THEORETICAL ELECTRICAL POWER OUTPUT PER UNIT VOLUME OF PVF_2 AND MECHANICAL-TO-ELECTRICAL CONVERSION EFFICIENCY AS FUNCTIONS OF FREQUENCY

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abstract

The electrical energy output per unit volume of poly(vinylidene fluoride) (PVF_2) is calculated using the d_{31} piezoelectric coefficient applicable to the bimorph bending mode of operation. Strains approaching the yield strain are considered. The capacitive portion of the source impedance is assumed to be cancelled by a suitable series inductor. Limitations caused by electrical breakdown are considered, because this LC resonance effect can cause voltages across the sample greatly in excess of the emf generated directly by the bending. At series resonance, the power output is limited by the resistive parts of the PVF_2 internal impedance and the inductor impedance. The output power per unit volume is calculated from published values for the frequency dependence of the lossy part of the permittivity for PVF_2. This power density is surprisingly large at frequencies in the kHz range just below the frequencies at which losses become large, and is calculated to be about 100 W/cm^3 at 1 kHz. Also, the mechanical-to-electrical conversion efficiency is calculated. This efficiency can exceed the electromechanical coupling coefficient of about 1% considerably. Its calculated value near 70% is limited only by electrical and mechanical losses.

PERTINENT MATERIAL PROPERTIES

A piezoelectric polymer has seven important design parameters for purposes of obtaining the greatest possible electrical output and efficiency from a mechanical driving source. The mechanical parameters are Young's modulus Y, yield strain δ_y, and mechanical quality factor Q_m, while the electrical ones are dielectric permittivity K, electric breakdown field E_b, and electrical quality factor Q_e. The seventh is the piezoelectric coefficient d_{31} relating electric field across the sheet (3 direction) to strain along the stretch direction (1) within the sheet.

Young's modulus for PVF_2 is the ratio of tensile stress to tensile strain which is quoted[1] as $Y=1.5x10^9$ N/m^2 or $2.2x10^5$ psi. This agrees well with our own measurements.[2] Young's modulus is a measure of stiffness, which is low for plastics, allowing designs which give the necessary flexibility for coupling efficiently to wind and water energy sources without requiring extremely thin sections.

The mechanical quality factor Q_m is the ratio Y'/Y" of the real to the imaginary part of Young's modulus, and is the inverse of the mechanical dissipation factor D_m. A high Q_m is important for good efficiency and even more for low losses which minimize internal heating. A value of 200 for Q_m has been reported for a 25% TrFE (trifluoroethylene) copolymer with vinylidene fluoride at room temperature and 3.5 Hz,[3] which seems to be the highest frequency below the MHz range for which Q_m results are reported.

The yield strain is the fractional change in length $\Delta L/L$ beyond which the polymer will not return to its original length when the applied stress is removed. This occurs[1,2] at 3% strain (δ_y=0.03) for PVF_2. The corresponding yield stress according to Hooke's law is $S_y=Y\delta_y=4.5x10^7$ N/m^2. This high yield strain is fortunate as it allows flexible designs and a large output voltage.

Each of the three above mechanical parameters has its electrical analog. The relative dielectric permittivity K relates the electric displacement D (C/m^2, or coulombs stored per square meter of electrode covering the dielectric) to the electric field E in volts per meter (V/m) or newtons per coulomb (N/C). For an ordinary nonpiezoelectric dielectric this relation is $D=K_0KE$, where $K_0=8.85x10^{-12}$ C^2/Nm^2 is the dielectric permittivity of vacuum. Measurements[4,5] on PVF_2 yield K=12 (neglecting K's small lossy component). This large value, 4 times that typical of plastics, produces relatively large current output for a given piezoelectrically induced electric field.

The electrical quality factor Q_e is the ratio K'/K'' of the real part K' to the imaginary part K'' of the dielectric permittivity K=K'-jK". A high Q_e reduces the source impedance of the piezoelectric generator system when the system includes an inductor as described below. It also reduces heating caused by dielectric losses. The value of Q_e depends on frequency, temperature, and composition (amount of TrFE in the copolymer) as well as on processing, but a value of 50 is typical for frequencies below 2 kHz.[6]

The electric breakdown field E_b in PVF_2 is near[7] $3x10^7$ V/m. This is much higher than the breakdown field in air, so the metal electrodes coating the PVF_2 sheets should stop short of the edge, leaving an uncoated fringe around the edge. The breakdown field in PVF_2 is much higher than can

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be reached by simply stressing a PVF_2 sheet to its elastic limit. It becomes a design limitation if the capacitive source reactance of the piezoelectric generator is resonated away by a series inductor (as discussed below), in which case the voltage across the PVF_2 sheet can be much greater than the piezoelectrically induced emf (electromotive force).

Finally, the piezoelectric strain coefficient d_{31}, quoted[1] as 25 pC/N (25×10^{-12} C/N), which was essentially verified by our measurements,[2] describes the polarization P or electric displacement D as being 25×10^{-12} C/m^2 in direction 3 perpendicular to the sheet for each N/m^2 of tensile or compressive stress along the stretch direction within the sheet. Some ceramics and single crystals have considerably higher piezoelectric coefficients, but their stiffness makes them impractical for wind, hydroelectric, or wave generator applications. Larger values of d_{31} are being attained with improved polymer materials, specifically with copolymers of vinylidene fluoride and trifluoroethylene.[8]

PIEZOELECTRIC GENERATOR CHARACTERISTICS

To calculate the power from a given generator design for a given blade deflection amplitude and oscillation frequency, we begin with the piezoelectric equations

$$\delta_{11}=S_{11}/Y_{1111}+d_{31}E_3, \quad (1)$$

$$D_3=K_0K_{33}E_3+d_{31}S_{11}. \quad (2)$$

For nonpiezoelectric materials for which $d_{31}=0$, these are just uncoupled mechanical and electrical equations. We have omitted pyroelectric and thermal expansion terms which are not very important.[9] They must be considered in an exact analysis because our operating frequencies are high enough that adiabatic rather than isothermal conditions obtain. The subscripts 1 and 3 refer to the directions described above. From here on, these subscripts will be omitted. We eliminate stress S from Eqs. (1) and (2) in favor of the more easily measured strain δ, and solve them simultaneously to obtain

$$D=K_0K(1-Yd^2/K_0K)E+Yd\delta\simeq K_0KE+Yd\delta. \quad (3)$$

Here Yd^2/K_0K is the dimensionless electromechanical coupling constant k^2, which is small (=0.0088) for PVF_2 and can be neglected in this context.

In a piezoelectric generator operating in a bending mode, each volume element obeys the above equations, and the entire generator is equivalent to a slab of piezoelectric material as represented in Fig. 1, with electroded surfaces of area wb, one grounded and the other connected to terminal T. If the generator is driven with constant mechanical amplitude and frequency, the slab can be replaced by the equivalent generator consisting of an ideal emf ε in series with a capacitor C and resistor R_e as derived below and shown in Fig. 1.

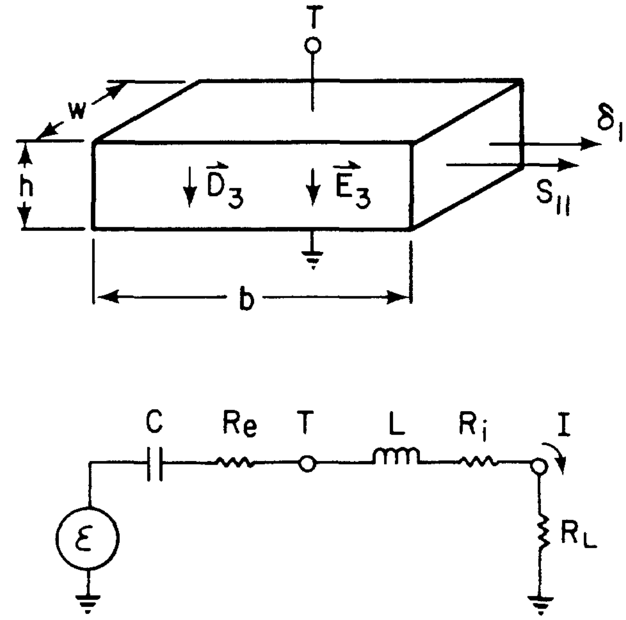

Figure 1. Slab of electroded piezoelectric polymer representing piezoelectric generator operating in d_{31} mode, and its equivalent circuit coupled at terminal T to a resonant inductor and a load R_L. Symbols are explained in text.

We assume a sinusoidal strain $\delta=\delta_m e^{j\omega t}$. The emf ε then is the open-circuit voltage found from Eq. (3) by setting D=0:

$$\varepsilon=hE=-(Yhd/K_0K)\delta, \quad (4)$$

where h is the thickness of the sheet.

The current is given by I=-wbdD/dt, so the short-circuit current I_s found by setting E=0 in Eq. (3) is

$$I_s=-j\omega wbYd\delta=\varepsilon/Z_s. \quad (5)$$

We see that while for a given maximum strain δ_m the emf is independent of the angular frequency ω, the current is proportional to ω, demonstrating the advantage of designing devices that oscillate at high frequency.

Finally, from Eqs. (4) and (5), the source impedance is

$$Z_s=\varepsilon/I_s=-jh/\omega K_0Kwb=-j/\omega C, \quad (6)$$

so the source impedance is simply the capacitive reactance of the slab's capacitance C, where the small lossy part of the dielectric permittivity has been neglected. This is justified if the generator is connected directly to a resistive load. The lossy component must be considered if the capacitive component of the source impedance is resonated away by a series inductor to increase the output as discussed below.

If the generator (or group of generators forced mechanically to oscillate in phase) is connected to an inductor of the correct value to give series resonance at the operating frequency, the new source impedance will just be the combined resistance $R_s=R_e+R_i$ of the generator resistance R_e and inductor resistance R_i shown in Fig. 1. From the value 50 chosen above for $Q_e=1/\omega CR_e$ for PVF_2 and choosing a value of 33 for $Q_i=\omega L/R_i$ of the series inductor at 1 kHz, the combined source Q value is $Q_s=20$. Thus R_s is 1/20 of the magnitude of the capacitive source reactance $X_c=-j/\omega C$, where the capacitance $C=K_0K'wb/h$ in accord with Eq. (6). Accordingly,

$$R_s=h/wbQ_sK_0K'\omega \tag{7}$$

is the source resistance of the generator when electrical resonance is employed.

We emphasize here that L is an actual inductor, and not an equivalent inductor employed to analyze piezoelectric resonance. The electrical resonance described above has approximate angular frequency $\omega=(LC)^{-1/2}$. Good designs should incorporate mechanical resonance at the same frequency to maximize amplitude of oscillation, but we assume here that this frequency is far from the piezoelectric resonance frequency of the material.

With the decreased source impedance given in Eq. (7), much larger currents are possible for the same emf and consequently the output power will increase accordingly. We must check to see whether this large current I can cause the breakdown field E_b to be exceeded. The field is the voltage which is approximately X_cI for large Q_s, divided by the thickness h. The current I is the emf ε from Eq. (4) divided by $R_s+R_L=R_s(1+r)$, where R_L is the load resistance and r is the ratio of load to source resistance. Since the magnitude of the ratio $X_c/R_s=Q_s$, we have, using Eq. (4):

$$E=Q_s\varepsilon/h(1+r)=Q_sYd\delta/K_0K'(1+r). \tag{8}$$

If δ is set at the yield strain 0.03, r at 0 (short circuited output), and other parameters given above are substituted into Eq. (8), we obtain $E_{max}=21.2\times10^7$ V/m. This is larger than the breakdown field[7] of $E_b=3\times10^7$ V/m, so breakdown must be considered as a possible limitation on output power, as described below. If electrical resonance is not employed, then maximum field occurs for open-circuited load, in which case Eq. (4) applies. The field magnitude corresponding to the emf at yield strain found from Eq. (4) is defined as

$$E_y=\varepsilon_y/h=Yd\delta_y/K_0K'=1.06\times10^7 \text{ V/m}, \tag{9}$$

so breakdown will not limit the power output if electrical resonance is not employed.

THEORETICAL ELECTRICAL POWER OUTPUT

For the electrically resonant case, the power W to the load per unit volume of PVF_2 is limited either by mechanical yield or electrical breakdown, depending on polymer parameters and the inductor and load resistances. For the yield-limited case,

$$W_y=I_y^2R_L/2wbh=[E_yh/R_s(1+r)]^2R_sr/2wbh \tag{10}$$
$$=Q_srK_0K'\omega E_y^2/2(1+r)^2,$$

using Eq. (7) for R_s. Note that for increasing load ratio r, W_y first increases linearly, peaks at r=1, and then decreases.

For the breakdown-limited case,

$$W_b=I_b^2R_L/2wbh=(E_b/X_C)^2R_sr/2wbh \tag{11}$$
$$=(E_b/Q_sR_s)^2R_sr/2wbh=rK_0K'\omega E_b^2/2Q_s.$$

Note that W_b increases linearly with r for all r. If there is a crossover from breakdown-limited to yield-limited power, it must occur at the r value r_x at which $W_b=W_y$. From Eqs. (10) and (11), this occurs at

$$r_x=Q_sE_y/E_b-1. \tag{12}$$

There are three possibilities for the load-dependence of the power output per unit volume.

First, if Eq. (12) indicates negative r_x, the yield-limited case of Eq. (10) is valid for all r, and the maximum power at r=1 is

$$W_{my}=Q_sK_0K'\omega E_y^2/8. \tag{13}$$

Second, if $0<r_x<1$, the maximum power is governed by Eq. (11) for $r<r_x$ and Eq. (10) for $r>r_x$, and the maximum power still occurs at r=1 and is still given by Eq. (13).

Third, if $r_x>1$, the maximum power is still governed by Eq. (11) or (10) as described above, but the maximum power occurs at $r=r_x$ and is given by

$$W_{mx}=(K_0K'\omega E_b/2)(E_y-E_b/Q_s). \tag{14}$$

For the parameters given above, we are in the third case, with $r_x=6.06$. For a frequency of 1 kHz, somewhat below the frequency at which K" starts increasing rapidly for PVF_2, $W_{mx}=91$ W/cm^3. This is a significant power level, and so possible overheating of the material must be considered in the design process, even after the operating power level is reduced to provide a factor of safety against yield and breakdown. The power curve as a function of load resistance is shown in Fig. 2.

If electrical resonance is not employed, the power is limited by yield and Eq. (13) applies for maximum power, with Q_s replaced by 2. For maximum power, the load resistance is numerically equal to the capacitive source impedance. The 2 occurs because the load is 90° out of phase with the source impedance. Thus the voltage across each element is $\varepsilon/2^{1/2}$ instead of $\varepsilon/2$, and the square of voltage which is proportional to power is twice as great. The maximum power for the above parameters is 19 W/cm^3. Thus adding a series inductor to provide electrical resonance increases the power per unit volume of PVF_2 almost fivefold.

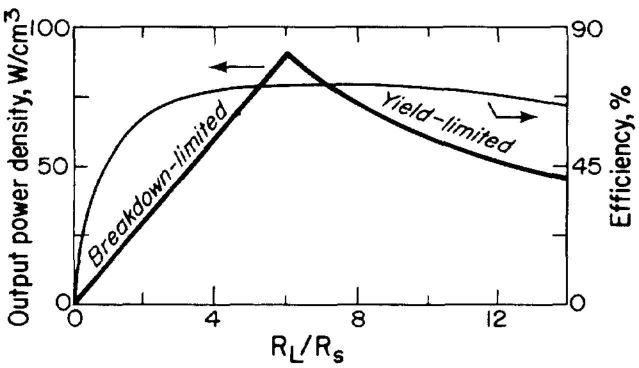

Figure 2. Theoretical maximum electrical power output density and efficiency for a PVF_2-based generator oscillated mechanically at 1 kHz.

EFFICIENCY

The efficiency is the power to the load, divided by the sum of the load power and the electrical and mechanical power losses, which must add up to the mechanical input power. In each case we consider power per unit volume. The three power and loss components have the same ratios independent of amplitude of oscillation, or of whether yield or breakdown limits power output. Accordingly, we can drop the subscript y from Eq. (10) and use Eq. (8) to obtain the output power W_o in terms of the strain δ:

$$W_o=Q_s r\omega Y^2 d^2 \delta^2/2K_0K'(1+r)^2. \quad (15)$$

The electrical loss power W_{el} is given by

$$W_{el}=W_oR_s/R_L=W_o/r \quad (16)$$

The mechanical loss power W_{ml} is the product of the angular frequency ω and the mean energy loss per radian, which in turn is the energy stored divided by the mechanical quality factor Q_m. The energy stored per unit volume, in a formula analogous to that for the energy stored in a spring, is $Y\delta^2/2$. Thus, W_{ml} becomes

$$W_{ml}=\omega Y\delta^2/2Q_m. \quad (17)$$

The efficiency η then can be written as

$$\eta=[1+W_{el}/W_o+W_{ml}/W_o]^{-1}=[1+r^{-1}+(r+1)^2/rF]^{-1} \quad (18)$$

in which F is a "Figure of merit" for the generator, given by

$$F=Q_mQ_sk^2 \quad (19)$$

and k^2 is the electromechanical coupling constant Yd^2/K_0K which is 0.0088 for PVF_2. For our chosen values of 200 for Q_m and 20 for Q_s, the figure of merit is 35.2, much larger than k^2 itself. At the load resistance ratio r_x=6.06 giving maximum power, the efficiency is 0.715, or 71.5%. As seen in Fig. 2, the efficiency is quite flat over a large range of r. It is maximum at $r_m=(F+1)^{1/2}$=6.02, which is coincidentally very near r_x.

If no series inductor is used to achieve resonance, the value of Q_s=2 discussed above must be used in Eq. (18). At maximum power of 19 W/cm^3, $r=Q_e$=50, and efficiency η=0.0633, or only 6.33%. The efficiency peaks at 36.0% for r_m=2.13, but here the power density is very low, only 1.6 W/cm^3.

If the generator system is synchronized to the line and its output is fed into the 60 Hz line instead of a load resistor R_L, the above power output and efficiency equations still hold so long as an inductor is used to achieve electrical resonance. One simply replaces R_L with V/I, where V is line voltage and I is current. At 60 Hz an inductor Q_i of only 20 can be expected, but Q_e remains at 50, so the combined Q_s becomes 14.3 and r_x in Eq. (12) becomes 4.05. Then the maximum power density from Eq. (14) becomes 5.1 W/cm^3 and the efficiency from Eq. (18) becomes 67%. At this r_x the ratio $\varepsilon/V=(R_s-R_L)/R_L=1+1/r_x$=1.25, so maximum power is achieved with a generator emf 25% higher than line voltage.

DISCUSSION

At frequencies near 1 kHz, electrical power outputs approaching 100 watts per cubic centimeter of PVF_2 at efficiencies near 70% can theoretically be obtained from mechanically-driven devices if the capacitive source impedance is resonated away by a series inductor. The load must be properly matched to the generator. The power output will be reduced by the product of the safety factors by which the device is operated below both the yield strain and electrical breakdown limits. Power and efficiency are tabulated below for different frequencies, with and without employing electrical resonance.

Freq., Hz	Resonant?	Power, W/cm^3	Efficiency, %
1000	yes	91	72
1000	no	19	6.3
60	yes	5.1	67

Table I. Maximum power density and corresponding efficiency for various operating conditions described more fully in text.

The only experimental test of these power output predictions is provided by data from three PVF_2-based wind generators which we developed, two rotating designs[10] and one oscillating design.[11] An output of 0.012 W/cm^3 was achieved[10] at 18.7 Hz and 144 V peak-peak output without employing electrical resonance. From Table 1 with output of 19 W/cm^3 reduced by the frequency ratio 18.7/1000, an output of 0.36 W/cm^3 is expected at yield strain amplitude. Our result is consistent with peak strain 18% of yield strain because from Eqs. (9) and (13) output is proportional to (strain)^2. This peak strain is close to the strain estimated from stroboscopic observation of the rotor. Construction and testing of generators operating at higher frequency with larger strain level and employing electrical resonance is planned, to provide a better test of predictions of Table I.

Piezoelectric polymers with better properties are being developed. The most promising is a copolymer of vinylidene fluoride (CH_2CF_2 monomer) with trifluoroethylene ($CHFCF_2$ monomer).[3,8] The 52%/48% copolymer has d_{31} as high as 49×10^{-12} C/N, twice as large as for PVF_2. Its dielectric permittivity K' is 19 instead of 12, and its Young's modulus Y is 1.04 instead of 1.5 in units of 10^9 N/m^2, so its electromechanical coupling constant k^2 is 0.0148 instead of 0.0088. These values predict improved efficiency and power density. From Eq. (14), noting that the E_y term is considerably larger than the E_b/Q_s term and substituting for E_y from Eq. (9), we see that the maximum power is proportional to ω and E_b and nearly proportional to Y, d, and δ_y, with only weak dependence on K' and Q_s. To increase efficiency, one should increase Q_m, Q_s, Y, and especially d, and should decrease K'.

CONCLUSIONS

Experiments should be made to determine whether the above power densities can actually be approached. Tests to determine lifetime against fatigue, electrode failure, depoling or other failure modes should be run, preferably over a wide temperature range. Temperature rise from heating caused by electrical and mechanical losses should be monitored.

Although the electromechanical coupling coefficient of PVF_2 is low compared to that of many ceramic and crystalline piezoelectrics, designs based on mechanical and electrical resonance should provide generators with high power output and efficiency. PVF_2 is preferable to these other materials in certain generator applications because of its much greater flexibility.

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