048.0	442.7	427.7	457.6	490.0
13	13.5	992.7	447.4	403.3	431.5	490.0

Prices

Nominal monthly prices for cull cows, open cows, steer calves (400-500 lbs), and heifer calves (400-500 lbs) from 1970 through 1993 were obtained from the Billings, Montana market. Real quarterly prices were calculated using the consumer price index (December 1993 = 100). The prices for cull cows and open cows were represented by the canner and utility cow prices, respectively.

Missing data points for steer and heifer calf prices were interpolated for the third quarter of 1970 and 1983, and a missing data point for cull cow price was

interpolated for the third quarter of 1980. Cull cow price data (canner price) were unavailable for the fourth quarter of 1992 and all four quarters of 1993. In order to have a complete data set from 1970 through 1993 for all relevant price series, these missing data were estimated recursively by a simple ordinary least squares procedure. The quarterly canner price data from the first quarter of 1970 through the third quarter of 1992 was regressed on independent explanatory variables (also in quarterly values from 1970 to 1993) to predict the canner price of the fourth quarter of 1992. Then the predicted value of the fourth quarter 1992 canner price was added to the sample and the regression was rerun to predict the canner price for the first quarter of 1993. In this manner each successive prediction was included in the sample to estimate the subsequent missing data point until all missing data were estimated and the canner price series was complete through 1993. Equations (10) through (14) show the successive regressions used to predict the five missing cull cow quarterly price data points.

$$(10)\quad PCCOW_4^{92} = \underset{(.4006)}{1.3790} + \underset{(27.842)}{.4541}CB_4^{92} - \underset{(-2.9809)}{.0032}ICS_4^{92} + U_1 \qquad \bar{R}^2 = .97$$

$$(11)\quad PCCOW_1^{93} = \underset{(.3703)}{1.2597} + \underset{(28.274)}{.4547}CB_1^{93} - \underset{(-2.9930)}{.0032}ICS_1^{93} + U_2 \qquad \bar{R}^2 = .97$$

$$(12)\quad PCCOW_2^{93} = \underset{(.3753)}{1.2562} + \underset{(28.604)}{.4547}CB_2^{93} - \underset{(-3.0241)}{.0032}ICS_2^{93} + U_3 \qquad \bar{R}^2 = .97$$

$$(13)\quad PCCOW_3^{93} = \underset{(.3831)}{1.2566} + \underset{(29.008)}{.4547}CB_3^{93} - \underset{(-3.0643)}{.0032}ICS_3^{93} + U_4 \qquad \bar{R}^2 = .97$$

(14) $PCCOW_4^{93} = 1.2560 + .4547CB_4^{93} - .0032ICS_4^{93} + U_5$

$(.3883) \quad (29.330) \qquad (-3.0898) \qquad \bar{R}_2 = .97$

where

$PCCOW_j^i$ = the price of canner cows where i and j describe the year and the relevant quarter, respectively;

CB_j^i = the canner/cutter carcass price of cow beef in the central U.S. market where i and j describe the year and relevant quarter, respectively;

ICS_j^i = the inventory of commercial cow slaughter in the U. S. where i and j describe the year and quarter, respectively;

U_i (i=1...5) = the error term distributed $N(0,\sigma^2)$; and

$\bar{R}^2$ = the adjusted coefficient of determination.

The t values are in parentheses below the estimated parameters.

From this data set, simple linear price relationships were estimated for each of the prices of cull cows, open cows, and heifer calves as a function of the price of steer calves. Available weekly market reports (also from Billings, Montana) on fourth quarter prices of pregnant stock cows from 1976 to 1993 were also compiled and converted into real prices (December, 1993 = 100). The relationship between pregnant cow prices and steer calf prices was estimated with the fourth quarter pregnant cow price as a linear function of the fourth quarter steer calf price and a quadratic function of the cow's age. The following equations describe the prices of open, cull, and pregnant cows and heifer calves as functions of the price of steers.

(15) $POCOW = .0136 + .5404(STRP) + U_1$

$(.3120) \quad (16.480) \qquad \bar{R}^2 = .9488$

(16) $PCCOW = -.0280 + .4772(STRP) + U_2$
$(-0.7392) \quad (16.366) \qquad \bar{R}^2 = .9369$

(17) $HFRP = -.0199 + .9000(STRP) + U_3$
$(-0.7291) \quad (42.636) \qquad \bar{R}^2 = .9885$

(18) $VPGCOW_t = 5.044 + 694.65(STRP) + 29.568(SAGE_t) - 5.295(SAGE_t)^2 + U_4$
$(-.0763) \quad (12.782) \quad (2.5429) \quad (-4.7176)$
$\bar{R}^2 = .7283$

where

POCOW = the price of open cows which is the utility cow price;

PCCOW = the price of cull cows which is the canner cow price;

HFRP = the price of 400 - 500 pound heifer calves;

STRP = the independent variable which is the price of 400-500 pound steer calves;

$VPGCOW_t$ = the fourth quarter value of a pregnant stock cow;

$SAGE_t$ = the reported age of the pregnant cow marketed at the end of period t, and is equal to t+1 years old;

$U_i (i=1...4)$ = the error term distributed $N(0,\sigma^2)$; and

$\bar{R}^2$ = the adjusted coefficient of determination.

The t values are in parentheses below the parameters.

When a pregnant cow is marketed in the fall on a per head basis (as a stock cow), her reported age is the age she will be by the subsequent spring. A cow marketed as a three year old cow is actually 2.5 years old, but this model measures her age at the midpoint of the period and labels her as two years old. Hence, reported selling age ($SAGE_t$) is t+1.

A buyer purchasing a cow as a pregnant cow is placing a value on her expected production that is a premium over her slaughter value. A cow's expected production in terms of calf weight is a function of her age. As she ages, her output, measured as calf weight, first increases and then declines. This function is well known by buyers, therefore the equation used to estimate the value of pregnant stock cows is specified as a quadratic function of age. The maximum value of a pregnant stock cow occurs at 2.8 years of age and decreases thereafter. Virtually no observations were available for older cows (over nine years of age), so it is not surprising that estimated values from this equation fall below the value of cull cows when cow age exceeds eight years. This problem is remedied by using the maximum of cull cow value or pregnant cow value for a given age. Therefore, in this model, a pregnant cow who has met all performance standards and who is to be marketed in the planned replacement period, T, is expected to be sold for the maximum of cull cow value or pregnant cow value.

Costs

The costs necessary to complete the analysis are based on a case study of an individual ranch in eastern Montana. Cows in their first period require different management practices than cows in period two to thirteen. Therefore separate costs of production budgets were prepared for one-year-old cows and cows older than one year of age. Table 6 shows the long-run average variable cost summary of cows one

Variable costs vary widely from ranch to ranch given the heterogenous nature of management practices and resource endowments. Therefore the relative high costs shown in Table 6 were varied (parameterized) to provide model results for varying cost situations.

Relevant variable costs not included in Table 6 are marketing costs; they are, however, included in the model. Marketing costs are incurred at the end of the period, when livestock are sold, and include transportation costs, commission costs, and weight shrinkage. These costs are assumed to be at the following levels: transportation cost is $.01/pound, commission cost is 1.5% of total revenue received from sales, and shrink is 4% of animal weight. Since these costs are functions of weight, they vary with the age of the animal under consideration.

Interest charges on the value of inputs and all variable costs (with the exception of marketing costs) were calculated assuming the costs were incurred at the midpoint of a period. The opportunity cost assessed on the value of the inputs was the real rate of interest (assumed to be 5%). Death loss was assumed to be uniformly distributed throughout a given period such that, on average, death occurred at the midpoint of the period.

Another cost relevant to the model is the opportunity cost of a replacement heifer. In this study, this cost is incurred when the cow is originally purchased and again when a cow is replaced due to either planned or unplanned replacement. It is referred to as the "acquisition cost" in the model. Acquisition cost is equal to the opportunity revenue, net of marketing costs, from a 490-pound replacement heifer.

The data describing production periods relative to cow age and calendar year, stochastic elements, prices, costs, and applicable interest rate, with all associated assumptions can now be incorporated into Burt's model in order to compute the optimal planned replacement age.

CHAPTER 5

RESULTS OF THE EMPIRICAL MODEL REPLACEMENT POLICY AND PROFITABILITY

This chapter specifies the empirical decision model by incorporating the relevant data into the theoretical model defined in Chapter 3. Variable relationships and the notation used to express these relationships are defined. The results of the decision model are then presented.

In order to incorporate the previously defined data with its associated notation into Burt's model, additional variables must be introduced. They are:

SRKFAC = the percentage of total animal weight net of shrinkage due to transportation to market.

CMFAC = the percentage of total revenue received from sales net of commission cost.

TRNRTE = transportation cost, measured in dollars per pound.

VWRHFR = the value of a weaned replacement heifer (net of all marketing costs) which is the opportunity cost of a replacement heifer calf. It is referred to as the acquisition cost of replacement, measured in dollars per head.

In reference to all previously defined variables (summarized in Table 7), the following relationships are given in (19) through (23).

(19) $H_t = PLB_t * PCSTW * SRKFAC * CMFAC * (PSTR) * [(STRWT_t * STRP) + (HFRWT_t * HFRP)] - [PLB_t * PCSTW * CALFWT_t * SRKFAC * TRNRTE]$

(20) $D_t = SRKFAC * CMFAC * COWWT_t * [(PCCOW * PC_t) + (POCOW * PO_t)] - TRNRTE * SRKFAC * COWWT * (PC_t + PO_t) - (1 - P_t) * VWRHFR$

(21) $C_T = [(PCCOW * COWWT_t * SRKFAC * CMFAC) - (TRNRTE * COWWT_t)] - VWRHFR$

or

(22) $C_T = [5.05 + 694.65 * STRP + 29.568 * SAGE_t - 5.2952 * (SAGE_t)^2] * CMFAC - (TRNRTE * COWWT_t) - VWRHFR$

where C_T is the maximum of (21) and (22).

(23) $R_t = (1 - PD_t) * H_t + D_t - (1 - PD_t) * [COST_t * (1 + i/2)]$

where

H_t = all of the revenue, net of marketing costs, that is generated by the cow in period t in the absence of replacement. Therefore, H_t includes all revenue generated by her calf sale in period t net of marketing costs. Thinking in terms of a single cow unit, calf revenue is the calf sales from the proportions of the cow that are pregnant, open, and culled by the end of period t. H_t also includes all probabilities associated with the calf;

D_t = the revenue generated by involuntary replacement in period t. D_t is a cow sale revenue and includes sales from the proportions of the cow that are open and culled, net of marketing costs, less the acquisition cost of a replacement cow. The probabilities associated with D measure the likelihood of receiving revenues from the sale of the proportions of the cow that are open and culled and the total likelihood of paying the acquisition cost from the proportions of the cow that are open, culled, and dead;

C_T = is the revenue generated by voluntary replacement of the cow. C_T can only occur in the planned replacement period T. It includes only the sale of a pregnant cow (who has satisfied all performance criterion), net of marketing costs, less the acquisition cost of a replacement cow. Equation

(21) defines C_T if the cow is marketed as a cull cow and (22) defines C_T if the cow is sold as a pregnant stock cow. This model specifies C_T as the maximum of cull cow value or pregnant cow value; and

R_t = the revenue net of, not only the marketing costs included in H_t and D_t, but also all other variable costs ($COST_t$). $(1 - PD_t)$ adjusts H_t and all other variable costs for death loss during the period t since it was assumed that the revenue and costs of a cow destined to die in t cancel each other. R_t is conditional upon the cow not being involuntarily replaced prior to period t.

Table 7. Summary of variable definitions/notations.

Production Periods	
t	a production period from November 1 in year n to October 31 in year n + 1.
Probabilities	
PO_t	the probability that a cow is open by the end of t.
PC_t	the probability that a cow is culled by the end of t.
PD_t	the probability that a cow will die in t.
P_t	the probability that a cow is pregnant and retained in the herd by the end of t.
$1 - P_t$	the probability that a cow will be replaced due to infertility, culling, or death by the end of t.
$1 - PD_t$	the probability that a cow will not die in t.
PLB_t	the probability that a calf will survive pregnancy and birth.
PCSTW	the probability that a calf will survive until weaning given $PLB_t = 1.0$.
PHFR	the probability that a calf is a heifer calf.
PSTR	the probability that a calf is a steer calf.
Weights	
$COWWT_t$	the weight of a cow by the end of t.
$CALFWT_t$	the average weight of a steer and heifer calf by the end of t.
$HFRWT_t$	the weight of a heifer calf by the end of t.
$STRWT_t$	the weight of a steer calf by the end of t.
WRHWT	the weight of a replacement heifer calf at the end of t.

Table 7. Summary of variable definitions/notations (continued).

Prices/Values	
STRP	the price per pound of a 400-500 pound steer calf.
HFRP	the price per pound of a 400-500 pound heifer calf.
PCCOW	the price per pound of a culled cow (canner cow price).
POCOW	the price per pound of an open cow (utility cow price).
CB	the canner/cutter carcass price of cow beef.
ICS	the inventory of commercial cow slaughter.
$VPGCOW_t$	the fourth quarter value of a pregnant stock cow.
VWRHFR	the value of a weaned replacement heifer which is the acquisition cost in the model.
Costs	
$COST_t$	the annual production cost accrued by a cow in t.
TRNRTE	the transportation cost.
CMFAC	the percentage of total sales revenue net of commission costs.
SRKFAC	the percentage of total weight net of shrink.
i	the real rate of interest.
Age	
Cow age	the age of a cow at the midpoint of period t.
TAGE	the age of a cow at the end of period t.
SAGE	the reported age of a stock cow marketed at the end of t.
Other	
β	the discount factor, 1/1 + i.

Using (19) through (23), equation (1) of Burt's general replacement model is specified by (24).

$$(24) \quad EPV_T = \beta R_1 + \beta^2 P_1 R_2 + \beta^3 P_1 P_2 R_3 + \ldots + \beta^T P_1 P_2 \ldots P_{T-1}(R_T + P_T C_T)$$

Burt defined C_T as the cost of voluntary replacement. However, in this study C_T, which is the disposal value less the acquisition cost, is a positive term of EPV_T.

Since all net revenue was assumed to be received at the end of a period, each period was discounted by the relevant powers of β to convert R_t to a present value. The probabilities in each term in (24) convert the measure of R_t, which is a conditional revenue, into an unconditional revenue that accounts for the probability that the asset will reach period t in a state of normal productivity. Therefore, the probability of reaching period T is $P_1 * P_2 * \ldots * P_{T-1}$ which can be referred to as the probability of the cow being in the herd at the beginning of T.

However, Burt stated that the expected value of net returns from an infinite planning horizon is the sum of the expected present value of returns from the first asset, EPV_T, and the expected present value of returns from all future assets. This was illustrated by (5) and is reintroduced in (25) which uses the specification of EPV_T from (24).

$$(25) \quad ERPV_T = \frac{EPV_T}{(1-\beta)\sum_{t=1}^{T} w_T} - VWRHFR$$

where

$ERPV_T$ = the expected present value of net returns generated by an infinite series of expected present values of net returns from a single asset (over a cycle of length T) or the capitalization of a weighted average of expected net revenues into perpetuity;

EPV_T = the expected present value of net returns from a single asset over its life to time T as defined by (24);

$\frac{1}{(1-\beta)\sum_{t=1}^{T} w_t}$ = the general discount rate, where the discount factor, β is weighted by the probability distribution of the assets life, L(T), as is shown in (3) and (4);

T = the production period t in which voluntary replacement occurs over an infinite planning horizon; and

VWRHFR = the acquisition cost of the original cow.

$ERPV_T$ was amortized in order to compute an annual constant return from each infinite series of possible cycle length T. This procedure is shown by (26).

(26) $AR_T = ERPV_T * i$

where

AR_T = the annualized return of the expected net present value of returns of cycle length T over an infinite planning horizon, and

i = the real rate of interest, which is assumed to be .05.

In order to find the optimal planned replacement policy, $ERPV_T$ was calculated (and amortized) for cycles of length T (T = 1, 2, ..., 13).

Results

This model specifies that the calculation of $ERPV_T$ is in the context of constant prices and costs. However, cattle prices vary widely over time; therefore prices are parameterized and results for varying price levels are obtained. Each price level is used as the constant price with which to maximize $ERPV_T$ with respect to T. In the defined price relationships, the price of steer calves is the independent variable used to estimate the prices of cull cows, open cows, heifer calves, and pregnant (stock) cows. Given these relationships it is obvious that a change in the price of steers precipitates changes in the prices of cull cows, open cows, heifer calves, and pregnant stock cows. Therefore it is necessary only to vary the steer calf price in order to

examine the results of the model under varying price levels. The standard price around which steer prices are varied is the mean fourth quarter steer calf price, which is $1.18 per pound over the data sample. A ten percent increase and a ten percent decrease in the mean steer calf price are specified.

These price levels are representative since the maximum and minimum fourth quarter real steer calf prices over the data sample were $1.84 and $0.80, respectively (December 1993 = 100). The price levels are shown in Table 8.

Table 8. Steer calf price levels (per pound).

Mean Steer Price	10% Increase in Steer Price	10% Decrease in Steer Price
$ 1.18	$ 1.29	$ 1.06

Cost levels are also parameterized. Variable production costs for a given ranch vary widely due to the heterogeneous nature of resource endowments and management practices. For these reasons the variable cost summary compiled for this study and shown in Table 6 is the standard from which costs are varied. Since the summary of variable costs listed in Table 6 are thought to represent a high cost producer, two other successively lower cost levels are specified. The costs shown in Table 6 are decreased by twenty and thirty percent, respectively. The resulting cost levels are exhibited in Table 9.

Table 9. Variable costs per cow, by cow age.

Cow Age (t)	High Variable Cost	20% Lower Variable Cost	30% Lower Variable Cost
1	$ 289.17	$ 231.34	$ 202.42
2 to 13	298.37	238.70	208.86

The optimal intended replacement age policy sought is one in which $ERPV_T$ and AR_T are maximized with respect to T. According to Burt, this method will satisfactorily derive an optimal replacement age policy only if $ERPV_T$ is unimodal or increasing over the range of T.

$ERPV_T$ and AR_T are calculated for each possible cycle length T (T=1, 2, ... 13). Each price level is specified with each level of variable costs. Using the prices and costs introduced in Tables 8 and 9, $ERPV_T$ and AR_T are presented for each product price/cost combination in Tables 10, 11, and 12.

Table 10. The expected present value of net returns and the amortized expected present value of net returns under a mean steer price with variable costs at varying levels.

	High Variable Costs		20% Lower Variable Costs		30% Lower Variable Costs	
T	Present Value	Amortized Present Value	Present Value	Amortized Present Value	Present Value	Amortized Present Value
1	$ 634.46	$ 31.72	$ 1779.58	$ 88.98	$ 2352.14	$ 117.61
2	1126.95	56.35	2287.33	114.37	2867.52	143.38
3	1276.45	63.82	2439.27	121.96	3020.68	151.03
4	1337.68	66.88	2500.96	125.01	3081.54	154.08
5	1361.73	68.09	2523.23	126.16	3103.98	155.20
6	1367.69	68.38	2527.88	126.39	3107.98	155.40
7	1365.21	68.26	2524.12	126.21	3103.57	155.18
8	1360.00	68.00	2517.76	125.89	3096.64	154.83
9	1355.48	67.77	2512.27	125.61	3090.67	154.53
10	1368.23	68.41	2524.27	126.21	3102.29	155.11
11	1371.22	68.56	2526.70	126.33	3104.44	155.22
12	1369.73	68.49	2524.82	126.24	3107.36	155.12
13	1366.85	68.34	2521.69	126.08	3099.11	154.96

Table 11. The expected present value of net returns and the amortized expected present value of net returns under a steer price 10% greater than the mean steer price with variable costs at varying levels.

	High Variable Costs		20% Lower Costs		30% Lower Costs	
T	Present Value	Amortized Present Value	Present Value	Amortized Present Value	Present Value	Amortized Present Value
1	$ 1209.03	$ 60.45	$ 2354.15	$ 117.71	$ 2926.71	$ 146.34
2	1798.21	89.91	2958.59	147.93	3538.78	176.94
3	1981.01	99.05	3143.83	157.19	3725.25	186.26
4	2058.66	102.93	3221.23	161.06	3802.52	190.13
5	2091.27	104.56	3252.77	162.64	3833.52	191.68
6	2101.22	105.06	3261.41	163.07	3841.51	192.08
7	2099.88	104.99	3258.79	162.94	3838.24	191.91
8	2094.04	104.70	3251.79	162.59	3830.67	191.53
9	2087.64	104.38	3244.43	162.22	3822.83	191.14
10	2093.20	104.66	3249.23	162.46	3827.25	191.36
11	2096.14	104.81	3251.62	162.58	3829.36	191.47
12	2094.26	104.71	3249.35	162.47	3826.90	191.34
13	2096.95	104.55	3245.79	162.29	3823.20	191.16

Table 12. The expected present value of net returns and the amortized expected present value of net returns under a steer price 10% less than the mean steer price with variable costs at varying levels.

	High Variable Costs		20% Lower Costs		30% Lower Costs	
T	Present Value	Amortized Present Value	Present Value	Amortized Present Value	Present Value	Amortized Present Value
1	$ 59.89	$ 2.99	$ 1205.01	$ 60.25	$ 1777.57	$ 88.88
2	455.69	22.78	1616.07	80.80	2196.26	109.81
3	571.88	28.59	1734.71	86.74	2316.12	115.81
4	616.71	30.84	1779.28	88.96	2360.57	118.03
5	632.20	31.61	1793.70	89.68	2374.45	118.72
6	634.15	31.71	1794.35	89.72	2374.44	118.72
7	630.54	31.53	1789.45	89.47	2368.91	118.45
8	625.96	31.30	1783.72	89.19	2362.60	118.13
9	631.31	31.57	1788.11	89.41	2366.51	118.33
10	643.26	32.16	1799.30	89.97	2377.32	118.87
11	646.30	32.31	1801.78	90.09	2379.52	118.98
12	645.19	32.26	1800.28	90.01	2377.83	118.89
13	642.76	32.14	1797.60	89.88	2375.01	118.75

A unimodal maximum for $ERPV_T$ and AR_T was not obtained in the results under the varying price/cost situations specified. Instead, a pattern of a bimodal maximum for $ERPV_T$ and AR_T over the range of T emerged. This pattern was consistent for each price/cost scenario specified in Tables 10, 11, and 12. The first maximum was achieved at $T = 6$ and the second at $T = 11$. Although the bimodal maxima were invariant with respect to varying price/cost situations, the values at the bimodal maxima varied under the different price and variable cost assumptions.

Although the mathematical analysis resulted in a bimodal maxima, a judgement can still be made with respect to an optimal replacement age. The differences in the maximums of AR_6 and AR_{11} for all price/cost combination specified are very small. Over all results, the greatest difference is $0.61 and the smallest difference is $0.06 indicating that there is little difference in the financial returns associated with replacement policies of six and eleven years. In addition, within each price/cost specification the variance in AR_T for T = 1, 2, ... 13 is very small. Although this model fails to derive a single optimal planned replacement age, it does suggest that voluntary replacement should occur between cow ages of six and eleven years and that the cost of delaying or speeding up a voluntary replacement decision within this range is minimal, given the model assumptions.

Although the price/cost levels imposed on the model are arbitrary, the results are useful as profitability indicators for producers who are analyzing long-run profit/loss under conditions similar to those imposed by this model. This study has not assumed an opportunity cost of the pasture in the variable cost summary. This implicitly implies that the grass harvested by a producer's cow has no alternative use other than to be harvested by another producer's cow. Therefore the annual return shown by AR_T is the return to the grass of the producer. Using USDA private fee estimates for rangeland rental (May 1994) for Montana and the management practices of this case study, a rancher's annual expected net revenue would need to be $90.00 per cow in order to recover the rangeland rental rate. At returns below $90.00 not all factors represented in the variable cost estimates would recover their full opportunity costs.

The results in Tables 10, 11, and 12 show that cattle ranching is profitable except in high cost with average to below average model price conditions. However, the $90.00 opportunity rental rate does not include the costs incurred by an owner/operator who makes the transition from pasturing his own cattle to pasturing cattle belonging to another producer. It is reasonable to assume that these conversion costs, if measurable, would reduce the opportunity cost associated with the rangeland use.

Unprofitable conditions can only be sustained in the short run. Except for the high cost/low price and high cost/mean price situations, all inputs to production recovered their opportunity costs, i. e., a return of at least $90.00 per cow annually. Although these two price/cost combinations did not provide full opportunity cost compensation, they did provide positive returns to the rangeland use.

The probability that a producer will face the planned replacement decision for an older cow is small because the probability that she will survive involuntary replacement is small. The cow that has achieved longevity with normal productivity is a special case given the odds of involuntary replacement she faces as she ages. Therefore the producer's greatest influence on the replacement decision is through management practices which affect the probabilities of a cow being pregnant, open, culled, and even dead over any given period.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The purpose of this study was to specify a procedure for individual ranchers to use to determine a long-run optimal replacement policy for beef cows. Optimal intended cow replacement in the face of productivity uncertainty resulting from stochastic elements was examined. An eastern Montana ranch with specific management practices was used to develop benchmark variable costs.

The asset replacement problem specific to cows and the need for a consistent procedure that could be used by individual cattle ranchers to determine a long-run cow replacement policy was identified. A short history of pertinent research was discussed.

Burt's general model addressing the problem of optimal replacement under risk was presented and explained. The stochastic elements that result in productivity uncertainty were defined. Static conditions imposed on the model included constant prices and costs, probabilities associated with the stochastic elements that are invariant with respect to time, and a constant herd size. An infinite planning horizon was specified.

Data used in the model were presented and production periods with respect to cow age and year were defined. The probabilities associated with the stochastic elements specific to cows were taken from work by Tronstad and Gum and others. The components of net revenue, which include animal weights, prices, and costs, were defined and discussed.

The results derived from the empirical decision model were presented. An empirical decision model was implemented by incorporating the data into the theoretical general asset replacement model. Several combinations of price/cost levels were examined to compute the respective optimal replacement ages and associated profitability measures.

Bimodal maxima, which was invariant with respect to T over all price/cost levels, were obtained for amortized returns. The profitability analyses indicate that there are insignificant differences in the net returns associated with replacement policies beyond six years. Although this model failed to derive a single optimal planned replacement age, it does suggest that voluntary replacement should occur between cow ages of six and eleven years, and that the cost of delaying or speeding up a voluntary replacement decision within this range is minimal.

The results indicate that cattle ranching is profitable except with mean to below mean cattle prices associated with high variable long-run average costs per cow.

This study has provided interesting insights about the determinants of an optimal replacement policy. Different management practices give different probability

measurements of stochastic events determining involuntary replacement and, therefore, planned replacement. Given the insensitivity of the bimodal maxima pattern to the different price/cost levels, there is a high likelihood that the probabilities assigned to the stochastic elements are the main determinants in the derivation of an optimal replacement age.

A useful direction of further research in this area would be to assimilate contemporary data relative to the probabilities associated with stochastic elements of a ranch in an eastern Montana environment in order to compute a long-run replacement policy. The data used for the stochastic elements were compiled in the southwestern United States, where the typical cow producer faces a different production environment than cow producers in eastern Montana. Depending on the sensitivity of the model, changes in the probabilities associated with the stochastic elements could conceivably affect the profitability and optimal replacement age found by this study. For individuals with data on their own cow herds, the model can provide a means to aid in determining cow replacement age specific to their cow-calf operation.

A second extension of considerable interest would be to assess the value of pregnancy testing. It is not uncommon for a producer not to pregnancy test his herd. If the probability of an open cow in the current period becoming pregnant in the subsequent period were available, results from a different culling criteria could be compared to the results of this study to determine the value of pregnancy testing.

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