PRESENTATION OF ALGEBRA BY PROGRAMMED TEXTBOOK OR TEACHING MACHINE

BY

WILLIAM J. BOOTH

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Montana State College
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# Table of Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>The Problem</td>
<td>1</td>
</tr>
<tr>
<td>Procedure</td>
<td>1</td>
</tr>
<tr>
<td>Definition of Terms</td>
<td>2</td>
</tr>
<tr>
<td>II. PSYCHOLOGICAL ASPECTS OF TEACHING MACHINES</td>
<td>10</td>
</tr>
<tr>
<td>Motivation</td>
<td>10</td>
</tr>
<tr>
<td>Programming</td>
<td>11</td>
</tr>
<tr>
<td>Some Results of Experimentation</td>
<td>16</td>
</tr>
<tr>
<td>Summary</td>
<td>22</td>
</tr>
<tr>
<td>III. STATUS OF PROGRAMMED LEARNING WITH TEACHERS AND PUPILS</td>
<td>24</td>
</tr>
<tr>
<td>Reaction to Present Applications</td>
<td>24</td>
</tr>
<tr>
<td>Attitudes Regarding the Effects on Education</td>
<td>26</td>
</tr>
<tr>
<td>Predicted Effects on Education</td>
<td>33</td>
</tr>
<tr>
<td>Summary</td>
<td>36</td>
</tr>
<tr>
<td>IV. PROGRAMMED PRESENTATION OF ALGEBRA</td>
<td>39</td>
</tr>
<tr>
<td>The New Algebra</td>
<td>39</td>
</tr>
<tr>
<td>Programming of Algebra</td>
<td>40</td>
</tr>
<tr>
<td>Summary</td>
<td>47</td>
</tr>
<tr>
<td>V. SUMMARY AND CONCLUSIONS</td>
<td>48</td>
</tr>
<tr>
<td>Summary</td>
<td>48</td>
</tr>
<tr>
<td>Conclusions</td>
<td>49</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>51</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Types of Teaching Aids and Devices</td>
<td>5</td>
</tr>
<tr>
<td>2. An Example of Linear Programming of Modern Algebra</td>
<td>42</td>
</tr>
<tr>
<td>3. An Example of Linear Programming of Conventional Algebra</td>
<td>44</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

The use of teaching machines and programmed learning has become an increasingly important and controversial subject in the past few years. Interest in the machines has been given stimulus by the critical shortage of skilled teachers and by the belief of many in education and psychology that automated teaching devices may be even more effective than a human teacher in cases where immediate reinforcement of learning for the student is desirable. Many educators believe the teaching machine to be the most valuable teaching aid since the textbook; some consider it to be merely a fad; and some fear, while others rejoice, that it may someday replace many of the less capable human teachers in our schools.

The Problem

There has been much said for and against the presentation of subject matter by teaching machines. The controversy regarding machine instruction and the writer's forthcoming assignment to the teaching of high school mathematics have suggested the problem considered in this paper—whether teaching machines or programmed textbooks would be effective and practicable in the teaching of algebra.

Procedure

The chief procedure was a review of literature from the fields of psychology and education. Literature was reviewed to determine the status
of the teaching machine and programmed learning per se. The findings of experimentation were studied and opinions of highly regarded experts in the field of educational psychology were considered. The importance of motivation and the effect produced by teaching machines in this area were also considered. Especially noted were claims regarding the effect of the machines on speed and ease of learning and the apparent advantages in the handling of individual differences.

Literature was also reviewed to determine the status of teaching machines and programmed learning in actual classroom use. The present applications of teaching machines were noted as were reported effects of their use in the improvement of learning and teaching. Opinions of experts in the field of education regarding the present and future applications of the machines and implications regarding their effect on the teaching profession were considered.

Definition of Terms

The teaching machine is not a new device; it dates back beyond 1926. Various types were developed and used by the armed forces during World War II. However, its introduction to the general public did not come until 1960. Since many of the terms and processes involved in the use of teaching machines and programmed learning are unfamiliar to many people, some of the more important ones are here defined.

Teaching Machine. The teaching machine is a device which presents a lesson consisting of information, actions, or objects in a prescribed sequence which are understood, learned, and retained by a learner completely
without the presence of a human instructor and in which there is an interaction of learner and machine.

Silvern lists the following, here paraphrased for brevity, as criteria for a teaching machine:

1. Instruction is provided without the presence of a human instructor.
2. Learning occurs at the learner's own rate.
3. The learner receives immediate knowledge of his progress.
4. There is a participative, overt interaction between the learner and the machine—two-way communication is provided.
5. Instructional material (subject matter) and the sequence are carefully controlled and consistent.
6. Reinforcement, or reward, is used to strengthen learning.

Basically the machine acts as a tutoring device for individual application. Many designs have been developed, but all seem to have certain operational features in common. Operation of a typical machine might be as follows:

1. Subject matter is presented to the learner visually, orally, or both.

2. A pertinent question is then presented to the learner. The solution may require mental and other activities such as pencil and paper calculation, using a tool, making measurements, or weighing a substance.

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2Ibid., p. 8.
3. When the learner's answer is prepared, he communicates it to the machine by writing in a space provided or by merely pressing a button to indicate his selection from a number of alternative choices.

4. The machine records the response and, if the answer was wrong, refers the learner to further explanatory material. If the answer was right, the machine proceeds to the next bit of information and further questioning.

5. The sequence in which the material is presented is under control of the machine.

6. The pace is set by the learner to match his individual abilities.

Many devices have been used for teaching but do not fulfill the stated requirements for classification as teaching machines. Porter categorizes various devices and states some important differences to be expected from each class as shown in Table 1. He says that the classifications are not perfect, inasmuch as many devices may fall into different categories depending on their use.

Many of the technical features of audio-visual devices such as motion picture projectors, film strip projectors, phonographs, tape recorders, and others, are often incorporated into teaching machines to serve in presenting material or in recording responses. Only those stimulus-response devices classified as immediate reinforcers, however, qualify as "teaching machines" under the criteria included in this paper.

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</tr>
</thead>
<tbody>
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<td></td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>Stimulus Devices</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>Auditory</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Audio-Visual</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Tactile</strong></td>
</tr>
<tr>
<td><strong>Response Devices</strong></td>
</tr>
<tr>
<td><strong>Data Manipulators</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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<td><strong>Data Gatherers</strong></td>
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<td></td>
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<td></td>
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<td></td>
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<tr>
<td><strong>Simulators</strong></td>
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<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Stimulus-Response Devices</strong></td>
</tr>
<tr>
<td><strong>Immediate Reinforcers</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
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The stimulus devices present information but do not provide for responses that can be recorded in any way. Therefore, there is no real assurance that learning is taking place.

The response devices can be used to gather and record responses, but they do not provide any stimulus information. Responses are related to the mechanical properties of the machines rather than to any subject matter.

Porter reports that the combination of stimulus devices and response devices is the more ideal means of involving the student in a learning situation:

Stimulus devices and response devices considered separately are truncated representations of the teaching-learning situation. However, when both aspects of learning, the presentation of cues and the opportunity for responding, are combined in one device, stimuli and responses are both placed under control of the teaching device and conditions for learning can be made optimum. Stimulus-response devices (or stimulus devices and response devices used in combination) are designed to present a sequence of stimuli (content) and provide the setting in which appropriate responses may be made and rehearsed (process). Thus the stimulus-response devices more nearly reproduce the characteristics of teaching which are required for efficient learning than do stimulus devices or response devices.

Programming. The most important part of the teaching machine is not the machine per se but the program of subject material which it contains. Generally the program is considered a part of the machine, but it is such an important part that it undoubtedly deserves individual consideration. Furthermore, the program is independently useful in the form of a programmed

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1Ibid., p. 129.
2Ibid., p. 129.
3Ibid., p. 129.
textbook.

Good programming requires step by step leadership through the problem:

Probably every teacher has at some time been baffled because a student could not see a point utterly obvious to the teacher. When this happens, a good teacher knows that he must back up and lead the student step by step until he can see the point for himself. In its simplest terms this indicates what is required for a good program.7

The two basic forms of programming are Skinner's 8 linear form and the branching form propounded by Crowder.9

Skinner's "linear" method breaks a subject into small "frames" with write-in answer blanks, followed by correct answers. To operate a machine with this type of programming, the student pulls a lever or turns a knob to make a frame appear in the window. After reading the subject material and the question which follows it, the student writes his answer in the space provided. Then he pulls the lever or turns the knob again. This moves his answer under a glass shield so that he cannot change it and exposes the correct answer along with the next frame. The method of response demanded by this type of program is known as the "constructed-answer" method.


Another linear type of teaching machine is Pressey's punchboard.10 The student reads the question from a mimeographed sheet and answers by punching a hole in the answer sheet which is fastened to a board. If the correct answer is chosen, the student's pencil will make a large hole; if the wrong answer, a small hole. This method of response is referred to as the "multiple-choice" method.

A rival school of anti-behavioral programmers led by Crowder prefers the "branching" method of programming. An article in Time magazine discusses this method as follows:

While Skinner deplores multiple-choice questions because they contain "plausible" errors that students may remember, Crowder bases his whole approach on multiple choices. Instead of small steps, Crowder programs big chunks of information followed by a question with alternate answers. Choosing a right answer wins the student an advanced frame; a wrong answer sends him to a remedial frame with an explanation of his error and perhaps a chiding comment. . . .11

Feedback. "Feedback" is a term borrowed from the field of electronics where it is used to describe the effect obtained when the emission from a loudspeaker placed too close to a microphone "feeds back" into the microphone thus multiplying itself and causing an annoying squeal.

Feedback in the educational process is the evidence of student reaction which returns to the teacher and helps him to know whether there is


understanding among the group. Poor feedback is generally considered evi-
dence that the students are not paying attention and that some change in
the method of presentation or in the subject matter is necessary to arouse
interest in learning. Fry discusses the importance of feedback and rate
of presentation of subject matter as follows:

One of the important elements for the teacher in accelerat-
ing human learning is feedback. Normally, teachers get feedback
from giving examinations, asking oral questions, and listening to
student discussions in class. By interpreting this feedback, the
teacher can intelligently vary the flow of information or further
instruction. One of the prime difficulties with the motion pic-
ture machine (or the lessons of a poor teacher) is that rates of
presentation and repetition are not varied to suit the student's
needs. Even under the best circumstances in group instruction
the rate of presentation may be too slow for the fast student and
too fast for the slow student. Teaching machines can almost com-
pletely correct this difficulty.12

Before determining the specific aptitudes of machine teaching as in
the presentation of algebra, it would be well to have a general view of
programmed learning and the machines which present it.

Teaching machines and programmed learning are the result of many
years of scientific research and experimentation by educational psycholo-
gists. Opinions of these people and some results of their experimentation
must be considered in order to determine the assets and liabilities of their
product as it is applied to the field of education in general.

12Fry, Edward, "Teaching Machines: The Coming Automation," Phi
CHAPTER II
PSYCHOLOGICAL ASPECTS OF TEACHING MACHINES

In ascertaining the effectiveness and practicability of the teaching machine for the presentation of educational subject matter, the prime consideration must be its effect on the student. The value of any teaching aid is highly contingent on whether it enhances the process of instruction either directly or indirectly.

Consideration is given here to motivation as a prime aspect of the learning process, to the programming of subject material to achieve motivation, and to some results derived from experimentation with teaching machines under experimental classroom conditions.

Motivation

An attempt to apply motivation to the methods and techniques of education would be meaningless without first explaining the term. The definition of motivation given by Good aptly fills the needs of this paper:

Motivation is the practical art of applying incentives and arousing interest for the purpose of causing a pupil to perform in a desired way; usually designates the act of choosing study materials of such a sort and presenting them in such a way that they appeal to the pupil's interests and cause him to attack the work at hand willingly and to complete it with sustained enthusiasm.¹

Incentives may be either extrinsic or intrinsic. Methods of influencing motivation that are most definitely extrinsic include honors lists,

grades, graduation requirements, honorary organizations, and similar schemes. Since these methods are an integral part of prevailing school organization, they are readily available for use by the teacher.

Marx² feels, however, that the skillful teacher will de-emphasize external motivators as soon as possible in order that full advantage can be taken of the intrinsically motivating effects of successful performance and achievement. Those desires which spring from within, or intrinsic motivators, are the ones which tend to have compelling and permanent force. Brown³ and others found that the attitude which a student has toward his studies may affect his scholastic performance to a greater degree than his native intelligence, study aids, or study habits. They found that the poorly motivated student lacks decisiveness of action, procrastinates, and is unwilling to conform to requirements and regulations.

Skinner⁴ rejects the notion that human learning must be rewarded by something external like a piece of candy or an academic prize. Learning itself can be reward enough. The more often a person is right and the quicker he knows it, the faster and better he learns.

Fry⁵ suggests that the most important reward a teacher can give is the knowledge of correct response. The student likes to know that he has

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answered a question correctly and that he is understanding. A strong positive correlation exists between frequency of rewards and assimilation of material. It has also been proven that more learning takes place when errors are corrected immediately.

Much of education is like sighting-in a rifle. Unless the person shooting learns immediately where the first shot went, he cannot make the necessary corrections for the second shot and could shoot all day without ever hitting the target. The need for immediate reinforcement is important in the learning process as well. Blyth says that "as far as the application of this principle is concerned, we do a much better job of teaching rats, pigeons, and football players than we do of teaching mathematicians and physicists."!

Stolurow says that "most mass media suffer from their failure to make specific cues and responses clear. Typically, they do not require an overt response which informs the learner of his progress."7 The teaching machine presents the information one item at a time and makes the critical cues obvious. If the student lacks understanding of a point, he can repeat immediately. The machine demands some form of response before supplying the correct answers.8

Crowder9 feels that the nature of the teaching machine itself solves

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7Stolurow, Lawrence M., "Teaching Machines--Fad or Fixture?" The Education Digest, 27:46-9, December, 1961, p. 9.

8Ibid., p. 9.

about 90 per cent of the motivational problem making discipline much less of a problem than in a conventional classroom. However, in an experiment at Roanoke, Virginia, some students complained of boredom while plugging away for long stretches on their programs. Programmers are now adding novelty and humor to the programs, and teachers break the work week with occasional blackboard sessions. In general, both students and teachers at Roanoke want more programmed classes.10

One student in a senior mathematics class in Roanoke was given a solid-geometry course on a Friday. He became so fascinated with it that he worked all weekend and did a semester's work in four days. When tested over the entire course, he made a grade of 100.11

Loehwing relates how Basic Systems, Inc., a company which turns out teaching machine programs, hired a group of 20 genuine juvenile delinquents from the New York City street corners to test the quality of their programs on. The experiments proved highly successful, but the company had to recruit a new group to continue its testing. "Every one of the 20," said David Padwa, president of the company, "having discovered the joy of learning, has reenrolled in high school."12

Control of the all-important problem of motivation seems to be one of the greatest assets of the teaching machine. However, comments from the Roanoke experiment emphasize the necessity of occasional variety in the

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11 <i>Ibid.</i>, p. 60.

program. Generally this variety should mean a move completely away from
the machines, but a certain amount of variety can be included in the ma-
cine programming.

Programming

The important part of a teaching machine is not the machine, itself,
but the programmed material it presents. The teaching machine is basically
only a glorified book cover to contain the subject matter program. The
programming represents the consolidated efforts and findings of many psy-
chologists over a period of about 30 years.

Komoski claims that the principles of programming are not difficult
to master:

Programming is not an esoteric art. A good teacher often can
become a good programmer. However, a certain amount of specialized
training is necessary. The principles of programming, as set down
by psychologists, are deceptively simple. The difficulty comes
when these principles must be put into practice. The task of pro-
gramming is not, as many people seem to believe, simply the task of
analyzing or breaking down subject matter into minute steps and
arranging these steps in a logical sequence.\(^\text{13}\)

Komoski feels that a major task now facing education is the deter-
mining of who will be trained as programmers and how they will be trained.
If independent learning through self-instruction becomes a reality, it will
be because of the quality of the people invested in programming rather than
the quantity of money invested in machines.\(^\text{14}\)

\(^{13}\) Komoski, P. Kenneth, "Programed Materials," The Nation's Schools,

\(^{14}\) Ibid., p. 75.
Although the printed program is basically a type of textbook, Leonard quotes Sullivan, one of the nation's top programmers as saying that "... the best programs bear little resemblance to any conventional teaching sequence. They represent a totally new way of organizing a field of knowledge. When I've finished a program, and it has been tested and revised again and again, it can out-teach me any time."\(^\text{15}\)

A good program will start the student at his present level of knowledge and gradually lead him on to new learning by short enough steps so that, theoretically, he never makes a mistake. Much pretesting and revising of the programs before they are marketed give assurance that the desired ends are attainable.

Blyth says that a good program "combines two ancient techniques in modified form: the Socratic method of teaching by asking questions, and the Cartesian method of analyzing a problem into its smallest parts and proceeding from the simple to the complex."\(^\text{16}\) All programs seem to abide by these principles, but there are differences in methods of presenting the material. The differences seem superficial to many, but in psychological circles there is some controversy over the relative merits of each method.

Crowder believes that multiple-choice methods are easiest to work with. Free response methods are more difficult to handle by automated tutoring processes. One of his hypotheses is that the teaching of meaningful material requires considerable covert activity on the part of the student, that is, "thinking," and that the form of the response that the

\(^{15}\) Leonard, op. cit., p. 67.

\(^{16}\) Blyth, op. cit., p. 119.
student makes to the teaching device is essentially irrelevant. 17

Skinner feels that the student should compose his answer rather than just recognize it. This complicates the problem of designing machines to fit the program, but Skinner does not believe that recognition training transfers well to reproduction. 18

Unfortunately, both the multiple-choice and the constructed-answer programs have flaws in technique. With only four choices for a multiple-choice question, the student can often make an educated guess at the correct answer without actually understanding the involved concept. With a constructed-answer type machine, if the student gives the wrong answer, the machine immediately gives him the correct answer instead of giving him an opportunity to reapply what he has learned in an effort to formulate the correct answer by himself. But, even with these faults, the machines seem to show up quite favorably under experimental conditions.

Some Results of Experimentation

Teaching machines, while available in many forms, for many subjects, and while being used in many schools throughout the nation, are still in the experimental stage. They are not widely used nor even widely available, but, as Finn says, "They have moved from what might be called the


laboratory phase into the field testing phase. Lab significant find
ings of scientifically managed experiments are presented here.

Hough reported on a study dealing with the comparative efficiency
of teaching machine instruction and the conventional lecture-discussion
method which was carried out at Temple University during the fall semester
of 1960. It involved 41 students from the Division of Secondary Education.
The first three weeks of the course "The Contemporary Secondary School"
surveys the historic development of the modern secondary school curriculum.
This portion of the course was programmed for machine presentation.

The students in the conventional class took notes and were to study
then in preparing for examinations. The experimental group used teaching
machines only and had no notes for review.

The group evaluation material was pre-tested on other students and
revised. Then it was tested for validity and reliability. Four tests were
used for equating the group. An experimental group of 20 students and a
central group of 20 students were then chosen so that there were no differ-
ences in mean scores or variability on the four tests. Group members were
asked to use no outside references and not to discuss the subject matter
with other members. The central group kept records of outside time spent
studying lecture notes. Care was taken to see that the same material was
taught to both groups and with the same emphasis.

An unannounced quiz was given after the first week. The second-week

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19Finn, James D., "Teaching Machines: Auto-Instructional Devices

20Hough, John B., "Research Vindication for Teaching Machines,"
quiz and the final post-test were announced, and records were kept on study time by the control group. No differences were noted on the second-week quiz and on the final test, but the unannounced first-week test significantly favored the experimental group.

Machine instruction was found to be significantly more efficient than the lecture-discussion method. The overall time saving of the teaching machine group was 47 per cent. Discounting outside study time on lecture notes by the control group, the saving of time was 44 per cent in favor of the machine group. The experimental group spent 305 minutes working with teaching machines. The control group spent 506 minutes in class and 72 minutes in home study of lecture notes.

Considering that the experimental group did not review for tests, the machine instruction was found to be significantly more effective than the lecture-discussion method.

In 1959 an experiment in teaching logic was begun at Hamilton College by Blyth and Jacobson. A teaching machine designed for individual use was used with the material presented on microfilm. Many advantages were found, but no disadvantages. Among the advantages were listed the following as quoted from Blyth:

1. We wasted no class time on routine checking or on drill.

2. We wasted no class time on unprepared students. We knew in advance exactly who had done the work on the programs and who had not. We believe that evidence of work on programmed material should become a ticket of admission to class.

\[21\text{Blyth, op. cit., p. 118-119.}\]
3. Examination of student work on the programs enabled us to prepare for a class period with prior knowledge of the points that needed further clarification.

4. In the classroom work we could presuppose a common background of experience with a large number of relevant examples encountered in the programs.

5. We wasted no time trying to locate and correct misconceptions students had acquired through unchecked practice.

6. We could usually count on a working command of basic concepts and principles. Class time could therefore be devoted to further development of the concepts and their application to new areas.

7. Classroom efficiency was increased by at least one-third. Instead of a standard three hours a week, we met two hours a week. Yet we were able to cover more material more thoroughly than ever before.

8. Individual differences were not entirely removed, but the range was greatly reduced, with every student moving closer to mastery. I had at least three students in a section of 20 who, in my judgment, would have failed the course in the preceding year. They passed with a very safe margin.

9. Fewer private conferences with individual students were needed. In the conferences that were necessary it was possible to diagnose the difficulty very quickly by analyzing the student's answers to the programmed materials. More often than not, such conferences led to revisions in the program in the hope of avoiding such troubles for other students.

10. The programs constitute an excellent diagnostic instrument in locating individual and class differences; an analysis of student answers provides an equally good diagnostic instrument in locating shortcomings in a program. After testing and revising programs a certain number of times, we should have programs which we know from experience will enable the vast majority of students to master complex materials in a minimum time.

11. There was a great increase in interest and improvement in morale.

12. During the summer of 1959 some of the materials prepared for Hamilton College students were used in a program for advanced studies for gifted junior and senior high-school students presented by the Board of Cooperative Services in Oneida County, New York, in another project aided by the
Fund for the Advancement of Education. Examination results compared favorably with those of college students.22

The experiment reported by Blyth is one of the few to be found where the teaching machine was actually used in its intended capacity as a teaching aid. Most experiments have pitted the machine as a mechanical teacher against the human teacher. However, even in this type of experimental setting the teaching machine has proven itself an invaluable aid to education—usually by being more efficient and more effective than its human opponent.

The experiment at Roanoke, Virginia, began as a test of programmed material against the human teachers. Programs in book form from Encyclopedia Britannica Films, Inc., are being used. Leonard says that the experiment "started out with a bang. In 1960, 34 eighth graders finished off a year's ninth-grade algebra in a half year with no homework—then tested out at a ninth-grade level."23

Leonard further states that a well perfected program can be used for children with widely varying ages and IQ's. Children of low IQ get just about as many right answers as do the children of more highly rated intelligence, but they cannot go as fast.24 The use of teaching machines might greatly reduce the percentage of children now classified as dull.

A report by the Pennsylvania Advisory Committee on Programmed Instruction of the Pennsylvania Department of Public Instruction on the

22Ibid., p. 118-119.
23Leonard, op. cit., p. 60.
24Ibid., p. 64.
effectiveness of programmed instruction lists indications of experimenta-
tion as follows:

Improved learning does take place.

Programmed learning is retained to a high degree.

The learner is stimulated to more sustained involvement in the learn-
ing process by actively and continually making responses to stimuli.

Each pupil can move at his own pace.

Understanding (i.e. "the ability to answer a variety of questions
different from those encountered during training but belonging to
the same general class") can be effectively developed by programmed
materials.

More learning takes place in less time.

The teacher is freed from such mechanical repetition and test con-
struction and correction.

Generally, small sequential steps result in more learning than do
long steps.

Multiple choice and constructed responses seem to have equal value.

The programmed approach develops self-reliance and self-discipline
in the learner.

The general consensus of opinion (sic) of those who have worked with
programmed instruction seems to indicate that success is maximized,
so that self-confidence is strengthened; both teachers and pupils
prefer teacher-pupil learning experiences to supplement programmed
instruction; teachers need better subject matter background when
using traditional teaching methods; reading ability is improved as
a result of the necessity imposed by the program; and on the secondary
school level, half-hour periods seem preferable to longer
periods.25

25A Guide to the Use of Programmed Instruction," The American
Summary

Motivation is one of the most important aspects of the learning process. Extrinsic motivators, such as grades, honor rolls, and honor societies, do not have the compelling and permanent force of intrinsic motivators which are the desires for personal satisfaction that exist deep within the individual. The immediate knowledge of correct response that is given by programmed instruction provides the type of reward necessary for intrinsic motivation. A strong positive correlation exists between frequency of rewards and intensity of learning. Learning is also enhanced by immediate correction of errors.

Students have covered a semester's work in as little as four days through sheer fascination with the program, but comments from other students suggest that variety should be presented in the programming as well as in teaching methods and school activities.

Programmed learning is a triumph of educational psychology. It is the result of the consolidated efforts of many psychologists and many years of study and experimentation. Now, with a little training, many good teachers can do much of their own programming. The program should be designed so that the questions teach rather than test. Theoretically, the students should not make any mistakes. Multiple-choice methods are easiest to work with, but constructed-answer methods might assure better understanding and better transfer.

Teaching machines are still in the field-testing stage, but their popularity is growing rapidly. Teachers are experimenting with them in progressive schools throughout the nation. Most of the experiments have
pitted the machine against the teacher instead of testing the improvement of instruction made possible by the machine as a teaching aid. In most experiments the machine has proven superior to the human teacher in both efficiency and effectiveness.

The close of the experimental era for programmed learning and teaching machines seems near at hand. Many companies are now producing programmed textbooks and programs for machines to be used for regular classroom instruction rather than for experimental purposes. A look at these programs and machine instruction from the viewpoint of teachers and pupils should prove interesting.
CHAPTER III
STATUS OF PROGRAMMED LEARNING WITH TEACHERS AND PUPILS

It has been stated that the teaching machine and the programmed textbook are still, generally, in the experimental stage. There is still much to be learned about the effects of programmed instruction. It is possible, for instance, that much of the reported motivational effect on students is due to the novelty of the new technique. Some schools, however, have been using them for over three years and are now beyond the experimental stage. Surely now many of the programs can be declared of proven value.

Since morale is such a strong factor in the teaching-learning process, a review of the opinions of teachers and pupils actually using the programs seems in order. Also, since general acceptance of programmed learning when proven of general value will be so essential to the total program of educational improvement, a review of the attitudes of people in the field of education regarding the effect of programmed learning and teaching machines on the field and on the educational process seems necessary.

Reaction to Present Applications

The nation's largest test of programmed learning is now in its second full school year. More than 2,000 students from grades nine through twelve are involved in at least one programmed class. Language and mathematics programs in book form by Encyclopedia Britannica Films, Inc., are being used. Leonard states that the experimental aura of the Roanoke, Virginia, experiment has faded, and that the teachers and students
have accepted the new procedure as a fact of school life. He describes this "school life" as follows:

Classes are of normal size. In a typical programmed math class, the students are working silently and steadily, reading a frame, writing an answer, moving the slider to check their answer, then going on to the next frame. An almost hypnotic silence pervades the room. Every now and then, a student raises his hand, and the teacher goes to help him or calls the student to his desk.¹

As previously stated, a few of the students at Roanoke complained of boredom after spending long periods with the programs.² But, Rushton³ says that they generally favor the new technique. They especially like the privilege of proceeding at their own rate of speed and the absence of homework.

Part of the Roanoke experiment started with teachers present in the rooms but forbidden to help the students who were working on programs. This method of approaching the determination of the value of the new technique was not well received. Rushton reports that the teachers' reactions were favorable toward the material but not toward the method used in experimenting:

They like the material. They approve of the way in which individual differences are cared for. In classes where they are not actively engaged in the day-to-day instruction, teachers do not like to be deprived of helping pupils learn. This is a different role and they simply don't like it. This is consistent with

²Ibid., p. 60.
a large number of pupils' reactions. Pupils, too, want help from teachers.\textsuperscript{4}

An eighth-grade mathematics teacher in Roanoke stated that she considers programmed instruction to be the salvation of the slow student as well as an excellent aid for the teacher.\textsuperscript{5}

One of the slow students involved in the Roanoke experiment is quoted by Leonard as saying that while he realizes that he is behind the rest of the class, he now understands mathematics for the first time.\textsuperscript{6}

Blyth says of the experiment in teaching logic at Hamilton College that the experience has convinced him that the most effective teaching aid available is a program of questions and answers designed for a teaching machine. He also feels that the most efficient method of presenting the program is in a machine designed for individual use and using microfilm.\textsuperscript{7}

Few remarks derogatory to programmed learning and teaching machines were noted in the literature reviewed. The few that were noted had been authored by people who had not been involved in any research programs using the new techniques.

Attitudes Regarding the Effects on Education

Attitudes regarding the effect of programmed learning and teaching

\textsuperscript{4}Ibid., p. 78.

\textsuperscript{5}Leonard, op. cit., p. 60.

\textsuperscript{6}Ibid., p. 60.

\textsuperscript{7}Blyth, John W., "Teaching Machines and Human Beings," The Educational Record, 41:116-125, April, 1960, p. 119.
machines on the educational process are overwhelmingly in favor of the new technique. Stoluwow considers the teaching machine a major break-through in education comparable to the book, radio, and television. He considers it also to be an excellent tool for experimentation inasmuch as the information it provides about teaching methods are free of uncontrolled variations due to differences among teachers. He also feels that the machine is more adequate in achieving the democratic ideal in education, because it has equal patience for fast and slow learners. It responds similarly to both the rich and the poor, to all races, creeds, and religious groups.\(^8\)

Blyth feels that the teaching machine provides superior private tutoring for every individual; superior because the programs are the work of experts and have multiple testing and revision. He further suggests that in fifteen minutes with a good sequence of questions and answers the average student "probably devotes more concentrated, alert, and active attention to the instructional material than he does in the course of an hour in the average class."\(^9\)

Regarding the time and labor to be saved for the teacher, Porter says that the machine offers advantages:

There can be no argument about the time- and labor-saving characteristics of devices which present a set of materials to a student and then score and record the results of his endeavors. The teacher is relieved of three tasks: making up and arranging the curriculum, reinforcing the student, and scoring tests.\(^{10}\)

\(^8\)Stoluwow, Lawrence H., "Teaching Machines--Fad or Fixture?" The Education Digest, 27:6-9, December, 1961, p. 6.

\(^9\)Blyth, op. cit., p. 124.

Pressey pointed out that labor-saving in education may become an economic necessity. Since education is a large-scale industry, it should use quantity production methods. This does not mean the mechanization of education but rather the freeing of teachers from clerical duties and drudgeries to give more time for guidance in learning.\(^\text{11}\)

Of the objections to programmed learning and teaching machines the two most frequently mentioned are the possible stultifying of creativity and the possible nationalization and secularization of the programs by vested interests of the companies producing them. Nordberg\(^\text{12}\) says that educational theory is already being partially shaped by the companies producing the 60 million standardized tests given yearly in the schools.

Programmers defend the new technique against the "weakening of creativity" objection by pointing out that a certain amount of conformity is necessary. There must be agreement on the performance of certain mathematical processes, the spelling and definition of words, certain precepts of science, and other factual matters. Another defense against this objection is the fact that psychologists and programmers are now experimenting with programs designed for the specific purpose of encouraging original and creative thinking and writing.\(^\text{13}\)


\(^{13}\)Leonard, op. cit., p. 70.
The possibility of nationalization and secularization of the programs actually does not present a new danger. The danger exists today in the textbook industry. Leonard says that "thus far, programming has shown no urge to interpret history and no tendency to fall under central control." So long as the present amount of competition in the publishing field exists and so long as care is used in the selection of teaching materials, the use of programmed instruction should create no new problem in this area.

Another objection to teaching machines is their cost which ranges from approximately $20 to well over $1,000. A $20 machine produced by Grolier, Inc., has an attachment which allows the student to write his answer on a roll of adding machine paper so that the programs (at about $10 each) can be used repeatedly. Some of the machines costing thousands of dollars are used in conjunction with electronic computers.

Programmed textbooks and programs for teaching machines vary in cost from about $1.95 to about $20.00. Many of these are made for one-time use only. Others, with additional provision of answer books, can be used again and again. Many other types of teaching aids are more expensive and are not used so effectively and so continuously.

For certain courses (mathematics, science, or modern languages) and for controlled experimentation, the National Defense Education Act will provide financial assistance in the purchase of programmed materials and devices. Still another objection to the automating of instruction is that

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14Ibid., p. 70.

the students through spending so much of their time working alone with their programs, will not be well adjusted to the process of working co-operatively with others. Regarding this self-learning procedure, Stolurow says:

If machines are widely used, the student may learn to study by himself and to play and work with others. Thus an important separation will be achieved. After all, what symphony was written by a group? What scientific theory was the product of a group? While groups have a very important place in education and students need experience with groups, might it not be better for the student to study alone and to use what he has learned in a group setting?16

Regarding the effect of programmed learning and teaching machines on the field of education, the very thought of "automated education" at first elicited a wave of protest and adverse criticism. Teachers envisioned themselves being replace by robots despite assurances from the people interested in the development of programming that the machines were merely another teaching aid—a device for turning pages. Textbook publishers, fearing the wrath of educators, were afraid to let anyone know that they were even interested in probing the possibility of programmed instruction.

Actually, the teacher's fear of replacement in many cases might be well founded. On reading between the lines, the assurances from programmers and educational psychologists seem weak in persuasive logic.

Regarding the replacement of teachers by teaching machines, Snider says that it is not very likely:

In the late Nineteenth Century, Thomas Edison expressed the conviction that his motion picture projector would do away with

16Stolurow, op. cit., p. 8.
the need for teachers, and sometime before then similar sentiments were expressed for the printed book. To date nothing has replaced the skilled teacher, and when we seriously consider the present goals of education, the likelihood of a completely automated educational system seems foolhardy. 17

However, Snider does feel that the teacher's role will change with the machine giving him time for a larger measure of individual attention for the students. 18

Exton 19 says that the machines will not replace teachers but will be used by them to increase their effectiveness by relieving them of such routine chores as oral drills, reviews, and tedious work correcting papers so that they will have more time to devote to lesson planning, individualized instruction, and creative aspects of teaching.

Many other authorities repeat that teaching machines are not devices for eliminating the teacher; rather they are a way of multiplying his effectiveness and implementing his instruction. The teacher would still have the important job of educating and of planning the instruction, but would not be tied down by tedious routine and the specific details of instruction and drill. Free to plan and determine individual needs, he could experiment with instructional methods, keep up-to-date in his field, and get to know better his students and his subject matter.

Crowder also reassures the teachers when he says that the teacher

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18 Ibid.

is still necessary as a classroom administrator and as an individual tutor to take care of the students whose problems were not anticipated in the programming. But, the handwriting on the wall appears when he further states that "... the machines add to the number of students a teacher can handle, and also make it possible to bring less-highly trained people into the classroom." Crowder feels that college educated housewives could act as non-certified teachers.

This point of view is further expounded by Kowitz who also suggests the probable use of non-certified personnel. He feels that it would be most inefficient to have skilled teachers spend time in servicing the machines, putting in programs, showing students how to use the machines, and doing other such routine duties.

What most authorities fail to mention in regard to teacher replacement is the probability that in the poorer school systems the machines would give the teacher more time not for creativity but for supervising the programmed instruction of 20 or 30 more students who would otherwise have composed another class requiring another teacher. Thus many teachers would probably be replaced by the machines and by non-certified personnel.

However, in many cases this ability of the teaching machine to replace human teachers, while a threat to the teaching field, can be a real

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21Ibid., p. 41.

22Ibid., p. 41.

blessing to the educational program in areas where the teacher supply is inadequate. Regarding this point of view, Blyth says:

Wherever instructional materials can be satisfactorily programmed for this method of presentation, mechanical tutors can free human tutors for other teaching tasks. This has obvious implications for any country either unable or unwilling to reward its teachers well enough to recruit them in adequate numbers. On this score alone the potentialities of the teaching machine are worth examining and exploiting as rapidly as possible if they prove their value.²⁴

Predicted Effects on Education

The future of programmed learning and teaching machines promises to be a bright one. The present lack of general acceptance seems to be due mainly to the lack of availability of machines and programs and to the lack of advertising. Predictions regarding future use of the new technique have been made by many experts in the field, all of whom are extremely optimistic.

Carpenter feels that educational implications of theory and research involving self-instruction devices are that:

1. Self-instructional devices promise to relieve the teacher of much time in presenting course content.

2. The teacher will devote more time to the management of media that promote understanding and application of knowledge.

3. The machine record of pupil performance will supply a valuable aid to the teacher for identifying specific learning difficulties.

4. Methods of evaluating student achievement are likely to be modified radically. Marking practices will be revised, since even

314

slow learners can master content by going at their own pace.

5. The teacher will be able to emphasize more individual counseling and guidance.

6. The management of space in school buildings will be modified. Allotted space will increase for private study; space for large lecture halls will probably decrease.

7. Secondary teachers will need greater concentration in the subjects they teach, as well as knowledge of related disciplines.

8. Research on self-instructional equipment will interest more academic psychologists in education.25

Fry suggests that the small schools with limited curriculum offerings will be able to offer a wider variety of subjects by the use of a machine-laboratory where one teacher can supervise a group of students learning different subjects.26

Regarding the production aspects of teaching machines and programs, Silvern predicted in 1960 that:

1. Interest in teaching machines will increase in business, military, and industrial fields as well as in education.

2. Pressure will be on to speed up programming, but experimentation and revision of programs will be necessary for a public that demands proof of quality.


3. Within a few years six or seven large reputable companies will have driven the smaller companies out of the market.

4. The low cost machines will be preferred over the computer-controlled systems.

5. A home market will develop after the machines have achieved acceptance in the schools.

6. Confidence will waver because of quacks and charlatans entering the field of programming.²⁷

Other responsible programmers also fear that the entire program might be set back if many poor programs enter the market. The Educational Testing Service of Princeton, New Jersey, is working on methods of testing programs. Criteria are being set up by a joint committee of the American Psychological Association, the American Educational Research Association, and the National Education Association's Department of Audio-Visual Instruction.²⁸

An extreme degree of faith in machine teaching is demonstrated by Blyth when he suggests that, since the machines are designed for individual use, large rooms and even school buildings might not be necessary. He says that "it might even turn out in the long run to be more economical to place a teaching machine in every home than to double the size of school buildings."²⁹

Predictions regarding the changing status of the teacher are widely


²⁸Leonard, op. cit., p. 70.

²⁹Blyth, op. cit., p. 124.
divergent with some authorities predicting the need for specialization and others for diversification. One authority suggests that the classroom teacher might become something on the order of a motivational or psychological diagnostician and a leader for group discussion. Most seem to feel that the introduction of technology to the teaching field will result in increased professional status for the teacher.

Summary

The teaching machine and programmed learning are generally considered to be in the experimental stage, although many schools have been using them for over three years. Thus far they have been used primarily for the presentation of factual materials. Almost everyone who has used the programmed materials for teaching or for learning is enthusiastic about the new technique. Part of this enthusiasm could be due to the novelty.

It must be remembered, however, that the new technique involves using the machines or programmed texts as teaching aids and not as teacher replacements. Most experimentation has pitted the programmed instruction against the teacher. Although this is not its intended use, it has proven to be more efficient and more effective than the human teacher. Programmed learning also more closely approaches democratic ideals, since it has infinite patience and can be made free of the prejudices of the human teacher. It promises to give the teacher freedom from clerical drudgeries to allow more time for creative teaching and private tutoring where needed.

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30 Leonard, op. cit., p. 69.
Unfortunately, in some schools this extra time might be used instead for the handling of additional students in enlarged classes. Even applied in this manner, however, the new method for presentation of instruction might prove a blessing for those schools and for those countries unable to afford good teachers.

It has been noted that most of the objections to programmed learning and teaching machines come from people who have not actually tried them and who are afraid of the possible effects they will have on the teaching profession. There are, of course, many bugs, both present and potential to be worked out of the new teaching method.

The principal objections to programmed learning and teaching machines are the possible stultifying effect on creativity, the possible nationalization and secularization of the programs, the cost of the machines and materials, and the isolationism involved in individual study.

Defenses offered against these objections are that programmers are now working on programs which will have as their specific purpose the promotion of creativity in thinking and writing; the danger of nationalization and secularization is existant today in textbooks, films, and other teaching materials; the cost of the cheaper machines is not so prohibitive when their utility is compared with that of other teaching aids; and use of the machine of the programmed text for presentation of subject matter will allow more time for group interaction.

The ability of the machine to out-teach the human teacher in many cases will offer a partial solution to the shortage of competent teachers. However, there is conflict in this area with some programmers insisting that the teacher will have to be much better prepared than now in the
subjects he teaches, while others feel that even non-certified personnel will be satisfactory.

A very successful future is predicted for the new technique, but as in any methodological innovation, there are inherent dangers. The industry must police itself carefully to prevent unscrupulous companies from flooding the market with poor programs.

So far programming has been most successful at teaching such factual subjects as spelling, grammar, technical skills, the sciences, and mathematics. Its application to the teaching of algebra is of greatest importance to this paper.
CHAPTER IV
PROGRAMMED PRESENTATION OF ALGEBRA

After investigating the general value of programmed learning and machine presentation, a look at the specific area of interest, algebra, seems in order. Because of the findings of research in the field of mathematics, there is no longer only one kind of algebra to present, and, as previously stated, there is more than one way of presenting it. A sampling of some of the methods in actual application to the teaching of algebra will aid in determining the practicability of the new technique.

The New Algebra

The "new" algebra is a highly theoretical presentation of the subject. It involves the use of set theory and abstract algebra.

Since the orbiting of Sputnik I, there has been a nationwide movement and demand for modernization of the mathematics and science curricula. Changes in the mathematics curriculum have involved new types of mathematics, new and expanded uses of conventional mathematics, and changes in emphasis on some topics in mathematics. The curriculum must provide the essential materials for future mathematicians while avoiding the premise that only these potential professional mathematicians are worth cultivating. It is hoped that the new type of algebra will help to attain this goal.

The bad part of the present mathematics curriculum is not the subject matter presented by the isolation of mathematics from other domains of knowledge and even of one branch of mathematics from the others.
Judicious use of sets and of the concepts of "abstract" algebra might provide more coherence and unity.  

Foremost among the national groups working on this problem are the School Mathematics Study Group, the University of Illinois Committee on School Mathematics, and the Ball State Teachers College Experimental Program. The SMSG project is financed by the National Science Foundation and involves at least 100 mathematicians and 100 high school teachers. They wrote experimental material for grades 7 through 12. This experimental material was used with about 42,000 students in 15 states by over 100 teachers.

Revised textbooks, enrichment pamphlets, and teachers' manuals based on this experimental material are now available. All publishers of importance in the field are now engaged in producing a new and updated mathematics series containing the basic principles and approaches that have proved most successful in the experimental programs.

Programming of Algebra

These new principles and approaches as applied to algebra are also available in programmed material. One source is a series of programmed textbooks from Science Research Associates, Inc. Set theory is used throughout the program. An example of this type of material as programmed

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in keeping with the philosophy of Skinner is shown in Table 1.

In a teaching machine, the answers (at the extreme right) and the subsequent questions would be covered while the student read the material in the first frame and tried to answer the question. Then the program sheet would be moved up to expose the correct answer and the next frame.

If the materials shown in Table 1 were part of a programmed textbook, a pasteboard slider would be used to cover the answers and the succeeding frames.
1. A secondary form of set is the subset. By removing an element from the set \( R = (0, 1, 2) \) we can form a new set. For example, removing the element of 1 from \( R = (0, 1, 2) \) forms the new set \( M = (0, ?) \).

2. Since every element in set \( M \) is also an element in set \( R \), we say that \( M \) is a subset of \( R \).
   
   If \( S = (1, 2) \), is \( S \) a subset of \( R \)?
   
   Yes

3. If \( Q = (1) \), is \( Q \) a subset of \( R \)?
   Is \( Q \) a subset of \( S \)?
   Yes

4. A subset which does not contain all the elements of a given set is called a proper subset of the given set. Thus, \( \text{apples} \) is a proper subset of \( \text{fruit} \).

   Is \( S \) a proper subset of \( R \)?
   Yes

5. Proper subsets of \( R = (0, 1, 2) \) are:
   
   \( (0, 1), (0, ?), (1, 2), (0), (1), \) and \( (?) \).

6. Another subset of \( R \) is formed when no elements are removed. Thus, the full set \( (0, 1, 2) \) is also a subset of \( R \). It is called an improper subset. Every set is a subset of itself.

   \( Q \) is a subset of \( S \). \( S \) is a subset of \( R \). \( R \) is a subset of \( ? \).

7. Still another set is formed by removing all of the elements from \( R \). This set is called the null set or empty set. The null set is a proper subset of every set except itself.

   \( R \) is an improper subset of \( ? \).
   The null set is an _empty_ subset of itself.

*For typographical convenience parentheses are used here instead of the usual braces.
Since the move for revision of the mathematics curriculum has been in the public limelight for an even shorter time than the teaching machine, it, too, is still in the experimental stages and has not yet achieved general acceptance. Consequently, most publishers still are producing materials of the more conventional nature. Programmed materials containing the conventional algebra are available from Grolier, Inc., from Encyclopedia Britannica Films, Inc., and others.

Programmed material from Encyclopedia Britannica Films, Inc., in textbook form is being used in the Roanoke, Virginia, experiment. The mathematics series being used in Roanoke begins with algebra and continues through the calculus. The Roanoke mathematics coordinator taught a programmed class of 21 seniors and says that during the year all of them finished axiomatic algebra and solid geometry. There was time left for all of the class to have an introduction to the calculus, and five of them finished the course. As previously stated, one boy finished a semester course in solid geometry in only four days. He received a grade of 100 on the course examination.³

An example of a conventional algebra dealing with an introduction to the algebraic treatment of fractions is given in Table 2. This example also follows the linear programming format and the constructed answer philosophy of Skinner. The method of use would be the same as that for the program presented in Table 1.

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TABLE 3. AN EXAMPLE OF LINEAR PROGRAMMING OF CONVENTIONAL ALGEBRA

1. The top number of a fraction is called the **numerator**.

   In the fraction $\frac{3}{4}$, 3 is the __?__.

2. The numerator of $\frac{1}{2}$ is __?__.

3. The bottom number of a fraction is called the **denominator**.

   In the fraction $\frac{1}{2}$, 2 is the __?__.

4. The denominator of $\frac{3}{5}$ is __?__.

5. The denominator of $\frac{x}{y}$ is __?__.

6. The value of a fraction is not changed when both the numerator and denominator are multiplied by the same number.

   Thus, $\frac{1}{2} \cdot \frac{5}{3} = \frac{5}{10} = \frac{1}{2}$

7. The value of a fraction is not changed when both the numerator and denominator are divided by the same number (other than zero).

   Thus, $\frac{3}{9} \div \frac{3}{3} = \frac{1}{3} = \frac{7}{9}$

8. $\frac{x}{y} \cdot \frac{a}{a} = \frac{ax}{ay}$ and $\frac{ax}{ay} \div \frac{a}{a} = \frac{7}{x}$
There is great variety in the types of programs that can be prepared by Crowder's\(^4\) branching technique. The simplest type is that in which each wrong answer refers the student to another page for further discussion of the problem and then back to the original choice page for another try. An example of this type of programming, starting, hypothetically from page five of such a program, follows:

Page 5

Reducing a fraction to its lowest terms means dividing out or cancelling the factors common to both the numerator and denominator.

<table>
<thead>
<tr>
<th>Answer</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>28</td>
</tr>
<tr>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

If \( \frac{ax}{bx} = \frac{a}{b} \), then \( \frac{aby}{xba} = ? \)

<table>
<thead>
<tr>
<th>Answer</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>by</td>
<td>42</td>
</tr>
<tr>
<td>ba</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>17</td>
</tr>
<tr>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

The student choosing the first answer turns to page 28 to find:

YOUR ANSWER: \( \frac{aby}{xba} = \frac{a}{x} \)

We were told to cancel the factors that are common to both the numerator and denominator. There is a factor "a" in both the numerator and denominator and, also, a factor "b" in both. The factor "y" is in the numerator only, and the factor "x" is in the denominator only. Since these two factors are not alike, they cannot cancel each other. Therefore, the answer must contain these two factors.

Now turn to page 5 and try again for the correct answer.

The student who chooses the second answer will be corrected when he turns

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We were told to cancel the factors that are common to both the numerator and denominator. Obviously the factor "b" is in both and can be cancelled since $\frac{b}{b} = 1$. Similarly, the factor "a" is in both and should be cancelled. This leaves only "y" in the numerator and "x" in the denominator. Since these are not identical, they cannot cancel each other and must remain in the answer.

Now return to page 5 and choose the right answer.

The student who chooses the correct answer will find:

YOUR ANSWER: $\frac{aby}{xba} = \frac{by}{ba}$

Now apply this cancellation method of reducing fractions to a specific problem we have:

<table>
<thead>
<tr>
<th>Answer</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3\cdot4\cdot5}{7\cdot1\cdot3} = \frac{5}{7}$</td>
<td>81</td>
</tr>
<tr>
<td>Reduced to lowest terms $\frac{2\cdot5\cdot3}{5\cdot10} = ?$</td>
<td>21</td>
</tr>
</tbody>
</table>

Since the programming technique is still in the experimental stage most schools have been reluctant to invest in the expensive materials. Those that have experimented and are now using programs have chosen to keep the amount of their investment low by purchasing the less expensive programs. Consequently, the linear programming is enjoying the greatest popularity at the present time in textbook form.
The branching technique, presented as illustrated herein, is also available in textbook form. A book of this type is referred to as a "scrambled text."

Summary

From the nationwide movement to modernize the mathematics curriculum have evolved new types of mathematics. These involve the use of sets and abstract algebra. Groups of the nation's foremost mathematicians and teachers are involved in modernization projects. The largest textbook publishing companies are now producing texts which incorporate the new principles.

Since many teachers of mathematics are not familiar with the new approach, it has not achieved universal acceptance. Publishers, therefore, are still producing materials for the conventional mathematics. This conventional form of mathematics in a programmed text is being used in the thus far very successful Roanoke, Virginia, experiment. Most of the schools that are experimenting with the new technique are using the less expensive materials to keep the investment in unproven materials within reasonable bounds.

A summary of the material presented in this paper should help the individual reader to decide whether the presentation of algebra by programmed textbook or teaching machine is practical, impractical, or deserving of further study.
CHAPTER V

SUMMARY AND CONCLUSIONS

The problem considered in this study was the practicability of applying teaching machines or programmed textbooks to the presentation of algebra. The principal procedure involved was a review of literature from the fields of psychology and education.

Summary

The review of literature shows that programmed materials are considered to be very strong in the area of intrinsic motivation which is one of the prime factors in the learning process. Programming is the result of many years of psychological research but is still in the field-testing stage. The teaching machine which is used to present the programmed material is designed to be used as a teaching aid, but, when pitted against the human teacher, it has proven to be more effective and efficient in the presentation of factual materials than the human teacher. It is also more democratic in that it can be made free of the prejudices of the human teacher, and it has infinite patience with the slow learner. Through its ability to present subject matter, the machine promises to relieve the teacher of this chore and give him more time for creative teaching.

Most of the objections voiced against programmed learning and the teaching machine seem to come from people who have not actually used the new techniques. All of the objections seem readily defensible except the objection that the teaching machines might replace the teachers. This
possible substitution seems quite likely in many cases. In some of these cases it might be used to help alleviate the shortage of qualified teachers.

Programming in application to the teaching of algebra has been phenomenally successful. Reports from many experiments indicate that the students are able to cover the material in about two-thirds or less of the time generally required in a conventional class, and that they understand the material better and retain it longer.

Materials are available for machines and in textbook form for both the conventional algebra and the "new algebra" which includes set theory and is highly theoretical.

Conclusions

The findings of this study seem to justify the following conclusions:

1. The new techniques of learning are rapidly gaining recognition but are slow in gaining general acceptance.

2. Interest and controversy are increasing as more teaching machines and programmed texts enter the schools.

3. More space is being allowed in new school plants for individual study booths.

4. Low cost machines and programmed texts are much more in demand for experimental purposes than the more expensive machines.

5. Algebra can be learned by students who were previously considered too dull to learn it.

6. The gifted student no longer has to be held back by his slower classmates.
7. The use of programmed materials does relieve the teacher of much work in subject matter presentation so that he can spend more time in planning for creative teaching and in guiding and counseling students.
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