Area Conceptions Sprout On Earth Day

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Area Conceptions Sprout On Earth Day

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On a breezy spring day, second and third grade students looked out their classroom windows and imagined what it might be like to build a garden with their teachers for Earth Day. The students had just read the book, *Mrs. Spitzer's Garden* (Pattou 2001), which tells the story of a teacher who plants a garden each school year. When she plants seeds, Mrs. Spitzer knows exactly how to help each plant thrive and grow to its potential. While reading the book, we substituted our names and school into the story so that students could imagine themselves in the plot. Our class had spent several weeks discussing seeds and plant growth, so we ended the story by discussing what is important when planting a garden. Several students discussed that plants need space to grow because they compete for food and water. This concept of space sparked our gardening and area measurement lesson.

With the adoption of the Common Core State Standards, many concepts related to area are covered in third grade. These include recognizing area as an attribute of a plane figure, understanding a square with a side length of one is a unit square, measuring area by tiling figures and counting the squares it takes to cover without gaps or overlaps, and also relating area to the operations of multiplication and addition. Area concepts take time and experience to work through and understand. Researchers (Battista 2007, Sarama and Clements 2009) have shown when students first encounter area they often find difficulty structuring square units in rows and columns. Students will overlap area units or leave gaps between them as well as create units that are non-uniform.

In response to these concerns, it has been documented (Miller 1984, Wolf 1995) that when students are allowed to use physical square units or tools they are more likely to develop strategies consistent with multiplicative rules and also develop mental imagery. To build spatial awareness, students need to solve problems through mathematical modeling allowing interaction with and sense making of their world (Lesh and Zawojewski 2007). For this lesson, we wanted to encourage students to conceptualize the idea of area beyond the formula. Based on these ideas, we decided to create a measurement lesson centered on gardening and plant growth. Students investigated area and perimeter by constructing rectangular gardens using the same perimeter of fence to see if and how this affected the area. We incorporated estimating, drawing, building, and checking to help students develop and refine their conceptions of area. This lesson took approximately three days of instruction, one day for part 1 of the lesson and two days for part 2.

The Lesson: Part 1

On the first day of the lesson, we spent fifteen minutes reading and discussing the story of Mrs. Spitzer's Garden or, in our case, Mrs. N's garden. After discussing the story, we recruited our students to help us design a classroom garden by posing the following task:

Mrs. N wants to plant a rectangular garden at school for her students. She has 16 feet of fence that she can use to make a garden. Before building, she needs to make a plan. She wonders how many garden possibilities she could make with 16 feet of fence. While getting supplies, Mrs. C tells Mrs. N that it doesn't matter how she lays the fence down, all the gardens will look the same. What do you think?

- *Draw: Take 5 minutes and think about the question. Can you draw a garden that will work?*

- *Build: If you want, grab a bag of popsicle sticks and see if there are any more gardens you can build. Make sure that you record your new garden on a piece of paper.*
- *Discuss: Compare your drawing with another group. Is it the same or different? How?*

Before students began the problem, we had a short discussion on the properties of a rectangle to aide in the construction of their gardens. Students were then asked to think about the first prompt independently for a few minutes. We asked them, "Can you draw a rectangular garden with 16 feet of fence for Mrs. N?". They were each given a piece of paper to write down their ideas and if possible draw a garden possibility. After the students made one garden we asked, "is this the only garden Mrs. N can make, or can you draw another?". After independently working, the students were paired together and given 16 sticks to help them generate ideas.

The purpose of the drawing phase was to get an idea of how students thought about number and space independently. Table 1, shown below, highlights common student strategies that we observed. Almost all of the students were able to draw their interpretation of a garden. Some students began by making lengths of fence and connecting them together until they had sixteen or more. This was problematic because students wanted the sides of the rectangle to connect but often needed more than sixteen feet due to their initial construction. Half of the class was able to reason about viable side length combinations. Some students thought about a possible combination and then drew individual units or hops to represent the lengths. Others understood that the side lengths should sum to 16 feet but either forgot the properties of a rectangle or did not draw the sides in proportion to one another. Only two or three students in the class were able to think of a correct solution as well as draw the side lengths in proportion to one another.

Table 1: Students' Drawings of Possible Gardens

<div> </div> <div><p>Drawing A</p><p>Student Response: <i>"I can't make a rectangle that will work. I drew the pieces of fence and counted them, but there are too many"</i></p><p>The student drew the fence by making one-foot lengths and connecting them together. This did not always result in a garden with perimeter of 16 feet.</p></div>	<div> </div> <div><p>Drawing B</p><p>Student Response: <i>"I thought of numbers that would make 16 and I know 4 + 4 + 4 + 4 = 16. Each hop represents a piece of fence."</i></p><p>The student thought about possible number combinations that would result in a rectangle with perimeter of 16 feet. The student then drew lengths or hops to show the 16 feet.</p></div>
<div> </div> <div><p>Drawing C</p></div>	<div> </div> <div><p>Drawing D</p></div>

<p>Student Response: <i>"I made a garden with a flower in the middle. I thought of four numbers that make 16 for each of the sides because $3 + 5 + 4 + 4 = 16$"</i></p> <p>The student thought about possible side length combinations that would result in 16 feet of fence but forgot about the properties of a rectangle.</p>	<p>Student Response: <i>"I thought of two numbers for the different sides that I could make 16 with. I know this works because $1 + 1 + 7 + 7 = 16$"</i></p> <p>The student correctly found rectangle side lengths that would result in a perimeter of 16 feet, but does not consider the lengths in relation to one another.</p>
<div data-bbox="545 774 1105 1209" data-label="Image"> </div> <p style="text-align: center;">Drawing E</p> <p>Student Response: <i>"I thought of two numbers that could be the side lengths, 6 and 2. I know this because $6 + 6 + 2 + 2 = 16$. The 6 is the longer side and the 2 is the shorter side"</i></p> <p>The student was able to correctly find rectangle side lengths that would result in a perimeter of 16 as well as draw the side lengths in proportion to one another.</p>	

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71 Since the goal of the lesson was to eventually investigate area, we thought it was
72 important for students to visualize and construct rectangles that were in proportion to one other.
73 The drawings showed us students' thinking and gave students ideas to work from but could be

confusing when comparing areas. To prepare for the second lesson, we asked students to document their findings using popsicle sticks.

With the addition of sticks, the task became easier and more accessible for all students. In figure 2, the student whose drawing was featured in Table 1 (strategy A) was unable to construct a drawing of a garden composed of 16 square feet but, when given sticks, was able to construct and draw a new rectangle. We found, once given sticks, many students began to draw rectangles to scale and required larger sheets of paper to record their findings.

Figure 1: Student's Garden

After students found a solution, they were asked to glue it down onto a poster board. To end the first lesson, everyone was asked to reflect on the question, "How many garden choices does Mrs. N have and have we made all of the possible choices?" as well as the strategies they used to construct gardens. Several students discussed what they called the "add and take away" strategy. They found that if they added one to each of the lengths and took away one from each of the widths then they could generate a new garden. Other students discussed that they saw that the length and the width always added to eight and tried to think of number combinations with a

sum of eight. We asked why they thought it added to eight and most were not sure. One student suggested that it was because we made it half way around and half of sixteen was eight.

Next we asked students, "Do you think we found all of the gardens?". The class initially came up with 7 gardens that had whole number side lengths. After some discussion, this number decreased to 4 because they realized that several of the gardens were the same but rotated (these representations are shown in Figure 2). Many students thought maybe there could be more, but were unsure of construction. A few students suggested that partial feet could be used to construct gardens such as 3.5 feet by 4.5 feet. As a class, we discussed that these numbers still fit the properties of a rectangle as well as added to a perimeter of 16. Because this lesson transitioned into area concepts, we discussed rational number solutions for perimeter but told students for our garden we would first explore only whole number side lengths.

Figure 2: Classroom Gardens

The Lesson: Part 2

The second piece of the lesson, which lasted about two days, focused on exploring area concepts. We began our discussion by recalling students' comments about space and sharing that

farmers and gardeners often create garden plots so they can insure that plants will have enough space to grow. We showed students a picture of a garden plot (1 x 1 popsicle square shown in Figure 3) in relation to their gardens and we posed the following problem:

Figure 3: Garden Plot

Mrs. N wants to grow as many plants as she can in her garden. She knows that each plant needs a certain amount of space so it can grow well (show students the picture of a garden plot, Figure 3). She wonders which garden would be the best for her to have the largest number of garden plots or if all the gardens are the same. Another teacher, Mrs.W, thinks all the gardens will have the same amount of space because they have a perimeter of 16 feet of fence. What do you think?

- Predict! Just by looking which garden do you think will have the most garden plots? Which one will have the least? Or will they all have the same?*
- Measure. Find a garden plot and use squares and/or popsicle sticks to help you measure the number of plots.*
- Record. Write on the chalkboard the number your group decided on.*
- Discuss. Talk with someone from another group about your measurements.*

Predictions

We asked the students initially, "How many garden plots do you think will fit in each garden?". The students were asked to write their initial predictions down on a notecard to share with the class. Surprisingly, the class was almost evenly distributed in their responses. Several students picked D as the biggest because it was the longest, and several others picked C because it was the fattest. Students that chose A or B picked them because they were somewhat long and somewhat wide. To our surprise, none of the children thought all of the gardens would contain the same number of garden plots. We also asked students to estimate how many garden plots would fit into each garden. Some students' estimates were pretty close such as estimating 9 plots would fit in D and some estimates were pretty high or low, such as estimating 50 plots would fit in C.

Measuring

We knew that structuring area units is often difficult for young students and they need tools, such as square tiles, to help them develop conceptions of area and space. To help develop students' thinking, we provided paper garden plots and additional popsicle sticks. Students were encouraged to use what they needed to help them solve the problem.

As we walked around, we saw four primary strategies implemented equally across the classroom. These strategies are highlighted in greater detail in Table 2. Some students began by grabbing as many square units as they could, covering their garden, and counting the total one by one. This was problematic because students often left gaps between plots or overlapped them. Other students grabbed sticks and connected them to make garden plots. This strategy was usually effective because students could imagine and construct the outlines of the plots. A couple groups grabbed only one square and moved it down a column and then across, using their finger

as a place marker. Students who used this strategy were often off in their measurement as well because they did not consider that fingers take up space. One group of students used squares to measure one row or one column and then skip counted up or across the rectangle. This particular group was usually effective at finding the correct area measurement.

Table 2: Students' Strategies to find Area

A photograph showing two young students sitting at a light-colored table. They are using numerous small, square green tiles to cover a rectangular area on the table. One student is pointing at the tiles while the other looks on. The tiles are arranged in a grid pattern, though some overlap is visible.	<p>Strategy 1</p> <p>Student Response: <i>"I put as many garden plots in the garden as I could"</i></p> <p>The students use the green tiles that represent garden plots to completely cover the rectangular space. Sometimes this method was problematic because students overlapped garden plots or left spaces in between garden plots.</p>
A photograph showing a rectangular area on a table covered with a green sheet of paper. A series of wooden popsicle sticks are laid out horizontally across the paper, spaced apart to create a row of rectangular sections. A yellow pencil lies on the paper next to the sticks. Some popsicle sticks are also visible on the table surface below the paper.	<p>Strategy 2</p> <p>Student Response: <i>"I used popsicle sticks to make garden plots like the one you showed us"</i></p> <p>Students use popsicle sticks to partition the rectangle into garden plots. This strategy was almost always effective for</p>

	<p>the student in finding the correct number of area plots.</p>
<p>Strategy 3</p> <p>Student Response: <i>"I took one square and put it in the corner of the rectangle. Then I put my finger down to remember my spot and put the square on the other side. I did this for all the rows and counted as I went".</i></p> <p>The student uses one garden plot and iterates it or flips it (using a finger to mark her place) to find the total number of garden plots. This was not always an effective strategy because the student often left large gaps with fingers.</p>
<p>Strategy 4</p> <p>Student Response: <i>"I put down four squares across the bottom. I saw three rows above, so I skip counted 4, 8, 12, 16"</i></p> <p>The student finds one row or one column of garden plots and repeats it across or</p>

	down (usually skip counting) to find the total number of garden plots.
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Recording and Discussion

After groups had each measured the gardens, they recorded their findings on the board. We saw that for B (2 x 6) and D (1 x 7) most responses were identical. Students said they could visualize where the garden plots would go. One student said that for garden B, she could see two squares sitting next to each other and then counted by two down the length of the rectangle. We then asked the students, "How do you know where a garden plot ends?". The students stated that they could see that a plot would stop at the end of a stick because it was the same length.

Since garden A (4 x 4) and garden C (3 x 5) seemed to have a variety of answers, we asked students to explain their strategies. Groups explained and/or physically showed what they did. As a class, we discussed if there was a way we could visualize gardens A or C like we had for B and D. We asked them, "how could we really know what the correct measurement was?". Students were asked to check their method by filling the entire garden with plots. The students became unsure when they got to the inner part of the wider gardens and it became harder for them to visualize the squares and to see the rows and columns like they had with the longer gardens. After several minutes of checking, we met back to discuss.

One group discussed that they only laid down as many squares as could fit in one row and then repeated the row over and over again until it reached the bottom. Other groups discussed that we could do the same thing with columns, and skip count across. One student even

exclaimed that either strategy would work for measuring the garden because 5×3 is the same as 3×5 . To visualize their description, we taped square units together so that students could imagine the row or column moving across the garden. The class liked this strategy and could visualize units in a row and also visualize it moving.

Lesson Wrap Up and Reflection

To end the lesson we revisited our initial question of which garden held the most plots and more importantly why. By examining their work, students could see that A (4×4) would hold the most garden plots. Students were asked to think about how they could solve this question for a larger space without laying out tiles. Several groups responded that we have to consider the length and the width and look at how many groups of rows we have. Some students made the connection to multiplication and said they could see the area was going to be the number in a row multiplied by the number of columns. Our goal for the lesson was for students to begin to visualize row and column structure. Because of this, we accepted students thinking about area in an additive sense or in a multiplicative sense. As the unit progressed, we thought more students would readily make the transition between the two. We ended the lesson by giving students containers with seeds and dirt to plant the garden.

In reflecting on the lesson, the construct of gardening allowed for fluidity between many mathematical concepts as well as a connection to the sciences. Because of the topic, students were invested and interested in the problem throughout the lesson and some even replicated the problem at home with their families. We felt that our students left this lesson with a wealth of experiences that will help them conceptualize area beyond far beyond the formula.

Figure 4: Students Planting A Garden

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