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Date June 30, 1971
AN ANALYSIS OF THE PREDICTIVE VALIDITY OF EIGHT PREDICTORS OF SUCCESS IN FIRST-YEAR ALGEBRA

by

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A professional paper submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

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ABSTRACT

The purpose of this study was to determine the predictive validity of eight predictors of success in first-year algebra. Data from seventy-nine algebra students in Great Falls, Montana, was analyzed by the multiple regression technique.

The results of the study indicated the grade in eighth grade mathematics is an excellent predictor of success in first-year algebra. The arithmetic section scores and the Composite Score of the Iowa Tests of Basic Skills proved to be only fair predictors of success as did consideration of all factors seven at a time. Intelligence quotient, teacher predictions and reading comprehension proved to be very poor as predictors of success in first-year algebra.

Recommendations were made to use only one or two factors in predicting algebra success and to do further research in the areas of teacher predictions, reading, and student attitude towards mathematics.
Chapter 1

INTRODUCTION

Each spring counselors and mathematics teachers are faced with the problem of deciding who should enroll in first-year algebra. These people have had little information to guide them in accurately advising students with respect to the choice of first-year algebra. Although there has been some research concerning predictors of success in first-year algebra not enough has been done to provide definitive answers in all cases.

In this chapter, the groundwork is laid for the study of various predictors of success in first-year algebra. The research topic is formally stated in addition to an outline of precisely what questions the research was to answer. An explanation of the importance of the study is included as is a general description of the research model that was used. The limitations inherent in the research are stated in order to place the study in its proper perspective. Several terms are defined to clarify their meaning with respect to this study.

STATEMENT OF THE PROBLEM

The problem of this study was to investigate the predictive validity of various predictors of success in first-year algebra. That is, the study was designed to determine how effective each of the following predictors was in forecasting success in a first-year algebra
The eight predictors studied were the student's grade in eighth grade mathematics, intelligence quotient, Arithmetic Concepts Score of the Iowa Tests of Basic Skills (ITBS), Problem Solving Score of the ITBS, Arithmetic Composite Score of the ITBS, Reading Comprehension Score of the ITBS, Composite Score of the ITBS, and the teacher's prediction of success.

IMPORTANCE OF THE STUDY

The task of deciding whether or not a particular student should enroll in a first-year algebra course has plagued teachers, counselors, and students for years. If teachers and counselors are going to wisely advise students with respect to the choice of first-year algebra, they need the most accurate predictors available. Not all predictors of success have been investigated and some that have need more verification before they can be used with confidence.

Today students are living in a world of increasing competitiveness and complexity. In order to successfully meet the challenges of today, the student needs to gain an increasing amount of knowledge. This requires him to make educational decisions early with respect to what courses will best serve his needs. Only with accurate predictors of success can a student wisely choose whether or not to take a first-year algebra course.
GENERAL QUESTIONS TO BE ANSWERED

This study was designed to answer the following questions concerning predictors of success in first-year algebra.

1. What is the predictive validity for success in first-year algebra of the grade in eighth grade mathematics?

2. What is the predictive validity for success in first-year algebra of the intelligence quotient?

3. What is the predictive validity for success in first-year algebra of the Arithmetic Concepts Score (in GEL) of the ITBS?

4. What is the predictive validity for success in first-year algebra of the Problem Solving Score (in GEL) of the ITBS?

5. What is the predictive validity for success in first-year algebra of the Arithmetic Composite Score (in GEL) of the ITBS?

6. What is the predictive validity for success in first-year algebra of the Reading Comprehension Score (in GEL) of the ITBS?

7. What is the predictive validity for success in first-year algebra of the Composite Score (in GEL) of the ITBS?

8. What is the predictive validity for success in first-year algebra of the teacher's prediction of success in first-year algebra.

9. What is the predictive validity for success in first-year algebra when the eight factors are considered seven at a time?
GENERAL PROCEDURE

The study involved seventy-nine first-year algebra students at East Junior High School and North Junior High School in Great Falls, Montana. The data collected for each student was that student's score on each of the eight predictors of success listed in the STATEMENT OF THE PROBLEM on pages one and two. This information was available from the Mathematics Department of East Junior High School and the permanent records at North Junior High School.

The data was then analyzed by the use of a statistical method known as multiple regression. The results of such a statistical analysis told the investigator the effectiveness of each of the eight factors as predictors of success in first-year algebra, as well as the effectiveness of the various combinations of these eight predictors. The necessary computations were done by a computer.

LIMITATIONS

The conclusions a person is able to deduce from this study are restricted due to the involvement of students at only two schools rather than many schools throughout the country. Before applying the results of this study to another school system, the reader has to make a comparison between the two school systems in order to determine if the conditions at the two systems are comparable enough to make this study applicable to that particular school system.
DEFINITION OF TERMS

The terms predictive validity, predictor of success, student, and success in first-year algebra are defined with respect to their meaning in this study.

Predictive Validity

In this study, predictive validity meant the accuracy with which a predictor of success forecasted success in first-year algebra for a student.

Predictor of Success

Predictor of success, as used in this study, was a measurable factor which was used to forecast a student's performance in first-year algebra. In this study, the term applied to the eight factors listed in the STATEMENT OF THE PROBLEM on pages one and two.

Student

The term student meant one of the seventy-nine first-year algebra students at East Junior High School and North Junior High School about whom data was collected.

Success in First-Year Algebra

The criterion for success in first-year algebra was the student's first semester grade. This grade was determined by the student's performance over the first semester. A student was considered to have
succeeded in first-year algebra if he received an A, B, C, or D for a first semester grade.

SUMMARY

The problem of the study was to investigate the predictive validity of eight predictors of success in first-year algebra.

The study involved seventy-nine students in Great Falls, Montana. The data collected was analyzed by the multiple regression technique.

The study is important because accurate information concerning the probability of success in first-year algebra is needed by teachers, counselors, and students to help make wise decisions concerning that class.

Because the study was limited to only two schools, the conclusions a person is able to deduce from the results are restricted.
Chapter 2

REVIEW OF RELATED LITERATURE

In this chapter, a review of previous research pertinent to the study of the predictive validity of predictors of success in first-year algebra is given. Five general areas are included. They are prognostic tests, grades, achievement tests, intelligence quotients, and verbal skills. Each of these five areas is subdivided to include the specific predictors that have been researched, as well as each predictor's actual effectiveness in predicting success in first-year algebra.

PROGNOSTIC TESTS

The two tests included in this section are the Orleans-Hanna Prognosis Test and the Iowa Algebra Aptitude Test (IAAT).

Orleans-Hanna Prognosis Test

Hanna and others (1969), using the first semester algebra grade and Lankton's First-Year Algebra Test, Revised Edition as the criterion of success, found the test to be very effective as a predictor of success in first-year algebra. The data, collected from over one thousand students in six states, was analyzed by the multiple regression technique. Similar results were reported by Osburn and Melton (1963).

Barnes and Asher (1962), however, found the test to be a rather mediocre predictor of success in first-year algebra. Their research
indicated the test was neither a help nor a hindrance in predicting success in first-year algebra. The study did not differ appreciably in procedure from the above study except in sample size. Barnes and Asher (1962) used the final algebra grade as the criterion of success and used multiple regression to analyze the data for the 192 members of the sample.

**Iowa Algebra Aptitude Test**

The Iowa Algebra Aptitude Test was found by Sabers and Feldt (1968) to be a highly effective predictor of success in first-year algebra. Their study used a teacher-made final examination and the final grade for the course as the criterion of success. Data for the 375 members of the sample was analyzed by multiple regression. Their study corroborated similar findings by Osburn and Melton (1963).

**GRADES**

Past researchers investigated three predictors of success which involved grades. They were the student's grade in eighth grade mathematics, the composite of the student's grades in his eighth grade major subjects and the teacher's prediction of the student's algebra grade.

**Eighth Grade Mathematics Grade**

The student's eighth grade mathematics grade was found to be an
excellent predictor of success by Rothenberger (1967) in a study that involved two hundred students. The two semester grades were used as the criterion of success. The correlation between the individual predictors and the criterion of success was determined by use of the Pearson product-moment correlation formula. Similar results were reported by Barnes and Asher (1962) and Callicutt (1961). In fact, all three groups of investigators reported the eighth grade mathematics grade to be the most effective predictor of those they studied.

One group of researchers (Hanna and others, 1969) reported the predictor to not be particularly effective. The actual results of that study showed the Orleans-Hanna Prognosis Test to have a coefficient of correlation in the .68 to .80 range, a good correlation. The grade in eighth grade mathematics was reported to have a predictive validity coefficient of .50. Actually, that difference was not great enough to make the statement that the test was a poor predictor of success in first-year algebra.

Composite of Eighth Grade Grades

In the most recent study that involved the composite of a student's major subject grades, Hanna, Bligh, and Lenke (1970) reported the factor was a good indicator of success in first-year algebra. A major subject was considered to be an academic course that met five times a week. The criterion of success was the combination of a mid-
year algebra test, the Lankton First-Year Algebra Test, Revised Edition, and the final course grade. The sample consisted of approximately five hundred students. To establish the relationship between each individual predictor and the criterion of success, the simple correlation statistic was used. Similar results were reported by Rothenberger (1967) and Callicutt (1961).

**Teacher's Prediction of Algebra Grade**

The one study involved with this predictor reported the teacher's prediction of a student's algebra grade was a poor predictor of success in first-year algebra (Hanna and others, 1969).

**ACHIEVEMENT TESTS**

Investigators have studied five tests as possible predictors of success in first-year algebra. The five tests were the Iowa Test of Basic Skills (ITBS), the Iowa Every-Pupil Tests of Basic Skills (L Form), the Iowa Test of Educational Development (ITED), the Stanford Achievement Test (Form N), and the Science Research Associates (SRA) Test.

**Iowa Test of Basic Skills**

In the one study that involved the ITBS only, two sections of the test were investigated, the arithmetic score and the composite score (Sabers and Feldt, 1968). The investigators reported both to be
very effective predictors of success in first-year algebra.

**Iowa Every-Pupil Tests of Basic Skills, L Form**

Barnes and Asher (1962) were the only ones who reported they used this particular achievement test. Their study indicated the test was a good predictor of success in first-year algebra.

**Iowa Test of Educational Development**

Sabers and Feldt (1968) were the only investigators who reported they had used this test. They used only two portions of the test, the composite score and the quantitative thinking score, and reported a good coefficient of correlation between those scores and success in first-year algebra.

**Stanford Achievement Test, Form N**

In the single study that investigated this test, Callicutt (1961) found the test was reasonably good as a predictor of success in first-year algebra. His criterion of success was the first-semester algebra grade. Data for the 150 members of the sample was analyzed by the Pearson product-moment correlation formula.

**Science Research Associates Test**

Investigation of the SRA Test as a possible predictor of success in first-year algebra was done by Rothenberger (1967). The study investigated the mathematical reasoning, mathematical concepts,
mathematical computations, and the composite scores of the test. Only the mathematical computation score was found to be an effective predictor of success in first-year algebra.

INTELLIGENCE QUOTIENTS

In an attempt to find out whether or not intelligence quotients were effective as predictors of success in first-year algebra, researchers investigated three tests. The tests were the Otis-Beta Intelligence Quotient, the Otis-Lennon Mental Ability, and the Lorge-Thorndike Intelligence Test, Level Four.

Otis-Beta Intelligence Quotient

The Otis-Beta Intelligence Quotient was found by Rothenberger (1967) to be an extremely poor predictor of success in first-year algebra. Similar results were reported by Barnes and Asher (1962).

Otis-Lennon Mental Ability

In the study by Hanna and others (1969), the Otis-Lennon Mental Ability was reported to be very ineffective as a predictor of success in first-year algebra.

Lorge-Thorndike Intelligence Test, Level Four

As a predictor of success in first-year algebra Callicutt (1961) reported the Lorge-Thorndike Intelligence Test was very poor.
VERBAL SKILLS

Within the area of verbal skills, two studies have been conducted. One involved reading and the other involved a test that measured several verbal skills.

Reading

In the single study that investigated reading as a predictor of success in first-year algebra, Barnes and Asher (1962) reported reading was absolutely no help.

Science Research Associates Primary Mental Abilities, Third Edition

In the study by Osburn and Melton (1963), five areas of the test were investigated. They were verbal, space, reasoning, numbers, and word fluency. All five factors were reported to be very poor predictors of success in first-year algebra. The researchers used a combination of three proficiency tests, a teacher-made algebra test, and a teacher-made final examination as the criterion of success. The sample consisted of 155 students. The method of determining the relationship between the independent variables and the criterion of success was not given.

SUMMARY

Researchers appeared to agree that of the predictors studied
the grade a student received in eighth grade mathematics was the best predictor of success in first-year algebra.

Investigation also showed that the composite of a student's grades in his eighth grade major subjects was a very good predictor of success. Similarly, there appeared to be consensus that the Orleans-Hanna Prognosis Test and the Iowa Algebra Aptitude Test were good predictors of how a student would do in first-year algebra.

There also seemed to be uniform agreement that of the predictors studied, intelligence quotients were the poorest predictors of success.

In the area of teacher predictions and verbal skills not enough research had been conducted to clearly indicate their effectiveness. The studies that were conducted in those areas reported poor correlation between those factors and success in first-year algebra.

Similarly, although single studies indicated various achievement tests as good predictors of success, more investigation was needed before definite trends could be ascertained.
Chapter 3

PROCEDURES

The problem under investigation was that of determining the predictive validity of eight factors as predictors of success in first-year algebra. The purpose of this chapter is to outline precisely what was done in the analysis of that topic.

The chapter includes a description of the population that was used and the sampling procedure that was used to select an unbiased sample from the population. The eight categories that were investigated are defined. In addition to describing how the data was organized, the manner in which the data was collected is also described. The questions the study answered were translated into statistical hypotheses that lent themselves to statistical analysis. An explanation of how the data was analyzed by the multiple regression technique is included.

POPULATION DESCRIPTION AND SAMPLING PROCEDURE

The population studied were those students who enrolled in first-year algebra for the 1970-71 school year at East Junior High School and North Junior High School in Great Falls, Montana. The population consisted of approximately three hundred students.

Seventy-nine students were selected from the population by the
basic stratified random sample method. Approximately eight students were selected from each of ten algebra classes by use of a table of random numbers.

DEFINITION OF CATEGORIES

The study involved eight different factors as predictors of success in first-year algebra. The factors were the student's grade in eighth grade mathematics, intelligence quotient, knowledge of arithmetic theory, arithmetic problem solving ability, overall arithmetic ability, reading comprehension, overall ability, and teacher's prediction of success.

Eighth Grade Mathematics Grade

The student's grade in eighth grade mathematics was the first semester grade. The grade was either an A, B, C, D, or F. This grade was a cumulative mark that reflected the student's performance over the first semester.

Intelligence Quotient

The intelligence quotient for each student was measured by the student's score on the Otis-Beta Intelligence Quotient Test. The students took the test during January of their seventh grade.

Knowledge of Arithmetic Theory

The Arithmetic Concepts Score of the ITBS was used as a
measure of the student's knowledge of arithmetic theory. This score, as well as the other ITBS scores that were used, were given in Grade Equivalent Levels (GEL). The ITBS was administered during January of the student's eighth grade.

**Arithmetic Problem Solving Ability**

As a measure of the student's ability to solve arithmetic problems, the Problem Solving Score of the ITBS was used.

**Overall Arithmetic Ability**

The Arithmetic Composite Score of the ITBS was used as a measure of the student's overall ability in arithmetic.

**Reading Comprehension**

In order to measure a student's general reading ability, the Reading Score of the ITBS was used.

**Overall Ability**

As a measure of a student's overall ability, the Composite Score of the ITBS was used.

**Teacher's Prediction of Success**

Each teacher indicated at the completion of each student's eighth grade whether or not that student should take first-year algebra. A recommendation that the student take first-year algebra was interpreted to mean the teacher predicted success in first-year algebra for
the student.

COLLECTION OF DATA

Data was collected for each member of the sample in each of the eight categories listed on pages one and two. This information was available in tabular form from the Mathematics Department at East Junior High School and the permanent records at North Junior High School in Great Falls, Montana. When recording this information, it was necessary to have the following system of coding in order to use the information in the computer.

The eighth grade mathematics grade and the first-year algebra grade were recorded on the basis that $A = 4.00$, $B = 3.00$, $C = 2.00$, $D = 1.00$, and $F = 0.00$.

The student's intelligence quotient was recorded in three spaces, such as 127 or 083.

The five ITBS scores were recorded in Grade Equivalent Levels, such as 10.5 or 07.3.

The teacher's prediction of success was recorded as a binary function with 1 meaning a prediction of success and 0 meaning a prediction of failure.

ORGANIZATION OF THE RESULTS

The results of the study are presented in tabular form. The
table shows the correlation between each predictor and the criterion of success, as well as the intercorrelations between the various predictors of success.

**STATISTICAL HYPOTHESES**

In order to answer the questions raised by this study on pages one and two, the following null hypotheses ($H_0$) and alternative hypotheses ($H_1$) were used.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the grade in eighth grade mathematics and the predictive validity that would occur by chance.

$H_1$: The predictive validity for success in first-year algebra of the grade in eighth grade mathematics is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the intelligence quotient score and the predictive validity that would occur by chance.

$H_2$: The predictive validity for success in first-year algebra of the intelligence quotient score is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the Arithmetic Composite Score of the ITBS and the predictive validity that would occur by chance.

$H_3$: The predictive validity for success in first-year algebra of the Arithmetic Composite Score of the ITBS is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the Arithmetic Concepts Score of the ITBS and the predictive validity that would occur by chance.
$H_4$: The predictive validity for success in first-year algebra of the Arithmetic Concepts Score of the ITBS is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the Problem Solving Score of the ITBS and the predictive validity that would occur by chance.

$H_5$: The predictive validity for success in first-year algebra of the Problem Solving Score of the ITBS is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the Reading Comprehension Score of the ITBS and the predictive validity that would occur by chance.

$H_6$: The predictive validity for success in first-year algebra of the Reading Comprehension Score of the ITBS is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the Composite Score of the ITBS and the predictive validity that would occur by chance.

$H_7$: The predictive validity for success in first-year algebra of the Composite Score of the ITBS is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the teacher's prediction of success and the predictive validity that would occur by chance.

$H_8$: The predictive validity for success in first-year algebra of the teacher's prediction of success is greater than the predictive validity that would occur by chance.

$H_0$: There is no difference between the predictive validity for success in first-year algebra of the eight factors considered seven at a time and the predictive validity that would occur by chance.

$H_9$: The predictive validity for success in first-year algebra of the eight factors considered seven at a time is greater than the predictive validity that would occur by chance.
ANALYSIS OF DATA

The analysis of the data was done in two steps. The first step, multiple regression, answered the questions posed by this study. The second step, the F-statistic, determined whether or not those results were statistically significant.

Multiple Regression

In addition to determining the relationship between each individual predictor of success and the criterion of success in first-year algebra, multiple regression also determined the relationship between the various combinations of the predictors of success. This relationship was denoted by $R^2$.

The technique also provided a regression equation which allows a student's success in first-year algebra to be predicted from various known factors. The accuracy of this prediction depended upon the strength of $R^2$. It should be noted that the regression equation is applicable only to members of the population under consideration and other populations with similar characteristics.

The multiple regression equation for this study was:

$$y' = b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6x_6 + b_7x_7 + b_8x_8 + k$$

where:

$y'$ = the predicted value of success in first-year algebra for a given member of the population
the multiple regression coefficients

$X_1$ = the values of each of the eight factors upon which the prediction of success was based

$k$ = a constant

Before the multiple regression equation could be used to predict a student's success in first-year algebra, the values of the $b_1$ had to be calculated. The values of the $X_1$ and $y'$ for each member of the sample were collected and used with an existing multiple regression program for the Sigma VII computer at Montana State University in Bozeman, Montana. The values calculated for the sample $b_1$ are estimates of the population $b_1$. In the regression equation, the $b_1$ play the role of weighing each of the eight prediction factors according to the factor's influence in predicting success in first-year algebra.

The following example is provided to facilitate understanding of multiple regression.

Suppose a person is interested in predicting success in first-quarter college calculus from a student's grade in high school senior mathematics and the student's intelligence quotient. The regression equation would be:

$y' = b_1X_1 + b_2X_2 + k$

where:

$y'$ = the predicted value of success in first-quarter college calculus for a given member of the population

$b_1$ = the multiple regression coefficient for $X_1$
$b_2 = \text{the multiple regression coefficient for } X_2$

$X_1 = \text{the student's grade in high school senior mathematics}$

$X_2 = \text{the student's intelligence quotient}$

$k = \text{a constant}$

After the values of the $y^*$, $X_1$, and $X_2$ are collected for the sample, suppose the values of $b_1$, $b_2$, and $k$ are calculated to be 0.045, 0.025, and 0.195, respectively.

A student who has an A (4.00) in his senior mathematics course and an intelligence quotient of 125 will have a predicted score in calculus of 3.50, as shown by the following calculations:

\[ y' = b_1 X_1 + b_2 X_2 + k \]

\[ y' = (0.045)(4.00) + (0.025)(125) + 0.195 \]

\[ y' = 0.180 + 3.305 + 0.195 \]

\[ y' = 3.50 \]

The regression equation that was used in this study is used as the equation in the above example except there are more $b_1$ and $X_1$ values to consider.

**F-Statistic**

In order to determine whether or not the coefficient of multiple regression ($R^2$) was significant and probably not due to a chance occurrence, the F-statistic was calculated from the following formula:

\[ F = \frac{R^2_{FM} - R^2_{RM}}{(1 - R^2_{FM})/df_n} \]

\[ (1 - R^2_{FM})/df_d \]
where:

\[ F = \text{the calculated value of the } F\text{-statistic} \]

\[ R^2_{FM} = \text{the coefficient of multiple regression for the full regression model} \]

\[ R^2_{RM} = \text{the coefficient of multiple regression for the restricted model} \]

\[ df_n = \text{the number of degrees of freedom for the numerator} \]

\[ df_d = \text{the number of degrees of freedom for the denominator} \]

The term "full regression model" in the definition of \( R^2_{FM} \) refers to the regression model that involved all the predictors of success—in this study there were eight predictors.

The restricted model referred to in the definition of \( R^2_{RM} \) refers to the regression model in which all the \( b_i \) were set equal to some arbitrary weight, say \( b_0 \). That is, the restricted model is the chance model. In the restricted model, the restricted regression equation was:

\[ y' = b_0 U + k \]

where:

\[ y' = \text{the predicted value of success in first-year algebra for any member of the population} \]

\[ b_0 = \text{the arbitrary weight of the } b_i \]

\[ U = \text{the unit factor} \]

\[ k = \text{a constant} \]
The terms $df_n$ and $df_d$ were calculated by the following formulas:

$$df_n = \text{the number of independent variables in the full regression model} - 1$$

$$df_d = \text{the number in the sample} - \text{the number of independent variables in the full regression model}$$

Once the value of the F-statistic was calculated, the calculated value was compared with the value of F in a table of the F-statistic. The table value of F depends upon three factors—$df_n$, $df_d$, and the level of significance. The level of significance, called an $\alpha$ level, was determined prior to making the above comparison. If the calculated F value exceeded the table value of F at the given $df_n$, $df_d$, and $\alpha$ values, then the null hypothesis ($H_0$) was rejected and the alternative hypothesis meant that at the given $\alpha$ level $R^2$ was not due to a chance occurrence.

The example used on page 24 is used to help clarify the use of the F-statistic.

Suppose the values calculated by the computer for $R^2_{FM}$ and $R^2_{RM}$ are 0.632 and 0.482, respectively. Also, assume there are twenty-seven students in the sample and the level of significance is $\alpha = 0.01$.

Since there are only two factors used as predictors of success, the number of independent variables in the full regression model is two. The values of $df_n$ and $df_d$ would be one and twenty-five, respectively, as shown by the following calculations.
dfₙ = the number of independent variables in the full regression model - 1

\[ = 2 - 1 \]

\[ = 1 \]

dfₙ = the number in the sample - the number of independent variables in the full regression model

\[ = 27 - 2 \]

\[ = 25 \]

The calculated value of F would be approximately 10.12, as shown by the following calculations.

\[
F = \frac{(R_{FM}^2 - R_{RM}^2)/df_n}{(1 - R_{FM}^2)/df_d}
\]

\[= \frac{(0.632 - 0.482)/1}{(1 - 0.632)/25} \]

\[= 0.150/1 = 0.150 \leq 10.12 \]

\[0.368/25 = 0.01472 \]

The table value of F at \( \alpha = 0.01 \) with dfₙ = 1 and df_d = 25 is 7.77. Since the calculated value of F exceeds the table value, the null hypothesis is rejected and the alternative hypothesis is accepted. This would mean a person could be ninety-nine per cent sure that the value of \( R^2 \) was significant and not due to chance occurrence.

**SUMMARY**

The population was those students at East Junior High School and North Junior High School in Great Falls, Montana, who enrolled in
first-year algebra during the 1970-71 school year. A sample of seventy-nine was selected by the basic stratified random sample method.

The eight categories under investigation were the student's grade in eighth grade mathematics, intelligence quotient, knowledge of arithmetic theory, arithmetic problem solving ability, overall arithmetic ability, reading comprehension, overall ability, and teacher's prediction of success.

The data that was necessary for this study was available in tabular form from the Mathematics Department at East Junior High School and the permanent records at North Junior High School.

The results of the study are presented in tabular form.

Multiple regression and the F-statistic were used to analyze the data.
Chapter 4

ANALYSIS OF THE DATA

In this chapter, the results of the multiple regression analysis are reported. These results are then used to answer the questions posed in this paper on page three. The prediction equation that resulted from the study is also given.

The correlation between each predictor of success and the criterion of success, as well as the intercorrelations among the various predictors of success, are listed in Table 1, page 29.

Since the correlation between each predictor and the criterion of success exceeds the critical F value for \( df_n = 9 \) and \( df_d = 70 \) at a .01 level of significance, the alternate hypotheses listed on pages nineteen and twenty were accepted in each case. Therefore, within the confidence interval given, the results do reflect an actual correlation between the predictors and the criterion of success and are not due to a chance occurrence.

The answers to questions one through nine on page three can be directly read from Column ten of Table 1 (see page 29).

THE PREDICTION EQUATION

A value of .5616 was calculated for the coefficient of multiple regression \( (R^2) \). This value is a measure of the relationship that
Table 1

Results of Multiple Regression Analysis

<table>
<thead>
<tr>
<th></th>
<th>Eighth Grade Grade</th>
<th>Intelligence Quo.</th>
<th>Arithmetic Theory</th>
<th>Problem Solving</th>
<th>Arithmetic Composite</th>
<th>Reading Comprehension</th>
<th>ITBS Composite</th>
<th>Teacher's Prediction</th>
<th>Comb. seven at a time</th>
<th>Criterion of success</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eighth Grade Grade</td>
<td>1.00</td>
<td>.461</td>
<td>.542</td>
<td>.552</td>
<td>.604</td>
<td>.552</td>
<td>.609</td>
<td>.398</td>
<td>.613</td>
<td>.715</td>
</tr>
<tr>
<td>Intelligence Quo.</td>
<td>1.00</td>
<td>.650</td>
<td>.481</td>
<td>.625</td>
<td>.595</td>
<td>.730</td>
<td>.284</td>
<td>.604</td>
<td>.467</td>
<td>.711</td>
</tr>
<tr>
<td>Arithmetic Theory</td>
<td>1.00</td>
<td>.622</td>
<td>.887</td>
<td>.660</td>
<td>.855</td>
<td>.358</td>
<td>.711</td>
<td>.489</td>
<td></td>
<td>.489</td>
</tr>
<tr>
<td>Problem Solving</td>
<td>1.00</td>
<td>.800</td>
<td>.370</td>
<td>.675</td>
<td>.457</td>
<td>.560</td>
<td>.534</td>
<td></td>
<td></td>
<td>.534</td>
</tr>
<tr>
<td>Arithmetic Composite</td>
<td>1.00</td>
<td>.572</td>
<td>.845</td>
<td>.457</td>
<td>.697</td>
<td>.569</td>
<td></td>
<td></td>
<td></td>
<td>.569</td>
</tr>
<tr>
<td>Reading Comprehension</td>
<td>1.00</td>
<td>.793</td>
<td>.397</td>
<td>.529</td>
<td></td>
<td>.364</td>
<td></td>
<td></td>
<td></td>
<td>.364</td>
</tr>
<tr>
<td>ITBS Composite</td>
<td>1.00</td>
<td>.434</td>
<td>.771</td>
<td>.550</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.550</td>
</tr>
<tr>
<td>Teacher's Prediction</td>
<td>1.00</td>
<td>.410</td>
<td>.391</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.391</td>
</tr>
<tr>
<td>Comb. seven at a time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>.564</td>
</tr>
<tr>
<td>Criterion of success</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[ R^2 = .5616 \]

Critical value of F at \( df_n = 9, df_d = 70, \) and \( \alpha = .01 \) is \( .274 \)
exists between each predictor of success and the criterion of success, as well as the intercorrelations among the various predictors of success. Using a one-tailed F-statistic, it was found that $R^2$ was significant at an $\alpha$ level of .01. Knowing the value of $R^2$ is of significant importance because the strength of the prediction equation depends upon the strength of $R^2$.

The prediction equation for this study is:

$$y' = 0.6314X_1 + 0.01729X_2 + (-0.1396)X_3 + (-0.00338)X_4 + 0.2353X_5 + 0.01869X_6 + (0.07187)X_7 + 0.1999X_8 + 0.1249X_9 + (-2.535)$$

where:

$y'$ = the predicted value of success in first-year algebra for a given member of the population

$X_1$ = the value of the grade in eighth grade mathematics

$X_2$ = the value of the intelligence quotient

$X_3$ = the value of the arithmetic theory score

$X_4$ = the value of the problem solving score

$X_5$ = the value of the arithmetic concepts score

$X_6$ = the value of the reading comprehension score

$X_7$ = the value of the composite score

$X_8$ = the value of the teacher prediction

$X_9$ = the value of the eight factors considered seven at a time

The high $R^2$ value indicates a great deal of intercorrelation among the predictors and this is borne out by an examination of the intercorrelations listed in Table 1 (see page 29). This is not a very
surprising fact when one examines the factors used to predict success in first-year algebra.

SUMMARY

Table 1, page 29, lists the results of the multiple regression analysis of the data. In each case, the value exceeded the critical F value so each of the null hypotheses were rejected and the alternate hypotheses were accepted. The answers to the question posed in the paper were also answered.
Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

In this chapter, the results of the study are interpreted with respect to their ability to predict success in first-year algebra and with respect to previous research done in the field. Suggestions for the use of the results and possibilities for further research are also given.

CONCLUSIONS

Each of the eight factors tested were found to have a significant correlation to success in first-year algebra at a level of significance of .01. A one-tailed F-test confirmed that the value of .5616 for the multiple regression coefficient ($R^2$) was also significant at an level of .01. Based on the strength of $R^2$, the prediction equation on page 30 is considered an accurate predictor of success in first-year algebra.

As had been reported in many previous studies, the grade in eighth grade mathematics proved to have the highest predictive validity (.715) of those factors studied. In the opinion of the researcher, this is due to the fact that the grade in mathematics is a reflection of all traits, measurable and immeasurable, that contribute to a student's overall ability in algebra.
Although the predictive validity of one's intelligent quotient (.467) was not the lowest of the factors studied as had been indicated by previous researchers, the predictive validity was below many of the factors studied.

The predictive validity of reading comprehension (.364) was the lowest of all factors studied. Only limited research has been done in the past with respect to this factor, but the study does seem to confirm earlier findings by others.

The four ITBS factors studied—arithmetic composite (.569), problem solving (.534), arithmetic theory (.489), and overall composite (.550)—along with considering all eight factors seven at a time (.564), have rather mediocre ability in predicting success in first-year algebra. As one would expect, there were very high intercorrelations among the ITBS arithmetic scores.

The teacher's ability to predict success in first-year algebra (.391) proved to be quite poor.

RECOMMENDATIONS

In reading the following recommendations, keep in mind that the results of this study are applicable only to schools similar to the ones studied.

Since the calculations involved in using the prediction equation given on page thirty are quite tedious and long, it would not be
advisable to use the full equation unless one had access to a computer.

Even then the relatively low correlations between all factors, except the grade in eighth grade mathematics, and the criterion of success would not seem to warrant the time and effort needed to make use of them.

It would seem more profitable to use the grade in eighth grade mathematics and a good prognostic test, such as the Orleans-Hanna or Iowa Algebra Aptitude Test, and develop a prediction equation using only those factors. The Orleans-Hanna and Iowa Algebra Aptitude Test were suggested since several other investigators have reported very good correlations between these tests and success in first-year algebra.

Since the predictive validity of the four scores used from the ITBS are all about the same, any one could be used if a larger prediction equation was desired. The predictive validity of intelligence quotient, teacher prediction, and reading comprehension appear too low to warrant their use.

Although this study confirmed an earlier study regarding the ineffectiveness of teacher predictions as a predictor of success in first-year algebra, more research should be done in this area before definite trends can be ascertained.

The low predictive validity of reading comprehension (.364), although reported in an earlier study, was surprising. So often it seems a student does poorly in algebra because he has reading problems,
especially with respect to word problems. More research should be done in this area. The research should first try to determine if the type of reading experienced in mathematics is the same as tested in standard reading comprehension tests. If not, a test is definitely needed to measure this skill.

As stated earlier, one reason the grade in eighth grade mathematics is so effective as a predictor of success in first-year algebra is that it may take into account many traits which are immeasurable or at least difficult to measure. One such area is a student's attitude towards mathematics. Such a trait could be measured if all teachers used the same scale to rate students with respect to attitude. A great deal more research should be done in this area.

SUMMARY

All the factors studied were shown to have positive correlations with success in first-year algebra that were real and not due to chance occurrence.

The grade in eighth grade mathematics was shown to have the highest predictive validity for first-year algebra.

The three arithmetic scores and the composite score of the ITBS, as well as the combination of all factors considered seven at a time, were shown to be only fair predictors of success in first-year algebra.
A student's intelligence quotient, ability to read and comprehend, and his teacher's prediction of success all had very low predictive validities.

Recommendations were made to develop prediction equations that made use of only a few of the proven factors rather than all eight.

More research was recommended in the areas of reading comprehension, teacher predictions, and attitudes of students.
LITERATURE CITED


