ALL-SKY POLARIZATION IMAGER DEPLOYMENT AT MAUNA LOA OBSERVATORY, HAWAII

by

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ii

APPROVAL

of a thesis submitted by

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citation, bibliographic style, and consistency, and is ready for submission to the Division of Graduate Education.

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An all-sky imaging polarimeter was deployed in summer 2008 to the Mauna Loa Observatory in Hawaii to study clear-sky atmospheric skylight polarization. The imager, designed at Montana State University, operates in five distinct wavebands in the visible region of the spectrum and is capable of imaging the overhead skylight hemisphere with a fisheye lens. This thesis describes the Mauna Loa deployment and presents an initial comparison of these data to those observed by Coulson with a zenith–slice polarimeter in the late 1970s and early 1980s. We show how the all-sky imaging technique yields additional insight to the nature of skylight polarization beyond what is observed in a single zenith scan.

It was found that the skylight polarization data collected compared well to that collected by Coulson. Furthermore, the polarization signatures obtained over the two week deployment were found to depend inherently on the underlying cloud cover at altitudes beneath the observatory. The different cloud topologies provided variable upwelling unpolarized light which entered the field of view of the instrument. As a result, the anticipated polarization signatures were reduced by variable amounts from this increased unpolarized radiation countering the strong skylight polarization band observed at 90° from the sun. Finally, to quantify the nature of the upwelling scattered radiation from the clouds to the variation in the degree of polarization, the maximum degree of polarization was fit to a decreasing exponential trend versus upwelling radiance as obtained from overhead satellites. Statistical correlation for these quantities was found to be favorable and this result can potentially yield useful prediction ability for skylight polarization signatures in clear-sky conditions.
INTRODUCTION

Polarization-Sensitive Imaging

Polarization imaging in the visible and near-infrared (NIR) wavelengths is an important subset of general polarimetry of interest to both military and civilian research. The ability to image a scene and discern the polarization-sensitive features in that image is useful in discriminating targets with polarization-varying characteristics [1]. Recently, the application of these traditional surveillance techniques—previously constrained primarily to the mid-wave and long-wavelength infrared bands because of a simpler phenomenology dominated by emission as opposed to reflection—to the visible and NIR wavelengths has been explored by the United States Air Force. However, due to complicated scattering and reflection properties of light in these wavelength regions, the signatures are significantly more complex than the longer-wavelength IR regions. In addition to the incident unpolarized light from the sun, light scattering from molecules and aerosols in the atmosphere influences the polarization signature of objects on the ground [2]. Therefore, in a better attempt to understand the nature of the polarization of the illuminating skylight, an all-sky imaging polarimeter was designed [2,3] to measure skylight polarization.

In addition to research centered on target polarization properties, there is also interest in using skylight polarization for retrieving aerosol properties [4,5,6,7]. For example, the atmospheric aerosol size distribution, an important quantity that indicates anthropogenic aerosol content affecting the Earth’s overall radiation budget, is itself not
an observable quantity; instead, several active and passive optical systems aim to estimate this quantity through an aerosol retrieval or inversion, where observable quantities such as aerosol extinction and optical thickness data from calibrated lidar and sky radiance measurements are used to estimate the size distribution given a number of assumed or modeled parameters. In conjunction with these measurements, calibrated polarization sky radiance measurements can be used to increase accuracy in present inversion techniques [4] which is critical in understanding the role of aerosols in potential climate change.

Critical to both of these applications is an understanding of the skylight polarization in an atmosphere largely devoid of anthropogenic aerosols such that effects attributed to aerosol loading may be distinguished exclusively. Localized continental aerosols made it less convenient to establish this baseline understanding in Bozeman MT; therefore, the instrument was deployed to the National Oceanic and Atmospheric Administration (NOAA) Mauna Loa Observatory on the island of Hawaii in May and June of 2008. This isolated, industry-free island location at an altitude above the aerosol and marine boundary layers provided an ideal location to study the scattering properties of atmospheric molecules and their subsequent polarization signatures. These measurements, in conjunction with the availability of in situ aerosol data available from the observatory and surface reflectance data from satellite images, allowed for a more complete characterization of measurements in differing aerosol or cloud environments.
Instrument Overview

The imaging polarimeter was designed chiefly as an instrument to be used for validation of the US Air Force’s polarized radiative transfer model, p-MODTRAN (Polarized Moderate Resolution Transmission code) [2]. The front lens array was designed to accommodate two fields of view—a telephoto assembly for regionalized scene imagery and a wide-angle, equidistance projection fisheye lens to image the sky dome.

Many different types of polarimeters have been designed and used to measure the polarization signatures of objects and different atmospheric phenomena. A number of these, such as the one deployed by Coulson [8], are single-pixel photodiode or photomultiplier tube scanning detectors [9] that require a scanning system to raster the target of interest or sky dome. The method utilized by the MSU imager captures the entire sky dome simultaneously, making it a better choice for rapidly capturing the polarization signature of the sky dome in changing cloud and aerosol conditions.

Recently, Horvath [10] used a polarimeter with fisheye lenses in conjunction with film cameras to make simultaneous all-sky polarization measurements to measure skylight polarization and clouds. This method, while allowing for rapid imaging, required chemically developing film images, which could introduce inconsistencies from image to image that would make radiometric calibration of this instrument impossible. Similarly, North and Duggin [11] employed a polarimeter with a 4-lens film camera on a tripod with a dome mirror to reflect the sky dome to the camera field of view. This
system exhibited similar issues with the inherent radiometric inconsistencies that result from using a film-based system. Voss and Liu [12] employed an all-sky polarimeter that used a fisheye lens with a CCD imager. While this solved the issues surrounding imaging with film, this system had polarizers mounted in a rotating filter wheel that required long exposure times that created polarization errors when atmospheric conditions varied rapidly (as with cloud motion), so this instrument was restricted to operation on clear days only. This imager also had distinct 10-nm bandwidth spectral filters that allowed better recognition of polarized spectral features that would not be allowed by the large spectral width of a film system.

Many of these difficulties were addressed by the design of the all-sky polarization imager used in this experiment. Two liquid-crystal variable retarders (LCVRs) were used to obtain sequential images at different polarization states, which were then used to infer the Stokes vector parameters at each pixel. Five 10-nm-wide channels centered through the visible and NIR at 450, 490, 530, 630, and 700 nm were used and rotated into place with a filter wheel. The fast switching time of the LCVRs allowed for a 4-image sequence at a single wavelength to be taken in less than 0.3 s. An entire five-wavelength sequence, including time for filter wheel rotation and focusing to the 1 megapixel CCD array, takes roughly 10-15 seconds. Rapid imaging at a single wavelength was a driving requirement to accurately capture clouds in motion and the LCVR design accomplishes this goal. Figure 1 illustrates the instrument schematic and operation in all-sky imaging mode.
Thesis Organization

This thesis is organized in the following fashion. Chapter 2 presents a basic explanation of polarized skylight using Rayleigh single scattering by molecules in the atmosphere, along with an overview of the skylight scattering geometry for the sky dome. Chapter 3 covers the purpose and history of the Mauna Loa Observatory and details some of the motivations for moving to this location to take skylight polarization data. Chapters 4 and 5 describe the experimental data obtained in this campaign and the steps needed to properly analyze polarization images in a quantitative manner.
BACKGROUND

Light Polarization Overview

Polarization is an intrinsic property to all electromagnetic waves and refers to the spatial orientation and evolution of the electric field vector about the axis of propagation as a function of time. Consider a plane wave in a non-absorbing medium with wave number $k$, angular frequency $\omega$, and amplitudes $E_x$ and $E_y$ corresponding to the maximum electric field magnitudes in the $\hat{x}$ and $\hat{y}$ directions respectively. Equation (2.1) illustrates the representation of this wave mathematically as a function of time $t$ and a phase differential between the orthogonal vectors of $\phi$. Note that over time, for a fixed location in the x-y plane (e.g. $z = 0$), the field magnitude vector will trace out a pattern dependent on the magnitudes in each direction and relative phase difference.

$$E = \hat{x} E_x \cos(\omega t - kz) + \hat{y} E_y \sin(\omega t - kz + \phi) \quad (2.1)$$

The most general case of polarized light is that of elliptical polarization, for which the magnitudes $E_x$ and $E_y$ and phase differential $\phi$ vary in some combination that traces an ellipse in the x-y plane. From this equation the two special cases of purely linear and purely circularly polarized light can be obtained. For linearly polarized light to occur, either the $x$ or $y$ directed electric field magnitude is zero, causing oscillation of the field vector solely in one plane, or the relative phase difference between the field components in the $x$ and $y$ direction is zero. This causes oscillation in a line that depends on the magnitude of the field in each of the principal directions. Circularly polarized light requires that the magnitudes of the field in each direction be equal and that the phase
difference between field components be exactly 90°. This results in the magnitude of the total electric field for a given z-plane over time tracing out a circle in the x-y plane.

Figure 2 illustrates the spatial variation of the electric field vector magnitude for unpolarized, linear, and circularly polarized light in the x-y plane for a fixed distance z.

Because of the inability to measure instantaneous phase of a field directly, measurements of polarization must utilize the intensity of the wave in question. Developed in 1852 but not popularized until the middle of the 1900s, the Stokes polarization parameters provide a concise way to represent the entire set of potential polarization states of an electromagnetic wave [13]. The 4-element Stokes vector illustrates how light is decomposed into real scalar coefficients denoting portions that are nonpolarized ($s_0$), horizontally or vertically polarized ($s_1$), polarized at an angle of 45° ($s_2$), and circularly polarized ($s_3$). These values are shown in the bottom portion of the previous figure for the states listed. In addition, the 4x4, 16-element Mueller matrix is used in conjunction with Stokes vectors to allow for the analysis of the effects of various
interspersed media on incident light waves with an approach analogous to that of a transfer function transforms an input. As will be shown, the polarization state of electromagnetic waves scattered from very small particles can be determined from the knowledge of the incident Stokes vector parameters and the calibrated Mueller matrix of the observing sensor.

Atmospheric Polarization

Atmospheric polarization signatures originate from two primary mechanisms. The first, single scattering by small spherical particulates, is explained using Mie scattering theory applied to collections of small scattering volumes of various size distributions. It is this theory that can be applied to scattering by atmospheric gas molecules, cloud particles, and hydrometeors to explain the predominant atmospheric phenomena such as sky colors, rainbows and glories [14]. This scattering theory applied to atmospheric molecules and aerosols gives rise to the polarization characteristics of the atmosphere that will be the dominant phenomenon discussed here.

The second process that affects skylight polarization signatures is multiple scattering of light in the atmosphere and from the ground back into the atmosphere. Multiple scattering, to a first order for the purposes of this thesis, contributes to the depolarization of skylight and in practice requires complex computational processing to solve the equations of radiative transfer that describe the true nature of the process. Furthermore, because of the extreme variability inherent in a multiple scattering process that stems from varying terrain, cloud cover, and atmospheric conditions, rarely is it
possible to provide a generalized solution to these equations for an arbitrary scene. Therefore, this treatment will focus on the empirical sky polarization signatures obtained from field measurements, and relating these data to the primary single-scattering mechanisms that are theoretically predictable and then use ancillary data from overhead satellites and co-located aerosol sensors to account for and quantify the effects of multiple scattering on the degree of skylight polarization.

**Single Scattering by a Spherical Particle**

For the purposes of this derivation, scattering from atmospheric particles will be considered from the standpoint of particles being spherical in nature. Strictly speaking, this assumption does not apply to asymmetric molecules; however, expressions for scattered fields from non-spherical particles do not generally have closed-form solutions. To remedy the effects of this on anticipated polarization signatures, a depolarization term will be introduced in the following section that has been experimentally determined for particular molecular compositions.

Naturally occurring atmospheric gaseous molecular constituents have effective diameters that range from 0.1 to 1 nm [9]. Therefore, at visible wavelengths of approximately 400-700 nm, the particle diameter is much less than the wavelength of incident light. This condition, commonly referred to as the Rayleigh scattering limit, is generally satisfied when the radius of the particle satisfies \( a \leq 0.03 \lambda \) [9], where \( a \) is the radius and \( \lambda \) is the wavelength of incident light. If further approximations are made to the expressions for the scattered field from these particles and the particles are constrained to the troposphere, and are thus non-ionized, and have a similar index of
refraction as the surrounding atmosphere, Mie scattering theory reduces into the approximate and intuitive Rayleigh solution to the scattering of light from particles much smaller than the wavelength.

Rayleigh scattering theory can be derived independently as a dipole scattering solution for visible light and particles discussed here [15], but here the more complete development from the Mie solution is given. Therefore, this derivation begins with the expression of the electric field scattered from a particle in terms of spherical coordinates. For this discussion, only the scattered fields are of interest as these are the fields that interact with the polarimeter. The incident wave is assumed to be a $\hat{z}$-directed plane wave with the electric field oriented in the $\hat{z}$-direction. Furthermore, it is assumed that the field is observed in the far-field from the scattering molecule, such that $r$, the distance from the particle, and $k$, the wave number of the scattered field, satisfy $kr \gg 1$.

With these assumptions and taking into account the approximations inherent for spherical Hankel functions $h_n^1(kr)$ with large values of $kr$, the scattered electric field in the transverse spherical coordinate $\hat{\theta}$ and $\hat{\phi}$ directions ($\hat{r}$ component goes asymptotically to 0 in the far-field) reduces to Equation (2.2) [16].

$$E_s(\theta, \phi) = E_o e^{jkr} \cos \phi S_2(\cos \theta) \hat{\theta} - E_o e^{jkr} \sin \phi S_1(\cos \theta) \hat{\phi} \left[ \frac{V}{m} \right] \quad (2.2)$$

In this equation, $E_o$ is a complex scattered field amplitude and $S_1$ and $S_2$ are the field amplitude scattering coefficients defined by Equation (2.3) and are functions of the spherical polar coordinate angle $\theta$. 

The scattering coefficients include internal coefficients $a_n$ and $b_n$, which can be solved for explicitly using the boundary conditions for the transverse electric and magnetic fields on the boundary of the sphere, and which are thus expressed in terms of $j_n(\rho)$ and $h_n^{(l)}(\rho)$, the spherical Bessel and Hankel functions, respectively [16,17]. The angularly dependent functions $\pi_n$ and $\tau_n$ are defined in terms of Legendre polynomials $P_n^l(\cos \theta)$, commonly used in cases involving spherical coordinate symmetry. Bohren and Huffman [16] show the full derivation of the explicit solutions for the scattering coefficients. A limited coverage of this derivation is found in Appendix A; however, the remaining treatment of these relations here will focus on the approximate values of these functions and constants for the limiting case of Rayleigh scattering. First, however, the field expressions can be manipulated in such a way that the electric field expression in Equation (2.2) can be used to express the parallel and perpendicular polarization components in terms of the incident and scattered components. The scattering geometry is shown in Figure 3.

\[
S_1(\cos \theta) = \sum_{n=1}^{\infty} \frac{2n + 1}{n(n + 1)} (a_n \pi_n(\cos \theta) + b_n \tau_n(\cos \theta)) \\
S_2(\cos \theta) = \sum_{n=1}^{\infty} \frac{2n + 1}{n(n + 1)} (a_n \tau_n(\cos \theta) + b_n \pi_n(\cos \theta))
\]
Writing out the two equations in the form of an amplitude scattering matrix for the geometry in Figure 3, Equation (2.4) illustrates the relationship between the orthogonal polarizations of both incident and scattered light.

\[
\begin{bmatrix}
E_{s\parallel} \\ E_{s\perp}
\end{bmatrix} = \frac{e^{jk(r-z)}}{-jkr} \begin{bmatrix}
S_2 & 0 \\ 0 & S_1
\end{bmatrix} \begin{bmatrix}
E_{i\parallel} \\ E_{i\perp}
\end{bmatrix}
\] (2.4)
Muller scattering matrix determined based on the irradiances (in units of $W/m^2$) corresponding to the fields expressed above. The transition from fields to irradiances is accomplished by generating the Stokes parameters for both incident and scattered waves according to the relations given in Equation (2.5) [13].

\[
\mathbb{I} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \langle E^*_\parallel E^*_\parallel + E^*_\perp E^*_\perp \rangle \\ \langle E^*_\parallel E^*_\perp - E^*_\perp E^*_\parallel \rangle \\ \langle E^*_\parallel E^*_\perp + E^*_\perp E^*_\parallel \rangle \\ j \langle E^*_\perp E^*_\perp - E^*_\parallel E^*_\parallel \rangle \end{bmatrix} \propto \begin{bmatrix} I_{0^o} + I_{90^o} \\ I_{0^o} - I_{90^o} \\ I_{45^o} - I_{135^o} \\ I_{LHCP} - I_{RHCP} \end{bmatrix} \tag{2.5}
\]

Here, the 4x1 Stokes vector $\mathbb{I}$ is shown to be a function of the two orthogonal scattered field components. When applied to Equation (2.4), these equations contain implicit relations of the amplitude scattering matrix containing $S_1$ and $S_2$ combined in various fashions. In order to relate the incident Stokes vector to the scattered Stokes vector, a 4x4 transformation Mueller matrix must be generated based on the combinations of $S_1$ and $S_2$ resulting from the calculations inferred by Equation (2.5). Because the amplitude scattering matrix has zeros for two elements, several cancellations occur in the top right and lower left portion of the matrix. This matrix, $\mathbb{P}$ takes the form shown in Equation (2.6).

\[
\mathbb{I}_s = \frac{1}{k^2 r^2} \mathbb{P} \mathbb{I}_i = \begin{bmatrix} S_{0i} \\ S_{1i} \\ S_{2i} \\ S_{3i} \end{bmatrix} = \frac{1}{k^2 r^2} \begin{bmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{12} & P_{11} & 0 & 0 \\ 0 & 0 & P_{33} & P_{34} \\ 0 & 0 & -P_{34} & P_{33} \end{bmatrix} \begin{bmatrix} S_{0i} \\ S_{1i} \\ S_{2i} \\ S_{3i} \end{bmatrix} \tag{2.6}
\]

The Mueller matrix elements shown in the above expression and are defined in Equation (2.7).
With the basic relationships between incident and scattered Stokes vectors now described by a scattering matrix containing an infinite series of spherical Bessel and Hankel function terms, the coefficients \( a_n \) and \( b_n \), and angularly dependent functions \( \tau_n \) and \( \pi_n \), a number of simplifications can be made when the particle size is very small compared to the wavelength of light. First, using the power series expansions for the first few terms of the spherical Bessel and Hankel functions, finite expressions for the \( a_n \) and \( b_n \) coefficients are obtained and illustrated in Equation (2.8), accurate to order \( x^5 \) where \( x \) is the size parameter equal to \( ka \) (\( a \) is the particle radius).

\[
\begin{align*}
P_{11} &= \frac{1}{2}(|S_2|^2 + |S_1|^2), \\
P_{12} &= \frac{1}{2}(|S_2|^2 - |S_1|^2) \\
P_{33} &= \frac{1}{2}(S_2^*S_1 + S_2S_1^*), \\
P_{34} &= \frac{j}{2}(S_2^*S_1 - S_2S_1^*)
\end{align*}
\]  

(2.7)

(2.8)

The parameter \( m \) is the relative index of refraction of the particle \((N_f)\) divided by the surrounding medium (typically air, \( N \)). Pausing here for a moment and returning to the Rayleigh scattering condition discussed previously, with a particle radius on the order of 1 nm at the mid-band instrument wavelength of 530 nm, the size parameter \( x \) will be roughly 0.012. Thus, for terms on the order of \( x^5 \), the contribution is 8 orders of magnitude beneath the contribution for parameters of order \( x \) alone. Thus, if only terms of order \( x^3 \) are kept, Equation (2.8) simplifies to a single term in \( a_1 \), the remaining terms
are ignored, and the series expansion for the Bessel and Hankel functions can be terminated at \( n=1 \).

Now applying the single remaining term to the expression in Equation (2.3) and applying the equations for \( \pi_1(\cos \theta) \) and \( \tau_1(\cos \theta) \) [16], the corresponding \( S_1 \) and \( S_2 \) amplitude scattering matrix parameters are determined as seen in Equation (2.9).

\[
S_1 = \frac{3}{2} a_1, \quad S_2 = \frac{3}{2} a_1 \cos \theta \quad (2.9)
\]

From these relationships, it is apparent that the scattered irradiance will only depend on the angle \( \theta \). Henceforth, to adapt to conventional atmospheric polarization studies, this angle is termed the \textit{scattering angle} and re-designated \( \alpha \) for the remainder of this thesis.

The Mueller matrix for the above conditions with this change of angular variable taken into consideration is shown as Equation (2.10)

\[
\mathbb{P} = \frac{9|a_1|^2}{4k^2 r^2} \begin{bmatrix}
\frac{1}{2}(1+\cos^2 \alpha) & \frac{1}{2}\sin^2 \alpha & 0 & 0 \\
\frac{1}{2}\sin^2 \alpha & \frac{1}{2}(1+\cos^2 \alpha) & 0 & 0 \\
0 & 0 & \cos \alpha & 0 \\
0 & 0 & 0 & \cos \alpha
\end{bmatrix} \quad (2.10)
\]

From this, it is easily seen that for an incident unpolarized wave with \( \mathbb{I}_i = [I_i \ 0 \ 0 \ 0]^T \), the far-field scattered irradiance becomes that of Equation (2.11), which follows the anticipated \( 1/\lambda^4 \) dependence with respect to wavelength and has a maximum intensity at a scattering angle of \( 0^\circ \) and \( 180^\circ \) respectively.

\[
I_s = \frac{8\pi^4 N^6 a^6}{\lambda^4 r^2} \left| m^2 - 1 \right|^2 (1 + \cos^2 \alpha) I_i \quad (2.11)
\]

With regard to the polarization components of an unpolarized incident wave, the degree of linear polarization (DoLP) is defined as the ratio of the square root of the sum
of the squares of the linear-polarization Stokes components to the total incident light irradiance [1]. Considering the above phase matrix, the expression for the degree of linear polarization for isotropic spherical-particle Rayleigh scattering reduces to Equation (2.12).

\[
\frac{I_{pol}}{I} = \sqrt{s_1^2 + s_2^2} = \frac{\sin^2 \alpha}{1 + \cos^2 \alpha}
\]  

(2.12)

Anisotropic Rayleigh Scattering

The Rayleigh scattering phase matrix for the molecular constituents of the atmosphere can also be modified to account for the anisotropy of these molecules. Hansen and Travis [14] arrive at a similar expression to that shown in Equation (2.12), where the parameter \( \delta \), the depolarization factor accounting for molecular asymmetries, has been introduced.

\[
DoLP_{anisotropic} = \frac{\sin^2 \alpha}{1 + \cos^2 \alpha + 2\delta/(1 - \delta)}
\]  

(2.13)

This depolarization factor varies for different molecular species from 0.02 to 0.09 [14] and is approximated for the standard mix of air at visible wavelengths by multiple references [9,18,19] as being on the order of 0.0279 to 0.0303.

This result indicates that as the depolarization factor decreases to zero, the maximum degree of linear polarization becomes 100% at an angle of 90° from the incident light direction. Typical values of the depolarization factor \( \delta \) mentioned above result in a maximum polarization of not 1, but rather on the order of 0.94, or 94% linearly polarized scattered light. In actual sky polarization observations, it is a combination of
this depolarization factor and multiple scattering that will drive the observed maximum degree of polarization down from the theoretical maximum [20]. The empirical magnitude of this effect, especially when viewing the atmosphere in a mostly aerosol-free (thereby ideally Rayleigh in nature) environment, is the primary focus of this thesis.

Multiple Scattering and Surface Scattering Effects

The Mauna Loa Observatory (MLO) is well suited for observing clear overhead skies; however, both the trade winds and volcanic activity at nearby Kilauea result in extremely variable amounts of cloud cover beneath the observatory, leading to multiple scattering of sunlight back onto molecules in the field of view (FOV) of the imager, which contributes to skylight depolarization (see Figure 4).

---

Kilauea, a volcano that has been continuously active since 1983, has produced significant amounts of sulfur dioxide and volcanic aerosols that combine with the trade winds to produce significant aerosol and volcanic cloud layers at altitudes predominantly below the MLO. The observatory usually has the significant advantage, however, in that contribution to total surface reflectance from other sources is minimal. Reflected
scattering from the sea surface accounts for 4-5% of total reflectance and contributions to
depolarization are minimal owing to the centralized observatory location on the island
[9]. Furthermore, the reflectivity of the surrounding dark brown and black lava
formations is comparably small such that it too can be ignored to first order. Shown in
Figure 5 are two photos of the surrounding landscape along the road to the observatory.
The low reflectivity of the lava made this site ideal for reduction of multiple scattering
effects from the surrounding terrain.

![Figure 5: Photos taken from the MLO access road looking (a) eastward (credit J. A. Shaw) and (b) northward towards Mauna Kea illustrating the brown and black surface.](image)

Thus, while being a highly unpredictable variable, the underlying cloud cover
surrounding the observatory is the remaining critical quantity in characterizing the
depolarization of the clear sky above when compared to what is expected from pure
Rayleigh scattering.

**Skylight Scattering Geometry**

If we consider the sun to be a point source of unpolarized light from infinity, then
the maximum degree of polarization will occur at a scattering angle $\alpha$ of $90^\circ$. With this
assumption, the unit vector direction $\hat{n}_s$ from the scattering volume $P$ is identical to that of the unit vector from the observer location. Figure 6a illustrates the scattering geometry convention. Furthermore, if the unit vector to the scattering volume $\hat{n}_p$ is rotated about solar vector $\hat{n}_s$ an arc of the maximum degree of linear polarization (DoLP) is created that passes through both $P$, the scattering volume in the principal plane (containing the sun, zenith, scattering volume, and the observer) and the rotated point $P'$ that still lies at a $90^\circ$ angle from the sun; however, this is a dihedral plane rotated by an angle $\gamma$ from the principal plane. Figure 6b illustrates the red arc of maximum DoLP as well as the green dihedral plane rotated from the principal plane. For a ground-based, all-sky imager such as this, it was expected to see arcs of this nature, projected into a 2-D plane with a fisheye lens, translate across the sky throughout the day as the sun moved through the sky.

![Figure 6: Derived from [21], (a) illustrates the scattering angle geometry used for the principal plane and (b) extrapolates this geometry to a full three-dimensional DoLP arc at a constant 90° angle from the sun.](image-url)
MAUNA LOA OBSERVATORY DEPLOYMENT

Mauna Loa Observatory

The Mauna Loa Observatory (MLO) is an atmospheric monitoring station situated on the north flank of the Mauna Loa volcano on the island of Hawaii in the Hawaiian island chain. Renowned for its role in the measurement of atmospheric aerosol and molecular constituents, MLO is especially known for the uninterrupted monitoring of atmospheric carbon dioxide (CO₂) levels and tracking the steady rise in concentrations since the mid 1900s. Shown below, this Google Earth rendering of the terrain illustrates the location of MLO geographically with reference to the active volcano Kilauea. Mauna Loa, while technically an active volcano, has not had any major volcanic activity since the last eruption in 1984.

Figure 7: Geography of the island of Hawaii. Note the location of the active Kilauea volcano to the south and east of the observatory.
Additionally, MLO historically has been a site in which cloud-free, low-aerosol polarized sky measurements have been made [9]. In the late 1970s and early 1980s, K. Coulson initiated a measurement campaign with a scanning zenith slice polarimeter [8] to obtain cross sections of the degree of polarization over a 5°-wide sun-zenith-observer slice (principal plane) of the sky dome. MLO is a favored location for this type of measurements due to an elevation of 3,394 m above sea level and frequent lack of overhead clouds. Clouds and aerosols tend to remain beneath the observatory until the late morning, only rising to the observatory level in early to mid-afternoon as the local heating of the surrounding volcanic landscape shifts the downslope winds upward, carrying clouds and aerosols with them. For this reason, and due to the *in situ* ability to quantify the aerosol changes throughout the day, MLO has been an ideally suited location to observe a nearly pure Rayleigh scattering atmosphere as it changes to one containing aerosols. Figure 8 illustrates the MLO observatory and the relative location of the instrument while operating there.

![Figure 8: (L) Mauna Loa Observatory as viewed from the CO₂ monitoring tower facing east (photo by Forrest Mims III). (R) Instrument deployed along a roughly East-West line with adjustable sun occulter to the left of the image (Photo by J. A. Shaw).](image)
The imager was deployed at MLO from 21 May 2008 to 3 June 2008 and operated continuously throughout the day from sunrise until sunset. To accomplish this, all calibration and test equipment was required to be on-site. Because the liquid crystal variable retarders require constant temperature control to maintain the set phase retardences, there was no way to ship the instrument overseas, a process taking more than a week, and maintain powered control. Once arriving at the site, the instrument was unpackaged and calibrated within the span of a day. Full-time data collection commenced on the afternoon of 21 May 2008. During the 14-day span of operation, the weather remained generally clear overhead with the exception of 4 days (May 21st, 22nd, 27th and 28th) when afternoon clouds encroached and surpassed the observatory’s elevation. On 29 May 2009, an instrument malfunction of the rotating filter wheel caused the afternoon data for that day to be corrupted. Therefore, nine days of observations were obtained with consistently clear overhead conditions throughout the day as seen by the naked eye.

Conditions at elevations below the observatory, however, fluctuated extensively in terms of aerosol content and cloud cover throughout the deployment. In terms of cloud cover, a persistent cloud bank was present for the first 5 days of the experiment on the western (leeward) side of the island. Fueled by the westerly trade winds, clouds and volcanic emissions from Kilauea wrapping along the southern portion of the island, there was significant and consistent haze and cloud cover at elevations approximately 1.8 to 2 km above sea level. Typical scenes for the morning and afternoon periods are shown as Figure 9 and Figure 10, respectively. Note the presence of clouds beneath the
observatory at sunrise and midmorning (Figure 9) as well as in the mid afternoon and at sunset in Figure 10.

Figure 9: Easterly view looking out over the saddle at sunrise (a) and mid-morning hours (b).

Figure 10: West to northwesterly views looking at mid afternoon (a) and sunset (b).
Figure 11 illustrates an image (a) of cloud and surface reflected radiance as captured by the Geostationary Operational Environmental Satellite (GOES-11) satellite and an image showing the substantial out gassing of the Kilauea vent (b).

Kilauea Eruption Status

Kilauea has been continuously active at various levels of significance since 3 January 1983 [22]. Beginning on 21 November 2007, the activity in both the Kilauea summit crater (Halema’uma’u) and the continuously-erupting Pu’u ‘O’o crater to the southeast of the summit began to increase significantly. A new channel of lava broke the surface at the southeast eruption site on 21 November 2007 and shortly after that in January and February 2008, sulfur dioxide (SO$_2$) emissions at the summit crater began to
rise steadily from a background rate of around 150-200 tonnes/day to around 600 to 1000 tonnes/day as plumes of SO$_2$ began to erupt from the crater floor. Sulfur dioxide molecules, with average diameter near 0.3 nm, satisfy the condition for Rayleigh scattering of visible light; however, the increased presence over these two months was indicative of increasing volcanic activity. On 19 March 2008 an existing SO$_2$ vent exploded in the summit crater floor which was followed by significant gas- and ash-laden eruptions through the last half of March and early April. Unlike SO$_2$, the ash plumes contain particulates of significantly larger sizes, typically less than 2 mm [22], as well as significant water vapor, leading to visible brown and white plumes that are lofted into the boundary layer. During the deployment, as indicated by Figure 12, this fissure at the Halema’uma’u crater was dispensing SO$_2$ at rates as high as nearly 2000 tonnes/day—an almost tenfold increase compared to the baseline levels observed from 2002-2006 [23]. While these data do not yield specific correlation to ash and water vapor emission rates for this time, they provide insight into the extreme variability of this eruption during the deployment.
This level of activity is significantly higher than what was present in the late 1970s and early 1980s when Coulson was performing his clear-sky observations at MLO during a quiescent period of Kilauea activity. Therefore, because of the significantly different underlying structure in cloud and aerosol content and distribution caused by increased water vapor and ash clouds, the upwelling radiance as seen by GOES would be dramatically increased for the duration of the 2008 MLO deployment. Furthermore, the recent surge in activity, which began in 2007 and continues at even higher levels at the volcano to this day, precludes the likelihood of being able to measure polarization signatures not affected by increased multiple scattering from underlying volcanic clouds and fog for some time to come.
SKY POLARIZATION IMAGING

All-sky Polarization Images

As stated, to obtain an image of the entire sky dome, a wide-angle fisheye lens was used. Specifically, a Nikon Nikkor 16mm focal length, equidistance-projection lens with a maximum aperture of f/2.8 was used to collect all-sky images. As discussed extensively in [2], this lens is the front end of an imaging system that incorporates field lenses to reduce wide-angle ray vignetting and ray incidence angles on the polarization optics (LCVRs and polarization analyzer), and a Nikon Micro-Nikkor 105 mm f/2.8 lens to scale the image to fit on the 1024x1024 pixel CCD array. Figure 13 illustrates the fisheye lens (a) and the micro lens re-imaging lens (b).

![Figure 13: (a) 16 mm focal length fisheye lens and (b) 105 mm micro lens](image)

An equidistance projection lens ideally maps the above hemisphere to circular contours of equivalent angle starting from the zenith at the origin to nominally the horizon at the farthest radial contour. Because of both angular calibration constraints and the presence of baffles on the lens (as shown in the above figure), the maximum full-angle field of view for this deployment was limited from ideally 180° to around 155°.
The equidistance fisheye projection equation is shown in Figure 14 illustrates the contour mapping. In this equation, $f_d$ is the number of pixels per degree of angle of the fisheye projection to the surface, $\phi$ is the angle relative to the zenith and $r$ is the radial location on the surface itself.

$$r = f_d \phi$$

To produce an all-sky image that is sensitive to linear polarization, as shown in Equation (2.8), at least the first three Stokes vectors are required. This imaging polarimeter produces estimates for all four Stokes parameters; however, as shown in [2,3,9], the anticipated amount of naturally produced circular polarization is negligible and for the purposes of this analysis will be ignored in favor of treatment of the linear polarization only. For each pixel of the CCD, the Stokes parameters $s_0$, $s_1$, and $s_2$ are used to produce the degree of polarization image. Figure 15 shows the combination of the two linear polarization Stokes components combining to form the DoLP image. Because
these images were taken near sunrise, the band of maximum DoLP is near the zenith and extends across the imager FOV. The zero-valued artifacts seen at the edges of the Stokes images are an instrument calibration effect caused by a slight lens tube misalignment over the optical path. These edge effects are removed during image processing, which is why they appear as zeros in right-hand DoLP image.

For the viewing geometry at MLO, the imager was placed in a configuration with the top of the FOV pointing essentially due west. The instrument position and orientation was kept as consistent as possible for the duration of the deployment. This was difficult because the system needed to be moved indoors at night to assure that the LCVR calibrations were never compromised, since the ambient temperature at night frequently approached 4°C (39°F) and it was unknown (and thereby not left to chance) if the temperature controllers could compensate for such cold temperatures. In general, after
looking at the results of individual images, it was clear that the daily deviation minimally affected the processing to within around +/- 6 pixels (around 1°).

The nominal layout of the instrument with respect to the observatory and the anticipated solar path through the FOV is shown in Figure 16. Images such as those in Figure 16b (for 700 nm) were produced on 1-2 minute intervals for each of the five wavelengths. In this morning image with clear sky overhead, we see the characteristic maximum DoLP arc stretch across the sky with flaring at the edges caused by distortion in the fisheye lens. The sun, still not in the FOV of the imager, moves across the sky from lower left to upper right, rotating the max-DoLP arc throughout the day to maintain a constant 90° separation.

Figure 16: Imager orientation (a) and observed solar path (b) shown for a morning DoLP image.
Fisheye Mapping Techniques

Using the relationship denoted in Figure 14 and the relative angular positioning of the sensor with respect to the cardinal directions, the CCD image plane was mapped to a true three-dimensional hemisphere representation of the polarized scattered light. The typical spherical and Cartesian relationships are shown in the figure below. Note that the fisheye mapping equation yields a direct result for the angular elevation \( \phi \) for each pixel given that the number of pixels per degree elevation, \( f_d \), is known. This number was estimated, through subsequent measurements of roof edges at Montana State University to be roughly 6.75 pixels per degree for this given field of view.

\[
\begin{align*}
    x &= r \cos(\phi) \cos(\theta) \\
    y &= r \cos(\phi) \sin(\theta) \\
    z &= r \sin(\phi)
\end{align*}
\]

Once the elevation angle was mapped and the relative sensor azimuth reference \( (\theta) \) was known, the fisheye projection was mapped to a unit hemisphere \( (r=1) \) by the above equations where \( x, y, \) and \( z \) are the scaled three-dimensional Cartesian coordinates for each pixel in the image.
Next, vector relationships were used to extract the true spatial locations of the maximum DoLP arc from the three-dimension fisheye projection (see Figure 18). First, the sun location for a given image was determined. Next, the location of the principal plane point at an angle of 90° from the sun was determined by adding 90° to the elevation angle and accounting for a discontinuity at angles greater than 90° when the arc wraps around the horizon. The locus of points at an angle of 90° from sun (blue arc below) was then computed by rotating the vector from the origin to the principal plane maximum DoLP point, \( \vec{R}_p \), about the vector from the origin to the sun, \( \vec{R}_s \). To assure that the constraints on the FOV were conserved, points along this arc with an elevation angle less than one-half the FOV were ignored. This arc then was sampled (typically on the order of 0.01 degrees per sample) to generate a vector of points along the arc in which the scattering angle was determined. Each of these \( i^{th} \) sample points, \( \vec{R}_{D_i} \), forms the basis for construction of a plane, termed for the purposes of this discussion a dihedral plane, that extends from this point through a plane containing the origin and the sun (the green curve in Figure 18 is in the lowest dihedral plane to the horizon). To generate the scattering angles required to analyze DoLP profiles versus scattering angle, a perpendicular to the dihedral plane, \( \vec{n}_{D_i} \), as shown in red below, was constructed via \( \vec{n}_{D_i} = \vec{R}_s \times \vec{R}_{D_i} \) and then the vector for each maximum DoLP arc dihedral location \( \vec{R}_{D_i} \) was rotated about the constructed normal \( \vec{n}_{D_i} \) to generate the scattering angles denoted by the green arc and referred to in Equation (2.8) as the angle \( \alpha \).
Figure 18: (a) The generalized solar scattering geometry for the band of maximum DoLP for a typical morning scene. (b) The overlay of the DoLP from a zenith-viewing downward perspective with vector references included.

Figure 18 illustrates the scattering geometry referenced above. Note that the blue arc corresponds to the maximum DoLP and the green arc corresponds to a dihedral plane of scattering angles at the maximum dihedral angle within the FOV. Once the scattering angle coordinates were computed for the green arc, a 3-D nearest-neighbor lookup was used to co-locate this coordinate in the fisheye projected Cartesian space. Due to the computational complexity of computing hundreds of sky sample coordinates in a given image and the sheer number of images taken during the deployment across each day and wavelength, an interpolation across absolute spatial coordinate was used to reduce runtime.
While this direct lookup method is required to locate the exact spatial coordinates to extract from the projected fisheye image, both the scattering and dihedral angles may be directly calculated from the fisheye projection. Because acquiring the coordinates for each contour still required a nearest neighbor lookup in at least two dimensions, this did not save in computational complexity; however, resulting angle images gave immediate insight to a particular viewing geometry.

To calculate these angles directly, the same initial fisheye referencing was done such that three pixel maps of the Cartesian $x$, $y$, and $z$ positions were obtained. The calculation of the scattering angle is straightforward in that the angle $\alpha$, as defined previously, is just the angle between the sun and a given position in the sky. If the 3-D Cartesian space is mapped to the CCD array size of 1024x1024 pixels, then for each Cartesian variable there are indices $m$ and $n$ corresponding to each pixel reference, where in this case $m=n$ because of the square array. The computation to produce an image of scattering angles is then $\alpha_{m,n} = \cos^{-1}(\vec{R}_s \cdot \vec{R}_{m,n})$ where $|\vec{R}_{m,n}| = \sqrt{x_{m,n}^2 + y_{m,n}^2 + z_{m,n}^2}$.

Similarly, a mapping of the fisheye space to the dihedral scattering angles was applied to produce an image of the dihedral scattering angles relative to the principal plane. Again, starting from the three Cartesian coordinates per pixel, each point is projected into the plane perpendicular to the principal plane (containing the maximum DoLP arc) by taking the cross-product of the solar reference vector with each pixel position vector as noted by $\vec{R}_{\perp P}(m,n) = \frac{\vec{R}_s \times \vec{R}_{m,n}}{|\vec{R}_s \times \vec{R}_{m,n}|}$. Next, a vector perpendicular to the principal plane was constructed by taking the cross-product of the vertical ($z$) axis with the solar vector denoted as $\vec{R}_{\perp P} = \hat{z} \times \vec{R}_s$. The angle between these two vectors...
describes the angle between the principal plane and the plane containing the maximum DoLP at that location on the $90^\circ$ arc, the origin, and the sun (the dihedral plane). To compute this angle, $\gamma_{m,n}$ for each pixel, accounting for common scattering angle references, the inverse cosine of the dot product is taken as follows: $\gamma_{m,n} = \pi - \cos^{-1}(\vec{R}_{s,p} \cdot \vec{R}_{m,n})$. Figure 19 illustrates the mapping of both of these angles for early morning data in which the maximum DoLP arc is near the zenith.

![Morning Geometry: Mapping to Scattering and Dihedral Angle](image)

Extraction of the principal and dihedral plane DoLP vs. scattering angles was completed by computing many dihedral angle cuts and computing the scattering angles along those planes. From this processing, each image was projected into a rectilinear space such that Equation (2.8) could be compared to the observed data for a given dihedral sky angle. Figure 20 shows a coarsely sampled hemisphere projection of the angular cuts in dihedral angle (a) for illustration purposes, and the subsequent projection of finely sampled cuts into the scattering angle vs. dihedral angle rectilinear space (b).
Section 5 of this thesis uses these rectilinear projections further to compare empirical results to the Rayleigh scattering theory presented previously.

Figure 20: (a) Hemispherical projection of DoLP with black lines indicating dihedral arcs across the bold black line indicative of the maximum DoLP arc. (b) The extraction and representation of the DoLP with respect to both dihedral and scattering angles.
RESULTS

Clear Sky Principal Plane Maximum Degree of Linear Polarization

The most concise way to present results for such a large quantity of data over multiple days is by plotting the daily principal plane maximum DoLP profile against solar elevation angle (or zenith angle). This metric allowed for consistent comparisons of the DoLP data as the solar geometry is an invariant independent variable as compared to the local time which changed daily throughout the experiment. Profiles for the nine clear days of measurements are shown in Figure 21. Note that three days in the middle of the experiment, 27 May through 29 May 2008, had significant clouds in the afternoon or had afternoon instrument malfunctions, and have been omitted from this analysis. All subsequent results neglected these days entirely and focused on the nine clear days.

Figure 21: Maximum DoLP principal plane profiles for each of the 5 sensor wavelengths.
The principal plane profiles show that there was significantly less polarization in the early afternoon than at corresponding elevation angles in the morning hours. Furthermore, there was a general trend of increased polarization at longer wavelengths. It was speculated that this trend was due to an overall decrease in multiple scattering from Rayleigh-sized particles in the longer wavelengths even though the radiance of scattering at longer wavelengths was reduced. While this resulted in less overall light at longer wavelengths, it was more polarized than scattering at shorter wavelengths.

Finally, these data illustrate the variability of the observations through the deployment. For example, on the 23rd and 24th of May the morning signatures were the highest for all wavelengths during the deployment; however, the corresponding afternoons for each of these days are remarkably consistent with the afternoon observations of the subsequent days. Figure 22 shows the respective mean and standard deviation in error bars for each of the wavelengths averaged over the nine clear days in the deployment in 3° bins of solar elevation angle.

These plots reiterate the decreased DoLP at all wavelengths in the early afternoon compared to similar solar geometries in the late morning. Of further note was the marked decline in DoLP from about 0° to 30° in solar elevation angle for the morning geometries. This effect was mirrored at all wavelengths and was most pronounced at the longer wavelengths of 630 and 700 nm. From roughly 30° to 60° there was found to be stabilization in DoLP, followed again by a sharp decline as the DoLP arc approached the horizon.
While this decline was anticipated due to the increased air mass and multiple scattering encountered when the DoLP arc nears the horizon, both the asymmetry between morning and afternoon and the structural dissimilarities in the two portions of the curve were deemed to be a function of both the cloud cover observed during the deployment and the topography of the island. Further research into the nature of these asymmetric profiles is ongoing.

**Comparisons to Previous Data**

Coulson [9] deployed a zenith-slice polarized radiometer that scanned across the solar principal plane (or sun’s vertical) in 5° increments. Operating in dual-channel mode
which measured the polarization at two wavelengths simultaneously, full zenith scans of
all eight of the instrument wavelengths from 320 to 800 nm could be completed every 24
minutes. For his published results data were taken in the morning, typically with solar
zenith angles not less than 20°. The morning was preferred by Coulson under the premise
that in the afternoon solar heating caused aerosols and orographic clouds to frequently
climb to and above the observatory level, thereby obscuring and depolarizing the
observable skylight through multiple scattering processes. This same type of morning
atmospheric evolution was observed during the 2008 deployment; however, clouds only
overtook the observatory on 2-3 days during the two-week long deployment.

Figure 23 summarizes the cloud and aerosol conditions both for Coulson’s 1977
deployment and the 2008 deployment of the imaging polarimeter. Of note in this figure
is the isolated and extremely clear day observed by Coulson on 19 February 1977 that is
referenced as the clearest day in a 30-year measurement of aerosol content [9]. For the
purposes of comparison, two additional days with conditions similar to the 2008 data—9
and 10 March 1977—were also used, as were data taken during an Asian dust storm on 2
May 1977 as a lower-bound on atmospheric clarity at MLO.

In assessing the quality of these data, the color scale indicates the relative clarity
of the atmosphere in terms of underlying cloud cover, observed atmospheric aerosol
layers, and the position of the aerosol and cloud boundary layer throughout the day. As
shown, during the 2008 deployment there was moderate to significant underlying cloud
cover, especially in the afternoon, for the duration of the deployment.
Two mornings, the 23rd and 24th of May, exhibited similar increased clarity and had the highest DoLP of the days in this deployment for early morning. The majority of the middle period of the experiment had moderate cloud and aerosol presence, with the exception of the 27th to the 29th of May, in which significant boundary layer presence above the observatory largely obscured measurements for parts of the early and middle afternoon. The last days contained consistently lower DoLP and significant underlying cloud coverage in the morning periods; however, the afternoon periods had coverage and DoLP that was consistent with the majority of the data taken through the deployment.

For comparing the all-sky imaging data to Coulson’s, the DoLP images were processed to locate the principal plane in each image and a slice was extracted from the image for the appropriate range of scattering angles. Localized averaging over a 3x3 pixel area was used to produce a profile of maximum DoLP versus solar elevation angle,
which was then compared to Coulson’s data. Figure 24 reproduces both the original Coulson results and the 2008 MSU results at the closest comparable wavelengths obtainable with the all-sky imager.

In Figure 24, all dashed lines refer to data taken in 1977 by Coulson at 800 and 365 nm. Data taken with the imager are shown with solid lines and were taken at 700 and 450 nm. Coulson repeatedly states that the data on 19 February 1977 represents the clearest sky conditions, both above and below the observatory, in 30 years of measurements. As a result, the maximum DoLP for that day was consistently above 80%. Note that while pure Rayleigh scattering, even with molecular anisotropy accounted for, predicts a maximum DoLP around 94%, the commonly observed maximum near sunrise is only 85%. Both the presence of atmospheric aerosols and, unique to MLO, multiple scattering from the underlying clouds contribute to this overall
depolarization [9]. Unfortunately, Coulson only reported data for approximately the first 50-60° of solar elevation angle, so afternoon trends that might have been of interest for aerosol studies were not included. Looking to Coulson’s 800-nm and MSU’s 700-nm imager data, good agreement was seen for 800-nm data on 9-10 March 1977 compared with 700-nm data on 23 May and 3 June 2008. For these two days in 2008, the DoLP principal plane measurements consisted of the two most comparable days to that of Coulson in terms of morning polarization values for the duration of the two-week deployment. Coulson qualitatively cites the 9th and 10th of March 1977 as having cloud undercast values of 30-50% cloudy on the 9th and 60-70% on the 10th of March. These values are consistent with visual observations made on the 23rd of May and 3rd of June 2008. Further quantitative comparisons are difficult since satellite measurements of the quality now available were not at the time of Coulson’s data.

Figure 25 shows the mean DoLP profiles from 700-nm images, binned into 2° elevation-angle partitions, for the nine days during the 2008 deployment that met clear overhead sky criteria. Both the profile shape and DoLP values are consistent over the length of the deployment compared to the most appropriate data available from Coulson (800 nm). The standard deviation of roughly ± 2% is very near the minimum instrument error tolerance for the MSU imager [2]. It should be noted that agreement at sunrise is not as good between the two Coulson days and the average of the imager data in the region where solar elevation angle is 10° or less. This effect is due to the rapid variation of DoLP with respect to elevation angle that occurs in this region and because the results shown below are binned in 2° partitions, which effectively decreases the DoLP near
sunrise. The true DoLP for this low elevation angle region tended towards that as shown in the previous figure for the 700 nm imager wavelengths and does follow the increase near sunrise.

![700 nm DoLP 9-Day Average](image)

**Figure 25:** Mean DoLP over 9 days of favorable aerosol and cloud conditions. The 1-σ standard deviation for a 2° bin in elevation angle is also shown as error bars.

Short-wave measurements, however, compared less favorably when 365 nm data from 1977 was compared to 450 nm data from 2008. The comparison, as illustrated in Figure 24b, is between two wavelengths that differ by 115 nm. While the long-wave comparisons appear to have little dependence on wavelength, the variation with conditions and wavelengths in the near-UV regions appear to have significant impact on the compared DoLP for 365 nm with respect to 450 nm. Even though these values differ on the order of 10-15% for the days compared, the trend observed at the 700 and 800 nm wavelengths is similar, in that the DoLP for 3 June 2008 is less than the DoLP for 23 May 2008.
To investigate this wavelength dependence, the Coulson data illustrated several sweeps of DoLP at given elevation angles for the wavelength extent of the sensor. Figure 26 shows the wavelength variation of Coulson’s data for 10° and 45° elevation angles on the exceptionally clear day of 19 February 1977. This figure illustrates the sharp fall-off of DoLP at short wavelengths. This decline in DoLP explains why comparisons at the longer 700 and 800 nm wavelengths agreed well for similar underlying cloud conditions, but did not agree for the 365 to 450 nm case. These wavelengths lie close enough to the linear downward trend in DoLP that the 115 nm difference in wavelength results in an at least a 10% reduction in DoLP when the 19 February 1977 data are used as a reference.

Figure 26 also shows Coulson’s DoLP versus wavelength during an Asian dust episode. Tending to occur during the spring months, significant quantities of dust from the Gobi desert spanning from China to Mongolia enter the troposphere and can move across the Pacific ocean all the way to the continental US and Hawaiian airspace [9]. The effects of this dust observed during such a storm in May 1977 are shown as the red curve in Figure 26. Note that the DoLP has decreased significantly throughout the long-wave visible and NIR spectral regions and that the overall DoLP signature has decreased significantly compared to the clear day at a given elevation angle of 45° degrees. The strong decrease in the NIR region suggests that the dust particle sizes were generally in the hundreds of nanometer size range [9]. This inference is supported by measurements taken in 1980 [24] and 1986 [25], where significant silicon and sulfur concentrations were found with sizes in the range from 0.5 to 1 µm. Also shown are Coulson’s data for
the average of 9 and 10 March 1977 at 10° and 45° degrees elevation for wavelengths of 365 and 800 nm. The standard deviation is shown for the average of the two days as error bars. Note that the error bars fit within the 10° and 365 nm data point. These data points, while not supplying an entire profile over wavelength, illustrate that the fall-off with decreasing wavelength is present for clear overhead sky conditions when Asian dust is not present—the predominant observed condition for the 2008 deployment. Two elevation angles are shown here, with the 10° angle in blue and the 45° angle shown in green.

Figure 26: Coulson maximum DoLP data plotted as a function of wavelength for a very clear day at 10° and 45° elevations and during a dust storm. The four individual data points indicate the mean DoLP for 9/10 March 1977 at 10° and 45° elevations for 365 and 800 nm.

The wavelength profiles in Figure 26 give a range of DoLP in which the 2008 imager data were expected to fall. Although no major Asian dust events were confirmed
by solar radiometer data during the 2008 deployment, slight dust layers were observed on the horizon during a period where Asian dust was forecast. Because no significant increase in optical thickness was observed; however, it can’t be confirmed that the dust layers encroached on the observatory or in the sky above the observatory. There were significant volcanic aerosols and underlying clouds created DoLP variations that could approach that of Coulson’s 1977 Asian dust scenario, so these curves serve as approximate boundaries in which the bulk of the clear sky data should lie. Selecting the data points closest to the 10° and 45° elevation angles from the 9 days of clear sky data, and then averaging over these days for each wavelength, a plot similar to Figure 26 was generated for the principal plane of the 2008 imager data (Figure 27).

Figure 27: Average DoLP vs. wavelength for the 2008 imager. Error bars indicate a 1-σ standard deviation from the daily mean. Coulson data is plotted behind for comparison.
Figure 27 plots the mean DoLP as a function of wavelength for the 10° and 45° elevation angles. Data from Figure 26 is repeated in light shading for comparison. Note that if a line is drawn for each elevation angle from the average data points for 800-nm Coulson data, through the 2008 mean imager data, to the Coulson 365 nm data, these lines would resemble the expected Rayleigh roll off and would fall within the 1-σ standard deviation of the 2008 imager measurements. Thus, from these aggregate data, the results obtained in 2008 were consistent with those obtained in 1977 for conditions where moderate to significant underlying clouds were present, but the atmosphere above the observatory was predominantly clear.

Ancillary Data Products and Sensors

Solar Radiometer Data

As also inferred by Coulson’s results, it was also found in the 2008 deployment that it is not the optical depth that factored the most in the overall depolarization of skylight at the MLO under consistently low-aerosol conditions. Optical depth, monitored on-site with a CIMEL solar radiometer as a part of NASA’s Aerosol Robotic Network (AERONET) project [26], remained remarkably stable throughout the clear-sky days for each of the wavelengths at which the imaging polarimeter operated; however, the DoLP varied dramatically independent of this stability. Figure 28 illustrates the total optical thickness or optical depth for the duration of the deployment. Note that the optical thickness attributed to molecular absorption and scattering—the Rayleigh optical thickness—is plotted for reference as dashed lines. As expected, this value remained
essentially constant throughout the deployment, with the aerosol optical thickness modulated around a level slightly above this baseline value.

Rayleigh optical thickness can be computed directly from a number of models that give a temperature and pressure profile as a function of altitude. For MLO, the elevation of 3,400 m above sea level significantly decreased the expected amount of molecular scattering. Whereas at sea level for a 530-nm wavelength the molecular optical thickness calculated for a tropical latitude of 15° N (close to MLO at 19° N) was 0.1135 \cite{18}, at the elevation of MLO this calculated value decreased significantly to 0.073. This compares favorably to the average AERONET Rayleigh optical depth of 0.067 over the course of the MLO deployment, considering that the calculation utilized a pre-determined model for a tropical locale and not a temperature or pressure profile from an \textit{in situ} radiosonde.

The variation in aerosol optical thickness was very small throughout the entire deployment relative to what would occur at sea level. Nevertheless, on days for which it was definitely determined the boundary layer overtook the observatory (with clouds observed overhead)—27 and 28 May 2008—there was an appreciable increase in the optical thicknesses across wavelengths. Some of the days, May 25\textsuperscript{th} and 31\textsuperscript{st}, for example, exhibited a rise in aerosol optical thickness in the afternoon but this rise was not correlated with clouds present above the observatory and therefore indicates that the aerosol-laden marine boundary layer overtook the observatory on those days. This plot also indicates the stable and consistent molecular optical thickness throughout the
deployment. The molecular optical depths varied minimally for the duration and these variations were on the order of $10^{-2}$ or less.

Figure 28: Total optical depth history for 2008 MLO deployment. The average molecular (Rayleigh) optical depth shown in (--) for each wavelength. Data shaded grey is indicative of substantial overhead cloud cover in the afternoon.

Shown for the 530 nm wavelength, Figure 29 shows aerosol thickness for three days of the deployment. The left and right plots represent days for which the optical thickness was essentially constant (23 May and 3 June 2008), but for which there were significant difference in DoLP. The middle day, 28 May 2008, had the boundary layer encroach in the early afternoon and the difference in aerosol optical thickness is more than three times that for when the above atmosphere is largely devoid of aerosols. It should also be noted that when clouds are in the path between the sun and the solar
radiometer, no data are taken, so the measured optical thickness indicates aerosol content only and results in the jagged nature of the middle curve as clouds were present.

Figure 29: The measured 530 nm aerosol optical thickness for three days in the deployment. Note the increase in aerosol thickness indicative of boundary layer presence and overhead clouds as indicated by the center shaded region.

GOES Satellite Imagery

Rather than the dominant DoLP variation arising from overhead aerosol thickness, it appears to have arisen from fluctuations in reflected surface radiance caused by underlying clouds. The upwelling reflected radiance from the surface of the earth and clouds beneath the observatory lead to multiple scattering that appears to be the most significant cause of DoLP fluctuation. Multiply scattered radiance is essentially unpolarized and thus, while it contributes to the overall radiance, it reduces the fraction of polarized light in the DoLP calculation.
The most widely available method to gauge the reflectance from the earth’s surface is to analyze downward-looking satellite data. While both the Advanced Very High Resolution Radiometer (AVHRR) and the Moderate Resolution Imaging Spectroradiometer (MODIS) satellites both provide visible and near-IR coverage with reasonably high spatial resolution, the Geostationary Operational Environmental Satellite (GOES) provides the most frequent visible band measurements at 15-minute intervals. Because the other satellites have non-geostationary orbits, the scan times and scan geometries vary from scan to scan. Thus, even though the 1-km spatial resolution of the GOES system is not as fine as that offered by MODIS or AVHRR, the repeatability of both the scanning geometry and scan intervals led to the use of data from GOES-11—the western hemisphere-centric satellite.

The principal observable from the GOES system used for this analysis was the effective upwelling radiance measured in the visible range. These data were obtained from the online repository at the NOAA Comprehensive Large Array-Data Stewardship System (CLASS) [27] and consisted of 10-bit digital imager counts, which were calibrated using a set of pre and post-launch calibration coefficients that accounted for both the initial imager status as well as an assumed instrument performance degradation while on orbit [28,29]. The resulting data used for analysis was a consistent area scan that encompasses a geodetic extent roughly ± 2.5° in latitude and longitude centered on the MLO on the Island of Hawaii. Images produced were roughly 400 by 600 pixels in size that yielded calibrated radiance in units of $W/m^2/sr/\mu m$ at each pixel.
To assess the correlation between upwelling radiance and DoLP, several images similar to those in Figure 30 were used to verify the decrease in DoLP as a function of upwelling radiance. Because the upwelling radiance reflected by the earth surface does not change appreciably from day to day over two weeks, the variable quantity of importance was deemed to be the reflected radiance from cloud layers below the observatory. This is shown in Figure 30 for two days of data collection, where on both days the overhead AOD at 700 nm was consistently near 0.03 to within 0.001; however, the cloud configuration differed significantly between the two days. The top row shows two DoLP images taken on different days at 1000 Hawaiian Standard Time (HST), with the maximum DoLP band just beginning to exit the FOV of the instrument near the horizon. Beneath the sky polarization images are two radiance images from the GOES-11 satellite stationed over the Pacific Ocean. From these images, it is clear that the overall DoLP is reduced significantly on 3 June 2008 relative to 23 May 2008, even though the overhead AOD is nearly identical on both days. With the GOES imagery, it is clear that, while a cloud bank had developed to the southwest of the island on both days (primarily due to Kilauea out gassing becoming trapped in the southwesterly trade winds), the overall morning reflected radiance is substantially less for 23 May than for 3 June.
Figure 30: Top row: All-sky DoLP images for (left) 23 May 2008 and (right) 3 June 2008. Bottom row: GOES a radiance image showing the entire island of Hawaii at the same times. All-sky polarization imager deployment location approximated by ‘x.’

Figure 31 shows the maximum DoLP plotted versus elevation angle over the course of the day for 23 May and 3 June 2008, along with several calculated DoLP profiles. The depolarization effect of underlying clouds is visible, especially for the morning hours, as this is nominally when the atmosphere around MLO is the least turbid and the underlying cloud impact becomes a relatively larger effect. However, this figure illustrates an additional effect not observed by Coulson since his polarimeter operated predominantly in the morning hours. Because the 2008 measurements were made near the summer solstice, the solar track through the day passed nearly exactly overhead from...
east to west, with a maximum zenith angle of 89°. As such, the maximum DoLP profile was expected to be symmetrical from morning to evening. However, it was apparent that additional effects were depressing the afternoon maximum DoLP.

To verify further that the AOD was not a significant factor in explaining the DoLP curve asymmetry, a single-scattering model was run with the polarized MODTRAN radiative transfer code over a parametric sweep in AOD. Shown as black curves in Figure 31, the MODTRAN results suggest that when only single-scattering conditions are accounted for and no underlying surface or cloud albedo model is assumed, the curves are symmetric for the late May and early June solar geometry encountered during the deployment. It should be noted that the solid black curve
indicates the MODTRAN results for an AOD similar to that which was encountered during the 2008 deployment, but the maximum DoLP calculated for this AOD is a result of the MODTRAN code being run in single-scattering mode only. Neither multiple scattering nor surface albedo are considered in this model; however, these results indicate that secondary processes are needed beyond optical depth to explain the measured asymmetric results.

Currently, work is underway on better explaining this asymmetric structure and large reduction in DoLP as compared to the single-scattering model results. Mie scattering codes that account for multiple scattering and surface reflection effects beyond those seen with MODTRAN have been obtained, and work to reconcile the 1st-order model deficiencies when compared to the observed data is ongoing.

Comparisons to Rayleigh Scattering Theory

As discussed previously, the fisheye lens mapping can be used to convert the flat CCD image plane to a full 3-dimensional representation of the sky dome. As such, the dihedral angle slices seen in Figure 20b represent profiles across the dihedral angle $\gamma$ along the abscissa of DoLP as a function of scattering angle $\alpha$ along the ordinate. As shown by Equation (2.8), this relationship is clearly defined for Rayleigh scattered particles. However, this 1st-order approximation does not represent the observed atmospheric conditions in that aerosols and clouds both decrease the DoLP beyond the predicted values even when the molecular asymmetry was accounted for by anticipated depolarization factors ($\delta$). The effects of the atmospheric depolarization factor upon the
incident solar irradiance have been studied extensively, with most emphasis coming in terms of determining the optical thicknesses associated with the molecular constituency of the atmosphere [9,19,18,30,31,32]. There was considerable disagreement in the calculation of the value of this factor throughout the 1980s; however, two of the most recent works on the matter by Bucholtz [18] and Bodhaine et. al [33] contain the most comprehensive techniques for accounting for the molecular anisotropy depolarization factor and thus these works, in conjunction with atmospheric profiles from [34], were used to calculate the Rayleigh scattering parameters that follow.

Using the fits to data repeated by Bodhaine et al. [33], the depolarization factor $\delta$ (also commonly seen in literature as $\Delta$ or $\rho_n$) was computed and is shown in Figure 32.

![Figure 32: Depolarization factor $\delta$ plotted as a function of wavelength with instrument filter bandwidths imposed along the curve.](image)

Note that the nominal value of 0.0279 suggested by Young [30] is within 2% of the calculated values, with the exception of the 450-nm filter, which has an error on the order of 3.7% as compared to the fitted value.

These computations suggest that for the clear Rayleigh sky devoid of aerosols, the maximum DoLP should be approximately 94% depending on the wavelength of observation. To compare these values of $\delta$ to those derived from the observed DoLP
data, the rectilinear DoLP profiles were processed to extract the principal plane profile (at a dihedral angle of 0°) and then fit to Equation (2.8) using a nonlinear least-squares solver. Because the daily instrument position varied and the instrument was subject to wind gusts that caused slight shifting in the image plane, the mapping from the fisheye frame of reference to the rectilinear profile in scattering angle often produced slight offsets that can be seen in the Figure 33 for the 530-nm imager wavelength. Note that the DoLP trend has a slight linear bias with respect to scattering angle compared to the flat dashed line on the left plot for a theoretical maximum scattering angle of 90°. This effect was believed to be largely introduced by improper referencing of the imager rather than by any radiative effect.

The nonlinear least-squares fit procedure was done with the three $\beta$ parameters shown above. Because the motion of the imager has introduced offsets, these were accounted for by adding scaling coefficients ($\beta_1$ and $\beta_2$) to the sinusoidal dependencies.
Figure 33: (a) Rectilinear DoLP projection and b) the cross-section in the principal plane as well as the fit and fit parameters obtained from a nonlinear least-squares fitting process. The shown plots are for the 530 nm imager wavelength.

Note further that to maintain consistent phase relationships with respect to $\beta$, a trigonometric identity has been used to replace the $\sin^2(\alpha)$ term with a $1-\cos^2(\alpha)$ term. The $\beta_3$ term is thus representative of the depolarization factor. For these data taken in the early morning on 24 May 2008, the effective value of the depolarization factor was 0.11, which was significantly higher than anticipated for a pure Rayleigh sky. For comparison, the data and fit are shown as blue and green lines in Figure 33b. The red line indicates the pure Rayleigh curve with a maximum around 94.5% and a $\delta$ of 0.0285. The sinusoidal phase terms were then used to compute the data shift amounts away from a maximum at 90°. By solving the equation $\beta_1 \alpha + \beta_2 = 90$ for $\alpha$, the scattering angle for maximum DoLP was calculated at 88.04°. This fit procedure thus also served to provide an alignment parameter that was used to correct for instrument motion and misalignment in the rectilinear domain. However, it should be noted that previous investigations [9]
have shown that adherence to theory with regard to predicting angular dependences—
especially the neutral point angular references—is highly dependent on atmospheric
turbidity and that decoupling the effects of different atmospheric conditions on small
shifts in maximum DoLP angular location is impossible without absolute and consistent
instrument location throughout the deployment.

The inflated value of the depolarization fit parameter as well as the fact that the
optical thickness observed was clearly above that attributed solely to molecular scattering
(see Figure 28, 530 nm for example) indicated again that the conditions throughout the
deployment were not ideal for observing a pure Rayleigh scattering atmosphere. The true
conditions were not only accounted for by an increased aerosol presence, but also by the
elevated surface albedo caused by significant cloud coverage beneath the observatory, as
has been suggested. A limited study by Bahethi and Fraser [19] also revealed the effects
of these quantities in conjunction with the inclusion or exclusion of the depolarization
parameter. Simulations using Mie scattering codes were required since inclusion of these
particles violated the Rayleigh scattering requirements specified previously. Done in
1980, this study also used higher molecular optical depths and the increased $\delta$ value of
0.0303 originally determined by Gucker [35]. Five distinct models were used to compare
different cases. The relevant models for comparison to the MLO data are models C, D,
and E. Models A and B showed that for a clear sky at sea level (effective optical
thicknesses were around 0.0924 neglecting the depolarization factor and 0.0973 when it
was accounted for) the depolarization factor has the greatest effect near the 90$^\circ$ scattering
angle where the DoLP is the highest. Models C and D were computed for much more
turbid atmospheres at sea level with total optical thicknesses of 0.4172 when the depolarization term was neglected and 0.4221 when the term was included. Model E included the same parameters as model D; however, it included upward-directed radiance as deflected by an assumed underlying Lambertian surface, thus considering albedo explicitly. For the data shown, the corresponding upward albedo was 0.25 corresponding to a reflectivity of 25%.

Figure 34: Comparison of models A-E for different conditions as adapted from [19]. Shown in blue and black are two curves from the 2008 MLO deployment at zenith angles of 60° at 0800 HST.

Figure 34 illustrates the five models for a solar zenith angle of 60°. Also included are two DoLP profiles taken on the mornings of 24 May (blue) and 3 June (black) 2008. Several interesting points are illustrated by this figure. First, as expected, the MLO data taken at an altitude of 3.4 km and smaller optical depth was clearly more polarized than simulated data at sea level with larger molecular and aerosol optical depths. Second, the effects of the depolarization term are lessened as the aerosol optical thickness increased,
as shown by models C and D. Bahethi and Fraser [19] and Coulson [9] both commented on this and conclude that the molecular depolarization term effect becomes much less significant as the aerosol air mass increases. Third, there was a clear correlation and reduction in DoLP when a 25% Lambertian surface reflectance was included in the calculation. This is a key result that illustrates the difficulty in extracting the root cause behind a reduction in DoLP as pertaining to cloud albedo or increased aerosol. Fourth, in looking to the two data profiles taken at MLO on days with comparable optical depths at the time of measurement, the separation in these profiles was consistent with the separation between models D and E caused only by an increase of albedo. Thus, it can be concluded that for days with essentially constant total optical depths, as was mostly the case during the 2008 MLO deployment, the most important characteristic in correlating a decrease in DoLP is the quantification of upwelling radiance caused by cloud cover below the observatory.

Underlying Cloud Cover and Upwelling Radiance Correlations to Depolarization

Determining the correlation between skylight depolarization and upwelling reflected radiance beyond the simple qualitative approaches done by Coulson was one of the goals of this research. Coulson lists cloud cover fractions that range over many tens of percents. For this deployment, given the overhead imagery available to quantitatively assess reflected radiance, a more rigorous approach was taken to correlate the DoLP depolarization as a direct function of cloud radiances. Several approaches were explored, including using the albedo as a direct variable of dependence and using satellite cloud-
detection and cloud-fraction algorithms; however, these approaches were hindered by effects of satellite viewing geometry as the sun moves through the sky.

Albedo and cloud fraction metrics are all derived from the radiance base measurement of the GOES-11 satellite. This radiance is measured from a stationary point in orbit above the earth. As the sun rises and sets, the angle of illumination on land changes dramatically and there is an angular-variation of reflection unique to not only the terrain surrounding the observatory on Hawaii, but also to the cloud conditions for a particular satellite image. This variation is commonly referred to as the bidirectional reflectance distribution function (BRDF) for a given object or surface [36]. The BRDF is a 4-dimensional function, where both the incident and exiting azimuth and elevation angles of light scattering from the surface are variables. While direct measurement of an object’s BRDF for laboratory-sized objects can be made, doing so for the full island of Hawaii is not practical, especially given that cloud coverage is highly variable.

The calculated albedo product referenced in [29] is simply a scaled variation on the calibrated radiance and not a generalized albedo that accounts for the BRDF angular variability; therefore, the compression of the large dynamic range associated with the radiance measurement onto a scale from zero to unity generally did not map well with greater-than-unity-albedo data points resulting from the conversion. Cloud percentages or fraction retrievals were explored also, but because of the difficulty in specifying what was or was not a cloud in varying illumination geometries; this approach was abandoned as well, in favor of using the calibrated upwelling radiance measurement directly. Note that this measurement does include an inherent angular dependence that yields a
\( \sim \cos^2 \phi \) dependence as the sun moves through elevation angles \( \phi \); however, given the nature of the stationary GOES-11 satellite this result is unavoidable.

Partitioning and Distribution of Upwelling Radiance Over the Island

Using the satellite measurements of upwelling radiance, it was determined that there was bias in the probability of clouds being present in certain areas surrounding the island. As shown previously, Kilauea ash plumes regularly added significant reflected radiance to the southwest portion of the island, with the trade winds pushing this around the south flank of Mauna Loa to the southeasterly side of the island. Cloud banks were also regularly sustained on both the east (windward) and west (leeward) sides of the island for multiple days during the deployment.

Because the DoLP moves through the sky dome throughout the day, it was hypothesized that DoLP reductions would correlate most highly with clouds in localized regions that change throughout the day. Consequently, the island was partitioned into seven distinct regions that varied in extent in 5 km steps outward from MLO to \( \pm 100 \) km around the island. These regions consisted of 4 quadrants arranged in a northeast-southeast-southwest-northwest fashion. The average radiance in each of these quadrants was calculated at each of the 20 successive expanding ranges outward from MLO. The average of the east, west, and all quadrants made up the remaining 3 regions. Figure 35 shows the \( \pm 50 \) and \( \pm 100 \) km borders, as well as the quadrant partitioning denoted by the dashed partitions in the \( \pm 50 \) km border.
While it was anticipated that this localization analysis would yield better correlation between radiance and DoLP than a simple mean of radiance over a large portion of the island, it was found that a particular region’s radiance was largely reflected in the mean over all four quadrants together. As such, the overall mean radiance was the driving metric to compare with skylight depolarization effects. Remaining to be decided, however, was the exact area over which this mean should be computed. As shown below, the relatively low radiance of the surrounding ocean relative to clouds and some land features made averaging over a larger extent than the island size reduce the dynamic range of the total radiance values, as it simply drove down the mean by adding small to insignificant radiances to the calculation while adding to the number of pixels. Actual determination of the ideal region for averaging was made by looking at the $R$-Squared
statistical test on a decaying exponential regression fit against the maximum DoLP in the principal plane as a function of total radiance for an increasing region of averaging. The scores are shown in Figure 36 for the seventh region of interest, which included all four quadrants averaged together.

These scores indicated that the ±50 km distance from the observatory consistently had scores greater than 0.85 (the score of an $R^2$-Squared test can vary from 0 to 1, with 1 being the best score that indicates 100% correlation of the fit to the original data). With consistent scores of around 0.85 or better, the ±50 km distance was selected as the optimal distance for the five wavelengths in question.

Figure 36: Fit test scores for the seventh region (average over entire extent) for expanding distances from MLO for each imager wavelength.
Final Regression Results

The fits used to locate the optimal distance were analyzed to assess the correlation of the DoLP reduction for an average upwelling radiance. Initially it was thought that the optical depth would factor in; however, owing to the remarkably consistent optical depths seen through the nine clear days, this additional parameter did not vary significantly. The data was fit with a nonlinear least-squares solver to a decreasing exponential fit, as this trend was observed in the data. Figure 37 illustrates the results of the fit of the DoLP data to the upwelling radiance.

Figure 37: DoLP fit to upwelling radiance for each imager wavelength. Note the $R^2$ test score in the lower left of each plot.
These plots indicate the decreasing trend observed in polarization as the upwelling radiance increases. There is notable spread in the DoLP for a given radiance, however. For the 700-nm case especially, this can be observed near 150 W/m²-sr-µm) where there is a sharp decrease in DoLP for a fixed value of radiance. This occurred to some extent on several occasions but not in as severe a fashion as observed on this plot by the noticeable severe curve which corresponds to a particular afternoon measurement. It was suspected that these precipitous drops in DoLP come from additional dependencies on cloud location beyond what a simple quadrant approach yielded. Furthermore, the averaging of the radiance over the extent of 100 km does not account for rapidly moving clouds on the periphery of successive images. For example, for cloudy conditions, a large radiance of 150 W/m²-sr-µm) was not unexpected. However, if between successive images there is cloud motion such that the average radiance does not change but the location of the radiance does, this factor would not be accounted for in the above data.

As a means of validating these fit results, the data used to generate the fits was put back into the model results to check for self-consistency. Figure 38 illustrates the results for two days during the deployment. Note that the model predicted the afternoon DoLP signatures well; however, the mid-morning structure observed from around 30° to 60° in solar elevation angle was not predicted by the model. The approximate error in the model prediction is roughly 5%.
Figure 38: Maximum DoLP profiles for two days at 700 nm and the results of predicting the DoLP profile with the parameterized exponential decay fit.
CONCLUSIONS

In May and June of 2008, an all-sky imaging polarimeter was successfully deployed to the Mauna Loa Observatory to measure skylight polarization. The isolated nature of the observatory at an altitude above the aerosol and marine boundary layers provided an ideal location to study clear-sky polarization signatures. Furthermore, because the surface surrounding the observatory consisted of dark lava, this location was well suited to study Rayleigh-scattering polarization signatures that were largely unaffected by multiple scattering from surface albedo. The intent of this experiment was to quantify to what extent the polarimeter could measure the Rayleigh scattering polarization signature and how well these measurements agreed with predicted first-order scattering theory in a location that has been used to previously verify such phenomena.

It was found that while the overhead sky remained clear and the overall optical thickness on such days varied very little throughout the day, the maximum degree of polarization signatures varied between 5 and 10 percent throughout the nine clear days of measurement depending on wavelength. Subsequent analysis indicated that while the reflected surface radiance of the island was low as expected, the upwelling radiation from clouds beneath the observatory as measured by the GOES-11 satellite was scattered into the instrument field of view causing an increase in the unpolarized light. The maximum DoLP is inversely proportional to the amount of unpolarized light, hence, as cloudiness and thus upwelling radiance increased, the maximum DoLP decreased accordingly. Furthermore, since the cloud conditions throughout the day varied extensively, the ability
to decouple first-order Rayleigh scattered polarization phenomenon from signatures incorporating multiply-scattered light became impossible.

The deployment did, however, result in a number of useful results. Experimental validation of both the instrument design and calibration was verified. The development of numerous all-sky polarization image processing techniques allowed for the fisheye imaging plane to be translated into an absolute three-dimensional reference space which was then used to help correct alignment and extract principal and dihedral angular slices. Such profiles showing the experimentally-obtained DoLP versus scattering angle were critical in comparing data to that in literature utilizing Mie scattering simulations yielding profiles in this reference space rather than elevation or azimuth angle as is the case with many instrument-based studies.

Comparisons to Coulson’s similar measurements made 30 years prior were favorable even though conditions were drastically different at during these two periods. Kilauea, an active volcano since 1983, has drastically affected aerosol content since Coulson’s data was taken. Furthermore, an intensely active period beginning in November 2007 resulted in a marked increase in volcanic aerosols culminating with large gaseous eruptions beginning in March of 2008 and continuing throughout the deployment. Such eruptions carried significant particulates contributing to cloud cover on the west and south parts of the island that again made comparisons to first-order Rayleigh data difficult. However, during Coulson’s campaign, there were days during which qualitative observations of cloud cover indicated similar conditions to those observed during the 2008 deployment. As such, while the clearest of days observed by
Coulson could not nor will be able to be repeated, due to volcanic activity, for some time, days with underlying clouds for both Coulson’s and the 2008 data have very consistent maximum DoLP signatures.

Finally, because the optical thicknesses of the clear days was similar throughout, a quantitative correlation between the upwelling radiance from surface scattering—dominated in variation by cloud cover variability—was obtained. With some differences owing only to localized cloud cover variability and scattering effects that are difficult to decouple given the data available, these data fit well to a decreasing exponential trend with respect to increasing upwelling radiance from underlying clouds. This represents some of the first experimentally-obtained correlative and quantitative data for the effective upwelling radiance and polarization variability in an otherwise overhead clear sky.

**Future Work**

Work in this area continues to explain the DoLP asymmetries occurring through the course of the day as well as to verify the observed trends with suitable Mie scattering simulations that can account for both optical thicknesses and upwelling radiance effects beyond simple Rayleigh scattering. Moving forward, this imager, in conjunction with an in situ solar radiometer, radiosonde station, and polarized lidar will allow for aerosol retrievals to be made with increased precision over the current capability.

Also, this imager has recently been deployed over the summer of 2009 at Montana State University in a new enclosure with an automated sun occulting system.
Such modifications allowed for unattended operation over the course of several months and the observation of several interesting effects such as solar halos and cloud formation that would otherwise be difficult to capture if the instrument was run in an as-needed fashion.


http://aeronet.gsfc.nasa.gov/

[27] NOAA Comprehensive Large Array-Data Stewardship System. [Online].
http://www.nsof.class.noaa.gov/saa/products/welcome


APPENDIX A:

SPHERICAL PARTICLE SCATTERING
I. Introductory Steps:

The full derivation for scattering from an arbitrary spherical particle requires the conversion of an incident plane wave (e.g., the sun’s rays incident on the earth’s atmosphere) into spherical harmonics. This procedure, illustrated in [16] is not trivial and requires the use of Legendre polynomials as the general solution to the differential equations that result from the $\theta$-directed terms and the use of spherical Bessel functions as the solution to the differential equations in the $\phi$-direction.

The expressions shown in this derivation also make use of the vector potentials $\mathbf{M}$ and $\mathbf{N}$, which are commonly used as vectors satisfying the required properties of an electromagnetic wave, but are computationally simpler to solve than the field equations of a wave directly. This derivation will begin with the expansion into spherical harmonics for each of the three waves involved in the scattering from a spherical particle—the incident, refracted or internal, and scattered waves.

II. Electric and Magnetic Fields at a Particle Interface:

The incident, internal, and scattered fields are denoted by subscripts $i$, $i_1$, and $s$ respectively below from [16]. The subscript $i$ is used to represent a difference in refractive index (i.e., material 1, the particle) from the surrounding medium (no subscript, typically with refractive index of free space or air).

\[
E_i = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( M_{\text{o}i} j_n (kr) - i N_{\text{ei}} j_n (kr) \right)
\]

\[
H_i = -\frac{k}{\omega \mu} E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( M_{\text{ei}} j_n (kr) + i N_{\text{oi}} j_n (kr) \right)
\]

\[
E_i = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( c_n M_{\text{ei}} (j_n (k, r)) - i d_n N_{\text{ei}} (j_n (k, r)) \right)
\]

\[
H_i = -\frac{k}{\omega \mu} E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( d_n M_{\text{ei}} (j_n (k, r)) + i c_n N_{\text{ei}} (j_n (k, r)) \right)
\]

\[
E_s = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( i a_n N_{\text{ei}} (h_n^{(1)} (kr)) - b_n M_{\text{ei}} (h_n^{(1)} (kr)) \right)
\]

\[
H_s = \frac{k}{\omega \mu} E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( i b_n N_{\text{ei}} (h_n^{(1)} (kr)) - a_n M_{\text{ei}} (h_n^{(1)} (kr)) \right)
\]

In this equation, $j_n$ is a spherical Bessel function and $i$ is the square root of -1. Note that to eschew confusion between this appendix and Section 2, $j$ will represent the Bessel function terms and $i$ will be the imaginary number. In Section 2, $j$ is the square root of -1 to keep electrical engineering conventions and because the $j_n$ terms seen in this appendix do not enter into the approximations in Section 2. The term $h_n^{(1)} (\rho)$ is a spherical Hankel function of the third kind and enters only into the scattered field expressions. From these
expressions, there are 4 scattering coefficients, \(a_n, b_n, c_n, \) and \(d_n\), that must be solved for. First, however, the four required vector potentials in \(M\) and \(N\) are shown below. Note that \(z_n(\xi)\) for each \(M\) and \(N\) will take on the value of the argument in parentheses shown above and either be a spherical Bessel or Hankel function depending on whether the field is incident, internal, or scattered.

\[
M_{o1n}(z_n(\rho)) = \cos \phi \pi_n(\cos \theta) z_n(\rho) \hat{e}_\theta - \sin \phi \tau_n(\cos \theta) z_n(\rho) \hat{e}_\phi \\
M_{e1n}(z_n(\rho)) = -\sin \phi \pi_n(\cos \theta) z_n(\rho) \hat{e}_\theta - \cos \phi \tau_n(\cos \theta) z_n(\rho) \hat{e}_\phi \\
N_{o1n}(z_n(\rho)) = \sin \phi n(n+1) \sin \theta \pi_n(\cos \theta) z_n(\rho) \hat{e}_\rho + \sin \phi \tau_n(\cos \theta) \frac{1}{\rho} \frac{d}{d\rho} [\rho \varphi_n(\rho)] \hat{e}_\theta \\
+ \cos \phi \pi_n(\cos \theta) \frac{1}{\rho} \frac{d}{d\rho} [\rho \varphi_n(\rho)] \hat{e}_\phi \\
N_{e1n}(z_n(\rho)) = \cos \phi n(n+1) \sin \theta \pi_n(\cos \theta) z_n(\rho) \hat{e}_\rho + \cos \phi \tau_n(\cos \theta) \frac{1}{\rho} \frac{d}{d\rho} [\rho \varphi_n(\rho)] \hat{e}_\theta \\
+ \sin \phi \pi_n(\cos \theta) \frac{1}{\rho} \frac{d}{d\rho} [\rho \varphi_n(\rho)] \hat{e}_\phi 
\]

In the above definitions, the angularly dependent functions \(\pi(\cos \theta)\) and \(\tau(\cos \theta)\) are defined in terms of the Legendre polynomials as defined below.

\[
\pi_n(\cos \theta) = \frac{P_n(\cos \theta)}{\sin \theta} \\
\tau_n(\cos \theta) = \frac{d P_n(\cos \theta)}{d\theta}
\]

III. Scattering Coefficients:

Assuming a sphere with radius \(a\) and index of refraction \(N_j\), the transverse electric and magnetic field boundary conditions can be used to produce the four equations needed to solve for each of the 4 scattering coefficients. The four boundary conditions are as follows below.

\[
E_{i\theta} + E_{s\theta} = E_{1\theta} \\
E_{i\phi} + E_{s\phi} = E_{1\phi} \\
H_{i\theta} + H_{s\theta} = H_{1\theta} \\
H_{i\phi} + H_{s\phi} = H_{1\phi}
\]

 Skipping significant algebraic manipulation, exploiting certain orthogonality conditions regarding Legendre polynomials, and defining the size parameter \(x = ka = \frac{2\pi Na}{\lambda}\) and \(m = \frac{k_1}{k} = \frac{N_1}{N}\), the four linear equations in \(a_n, b_n, c_n, \) and \(d_n\) are obtained as shown below.
Solving these four equations and then making the assumption that the surrounding permeability $\mu = \mu_1$, and substituting the Riccati-Bessel functions $\psi_n = \rho j_n(\rho)$ and $\xi_n(\rho) = \rho h_n^{(1)}(\rho)$ into these solutions, the scattering coefficients can be explicitly determined. Because we are concerned only with the scattered wave for the purposes of this discussion, only the scattered wave coefficients are listed below.

$$a_n = \frac{m \psi_n(mx)\psi'_n(x) - \psi_n(x)\psi'_n(mx)}{m \psi_n(mx)\xi'_n(x) - \xi_n(x)\psi'_n(mx)}$$

$$b_n = \frac{\psi_n(mx)\psi'_n(x) - m \psi_n(x)\psi'_n(mx)}{\psi_n(mx)\xi'_n(x) - m \xi_n(x)\psi'_n(mx)}$$

IV. Far-Field Asymptotic Approximations:

For large distances from the scattered molecule at visible wavelengths, the spherical Hankel function and the derivative may be approximated by the asymptotic expression given by the equations below

$$h_n^{(1)}(kr) \sim \frac{(-i)^n e^{ikr}}{ikr} \text{ for } kr \gg n^2$$

$$\frac{dh_n^{(1)}}{d\rho} \sim \frac{(-i)^n e^{ikr}}{\rho}$$

Using the expressions for the scattered transverse ($\theta$ and $\phi$) field components, the appropriate components of the vector potentials $M$ and $N$, the above approximations, and the fact that terms with $\frac{1}{(kr)^2} \ll \frac{1}{kr}$ and can be neglected, the expressions for the transverse electric field can be obtained as

$$E_z(\theta, \phi) = E_o \frac{e^{ikr}}{-ikr} \cos \phi S_2(\cos \theta)\hat{\theta} - E_o \frac{e^{ikr}}{-ikr} \sin \phi S_1(\cos \theta)\hat{\phi} \left[ \frac{V}{m} \right]$$

where

$$S_1(\cos \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left( a_n \pi_n(\cos \theta) + b_n \tau_n(\cos \theta) \right)$$

$$S_2(\cos \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left( a_n \tau_n(\cos \theta) + b_n \pi_n(\cos \theta) \right)$$
V. Power Series Expansion of Bessel and Hankel Functions:

The last step required to bring the above equations in line with those expressed in Section 2, Equation (2.8), is to expand the infinite sum over $n$ and then truncate insignificant terms. First, the spherical Hankel function $h_n^{(1)}(\rho)$ can be expressed as a combination of two spherical Bessel functions in the following fashion

$$h_n^{(1)}(\rho) = j_n(\rho) + iy_n(\rho).$$

The power series expansion of these spherical Bessel functions is shown as:

$$j_n(\rho) = \frac{\rho^n}{1 \cdot 3 \cdot 5 \ldots (2n + 1)} \left[ 1 - \frac{1}{2} \frac{\rho^2}{1!(2n + 3)} + \frac{\left(\frac{1}{2} \rho^2\right)^2}{2!(2n + 3)(2n + 5)} - \ldots \right]$$

$$y_n(\rho) = \frac{1 \cdot 3 \cdot 5 \ldots (2n - 1)}{\rho^{n+1}} \left[ 1 - \frac{1}{2} \frac{\rho^2}{1!(1 - 2n)} + \frac{\left(\frac{1}{2} \rho^2\right)^2}{2!(1 - 2n)(3 - 2n)} - \ldots \right]$$

These are the expansions used to solve for $\psi_{n=1,2}$, $\xi_{n=1,2}$ which yield $a_{n=1,2}$ and $b_{n=1,2}$ shown in Equation (2.8). From here, the derivation follows exactly in that of Section 2 wherein all terms except $a_1$ are insignificant in the Rayleigh approximation.