Biofilm growth in a homogeneous porous medium
by Sunil Kumar Tiwari

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mathematics
Montana State University

Abstract:
As biofilms intervene in a porous medium, they affect the porosity and permeability, which in turn alters the hydrodynamics. A biofilm growth model is presented which is suitable for microscale simulations of biofilm activity in a porous medium. The model is then used to predict the changing porosity and permeability. The predictions are compared to experimental data and finally these calculated properties are used to simulate flow in a biofilm infested porous medium.

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MONTANA STATE UNIVERSITY
Bozeman, Montana

January 1997

APPROVAL

of a thesis submitted by

Sunil Kumar Tiwari

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ACKNOWLEDGEMENTS

I am indebted to my advisor, Dr. Kenneth L. Bowers, for his guidance, support and encouragement throughout my association with him. In particular, his criticisms and suggestions during the writing of this thesis have been invaluable. My appreciation is also extended to professors Jack Dockery and Gary Bogar for their careful reading of this manuscript and their technical guidance.

I am grateful to Dr. Ken Tiahrt and Dr. John Lund for giving me the opportunity to continue my graduate studies at Montana State University. Thanks are also due to the rest of the faculty and my fellow graduate students who made my stay here such an enjoyable and intellectually rewarding experience. Special appreciation is due to the office staff of the Department of Mathematical Sciences who were very helpful.

Finally, I wish to thank my wife, Kala, for her patience, encouragement and support during these past years.

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ABSTRACT

As biofilms intervene in a porous medium, they affect the porosity and permeability, which in turn alters the hydrodynamics. A biofilm growth model is presented which is suitable for microscale simulations of biofilm activity in a porous medium. The model is then used to predict the changing porosity and permeability. The predictions are compared to experimental data and finally these calculated properties are used to simulate flow in a biofilm infested porous medium.

CHAPTER 1

Introduction

The goal of this dissertation is to develop a mathematical model to describe the change in the hydrodynamic properties of a porous medium due to biofilm growth. A one-dimensional mathematical model called the Biofilm Growth Model (BGM) has been developed which describes the growth of biofilm on a surface. The results of the one-dimensional BGM and its zero-dimensional version have been compared. Also the results of the zero-dimensional BGM and the zero-dimensional versions of two existing models called the Biofilm Accumulation Model (BAM) and Rittman's model have been compared. Different model equations relating the porosity and the permeability of a porous medium have been discussed and some of the relations have been compared with the experimental data from [5], [6]. A complete porous media model describing the effect of biofilm growth on porosity, permeability, and hence the flow, has been presented. Finally, the numerical results of the one-dimensional porous media flow model simulations have been presented and these numerical results are compared with the experimental data from [5], [6].

Controlled, artificially grown biofilm in porous media provides significant opportunities for improving the performance of industrial and environmental processes. The petroleum industry, for example, uses biofilm for deliberate plugging of parts of the oil reservoir to enhance oil recovery. A controlled biofilm accumulation in high permeability zones can be used to prevent injection water from reaching the production well. This can be accomplished by injecting cells and nutrients into the oil-bearing formation, [16], [19]. Also, controlling biofilm accumulation is important

to both injection and production well operation in order to avoid unwanted formation plugging near the well bore. The environmental industry likewise uses deliberate plugging of pore channels between spilled contaminants and lakes or rivers to prevent the contamination of these water resources. Subsurface biofilms also offer the potential for biotransformation of organic compounds providing an in situ method (Figure 1) for treating contaminated groundwater supplies, [4], [26]. The mining industry is developing methods for microbially enhanced leaching of metals from ores and recovery of metals from solutions, [11]. An efficient use of biofilm by engineers

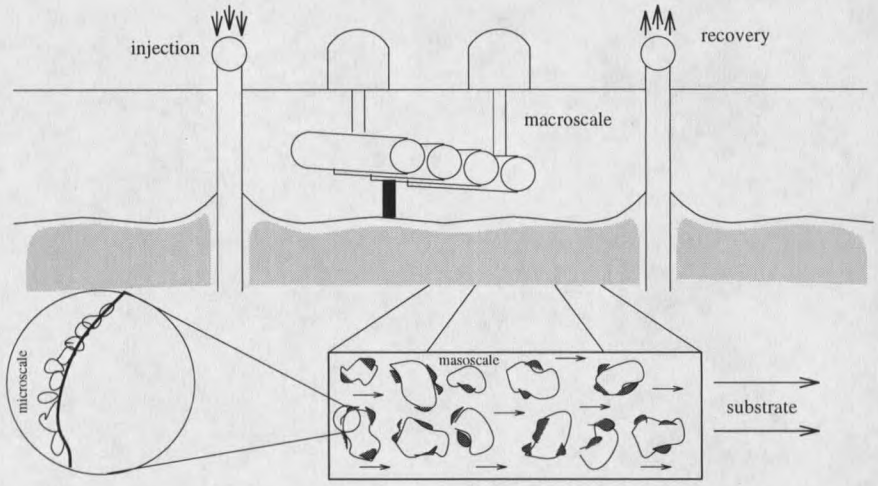

Figure 1: Injection/recovery scheme for enhancing in situ bioremediation of contaminated aquifer. Inset shows the growing biofilm in porous media.

requires an improved understanding of the interrelationship between porous media hydrodynamic properties and the accumulation rate and spatial distribution of the biofilm.

The pressure driven flow between two points in an isotropic homogeneous porous medium depends on the pressure gradient between the two points and the viscosity of the liquid flowing through it as well as the porosity and the permeability of the medium, [2], [7], [20]. If the pressure gradient between the two points and the viscosity of the liquid are assumed to be constant then the porosity and the

permeability dictate the flow rate. If one assumes the presence of an initial thin layer of biofilm on the pore surface and a high concentration of substrate (food for the bacteria) in the liquid flowing through the porous medium then it is reasonable to expect a decrease in the porosity and the permeability of the medium due to the biofilm growth in it. How fast does the biofilm grow? How does the flow rate change? In this work, a mathematical model has been developed that should help in the understanding of these issues.

In order to derive a mathematical model to study the effect of biofilm growth on porous media flow, one must combine the system of flow equations with a system of biofilm growth equations. A one-dimensional mathematical model called the Biofilm Growth Model (BGM) has been developed in Chapter 2 which describes the growth of a biofilm on a surface. The one-dimensional Biofilm Accumulation Model (BAM, [14], originally developed in [25]) and Rittman's model (originally developed in [12]) have also been described in Chapter 2. Based on Rittman's zero-dimensional model, the zero-dimensional versions of BAM and BGM have also been derived. A zero-dimensional model ignores the spatial dependence of the variables, namely the substrate concentration in the biofilm and the volume fraction of the active and inactive bacteria in the biofilm and describes the change in the spatial average value of these variables with respect to time.

A comparison, undertaken in Chapter 3, of the numerical solutions of one-dimensional BGM and zero-dimensional BGM show that (*i*) the qualitative behavior of the solutions from both the models are very similar and they are also quantitatively close, and (*ii*) the zero-dimensional model equations are comparatively easy to solve and the computation time is much less than the computation time for the one-dimensional model. The zero-dimensional model lacks some of the features which the one-dimensional model possesses. For example, the substrate concentration or the

active biomass volume fraction at a certain point in the biofilm can not be computed with a zero-dimensional model. However, since we intend to use the biofilm growth model in a porous medium setting, the zero-dimensional BGM is chosen over the one-dimensional BGM. This is because the zero-dimensional model efficiently describes the average change in the substrate concentration and the biofilm thickness which suffices our need. After a comparison of the three zero-dimensional models (BGM, BAM, and Rittman's model), also completed in Chapter 3, the zero-dimensional BGM has been chosen over the other two models primarily because BGM assumes the bulk volume (in the case of a porous medium it is the pore volume) to be a variable as opposed to the other two models (BAM and Rittman's model) in which the bulk volume is assumed to be a constant. This is necessary in order to study the effect of biofilm growth on porosity and hence permeability.

The relation between porosity and permeability of a porous medium has been investigated by many researchers in the past. A collection of experiment-based algebraic relations between porosity and permeability and a list of references can be found in the Chapter 3 of [9]. Some of the relations which are relevant for the models developed in this dissertation are described here in Chapter 4 and the validity of two of such formulas has been checked against the experimental data given in [5], [6]. Finally in Chapter 4, the complete model describing biofilm growth in a porous medium and its effect on porosity and the permeability of the medium is formulated.

The complete model describing the biofilm growth in a homogeneous porous medium and its effect on the one-dimensional incompressible fluid flow through the medium is numerically solved in Chapter 5 and numerical solutions are presented. The model equations are solved for a short bed (5 cm long) of spherical balls and a long bed (60 cm long) of spherical balls. The numerical results for a short bed experiment have been compared with the experimental data from [5], [6]. Lastly, the

change in the variables over time is predicted for a long bed experiment.

CHAPTER 2

Biofilm Models

Introduction

What follows is the discussion of biofilm growth on a surface. In addition, the derivation of one-dimensional and zero-dimensional mathematical models which describe these physical phenomena is given.

Consider a water filled tank with an initial biofilm thickness on one of its walls. Assume that the substrate (food or nutrient) for the bacteria is dissolved in the water and diffuses into the biofilm. The bacteria in the biofilm will consume the substrate and will multiply. The growth of the biofilm (increase in thickness) depends on the volume fraction of the active biomass, the substrate concentration in the biofilm, the substrate concentration in the bulk liquid in the tank, the rate of the consumption of the substrate by the bacteria, the diffusion rate of the substrate in the biofilm, and possibly other factors. In fact, the growth of the biofilm is a very complicated phenomenon and to produce an accurate mathematical model is not easy. This physical system is shown in Figure 2, where $L(t)$ is the biofilm thickness at time t, $u(L,t)$ is the biomass velocity at the film-water interface and Q is the volumetric flow rate of the influent fluid.

A similar problem has been discussed in [12] and [25] and one-dimensional models have been derived that describe the growth of the thickness of a biofilm with respect to time and space. The model developed in [12] is also called **Rittman's**

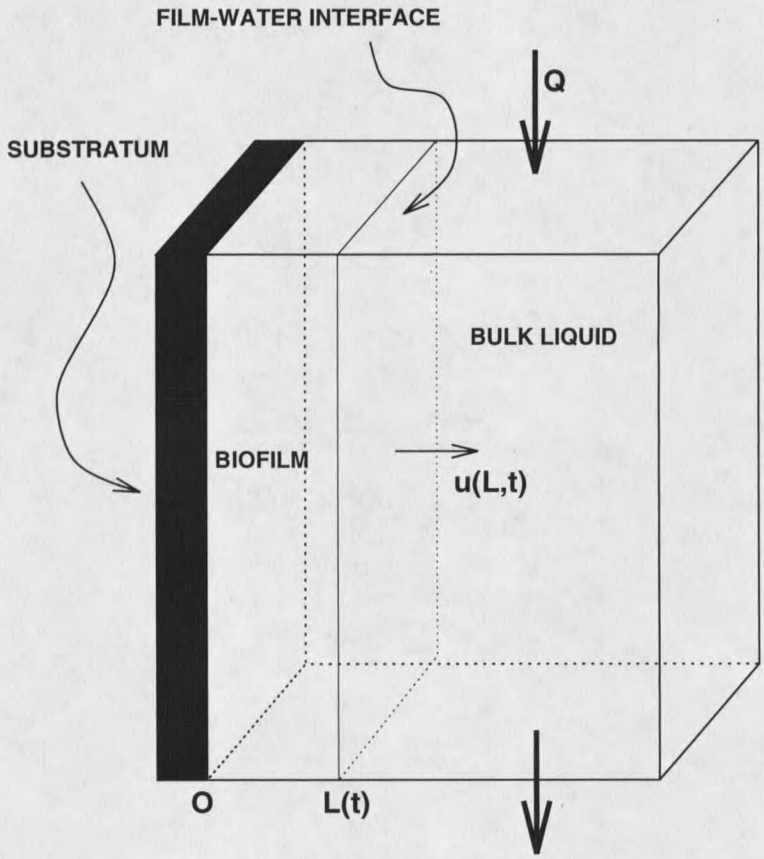

Figure 2: Biofilm on a surface

Model. The Center for Biofilm Engineering at Montana State University, has developed a computer simulator called the **Biofilm Accumulation Model, (BAM)**, [14] which studies the growth of biofilm on a flat surface. The model equations are based on [25]. Both of these models assume the volume of the bulk liquid is constant. Also some of the terms in the model equations are not consistent with the underlying assumptions. Hence a new model called the **Biofilm Growth Model (BGM)** is developed here which considers the volume of the bulk liquid to be a time-dependent function instead of a constant. The equations described in these models cannot be used directly in a problem where biofilm grows in a porous media because of the simplified assumptions of these models and the complicated geometry of the pore surface, however the idea of the zero-dimensional model discussed in [12] is very useful here. In both Rittman's model and BAM, different types of bacteria are consuming different types of substrates and are growing simultaneously. But, for now, in order to keep the notation clean and the problem simple and explainable, we shall restrict ourselves to only one bacteria consuming only one substrate.

Rittman's Model

One-dimensional Model

In this model, originally proposed in [12], the microbial interactions and diffusion phenomena are described by two sets of mathematical equations. The first set is called diffusional equations, the other one transport equations. They are derived from basic mass balances. The following a *priori* assumptions have been made to develop this model:

- The biofilm is homogeneous and can be treated as a continuum.
- Changes occur only in the direction normal to the biofilm surface.

- There is a laminar diffusional sublayer of constant thickness in the bulk liquid.
- The biofilm is composed entirely of active or inactive biomass.

This model predicts four time-dependent functions, two of which depend on the spatial variable y as well. These functions and their fundamental units of length (L) and substrate mass (M_s) are given in Table 1.

Variables	Physical quantity	Units
$L(t)$	biofilm thickness	L
$S_b(t)$	bulk substrate concentration	$M_s L^{-3}$
$S(y,t)$	substrate concentration in the biofilm	$M_s L^{-3}$
$f(y,t)$	volume fraction of active biomass in the biofilm	dimensionless

Table 1: Unknown dependent variables and their fundamental units in Rittman's one-dimensional model

Other variables and the parameters used in the development of this model and their fundamental units of length (L), time (T), substrate mass (M_s), and biomass (M_x), are given in Table 2.

A visualization of the physical system to be modeled is given in Figure 3 where σ is the surface area of the film-water interface.

Diffusional Equations The mass balance equation for the total substrate in the bulk liquid, $V_L S_b$, has the form

$$V_L \frac{d}{dt}(S_b(t)) = Q(S_0 - S_b(t)) - \sigma J(t), \tag{2.1}$$

where V_L is the volume (L^3), Q represents the bulk liquid flow rate ($L^3 T^{-1}$), σ is the surface area of the film-water interface (L^2), S_0 refers to the influent substrate concentration ($M_s L^{-3}$) and $J(t)$ refers to the substrate flux through the laminar diffusional sublayer ($M_s L^{-2} T^{-1}$). V_L, Q, σ and S_0 are assumed constant. According

Parameters	Physical quantity	Units
V_L	volume of the bulk liquid	L^3
Q	volumetric flow rate of the bulk liquid	L^3T^{-1}
S_0	substrate concentration in the influent fluid	M_sL^{-3}
σ	area of the film-water interface	L^2
D	diffusivity coefficient of the substrate through the laminar diffusional sublayer	L^2T^{-1}
L_l	thickness of laminar diffusional sublayer	L
d	diffusivity coefficient of the substrate inside the biofilm	L^2T^{-1}
V_r	maximum specific growth rate	$M_sM_x^{-1}T^{-1}$
K	Monod constant	M_sL^{-3}
ρ	biomass density	M_xL^{-3}
b	inactivation coefficient of bacteria	T^{-1}
Y	yield coefficient	$M_xM_s^{-1}$
G_s	global shear stress coefficient	T
B'	average shear stress coefficient	T^{-1}
f_d	biodegradable fraction of the biomass	dimensionless
Variables	**Physical quantity**	**Units**
$J(t)$	influx rate of the substrate into the biofilm	$M_sL^{-2}T^{-1}$
$u(y,t)$	velocity of the biomass particle at (y,t)	LT^{-1}
$G(y,t)$	active biomass quantity in the interval $[0,y]$	M_xL^{-2}
$c(y,t)$	net growth of active biomass	$M_xL^{-3}T^{-1}$
$R(y,t)$	reaction rate	$M_sL^{-3}T^{-1}$
$b'(y)$	shear function along y-axis	T^{-1}
$f_0(t)$	volume fraction at the substratum	dimensionless
$\bar{G}(y,t)$	inactive biomass quantity in the interval $[0,y]$	M_xL^{-2}
$\bar{f}(y,t)$	volume fraction of inactive biomass in the biofilm	dimensionless
$\bar{c}(y,t)$	net growth of inactive biomass	$M_xL^{-3}T^{-1}$
$B(y,t)$	total biomass amount	M_xL^{-2}

Table 2: Parameters and the variables used in Rittman's model and their fundamental units